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02

# PHYSICS

# ELECTROSATICS CHARGE & FIELD

CBSE REVISION MODULE

## SAMPLE QUESTIONS

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CBSE

ELECTROSATICS CHARGE & FIELD



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PHYSICS REVISION MODULE

# ELECTROSATICS CHARGE & FIELD

## Electric Charges & Fields

**Quantization of charge**

All charges must be a multiple of charge (e)
$Q = ne$
Where $n$ is an integer

**Conservation of charge**

It is not possible to create or destroy net charge of an isolated system.

**Electric charges**

- Two kinds of charges + ve and – ve
- S.I unit is coulomb (C)
  1 C = charge flowing through a wire in 1 sec if current in it is 1A
  $1e = 1.602 \times 10^{-19}$C

**Coulomb's Law**

In 1985, Charles Augustin de Coulomb gave Coulomb's law

- Force between two charged particles

$\vec{F} = \frac{k\, q_1\, q_2 \vec{r}}{r^3} = \frac{k q_1\, q_2 \hat{r}}{r^2}$, force is attractive when $q_1$ and $q_2$ are of opposite sign, otherwise repulsive

$|\vec{F}| = \frac{k\, q_1\, q_2}{r^2}$

$k = \frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9 \text{Nm}^2\text{C}^{-2}$

- Force between multiple charges

$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \ldots\ldots = \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{n} \frac{q_i}{r_{1i}^2} \hat{r}_{1i}$

where is the unit vector along charge particle under consideration
$\hat{r}_{1i}$ = distance between Ist and $i^{th}$ particle

"The electric force with which two charges attract or repel one another are not affected by the presence of other charges". It is known as Superposition Principle.

**Electric flux**

$\Delta\phi = \vec{E}.\Delta\vec{S}$, $\Delta\vec{S}$ = area vector
$\vec{E}$ = electric field
$\phi = \int \vec{E}.\vec{ds}$

**Gauss's Theorem**

$\oint \vec{E}.\vec{ds} = q_{in}/\varepsilon_0 = \phi$
where, $q_{in}$ = net charge enclosed
$\phi$ = flux through a closed surface

**Applications of Gauss's Law**

- Electric Field due to linear charge distribution
  $E = \lambda/2\pi\, \varepsilon_0 r$,
  $\lambda$ = linear charge density
- ElectricField due to a plane sheet of charge
  $E = \sigma/2\varepsilon_0$,
  $\sigma$ = charge per unit area
- Electric Field due to a charged conducting plate
  $E = \sigma/\varepsilon_0$,
  $\sigma$ = charge per unit area
- Electric Field inside a conductor = 0
- Electric field due to a thin spherical shell at a point
  Outside the shell, $E = Q/4\pi\varepsilon_0 r^2$
  Inside the shell, $E = 0$
  On the surface, $E = Q/4\pi\varepsilon_0 R^2$

Electric field due to uniformly charged spherical shell at :
- Outside point,
  $E = Q/4\pi\varepsilon_0 r^2$
- Internal point,
  $E = Q/4\pi\varepsilon_0 R^3$
- The surface,
  $E = Q/4\pi\varepsilon_0 R^2$

**Electric Field**

Electric field lines
- Always normal to conducting surface
- Lines originating from +ve charge
- Terminating at –ve charge
- Never intersect each other
- Do not form closed loop
- Are imaginary lines.

Electric field due to point charge

$\vec{E} = \frac{\vec{F}}{q} = k\, \frac{Q}{r^2}$ (N/C)

$\because$ unit = N/C

- Due to discrete distribution of charges
  $\vec{E} = \sum_i^n \vec{E}_i$
- Due to continuous distribution of charges
  $\vec{E} = k \int \frac{dQ}{r^3} \vec{r}$
  $|\vec{E}| = k \int \frac{dQ}{r^2}$

**Electric Dipole moment, $\vec{p} = q\vec{d}$**

- Electric field due to dipole at axial position,
  $\vec{E} = \frac{1}{4\pi\, \varepsilon_0} \left(\frac{2p}{r^3}\right) \hat{r}$
- Electric field due to dipole at an equatorial position,
  $|E| = \frac{1}{4\pi\, \varepsilon_0} \left(\frac{p}{r^3}\right)$
- Torque on an electric dipole placed in an electric field $(\vec{E})$
  $\vec{\tau} = \vec{p} \times \vec{E} = p\, E \sin\theta$

Trace the Mind Map
- First Level
- Second Level
- Third Level

## Syllabus

- *Electric charges; Conservation of charge; Coulomb's law—force between two point charges, forces between multiple charges; Superposition principle and continuous charge distribution.*
- *Electric field; Electric field due to a point charge; Electric field lines; Electric dipole, Electric field due to a dipole; Torque on a dipole in uniform electric field.*
- *Electric flux; Statement of Gauss's theorem and its applications to find the field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).*

## Trend Analysis

| List of Concepts | 2018 | | 2019 | | 2020 | |
|---|---|---|---|---|---|---|
| | D | Delhi Outside | D | Delhi Outside | D | Delhi Outside |
| Electric Field and Dipole | | | | 1 Q (1 M) | | |
| | | | 2 Q (2 M) | | 1 Q (2 M) | |
| | 2Q (3 M) | | 1 Q (3 M) | | 1 Q (3 M) | |
| | | | 1 Q (5 M) | | | |
| Gauss's Theorem and Its Application | | | | | 2 Q (1 M) | |
| | | | | | | |
| | | | | 2 Q (3 M) | | 2 Q (3 M) |
| | 2 Q (5 M) | | | | 1 Q (5 M) | |

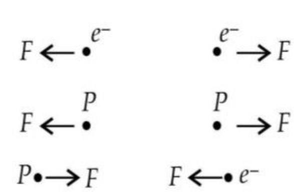
# TOPIC-1
## Electric Field and Dipole



## Revision Notes

### Electric Charge

- Electric charge is the property of a matter due to which, it experiences a force when placed in an electromagnetic field.
- Point charge is an accumulation of the electric charges at a point, without spatial extent.
- Electrons are the smallest and lightest fundamental particles in an atom having negative charge as these are surrounded by invisible force known as electrostatic field.
- Protons are comparatively larger and heavier than electrons with positive electrical charge which is similar in strength as electrostatic field in an electron with opposite polarity.
- Two electrons or two protons will tend to repel each other as they carry like charges, negative and positive respectively.




- The electron and proton will get attracted towards each other due to their unlike charges.

- The charge present on the electron is equal and opposite to charge on the proton.

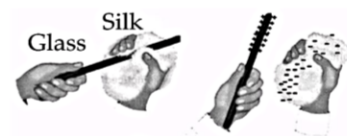
  Charge on a proton $= +1.6 \times 10^{-19}$ C

  and charge on an electron $= -1.6 \times 10^{-19}$ C

**Electrostatic Charge**

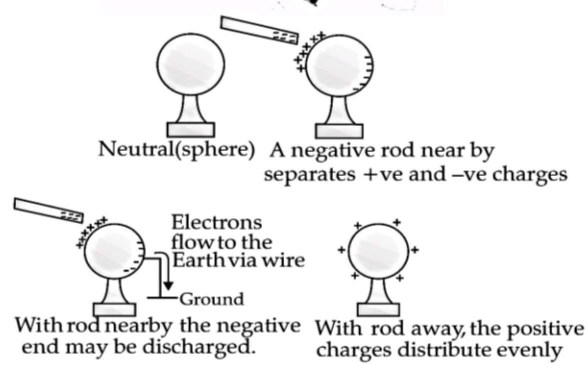
- Electrostatic charge means the charge is at rest.
- Electrostatic charge is a fundamental physical quantity like length, mass and time.
- Charge on a body is expressed as $q = \pm ne$
- The magnitude of charge is independent of the speed of the particle.
- Based on the flow of charge across them, materials are classified as:
  - Conductors - Allow electric charge to flow freely, *e.g.* metals
  - Semi-conductors - Behave as the conductor or insulator depending on the number of free electrons and holes availability. e.g. silicon
  - Insulators - Do not allow electric charge to flow, *e.g.* rubber, wood, plastic, etc.
- **Net charge on a body is given by:**
  - Charging by friction - Charging insulators
  - Charging by conduction - Charging metals and other conductors
  - Charging by induction - Wireless charging

**Charging by Rubbing**




- On rubbing a glass rod and silk cloth piece together, glass rod gets positively charged whereas silk cloth gets negatively charged.
- If a plastic rod is rubbed with wool, it becomes negatively charged.

**Charging by Induction**




- Charging by induction means charging without contact.
- If a negatively charged rod is brought near neutral metal with insulator mounting, it repels free electrons and attracts positive charges on metal.
- If far end is connected to Earth by a wire, electrons will flow towards ground while positive charges are kept captive by the rod.
- When the rod is removed, the captive positive charge is distributed evenly.

**Properties of Electric Charge**

**Addition of charges**

- If a system contains three point charges $q_1$, $q_2$ and $q_3$, then the total charge of the system will be the algebraic addition of $q_1$, $q_2$ and $q_3$, *i.e.*, charges will add up.

$$q = q_1 + q_2 + q_3$$

**Conservation of charges**

- Electric charge is always conserved. It is the sum of positive and negative charges present in an isolated system, which remains constant.
- Charge can neither be created nor destroyed in the process, but only exists in positive-negative pairs.

**Quantization of charges**

- Electric charge is always quantized *i.e.*, electric charge is always an integral multiple of charge '$e$
- Net charge $q_{net}$ of an object having $N_e$ electrons, $N_p$ protons and $N_n$ neutrons is:

  $q_{net} = -eN_e + eN_p + 0N_n = e(N_p - N_e) = \pm ne$
- Neutron ($n$): $m = 1.675 \times 10^{-27}$ kg; $q = 0$
- Proton ($p$): $m = 1.673 \times 10^{-27}$ kg; $q = +1.6 \times 10^{-19}$ C
- Electron ($e$): $m = 9.11 \times 10^{-31}$ kg; $q = -1.6 \times 10^{-19}$ C

**Coulomb's Law**

- The force of attraction or repulsion between two point charges $q_1$ and $q_2$ separated by a distance $r$ is directly proportional to product of magnitude of charges and inversely proportional to square of distance between charges, written as:

$$F = k\frac{|q_1||q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

where,
$F$ = Force of attraction/repulsion between charges $q_1$ and $q_2$.
$q_1, q_2$= Magnitudes of charge 1 and charge 2 respectively
$r$ = Distance between charges $q_1, q_2$
$k$ = Constant whose value depends on medium where charges are kept. $k = \dfrac{1}{4\pi\varepsilon}$

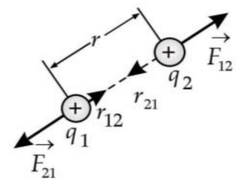
As $\varepsilon = K'\varepsilon_0$, $k = \dfrac{1}{4\pi K'\varepsilon_0}$

$\varepsilon_0$ = Permittivity of vacuum = $8.854 \times 10^{-12}$ F/m
$K'$ = Relative permittivity of medium or dielectric constant.

- For vacuum, relative permittivity, $K' = 1$,
- As $\varepsilon = K'\varepsilon_0$, therefore force of attraction/repulsion between two electric charges $q_1, q_2$ placed in vacuum and medium will be:

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2} \text{ (vacuum) and } F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\cdot\frac{q_1 q_2}{r^2} \text{ (medium)}$$

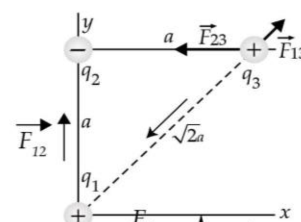
- The unit coulomb (C) is derived from the SI unit ampere (A) of the electric current.
- Current is the rate $\dfrac{dq}{dt}$ at which charge moves past a point or through a region, $i = \dfrac{dq}{dt}$, hence 1 C=(1 A) × (1 s).
- The vector form of Coulomb force with $\hat{r}_{12}$ = unit vector from $q_1$ to $q_2$ is given as:




$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{12} \text{ and } \vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

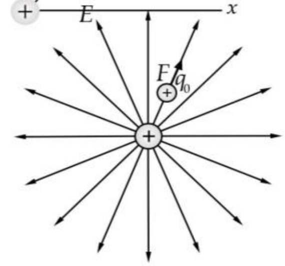
$$\Rightarrow \quad \vec{F}_{21} = -\vec{F}_{12}$$

**Principle of Superposition**



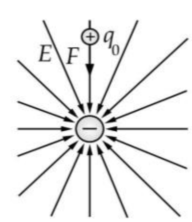

- The force on any charge due to other charges at rest is the vector sum of all the forces on that charge due to the other charges, taken one at a time.
- The individual forces are unaffected due to presence of other charges.
- Force exerted by $q_1$ on $q_3 = \vec{F}_{13}$
- Force exerted by $q_2$ on $q_3 = \vec{F}_{23}$
- Net force exerted on $q_3$ is vector sum of $\vec{F}_{13}$ and $\vec{F}_{23}$

**Electric field**




- The space around a charge up to which its electric force can be experienced is called electric field.
- If a test charge $q_0$ is placed at a point where electric field is $E$, then force on the test charge is $F = q_0 E$
- The electric field strength due to a point source charge '$q$' at an observation point 'A' at a distance '$r$' from the source charge is given by:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^3}\vec{r} \text{ or } E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

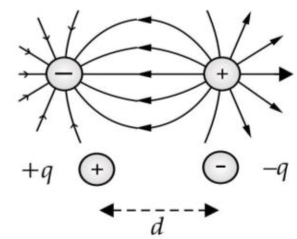



- The unit of electric field is N/C.
- Electric field inside the cavity of a charged conductor is zero.
- If a charged/uncharged conductor is placed in an external field, the field in conductor is zero.
- In case of charged conductor, electric field is independent of the shape of conductor.

**Electric field lines**

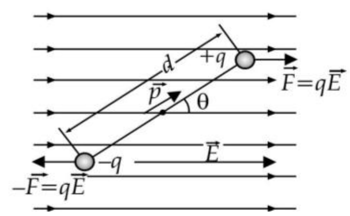
- Electric field lines are imaginary lines which originates from the positive charge and towards negative charge.
- Direction of electric field lines around positive charge is imagined by positive test charge $q_0$ located around source charge.
- Electric field has same direction as force on positive test charge.
- Electric field lines linked with negative charge are directed inward described by force on positive test charge $q_0$.
- The electric field lines never intersect each other.
- Strength of electric field is encoded in density of field lines.

**Electric Dipole**




- The system formed by two equal and opposite charges separated by a small distance is called an electric dipole.
- The electric field exists due to a dipole.
- The force on a dipole in a uniform electric field is zero in both stable as well as unstable equilibrium.
- The potential energy of a dipole in an uniform electric field is minimum for a stable equilibrium and maximum for an unstable equilibrium.

**Torque on a dipole**




- In a dipole, when net force on dipole due to electric field is zero and center of mass of dipole remains fixed, the forces on charged ends produce net torque τ about its center of mass.

$$\tau = F\, d\sin\theta = qE\, d\sin\theta = pE\sin\theta$$

$$\vec{\tau} = \vec{p}\times\vec{E}$$

  - If $\theta = 0^\circ$ or $360^\circ$, dipole exists in stable equilibrium state.
  - If $\theta = 180^\circ$, dipole exists in an unstable equilibrium state.
- In uniform electric field, dipole experiences torque, net force on dipole is zero.
- In uniform electric field, dipole experiences a rotatory motion.
- In non-uniform electric field, dipole experiences torque and net force.
- In non-uniform electric field, dipole experiences rotatory and translatory motion.
  - The torque aligns dipole with electric field and it becomes zero.
  - The direction of torque is normal to the plane going inward.

**Electric Dipole Moment**

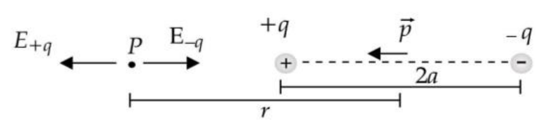
- Dipole moment is a vector quantity whose unit is coulomb-metre (Cm).
- Dipole moment vector of electric dipole is $\vec{p} = \vec{q}\times 2a$ between pair of charges $q$, $-q$, along the line, seperated by distance $2a$.

**Electric field due to a dipole**

- For point $P$ at distance $r$ from centre of dipole on charge $q$, for $r >> a$, total field at point $P$ is

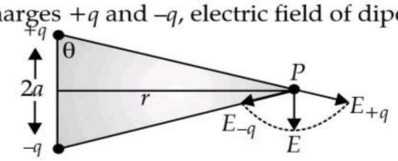
$$E = \frac{4qa}{4\pi\varepsilon_0 r^3}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3} \text{ (if } a << r)$$




- For point $P$ on the equatorial plane due to charges $+q$ and $-q$, electric field of dipole at a large distance,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$




## Know the Terms

- 1 coulomb: When two point charges placed at a distance of 1 m in vacuum, repel/attract each other with force of $9 \times 10^9$ N, the charge on each is known as 1 coulomb.
- Electric line of force: It is a curve drawn in such a way that the tangent at each point to curve gives the direction of the net field at that point.

## Know the Formulae

- Coulomb's force: $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r^2}$;

  where all alphabets have their usual meanings.
- Electric field due to point charge $q$: $E = \dfrac{k|q|}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{r^2}$
- Electric field due to a dipole at a point on the dipole axis: $E = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{2p}{r^3}$ $(r >>> a)$
- Electric field at a point on equatorial plane: $E = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{p}{r^3}$ $(r >>> a)$
- Torque on an electric dipole placed in an electric field, $\tau = pE\sin\theta$

## Mnemonics

**Concept:** Characteristics of Electric field lines

**Mnemonics:** **I**ndia **S**tarts **P**laying **N**ight **C**ricket **T**ournament **D**aily with **N**ew **I**nspiration.

How to remember all the 5 characteristics ?

Remember this sentence.

India Starts Playing Night Cricket Tournament Daily with New Inspiration.

- India → Imaginary line
- Starts → Start from
- Playing → Positive charge
- Night → Never forms
- Cricket → closed loops
- Tournament → Tangent at any point
- Daily → Direction of field
- New → Never
- Inspiration → Intersects

## Objective Type Questions

**(1 mark each)**

### [A] Very Short Answer Type Questions

**AI Q. 1. Draw the equipotential surfaces for an electric dipole.** **[CBSE Delhi Outside Set 1 2019]**

**Ans.**

1

(Even if a student mentions or draws equatorial plane, award 1 mark.)

**[CBSE Marking Scheme, 2019]**

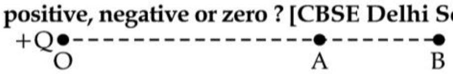
**AI Q. 2. Does the charge given to a metallic sphere depend on whether it is hollow or solid ? Give reason for your answer.** **[CBSE Delhi Set 1 2017]**

**Ans.** No. ½

Because the charge resides only on the surface of the conductor. ½

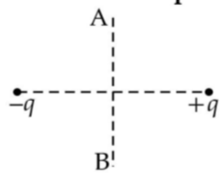
**[CBSE Marking Scheme, 2017]**

**Q. 3. A point charge + Q is placed at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero ? [CBSE Delhi Set 1 2016]**



**Ans.** Positive. **[CBSE Marking Scheme, 2016] 1**

**Q. 4. A charge '$q$' is moved form a point A above a dipole of dipole moment '$p$' to a point B below the dipole in equatorial plane without acceleration. Find the work done in the process.**




**[CBSE Delhi Outside Set 1 2016]**

**Ans.** No work is done.

$W = qV_{AB} = q \times 0 = 0$ **1**

**[CBSE Marking Scheme, 2016]**

**Q. 5. A plastic rod and a glass rod are rubbed with fur and silk respectively. What will happen if they are brought close to each other ?** U

**Ans.** The will attract each other.

[**Explanation:** If a plastic rod is rubbed with fur, it becomes negatively charged. If glass rod is rubbed with silk, it becomes negatively charged. Since they are being oppositely charged, they will be attracted.] **1**

**Q. 6. What is the name of the electrical force acting between two charges at rest ?** U

**Ans.** Electrostatic force.

[Coulomb force acts between two charges at rest. Coulomb force is also known as electrostatic force since the force is acting between two static charges.] **1**

**Q. 7. Name the law which states "force is directly proportional to the product of charges and inversely proportional to square of separation between them" ?** U

**Ans.** Coulomb's law. **1**

**Q. 8. Do electrostatic field lines form closed loops for point charges ?**

**Ans.** No. electrostatic field lines form closed loops for point charges. **1**

**Q. 9. If $E_1$ and $E_2$ be the electric field strength of a short dipole on its axial line and on its equatorial line respectively, then what is the relation between $E_1$ and $E_2$ ?** U

**Ans.** $E_1 = 2E_2$ **1**

[Explanation: Electric field at a point on equatorial line of a dipole is $Kp/r^3$ and that on axial line is $2Kp/r^3$.]

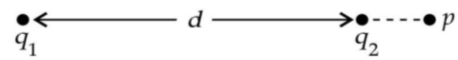
**Commonly Made Error**

- Students sometimes get confused and cannot remember the correct relation between electric field at a point on equatorial line of a dipole and that on axial line.

**Answering Tip**

- Remember that the equatorial line is the perpendicular line at a point half-way of a dipole. So, electric field at a point on the equatorial line will be half of the electric field at a point on the axial line.

AI **Q. 10. Two point charges '$q_1$' and '$q_2$' are placed at a distance '$d$' apart as shown in the figure. The electric field intensity is zero at a point 'P' on the line joining them as shown. Write two conclusions that you can draw from this.**




**Ans. (i)** The two point charges ($q_1$ and $q_2$) should be of opposite nature. ½

**(ii)** Magnitude of charge $q_1$ must be greater than that of charge $q_2$. ½

AI **Q. 11. Two equal balls having equal positive charge '$q$' coulombs are suspended by two insulating strings of equal length. What would be the effect on the force when a plastic sheet is inserted between the two ?**

**Ans.** The force decreases. **1**

**Detailed Answer:**

The force given as $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{\left(d - t + t\sqrt{K}\right)^2}$ will decrease when a plastic is introduced, as the dielectric constant $K$ for plastic is more than 1.

**Q. 12. When is the torque on a dipole in an electric field maximum?** U

**Ans.** The torque on an electric dipole is maximum when it is held perpendicular to the field. **1**

## [B] ASSERTION REASON TYPE QUESTIONS

**Following questions are having two statements one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.**

**(a) Both A and R are true and R is the correct explanation of A**

**(b) Both A and R are true but R is NOT the correct explanation of A**

**(c) A is true but R is false**

**(d) A is false and R is also false**

**Q. 1. Assertion (A):** In a non-uniform electric field, a dipole will have translatory as well as rotatory motion.

**Reason (R):** In a non-uniform electric field, a dipole experiences a force as well as torque. **1**

**[CBSE SQP 2020-21]**

**Ans.** (a) Both A and R are true and R is the correct explanation of A.

**Detailed Answer:**

When an electric dipole placed in a uniform electric field at an angle θ with the field, the dipole experiences a torque.

The torque produced by two parallel forces $qE$ acting as couple = τ

$$\tau = qE\,(2l \sin\theta)$$

In case of non-uniform field, force acting on both the ends of the dipole will not be equal. So, there will be a combination of couple and a net force. In this way, dipole will have both rotational as well as linear motion.

So, both assertion and reason are true. Reason also explains the assertion.

**Q. 2. Assertion (A):** The basic difference between magnetic lines of force and electric lines of force is electric lines of force are discontinuous and magnetic lines of force are continuous.

**Reason (R):** Magnetic lines of force exist in a magnet but no electric lines of force exists in a charged body.

**Ans. Correct option: (a)**

**Explanation:** Let us consider an electric dipole. The electric lines of force exists outside only and not inside the dipole.

Let us now consider a magnetic dipole. The magnetic lines of force exist outside as well as inside the dipole.

So, it can be said that magnetic lines of force are continuous and electric lines of force are discontinuous.

So assertion and reason both are true and reason explains the assertion too.

**Q. 3. Assertion (A):** Electric lines of force cross each other.

**Reason (R):** The resultant electric field at a point is the superimposition of the electric fields at that point.

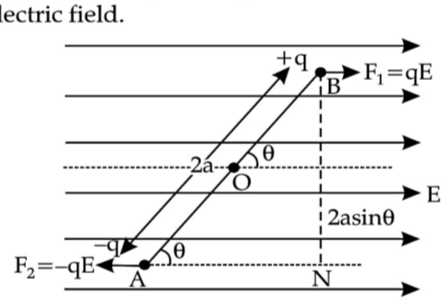
**Ans. Correction option: (d)**

**Explanation:** Electric lines of force never cross each other. If they cross each other then at that point we get two directions of electric field at that point, which is not possible. So, the assertion is false. The resultant electric field at a point is a vector sum of the electric fields at that.

## Short Answer Type Question-I (2 marks each)

**Q. 1. Derive the expression for the torque acting on an electric dipole, when it is held in a uniform electric field. Identify the orientation of the dipole in the electric field, in which it attains a stable equilibrium.** A **[CBSE Delhi Set 2020]**

**Ans.** An electric dipole AB consisting of charge $+q$ and $-q$ and of length $2a$ is placed in uniform electric field E making an angle θ with the direction of electric field.




Force acting on $-q$ is $-qE$

Force acting on $+q$ is $qE$

These two forces are equal and opposite to each other. Hence, a torque on the dipole is developed.

Torque = Force × perpendicular distance between the forces

Or, $\tau = qE \times 2a\sin\theta$

Or, $\tau = (q \times 2a)E \sin\theta$

∴ $\tau = pE \sin\theta$ (where $p$ is dipole moment)

Dipole will attain stable equilibrium when it will be oriented along the direction of electric field.

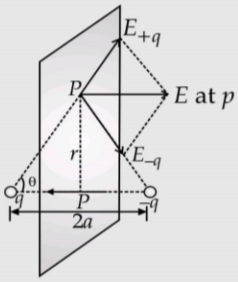
## Short Answer Type Questions-II (3 marks each)

AI **Q. 1. (i) Derive the expression for electric field at a point on the equatorial line of an electric dipole.**

**(ii) Depict the orientation of the dipole in (i) stable, (ii) unstable equilibrium in a uniform electric field.** **[CBSE Delhi Set-I 2018]**

**Ans. (i)** Derivation of expression of electric field on equatorial line of the dipole 2

**(ii)** Depiction of orientation for stable and unstable equilibrium. ½+½




**(i)** Let the point '$P$' be at a distance '$r$' from the mid point of the dipole.

$$E_{+q} = \frac{q}{4\pi\varepsilon_0\left(r^2+a^2\right)} \qquad ½$$

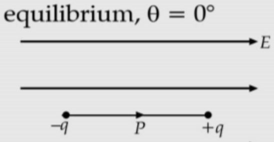
$$E_{-q} = \frac{q}{4\pi\varepsilon_0\left(r^2+a^2\right)} \qquad ½$$

Both are equal and their directions are as shown in the figure, hence net electric field,

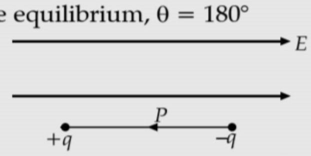
$$E_{+q} = \left[-\left(E_{+q}+E_{-q}\right)\cos\theta\right]\hat{p} \qquad ½$$

$$E_p = -\frac{2qa}{4\pi\varepsilon_0\left(r^2+a^2\right)^{3/2}}\hat{p} \qquad ½$$

**(ii)** In stable equilibrium, $\theta = 0°$ ½




In unstable equilibrium, $\theta = 180°$ ½




**[CBSE Marking Scheme, 2017]**

**Commonly Made Error**

- Students are often confused with the two terms: equatorial line and axial line.

**Answering Tip**

- Axial line is the line joining the two charges of a dipole. The line perpendicular to the axial line passing through the centre of the dipole is called an equatorial line.

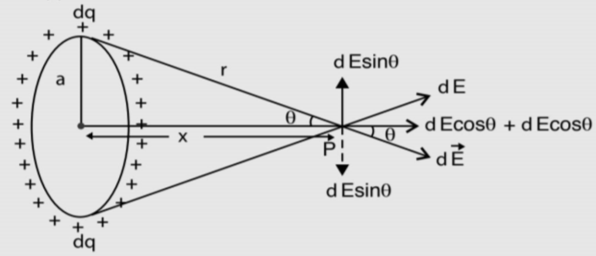
**AI Q. 2. A charge is distributed uniformly over a ring of radius '$a$'. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence show that for points at large distances from the ring, it behaves like a point charge.**

A **[CBSE Delhi Set-I 2016]**

**Ans.** Obtaining an expression for electric field intensity 2

Showing behaviour at large distance 1

**(i)**




½

Net Electric Field at point $P = \int_0^{2\pi a} dE\cos\theta$

$dE$ = Electric field due to a small element having charge $dq$

$$= \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} \qquad ½$$

Let $\lambda$ = Linear charge density

$$= \frac{dq}{dl} \qquad ½$$

$$dq = \lambda dl$$

Hence $E = \int_0^{2\pi a}\frac{1}{4\pi\varepsilon_0}.\frac{\lambda dl}{r^2}\times\frac{x}{r}$, where $\cos\theta = \frac{x}{r}$

$$= \frac{\lambda x}{4\pi\varepsilon_0 r^3}(2\pi a) \qquad ½$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(x^2+a^2)^{\frac{3}{2}}},$$

where total charge $Q = \lambda \times 2\pi a$

**(ii)** At large distance *i.e.*, $x >> a$ ½

$$E \simeq \frac{1}{4\pi\varepsilon_0}.\frac{Q}{x^2} \qquad ½$$

This is the electric field due to a point charge at distance $x$.

(**NOTE:** Award two marks for this question, if a student attempts this question but does not give the complete answer)

**[CBSE Marking Scheme, 2016]**

**Q. 3. A thin circular ring of radius $r$ is charged uniformly so its linear charge density becomes $\lambda$. Derive an expression for the electric field at a point P at a distance $x$ from it along the axis of the ring. Hence, prove that for large distance ($x >> r$), the ring behaves as a point charge.**

A **[CBSE Delhi 1 2020]**

**Ans.** Try Yourself. See Q. No. 2 3

**Q. 4. (a) Draw the equipotential surfaces due to an electric dipole.**

**(b) Derive as expression for the electric field due to a dipole of dipole moment $\vec{P}$ at a point on its perpendicular bisector.** A&U **[CBSE Delhi 3 2019]**

**Ans. (a)** Try yourself. Refer Q.No. 1 of 1 mark VSATQ.

**(b)** Try yourself. Refer Q.No. 1(i) of 3 marks SATQ.

**Q. 5. (i) Obtain the expression for the torque $\vec{\tau}$ experienced by an electric dipole of dipole moment $\vec{P}$ in a uniform electric field $\vec{E}$.**

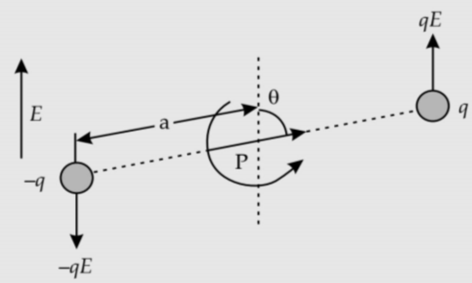
**(ii) What will happen if the field were not uniform?**

A&U **[CBSE Delhi Set-III 2017]**

**Ans. (i)** Obtaining of the expression for torque experienced by an electric dipole **2**

**(ii)** Effect of non-uniform electric field **1**

**(i)**




Force on $+q$, $\vec{F} = q\vec{E}$

Force on $-q$, $\vec{F} = -q\vec{E}$

Magnitude of torque

$$\tau = qE \times 2a \sin\theta$$

$$= 2qa\, E \sin\theta$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$ **2**

**(ii)** If the electric field is non uniform, the dipole experiences a translatory force as well as a torque. **1**

**[CBSE Marking Scheme, 2017]**

**Q. 6. A charge is distributed uniformly over a ring of radius '*a*'. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence show that for points at large distances from the ring, it behaves like a point charge.**

A **[CBSE Delhi Set-I 2016]**

**Ans.** Try Yourself See Q. No. 2 of 3 marks SATQ. **3**

**Q. 7. Define an equipotential surface. Draw equipotential surfaces:**

**(i) In the case of a single point charge and**

**(ii) In a constant electric field in Z-direction.**

**Why the equipotential surfaces about a single charge are not equidistant?**

**(iii) Can electric field exist tangential to an equipotential surface? Give reason.**

R&U **[CBSE Delhi Outside Set-I 2016]**

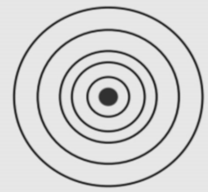
**Ans.** Definition: ½

**(i)** Diagram of Equipotential Surface ½

**(ii)** Diagram and reason ½ + ½

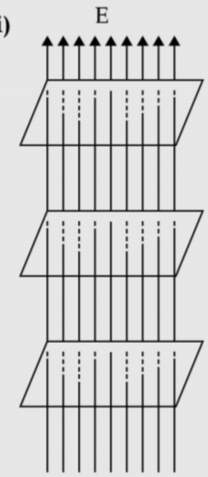
**(iii)** Answer and Reason ½ + ½

**(i)** Surface with a constant value of potential at all points on the surface. ½



½

**(ii)**




½

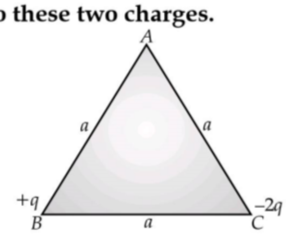
$V \propto \dfrac{1}{r}$ ½

**(iii)** No ½

If the field lines are tangential, work will be done in moving a charge on the surface which goes against the definition of equipotential surface. ½

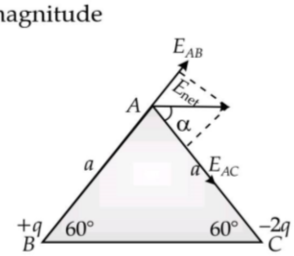
**[CBSE Marking Scheme, 2016]**

AI **Q. 8. Two point charges $+q$ and $-2q$ are placed at the vertices '*B*' and '*C*' of an equilateral triangle *ABC* of side '*a*' as given in the figure. Obtain the expression for (i) the magnitude and (ii) the direction of the resultant electric field at the vertex A due to these two charges.**




A

**Ans. (i)** The magnitude




½

$$\left|\vec{E}_{AB}\right| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2} = E \qquad ½$$

$$\left|\vec{E}_{AC}\right| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{a^2} = 2E \qquad ½$$

$$E_{net} = \sqrt{(2E)^2 + (E^2) + 2\times 2E\times E\cos 120°}$$

$$E_{net} = \sqrt{(2E)^2 + E^2 + 2\times 2E\times E\times\left(-\frac{1}{2}\right)}$$

$$E_{net} = \sqrt{4E^2 + E^2 - 2E^2}$$

$$E_{net} = E\sqrt{3} = \frac{1}{4\pi\varepsilon_0}\frac{q\sqrt{3}}{a^2} \qquad ½$$

**(ii)** The direction of resultant electric field at vertex $A$

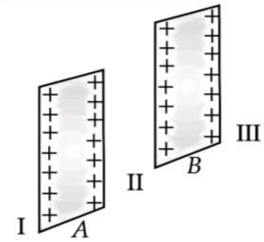
$$\tan\alpha = \frac{E_{AB}\sin 120°}{E_{AC} + E_{AB}\cos 120°} \qquad ½$$

$$\tan\alpha = \frac{E\times\frac{\sqrt{3}}{2}}{2E + E\times\left(\frac{-1}{2}\right)} = \frac{1}{\sqrt{3}}$$

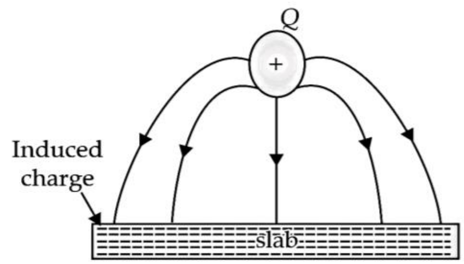
$$\alpha = 30° \text{ (with side } AC) \qquad ½$$

**AI Q. 9. (i) A point charge (+$Q$) is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines between the charge and the plate.**

**(ii) Two infinitely large plane thin parallel sheets having surface charge densities $\sigma_1$ and $\sigma_2$ ($\sigma_1 > \sigma_2$) are shown in the figure. Write the magnitudes and directions of the fields in the regions marked II and III.**




**Ans. (i)**




1

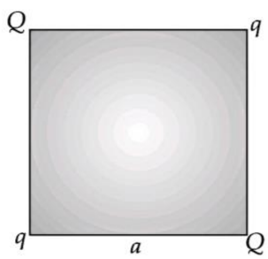
**(ii) (a)** For region II, $E_{II} = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2)$ ½

towards right side from sheet A to sheet B. ½

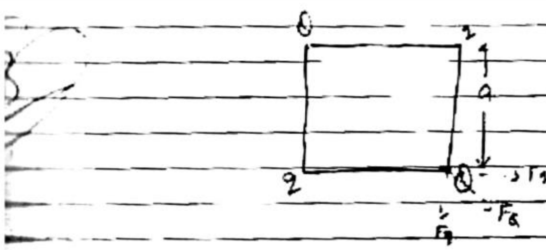
**(b)** For region III, $E_{III} = \frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2)$ ½

towards right side away from the two sheets. ½

**Q. 10. Four point charges $Q$, $q$, $Q$ and $q$ are placed at the corners of a square of side '$a$' as shown in the figure. Find the resultant electric force on a charge $Q$** **A [CBSE Board Paper, 2018]**




**Ans.**



Force on Q acts due to 2 q and Q

as the force due to both q are equal with angle of 90°

⇒ ~~Resulta~~ Resultant Force F is given by

$\sqrt{F^2 + F^2} = \sqrt{2}F$ (acts along diagonal)

where $F = \frac{KQq}{a^2}$

Now, force due to Q acts along diameter

$$\Rightarrow F_Q = \frac{KQ^2}{(\sqrt{2}a)^2} = \frac{KQ^2}{2a^2}$$

$\Rightarrow$ Net force $= \sqrt{2}F + F_Q$ (as they act along diameter)

$$\Rightarrow \frac{\sqrt{2}Qq K}{a^2} + \frac{KQ^2}{2a^2}$$

$$= \frac{K(2\sqrt{2}Qq + Q^2)}{2a^2}$$

$$= \frac{KQ}{2a^2}\left[2\sqrt{2}q + Q\right] \text{ N}$$

along the diameter away from charge Q.

where $K = \frac{1}{4\pi\varepsilon_0}$

[Topper's Answer, 2018]

## Long Answer Type Questions

**(5 marks each)**

**Q. 1. (a) Define an ideal electric dipole. Give an example.**

**(b) Derive an expression for the torque experienced by an electric dipole in a uniform electric field. What is net force acting on this dipole?**

**(c) An electric dipole of length 2 cm is placed with its axis making an angle of 60° with respect to uniform electric field of $10^5$ N/C. If it experiences a torque of $8\sqrt{3}$ Nm, calculate the**

**(i) magnitude of charge on the dipole, and (ii) its potential energy.** R & A

**[CBSE SQP 2020-2021]**

**Ans.** Definition of ideal dipole and example ½ + ½

Derivation of torque 2

Putting values in correct formula and solving value of charge and potential energy

$Q = 8 \times 10^{-3}$ C 1

$U = -8$J 1

**(a)** Ideal electric dipole: An ideal electric dipole consists of two very large charges $+q$ and $-q$ separated by a very small distance. An ideal dipole has almost no size. ½

Example: Molecules of water, ammonia, etc are examples of ideal electric dipoles. ½

**(b)** Consider an electric dipole consisting of two charges $+q$ and $-q$ and separated by a distance $2l$ is placed in a uniform electric field $\vec{E}$ so that that dipole makes an angle θ with the electric field.

Force acting on $-q$ is $-q\vec{E}$

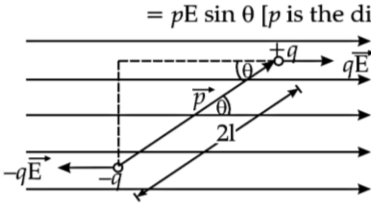
Force acting on $+q$ is $+q\vec{E}$

The magnitude of torque developed $= q$E × perpendicular distance between the forces

$= q\text{E} \times 2l \sin\theta$

$= q \times 2l\,\text{E} \sin\theta$

$= p\text{E}\sin\theta$ [$p$ is the dipole moment]




2

**(c)** Torque $= q \times 2l \times \text{E}\sin\theta$

Or, $8\sqrt{3} = q \times 0.02 \times 10^5 \times \sin 60°$

∴ $q = 8 \times 10^{-3}$ C 1

Potential energy $= -q \times 2l \times \text{E}\cos\theta$

$= -8 \times 10^{-3} \times 0.02 \times 10^5 \times \cos 60°$

$= -8$ J 1

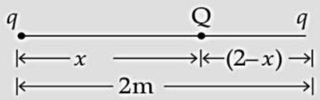
**Q. 2. (a) Derive an expression for the electric field at any point on the equatorial line of an electric dipole.**

**(b) Two identical point charges, $q$ each, are kept 2 m apart in air. A third point charge Q of unknown magnitude and sign is placed on the line joining the charges such that the system remains in equilibrium. Find the position and nature of Q.**

A **[CBSE Delhi Set-I 2019]**

**Ans.** (a) Derivation for the expression of the electric field on the equatorial line **3**
(b) Finding the position and nature of Q **1 + 1**

(a) Try it yourself. See Q. No. 1(i) of 3 marks questions. **3**

(b) 

½

System is in equilibrium, therefore net force on each charge of system will be zero.
For the total force on 'Q' to be zero

$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{(2-x)^2} \quad ½$$

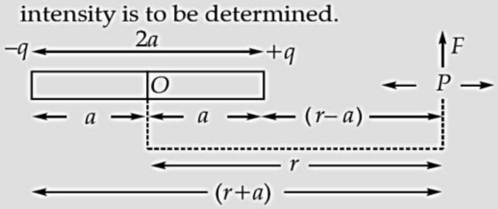
$$x = 2 - x$$
$$2x = 2 \quad ½$$
$$x = 1\text{ m}$$

(Give full credit of this part, if a student writes directly 1m by observing the given condition)
For the equilibrium of charge "$q$", the nature of charge Q must be opposite to the nature of charge $q$.
**[CBSE Marking Scheme 2019]** ½

**Q. 3. (a) Derive an expression for the electric field E due to a dipole of length '2*a*' at a point distant *r* from the centre of the dipole on the axial line.**
**(b) Draw a graph of E versus *r* for *r* >> *a*.**
**(c) If this dipole were kept in a uniform external electric field $E_0$, diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases.**
A&U **[CBSE Delhi Outside Set-I 2017]**

**Ans.** (a) Derivation of E along the axial line of dipole **2**
(b) Graph between $E$ vs $r$ **1**
(c) (i) Diagrams for stable and unstable equilibrium of dipole ½ + ½
(ii) Torque on the dipole in the two cases ½ + ½

(a) Find the expression for electric field intensity in an axial position due to electric dipole. R
Consider an electric dipole whose length is $2a$ and centre at $O$. From the mid-point $O$, consider a point $P$ at a distance $r$, where the electric field intensity is to be determined.




We have $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$

Case I, $E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(r-a)^2}$ (due to positive charge)

Case II, $E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(r+a)^2}$ (due to negative charge)

Then, the net electric field $E = E_1 - E_2$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)^2-(r-a)^2}{(r-a)^2.(r+a)^2}\right]$$

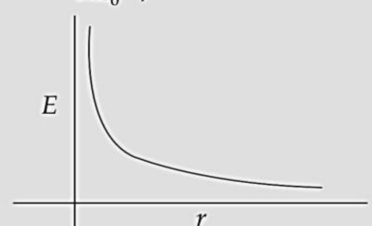
$$E = \frac{q}{4\pi\varepsilon_0}\cdot\frac{4ar}{(r^2-a^2)^2}$$

for, $r^2 >>> a^2$

$$\therefore \quad E = \frac{q}{4\pi\varepsilon_0}\left|\frac{4ar}{r^4}\right|$$

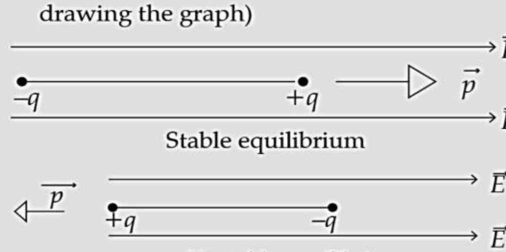
$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\times q\times 2a\times r}{r^4}$$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\vec{p}}{r^3}\text{ N/C} \quad [\because p = 2qa] \; 2$$

(b) 

**1**

(**Note:** Award ½ mark if the student just writes: For short Dipole $= \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$ without drawing the graph)

(c) 

Stable equilibrium ½

Unstable equilibrium ½

(Note: Award ½ mark only if the student does not draw the diagrams but just writes:

(i) For stable Equilibrium: $\vec{p}$ is parallel to $\vec{E}$.

(ii) For unstable equilibrium: $\vec{p}$ is antiparallel to $\vec{E}$.
Torque = 0 for (i) as well as case (ii).

(Also accept, $\vec{\tau} = \vec{p}\times\vec{E}$ or $\tau = pE\sin\theta$) ½+½

**[CBSE Marking Scheme, 2017]**

**Commonly Made Error**

- Several candidates derive an expression for intensity of electric field $E$ at a point in the broadside position *i.e.*, coaxial position, instead of that in the end-on position as required. Some candidates do not understand which derivation to write. Hence they write both the derivations. Many candidates are not able to draw the correctly labelled diagram.

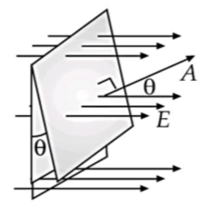
**Answering Tips**

- Students should derive the expression by using proper vector diagram and find the resultant by vectorial method.

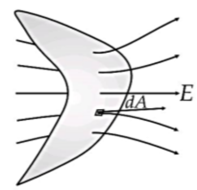
# TOPIC-2
## Gauss's Theorem and its Applications

## Revision Notes

### Electric Flux



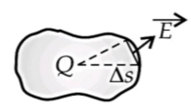

- Electric flux is proportional to algebraic number of electric field lines passing through the surface, outgoing lines with positive sign, incoming lines with negative sign.
- Due to arbitrary arrangement of electric field lines, electric flux can be quantify as $\phi_E = EA$
- If vector $A$ is perpendicular to surface, magnitude of vector $A$ parallel to electric field is $A\cos\theta$
  $A_{II} = A\cos\theta$
  $\phi_E = EA_{II} = EA\cos\theta$
- In non-uniform electric field, the flux will be $\phi_E = \int EdA$



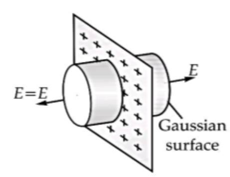

### Continuous Charge Distribution

- It is a system in which the charge is uniformly distributed over the material. In this system, infinite number of charges are closely packed and have minor space among them. Unlike the discrete charge system, the continuous charge distribution is uninterrupted and continuous in the material. There are three types of continuous charge distribution system.
- For linear charge distribution ($\lambda$), $\vec{F} = \frac{q_0}{4\pi\varepsilon_0}\int_x \frac{\lambda}{r^2}dx\hat{r}$ (Where, $\lambda$ = linear charge density)
- For surface charge distribution ($\sigma$), $\vec{F} = \frac{q_0}{4\pi\varepsilon_0}\int_s \frac{\sigma}{r^2}dS\hat{r}$ (Where, $\sigma$ = surface charge density)
- For volume charge distribution ($\rho$), $\vec{F} = \frac{q_0}{4\pi\varepsilon_0}\int_V \frac{\rho}{r^2}dV\hat{r}$ (Where, $\rho$ = volume charge density)

### Gauss theorem




- The net outward normal electric flux through any closed surface of any shape is equal to $1/\varepsilon_0$ times to net charge enclosed by the surface.
- The electric field flux at all points on Gaussian surface is $\phi = E\oint dA = \frac{q}{\varepsilon_0}$




- If there is a positive flux, net positive charge is enclosed.
- If there is a negative flux, net negative charge is enclosed.
- If there is zero flux, no net charge is enclosed.
- The expression for electric field due to a point charge on Gaussian surface is

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

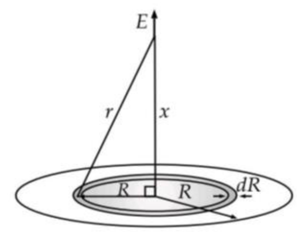
- In an insulating sheet, charge remains in the sheet, so electric field, $E = \frac{\sigma}{2\varepsilon_0}$
- Gauss theorem works in cases of cylindrical, spherical and rectangular symmetries.
- The field outside the wire points radially outward which depends on distance from wire, $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{n}$, where, $\lambda$ is linear density of charge.
- Closed surface : It is a surface which divides the space inside and outside region, where one can't move from one region to another without crossing the surface.
- Gaussian surface : It is a hypothetical closed surface having similar symmetry as problem on which we are working.
- Electrostatic Shielding : It is the phenomenon of protecting a certain region of space from external electric field.

- Dielectric : The non-conducting material in which charges are easily produced on the application of electric field is called dielectric. *e.g.* Air, $H_2$ gas, glass, mica, paraffin wax, transformer oil, etc.

## Key Formulae

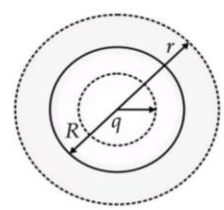
- Electric flux through an area $A$: $\phi = E.A = EA\cos\theta$
- Electric flux through a Gaussian surface: $\phi = \oint E.dS$
- Gauss's Law: $\phi = \dfrac{q_{enc}}{\varepsilon_0}$
- Electric Field due to an infinite line of charge: $E = \dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{2k\lambda}{r}$

  where, $E$ = electric field [N/C], $\lambda$ = charge per unit length [C/m]
  $\varepsilon_0$ = permittivity of free space = $8.85 \times 10^{-12}$ [$C^2/N\,m^2$], $r$ = distance (m), $k = 9 \times 10^9$ N $m^2 C^{-2}$
- Electric field due to a ring at a distance $x$ is: $E = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{qx}{(r^2+x^2)^{3/2}}$




- When, $x >>> r$: $E = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{x^2}$
- When $x <<< r$: $E = 0$
- Electric field due to a charged disc: $E = \dfrac{\sigma}{2\varepsilon_0}\left[1 - \dfrac{x}{\sqrt{R^2+x^2}}\right]$

  where ,
  $E$ = electric field [N/C] — $\sigma$ = charge per unit area [$C/m^2$]
  $\varepsilon_0 = 8.85 \times 10^{-12}$ [$C^2/Nm^2$] — $x$ = distance from charge [m]
  $R$ = radius of the disc [m]
- Electric field due to a thin infinite sheet: $\vec{E} = \dfrac{\sigma}{2\varepsilon_0}\hat{n}$




- Electric field inside a spherical shell: $\vec{E} = 0$ for $r < R$
- Electric field outside a spherical shell $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\hat{r}$ or $E = \dfrac{kq}{r^2}$ for $r > R$

  where, $r$ = distance from the centre of shell to the point of measurement in m.

## How is it done on the GREENBOARD?

**AI Q. 1. Given a uniform electric field $\vec{E} = 5 \times 10^3\ \hat{i}$ N/C. Find the flux of this field through a square of side 10 cm on a side whose plane is parallel to the $y$-$z$ plane. What would be the flux through the same square if the plane makes a 30° angle with the $x$-axis ?** A

**Solution:**

**Step I: Given :**

$E = 5 \times 10^3\ \hat{i}$ N/C along (+) positive direction of $x$-axis.

Surface area, $A$ = 10 cm × 10 cm
= 0.10 m × 0.10 m
= $10^{-2}$ $m^2$

**Step II:** In case of plane parallel to $y$-$z$ plane, normal to plane will be along $x$-axis, so $\phi = 0°$

Electric flux will be calculated using $\phi = |\vec{E}|\ A \cos\theta$ ½
= $5 \times 10^3 \times 10^{-2} \times \cos 0°$
= 50 $Nm^2/C$ ½

**Step III:** Since plane is making an angle of 30° with $x$-axis, so normal to its plane will make 60° with $x$-axis, so $\theta = 60°$

Now finding Electric flux again with $\phi = |\vec{E}|\,A \cos\theta$
= $5 \times 10^3 \times 10^{-2} \times \cos 60°$ ½
= 25 $Nm^2/C$ ½

## Objective Type Questions

(1 mark each)

### [A] Very Short Answer Type Questions

**Q. 1. What will be the ratio of the surface charge density of the inner surface to that of the outer surface of a hollow conducting sphere if a point charge is placed at the centre of the hollow conducting sphere having internal radius '*r*' and outer radius '2*r*' ?** U **[CBSE Delhi Set-I 2020/Modified]**

**Ans.** 4 : 1.

[**Explanation:** If $+q$ charge is placed at the centre of the hollow sphere, charge on inner surface will be $-q$. So, charge density at the inner surface $= -q/4\pi r^2$.

Charge on outer surface will be $+q$. So, charge density at the inner surface $= q/4\pi(2r)^2$

So, the required ratio $= [-q/4\pi r^2]/[q/4\pi(2r)^2]$

$= -4 : 1$

**Q. 2. State Gauss's law in electrostatics.**

R **[Delhi Set-I, II, III 2016]**

**Ans. Gauss's law in electrostatics:** "The surface integral of electrostatic field $\vec{E}$ produced by any source over any closed surface $S$ enclosing a volume $V$ in vacuum, *i.e.*, total electric flux over the closed surface $S$ in vacuum, is $\frac{1}{\varepsilon_0}$ times the total charge ($Q$) contained inside $S$, *i.e.*,

$$\phi_E = \oint \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$ 1

**[CBSE Marking Scheme, 2016]**

**Commonly Made Error**

- Most of the candidates are unable in stating Gauss's law correctly. Key words like net charges, closed surface etc. are missed by students. Some candidates write magnetic flux instead of electric flux.

**Answering Tip**

- Student should keep the precaution about the keywords while doing the practice of Gauss law.

**Q. 3. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased?**

U **[CBSE Delhi Set-I 2016]**

**Ans.** Electric flux remains unaffected.

[**Note:** (As per the Hindi translation), change in Electric field is being asked, hence give credit if student write answer as decreases] 1

**[CBSE Marking Scheme, 2016]**

AI **Q. 4. What is the electric flux through a cube of side 1 cm which encloses an electric dipole ?** U

**Ans.** Zero (as net charge enclosed by the surface is zero.) **[CBSE Marking Scheme, 2015] 1**

**Detailed Answer:**

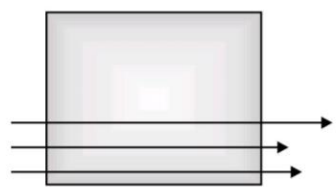
In a cubic surface, the net electric charge will be zero since dipole carries equal and opposite charges. It is observed that the net electric flux through closed cubic surface will be

$$= \frac{\text{Charge enclosed}}{\varepsilon_0}$$

and because the charge enclosed is zero,

$\therefore$ electric flux is also zero. 1

**Q. 5. A square surface of side L metres is in the plane of the paper. A uniform electric field E volt m$^{-1}$ is also in the plane of the paper, is limited only to the lower half of the square surface as shown in the figure. What is the electric flux (in SI unit) associated with the surface?** U



**Ans.** The electric flux will be zero. 1

[**Explanation:** As the electric field lines are parallel to the surface of the square, so there will not be any field lines crossing the surface. Hence the electric flux through the surface will be zero.]

**Commonly Made Error**

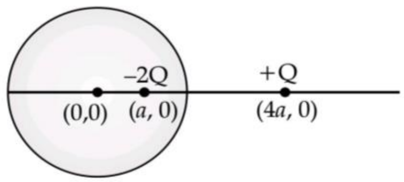
- Most of the students are unable to find the electric field from Gauss 's law properly.

**Answering Tip**

- The given figure should be carefully studied to know that the electric field lines are parallel to the surface and elecltric field lines will cross the surface.

AI **Q. 6. Two charges of magnitudes –2Q and +Q are located at points (*a*, 0) and (4*a*, 0) respectively. What is the electric flux due to these charges through a sphere of radius '3*a*' with its centre at the origin ?** U

**Ans.** Gauss's theorem states that the electric flux through a closed surface enclosing a charge is equal to $\left(\frac{1}{\varepsilon_0}\right)$ times the magnitude of the charge enclosed.




The sphere encloses a charge of – 2Q, so

$$\phi = \frac{2Q}{\varepsilon_0}$$ **1**

**Q. 7. On which factors does the electric flux through a closed Gaussian surface depend upon ?** R **[CBSE Delhi Set-I 2020/Modified]**

**Ans.** Net charge enclosed by the surface and the permittivity of the medium. **1**

[Gauss's laws mathematical expression is $\phi = Q_{ENCLO.} / \varepsilon_0$]

**Q. 8. What will be the changes in outward electric flux if the radius of Gaussian spherical surface of radius R enclosing a charge Q is doubled ?** A

**Ans.** Flux will remain same.

[**Explanation:** According to Gauss's law, the flux depends on the charge enclosed and the permittivity of the medium. It does not depend on the surface area.] **1**

**Q. 9. Is Gauss's law valid for an open surface?** U

**Ans.** No. Gauss's law is not valid for any open surface. **1**

**Q. 10. A charge Q is kept at the centre of the cube. What is the electric flux through the two opposite faces of the cube?** A

**Ans.** Electric flux $(\phi) = \frac{1}{\varepsilon_0} \times \frac{2}{6} = \frac{1}{3\varepsilon_0}$ **1**

**Q. 11. Is electric flux a scalar or a vector quantity? Give reason.** U

**Ans** The electric flux is the number of electric field lines crossing an imaginary area. So, it is a scalar quantity. **1**

## [B] ASSERTION REASON TYPE QUESTIONS

Following questions are having two statements one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

**(a)** Both A and R are true and R is the correct explanation of A

**(b)** Both A and R are true but R is NOT the correct explanation of A

**(c)** A is true but R is false

**(d)** A is false and R is also false

**Q. 1. Assertion (A):** If two spherical conductors of different radii have the same surface charge densities , then their electric field intensities will be equal.

**Reason (R):** Surface charge density = $\frac{\text{Total charge}}{\text{area}}$

**Ans.** Correct option is (b)

**Explanation:** If $\sigma$ be the surface charge density of the two spheres of radius $r$ and R, then Electric fields for the two spheres are respectively:

$$E_1 = \frac{\kappa 4\pi r^2 \sigma}{r^2} = \kappa 4\pi\sigma$$

$$E_2 = \frac{\kappa 4\pi R^2 \sigma}{R^2} = \kappa 4\pi\sigma$$

So electric field intensities are equal. The assertion is true.

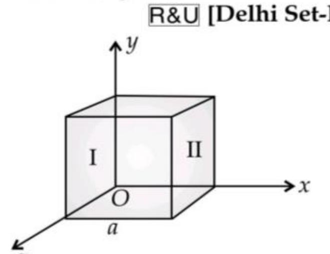
Surface charged density is charge per unit area = Total charge/area.

So reason is also true.

But the reason does not explain the assertion.

## Short Answer Type Questions-I (2 marks each)

AI **Q. 1. Given the electric field in the region $\vec{E} = 2x\hat{i}$, find the net electric flux through the cube and the charge enclosed by it.** R&U **[Delhi Set-I, II, III 2015]**




**Ans.** From the given diagram,

Only the face perpendicular to the direction of $x$-axis, contribute to the electric flux. The remaining faces of the cube gives zero contribution. ½

Total flux, $\phi = \phi_I + \phi_{II}$

$$= \oint_I \vec{E}.\vec{ds} + \oint_{II} \vec{E}.\vec{ds}$$ ½

$$= 0 + 2(a).a^2$$ ½

$$= 2a^3$$ ½

Charge enclosed, $q = \varepsilon_0\phi = 2a^3\varepsilon_0$

## Short Answer Type Questions-II (3 marks each)

**Q. 1. A hollow conducting sphere of inner radius $r_1$ and outer radius $r_2$ has a charge Q on its surface. A point charge $-q$ is also placed at the centre of the sphere.**

**(a) What is the surface charge density on the (i) inner and (ii) outer surface of the sphere ?**

**(b) Use Gauss' law of electrostatics to obtain the expression for the electric field at a point lying outside the sphere.** U & A **[CBSE Delhi Outside Set-I 2020]**

**Ans. (a)** Charge placed at the centre of the hollow sphere is $-q$. Hence, a charge of magnitude $+q$ will be induced to the inner surface. Therefore, total charge on the inner surface of the shell is $+q$. Surface charge density at the inner surface,

$$\sigma_i = \frac{\text{Total charge}}{\text{Inner surface area}} = \frac{+q}{4\pi r_1^2} \qquad \mathbf{1}$$

A charge of $-q$ is induced on the outer surface of the sphere. A charge of magnitude $Q$ is placed on the outer surface of the sphere. Therefore, total charge on the outer surface of the sphere is $Q - q$. Surface charge density at the outer surface,

$$\sigma_{\text{outer}} = \frac{\text{Total charge}}{\text{Outer surface area}} = \frac{Q-q}{4\pi r_2^2} \qquad \mathbf{1}$$

**(b)** Electric field at point lying outside the sphere at a distance $r$ from the centre of the sphere:

Applying Gauss theorem

$$\text{Flux} = \phi = \frac{\text{Charge enclosed}}{\varepsilon_o}$$

Or, $E \times 4\pi r^2 = \dfrac{Q-q}{\varepsilon_0}$

$$\therefore E = \frac{Q-q}{4\pi r^2 \varepsilon_0} \qquad \mathbf{1}$$

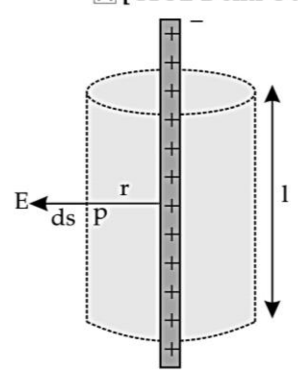
**AI Q. 2. An infinitely long thin straight wire has a uniform linear charge density λ.**

**(a) Obtain the expression for the electric field (E) at a point lying at a distance $x$ from the wire, using Gauss' law.**

**(b) Show graphically the variation of this electric field E as a function of distance $x$ from the wire.**

**A [CBSE Delhi Outside Set-I 2020]**

**Ans.**




**(a)** Electric field due to an infinitely long straight wire having uniform linear charge density $\lambda$:

$x$ = Distance of the point P from the wire where the electric field is to be evaluated

$E$ = Electric field at the point $P$

A Gaussian cylinder of length $l$, radius $x$ is considered.

An infinitesimally small area $ds$ on the Gaussian surface is considered.

Electric field is same at all points on the curved surface of the cylinder and directed radially outward. So, $E$ and $ds$ are along the same direction.

The total electric flux ($\phi$) through curved surface $= \int E ds \cos\theta$

Since $E$ and $ds$ are along the same direction, so $\theta = 0°$

So, $\phi = E(2\pi x l)$

The net charge enclosed by Gaussian surface is, $q = \lambda l$

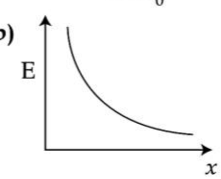
$\therefore$ By Gauss's law,

$$\phi = \frac{1}{\varepsilon_0} q$$

Or, $\therefore \dfrac{1}{\varepsilon_0} q = E(2x\pi l)$

$$\therefore E = \frac{\lambda}{2\pi x \varepsilon_0}$$

**(b)**



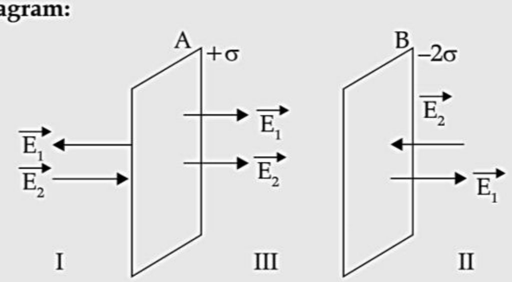

**Q. 3. Two large charged plane sheets of charge densities σ and – 2σ C/m² are arranged vertically with a separation of $d$ between them. Deduce expressions for the electric field at points**

**(i) to the left of the first sheet, (ii) to the right of the second sheet, and (iii) between the two sheets.**

**A [CBSE Delhi Outside Set-I 2019]**

| | | |
|---|---|---|
| **Ans.** Diagram | | **1+½** |
| | Deducing electric field expression | |
| **(i)** | To the left of first sheet | ½ |
| **(ii)** | To the right of second sheet | ½ |
| **(iii)** | Between the two sheets | ½ |

**Diagram:**




**1½**

Electric field in the region left of first sheet

$$E_I = E_1 + E_2$$

$$E_I = \frac{\sigma}{\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$$

$$E_I = +\frac{\sigma}{2\varepsilon_0} \qquad ½$$

It is towards right,

Electric field in the region to the right of second sheet,

$$E_{II} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{\varepsilon_0}$$

$$E_{II} = -\frac{\sigma}{2\varepsilon_0} \qquad ½$$

It is towards left,

Electric field between the two sheets

$$E_{III} = E_I + E_{II}$$

$$E_{III} = \frac{\sigma}{\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$E_{III} = \frac{3\sigma}{2\varepsilon_0}$$ ½

Electric field is towards the right.

**[CBSE Marking Scheme, 2019]**

**AI Q. 4. A spherical conducting shell of inner radius $r_1$ and outer radius $r_2$ has a charge Q.**

**(a) A charge $q$ is placed at the centre of the shell. Find out the surface charge density on the inner and outer surfaces of the shell.**

**(b) Is the electric field inside a cavity (with no charge) zero-independent of the fact whether the shell is spherical or not? Explain.**

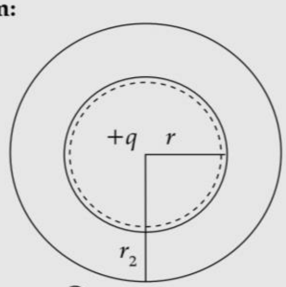
**A&U [CBSE Delhi Outside Set-I 2019]**

**Ans.** Diagram ½

Finding the surface charge density in the inner and outer surface of the shell **1+½**

Electric field in the cavity **1**

**(a) Diagram:**




Q + q ½

The surface charge density on inner surface of the shell is $\sigma_1 = -\frac{q}{4\pi r_1^2}$ **1**

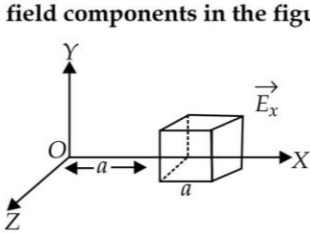
The surface charge density on outer shell is

$$\sigma_2 = -\frac{Q+q}{4\pi r_2^2}$$ ½

**(b)** Consider a Gaussian surface inside the shell, net flux is zero since $q_{net} = 0$. According to Gauss's law, it is independent of shape and size of shell. **1**

**[CBSE Marking Scheme, 2019]**

**Q. 5. Define electric flux and write its SI unit. The electric field components in the figure shown are:**




**$E_x = \alpha x$, $E_y = 0$, $E_z = 0$ where $\alpha = \frac{100\,N}{C\,m}$. Calculate the charge within the cube, assuming $a = 0.1$ m.**

**A & E [Delhi Outside Comptt. I, II, III 2018]**

**Ans. Definition of Electric flux** **1**

**SI unit** **½**

**Formula (Gauss's Law)** **½**

**Calculation of Charge within the cube** **1**

Electric Flux is the dot product of electric field and area vector. **1**

Also accept,

$$\phi = \int \vec{E} \cdot \vec{ds}$$

**SI Unit:** $Nm^2/C$ or volt-metre ½

For a given case,

$$\phi = \phi_1 + \phi_2 = [E_x(at\, x = 2a) - E_x(at\ x = a)]a^2$$

$$= [\alpha(2a) - \alpha(a)]a^2$$

$$= \alpha a^3$$

$$= 100 \times (0{\cdot}1)^3 = 0{\cdot}1\ Nm^2/C$$ ½

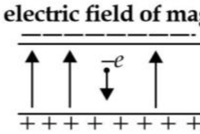
But $\phi = \frac{q}{\varepsilon_0}$

$\therefore$ $q = \varepsilon_0 \phi = 8{\cdot}854 \times 10^{-12} \times 10^{-1}$ C

$= 0.8854$ pC **1**

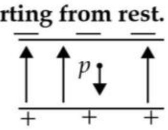
**[CBSE Marking Scheme, 2018]**

**AI Q. 6. An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude $2.0 \times 10^4$ N/C**




**(a)**

**Calculate the time it takes to fall through this distance starting from rest.**




**(b)**

**If the direction of the field is reversed (Fig. b) keeping its magnitude unchanged, calculate the time taken by a proton to fall through this distance starting from rest.**

**A & E [Delhi Outside Comptt. I, II, III 2018]**

**Ans. Relevant formulae** **1**

**Calculation of time taken by the electron** **1**

**Calculation of time taken by the proton** **1**

We have

Force = $qE$

Acceleration,

$$a = \frac{qE}{m}$$ ½

Also

$$s = \frac{1}{2}at^2 \quad \text{as } u = 0$$

$$\therefore \quad t = \sqrt{\frac{2s}{a}}$$ ½

**(i)** For the electron,

$$a = \frac{eE}{m}$$

$$\therefore \quad t = \sqrt{\frac{3\times10^{-2}\times9.1\times10^{-31}}{1.6\times10^{-19}\times2.0\times10^{4}}}$$

$= 2.92$ ns **1**

**(ii)** For proton,

$$t = \sqrt{\frac{2\times1.5\times10^{-2}\times1.67\times10^{-27}}{1.6\times10^{-19}\times2\times10^{4}}}$$

$= 0.125$ μs **1**

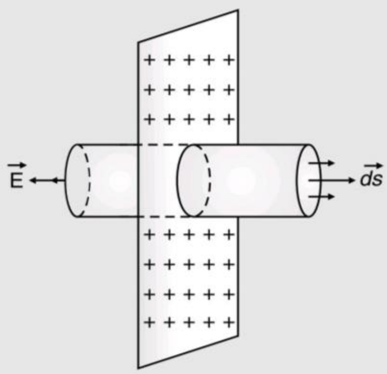
**[CBSE Marking Scheme, 2018]**

**AI Q. 7. Use Gauss's law to find the electric field due to a uniformly charged infinite plane sheet. What is the direction of field for positive and negative charge densities?**

R&U **[CBSE Delhi Outside Set-I 2016]**

**Ans.** Derivation for electric field due to infinite plane Sheet of charge **2**

Directions of field **½ + ½**




Symmetry of situation suggests that $\vec{E}$ is perpendicular to the plane. A Gaussian surface is considered through P like a cylinder of flat caps parallel to the plane and one cap passing through P. The plane being the plane of symmetry for the Gaussian surface.

$$\oint \vec{E}.\vec{ds} = \int_{\text{through caps}} \vec{E}.\vec{ds}$$

$\vec{E} \perp \vec{ds}$ for all over curved surface and hence $\vec{E}.\vec{ds} = 0$

$$\int_{\text{caps}} E ds = 2E\Delta s$$

$\Delta s$ = area of each cap

By Gauss' law

$$\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0} = \frac{\sigma\Delta s}{\varepsilon_0}$$

$$\therefore \quad 2E\Delta s = \frac{\sigma\Delta s}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

If $\sigma$ is positive $\vec{E}$ points normally outwards/away from the sheet

If $\sigma$ is (–)ve, $\vec{E}$ points normally inwards/towards the sheet. **[CBSE Marking Scheme, 2016]**

**AI Q. 8. A charge $Q$ is distributed uniformly over a metallic sphere of radius $R$. Obtain the expression for the electric field ($E$) and electric potential ($V$) at a point $0 < x < R$.**

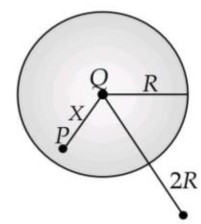
**Show or plot the variation of $E$ and $V$ with $x$ for $0 < x < 2R$.** A & E

**Ans.** Expression for electric field **1½**

Expression for potential **½**

Plot of graph (E vs $r$) **½**

Plot of graph (V vs $r$) **½**




**½**

By Gauss's law,

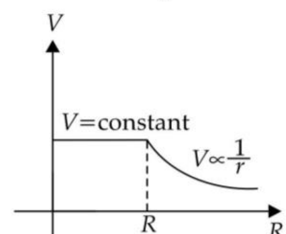
$$\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$$ **½**

$q = 0$ in interval $0<x<R$

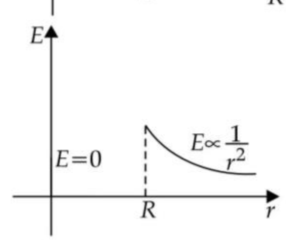
$E = 0$ **½**

or $\quad E = -\frac{dV}{dr}$ **½**

Hence, $V$ = constant $= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$




**½**




**½**

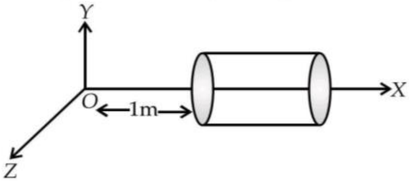
[Even if a student draws $E$ and $V$ for $0<r<R$ award ½ + ½ mark]

**AI Q. 9. A hollow cylindrical box of length 1 m and area of cross-section 25 cm² is placed in a three dimensional co-ordinate system as shown in the figure. The electric field in the region is given by**

$\vec{E} = 50\,x\,\hat{i}$, **where $E$ is in $NC^{-1}$ and $x$ is in metres. Find**

**(i) Net flux through the cylinder.**

**(ii) Charge enclosed by the cylinder.**



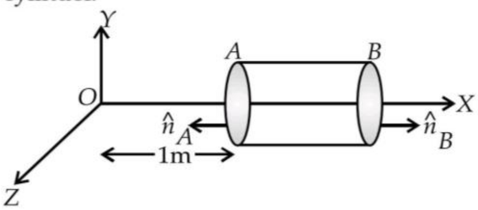

A & E

**Ans. (i) Given:**

$$\vec{E} = 50x\,\hat{i}$$

and $\Delta S = 25 \text{ cm}^2$ or $25 \times 10^{-4}\, m^2$

As the electric field is only along the $x$-axis, hence, flux will pass only through the cross-section of cylinder.




½

Magnitude of electric field at cross-section A,

$$E_A = 50 \times 1 = 50 \text{ N/C}.$$

Magnitude of electric field at cross-section $B$,

$$E_B = 50 \times 2 = 100 \text{ N/C}$$

The corresponding electric fluxes are

$$\oint \phi_A = \vec{E}\,.\,\vec{\Delta S}$$

$$= 50 \times 25 \times 10^{-4} \times \cos 180°$$

$$= -0.125 \text{ Nm}^2/\text{C}^2 \quad ½$$

$$\oint \phi_B = \vec{E}\,.\,\vec{\Delta S}$$

$$= 100 \times 25 \times 10^{-4} \times \cos 0°$$

$$= 0.25 \text{ Nm}^2/\text{C}^2 \quad ½$$

So, the net flux through the cylinder,

$$\oint \phi = \oint \phi_A + \oint \phi_B$$

$$= -0.125 + 0.25$$

$$= 0.125 \text{ Nm}^2/\text{C}^2 \quad ½$$

**(ii)** Using the Gauss's law,

$$\phi = \oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad 0.125 = \frac{q}{8.85 \times 10^{-12}}$$

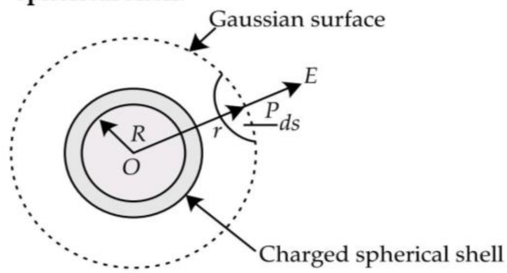
$$\Rightarrow \quad q = 8.85 \times 10^{-12} \times 0.125$$

$$= 1.107 \times 10^{-12} \text{ C}. \quad 1$$

**AI Q. 10. Using Gauss's law deduce the expression for the electric field due to a uniformly charged spherical conducting shell of radius $R$ at a point (i) outside, and (ii) inside the shell.**

**Plot a graph showing variation of electric field as function of $r > R$ and $r < R$ ($r$ being the distance from the centre of the shell).** A **[Imp]**

**Ans. Electric field due to a uniformly charged thin spherical shell:**




**(i) When point P lies outside the spherical shell:** Suppose that we have to calculate electric field at the point P at a distance $r$ ($r > R$) from its centre. Draw the Gaussian surface through point $P$ so as to enclose the charged spherical shell. The Gaussian surface is a spherical shell of radius $r$ and centre $O$. **1**

Let $\vec{E}$ be the electric field at point $P$, then the electric flux through area element $d\vec{s}$ is given by,

$$\Delta\phi = \vec{E}\,.\,\vec{\Delta S}$$

Since $\vec{\Delta S}$ is also along normal to the surface,

$$\Delta\phi = E.\,dS$$

∴ Total electric flux through the Gaussian surface is given by.

$$\phi = \oint_s E dS = E \oint_s dS$$

Now, $\oint_s dS = 4\pi r^2$

$$\therefore \quad \phi = E \times 4\pi r^2 \quad \ldots\text{(i)}$$

Since the charge enclosed by the Gaussian surface is $q$, according to the Gauss's law,

$$\phi = \frac{q}{\varepsilon_0} \quad \ldots\text{(ii)}$$

From equations (i) and (ii), we obtain

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

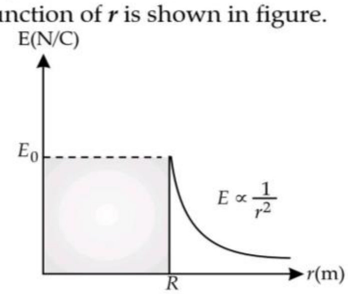
$$E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2} \quad \text{(for } r > R\text{)} \quad \mathbf{1}$$

**(ii) When point P lies inside the spherical shell:** In such a case, the Gaussian surface encloses no charge. According to the Gauss's law,

$$E \times 4\pi r^2 = 0$$

*i.e.,* $E = 0 \quad (r < R)$

A graph showing the variation of electric field as a function of $r$ is shown in figure.




**1**

## Long Answer Type Questions (5 marks each)

**Q. 1. (a) State Gauss's law on electrostatics and derive an expression for the electric field due to a long, straight and thin uniformly charged wire (linear charge density $\lambda$) at a point lying at a distance $r$ from the wire.**

**(b) The magnitude of electric field (in $NC^{-1}$) in a region varies with the distance $r$ (in m) as $E = 10r + 5$. By how much, does the electric potential increase in moving from point at $r = 1$ m to a point at $r = 10$ m.** A **[CBSE Delhi Set 1 2020]**

**Ans. (a) Gauss's Law:** The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the electrical permittivity of free space.

$$\phi = \frac{Q}{\varepsilon_0}$$ ½

Electric field due to a long, straight and thin uniformly charged wire:

Try it youself: See Q. No. 2(a) of 3 marks SATQ-II. 2

**(b)** $$E = -\frac{dV}{dr}$$

$\therefore$ $$|V| = \int_1^{10} E\,dr$$

or, $$|V| = \int_1^{10} E\,dr$$ 1

or, $$|V| = \int_1^{10} (10r + 5)dr$$ 1

1

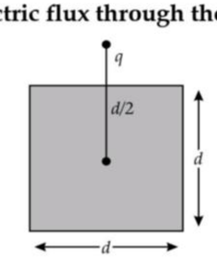
or, $$|V| = [5r^2 + 5r]_1^{10}$$

or, $$|V| = 540 \text{ V}$$

Thus, electric potential changes by 540 V. ½

**Q. 2. (a) Define electric flux. Is it a scalar or a vector quantity?**

**A point charge $q$ is at a distance of $\frac{d}{2}$ directly above the centre of a square of side $d$, as shown in the figure. Use Gauss' law to obtain the expression for the electric flux through the square.**




**(b) If the point charge is now moved to a distance '$d$' from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected.** A&U **[CBSE Delhi 2018]**

**Ans. (a)** Definition of electric flux 1

Stating scalar/ vector ½

Gauss's Theorem ½

Derivation of the expression for electric flux 1

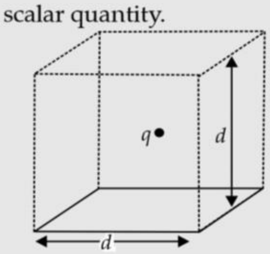
**(ii)** Explanation of change in electric flux 2

**(i)** Electric flux through a given surface is defined as the dot product of electric field and area vector over that surface. 1

Alternatively, $\phi = \int_s \vec{E}.d\vec{s}$

Electric flux, through a surface equals the surface integral of the electric field over that surface. ½

It is a scalar quantity. ½




Constructing a cube of side '$d$' so that charge '$q$' gets placed within of this cube (Gaussian surface) According to Gauss's law the Electric flux,

$$\phi = \frac{\text{Charge enclosed}}{\varepsilon_0}$$

$$= \frac{q}{\varepsilon_0}$$ 1½

This is the total flux through all the six faces of the cube.

Hence electric flux through the square

$$\frac{1}{6} \times \frac{q}{\varepsilon_0} = \frac{q}{6\varepsilon_0}$$ ½

**(ii)** If the charge is moved to a distance $d$ and the side of the square is doubled, the cube will be constructed to have a side $2d$ but the total charge enclosed in it will remain the same. Hence the total flux through the cube and therefore the flux through the square will remain the same as before. [Deduct 1 mark if the student just writes No change/not affected without giving any explanation.] 1+1

AI **Q. 3. (a) Use Gauss' law to derive the expression for the electric field $\vec{E}$ due to a straight uniformly charged infinite line of charge density $\lambda$ C/m.**

**(b) Draw a graph to show the variation of E with perpendicular distance $r$ from line of charge.**

**(c) Find the work done in bringing a charge $q$ from perpendicular distance $r_1$ to $r_2$ ($r_2 > r_1$).**

A&U **[CBSE Delhi 2018]**

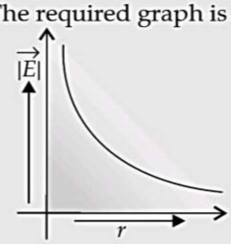
**Ans. (a)** Derivation of the expression for electric field. **3**

**(b)** Graph to show the required variation of the electric field. **1**

**(c)** Calculation of work done **1**

**(a)** Try it Yourself See Q. No. 2(a) of 3 marks questions.

**(b)** The required graph is as shown: **1**




**(c)** Work done in moving the charge '*q*' through a small displacement '*dr*'

$$dW = \vec{F}.d\vec{r}$$

$$dW = q\vec{E}.d\vec{r}$$

$$= qEdr\cos 0°$$

$$dW = q\times\frac{\lambda}{2\pi\varepsilon_0 r}dr \quad ½$$

Work done in moving the given charge from $r_1$ to $r_2$ $(r_2 > r_1)$

$$W = \int_{r_1}^{r_2} dW = \int_{r_1}^{r_2}\frac{\lambda q dr}{2\pi\varepsilon_0 r}$$

$$W = \frac{\lambda q}{2\pi\varepsilon_0}[\log_e r_2 - \log_e r_1] \quad ½$$

$$W = \frac{\lambda q}{2\pi\varepsilon_0}\left[\log_e\frac{r_2}{r_1}\right]$$

**[CBSE Marking Scheme, 2018]**

**AI Q. 4. Use Gauss's theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet with surface charge densidty σ.**

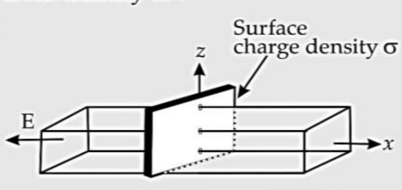
**(b) An infinitely large thin plane sheet has a uniform surface charge density +σ. Obtain the expression for the amount of work done in bringing a point charge *q* from infinity to a point, distant *r*, in front of the charged plane sheet.**

**A [CBSE Delhi Outside Set-I 2017]**

**Ans. (a)** Using Gauss's theorem to find *E* due to an infinite plane sheet of charge **3**

**(b)** Expression for the work done to bring charge *q* from infinity to *r* **2**

**(a)**




**2**

$$\oint E.ds = \frac{q}{\varepsilon_0}$$

The electric field *E* points outwards normal to the sheet. The field lines are parallel to the Gaussian surface except for surfaces 1 and 2.

Hence the net flux $= \oint E.ds = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} = 2EA$

where *A* is the area of each of the surface 1 and 2. **1**

$$\therefore \oint E.ds = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} = 2EA$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

**1**

**(b)** $$W = q\int_{\infty}^{r}\vec{E}.\vec{dr} \quad ½$$

$$= q\int_{\infty}^{r}(-Edr) \quad ½$$

$$= -q\int_{\infty}^{r}\left(\frac{\sigma}{2\varepsilon_0}\right)dr \quad ½$$

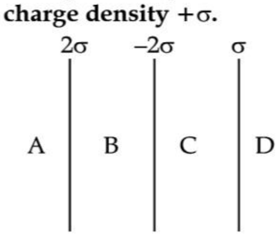
$$= \frac{q\sigma}{2\varepsilon_0}|\infty - r|$$

$$= (\infty)$$

**[CBSE Marking Scheme, 2017] ½**

**AI Q. 5. (a) State Gauss's law in electrostatics. Show that with help of suitable figure that outward flux due to a point charge Q, in vacuum within Gaussian surface, is independent of its size and shape.**

**(b) In the figure there are three infinite long thin sheets having surface charge density +2σ, – 2σ and +σ respectively. Give the magnitude and direction of electric field at a point to the left of sheet of charge density +2σ and to the right of sheet of charge density +σ.**




**R&U [CBSE SQP 2020-21]**

**Ans. (a)** Statement of Gauss law **1**

Proof of outward flux due to a point charge Q ,in vacuum within gaussian surface, is independent of its size and shape **2**

**(b)** Net electric field towards left = $\frac{\sigma}{\varepsilon}$ **1**

Net electric field towards right = $\frac{\sigma}{\varepsilon}$ **1**

**(a) Gauss law:** Total electric flux over a closed surface in vacuum is $\frac{1}{\varepsilon_0}$ times of the total charge enclosed by the surface. **1**

**Proof:** consider that a point charge $+q$ is placed at O within a closed surface.

Let $\vec{E}$ be the electric field produced.

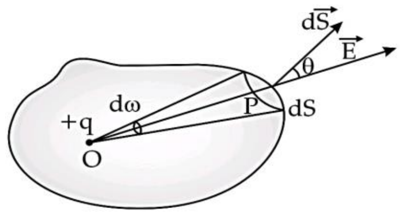
At P a small distance $ds$ is considered. OP = $r$

Electric flux through $\vec{ds}$ is $d\phi = \vec{E}.\vec{ds}$

Electric field at P is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q\vec{r}}{r^3}$$




$$\therefore \quad d\phi = \vec{E}.\vec{ds} = \frac{1}{4\pi\varepsilon_0}\frac{q\,\vec{r}.\vec{ds}}{r^3}$$

$$d\phi = \frac{q}{4\pi\varepsilon_0}d\omega$$

[$d\omega$ is the solid angle subtended by ds at 0.] **1**

Total electric flux = $\phi = \oint \vec{E}.\vec{ds}$

Or, $\phi = \frac{q}{4\pi\varepsilon_0}\oint dw$

Or, $\phi = \frac{q}{4\pi\varepsilon_0}\times 4\pi$ [sin $\oint dw = 4\pi$ ]

$\therefore \quad \phi = \frac{q}{\varepsilon_0}$

So, the flux is independent of shape and size. **1**

**(b)** At point A, electric field = $\vec{E_A}$

$$= \frac{2\sigma}{2\varepsilon_0}(-\hat{i}) + \frac{2\sigma}{2\varepsilon_0}(\hat{i}) + \frac{\sigma}{2\varepsilon_0}(-\hat{i})$$

$$= \frac{\sigma}{2\varepsilon_0}(-\hat{i})$$

At point B, electric filed = $\vec{E_B}$

$$= \frac{\sigma}{2\varepsilon_0}(\hat{i}) + \frac{2\sigma}{2\varepsilon_0}(-\hat{i}) + \frac{2\sigma}{2\varepsilon_0}(\hat{i})$$

$$= \frac{\sigma}{2\varepsilon_0}(\hat{i})$$ **1**

So, net electric field to the left of the sheet having charge density $2\sigma$ is $\frac{\sigma}{2\varepsilon_0}$ towards left.

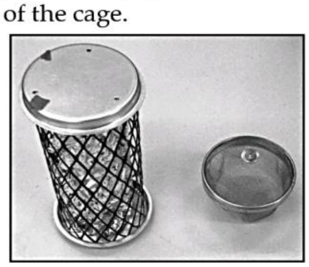
Similarly net electric field to the right of the sheet having charge density $\sigma$ is $\frac{\sigma}{2\varepsilon_0}$ towards right. **1**

## Visual Case-based Questions

**(1×4=4 marks)**

**Attempt any 4 sub-parts out of 5. Each question carries 1 mark.**

**1. Faraday Cage:** A Faraday cage or Faraday shield is an enclosure made of conducting material. The fields within a conductor cancel out with any external fields, so the electric field within the encloser is zero. These Faraday cages act as big hollow conductors you can put things in to shield them from electrical fields. Any electrical shocks the cage receivers, pass harmlessly around the outside of the cage.



**(i)** Which of the following material can be used to make a Faraday cage ?

**(a)** Plastic **(b)** Glass
**(c)** Copper **(d)** Wood

**Ans. (c)** Copper

**Detailed Answer:**

A Faraday cage or Faraday shield is an enclosure made of a conducting material.

Since copper is the only metal given in the list of options, copper is the correct answer.

**(ii)** Example of a real-world Faraday cage is :

**(a)** car **(b)** plastic box
**(c)** lighting rod **(d)** metal rod

**Ans. (a)** Car

**Detailed Answer:**

Cars are example Faraday Cages in the real world. Cars can help keep ourselves safe from lightning. Its metal body acts as a Faraday Cage.

**(iii)** What is the electrical force inside a Faraday cage when it is struck by lightning ?

**(a)** The same as the lightning
**(b)** Half that of the lightning
**(c)** Zero
**(d)** A quarter of the lightning

**Ans.** (c) zero

**Detailed Answer:**

The field within a conductor cancel out with any external fields, so the electric field within the enclosure is zero.

**(iv)** If isolated point charge $+q$ is placed inside the Faraday cage. Its surface must have charge equal to :

**(a)** Zero **(b)** $+q$

**(c)** $-q$ **(d)** $+2q$

**Ans.** (c) $-q$

**Detailed Answer:**

If a charge is placed inside an ungrounded Faraday shield without touching the walls of the internal face of the shield becomes charged with $-q$, and $+q$ accumulates on the outer face of the shield. If the cage is grounded, the excess charges will be neutralized by the ground connection.

**(v)** A point charge of 2C is placed at centre of Faraday cage in the shape of cube with surface of 9 cm edge. The number of electric field lines passing through the cube normally will be :

**(a)** $1.9 \times 10^5$ Nm²/C entering the surface

**(b)** $1.9 \times 10^5$ Nm²/C leaving the surface

**(c)** $2.0 \times 10^5$ Nm²/C entering the surface

**(d)** $2.0 \times 10^5$ Nm²/C leaving the surface

**Ans.** **(d)** $2.0 \times 10^5$ Nm²/C leaving the surface

**Detailed Answer:**

The number of electric field lines passing through the cube normally and leaving the surface = $Q/\varepsilon_0$

$$Q = 2\mu C = 2 \times 10^{-6}\ C$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$$

$$\therefore \quad Q/\varepsilon_0 = 2.2 \times 10^5\ C^2/Nm^2$$

❑❑