AEP STUDY CIRCLE
IIT-NEET-CBSE
ACCENTS EDUCATIONAL PROMOTERS

23 YEARS OF LEGACY

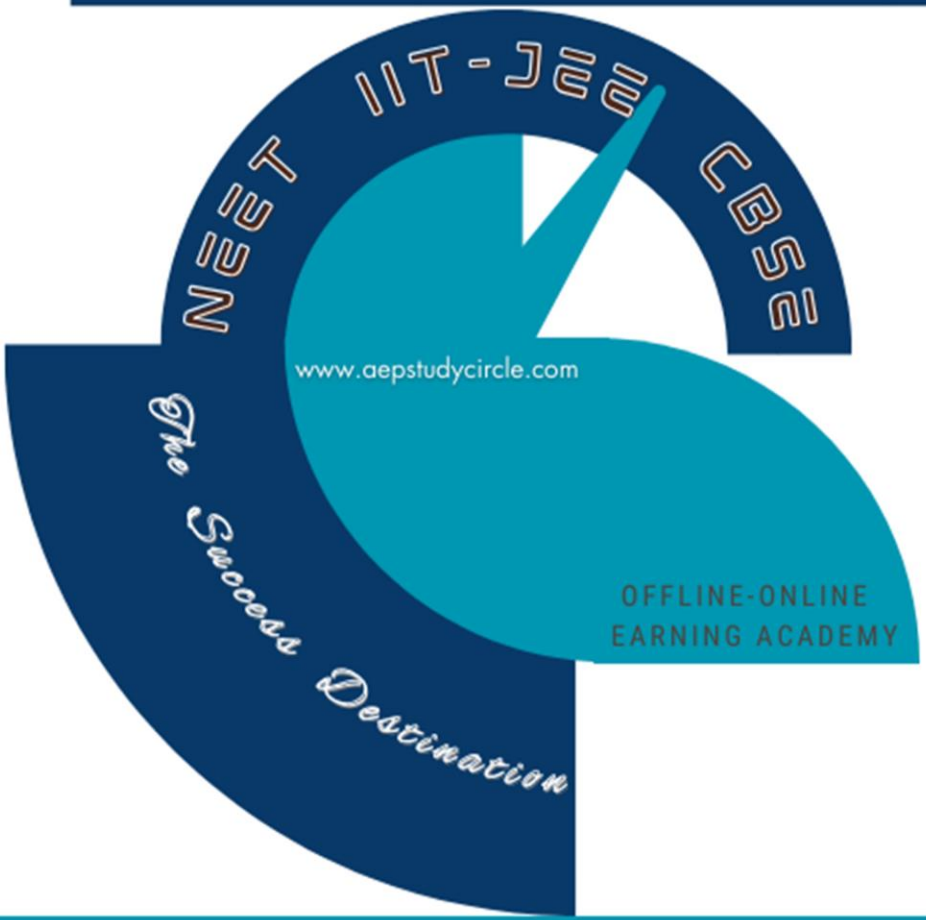
CBSE-X

qUADRATIC eQUATIONS

Revision module

MATHEMATICS

Ace your Class 10th Mathematics with our specialized Revision Material focusing on Quadratic Equations. This comprehensive module is designed to streamline your review process, covering essential concepts, solving techniques, and practical applications.



+91-9939586130
+91-7739650505
2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122-JH

QUADRATIC EQUATIONS

CBSE-X

Revision Module

Conceptual Review: Dive into a thorough review of quadratic equations, exploring fundamental concepts such as roots, coefficients, and discriminants. Strengthen your understanding of the properties and behavior of quadratic functions.

Solving Techniques: Master solving quadratic equations through various methods—factoring, completing the square, and using the quadratic formula. Our revision material guides you through step-by-step procedures to enhance your problem-solving skills.

Real-world Applications: Connect theoretical knowledge to practical scenarios. Explore how quadratic equations model real-life situations, from projectile motion in physics to financial applications in economics.

Problem-solving Practice: Hone your problem-solving skills with targeted practice questions. Covering a spectrum of difficulty levels, our revision material provides diverse exercises to reinforce your understanding and boost your confidence in tackling quadratic equations.

Exam-oriented Approach: Tailored to align with the CBSE examination pattern, our revision material ensures you are well-prepared for questions related to quadratic equations in the Class 10th board exams. Boost your confidence with targeted, exam-focused content.

Online Accessibility: Access your revision material anytime, anywhere. Our digital platform offers the flexibility needed for efficient and personalized learning, allowing you to tailor your study schedule for optimal results.

HELPLINE : +91-9939586130 // +91-7739650505

2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122-JH

QUADRATIC EQUATIONS

CBSE-X

Revision Module

Basic Concepts

- **Quadratic Equation:** An equation, which is written or may be written in the form $ax^2 + bx + c = 0$ where a, b, c are real numbers and $a \neq 0$ is called quadratic equation.

  For example, $2x^2 + 5x + 7 = 0$, $x^2 + 7 = 0$, $3x^2 + 5x = 9$; $\frac{5}{7}x^2 - 7 = 0$ are quadratic equations.

  In other way we can say that any equation of the form $p(x) = 0$, where $p(x)$ is a polynomial of degree 2 is a quadratic equation.

- **Roots/Solutions of a Quadratic Equation:** A real number α is called a root or solution of the quadratic equation $p(x) = 0$ iff α satisfy the quadratic equation $p(x) = 0$ *i.e.* $p(\alpha) = 0$.

  In simplest way, we can say a real number α is called root or solution of the quadratic equation $ax^2 + bx + c = 0, a \neq 0$ if $a\alpha^2 + b\alpha + c = 0$.

  For example, 3 is the root of quadratic equation $x^2 - 5x + 6 = 0$ because

  $$3^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 0$$

  Also, 2 is the root of quadratic equation $x^2 - 5x + 6 = 0$ because $2^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 0$.

  Therefore, 2, 3 are roots or solutions of quadratic equation $x^2 - 5x + 6 = 0$.

  In previous chapter, we have studied that a quadratic polynomial can have at most two zeros. Similarly a quadratic equation can have atmost two real roots or solutions. Sometimes, they may be equal.

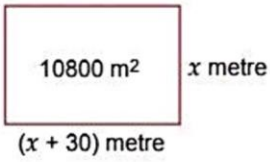
  For example: $x^2 - 4x + 4 = 0$ have two equal roots 2, 2.

- **Formulation of Quadratic Equation:** We come across a quadratic equation during solving several problems related to practical life and in different fields of mathematics. Following example will illustrate the formations of quadratic equations according to given situation.

  For example: Suppose you own a plot of land and want to build a house there. Let the area of the plot be 10800 m^2.

  You want your house to have a length 30 m more than its breadth. How will you optimise the area of land you have? What will be the length and breadth of house? For this you have to formulate the given situation algebraically.

  Suppose the breadth of the hall is x metres then its length should be $(x + 30)$ metre. We can depict this information pictorially as shown in figure.



Now, area of the hall = length × breadth

$\Rightarrow \quad 10800 = (x + 30).\, x$

$\Rightarrow \quad x^2 + 30x - 10800 = 0$, which is a quadratic equation.

❒ **Solution of a Quadratic Equation by Factorisation Method:**

In previous classes we have discussed how to factorise quadratic polynomials. In this topic, we will apply the same method of factorisation to solve or to find out roots of a quadratic equation.

For example: Suppose, we have to solve or find out roots of quadratic equation $x^2 - x - 20 = 0$.

We have $x^2 - x - 20 = 0$

$\Rightarrow \quad x^2 - 5x + 4x = 20 = 0$

$\Rightarrow \quad x(x - 5) + 4(x - 5) = 0$

$\Rightarrow \quad (x - 5)\,(x + 4) = 0$

$\Rightarrow \quad x - 5 = 0 \text{ or } x + 4 = 0$

$\Rightarrow \quad x = 5 \text{ or } x = -4$

Therefore $x = 5$ and $x = -4$ are the roots of quadratic equation $x^2 - x - 20 = 0$

In this way to solve quadratic equations, we follow following algorithm:

Step I: Obtain given quadratic equation in the form $ax^2 + bx + c = 0$.

Step II: Split the mid term in two parts such that their sum is equal to mid term and product is equal to the product of the term containing x^2 and constant term.

Step III: Factorise the quadratic equation obtained in step II by grouping method as in example given above.

Step IV: Factors obtained in step III are then equated with zero and the values of x are found.

❒ **Quadratic Formula (Sridharacharya's formula):** In order to solve the quadratic equation by factorisation method sometimes it is difficult or not possible to split up middle term according to process. Therefore such equations cannot be solved by using factorisation method. To solve such a quadratic equation a formula is obtained called "Sridharacharya formula", which is given as $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for quadratic equation $ax^2 + bx + c = 0$.

Obviously, for real value of x, $b^2 - 4ac \geq 0$; "$b^2 - 4ac$" is called discriminant and represented by D.

Thus if $D = b^2 - 4ac \geq 0$, then the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a}, \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

This formula for finding the roots of a quadratic equation is known as the "Quadratic formula" or "Sridharacharya formula".

❒ **Nature of Roots:** From quadratic formula, we know that the roots of quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Case-I: If $D = b^2 - 4ac = 0$, then

$$x = \frac{-b + 0}{2a} \text{ and } \frac{-b - 0}{2a}$$

$\Rightarrow \quad x = \dfrac{-b}{2a} \text{ and } \dfrac{-b}{2a}$

i.e., if $D = 0$, quadratic equation has two equal real roots.

Case-II: If $D = b^2 - 4ac = l$ (say) > 0, then

$$x = \frac{-b+l}{2a} \text{ and } \frac{-b-l}{2a}$$

i.e., if $D > 0$, quadratic equation have two distinct real roots.

Case-III: If $D = b^2 - 4ac < 0$

Then $\sqrt{b^2-4ac} = \sqrt{-\text{ve number}}$, since –ve number can never be square of any real number.

$\Rightarrow$ $\sqrt{-\text{ve number}}$ can not be obtained as real number.

Thus the value of $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ can not be obtained as real number

i.e. if $D < 0$, equation has no real roots.

Important Facts/Tips

(*i*) $ax^2 + bx + c = 0, a \neq 0$ is called the standard form of quadratic equation.

(*ii*) If α, β are roots of a quadratic equation then equation is given as $k\{x^2 - (\alpha+\beta)x + \alpha\beta\} = 0$, where k may be any real number.

(*iii*) Irrational roots of a quadratic equation always occur with its conjugate.

For example, if $a + b\sqrt{2}$ is root of a quadratic equation then $a - b\sqrt{2}$ is its other root.

(*iv*) The zeros of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.

(*v*) If we can factorise $ax^2 + bx + c; a \neq 0$, into product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.

Selected NCERT Questions

1. Find the roots of the quadratic equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$.

Sol. The given quadratic equation is

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

By applying mid term splitting, we get

$$\sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$$

$$\Rightarrow \quad \sqrt{2}x(x+\sqrt{2}) + 5(x+\sqrt{2}) = 0 \Rightarrow (\sqrt{2}x+5)(x+\sqrt{2}) = 0$$

$$\Rightarrow \quad x = \frac{-5}{\sqrt{2}}, -\sqrt{2} \quad \text{or} \quad \frac{-5\sqrt{2}}{2}, -\sqrt{2}$$

2. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

[*CBSE* 2020(30/1/2)]

Sol. Let the speed of train be x km/h.

$$\therefore \quad \frac{480}{x-8} - \frac{480}{x} = 3 \qquad 1$$

$$\Rightarrow \quad x^2 - 8x - 1280 = 0 \qquad 1$$

$$(x - 40)(x + 32) = 0$$

$$x = 40, -32 \text{ (Rejected)} \qquad 1$$

$\therefore$ Speed of train = 40 km/h **[*CBSE Marking Scheme* 2020(30/1/2)]**

3. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in the two subjects.

Sol. Let Shefali's marks in Mathematics be x.

Therefore, Shefali's marks in English is $(30 - x)$.

Now, according to question,

$$(x + 2)(30 - x - 3) = 210 \Rightarrow (x + 2)(27 - x) = 210$$

$$\Rightarrow 27x - x^2 + 54 - 2x = 210 \Rightarrow 25x - x^2 + 54 - 210 = 0$$

$$\Rightarrow 25x - x^2 - 156 = 0 \Rightarrow -(x^2 - 25x + 156) = 0$$

$$\Rightarrow x^2 - 25x + 156 = 0 \Rightarrow x^2 - 13x - 12x + 156 = 0$$

$$\Rightarrow x(x - 13) - 12(x - 13) = 0 \Rightarrow (x - 13)(x - 12) = 0$$

Either $x - 13 = 0$ or $x - 12 = 0$

$\Rightarrow x = 13$ or $x = 12$

$\therefore$ Shefali's marks in Mathematics = 13, marks in English = 30 – 13 = 17

or Shefali's marks in Mathematics = 12, marks in English = 30 – 12 = 18.

4. Find the roots of the following quadratic equation by applying the quadratic formula.

$$4x^2 + 4\sqrt{3}x + 3 = 0$$

Sol. We have, $4x^2 + 4\sqrt{3}x + 3 = 0$

Here, $a = 4, b = 4\sqrt{3}$ and $c = 3$

Therefore, $D = b^2 - 4ac$

$$= (4\sqrt{3})^2 - 4 \times 4 \times 3 = 48 - 48 = 0$$

$\because$ $D = 0$, roots exist and are equal.

Thus, $$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-4\sqrt{3} \pm 0}{2 \times 4} = \frac{-\sqrt{3}}{2}$$

Hence, the roots of given equation are $\frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$.

5. Find the roots of the following quadratic equation by factorisation:

$$2x^2 - x + \frac{1}{8} = 0$$

Sol. We have, $2x^2 - x + \frac{1}{8} = 0$

$$\Rightarrow \frac{16x^2 - 8x + 1}{8} = 0 \Rightarrow 16x^2 - 8x + 1 = 0$$

$$\Rightarrow 16x^2 - 4x - 4x + 1 = 0 \Rightarrow 4x(4x - 1) - 1(4x - 1) = 0$$

$$\Rightarrow (4x - 1)(4x - 1) = 0$$

So, either $4x - 1 = 0$ or $4x - 1 = 0$

$$x = \frac{1}{4} \text{ or } x = \frac{1}{4}$$

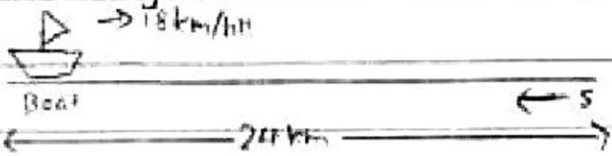
Hence, the roots of the given equation are $\frac{1}{4}$ and $\frac{1}{4}$.

6. A motor boat whose speed is 18 km/h. in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. [*CBSE* 2018(30/1/1)]

Sol.

23) (Choice 1)

Representative diagram:



Given that:

Speed of boat = 18 km/hr in still water.

Speed of stream = S (variable, must find)

Distance upstream = 24 km

Time upstream = 1 hr more than time downstream.

We know, $\text{Speed} = \frac{\text{Distance}}{\text{Time}} \rightarrow \text{Time} = \frac{\text{Distance}}{\text{Speed}}$.

Time upstream $= \frac{24}{18-S}$, Time downstream $= \frac{24}{18+S}$.

$$\rightarrow \frac{24}{18-S} = 1 + \frac{24}{18+S}.$$

$$\frac{24}{18-S} = \frac{18+S+24}{18+S}.$$ (cross-multiplying)

$$24(18+S) = (42+S)(18-S)$$

$$432+24S = 756+18S-42S-S^2$$

$$\rightarrow S^2+42S+24S+432-756=0.$$

$$S^2+48S+(-324)=0.$$

$$S^2+54S-6S-324=0.$$

$$S(S+54)-6(S+54)=0.$$

$$(S-6)(S+54)=0.$$

Now, either $S-6=0$ or $S+54=0$.

$\rightarrow S=6$ $\rightarrow S=-54$.

So speed = 6 or -54 km/hr.

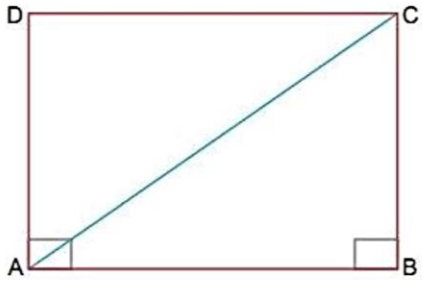
But speed cannot be negative.

$\Rightarrow$ **Speed of the stream is 6 km/hr.**

[*Topper's Answer* 2018(30/1/1)]

7. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Sol. Let $ABCD$ be the rectangular field. Let the shorter side BC of the rectangle $= x$ metres.



According to question,

Diagonal of the rectangle, $AC = (x + 60)$ metres

side of the rectangle, $AB = (x + 30)$ metres

By Pythagoras theorem, $AC^2 = AB^2 + BC^2$

$\therefore \quad (x + 60)^2 = (x + 30)^2 + x^2$

or $\quad x^2 + 120x + 3600 = x^2 + 60x + 900 + x^2$

$\therefore \quad (2x^2 - x^2) + (60x - 120x) + 900 - 3600 = 0$

or $\quad x^2 - 60x - 2700 = 0$

or $\quad (x - 90)(x + 30) = 0$

$\therefore$ Either $x - 90 = 0$ or $x + 30 = 0$

$\Rightarrow \quad x = 90$ or $x = -30$ (But side cannot be negative)

So, the shorter side of rectangle = 90 m

and longer side of rectangle = 120 m

8. The sum of the reciprocals of Rehman's age (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Sol. Let the present age of Rehman be x years.

So, 3 years ago, Rehman's age $= (x - 3)$ years

And 5 years from now, Rehman's age $= (x + 5)$ years

Now, according to question, we have

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3} \qquad \Rightarrow \quad \frac{2x+2}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow \quad 6x + 6 = (x-3)(x+5) \qquad \Rightarrow \quad 6x + 6 = x^2 + 5x - 3x - 15$$

$$\Rightarrow \quad x^2 + 2x - 15 - 6x - 6 = 0 \qquad \Rightarrow \quad x^2 - 4x - 21 = 0$$

$$\Rightarrow \quad x^2 - 7x + 3x - 21 = 0 \qquad \Rightarrow \quad x(x-7) + 3(x-7) = 0$$

$$\Rightarrow \quad (x-7)(x+3) = 0 \qquad \Rightarrow \quad x = 7 \quad \text{or} \quad x = -3$$

But $x \neq -3$ (age cannot be negative)

Therefore, present age of Rehman = 7 years.

9. Find the value of k for which the quadratic equation $kx(x - 2) + 6 = 0$ has two equal roots.
[*CBSE* 2019(30/4/1)]

Sol. Given equation $kx^2 - 2kx + 6 = 0$

For two equal roots

$$D = 0$$

$$\Rightarrow \quad b^2 - 4ac = 0$$

$$\Rightarrow \quad 4k^2 - 4k \times 6 = 0$$

$$\Rightarrow \quad 4k(k-6) = 0$$

$$\Rightarrow \quad k = 6 \qquad [k \neq 0, \text{ as if } k = 0 \text{ then the given equation is not a valid equation.}]$$

10. The difference of square of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Sol. Let smaller and larger number be x and y respectively.

From question

$$y^2 - x^2 = 180 \quad \ldots(i)$$

Also, $$x^2 = 8y \quad \ldots(ii)$$

From (i) and (ii)

$$y^2 - 8y = 180$$

$$\Rightarrow \quad y^2 - 8y - 180 = 0$$

By applying quadratic formula, we get

$$y = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 1 \times (-180)}}{2}$$

$$= \frac{8 \pm \sqrt{64 + 720}}{2} = \frac{8 \pm \sqrt{784}}{2}$$

$$= \frac{8 \pm 28}{2} = \frac{36}{2} \text{ or } \frac{-20}{2}$$

$$= 18 \text{ or } -10$$

If $y = 18$, $\quad x^2 = 8 \times 18 = 144 \quad \Rightarrow \quad x = \pm 12$

Required two numbers are 18, 12 or 18, –12.

Also, $y = -10$ is not possible because $x^2 = 8 \times (-10) = -80$ (negative number which is not possible)

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. The product of three consecutive integers is equal to 6 times the sum of three integers. If the smallest integer is x, which of the following equations represent the above situation?

(a) $2x^2 + x - 9 = 0$ (b) $2x^2 - x + 9 = 0$

(c) $x^2 + 2x + 18 = 0$ (d) $x^2 + 2x - 18 = 0$

2. Consider the equation $kx^2 + 2x = c(2x^2 + b)$. For the equation to be quadratic, which of these cannot be the value of k? ***[Competency Based Question]***

(a) $2c$ (b) $3c$ (c) $4c$ (d) $2c + 2b$

3. Which of the following is not a quadratic equation? ***[NCERT Exemplar]***

(a) $2(x-1)^2 = 4x^2 - 2x + 1$ (b) $2x - x^2 = x^2 + 5$

(c) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$ (d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

4. The roots of the quadratic equation $x^2 - 0.04 = 0$ are ***[CBSE 2020(30/2/1)]***

(a) ± 0.2 (b) ± 0.02 (c) 0.4 (d) 2

5. Which of the following equations has 2 as a root? ***[NCERT Exemplar]***

(a) $x^2 - 4x + 5 = 0$ (b) $x^2 + 3x - 12 = 0$

(c) $2x^2 - 7x + 6 = 0$ (d) $3x^2 - 6x - 2 = 0$

6. Which of the following equations has the sum of its roots as 3? ***[NCERT Exemplar]***

(a) $2x^2 - 3x + 6 = 0$ (b) $-x^2 + 3x - 3 = 0$

(c) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ (d) $3x^2 - 3x + 3 = 0$

7. Rahul follows the below steps to find the roots of the equation $3x^2 - 11x - 20 = 0$, by splitting the middle term.

Step 1: $3x^2 - 11x - 20 = 0$ Step 2: $3x^2 - 15x + 4x - 20 = 0$

Step 3: $3x(x - 5) + 4(x - 5) = 0$ Step 4: $(3x - 4)(x - 5) = 0$

Step 5: $x = \frac{4}{3}$ and 5

In which step did Rahul make the first error? [*Competency Based Question*]

(*a*) Step 1 (*b*) Step 2 (*c*) Step 3 (*d*) Step 4

8. If the list price of a toy is reduced by ₹ 2, a person can buy 2 toys more for ₹ 360. The original price of the toy is

(*a*) ₹18 (*b*) ₹ 20 (*c*) ₹ 19 (*d*) ₹ 21

9. The quadratic equation whose one rational root is $3 + \sqrt{2}$ is

(*a*) $x^2 - 7x + 5 = 0$ (*b*) $x^2 + 7x + 6 = 0$

(*c*) $x^2 - 7x + 6 = 0$ (*d*) $x^2 - 6x + 7 = 0$

10. A student is trying to find the roots of $3x^2 - 10x - 8 = 0$ by splitting the middle term as follows:

Step 1: $3x^2 - 10x - 8 = 0$

Step 2: $3x^2 - mx + nx - 8 = 0$

What could be the values of m and n? [*Competency Based Question*]

(*a*) $m = 8$ and $n = 2$ (*b*) $m = -8$ and $n = -2$

(*c*) $m = 12$ and $n = 2$ (*d*) $m = -12$ and $n = -2$

11. The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is [*CBSE* 2020(30/5/1)]

(*a*) 4 (*b*) ±4 (*c*) –4 (*d*) 0

12. If the difference of roots of the quadratic equation $x^2 + kx + 12 = 0$ is 1, the positive value of k is

(*a*) – 7 (*b*) 7 (*c*) 4 (*d*) 8

13. Which of the following equations has two distinct real roots? [*NCERT Exemplar*]

(*a*) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ (*b*) $x^2 + x - 5 = 0$

(*c*) $x^2 + 3x + 2\sqrt{2} = 0$ (*d*) $5x^2 - 3x + 1 = 0$

14. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has [*NCERT Exemplar*]

(*a*) two distinct real roots (*b*) two equal real roots

(*c*) no real root (*d*) more than two real roots

15. Which of the following equations has no real roots? [*NCERT Exemplar*]

(*a*) $x^2 - 4x + 3\sqrt{2} = 0$ (*b*) $x^2 + 4x - 3\sqrt{2} = 0$

(*c*) $x^2 - 4x - 3\sqrt{2} = 0$ (*d*) $3x^2 + 4\sqrt{3}x + 4 = 0$

16. The quadratic equation $x^2 - 4x + k = 0$ has distinct real roots if [*CBSE* 2020(30/4/1)]

(*a*) $k = 4$ (*b*) $k > 4$

(*c*) $k = 16$ (*d*) $k < 4$

17. Consider the equation $px^2 + qx + r = 0$. Which conditions are sufficient to conclude that the equation has real roots?

(*a*) $p < 0, q > 0$ (*b*) $p > 0, q < 0$

(*c*) $p > 0, r > 0$ (*d*) $p > 0, r < 0$

18. If the equation $x^2 - mx + 1 = 0$ does not possess real roots, then
(a) $m > 2$ (b) $m < -2$ (c) $-2 < m < 2$ (d) $-3 < m < 3$

Answers

1. (d)	2. (a)	3. (c)	4. (a)	5. (c)	6. (b)	7. (d)
8. (b)	9. (d)	10. (c)	11. (b)	12. (b)	13. (b)	14. (c)
15. (a)	16. (d)	17. (d)	18. (c)			

Very Short Answer Questions

Each of the following questions are of 1 *mark.*

1. Find the value of k for which $x = 2$ is a solution of the equation $kx^2 + 2x - 3 = 0$.
[*CBSE* 2019(30/5/1)]

Sol. $k(2)^2 + 2(2) - 3 = 0$ ½

$k = -\frac{1}{4}$ ½

[*CBSE Marking Scheme* 2019(30/5/1)]

2. If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k.
[*CBSE* 2018(30/1)]

Sol.

1) $x^2 - 2kx - 6 = 0$. let α be other root.

Product $= \frac{c}{a} = \frac{-6}{1} = -6$.

$3 \times \alpha = -6$

$\alpha = -2$.

Sum $= \frac{-b}{a} = -\frac{(-2k)}{1} = 2k$.

$\Rightarrow 3 + (-2) = 2k$

$1 = 2k$, $k = \frac{1}{2}$.

Value of k is ½

[*Topper's Answer* 2018]

3. Write the discriminant of the quadratic equation: $(x + 5)^2 = 2(5x - 3)$. [*CBSE* 2019(30/3/1)]

Sol.

3. $(x+5)^2 = 2(5x-3)$

$\Rightarrow x^2 + 25 + 10x = 10x - 6$

$\Rightarrow x^2 + 31 = 0$.

$a = 1$, $b = 0$, $c = 31$

Discriminant $= b^2 - 4ac$

$= 0^2 - 4 \times 1 \times 31$

$= 0 - 124$

$= -124$

[*Topper's Answer* 2019]

4. If a and b are the roots of the equation $x^2 + ax - b = 0$, then find a and b.

Sol. Sum of the roots $= a+b = -\frac{B}{A} = -a$

Product of the roots $= ab = \frac{C}{A} = -b$

$\Rightarrow \quad a + b = -a$ and $ab = -b$

$\Rightarrow \quad 2a = -b$ and $a = -1$

$\Rightarrow \quad b = 2$ and $a = -1$

5. Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other? **[*CBSE* 2019(30/1/1)]**

Sol. Given quadratic equation be

$$3x^2 - 10x + k = 0$$

Since its roots are reciprocal to each other.

$\therefore$ Let one root be α, therefore other will be $\frac{1}{\alpha}$.

Now, product of roots $= \frac{c}{a}$.

$\Rightarrow \quad \alpha \times \frac{1}{\alpha} = \frac{k}{3} \quad \Rightarrow \quad 1 = \frac{k}{3} \Rightarrow k = 3$

6. For what values of k does the quadratic equation $4x^2 - 12x - k = 0$ have no real roots?

[*CBSE* 2019(30/4/1)]

Sol. For non-real roots, $D < 0 \quad \Rightarrow \quad 144 - 4 \times 4 \times (-k) < 0$ ½

$16k < -144$

$k < -9$ ½

[*CBSE Marking Scheme* 2019 (30/4/1)]

Short Answer Questions–I

Each of the following questions are of 2 marks.

1. Solve for x: $6x^2 + 11x + 3 = 0$ **[*CBSE* 2020(30/4/1)]**

Sol. Given equation,

$6x^2 + 11x + 3 = 0 \quad \Rightarrow \quad 6x^2 + 9x + 2x + 3 = 0$

$\Rightarrow \quad 3x(2x + 3) + 1(2x + 3) = 0 \quad \Rightarrow \quad (2x + 3)(3x + 1) = 0$

$\Rightarrow \quad 2x + 3 = 0$ or $3x + 1 = 0 \quad \Rightarrow \quad x = \frac{-3}{2}, \frac{-1}{3}$

2. Solve for x: $\sqrt{2x + 9} + x = 13$ **[*CBSE (AI)* 2016(30/2)]**

Sol.

16) $\sqrt{2x+9} + x = 13$

$\sqrt{2x+9} = 13 - x$

$2x + 9 = (13 - x)^2 \Rightarrow$

$2x + 9 = 169 + x^2 - 26x \Rightarrow x^2 + 169 - 26x - 9 - 2x = 0$

$x^2 - 28x + 160 = 0$

$x^2 - 20x - 8x + 160 = 0$

$x(x - 20) - 8(x - 20) = 0$

$(x - 8)(x - 20) = 0$

either $= \boxed{x = 8}$ or $\boxed{x = 20}$

[*Topper's Answer* 2016]

3. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b. **[CBSE Delhi 2016]**

Sol. Let us assume the quadratic equation be $Ax^2 + Bx + C = 0$.

Sum of the roots $= -\frac{B}{A}$

$\Rightarrow \quad \frac{-7}{a} = \frac{2}{3} - 3 \Rightarrow a = 3$

Product of the roots $= \frac{C}{A}$

$\Rightarrow \quad \frac{b}{a} = \frac{2}{3} \times (-3) \quad \Rightarrow \quad \frac{b}{a} = -2$

$\Rightarrow \quad b = 3 \times (-2) \quad \Rightarrow \quad b = -6$

4. Find the roots of the quadratic equation $-x^2 + 7x - 10 = 0$ by using quadratic formula. **[NCERT Exemplar]**

Sol. Given quadratic equation

$$-x^2 + 7x - 10 = 0$$

Its Discriminant $(D) = b^2 - 4ac$

$\Rightarrow \quad D = (7)^2 - 4 \times (-1) \times (-10) = 9$

$\therefore \quad x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-7 \pm \sqrt{9}}{2 \times (-1)} = \frac{-7 \pm 3}{-2}$

$\Rightarrow \quad x = \frac{-7+3}{-2}, \frac{-7-3}{-2} = \frac{-4}{-2}, \frac{-10}{-2}$

$\Rightarrow \quad x = 2, 5$

$\therefore \quad$ Roots are 2 and 5.

5. Solve the quadratic equation $2x^2 + ax - a^2 = 0$ for x. **[CBSE Delhi 2014]**

Sol. $2x^2 + ax - a^2 = 0$

Here, $a = 2, b = a$ and $c = -a^2$

Using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ we get}$$

$$x = \frac{-a \pm \sqrt{a^2 - 4 \times 2 \times (-a^2)}}{2 \times 2} = \frac{-a \pm \sqrt{9a^2}}{4} = \frac{-a \pm 3a}{4}$$

$$x = \frac{-a+3a}{4} = \frac{a}{2}, \quad x = \frac{-a-3a}{4} = -a \quad \Rightarrow \quad x = \frac{a}{2}, -a$$

6. Does there exist a quadratic equation whose co-efficients are rational but both of its roots are irrational? Justify your answer.

Sol. Yes, $x^2 - 4x + 1 = 0$ is a quadratic equation with rational co-efficients.

Its roots are $\frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times 1}}{2} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$, which are irrational.

7. Find the value of k for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ has real and equal roots. **[CBSE Delhi 2017]**

Sol. Given quadratic equation:

$$x^2 + k(2x + k - 1) + 2 = 0$$

$\Rightarrow \quad x^2 + 2kx + (k^2 - k + 2) = 0$

For equal roots, $b^2 - 4ac = 0$

$\Rightarrow \quad 4k^2 - 4k^2 + 4k - 8 = 0$

$\Rightarrow \quad 4k = 8 \quad \Rightarrow \quad k = 2$

Short Answer Questions–II

Each of the following questions are of 3 marks.

1. In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced by 200 km/h and time of flight increased by 30 minutes. Find the original duration of flight. **[*CBSE* 2020(30/1/1)]**

Sol. Let original speed of the aircraft be x km/h.

$\therefore$ Time taken to cover a flight of 600 km $= \frac{600}{x}$ h

When speed is reduced by 200 km/h

$\therefore$ New speed $= (x - 200)$ km/h

$\therefore$ Time taken to cover 600 km $= \frac{600}{x-200}$ h

ATQ, $\frac{600}{x-200} - \frac{600}{x} = \frac{30}{60}$

$\Rightarrow \quad 600\left(\frac{1}{x-200} - \frac{1}{x}\right) = \frac{1}{2} \quad \Rightarrow \quad 600\left(\frac{x - x + 200}{x(x-200)}\right) = \frac{1}{2}$

$\Rightarrow \quad 120000 \times 2 = x^2 - 200x \quad \Rightarrow \quad x^2 - 200x - 240000 = 0$

$\Rightarrow \quad x = \frac{200 \pm \sqrt{40000 + 960000}}{2}$

$\Rightarrow \quad x = \frac{200 \pm \sqrt{1000000}}{2} = \frac{200 \pm 1000}{2}$

$\therefore \quad x = \frac{200 + 1000}{2}$ (Negative sign neglected)

$x = \frac{1200}{2} = 600 \quad \Rightarrow \quad x = 600$ km/h

$\therefore$ Original duration of flight $= \frac{600}{x} = \frac{600}{600}$ h $= 1$ hour

2. A two digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number. **[*CBSE* (*F*) 2016]**

Sol. Let the ten's digit be x and unit's digit $= y$.

$\therefore$ Number $= 10x + y$

ATQ, $10x + y = 4(x + y)$

$\Rightarrow \quad 6x = 3y \quad \Rightarrow \quad 2x = y \quad ...(i)$

Again $10x + y = 3xy$

$10x + 2x = 3x\,(2x) \quad \Rightarrow \quad 12x = 6x^2$ [From equation (i)]

$\Rightarrow \quad 6x^2 - 12x = 0 \quad \Rightarrow \quad 6x(x - 2) = 0$

$\Rightarrow \quad x = 2$ (rejecting $x = 0$)

From (i), $2x = y$

$\Rightarrow \quad y = 4$

$\therefore$ The required number is $10x + y = 10 \times 2 + 4 = 24$.

3. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time it had to increase its speed by 100 km/h. Find its usual speed.

[*CBSE* 2018]

Sol.

16) Given distance is 1500 km.

Usual speed = s.

We know, speed = $\frac{\text{distance}}{\text{time}}$ $\Rightarrow$ time = $\frac{\text{distance}}{\text{speed}}$.

$\rightarrow$ From question, $\frac{1500}{100+s} + \frac{1}{2} = \frac{1500}{s}$ [half an hour late]. (30 mins = 0.5 hr).

$\frac{1500}{100+s} = \frac{1500}{s} - \frac{1}{2}$.

$\frac{1500}{100+s} = \frac{3000-s}{2s}$.

Cross multiplying,

$3000s = 300000 - 100s + 3000s - s^2$.

$s^2 + 100s - 300000 = 0$.

$s^2 + 600s - 500s - 300000 = 0$.

$s(s+600) - 500(s+600) = 0$

$(s-500)(s+600) = 0$

$\Rightarrow$ $s - 500 = 0$ or $s + 600 = 0$

$\rightarrow s = 500$ km/h $\rightarrow s = -600$ km/h.

$\Rightarrow$ $s = 500$ or -600 km/hr.

But speed cannot be negative.

$\Rightarrow$ The usual speed of the plane is 500 km/hr. [*Topper's Answer* 2018]

4. Solve the quadratic equation $(x-1)^2 - 5(x-1) - 6 = 0$.

Sol. We have,

$(x-1)^2 - 5(x-1) - 6 = 0$

$\Rightarrow$ $y^2 - 5y - 6 = 0$, where $y = (x-1)$

$\Rightarrow$ $y^2 - 6y + y - 6 = 0$

$\Rightarrow$ $y(y-6) + 1(y-6) = 0$

$\Rightarrow$ $(y-6)(y+1) = 0$

$\Rightarrow$ $y = -1, 6$

$\Rightarrow$ $x - 1 = -1$, or $x - 1 = 6$

$\Rightarrow$ $x = 0, 7$

$\therefore$ $x = 0$ or 7

5. Solve for x: $x^2 + 5x - (a^2 + a - 6) = 0$ **[*Competency Based Question*] [*CBSE (F)* 2015]**

Sol.

$$x^2 + 5x - (a^2 + a - 6) = 0$$
$$x^2 + 5x - (a^2 + 3a - 2a - 6) = 0$$
$$x^2 + 5x - [a(a+3) - 2(a+3)] = 0$$
$$x^2 + 5x - (a-2)(a+3) = 0$$

$\therefore \quad x^2 + (a+3)x - (a-2)x - (a-2)(a+3) = 0$

$x[x + (a+3)] - (a-2)[x + (a+3)] = 0$

$[\{x + (a+3)\}\{x - (a-2)\}] = 0$

$\therefore \quad x = -(a+3) \text{ or } x = (a-2)$

$\Rightarrow \quad x = -(a+3), (a-2)$

Alternative method

$x^2 + 5x - (a^2 + a - 6) = 0$

$\therefore \quad x = \dfrac{-5 \pm \sqrt{5^2 - 4 \times 1 \times [-(a^2 + a - 6)]}}{2 \times 1}$

$= \dfrac{-5 \pm \sqrt{25 + 4a^2 + 4a - 24}}{2} = \dfrac{-5 \pm \sqrt{4a^2 + 4a + 1}}{2}$

$= \dfrac{-5 \pm \sqrt{(2a)^2 + 2.(2a).1 + 1^2}}{2} = \dfrac{-5 \pm \sqrt{(2a+1)^2}}{2}$

$= \dfrac{-5 \pm (2a+1)}{2} = \dfrac{-5 + 2a + 1}{2}, \dfrac{-5 - 2a - 1}{2}$

$= \dfrac{2a-4}{2}, \dfrac{-2a-6}{2} = (a-2), -(a+3)$

6. Using quadratic formula solve the following quadratic equation:

$p^2x^2 + (p^2 - q^2)x - q^2 = 0$ **[*Competency Based Question*]**

Sol. We have, $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we have

$a = p^2, b = p^2 - q^2$ and $c = -q^2$

$\therefore \quad D = b^2 - 4ac = (p^2 - q^2)^2 - 4 \times p^2 \times (-q^2)$

$= (p^2 - q^2)^2 + 4p^2q^2 = (p^2 + q^2)^2 > 0$

So, the given equation has real roots given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(p^2 - q^2) + (p^2 + q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$$

and $$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(p^2 - q^2) - (p^2 + q^2)}{2p^2} = \frac{-2p^2}{2p^2} = -1$$

Hence, roots are $\dfrac{q^2}{p^2}$ and -1.

7. Solve for x: $\dfrac{1}{x+1} + \dfrac{2}{x+2} = \dfrac{4}{x+4}, x \neq -1, -2, -4$ **[*CBSE (AI)* 2016]**

Sol. $\dfrac{1}{x+1} + \dfrac{2}{x+2} = \dfrac{4}{x+4}$

$\Rightarrow \quad \dfrac{x + 2 + 2(x+1)}{(x+1)(x+2)} = \dfrac{4}{x+4}$

$\Rightarrow \quad (x+4)(x+2+2x+2) = 4(x+1)(x+2)$

$\Rightarrow \quad (x+4)(3x+4) = 4(x^2 + 3x + 2)$

$\Rightarrow \quad x^2 - 4x - 8 = 0$

$\Rightarrow \quad x = \dfrac{4 \pm \sqrt{16 + 32}}{2} = \dfrac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$

8. If the roots of the quadratic equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that $2a = b + c$. **[CBSE (AI) 2016(30/2)]**

Sol.

19) $(a-b)x^2+(b-c)x+(c-a)=0$

The root are equal, then $D=0$

Comparing eqn by $ax^2+bx+c=0$

$a=(a-b)$; $b=(b-c)$; $c=c-a$

$D = b^2-4ac$

$=(b-c)^2-4\times(a-b)(c-a)$

Now, $D=0$

$(b-c)^2-4(a-b)(c-a)=0$

$b^2+c^2-2bc-4(ac-a^2-bc+ab)=0$

$b^2+c^2-2bc-4ac+4a^2+4bc-4ab=0$

$4a^2+b^2+c^2+2bc-4ab-4ac=0$

$\Rightarrow (-2a+b+c)^2=0$ $\quad[a^2+b^2+c^2+2ab+2bc+2ca=(a+b+c)^2]$

$-2a+b+c=0$

$\boxed{b+c=2a}$

Hence proved.

[Topper's Answer 2016]

9. Find the values of k, for which the quadratic equation $(k + 4)x^2 + (k + 1)x + 1 = 0$ has equal roots. **[CBSE 2020(30/3/1)]**

Sol. For equal roots $(k + 1)^2 - 4(k + 4) \times 1 = 0$ **1**

$\Rightarrow \quad k^2 - 2k - 15 = 0$ **1**

$\Rightarrow \quad (k + 3)(k - 5) = 0$ **½**

$\Rightarrow \quad k = -3, 5$ **½**

[CBSE Marking Scheme 2020 (30/3/1)]

10. Find the positive value(s) of k for which both quadratic equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have real roots. **[CBSE (F) 2016]**

Sol. (*i*) For $x^2 + kx + 64 = 0$ to have real roots

$$k^2 - 4(1)(64) \geq 0 \qquad i.e., \quad k^2 - 256 \geq 0$$

$\Rightarrow \quad (k - 16)(k + 16) \geq 0 \Rightarrow k \leq -16$ or $k \geq 16$

(*ii*) For $x^2 - 8x + k = 0$ to have real roots

$$(-8)^2 - 4(k) \geq 0 \quad i.e., \quad 64 - 4k \geq 0 \Rightarrow k \leq 16$$

For (*i*) and (*ii*) to hold simultaneously $k = 16$.

11. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, show that $c^2 = a^2(1 + m^2)$. **[Competency Based Question] [CBSE Delhi 2017]**

Sol. The given equation is $(1 + m^2)x^2 + (2mc)x + (c^2 - a^2) = 0$

Here, $A = 1 + m^2$, $B = 2mc$ and $C = c^2 - a^2$

Since the given equation has equal roots, therefore $D = 0 \Rightarrow B^2 - 4AC = 0$.

$\Rightarrow \quad (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$\Rightarrow \quad 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$

$\Rightarrow \quad m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$ [Dividing throughout by 4]

$\Rightarrow \quad -c^2 + a^2(1 + m^2) = 0 \quad \Rightarrow \quad c^2 = a^2(1 + m^2)$

Hence proved.

12. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ in x are equal, then show that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$. **[CBSE (*AI*) 2017]**

Sol. For equal roots $D = 0$

Therefore $4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$

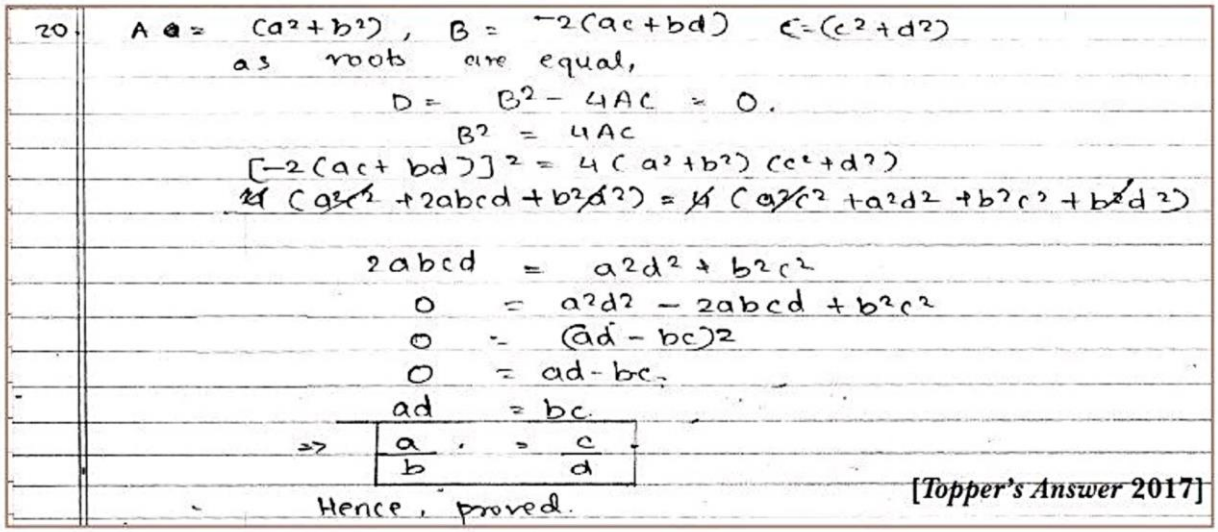
$\Rightarrow \quad 4[a^4 + b^2c^2 - 2a^2bc - b^2c^2 + ac^3 + ab^3 - a^2bc] = 0$

$\Rightarrow \quad a(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow \quad$ Either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

13. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$. **[*CBSE (AI)* 2017(30/3)]**

Sol.


20. A = (a^2+b^2), B = $-2(ac+bd)$ C = (c^2+d^2)
as roots are equal,
$D = B^2 - 4AC = 0$.
$B^2 = 4AC$
$[-2(ac+bd)]^2 = 4(a^2+b^2)(c^2+d^2)$
$4(a^2c^2 + 2abcd + b^2d^2) = 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$
$2abcd = a^2d^2 + b^2c^2$
$0 = a^2d^2 - 2abcd + b^2c^2$
$0 = (ad - bc)^2$
$0 = ad - bc$
$ad = bc$
$\Rightarrow \frac{a}{b} = \frac{c}{d}$
Hence, proved.

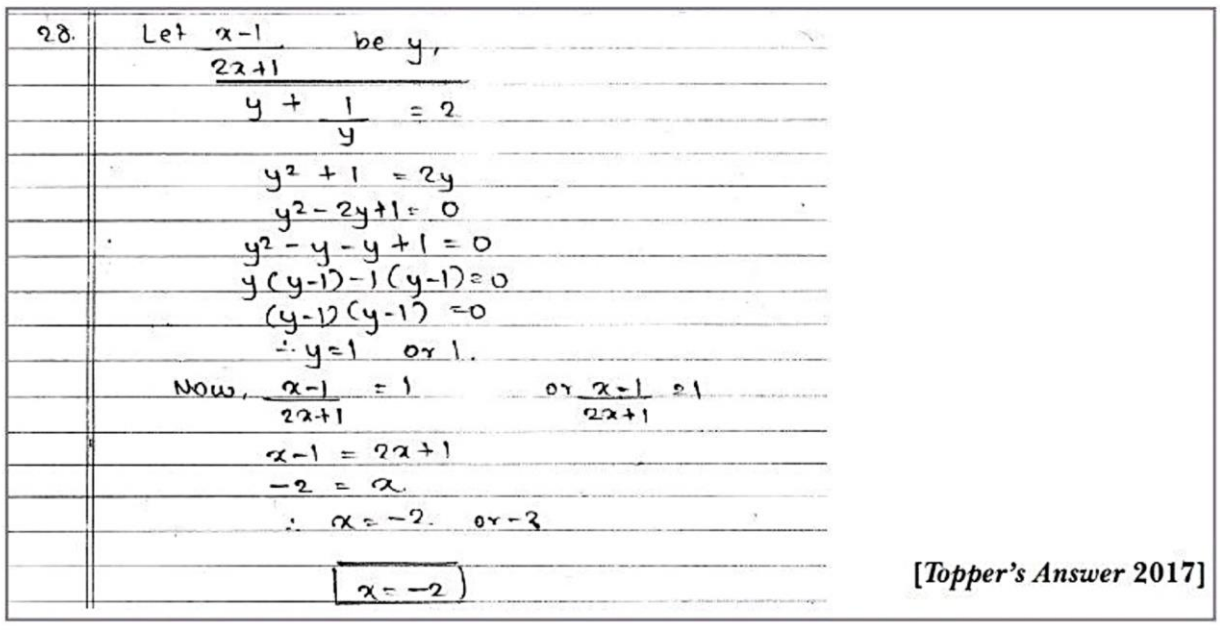
[*Topper's Answer* 2017]

Long Answer Questions

Each of the following questions are of 5 marks.

1. Solve for x: $\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 2$, where $x \neq -\frac{1}{2}, 1$. **[*CBSE (AI)* 2017(30/3)]**

Sol.


28. Let $\frac{x-1}{2x+1}$ be y,
$y + \frac{1}{y} = 2$
$y^2 + 1 = 2y$
$y^2 - 2y + 1 = 0$
$y^2 - y - y + 1 = 0$
$y(y-1) - 1(y-1) = 0$
$(y-1)(y-1) = 0$
$\therefore y = 1$ or 1.
Now, $\frac{x-1}{2x+1} = 1$ or $\frac{x-1}{2x+1} = 1$
$x - 1 = 2x + 1$
$-2 = x$
$\therefore x = -2$ or -3
$x = -2$

[*Topper's Answer* 2017]

2. Solve the following quadratic equation:

$9x^2 - 9(a+b)x + [2a^2 + 5ab + 2b^2] = 0$ **[*Competency Based Question*] [*CBSE (F) 2016*]**

Sol. Consider the equation $9x^2 - 9(a+b)x + [2a^2 + 5ab + 2b^2] = 0$

Now comparing with $Ax^2 + Bx + C = 0$, we get

$A = 9, B = -9(a+b)$ and $C = [2a^2 + 5ab + 2b^2]$

Now discriminant,

$$\begin{aligned} D &= B^2 - 4AC \\ &= \{-9(a+b)\}^2 - 4 \times 9(2a^2 + 5ab + 2b^2) \\ &= 9^2(a+b)^2 - 4 \times 9(2a^2 + 5ab + 2b^2) \\ &= 9\{9(a+b)^2 - 4(2a^2 + 5ab + 2b^2)\} \\ &= 9\{9a^2 + 9b^2 + 18ab - 8a^2 - 20ab - 8b^2\} \\ &= 9\{a^2 + b^2 - 2ab\} = 9(a-b)^2 \end{aligned}$$

Now using the quadratic formula,

$$x = \frac{-B \pm \sqrt{D}}{2A}, \text{ we get } x = \frac{9(a+b) \pm \sqrt{9(a-b)^2}}{2 \times 9}$$

$$\Rightarrow \quad x = \frac{9(a+b) \pm 3(a-b)}{2 \times 9} \quad \Rightarrow \quad x = \frac{3(a+b) \pm (a-b)}{6}$$

$$\Rightarrow \quad x = \frac{(3a+3b)+(a-b)}{6} \text{ and } x = \frac{(3a+3b)-(a-b)}{6}$$

$$\Rightarrow \quad x = \frac{(4a+2b)}{6} \text{ and } x = \frac{(2a+4b)}{6}$$

$$\Rightarrow \quad x = \frac{2a+b}{3} \text{ and } x = \frac{a+2b}{3} \text{ are required solutions.}$$

3. At t minutes past 2 pm, the time needed by the minute hand of clock to show 3 pm was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t. [*Competency Based Question*] [*NCERT Exemplar*]

Sol. As we know that total time taken by the minute hand to run from 2 pm to 3 pm = 60 minutes.

According to the question

$$t + \left(\frac{t^2}{4} - 3\right) = 60$$

$$\Rightarrow \quad 4t + t^2 - 12 = 240 \quad \Rightarrow \quad t^2 + 4t - 252 = 0$$

$$\Rightarrow \quad t^2 + 18t - 14t - 252 = 0 \quad \Rightarrow \quad t(t+18) - 14(t+18) = 0$$

$$\Rightarrow \quad (t+18)(t-14) = 0$$

$$\Rightarrow \quad t = 14 \text{ or } -18$$

But time can not be negative.

$$\therefore \quad t = 14 \text{ minutes}$$

4. Seven years ago Varun's age was five times the square of Swati's age. Three years hence, Swati's age will be two-fifth of Varun's age. Find their present ages.

Sol. Seven years ago, let Swati's age be x years. Then, seven years ago Varun's age was $5x^2$ years.

$\therefore$ Swati's present age $= (x+7)$ years

Varun's present age $= (5x^2 + 7)$ years

Three years hence,

Swati's age $= (x + 7 + 3)$ years $= (x + 10)$ years

Varun's age $(5x^2 + 7 + 3)$ years $= (5x^2 + 10)$ years

According to the question,

$$x + 10 = \frac{2}{5}(5x^2 + 10) \quad \Rightarrow \quad x + 10 = \frac{2}{5} \times 5(x^2 + 2)$$

$$\Rightarrow \quad x + 10 = 2x^2 + 4 \quad \Rightarrow \quad 2x^2 - x - 6 = 0$$

$$\Rightarrow \quad 2x^2 - 4x + 3x - 6 = 0 \quad \Rightarrow \quad 2x(x - 2) + 3(x - 2) = 0$$

$$\Rightarrow \quad (2x + 3)(x - 2) = 0$$

$$\Rightarrow \quad x - 2 = 0 \text{ or } 2x + 3 = 0$$

$$\Rightarrow \quad x = 2 \qquad [\because 2x + 3 \neq 0 \text{ as } x > 0]$$

Hence, Swati's present age $= (2 + 7)$ years $= 9$ years

and Varun's present age $= (5 \times 2^2 + 7)$ years $= 27$ years

5. *A* takes 6 days less than *B* to do a work. If both *A* and *B* working together can do it in 4 days, how many days will *B* take to finish it? **[*CBSE* 2017(30/3)]**

Sol.

29. Let B complete a work in x days.
Then A takes $x-6$ days to complete it.
Together they complete it in 4 days.
According to work done per day,

$$\frac{1}{x-6} + \frac{1}{x} = \frac{1}{4}$$

$$\frac{x + x - 6}{x(x-6)} = \frac{1}{4}$$

$$4(2x - 6) = x(x-6)$$

$$8x - 24 = x^2 - 6x$$

$$\therefore x^2 - 14x + 24 = 0$$

$$x^2 - 12x - 2x + 24 = 0$$

$$x(x-12) - 2(x-12) = 0$$

$$(x-2)(x-12) = 0$$

$\therefore x = 2$ or 12

$x = 2$ is not possible because then $x - 6$ is $\not\in 4$)

$\therefore \quad x = 12$.

So, B takes 12 days to finish the work.

[*Topper's Answer* 2017]

6. One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels. **[*Competency Based Question*]**

Sol. Let x be the total number of camels.

Then, number of camels in the forest $= \frac{x}{4}$

number of camels on mountains $= 2\sqrt{x}$

and number of camels on the bank of river $= 15$

Thus, total number of camels $= \frac{x}{4} + 2\sqrt{x} + 15$

Now by hypothesis, we have

$$\frac{x}{4} + 2\sqrt{x} + 15 = x \quad \Rightarrow \quad 3x - 8\sqrt{x} - 60 = 0$$

Let $\sqrt{x} = y$, then $x = y^2$

$\Rightarrow \quad 3y^2 - 8y - 60 = 0 \quad \Rightarrow \quad 3y^2 - 18y + 10y - 60 = 0$

$\Rightarrow \quad 3y(y - 6) + 10(y - 6) = 0 \quad \Rightarrow \quad (3y + 10)(y - 6) = 0$

$\Rightarrow \quad y = 6 \quad \text{or} \quad y = -\frac{10}{3}$

Now, $y = -\frac{10}{3} \quad \Rightarrow \quad x = \left(-\frac{10}{3}\right)^2 = \frac{100}{9} \quad (\because x = y^2)$

But, the number of camels cannot be a fraction.

$\therefore \quad y = 6$

$\Rightarrow \quad x = 6^2 = 36$

Hence, the number of camels = 36

7. If Zeba was younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now? [*NCERT Exemplar*]

Sol. Let the present age of Zeba be x years.

Age before 5 years = $(x - 5)$ years

According to given condition,

$\Rightarrow \quad (x - 5)^2 = 5x + 11 \quad \Rightarrow \quad x^2 + 25 - 10x = 5x + 11$

$\Rightarrow \quad x^2 - 10x - 5x + 25 - 11 = 0 \quad \Rightarrow \quad x^2 - 15x + 14 = 0$

$\Rightarrow \quad x^2 - 14x - x + 14 = 0 \quad \Rightarrow \quad x(x - 14) - 1(x - 14) = 0$

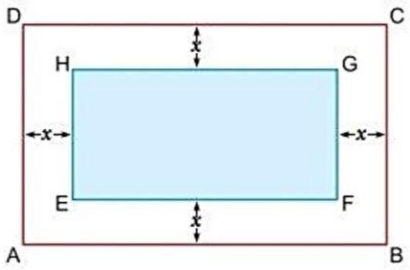
$\Rightarrow \quad (x - 1)(x - 14) = 0 \quad \Rightarrow \quad x - 1 = 0 \quad \text{or} \quad x - 14 = 0$

$x = 1 \quad \text{or} \quad x = 14$

But present age cannot be 1 year.

$\therefore$ Present age of Zeba is 14 years.

8. In a rectangular park of dimensions 50 m × 40 m, a rectangular pond is constructed so that the area of grass strip of uniform width surrounding the pond would be 1184 m^2. Find the length and breadth of the pond. [*NCERT Exemplar, CBSE (F) 2017*]



Sol. Let $ABCD$ be rectangular lawn and $EFGH$ be rectangular pond. Let x m be the width of grass area, which is same around the pond.

Given, Length of lawn = 50 m

Width of lawn = 40 m

$\Rightarrow$ Length of pond = $(50 - 2x)$ m

Breadth of pond = $(40 - 2x)$ m

Also given,

Area of grass surrounding the pond = 1184m^2

$\Rightarrow$ Area of rectangular lawn – area of pond = 1184 m^2

$\Rightarrow \quad 50 \times 40 - \{(50 - 2x) \times (40 - 2x)\} = 1184$

$\Rightarrow \quad 2000 - (2000 - 80x - 100x + 4x^2) = 1184$

$\Rightarrow \quad 2000 - 2000 + 180x - 4x^2 = 1184$

$\Rightarrow \quad 4x^2 - 180x + 1184 = 0 \quad \Rightarrow \quad x^2 - 45x + 296 = 0$

$\Rightarrow \quad x^2 - 37x - 8x + 296 = 0 \quad \Rightarrow \quad x(x - 37) - 8(x - 37) = 0$

$\Rightarrow \quad (x - 37)(x - 8) = 0 \quad \Rightarrow \quad x - 37 = 0 \text{ or } x - 8 = 0$

$\Rightarrow \quad x = 37 \text{ or } x = 8$

$x = 37$ is not possible as in this case length of pond becomes $50 - 2 \times 37 = -24$ (not possible).

Hence, $x = 8$ is acceptable.

$\therefore$ Length of pond $= 50 - 2 \times 8 = 34$ m

Breadth of pond $= 40 - 2 \times 8 = 24$ m

9. ₹ 9,000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got ₹ 160 less. Find the original number of persons.

[*CBSE* 2020(30/4/1)]

Sol. Let the original number of persons be x.

$\therefore$ Each person will get amount $= ₹ \dfrac{9000}{x}$

When 20 persons will increase

$\therefore$ Each person will get $= ₹ \dfrac{9000}{x+20}$

ATQ,

$$\frac{9000}{x} - \frac{9000}{x+20} = 160$$

$\Rightarrow \quad 9000\left(\frac{1}{x} - \frac{1}{x+20}\right) = 160 \quad \Rightarrow \quad 9000\left(\frac{x+20-x}{x(x+20)}\right) = 160$

$\Rightarrow \quad 180000 = 160(x^2 + 20x)$

$\Rightarrow \quad x^2 + 20x = 1125 \quad \Rightarrow \quad x^2 + 20x - 1125 = 0$

$\Rightarrow \quad x^2 + 45x - 25x - 1125 = 0 \quad \Rightarrow \quad x(x + 45) - 25(x + 45) = 0$

$\Rightarrow \quad (x - 25)(x + 45) = 0$

$\Rightarrow \quad x - 25 = 0 \qquad (\text{but } x + 45 \neq 0 \Rightarrow x \neq -45)$

$\therefore \quad x = 25$

$\therefore$ Original number of persons is 25.

10. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank? [*CBSE* (*AI*) 2017]

Sol. Let, time taken by faster tap to fill the tank be x hours.

Therefore, time taken by slower tap to fill the tank $= (x + 3)$ hours

Since the faster tap takes x hours to fill the tank.

$\therefore$ Portion of the tank filled by the faster tap in one hour $= \dfrac{1}{x}$

Portion of the tank filled by the slower tap in one hour $= \dfrac{1}{x+3}$

Portion of the tank filled by the two tap together in one hour $= \dfrac{1}{\frac{40}{13}} = \dfrac{13}{40}$

According to question,

$\Rightarrow \quad \dfrac{1}{x} + \dfrac{1}{x+3} = \dfrac{13}{40} \quad \Rightarrow \quad \dfrac{x+3+x}{x(x+3)} = \dfrac{13}{40}$

$\Rightarrow \quad 40(2x + 3) = 13x(x + 3) \quad \Rightarrow \quad 80x + 120 = 13x^2 + 39x$

$\Rightarrow \quad 13x^2 - 41x - 120 = 0 \quad \Rightarrow \quad 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow \quad 13x(x-5) + 24(x-5) = 0 \quad \Rightarrow \quad (x-5)(13x+24) = 0$

Either $x - 5 = 0$ or $13x + 24 = 0$

$\Rightarrow \quad x = 5$ or $x = \frac{-24}{13}$

$\Rightarrow \quad x = 5$ [$\because$ x cannot be negative]

Hence, time taken by faster tap to fill the tank $= x = 5$ hours

and time taken by slower tap $= x + 3 = 5 + 3 = 8$ hours.

11. A two digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

Sol. Let the digit at tens place be x.

Then, digit at unit place $= \frac{18}{x}$

$\therefore \quad$ Number $= 10x + \frac{18}{x}$

and number obtained by interchanging the digits $= 10 \times \frac{18}{x} + x$

According to question,

$\left(10x + \frac{18}{x}\right) - 63 = 10 \times \frac{18}{x} + x \Rightarrow \left(10x + \frac{18}{x}\right) - \left(10 \times \frac{18}{x} + x\right) = 63$

$\Rightarrow \quad 10x + \frac{18}{x} - \frac{180}{x} - x = 63 \quad \Rightarrow \quad 9x - \frac{162}{x} - 63 = 0$

$\Rightarrow \quad 9x^2 - 63x - 162 = 0 \quad \Rightarrow \quad x^2 - 7x - 18 = 0$

$\Rightarrow \quad x^2 - 9x + 2x - 18 = 0 \quad \Rightarrow \quad x(x-9) + 2(x-9) = 0$

$\Rightarrow \quad (x-9)(x+2) = 0 \quad \Rightarrow \quad x = 9$ or $x = -2$

$\Rightarrow \quad x = 9$ [$\because$ a digit can never be negative]

Hence, the required number $= 10 \times 9 + \frac{18}{9} = 92$.

Case Study-based Questions

Each of the following questions are of 4 marks.

1. Read the following and answer any four questions from (*i*) to (*v*).

Raj and Ajay are very close friends. Both the families decide to go to Ranikhet by their own cars. Raj's car travels at a speed of x km/h while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete the journey of 400 km. ***[CBSE Question Bank]***

(i) What will be the distance covered by Ajay's car in two hours?

(a) $2(x+5)$ km (b) $(x-5)$ km

(c) $2(x+10)$ km (d) $(2x+5)$ km

(ii) The quadratic equation in terms of speed of Raj's car is

(a) $x^2 - 5x - 500 = 0$ (b) $x^2 + 4x - 400 = 0$

(c) $x^2 + 5x - 500 = 0$ (d) $x^2 - 4x + 400 = 0$

(iii) What is the speed of Raj's car?

(a) 20 km/hour (b) 15 km/hour (c) 25 km/hour (d) 10 km/hour

(iv) How much time Ajay took to travel 400 km?

(a) 20 hours (b) 40 hours (c) 25 hours (d) 16 hours

(v) What is the speed of Ajay's car?

(a) 15 km/h (b) 20 km/h (c) 25 km/h (d) 30 km/h

Sol. We have speed of Raj's Car $= x$ km/h

Therefore, speed of Ajay's Car $= (x+5)$km/h

Now, time taken to complete the journey by Raj $= \frac{400}{x}$

and time taken to complete the journey by Ajay $= \frac{400}{x+5}$

According to question, Raj took 4 hours more than Ajay.

$\therefore \quad \frac{400}{x} - \frac{400}{x+5} = 4 \Rightarrow 400\left(\frac{x+5-x}{x(x+5)}\right) = 4$

$\Rightarrow \quad 500 = x(x+5) = x^2 + 5x$

$\Rightarrow \quad x^2 + 5x - 500 = 0$

$\Rightarrow \quad x^2 + 25x - 20x - 500 = 0$

$\Rightarrow \quad x(x+25) - 20(x+25) = 0$

$\Rightarrow \quad (x+25)(x-20) = 0$

$\Rightarrow \quad x = 20, x \neq -25$ (Because speed can never be negative)

$\therefore \quad x = 20$ km/h

(i) Distance covered by Ajay's car in two hours

$= \text{Speed} \times \text{Time}$

$= (x+5) \times 2$

$= 2(x+5)$ km

$\therefore$ Option (a) is correct.

(ii) We get the quadratic equation $x^2 + 5x - 500 = 0$ (Described above) for the speed of Raj's car.

$\therefore$ Option (c) is correct.

(iii) Speed of the Raj's car $= x$ km/h

$= 20$ km/h

$\therefore$ Option (a) is correct.

(*iv*) Time taken by Ajay to travel 400 km $= \frac{400}{x+5}$

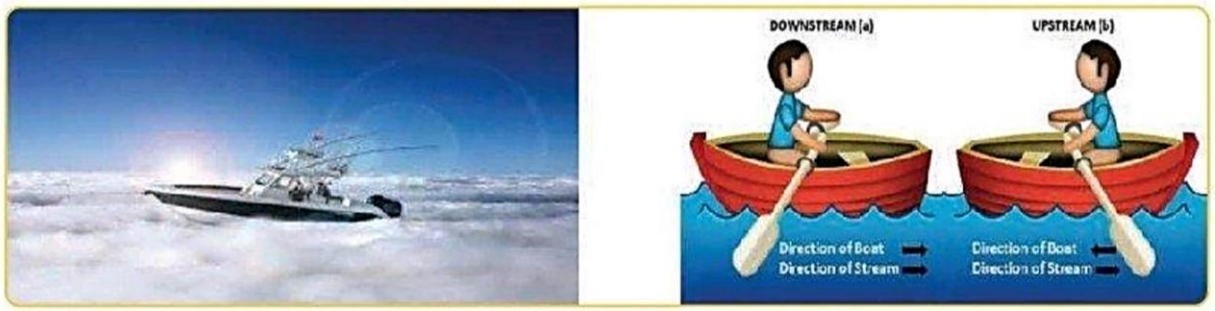
$= \frac{400}{20+5} = \frac{400}{25} = 16$ hours

$\therefore$ Option (*d*) is correct.

(*v*) Speed of Ajay's car = $(x + 5)$ km/h = $20 + 5 = 25$ km/h

$\therefore$ Option (*c*) is correct.

2. The speed of a motor boat is 20 km/h. For covering the distance of 15 km the boat took 1 hour more for upstream than downstream.



Based on above information answer the following questions.

(*i*) What is the speed of current ?

(*ii*) (*a*) Which is the correct quadratic equation for the speed of the current?

(*b*) How much time boat took in downstream?

Sol. (*i*) We have downstream speed of motor boat = $(20 + x)$ km/h

and upstream speed of motor boat = $(20 - x)$ km/h

where x km/h is the speed of the stream.

We have,

$$\frac{15}{20-x} - \frac{15}{20+x} = 1$$

$$\Rightarrow \quad \frac{15(20+x-20+x)}{(20-x)(20+x)} = 1$$

$$\Rightarrow \quad 15 \times 2x = 400 - x^2$$

$$\Rightarrow \quad x^2 + 30x - 400 = 0$$

$$\Rightarrow \quad x^2 + 40x - 10x - 400 = 0$$

$$\Rightarrow \quad x(x + 40) - 10(x + 40) = 0$$

$$\Rightarrow \quad (x + 40)(x - 10) = 0$$

$$\Rightarrow \quad x = 10 \qquad \left\{ \begin{array}{l} x \neq -40 \\ \because \text{ Speed can never be negative} \end{array} \right\}$$

$\therefore \quad x = 10$ km/h

(*ii*) (*a*) The quadratic equation $x^2 + 30x - 400 = 0$ is for the speed of the current (Stream).

(*b*) Time taken by boat in downstream $= \frac{15}{20+x}$ hour

$= \frac{15}{20+10} = \frac{15}{30} = \frac{1}{2}$ hour

$= 30$ minutes

PROFICIENCY EXERCISE

■ Objective Type Questions: [1 mark each]

1. Choose and write the correct option in each of the following questions.

(*i*) A sum of ₹4000 was divided among x persons. Had there been 10 more persons, each would have got ₹80 less. Which of the following represents the above situation?

(*a*) $x^2 + 10x + 500 = 0$ (*b*) $8x^2 + 10x - 400 = 0$

(*c*) $x^2 + 10x - 500 = 0$ (*d*) $8x^2 + 10x + 400 = 0$

(*ii*) What is the smallest positive integer value of k such that the roots of the equation $x^2 - 9x + 18 + k = 0$ can be calculated by factorising the equation?

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

(*iii*) If the roots of $ax^2 + bx + c = 0$ are equal, then the value of c is

(*a*) $\frac{-b}{2a}$ (*b*) $\frac{b}{2a}$ (*c*) $\frac{-b^2}{4a}$ (*d*) $\frac{b^2}{4a}$

(*iv*) The product of three consecutive integers is equal to 6 times the sum of three integers. If the smallest integer is x, which of the following equations represent the above situation?

(*a*) $2x^2 + x - 9 = 0$ (*b*) $2x^2 - x + 9 = 0$ (*c*) $x^2 + 2x + 18 = 0$ (*d*) $x^2 + 2x - 18 = 0$

(*v*) For what value of k, the roots of the quadratic equation $3x^2 + 2kx + 27 = 0$ are real and equal?

(*a*) $k = \pm 4$ (*b*) $k = \pm 3$ (*c*) $k = \pm 6$ (*d*) $k = \pm 9$

(*vi*) If the discriminant of a quadratic equation is less than zero then it has

(*a*) equal roots (*b*) real roots (*c*) no real roots (*d*) can't be determined

■ Very Short Answer Questions: [1 mark each]

2. If one root of the quadratic equation $6x^2 - x - k = 0$ is $\frac{2}{3}$, then find the value of k.
[*CBSE* (*F*) 2017]

3. If one root of $5x^2 + 13x + k = 0$ is the reciprocal of the other root, then find value of k.
[*CBSE* 2018 (*C*)]

4. Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0$. **[CBSE 2019(30/2/1)]**

5. For what values of 'a' the quadratic equation $9x^2 - 3ax + 1 = 0$ has equal roots?
[*CBSE* 2019 (*C*) (30/1/1)]

6. Find the value(s) of k for which the quadratic equation $3x^2 + kx + 3 = 0$ has real and equal roots.
[*CBSE* 2019(30/5/1)]

7. For what values of k does the quadratic equation $4x^2 - 12x - k = 0$ have no real roots?
[*CBSE* 2019(30/4/2)]

■ Short Answer Questions–I: [2 marks each]

8. State whether the equation $(x + 1)(x - 2) + x = 0$ has two distinct real roots or not. Justify your answer.

9. Is 0.3 a root of the equation $x^2 - 0.9 = 0$? Justify.

10. Find the value of k, for which $x = 2$ is a solution of the equation $kx^2 + 2x - 3 = 0$.
[*CBSE* 2019(30/5/1)]

11. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the value of k. **[CBSE (F) 2014, (AI) 2016]**

12. Find the roots of the quadratic equation:

$$x^2 - 3\sqrt{5}x + 10 = 0 \text{ by factorisation method.}$$

13. Find the roots of the quadratic equation:

$$2x^2 + \frac{5}{3}x - 2 = 0 \text{ using quadratic method.}$$

14. If $(x^2 + y^2)(a^2 + b^2) = (ax + by)^2$, prove that $\frac{x}{a} = \frac{y}{b}$. **[Competency Based Question]**

■ Short Answer Questions–II: [3 marks each]

15. Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other. **[CBSE (AI) 2017]**

16. Solve for x : $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}, x \neq 1, 2, 3.$ **[CBSE (AI) 2016]**

17. Solve for x: $2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5; x \neq -3, \frac{1}{2}$ **[CBSE (F) 2014]**

18. Solve for x: $\frac{14}{x+3} - 1 = \frac{5}{x+1}; \neq -3, -1$ **[CBSE Delhi 2014]**

19. Solve for x: $4x^2 - 4a^2x + (a^4 - b^4) = 0$ **[CBSE Delhi 2015]**

20. Sum of the areas of two squares is 157 m^2. If the sum of their perimeters is 68 m, find the sides of the two squares. **[CBSE 2019, (30/4/2)]**

21. Find the dimensions of a rectangular park whose perimeter is 60 m and area 200 m^2. **[CBSE 2019(30/4/2)]**

22. Find that non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k-1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation. **[CBSE Delhi 2015]**

23. Find the value of k for which the quadratic equation $(k+1)x^2 - 6(k+1)x + 3(k+9) = 0, k \neq -1$ has equal roots. **[CBSE 2019 (C) (30/1/3)]**

24. If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find the value of k. **[CBSE (F) 2014]**

■ Long Answer Questions: [5 marks each]

25. Solve for x: $\frac{x-3}{x-4} + \frac{x-5}{x-6} = \frac{10}{3}; x \neq 4, 6$ **[CBSE (AI) 2014]**

26. Find x in terms of a, b and c: $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a, b, c.$ **[CBSE Delhi 2016]**

27. Solve for x:

$$\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}, \; x \neq -1, -\frac{1}{5}, -4$$ **[CBSE (AI) 2017]**

28. Solve for x:

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}; x \neq 0, x \neq \frac{-2a-b}{2}, a, b \neq 0$$ **[CBSE 2019(30/4/3)]**

29. Solve the following equation for x: **[CBSE 2019(C)(30/1/1)]**

$$\frac{1}{x+1} + \frac{2}{x+2} = \frac{7}{x+5}, x \neq -1, -2, -5$$

30. The sum of the squares of two consecutive even numbers is 340. Find the numbers. **[*CBSE (F)* 2014]**

31. Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number. **[*NCERT Exampler*]**

32. At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha. **[*NCERT Exampler*]**

33. There is a square field whose side is 44 m. A square flower bed is prepared in its centre leaving a gravel path all round the flower bed. The total cost of laying the flower bed and gravelling the path at ₹2.75 and ₹1.50 per m^2 respectively, is ₹4904. Find the width of gravel path.

34. A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/h more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

35. In a class test, the sum of the marks obtained by Puneet in Mathematics and Science is 28. Had he got 3 marks more in Mathematics and 4 marks less in Science, the product of their marks, would have been 180. Find his marks in two subjects. **[*Competency Based Question*]**

36. A faster train takes one hour less than a slower train for a journey of 200 km. If the speed of slower train is 10 km/h less than that of faster train, find the speeds of two trains. **[*CBSE* 2018(*C*)]**

37. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately. **[*Competency Based Question*] [*CBSE* 2019(30/1/1)]**

38. The total cost of a certain length of a piece of cloth is ₹200. If the piece was 5 m longer and each metre of cloth costs ₹2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre? **[*CBSE* 2019(30/2/2)]**

Answers

1. (*i*) (*c*) (*ii*) (*a*) (*iii*) (*d*) (*iv*) (*d*) (*v*) (*d*) (*vi*) (*c*)

2. $k = 2$ **3.** $k = 5$ **4.** No real root **5.** $a = \pm 2$ **6.** $k = \pm 6$

7. $k < -9$ **8.** Yes, it has two distinct real roots **9.** No **10.** $k = -\frac{1}{4}$

11. $k = \frac{7}{4}$ **12.** $\sqrt{5}, 2\sqrt{5}$ **13.** $x = \frac{-3}{2}, \frac{2}{3}$ **15.** $p = 3$ **16.** $x = 0, 4$

17. $x = -\frac{1}{5}$ or -10 **18.** $x = 1$ and 4 **19.** $x = \frac{a^2+b^2}{2}, \frac{a^2-b^2}{2}$

20. 6 m and 11 m **21.** Length = 20 m and Breadth = 10 m

22. $k = 3, x = \frac{1}{2}, \frac{1}{2}$ **23.** $k = 3$ **24.** $k = 1$ **25.** $\frac{9}{2}$ and 7

26. $x = \frac{2ab - ac - bc}{a + b - 2c}, 0$ **27.** $\frac{-11}{17}, 1$ **28.** $x = -a$ or $-\frac{b}{2}$

29. $x = 1, x = \frac{-3}{2}$ **30.** 12 and 14 **31.** 12

32. Nisha's age is 5 years, Asha's age is 27 years **33.** 2 m

34. 45 km/h **35.** Mathematics: 9 and Science: 19 or Mathematics: 12 and Science: 16

36. Speed of faster train = 50 km/h and speed of slower train = 40 km/h

37. 5 hours, 3 hours **38.** l = 20 m, ₹10 per metre

Self-Assessment

Time allowed: 1 hour | Max. marks: 40

SECTION A

1. Choose and write the correct option in the following questions. (3 × 1 = 3)

(*i*) A student solved a quadratic equation and obtains the roots as –4 and 3. Part of the student's work to verify the root is shown: $(-4)^2 + 2(-4) - 9 = 0$. Based on the student's work, which of these is correct? [*Competency Based Question*]

(*a*) The student calculated the roots of the equation that can be obtained by adding 1 to the equation that the student solved.

(*b*) The student calculated the roots of the equation that can be obtained by adding –1 to the equation that the student solved.

(*c*) The student calculated the roots correctly.

(*d*) The student calculated the roots correctly but should replace $2(-4)$ in his work with $2(3)$.

(*ii*) The roots of the quadratic equation $x^2 - 9x + 20 = 0$ are

(*a*) –4, 5 (*b*) –4, –5 (*c*) 4, 5 (*d*) 4, –5

(*iii*) The smallest positive value of k for which the equation $x^2 + kx + 9 = 0$ has real roots is

(*a*) – 6 (*b*) 6 (*c*) 36 (*d*) 3

2. Solve the following questions. (2 × 1 = 2)

(*i*) Show that $x = -2$ is a solution of $3x^2 + 13x + 14 = 0$.

(*ii*) Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other. [CBSE 2019(30/1/1)]

SECTION B

■ **Solve the following questions.** (4 × 2 = 8)

3. What are the roots of the equation $4x^2 - 2x - 20 = x^2 + 9x$?

4. Solve for x: $\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$, $x \neq -2, \frac{3}{2}$ [*CBSE Delhi* 2017(*C*)]

5. Determine the condition for one root of the quadratic equation $ax^2 + bx + c = 0$ to be thrice the other.

6. Find the value of k such that the equation $(k-12)x^2 + 2(k-12)x + 2 = 0$ has equal roots. [*CBSE Delhi* 2017(*C*)]

■ **Solve the following questions.** (4 × 3 = 12)

7. Solve for x: $3\left(\frac{3x-1}{2x+3}\right) - 2\left(\frac{2x+3}{3x-1}\right) = 5; x \neq \frac{1}{3}, -\frac{3}{2}$ [*CBSE (F)* 2014]

8. Solve for x: $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$; $a + b + x \neq 0, a, b, x \neq 0$ [*Competency Based Question*]

HELPLINE : +91-9939586130 // +91-7739650505
2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122-JH

9. The difference of two natural numbers is 3 and the difference of their reciprocals is $\frac{3}{28}$. Find the numbers. **[*CBSE Delhi* 2014]**

10. Find the value of c for which the quadratic equation $4x^2 - 2(c + 1)x + (c + 1) = 0$ has equal roots, which are real. **[*CBSE Delhi* 2017(*C*)]**

■ **Solve the following questions.** **(3 × 5 = 15)**

11. The sum of the areas of two squares is 640 m^2. If the difference of their perimeters is 64 m, find the sides of the squares. **[*CBSE* 2019(30/4/3)]**

12. Two water taps together can fill a tank in 9 hours 36 minutes. The tap of larger diameter takes 8 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. **[*CBSE* (*F*) 2016]**

13. A motorboat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream. **[*CBSE* (*AI*) 2016]**

Answers

1. (*i*) (*a*) (*ii*) (*c*) (*iii*) (*b*)

2. (*ii*) $k = 3$ **3.** $-\frac{4}{3}, 5$ **4.** 5, –1 **5.** $3b^2 = 16\,ac$ **6.** $k = 14$ **7.** 0, –7

8. $x = -a, -b$ **9.** 7 and 4 **10.** $c = -1, 3$ **11.** 24 m and 8 m **12.** 24 hours, 16 hours

13. 8 km/h