



## KINETIC THEORY OF GASES

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**Topic Coverage:** The revision module comprehensively covers the Kinetic Theory of Gases, encompassing principles such as gas laws, kinetic interpretation of temperature, and the behavior of ideal gases – aligning with the CBSE Class 11 Physics curriculum.



**Concise and Clear:** The content is presented in a concise and clear manner, focusing on the most critical aspects of the Kinetic Theory of Gases. It serves as an effective tool for quick revision before exams.



**Important Formulas:** The module includes a compilation of important formulas and equations related to the Kinetic Theory of Gases. This aids in quick recall and application during problem-solving.



**Application-Based Questions:** Engage in practice with a set of application-based questions to enhance your problem-solving skills and understand the real-world applications of kinetic concepts in gas behavior.



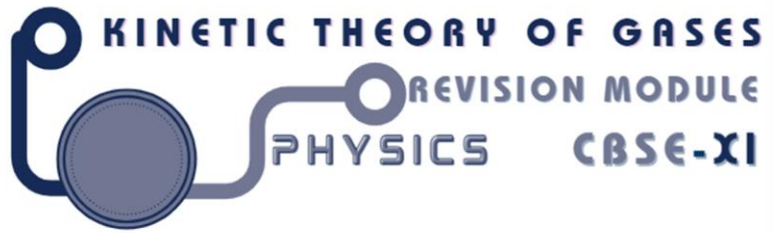
**Concept Reinforcement:** Reinforce your understanding of the underlying concepts through targeted explanations and examples, ensuring a solid foundation in the Kinetic Theory of Gases.



**Exam-Style Questions:** Familiarize yourself with the types of questions that could appear in the CBSE Class 11 Physics exam. Solve exam-style questions to refine your exam-taking strategies.



**Online Accessibility:** The revision module is accessible online, allowing you to study anytime anywhere. This flexibility enables you to integrate revision seamlessly into your schedule.



**SECTION A**

**NCERT EXERCISES**

**13.1** Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be  $3\text{\AA}$ .

**Sol.** Given, Diameter,  $d = 3\text{\AA}$ ,  $r = d/2 = 1.5\text{\AA}$   
 $= 1.5 \times 10^{-8} \text{ cm}$

$V =$  molecular volume  $= \frac{4}{3}\pi r^3 N$  (Where  $N =$  Avogadro's number)

$$= \frac{4}{3} \times \left(\frac{22}{7}\right) (1.5 \times 10^{-8})^3 \times (6.023 \times 10^{23})$$

$$= 8.52 \text{ cc}$$

$V' =$  actual volume occupied by 1 mole of  $\text{O}_2$  at STP = 22400 cc

$$\therefore \frac{V}{V'} = \frac{8.52}{22400} = 3.8 \times 10^{-4}$$

**13.2** Molar volume is the volume occupied by 1 mole of any ideal gas at standard temperature and pressure (STP) which are 1 atmospheric pressure and  $0^\circ\text{C}$ . Show that it is 22.4 litres.

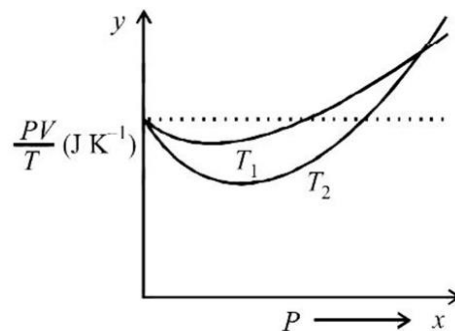
**Sol.** For one mole of an ideal gas,  $PV = RT$

$$\therefore V = \frac{RT}{P} \text{ [Where, } R = 8.31 \text{ J mol}^{-1}\text{K}^{-1},$$

$$T = 273 \text{ K, } P = 1 \text{ atmosphere} = 1.013 \times 10^5 \text{ Nm}^{-2}]$$

$$= \frac{8.31 \times 273}{1.013 \times 10^5} = 0.0224 \text{ m}^3 = 22.4 \text{ litres.}$$

**13.3** Figure shows plot of  $PV/T$  versus  $P$  for  $1.00 \times 10^{-3} \text{ kg}$  of oxygen gas at two different temperatures.



- What does the dotted plot signify ?
- Which is true :  $T_1 > T_2$  or  $T_1 < T_2$  ?
- What is the value of  $PV/T$  where the curves meet on the y-axis ?
- If we obtained similar plots for  $1.00 \times 10^{-3} \text{ kg}$  of hydrogen, would we get the same value of  $PV/T$  at the point where the curves meet on the y-axis ? If not, what mass of hydrogen yields the same value of  $PV/T$  (for low pressure high temperature region of the plot)? (Molecular mass of  $\text{H}_2 = 2.02 \text{ u}$ , of  $\text{O}_2 = 32.0 \text{ u}$ ,  $R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$ .)

**Sol.** (a) For dotted plot,  $\frac{PV}{T} = \text{constant}$  which is true for ideal gas.

(b) As the temperature decreases, the deviation from ideal behaviour increases. Hence as deviation for  $T_2$  is more than  $T_1$ , Hence,  $T_1 > T_2$

(c) Required value of  $\frac{PV}{T} = nR$

$$= \frac{10^{-3} \times 10^3}{32} \times 8.314 \text{ JK}^{-1} = 0.26 \text{ JK}^{-1}$$

- (d) No, because the no. of moles for hydrogen will be different from that for oxygen.

The required mass of hydrogen can be

$$\text{found from } \frac{m \times 10^3}{2} \times 8.314 = 0.26$$

$$\Rightarrow m = \frac{0.26 \times 2}{8.314} \times 10^{-3} \text{ kg} = 6.3 \times 10^{-5} \text{ kg}$$

- 13.4** An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17°C. Estimate the mass of oxygen taken out of the cylinder. ( $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$ , molecular mass of  $\text{O}_2 = 32 \text{ cc}$ )

**Sol.** Initially in the  $\text{O}_2$  cylinder,

$$V_1 = 30 \text{ litre} = 30 \times 10^{-3} \text{ m}^3$$

$$P_1 = 15 \text{ atm} = 15 \times 1.01 \times 10^5 \text{ Pa,}$$

$$T_1 = 27 + 273 = 300^\circ \text{K}$$

If  $n_1$  be the moles of  $\text{O}_2$  gas in the cylinder,

$$P_1 V_1 = n_1 R T_1$$

$$\therefore n_1 = \frac{(15 \times 1.01 \times 10^5) \times (30 \times 10^{-3})}{8.3 \times 300} = 18.253$$

$M =$  molecular weight of  $\text{O}_2 = 32 \text{g}$ ; Initial cylinder mass =  $m_1 = n_1 M = 18.253 \times 32 = 584.1 \text{g}$

Let,  $n_2$  – moles of  $\text{O}_2$  left in the cylinder

$$\therefore n_2 = \frac{P_2 V_2}{R T_2} \text{ [Where, } V_2 = 30 \times 10^{-3} \text{ m}^3,$$

$$P_2 = 11 \times 1.01 \times 10^5 \text{ Pa, } T_2 = 17 + 273 = 290^\circ \text{K}]$$

$$= \frac{(11 \times 1.01 \times 10^5)(30 \times 10^{-3})}{8.3 \times 290} = 13.847$$

$$\therefore m_2 = \text{final mass of } \text{O}_2 \text{ in cylinder} \\ = 13.847 \times 32 = 453.1 \text{g}$$

$\therefore$  Mass of Oxygen taken out of the cylinder.

$$m_1 - m_2 = 584.1 - 453.1 = 131 \text{g}$$

- 13.5** An air bubble of volume  $1 \text{ cm}^3$  rises from the bottom of a lake 40 m deep at a temperature of  $12^\circ \text{C}$ . To what volume does it grow when it reaches the surface, which is at a temperature of  $35^\circ \text{C}$ ? Given  $1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$ .

**Sol.** Given,  $V_1 = 1.0 \text{ cm}^3 = 1 \times 10^{-6} \text{ m}^3$ ,

$$T_1 = 12^\circ \text{C} = 285 \text{ K, } P_1 = 1 \text{ atm} + h_1 \rho g$$

$$= 1.01 \times 10^5 + 40 \times 10^3 \times 9.8 = 493000 \text{ Pa}$$

Let,  $V_2 =$  volume of bubble at the surface of

$$\text{the lake } \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\therefore V_2 = \frac{P_1 V_1 T_2}{T_1 P_2} \text{ [Where, } T_2 = 35^\circ \text{C} = 308 \text{K,}$$

$$P_2 = 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}]$$

$$= \frac{(493000) \times (1.0 \times 10^{-6}) \times 308}{285 \times 1.01 \times 10^5}$$

$$= 5.275 \times 10^{-6} \text{ m}^3$$

- 13.6** Estimate the total number of air molecules (inclusive of  $\text{O}_2$ ,  $\text{N}_2$ , water vapour and other constituents) in a room of capacity  $25 \text{ m}^3$  at a temperature of  $27^\circ \text{C}$  and 1 atm pressure. [Boltzmann constant =  $1.38 \times 10^{-23} \text{ JK}^{-1}$ ]

**Sol.** Given,  $V = 25 \text{ m}^3$ ,  $T = 27 + 273 = 300 \text{K}$ ,  
 $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$ .

$$\text{Now, } PV = nRT = n(Nk)T = N'kT$$

where,  $nN = N' =$  total no. of air molecules in the given gas

$$N' = \frac{PV}{kT} = \frac{(1.01 \times 10^5) \times 25}{(1.38 \times 10^{-23}) \times 300} = 6.10 \times 10^{26}$$

- 13.7** Estimate the average thermal energy of a helium atom at (i) room temperature ( $27^\circ \text{C}$ ) (ii) the temperature of the surface of the sun ( $6000 \text{K}$ ) (iii) the temperature of 10 million kelvin (the typical core temperature in the case of a star).

**Sol.** (i) Given,  $T = 27^\circ \text{C} = 300 \text{K}$

$$\text{Average thermal energy} = \frac{3}{2} kT$$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.2 \times 10^{-21} \text{ J}$$

(ii) At  $T = 6000\text{K}$ ; Avg. thermal energy

$$= \frac{3}{2}kT = \frac{3}{2} \times (1.38 \times 10^{-23}) \times 6000$$

$$= 1.24 \times 10^{-19} \text{ J}$$

(iii) At  $T = 10$  million K,  $T = 10^7\text{K}$

$$\text{Avg. thermal energy} = \frac{3}{2}kT$$

$$= \frac{3}{2} \times (1.38 \times 10^{-23}) \times 10^7 = 2.1 \times 10^{-16} \text{ J.}$$

**13.8** Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains oxygen (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is  $v_{rms}$  the largest?

**Sol.** At same temperature and pressure, all the vessels have same volume.

By Avogadro's law, three vessels contain equal number of molecules

$$\text{Avogadro's no. } N = 6.023 \times 10^{23}$$

$$\text{As } v_{rms} = \sqrt{\frac{3kT}{m}}$$

$$\therefore v_{rms} \propto \frac{1}{\sqrt{m}}, \text{ therefore, at a given}$$

temperature, rms speed of molecules will not be the same in the three cases. As neon has the smallest mass, therefore, rms speed will be the largest in case of neon.

**13.9** At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at  $-20^\circ\text{C}$ ? (Atomic mass of Ar =  $39.9 u$  and He =  $4u$ ).

**Sol.** Given,  $M = 39.9$ ,  $M' = 4$ ,  
 $T = -20 + 273 = 253^\circ\text{K}$

Let  $C$ ,  $C'$  be the rms velocity of Ar and He at temperatures  $T$  and  $T'$  K respectively.

$$\text{Now, } C = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{39.9}}$$

$$\text{and } C' = \sqrt{\frac{3RT'}{M'}} = \sqrt{\frac{3R \times 253}{4}}$$

Since,  $C = C'$ ,

$$\therefore \sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R \times 253}{4}}$$

$$\text{or, } T = 2523.7 \text{ K}$$

**13.10** Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2 atm and temperature  $17^\circ\text{C}$ . Take the radius of a nitrogen molecule to be roughly  $1\text{\AA}$ . Compare the collision time with the time, the molecule moves freely between two successive collisions. (Molecule mass of nitrogen =  $28u$ )

**Sol.** Given,  $P = 2 \text{ atm} = 2 \times 1.013 \times 10^5 \text{ Nm}^{-2}$ ;

$$T = 17^\circ\text{C} = 290 \text{ K}, \sigma = 2 \times 1 = 2\text{\AA}$$

$$= 2 \times 10^{-10} \text{ m}$$

$$k = 1.38 \times 10^{-23} \text{ J molecule}^{-1}\text{K}^{-1};$$

$$M = 28 \times 10^{-23} \text{ kg}$$

$$\text{Mean free path, } \lambda = \frac{kT}{\sqrt{2}\pi\sigma^2 P}$$

$$= \frac{1.38 \times 10^{-23} \times 290}{1.414 \times 3.14 \times (2 \times 10^{-10})^2 \times (2.026 \times 10^5)}$$

$$\Rightarrow \lambda = 1.11 \times 10^{-7} \text{ m}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 290}{28 \times 10^{-3}}} = 508.24 \text{ m/s}$$

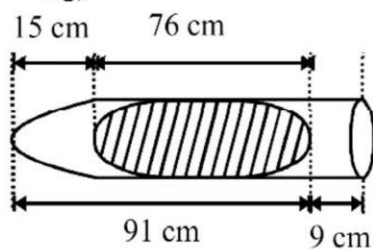
Collision frequency = no. of collisions per

$$\text{second} = \frac{508.24}{1.11 \times 10^{-7}} = 4.58 \times 10^9$$

**ADDITIONAL EXERCISES**

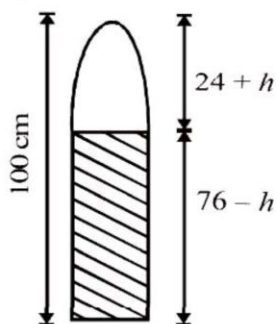
**3.11** A metre long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread, which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom?

**Sol.** In horizontal position, the bore will leave an air-column of 9 cm as shown in figure (a) so as to balance the atmospheric pressure (= 76 cm of Hg)



**Fig (a)**

$\therefore P_1 = 76 \text{ cm}$  &  
 Volume of air column =  $15 \text{ cm}^3$   
 assuming area of cross section of tube =  $1 \text{ cm}^2$   
 Total length of air column in the tube = 24 cm.



**Fig (b)**

In vertical position as shown in figure (b) if  $h'$  cm of mercury flows out to balance the atmospheric pressure then new pressure on mercury column =  $76 - (76 - h) = h \text{ cm}$  of Hg  
 The new volume of air column =  $24 + h$ .  
 Since the temperature remains constant during the transition from horizontal to vertical position.

$$\begin{aligned} \therefore P_1 V_1 &= P_2 V_2 \\ 76 \times 15 &= h \times (24 + h) \\ \text{or } h^2 + 24h - 1140 &= 0 \\ \Rightarrow h &= 23.8 \text{ cm} \end{aligned}$$

Hence, 23.8 cm of mercury flows out from the tube in vertical position.

**3.12** From a certain apparatus, the diffusion rate of  $\text{H}_2$  has an average value of  $28.7 \text{ cm}^3\text{s}^{-1}$ .

The diffusion of another gas under the same condition is measured to have an average rate of  $7.2 \text{ cm}^3\text{s}^{-1}$ . Identify the gas.

[Hint : Use Graham's law of diffusion :  $R_1/R_2 = (M_2/M_1)^{1/2}$ , where  $R_1, R_2$  are diffusion rates of gases 1 and 2, and  $M_1$  and  $M_2$  their respective molecular masses. The law is a simple consequence of kinetic theory.]

**Sol.** According to Graham's law of diffusion,

$$\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$$

where,  $r_1$  = diffusion rate of  $\text{H}_2 = 28.7 \text{ cm}^3\text{s}^{-1}$   
 $r_2$  = diffusion rate of unknown gas =  $7.2 \text{ cm}^3\text{s}^{-1}$   
 $M_1$  = molecular wt. of  $\text{H}_2 = 2u$

$$\therefore \frac{28.7}{7.2} = \sqrt{\frac{M_2}{2}} \Rightarrow M_2 = 32$$

As  $M_2 = 32$ , which is the molecular mass of  $\text{O}_2$  gas hence the gas is Oxygen.

**13.13** A gas in equilibrium has uniform density and pressure throughout its volume. This is strictly true only if there are no external influences. A gas column under gravity, for example, does not have uniform density (and pressure). As you might expect, its density decreases with height. The precise dependence is given by the so-called 'law of atmospheres'

$n_2 = n_1 \exp [mg(h_2 - h_1)/k_B T]$  where  $n_2, n_1$  refer to number density at heights  $h_2$  and  $h_1$  respectively. Use this relation to derive the equation for sedimentation equilibrium of a suspension in a liquid column :

$n_2 = n_1 \exp[-mg N_A(\rho - P)(h_2 - h_1)/(\rho RT)]$  where  $\rho$  is the density of the suspended particle, and  $\rho'$  that of surrounding medium. [ $N_A$  is Avogadro's number, and  $R$  the universal gas constant.]

[Hint : Use Archimedes principle to find the apparent weight of the suspended particle.]

**Sol.** According to Archimedes' Principle :  
 Apparent weight of the suspended particle ( $mg$ ) = Actual wt - (weight of an equal volume of the liquid)

$$(mg)' = mg - \left( \frac{m}{\rho} \rho' g \right)$$

(∴ Volume of particle =  $m/\rho$  and weight = Vol.  $\times$  density  $\times$   $g$ )

$$(mg') = mg (1 - \rho'/\rho) \quad \dots(1)$$

Given equation is

$$n_2 = n_1 \exp \left[ -\frac{mg}{kT} (h_2 - h_1) \right] \quad \dots(2)$$

Replacing  $mg$  by the apparent weight ( $mg'$ )

From equations (1) and (2), we get

$$n_2 = n_1 \exp \left[ \frac{-mgN_A}{RT} \left( 1 - \frac{\rho'}{\rho} \right) (h_2 - h_1) \right] \quad (\because h = R/N_A)$$

This is the required equation.

**13.14** Given below are densities of some solids and liquids. Give rough estimates of the size of their atoms :

Substance	Atomic Mass ( $u$ )	Density ( $10^3 \text{ kg m}^{-3}$ )
Carbon (diamond)	12.01	2.22
Gold	197.00	19.32
Nitrogen (liquid)	14.01	1.00
Lithium	6.94	0.53
Fluorine (liquid)	19.00	1.14

[Hint : Assume the atoms to be 'tightly packed' in a solid or liquid phase, and use the known value of Avogadro's number. You should, however, not take the actual numbers you obtain for various atomic sizes too literally. Because of the crudeness of the tight packing approximation, the results only indicate that atomic sizes are in the range of a few Å].

**Sol.** For carbon, Atomic mass =  $\frac{4}{3}\pi R^3 \times$  density

$$\left( \because \text{Volume} = \frac{4}{3}\pi R^3 \right)$$

$$\Rightarrow 12.01 \times 1.67 \times 10^{-27} \text{ kg}$$

$$= \frac{4}{3} \times 3.14 \times R^3 \times 2.22 \times 10^3 \text{ kg m}^{-3}$$

$$\therefore R^3 = 2.15 \times 10^{-30} \text{ m}^3$$

$$\Rightarrow R = 1.29 \times 10^{-10} \text{ m} = 1.29 \text{ \AA}$$

Similarly for Gold,  $R = 1.59 \text{ \AA}$

Liquid  $N_2$ ,  $R = 1.77 \text{ \AA}$

Lithium,  $R = 1.73 \text{ \AA}$

Liquid fluorine,  $R = 1.88 \text{ \AA}$

## SECTION B

## PRACTICE QUESTIONS

### VERY SHORT ANSWER QUESTIONS

- For ideal behavior of a gas, state the condition in terms of density of the gas.
- The velocities of three molecules A, B, C of a gas are  $3V$ ,  $4V$ , and  $5V$  respectively. Calculate the R.M.S velocity?
- The ratio of vapour densities of two gases at the same temperature is  $4 : 9$ . Calculate the ratio of their r.m.s velocities of molecules.
- A container filled with Helium gas at  $300 \text{ K}$  is heated to  $600 \text{ K}$ . The coefficient of thermal expansion of the container is negligible. Calculate the change in the average K.E. of the helium atoms.

- The pressure of a gas at  $-173^\circ\text{C}$  is 1 atmosphere, what should be the temperature of the gas when the pressure is 2 atmosphere if the volume of the gas is kept constant?
- The absolute temperature of a gas is increased 2 times. What will be the increase in r.m.s velocity of the gas molecules?
- Nitrogen and hydrogen gas are taken at the same temperature. What will be the ratio of the kinetic energies of nitrogen and hydrogen molecule when nitrogen is 14 times heavier than hydrogen?
- $2 \text{ cm}^3$  of oxygen and  $2 \text{ cm}^3$  of nitrogen gas are taken at N.T.P. Which will have larger number of molecules?

9. There are  $N$  no. of molecules of a gas in a container. It is increased to  $2N$ . Explain, with reason, how the pressure, total energy and r.m.s speed of the gas will change?
10. When a scooter runs for a long time, the air pressure in its tyres increases slightly.
11. On what factors does the average kinetic energy of gas molecules depend : Nature of the gas, temperature, volume?
12. Obtain the dimensional formula for  $R$  used in the ideal gas equation  $PV = RT$ .
13. What is the relation between ratio of specific heats  $\gamma$  and degree of freedom  $f$  of an ideal gas ?
14. At what temperature does all molecular motion cease ?
15. Compare the mean kinetic energy of one mole of Helium and Nitrogen at the same temperature.
16. At  $0K$ , which of the following properties of a gas will be zero. K.E., P.E., Vibrational energy, Density?
17. If in an isothermal process, pressure is decreased then what will be its effect on mean free path ?
9. Derive Gay Lussac's law on the basis of kinetic theory.
10. Derive Avogadro's law from kinetic theory of gases.
11. Two ideal gases at absolute temperature  $T_1$  and  $T_2$  are mixed. There is no loss of energy. Find the temperature of the mixture if the masses of the molecules are  $m_1$  and  $m_2$  and the no. of molecules are  $n_1$  and  $n_2$  respectively.
12. Draw  $P - V$  curves showing deviations from ideal behaviour for a given mass of a gas for two different temperatures.
13. When air is pumped into a cycle tyre the volume and pressure of the air in the tyre both are increased. What about Boyle's law in this case ?
14. Why temperature less than absolute zero is not possible ?
15. A box contains equal number of molecules of hydrogen and oxygen. If there is a fine hole in the box, which gas will leak rapidly ? Why?
16. Two gases, each at temperature  $T$ , volume  $V$  and pressure  $P$  are mixed such that the temperature and volume of the mixture are  $T$  and  $V$  respectively. What would be the pressure of the mixture ? Justify your answer on the basis of kinetic theory.

### SHORT ANSWER QUESTIONS

1. State the postulates of the kinetic theory of gases.
2. The rms speed of hydrogen molecule at a certain temperature  $T$  is  $V$ . If the temperature of the gas becomes double and the molecule dissociates into atoms, what will be the speed of atomic hydrogen?
3. When a gas filled in a closed vessel is heated through  $2^\circ C$ , its pressure increases by  $0.2\%$ . What is the initial temperature of the gas.
4. The volume of vessel  $A$  is 4 times the volume of vessel  $B$  and both of them are filled with the same gas. If the pressure and temperature of gas  $A$  is 4 times as that of gas  $B$ , then calculate the ratio of no. of molecules of  $A$  and  $B$ .
5. Derive a relation between pressure and kinetic energy of a gas.
6. Show that average K.E. of each molecule of a gas doesn't depend on the mass of the molecule, but depends on the temperature of the gas.
7. Derive Boyle's law on the basis of kinetic theory.
8. Derive Charles' law on the basis of kinetic theory.
17. Calculate the value of the gas constant for one mole of an ideal gas. Given that one mole of a gas at S.T.P. has a volume of  $22.4 L$ .
18. The pressure of a gas increases on increase of its temperature and also on decrease of its volume. How are these cases different on the basis of kinetic theory of gases?

### LONG ANSWER QUESTIONS

1. Derive an expression for the pressure due to an ideal gas.
2. From kinetic theory of gases, explain kinetic interpretation of temperature and absolute zero of temperature.
3. Explain the meaning of mean speed, rms speed and most probable speed of a gas.
4. Calculate the specific heats of mono, di and triatomic gases from the law of equipartition of energy.
5. Calculate the expression for mean free path of an ideal gas.



**PRACTICE QUESTIONS SOLUTIONS**

**VERY SHORT ANSWER QUESTIONS**

1. For ideal gas,  $P = \rho RT$   
 For ideal behavior,  $\rho$  should be low.

$$2. C = \sqrt{\frac{C_A^2 + C_B^2 + C_C^2}{3}}$$

$$= \sqrt{\frac{(3V)^2 + (4V)^2 + (5V)^2}{3}} = \sqrt{\frac{50}{3}}V = 4.08V$$

$$3. \frac{C_1}{C_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{d_2}{d_1}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

$$\therefore C_1 : C_2 = 3 : 2$$

$$4. \text{Average K.E.} = \frac{3}{2}kT$$

$\therefore$  Change in average K.E.

$$= \frac{3}{2}k(600 - 300) = \frac{3}{2} \times 300 \times 1.38 \times 10^{-23}$$

$$\text{Joule} = 6.21 \times 10^{-21} \text{ Joule}$$

5. When  $V$  is constant,  $P \propto T$

$$\therefore \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\Rightarrow T_2 = \frac{P_2 T_1}{P_1} = \frac{2 \times (273 - 173)}{1} = 200 \text{ K}$$

$$= (200 - 273)^\circ\text{C} = -73^\circ\text{C}$$

6.  $C \propto \sqrt{T}$   $\therefore$  New r.m.s velocity will be  $\sqrt{2}C$

$$\therefore \text{Increase in r.m.s velocity} = \sqrt{2}C - C$$

$$= (\sqrt{2} - 1)C = (1.414 - 1)C = 0.414C$$

$$7. \therefore \text{K.E./molecule} = \frac{3}{2}kT$$

$\therefore$  Both will have the same K.E. and ratio will be 1 : 1.

8. Both will have the same number of molecules as Avogadro's hypothesis says equal volumes of all gases contain equal number of molecules.

$$9. \therefore \text{Pressure} = P = \frac{1}{3}\rho C^2 = \frac{1}{3} \frac{mn}{V} C^2$$

$\therefore P \propto$  no. of molecules.

As it becomes double,

$\therefore$  Pressure will also be doubled.

$$\text{K.E.} = \frac{1}{2}mnC^2$$

$\therefore$  It will also become double.

$$\text{R.M.S speed, } C \propto \sqrt{T}$$

It doesn't depend on the no. of molecules. So, it remains unchanged.

10. On running the scooter, the work done against friction is converted to heat.  $\therefore P \propto T$ , so, as the temperature of the tyre increases, pressure will also increase.

11. The average kinetic energy of the molecules of a gas depends only on temperature i.e.,

$$E_{av} = f(T).$$

12. Since  $PV = RT$

$$\text{So } R = \frac{PV}{T} = \frac{FV}{AT} \text{ or } R = \frac{[\text{MLT}^{-2}][L^3]}{[L^2][K]}$$

$$\text{or } R = [\text{ML}^2\text{T}^{-2}\text{K}^{-1}]$$

$$13. \gamma = 1 + \frac{2}{f}$$

14. Absolute zero.

15. Since mean kinetic energy per mole depends only on temperature, hence it will be same for Helium and Nitrogen.

16. Since K.E.  $\propto T$  and vibrations are excited only at high temperature, hence K.E. of vibrational energy will be zero at 0K.

17. As mean free path  $\lambda \propto \frac{1}{P}$  [at constant  $T$ ]

Hence on decreasing pressure,  $\lambda$  will increase.

**SHORT ANSWER QUESTIONS**

1. Postulates of kinetic theory : -

(i) A gas consists of a very large number of molecules which are perfect elastic spheres and are identical in all respects for a given gas and are different for different gases.

(ii) The molecules of a gas are in a state of continuous, rapid and random motion.

(iii) The size of the gas molecules is very small as compared to the distance between them. So, volume of all molecules of the gas is negligible than the volume of the gas.

- (iv) The molecules do not exert any force of attraction or repulsion on each other, except during collision.
- (v) The collisions of the molecules with themselves and with the walls of the container are perfectly elastic.
- (vi) Molecular density is uniform throughout the gas.
- (vii) A molecule moves along a straight line between two successive collisions.
- (viii) The collisions are almost instantaneous.

2. R.M.S speed,  $C = \sqrt{\frac{3RT}{M}} = V$

New r.m.s speed  $C' = \sqrt{\frac{3R2T}{M/2}}$

[Mass of atomic hydrogen is half that of its molecular mass]

$C' = 2\sqrt{\frac{3RT}{M}} = 2V$

∴ R.M.S speed becomes double.

3.  $P_2 = \left(P + \frac{0.2}{100}P\right)$  and  $T_2 = (T + 2)$

∴  $\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{P}{T} = \frac{P + \frac{0.2}{100}P}{T + 2}$

$\Rightarrow \frac{P}{T} = \frac{P(1.002)}{T + 2} \Rightarrow T + 2 = 1.002 T$

$\Rightarrow 2 = 1.002 T - T = 0.002T$

$\Rightarrow T = \frac{2}{0.002} = 1000 K$

4.  $V_A = 4 V_B$ ,  $T_A = 4T_B$  and  $P_A = 4P_B$

From ideal gas equation,  $\frac{P_A V_A}{T_A} = n_A R$  and

$\frac{P_B V_B}{T_B} = n_B R$

Dividing we get,  $\frac{n_A}{n_B} = \frac{P_A V_A}{T_A} \times \frac{T_B}{P_B V_B}$ ;

$\frac{n_A}{n_B} = \frac{4 \times 4}{4} = 4 \quad \therefore n_A : n_B = 4 : 1$

5. From kinetic theory, pressure of a gas

$P = \frac{1}{3}\rho C^2$ , where  $\rho$  = density of the gas, Mean

K.E/unit volume of the gas  $= E = \frac{1}{2}\rho C^2$

[∵ Mass/unit volume = density =  $\rho$ ]

∴  $\frac{P}{E} = \frac{1/3\rho C^2}{1/2\rho C^2} = \frac{2}{3} \quad \therefore \boxed{P = \frac{2}{3}E}$

6. Consider one mole of an ideal gas at absolute temperature  $T$ , volume  $V$  and molecular weight  $M$ . If  $m$  be the mass of each molecule and  $N$  is the Avogadro's number then  $M = m \times N$

Pressure,  $P = \frac{1}{3}\rho C^2 = \frac{1}{3}\frac{M}{V}C^2$

∴  $PV = \frac{1}{3}MC^2$

From ideal gas equation  $PV = RT$

∴  $\frac{1}{3}MC^2 = RT \quad \therefore \frac{1}{2}MC^2 = \frac{3}{2}RT$

$\Rightarrow \frac{1}{2}mNC^2 = \frac{3}{2}RT \Rightarrow \frac{1}{2}mC^2 = \frac{3}{2}\frac{R}{N}T$

∴ K.E/molecule  $= \frac{1}{2}mC^2$

$= \frac{3}{2}\left(\frac{R}{N}\right)T = \frac{3}{2}kT$ ,

where,  $k = \frac{R}{N}$  = Boltzmann's constant.

∴ K.E  $\propto T$

7. From kinetic theory, pressure due to an ideal gas,

$P = \frac{1}{3}\rho C^2 = \frac{1}{3}\frac{M}{V}C^2 \Rightarrow PV = \frac{1}{3}MC^2$

$\Rightarrow PV \propto C^2$

But  $C \propto \sqrt{T}$  (from kinetic interpretation of temperature) ∴  $PV \propto T$

When  $T$  is constant,  $PV = \text{constant}$

∴  $P \propto \frac{1}{V}$  or  $V \propto \frac{1}{P}$  when  $T$  is constant, which is Boyle's law.

8. From kinetic theory, pressure due to an ideal

gas,  $P = \frac{1}{3}\rho C^2 = \frac{1}{3}\frac{M}{V}C^2 \Rightarrow V = \frac{1}{3}\frac{M}{P}C^2$

$V \propto C^2$  for a given mass of a gas at constant

pressure. But  $C^2 \propto T$  (from kinetic theory)

∴  $V \propto T$  at constant pressure, which is Charles' law.

9. From kinetic theory,  $P = \frac{1}{3} \rho C^2 = \frac{1}{3} \frac{M}{V} C^2$

For a given mass of a gas (i.e.  $M = \text{constant}$ ) and at constant volume ( $V = \text{constant}$ )

$P \propto C^2$  but  $C^2 \propto T$  (from kinetic theory)

$\therefore P \propto T$  which is Gay Lussac's law.

10. Consider two gases A and B at same temperature  $T$ , pressure  $P$  and having same volume  $V$ .

Let  $M_A, M_B =$  Total mass of gas A and B;  $C_A, C_B =$  R.M.S velocities of gas A and B

$m_A, m_B =$  mass of each molecule of A and B;  $n_A, n_B =$  no. of molecules of A and B respectively.

$\therefore M_A = n_A m_A$  and  $M_B = n_B m_B$

From kinetic theory, pressure of gas A and B

are,  $P = \frac{1}{3} \frac{M_A}{V} C_A^2$  and  $P = \frac{1}{3} \frac{M_B}{V} C_B^2$

$\therefore P = \frac{1}{3} \frac{M_A}{V} C_A^2 = \frac{1}{3} \frac{M_B}{V} C_B^2$

$\Rightarrow M_A C_A^2 = M_B C_B^2$  ..... (1)

$\therefore$  Both the gases are at same temperature  $T$ .  
 $\therefore$  Average K.E./molecule of both the gases will be equal.

$\therefore \frac{1}{2} m_A C_A^2 = \frac{1}{2} m_B C_B^2$

$\Rightarrow m_A C_A^2 = m_B C_B^2$  ..... (2)

Dividing (1) by (2),  $\frac{M_A C_A^2}{m_A C_A^2} = \frac{M_B C_B^2}{m_B C_B^2}$

$\Rightarrow \frac{n_A m_A}{m_A} = \frac{n_B m_B}{m_B} \Rightarrow n_A = n_B$

$\therefore$  All gases contain equal no. of molecules under same condition of pressure, volume and temperature, which is Avogadro's law.

11. Average K.E./molecule of a gas =  $\frac{3}{2} kT$ . Before mixing, the average K.E. of the molecules of the

two gases =  $\frac{3}{2} k n_1 T_1 + \frac{3}{2} k n_2 T_2$ , where  $n_1$  and  $n_2$  are no. of molecules of gas 1 and 2 respectively.

After mixing, the average K.E. of both the gases

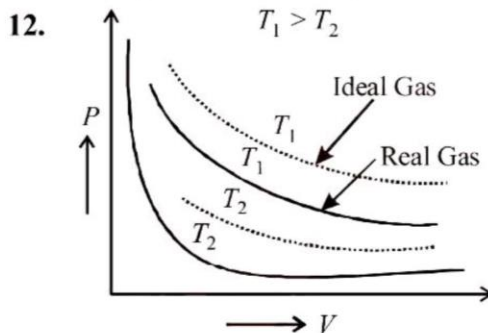
=  $\frac{3}{2} k (n_1 + n_2) T$

$T$  is the absolute temperature of the mixture.

$\therefore$  There is no loss of energy.

$\therefore \frac{3}{2} k (n_1 + n_2) T = \frac{3}{2} k n_1 T_1 + \frac{3}{2} k n_2 T_2$

$\therefore T = \frac{n_1 T_1 + n_2 T_2}{(n_1 + n_2)}$



13. When air is pumped into a cycle tyre, a large number of molecules are pumped in, and Boyle's law is applicable in a situation in which the number of molecules remain constant.

14. According to the kinetic interpretation of temperature,

Absolute temperature  $\propto$  average K.E. of molecules.

As the heat is removed, the temperature falls and velocity of molecules decreases. At absolute zero, the molecular motion ceases i.e., the kinetic energy becomes zero. As kinetic energy cannot be negative, so no further decrease in kinetic energy is possible. Hence temperature cannot be decreased below 0K.

15. As  $v_{\text{rms}} \propto \frac{1}{\sqrt{M}}$

Hence hydrogen gas will leak more rapidly because of its smaller molecular mass.

16. From kinetic theory,  $P = \frac{1}{3} \frac{M}{V} \overline{v^2}$

But  $\overline{v^2} \propto T \therefore P \propto \frac{MT}{V}$

As both  $T$  and  $V$  remain unchanged but mass  $M$  is doubled, so the pressure of mixture gets doubled i.e., it is equal to  $2P$ .

17. Volume of 1 mole of a gas at S.T.P. = 22.4 L  
 =  $22.4 \times 10^{-3} \text{ m}^3$

$\therefore R = \frac{PV}{T}$

=  $\frac{0.76 \times 13.6 \times 10^3 \times 9.8 \times 22.4 \times 10^{-3}}{273}$

= 8.31 J/mole / K

18. When the volume of a gas is decreased, the space for the given number of molecules of the gas decreases and no. of molecules per unit volume increases. So, more molecules collide with the walls of the container per second and a larger momentum is transferred, to the walls per second. Therefore, pressure of the gas increases.

When the temperature of the gas is increased, the molecules gain more kinetic energy. Hence they start moving faster and collide more frequently with the walls of the container. Energy and hence transfer of momentum per second to the wall of the container increases. Hence, pressure increases.

### LONG ANSWER QUESTIONS

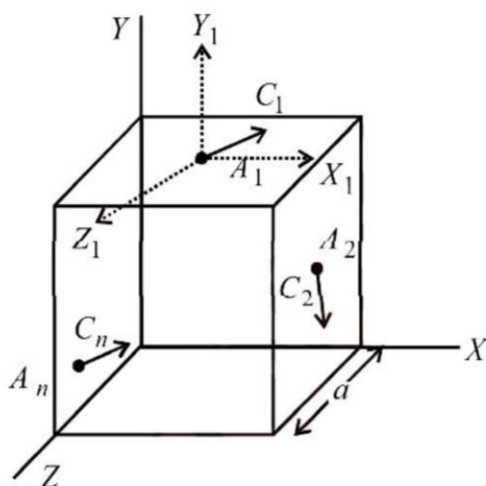
1. Let us consider, an ideal gas in a cubical container of side  $a$ .

$V$  = volume of the container =  $a^3$ ,  $n$  = no. of molecules,  $m$  = mass of each molecule

Total mass =  $M = m \times n$

$C_1, C_2, C_3$  = velocities of  $A_1, A_2, \dots, A_n$

$(x_1, y_1, z_1), (x_2, y_2, z_2), \dots, (x_n, y_n, z_n)$  are the components of  $C_1, C_2, \dots, C_n$  along  $x, y, z$  axes respectively.



Initial momentum of  $A_1$  along  $OX = mx_1$ ;  
 Final momentum of  $A_1$  after collision along  $OX = -mx_1$

Change in momentum =  $-mx_1 - mx_1 = -2mx_1$ ,  
 According to law of conservation of momentum. Momentum transferred by the wall to  $A_1 = 2mx_1$

$\therefore$  Time between two successive collisions  
 =  $\frac{2a}{x_1}$ ; No. of collisions/s =  $\frac{1}{t} = \frac{x_1}{2a}$

Momentum transferred in one second by

$$A_1 = 2mx_1 \times \frac{x_1}{2a} = \frac{mx_1^2}{a}$$

Force exerted by molecule  $A_1$  on the wall

$$= f_1 = \frac{mx_1^2}{a}$$

$$\therefore f_2 = \frac{mx_2^2}{a}, f_3 = \frac{mx_3^2}{a}, \dots, f_n = \frac{mx_n^2}{a}$$

$\therefore$  Total force on the wall by all the molecules  
 =  $F_x = f_1 + f_2 + \dots + f_n$

$$= \frac{mx_1^2}{a} + \frac{mx_2^2}{a} + \dots + \frac{mx_n^2}{a}$$

$$= \frac{m}{a}(x_1^2 + x_2^2 + \dots + x_n^2)$$

Pressure exerted on the wall =  $P_x = \frac{F_x}{a^2}$

$$= \frac{m}{a^3}(x_1^2 + x_2^2 + \dots + x_n^2);$$

$$P_y = \frac{m}{a^3}(y_1^2 + y_2^2 + \dots + y_n^2);$$

$$P_z = \frac{m}{a^3}(z_1^2 + z_2^2 + \dots + z_n^2)$$

$\therefore$  The molecular density is uniform.

$$\therefore P_x = P_y = P_z = P \Rightarrow P_x + P_y + P_z = 3P$$

$$\therefore P = \frac{P_x + P_y + P_z}{3}$$

$$P = \frac{1}{3} \left[ \frac{m}{a^3}(x_1^2 + x_2^2 + \dots + x_n^2) + \frac{m}{a^3} \right.$$

$$\left. (y_1^2 + y_2^2 + \dots + y_n^2) + \frac{m}{a^3}(z_1^2 + z_2^2 + \dots + z_n^2) \right]$$

$$= \frac{m}{3a^3} [(x_1^2 + y_1^2 + z_1^2) + (x_2^2 + y_2^2 + z_2^2)$$

$$+ \dots + (x_n^2 + y_n^2 + z_n^2)]$$

$$= \frac{m}{3V}(C_1^2 + C_2^2 + \dots + C_n^2)$$

$$= \frac{mn}{3V} \left( \frac{C_1^2 + C_2^2 + \dots + C_n^2}{n} \right) = \frac{M}{3V} C^2$$

Where  $C$  = R.M.S velocity

$$= \sqrt{\frac{C_1^2 + C_2^2 + \dots + C_n^2}{n}} \Rightarrow \boxed{P = \frac{1}{3} \rho C^2},$$

where,  $\frac{M}{V} = \rho$  = density of the gas

2. From Kinetic Theory of gases, Pressure of 1

mole of an ideal gas =  $P = \frac{1}{3} \frac{M}{V} C^2$

$$\Rightarrow PV = \frac{1}{3} MC^2 \quad [\because PV = RT]$$

$$\Rightarrow RT = \frac{1}{3} MC^2 \Rightarrow C^2 = \frac{3RT}{M}$$

$$\Rightarrow C^2 \propto T \Rightarrow C \propto \sqrt{T}$$

When  $T = 0$ ,  $C = 0$

$\therefore$  Absolute zero is that temperature at which the r.m.s speed of the molecules become zero.

$\therefore$  **The molecular motion ceases at that temperature.**

3. **Mean speed** is the average speed with which a molecule of a gas moves.

If  $v_1, v_2, \dots, v_n$  are the individual speeds of the molecules then, mean speed

$$= v = \frac{v_1 + v_2 + \dots + v_n}{n} = \sqrt{\frac{8kT}{m\pi}};$$

**r.m.s speed** is the square root of the mean of

the squares of the random velocities of the individual molecules of a gas.

$$C_{\text{rms}} = \sqrt{\frac{C_1^2 + C_2^2 + \dots + C_n^2}{n}} = \sqrt{\frac{3kT}{m}}$$

**Most probable speed** of the molecules of a gas is that speed with which the maximum fraction of total no. of molecules moves.

$$\text{Most probable speed} = v_{\text{max}} = \sqrt{\frac{2kT}{m}}$$

4. For monoatomic gases, there is only three degree of freedom.

K.E./molecule/degree of freedom =  $\frac{1}{2} kT$  (From law of equipartition of energy)

$\therefore$  For three degrees of freedom,

$$\text{K.E.} = \frac{3}{2} kT$$

$\therefore$  Total K.E. of 1 gm mole of the gas

$$= \frac{3}{2} kT \times N = \frac{3}{2} RT, \text{ where } R = kN$$

If  $dQ$  is the small amount of heat given to increase the temperature by  $dT$  of 1 gm mole of a gas at constant volume, then  $dQ = 1 \cdot C_v \cdot dT$

$$\therefore C_v = \frac{dQ}{dT} = \frac{d}{dT} \left( \frac{3}{2} RT \right) = \frac{3}{2} R$$

$$\begin{aligned} \therefore C_p - C_v = R &\Rightarrow C_p = C_v + R \\ &= \frac{3}{2} R + R = \frac{5}{2} R \end{aligned}$$

$$\therefore \gamma = \frac{C_p}{C_v} = \frac{(5/2)R}{(3/2)R} = \frac{5}{3}$$

For diatomic gases, there are 5 degrees of freedom.

$\therefore$  Energy associated with one gm mole of a

$$\text{diatomic gas} = E = 5 \times \frac{1}{2} kT \times N = \frac{5}{2} RT$$

$$\therefore C_v = \frac{dQ}{dT} = \frac{d}{dT} \left( \frac{5}{2} RT \right) = \frac{5}{2} R$$

$$\therefore C_p = C_v + R = \frac{5}{2} R + R = \frac{7}{2} R$$

$$\therefore \gamma = \frac{C_p}{C_v} = \frac{(7/2)R}{(5/2)R} = \frac{7}{5}$$

For triatomic gases with linear molecule, there are seven degrees of freedom.

$\therefore$  Total energy associated with 1 gm

$$\text{molecule of such a gas} = 7 \times \frac{1}{2} kT \times N = \frac{7}{2} RT$$

$$\therefore C_v = \frac{d}{dT} \left( \frac{7}{2} RT \right) = \frac{7}{2} R$$

$$\therefore C_p = C_v + R = \frac{7}{2} R + R = \frac{9}{2} R$$

$$\therefore \gamma = \frac{C_p}{C_v} = \frac{(9/2)R}{(7/2)R} = \frac{9}{7}$$

For non linear triatomic molecule, there are six degrees of freedom.

$\therefore$  Total energy associated with one gm molecule of such a gas

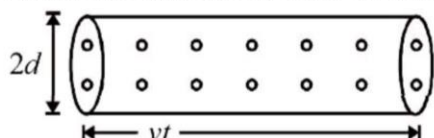
$$= 6 \times \frac{1}{2} kT \times N = \frac{6}{2} RT = 3RT$$

$$\therefore C_v = \frac{d}{dT}(3RT) = 3R$$

$$\therefore C_p = C_v + R = 3R + R = 4R$$

$$\therefore \gamma = \frac{C_p}{C_v} = \frac{4R}{3R} = \frac{4}{3}$$

5. Let us consider a gas possessing  $n$ -molecules per unit volume. If  $d$  is the diameter of each molecule, then the moving molecule will collide with all those molecules whose centres be within the distance  $d$  from its centre.



Number of collisions made by a moving molecule in 1 sec =  $\pi d^2 v n$

$\therefore$  Mean free path  $\lambda$

$$= \frac{\text{Total distance traversed in 1 sec}}{\text{No of collisions suffered in 1 sec}}$$

$$= \frac{v}{\pi d^2 v n} = \frac{1}{\pi d^2 n}$$

Maxwell made correction to the term and found

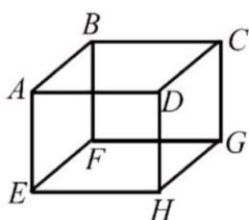
$$\lambda = \frac{1}{\sqrt{2} \pi d^2 n}$$

## SECTION C

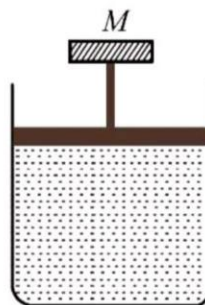
## NCERT EXEMPLAR QUESTIONS

### MULTIPLE CHOICE QUESTIONS (MCQs)

- A cubic vessel (with face horizontal + vertical) contains an ideal gas at NTP. The vessel is being carried by a rocket which is moving at a speed of  $500 \text{ m s}^{-1}$  in vertical direction. The pressure of the gas inside the vessel as observed by us on the ground
  - remains the same because  $500 \text{ ms}^{-1}$  is very much smaller than  $v_{\text{rms}}$  of the gas
  - remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules and the walls
  - will increase by a factor equal to  $(v_{\text{rms}}^2 + (500)^2) / v_{\text{rms}}^2$  where  $v_{\text{rms}}$  was the original mean square velocity of the gas
  - will be different on the top wall and bottom wall of the vessel
- 1 mole of an ideal gas is contained in a cubical volume  $V$ ,  $ABCDEFGH$  at  $300\text{K}$  (figure). One face of the cube ( $EFGH$ ) is made up of a material which totally absorbs any gas molecule incident on it. At any given time,

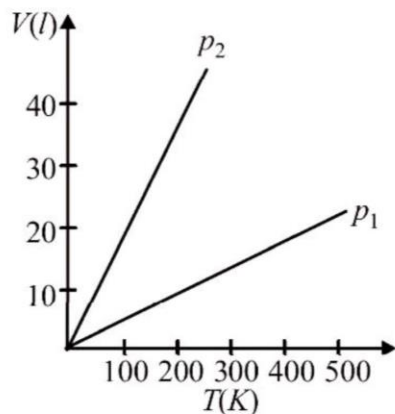


- the pressure on  $EFGH$  would be zero
  - the pressure on all the faces will be equal
  - the pressure of  $EFGH$  would be double the pressure on  $ABCD$
  - the pressure on  $EFGH$  would be half that on  $ABCD$
- Boyle's law is applicable for an
    - adiabatic process
    - isothermal process
    - isobaric process
    - isochoric process
  - A cylinder containing an ideal gas is in vertical position and has a piston of mass  $M$  that is able to move up or down without friction (figure). If the temperature is increased



- both  $p$  and  $V$  of the gas will change
- only  $p$  will increase according to Charles' law
- $V$  will change but not  $p$
- $p$  will change but not  $V$

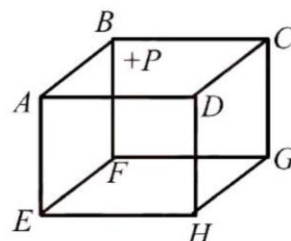
5. Volume versus temperature graphs for a given mass of an ideal gas are shown in figure. At two different values of constant pressure. What can be inferred about relation between  $p_1$  and  $p_2$ ?



- (a)  $p_1 > p_2$                       (b)  $p_1 = p_2$   
 (c)  $p_1 < p_2$                       (d) Data is insufficient
6. 1 mole of  $H_2$  gas is contained in a box of volume  $V = 1.00 \text{ m}^3$  at  $T = 300 \text{ K}$ . The gas is heated to a temperature of  $T = 3000 \text{ K}$  and the gas gets converted to a gas of hydrogen atoms. The final pressure would be (considering all gases to be ideal)
- (a) same as the pressure initially  
 (b) 2 times the pressure initially  
 (c) 10 times the pressure initially  
 (d) 20 times the pressure initially
7. A vessel of volume  $V$  contains a mixture of 1 mole of hydrogen and 1 mole oxygen (both considered as ideal). Let  $f_1(v)dv$ , denote the fraction of molecules with speed between  $v$  and  $(v + dv)$  with  $f_2(v)dv$ , similarly for oxygen. Then,
- (a)  $f_1(v) + f_2(v) = f(v)$  obeys the Maxwell's distribution law  
 (b)  $f_1(v)$ ,  $f_2(v)$  will obey the Maxwell's distribution law separately  
 (c) neither  $f_1(v)$ , nor  $f_2(v)$  will obey the Maxwell's distribution law  
 (d)  $f_2(v)$  and  $f_1(v)$  will be the same
8. An inflated rubber balloon contains one mole of an ideal gas, has a pressure  $p$ , volume  $V$  and temperature  $T$ . If the temperature rises to  $1.1 T$ , and the volume is increased to  $1.05 V$ , the final pressure will be
- (a)  $1.1 p$                               (b)  $p$   
 (c) less than  $p$                       (d) between  $p$  and  $1.1 p$

**MULTIPLE CHOICE QUESTIONS (MCQs)**  
**(ONE OR MORE THAN ONE OPTION CORRECT)**

1.  $ABCDEFGH$  is a hollow cube made of an insulator (figure) face  $ABCD$  has positive charge on it. Inside the cube, we have ionised hydrogen. The usual kinetic theory expression for pressure

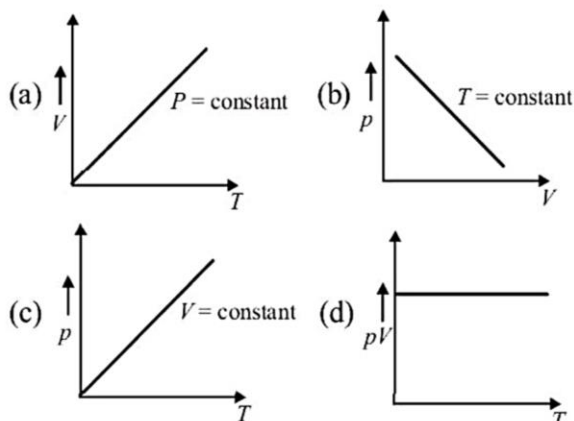


- (a) will be valid  
 (b) will not be valid, since the ions would experience forces other than due to collisions with the walls  
 (c) will not be valid, since collisions with walls would not be elastic  
 (d) will not be valid because isotropy is lost
2. Diatomic molecules like hydrogen have energies due to both translational as well as rotational motion. From the equation in kinetic theory

$$pV = \frac{2}{3} E, E \text{ is}$$

- (a) the total energy per unit volume  
 (b) only the translational part of energy because rotational energy is very small compared to the translational energy  
 (c) only the translational part of the energy because during collisions with the wall pressure relates to change in linear momentum  
 (d) the translational part of the energy because rotational energies of molecules can be of either sign and its average over all the molecules is zero.
3. In a diatomic molecule, the rotational energy at a given temperature
- (a) obeys Maxwell's distribution  
 (b) have the same value for all molecules  
 (c) equals the translational kinetic energy for each molecule  
 (d) is  $(2/3)$ rd the translational kinetic energy for each molecule

4. Which of the following diagrams (figure) depicts ideal gas behaviour ?



5. When an ideal gas is compressed adiabatically, its temperature rises the molecules on the average have more kinetic energy than before. The kinetic energy increases,
- because of collision with moving parts of the wall only
  - because of collisions with the entire wall
  - because the molecules gets accelerated in their motion inside the volume
  - because of redistribution of energy amongst the molecules

### VERY SHORT ANSWER QUESTIONS

- Calculate the number of atoms in 39.4 g gold. Molar mass of gold is  $197 \text{ g mole}^{-1}$ .
- The volume of a given mass of a gas at  $27^\circ\text{C}$ , 1 atm is 100 cc. What will be its volume at  $327^\circ\text{C}$ ?
- The molecules of a given mass of a gas have root mean square speeds of  $100 \text{ ms}^{-1}$  at  $27^\circ\text{C}$  and 1.00 atmospheric pressure. What will be the root mean square speeds of the molecules of the gas at  $127^\circ\text{C}$  and 2.0 atmospheric pressure?
- Two molecules of a gas have speeds of  $9 \times 10^6 \text{ ms}^{-1}$  and  $1 \times 10^6 \text{ ms}^{-1}$ , respectively. What is the root mean square speed of these molecules.
- A gas mixture consists of 2.0 moles of oxygen and 4.0 moles of neon at temperature  $T$ . Neglecting all vibrational modes, calculate the total internal energy of the system. (Oxygen has two rotational modes.)
- Calculate the ratio of the mean free paths of the molecules of two gases having molecular diameters  $1\text{\AA}$  and  $2\text{\AA}$ . The gases may be considered under identical conditions of temperature, pressure and volume.

### SHORT ANSWER QUESTIONS

- The container shown in figure has two chambers, separated by a partition, of volumes  $V_1 = 2.0 \text{ L}$  and  $V_2 = 3.0 \text{ L}$ .

$p_1, V_1$	$p_2, V_2$
$\mu_1$	$\mu_2$

The chambers contain  $\mu_1 = 4.0$  mole and  $\mu_2 = 5.0$  mole of a gas at pressures  $p_1 = 1.00 \text{ atm}$  and  $p_2 = 2.00 \text{ atm}$ . Calculate the pressure after the partition is removed and the mixture attains equilibrium.

- A gas mixture consists of molecules of  $A$ ,  $B$  and  $C$  with masses  $m_A > m_B > m_C$ . Rank the three types of molecules in decreasing order of
  - average KE
  - rms speeds
- We have 0.5 g of hydrogen in a cubic chamber of size 3 cm kept at NTP. The gas in the chamber is compressed keeping the temperature constant till a final pressure of 100 atm. Is one justified in assuming the ideal gas law, in the final state? (Hydrogen molecules can be consider as spheres of radius  $1\text{\AA}$ ).
- When air is pumped into a cycle tyre the volume and pressure of the air in the tyre both are increased. What about Boyle's law in this case?
- A balloon has 5.0 mole of helium at  $7^\circ\text{C}$ . Calculate
  - the number of atoms of helium in the balloon.
  - the total internal energy of the system.
- Calculate the number of degrees of freedom of molecules of hydrogen in 1 cc of hydrogen gas at NTP.
- An insulated container containing monoatomic gas of molar mass  $m$  is moving with a velocity  $v_0$ . If the container is suddenly stopped, find the change in temperature.

### LONG ANSWER QUESTIONS

- Explain why
  - there is no atmosphere on moon
  - there is fall in temperature with altitude



2. Consider an ideal gas with following distribution of speeds.

Speed m/s	% of molecules
200	10
400	20
600	40
800	20
1000	10

- (a) Calculate  $v_{\text{rms}}$  and hence  $T$ . ( $m = 3.0 \times 10^{-26}$  kg)  
 (b) If all the molecules with speed 1000 m/s escape from the system, calculate new  $v_{\text{rms}}$  and hence  $T$ .
3. Ten small planes are flying at a speed of 150 km/h in total darkness in an air space that is  $20 \times 20 \times 1.5$  km<sup>3</sup> in volume. You are in one of the planes, flying at random within this space with no way of knowing where the other planes are. On the

average about how long a time will elapse between near collision with your plane. Assume for this rough computation that a safety region around the plane can be approximated by a sphere of radius 10 m.

4. A box of 1.00 m<sup>3</sup> is filled with nitrogen at 1.50 atm at 300 K. The box has a hole of an area 0.010 mm<sup>2</sup>. How much time is required for the pressure to reduce by 0.10 atm, if the pressure outside is 1 atm.
5. Consider a rectangular block of wood moving with a velocity  $v_0$  in a gas at temperature  $T$  and mass density  $\rho$ . Assume the velocity is along  $x$ -axis and the area of cross-section of the block perpendicular to  $v_0$  is  $A$ . Show that the drag force on the block is  $4rAv_0\sqrt{\frac{kT}{m}}$ , where,  $m$  is the mass of the gas molecule.

## NCERT EXEMPLAR SOLUTIONS

### MULTIPLE CHOICE QUESTIONS (MCQs)

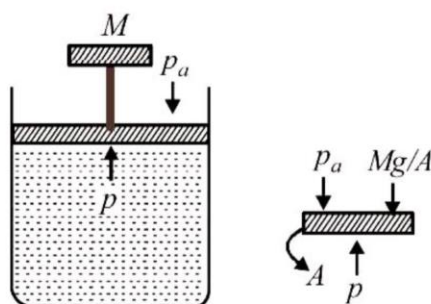
1. (b) As the relative velocity of molecule with respect to the walls of container does not change in rocket, due to the mass of a molecule is negligible with respect to the mass of whole system and system of gas moves as a whole and ( $g = 0$ ) on molecule energy where.  
 Hence pressure of the gas inside the vessel, as observed by us, on the ground remain the same.
2. (d) Pressure on the wall due to force exerted by molecule on walls due to its rate of transfer of momentum to wall.  
 In an ideal gas, when a molecule collides elastically with a wall, the momentum transferred to each molecule will be twice the magnitude of its normal momentum is  $2mv$ . For the wall  $EFGH$ , absorbs those molecules.  
 Which strike to it so rate of change in momentum to it become only  $mv$  so the pressure of  $EFGH$  would half of  $ABCD$ .
3. (b) Boyle's law is applicable at constant temperature and temperature remains constant in isothermal process.  
 For ideal gas,  $pV = nRT = \text{constant}$

So,  $pV = \text{constant}$  (at constant temperature)

$$p \propto \frac{1}{V}$$

So, this process can be called as isothermal process.

4. (c) Let us consider the given diagram where an ideal gas is contained in a cylinder, having a piston of mass  $M$ . The pressure on gas does not change.



The pressure inside the gas will be

$$p = p_a + Mg/A$$

where,  $p_a =$  atmospheric pressure

$A =$  area of cross-section of the piston.

$Mg =$  weight of piston

Hence,  $p = \text{constant}$ .

As the piston and cylinder is frictionless so the equation for ideal gas

$pV = nRT$ , volume ( $V$ ) increases at constant pressure.

as  $p, R, n$  are constant so,

$$V \propto T$$

so on increasing temperature of system its volume increased but  $p$  will remain constant.

5. (a) As we know that an ideal gas equation, as the pressure and quantity of gas in system are constant

$$pV = nRT \Rightarrow V = \left(\frac{nR}{p}\right)T$$

$$V \propto \frac{T}{p} \text{ as } n, R \text{ are constant } \frac{V}{T} \propto \left(\frac{1}{p}\right)$$

Slope of the  $V-T$  graph,

$$m = \frac{dV}{dT} = \frac{nR}{p} \quad [m = \text{slope of } V-t \text{ graph}]$$

$$\frac{dV}{dT} \propto \frac{1}{p} \text{ or } m \propto \frac{1}{p} \quad [\because nR = \text{constant}]$$

$$\text{So, } p \propto \frac{1}{m}$$

$$\text{hence, } \frac{p_1}{p_2} = \frac{m_2}{m_1} < 1$$

where,  $m_1$  is slope of the graph corresponding to  $p_1$  and similarly  $m_2$  is slope corresponding to  $p_2$ . So slope of  $p_1$  is smaller than  $p_2$ . Hence, ( $p_1 > p_2$ ).

6. (d) Pressure exerted by gas is due to rate of change of momentum ( $p$ ) imparted by particles to wall.

When the molecules breaks into atoms, the number of moles would become twice.

From ideal gas equation,

$$pV = nRT$$

where,  $p$  = Pressure of gas,  $n$  = Number of moles

$R$  = Gas constant,  $T$  = temperature

As gases breaks number of moles becomes twice of initial, so  $n_2 = 2n_1$

$$\text{So, } p \propto nT$$

$$\text{or } \frac{p_2}{p_1} = \frac{n_2 T_2}{n_1 T_1} = \frac{(2n_1)(3000)}{n_1(300)} = 20$$

$$\text{So, } p_2 = 20p_1$$

Hence, final pressure of the gas would be 20 times to the initial pressure.

7. (b) For a function  $f_1(v)$  the number of molecules ( $n$ ) which will have their speeds between  $v$  and  $v + dv$ .

For each function  $f_1(v)$  and  $f_2(v)$  number of molecules remain same 1 mole each but due to mass difference their speed will be different.

Hence both gases of each function  $f_1(v)$  and  $f_2(v)$  will obey the Maxwell's distribution law separately.

8. (d) As we know that an ideal gas equation,

$$pV = nRT$$

where,  $n$  = Number of moles,  $p$  = Pressure,

$V$  = Volume,  $R$  = Gas constant,

$T$  = Temperature

$$n = \frac{pV}{RT}$$

If  $n, R$  are constant for the system or as number of moles of the gas remains fixed, hence, we can write

$$\frac{pV}{T} = \text{constant}$$

$$\text{or } \frac{p_1 V_1}{RT_1} = \frac{p_2 V_2}{RT_2}$$

$$p_2 = (p_1 V_1) \left( \frac{T_2}{V_2 T_1} \right)$$

$$= \frac{(p)(V)(1.1 T)}{(1.05)V(T)} \quad [p_1 = p, V_2 = 1.05 V \text{ and } T_2 = 1.1 T]$$

$$= p \times \left( \frac{1.1}{1.05} \right)$$

$$= p(1.0476)$$

So, final pressure  $p_2$  will lies between  $p$  and  $1.1p$ .

### MULTIPLE CHOICE QUESTIONS (MCQs)

#### (ONE OR MORE THAN ONE OPTION CORRECT)

1. (b, d)

Due to presence of hydrogen ions external positive charge on the wall  $ABCD$ . The usual expression for pressure on the basis of kinetic theory will not be valid as ions would also experience electrostatic forces other than the forces due to collisions with the walls of the container. Due to presence of positive ion in place of hydrogen molecules, the isotropy is also lost.

2. (c) According to the postulate of kinetic theory of gases it assumed that pressure on the gas molecule is due to only perpendicular forces on wall due to motion of molecules. They do not exert any parallel force, hence there will not be any type of rotation present.  
 So the pressure on the wall produces due to only change in translational motion.

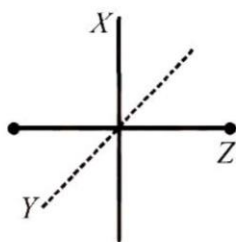
$$\text{Hence, } pV = \frac{2}{3} E$$

[where  $p$  = pressure,  $V$  = volume]  
 $E$  is representing the only translational part of energy.

3. (a,d) Consider a diatomic molecule along  $z$ -axis is so its rotational energy about  $z$ -axis is zero.  
 So, the total energy associated with the diatomic molecule is

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2 + \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2$$

In the above expression the independent term is (5) and it contains translational kinetic energy  $\left(\frac{1}{2}mv^2\right)$  corresponding to velocity in each  $x$ ,  $y$  and  $z$ -directions as well as rotation KE  $\left(\frac{1}{2}I\omega^2\right)$  associated with axis of rotations  $x$  and  $y$ .



As we can predict velocities of molecules by Maxwell's distribution, hence the above expression also obeys Maxwell's distribution.

As 2 rotational and 3 translational energies are associated with each molecule.

So the rotational energy at a given

temperature is  $\left(\frac{2}{3}\right)$ rd of its translational KE of each molecule.

4. (a, c)  
 As we know that,  
 Gas equation for ideal gas  
 $pV = nRT$ , ... (i)  
 where,  $n$  and  $R$  are constant for a system

- (a) If pressure  $p = \text{constant}$ , from (i)  
 $V(\text{Volume}) \propto T(\text{Temperature})$   
 Hence verifies option (a).  
 (b) If  $T = \text{constant}$ , from (i)  
 $pV = \text{constant}$ .  
 Hence the graph must be rectangular hyperbola.  
 So, reject option (b).  
 (c) If  $V = \text{constant}$ ,  
 from (i),  $p \propto T$   
 Hence, the graph is straight line passes through the origin.  
 So, verifies the option (c).  
 (d) From graph (i),  $pV \propto T$

$$\text{So, } \frac{pV}{T} = (\text{slope is constant})$$

at  $p = 0$ , constant  $K = 0$ ,  $V = 0$

Hence, the graph through origin  
 Hence reject option (d).

5. (a) As the ideal gas compress, then the mean free path becomes smaller so the number of collision per second between the molecules.

When the gas is compressed adiabatically, the total work done on the gas increases its internal energy so wall increases which increase the temperature of gas which in turn increase the KE of gas molecules and hence, the collisions between molecules also increases.

### VERY SHORT ANSWER QUESTIONS

1. As given that,  
 molar mass of gold = 197 g/mol  
 Molar Mass i.e. mass of Avogadro's number of atoms =  $6.023 \times 10^{23}$  atoms  
 So, 197 gm gold contains =  $6.023 \times 10^{23}$  atoms  
 Number of atoms 1gm gold contain

$$= \frac{6.023 \times 10^{23}}{197} \text{ atoms}$$

Number of gold atoms 39.4 gm gold contains

$$= \frac{6.023 \times 10^{23} \times 39.4}{197}$$

$$= \frac{12.046 \times 10^{23}}{10} = 1.2046 \times 10^{23} \text{ atoms}$$

Number of gold atoms.

2. As given that, for ideal gas equation  
 $p_1 = 1 \text{ atm}, p_2 = 1 \text{ atm}, V_1 = 100 \text{ cc}, V_2 = ?$   
 $T_1 = 273 + 27 = 300 \text{ K}$   
 $T_2 = 273 + 327 = 600 \text{ K}$

For ideal gas the relation is,  $pV \propto T$

$$\frac{pV}{T} = \text{constant}$$

[ $V$  = Volume of gas,  $T$  = Temperature of gas]

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\Rightarrow V_2 = V_1 \left( \frac{T_2}{T_1} \right) \times \left( \frac{P_1}{P_2} \right)$$

$$\text{But, } V_2 = V_1 \left( \frac{600}{300} \right) \times \left[ \frac{1}{1} \right]$$

$$V_2 = 2V_1$$

$$V_2 = 2 \times 100 = 200 \text{ cc}$$

So, unit of ( $p_1, p_2$ ) and ( $V_1, V_2$ ) must same separately but unit of  $T$  must be in only on Kelvin scale.

3. As we know that for a given mass of a gas the root mean square velocity is

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

where,  $R$  is gas constant,  $T$  is temperature in Kelvin,  $M$  is molar mass of gas for a gas  $M$  is constant.

$$\text{So, } v_{\text{rms}} \propto \sqrt{T}$$

$$\frac{(v_{\text{rms}})_1}{(v_{\text{rms}})_2} = \sqrt{\frac{T_1}{T_2}} \quad \dots(i)$$

As given that,  $(v_{\text{rms}})_1 = 100 \text{ m/s}, (v_{\text{rms}})_2 = ?$

$$T_1 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$$

$$T_2 = 127^\circ\text{C} = 127 + 273 = 400 \text{ K}$$

By putting the value of  $T_1, T_2, (v_{\text{rms}})_1$  in eq. (i),

$$\frac{100}{(v_{\text{rms}})_2} = \sqrt{\frac{300}{400}} = \frac{\sqrt{3}}{2}$$

$$\text{So, } (v_{\text{rms}})_2 = \frac{2 \times 100}{\sqrt{3}} = \frac{200}{\sqrt{3}} \text{ m/s}$$

4. For  $n$ -molecules,  
 As we know that the r.m.s. velocity is

$$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2}{n}}$$

where  $v_1, v_2, v_3, \dots, v_n$  are individual velocities of  $n$ -molecules of the gas.

According to question, root mean square speed for two molecules.

$$\text{So, } v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2}{2}}$$

As given that,  $v_1 = 9 \times 10^6 \text{ m/s}$  and  $v_2 = 1 \times 10^6 \text{ m/s}$

$$\therefore v_{\text{rms}} = \sqrt{\frac{(9 \times 10^6)^2 + (1 \times 10^6)^2}{2}}$$

$$= \sqrt{\frac{81 \times 10^{12} + 1 \times 10^{12}}{2}}$$

$$= \sqrt{\frac{(81+1) \times 10^{12}}{2}}$$

$$= \sqrt{\frac{82 \times 10^{12}}{2}} = \sqrt{41} \times 10^6 \text{ m/s}$$

5. To find total energy for a given molecule of a gas, we must find the number of degree of freedom.

The molecule of oxygen has 2 atom.

So it has  $(3T + 2R) = 5$  degree of freedom.

So  $O_2$  is a diatomic gas having 5 degrees of freedom.

Energy (total internal) per mole of the gas

$$= \frac{5}{2} RT$$

[ $R$  = Universal gas constant,  $T$  = temperature]

For 2 moles of the gas total internal energy

$$= 2 \times \frac{5}{2} RT = 5RT \quad \dots(i)$$

Neon (Ne) gas is a monoatomic so its degrees of freedom is only 3. Hence total internal energy

$$\text{per mole} = \frac{3}{2} RT$$

so total internal energy of 4 moles of Ne.

$$\text{Energy} = 4 \times \frac{3}{2} RT = 6RT \quad \dots(ii)$$

[Using Eqs. (i) and (ii)]

Hence total internal energy of 2 mole oxygen and 4 mole Ne.

$$\therefore \text{Total energy} = 5RT + 6RT = 11RT.$$

6. As we know that,

Mean free path of a molecule is

$$\lambda = \frac{1}{\sqrt{2} d^2 n}$$

where,  $n$  = number of molecules/ volume

$d$  = diameter of the molecule

As both the gases are identical so  $n$  will be constant,

$$\text{Hence, } \lambda \propto \frac{1}{d^2}$$

As given that,  $d_1 = 1\text{\AA}$ ,  $d_2 = 2\text{\AA}$

$$\text{As } \lambda_1 \propto \frac{1}{d_1^2} \text{ and } \lambda_2 \propto \frac{1}{d_2^2}$$

$$\text{So, } \frac{\lambda_1}{\lambda_2} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$$

Hence,  $\lambda_1 : \lambda_2 = 4 : 1$

### SHORT ANSWER QUESTIONS

1. As given that,

$$V_1 = 2.0 \text{ L}, V_2 = 3.0 \text{ L}, T_1 = T, T_2 = T$$

$$\mu_1 = 4.0 \text{ mole}, \mu_2 = 5.0 \text{ mole}$$

$$p_1 = 1.00 \text{ atm}, p_2 = 2.00 \text{ atm}$$

For ideal gas equation,  $pV = \mu RT$

So, for ideal gases in chamber 1,  $p_1 V_1 = \mu_1 RT_1$

For ideal gases in chamber 2,  $p_2 V_2 = \mu_2 RT_2$

When the partition between gases removed then get mixed without any loss of energy. The mixture now attains a common equilibrium pressure and the total volume of the system is sum of the volume of individual chambers  $V_1$  and  $V_2$ .

$$\text{So, } \mu = \mu_1 + \mu_2, V = V_1 + V_2$$

By kinetic theory of gases,

The kinetic translational energy for  $l$  mole

$$pV = \frac{2}{3} E \quad (\text{per mole})$$

So the KE by gas of  $\mu_1$  moles,

$$p_1 V_1 = \frac{2}{3} \mu_1 E_1 \quad \dots(i)$$

Then the KE by gas of  $\mu_2$  moles,

$$p_2 V_2 = \frac{2}{3} \mu_2 E_2 \quad \dots(ii)$$

By adding above two equation.

So, total energy is,

$$(p_1 V_1 + p_2 V_2) = \frac{2}{3} (\mu_1 E_1 + \mu_2 E_2)$$

From the above relation the combined effect is

$$pV = \frac{2}{3} E_{\text{total}} \text{ per mole} = \frac{2}{3} \mu E \text{ per mole}$$

$$p(V_1 + V_2) = \frac{2}{3} \times \left[ \frac{3}{2} (p_1 V_1 + p_2 V_2) \right]$$

$$\therefore \mu_1 E_1 + \mu_2 E_2 = \frac{3}{2} (p_1 V_1 + p_2 V_2)$$

$$p = \frac{p_1 V_1 + p_2 V_2}{V_1 + V_2} = \left( \frac{100 \times 2.0 + 2.00 \times 3.0}{2.0 + 3.0} \right) \text{ atm} = \frac{(2+6)}{5} = 1.6 \text{ atm}$$

2. (a) The average KE will be same as condition of temperature and pressure are the same.  
 (b) For the rms speed, the temperature and pressure are the same.

$$v_{\text{rms}} = \sqrt{\frac{3pV}{M}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{mN}} = \sqrt{\frac{3k_B T}{m}}$$

where,  $M$  = molar mass of the gas

$m$  = mass of each molecular of the gas

$R$  = gas constant

$T$  = absolute temperature (same for all)

$$\text{So, } v_{\text{rms}} \propto \sqrt{\frac{1}{m}}$$

$$m_A > m_B > m_C \quad (\text{given})$$

$$\text{Hence, } (v_{\text{rms}})_C > (v_{\text{rms}})_B > (v_{\text{rms}})_A$$

3. Consider hydrogen molecules as spheres of radius  $1\text{ \AA}$ .

$$\text{So, } r = 1\text{ \AA} = \text{radius} = 10^{-10}\text{ m}$$

$$\text{Volume of hydrogen molecules} = \frac{4}{3}\pi r^3$$

$$\begin{aligned}\text{Volume of 1 molecule} &= \frac{4}{3}(3.14)(10^{-10})^3 \\ &= 4 \times 10^{-30}\text{ m}^3\end{aligned}$$

Number of moles is 0.5 g of  $\text{H}_2$  gas

$$= \frac{\text{Mass}}{\text{Molecular mass}}$$

$$= \frac{0.5}{2} = 0.25\text{ mole (H}_2\text{ has 2 mole)}$$

Molecules of  $\text{H}_2$  present

$$\begin{aligned}&= \text{Number of moles of H}_2\text{ present} \times 6.023 \times 10^{23} \\ &= 0.25 \times 6.023 \times 10^{23}\end{aligned}$$

$\therefore$  Volume of  $\text{H}_2$  molecules

= number of molecules  $\times$  volume of each molecule

$$\begin{aligned}&= 0.25 \times 6.023 \times 10^{23} \times 4 \times 10^{-30}\text{ m}^3 \\ &= 6.023 \times 10^{23} \times 10^{-30} \approx 6 \times 10^{-7}\text{ m}^3\end{aligned}$$

Now for ideal gas at constant temperature is considered to be constant

$$p_i V_i = p_f V_f$$

$$V_f = \left(\frac{p_i}{p_f}\right) V_i = \left(\frac{1}{100}\right) (3 \times 10^{-2})^3$$

$$= \frac{27 \times 10^{-6}}{10^2} = 2.7 \times 10^{-7}\text{ m}^3 \quad \dots(\text{ii})$$

( $\therefore$  Volume of cube  $V_i = (\text{Side})^3$  and  $p_i = 1\text{ atm}$  at NTP.)

Hence on compression the volume of the gas is of the order of the molecular volume [from Eq. (i) and Eq. (ii)]. If the intermolecular forces will play the role as in kinetic theory of gas, molecules do not interact each other so gas will deviate from ideal gas behaviour.

4. According to Boyle's law, at a constant temperature the volume of gas is inversely proportional to pressure i.e. it is valid only for constant mass of gas.

When air is pumped, more molecules are pumped and Boyle's law is stated for situation where number of molecules remains constant.

In that case, as the number of air molecules keep increasing. Hence this is a case of variable mass. Boyle's law (and even Charles's law) is only applicable in situations, where number of gas molecules remains fixed.

Hence, Boyle's law is not applicable in this case.

5. As given that

Number of moles of helium,  $n = 5\text{ mole}$ .

$$T = 7^\circ\text{C} = 7 + 273 = 280\text{ K}$$

- (a) Number of atoms (He is monoatomic) in 5 mole

$$\begin{aligned}&= \text{Number of moles} \times \text{Avogadro's number} \\ &= 5 \times 6.023 \times 10^{23}\text{ atoms} \\ &= 30.115 \times 10^{23}\text{ atoms} \\ &= 3.0115 \times 10^{24}\text{ atoms.}\end{aligned}$$

- (b) He-atom is monoatomic so degree of freedom is 3.

Average kinetic energy per molecule

$$= \frac{3}{2} k_B T$$

where,  $k_B =$  Boltzmann constant

$\therefore$  Net energy ( $E$ ) of all the atoms

= Total internal energy

$$= \frac{3}{2} k_B T \times \text{number of He-atoms}$$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 280 \times 3.01150 \times 10^{24}$$

Hence net energy of 15 mole of He

$$= 1.74 \times 10^4\text{ J}$$

6. Hydrogen molecule is diatomic so it has 3 translational degree of freedom and 2 rotational. So degree of freedom in  $\text{H}_2$  molecule =  $3 + 2 = 5$ .

volume occupied by 1 mole

= 1 mole of the gas at NTP

$$= 22400\text{ mL} = 22400\text{ cc}$$

$\therefore$  Number of molecules in 1 cc of hydrogen,

For 22400 cc  $\text{H}_2$  gas at STP has =  $6.023 \times 10^{23}$  molecule

So, for 1 cc  $\text{H}_2$  gas at STP has

$$= \frac{6.023 \times 10^{23}}{22400} = 2.688 \times 10^{19}$$

$\text{H}_2$  is a diatomic gas, having a total of 5 degrees of freedom (3 translational + 2 rotational)

So, total degrees of freedom possessed by all the molecules =  $5 \times 2.688 \times 10^{19} = 1.344 \times 10^{20}$ .

7. By the kinetic interpretation of temperature, absolute temperature of a given sample of a gas is proportional to the total translational kinetic energy of its molecules.

So, any change in absolute temperature of a gas will contribute to corresponding change in translational KE and vice-versa.

As the gas is monoatomic so its degree of freedom will be due to only translational motion, which is three.

$$\text{So if KE per molecule} = \frac{3}{2} RT$$

When, the insulated container stops suddenly its total KE is transferred to gas molecules in the form of translational KE, so increasing in the absolute temperature of gas let it be  $\Delta T$  if  $n$  is moles of gas.

If  $\Delta T$  = change in absolute temperature.

Then, KE of molecules increased due to velocity

$$\text{so KE} = \frac{1}{2} (mn)v_0^2 \quad \dots(i)$$

where,  $n$  = number of moles,  $m$  = molar mass of the gas

$$\text{Increase in translational (KE)} = n \frac{3}{2} R(\Delta T) \quad \dots(ii)$$

According to kinetic theory Eqs. (i) and (ii) are equal

$$\frac{1}{2} (mn)v_0^2 = n \frac{3}{2} R(\Delta T)$$

$$(mn)v_0^2 = n3R(\Delta T)$$

$$\Delta T = \frac{(mn)v_0^2}{3nR} = \frac{mv_0^2}{3R}$$

### LONG ANSWER QUESTIONS

1. (a) The moon has small gravitational force (pull) and hence, the escape velocity is small. The value of escape velocity for the moon is 2.38 km/sec.

As the moon is in the proximity of the earth as seen from the sun, the moon has the same amount of heat per unit area as that of the earth.

The air molecules have large range of speeds. Even though the rms speed of the air molecules is smaller than the escape velocity on the moon, a significant number of molecules have speed greater than escape velocity and they escape.

So, rest of the molecules arrange the speed distribution for the equilibrium temperature. Again a significant number of molecules escape as their speed exceed escape speed. So, over a long time the moon has lost most of its atmosphere.

$$\begin{aligned} \text{At 300 K, } v_{\text{rms}} &= \sqrt{\frac{3k_B T}{m}} \\ &= \sqrt{\frac{3 \times 138 \times 10^{-23} \times 300}{1.67 \times 10^{-24}}} \\ &= 2.72 \text{ km/s} \\ &(\because m = 1.67 \times 10^{-24} \text{ kg}) \end{aligned}$$

As acceleration due to gravity of moon is  $\frac{1}{6}$  th. So, the escape velocity ( $v_{\text{es}}$ ) on moon

$$= \sqrt{2gR} = 2.38 \text{ km/sec.}$$

Due to gravitational force,  $v_{\text{rms}}$  is greater than escape velocity. So, molecule of air can escape out.

- (b) The temperature of atmospheric is due to the KE of air molecule.

As the molecules move higher, their potential energy increases and hence, kinetic energy decreases and hence temperature reduces.

At greater height more volume is available and gas expands and hence, some cooling takes place.

2. (a) As we know that

$$\begin{aligned} v_{\text{rms}}^2 &= \frac{\sum_i n_i v_i^2}{\sum_i n_i} \\ &= \frac{n_1 v_1^2 + n_2 v_2^2 + \dots + n_n v_n^2}{n_1 + n_2 + n_3 + \dots + n_n} \end{aligned}$$

where,  $v_{\text{rms}}$  is the rms speed for all molecules collectively. So,

$$v_{\text{rms}} = \left( \frac{\sum_i n_i v_i^2}{\sum_i n_i} \right)^{\frac{1}{2}}$$

$$= \sqrt{\frac{n_1 v_1^2 + n_2 v_2^2 + n_3 v_3^2 + \dots + n_n v_n^2}{n_1 + n_2 + n_3 + \dots + n_n}}$$

For five (5) molecules, then

$$= \sqrt{\frac{n_1 v_1^2 + n_2 v_2^2 + n_3 v_3^2 + n_4 v_4^2 + n_5 v_5^2}{n_1 + n_2 + n_3 + n_4 + n_5}}$$

$$= \sqrt{\frac{10 \times (200)^2 + 20 \times (400)^2 + 40 \times (600)^2 + 20 \times (800)^2 + 10 \times (1000)^2}{100}}$$

$$= \sqrt{\frac{10000 \times (4 + 32 + 144 + 128 + 100) \times 10}{100}}$$

$$= \sqrt{\frac{10^5 \times 408}{100}} = \sqrt{408 \times 1000} = 639 \text{ m/s}$$

Now according to kinetic theory of gasses

$$\frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} k_B T$$

[ $k_B$  = Boltzmann constant,  $m$  = mass of gaseous molecules]

$$T = \frac{1}{3} \frac{m v_{\text{rms}}^2}{k_B}$$

$$= \frac{1}{3} \times \frac{30 \times 10^{-26} \times 4.08 \times 10^5}{1.38 \times 10^{-23}}$$

$$= \frac{204 \times 10^{-23} \times 10^2}{69 \times 10^{-23}}$$

$$= 2.96 \times 10^2 \text{ K} = 296 \text{ K.}$$

- (b) When all the molecules with speed 1000 m/s escape out then

$$v_{\text{rms}}^2 = \frac{10 \times (200)^2 + 20 \times (400)^2 + 40 \times (600)^2 + 20 \times (800)^2}{90}$$

$$= \frac{10 \times 100^2 \times (1 \times 4 + 2 \times 16 + 4 \times 36 + 2 \times 64)}{90}$$

$$v_{\text{rms}} = \sqrt{10000 \times \frac{308}{9}} = \frac{100}{3} \sqrt{308}$$

$$= 33.33 \times 17.55$$

$$v_{\text{rms}} = 585 \text{ m/s}$$

$$\text{So, } T = \frac{1}{3} \frac{m v_{\text{rms}}^2}{k}$$

$$= \frac{1}{3} \times \frac{3 \times 10^{-26} \times (585)^2}{1.38 \times 10^{-23}}$$

$$= \frac{(585)^2}{138} \times 10^{-24+23}$$

$$= \frac{(585)^2}{138} \times 10^{-1} = 248.04 \text{ K}$$

3. When planes can be considered as motion of molecule in confined space this situation can be considered as the time of relaxation mean free path ( $\lambda$ ) based on kinetic theory of gases. Mean free path ( $\lambda$ ) is the distance between two successive collisions, which we will consider here as the distance travelled by the plane before it just avoids the collision safe radius is equivalent to radius of the atom.

Hence, the required time,

$$\lambda = \text{mean free path} = \frac{1}{\sqrt{2} \pi d^2 n}$$

$$\text{time} = \frac{\text{distance } (\lambda)}{\text{speed}}, \quad t = \frac{\lambda}{v}$$

$$\text{So, } t = \frac{1}{\sqrt{2} \pi d^2 n} \times \frac{1}{v} \quad \dots(i)$$

Number of particles per unit volume

$$n = \frac{\text{Number of aeroplanes } (N)}{\text{Volume } (V)}$$

$$= \frac{10}{20 \times 20 \times 1.5} = 0.0167 \text{ km}^{-3}$$

As given that  $d = 2 \times 10 \text{ m} = 20 \text{ m} = 20 \times 10^{-3} \text{ km}$   
 $V = 150 \text{ km}^3/\text{hr}$

$$t = \frac{1}{\sqrt{2} \pi d^2 n} \times \frac{1}{v} \quad [v = \text{velocity of aeroplane}]$$



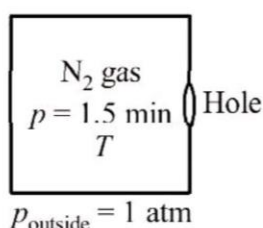
By putting the values of  $d$ ,  $v$ ,  $n$  in equation (i)

$$t = \frac{1}{\sqrt{2} \times 3.14 \times (20)^2 \times 0.0167 \times 10^{-6} \times 150}$$

$$= \frac{10^6}{1776.25 \times 2.505} = \frac{10^6}{4449.5}$$

$$= 224.74 \text{ hrs.} \approx 225 \text{ hrs.}$$

4. As given that volume of the box,  $V = 1.00 \text{ m}^3$   
 Area ( $a$ ) =  $0.010 \text{ mm}^2 = .01 \times 10^{-6} \text{ m}^2$



Temperature outside = Temperature inside  
 Initial pressure inside the box ( $p_1$ ) = 1.50 atm.  
 For outside ( $p_2$ ) = 1 atm  
 Final pressure inside the box ( $p'_1$ ) = 0.10 atm  
 So after time  $T$  pressure reduced by 0.1 and becomes ( $p'_2$ ) =  $1.5 - 0.1 = 1.4 \text{ atm}$ .  
 Assuming,  
 $v_{ix}$  = Speed of nitrogen molecule inside the box along  $x$ -direction.  
 $n_i$  = Number of molecules per unit volume in a time interval of  $\Delta T$ , all the particles at a distance ( $v_{ix}\Delta t$ ) will collide the hole and the wall, the particle colliding along the hole will escape out reducing the pressure in the box.  
 Let area of the wall, number of particles colliding in time  $\Delta t$  on a wall of cube

$$\Delta t = \frac{1}{2} \rho n_i (v_{ix} \Delta t) A$$

$\frac{1}{2}$  is the factor because all the particles along  $x$ -direction are behaving randomly. Hence, half of these are colliding against the walls on either side.

Inside the box,

$$v_{ix}^2 + v_{iy}^2 + v_{iz}^2 = v_{rms}^2 \quad (\text{N}_2 \text{ molecule})$$

$$v_{ix}^2 = \frac{v_{rms}^2}{3} \quad \dots(i) \quad (\because v_{ix} = v_{iy} = v_{iz})$$

$$\text{K.E. of gas molecule} = \frac{3}{2} k_B T$$

$$\text{or } \frac{1}{2} m v_{rms}^2 = \frac{3}{2} k_B T$$

[ $v_{rms}$  = Root mean square velocity,  
 $k_B$  = Boltzmann constant,  $T$  = temperature]

$$v_{rms}^2 = \frac{3k_B T}{m}$$

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

From (i) equation,

$$\text{Now, } v_{ix}^2 = \frac{v_{rms}^2}{3} = \frac{1}{3} \times \frac{3k_B T}{m}$$

$$\text{or } v_{ix}^2 = \frac{k_B T}{m} \quad \text{or } v_{ix} = \sqrt{\frac{k_B T}{m}}$$

$\therefore$  Number of  $\text{N}_2$  gas molecule striking to hole in  $\Delta t$  time (outward)

$$\Delta t = \frac{1}{2} \rho n_1 \sqrt{\frac{k_B T}{m}} \Delta t A$$

The number of air molecule striking to hole in  $\Delta t$

$$\text{inward} = \frac{1}{2} \rho n_2 \sqrt{\frac{k_B T}{m}} (\Delta t) a$$

If particles collide along hole, they move out. Similarly, outer particles colliding along hole will move in.

If  $a$  = area of hole

Then, net number of particle flow in  $\Delta t$  time going outward,

$$\Delta t = \frac{1}{2} \rho n_1 \sqrt{\frac{k_B T}{m}} \Delta t a - \frac{1}{2} \rho n_2 \sqrt{\frac{k_B T}{m}} \Delta t a$$

$$= \frac{1}{2} (\rho n_1 - \rho n_2) \sqrt{\frac{k_B T}{m}} \Delta t a$$

[Temperature inside and outside the box are equal]

$$pV = \mu RT \Rightarrow \mu = \frac{pV}{RT}$$

$$\rho n_1 = \frac{N \text{ (total no. of molecule in box)}}{\text{Volume of box}} = \left( \frac{\mu N_A}{V} \right)$$

$$\rho n_1 = \frac{P_1 N_A}{RT} \quad (\text{per unit volume})$$

$$\text{and } \rho n_2 = \frac{P_2' N_A}{RT} \quad (\text{per unit volume})$$

Now, number of molecules gone out

$$= \rho n_1 V - \rho n_2 V$$

$$= \frac{1}{2}(\rho n_1 - \rho n_2) \sqrt{\frac{k_B T}{m}} \tau a$$

$$[\therefore \rho n_1 - \rho n_2 = \frac{P_1 N_A}{RT} - \frac{P_2 N_A}{RT}]$$

$$= \frac{N_A}{RT} (P_1 - P_2)$$

$$= \frac{1}{2} (P_1 - P_2) \frac{N_A}{RT} \sqrt{\frac{k_B T}{m}} \tau a$$

Equating equation (i) & (ii),

$$\frac{1}{2} (\rho_1 - \rho_2) \frac{N_A}{RT} \sqrt{\frac{k_B T}{m}} \tau a = \frac{N_A V}{RT} (\rho_1 - \rho_2')$$

$$\text{So, } \tau = 2 \left( \frac{P_1 - P_2'}{P_1 - P_2} \right) \frac{V}{a} \sqrt{\frac{m}{k_B T}}$$

Putting the values from the given values,

$$\tau = 2 \left( \frac{1.5 - 1.4}{1.5 - 1.0} \right) \frac{1 \times 1.00}{0.01 \times 10^{-6}} \sqrt{\frac{46.7 \times 10^{-27}}{1.38 \times 10^{-23} \times 300}}$$

$$= 2 \left( \frac{0.1}{0.5} \right) \frac{1}{10^{-8}} \sqrt{\frac{4.7}{1.38 \times 3}} \times 10^{-6}$$

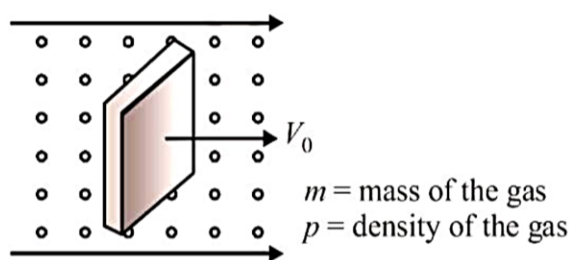
$$= 2 \left( \frac{1}{5} \right) 1 \times 10^8 \times 10^{-2} \times \sqrt{\frac{46.7}{4.14}}$$

$$= \frac{2}{5} \times 10^5 \sqrt{\frac{45.7}{4.14}} = \frac{2}{5} \times 10^5 \sqrt{11.28}$$

$$= \frac{2}{5} \times 3.358 \times 10^5 = \frac{6.717}{5} \times 10^5$$

$$= 1.34 \times 10^5 \text{ seconds}$$

5. Let us consider the drawn diagram



Let  $\rho_m =$  number of molecules per unit volume,  
 $\rho_m$  is also a molecular density per unit volume.

$v_{\text{rms}}$  = rms speed of the gas molecule

When block is moving with speed  $v_0$  in gas the molecules of gas strike to front face in opposite direction and on back face in same direction as  $v \gg v_0$  (box) so relative speed of molecules w.r.t. front face of block =  $(v + v_0)$

and relative speed on back face =  $(v - v_0)$

Coming head on change in momentum transferred to block a molecule per collision

$$= 2m(v + v_0)$$

Change in momentum by a molecule on back side =  $2m(v - v_0)$

where  $m =$  mass of molecule.

Number of collision in time

$$\Delta t = \frac{1}{2} (v + v_0) \rho_m \Delta t A$$

where,  $A =$  Area of cross-section of block and factor of  $1/2$  appears due to particles moving towards block.

$\therefore$  Momentum transferred in time

$$\Delta t = m(v + v_0)^2 \rho_m A \Delta t \quad \text{from front surface.}$$

Similarly, momentum transferred in time

$$\Delta t = m(v - v_0)^2 \rho_m A \Delta t \quad \text{(from back surface)}$$

$\therefore$  Net dragging force

$$= m \rho_m A [(v + v_0)^2 - (v - v_0)^2] \quad \text{(from front)}$$

$$= m \rho_m A (4v v_0) = (4m \rho_m A v) v_0$$

$$= (4 \rho_m A v) v_0 m$$

... (i)

So magnitude of dragging force due to gas molecule =  $(4m v v_0 \rho_m)$

KE of gas molecule :

If  $v =$  velocity along  $x$ -axis

Then, we can write

$$\text{KE} = \frac{1}{2} m v^2 = \frac{1}{2} k_B T$$

$$v = \sqrt{\frac{k_B T}{m}}$$

[where  $k_B =$  Boltzmann constant, KE = Kinetic energy,  $T =$  Temperature]

$\therefore$  From Eq. (i),

Dragging force becomes ( $F_D$ )

$$= (4m \rho_m A v) v_0 = 4m \rho_m A \sqrt{\frac{k_B T}{m}} v_0$$

$$\text{So, } F_D = 4m A \rho_m v_0 \sqrt{\frac{k_B T}{m}}$$