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23 YEARS OF LEGACY

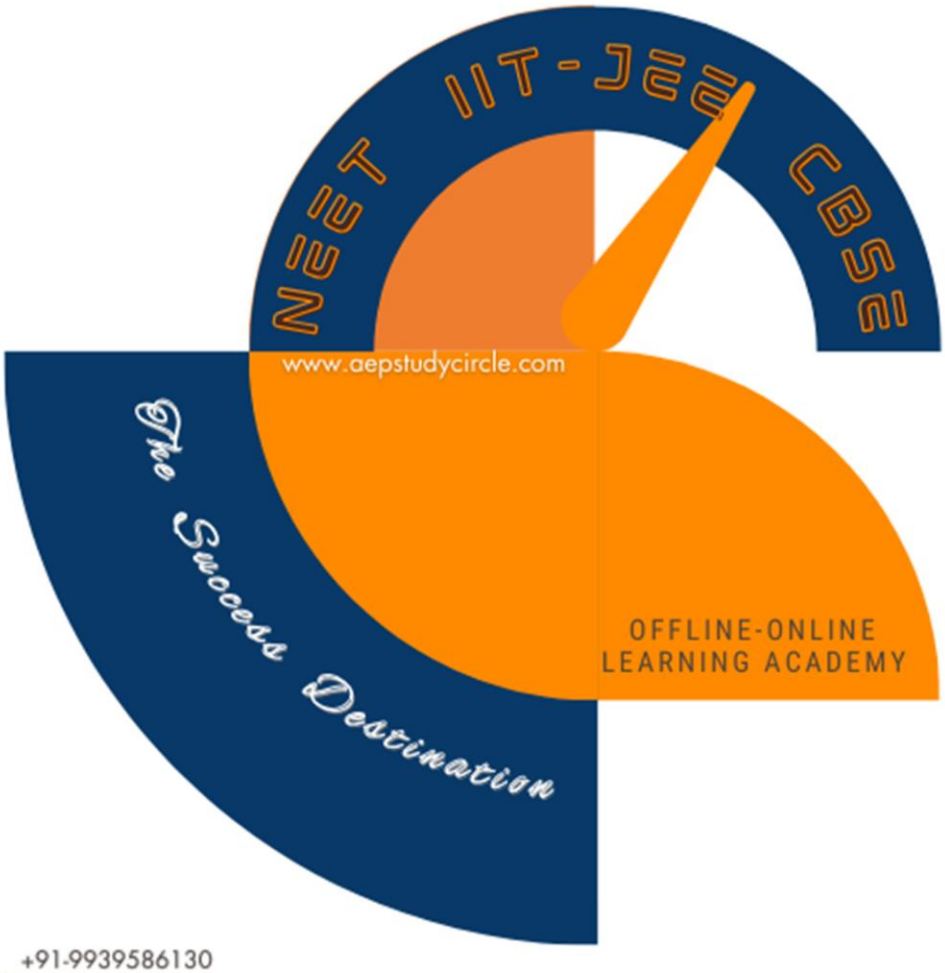
CBSE-X MATHEMATICS

EXAM DRILL : QUESTIONS AND SOLUTIONS

QUADRATIC EQUATIONS

Revision module 04

Prepare for excellence in CBSE Class 10th Mathematics with our best-in-class Topicwise Revision Module focusing on Quadratic Equations. This module is meticulously designed to provide a targeted, thorough, and efficient review of essential concepts, problem-solving techniques, and real-world applications..



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EXAM DRILL

FOR BOARD EXAM

(i) All the questions are compulsory.
(ii) The question paper consists of 35 questions.
(iii) Section A comprises of 16 questions of 1 mark each and 4 case study based questions of 4 marks each. Section B comprises of 5 questions of 2 marks each. Section C comprises of 6 questions of 3 marks each. Section D comprises of 4 questions of 5 marks each.
(iv) Use of calculators is not permitted.

Time: 3 hrs. Max. Marks: 80

SECTION - A

Multiple Choice Type Questions

1. If $\alpha = \frac{-b+\sqrt{b^2-12c}}{6}$ and $\beta = \frac{-b-\sqrt{b^2-12c}}{6}$ be two roots of the quadratic equation $kx^2 + bx + c = 0$, then find the value of k.

(a) 6 (b) 3 (c) 2 (d) 4

2. For which of the following values of p, the equation $9x^2 + 3px + 4 = 0$ has real and equal roots ?

(a) $p = \pm 9$ (b) $p = \pm 4$
(c) $p = \pm 12$ (d) $p = \pm 3$

3. For which of the following conditions the equation $m^2x^2 + 2mcx = (a^2 - c^2) - x^2$ has equal roots?

(a) $c^2/a^2 = 1 - m^2$ (b) $c^2 = a(1 + m^2)$
(c) $a = c\sqrt{1-m^2}$ (d) $c^2 = a^2(1 + m^2)$

4. $\sqrt{20+\sqrt{20+\sqrt{20+.....\infty}}} =$

(a) 4 (b) 5 (c) 9 (d) 20

5. Find the roots of $a^2x^2 - (a^2b^2 + 1)x + b^2 = 0$.

(a) a, b (b) $\frac{1}{a^2}, b^2$ (c) a^2b^2, b^2 (d) $\frac{1}{b^2}, a^2$

6. The nature of the roots of quadratic equation $21x^2 - 2x + 1/21 = 0$ is

(a) real and distinct (b) real and repeated
(c) not real (d) none of these

Fill in the Blanks

7. If the equation $x(x + 2c) = -ab$ has real and unequal roots, then the equation $x^2 - 2(a + b)x + c^2 = -(a^2 + b^2 + c^2)$ has _____ roots.

8. The value(s) of k, if $(k + 4)x^2 + (k + 1)x + 1 = 0$ has equal roots is/are _____ .

9. If 1 is a root of quadratic equation $ax^2 - 5(a - 1)x - 1 = 0$, then the value of a is _____ .

10. The roots of the equation $p^2q^2x^2 - q^2x - p^2x + 1 = 0$, are _____ .

VSA Type Questions

11. Two numbers differ by 4 and their product is 45. Find the numbers.

12. If a number is added to twice its square, then the resultant is 21. Find the number.

13. Find the value of k for which the quadratic equation $3x^2 + 7x + k = 0$ has real and equal roots.

14. Find the value of p for which the quadratic equation $x(x - 4) + p = 0$ has real and equal roots.

15. If 2 is a root of the equation $x^2 + kx + 12 = 0$ and the equation $x^2 + kx + q = 0$ has equal roots, find the value of q.

16. Check whether the quadratic equation $x^2 - x + 2 = 0$ has real roots or not. If yes, find the roots.

Case Study-Based Questions

Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. Nature of Roots

A quadratic equation can be defined as an equation of degree 2. This means that the highest exponent of the polynomial in it is 2. The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$. Every quadratic equation has two roots depending on the nature of its discriminant, $D = b^2 - 4ac$.

Based on the above information, answer the following questions.

(i) Which of the following quadratic equation have no real roots?
(a) $-4x^2 + 7x - 4 = 0$ (b) $-4x^2 + 7x - 2 = 0$
(c) $-2x^2 + 5x - 2 = 0$ (d) $3x^2 + 6x + 2 = 0$

(ii) Which of the following quadratic equation have rational roots?
(a) $x^2 + x - 1 = 0$ (b) $x^2 - 5x + 6 = 0$
(c) $4x^2 - 3x - 2 = 0$ (d) $6x^2 - x + 11 = 0$

(iii) Which of the following quadratic equation have irrational roots?
(a) $3x^2 + 2x + 2 = 0$ (b) $4x^2 - 7x + 3 = 0$
(c) $6x^2 - 3x - 5 = 0$ (d) $2x^2 + 3x - 2 = 0$

(iv) Which of the following quadratic equations have equal roots?
(a) $x^2 - 3x + 4 = 0$ (b) $2x^2 - 2x + 1 = 0$
(c) $5x^2 - 10x + 1 = 0$ (d) $9x^2 + 6x + 1 = 0$

(v) Which of the following quadratic equations has two distinct real roots?
(a) $x^2 + 3x + 1 = 0$ (b) $-x^2 + 3x - 3 = 0$
(c) $4x^2 + 8x + 4 = 0$ (d) $3x^2 + 6x + 4 = 0$

18. Quadratic in Day to Day Life

In our daily life we use quadratic formula as for calculating areas, determining a product's profit or formulating the speed of an object and many more.
Based on the above information, answer the following questions.

(i) If the roots of the quadratic equation are 2, –3, then find its equation.

(ii) If one root of the quadratic equation $2x^2 + kx + 1 = 0$ is $-1/2$, then find the value of k.

(iii) Find the roots of $16x^2 - 9 = 0$.

(iv) If $(x - 2)^2 + 19 = 0$ equals to $ax^2 + bx + c = 0$, then find the value of $a + b + c$.

(v) If one root of a quadratic equation is $\frac{1+\sqrt{5}}{7}$, then find its other root.

19. Factorization Method

Amit is preparing for his upcoming semester exam. For this, he has to practice the chapter of Quadratic Equations. So he started with factorization method. Let two linear factors of $ax^2 + bx + c$ be $(px + q)$ and $(rx + s)$.

$$\therefore \quad ax^2 + bx + c = (px + q)(rx + s)$$
$$= prx^2 + (ps + qr)x + qs.$$

(i) Find the roots of $6x^2 + x - 2 = 0$.

(ii) Solve for x : $2x^2 + x - 300 = 0$

(iii) Find the roots of $x^2 - 8x + 16 = 0$.

(iv) Find the roots of $6x^2 - 13x + 5 = 0$.

(v) Solve for x : $100x^2 - 20x + 1 = 0$

20. Basic Concepts of Quadratic Equation

If $p(x)$ is a quadratic polynomial *i.e.*, $p(x) = ax^2 + bx + c$, $a \neq 0$, then $p(x) = 0$ is called a quadratic equation.
Now, answer the following questions.

(i) Which of the following is correct about the quadratic equation $ax^2 + bx + c = 0$?
(a) a, b and c are real numbers, $c \neq 0$
(b) a, b and c are rational numbers, $a \neq 0$
(c) a, b and c are integers, a, b and $c \neq 0$
(d) a, b and c are real numbers, $a \neq 0$

(ii) The degree of a quadratic equation is
(a) 1 (b) 2
(c) 3 (d) other than 1

(iii) Which of the following is a quadratic equation?
(a) $x(x + 3) + 7 = 5x - 11$
(b) $(x - 1)^2 - 9 = (x - 4)(x + 3)$
(c) $x^2(2x + 1) - 4 = 5x^2 - 10$
(d) $x(x - 1)(x + 7) = x(6x - 9)$

(iv) Which of the following is incorrect about the quadratic equation $ax^2 + bx + c = 0$?
(a) If $a\alpha^2 + b\alpha + c = 0$, then $x = -\alpha$ is the solution of the given quadratic equation.
(b) The additive inverse of zeroes of the polynomial $ax^2 + bx + c$ is the roots of the given equation.
(c) If α is a root of the given quadratic equation, then its other root is $-\alpha$.
(d) All of these

(v) Which of the following is not a method of finding solutions of the given quadratic equation?
(a) Factorisation method
(b) Completing the square method
(c) Formula method
(d) None of these

SECTION - B

21. The altitude of a right-angled triangle is 8 more than its base. If the hypotenuse is 40 cm, then, find the length of the base.

22. Divide 12 into two parts such that the sum of their squares is 74.

23. The product of two positive numbers that differ by 7 is 408, then find the sum of those two numbers.

24. Find the value of c for which the quadratic equation $4x^2 - 2(c+1)x + (c+4) = 0$ has equal roots.

25. Solve for x by factorisation method :

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}; \; x \neq 0, -\left(a + \frac{b}{2}\right)$$

SECTION - C

26. Solve for x : $\frac{6}{x} - \frac{2}{x-1} = \frac{1}{x-2}$; $x \neq 0, 1, 2$

OR

₹ 6500 is divided equally among a certain number of persons. If there are 15 more persons, each will get ₹ 30 less. Find the original number of persons.

27. A farmer wishes to grow vegetables in a 100 m^2 rectangular garden. Since he has only 30 m barbed wire, he fences three sides of the rectangular garden letting compound wall of his house act as the fourth side fence. Find the dimensions of his garden.

OR

Three eighth of the students of a class opted for visiting an old age home. Sixteen students opted for having a nature walk. Square root of total number of students in the class opted for tree plantation in the school. The number of students who visited an old age home is same as the number of students who went for a nature walk and did tree plantation. Find the total number of students.

28. The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

29. If the list price of a toy is reduced by ₹ 2, a person can buy 2 toys more for ₹ 360. Find the original price of the toy.

30. Find the value of p for which the quadratic equation $(2p+1)x^2 - (7p+2)x + (7p-3) = 0$ has equal roots. Also, find these roots.

31. The numerator of a fraction is 4 less than its denominator. If 1 is added to the denominator, the fraction is decreased by 1/18. Find the fraction.

SECTION - D

32. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 sq. m more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park.

OR

A piece of cloth costs ₹ 200. If the piece of cloth was 5 m longer and each metre of cloth costed ₹ 2 less, the total cost of the piece would have remained unchanged. How long is the piece of cloth and what is its original rate per metre?

33. A certain group of students uses the internet services for a monthly charge of ₹ 4800. If 4 more students join the group, each person would pay ₹ 200 less. Find the number of students in the group in the beginning.

OR

A train travels 180 km at a uniform speed. If the speed had been 9 km/hour more, it would have taken 1 hour less for the same journey. Find the uniform speed of the train.

34. The sum of the areas of two squares is 640 m^2. If the difference in their perimeters be 64 m, find the sides of the two squares.

OR

The distance between Mumbai and Pune is 192 km. Travelling by Deccan Queen, it takes 48 minutes less than another train. Calculate the speed of the Deccan Queen if the speed of the two trains differ by 20 km/hr.

35. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/hr than the usual speed. Find the usual speed of the plane.

EXAM DRILL

SOLUTIONS

1. (b) : Given α and β be roots of the equation $kx^2 + bx + c = 0$.

We have, $\alpha = \frac{-b+\sqrt{b^2-12c}}{6}$ and $\beta = \frac{-b-\sqrt{b^2-12c}}{6}$

$\therefore \quad 2k = 6 \Rightarrow k = 3$

2. (b) : We have, $9x^2 + 3px + 4 = 0$

Here, $a = 9$, $b = 3p$ and $c = 4$.

$\therefore \quad D = b^2 - 4ac = (3p)^2 - 4(9)(4) = 9p^2 - 144$

The equation has real and equal roots, so $D = 0$

$\Rightarrow \quad 9p^2 - 144 = 0 \Rightarrow p^2 = \frac{144}{9} \Rightarrow p^2 = 16$

$\Rightarrow \quad p = \pm 4$

3. (d) : We have, $m^2x^2 + 2mcx = (a^2 - c^2) - x^2$

$\Rightarrow \quad (m^2 + 1)x^2 + 2mcx - a^2 + c^2 = 0$

Here, $A = m^2 + 1$, $B = 2mc$ and $C = -a^2 + c^2$.

$\therefore \quad D = B^2 - 4AC = 4m^2c^2 - 4(m^2 + 1)(c^2 - a^2)$

$= 4m^2c^2 - 4m^2c^2 + 4a^2m^2 - 4c^2 + 4a^2 = 4(a^2 - c^2 + a^2m^2)$

Since, the equation has equal roots, so $D = 0$

$\Rightarrow \quad 4(a^2 - c^2 + a^2m^2) = 0 \Rightarrow c^2 = a^2(1 + m^2)$

4. (b) : Let $x = \sqrt{20+\sqrt{20+\sqrt{20+\ldots\infty}}} \Rightarrow x = \sqrt{20+x}$

Squaring on both sides, we get

$x^2 = 20 + x \Rightarrow x^2 - x - 20 = 0$

$\Rightarrow \quad (x - 5)(x + 4) = 0 \Rightarrow x = 5$ or $x = -4$

But x is a positive quantity.

$\therefore \quad x = 5$

5. (b) : Given, $a^2x^2 - (a^2b^2 + 1)x + b^2 = 0$

$\Rightarrow \quad a^2x^2 - a^2b^2x - x + b^2 = 0 \Rightarrow a^2x(x - b^2) - 1(x - b^2) = 0$

$\Rightarrow \quad (a^2x - 1)(x - b^2) = 0$

$\Rightarrow \quad a^2x - 1 = 0$ or $x - b^2 = 0 \Rightarrow x = 1/a^2$ or $x = b^2$

$\therefore \quad 1/a^2, b^2$ are the required roots.

6. (b) : We have, $21x^2 - 2x + 1/21 = 0$

$\Rightarrow \quad 441x^2 - 42x + 1 = 0$

Here, $a = 441$, $b = -42$ and $c = 1$.

$\therefore \quad D = b^2 - 4ac = (-42)^2 - 4(441)(1) = 1764 - 1764 = 0$

Hence, both roots are real and repeated.

7. We have, $x(x + 2c) = -ab \Rightarrow x^2 + 2cx + ab = 0$ ...(i)

(i) has real and unequal roots, so $D = b^2 - 4ac > 0$

$\Rightarrow \quad 4c^2 - 4ab > 0 \Rightarrow c^2 > ab$

Also, we have $x^2 - 2(a + b)x + 2c^2 + a^2 + b^2 = 0$ ...(ii)

Here, $D = 4(a + b)^2 - 4(2c^2 + a^2 + b^2)$

$= 4(a^2 + b^2 + 2ab - 2c^2 - a^2 - b^2) = 8(ab - c^2) < 0 \quad [\because c^2 > ab]$

So, (ii) has no real roots.

8. For equal roots, discriminant = 0

$\therefore \quad (k + 1)^2 - 4(k + 4)(1) = 0$

$\Rightarrow \quad k^2 + 2k + 1 - 4k - 16 = 0 \Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow \quad (k - 5)(k + 3) = 0 \Rightarrow k = 5$ or $k = -3$

9. Given, $x = 1$ is root of the given equation, so it will satisfy the given equation.

$\therefore \quad a(1)^2 - 5(a - 1) \times 1 - 1 = 0$

$\Rightarrow \quad a - 5a + 5 - 1 = 0 \Rightarrow -4a = -4 \Rightarrow a = \frac{-4}{-4} = 1$

10. We have, $p^2q^2x^2 - q^2x - p^2x + 1 = 0$

$\Rightarrow \quad q^2x(p^2x - 1) - 1(p^2x - 1) = 0$

$\Rightarrow \quad (p^2x - 1)(q^2x - 1) = 0 \Rightarrow x = \frac{1}{p^2}$ or $x = \frac{1}{q^2}$

11. Let the numbers be x and $(x + 4)$.

According to the question, $x(x + 4) = 45$

$\Rightarrow \quad x^2 + 4x - 45 = 0 \Rightarrow x^2 + 9x - 5x - 45 = 0$

$\Rightarrow \quad x(x + 9) - 5(x + 9) = 0$

$\Rightarrow \quad (x + 9)(x - 5) = 0 \Rightarrow x + 9 = 0$ or $x - 5 = 0$

$\Rightarrow \quad x = -9$ or $x = 5$

If $x = -9$, numbers are $-9, -9 + 4$ *i.e.*, $-9, -5$

If $x = 5$, numbers are $5, 5 + 4$ *i.e.*, $5, 9$

12. Let the number be x.

According to question, $x + 2x^2 = 21$

$\Rightarrow \quad 2x^2 + x - 21 = 0 \Rightarrow 2x^2 - 6x + 7x - 21 = 0$

$\Rightarrow \quad 2x(x - 3) + 7(x - 3) = 0$

$\Rightarrow \quad (x - 3)(2x + 7) = 0 \Rightarrow x = 3$ or $x = \frac{-7}{2}$

13. The given quadratic equation is $3x^2 + 7x + k = 0$ ...(i)

Here, $a = 3$, $b = 7$ and $c = k$.

$\therefore \quad D = b^2 - 4ac = (7)^2 - 4(3)(k) = 49 - 12k$

$\because$ Equation (i) has real and equal roots, so $D = 0$.

$\Rightarrow \quad 49 - 12k = 0 \Rightarrow 12k = 49 \Rightarrow k = \frac{49}{12}$

14. The given quadratic equation is

$x(x - 4) + p = 0 \Rightarrow x^2 - 4x + p = 0$

Here, $a = 1$, $b = -4$ and $c = p$.

For real and equal roots : $D = b^2 - 4ac = 0$

$\Rightarrow$ $(-4)^2 - 4(1)(p) = 0$

$\Rightarrow$ $16 - 4p = 0 \Rightarrow 4p = 16 \Rightarrow p = 4$

15. Since 2 is a root of the equation $x^2 + kx + 12 = 0$.

$\therefore$ $(2)^2 + k(2) + 12 = 0 \Rightarrow 4 + 2k + 12 = 0 \Rightarrow 2k + 16 = 0$

$\Rightarrow$ $k = -16/2 \Rightarrow k = -8$

Putting $k = -8$ in the equation $x^2 + kx + q = 0$, we get

$x^2 - 8x + q = 0$ ...(i)

The equation (i) will have equal roots, if discriminant = 0

$\Rightarrow$ $(-8)^2 - 4(1)q = 0$

$\Rightarrow$ $64 - 4q = 0 \Rightarrow q = 64/4 \Rightarrow q = 16$

16. We have, $x^2 - x + 2 = 0$

Here, $a = 1$, $b = -1$ and $c = 2$

$\therefore$ $D = b^2 - 4ac = (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7 < 0$

$\therefore$ The given quadratic equation does not have real roots.

17. (i) (a) : To have no real roots, discriminant ($D = b^2 - 4ac$) should be < 0.

(a) $D = 7^2 - 4(-4)(-4) = 49 - 64 = -15 < 0$

(b) $D = 7^2 - 4(-4)(-2) = 49 - 32 = 17 > 0$

(c) $D = 5^2 - 4(-2)(-2) = 25 - 16 = 9 > 0$

(d) $D = 6^2 - 4(3)(2) = 36 - 24 = 12 > 0$

(ii) (b) : To have rational roots, discriminant ($D = b^2 - 4ac$) should be > 0 and also a perfect square.

(a) $D = 1^2 - 4(1)(-1) = 1 + 4 = 5$, which is not a perfect square.

(b) $D = (-5)^2 - 4(1)(6) = 25 - 24 = 1$, which is a perfect square.

(c) $D = (-3)^2 - 4(4)(-2) = 9 + 32 = 41$, which is not a perfect square.

(d) $D = (-1)^2 - 4(6)(11) = 1 - 264 = -263$, which is not a perfect square.

(iii) (c) : To have irrational roots, discriminant ($D = b^2 - 4ac$) should be > 0 but not a perfect square.

(a) $D = 2^2 - 4(3)(2) = 4 - 24 = -20 < 0$

(b) $D = (-7)^2 - 4(4)(3) = 49 - 48 = 1 > 0$ and also a perfect square.

(c) $D = (-3)^2 - 4(6)(-5) = 9 + 120 = 129 > 0$ and not a perfect square.

(d) $D = 3^2 - 4(2)(-2) = 9 + 16 = 25 > 0$ and also a perfect square.

(iv) (d) : To have equal roots, discriminant ($D = b^2 - 4ac$) should be = 0.

(a) $D = (-3)^2 - 4(1)(4) = 9 - 16 = -7 < 0$

(b) $D = (-2)^2 - 4(2)(1) = 4 - 8 = -4 < 0$

(c) $D = (-10)^2 - 4(5)(1) = 100 - 20 = 80 > 0$

(d) $D = 6^2 - 4(9)(1) = 36 - 36 = 0$

(v) (a) : To have two distinct real roots, discriminant ($D = b^2 - 4ac$) should be > 0.

(a) $D = 3^2 - 4(1)(1) = 9 - 4 = 5 > 0$

(b) $D = 3^2 - 4(-1)(-3) = 9 - 12 = -3 < 0$

(c) $D = 8^2 - 4(4)(4) = 64 - 64 = 0$

(d) $D = 6^2 - 4(3)(4) = 36 - 48 = -12 < 0$

18. (i) Roots of the quadratic equation are 2 and –3.

$\therefore$ The required quadratic equation is

$(x - 2)(x + 3) = 0 \Rightarrow x^2 + x - 6 = 0$

(ii) We have, $2x^2 + kx + 1 = 0$

Since, $-1/2$ is the root of the equation, so it will satisfy the given equation.

$$\therefore \quad 2\left(-\frac{1}{2}\right)^2 + k\left(-\frac{1}{2}\right) + 1 = 0 \Rightarrow 1 - k + 2 = 0 \Rightarrow k = 3$$

(iii) We have, $16x^2 - 9 = 0$ ...(i)

$$\Rightarrow \quad x^2 = \frac{9}{16} \Rightarrow x = \frac{\pm 3}{4}$$

$\Rightarrow$ Roots of (i) are $\frac{3}{4}$ and $\frac{-3}{4}$.

(iv) The given equation is $(x - 2)^2 + 19 = 0$

$\Rightarrow$ $x^2 - 4x + 4 + 19 = 0 \Rightarrow x^2 - 4x + 23 = 0$

(v) If one root of a quadratic equation is irrational, then its other root is also irrational and also its conjugate *i.e.*, if one root is $p + \sqrt{q}$, then its other root is $p - \sqrt{q}$.

19. (i) We have, $6x^2 + x - 2 = 0$

$\Rightarrow$ $6x^2 - 3x + 4x - 2 = 0$

$\Rightarrow$ $(3x + 2)(2x - 1) = 0$

$$\Rightarrow \quad x = \frac{1}{2}, \frac{-2}{3}$$

(ii) $2x^2 + x - 300 = 0$

$\Rightarrow$ $2x^2 - 24x + 25x - 300 = 0$

$\Rightarrow$ $(x - 12)(2x + 25) = 0$

$$\Rightarrow \quad x = 12, \frac{-25}{2}$$

(iii) $x^2 - 8x + 16 = 0$

$\Rightarrow$ $(x - 4)^2 = 0 \Rightarrow (x - 4)(x - 4) = 0 \Rightarrow x = 4, 4$

(iv) $6x^2 - 13x + 5 = 0$

$\Rightarrow$ $6x^2 - 3x - 10x + 5 = 0 \Rightarrow (2x - 1)(3x - 5) = 0$

$$\Rightarrow \quad x = \frac{1}{2}, \frac{5}{3}$$

(v) $100x^2 - 20x + 1 = 0$

$$\Rightarrow \quad (10x - 1)^2 = 0 \quad \Rightarrow \quad x = \frac{1}{10}, \frac{1}{10}$$

20. **(i) (d)** **(ii) (b)**

(iii) (a) : $x(x+3)+7=5x-11$

$\Rightarrow x^2+3x+7=5x-11$

$\Rightarrow x^2-2x+18=0$ is a quadratic equation.

(b) $(x-1)^2-9=(x-4)(x+3)$

$\Rightarrow x^2-2x-8=x^2-x-12$

$\Rightarrow x-4=0$ is not a quadratic equation.

(c) $x^2(2x+1)-4=5x^2-10$

$\Rightarrow 2x^3+x^2-4=5x^2-10$

$\Rightarrow 2x^3-4x^2+6=0$ is not a quadratic equation.

(d) $x(x-1)(x+7)=x(6x-9)$

$\Rightarrow x^3+6x^2-7x=6x^2-9x$

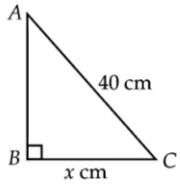
$\Rightarrow x^3+2x=0$ is not a quadratic equation.

(iv) (d) **(v) (d)**

21. Let ΔABC is the given triangle.

Let base, $BC = x$ cm, then altitude, $AB = (x+8)$ cm

By Pythagoras theorem, we have



$(AB)^2+(BC)^2=(AC)^2$

$\Rightarrow (x+8)^2+x^2=40^2$

$\Rightarrow x^2+64+16x+x^2=1600$

$\Rightarrow 2x^2+16x-1536=0$

$\Rightarrow x^2+8x-768=0$

$\Rightarrow x^2+32x-24x-768=0 \Rightarrow x(x+32)-24(x+32)=0$

$\Rightarrow (x+32)(x-24)=0 \Rightarrow x=-32$ or $x=24$

But side of a triangle can't be negative.

$\therefore \quad x=24$

22. Let the first part be x, then the second part will be $12-x$.

According to the given condition,

$x^2+(12-x)^2=74 \Rightarrow x^2+144+x^2-24x-74=0$

$\Rightarrow 2x^2-24x+70=0 \Rightarrow x^2-12x+35=0$

$\Rightarrow x^2-7x-5x+35=0 \Rightarrow x(x-7)-5(x-7)=0$

$\Rightarrow (x-7)(x-5)=0 \Rightarrow x-7=0$ or $x-5=0$

$\Rightarrow x=7$ or $x=5$

$\therefore$ Two parts of 12 are 7 and 5.

23. Let one number be x, then other number will be $x-7$.

According to question, $x(x-7)=408 \Rightarrow x^2-7x-408=0$

$\Rightarrow x^2-24x+17x-408=0 \Rightarrow x(x-24)+17(x-24)=0$

$\Rightarrow (x-24)(x+17)=0 \Rightarrow x=24$ or $x=-17$ (rejected)

Thus, one number is 24 and other number is 17.

Sum of numbers $= 24+17=41$

24. Given, $4x^2-2(c+1)x+(c+4)=0$

Here, $A=4$, $B=-2(c+1)$ and $C=c+4$

Now, $D=B^2-4AC$

$=\{-2(c+1)\}^2-4\times4\times(c+4)=4(c^2+2c+1)-16(c+4)$

$=4c^2+8c+4-16c-64=4c^2-8c-60$

For equal roots, $D=0$

$\therefore \quad 4c^2-8c-60=0 \Rightarrow c^2-2c-15=0$

$\Rightarrow (c+3)(c-5)=0 \Rightarrow c=-3$ or $c=5$

25. Given, $\dfrac{1}{2a+b+2x}=\dfrac{1}{2a}+\dfrac{1}{b}+\dfrac{1}{2x}$

$\Rightarrow \dfrac{1}{2a+b+2x}-\dfrac{1}{2x}=\dfrac{1}{2a}+\dfrac{1}{b}$

$\Rightarrow \dfrac{2x-2a-b-2x}{2x(2a+b+2x)}=\dfrac{b+2a}{2ab}$

$\Rightarrow \dfrac{-(2a+b)}{2x(2a+b+2x)}=\dfrac{b+2a}{2ab} \Rightarrow \dfrac{-1}{x(2a+b+2x)}=\dfrac{1}{ab}$

$\Rightarrow 2x^2+2ax+bx+ab=0 \Rightarrow 2x(x+a)+b(x+a)=0$

$\Rightarrow (x+a)(2x+b)=0 \Rightarrow x=-a$ or $x=\dfrac{-b}{2}$

26. Given, $\dfrac{6}{x}-\dfrac{2}{x-1}=\dfrac{1}{x-2} \Rightarrow \dfrac{6x-6-2x}{x(x-1)}=\dfrac{1}{x-2}$

$\Rightarrow \dfrac{4x-6}{x^2-x}=\dfrac{1}{x-2} \Rightarrow 4x^2-6x-8x+12=x^2-x$

$\Rightarrow 4x^2-14x+12=x^2-x$

$\Rightarrow 3x^2-13x+12=0 \Rightarrow 3x^2-9x-4x+12=0$

$\Rightarrow 3x(x-3)-4(x-3)=0 \Rightarrow (x-3)(3x-4)=0$

$\Rightarrow x-3=0$ or $3x-4=0$

$\Rightarrow x=3$ or $x=4/3$

OR

Let the number of persons in 1st condition is x and in 2nd condition is $(x+15)$.

Amount to be divided = ₹ 6500

According to the question, $\dfrac{6500}{x}-\dfrac{6500}{x+15}=30$

$\Rightarrow \dfrac{6500x+97500-6500x}{x(x+15)}=\dfrac{30}{1}$

$\Rightarrow 30x^2+450x=97500 \Rightarrow 30x^2+450x-97500=0$

$\Rightarrow x^2+15x-3250=0 \Rightarrow x^2+65x-50x-3250=0$

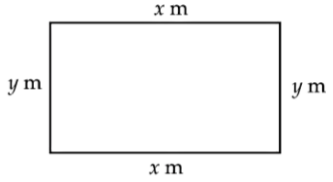
$\Rightarrow x(x+65)-50(x+65)=0 \Rightarrow (x+65)(x-50)=0$

$\Rightarrow x+65=0$ or $x-50=0 \Rightarrow x=-65$ or $x=50$

$\because$ Number of persons cannot be negative

$\therefore$ Original number of persons = 50.

27. Let the length of one side of garden be x m and other side be y m. Then,



$x+y+x=30$

$\Rightarrow y=30-2x$ ...(i)

Given, area of the vegetable garden $= 100\ m^2$

$\Rightarrow xy=100$

$\Rightarrow x(30-2x)=100$ [Using (i)]

$\Rightarrow 30x-2x^2=100 \Rightarrow 15x-x^2=50$

$\Rightarrow x^2-15x+50=0 \Rightarrow x^2-10x-5x+50=0$

$\Rightarrow$ $(x-10)(x-5)=0 \Rightarrow x=5$ or 10

When $x=5$, then $y=30-2\times5=20$ [Using (i)]

When $x=10$, then $y=30-2\times10=10$ [Using (i)]

Hence, the dimensions of the vegetable garden are 5 m and 20 m or 10 m and 10 m.

OR

Let x be the total number of students of the class.

Number of students opted for visiting an old age home $=\frac{3}{8}x$.

Number of students opted for having a nature walk = 16.

Number of students opted for tree plantation in the school $=\sqrt{x}$.

According to the given condition,

$$\frac{3}{8}x=16+\sqrt{x} \Rightarrow 3x=128+8\sqrt{x}$$

$\Rightarrow$ $3y^2=128+8y$, where $\sqrt{x}=y$

$\Rightarrow$ $3y^2-8y-128=0 \Rightarrow 3y^2-24y+16y-128=0$

$\Rightarrow$ $3y(y-8)+16(y-8)=0 \Rightarrow (y-8)(3y+16)=0$

$\Rightarrow$ $y-8=0$ or $3y+16=0$

$\Rightarrow$ $y=8$ or $y=-\frac{16}{3} \Rightarrow \sqrt{x}=8 \quad \left[\because \sqrt{x}\neq-\frac{16}{3}\right]$

$\Rightarrow$ $x=64$

Hence, the total number of students of the class is 64.

28. Let the numerator of the fraction = x

Then denominator of the fraction = $2x+1$

$\therefore$ Fraction $=\frac{x}{2x+1}$ and its reciprocal $=\frac{2x+1}{x}$

According to given condition, $\frac{x}{2x+1}+\frac{2x+1}{x}=2\frac{16}{21}$

$\Rightarrow$ $\frac{x^2+4x^2+1+4x}{2x^2+x}=\frac{58}{21} \Rightarrow \frac{5x^2+1+4x}{2x^2+x}=\frac{58}{21}$

$\Rightarrow$ $116x^2+58x=105x^2+84x+21$

$\Rightarrow$ $116x^2+58x-105x^2-84x-21=0$

$\Rightarrow$ $11x^2-26x-21=0 \Rightarrow 11x^2-33x+7x-21=0$

$\Rightarrow$ $11x(x-3)+7(x-3)=0 \Rightarrow (x-3)(11x+7)=0$

$\Rightarrow$ $x-3=0$ or $11x+7=0 \Rightarrow x=3$ or $x=-7/11$

$\therefore$ $x=3$ (Neglecting negative value)

$\therefore$ Fraction $=\frac{x}{2x+1}=\frac{3}{6+1}=\frac{3}{7}$

29. Let the original price of the toy = ₹ x

Then the reduced price of the toy = ₹ $(x-2)$

According to the question,

$$\frac{360}{x-2}-\frac{360}{x}=2 \quad \left(\because \text{Number of toys}=\frac{\text{Total amount}}{\text{Price of 1 toy}}\right)$$

$\Rightarrow$ $\frac{360x-360x+720}{x(x-2)}=2 \Rightarrow \frac{720}{x(x-2)}=\frac{2}{1}$

$\Rightarrow$ $x(x-2)=360$

$\Rightarrow$ $x^2-2x-360=0 \Rightarrow x^2-20x+18x-360=0$

$\Rightarrow$ $x(x-20)+18(x-20)=0 \Rightarrow (x-20)(x+18)=0$

$\Rightarrow$ $x-20=0$ or $x+18=0$

$\Rightarrow$ $x=20$ or $x=-18$

$\therefore$ $x=20$ [$\because$ Price cannot be negative]

$\therefore$ Original price of the toy = ₹ 20

30. Given, $(2p+1)x^2-(7p+2)x+(7p-3)=0$ ...(i)

$\because$ Roots are equal. $\therefore$ $D=0$

$\Rightarrow$ $(-(7p+2))^2-4(2p+1)(7p-3)=0$

$\Rightarrow$ $49p^2+4+28p-4(14p^2+7p-6p-3)=0$

$\Rightarrow$ $49p^2+28p+4-56p^2-4p+12=0$

$\Rightarrow$ $7p^2-24p-16=0 \Rightarrow 7p^2+4p-28p-16=0$

$\Rightarrow$ $p(7p+4)-4(7p+4)=0 \Rightarrow (p-4)(7p+4)=0$

$\Rightarrow$ $p=4$ or $p=\frac{-4}{7}$

When $p=4$, (i) becomes $9x^2-30x+25=0$

$\Rightarrow$ $(3x)^2-2(3x)(5)+(5)^2=0$

$\Rightarrow$ $(3x-5)^2=0 \Rightarrow x=\frac{5}{3},\frac{5}{3}$

When $p=\frac{-4}{7}$, (i) becomes

$$\frac{-x^2}{7}+2x-7=0 \Rightarrow x^2-14x+49=0$$

$\Rightarrow$ $(x-7)^2=0 \Rightarrow x=7,7$

Thus, equal roots of given equation are either 5/3 or 7.

31. Let the denominator of the fraction = x

$\therefore$ Numerator of the fraction = $x-4$

$\Rightarrow$ Fraction $=\frac{x-4}{x}$

According to question,

$\frac{x-4}{x+1}=\frac{x-4}{x}-\frac{1}{18} \Rightarrow \frac{x-4}{x}-\frac{x-4}{x+1}=\frac{1}{18}$

$\Rightarrow$ $(x-4)\left[\frac{1}{x}-\frac{1}{x+1}\right]=\frac{1}{18} \Rightarrow (x-4)\left[\frac{x+1-x}{x(x+1)}\right]=\frac{1}{18}$

$\Rightarrow$ $18(x-4)=x(x+1) \Rightarrow 18x-72=x^2+x$

$\Rightarrow$ $x^2-17x+72=0 \Rightarrow x^2-9x-8x+72=0$

$\Rightarrow$ $x(x-9)-8(x-9)=0 \Rightarrow (x-8)(x-9)=0$

$\Rightarrow$ $x=8$ or $x=9$

But $x=8$ is not possible $\therefore$ $x=9$

Hence, the fraction $\frac{x-4}{x}$ is $\frac{5}{9}$.

32. Let breadth of rectangular park = x m

Then, length of rectangular park = $(x+3)$m

Now, area of rectangular park = $x(x+3)=(x^2+3x)$m^2

Given, base of triangular park = Breadth of the rectangular park

$\therefore$ Base of triangular park = x m

and also it is given that altitude of triangular park = 12 m

$\therefore$ Area of triangular park = $\frac{1}{2} \times x \times 12 = 6x$ m^2

According to the question,

Area of rectangular park = 4 + Area of triangular park

$\Rightarrow x^2 + 3x = 4 + 6x \Rightarrow x^2 + 3x - 6x - 4 = 0$

$\Rightarrow x^2 - 3x - 4 = 0 \Rightarrow x^2 - 4x + x - 4 = 0$

$\Rightarrow x(x - 4) + 1(x - 4) = 0 \Rightarrow (x - 4)(x + 1) = 0$

$\Rightarrow x - 4 = 0$ or $x + 1 = 0 \Rightarrow x = 4$ or $x = -1$

Since, breadth cannot be negative.

$\therefore x = 4$

Hence, breadth of the rectangular park = 4 m

and length of the rectangular park = $x + 3 = 4 + 3 = 7$ m.

OR

Let the length of piece of cloth = x m

Increased length of piece of cloth = $(x + 5)$m

Total cost of piece of cloth = ₹ 200

According to the question,

$$\frac{200}{x} - \frac{200}{x+5} = 2 \qquad \left[\because \text{Rate per metre} = \frac{\text{Total cost}}{\text{Length}}\right]$$

$$\Rightarrow \frac{200x + 1000 - 200x}{x(x+5)} = 2$$

$\Rightarrow 1000 = 2x^2 + 10x \Rightarrow 2x^2 + 10x - 1000 = 0$

$\Rightarrow x^2 + 5x - 500 = 0 \Rightarrow x^2 + 25x - 20x - 500 = 0$

$\Rightarrow x(x + 25) - 20(x + 25) = 0 \Rightarrow (x + 25)(x - 20) = 0$

$\Rightarrow x + 25 = 0$ or $x - 20 = 0 \Rightarrow x = -25$ or $x = 20$

But, length can never be negative.

$\therefore$ Length of cloth = 20 m

and rate per metre = ₹ $\frac{200}{20}$ = ₹ 10.

33 Let the number of students in the group in the beginning be x.

Total internet service charges for x students = ₹ 4800

$\therefore$ Internet service charges for each student = ₹ $\frac{4800}{x}$

It is given that 4 more students join the group.

$\therefore$ The number of students in group for internet service = $(x + 4)$

Now, the internet service charges for each student = ₹ $\frac{4800}{x+4}$

According to question, $\frac{4800}{x} - \frac{4800}{x+4} = 200$

$$\Rightarrow \frac{4800x + 19200 - 4800x}{x(x+4)} = 200$$

$\Rightarrow 19200 = 200(x^2 + 4x) \Rightarrow 96 = x^2 + 4x$

$\Rightarrow x^2 + 4x - 96 = 0 \Rightarrow x^2 + 12x - 8x - 96 = 0$

$\Rightarrow x(x + 12) - 8(x + 12) = 0 \Rightarrow (x - 8)(x + 12) = 0$

$\Rightarrow x - 8 = 0$ or $x + 12 = 0 \Rightarrow x = 8$ or $x = -12$

But number of students cannot be negative

$\therefore x = 8$

Hence, the number of students in the group in the beginning is 8.

OR

Let the speed of the train be x km/hour.

When the speed is 9 km/hour more, then the new speed of the train is $(x + 9)$ km/hour.

Time taken by the train with speed x km/hour for a journey of 180 km = $\frac{180}{x}$ hours

Time taken by the train with new speed $(x + 9)$ km/hour for a journey of 180 km = $\frac{180}{(x+9)}$ hours

According to the question, $\frac{180}{x} - \frac{180}{x+9} = 1$

$$\Rightarrow 180\left[\frac{1}{x} - \frac{1}{x+9}\right] = 1 \Rightarrow 180\left[\frac{x+9-x}{x(x+9)}\right] = 1$$

$\Rightarrow 180 \times 9 = x(x + 9) \Rightarrow x^2 + 9x - 1620 = 0$

$\Rightarrow x^2 + 45x - 36x - 1620 = 0 \Rightarrow x(x + 45) - 36(x + 45) = 0$

$\Rightarrow (x + 45)(x - 36) = 0 \Rightarrow x + 45 = 0$ or $x - 36 = 0$

$\Rightarrow x = -45$ or $x = 36$

But, speed can't be negative.

$\therefore x = 36$

Hence, the uniform speed of the train is 36 km/hour.

34. Let x and y be the sides of two squares, respectively such that $x > y$, where x is the side of the first square and y is the side of the second square.

$\therefore$ Area of the first square + Area of the second square = 640 m^2

$\Rightarrow x^2 + y^2 = 640$ ...(i)

Again, it is given that the difference of their perimeters = 64 m

$\Rightarrow 4x - 4y = 64 \Rightarrow x = 16 + y$ ...(ii)

From (i) and (ii), we have, $(16 + y)^2 + y^2 = 640$

$\Rightarrow 256 + y^2 + 32y + y^2 = 640 \Rightarrow 2y^2 + 32y - 384 = 0$

$\Rightarrow y^2 + 16y - 192 = 0 \Rightarrow y^2 + 24y - 8y - 192 = 0$

$\Rightarrow y(y + 24) - 8(y + 24) = 0 \Rightarrow (y + 24)(y - 8) = 0$

$\Rightarrow y + 24 = 0$ or $y - 8 = 0 \Rightarrow y = -24$ or $y = 8$

But, side of a square can't be negative. $\therefore y = 8$

When $y = 8$, then from (ii), we get $x = 16 + 8 = 24$.

Hence, the sides of the two squares are 24 m and 8 m respectively.

OR

Let the speed of Deccan Queen = x km/hr

and speed of other train = $(x - 20)$ km/hr

Time taken by Deccan Queen = $\frac{192}{x}$ hr

and time taken by other train = $\frac{192}{(x-20)}$ hr

According to the question, $\frac{192}{(x-20)} - \frac{192}{x} = \frac{48}{60}$ or $\frac{4}{5}$

$\Rightarrow \quad \frac{192x - 192x + 3840}{x(x-20)} = \frac{4}{5}$

$\Rightarrow \quad 5(3840) = 4x(x - 20) \Rightarrow 19200 = 4x^2 - 80x$

$\Rightarrow \quad 4x^2 - 80x - 19200 = 0 \Rightarrow x^2 - 20x - 4800 = 0$

$\Rightarrow \quad x^2 - 80x + 60x - 4800 = 0 \Rightarrow x(x - 80) + 60(x - 80) = 0$

$\Rightarrow \quad (x - 80)(x + 60) = 0 \Rightarrow x - 80 = 0$ or $x + 60 = 0$

$\Rightarrow \quad x = 80$ or $x = -60$

As speed can never be negative. $\therefore \; x = 80$

$\therefore$ Speed of Deccan Queen = 80 km/hr.

35. Let the usual speed of the plane be x km/hr.

$\therefore$ Time taken to travel 1500 km at x km/hr

$= \frac{1500}{x}$ hour

Increased speed of the plane = $(x + 250)$ km/hr

$\therefore$ Time taken to travel 1500 km at $(x + 250)$ km/hr

$= \frac{1500}{x+250}$ hour

According to question,

$\frac{1500}{x} - \frac{1500}{x+250} = \frac{30}{60} \Rightarrow 1500\left(\frac{x+250-x}{x(x+250)}\right) = \frac{1}{2}$

$\Rightarrow \quad 2 \times 1500 \times 250 = x^2 + 250x$

$\Rightarrow \quad x^2 + 250x - 750000 = 0$

$\Rightarrow \quad x^2 + 1000x - 750x - 750000 = 0$

$\Rightarrow \quad x(x + 1000) - 750(x + 1000) = 0$

$\Rightarrow \quad (x + 1000)(x - 750) = 0$

$\Rightarrow \quad x = 750$ or $x = -1000$ (But speed can't be negative)

$\therefore \quad x = 750$

Hence, usual speed of the plane is 750 km/hr.