

SURFACE AREA AND VOLUME

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**SURFACE AREA
VOLUME**

MATHEMATICS



Geometric Solids Mastery: Immerse yourself in the study of geometric solids, including cubes, cuboids, cylinders, cones, and spheres. Understand the properties and characteristics of each solid, laying the groundwork for surface area and volume calculations.



Surface Area Computation: Master the art of calculating the surface area of various solids. From the faces of cubes to the curved surfaces of cylinders, our study module guides you through step-by-step procedures, ensuring a comprehensive understanding.



Volume Calculation Techniques: Explore the world of volumes, delving into the methods for calculating the space occupied by different solids. From the cubic volume of prisms to the conical and spherical volumes, grasp the intricacies of volume computation.



Conceptual Clarity: Clarify fundamental concepts related to surface area and volume, ensuring a strong conceptual foundation. Understand the relationship between different dimensions and how changes in one parameter affect surface area and volume.



Problem-solving Practice: Hone your problem-solving skills with a diverse range of practice questions. Covering various difficulty levels, our study module provides targeted exercises to reinforce your understanding and boost confidence.



Exam-oriented Approach: Tailored to align with the CBSE examination pattern, our study module ensures you are well-prepared for questions related to surface area and volume in the Class 9th board exams. Boost your confidence with targeted, exam-focused content.



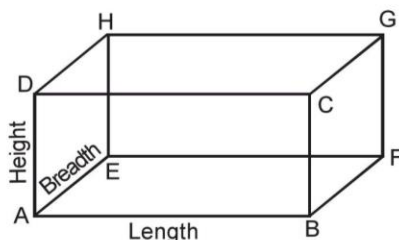
Online Accessibility: Access your study module anytime, anywhere. Our digital platform offers the flexibility needed for efficient and personalized learning, allowing you to tailor your study schedule for optimal results.

Surface area VOLUMES

Syllabus Reference

➤ SURFACE AREA OF A CUBOID AND A CUBE

- 1. Cuboid:** A cuboid is a solid bounded by six rectangular plane regions.



ABCDEFGH is a cuboid of length AB, breadth AE and height AD. Each rectangular region is called a face of the cuboid.

The combined area of all the six faces of a cuboid is called the surface area of the cuboid.

∴ Total surface area of a cuboid

$$= 2(lb + bh + hl)$$

Lateral surface area of a cuboid = $2(l + b) \times h$

Length of a diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

- 2. Cube:** A cuboid whose length, breadth and height are all equal, is called a cube. Each edge of a cube is called its side.

Total surface area of a cube = $6 \times (\text{side})^2$

Lateral surface area of a cube = $4 \times (\text{side})^2$

(i) Length of a diagonal of a cube = $\sqrt{3} \times \text{side}$

(ii) Units of area is square units.

(iii) Some useful conversions

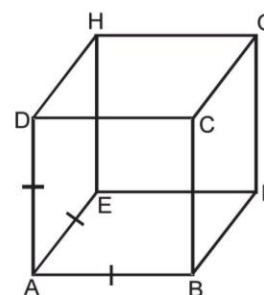
$$1 \text{ cm}^2 = 100 \text{ mm}^2$$

$$1 \text{ dm}^2 = 100 \text{ cm}^2$$

$$1 \text{ m}^2 = 100 \text{ dm}^2 = 10000 \text{ cm}^2$$

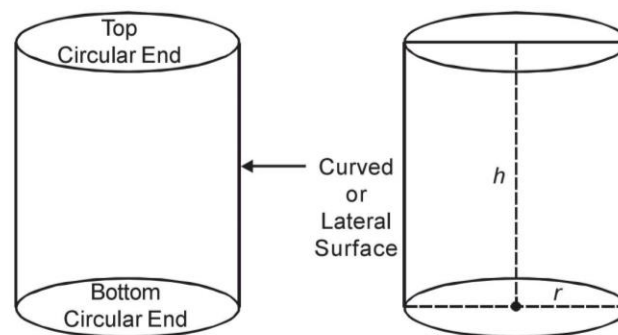
$$1 \text{ km}^2 = 1000000 \text{ m}^2$$

$$1 \text{ hectare} = 1 \text{ hm}^2 = 10000 \text{ m}^2$$



➤ SURFACE AREA OF A RIGHT CIRCULAR CYLINDER

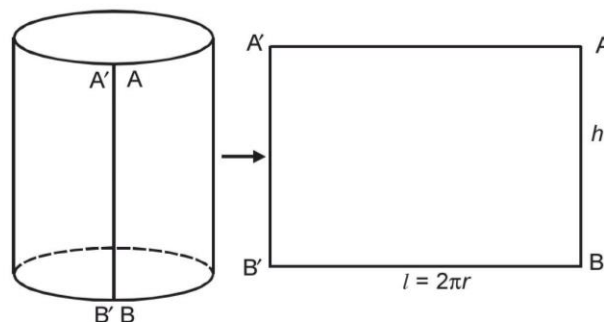
- 1. Definition of cylinder:** A solid generated by the revolution of a rectangle about one of its sides is called a right circular cylinder.



If the line joining the centres of circular ends of cylinder is not perpendicular to the circular ends, then the cylinder is not a right circular cylinder.

2. Curved or lateral surface area of a cylinder:

Curved surface (or lateral surface) area of a cylinder = $l \times b = 2\pi r \times h = 2\pi rh$



3. Total surface area of a cylinder: Consider a right circular cylinder with radius r and height h . The total surface area of this cylinder is equal to the sum of the areas of top and bottom of the cylinder and lateral surface area of the cylinder.

$$\begin{aligned} \therefore \text{Total surface area of the cylinder} &= \pi r^2 + \pi r^2 + 2\pi rh \\ &= 2\pi r^2 + 2\pi rh \\ &= 2\pi r (r + h) \end{aligned}$$

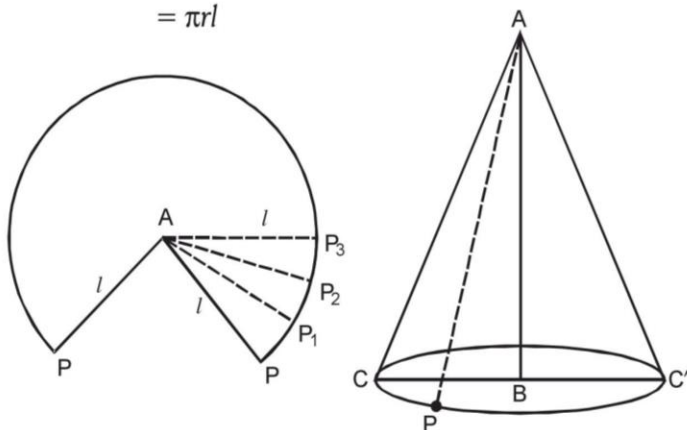
➤ **RIGHT CIRCULAR CONE**

When a right-angled triangle revolves around one of its side containing right angle, then the solid so generated is called a right circular cone.

Surface area of a right circular cone:

Area of whole circular part

$$\begin{aligned} &= \text{Sum of the areas of all triangles} \\ &= \frac{1}{2} P P_1 l + \frac{1}{2} P_1 P_2 l + \frac{1}{2} P_2 P_3 l + \dots \\ &= \frac{1}{2} \times (P P_1 + P_1 P_2 + P_2 P_3 + \dots) \times l \\ &= \frac{1}{2} \times 2\pi r \times l \\ [\because PP_1 + P_1P_2 + P_2P_3 + \dots &= \text{circumference of the base of the cone}] \\ &= \pi r l \end{aligned}$$



Thus, curved or lateral surface area of a right circular cone = $\pi r l$, where r is the radius of the base and l is the slant height of the cone given by

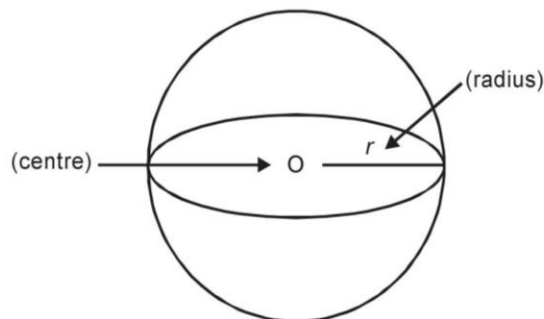
$$l = \sqrt{r^2 + h^2}$$

Total surface area of a right circular cone

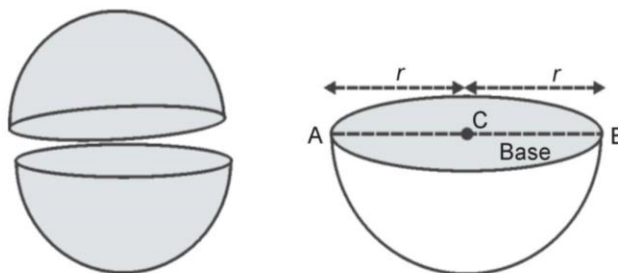
$$\begin{aligned} &= \text{area of the base} + \text{curved surface area} \\ &= \pi r^2 + \pi r l \\ &= \pi r (r + l) \end{aligned}$$

➤ **A SPHERE**

A sphere is a three-dimensional figure or solid figure, which is made-up of all points in the space.

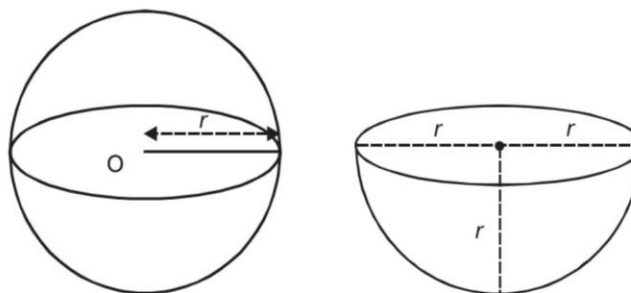


1. Hemisphere: A plane exactly through the middle (the centre) of a sphere divides the sphere into two equal parts, each of which is called a hemisphere.



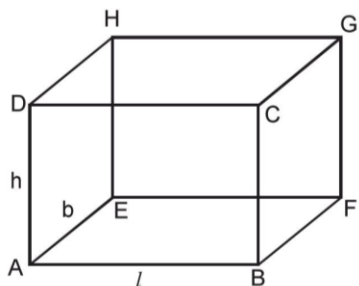
2. Surface area of a sphere and a hemisphere:

Surface area of a sphere of radius ' r ' = $4\pi r^2$
 Curved surface area of a hemisphere of radius ' r ' = $2\pi r^2$
 Total surface area of a hemisphere of radius ' r ' = $3\pi r^2$



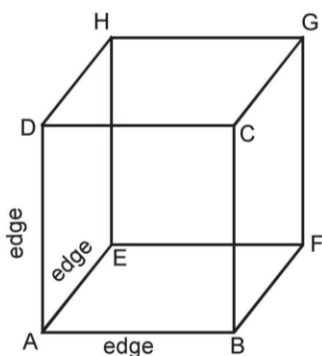
❖ **VOLUME OF A CUBOID**

The space occupied by a solid is called its volume.
 Volume of the cuboid = length \times breadth \times height
 $= l \times b \times h = (l \times b) \times h$
 $= \text{area of the base} \times \text{height}$



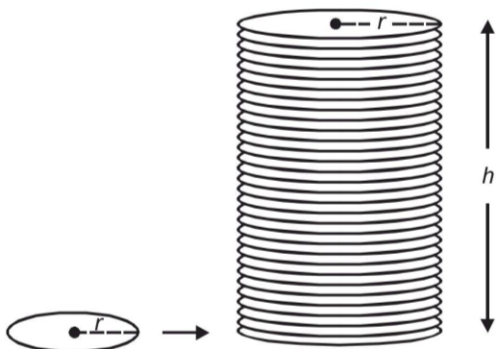
Volume of the cube = side \times side \times side

or
 $= \text{edge} \times \text{edge} \times \text{edge}$
 $1\text{m}^3 = 1 \text{ kilolitre} = 1000 \text{ litres}$
 $1000 \text{ cm}^3 = 1 \text{ litre}$



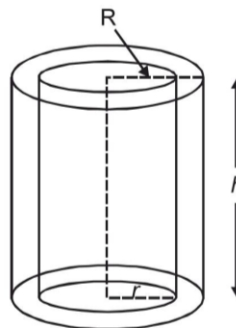
❖ **VOLUME OF A CYLINDER**

Let us take a number of one rupee coins of radius r and stack them up vertically as shown in the figure, to form a right circular cylinder of height h , then
 Volume of cylinder = Area of base (one rupee coin) \times Height
 $= \pi r^2 \times h = \pi r^2 h$



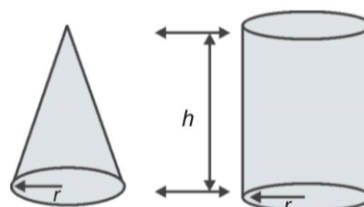
Volume of a hollow cylinder: Let r and R be the internal and external radii of a hollow cylinder and h be its height.

\therefore Volume of the material = Exterior volume
 - Interior volume
 $= \pi R^2 h - \pi r^2 h$
 $= \pi(R^2 - r^2) \times h$

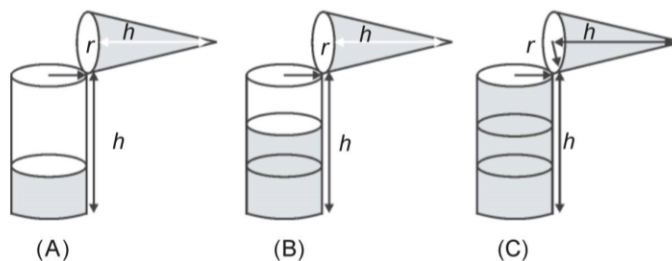


❖ **VOLUME OF A RIGHT CIRCULAR CONE**

Take a hollow cone and a hollow cylinder of same base radius and same height. Fill the cone with sand upto the brim and empty it into the cylinder. Repeat this process two times more.



Now, the cylinder will be filled upto the brim.



Thus, 3 times the volume of the cone of radius r and height h gives volume of a cylinder of radius r and height h .

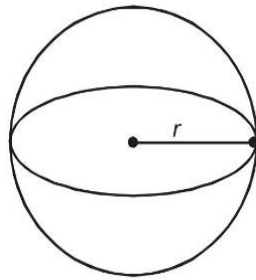
Hence, volume of a cone with radius ' r ' and height ' h '

$$= \frac{1}{3} \pi r^2 h$$

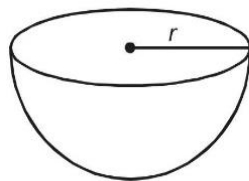
$$= \frac{1}{3} \times \text{area of the base} \times \text{height}$$

➤ **VOLUME OF A SPHERE**

Volume of a sphere with radius 'r' is given by $\frac{4}{3}\pi r^3$ and volume of a hemisphere with radius 'r' is given by $\frac{2}{3}\pi r^3$.



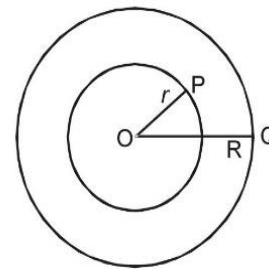
Volume of a sphere = $\frac{4}{3}\pi r^3$



Volume of a hemisphere = $\frac{2}{3}\pi r^3$

Volume of a spherical shell = $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$,

where r is the inner radius and R is the outer radius.



NCERT & BOARD QUESTIONS CORNER
 (Remembering & Understanding Based Questions)

Very Short Answer Type Questions

1. Find the total surface area of a cuboid obtained by joining two cubes of edge 'a' units side by side.

Sol. Total surface area of cuboid
 $= 2(lb + bh + hl)$
 $= 2(2a \times a + a \times a + a \times 2a)$
 $[\because \text{length of cuboid} = 2 \times a]$
 $= 2(5a^2) = 10a^2$

2. Find the total surface area of a hemisphere.

Sol. $3\pi r^2$

3. Find each edge of the cube whose volume is 216 cm^3 .

Sol. (edge)³ = volume = 216
 (edge)³ = $6 \times 6 \times 6$
 edge = 6 cm

4. Find the number of bricks, each measuring 25 cm by 15 cm by 8 cm required to built a wall 32 m long, 3 m high and 40 cm thick.

Sol. Required number of bricks = $\frac{300 \times 3200 \times 40}{15 \times 25 \times 8}$
 $= 12800$

5. If the lateral surface area of a cube is 1600 cm^2 , then find its edge.

Sol. Let a cm be edge of cube
 $\therefore 4a^2 = 1600$
 $a^2 = 400$
 $\Rightarrow a = 20 \text{ cm}$

6. Find the number of cubes of 3 cm side which can be made from cuboid of dimensions $l = 18 \text{ cm}$, $b = 15 \text{ cm}$ and $h = 2 \text{ cm}$.

Sol. Number of cubes = $\frac{\text{Volume of cuboid}}{\text{Volume of cube}}$
 $= \frac{18 \times 15 \times 2 \text{ cm}^3}{3 \times 3 \times 3 \text{ cm}^3} = 20$

7. If the volume of a cube is $3\sqrt{3} a^3$, then find the total surface area of the cube.

Sol. Let x be edge of a cube.
 \therefore We have, $x^3 = 3\sqrt{3} a^3$
 $\Rightarrow x^3 = (\sqrt{3}a)^3$
 $\Rightarrow x = \sqrt{3} a$

$$\begin{aligned} \text{Total surface area} &= 6x^2 \\ &= 6(\sqrt{3}a)^2 \\ &= 6 \times 3a^2 = 18a^2 \end{aligned}$$

- 8. Find the length of the longest rod that can be put in a hall of dimensions 23 m × 10 m × 10 m.**

Sol. The length of the longest rod
 = diagonal of the cuboid
 $= \sqrt{23^2 + 10^2 + 10^2}$
 $= \sqrt{529 + 100 + 100}$
 $= \sqrt{729} = 27 \text{ m}$

- 9. Find the total surface area of a right circular cylinder with radius 3 cm and height 11 cm.**

Sol. T.S.A. of cylinder = $2\pi r^2 + 2\pi rh$
 $= 2\pi r(r + h)$
 $= 2 \times \frac{22}{7} \times 3(3 + 11)$
 $= 2 \times \frac{22}{7} \times 3 \times 14$
 $= 264 \text{ cm}^2$

- 10. The curved surface area of a cone is 12320 sq. cm, if the radius of its base is 56 cm, find its height.**

Sol. Here, radius of base of a cone (r) = 56 cm
 And, curved surface area = 12320 cm^2
 $\therefore \pi rl = 12320$
 $l = \frac{12320}{\pi r} = \frac{12320 \times 7}{22 \times 56}$
 $= 70 \text{ cm}$

Again, we have

$$\begin{aligned} r^2 + h^2 &= l^2 \\ \therefore h^2 &= l^2 - r^2 = 70^2 - 56^2 \\ &= 4900 - 3136 = 1764 \\ \Rightarrow h &= \sqrt{1764} = 42 \text{ cm} \end{aligned}$$

Hence, the height of the cone is 42 cm.

- 11. A metallic sphere is of radius 4.9 cm. If the density of the metal is 7.8 g/cm³, find the mass of the sphere. ($\pi = \frac{22}{7}$).**

Sol. Here, radius of metallic sphere (r) = 4.9 cm

$$\begin{aligned} \therefore \text{Volume of metallic sphere} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times (4.9)^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times 4.9 \times 4.9 \times 4.9 \\ &= 493 \text{ cm}^3 \end{aligned}$$

Density of metal used = 7.8 g/cm³

Mass of the sphere = Volume × Density
 $= 493 \times 7.8 = 3845.4 \text{ g}$

- 12. If the radius of a sphere is 2r, then find its volume.**

Sol. Radius of the sphere (R) = $2r$ units

$$\begin{aligned} \therefore \text{Volume of sphere} &= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(2r)^3 \\ &= \frac{32}{3}\pi r^3 \text{ cubic units.} \end{aligned}$$

- 13. The volume of a solid hemisphere is 1152 π cm³. Find its curved surface area.**

Sol. Here, volume of hemisphere = $1152 \pi \text{ cm}^3$

$$\therefore \frac{2}{3}\pi r^3 = 1152 \pi$$

$$\Rightarrow r^3 = \frac{1152 \times 3}{2} = 1728$$

$$\Rightarrow r^3 = (12)^3$$

$$\Rightarrow r = 12 \text{ cm}$$

Now, curved surface area = $2\pi r^2$
 $= 2 \times \pi \times (12)^2$
 $= 288\pi \text{ cm}^2$

- 14. Find the diameter of a cylinder whose height is 5 cm and numerical value of volume is equal to numerical value of curved surface area.**

Sol. Here, height of cylinder (h) = 5 cm

According to the statement of the question, we have

$$\pi r^2 h = 2\pi r h$$

$$r = 2 \text{ cm}$$

Thus, diameter of the base of the cylinder is 2×2 i.e., 4 cm.

Short Answer Type - I Questions

- 15. A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?**

Sol. Volume of one matchbox
 $= 4 \text{ cm} \times 2.5 \text{ cm} \times 1.5 \text{ cm}$
 $= 15 \text{ cm}^3$

Now, volume of the packet containing 12 such matchboxes $= 12 \times 15 = 180 \text{ cm}^3$.

- 16. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? ($1 \text{ m}^3 = 1000 \text{ L}$)**

Sol. Volume of the cuboidal water tank
 $= 6 \text{ m} \times 5 \text{ m} \times 4.5 \text{ m}$
 $= 135 \text{ m}^3$

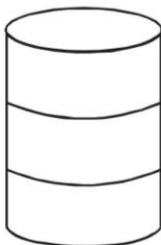
\therefore Capacity of the cuboidal water tank in litres
 $= 135 \times 1000$
 $= 135000 \text{ litres.}$

- 17. A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?**

Sol. Here, volume of the liquid $= 380 \text{ m}^3$
 \Rightarrow Volume of the cuboidal vessel $= 380 \text{ m}^3$
 $\Rightarrow l \times b \times h = 380 \text{ m}^3$
 $\Rightarrow 10 \text{ m} \times 8 \text{ m} \times h = 380 \text{ m}^3$
 $\Rightarrow h = \frac{380}{10 \times 8} = 4.75 \text{ m}$

Hence, the required height is 4.75 m.

- 18. You see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.**



Sol. Given that: Diameter of the base $= 20 \text{ cm}$
 \therefore Radius of the base $= 10 \text{ cm}$
 Height of the frame including folding margin of 2.5 cm $= 30 \text{ cm} + 2.5 \text{ cm} + 2.5 \text{ cm} = 35 \text{ cm}$
 \therefore Cloth required $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 10 \times 35$$

$$= 2200 \text{ cm}^2$$

Hence, the required area of the cloth for covering the lampshade is 2200 cm^2 .

- 19. Curved surface area of a right circular cylinder is 4.4 m^2 . If the radius of the base of the cylinder is 0.7 m , find its height.**

Sol. Radius of the base of a cylinder $= 0.7 \text{ m}$
 And curved surface area of the cylinder $= 4.4 \text{ m}^2$
 $\Rightarrow 2\pi rh = 4.4$

$$\Rightarrow 2 \times \frac{22}{7} \times 0.7 \times h = 4.4$$

$$\Rightarrow h = \frac{4.4 \times 7}{2 \times 22 \times 0.7} = 1 \text{ m}$$

Hence, the required height of the cylinder is 1 m.

- 20. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?**

Sol. Here, breadth of the river $= 40 \text{ m}$
 Depth of the river $= 3 \text{ m}$
 and length per hour of the river $= 2 \text{ km}$
 or length per minute of the river $= \frac{2000}{60} \text{ m}$

$$\therefore \text{Water flowing per minute} = \frac{2000}{60} \times 40 \times 3$$

$$= 4000 \text{ m}^3.$$

- 21. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?**

Sol. Here, diameter (d) $= 3.5 \text{ m}$
 \therefore Radius (r) $= \frac{3.5}{2} = \frac{7}{4} \text{ m}$
 Height (h) $= 12 \text{ m}$

Now, capacity of the conical pit

$$\begin{aligned} &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 12 \\ &= \frac{77}{2} \text{ m}^3 = 38.5 \text{ m}^3 \\ &= 38.5 \text{ kilolitres.} \end{aligned}$$

- 22. If the volume of a right circular cone of height 9 cm is $48\pi \text{ cm}^3$, find the diameter of its base.**

Sol. Given, height of the cone (h) = 9 cm
 Volume of the cone = $48\pi \text{ cm}^3$

$$\therefore \frac{1}{3}\pi r^2 h = 48\pi$$

$$\Rightarrow \frac{1}{3}\pi r^2 \times 9 = 48\pi$$

$$\Rightarrow r^2 = \frac{48\pi \times 3}{\pi \times 9} = 16$$

$$\Rightarrow r = \sqrt{16} = 4 \text{ cm}$$

Hence, the required diameter of the base
 $= 2 \times 4 = 8 \text{ cm.}$

- 23. Find the surface area of a sphere of radius:**
(i) 10.5 cm (ii) 5.6 cm (iii) 14 cm

Sol. (i) Here, radius (r) = 10.5 cm

$$\begin{aligned} \therefore \text{Surface area of the sphere} &= 4\pi r^2 \\ &= 4 \times \frac{22}{7} \times 10.5 \times 10.5 \\ &= 1386 \text{ cm}^2 \end{aligned}$$

(ii) Here, radius (r) = 5.6 cm

$$\begin{aligned} \therefore \text{Surface area of the sphere} &= 4\pi r^2 \\ &= 4 \times \frac{22}{7} \times 5.6 \times 5.6 \\ &= 394.24 \text{ cm}^2 \end{aligned}$$

(iii) Here, radius (r) = 14 cm

$$\begin{aligned} \therefore \text{Surface area of the sphere} &= 4\pi r^2 \\ &= 4 \times \frac{22}{7} \times 14 \times 14 \\ &= 2464 \text{ cm}^2. \end{aligned}$$

- 24. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.**

Sol. Surface area of a spherical balloon of radius 7 cm
 $= 4\pi (7)^2 = 4\pi (49) \text{ cm}^2$
 Surface area of a spherical balloon of radius 14 cm
 $= 4\pi (14)^2 = 4\pi (196) \text{ cm}^2$

Now, required ratio of surface areas

$$\begin{aligned} &= \frac{4\pi(49)}{4\pi(196)} = \frac{49}{196} = \frac{1}{4} \\ &= 1:4. \end{aligned}$$

- 25. Find the volume of a sphere whose surface area is 154 cm^2 .**

Sol. Here, surface area of a sphere = 154 cm^2

$$\therefore 4\pi r^2 = 154$$

$$\Rightarrow r^2 = \frac{154 \times 7}{4 \times 22} = \frac{49}{4}$$

$$\Rightarrow r = \frac{7}{2} \text{ cm}$$

Now, required volume of the sphere

$$\begin{aligned} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \\ &= \frac{539}{3} = 179\frac{2}{3} \text{ cm}^3. \end{aligned}$$

- 26. Find the volume of a sphere whose radius is:**
(i) 7 cm (ii) 0.63 m

Sol. (i) Here, radius of the sphere (r) = 7 cm

$$\begin{aligned} \therefore \text{Required volume of the sphere} &= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \\ &= \frac{4312}{3} = 1437\frac{1}{3} \text{ cm}^3 \end{aligned}$$

(ii) Here, radius of the sphere (r) = 0.63 m

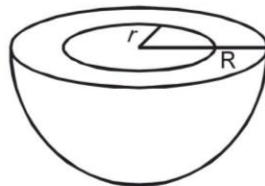
$$\begin{aligned} \therefore \text{Required volume of the sphere} &= \frac{4}{3} \times \frac{22}{7} \times 0.63 \times 0.63 \times 0.63 \\ &= 1.047816 \\ &\approx 1.05 \text{ m}^3 \text{ (approx.).} \end{aligned}$$

- 27. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.**

Sol. Here, inner radius of the bowl (r) = 5 cm
 Thickness of the steel sheet = 0.25 cm
 \therefore Outer radius of the bowl (R) = 5.25 cm
 Now, outer curved surface area of the bowl
 $= 2\pi R^2$
 $= 2 \times \frac{22}{7} \times (5.25)^2$
 $= 173.25 \text{ cm}^2$.

- 28. A hemispherical tank is made-up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.**

Sol. Here, inner radius (r) = 1 m = 100 cm
 Thickness of iron sheet = 1 cm
 \therefore Outer radius (R) = 101 cm
 Now, volume of the iron used
 $= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3$
 $= \frac{2}{3}\pi(R^3 - r^3)$
 $= \frac{2}{3} \times \frac{22}{7} (101^3 - 100^3)$
 $= \frac{2}{3} \times \frac{22}{7} \times 30301$
 $= 63487.81 \text{ cm}^3$
 $= \frac{63487.81}{1000000} \text{ m}^3$
 $= 0.06348781 \text{ m}^3$
 $\approx 0.06349 \text{ m}^3$ (approx.)



- 29. How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?**

Sol. Here, radius of the hemispherical bowl = $\frac{10.5}{2}$ cm

$$\begin{aligned} \therefore \text{Capacity of the hemispherical bowl} &= \frac{2}{3}\pi r^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times \frac{10.5}{2} \times \frac{10.5}{2} \times \frac{10.5}{2} \\ &= 303.188 \text{ cm}^3 \\ &= \frac{303.188}{1000} \text{ litres} \\ &= 0.303188 \text{ litres} \\ &\approx 0.303 \text{ litres (approx.)} \end{aligned}$$

- 30. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?**

Sol. Here, radius of spherical capsule (r) = $\frac{3.5}{2}$ mm
 \therefore Capacity of the capsule = $\frac{4}{3}\pi r^3$
 $= \frac{4}{3} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times \frac{3.5}{2}$
 $= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4}$
 $= \frac{539}{24} = 22.46 \text{ mm}^3$.

- 31. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?**

Sol. Let R be the radius of the earth and r be the radius of moon.

$$\text{Now, } 2r = \frac{1}{4} (2R)$$

$$\Rightarrow r = \frac{R}{4}$$

$$\begin{aligned} \text{Now, } \frac{\text{Volume of the moon}}{\text{Volume of the earth}} &= \frac{\frac{4}{3}\pi \left(\frac{R}{4}\right)^3}{\frac{4}{3}\pi(R)^3} \\ &= \frac{1}{64} \frac{R^3}{R^3} = \frac{1}{64} \end{aligned}$$

Short Answer Type - II Questions

- 32.** The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white-washing the walls of the room and the ceiling at the rate of ₹ 7.50 per m².

Sol. Here, length of a room = 5 m

Breadth of a room = 4 m

Height of a room = 3 m

$$\begin{aligned} \text{Now, area of four walls and the ceiling} &= 2(l + b) \times h + lb \\ &= 2(5 + 4) \times 3 + 5 \times 4 \\ &= 54 + 20 \\ &= 74 \text{ m}^2 \end{aligned}$$

Total cost of white-washing at the rate of ₹ 7.50 per m²

$$\begin{aligned} &= ₹ 7.50 \times 74 \\ &= ₹ 555. \end{aligned}$$

- 33.** A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine:

- (i) the area of the sheet required for making the box.
(ii) the cost of sheet for it, if a sheet measuring 1 m² costs ₹ 20.

Sol. (i) Given, length of the box = 1.5 m

Breadth of the box = 1.25 m

Height of the box = 65 cm = 0.65 m

Now, area of the sheet required for making the box which is open at the top

$$\begin{aligned} &= 2(l + b) \times h + lb \\ &= 2(1.5 + 1.25) \times 0.65 + 1.5 \times 1.25 \\ &= 3.575 + 1.875 \\ &= 5.45 \text{ m}^2 \end{aligned}$$

(ii) Total cost of the sheet at the rate of ₹ 20 per m²
= ₹ 20 × 5.45 = ₹ 109.

- 34.** A godown measures 40 m × 25 m × 10 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown.

Sol. Volume of the godown = 40 × 25 × 10 m³

Volume of each wooden crate

$$= 1.5 \times 1.25 \times 0.5 \text{ m}^3$$

∴ Maximum number of wooden crates

$$\begin{aligned} &= \frac{40 \times 25 \times 10}{1.5 \times 1.25 \times 0.5} \\ &= \frac{400}{15} \times \frac{2500}{125} \times \frac{100}{5} \\ &= 10666.67 \end{aligned}$$

Hence, the maximum number of wooden crates to be occupied are 10666.

- 35.** A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.

- (i) Which box has the greater lateral surface area and by how much?
(ii) Which box has the smaller total surface area and by how much?

Sol. (i) Lateral surface area of the cubical box with each edge 10 cm = 4 × (10)² = 400 cm²

Lateral surface area of the cuboidal box with dimensions 12.5 cm × 10 cm × 8 cm

$$\begin{aligned} &= 2(12.5 + 10) \times 8 \\ &= 360 \text{ cm}^2 \end{aligned}$$

Thus, the lateral surface area of the cubical box is 40 cm² [400 cm² – 360 cm² = 40 cm²] greater than that of the cuboidal box.

(ii) Total surface area of the cubical box with each edge 10 cm = 6 × (10)² = 600 cm²

Total surface area of the cuboidal box with dimensions 12.5 cm × 10 cm × 8 cm

$$\begin{aligned} &= 2(12.5 \times 10 + 10 \times 8 + 8 \times 12.5) \\ &= 2(125 + 80 + 100) \\ &= 610 \text{ cm}^2 \end{aligned}$$

Thus, the total surface area of the cubical box is 10 cm² less than that of the cuboidal box.

- 36.** A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.

- (i) What is the area of the glass?
(ii) How much of tape is needed for all the 12 edges?

Sol. (i) Given that, length of greenhouse = 30 cm
 Breadth of greenhouse = 25 cm
 Height of greenhouse = 25 cm

$$\begin{aligned} \therefore \text{Area of the glass} &= 2(lb + bh + hl) \\ &= 2(30 \times 25 + 25 \times 25 + 25 \times 30) \\ &= 2(750 + 625 + 750) \\ &= 4250 \text{ cm}^2 \end{aligned}$$

(ii) Required length of the tape
 $= 4(l + b + h)$
 $= 4(30 + 25 + 25)$
 $= 4(80) = 320 \text{ cm.}$

37. The paint in a certain container is sufficient to paint an area equal to 9.375 m^2 . How many bricks of dimensions $22.5 \text{ cm} \times 10 \text{ cm} \times 7.5 \text{ cm}$ can be painted out of this container?

Sol. Given that, the dimensions of a brick are $22.5 \text{ cm} \times 10 \text{ cm} \times 7.5 \text{ cm}$.

$$\begin{aligned} \therefore \text{Total surface area of a brick} &= 2(lb + bh + hl) \\ &= 2(22.5 \times 10 + 10 \times 7.5 + 7.5 \times 22.5) \\ &= 2(225 + 75 + 168.75) \\ &= 2(468.75) = 937.50 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Now, number of bricks} &= \frac{9.375 \text{ m}^2}{937.50 \text{ cm}^2} \\ &= \frac{9.375 \times 10000 \text{ cm}^2}{937.50 \text{ cm}^2} \\ &= 100. \end{aligned}$$

38. A wall of length 10 m is to be built across an open ground. The height of the wall is 5 m and thickness of the wall is 42 cm. If this wall is to be built with brick of dimensions $42 \text{ cm} \times 12 \text{ cm} \times 10 \text{ cm}$, then how many bricks would be required?

Sol. Here, length of the wall (L) = 10 m = 1000 cm
 Breadth of the wall (B) = 42 cm
 Height of the wall (H) = 5 m = 500 cm
 \therefore Volume of the wall = $L \times B \times H$
 $= 1000 \times 42 \times 500 \text{ cm}^3$
 Volume of each brick = $42 \times 12 \times 10 \text{ cm}^3$

$$\text{Thus, no. of bricks} = \frac{1000 \times 42 \times 500}{42 \times 12 \times 10}$$

$$\begin{aligned} &= \frac{50000}{12} = 4166.67 \\ &\cong 4167 \end{aligned}$$

Hence, the required number of bricks is 4167.

39. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Sol. Volume of cuboidal tank = lbh
 $\Rightarrow lbh = 2.5 \times b \times 10 \text{ m}^3$
 $\therefore 2.5 \times b \times 10 = \frac{50000}{1000}$
 (1000 litres = 1 m^3)
 $b = 2 \text{ m}$

40. A storage tank is in the form of a cube. When it is full of water the volume of water is 15.625 m^3 . If the present depth of water is 1.3 m. Find the volume of water used.

Sol. Volume of the tank = 15.625 m^3
 (Side)³ = Volume of the tank
 Side = $\sqrt[3]{\text{Volume}} = \sqrt[3]{15.625}$
 Side = $\sqrt[3]{\frac{15625}{1000}} = \sqrt[3]{\frac{5^3 \times 5^3}{10^3}}$
 $= \frac{5 \times 5}{10} = 2.5 \text{ m}$

$$\begin{aligned} \text{Water left in tank} &= l \times b \times h \\ &= 2.5 \times 2.5 \times 1.3 \\ &= 6.25 \times 1.3 \\ &= 8.125 \text{ m}^3 \end{aligned}$$

41. Find the number of bricks, each measuring $25 \text{ cm} \times 12.5 \text{ cm} \times 7.5 \text{ cm}$, required to construct a wall 6 m deep, 5 m high and 0.5 m thick, while the cement and sand mixture occupies $\frac{1}{20}$ of the volume of the wall.

Sol. Volume of the wall = $6 \times 5 \times 0.5 = 15 \text{ m}^3$
 Volume occupied by cement and sand mixture
 $= \frac{1}{20} \times 15 \text{ m}^3 = 0.75 \text{ m}^3$

$$\therefore \text{Volume of the wall occupied by bricks} = 15 - 0.75 = 14.25 \text{ m}^3$$

$$\begin{aligned} \text{Volume of each brick} &= \frac{25}{100} \times \frac{12.5}{100} \times \frac{7.5}{100} \text{ m}^3 \\ &= \frac{1}{4} \times \frac{1}{8} \times \frac{3}{40} = \frac{3}{1280} \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{Now, number of bricks used} &= \frac{14.25}{\frac{3}{1280}} \\ &= \frac{14.25 \times 1280}{3} \\ &= \frac{18240}{3} = 6080 \end{aligned}$$

Hence, the required number of bricks used are 6080.

- 42. The curved surface area of a right circular cylinder of height 14 cm is 88 cm². Find the diameter of the base of the cylinder.**

Sol. Height of the cylinder (h) = 14 cm
 Curved surface area of cylinder = $2\pi rh$
 $\therefore 2\pi rh = 88 \text{ cm}^2$

$$2 \times \frac{22}{7} \times r \times 14 = 88$$

$$\Rightarrow r = \frac{88 \times 7}{2 \times 22 \times 14} = 1 \text{ cm}$$

$$\therefore \text{Diameter } d = 2r = 2 \text{ cm}$$

- 43. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m².**

Sol. Given that: Diameter of the roller = 84 cm
 \therefore Radius of the roller = 42 cm
 Length of the roller = 120 cm
 Now, curved surface area of the roller = $2\pi rh$
 $= 2 \times \frac{22}{7} \times 42 \times 120$
 $= 31680 \text{ cm}^2$

Number of complete revolutions to move once over a playground = 500

$$\begin{aligned} \therefore \text{Area of the playground} &= 500 \times 31680 \\ &= 15840000 \text{ cm}^2 \\ &= 1584 \text{ m}^2. \end{aligned}$$

- 44. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?**

Sol. Here, diameter of the cylindrical tank = 140 cm
 \therefore Radius of the cylindrical tank = 70 cm
 $= \frac{70}{100} = \frac{7}{10} \text{ m}$

Height of the cylindrical tank = 1 m

Now, sheet required = total surface area of the cylinder

$$\begin{aligned} &= 2\pi r(r + h) \\ &= 2 \times \frac{22}{7} \times \frac{7}{10} \left(\frac{7}{10} + 1 \right) \\ &= 2 \times \frac{22}{7} \times \frac{7}{10} \times \frac{17}{10} \\ &= 7.48 \text{ m}^2 \end{aligned}$$

Hence, 7.48 m² of the sheet is required to make a closed cylindrical tank.

- 45. Find:**

(i) **the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.**

(ii) **how much steel was actually used, if $\frac{1}{12}$ of the steel actually used was wasted in making the tank.**

Sol. (i) Given that: Diameter of the cylindrical petrol tank = 4.2 m

\therefore Radius of the cylindrical petrol tank = 2.1 m

Height of the cylindrical petrol tank = 4.5 m

\therefore Lateral or curved surface area

$$\begin{aligned} &= 2\pi rh \\ &= 2 \times \frac{22}{7} \times 2.1 \times 4.5 \\ &= 59.4 \text{ m}^2 \end{aligned}$$

(ii) Total surface area of the petrol storage tank

$$\begin{aligned} &= 2\pi r(r + h) \\ &= 2 \times \frac{22}{7} \times 2.1 (2.1 + 4.5) \\ &= 2 \times \frac{22}{7} \times 2.1 \times 6.6 \\ &= 87.12 \text{ m}^2 \end{aligned}$$

Let the required steel used be $x \text{ m}^2$

$$\therefore x - \frac{1}{12}x = 87.12$$

$$\Rightarrow \frac{11}{12}x = 87.12$$

$$\Rightarrow x = 87.12 \times \frac{12}{11}$$

$$\Rightarrow x = 95.04$$

Hence, the steel required for making the petrol storage tank is 95.04 m^2 .

- 46. A metal pipe is 77 cm long. The inner diameter of a cross-section is 4 cm, the outer diameter being 4.4 cm (see fig.). Find its:**

- (i) inner curved surface area
(ii) outer curved surface area
(iii) total surface area.



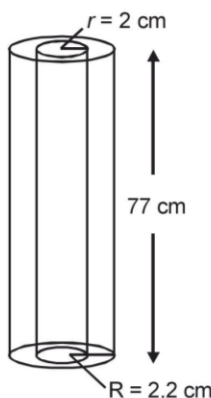
Sol. Here, $r = 2 \text{ cm}$, $R = 2.2 \text{ cm}$ and $h = 77 \text{ cm}$

(i) Inner curved surface area $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 2 \times 77 = 968 \text{ cm}^2$$

(ii) Outer curved surface area $= 2\pi Rh$

$$= 2 \times \frac{22}{7} \times 2.2 \times 77 = 1064.8 \text{ cm}^2$$



(iii) Total surface area

$$= 968 + 1064.8 + 2\pi (R^2 - r^2)$$

$$= 968 + 1064.8 + 2 \times \frac{22}{7} (2.2^2 - 2^2)$$

$$= 968 + 1064.8 + 2 \times \frac{22}{7} \times 0.84$$

$$= 968 + 1064.8 + 5.28$$

$$= 2038.08 \text{ cm}^2$$

- 47. The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find:**

- (i) its inner curved surface area,
(ii) the cost of plastering this curved surface at the rate of ₹ 40 per m^2 .

Sol. Here, inner diameter of the circular well $= 3.5 \text{ m}$

Inner radius of the circular well $= \frac{3.5}{2} \text{ m}$

Depth of the circular well $= 10 \text{ m}$

(i) Inner curved surface area $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{3.5}{2} \times 10 = 110 \text{ m}^2$$

(ii) Total cost of plastering the well at the rate of ₹ 40 per $\text{m}^2 = ₹ 40 \times 110 = ₹ 4400$

- 48. The total surface area of a solid right circular cylinder is 231 cm^2 . If curved surface area is two-third of the total surface area, find the radius of base.**

Sol. Curved surface area $= \frac{2}{3} \times 231 = 154 \text{ cm}^2$

Total surface area $= 231 \text{ cm}^2$

\Rightarrow Curved surface area $+ 2 \times$ area of base $= 231$

$\Rightarrow 2 \times$ area of base $= 231 - 154 = 77$

$$\Rightarrow 2 \times \left(\frac{22}{7}\right) \times r^2 = 77$$

$$r^2 = \frac{77 \times 7}{2 \times 22} = \frac{7 \times 7}{2 \times 2}$$

$$\Rightarrow r = \frac{7}{2} = 3.5 \text{ cm}$$

- 49. The pillars of a temple are cylindrically shaped. If each pillar has a circular base of radius 20 cm and height 10 m, how much concrete mixture would be required to build 14 such pillars?**

Sol. Radius of the base of the cylinder

$$(r) = 20 \text{ cm} = 0.2 \text{ m}$$

Height of the cylindrical pillar

$$(h) = 10 \text{ m}$$

\therefore Volume of each pillar $= \pi r^2 h$

$$= \frac{22}{7} \times 0.2 \times 0.2 \times 10 \text{ m}^3$$

$$= \frac{8.8}{7} \text{ m}^3$$

Now, required concrete mixture
 = Volume of 14 pillars
 \therefore Volume of 14 pillars
 = Volume of one pillar \times 14
 = $\frac{8.8}{7} \times 14 \text{ m}^3 = 17.6 \text{ m}^3$

\therefore 14 pillars would need 17.6 m^3 of concrete mixture.

- 50. Water flows at the rate of 5 m per minute through a cylindrical pipe, whose diameter is 7 cm. How long it will take to fill the conical vessel having base diameter 21 m and depth 12 m?**

Sol. For pipe, $d = 7 \text{ cm}$, $r = 0.035 \text{ m}$
 Volume of water flowing per minute = $\pi r^2 h$
 = $\frac{22}{7} \times 0.035 \text{ m} \times 0.035 \text{ m} \times 5 \text{ m}$
 = 0.01925 m^3

For conical vessel,

$$\text{Volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 12$$

$$= 1386 \text{ m}^3$$

$$\text{Time taken to fill vessel} = \frac{1386 \text{ m}^3}{0.01925 \text{ m}^3/\text{minutes}}$$

$$= 72000 \text{ minutes}$$

- 51. A rectangular sheet of paper 66 cm \times 20 cm is rolled along its length to form a cylinder. Find the radius and volume of the cylinder.**

Sol. $l = 66 \text{ cm} = 2\pi r$
 $\therefore r = \frac{66 \times 7}{2 \times 22} = \frac{21}{2} \text{ cm}$
 $b = h = 20 \text{ cm}$
 $\therefore V = \pi r^2 h = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 20$
 = 330×21
 = 6930 cm^3

- 52. We want to make a closed cylindrical storage tank of height 2 m and base diameter 210 cm from a metal sheet. How many square metres of the sheet are required for the purpose?**

Sol. $d = 210 \text{ cm}$
 $r = 105 \text{ cm} = 1.05 \text{ m}$
 and $h = 2 \text{ m}$
 Sheet required = Total surface area of cylinder
 = $2\pi r(r + h)$
 = $2 \times \frac{22}{7} \times 1.05 (1.05 + 2)$
 = $\frac{44}{7} \times 1.05 \times 3.05$
 = 20.13 m^2

- 53. A soft drink is available in two packs:**

- (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and
 (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

Sol. (i) Volume of first (tin can) container
 = $l \times b \times h$
 = $5 \times 4 \times 15$
 = 300 cm^3
 (ii) Volume of second (plastic cylinder) container
 = $\pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10$
 = 385 cm^3

Hence, the cylindrical container has 85 cm^3 greater capacity.

- 54. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.**

Sol. Here, diameter of the pencil = 7 mm
 \therefore Radius of the pencil = $\frac{1}{2} \times \frac{7}{10} = \frac{7}{20} \text{ cm}$
 Length of the pencil = 14 cm
 \therefore Volume of the pencil = $\pi r^2 h$
 = $\frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 14$
 = 5.39 cm^3

Also, diameter of the graphite = 1 mm

$$\therefore \text{Radius of the graphite} = \frac{1}{2} \times \frac{1}{10} = \frac{1}{20} \text{ cm}$$

$$\begin{aligned} \text{Volume of the graphite} &= \pi r^2 h \\ &= \frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 14 \\ &= 0.11 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Thus, volume of the wood used} &= 5.39 - 0.11 \\ &= 5.28 \text{ cm}^3 \end{aligned}$$

Hence, the required volume of wood is 5.28 cm^3 and volume of the graphite is 0.11 cm^3 .

- 55. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?**

Sol. Here, diameter of cylindrical bowl = 7 cm

$$\therefore \text{Radius of cylindrical bowl } (r) = \frac{7}{2} \text{ cm}$$

Height of cylindrical bowl (h) = 4 cm

$$\begin{aligned} \therefore \text{Volume of cylindrical bowl (volume of soup} \\ \text{per patient)} &= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 4 = 154 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Now, soup required for 250 patients} \\ &= 250 \times 154 \\ &= 38500 \text{ cm}^3 \\ &= \frac{38500}{1000} \text{ litres} \\ &= 38.5 \text{ litres} \end{aligned}$$

- 56. The volume of cylindrical pipe is 748 cm^3 . Its length is 0.14 m and its internal radius is 0.09 m. Find thickness of pipe.**

Sol. Internal radius (r) of cylindrical pipe = 0.09 m
 = 9 cm

$$\begin{aligned} \text{Length (height) of cylindrical pipe } (h) &= 0.14 \text{ m} \\ &= 14 \text{ cm} \end{aligned}$$

Let external radius of the cylindrical pipe be R cm.

$$\text{Volume of cylindrical pipe} = 748 \text{ cm}^3$$

$$\Rightarrow \pi(R^2 - r^2)h = 748$$

$$\Rightarrow \frac{22}{7}(R^2 - 9^2)14 = 748$$

$$\Rightarrow R^2 - 81 = \frac{748 \times 7}{22 \times 14} = 17$$

$$\begin{aligned} \Rightarrow R^2 &= 81 + 17 \\ &= 98 \end{aligned}$$

$$\begin{aligned} \Rightarrow R &= \sqrt{98} \\ &= 7\sqrt{2} \text{ cm} \\ &= 9.9 \text{ cm} \end{aligned}$$

Thus, thickness of the pipe = $9.9 - 9 = 0.9$ cm

- 57. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. (use $\pi = 3.14$)**

Sol. Given, radius of the base (r) = 6 m

Height of the cone (h) = 8 m

$$\begin{aligned} \therefore \text{Slant height } (l) &= \sqrt{r^2 + h^2} \\ &= \sqrt{(6)^2 + (8)^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} = 10 \text{ m} \end{aligned}$$

$$\begin{aligned} \therefore \text{Curved surface area of the cone} \\ &= \pi r l \\ &= 3.14 \times 6 \times 10 \\ &= 188.4 \text{ m}^2 \end{aligned}$$

Width of the tarpaulin = 3 m

And stitching margins + wastage required = 20 cm

\therefore Total length of the tarpaulin required

$$\begin{aligned} &= \frac{188.4}{3} + \frac{20}{100} \\ &= 62.8 \text{ m} + 0.20 \text{ m} \\ &= 63 \text{ m} \end{aligned}$$

- 58. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is ₹12 per m^2 , what will be the cost of painting all these cones? (use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$)**

Sol. Here, diameter of the base of a cone = 40 cm
∴ Radius of the base of the cone (r) = 20 cm

$$\begin{aligned} &= \frac{20}{100} \\ &= 0.2 \text{ m} \end{aligned}$$

Height of the cone (h) = 1 m

$$\begin{aligned} \therefore \text{Slant height } (l) &= \sqrt{r^2 + h^2} \\ &= \sqrt{(0.2)^2 + (1)^2} \\ &= \sqrt{0.04 + 1} \\ &= \sqrt{1.04} = 1.02 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Now, curved surface area of each cone} &= \pi r l \\ &= 3.14 \times 0.2 \times 1.02 \\ &= 0.64056 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Curved surface area of 50 such cones} &= 50 \times 0.64056 \\ &= 32.028 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Total cost of painting all these cones at the rate of} & \text{ ₹ 12 per m}^2 = \text{ ₹ } 12 \times 32.028 \\ &= \text{ ₹ } 384.34 \end{aligned}$$

59. Find the amount of water displaced by a solid spherical ball of diameter:

(i) **28 cm**

(ii) **0.21 m**

Sol. (i) Here, radius of spherical ball (r) = 14 cm

$$\begin{aligned} \text{Required volume of water displaced} &= \text{Volume of sphere} \\ &= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14 \\ &= 11498.67 \text{ cm}^3 \end{aligned}$$

(ii) Here, radius of spherical ball (r) = $\frac{21}{200}$ m

$$\begin{aligned} \text{Required volume of water displaced} &= \text{Volume of sphere} \end{aligned}$$

$$\begin{aligned} &= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{200} \times \frac{21}{200} \times \frac{21}{200} \\ &= \frac{4851}{1000000} \text{ m}^3 \\ &= 0.004851 \text{ m}^3. \end{aligned}$$

60. A dome of a building is in the form of a hemisphere. From inside, it was whitewashed at the cost of ₹ 498.96. If the cost of white-washing is ₹ 2.00 per square metre, find the:

(i) **inside surface area of the dome,**

(ii) **volume of the air inside the dome.**

Sol. (i) Here, cost of whitewashing from inside of a hemispherical dome = ₹ 498.96

Rate of white washing = ₹ 2 per sq. m

$$\begin{aligned} \therefore \text{Inside surface area of the dome} &= \frac{498.96}{2} \\ &= 249.48 \text{ m}^2 \end{aligned}$$

$$\therefore 2\pi r^2 = 249.48$$

$$\Rightarrow r^2 = \frac{249.48 \times 7}{2 \times 22}$$

$$= 39.69$$

$$\Rightarrow r = \sqrt{39.69}$$

$$= 6.3 \text{ m}$$

(ii) Now, volume of the dome = $\frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 6.3 \times 6.3 \times 6.3$$

$$= 523.908$$

$$= 523.9 \text{ m}^3 \text{ (approx.)}$$

61. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm³?

Sol. Radius of metallic ball $r = \frac{4.2}{2} \text{ cm} = 2.1 \text{ cm}$

$$\begin{aligned} \text{Volume of the metallic ball} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1 \\ &= 38.808 \text{ cm}^3 \\ \text{Density of the metal} &= 8.9 \text{ g per cm}^3 \\ \therefore \text{Mass of 1 cm}^3 \text{ of metal} &= 8.9 \text{ g} \\ \text{Mass of the ball} &= 38.808 \text{ cm}^3 \times 8.9 \\ &= 345.39 \text{ g (approx.)} \end{aligned}$$

62. Twenty seven solid iron spheres, each of radius 2 cm are melted to form a new solid sphere. What will be the surface area of the new sphere?

Sol. Radius of each small iron sphere (r) = 2 cm
 Let the radius of big iron sphere be R cm.
 Now, Volume of big sphere
 = Volume of 27 small spheres

$$\begin{aligned} \frac{4}{3}\pi R^3 &= \frac{4}{3}\pi r^3 \times 27 \\ R^3 &= 2 \times 2 \times 2 \times 27 \end{aligned}$$

$$\begin{aligned} \therefore R &= 6 \text{ cm} \\ \therefore \text{Surface area of new sphere} &= 4\pi R^2 \\ &= 4\pi \times 6 \times 6 \text{ cm}^2 \\ &= 144\pi \text{ cm}^2 \end{aligned}$$

63. A hemispherical dome of a building needs to be painted. If the circumference of the base of the dome is 17.6 m, find the cost of painting it, given the cost of painting is ₹ 5 per 100 cm².

Sol. Circumference of the dome = 17.6 m
 $\therefore 2\pi r = 17.6$
 $r = \frac{17.6 \times 7}{2 \times 22}$
 $= 2.8 \text{ m}$
 The curved surface area of the dome = $2\pi r^2$
 $= 2 \times \frac{22}{7} \times 2.8 \times 2.8 \text{ m}^2$
 $= 49.28 \text{ m}^2$

Now, cost of painting 100 cm² is ₹ 5.
 \therefore Cost of painting 1 m² = ₹ 500
 \therefore Cost of painting the whole dome
 $= ₹ (500 \times 49.28)$
 $= ₹ 24640$

64. The radius of sphere is 5 cm. If the radius is increased by 20%. Find by how much percent volume is increased.

Sol. Let r_1 = Radius of sphere = 5 cm
 r_2 = New radius = 5 + 20% of 5 = 6 cm

$$\begin{aligned} \text{Difference in volumes} &= \frac{4}{3}\pi (r_2^3 - r_1^3) \\ &= \frac{4}{3}\pi (6^3 - 5^3) \\ &= \frac{4}{3}\pi (216 - 125) \\ &= \frac{4}{3}\pi (91) \end{aligned}$$

Percent increase in volume

$$\begin{aligned} &= \frac{\frac{4}{3}\pi (91)}{\frac{4}{3}\pi (5)^3} \times 100\% = \frac{364}{5} \\ &= 72\frac{4}{5}\% \text{ or } 72.8\% \end{aligned}$$

65. A hollow cube of side 4 cm contains a solid sphere touching its sides. Find the volume of gaps between sphere and walls of cube.

Sol. Volume of cube = (side)³ = (4)³ = 64 cm³
 Diameter of sphere = 4 cm
 Radius of sphere = $\frac{4}{2} = 2 \text{ cm}$
 Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 2 \times 2 \times 2$
 $= \frac{32}{3}\pi = 33.52 \text{ cm}^3$
 Volume of gap = 64.00 - 33.52
 $= 30.48 \text{ cm}^3$

Long Answer Type Questions

66. It costs ₹ 2200 to paint the inner curved surface of a 10 m deep well. If the rate cost of painting is of ₹ 20 per m², find:

- (a) inner curved surface area
 (b) diameter of the well
 (c) capacity of the well.

Sol. Depth of well (h) = 10 m

Cost of painting inner curved surface is ₹ 20 per m² and total cost is ₹ 2200

$$\therefore \text{Curved surface area} = \frac{2200}{20} = 110 \text{ m}^2$$

$$2\pi rh = 110$$

$$r = \frac{110}{2\pi h} = \frac{110 \times 7}{2 \times 22 \times 10} \\ = 1.75 \text{ m} = 175 \text{ cm}$$

Now, volume of the well = $\pi r^2 h$

$$= \frac{22}{7} \times 1.75 \times 1.75 \times 10 \\ = 96.25 \text{ m}^3$$

Hence, inner curved surface area is 110 m², diameter of the well is 2×1.75 i.e., 3.5 m and capacity of the well is 96.25 m³.

67. Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is ₹ 4 for 1000 cm², find the cost of cardboard required for supplying 250 boxes of each kind.

Sol. Total surface area of smaller box

$$= 2(lb + bh + hl) \\ = 2(15 \times 12 + 12 \times 5 + 5 \times 15) \\ = 2(180 + 60 + 75) = 630 \text{ cm}^2$$

Extra area of cardboard required for overlapping

$$= \frac{5}{100} \times 630 = 31.50 \text{ cm}^2$$

\therefore Total area of cardboard required for each smaller box

$$= 630 + 31.50 \\ = 661.50 \text{ cm}^2$$

Total surface area of bigger box

$$= 2(lb + bh + hl) \\ = 2(25 \times 20 + 20 \times 5 + 5 \times 25) \\ = 2(500 + 100 + 125) \\ = 2(725) \\ = 1450 \text{ cm}^2$$

Extra area of cardboard required for overlapping

$$= \frac{5}{100} \times 1450 = 72.50 \text{ cm}^2$$

\therefore Total area of cardboard required for each bigger box

$$= 1450 \text{ cm}^2 + 72.50 \text{ cm}^2 \\ = 1522.50 \text{ cm}^2$$

Total area of cardboard required for 250 such boxes each smaller and bigger

$$= 250 \times (661.50 + 1522.50) \\ = 250 \times 2184 \\ = 546000 \text{ cm}^2$$

Now, total cost of cardboard at the rate of ₹ 4 for 1000 cm²

$$= ₹ \frac{4}{1000} \times 546000 \\ = ₹ 2184$$

68. Circumference of the base of a cylinder, open at the top, is 132 cm. The sum of radius and height is 41 cm. Find cost of polishing the outer surface area of cylinder at the rate ₹ 10 per square dm (decimetre).

$$\left(\text{Use } \pi = \frac{22}{7} \right)$$

Sol. Circumference = $2\pi r = 132$ cm

$$r = \frac{132}{2\pi} = \frac{132}{2 \times 22} \times 7 = 21 \text{ cm}$$

According to question,

$$r + h = 41 \text{ cm}$$

$$h = 41 - r = 41 - 21 = 20 \text{ cm}$$

$$\begin{aligned} \text{Surface area of cylinder open at top} &= 2\pi rh + \pi r^2 \\ &= 2 \times \frac{22}{7} \times 21 \times 20 + \frac{22}{7} \times 21 \times 21 \\ &= 66 \times 40 + 66 \times 21 \\ &= 2640 + 1386 \\ &= 4026 \text{ cm}^2 = 40.26 \text{ dm}^2 \end{aligned}$$

$$\text{Cost of polishing} = 40.26 \times 10 = ₹ 402.60$$

- 69. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 5 cm. Find the volume of the solid so obtained. If it is now revolved about the side 12 cm, then what would be the ratio of the volumes of the two solids obtained in two cases?**

Sol. Here, right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 5 cm.

$$\therefore \text{Radius of the base of cone} = 12 \text{ cm}$$

$$\text{Height of the cone} = 5 \text{ cm}$$

$$\begin{aligned} \therefore \text{Volume of the cone} &= \frac{1}{3} \pi (12)^2 (5) \\ &= \frac{\pi}{3} \times 720 \text{ cm}^3 \end{aligned}$$

Again, right triangle ABC is now revolved about the side 12 cm.

$$\therefore \text{Radius of the base of cone} = 5 \text{ cm}$$

$$\text{Height of the cone} = 12 \text{ cm}$$

$$\begin{aligned} \therefore \text{Volume of the cone} &= \frac{1}{3} \pi (5)^2 (12) \\ &= \frac{\pi}{3} \times 300 \text{ cm}^3 \end{aligned}$$

Now, the required ratio of their volumes

$$\begin{aligned} &= \frac{\pi}{3} \times 720 : \frac{\pi}{3} \times 300 \\ &= 12 : 5 \end{aligned}$$

- 70. A right triangle of hypotenuse 13 cm and one of its sides 12 cm is made to revolve taking side 12 cm as its axis. Find the volume and curved surface area of the solid so formed.**

Sol. Here, hypotenuse and one side of a right triangle are 13 cm and 12 cm respectively.

$$\begin{aligned} \therefore \text{Third side} &= \sqrt{(13)^2 - (12)^2} \\ &= \sqrt{169 - 144} \\ &= \sqrt{25} = 5 \text{ cm} \end{aligned}$$

Now, given triangle is revolved, taking 12 cm as its axis

$$\therefore \text{Radius of the cone } (r) = 5 \text{ cm}$$

$$\text{Height of the cone } (h) = 12 \text{ cm}$$

$$\text{Slant height of the cone } (l) = 13 \text{ cm}$$

$$\begin{aligned} \therefore \text{Curved surface area} &= \pi r l = \pi (5)(13) \\ &= 65 \pi \text{ cm}^2 \end{aligned}$$

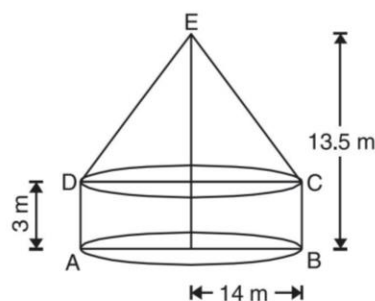
$$\begin{aligned} \text{Volume of the cone} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi \times 5 \times 5 \times 12 \\ &= 100 \pi \text{ cm}^3 \end{aligned}$$

Hence, the volume and curved surface area of the solid so formed are $100 \pi \text{ cm}^3$ and $65 \pi \text{ cm}^2$ respectively.

- 71. A tent is in shape of a right circular cylinder upto a height of 3 m and a cone above it. The maximum height of the tent above ground is 13.5 m. Calculate the cost of painting the inner side of the tent at the rate of ₹ 3 per sq. m, if the radius of the base is 14 m.**

Sol. Curved surface area of cylinder = $2\pi rh$

$$\begin{aligned} &= 2 \times \frac{22}{7} \times 14 \times 3 \\ &= 264 \text{ m}^2 \end{aligned}$$



$$\text{Height of cone} = 13.5 - 3 = 10.5 \text{ cm}$$

$$\begin{aligned} \text{Now, } l &= \sqrt{r^2 + h^2} = \sqrt{14^2 + 10.5^2} \\ &= \sqrt{196 + 110.25} = \sqrt{306.25} = 17.5 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Curved surface area of cone} &= \pi r l \\ &= \frac{22}{7} \times 14 \times 17.5 = 770 \text{ m}^2 \end{aligned}$$

Total inner curved surface area
 $= 770 + 264 = 1034 \text{ m}^2$

Cost per $\text{m}^2 = ₹ 3$

\Rightarrow Total cost $= 1034 \times 3 = ₹ 3102$

72. A hemispherical bowl of internal and external diameters 6 cm and 10 cm is melted and formed into a right circular cylinder of radius 14 cm. Find the height of the cylinder.

Sol. Outer radius $R = 5 \text{ cm}$

Inner radius $r = 3 \text{ cm}$

Volume of hemisphere $= \frac{2}{3} \pi \times (5^3 - 3^3)$

Volume of cylinder $= \pi \times 14 \times 14 \times h$

Volume of hemisphere = Volume of cylinder

$\Rightarrow \frac{2}{3} \times \pi \times (5^3 - 3^3) = \pi \times 14 \times 14 \times h$

$h = \frac{2}{3} \times \frac{\pi(125-27)}{\pi \times 14 \times 14}$

$= \frac{2 \times 98}{3 \times 14 \times 14}$

\therefore Height of cylinder $= \frac{1}{3} \text{ cm}$.

APPLICATION BASED QUESTIONS (Solved)

1. The cost of papering the walls of a room 12 m long at the rate of ₹ 1.35 per m^2 is ₹ 340.20 and the cost of matting the floor at the rate of 85 paise per m^2 is ₹ 91.80. Find the height of the room.

Sol. Here, length of the room (l) = 12 m

Let breadth and height of the room be $b \text{ m}$ and $h \text{ m}$ respectively

\therefore Area of four walls of the room $= 2(12 + b)h$

Since cost of papering the walls per m^2 is ₹ 1.35

\therefore Total cost of papering the walls

$= [2(12 + b)h] \times 1.35$

According to the statement of the question, we have

$2(12 + b)h \times (1.35) = 340.20$

$2.7 h (12 + b) = 340.20$

$(12 + b)h = \frac{340.20}{2.7}$
 $= 126 \quad \dots(i)$

Also, cost of matting the floor per m^2 is ₹ 0.85

\therefore Total cost of matting the floor

$= 12 \times b \times 0.85$

$= 10.2b$

According to the statement of the question, we have

$10.2b = 91.80$

$\Rightarrow b = \frac{91.80}{10.2} = 9 \quad \dots(ii)$

From (i) and (ii), we obtain

$(12 + 9)h = 126$

$\Rightarrow h = \frac{126}{21} = 6$

Hence, the height of the room is 6 m.

2. A cylindrical roller of length 2.5 m and diameter 3 m when rolled on a road was found to cover the area of 33000 m^2 . How many revolution did it make?

Sol. Here, radius of the cylindrical roller $= \frac{3}{2} = 1.5 \text{ m}$

Length (Height) of the roller (h) = 2.5 m

Area covered in one revolution

$=$ curved surface area of cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 1.5 \times 2.5 \text{ m}^2$

∴ Number of revolutions required to cover 33000 m²

$$= \frac{33000}{2 \times \frac{22}{7} \times 1.5 \times 2.5}$$

$$= \frac{33000}{2 \times 22 \times 1.5 \times 2.5}$$

$$= 1400$$

- 3. A mansion has 12 cylindrical pillars each having radius 50 cm and height 3.5 m. Find the cost to paint the curved surface of the pillars at ₹ 50 per m².**

Sol. Here, radius of each cylindrical pillar (r) = 50 cm
 $= \frac{1}{2}$ m

Height of each cylindrical pillar (h) = 3.5 m

Now, curved surface area = $2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{1}{2} \times 3.5$$

$$= 11 \text{ m}^2$$

Total surface area to be painted = 12×11
 $= 132 \text{ m}^2$

Cost of painting 1 m² curved surface area = ₹ 50

∴ Total cost of painting the pillars = ₹ 132×50
 $= ₹ 6600$

- 4. The height, curved surface area and volume of a cone are h , c and V respectively. Prove that: $3\pi Vh^3 - c^2h^2 + 9V^2 = 0$**

Sol. We know that:

$l^2 = r^2 + h^2$, where l is the slant height and r is the radius

Curved surface area (c) = πrl

and volume of the cone (V) = $\frac{1}{3}\pi r^2h$

L.H.S. = $3\pi Vh^3 - c^2h^2 + 9V^2$

$$= 3\pi \left(\frac{1}{3}\pi r^2h \right) h^3 - \pi^2 r^2 l^2 h^2 + 9 \left(\frac{1}{3}\pi r^2h \right)^2$$

$$= \pi^2 r^2 h^4 - \pi^2 r^2 (r^2 + h^2) h^2 + 9 \times \frac{1}{9} \pi^2 r^4 h^2$$

$$= \pi^2 r^2 h^4 - \pi^2 r^4 h^2 - \pi^2 r^2 h^4 + \pi^2 r^4 h^2$$

$$= 0$$

- 5. The radius of a sphere is 14 cm. If the radius is increased by 50%, find by how much percent its volume is increased.**

Sol. Here, the radius of sphere (r) = 14 cm

$$\therefore \text{Volume of the sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(14)^3$$

$$= \frac{4}{3}\pi(2744) \text{ cm}^3$$

Now, radius is increased by 50%

$$\therefore \text{New radius of sphere} = 14 + \frac{50}{100} \times 14$$

$$= 14 + 7$$

$$= 21 \text{ cm}$$

$$\therefore \text{New volume of sphere so formed} = \frac{4}{3}\pi(21)^3$$

$$= \frac{4}{3}\pi(9261) \text{ cm}^3$$

Thus, total increase in volume

$$= \frac{4}{3}\pi(9261) - \frac{4}{3}\pi(2744)$$

$$= \frac{4}{3}\pi(9261 - 2744)$$

$$= \frac{4}{3}\pi(6517) \text{ cm}^3$$

Hence, percent increase in volume

$$= \frac{\frac{4}{3}\pi(6517)}{\frac{4}{3}\pi(2744)} \times 100$$

$$= \frac{651700}{2744}$$

$$= 237.5\%$$

ANALYZING, EVALUATING & CREATING TYPE QUESTIONS (Solved)

1. To maintain beauty of a monument, the students of the school cleaned and painted the dome of the monument. The monument is in the form of a hemisphere. From inside, it was white washed by the students whose area is 249.48 m^2 . Find the volume of the air inside the dome. If white washing costs ₹2 per m^2 , how much does it cost?

Sol. Here, dome of the monument is hemispherical in shape, which was white washed by the students. Now, total area to be white washed = 249.48 m^2
 Cost of white washing = ₹ 2 per m^2
 \therefore Total cost of white washing = ₹ 2×249.48
 = ₹ 498.96

Also, $2\pi r^2 = 249.48$

$\Rightarrow r^2 = \frac{249.48 \times 7}{2 \times 22}$

$r^2 = 158.76$

$\therefore r = \sqrt{158.76} = 12.6 \text{ m}$

Volume of the air inside the dome
 = Volume of hemisphere
 = $\frac{2}{3}\pi r^3$
 = $\frac{2}{3} \times \frac{22}{7} \times 12.6 \times 12.6 \times 12.6$
 = 4191.264 m^3

2. Salim provides water to a village, having a population of 4000 which requires 150 litres of water per head per day. He has storage tank measuring $20 \text{ m} \times 15 \text{ m} \times 6 \text{ m}$. For how many days will the water of his tank last? He increased the rate for providing water as the dependence of villagers increased on him.

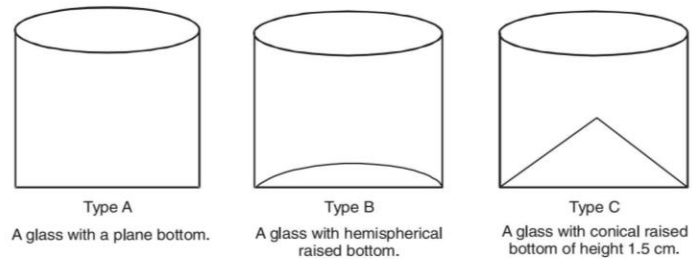
Sol. Here, the population of the village = 4000
 Requirement of water per head per day = 150 litres
 \therefore Total requirement of water per day = 4000×150 litres
 = 600000 litres

Volume of water tank = $20 \times 15 \times 6$
 = 1800 m^3
 = 1800×1000 litres

Now, number of days for which water of the tank will last = $\frac{1800 \times 1000}{600000} = 3$ days

Hence, water tank can serve for 3 days.

3. Naresh, a juice seller has set up his juice shop. He has three types of glasses (see figure) of inner diameter 5 cm to serve the customers. The height of the glasses is 10 cm.



He decided to serve the customer in 'A' type of glasses. (Take $\pi = 3.14$)

- (i) Find the volume of each type of glass.
 (ii) Which glass has the minimum capacity?
 (iii) Which mathematical concept is used in above problem?

Sol. (i) Volume of glass A = $\pi r^2 h$
 = $3.14 \times 2.5 \times 2.5 \times 10$
 = 196.25 cm^3

Volume of hemisphere in glass B = $\frac{2}{3}\pi r^3$
 = $\frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5$
 = 32.71 cm^3

\therefore Volume of glass B = Volume of glass A
 - Volume of hemisphere
 = $196.25 - 32.71$
 = 163.54 cm^3

Now, Volume of cone of glass C = $\frac{1}{3}\pi r^2 h$
 = $\frac{1}{3} \times 3.14 \times 2.5 \times 2.5 \times 1.5$
 = 9.81 cm^3

$$\begin{aligned}\text{Volume of glass C} &= 196.25 - 9.81 \\ &= 186.44 \text{ cm}^3\end{aligned}$$

(ii) The glass of type B has minimum capacity of 163.54 cm^3 .

(iii) Volume of solid figures.

4. A solid cube has been cut into two cuboids of equal volumes. Find the ratio of the total surface area of one of the cuboids to that of the given cube.

Sol. Let the edge of the given cube be x units.

Now, given cube has been cut into two cuboids of equal volumes.

\therefore Dimensions of each cuboid are x by x by $\frac{x}{2}$

Surface area of given cube = $6x^2$ sq. units

$$\begin{aligned}\text{Surface area of one of the cuboids} &= 2(lb + bh + hl) \\ &= 2\left(x \times x + x \times \frac{x}{2} + \frac{x}{2} \times x\right) \\ &= 2(2x^2) = 4x^2 \text{ sq. units}\end{aligned}$$

$$\begin{aligned}\text{Thus, the required ratio} &= 4x^2 : 6x^2 \\ &= 2 : 3\end{aligned}$$

5. Find the percentage decrease in the curved surface area of a sphere, if its diameter is decreased by 25%.

Sol. Here, diameter of the sphere is decreased by 25%
 \Rightarrow Radius of the sphere is also decreased by 25%

$$\begin{aligned}\therefore \text{Decreased radius 'r'} &= r - \frac{25}{100} \text{ of } r \\ &= \frac{3}{4}r \text{ units}\end{aligned}$$

Now, original curved surface area of the sphere
 $= 4\pi r^2$

New curved surface area of the sphere

$$= 4\pi\left(\frac{3}{4}r\right)^2 = 4\pi\frac{9}{16}r^2$$

So, decrease in curved surface area

$$= 4\pi r^2 - \frac{9\pi r^2}{4} = \frac{7\pi r^2}{4}$$

Thus, percentage decrease in curved surface area

$$\begin{aligned}&= \frac{7\pi r^2}{4\pi r^2} \times 100 \\ &= \frac{700}{16} = 43\frac{3}{4}\%\end{aligned}$$

6. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is ₹ 12 per m^2 , what will be the cost of painting all these cones?

(use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$)

Sol. Here, diameter of the base of a cone = 40 cm

\therefore Radius of the base of the cone (r) = 20 cm

$$\begin{aligned}&= \frac{20}{100} \\ &= 0.2 \text{ m}\end{aligned}$$

Height of the cone (h) = 1 m

$$\begin{aligned}\therefore \text{Slant height (l)} &= \sqrt{r^2 + h^2} \\ &= \sqrt{(0.2)^2 + (1)^2} = \sqrt{0.04 + 1} \\ &= \sqrt{1.04} = 1.02 \text{ m}\end{aligned}$$

Now, curved surface area of each cone = πrl

$$\begin{aligned}&= 3.14 \times 0.2 \times 1.02 \\ &= 0.64056 \text{ m}^2\end{aligned}$$

Curved surface area of 50 such cones

$$\begin{aligned}&= 50 \times 0.64056 \\ &= 32.028 \text{ m}^2\end{aligned}$$

Total cost of painting all these cones at the rate of ₹ 12 per m^2 = ₹ 12 \times 32.028 = ₹ 384.34

7. Water is flowing at the rate of 15 km/h through a cylindrical pipe of radius 7 cm into a rectangular tank which is 50 m long and 44 m wide. In how many hours will the water level in the tank raise by 21 cm?

Sol. Let n hours will be required to raise the water level in the tank by 21 cm.

∴ Volume of water in the tank

$$= 50 \times 44 \times \frac{21}{100} = 22 \times 21 \text{ m}^3$$

Volume of water passing through the pipe in one hour = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15 \times 1000 \text{ m}^3$$

∴ Volume of water passing through the pipe in n hours

$$= \frac{22 \times 7 \times 15 \times n}{10} \text{ m}^3$$

According to the statement of the question, we have

$$\frac{22 \times 7 \times 15 \times n}{10} = 22 \times 21$$

$$n = \frac{22 \times 21 \times 10}{22 \times 7 \times 15} = 2$$

Hence, the time required is 2 hours.

8. A cylindrical pipe empties a spherical tank,

full of water, at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to empty half of the tank, if radius of the tank is 3 m?

Sol. Here, radius of the tank = 3 m

$$\begin{aligned} \therefore \text{Volume of the tank} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \text{ m}^3 \\ &= \frac{264}{7} \text{ m}^3 \\ &= \frac{264 \times 1000}{7} \text{ litres} \end{aligned}$$

$$\begin{aligned} \text{Now, water taken out in one second} &= 3\frac{4}{7} \\ &= \frac{25}{7} \text{ litres} \end{aligned}$$

$$\begin{aligned} \text{Volume of half of the tank} &= \frac{1}{2} \times \frac{264 \times 1000}{7} \\ &= \frac{132 \times 1000}{7} \text{ litres.} \end{aligned}$$

Thus, $\frac{132000}{7}$ litres of water will be emptied

$$\begin{aligned} \text{by the cylindrical pipe in} &= \frac{132000 \times 7}{7 \times 25} \text{ sec} \\ &= 5280 \text{ sec} \\ &= \frac{5280}{60} \text{ min} \\ &= 88 \text{ minutes.} \end{aligned}$$

ASSIGNMENT - I

- Q.1.** The total surface area of cylinder with radius = r and height $t = h$ is:
 (a) $\pi r(r+h)$ (b) $2\pi r(r+h)$
 (c) $2\pi rh$ (d) $\pi r^2 h$
- Q.2.** If the length of the diagonal of a cube is $6\sqrt{3}$ cm, then the length of edge of the cube is:
 (a) 3 cm (b) 5 cm
 (c) 6 cm (d) 9 cm
- Q.3.** The length of diagonal of a cuboid of dimensions length = l , breadth = b and height = h is:
 (a) $\sqrt{l^2 + b + h^2}$ (b) $\sqrt{l^2 + b^2 + h^2}$
 (c) $\sqrt{l + b + h^2}$ (d) $\sqrt{l + b + h}$
- Q.4.** The radius of a cylinder is $2R$ and height = h , then its volume will be:
 (a) $2\pi R^2 h$ (b) $3\pi R^2 h$
 (c) $4\pi R^2 h$ (d) $6\pi R^2 h$
- Q.5.** The surface area of a cube whose edge equals to 3 cm is:
 (a) 64 cm^2 (b) 44 cm^2
 (c) 52 cm^2 (d) 54 cm^2
- Q.6.** The total surface area of a cube is 726 cm^2 . Find the length of its edge.
 (a) 10 cm (b) 11 cm
 (c) 8 cm (d) 9 cm
- Q.7.** What is the lateral surface area of a cuboid of length 20 cm, breadth 10 cm and height 40 cm?
 (a) 2400 cm^2 (b) 240 cm^2
 (c) 2800 cm^2 (d) 720 cm^2
- Q.8.** The lateral surface area of a cube is 256 m^2 . The volume of the cube is:
 (a) 512 m^3 (b) 64 m^3
 (c) 216 m^3 (d) 256 m^3
- Q.9.** The length of the longest rod that can be fitted in a cubical vessel of edge 10 cm long is:
 (a) 10 cm (b) $10\sqrt{2}$ cm
 (c) $10\sqrt{3}$ cm (d) 20 cm
- Q.10.** A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube?
- Q.11.** Two equal cubes with sides 6 cm are placed one above the other, forming a cuboid. Find the total surface area of the cuboid thus formed.
- Q.12.** The diameter of a garden roller is 1.4 m and it is 2 m long. Calculate the curved surface area of a garden roller.
- Q.13.** If the curved surface area of a cylinder is 94.2 cm^2 and height is 5 cm, then find the radius of its base and volume of the cylinder (use $\delta = 3.14$).
- Q.14.** The cost of painting the outer curved surface of a cylinder of at ₹ 1.50 per cm^2 is ₹ 660. If the height of the cylinder is 2 m, then find the curved surface of the base of cylinder.
- Q.15.** Find the number of planks of dimensions (4 m × 50 cm × 20 cm) that can be stored in a pit which is 16 m long, 12 m wide and 40 m deep.

ASSIGNMENT - II

- Q.1.** The total surface area of a cone whose radius is r and slant height l is:
 (a) $\pi r (r + l)$ (b) $\pi r (r + 2l)$
 (c) $2\pi r (r + l)$ (d) $\pi r (2r + l)$
- Q.2.** The volume of a hemisphere with radius r is:
 (a) $\frac{4}{3}\pi r^3$ (b) $4\pi r^3$
 (c) $2\pi r^3$ (d) $\frac{2}{3}\pi r^3$
- Q.3.** The total surface area of a cube is 96 cm^2 . The side of the cube is:
 (a) 3 cm (b) 5 cm
 (c) 4 cm (d) 6 cm
- Q.4.** The length of the longest pole that can be put in a room of dimensions $(l \times b \times h)$ is:
 (a) $\sqrt{l+b^2+h^2}$ (b) $\sqrt{l^2+b^2+h^2}$
 (c) $\sqrt{l^2+b+h^2}$ (d) $\sqrt{l+b+h^2}$
- Q.5.** A cylinder and a right circular cone are having the same base and same height. The volume of the cylinder is how many times the volume of the cone?
 (a) 4 times (b) 3 times
 (c) 5 times (d) 2.5 times
- Q.6.** The radius of a sphere is $2r$, and then its volume will be:
 (a) $\frac{16}{3}\pi r^3$ (b) $\frac{32}{3}\pi r^3$
 (c) $\frac{64}{3}\pi r^3$ (d) $\frac{8}{3}\pi r^3$
- Q.7.** In a cylinder, radius is doubled and height is halved, then curved surface area will be:
 (a) halved (b) doubled
 (c) same (d) four times
- Q.8.** The radii of two cylinders are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 3. The ratio of their volumes is:
 (a) 10 : 17 (b) 20 : 27
 (c) 17 : 27 (d) 20 : 37
- Q.9.** The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. The ratios of the surface areas of the balloon in the two cases is:
 (a) 1 : 4 (b) 1 : 3
 (c) 2 : 3 (d) 2 : 1
- Q.10.** A solid cylinder has a total surface area 462 cm^2 and the radius 7 cm, then find the height of a cylinder.
- Q.11.** Find the ratio of the volume of a right circular cylinder and a right circular cone of the same base and height.
- Q.12.** Three solid spheres of iron whose diameters are 2 cm, 12 cm and 16 cm respectively, are melted into a single solid sphere. Find the radius of the solid sphere.
- Q.13.** A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.
- Q.14.** The radius and the height of a right circular cone are in the ratio 5 : 12, respectively. If its volume is 314 m^3 , then find its slant height and the radius.
- Q.15.** Twenty seven solid iron sphere, each of radius r and surface area S are melted to form a sphere with surface area S' . Find the:
 (i) radius r' of the new sphere (ii) ratio of S and S'