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CBSE-X
MATHEMATICS

Topic Coverage: The revision module thoroughly covers Real Numbers, encompassing prime factorization, Euclid's Division Lemma, Fundamental Theorem of Arithmetic, and the properties of irrational numbers – aligning with the CBSE Class 10 Mathematics curriculum.

Concise and Clear: The content is presented in a concise and clear manner, emphasizing the most critical aspects of Real Numbers. It serves as an efficient tool for rapid revision before exams.

Important Properties and Theorems: The module includes a compilation of important properties and theorems related to Real Numbers, aiding in quick recall and application during problem-solving.

Application-Based Questions: Engage in practice with a set of application-based questions to enhance your problem-solving skills and understand the real-world applications of real number concepts.

Concept Reinforcement: Reinforce your understanding of the underlying concepts through targeted explanations and examples, ensuring a solid foundation in Real Numbers.

Exam-Style Questions: Familiarize yourself with the types of questions that could appear in the CBSE Class 10 Mathematics exam. Solve exam-style questions to refine your exam-taking strategies.

Online Accessibility: The revision module is accessible online, allowing you to study anytime anywhere. This flexibility enables you to integrate revision seamlessly into your schedule.

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Basic Concepts

- **Fundamental theorem of Arithmetic:** Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique apart from the order in which the prime factors occur.
- If x is a positive prime, then $\sqrt{x}$ is an irrational number.

  For example, 7 is a positive prime $\Rightarrow$ $\sqrt{7}$ is an irrational number.
- **Some Important Facts/Tips:**

  (*i*) If 'p' is a prime and p divides a^2, then p divides 'a' also, where a is positive integer.

  For example: 3 divides 36 *i.e.*, 6^2

  $\Rightarrow$ 3 divides 6.

  (*ii*) For any two positive integers a and b; HCF $(a, b) \times$ LCM $(a, b) = a \times b$

  For example: $a = 6, b = 4$

  $$\text{LCM of } (6, 4) = 12$$
  $$\text{HCF of } (6, 4) = 2$$
  $$\text{LCM } (6, 4) \times \text{HCF } (6, 4) = 6 \times 4$$
  $$12 \times 2 = 6 \times 4 = 24$$

  (*iii*) The sum or difference of a rational and an irrational number is irrational.

  (*iv*) The product and quotient of a non-zero rational number and an irrational number is irrational.

Selected NCERT Questions

1. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol. For the maximum number of columns, we have to find the HCF of 616 and 32.

$$616 = 2 \times 2 \times 2 \times 7 \times 11$$
$$= 2^3 \times 7 \times 11$$
$$32 = 2 \times 2 \times 2 \times 2 \times 2$$
$$= 2^3 \times 2^2$$
$$\text{HCF of } 616, 32 = 2^3 = 8$$

2	616
2	308
2	154
7	77
	11

2	32
2	16
2	8
2	4
	2

Hence, maximum number of columns is 8.

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2. Check whether 6^n can end with the digit 0 for any natural number n.

Sol. If the number 6^n, for any n, were to end with the digit zero, then it would be divisible by 5. That is, the prime factorisation of 6^n would contain the prime 5. But $6^n = (2 \times 3)^n = 2^n \times 3^n$ so the primes in factorisation of 6^n are 2 and 3. So the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes except 2 and 3 in the factorisation of 6^n. So there is no natural number n for which 6^n ends with digit zero.

3. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers?

Sol. We have, $7 \times 11 \times 13 + 13 = 1001 + 13 = 1014$

$$1014 = 2 \times 3 \times 13 \times 13$$

So, it is the product of more than two prime numbers. 2, 3 and 13.

Hence, it is a composite number.

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5040 + 5 = 5045$$

$$\Rightarrow \quad 5045 = 5 \times 1009$$

It is the product of prime factor 5 and 1009.

Hence, it is a composite number.

4. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start from the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Sol. To find the time after which they meet again at the starting point, we have to find LCM of 18 and 12 minutes. We have

$$18 = 2 \times 3^2$$

and $\quad 12 = 2^2 \times 3$

Therefore, LCM of 18 and 12 $= 2^2 \times 3^2 = 36$

So, they will meet again at the starting point after 36 minutes.

2	18
3	9
3	3
	1

2	12
2	6
3	3
	1

5. Show that $5 - \sqrt{3}$ is irrational.

Sol. Let us assume, to the contrary, that $5 - \sqrt{3}$ is rational.

That is, we can find coprime a and b ($b \neq 0$) such that $5 - \sqrt{3} = \frac{a}{b}$.

Therefore, $5 - \frac{a}{b} = \sqrt{3}$.

Rearranging this equation, we get $\sqrt{3} = 5 - \frac{a}{b} = \frac{5b - a}{b}$.

Since a and b are integers, we get $\frac{5b - a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{3}$ is rational.

So, we conclude that $5 - \sqrt{3}$ is irrational.

6. Prove that $3 + 2\sqrt{5}$ is an irrational number.

Sol. Let $\quad 3 + 2\sqrt{5}$ is a rational number.

$\Rightarrow \quad 3 + 2\sqrt{5} = \frac{p}{q}$, where p, q are integers and $q \neq 0$

$\Rightarrow \quad 2\sqrt{5} = \frac{p}{q} - 3 \quad \Rightarrow \quad 2\sqrt{5} = \frac{p - 3q}{q}$

$\Rightarrow \quad \sqrt{5} = \frac{p - 3q}{2q} \quad$ ...(i)

Since, p, q, 2 and –3 are integers, p, $-3q$, $2q$ are also integers.

Also, $2 \neq 0$, $q \neq 0$ $\Rightarrow$ $2q \neq 0$

[$\because$ Product of two non-zero numbers can never be zero]

Therefore, RHS of (*i*) is rational number and LHS = $\sqrt{5}$ is an irrational number.

But this is not possible.

So, our assumption is wrong.

Hence, $3 + 2\sqrt{5}$ is irrational number.

7. Prove that $7\sqrt{5}$ is an irrational number.

Sol. Let $7\sqrt{5}$ be a rational number.

$\Rightarrow$ $7\sqrt{5} = \frac{p}{q}$, where p, q are integers and $q \neq 0$

$\Rightarrow$ $\sqrt{5} = \frac{p}{7q}$ ...(*i*)

$\because$ p, 7, q are integers $\Rightarrow$ p, $7q$ are integers

Also $7 \neq 0, q \neq 0,$ $\Rightarrow$ $7q \neq 0$

Therefore RHS of (*i*) is rational number but LHS = $\sqrt{5}$ is irrational, which is contradiction.

Hence, $7\sqrt{5}$ is an irrational number.

8. Prove that $6 + \sqrt{2}$ is an irrational number.

Sol. Let $6 + \sqrt{2}$ be a rational number.

$\Rightarrow$ $6 + \sqrt{2} = \frac{p}{q}$, where p, q are integers and $q \neq 0$.

$\Rightarrow$ $\sqrt{2} = \frac{p}{q} - 6$ $\Rightarrow$ $\sqrt{2} = \frac{p - 6q}{q}$ ...(*i*)

$\because$ p, q, –6 are integers. $\Rightarrow$ $p - 6q$, q are integers.

Also, $q \neq 0$

Therefore, RHS of (*i*) is rational number but LHS = $\sqrt{2}$ is irrational, which is contradiction.

Hence, $6 + \sqrt{2}$ is irrational.

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. $n^2 - 1$ is divisible by 8 if n is *[NCERT Exemplar]*

(*a*) an integer (*b*) a natural number (*c*) an odd integer (*d*) an even integer

2. The product of three consecutive integers is divisible by

(*a*) 5 (*b*) 6 (*c*) 7 (*d*) none of these

3. The largest number which divides 615 and 963 leaving remainder 6 in each case is

(*a*) 82 (*b*) 95 (*c*) 87 (*d*) 93

4. The largest number which divides 70 and 125 leaving remainders 5 and 8 respectively is *[NCERT Exemplar]*

(*a*) 13 (*b*) 65 (*c*) 875 (*d*) 1750

5. **If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers, then LCM (a, b) is** *[NCERT Exemplar]*
(*a*) xy (*b*) xy^2 (*c*) x^3y^3 (*d*) x^2y^2

6. **If HCF (26, 169) = 13 then LCM (26, 169) is**
(*a*) 26 (*b*) 52 (*c*) 338 (*d*) 13

7. **The HCF and the LCM of 12, 21, 15 respectively are** *[CBSE 2020 (30/1/1)]*
(*a*) 3, 140 (*b*) 12, 420 (*c*) 3, 420 (*d*) 420, 3

8. **The product of two irrational numbers is**
(*a*) always irrational (*b*) always rational
(*c*) rational or irrational (*d*) one

9. **$3.\overline{27}$ is**
(*a*) an integer (*b*) a rational number (*c*) a natural number (*d*) an irrational number

10. **If 3 is the least prime factor of number a and 7 is the least prime factor of number b, then the least prime factor of $(a + b)$ is** *[Competency Based Question]*
(*a*) 2 (*b*) 3 (*c*) 5 (*d*) 10

11. **Which of these is a RATIONAL number?** *[CBSE Question Bank]*
(*a*) 3π (*b*) $5\sqrt{5}$ (*c*) 0.346666... (*d*) 0.345210651372849...

12. **Which of these numbers can be expressed as a product of two or more prime numbers?**
(*i*) 15 (*ii*) 34568 (*iii*) (15 × 13) *[CBSE Question Bank]*
(*a*) only (*ii*) (*b*) only (*iii*) (*c*) only (*i*) and (*ii*) (*d*) all-(*i*), (*ii*) and (*iii*)

13. **A number of the form 8^n, where n is a natural number greater than 1, cannot be divisible by** *[CBSE Question Bank]*
(*a*) 1 (*b*) 40 (*c*) 64 (*d*) 2^{2n}

14. **1245 is a factor of the numbers p and q.**
Which of the following will always have 1245 as a factor?
(*i*) $p + q$ (*ii*) $p \times q$ (*iii*) $p \div q$ *[Competency Based Question]*
(*a*) only (*ii*) (*b*) only (*i*) and (*ii*) (*c*) only (*ii*) and (*iii*) (*d*) all-(*i*), (*ii*) and (*iii*)

Answers

1. (*c*)	**2.** (*b*)	**3.** (*c*)	**4.** (*a*)	**5.** (*c*)	**6.** (*c*)	**7.** (*c*)
8. (*c*)	**9.** (*b*)	**10.** (*a*)	**11.** (*c*)	**12.** (*d*)	**13.** (*b*)	**14.** (*b*)

Very Short Answer Questions

Each of the following questions are of 1 mark.

1. **What is the HCF of the smallest composite number and the smallest prime number?** *[CBSE 2018]*

Sol.

1) Smallest prime = 2
Smallest composite = 4
HCF (2,4) = 2.
The HCF of the smallest prime and smallest composite is 2.

[Topper's Answer 2018]

2. If HCF (336, 54) = 6. Find LCM (336, 54). [*CBSE* 2019(30/2/1)]

Sol. LCM (336, 54) $= \frac{336 \times 54}{6}$ ½

$= 336 \times 9 = 3024$ ½

[*CBSE Marking Scheme* 2019(30/2/1)]

3. If a is an odd number, b is not divisible by 3 and LCM of a and b is P, what is the LCM of $3a$ and $2b$?

Sol. ∵ a is odd ⇒ factors of a can be : 1, 3, 5, 7 ...(*i*)

b is not divisible by 3 ⇒ factors of b can be : 1, 2, 4, 5, 7 ...(*ii*)

LCM $(a, b) = P$ [Given]

Factors of $3a = 3(1, 3, 5, 7, ...)$ [From (*i*)]

Factors of $2b = 2(1, 2, 4, 5, 7, ...)$ [From (*ii*)]

LCM ($3a$ and $2b$) $= 3 \times 2 \times$ LCM $(a, b) = 6P$

4. Two positive integers p and q can be expressed as $p = a^2b^3$ and $q = a^3b^3$, a and b are prime numbers. What is the LCM of p and q?

Sol. LCM $(p, q) = a^3b^3$ [Highest power of the variables]

Short Answer Questions–I

Each of the following questions are of 2 marks.

1. Explain whether 3 × 12 × 101 + 4 is prime number or a composite number.

Sol. We have,

$$3 \times 12 \times 101 + 4 = 4(3 \times 3 \times 101 + 1)$$
$$= 4(909 + 1)$$
$$= 4(910)$$
$$= 2 \times 2 \times 2 \times 5 \times 7 \times 13 \text{ is a composite number}$$

(∵ Product of more than two prime factors)

2. Check whether 12^n can end with the digit 0 for any natural number n. [*CBSE* 2020(30/5/1)]

Sol. Prime factors of 12 are 2 × 2 × 3. 1

Since 5 is not a factor, so 12^n can not end with 0. 1

[*CBSE Marking Scheme* 2020(30/5/1)]

3. Find the HCF of 612 and 1314 using prime factorisation. [*CBSE* 2019(30/5/3)]

Sol.

2	612
2	306
3	153
3	51
	17

2	1314
3	657
3	219
	73

$612 = 2 \times 2 \times 3 \times 3 \times 17$

$1314 = 2 \times 3 \times 3 \times 73$

HCF $= 2 \times 3 \times 3 = 18$

4. Given that $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number. [*CBSE* 2018(30/1)]

Sol.

7) Given, $\sqrt{2}$ is irrational

To prove: $5+3\sqrt{2}$ is irrational.

Proof: Let us assume $5+3\sqrt{2}$ is rational. so it is in form $\frac{a}{b}$. ~~[a, b ∈ R, b≠0]~~ [a, b ∈ Z, b ≠ 0, HCF(a, b) = 1]

$\Rightarrow 5+3\sqrt{2} = \frac{a}{b}$

$3\sqrt{2} = \frac{a}{b} - 5.$

$3\sqrt{2} = \frac{a-5b}{b}$

$\sqrt{2} = \frac{a-5b}{3b}.$

This shows that $\sqrt{2}$ is ~~irrational~~ rational ($a-5b$ and $3b$ are integers).

But we know that $\sqrt{2}$ is irrational.

This contradicts our assumption that $5+\sqrt{2}\times 3$ is rational.

$\Rightarrow$ $5+3\sqrt{2}$ is irrational, hence proved.

[*Topper's Answer* 2018]

5. Find the smallest natural number by which 1200 should be multiplied so that the square root of the product is a rational number.

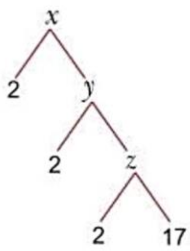
Sol. We have, $1200 = 4 \times 3 \times 10 \times 10$

$$= 4 \times 3 \times 2 \times 5 \times 2 \times 5 = 4 \times 3 \times (2 \times 5)^2$$

$$= 2^2 \times 3 \times 2^2 \times 5^2 = 2^4 \times 3 \times 5^2$$

Hence, the required smallest natural number is 3.

6. Find the value of x, y and z in the given factor tree. Can the value of 'x' be found without finding the value of 'y' and 'z'? If yes, explain.



Sol. $z = 2 \times 17 = 34;\ y = 34 \times 2 = 68$

and $x = 2 \times 68 = 136$

Yes, value of x can be found without finding value of y or z as

$x = 2 \times 2 \times 2 \times 17$ which are prime factors of x.

Short Answer Questions–II

Each of the following questions are of 3 marks.

1. Write the smallest number which is divisible by both 306 and 657.

Sol. Here, to find the required smallest number we will find LCM of 306 and 657.

$306 = 2 \times 3^2 \times 17$

$657 = 3^2 \times 73$

$\text{LCM} = 2 \times 3^2 \times 17 \times 73 = 22338$

2	306
3	153
3	51
	17

3	657
3	219
	73

2. Express the number $0.3\overline{178}$ in the form of rational number $\frac{a}{b}$.

Sol. Let $x = 0.3\overline{178}$

Then $x = 0.3178178178...$ ... (*i*)

$10x = 3.178178178...$ ... (*ii*)

$10000x = 3178.178178...$ ... (*iii*)

On subtracting (*ii*) from (*iii*), we get

$$9990x = 3175 \quad \Rightarrow \quad x = \frac{3175}{9990} = \frac{635}{1998}$$

$$\therefore \quad 0.3\overline{178} = \frac{635}{1998}$$

3. **Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers.** [*CBSE* 2018(30/1)]

Sol. (3) Numbers: 404, 96. To find: HCF and LCM.

2 | 404, 96
2 | 202, 48
  101, 24

⇒ Their HCF is 4.

404 = 2 × 202 = 2 × 2 × 101

96 = 2 × 48 = 2 × 2 × 24 = 2 × 2 × 2 × 12 = 2 × 2 × 2 × 2 × 6 = 2 × 2 × 2 × 2 × 2 × 3

$404 = 2^2 \times 101$

$96 = 2^5 \times 3$

HCF = greatest common factor = $2^2 = 4$.

LCM = $2^5 \times 3 \times 101$

= 96 × 101

= 9696

Product of two numbers = 96 × 404

= 38784

Product of HCF & LCM = 9696 × 4

= 38784.

Hence, HCF × LCM = product of two numbers. [*Topper's Answer* 2018)]

4. **Find the largest number which on dividing 1251, 9377 and 15628 leaves remainders 1, 2 and 3 respectively.** [*NCERT Exemplar, CBSE* 2019(30/3/1)]

Sol. 1251 – 1 = 1250, 9377 – 2 = 9375, 15628 – 3 = 15625 **1**

Required largest number = HCF (1250, 9375, 15625)

$$1250 = 2 \times 5^4$$
$$9375 = 3 \times 5^5$$
$$15625 = 5^6$$ **1½**

∴ HCF (1250, 9375, 15625) = 5^4 = 625 **½**

[*CBSE Marking Scheme* 2019 (30/3/1)]

Long Answer Questions

Each of the following questions are of 5 marks.

1. **Prove that $(\sqrt{2} + \sqrt{5})$ is irrational.** [*CBSE* 2020(30/3/1)]

Sol. On the contrary, let $\sqrt{2} + \sqrt{5}$ is rational, *i.e.*, $\sqrt{2} + \sqrt{5} = \frac{a}{b}$, where a and b are co-prime and $b \neq 0$.

$$\Rightarrow \quad \sqrt{5} = \frac{a}{b} - \sqrt{2}$$

Squaring both sides, we have

$$5 = \left(\frac{a}{b} - \sqrt{2}\right)^2 \quad \Rightarrow \quad 5 = \frac{a^2}{b^2} - 2\sqrt{2}\,\frac{a}{b} + 2$$

$$\Rightarrow \quad 2\sqrt{2}\,\frac{a}{b} = \frac{a^2}{b^2} - 3 \quad \Rightarrow \quad \sqrt{2} = \frac{a^2}{b^2} \times \frac{b}{2a} - \frac{3b}{2a}$$

$$\Rightarrow \quad \sqrt{2} = \frac{-3b}{2a} + \frac{a}{2b}$$

$\downarrow \qquad \downarrow$

Irrational = Rational, which is not possible as Irrational ≠ Rational.

This contradicts our assumption.

Thus, $\sqrt{2}+\sqrt{5}$ is an irrational number. Proved

2. Prove that $\sqrt{3}$ is an irrational number. **[*CBSE* 2019(30/3/1)]**

Sol.

19. Let us assume, if possible, that $\sqrt{3}$ is rational

Then, $\sqrt{3}$ can be expressed as $\frac{p}{q}$ where $(q \neq 0)$ and p, q are coprimes [$HCF(p,q)=1$]

$\therefore \sqrt{3} = \frac{p}{q}$ [$p,q \in Z$; $HCF(p,q)=1$]

On squaring both sides,

$3 = \frac{p^2}{q^2}$

$\Rightarrow p^2 = 3q^2$ ——(1)

3 divides p^2

$\therefore$ 3 divides p.

Then, p can be written as;

$p = 3a$ for some integer 'a'

On squaring,

$p^2 = 9a^2$

Put $p^2 = 3q^2$ from (1)

$\Rightarrow 3q^2 = 9a^2$

$\Rightarrow q^2 = 3a^2$.

3 divides q^2

$\therefore$ 3 divides q.

$\therefore$ 3 divides both p and q, 3 is a common factor of p and q

But, p and q are co-primes.

Therefore, our assumption is wrong

$\therefore \sqrt{3}$ is irrational.

[*Topper's Answer* 2019]

3. Show that there is no positive integer *n* for which $\sqrt{n-1}+\sqrt{n+1}$ is rational. **[*HOTS*]**

Sol. Let there be a positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ be a rational number.

$$\sqrt{n-1}+\sqrt{n+1} = \frac{p}{q}; \text{ where } p, q \text{ are integers and } q \neq 0 \qquad ...(i)$$

$\Rightarrow \quad \dfrac{1}{\sqrt{n-1}+\sqrt{n+1}} = \dfrac{q}{p}$

$\Rightarrow \quad \dfrac{\sqrt{n-1}-\sqrt{n+1}}{(\sqrt{n-1}+\sqrt{n+1})\times(\sqrt{n-1}-\sqrt{n+1})} = \dfrac{q}{p}$

$\Rightarrow \quad \dfrac{\sqrt{n-1}-\sqrt{n+1}}{(n-1)-(n+1)} = \dfrac{q}{p} \qquad \Rightarrow \quad \dfrac{\sqrt{n-1}-\sqrt{n+1}}{n-1-n-1} = \dfrac{q}{p}$

$\Rightarrow \quad \dfrac{\sqrt{n+1}-\sqrt{n-1}}{2} = \dfrac{q}{p}$

$\Rightarrow \quad \sqrt{n+1}-\sqrt{n-1} = \dfrac{2q}{p}$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$$\sqrt{n-1}+\sqrt{n+1}+\sqrt{n+1}-\sqrt{n-1} = \frac{p}{q}+\frac{2q}{p}$$

$\Rightarrow \quad 2\sqrt{n+1} = \dfrac{p^2+2q^2}{pq}$

$\Rightarrow \quad \sqrt{n+1} = \dfrac{p^2+2q^2}{2pq}$

$\Rightarrow \quad \sqrt{n+1}$ is rational number as $\dfrac{p^2+2q^2}{2pq}$ is rational.

$\Rightarrow \quad \sqrt{n+1}$ is perfect square of positive integer. ...(A)

Again subtracting (*ii*) from (*i*), we get

$$\sqrt{n-1}+\sqrt{n+1}-\sqrt{n+1}+\sqrt{n-1} = \frac{p}{q}-\frac{2q}{p}$$

$\Rightarrow \quad 2\sqrt{n-1} = \dfrac{p^2-2q^2}{pq}$

$\Rightarrow \quad \sqrt{n-1}$ is rational number as $\dfrac{p^2-2q^2}{2pq}$ is rational.

$\Rightarrow \quad \sqrt{n-1}$ is also perfect square of positive integer. ...(B)

From (A) and (B)

$\sqrt{n+1}$ and $\sqrt{n-1}$ are perfect squares of positive integer. It contradict the fact that two perfect squares differ at least by 3.

Hence, there is no positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.

4. Let a, b, c, k be rational numbers such that k is not a perfect cube. **[*HOTS*]**

If $a + bk^{1/3} + ck^{2/3}$ then prove that $a = b = c = 0$.

Sol. Given, $a + bk^{1/3} + ck^{2/3} = 0$ ...(*i*)

Multiplying both sides by $k^{1/3}$, we have

$$ak^{1/3} + bk^{2/3} + ck = 0 \quad \text{...}(ii)$$

Multiplying (*i*) by b and (*ii*) by c and then subtracting, we have

$\Rightarrow \quad (ab + b^2k^{1/3} + bck^{2/3}) - (ack^{1/3} + bck^{2/3} + c^2k) = 0$

$\Rightarrow \quad (b^2 - ac)k^{1/3} + ab - c^2k = 0$

$\Rightarrow \quad b^2 - ac = 0$ and $ab - c^2k = 0$ [Since $k^{1/3}$ is irrational]

$\Rightarrow \quad b^2 = ac \quad \text{and} \quad ab = c^2k$

$\Rightarrow \quad b^2 = ac \quad \text{and} \quad a^2b^2 = c^4k^2$

$\Rightarrow \quad a^2(ac) = c^4k^2$ [By putting $b^2 = ac$ in $a^2b^2 = c^4k^2$]

$\Rightarrow \quad a^3c - k^2c^4 = 0$

$\Rightarrow \quad (a^3 - k^2c^3)c = 0$

$\Rightarrow \quad a^3 - k^2c^3 = 0$ or $c = 0$

Now, if $a^3 - k^2c^3 = 0$

$\Rightarrow \quad k^2 = \frac{a^3}{c^3} \quad \Rightarrow \quad (k^2)^{1/3} = \left(\frac{a^3}{c^3}\right)^{1/3}$

$\Rightarrow \quad k^{2/3} = \frac{a}{c}$

This is impossible as $k^{2/3}$ is irrational and $\frac{a}{c}$ is rational.

$\therefore \quad a^3 - k^2c^3 \neq 0$

From other condition $c = 0$.

Substituting $c = 0$ in $b^2 - ac = 0$, we get $b = 0$

Substituting $b = 0$ and $c = 0$ in $a + bk^{1/3} + ck^{2/3} = 0$, we get $a = 0$

Hence, $a = b = c = 0$

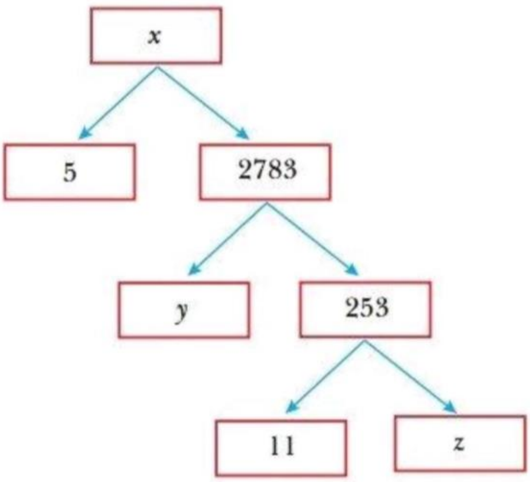
Case Study-based Questions

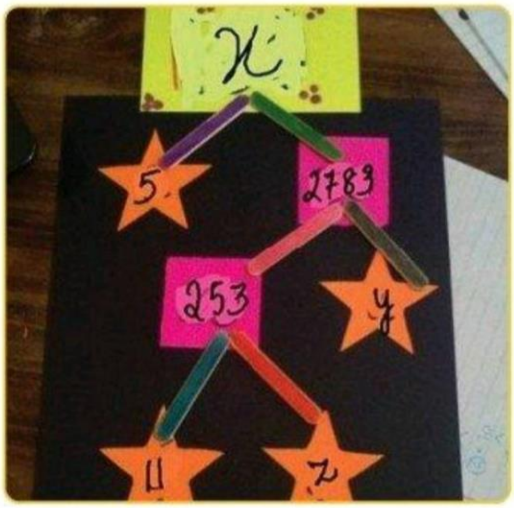
Each of the following questions are of 4 marks.

1. **Read the following and answer any four questions from (*i*) to (*v*).**

A Mathematics Exhibition is being conducted in your School and one of your friends is making a model of a factor tree. He has some difficulty and asks for your help in completing a quiz for the audience. **[*CBSE Question Bank*]**

Observe the following factor tree and answer the following:





(*i*) What will be the value of *x*?

(*a*) 15005 (*b*) 13915 (*c*) 56920 (*d*) 17429

(*ii*) What will be the value of *y*?

(*a*) 23 (*b*) 22 (*c*) 11 (*d*) 19

(*iii*) What will be the value of *z*?

(*a*) 22 (*b*) 23 (*c*) 17 (*d*) 19

(*iv*) According to Fundamental Theorem of Arithmetic 13915 is a

(*a*) Composite number (*b*) Prime number

(*c*) Neither prime nor composite (*d*) Even number

(*v*) The prime factorisation of 13915 is

(*a*) $5 \times 11^3 \times 13^2$ (*b*) $5 \times 11^3 \times 23^2$ (*c*) $5 \times 11^2 \times 23$ (*d*) $5 \times 11^2 \times 13^2$

Sol. (*i*) From the factor tree it is clear that

$$x = 5 \times 2783 = 13915$$

Hence option (*b*) is correct.

(*ii*) From the factor tree

$$y = \frac{2783}{253} = 11$$

Hence option (*c*) is correct.

(*iii*) From the factor tree

$$z = \frac{253}{11} = 23$$

Hence option (*b*) is correct.

(*iv*) ∵ The given number 13915 is not an even number and have more than two factors.

∴ According to fundamental theorem of arithmetic 13915 is a composite number.

Hence option (*a*) is correct.

(*v*) The prime factorisation of 13915

$$= 5 \times 11 \times 11 \times 23$$

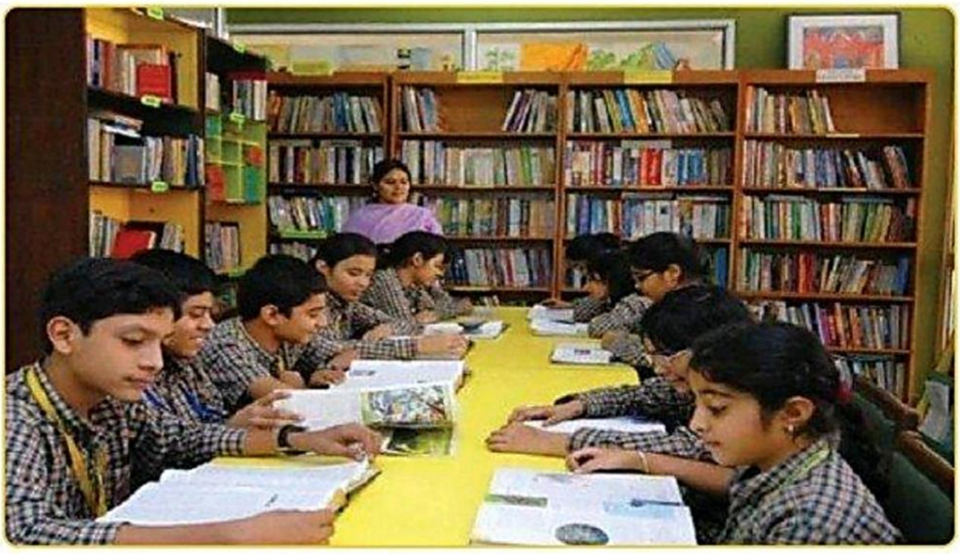
$$= 5 \times 11^2 \times 23$$

Hence option (*c*) is correct.

5	13915
11	2783
11	253
	23

2. Read the following and answer any four questions from (*i*) to (*v*).

To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections- section A and section B of grade X. There are 32 students in section A and 36 students in section B. ***[CBSE Question Bank]***



(i) What is the minimum number of books you will require for the class library, so that they can be distributed equally among students of Section A or Section B?

(a) 144 (b) 128 (c) 288 (d) 272

(ii) If the product of two positive integers is equal to the product of their HCF and LCM is true then, the HCF (32, 36) is

(a) 2 (b) 4 (c) 6 (d) 8

(iii) 36 can be expressed as a product of its primes as

(a) $2^2 \times 3^2$ (b) $2^1 \times 3^3$ (c) $2^3 \times 3^1$ (d) $2^0 \times 3^0$

(iv) $7 \times 11 \times 13 \times 15 + 15$ is a

(a) Prime number (b) Composite number

(c) Neither prime nor composite (d) None of the above

(v) If p and q are positive integers such that $p = ab^2$ and $q = a^2b$, where a, b are prime numbers, then the LCM (p, q) is

(a) ab (b) a^2b^2 (c) a^3b^2 (d) a^3b^3

Sol. (i) Minimum number of books required to distribute equally among students of both the sections = LCM(32, 36)

2	32, 36
2	16, 18
	8, 9

LCM (32, 36) = $2 \times 2 \times 8 \times 9 = 288$

Hence option (c) is correct.

(ii) It is given that

Product of two positive integers = HCF × LCM

So, $$\text{HCF} = \frac{\text{Product of two integers}}{\text{LCM}}$$

$$= \frac{32 \times 36}{288} = 4$$

Hence option (b) is correct.

(iii) Prime factorisation of 36 is

2	36
2	18
3	9
3	3
	1

$$36 = 2 \times 2 \times 3 \times 3$$
$$= 2^2 \times 3^2$$

Hence option (a) is correct.

(iv) Given expression is $7 \times 11 \times 13 \times 15 + 15$

$$= 15(7 \times 11 \times 13 + 1)$$
$$= 15 \times 1002$$

So, it is composite number.

Hence option (b) is correct.

(v) Given $p = ab^2$ and $q = a^2b$, where a, b are prime numbers.

$\because$ LCM of p and q is the highest power of the variables.

$\therefore$ LCM $(p, q) = a^2b^2$

Hence option (b) is correct.

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3. Read the following and answer any four questions from (*i*) to (*v*).

A seminar is being conducted by an Educational Organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. **[*CBSE Question Bank*]**

(*i*) In each room the same number of participants are to be seated and all of them being in the same subject, hence maximum number of participants that can accommodated in each room are

(*a*) 14 (*b*) 12 (*c*) 16 (*d*) 18

(*ii*) What is the minimum number of rooms required during the event?

(*a*) 11 (*b*) 31 (*c*) 41 (*d*) 21

(*iii*) The LCM of 60, 84 and 108 is

(*a*) 3780 (*b*) 3680 (*c*) 4780 (*d*) 4680

(*iv*) The product of HCF and LCM of 60,84 and 108 is

(*a*) 55360 (*b*) 35360 (*c*) 45500 (*d*) 45360

(*v*) 108 can be expressed as a product of its primes as

(*a*) $2^3 \times 3^2$ (*b*) $2^3 \times 3^3$ (*c*) $2^2 \times 3^2$ (*d*) $2^2 \times 3^3$

Sol. (*i*) Maximum number of participants that can be accommodated in each room

$= \text{HCF}(60, 84, 108)$

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$

$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$

So, $\text{HCF}(60, 84, 108) = 2^2 \times 3$

$= 2 \times 2 \times 3 = 12$

2	60
2	30
3	15
5	5
	1

2	84
2	42
3	21
7	7
	1

2	108
2	54
3	27
3	9
3	3
	1

Hence option (*b*) is correct.

(*ii*) Minimum number of rooms required during the event

$$= \frac{\text{Sum of all the students}}{\text{HCF of participants}} = \frac{60+84+108}{12} = 21$$

Hence option (*d*) is correct.

(*iii*) $\text{LCM}(60, 84, 108) = 2 \times 2 \times 3 \times 5 \times 7 \times 9$

$= 3780$

2	60, 84, 108
2	30, 42, 54
3	15, 21, 27
	5, 7, 9

Hence option (*a*) is correct.

(*iv*) The product of HCF and LCM of 60, 84 and 108

$= \text{HCF} \times \text{LCM}$

$= 12 \times 3780 = 45360$

Hence option (*d*) is correct.

(*v*) Prime factorisation of 108

$= 2 \times 2 \times 3 \times 3 \times 3$

$= 2^2 \times 3^3$

Hence option (*d*) is correct.

2	108
2	54
3	27
3	9
	3

PROFICIENCY EXERCISE

Objective Type Questions: [1 mark each]

1. Choose and write the correct option in each of the following questions.

(*i*) For some integer a, every odd integer is of the form

(*a*) $2a + 1$ (*b*) $2a$ (*c*) $a + 1$ (*d*) a

(*ii*) If the LCM of p and 18 is 36 and the HCF of p and 18 is 2 then p is equal to

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 1

(*iii*) Which of the following is an irrational number?

(*a*) $\frac{\sqrt{2}}{\sqrt{8}}$ (*b*) $\frac{\sqrt{63}}{\sqrt{7}}$ (*c*) $\frac{\sqrt{5}}{\sqrt{20}}$ (*d*) $\frac{\sqrt{3}}{3\sqrt{5}}$

(*iv*) Is $9 + \sqrt{2}$ an irrational number?

(*a*) Yes, because if $9+\sqrt{2}=\frac{a}{b}$, where a and b are integers and $b \neq 0$, then $\sqrt{2}=\frac{a-9b}{b}$, but $\sqrt{2}$ is an irrational number. So, $9+\sqrt{2} \neq \frac{a}{b}$.

(*b*) Yes, because if $9+\sqrt{2}=\frac{a}{b}$, where a and b are integers and $b \neq 0$, then $\sqrt{2}=\frac{9b+a}{b}$, but $\sqrt{2}$ is an irrational number. So, $9+\sqrt{2} \neq \frac{a}{b}$.

(*c*) No, because if $9+\sqrt{2}=\frac{a}{b}$, where a and b are integers and $b \neq 0$, then $\sqrt{2}=\frac{9b-a}{b}$, but $\sqrt{2}$ is an irrational number. So, $9+\sqrt{2} \neq \frac{a}{b}$.

(*d*) No, because if $9+\sqrt{2}=\frac{a}{b}$, where a and b are integers and $b \neq 0$, then $\sqrt{2}=\frac{9b+a}{b}$, but $\sqrt{2}$ is an irrational number. So, $9+\sqrt{2} \neq \frac{a}{b}$.

(*v*) The LCM of smallest two digit composite number and smallest composite number is

[*CBSE Sample Question Paper* 2020]

(*a*) 12 (*b*) 4 (*c*) 20 (*d*) 44

Very Short Answer Questions: [1 mark each]

2. Arnav has 40 cm long red and 84 cm long blue ribbon. He cuts each ribbon into pieces such that all pieces are of equal length. What is the length of each piece?

3. The LCM of two numbers is 9 times their HCF. The sum of LCM and HCF is 500. Find the HCF of two numbers. **[CBSE 2019 (*C*) (30/1/1)]**

4. Write whether $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplification gives an irrational or a rational number. **[*CBSE* 2018 (*C*) (30/1)]**

5. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$. **[*CBSE* 2019(30/4/2)]**

Short Answer Questions-I: [2 marks each]

6. Write whether every positive integer can be of the form $4q + 2$, where q is an integer. Justify your answer.

7. Can the numbers 4^n, n being a natural number end with the digit 5? Give reasons.

8. The HCF and LCM of two numbers are 9 and 360 respectively if one number is 45, find the other number. **[*CBSE Sample Question Paper* 2019]**

9. On a morning walk, three persons step out together and their steps measure 30 cm, 36 cm and 40 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps? **[*CBSE* 2019(30/3/1)]**

10. Show that $7-\sqrt{5}$ is irrational number, given that $\sqrt{5}$ is irrational number. **[*CBSE Sample Question Paper* 2019]**

Short Answer Questions-II: [3 marks each]

11. If the HCF (210, 55) is expressible in the form $210 \times 5 - 55y$, find y.

12. Three bulbs red, green and yellow flash at intervals of 80 seconds, 90 seconds and 110 seconds. All three flash together at 8:00 am. At what time will the three bulbs flash altogether again?

13. Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively.

14. Show that 9^n cannot end with digit 0 for any $n \in N$.

15. Prove that $\sqrt{2}$ is an irrational number.

16. Prove that $2 + 5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number. **[*CBSE* 2019(30/2/1)]**

Long Answer Questions: [5 marks each]

17. Prove that $\sqrt{5}$ is an irrational number and hence show that $3+\sqrt{5}$ is also an irrational number.

18. Show that $\sqrt{p}+\sqrt{q}$ is an irrational number, where p, q are primes.

Answers

1. (*i*) (*a*) (*ii*) (*c*) (*iii*) (*d*) (*iv*) (*a*) (*v*) (*c*)
2. 4 cm **3.** HCF = 50 **4.** Rational
5. 1.7, Any rational number between 1.41 and 1.73
6. No, because an integer can be written in the form $4q$, $4q + 1$, $4q + 2$, $4q + 3$.
7. No, because $4^n = (2 \times 2)^n = 2^n \times 2^n$, so the only primes in the factorisation of 4^n are 2 only, and not 5.
8. 72 **9.** 360 cm **11.** 19 **12.** 10:12 AM **13.** 63

Self-Assessment

Time allowed: 1 hour | Max. marks: 40

SECTION A

1. Choose and write the correct option in the following questions. (3 × 1 = 3)

(*i*) The sum of exponents of prime factors in the prime factorisation of 196 is

(*a*) 3 (*b*) 4 (*c*) 5 (*d*) 2

(*ii*) The LCM and HCF of two rational numbers are equal then the numbers must be

(*a*) prime (*b*) composite (*c*) not equal (*d*) equal

(*iii*) The product of a non zero rational and an irrational number is [*NCERT Exemplar*]

(*a*) always irrational (*b*) always rational

(*c*) rational or irrational (*d*) one

2. Solve the following questions. (2 × 1 = 2)

(*i*) If two positive integers p and q are written as $p = a^2b^3$ and $q = a^3b$; a, b are prime numbers then find HCF (p, q).

(*ii*) Given that HCF (135, 225) = 45, find the LCM (135, 225). [*CBSE* 2020(30/4/1)]

SECTION B

■ **Solve the following questions.** (4 × 2 = 8)

3. What is the least number that is divisible by all the numbers from 1 to 10?

4. Find the sum of $0.\overline{68} + 0.\overline{73}$.

5. Show that $5 + 2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number. [*CBSE* 2020(30/5/1)]

6. Given that $\sqrt{2}$ is irrational, prove that $(5+3\sqrt{2})$ is an irrational number. [*CBSE* 2018]

■ **Solve the following questions.** (4 × 3 = 12)

7. Given that $\sqrt{5}$ is irrational, prove that $2\sqrt{5} - 3$ is an irrational number. [*CBSE Sample Question Paper* 2021]

8. Find the LCM and HCF of 12, 15 and 21 by applying the prime factorisation method.

9. Find the LCM of $x^2 - 4$ and $x^4 - 16$.

10. Show that $3\sqrt{2}$ is an irrational number.

■ **Solve the following questions.** (3 × 5 = 15)

11. 144 cartons of coke cans and 90 cartons of pepsi cans are to be stacked in a canteen. If each stack is of the same height and is to contain carton of same drink. What would be the greatest number of cartons in each stack?

12. 105 donkeys, 140 cows and 175 goats have to be taken across a river. There is only one boat which will have to make many trips in order to do so. The lazy boatman has his own conditions for transporting them. He insists that he will take the same number of animals in every trip and they have to be of the same kind. He will naturally like to take the largest possible number each times, find how many animals went in each trip?

13. Prove that $\sqrt{7}$ is an irrational number.

Answers

1. (*i*) (*b*) (*ii*) (*d*) (*iii*) (*a*) **2.** (*i*) a^2b (*ii*) 675 **3.** 2520

4. $1.\overline{42}$ **8.** LCM = 420; HCF = 3 **9.** $(x^2 + 4)(x^2 - 4)$ **11.** 18 **12.** 35