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In this Chapter...

Elasticity

The property of matter by virtue of which it regains its original shape and size, when the deforming forces have been removed is called elasticity.

Plasticity

The property of a body by virtue of which it does not regain its original shape and size even after the removal of deforming force, is called plasticity.

Stress

When a deforming force is applied on a body, it changes the configuration of the body by changing the normal positions of the molecules or atoms of the body. As a result, an internal restoring force comes into play which tends to bring the body back to its initial configuration.

The internal restoring force acting per unit area of a deformed body is called **stress**.

$$\text{i.e. Stress} = \frac{\textbf{Restoring force (F)}}{\textbf{Area of cross-section (A)}}$$

If there is no permanent change in the configuration of the body, i.e. in the absence of plastic behaviour of the body, the restoring force is equal and opposite to the external deforming force applied.

Thus, quantitatively, stress can be given as

$$\text{Stress, } \mathbf{S} = \frac{\textbf{External deforming force}}{\textbf{Area of cross-section}}$$

Its SI unit is N/m^2 or pascal (**Pa**) and in CGS system unit is dyne/$\mathbf{cm}^2$. The dimensional formula of stress is $[\mathbf{ML^{-1}T^{-2}}]$

On the basis of applied forces on the body, the stress can be classified as

1. Normal Stress or Longitudinal Stress

It is defined as the restoring force per unit area, acts perpendicular to the surface of the body. It is of two types which are given below

(i) Tensile Stress

When two equal and opposite forces are applied at the ends of a circular rod to increase its length, then the restoring force per unit area of cross-section is known as tensile stress.

$$\text{Tensile stress} = \frac{\mathbf{F}}{\mathbf{A}}$$

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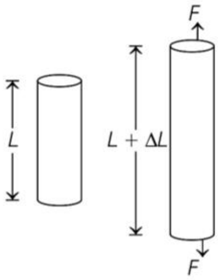
In case of tensile stress, there is an increase in length of the body. Consider a rod of length L, if two equal forces F are applied in the direction as shown in figure, then the final length of the rod becomes $\mathbf{L + \Delta L}$.

Thus, increment in the length of the rod is $\mathbf{\Delta L}$.



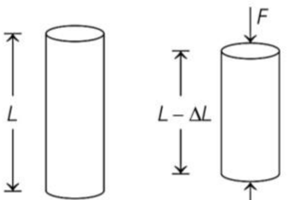
Tensile stress on a circular rod

(ii) Compressive Stress

When two equal and opposite forces are applied at the ends of a rod to decrease its length or compress it, then the restoring force per unit area of cross-section of the rod is known as compressive stress.

$$\text{Compressive stress} = \frac{\mathbf{F}}{\mathbf{A}}$$

In case of compressive stress, there is a decrease in the length of a body. If a rod of length L, the two equal forces $\mathbf{F}$ are applied in the direction as shown in figure, then the final the length of the rod becomes $\mathbf{L - \Delta L}$. Thus, decrease in length of the rod is $\mathbf{\Delta L}$.



Compressive stress on a circular rod

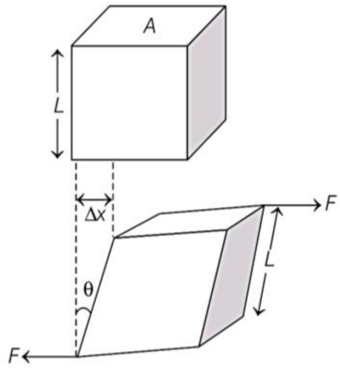
Under tensile stress or compressive stress, the net force acting on an object is zero but the object is deformed.

2. Tangential or Shearing Stress

When a deforming force acts tangentially to the surface of a body, it produces a change in the shape of the body without any change in volume. This tangential force applied per unit area of cross-section is known as tangential stress.

$$\text{Tangential stress} = \frac{\mathbf{F}}{\mathbf{A}}$$

In case of tangential stress, the deforming force F is applied on top surface of the cubical body in tangential direction due to which the upper face is deformed by an angle θ from its original position is shown in figure.



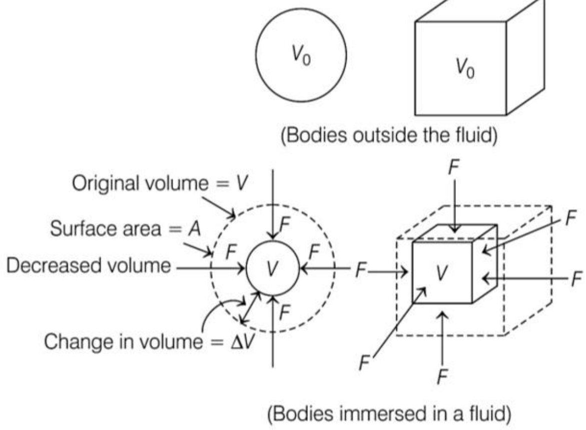
Deforming force on the surface of a body

3. Hydraulic or Bulk Stress

If a body is subjected to a uniform force from all sides, then the corresponding stress is called hydraulic stress or bulk stress. There is a change in volume of the body but not change in geometrical shape.

$$\text{Bulk stress} = \frac{\mathbf{F}}{\mathbf{A}}$$

In case of hydraulic stress, the force F is applied perpendicular to every point on the surface of body due to which the change in volume $\mathbf{\Delta V}$ of a body occurs.



Hydraulic stress on different surfaces

The hydraulic stress is also known as **volumetric stress**.

Strain

When a deforming force acts on a body, the body undergoes a change in its shape and size. The ratio of the change in configuration of the body to the original configuration is called strain.

$$\text{Strain} = \frac{\textbf{Change in configuration}}{\textbf{Original configuration}}$$

Strain is the ratio of two like quantities, so it has no unit and dimension.

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The strain can be three types which are classifed as given below

1. Longitudinal Strain

It is defined as the change in length per unit original length, when the body is deformed by external forces.

$$\text{Longitudinal strain} = \frac{\textbf{Change in length}}{\textbf{Original length}} = \frac{\Delta L}{L}$$

2. Volumetric Strain

It is defined as the change in volume per unit original volume, when the body is deformed by external forces.

$$\text{Volumetric strain} = \frac{\textbf{Change in volume}}{\textbf{Original volume}} = \frac{\Delta V}{V}$$

3. Shear Strain

If the deforming forces produce a change in the shape of the body, then the strain is called shear strain.

$$\text{Shear strain, } \theta = \tan\theta = \frac{\Delta L}{L}$$

Hooke's Law

It states that, "Within elastic limit, the stress developed is directly proportional to the strain produced in a body."

i.e. $\text{Stress} \propto \text{Strain}$

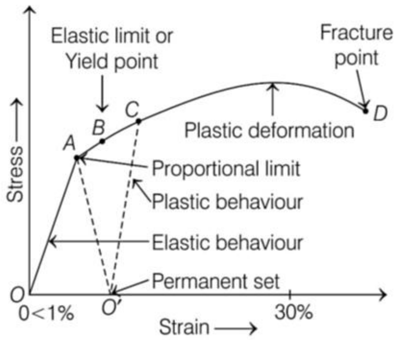
$\Rightarrow$ $\text{Stress} = E \times \text{Strain}$

or $$E = \frac{\text{Stress}}{\text{Strain}}$$

where E is a constant and is known as **modulus of elasticity** of the material of the body.

Stress-Strain Curve

When a wire is stretched by a load, then a typical graph is obtained as shown below



A typical stress-strain curve

(i) In the stress *versus* strain graph, the stress is found to be proportional to strain (% elongation) up to point A. Thus, Hooke's law is fully obeyed in this region, the point A is known as point of **proportional limit**.

(ii) When stress is increased beyond A, then for small stress, there is a large strain in the wire upto point B.

(iii) When the load is gradually removed between points O to B, the wire return to its original length. The wire regains its original dimension only when load applied is less than or equal to a certain limit. This limit is called **elastic limit**. The point B on stress-strain curve is known as **elastic limit** or **yield point.**

The material of the wire in the region OB shows the elastic behaviour, hence known as elastic region.

(iv) If the stress or load increases beyond point B, the strain further increases. This increase in strain represented by BC part of the curve. Now, if the load is removed, the wire does not regain its original length.

In other words, there is permanent strain and this permanent strain in the wire is known as **permanent set**.

(v) Now, as the stress beyond C is increased, there is large strain in the wire. The wire breaks at point D which is also known as **fracture point**.

The material of the wire from point C to point D shows the **plastic behaviour** or **plastic deformation**. The stress needed to cause the actual fracture of the material is known as **breaking stress** or **ultimate tensile strength**.

On the basis of elastic and plastic properties, materials can be classified in two ways which are as given below

(i) **Ductile Materials** The materials which have large plastic range of extension are called ductile materials. Such materials undergo an irreversible increase in length before snapping, so they can be drawn into thin wires, e.g. copper, silver, iron, aluminium, etc.

(ii) **Brittle Materials** The materials which have very small range of plastic extension are called brittle materials. Such materials break as soon as the stress is increased beyond the elastic limit, e.g. cast iron, glass, ceramics, etc.

Malleability

When a solid is compressed, a stage is reached beyond which it cannot regain its original shape after the deforming force is removed.

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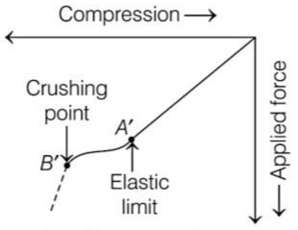
This is the elastic limit point $\mathbf{A'}$ for compression. The solid then behaves like a plastic body.



Load compression curve for a metal

The yield point $\mathbf{B'}$ obtained under compression is called **crushing point**. After this stage, metals are said to be malleable. i.e. They can be hammered or rolled into thin sheets. e.g. gold, silver, lead, etc.

Elastomers

The materials which can be elastically stretched to large values of strain are called elastomers.

Elastic Modulus

Young's Modulus of Elasticity

Within the elastic limit, the ratio of longitudinal stress to the longitudinal strain is called Young's modulus of elasticity.

i.e. Young's modulus, $Y = \dfrac{\textbf{Longitudinal stress}}{\textbf{Longitudinal strain}}$

$$\mathbf{Y} = \frac{\textbf{Tensile (or compressive) stress } (\sigma)}{\textbf{Longitudinal strain } (\varepsilon)}$$

Young's modulus for a wire of length L is given by

$$\mathbf{Y} = \frac{\mathbf{FL}}{\mathbf{A\Delta L}}$$

If $\quad \mathbf{L = 1}\text{ m}, \mathbf{A = 1}\text{ m}^2$

and $\quad \mathbf{\Delta L = 1}\text{ m, then } \mathbf{Y = F}$

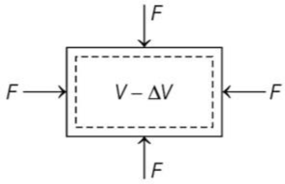
Thus, Young's modulus of elasticity is equal to the force required to extend a wire of unit length and unit area of cross-section by a unit amount.

Bulk Modulus of Elasticity

Within the elastic limit, the ratio of normal stress to the volumetric strain is called bulk modulus of elasticity. In other words, the ratio of hydraulic stress to the hydraulic strain is called bulk modulus.

Consider a body of volume V and surface area A.

Suppose a force F acts uniformly over the whole surface of the body and it decreases the volume by $\mathbf{\Delta V}$ as shown in figure.



Bulk modulus of elasticity

The Bulk modulus of elasticity is given by

$$\mathbf{B} = \frac{\textbf{Normal stress}}{\textbf{Volumetric strain}} = \frac{\mathbf{F/A}}{\mathbf{\Delta V/V}}$$

$\therefore$ Bulk modulus, $\mathbf{B} = \dfrac{\mathbf{-F}}{\mathbf{A}}\dfrac{\mathbf{V}}{\mathbf{\Delta V}}$

$$\Rightarrow \qquad \mathbf{B} = -\frac{\mathbf{pV}}{\mathbf{\Delta V}}$$

where, $\mathbf{p} = \dfrac{\mathbf{F}}{\mathbf{A}}$ is the normal pressure.

Compressibility

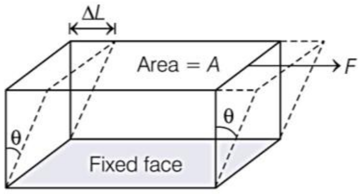
The reciprocal of the Bulk modulus of a material is called its compressibility.

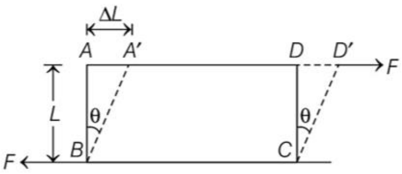
Compressibility, $\mathbf{K} = \dfrac{1}{\mathbf{B}} = \dfrac{\mathbf{-\Delta V}}{\mathbf{pV}}$

SI unit of compressibility $= \mathbf{N^{-1}m^2}$

and CGS unit of compressibility $= \mathbf{dyne^{-1}cm^2}$.

The dimensional formula of compressibility is $[\mathbf{M^{-1}LT^2}]$.





According to diagram, by displacing its upper face through distance $\mathbf{AA' = \Delta L}$

Let $\quad \mathbf{AB = DC = L}$ and $\angle \mathbf{ABA' = \theta}$,

$$\eta = \frac{\mathbf{F/A}}{\theta} = \frac{\mathbf{F}}{\mathbf{A\theta}}$$

$$\text{Shear strain, } \theta \approx \tan\theta = \frac{\mathbf{AA'}}{\mathbf{AB}} = \frac{\mathbf{\Delta L}}{\mathbf{L}}$$

$$\text{Shear modulus, } \eta = \frac{\mathbf{F}}{\mathbf{A}} \cdot \frac{\mathbf{L}}{\mathbf{\Delta L}}$$

Factors Affecting Elasticity of Material

Factors affect the elasticity of a material which are as given below

(i) **Hammering and Rolling** In both of these processes, the crystal grains are broken into small units and the elasticity of the material increases.

(ii) **Annealing** This process results in the formation of larger crystal grains and elasticity of the material decreases.

(iii) **Presence of Impurities** Depending on the nature of impurity, the elasticity of material can be increased or decreased.

(iv) **Temperature** Elasticity of most of the materials decreases with increase in the temperature but elasticity of invor steel (alloy) does not change with the change in temperature.

Elastic After Effect

When the deforming force is removed from the elastic bodies, the bodies tend to return to their respective original state. It has been found that, some bodies return to their original state immediately, others take appreciably long time to do so. The delay in regaining the original position is known as elastic after effect.

Elastic Fatigue

The loss in the elastic strength of a material caused due to repeated alternating strains to which the material is subjected, is called elastic fatigue.

Applications of Elastic Behaviour of Solids

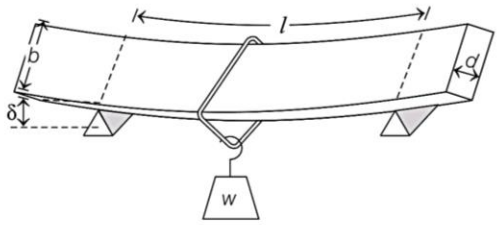
Some applications of elastic behaviour of solids which are as given below

(i) Any metallic part of a machinery is never subjected to a stress beyond the elastic limit of the material.

(ii) The thickness of metallic ropes used in cranes to lift and move heavy weights is decided on the basis of the elastic limit of the rope and the factor of safety.

(iii) In designing a bridge or beam that has to be designed such that it can withstand the load of the following traffic, the force of winds and its own weight.

When the beam is loaded in the middle with a load w, then it gets depressed by an amount δ given by



$$\text{Depression, } \delta = \frac{wl^3}{4\,Ybd^3}$$

where, Y = Young's modulus of elasticity.

(iv) Maximum height of a mountain on earth (~10 km) can be estimated from the elastic behaviour of earth.

Solved Examples

Example 1. Consider a solid cube which is subjected to a pressure of 6×10^5 N/m^2. Due to this pressure, each side of the cube is shortened by 2%. Find out the volumetric strain of the cube.

Sol. Let L be the initial length of the each side of the cube.

Volume, $V = L\times L\times L = L^3$

$= \text{Initial volume } (V_i \text{ say})$

If the each side of the cube is shortened by 2%, then final length of the cube $= L - 2\%$ of L

$$= \left(L - \frac{2L}{100}\right) = L\left(1 - \frac{2}{100}\right)$$

$\therefore$ Final volume, $V_f = L^3\left(1-\frac{2}{100}\right)^3 = V\left(1-\frac{2}{100}\right)^3$

Change in volume, $\Delta V = V_f - V_i = V\left(1-\frac{2}{100}\right)^3 - V$

$$= V\left[\left(1-\frac{2}{100}\right)^3 - 1\right]$$

$$\frac{\Delta V}{V} = \left(1-\frac{2}{100}\right)^3 - 1 \simeq \left[1 - \frac{2\times3}{100}\right] - 1$$

$[\because (1-x)^n \simeq 1 - nx \text{ for } x << 1]$

$\therefore$ Volumetric strain $= \frac{\Delta V}{V} = 1 - 0.06 - 1 = 0.06$

(take positive sign)

Example 2. If a wire of length 4 m and cross-sectional area of $2\ m^2$ is stretched by a force of 3 kN, then determine the change in length due to this force. (Take, Young's modulus of material of wire $= 110\times10^9$ N/m^2)

Sol. Given, area of cross-section, $A = 2\ m^2$

Force, $F = 3\text{ kN} = 3\times10^3$ N

Length, $L = 4$ m

Young's modulus, $Y = 110\times10^9$ N / m^2

Applying $Y = \frac{FL}{A\Delta L}$

$\Rightarrow \Delta L = \frac{FL}{AY} = \frac{3\times10^3\times4}{2\times110\times10^9}$

$= 0.0545\times10^{-6}$ m

$\Delta L = 54.5\times10^{-3}$ mm

Example 3. In a physics department, a Foucault pendulum consists of a 130 kg steel ball which swings at the end of a 8.0 m long steel cable having the diameter of 3.0 mm. If the ball was first hung from the cable, then determine how much did the cable stretch. (Take, $Y = 20\times10^8$ N/m^2)

Sol. Given, diameter, $D = 3.0\text{ mm} = 3.0\times10^{-3}$ m

Length, $L = 8.0$ m and mass, $m = 130$ kg

Radius, $r = \frac{D}{2} = \frac{3.0\times10^{-3}}{2} = 1.5\times10^{-3}$ m

The area of cross-section of the cable

$$A = \pi r^2 = \pi\times(1.5\times10^{-3})^2 = 7.065\times10^{-6}\ m^2$$

Thus, $F = w = mg = 130\times9.8$

$F = 1274$ N

We know that, $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L} \Rightarrow \Delta L = \frac{LF}{AY}$

Change in length, $\Delta L = \frac{8.0\times1274}{7.065\times10^{-6}\times20\times10^8}$

$= 0.72\text{ m} = 720$ mm

Example 4. The ball of 200 g is attached to the end of a string of an elastic material (say rubber) and having length and cross-sectional area of 51 cm and 22 mm^2, respectively. Find the Young's modulus of this material if string is whirled round, horizontally at a uniform speed of 50 rpm in a circle of diameter 104 cm.

Sol. Given, mass of the ball, $M = 200\text{ g} = 0.2$ kg

Area of cross-section, $A = 22\text{ mm}^2 = 22\times10^{-6}\ m^2$

Radius of the circle, $r = \frac{D}{2} = \frac{104}{2} = 52\text{ cm} = 0.52$ m

Length of the string, $l = 51\text{ cm} = 0.51$ m

Revolution per second, $N = 50\times60\text{ rps} = 3000$ rps

Certain petal force, $F = mr\omega^2 = 0.2\times0.52\times(2\pi\times N)^2$

$F = 36.95\times10^6$ N

Change in length, Δl

= Radius of the circle – Length of the string

$= 0.52 - 0.51$

$\Delta l = 0.01$ m

Young's modulus of the material,

$$Y = \frac{F}{A}\frac{l}{\Delta l} = \frac{36.95\times10^6}{22\times10^{-6}}\times\frac{0.51}{0.01} = 85.67\times10^{12}\ Nm^{-2}$$

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Example 5. What will be the decrease in volume of $100\ cm^3$ of water under pressure of 100 atm, if the compressibility of water is 4×10^{-5} per unit atmospheric pressure?

Sol. Bulk modulus, $B = \frac{1}{\text{Compressibility}} = \frac{1}{K}$

$$= \frac{1}{4\times10^{-5}}$$

$$= 0.25\times10^5 \text{atm}$$

$$= 0.25\times10^5\times1.013\times10^5\ N/m^2$$

$$= 2.533\times10^9\ N/m^2$$

Volume, $V = 100\,cm^3 = 10^{-4}\,m^3$

Pressure, $p = 100\ atm = 100\times1.013\times10^5\ N/m^2$

$$= 1.013\times10^7 N/m^{-2}$$

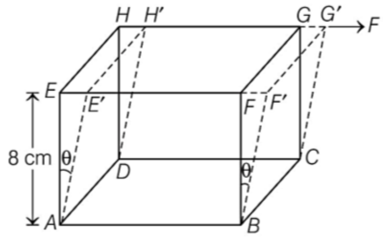
Now, apply $\frac{1}{B} = K = \frac{\Delta V}{pV}$

$\therefore \quad \Delta V = \frac{pV}{B} = \frac{1.013\times10^7\times10^{-4}}{2.533\times10^9}$

$$\Delta V = 0.4\times10^{-6}\ m^3 = 0.4\ cm^3$$

Example 6. Consider an Indian rubber cube having modulus of rigidity of 2×10^7 dyne/cm^2 and of side 8 cm. If one side of the rubber is fixed, while a tangential force equal to the weight of 300 kg is applied to the opposite face, then find out the shearing strain produced and distance through which the strain side moves.

Sol. Given, modulus of rigidity, $\eta = 2\times10^7$ dyne/cm^2



Side of the cube, $l = 8$ cm

Area, $A = l^2 = 64\ cm^2$

Force or load, $F = 300$ kgf

$$= 300\times1000\times981 \text{ dyne}$$

As, $\quad \eta = \frac{F}{A\theta} \quad$ ...(i)

$\Rightarrow \quad \theta = \frac{F}{A\eta}$

$$\theta = \frac{300\times1000\times981}{64\times2\times10^7}$$

$$\simeq 0.23 \text{ rad}$$

As, $\quad \eta = \frac{F}{A}\frac{l}{\Delta l} \quad$ ...(ii)

$\Rightarrow \quad \frac{\Delta l}{l} = \theta \quad$ [from Eqs. (i) and (ii)]

$\Rightarrow \quad \Delta l = l\theta = 8\times0.23$

$$\Delta l = 1.84\ cm$$

Chapter Practice

PART 1

Objective Questions

• Multiple Choice Questions

1. The property of a body, by virtue of which it tends to regain its original size and shape of a body when applied force is removed, is known as

(a) fluidity (b) elasticity
(c) plasticity (d) rigidity

2. The maximum load a wire can withstand without breaking, when its length is reduced to half of its original length, will **[NCERT Exemplar]**

(a) be double (b) be half
(c) be four times (d) remain same

3. A uniform bar of square cross-section is lying along a frictionless horizontal surface. A horizontal force is applied to pull it from one of its ends, then the

(a) bar is under same stress throughout its length
(b) bar is not under any stress because force has been applied only at one end
(c) bar simply moves without any stress in it
(d) stress developed gradually reduces to zero at the end of the bar, where no force is applied

4. A spring is stretched by applying a load to its free end. The strain produced in the spring is **[NCERT]**

(a) volumetric (b) shear
(c) longitudinal and shear (d) longitudinal

5. A wire is stretched to double its length. The strain is

(a) 2 (b) 1 (c) zero (d) 0.5

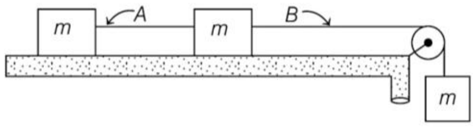
6. A wire of diameter 1 mm breaks under a tension of 1000 N. Another wire of same material as that of the first one, but of diameter 2 mm breaks under a tension of

(a) 500 N (b) 1000 N
(c) 10000 N (d) 4000 N

7. A uniform cube is subjected to volume compression. If each side is decreased by 1%, then bulk strain is

(a) 0.01 (b) 0.06
(c) 0.02 (d) 0.03

8. Three blocks are connected with wires **A** and **B** of same cross-section area **x** and Young's modulus **Y**. All three blocks are of mass **m** each.



With reference to the given situation, which of the following expression are correct?

I. Tension in wire $\mathbf{A} = \frac{2}{3}\,\mathbf{mg}$

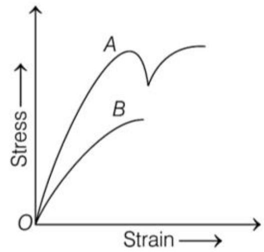
II. Tension in wire $\mathbf{B} = \frac{2}{3}\,\mathbf{mg}$

III. Stress in wire $\mathbf{A} = \frac{2\mathbf{mg}}{3\mathbf{x}}$

IV. Strain in wire $\mathbf{B} = \frac{2\mathbf{mg}}{3\mathbf{xY}}$

(a) Both I and II (b) Both II and III
(c) Both III and IV (d) Both II and IV

9. Stress-strain curves for the materials **A** and **B** are shown below



Then,

(a) **A** is brittle material (b) **B** is ductile material
(c) **B** is brittle material (d) Both (a) and (b)

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10. Which of the following statement (s) is/are correct regarding to elastomers?

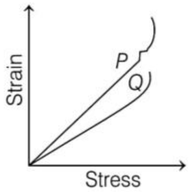
I. They can be elastically stretched to a large value of strain.

II. These materials do not obey Hooke's law.

III. Young's modulus of elastomers is very large.

(a) Both I and II (b) Both II and III
(c) Both I and III (d) I, II and III

11. In plotting stress *versus* strain curves for two materials *P* and *Q*, a student by mistake puts strain on the *Y*-axis and stress on the *X*-axis as shown in the figure. Then, which of the following statement(s) is/are correct?



(a) **P** has more tensile strength than *Q*.
(b) *P* is more ductile than *Q*.
(c) *P* is more brittle than *Q*
(d) The Young's modulus of *P* is more than that of *Q*.

12. On applying a stress of 20×10^8 Nm^{-2}, the length of a perfectly elastic wire is doubled. Its Young's modulus will be

(a) 40×10^8 Nm^{-2} (b) 20×10^8 Nm^{-2}
(c) 10×10^8 Nm^{-2} (d) 5×10^8 Nm^{-2}

13. A wire of length 2 m is made from 10 cm^3 of copper. A force **F** is applied, so that its length increases by 2 mm. Another wire of length 8 m is made from the same volume of copper. If the force **F** is applied to it, its length will increase by

(a) 0.8 cm (b) 1.6 cm (c) 2.4 cm (d) 3.2 cm

14. In steel, the Young's modulus and the strain at the breaking point are 2×10^{11} Nm^{-2} and 0.15, respectively. The stress at the breaking point for steel is

(a) 1.33×10^{11} Nm^{-2} (b) 1.33×10^{12} Nm^{-2}
(c) 7.5×10^{-13} Nm^{-2} (d) 3×10^{10} Nm^{-2}

15. A copper and a steel wire of the same diameter are connected end-to-end. A deforming force *F* is applied to this composite wire which causes a total elongation of 1 cm. The two wires will have **[NCERT]**

(a) the same stress and strain
(b) different stress and strain
(c) the same strain but different stress
(d) the same stress but different strain

16. When a pressure of 100 atm is applied on a spherical ball of rubber, then its volume reduces to 0.01%. The bulk modulus of the material of the rubber (in dyne **cm**$^{-2}$) is

(a) 10×10^{12} (b) 100×10^{12}
(c) 1×10^{12} (d) 20×10^{12}

17. Which of the following statement(s) is/are incorrect?

(a) The bulk modulus for solid is much larger than for liquids.
(b) Gases are least compressible.
(c) For a system in equilibrium, the value of bulk modulus is always positive .
(d) The SI unit of bulk modulus is same as that of pressure.

18. Over bridges are constructed with steel but not with aluminium because steel is

(a) more elastic than aluminium
(b) less elastic than aluminium
(c) more plastic than aluminium
(d) less plastic than aluminium

19. A metal bar is supported at two ends. If metal bar is loaded at centre with a heavy load, the depression in bar at the centre is proportional to (**Y** = Young's modulus of bar)

(a) $\frac{1}{Y^2}$ (b) $\frac{1}{Y}$
(c) **Y** (d) Y^2

20. If the load hanging from middle position of a metal beam is increased to double, then depression in the bar at the centre is

(a) increased to four times
(b) decreased to four times
(c) increased to double
(d) decreased to half

• Assertion-Reasoning MCQs

Direction (Q. Nos. 21-26) *Each of these questions contains two statements Assertion (A) and Reason (R). Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.*

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true, but R is not the correct explanation of A
(c) A is true, but R is false
(d) A is false and R is also false

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21. **Assertion** Spring balance shows incorrect readings after using it for a long time.

Reason Spring in the spring balance loses its elastic strength over the period of time.

22. **Assertion** When a solid sphere is placed in the fluid under high pressure, then it is compressed uniformly on all sides.

Reason The force applied by fluids acts in perpendicular direction at each point of surface.

23. **Assertion** The strain produced by a hydraulic pressure is volumetric in nature.

Reason It is a ratio of change in volume ΔV to the original volume **V**.

24. **Assertion** Young's modulus for a perfectly plastic body is zero.

Reason For a perfectly plastic body, restoring force is zero.

25. **Assertion** Gases have large compressibility.

Reason Compressibility is defined as the fractional change in volume per unit decrease in pressure.

26. **Assertion** Maximum height of a mountain on earth is ~**10** km.

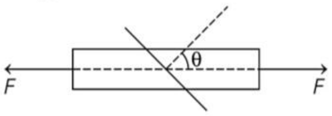
Reason A mountain base is not under uniform compression and provides some shearing stress to rock under which it can flow.

• Case Based MCQs

27. **Direction** *Read the following passage and answer the questions that follows*

Restoring Force due to Stress

When a bar of cross-section **A** is subjected to equal and opposite tensile forces at its ends, then a restoring force equal to the applied force normal to its cross-section comes into existence. This restoring force per unit area of cross-section is known as **tensile stress**. While when the deforming force acts tangentially to the surface, then this tangential force applied per unit area of cross-section is known as **tangential stress**. Consider a plane section of the bar, whose normal makes an angle θ with the axis of the bar.



(i) Which of the following property of the bar does not change due to this force?
(a) Area (b) Volume
(c) Shape (d) Size

(ii) What is the tensile stress on this plane?
(a) $(F/A)\cos^2\theta$ (b) F/A
(c) $(F/A)\tan\theta$ (d) $(F/A)\sec^2\theta$

(iii) What is the shearing stress on this plane?
(a) $\frac{F}{2A}\sin 2\theta$ (b) $\frac{F}{A}\cos 2\theta$
(c) $\frac{F}{2A}\cos^2\theta$ (d) $\frac{F}{4A^2}$

(iv) For what value of θ is the tensile stress maximum?
(a) 0° (b) 90° (c) 45° (d) 30°

(v) For what value of θ is the shearing stress maximum?
(a) 45° (b) 30 (c) 90° (d) 60°

PART 2

Subjective Questions

• Short Answer (SA) Type Questions

1. Calculate the value of stress in a wire of steel having radius of 2 mm of 10 kN of force is applied on it.

2. A steel cable with a radius of 1.5 cm supports a chair lift at a ski area. If the maximum stress is not to exceed 10^8 N/m^2, then what is the maximum load the cable can support? **[NCERT]**

3. A wire of length 2.5 m has a percentage strain of 0.012% under a tensile force. Determine the extension in the wire.

4. If the angle of shear is 30° for a cubical body and the change in length is 250 cm, then what must be the volume of this cubical body?

5. The ratio of stress/strain remains constant for a small deformation. What happens to this ratio, if deformation is made very large?

6. A wire is replaced by another wire of same length and material but of twice diameter.

(i) What will be the effect on the increase in its length under a given load?

(ii) What will be the effect on the maximum load which it can bear?

7. A wire of length **L** and radius **r** is clamped rigidly at one end. When the other end of the wire is pulled by a force **f**, its length increases by **l**. Another wire of the same material of length **2L** and radius **2r**, is pulled by a force 2*f*. Find the increase in length of this wire.

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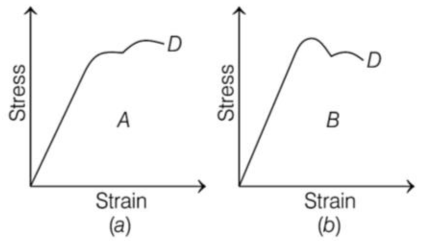
8. Two wires made of same material are subjected to forces in the ratio 1 : 4. Their lengths are in the ratio 2 : 1 and diameters in the ratio 1 : 3. What is the ratio of their extensions?

9. The stress-strain graphs for materials *A* and *B* are shown in Figs. (a) and (b).



The graphs are drawn to the same scale.

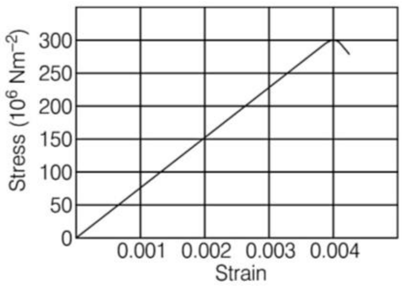
(i) Which of the materials has greater Young's modulus?

(ii) Which of the two is the stronger material? **[NCERT]**

10. A wire elongates by **l mm** when a load **w** is hanged from it. If the wire goes over a pulley and two weights **w** each are hung at the two ends, then what will be the elongation (in **mm**) of the wire?

11. The Young's modulus for steel is much more than that for rubber. For the same longitudinal strain, which one will have greater tensile stress? **[NCERT Exemplar]**

12. Figure shows the strain-stress curve for a given material. What are (i) Young's modulus and (ii) approximate yield strength for this material? **[NCERT]**



13. Calculate the percentage increase in length of a wire of diameter 2.5 mm stretched by a force of 100 kg weight. Young's modulus of elasticity of wire is 12.5×10^{11} dyne/sq cm.

14. A solid sphere of radius **R** made of a material of bulk modulus **B** is surrounded by a liquid in a cylindrical container. A massless piston of area **A** floats on the surface of the liquid. When a mass **M** is placed on the piston to compress the liquid, find fractional change in the radius of the sphere.

15. The Mariana trench is located in the Pacific ocean and at one place, it is nearly 11 km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa.

A steel ball of initial volume 0.32 m^3 is dropped into the ocean and falls to the bottom of trench.

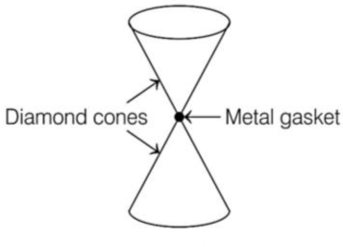
What is the change in the volume of the ball when it reaches to the bottom, if the Bulk modulus of steel is 1.6×10^{11} N/m^2? **[NCERT]**

16. To what depth must a rubber ball be taken in deep sea, so that its volume is decreased by 0.1%? (The Bulk modulus of rubber is 9.8×10^8 N/m^2 and the density of seawater is 10^3 kg/m^3) **[NCERT Exemplar]**

17. The maximum stress that can be applied to the material of a wire used to suspend on elevator is 1.3×10^8 Nm^{-2}. If the mass of the elevator is 900 kg and it moves up with an acceleration of 2.2 ms^{-2}, then what is the minimum diameter of the wire?

18. Two strips of metal are riveted together at their ends by four rivets, each of diameter 6 mm. What is the maximum tension that can be exerted by the riveted strip, if the shearing stress on the rivet is not to exceed 6.9×10^7 Pa? Assume that, each rivet is to carry one-quarter of the load. **[NCERT]**

19. Anvils made of single crystals of diamond, with the shape as shown in figure are used to investigate the behaviour of materials under very high pressure. Flat faces at the narrow end of the anvil have a diameter of 0.5 mm and the wide ends are subjected to a compressional force of 50000 N. What is the pressure at the tip of the anvil? **[NCERT]**



20. After a fall, a 95 kg rock climber finds himself dangling from the end of a rope that had been 15 m long and 9.6 mm in diameter but has stretched by 2.8 cm. For the rope, calculate.

(i) the strain, (ii) the stress and

(iii) the modulus of elasticity.

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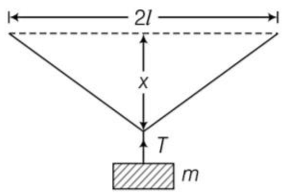
• Long Answer (LA) Type Questions

21. A steel wire of length $2l$ and cross-sectional area A is stretched within elastic limit as shown in figure. Calculate the strain and stress in the wire.



22. A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area 4.0×10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper? **[NCERT]**

23. A uniform heavy rod of weight w, cross-sectional area A and length l is hanging from a fixed support. Young's modulus of the material of the rod is Y. Neglecting the lateral contraction, find the elongation produced in the rod.

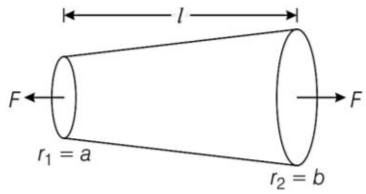
24. A mild steel wire of length 1 m and cross-sectional area 0.5×10^{-2} cm^2 is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100 g is suspended from the mid-point of the wire. Calculate the depression at the mid-point. (Take, Young's modulus for steel, $Y = 2 \times 10^{11}$ Pa) **[NCERT]**

25. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2 m long. Those at each end are of copper and the middle one is of iron. Determine the ratio of their diameters, if each wire have the same tension. Young's modulus of elasticity for copper and steel are 110×10^9 N/m^2 and 190×10^9 N/m^2, respectively. **[NCERT]**

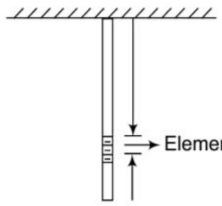
26. A 14.5 kg mass, fastened to one end of a steel wire of unstretched length 1 m is whirled in a vertical circle with an angular frequency of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire, when the mass is at the lowest point of its path. **[NCERT]**

27. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50000 kg. The inner and outer radii of each column are 30 cm and 60 cm, respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. (Take, Young's modulus, $Y = 2.0 \times 10^{11}$ Pa) **[NCERT]**

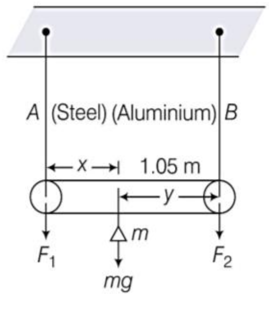
28. A slightly tappering wire of length l and end radii a and b on both sides is subjected to the stretching forces F on both sides as shown in figure. If Y is the Young's modulus of the wire, then calculate the extension produced in the wire.



29. Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in figure below. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Young's modulus of steel is 2.0×10^{11} Pa. Compute the elongations of steel and brass wires. (Take, 1 Pa = 1 N/m^2) **[NCERT]**



30. A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire **A**) and aluminium (wire **B**) of equal lengths as shown in figure below. The cross-sectional areas of wires **A** and **B** are 1 mm^2 and 2 mm^2, respectively. Young's modulus of elasticity for steel and aluminium are 2×10^{11} and 7×10^{10} N/m^2, respectively.



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At what point along the rod should a mass **m** be suspended in order to produce (i) equal stresses and (ii) equal strains in both steel and aluminium wires. **[NCERT]**

31. A rubber string **10 m** long is suspended from a rigid support at its one end. Calculate the extension in the string due to its own weight. The density of rubber is $\mathbf{1.5 \times 10^3}$ kg/m^3 and Young's modulus for the rubber is $\mathbf{5 \times 10^6}$ $\mathbf{N/m^2}$.

The breaking stress for a metal is $\mathbf{7.8 \times 10^9}$ $\mathbf{N/m^2}$. Calculate the maximum length of the wire made of this metal which may be suspended without breaking. The density of metal $= \mathbf{7.8 \times 10^3}$ $\mathbf{kg/m^3}$.

32. What is the density of water at a depth, where pressure is **80.0 atm**? (Take, density at the surface is $\mathbf{1.03 \times 10^3}$ $\mathbf{kg/m^3}$ and compressibility of water is $\mathbf{45.8 \times 10^{-11}}$ $\mathbf{Pa^{-1}}$) **[NCERT]**

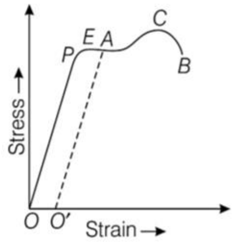
33. Compute the Bulk modulus of water from the following data; initial volume = 100.0 L, pressure increase = 100.0 atm (1 atm = 1.013 $\times 10^5$ Pa), final volume = 100.5 L. Compare the Bulk modulus of water with that of air (at constant temperature). Explain in simple terms, why the ratio is so large. **[NCERT]**

• Case Based Questions

34. Direction *Read the following passage and answer the questions that follows*

Stress-Strain Curve

The stress-strain graph for a metal wire is shown in figure. Upto the point E, the wire returns to its original state O along the curve EPO, when it is gradually unloaded. Point B corresponds to the fracture of the wire.



(i) Upto which point on the curve is Hooke's law obeyed? This point is sometimes called proportionality limit.

(ii) Which point on the curve corresponds to elastic limit and yield point of the wire?

(iii) Indicate the elastic and plastic regions of the stress-strain graph.

(iv) Describe what happens when the wire is loaded upto a stress corresponding to the point A on the graph and then unloaded gradually. In particular, explain the dotted curve.

(v) How the graph from C to B is different from the rest? Upto what stress can the wire be subjected without causing fracture?

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Chapter Test

Multiple Choice Questions

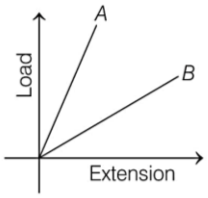
1. If a force is applied to a plastic substance, then they have

I. no gross tendency to regain their original shape.
II. permanently deformed.
III. tendency to regain their original shape.
IV. not permanently deformed.

Which of the following statement(s) is/are correct?

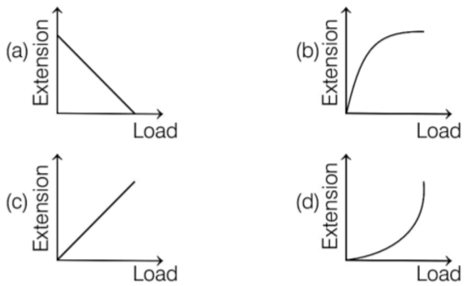
(a) Only I (b) Both I and II
(c) Only III (d) Both III and IV

2. In the given figure, if the dimension of the wire are the same and materials are different, Young's modulus is more for



(a) A (b) B
(c) Both (d) None of these

3. Within the limit of elasticity, which of the following graph obey Hooke's law?



4. A cube of aluminium of side 0.1 m is subjected to a shearing force of 100 N. The top face of the cube is displaced through 0.02 cm with respect to the bottom face. The shearing strain would be

(a) 0.02 (b) 0.1 (c) 0.005 (d) 0.002

5. One end of a uniform wire of length L and of weight w is attached rigidly to a point in the roof and a weight w_1 is suspended from its lower end. If S is the area of cross-section of the wire, the stress in the wire at a height $3L/4$ from its lower end is

(a) $\frac{w_1}{S}$ (b) $\frac{w_1 + (w/4)}{S}$
(c) $\frac{w_1 + (3w/4)}{S}$ (d) $\frac{w_1 + w}{S}$

Short Answer Type Questions

6. Find the change in volume which 1cc of water at the surface will undergo, when it is taken to the bottom of the lake 100m deep. (Take, volume elasticity is 22000 atm)
(**Ans.** 4.5×10^{-5}cc)

7. The star Sirius has a mass of 7×10^{30} kg, its distance from the earth is 8×10^{16} m and the mass of the earth is 6×10^{24} kg. Calculate the cross-section of a steel cable that can withstand the gravitational pull between the Sirius and the earth. (Take, $G = 6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$ and breaking stress $= 10^{10} \text{Nm}^{-2}$) (**Ans.** 44 m^2)

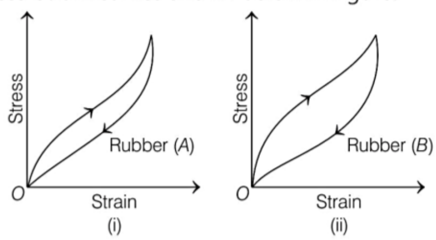
8. A metal bar of length L and area of cross-section A, is rigidly clamped between two walls. The Young's modulus of its material is Y and the coefficient of linear expansion is α. The bar is heated, so that its temperature is increased from 0 to θ°C. Find the force exerted at the ends of the bar.

9. Assume that if the shear stress in steel exceeds about 4×10^8 N / m^2, the steel reptures. Determine the shearing force necessary to (i) shear a steel bolt 1.00 cm in diameter and (ii) punch a 1 cm diameter hole in a steel plate 0.5 cm thick.
(**Ans.** (i) 3.14×10^4 N and (ii) 6.28×10^4 N)

Long Answer Type Questions

10. Four identical cylindrical columns of steel support a big structure of mass 50000 kg. The inner and outer radii of each column are 30 cm and 40 cm, respectively. Assume the load distribution to be uniform, calculate the compressional strain of each column. The Young's modulus of steel is 2.0×10^{11}Pa. (**Ans.** 2.8×10^{-6})

11. Two different types of rubber are found to have the stress-strain curves shown below in figure.



(i) In which significant ways, do these curves differ from the stress-strain curve of a metal wire?
(ii) A heavy machine is to be installed in a factory. To absorb vibrations of the machine, a block of rubber is placed between the machinery and the floor. Which of the two rubbers A and B would you prefer to use for this purpose? Why?
(iii) Which of the two rubber materials would you choose for a car tyre?

Answers

Multiple Choice Questions

1. (b) 2. (a) 3. (c) 4. (d) 5. (c)

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EXPLANATIONS

PART 1

(*b*) The property of a body, by virtue of which it tends to regain its original size and shape when the applied force is removed, is known as elasticity and the deformation caused is known as elastic deformation.

(*d*) We know that,

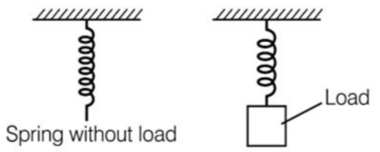
$$\text{Maximum stress} = \frac{\textbf{Maximum force}}{\textbf{Area of cross - section}}$$

Area of cross-section remains same even, if the length of the wire changes.

Hence, maximum force will be same when length changes.

(*d*) When a horizontal force is applied on uniform bar to pull it, then an acceleration is produced in the each cross-section of rod. Hence, each section of rod experiences a tension which is zero at other end. Therefore, stress in the rod developed gradually reduces to zero at the end of the bar, where no force is applied.

(*c*) Consider the diagram, where a spring is stretched by applying a load to its free end. Clearly, the length and shape of the spring changes.



The change in length corresponds to longitudinal strain and change in shape corresponds to shearing strain.

(*b*) Initial length = **L** and final length = **2L**

$$\text{Strain} = \frac{\textbf{Initial length}}{\textbf{Original length}} = \frac{2L - L}{L} = 1$$

(*d*) As wire are of some material, so stress produced is same.

$\because \quad F \propto A$ or $F \propto d^2$

Given, $F_1 = 1 \times 10^{-3}$ N, $d_2 = 2$ mm $= 2 \times 10^{-3}$

$d_1 = 1$ mm $= 1 \times 10^{-3}$ m

$$\Rightarrow \quad \frac{F_2}{F_1} = \left(\frac{2 \times 10^{-3}}{1 \times 10^{-3}}\right)^2 = 4$$

$$\therefore \quad F_2 = F_1 \times 4 = 4000 \text{ N}$$

(*d*) Volume of cube, $V = L^3$

$\therefore$ Percentage change in $V = 3 \times$ (Percentage change in L)

$$= 3(1\%) = 3\%$$

$$\therefore \quad \Delta V = 3\% \text{ of } V$$

$$\Rightarrow \quad \text{Volumetric strain} = \frac{\Delta V}{V} = \frac{3}{100} = 0.03$$

(*d*) Let, cross-section area of wires **A** and **B** = **x**

and Young's modulus of wires = **Y**

$$\text{Tension in wire } B = \frac{m \cdot (m + m)}{m + (m + m)} \cdot g = \frac{2}{3} mg \qquad \text{...(i)}$$

where, **g** is gravitational acceleration.

[$\because$ Tension = Force = Mass $\times$ Acceleration]

$$\because \quad \text{Stress} = \frac{\text{Force}}{\text{Area}}$$

$$\text{Stress in } B = \frac{\text{Force (tension)}}{\text{Cross-section area of wire } B} = \frac{2mg}{3x} \qquad \text{...(ii)}$$

[using Eq. (i)]

$$\text{Young's modulus, } Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\therefore \text{Strain in wire } B = \frac{\text{Stress}}{Y} = \frac{2mg}{3xY} \qquad \text{[using Eq. (ii)]}$$

$$\text{Tension in wire } A = \frac{m \cdot m}{m + (m + m)} g = \frac{mg}{3}$$

$$\text{Similarly, stress in wire } A = \frac{mg}{3x}$$

So, statements II and IV are correct but I and III are incorrect.

(*c*) **B** is brittle as there is no plastic region. However, **A** is ductile as it has large plastic range of extension.

(*a*) Elastomers are those materials which can be elastically stretched to a large value of strain. Elastic region for them is very large but they do not obey Hooke's law.

Thus, Young's modulus of elastomers are very small.

So, statements I and II are correct but III is incorrect.

(*b*) From given graph, **P** has more strain for same stress as on **Q**, so **P** is more ductile than **Q**.

Thus, the statement given in option (b) is correct, rest are incorrect.

(*b*) Given, stress, $F = 20 \times 10^8 \text{ Nm}^{-2}$

$$\text{Young's modulus} = \frac{\text{Stress}}{\text{Strain}}$$

As the length of wire gets doubled, therefore strain = **1**.

$$\left(\because \textbf{ Strain} = \frac{\textbf{Change in length}}{\textbf{Original length}} = \frac{2L - L}{L} = 1\right)$$

$$\therefore Y = \text{Stress} = 20 \times 10^8 \text{ N m}^{-2}$$

(*d*) Given, change in length of wire, $l_1 = 2$ mm,

Length of wire, $L_1 = 2$ m

and length of another wire, $L_2 = 8$ m

$$\text{Change in length, } l = \frac{FL}{AY} = \frac{FL^2}{VY}$$

where, **Y** is Young's modulus.

$$\therefore \quad l \propto L^2$$

(as **V**, **Y** and **F** are constants)

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$$\frac{l_2}{l_1} = \left[\frac{L_2}{L_1}\right]^2 = \left(\frac{8}{2}\right)^2 = 16$$

$\Rightarrow$ $l_2 = 16 l_1 = 16 \times 2$ mm

$= 32$ mm $= 3.2$ cm

(*d*) Given, Young's modulus, $\mathbf{Y = 2 \times 10^{11}}$ $\mathrm{Nm^{-2}}$,

Strain = 0.15

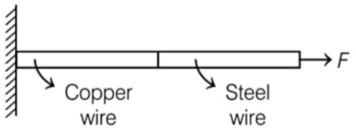
$\because \quad Y = \frac{\text{Stress}}{\text{Strain}}$

$\therefore$ Stress $= \mathbf{Y \times Strain = 2 \times 10^{11} \times 0.15 = 0.3 \times 10^{11}}$

$= 3 \times 10^{10}$ $\mathrm{Nm^{-2}}$

(*d*) Consider the diagram, where a deforming force **F** is applied to the combination.

For steel wire, $Y_{steel} = \frac{\textbf{Stress}}{\textbf{Strain}} = \frac{\mathbf{F / A}}{\textbf{Strain}}$



where, **F** is tension in each wire and **A** is cross-section area of each wires.

As **F** and **A** are same for both the wires, hence stress will be same for both the wires.

$(\text{Strain})_{steel} = \frac{\text{Stress}}{Y_{steel}}, (\text{Strain})_{copper} = \frac{\text{Stress}}{Y_{copper}}$

As, $\quad Y_{steel} \neq Y_{copper}$

Hence, the two wires will have different strain.

(*c*) Given, 1 atm $= 10^5$ $\mathrm{Nm^{-2}}$

$\therefore$ 100 atm $= 10^7$ $\mathrm{Nm^{-2}}$ and $\Delta V = 0.01\%\ V$

$\therefore \quad \frac{\Delta V}{V} = 0.0001$

$$B = \frac{p}{\Delta V / V} = \frac{10^7}{0.0001} = 1 \times 10^{11}\ \mathrm{Nm^{-2}}$$

$= 1 \times 10^{12}$ dyne/cm^2

(*b*) Statement given in option (b) is incorrect and it can be corrected as

Gases are about a million times compressible than solids, so solids are least compressible.

Rest statements are correct.

(*a*) A bridge has to be designed such that it can withstand the load of flowing traffic, the force of winds and its own weight. Since, steel is more elastic than aluminium. So, it can withstand the load of traffic. Thus, over bridges are constructed with steel but not with aluminium.

(*b*) A beam of length **l**, breadth **b** and depth **d** when loaded at the centre by a load **w** depresses by an amount given by

$$\delta = \frac{wl^3}{4bd^3Y}, \text{ i.e. } \delta \propto \frac{1}{Y}$$

(*c*) Since, depression δ in the bar at centre is directly proportional to load.

i.e. $\quad \delta \propto w$

$$\frac{\delta_2}{\delta_1} = \frac{w_2}{w_1} = \frac{2w_1}{w_1} = 2 \quad \therefore \quad \delta_2 = 2\delta_1$$

Hence, depression in the bar at the centre is increased to double.

(*a*) Spring balance shows incorrect reading after using it for a long time as with time the spring in it loses its elastic strength. This phenomenon is knwon as elastic fatigue.

Therefore, both A and R are true and R is the correct explanation of A.

(*a*) If a solid sphere placed in the fluid under high pressure, then it is compressed uniformly on all sides.

The force applied by the fluids acts in perpendicular direction at each point of the surface and the body is said to be under hydraulic compression.

This leads to decrease in its volume without any change in its geometrical shape.

Therefore, both A and R are true and R is the correct explanation of A.

(*b*) The strain produced by a hydraulic pressure is called volumetric strain as pressure creates a normal force on every point and is defined as the ratio of change in volume ΔV to the original volume V.

i.e. $\quad$ Volume strain $= \frac{\Delta V}{V}$

Therefore, both A and R are true but R is not the correct explanation of A.

(*a*) Young's modulus of a material, $Y = \frac{\textbf{Stress}}{\textbf{Strain}}$

$\therefore \quad$ Stress $= \frac{\textbf{Restoring force F}}{\textbf{Area A}}$

As, restoring force is zero for a plastic body.

$\therefore \quad Y = 0$

Therefore, both A and R are true and R is the correct explanation of A.

(*a*) Molecules in gases are very poorly attracted by their neighbouring molecules.

Since, compressibility is defined as the fractional change in volume per unit increase or decrease in pressure.

$$K = 1/B = -(1/\Delta p) \times (\Delta V/V)$$

where, **B** is bulk modulus and Δp change in pressure.

So, in gases, fractional change in volume with per unit increase or decrease in pressure is not very prominent.

Thus, they have large compressibility.

Therefore, both A and R are true and R is the correct explanation of A.

(*a*) As a mountain base is not under uniform compression and this provides some shearing stress to the rocks under which they can flow.

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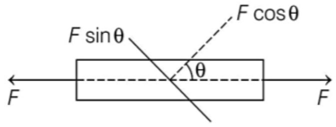
Thus, mathematically, it has been calculated that under the elastic limit, maximum height of a mountain is ~10 km.

Therefore, both A and R are true and R is the correct explanation of A.

(i) (*b*) This force produces a change in shape of the body. Hence, volume remains constant.

(ii) (*a*) The resolved part of **F** along the normal is the tensile force on this plane and the resolved part parallel to the plane is the shearing force on the plane.



$\because$ Area of plane section = $A\sec\theta$

Tensile stress $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{F\cos\theta}{A\sec\theta} = \dfrac{F}{A}\cos^2\theta$ $\left(\because \sec\theta = \dfrac{1}{\cos\theta}\right)$

(iii) (*a*) Shearing stress $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{F\sin\theta}{A\sec\theta}$

$$= \frac{F}{A}\sin\theta\cos\theta = \frac{F}{2A}\sin 2\theta$$

(iv) (*a*) Tensile stress will be maximum, when $\cos^2\theta$ is maximum, i.e. $\cos\theta = 1$ or $\theta = 0°$.

(v) (*a*) Given, shearing stress will be maximum, when $\sin 2\theta$ is maximum, i.e. $\sin 2\theta = 1$ or $2\theta = 90°$ or $\theta = 45°$.

PART 2

Given, force, $F = 10\text{ kN} = 1\times10^4$ N

Radius, $r = 2\text{ mm} = 2\times10^{-3}$ m

Area, $A = \pi r^2 = \pi\times(2\times10^{-3})^2 = 12.56\times10^{-6}\text{ m}^2$

Stress $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{1\times10^4\text{ N}}{12.56\times10^{-6}\text{ m}^2}$

$= 0.0796\times10^{10} = 7.96\times10^8\text{ N/m}^2$

Given, radius of steel cable, $r = 1.5\text{ cm} = 1.5\times10^{-2}$ m

Maximum stress $= 10^8\text{ N/m}^2$

Area of cross-section of steel cable, $A = \pi r^2$

$= 3.14\times(1.5\times10^{-2})^2\text{ m}^2$

$= 3.14\times2.25\times10^{-4}\text{ m}^2$

Maximum stress $= \dfrac{\text{Maximum force}}{\text{Area of cross - section}}$

$\Rightarrow$ Maximum force = Maximum stress × Area of cross-section

$= 10^8\times(3.14\times2.25\times10^{-4})$ N

$= 7.065\times10^4$ N

$= 7.1\times10^4$ N

Given, original length, $L = 2.5$ m

Strain $= \dfrac{\Delta L}{L} = 0.012\% = \dfrac{0.012}{100}$

$\Delta L = \text{Strain}\times L$

or $\Delta L = \text{Extension} = \dfrac{0.012}{100}\times L$

$= \dfrac{0.012\times2.5}{100} = 3\times10^{-4}$ m

$= 0.3$ mm

Given, angle of shear, $\theta = 30°$

and change in length, $\Delta L = 250\text{ cm} = 2.5$ m

$\therefore$ Shear strain, $\tan\theta = \dfrac{\Delta L}{L} \Rightarrow \tan 30° = \dfrac{2.5}{L}$

$L = \dfrac{2.5}{\tan 30°} = \dfrac{2.5}{0.577} = 4.332$ m

Volume, $V = L^3 = 81.309\text{ m}^3$

When the deformation is sufficient enough such that it exceeds the elastic limit, the strain increases more rapidly than stress. Hence, ratio of stress/strain decreases.

Young's modulus of wire,

$$Y = \frac{Mgl}{\pi r^2\Delta l} = \frac{4Mgl}{\pi D^2\Delta l}$$

(i) Elongation, $\Delta l = \dfrac{4Mgl}{\pi D^2 Y}$

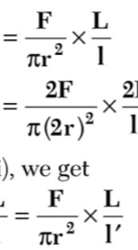
or $\Delta l \propto \dfrac{1}{D^2}$

So, if diameter is increased to two times, then the elongation or increase in length will become one-fourth.

(ii) Also, load, $Mg = \dfrac{\pi D^2 Y\Delta l}{4l} \Rightarrow Mg \propto D^2$

So, when diameter changes by a factor of two times, the wire can bear four times the original load.

The situation is as shown in the figure



Now, Young's modulus, $Y = \dfrac{F}{A}\times\dfrac{L}{l}$

For first wire, $Y = \dfrac{F}{\pi r^2}\times\dfrac{L}{l}$ ...(i)

For second wire, $Y = \dfrac{2F}{\pi(2r)^2}\times\dfrac{2L}{l'} = \dfrac{F}{\pi r^2}\times\dfrac{L}{l'}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{F}{\pi r^2}\times\frac{L}{l} = \frac{F}{\pi r^2}\times\frac{L}{l'}$$

$\therefore \quad l = l'$

($\because$ Both wires are of same material, hence Young's modulus will be same)

According to Hooke's law,

Modulus of elasticity, $E = \frac{F}{\pi r^2} \times \frac{l}{\Delta l}$ or $\Delta l = \frac{Fl}{\pi r^2 E}$

or $\quad \Delta l \propto \frac{Fl}{r^2} \quad$ ($\because$ E is same for two wires)

$$\therefore \quad \frac{\Delta l_1}{\Delta l_2} = \frac{F_1}{F_2} \times \frac{l_1}{l_2} \times \frac{r_2^2}{r_1^2} = \frac{1}{4} \times \frac{2}{1} \times \left(\frac{3}{1}\right)^2 = \frac{9}{2}$$

So, $\quad \Delta l_1 : \Delta l_2 = 9 : 2.$

Hence, the ratio of their extensions is 9 : 2.

(i) In the two graphs, the slope of graph in Fig. (a) is greater than the slope of graph in Fig. (b), so material *A* has greater Young's modulus.

(ii) Material *A* is stronger than material *B* because it can withstand more load without breaking. For material *A*, the break even point (*D*) is higher.

According to Hooke's law,

Modulus of elasticity, $E = \frac{w}{A} \times \frac{L}{l} \Rightarrow l = \frac{wL}{AE}$

where, L = original length of the wire

and $\quad A$ = cross-sectional area of the wire.

$\therefore$ Elongation, $\Delta l = \frac{wL}{E} \quad$ ...(i)

On either side of the wire, tension is w and length is $l/2$.

$$\Delta l = \frac{wL/2}{AE} = \frac{wL}{2AE} = \frac{l}{2} \quad \text{[from Eq. (i)]}$$

$\therefore$ Total elongation in the wire $= \frac{l}{2} + \frac{l}{2} = l$

Young's modulus, $Y = \frac{\text{Stress}}{\text{Longitudinal strain}}$

For same longitudinal strain, $Y \propto$ stress

$$\therefore \quad \frac{Y_{steel}}{Y_{rubber}} = \frac{(\text{stress})_{steel}}{(\text{stress})_{rubber}} \quad \text{...(i)}$$

But $\quad Y_{steel} > Y_{rubber}$

$$\therefore \quad \frac{Y_{steel}}{Y_{rubber}} > 1$$

Therefore, from Eq. (i), we get

$$\frac{(\text{stress})_{steel}}{(\text{stress})_{rubber}} > 1 \quad \text{or} \quad (\text{stress})_{steel} > (\text{stress})_{rubber}$$

Therefore, steel will have greater tensile stress than rubber.

(i) Young's modulus of the given material, Y

= Slope of strain-stress curve

$$Y = \frac{150 \times 10^6}{0.002}$$
$$= 75 \times 10^9 \text{ N/m}^2$$
$$= 7.5 \times 10^{10} \text{ N/m}^2$$

(ii) Yield strength of the given material

= Maximum stress

$= 300 \times 10^6$ N/m^2

$= 3 \times 10^8$ N/m^2

Given, diameter = 2.5 mm = 0.25 cm or r = 0.125 cm

$$\therefore \quad A = \pi r^2 = \frac{22}{7} \times (0.125)^2 \text{ sq. cm}$$

$$F = 100 \text{ kg} = 100 \times 1000 \text{ g}$$
$$= 100 \times 1000 \times 980 \text{ dyne}$$
$$Y = 12.5 \times 10^{11} \text{ dyne/sq. cm}$$

As, $\quad Y = \frac{F \times l}{A \times \Delta l}$

$$\therefore \quad \frac{\Delta l}{l} = \frac{F}{AY}$$

Hence, % increase in length

$$= \frac{\Delta l}{l} \times 100 = \frac{F}{AY} \times 100$$
$$= \frac{(100 \times 1000 \times 980) \times 7 \times 100}{22 \times (0.125)^2 \times 12.5 \times 10^{11}}$$
$$= 0.1812\,\%$$

When mass M is placed on the piston, the excess pressure, $p = Mg/A$. As this pressure is equally applicable from all the directions on the sphere, hence there will be decrease in volume due to decrease in radius of sphere.

Volume of the sphere, $V = \frac{4}{3}\pi R^3$.

Differentiating it, we get

$$\Delta V = \frac{4}{3}\pi(3R^2)\Delta R$$
$$= 4\pi R^2 \Delta R$$

$$\therefore \quad \frac{\Delta V}{V} = \frac{4\pi R^2 \Delta R}{\frac{4}{3}\pi R^3} = \frac{3\Delta R}{R}$$

We know that, $B = \frac{p}{\Delta V/V} = \frac{Mg}{A} \Big/ \frac{3\Delta R}{R}$

or $\quad \frac{\Delta R}{R} = \frac{Mg}{3BA}$

Depth, $h = 11$ km $= 11 \times 10^3$ m

Pressure at the bottom of the trench, p

$= 1.1 \times 10^8$ Pa

Initial volume of the ball, $V = 0.32$ m^3

Bulk modulus of steel, $B = 1.6 \times 10^{11}$ N/m^2

We know that, $B = \frac{p}{(\Delta V/V)} = \frac{pV}{\Delta V}$

$$\Delta V = \frac{pV}{B} = \frac{1.1 \times 10^8 \times 0.32}{1.6 \times 10^{11}}$$
$$\Delta V = 2.2 \times 10^{-4} \text{ m}^3$$

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Bulk modulus of rubber, $B = 9.8 \times 10^8$ N/m^2

Density of seawater, $\rho = 10^3$ kg/m^3

Percentage decrease in volume,

$$\left(\frac{\Delta V}{V} \times 100\right) = 0.1 \text{ or } \frac{\Delta V}{V} = \frac{0.1}{100} \text{ or } \frac{\Delta V}{V} = \frac{1}{1000}$$

Let the rubber ball be taken up to depth **h**.

$\because$ Change in pressure, $p = h\rho g$

$\therefore$ Bulk modulus, $B = \frac{p}{(\Delta V/V)} = \frac{h\rho g}{(\Delta V/V)}$

or $h = \frac{B \times (\Delta V/V)}{\rho g} = \frac{9.8 \times 10^8 \times \frac{1}{1000}}{10^3 \times 9.8} = 100$ m

Given,, $m = 900$ kg, $a = 2.2$ ms^{-2}

and maximum stress $= 1.3 \times 10^8$ Nm^{-2}

As the elevator moves up, the tension in the wire,

$$F = mg + ma = m(g + a) = 900 \times (9.8 + 2.2) = 10800 \text{N}$$

Stress in the wire $= \frac{F}{A} = \frac{F}{\pi r^2}$

Clearly, when the stress is maximum, **r** is minimum.

$\therefore$ Maximum stress $= \frac{F}{\pi r_{min}^2}$

or $r_{min}^2 = \frac{F}{\pi \times \text{Maximum stress}}$

$= \frac{10800}{3.14 \times 1.3 \times 10^8} = 0.2645 \times 10^{-4}$ m

or $r_{min} = 0.5142 \times 10^{-2}$ m

Minimum diameter

$= 2 r_{min} = 2 \times 0.5142 \times 10^{-2}$

$= 1.0284 \times 10^{-2}$ m

Given, diameter of each rivet, $D = 6$ mm

$\therefore$ Radius, $r = \frac{D}{2} = 3$ mm $= 3 \times 10^{-3}$ m

Maximum shearing stress on each rivet $= 6.9 \times 10^7$ Pa

Let **w** be the maximum load that can be subjected to the riveted strip, as each rivet carry one-quarter of the load.

Therefore, load on each rivet $= \frac{w}{4}$

Maximum shearing stress $= \frac{\text{Maximum shearing force}}{\text{Area}}$

$\therefore$ $6.9 \times 10^7 = \frac{w/4}{\pi r^2}$

or $w = 6.9 \times 10^7 \times 4\pi r^2$

or $w = 6.9 \times 10^7 \times 4 \times 3.14 \times (3 \times 10^{-3})^2$

$= 6.9 \times 4 \times 3.14 \times 9 \times 10 = 7.8 \times 10^3$ N

Given, compressional force, $F = 50000$ N

Diameter, $D = 0.5$ mm $= 5 \times 10^{-4}$ m

$\therefore$ Radius, $r = \frac{D}{2} = 2.5 \times 10^{-4}$ m

Pressure at the tip of the anvil, $p = \frac{\text{Force}}{\text{Area}}$

$\therefore$ $p = \frac{F}{\pi r^2} = \frac{50000}{3.14 \times (2.5 \times 10^{-4})^2} = 2.5 \times 10^{11}$ Pa

Given, $L = 1500$ cm is the unstretched length of the rope, and $\Delta L = 2.8$ cm is the amount of length stretches.

(i) Strain $= \frac{\Delta L}{L} = \frac{2.8 \text{ cm}}{1500 \text{ cm}} = 1.9 \times 10^{-3}$

Stress = Force/Area

Force, **F** = force of gravity on the rock climber

$= mg = 95 \times 9.8$N

Area, $A = \pi r^2 = \pi \times \left(\frac{D}{2}\right)^2 = \pi \times \left(\frac{9.6}{2} \times 10^{-3} \text{m}\right)^2$

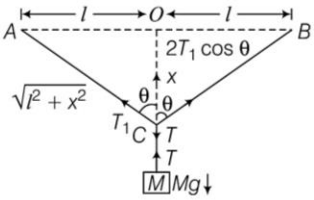
(ii) Stress $= \frac{95 \times 9.8}{\pi \times (4.8)^2 \times 10^{-6}} \simeq 1.29 \times 10^7$ N/m^2

(iii) Modulus of elasticity,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{1.3 \times 10^7}{1.9 \times 10^{-3}} = 6.84 \times 10^9 \text{ N/m}^2$$

Total length, $L = 2l$.

Increase in length of the wire, when it is stretched from its mid-point is as shown in the figure.



From Pythagoras theorem, $BC^2 = l^2 + x^2$

$BC = \sqrt{l^2 + x^2}$

Similarly, $AC = \sqrt{l^2 + x^2}$

Change in length of the wire,

$\Delta L = (AC + CB) - AB$

$= (\sqrt{l^2 + x^2} + \sqrt{l^2 + x^2}) - 2l$

$= 2(l^2 + x^2)^{1/2} - 2l = 2l\left(1 + \frac{x^2}{l^2}\right)^{1/2} - 2l$ ...(i)

Since $x << l$, so using binomial expansion, we have

$$\left(1 + \frac{x^2}{l^2}\right)^{1/2} = \left(1 + \frac{x^2}{2l^2}\right)$$

(neglecting terms containing higher powers of *x*)

$\therefore$ $\Delta L = 2l\left(1 + \frac{x^2}{2l^2}\right) - 2l = \frac{x^2}{l}$

Hence, strain $= \frac{\Delta L}{L} = \frac{x^2}{l \times 2l} = \frac{x^2}{2l^2}$

$\because$ $T = 2T_1 \cos\theta$

$\therefore$ $T_1 = \frac{Mg}{2\cos\theta}$ $(\because T = Mg)$

Putting $\cos\theta = \dfrac{x}{\sqrt{l^2+x^2}}$

$$T_1 = \frac{Mg}{2x}(\sqrt{l^2+x^2}) = \frac{Mgl}{2x}\left(1+\frac{x^2}{l^2}\right)^{1/2}$$

$$= \frac{Mgl}{2x}\left(1+\frac{x^2}{2l^2}\right) \quad [\text{using } (1+x)^x = 1+xx]$$

$\because$ $x << l$ $\therefore$ $\dfrac{x^2}{2l^2} \to 0$ Thus, $1+\dfrac{x^2}{2l^2} = 1$

$\therefore$ $T_1 = \dfrac{Mgl}{2x}$

Stress in the wire $= \dfrac{T_1}{A} = \dfrac{Mgl}{2xA}$

Given, **for steel wire**, length, $l_1 = 4.7$ m

Area of cross-section, $A_1 = 3.0\times 10^{-5}$ m^2

and **for copper wire**, length, $l_2 = 3.5$ m

Area of cross-section, $A_2 = 4.0\times 10^{-5}$ m^2

Let **F** be the given load under which steel and copper wires be stretched by the same amount Δl.

Young's modulus, $Y = \dfrac{F/A}{\Delta l/l} = \dfrac{F\times l}{A\times \Delta l}$

For steel, $Y_s = \dfrac{F\times l_1}{A_1\times \Delta l}$ ...(i)

For copper, $Y_c = \dfrac{F\times l_2}{A_2\times \Delta l}$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{Y_s}{Y_c} = \frac{F\times l_1}{A_1\times\Delta l}\times\frac{A_2\times\Delta l}{F\times l_2}$$

$$= \frac{l_1}{l_2}\times\frac{A_2}{A_1} = \frac{4.7}{3.5}\times\frac{4.0\times10^{-5}}{3.0\times10^{-5}}$$

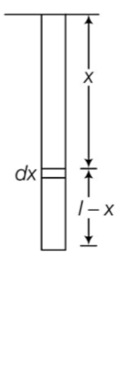
$$\frac{Y_s}{Y_c} = \frac{18.8}{10.5} = 1.79 = 1.8$$

The ratio of the Young's modulus of steel to that of copper is 1.8.

As shown in figure, consider a small element of thickness **dx** at distance **x** from the fixed support. Force acting on the element **dx**,

F = Weight of length $(l-x)$ of the rod

$$= \frac{w}{l}(l-x)$$



Elongation of the element

$$= \text{Original length}\times\frac{\text{Stress}}{Y}$$

$$= dx\times\frac{F/A}{Y} = \frac{w}{l\,Ay}(l-x)\,dx$$

Total elongation produced in the rod

$$= \frac{w}{l\,AY}\int_0^l (l-x)\,dx = \frac{w}{l\,Ay}\left(lx-\frac{x^2}{2}\right)_0^l$$

$$= \frac{w}{l\,Ay}\left(l^2-\frac{l^2}{2}\right) = \frac{wl}{2Ay}$$

Given, length, $l = 1$ m

Area of cross-section, $A = 0.5\times10^{-2}$ cm^2

$= 0.5\times10^{-6}$ m^2

Mass, $m = 100$ g $= 0.1$ kg

$\therefore$ Load, $w = mg = 0.1\times 9.8$ N $= 0.98$ N

Young's modulus for steel, $Y = 2\times10^{11}$ Pa

Area, $A = \pi r^2$ or $r^2 = \dfrac{A}{\pi} = \dfrac{0.5\times10^{-6}}{\pi}$

Depression in a wire, when a load is suspended at its centre,

$$\delta = \frac{wl^3}{12\pi r^4 Y} = \frac{0.98\times(1)^3}{12\pi\times\left(\dfrac{0.5\times10^{-6}}{\pi}\right)^2\times2\times10^{11}}$$

$$\delta = \frac{0.98\times\pi}{12\times0.25\times2\times10^{-1}} = 5.12 \text{ m}$$

Young's modulus of copper, $Y_1 = 110\times10^9$ N/m^2

Young's modulus of steel, $Y_2 = 190\times10^9$ N/m^2

Let d_1 and d_2 be the diameters of copper and steel wires. Since, tension in each wire is same, therefore each wire has same extension. As each wire is of same length, hence each wire has same strain.

Young's modulus, $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{F/A}{\text{Strain}}$

or $Y = \dfrac{F}{\left(\dfrac{\pi d^2}{4}\right)\times\text{Strain}} = \dfrac{4F}{\pi d^2\times\text{Strain}}$

$\because$ $Y \propto \dfrac{1}{d^2} \Rightarrow d^2 \propto \dfrac{1}{Y}$

$\therefore$ $\dfrac{d_1^2}{d_2^2} = \dfrac{Y_2}{Y_1}$

or $\dfrac{d_1}{d_2} = \sqrt{\dfrac{Y_2}{Y_1}} = \sqrt{\dfrac{190\times10^9}{110\times10^9}}$

$$= \sqrt{\frac{19}{11}} = \sqrt{1.73} = 1.31$$

$\therefore$ $d_1 : d_2 = 1.31 : 1$

Hence, the ratio of their diameters is 1.31 : 1.

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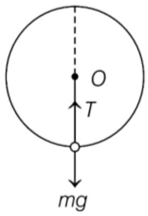
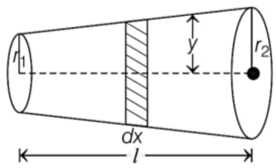
Given, mass, $m = 14.5$ kg
Length of wire, $l = 1$ m
Angular frequency, $\nu = 2$ rev/s
Angular velocity, $\omega = 2\pi\nu$
$= 2\pi \times 2 = 4\pi$ rad/s



Area of cross-section of wire, $A = 0.065\ cm^2$
$= 6.5 \times 10^{-6}\ m^2$
Young's modulus for steel, $Y = 2 \times 10^{11}\ N/m^2$.
At lowest point of the vertical circle,
$$T - mg = ml\omega^2$$
or $T = mg + ml\omega^2$
$= (14.5 \times 9.8) + 14.5 \times 1 \times (4\pi)^2$
$= 14.5(9.8 + 16\pi^2)$
$= 14.5(9.8 + 16 \times 9.87)$ $(\because \pi^2 = 9.87)$
$= 14.5 \times 167.72\ N = 2431.94\ N$
Young's modulus, $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{(T/A)}{\Delta l/l} = \frac{Tl}{A \cdot \Delta l}$
$\therefore \quad \Delta l = \frac{T \cdot l}{A \cdot Y} = \frac{2431.94 \times 1}{6.5 \times 10^{-6} \times 2 \times 10^{11}}$
$= 1.87 \times 10^{-3}\ m = 1.87\ mm$
Given, total mass supported by cylindrical columns,
$m = 50000$ kg
$\because$ Total weight supported by cylindrical columns
$= mg = 50000 \times 9.8 = 490000\ N$
$\therefore$ Load acting on each cylindrical support,
$$F = \frac{mg}{4} = \frac{490000}{4}\ N$$
$= 122500\ N$
Inner radius of each column, $r_1 = 30\ cm = 0.3\ m$
Outer radius of each column, $r_2 = 60\ cm = 0.6\ m$
$\therefore$ Area of cross-section of each cylindrical column,
$$A = \pi r_2^2 - \pi r_1^2 = \pi(r_2^2 - r_1^2)$$
$= 3.14[(0.6)^2 - (0.3)^2]$
$= 3.14 \times 0.27\ m^2$
Young's modulus, $Y = \frac{\text{Compressional stress}}{\text{Compressional strain}}$
$\Rightarrow$ Compressional strain $= \frac{\text{Compressional stress}}{\text{Young's modulus}}$

$= \frac{F/A}{Y} = \frac{F}{AY}$
$(\because$ Young's modulus, $Y = 2 \times 10^{11}$ Pa)
$= \frac{122500}{(3.14 \times 0.27) \times 2 \times 10^{11}}$
$= 0.722 \times 10^{-6} = 7.22 \times 10^{-7}$
If R be the rate of change of radius per length
$$R = \frac{r_2 - r_1}{l} = \frac{b - a}{l} \quad \ldots(i)$$
Consider an element of thickness dx at a distance of l from narrow end of the wire.



Radius of element $y = r_1 + Rx = a + Rx$ ...(ii)
From Young's modulus, the extension in the element,
$$dl = \frac{Fdx}{AY} = \frac{Fdx}{(\pi y^2)Y}$$
Integrating both sides, we get
$$\int dl = \int_0^l \frac{Fdx}{\pi Y y^2}$$
As, $y = a + Rx$ [using Eq. (ii)]
$\Rightarrow \quad dy = Rdx$
$\Rightarrow \quad l = \frac{F}{\pi Y}\int_a^{a+Rl} \frac{dy}{Ry^2}$
$= \frac{F}{\pi YR}\left(\frac{-1}{y}\right)_a^{a+Rl}$
$= \frac{F}{\pi YR}\left[\frac{1}{a} - \frac{1}{(a+Rl)}\right]$
$= \frac{F}{\pi YR}\left[\frac{a + Rl - a}{a(a+Rl)}\right]$
$= \frac{Fl}{\pi Ya\left[a + \left(\frac{b-a}{l}\right)l\right]}$ [using Eq. (i)]
$= \frac{Fl}{\pi abY}$
Given, diameter of wires, $2r = 0.25$ cm
$\therefore \quad r = 0.125\ cm = 1.25 \times 10^{-3}\ m$
For steel wire
Load, $F_1 = (4 + 6)$ kg-f $= 10 \times 9.8\ N = 98\ N$
Length of steel wire, $l_1 = 1.5$ m
Young's modulus, $Y_1 = 2.0 \times 10^{11}$ Pa
Young's modulus, $Y = \frac{F_1 \times l_1}{A_1 \times \Delta l_1}$

$\therefore$ Change in length, $\Delta l_1 = \frac{F_1 \times l_1}{A_1 \times Y_1} = \frac{F_1 \times l_1}{\pi r_1^2 \times Y_1}$

$= \frac{98 \times 1.5}{3.14 \times (1.25 \times 10^{-3})^2 \times 2.0 \times 10^{11}}$

$= 1.5 \times 10^{-4}$ m

For brass wire,

Load, $F_2 = 6$ kg-f $= 6 \times 9.8$ N $= 58.8$ N

Length of brass wire, $l_2 = 1.0$ m

Young's modulus, $Y_2 = 0.91 \times 10^{11}$ Pa

Change in length, $\Delta l_2 = \frac{F_2 \times l_2}{\pi r_2^2 \times Y_2}$

$= \frac{58.8 \times 1.0}{3.14 \times (1.25 \times 10^{-3})^2 \times 0.91 \times 10^{11}}$

$= 1.3 \times 10^{-4}$ m

Let the length of wires **A** and **B** is equal to **L** and their area of cross-section be A_1 and A_2, respectively.

Given, $A_1 = 1\ \text{mm}^2 = 1 \times 10^{-6}\ \text{m}^2$

$A_2 = 2\ \text{mm}^2 = 2 \times 10^{-6}\ \text{m}^2$

$Y_{steel} = 2 \times 10^{11}\ \text{N/m}^2$

$Y_{Al} = 7.0 \times 10^{10}\ \text{N/m}^2$

Let F_1 and F_2 be the tensions in the two wires, respectively.

(i) When equal stresses are produced, then

$\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$\Rightarrow \frac{F_1}{F_2} = \frac{A_1}{A_2} = \frac{1 \times 10^{-6}}{2 \times 10^{-6}}$

$\Rightarrow \frac{F_1}{F_2} = \frac{1}{2}$ ...(i)

Let mass **m** be suspended at distance **x** from steel wire **A**.

Taking moment of forces about the point of suspension of mass from the rod, we get

$F_1 \times x = F_2 \times (1.05 - x)$

or $\frac{F_1}{F_2} = \frac{(1.05 - x)}{x}$ ...(ii)

From Eqs. (i) and (ii), we get

$\frac{1}{2} = \frac{(1.05 - x)}{x}$

$x = 2.10 - 2x$

or $3x = 2.10$

or $x = 0.70$ m

$\therefore$ The mass *m* must be suspended at a distance 0.70 m from steel wire **A**.

(ii) Young's modulus, $Y = \frac{\text{Stress}}{\text{Strain}}$

$\therefore \quad \text{Strain} = \frac{\text{Stress}}{Y} = \frac{F/A}{Y}$

For steel wire A, $(\text{Strain})_{steel} = \frac{F_1}{A_1 Y_1}$

For aluminium wire *B*,

$(\text{Strain})_{aluminium} = \frac{F_2}{A_2 Y_2}$

When equal strains are produced in both wires, then

$\frac{F_1}{A_1 Y_1} = \frac{F_2}{A_2 Y_2}$

or $\frac{F_1}{F_2} = \frac{A_1 Y_1}{A_2 Y_2}$ ...(iii)

$\therefore$ From Eqs. (ii) and (iii), we get

$\frac{(1.05 - x)}{x} = \frac{A_1 Y_1}{A_2 Y_2}$

$= \frac{1 \times 10^{-6}}{2 \times 10^{-6}} \times \frac{2 \times 10^{11}}{7 \times 10^{10}}$

$\frac{(1.05 - x)}{x} = \frac{10}{7}$

$10x = 7.35 - 7x$

$\Rightarrow 17x = 7.35$ or $x = \frac{7.35}{17}$

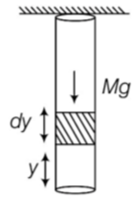
$x = 0.43$ m

$\therefore$ The mass **m** must be suspended at a distance 0.43 m from the steel with **A**.

Given, $l = 10$ m, $\rho = 1.5 \times 10^3\ \text{kg/m}^3$

$Y = 5 \times 10^6\ \text{N/m}^2$

We know that, $Y = \frac{F l}{A \Delta l}$



Efficient force $= Mg$

Consider a small length dy at a distance **y** from free end.

The length above this, $(l - y)$ will experience a force of

$F_{dy} = \frac{M}{l} g dy$

$\therefore$ Extension, $dl = \frac{F l}{A Y}$

$\Rightarrow \quad dl = \frac{(l - y)}{AY} \cdot \frac{M}{l} g dy = \frac{Mg}{l AY}(l - y) dy$

Net extension due to its own weight $= \int dl$

$= \frac{Mg}{AYl}\int_0^l (l - y) dy = \frac{Mg}{l AY}\left[ly - \frac{y^2}{2}\right]_0^l = \frac{Mgl}{2AY}$

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Net extension $= \frac{\mathbf{Mgl}}{\mathbf{2AY}} = \frac{\mathbf{Mgl^2}}{\mathbf{2YV}} = \frac{\rho \mathbf{gl^2}}{\mathbf{2Y}}$

Extension of rubber string

$$= \frac{1.5\times10^3\times10\times10^2}{2\times5\times10^6} = 0.15 \text{ m}$$

Breaking stress for a metal $= 7.8\times10^9 \text{ N/m}^2$

Density $= 7.8\times10^3 \text{ kg/m}^3$

Stress $= \frac{\text{Force}}{\text{Area}} = \frac{\mathbf{Mg}}{\mathbf{A}} = \frac{\mathbf{Mgl}}{\mathbf{Al}} = \frac{\mathbf{Mgl}}{\text{Volume}} = \rho gl$

If ρgl > Breaking stress, the wire will break.

$\therefore \quad l \le \frac{7.8\times10^9}{\rho g}, l \le \frac{7.8\times10^9}{7.8\times10^3\times10}$

i.e. $\quad l \le 10^5 \text{ m}$

Maximum length of wire $= 10^5$ m

Density of water at the surface, $\rho_0 = 1.03\times10^3 \text{ kg/m}^3$

Pressure, $p = 80.0 \text{ atm} = 80.0\times1.013\times10^5 \text{ Pa}$

$(\because 1 \text{ atm} = 1.013\times10^5 \text{ Pa})$

Compressibility of water $\left(\frac{1}{B}\right) = 45.8\times10^{-11} \text{ Pa}^{-1}$

Let **V** and **V′** be the volumes of certain mass of water at the surface and at a given depth. The density of water at the given depth be ρ'.

Volume of water at the surface, $V = \frac{m}{\rho}$

At the given depth, $V' = \frac{m}{\rho'}$

$\therefore$ Change in volume, $\Delta V = V - V' = m\left(\frac{1}{\rho} - \frac{1}{\rho'}\right)$

Volumetric strain $= \frac{\Delta V}{V} = m\left(\frac{1}{\rho} - \frac{1}{\rho'}\right)\times\frac{\rho}{m}$

$$= \left(1 - \frac{\rho}{\rho'}\right)$$

Compressibility $= \frac{1}{\textbf{Bulk modulus (B)}}$

$$= \frac{1}{\frac{\Delta p}{(\Delta V/V)}} = \frac{\Delta V}{\Delta p V}$$

$$45.8\times10^{-11} = \left(1 - \frac{\rho}{\rho'}\right)\times\frac{1}{80\times1.013\times10^5}$$

$\Rightarrow 45.8\times10^{-11}\times80\times1.013\times10^5 = 1 - \frac{1.03\times10^3}{\rho'}$

$\Rightarrow 3.712\times10^{-3} = 1 - \frac{1.03\times10^3}{\rho'}$

$\Rightarrow \qquad \frac{1.03\times10^3}{\rho'} = 1 - 3.712\times10^{-3}$

or $\rho' = \frac{1.03\times10^3}{1-0.003712} = 1.034\times10^3 \text{kg/m}^3$

Given, initial volume, $V_1 = 100.0$ L

Final volume, $V_2 = 100.5$ L

$\therefore$ Increase in volume, $\Delta V = V_2 - V_1$

$= 100.5 - 100.0 = 0.5$ L

$= 0.5\times10^{-3} \text{ m}^3 \qquad (\because 1 \text{ L} = 10^{-3}\text{m}^3)$

Increase in pressure,

$\Delta p = 100.0$ atm

$= 100.0\times1.013\times10^5$ Pa

$(\because 1 \text{ atm} = 1.013\times10^5 \text{Pa})$

$= 1.013\times10^7$ Pa

Bulk modulus of water,

$$B_w = \frac{\Delta p}{(\Delta V/V)}$$

$$= \frac{\Delta p V}{\Delta V} = \frac{1.013\times10^7\times100\times10^{-3}}{0.5\times10^{-3}}$$

$$= \frac{10.13}{5}\times10^9$$

$$= 2.026\times10^9 \text{ Pa}$$

Bulk modulus of air, $B_a = 1.0\times10^5$ Pa

$\therefore \quad \frac{\textbf{Bulk modulus of water } (B_w)}{\textbf{Bulk modulus of air } (B_a)} = \frac{2.026\times10^9}{1.0\times10^5}$

$$= 2.026\times10^4$$

This ratio is too large as gases are more compressible than those of liquids. In liquids, interatomic forces are more strong than that for gases.

(i) Upto point **P**, stress ∝ strain. So, Hooke's law is obeyed upto this point.

(ii) Point *E* corresponds to elastic limit and yield point of wire, as it returns to original state at *O* along *EPO*, when gradually unloaded.

(iii) The graph from *O* to *E* shows elastic region and from *E* to *B* shows plastic region.

(iv) Upto point *P*, stress is proportional to strain. From *P* to *E*, strain increases more than stress. Here, Hooke's law is not obeyed. When wire is unloaded at point *A* beyond *E*, it does not retrace the curve along *AEPO* but follows the dotted curve along *AEPO*. So, a strain **OO′** is left for zero stress on wire.

(v) Between points *C* and *B*, the wire virtually flows out, i.e. the strain increases even when the wire is being unloaded. Fracture takes place at point *B*. The stress can be applied to the value corresponding to point *C* without causing fracture.

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