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THEOREMS

SIMILAR TRIANGLES

▶ 1. THEOREM :1—[BASIC PROPORTIONALITY THEOREM - (THALES THEOREM)]

Statement – In a triangle, a line drawn parallel to one side to intersect the other side in distinct point divide the other two side in the same ratio.

Or

If a line drawn parallel to one side of a Δ divides the other two side in the same ratio.

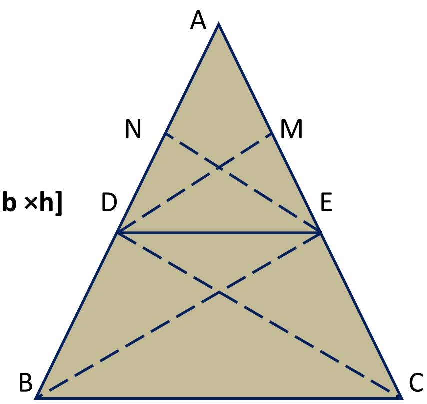
Given: – In a **Δ** ABC, DE is drawn ∥ to BC

To prove: - $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: - we draw DM ⊥ AC and EN ⊥ AD and join BE and DC

Proof: - We know that, **Δ**'s on the same base (DE) and between same ∥,s (BD ∥ CE) are equal in area.

∴ **ar.ΔBED = ar. Δ DCE ---------------------------- (I)**



Now $\frac{ar.\ \Delta ADE}{ar.\ \Delta CDE} = \frac{\frac{1}{2} \times DM \times AE}{\frac{1}{2} \times DM \times CE} = \frac{AE}{CE}$ ----- (II) [∵area of **Δ = ½ × b ×h]**

Again, $\frac{ar.\Delta ADE}{ar.\ \Delta DBE} = \frac{\frac{1}{2} \times EN \times AD}{\frac{1}{2} \times EN \times BD} = \frac{AD}{BD}$ -------- (III)

But, ar. Δ BED = ar. Δ CDE ------------------ -- [from (I)]

∴ $\frac{ar.\ \Delta ADE}{ar.\ \Delta CDE} = \frac{AD}{DB}$ ------- (IV)

From (III) and (IV), we have

$$\frac{AE}{CE} = \frac{AD}{DB}$$

Hence Proved.

1st Corollary – $\frac{AD}{DB} = \frac{AE}{EC}$ [Proved above]

Adding 1 to both sides, WE get

$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$

⇒ $\frac{AD + DB}{DB} = \frac{AE + EC}{CE}$ ∴ $\frac{AB}{BA} = \frac{AE}{EC}$

2nd Corollary - $\frac{AD}{DB} = \frac{AE}{EC}$ [proved above]

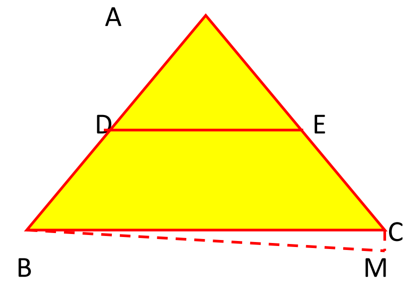
$\frac{DB}{AD} = \frac{EC}{AC}$

adding 1 to both sides.

$\frac{DB}{AD} + 1 = \frac{EC}{AC} + 1$

⇒ $\frac{DB + AD}{AD} = \frac{EC + AC}{AC}$ ∴ $\frac{AB}{AD} = \frac{AC}{AE}$

Converse of B.P.T → ***If line divides any two side of a triangle in the same ratio than it is parallel to 3rd side.***



Given: - In a **Δ** ABC, DE is a straight-line meeting AB & AC resp. at D & E such that $\frac{AD}{DB} = \frac{AE}{EC}$

To prove: - DE ∥ BC

Construction: - Consider that, DE ∦ BC, but it is ∥ to BM
AC is produced to intersect BM at M.

Proof: - In **Δ** ABM, DE ∥ BM [by assumption]

∴ $\frac{AD}{DB} = \frac{AE}{EM}$ ---------------------------------- ------ (I) [If a line drawn on other of a **Δ**, divides the other sides in the same ratio.(B.P.T)]

But $\frac{AD}{DB} = \frac{AE}{EC}$ --- (II) (given)

From (I) and (II), we get

$\frac{AE}{EM} = \frac{AE}{EC}$

EM = EC

Which is possible only when M coincides with C,

i.e., BC = BM , which contradicts our supposition.

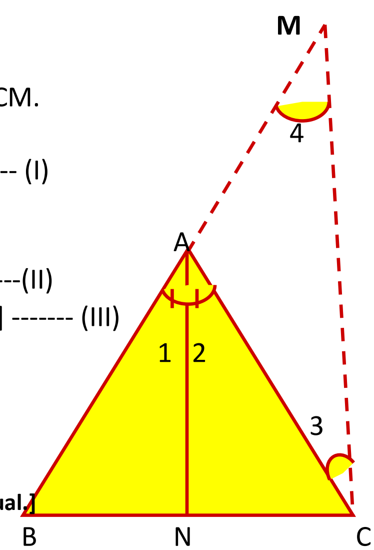
Hence, **DE || BC** **Hence proved.**

3. Internal bisector theorem: - The bisector of an angle of a triangle, divides the opposite side in ratio of side containing the angle.

Given: - Δ ABC, in which AN is an internal bisector of ∠ A meets BC at N.

To Prove: - $\frac{AB}{AC} = \frac{BN}{NC}$

Construction: - Through C we draw MC ∥ AN. Also, BM is produced to M to meet CM.



Proof: - Since AN ∥ CM and BM is transversal

∴ ∠ 1 = ∠ 2 (corresponding angles) ----- (I)

Again, since AN ∥ CM and AC is transversal

∴ ∠ 2 = ∠ 3 (alternate ∠ 's) ----------------(II)

But, **∠ 1 = ∠ 2** [∵ AN is internal bisector of ∠ A (given)] ------- (III)

From (I), (II) and (III)

∠ 3 = ∠ 4

Now in Δ ACM,

∠ 3 = ∠ 4 [proved above]

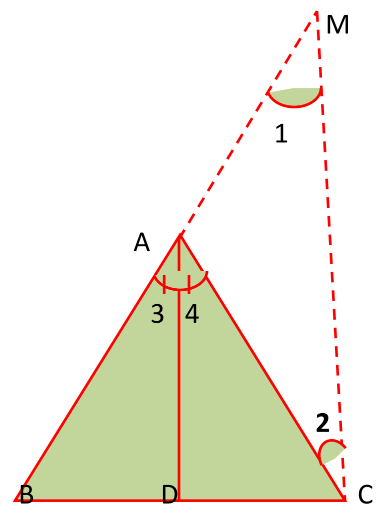
∴ AC = AM [**Sides opposite to equal ∠'s are also equal.**]

Now in Δ MBC, AN ∥ MC. (By construction.)

∴ $\frac{AB}{AM} = \frac{BN}{NC}$ [BY (BPT)]

or, $\frac{AB}{AC} = \frac{BN}{NC}$ **[∵ AM = AC (proved above)] Hence Proved.**

4. Converse: - If a line through one vertex of a triangle divides the opposite side in the ratio of other two side, then the line bisects the angle of the vertex.



Given: - A Δ ABC in which AD is a line from vertex A such that $\frac{AB}{AC} = \frac{BD}{DC}$

To prove: - AD is bisector of ∠ BAC or ∠ A.

Construction: - Through C, we draw MC ∥ AD and produce AB to meet MC at M.

Proof: - In Δ BCM, AD ∥ MC (By construction.)

∴ $\frac{AB}{AM} = \frac{BD}{DC}$ [By (BPT)]

But, $\frac{AB}{AC} = \frac{BD}{DC}$

∴ $\frac{\cancel{AB}}{AM} = \frac{\cancel{AB}}{AC}$

AM = AC

Now in ACM

AC = AM [Proved above]

∴ ∠ 1 = ∠ 2 [angles opposite to equal side are also equal] ------- (I)

Now AD ∥ MC (by construction.) and AC is a transversal.

∴ ∠ 2 = ∠ 4 (alternate ∠'s) ------- (II)

Also, AD ∥ MC (by construction.) and BM is a transversal.

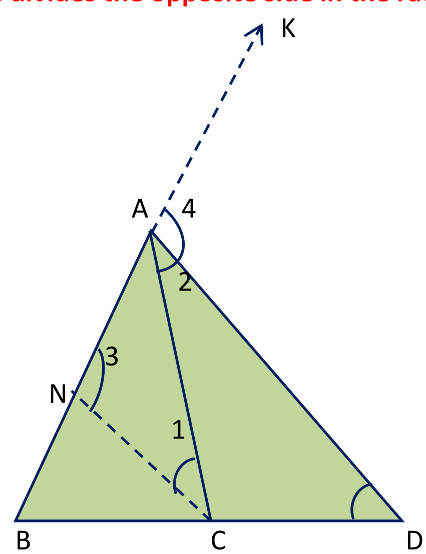
∴ ∠ 3 = ∠ 1 ------- (III) [corresponding ∠'s]

From (I), (II) and (III),

∠ 3 = ∠ 4

∴ **AD is bisector of ∠ BAC, hence proved.**

5. External Bisector Theorem – The external bisector of an angle of a triangle divides the opposite side in the ratio of sides containing the angle.



Given: - In Δ ABC, AQ is an eternal bisector of exterior ∠ CAK.

To prove: - $\frac{AB}{AC} = \frac{BD}{CD}$

Construction: - Through C we draw CN ∥ AD

Proof: - Since, NC ∥ AD and AC is a transversal (By construction.)

∴ ∠ 1 = ∠ 2 (alternate∠'s)

Again NC ∥ AD and AB is a transversal

∴ ∠ 3 = ∠ 4 (Corresponding ∠,s)

But, ∠ 2 = ∠ 4 (Given)

∴ ∠ 3 = ∠ 1

Now, In Δ ANC

∠ 3 = ∠ 1 [Proved above]

∴ AN = AC [Sides opposite to equal ∠'s are also equal.]

Now, In Δ ADB, AD ∥ NC (by construction.)

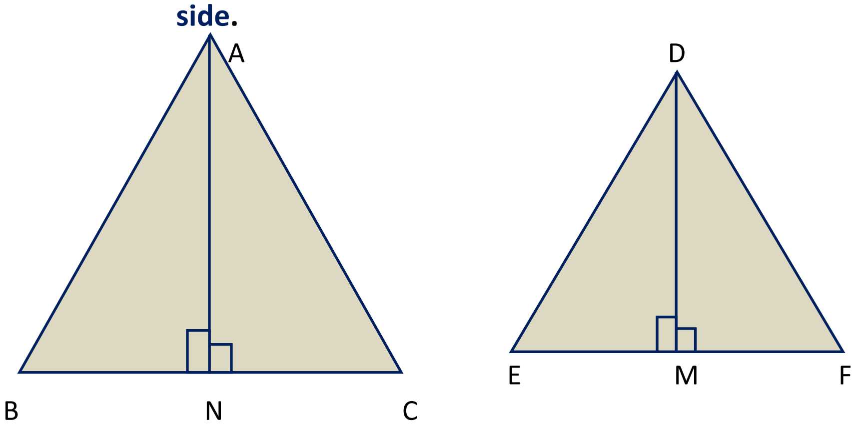
∴ $\frac{AB}{AN} = \frac{BD}{CD}$ [∵ a line drawn ∥ to one side of a Δ divides the other side in same ratio]

But, AN = AC [Proved above]

∴ $\frac{AB}{AC} = \frac{BD}{CD}$

Hence proved

6. Area Theorem: - The ratio of areas of two similar Δ's is equal to ratio of square on their corresponding side.



Given: - Δ ABC ~ Δ DEF

To prove: - $\frac{ar.\ \Delta ABC}{ar.\ \Delta DEF} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$

Proof: - $\frac{ar.\Delta ABC}{ar.\ \Delta DEF} = \frac{\frac{1}{2} \times BC \times AN}{\frac{1}{2} \times EF \times DM} = \frac{BC}{EF} \times \frac{AN}{DM}$ -------- (I)

Now, In Δ ANB and Δ DME

∠ ANB = ∠ DME [each 90^0]

∠ B = ∠ E [∵Δ ABC ~ ΔDEF]

∴ ΔANB ~ ΔDME [by AA similarity]

∴ $\frac{AB}{DE} = \frac{AN}{DM}$ [If two Δ's are similar then the ratio of corresponding sides are same.]

Also, $\frac{BC}{EF} = \frac{AB}{DE}$ [∵Δ ABC ~ Δ DEF] ---- (II)

From (I) and (II)

∴ $\frac{AN}{MD} = \frac{BC}{EF}$

$\frac{ar.\ \Delta ABC}{ar.\ \Delta DEF} = \frac{BC}{EF} \times \frac{BC}{EF} = \frac{BC^2}{EF^2}$ ----------- (A)

Similarly, we prove that

$\frac{ar.\ \Delta ABC}{ar.\ \Delta DEF} = \frac{AB^2}{DE^2}$ ------------ (B)

Also, $\frac{ar.\ \Delta ABC}{ar.\ \Delta DEF} = \frac{AC^2}{DF^2}$ ------- (C)

From (A), (B) and (C)

$\frac{ar.\ \Delta ABC}{ar.\ \Delta DEF} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$ **Hence proved**

7. Characteristics property of similar triangles –

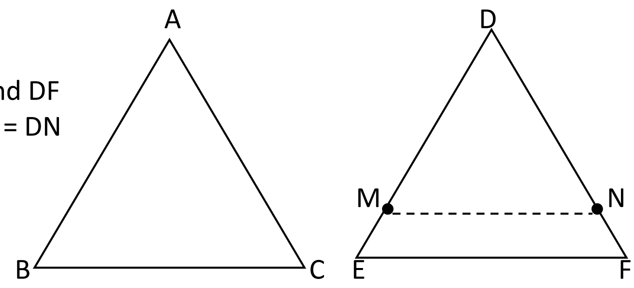
(I) A – A Similarity

1. **If in a two Δ's corresponding angles are equal. i.e. the triangles are equiangular, than the triangles are similar.**

Given: - In Δ's ABC and DEF. ∠ A = ∠ D, ∠ B = ∠ E and ∠ C = ∠ F.

To prove: - Δ ABC ~ Δ DEF

Construction: - We draw mark point M and N on DE and DF respectively such that AB = DM and AC = DN and join MN



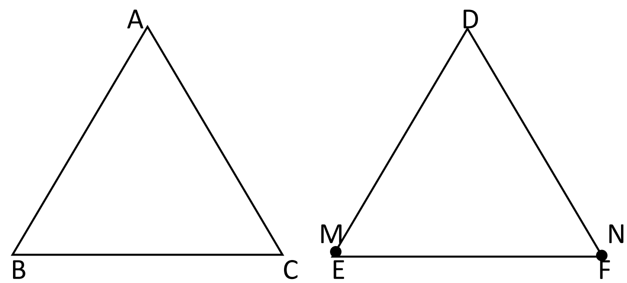
Proof: - Three cases arise

Case – I: AB = ED, Here M coincides with E

Case – II: AB < DE, Here M lies on DE

Case – III: AB > DE, Here M lies on DE produced.

► Case – I: - **AB = ED, here, M coincides with E**



In Δ ABC and Δ DEF

AB = DE [By construction]

∠ A = ∠ D [Given]

AC = DE [∵ N coincides with F]

∴ Δ ABC ≅ Δ DEF [by SAS]

∴ BC = EF [CPCTC]

$\frac{BC}{EF} = 1$

Similarly, $\frac{AC}{DF} = 1$ and $\frac{AB}{DE} = 1$

∴ $\frac{BC}{EF} = \frac{AC}{DF} = \frac{AB}{DC}$

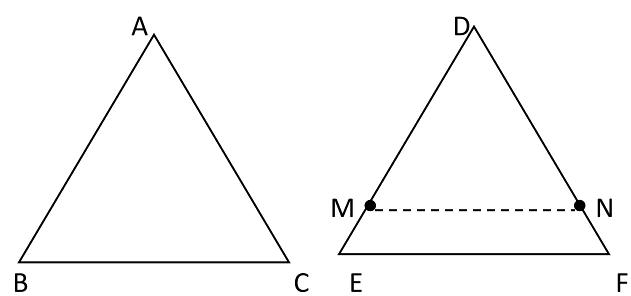
Also, ∠ A = ∠ D, ∠ B = ∠ E and ∠ C = ∠ F [Given]

∴ Δ ABC ~ Δ DEF **Hence proved**

► **Case – II**

AB < DE, Here M lies on DE,

In ΔABC and Δ DMN,



AB = DM (by construction.)

∠ A = ∠ D (given)

AC = DN (by construction.)

∴ Δ ABC ≅ Δ DMN (by SAS)

∴ ∠ B = ∠ DMN (CPCTC)

But, ∠ B = ∠ E (given)

∴ ∠ DMN = ∠ E

But these are corresponding angle and are equal

∴ MN ∥ EF

Now, in Δ DEF, MN ∥ EF (Proved above)

$\frac{DM}{DE} = \frac{DN}{DF}$ (∵ a line drawn ∥ to one side of a ∆, divides the other side in same ratio)

But, DN = AC and DM = AB (by construction.)

∴ $\frac{AB}{DE} = \frac{AC}{DF}$ -------- (I)

Similarly, we can prove that

$\frac{AB}{DE} = \frac{BC}{EF}$ -------- (II)

From (I) and (II)

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Also, ∠ A = ∠ D, ∠ B = ∠ E and ∠ C = ∠ F (given)

∴ ∆ ABC ~ ∆DEF **Hence proved**

Case – III

AB > DE, Here M lies on DE produced.

In ∆ABC and ∆ DMN

AB = DM (by construction.)

∠ A = ∠ D (given)

AC = DN (by construction.)

∴ ∆ ABC ≅ ∆ DMN (by SAS)

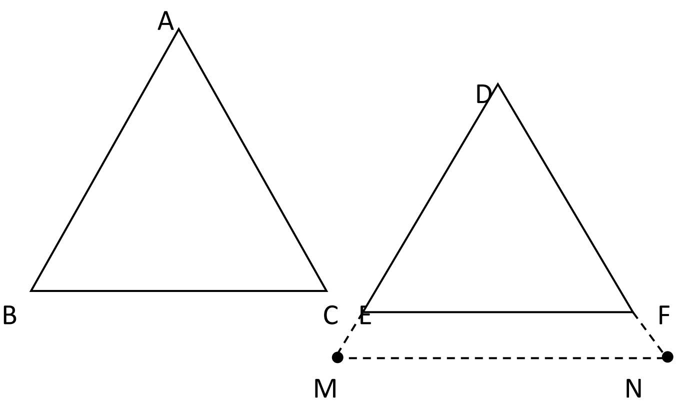
∴ ∠ B = ∠ DMN (CPCTC)

But, ∠ B = ∠DEF (given)

∴ ∠ DMN = ∠ DEF

But these are corresponding side and are equal

∴ EF ∥ MN



Now, In ∆DEF, EF ∥ MN (Proved above)

∴ $\frac{DM}{DE} = \frac{DN}{DF}$ (by corollary of BPT)

But, AB = DM and AC = DN (by construction.)

∴ $\frac{AB}{DE} = \frac{AC}{DF}$ -------- (I)

Similarly, we can prove that

$\frac{AB}{DE} = \frac{BC}{EF}$ -------- (II)

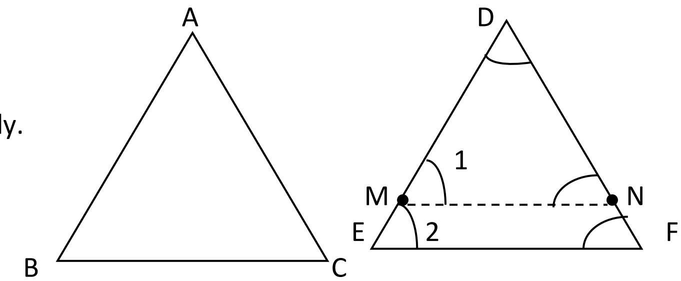
From (I) and (II)

$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Also, ∠ A = ∠ D, ∠ B = ∠ E and ∠ C = ∠ F (given) ∴ ∆ABC ~ ∆ DEF **Hence proved**

II. Characteristic property of similar Δ :- → S-S-S Similarity

If Corresponding sides of two triangles are proportional than they are similar



Given: - In Δ ABC and Δ DEF

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

To prove: - ΔABC ~ ΔDEF

Construction: - We mark point M and N on DE and DF respectively.
Such that AB = DM and AC = DN. Join MN

Proof: - Since $\frac{AB}{DE} = \frac{AC}{DF}$ (Given)

But, AB = DM and AC = DN (by construction)

∴ $\frac{DM}{DE} = \frac{DN}{DF}$

Now In Δ DEF, $\frac{DM}{DE} = \frac{DN}{DF}$ (proved above)

∴ MN ∥ EF [by the converse of BPT]

∴ ∠1 = ∠2 (Corresponding∠'s)

Now, In Δ DMN and Δ DEF

∠1 = ∠2 (proved above)

∠D = ∠d (Common)

∴ ΔDMN ~ Δ DEF (by AA similarity) --------------- (A)

$\frac{DM}{DE} = \frac{MN}{EF}$ [If the Δ's are similar then their corresponding sides are in the same ratio.]

$\frac{AB}{DE} = \frac{MN}{EF}$ [∵ AB = DM]

But, $\frac{AB}{DE} = \frac{BC}{EF}$

∴ $\frac{BC}{\cancel{EF}} = \frac{MN}{\cancel{EF}}$

In ΔABC and Δ DMN

BC = MN (proved above)

AB = DM (by construction)

And AC = DN (by construction)

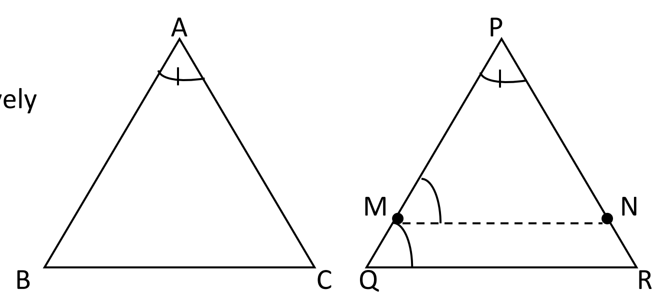
∴ Δ ABC ≅ Δ DMN (by SSS congruency) --------------- (B)

From (A) and (B)

ΔABC ~ ΔDEF **Hence proved**

III. Characteristic property of similar Δ:- → S-A-S Similarity –

If in two Δ's one pair of corresponding sides are proportional and the included angles are equal than the two Δ's are similar.



Given: - In Δ's ABC and PQR

$\frac{AB}{PQ} = \frac{AC}{PR}$ and ∠A = ∠P

To prove: - Δ ABC ~ Δ PQR

Construction: - We mark point M and N on PQ and PR respectively such that AB = DM and AC = DN and join MN.

Proof: - In Δ ABC and Δ PMN

AB = PM (by construction)

∠A = ∠P (given)

AC = PN (By construction)

∴ ΔABC and ≅ Δ PMN (by SAS)

ΔABS ~ΔPMN [∵ congruent Δ's are always similar] -------------- (A)

Now, $\frac{AB}{PQ} = \frac{AC}{PR}$

But, AB = DM and AC = DN

∴ $\frac{PM}{PQ} = \frac{PN}{PR}$

Again, In Δ PQR $\frac{PM}{PQ} = \frac{PN}{PR}$ (proved above)

∴ MN ∥ QR (by the converse of BPT)

∠1 = ∠2 (Corresponding∠'s)

Now, In Δ PMN and Δ PQR

∠1 = ∠2 (proved above)

∠P = ∠P (common)

∴ ΔPMN ~ Δ PQR (by AA) -------------- (B)

From (A) and (B)

ΔABC ~ ΔPQR **Hence proved**

8. Pythagoras Theorem: -

In a triangle, the square on the hypotenuse is equal to the sum of square of other two sides.

Given: - In rt ΔABC rt. ∠ ed. at A

To prove: - $BC^2 = AB^2 + AC^2$

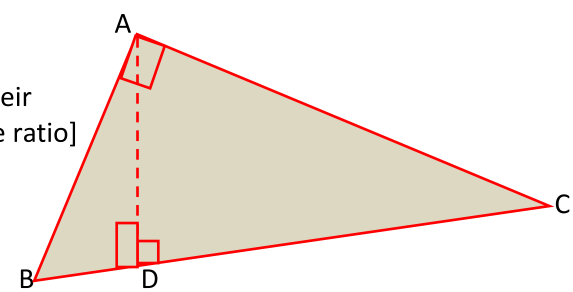
Construction: - We draw AB ⊥ BC

Proof: - In Δ ABD and ABC

∠B = ∠B (Common)

∠BAD = ∠ADB (each 90^0)

∴ Δ ABD ~ Δ ABC (by AA similarity)



∴ $\frac{AB}{BC} = \frac{BD}{AB}$ [If two triangles are similar than their corresponding side are in the same ratio]

$AB^2 = BD \times BC$ ----------- (I)

Again, In Δ ADC and ΔABC

∠C = ∠C (common)

∠ADC = ∠BAC (each 90^0)

∴ Δ ADC ~ Δ ABC (by AA similarity)

$\frac{AC}{BC} = \frac{DC}{AC}$ [if two Δ's are similar than their corresponding side are in the same ratio]

$AC^2 = BC \times DC$ ------------- (II)

Adding (I) and (II)

$AB^2 + AC^2 = BC \times DC + BC \times BD$

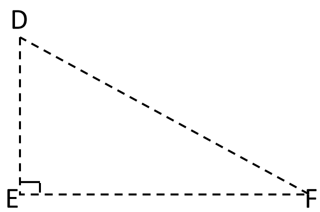
$AB^2 + AC^2 = BC (DC + BD)$

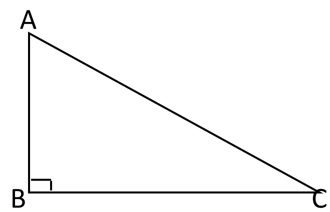
$AB^2 + AC^2 = BC \times BC$

∴ $BC^2 = AB^2 + AC^2$. **Hence proved**

9. Converse of Pythagoras theorem: -

In a triangle if the square of one side is equal to the sum of square of other two sides than the angle opposite to The 1st side is a right angle.





Given: - In $\triangle ABC$, $AC^2 = AB^2 + BC^2$

To prove: - Δ ABC is a rt. $\angle^{ed}$ Δ

Construction: - We draw rt. $\angle^{ed.}$ Δrt. $\angle^{ed.}$ at E such that AB = DE and EF = BC

Proof: - In rt. ΔABC, rt. $\angle^{ed.}$ at E (by construction)

$\therefore$ $DF^2 = DE^2 + EF^2$ (by Pythagoras theorem)

$DF^2 = AB^2 + BC^2$ [$\because$ AB = DE and BC = EF (by construction)] ------ (I)

But, $AC^2 = AB^2 + BC^2$ ------- (II)

From (I) and (II)

$DF^2 = AC^2$

$DF = \sqrt{AC^2}$

DF = AC

Now, In Δ DEF and ABC

DE = AB (by construction)

EF = BC (by Construction)

DE = AB (proved above)

$\therefore$ Δ DEF $\cong$ ΔABC (by SSS congruence)

$\therefore$ $\angle$ B = $\angle$ E (CPCPTC)

But, $\angle E = 90^0$ (by construction)

$\therefore$ $\angle B = 90^0$

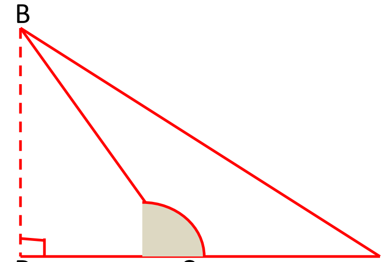
Hence Δ ABC is a rt. $\angle^{ed.}$ triangle

Proved

10. Obtuse Angle Theorem: -

In aΔ ABC, $\angle$ ACB is greater than 90^0 and side AC is produced to D such that segment BD $\perp$ AD.

Prove that $AB^2 = BC^2 + AC^2 + 2\ CA \times CD$



Given: - ΔABC is a Δ in which $\angle$ BCA > 90^0 also AC is produced to D such that BD $\perp$ AD.

To prove: - $AB^2 = BC^2 + AC^2 + 2\ CA \times CD$

Proof: - In rt. ▯ ABD, rt. $\angle^{ed.}$ at D

$\therefore$ $AB^2 = BD^2 + AD^2$ (by Pythagoras theorem)

$AB^2 = BD^2 + (DC + AC)^2$

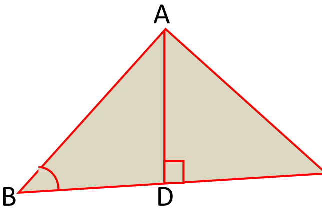
$AB^2 = DB^2 + DC^2 + AC^2 + 2\ DC \times AC$

$AB^2 = (BD^2 + DC^2) + AC^2 + 2\ DC \times AC$

Hence Proved

11. Acute Angle Theorem: -

In Δ ABC, $\angle$ B < 90^0 AD is drawn perpendicular to BC. Prove that $AC^2 = AB^2 + BC^2 - 2\ BC \times BD$



Given: - ABC is a ▯ in which $\angle$ A = 90^0 also AD $\perp$ BC

To Prove: - $AC^2 = AB^2 + BC^2 - 2\ BC \times BD$

Proof: - In rt. ▯ ADC rt. $\angle^{ed.}$ at D

$\therefore$ $AC^2 = AD^2 + DC^2$ (by Pythagoras theorem)

$AC^2 = AD^2 + (BC - DB)^2$

$AC^2 = AD^2 + BC^2 + BD^2 - 2\ BC \times BD$

$AC^2 = (AD^2 + BD^2) + BC^2 - 2\ BC \times BD$

$AC^2 = AB^2 + BC^2 - 2\ BC \times BD$ [in rt. Δ ABC, $AB^2 = AD^2 + BD^2$, By Pythagoras theorem]

Hence proved