

CBSE

IIT-JEE

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NEET

The Success Destination...

TS SIMILAR
TRIANGLES

SIMILAR TRIANGLES

CONTENTS

- Concept of Similarity
- Thales Theorem
- Criteria for Similarity of Triangles
- Area of Two Similar Triangles
- Pythagoras Theorem
- Some Important Theorems

CONCEPT OF SIMILARITY

Geometric figures having the same shape but different sizes are known as similar figures. Two congruent figures are always similar but similar figures need not be congruent.

Illustration 1 :

Any two line segments are always similar but they need not be congruent. They are congruent, if their lengths are equal.

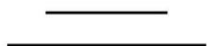


Illustration 2 :

Any two circles are similar but not necessarily congruent. They are congruent if their radii are equal.



Illustration 3 :

- (i) Any two square are similar (see fig. (i))

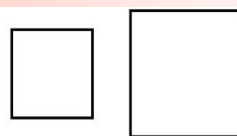


Fig.(i)

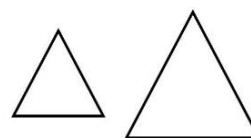


Fig.(ii)

- (ii) Any two equilateral triangles are similar (see fig. (ii))

SIMILAR POLYGONS

Definition

Two polygons are said to be similar to each other, if

- their corresponding angles are equal, and
- the lengths of their corresponding sides are proportional.

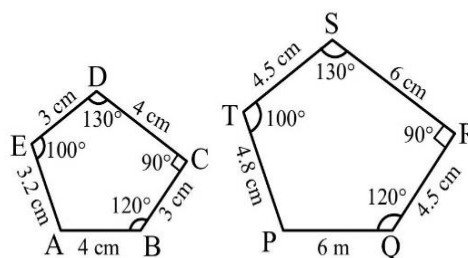
If two polygons ABCDE and PQRST are similar, then from the above definition it follows that :

Angle at A = Angle at P, Angle at B = Angle at Q,
 Angle at C = Angle at R, Angle at D = Angle at S,
 Angle at E = Angle at T

and, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DE}{ST}$

If two polygons ABCDE and PQRST, are similar, we write $ABCDE \sim PQRST$.

Here, the symbol ' \sim ' stands for is similar to.



SIMILAR TRIANGLE AND THEIR PROPERTIES

Definition

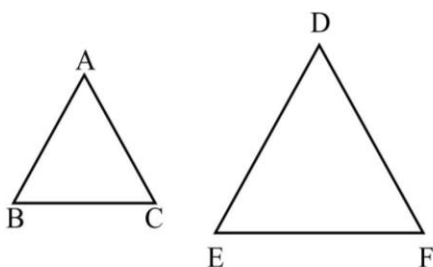
Two triangles are said to be similar, if their

- (i) corresponding angles are equal and,
- (ii) corresponding sides are proportional.

Two triangles ABC and DEF are similar, if

(i) $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$ and,

(ii) $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$



SOME BASIC RESULTS ON PROPORTIONALITY

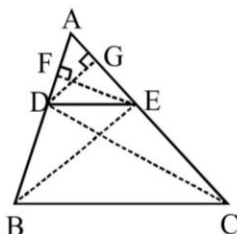
Basic Proportionality Theorem or Thales Theorem

If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

Given : A triangle ABC in which DE || BC, and intersects AB in D and AC in E.

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE, CD and draw EF ⊥ BA and DG ⊥ CA.



Proof : Since EF is perpendicular to AB. Therefore, EF is the height of triangles ADE and DBE.

Now, $\text{Area}(\triangle ADE) = \frac{1}{2} (\text{base} \times \text{height}) = \frac{1}{2} (AD \cdot EF)$

and, $\text{Area}(\triangle DBE) = \frac{1}{2} (\text{base} \times \text{height}) = \frac{1}{2} (DB \cdot EF)$

$$\therefore \frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle DBE)} = \frac{\frac{1}{2}(AD \cdot EF)}{\frac{1}{2}(DB \cdot EF)} = \frac{AD}{DB} \dots (i)$$

Similarly, we have

$$\frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle DEC)} = \frac{\frac{1}{2}(AE \cdot DG)}{\frac{1}{2}(EC \cdot DG)} = \frac{AE}{EC} \dots (ii)$$

But, $\triangle DBE$ and $\triangle DEC$ are on the same base DE and between the same parallels DE and BC.

$\therefore \text{Area}(\triangle DBE) = \text{Area}(\triangle DEC)$

$$\Rightarrow \frac{1}{\text{Area}(\triangle DBE)} = \frac{1}{\text{Area}(\triangle DEC)}$$

[Taking reciprocals of both sides]

$$\Rightarrow \frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle DBE)} = \frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle DEC)}$$

[Multiplying both sides by Area (ΔADE)]

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC} \quad \text{[Using (i) and (ii)]}$$

Corollary : If in a $\triangle ABC$, a line $DE \parallel BC$, intersects AB in D and AC in E, then :

(i) $\frac{AB}{AD} = \frac{AC}{AE}$

(ii) $\frac{AB}{DB} = \frac{AC}{EC}$

Proof : (i) From the basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{DB}{AD} = \frac{EC}{AE} \quad \text{[Taking reciprocals of both sides]}$$

$$\Rightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE} \quad \text{[Adding 1 on both sides]}$$

$$\Rightarrow \frac{AD + DB}{AD} = \frac{AE + EC}{AE} \Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

(ii) From the basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{DE}{EC}$$

$$\Rightarrow \frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

[Adding 1 on both sides]

$$\Rightarrow \frac{AD+DB}{DB} = \frac{AE+EC}{EC} \Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$$

So, if in a $\triangle ABC$, $DE \parallel BC$, and intersect AB in D and AC in E , then we have

$$(i) \frac{AD}{DB} = \frac{AE}{EC} \quad (ii) \frac{DB}{AD} = \frac{EC}{AE}$$

$$(iii) \frac{AB}{AD} = \frac{AC}{AE} \quad (iv) \frac{AD}{AB} = \frac{AE}{AC}$$

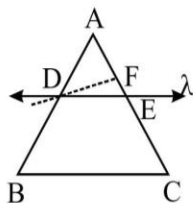
$$(v) \frac{AB}{DB} = \frac{AC}{EC} \quad (vi) \frac{DB}{AB} = \frac{EC}{AC}$$

◆ **Converse of Basic Proportionality Theorem**

If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Given : A $\triangle ABC$ and a line l intersecting AB in D and AC in E , such that $\frac{AD}{DB} = \frac{AE}{EC}$

To prove : $l \parallel BC$ i.e. $DE \parallel BC$



Proof : If possible, let DE be not parallel to BC . Then, there must be another line parallel to BC . Let $DF \parallel BC$.

Since $DF \parallel BC$. Therefore from Basic Proportionality Theorem, we get

$$\frac{AD}{DB} = \frac{AF}{FC} \quad \dots (i)$$

$$\text{But, } \frac{AD}{DB} = \frac{AE}{EC} \quad (\text{Given}) \quad \dots (ii)$$

From (i) and (ii), we get

$$\frac{AF}{FC} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AF}{FC} + 1 = \frac{AE}{EC} + 1 \quad [\text{Adding 1 on both sides}]$$

$$\Rightarrow \frac{AF+FC}{FC} = \frac{AE+EC}{EC}$$

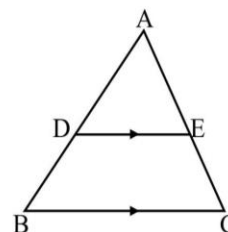
$$\Rightarrow \frac{AC}{FC} = \frac{AC}{EC} \Rightarrow FC = EC$$

This is possible only when F and E coincide i.e. DF is the line l itself. But, $DF \parallel BC$. Hence, $l \parallel BC$.

◆ **EXAMPLES** ◆

Ex.1 D and E are points on the sides AB and AC respectively of a $\triangle ABC$ such that $DE \parallel BC$.

Find the value of x , when



(i) $AD = 4$ cm, $DB = (x - 4)$ cm, $AE = 8$ cm and $EC = (3x - 19)$ cm

(ii) $AD = (7x - 4)$ cm, $AE = (5x - 2)$ cm, $DB = (3x + 4)$ cm and $EC = 3x$ cm.

Sol. (i) In $\triangle ABC$, $DE \parallel BC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad (\text{By thales theorem})$$

$$\frac{4}{x-4} = \frac{8}{3x-19}$$

$$4(3x - 19) = 8(x - 4)$$

$$12x - 76 = 8x - 32$$

$$4x = 44$$

$$x = 11$$

(ii) In $\triangle ABC$, $DE \parallel BC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad (\text{By thales theorem})$$

$$\frac{7x-4}{3x+4} = \frac{5x-2}{3x}$$

$$21x^2 - 12x = 15x^2 - 6x + 20x - 8$$

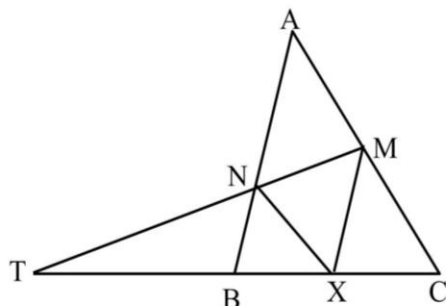
$$6x^2 - 26x + 8 = 0$$

$$3x^2 - 13x + 4 = 0$$

$$(x - 4)(3x - 1) = 0 \Rightarrow x = 4, 1/3$$

Ex.2 Let X be any point on the side BC of a triangle ABC. If XM, XN are drawn parallel to BA and CA meeting CA, BA in M, N respectively; MN meets BC produced in T, prove that $TX^2 = TB \times TC$.

Sol. In $\triangle TXM$, we have



$XM \parallel BN$

$$\therefore \frac{TB}{TX} = \frac{TM}{TN} \quad \dots (i)$$

In $\triangle TMC$, we have

$XN \parallel CM$

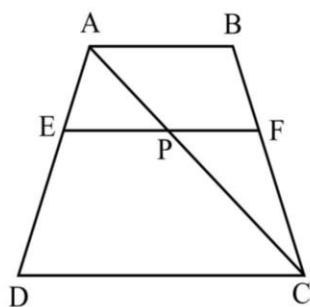
$$\therefore \frac{TX}{TC} = \frac{TN}{TM} \quad \dots (ii)$$

From equations (i) and (ii), we get

$$\frac{TB}{TX} = \frac{TX}{TC}$$

$$\Rightarrow TX^2 = TB \times TC$$

Ex.3 In fig., $EF \parallel AB \parallel DC$. Prove that $\frac{AE}{ED} = \frac{BF}{FC}$.



Sol. We have,

$EF \parallel AB \parallel DC$

$\Rightarrow EP \parallel DC$

Thus, in $\triangle ADC$, we have

$EP \parallel DC$

Therefore, by basic proportionality theorem, we have

$$\frac{AE}{ED} = \frac{AP}{PC} \quad \dots (i)$$

Again, $EF \parallel AB \parallel DC$

$\Rightarrow FP \parallel AB$

Thus, in $\triangle CAB$, we have

$FP \parallel BA$

Therefore, by basic proportionality theorem, we have

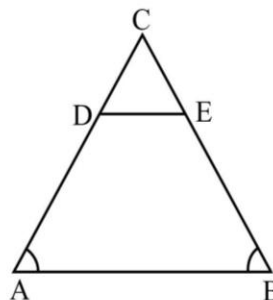
$$\frac{BF}{FC} = \frac{AP}{PC} \quad \dots (ii)$$

From (i) and (ii), we have

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Ex.4 In figure, $\angle A = \angle B$ and $DE \parallel BC$. Prove that $AD = BE$

Sol.



$\angle A = \angle B$ (given)

$\Rightarrow BC = AC \quad \dots (i)$

(Sides opposite to equal angles are equal)

Now, $DE \parallel AB$

$$\Rightarrow \frac{CD}{DA} = \frac{CE}{EB}$$

(By basic proportionality theorem)

$$\Rightarrow \frac{CD}{DA} + 1 = \frac{CE}{EB} + 1$$

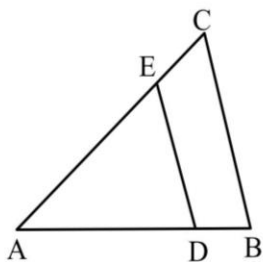
(Adding 1 on both sides)

$$\Rightarrow \frac{CD+DA}{DA} = \frac{CE+EB}{EB} \Rightarrow \frac{CA}{DA} = \frac{CE}{EB}$$

$$\Rightarrow \frac{AC}{AD} = \frac{BC}{BE} \Rightarrow \frac{AC}{AD} = \frac{AC}{BE}$$

$$\Rightarrow \frac{1}{AD} = \frac{1}{BE} \Rightarrow AD = BE$$

Ex.5 In fig., $DE \parallel BC$. If $AD = 4x - 3$, $DB = 3x - 1$, $AE = 8x - 7$ and $EC = 5x - 3$, find the value of x .



Sol. In $\triangle ABC$, we have

$DE \parallel BC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad [\text{By Thale's Theorem}]$$

$$\Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

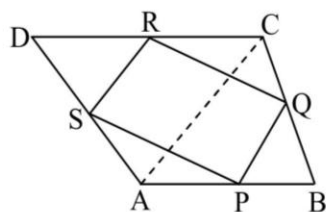
$$\Rightarrow (4x-3)(5x-3) = (3x-1)(8x-7)$$

$$x = 1$$

Ex.6 Prove that the line segment joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.

Sol. **Given :** A quadrilateral ABCD in which P, Q, R, S are the midpoints of AB, BC, CD and DA respectively.

To prove : PQRS is a parallelogram.



Construction : Join AC.

Proof : In $\triangle ABC$, P and Q are the midpoints of AB and BC respectively.

$$\therefore PQ \parallel AC \quad \dots(i)$$

In $\triangle DAC$, S and R are the midpoints of AD and CD respectively.

$$\therefore SR \parallel AC \quad \dots(ii)$$

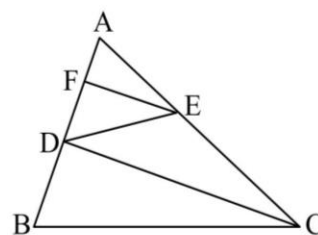
From (i) and (ii), we get $PQ \parallel SR$.

Similarly, $PS \parallel QR$.

Hence, PQRS is a parallelogram

Ex.7 In fig. $DE \parallel BC$ and $CD \parallel EF$. Prove that $AD^2 = AB \times AF$.

Sol. In $\triangle ABC$, we have



$DE \parallel BC$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} \quad \dots(i)$$

In $\triangle ADC$, we have

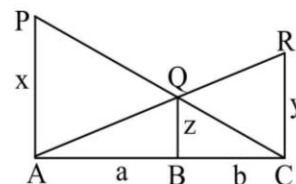
$FE \parallel DC$

$$\Rightarrow \frac{AD}{AF} = \frac{AC}{AE} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{AB}{AD} = \frac{AD}{AF} \Rightarrow AD^2 = AB \times AF$$

Ex.8 In the given figure PA, QB and RC each is perpendicular to AC such that $PA = x$, $RC = y$, $QB = z$, $AB = a$ and $BC = b$. Prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.



Sol. $PA \perp AC$ and $QB \perp AC \Rightarrow QB \parallel PA$.

Thus, in $\triangle PAC$, $QB \parallel PA$. So, $\triangle QBC \sim \triangle PAC$

$$\therefore \frac{QB}{PA} = \frac{BC}{AC} \Rightarrow \frac{z}{x} = \frac{b}{a+b} \quad \dots(i)$$

[By the property of similar Δ]

In $\triangle RAC$, $QB \parallel RC$. So, $\triangle QBC \sim \triangle RAC$

$$\therefore \frac{QB}{RC} = \frac{AB}{AC} \Rightarrow \frac{z}{y} = \frac{a}{a+b} \quad \dots(ii)$$

[By the property of similar Δ]

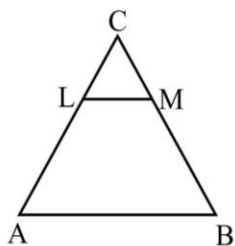
From (i) and (ii), we get

$$\frac{z}{x} + \frac{z}{y} = \left(\frac{b}{a+b} + \frac{a}{a+b} \right) = 1$$

$$\Rightarrow \frac{z}{x} + \frac{z}{y} = 1 \Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

Hence, $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.

Ex.9 In fig., $LM \parallel AB$. If $AL = x - 3$, $AC = 2x$, $BM = x - 2$ and $BC = 2x + 3$, find the value of x .



Sol. In $\triangle ABC$, we have

$LM \parallel AB$

$$\therefore \frac{AL}{LC} = \frac{MB}{MC} \quad [\text{By Thale's Theorem}]$$

$$\Rightarrow \frac{AL}{AC - AL} = \frac{BM}{BC - BM}$$

$$\Rightarrow \frac{x - 3}{2x - (x - 3)} = \frac{x - 2}{(2x + 3) - (x - 2)}$$

$$\Rightarrow \frac{x - 3}{x + 3} = \frac{x - 2}{x + 5}$$

$$\Rightarrow (x - 3)(x + 5) = (x - 2)(x + 3)$$

$$\Rightarrow x^2 + 2x - 15 = x^2 + x - 6$$

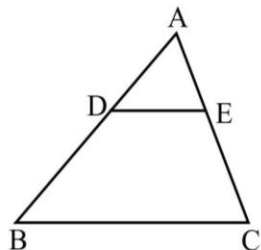
$$\Rightarrow x = 9$$

Ex.10 In a given $\triangle ABC$, $DE \parallel BC$ and $\frac{AD}{DB} = \frac{3}{4}$. If $AC = 14$ cm, find AE .

Sol. In $\triangle ABC$, we have

$DE \parallel BC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad [\text{By Thales Theorem}]$$



$$\Rightarrow \frac{AD}{DB} = \frac{AE}{AC - AE}$$

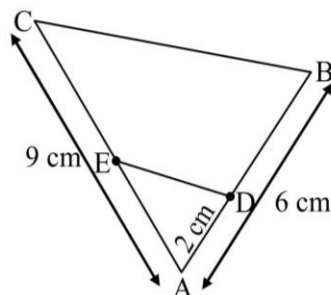
$$\Rightarrow \frac{3}{4} = \frac{AE}{14 - AE} \quad [\because AC = 14]$$

$$\Rightarrow 3(14 - AE) = 4AE$$

$$\Rightarrow 42 - 3AE = 4AE$$

$$\Rightarrow 42 = 7AE \Rightarrow AE = \frac{42}{7} = 6 \text{ cm}$$

Ex.11 In figure, $DE \parallel BC$. Find AE .



Sol. Let $AE = x$ cm

Then $EC = (9 - x)$ cm

$AD = 2$ cm

$DB = (6 - 2)$ cm = 4 cm

$$\text{We have } \frac{AE}{BE} = \frac{AD}{DB}$$

[By Basic Proportionality Theorem]

$$\Rightarrow \frac{x}{9 - x} = \frac{2}{4} \Rightarrow 4x = 2(9 - x)$$

$$\Rightarrow 6x = 18 \Rightarrow x = 3$$

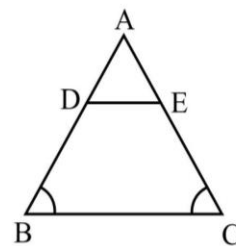
Hence, $AE = 3$ cm

Ex.12 In figure, ABC is a triangle in which $AB = AC$. Points D and E are points on the sides AB and AC respectively such that $AD = AE$. Show that the points B, C, E and D are concyclic.

Sol. In order to prove that the points B, C, E and D are concyclic, it is sufficient to show that $\angle ABC + \angle CED = 180^\circ$ and $\angle ACB + \angle BDE = 180^\circ$.

In $\triangle ABC$, we have

$AB = AC$ and $AD = AE$



$$\Rightarrow AB - AD = AC - AE$$

$$\Rightarrow DB = EC$$

Thus, we have

$$AD = AE \text{ and } DB = EC$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow DE \parallel BC$$

[By the converse of Thale's Theorem]

$$\Rightarrow \angle ABC = \angle ADE \text{ [Corresponding angles]}$$

$$\Rightarrow \angle ABC + \angle BDE = \angle ADE + \angle BDE$$

[Adding $\angle BDE$ on both sides]

$$\Rightarrow \angle ABC + \angle BDE = 180^\circ$$

$$\Rightarrow \angle ACB + \angle BDE = 180^\circ$$

[$\ominus AB = AC \therefore \angle ABC = \angle ACB$]

Again, $DE \parallel BC$

$$\Rightarrow \angle ACB = \angle AED$$

$$\Rightarrow \angle ACB + \angle CED = \angle AED + \angle CED$$

[Adding $\angle CED$ on both sides]

$$\Rightarrow \angle ACB + \angle CED = 180^\circ$$

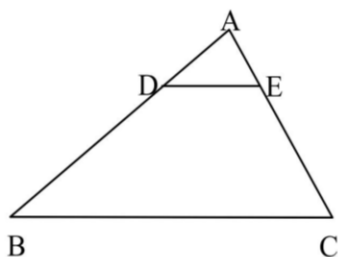
$$\Rightarrow \angle ABC + \angle CED = 180^\circ \text{ [}\ominus \angle ABC = \angle ACB\text{]}$$

Thus, BDEC is quadrilateral such that

$$\angle ACB + \angle BDE = 180^\circ \text{ and}$$

$$\angle ABC + \angle CED = 180^\circ$$

Ex.13 In fig., $\frac{AD}{DB} = \frac{1}{3}$ and $\frac{AE}{AC} = \frac{1}{4}$. Using converse of basic proportionality theorem, prove that $DE \parallel BC$.



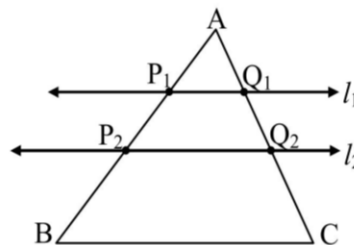
Sol. $\frac{AE}{AC} = \frac{1}{4}$

$$\Rightarrow \frac{AC}{AE} = 4 \Rightarrow \frac{AC}{AE} - 1 = 3$$

$$\Rightarrow \frac{AC - AE}{AE} = 3 \Rightarrow \frac{EC}{AE} = 3 \Rightarrow \frac{AE}{EC} = \frac{1}{3}$$

Ex.14 Using basic proportionality theorem, prove that the lines drawn through the points of trisection of one side of a triangle parallel to another side trisect the third side.

Sol.



$$l_1 \parallel BC, l_2 \parallel BC$$

$$\text{and } AP_1 = P_1P_2 = P_2B \text{ (given)}$$

$$= \frac{1}{3} AB.$$

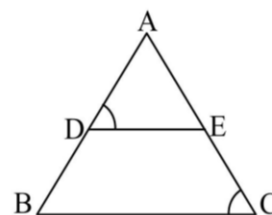
$$\text{To prove, } AQ_1 = Q_1Q_2 = Q_2B$$

$$= \frac{1}{3} AC.$$

$$\text{Proof } \frac{AQ_1}{AC} = \frac{AP_1}{AB} = \frac{\frac{1}{3} AB}{AB}$$

$$\Rightarrow \frac{AQ_1}{AC} = \frac{1}{3} \Rightarrow AQ_1 = \frac{1}{3} AC$$

Ex.15 In the given figure, $\frac{AD}{DB} = \frac{AE}{EC}$ and $\angle ADE = \angle ACB$. Prove that $\triangle ABC$ is an isosceles triangle.



Sol. We have,

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow DE \parallel BC$$

[By the converse of Thale's theorem]

$$\therefore \angle ADE = \angle ABC \text{ (corresponding } \angle\text{s)}$$

$$\text{But, } \angle ADE = \angle ACB \text{ (given)}$$

$$\therefore \angle ABC = \angle ACB.$$

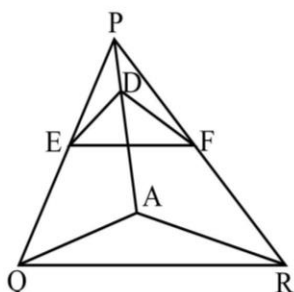
$$\text{So, } AB = AC \text{ [sides opposite to equal angles]}$$

Hence, $\triangle ABC$ is an isosceles triangles.

Ex.16 In fig., if $DE \parallel AQ$ and $DF \parallel AR$. Prove that $EF \parallel QR$. [NCERT]

Sol. In $\triangle PQA$, we have

$$DE \parallel AQ \quad [\text{Given}]$$



Therefore, by basic proportionality theorem, we have

$$\frac{PE}{EQ} = \frac{PD}{DA} \quad \dots(i)$$

In $\triangle PAR$, we have

$$DF \parallel AR \quad [\text{Given}]$$

Therefore, by basic proportionality theorem, we have

$$\frac{PD}{DA} = \frac{PF}{FR} \quad \dots(ii)$$

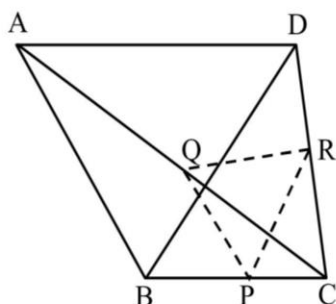
From (i) and (ii), we have

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\Rightarrow EF \parallel QR$ [By the converse of Basic Proportionality Theorem]

Ex.17 Two triangles ABC and DBC lie on the same side of the base BC . From a point P on BC , $PQ \parallel AB$ and $PR \parallel BD$ are drawn. They meet AC in Q and DC in R respectively. Prove that $QR \parallel AD$.

Sol. **Given :** Two triangles ABC and DBC lie on the same side of the base BC . Points P , Q and R are points on BC , AC and CD respectively such that $PR \parallel BD$ and $PQ \parallel AB$.



To Prove : $QR \parallel AD$

Proof : In $\triangle ABC$, we have

$$PQ \parallel AB$$

$$\therefore \frac{CP}{PB} = \frac{CQ}{QA} \quad \dots(i)$$

[By Basic Proportionality Theorem]

In $\triangle BCD$, we have

$$PR \parallel BD$$

$$\therefore \frac{CP}{PB} = \frac{CR}{RD} \quad \dots(ii)$$

[By Thale's Theorem]

From (i) and (ii), we have

$$\frac{CQ}{QA} = \frac{CR}{RD}$$

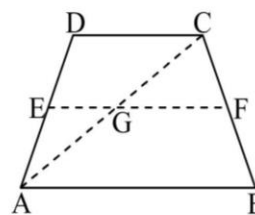
Thus, in $\triangle ACD$, Q and R are points on AC and CD respectively such that

$$\frac{CQ}{QA} = \frac{CR}{RD}$$

$\Rightarrow QR \parallel AD$ [By the converse of Basic proportionality theorem]

Ex.18 $ABCD$ is a trapezium with $AB \parallel DC$. E and F are points on non-parallel sides AD and BC respectively such that $EF \parallel AB$. Show that

$$\frac{AE}{ED} = \frac{BF}{FC}$$



Sol. **Given :** A trap. $ABCD$ in which $AB \parallel DC$. E and F are points on AD and BC respectively such that $EF \parallel AB$.

To prove : $\frac{AE}{ED} = \frac{BF}{FC}$

Construction : Join AC , intersecting EF at G .

Proof : $EF \parallel AB$ and $AB \parallel DC$

$\Rightarrow EF \parallel DC$

Now, in $\triangle ADC$, $EG \parallel DC$

$$\therefore \frac{AE}{ED} = \frac{AG}{GC} \quad \dots(i) \text{ [By Thale's theorem]}$$

Similarly, in $\triangle CAB$, $GF \parallel AB$.

$$\therefore \frac{AG}{GC} = \frac{BF}{FC} \quad \dots(ii)$$

$$[\ominus \frac{GC}{AG} = \frac{FC}{BF} \text{ by Thale's theorem}]$$

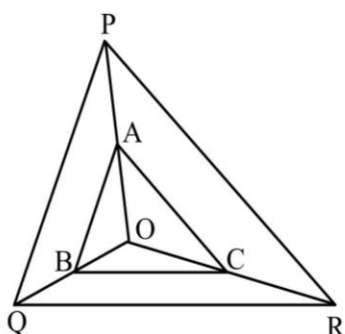
From (i) and (ii), we get

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Ex.19 In fig., A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$. [NCERT]

Sol. In $\triangle OPQ$, we have

$$AB \parallel PQ$$



$$\Rightarrow \frac{OA}{AP} = \frac{OB}{BQ} \quad \dots(i)$$

In $\triangle OQR$, we have

$$BC \parallel QR$$

$$\Rightarrow \frac{OB}{BQ} = \frac{OC}{CR} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{OA}{AP} = \frac{OC}{CR}$$

Thus, A and C are points on sides OP and OR respectively of $\triangle OPR$, such that

$$\frac{OA}{AP} = \frac{OC}{CR}$$

$\Rightarrow AC \parallel PR$ [Using the converse of BPT]

Ex.20 Any point X inside $\triangle DEF$ is joined to its vertices. From a point P in DX, PQ is drawn parallel to DE meeting XE at Q and QR is drawn parallel to EF meeting XF in R. Prove that $PR \parallel DF$.

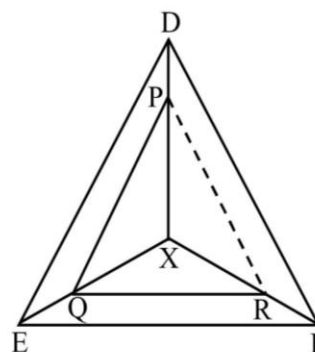
Sol. A $\triangle DEF$ and a point X inside it. Point X is joined to the vertices D, E and F. P is any point on DX. $PQ \parallel DE$ and $QR \parallel EF$.

To Prove : $PR \parallel DF$

Construction : Join PR.

Proof : In $\triangle XED$, we have

$$PQ \parallel DE$$



$$\therefore \frac{XP}{PD} = \frac{XQ}{QE} \quad \dots(i) \text{ [By Thale's Theorem]}$$

In $\triangle XEF$, we have

$$QR \parallel EF$$

$$\therefore \frac{XQ}{QE} = \frac{XR}{RF} \quad \dots(ii) \text{ [By Thale's Theorem]}$$

From (i) and (ii), we have

$$\frac{XP}{PD} = \frac{XR}{RF}$$

Thus, in $\triangle XFD$, points R and P are dividing sides XF and XD in the same ratio. Therefore, by the converse of Basic Proportionality Theorem, we have, $PR \parallel DF$

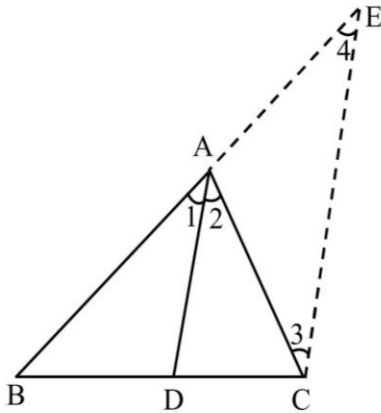
Theorem 1 :

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

Given : A $\triangle ABC$ in which AD is the internal bisector of $\angle A$ and meets BC in D.

$$\text{To Prove : } \frac{BD}{DC} = \frac{AB}{AC}$$

Construction : Draw $CE \parallel DA$ to meet BA produced in E .



Proof : Since $CE \parallel DA$ and AC cuts them.

$$\therefore \angle 2 = \angle 3 \quad [\text{Alternate angles}] \dots(i)$$

$$\text{and, } \angle 1 = \angle 4 \quad [\text{Corresponding angles}] \dots(ii)$$

$$\text{But, } \angle 1 = \angle 2 \quad [\because AD \text{ is the bisector of } \angle A]$$

From (i) and (ii), we get

$$\angle 3 = \angle 4$$

Thus, in $\triangle ACE$, we have

$$\angle 3 = \angle 4$$

$$\Rightarrow AE = AC \quad \dots(iii)$$

[Sides opposite to equal angles are equal]

Now, in $\triangle BCE$, we have

$$DA \parallel CE$$

$$\Rightarrow \frac{BD}{DC} = \frac{BA}{AE}$$

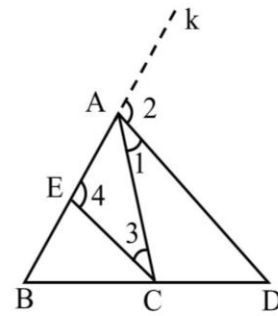
$$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$$

[$\because BA = AB$ and $AE = AC$ (From (iii))]

$$\text{Hence, } \frac{BD}{DC} = \frac{AB}{AC}$$

Theorem 2 :

The external bisector of an angle of a triangle divides the opposite side externally in the ratio of the sides containing the angle.

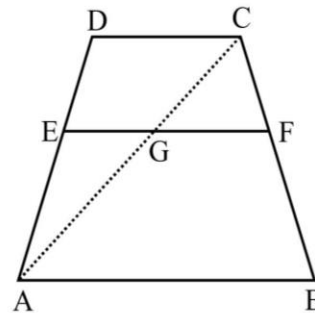


Ex.21 Prove that any line parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.

Sol. **Given :** A trapezium $ABCD$ in which $DC \parallel AB$ and EF is a line parallel to DC and AB .

$$\text{To Prove : } \frac{AE}{ED} = \frac{BF}{FC}$$

Construction : Join AC , meeting EF in G .



Proof : In $\triangle ADC$, we have

$$EG \parallel DC$$

$$\Rightarrow \frac{AE}{ED} = \frac{AG}{GC} \quad [\text{By Thale's Theorem}] \dots(i)$$

In $\triangle ABC$, we have

$$GF \parallel AB$$

$$\Rightarrow \frac{AG}{GC} = \frac{BF}{FC} \quad [\text{By Thale's Theorem}] \dots(ii)$$

From (i) and (ii), we get

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Ex.22 Prove that the line drawn from the mid-point of one side of a triangle parallel of another side bisects the third side.

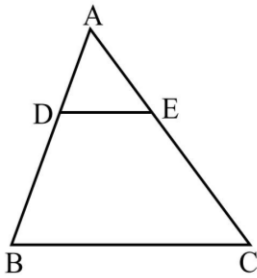
Sol. **Given :** A $\triangle ABC$, in which D is the mid-point of side AB and the line DE is drawn parallel to BC , meeting AC in E .

To Prove : E is the mid-point of AC i.e.,
 AE = EC.

Proof : In $\triangle ABC$, we have

DE \parallel BC

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC} \text{ [By Thale's Theorem](i)}$$



But, D is the mid-point of AB.

$$\Rightarrow AD = DB$$

$$\Rightarrow \frac{AD}{DB} = 1 \quad \text{....(ii)}$$

From (i) and (ii), we get

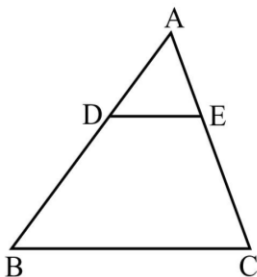
$$\frac{AE}{EC} = 1$$

$$\Rightarrow AE = EC$$

Hence, E bisects AC.

Ex.23 Prove that the line joining the mid-point of two sides of a triangle is parallel to the third side. [NCERT]

Sol. **Given :** A $\triangle ABC$ in which D and E are mid-point of sides AB and AC respectively.



To Prove : DE \parallel BC

Proof : Since D and E are mid-points of AB and AC respectively.

$$\therefore AD = DB \text{ and } AE = EC$$

$$\Rightarrow \frac{AE}{DB} = 1 \text{ and } \frac{AE}{EC} = 1$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

Thus, the line DE divides the sides AB and AC of $\triangle ABC$ in the same ratio. Therefore, by the converse of Basic Proportionality Theorem, we have

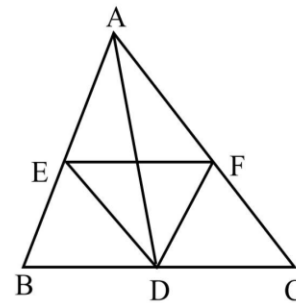
DE \parallel BC

Ex.24 AD is a median of $\triangle ABC$. The bisector of $\angle ADB$ and $\angle ADC$ meet AB and AC in E and F respectively. Prove that EF \parallel BC.

Sol. **Given :** In $\triangle ABC$, AD is the median and DE and DF are the bisectors of $\angle ADB$ and $\angle ADC$ respectively, meeting AB and AC in E and F respectively.

To Prove : EF \parallel BC

Proof : In $\triangle ADB$, DE is the bisector of $\angle ADB$.



$$\therefore \frac{AD}{DB} = \frac{AE}{EB} \quad \text{....(i)}$$

In $\triangle ADC$, DF is the bisector of $\angle ADC$.

$$\therefore \frac{AD}{DC} = \frac{AF}{FC}$$

$$\Rightarrow \frac{AD}{DB} = \frac{AF}{FC} \left[\begin{array}{l} \ominus AD \text{ is the median} \\ \therefore BD = DC \end{array} \right] \text{....(ii)}$$

From (i) and (ii), we get

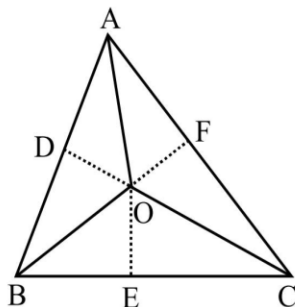
$$\frac{AE}{EB} = \frac{AF}{FC}$$

Thus, in $\triangle ABC$, line segment EF divides the sides AB and AC in the same ratio.

Hence, EF is parallel to BC.

Ex.25 O is any point inside a triangle ABC. The bisector of $\angle AOB$, $\angle BOC$ and $\angle COA$ meet the sides AB, BC and CA in point D, E and F respectively. Show that $AD \times BE \times CF = DB \times EC \times FA$.

Sol. In $\triangle AOB$, OD is the bisector of $\angle AOB$.



$$\therefore \frac{OA}{OB} = \frac{AD}{DB} \quad \dots(i)$$

In $\triangle BOC$, OE is the bisector of $\angle BOC$.

$$\therefore \frac{OB}{OC} = \frac{BE}{EC} \quad \dots(ii)$$

In $\triangle COA$, OF is the bisector of $\angle COA$

$$\therefore \frac{OC}{OA} = \frac{CF}{FA} \quad \dots(iii)$$

Multiplying the corresponding sides of (i), (ii) and (iii), we get

$$\frac{OA}{OB} \times \frac{OB}{OC} \times \frac{OC}{OA} = \frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA}$$

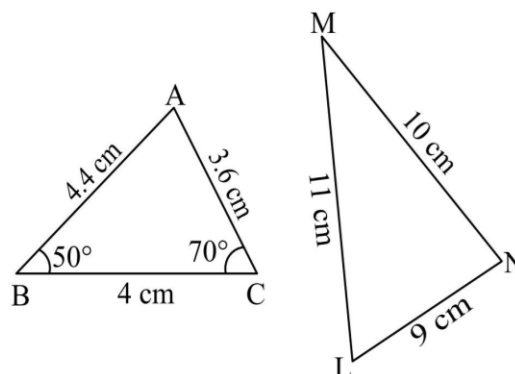
$$\Rightarrow 1 = \frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA}$$

$$\Rightarrow DB \times EC \times FA = AD \times BE \times CF$$

$$\Rightarrow AD \times BE \times CF = DB \times EC \times FA$$

❖ **EXAMPLES** ❖

Ex.26 In figure, find $\angle L$.



Sol. In $\triangle ABC$ and $\triangle LMN$,

$$\frac{AB}{LM} = \frac{4.4}{11} = \frac{2}{5}$$

$$\frac{BC}{MN} = \frac{4}{10} = \frac{2}{5}$$

$$\text{and } \frac{CA}{NL} = \frac{3.6}{9} = \frac{2}{5}$$

$$\Rightarrow \frac{AB}{LM} = \frac{BC}{MN} = \frac{CA}{NL}$$

$$\Rightarrow \triangle ABC \sim \triangle LMN \quad (\text{SSS similarity})$$

$$\Rightarrow \angle L = \angle A$$

$$= 180^\circ - \angle B - \angle C$$

$$= 180^\circ - 50^\circ - 70^\circ = 60^\circ$$

$$\Rightarrow \angle L = 60^\circ$$

Ex.27 Examine each pair of triangles in Figure, and state which pair of triangles are similar. Also, state the similarity criterion used by you for answering the question and write the similarity relation in symbolic form.

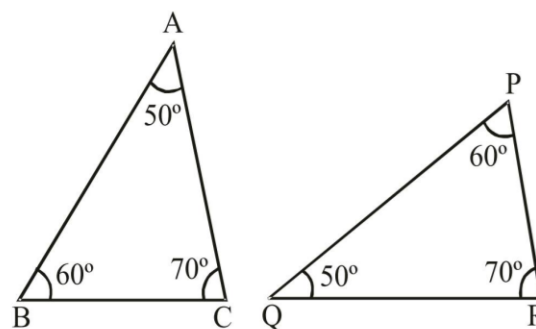


Figure (i)

➤ **CRITERIA FOR SIMILARITY OF TRIANGLES**

❖ **Equiangular Triangles :**

Two triangles are said to be equiangular, if their corresponding angles are equal.

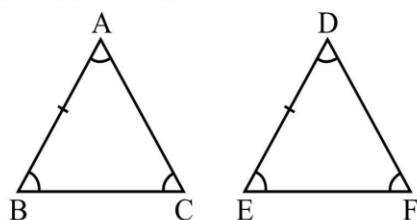
If two triangles are equiangular, then they are similar.

Two triangles ABC and DEF such that

$$\angle A = \angle D, \angle B = \angle E \text{ and } \angle C = \angle F.$$

Then $\triangle ABC \sim \triangle DEF$ and

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$



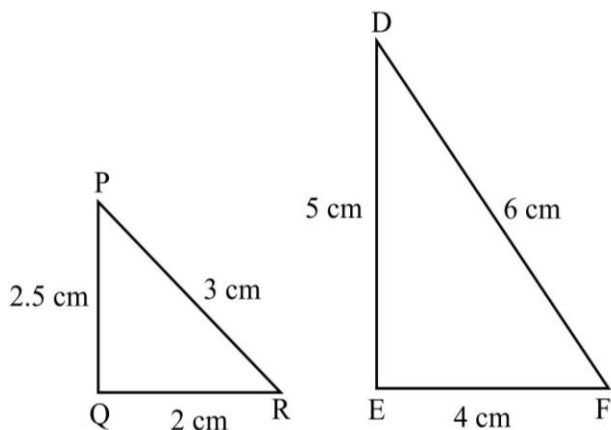


Figure (ii)

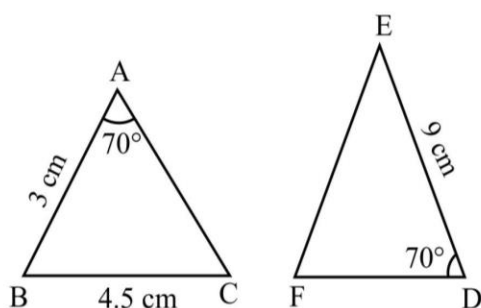


Figure (iii)

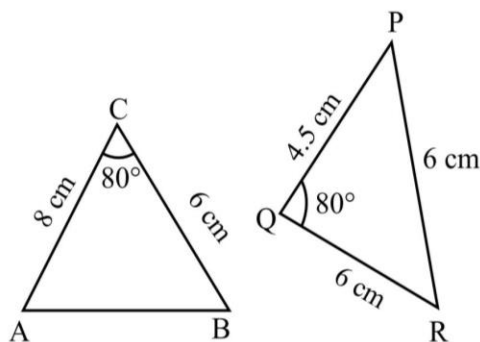


Figure (iv)

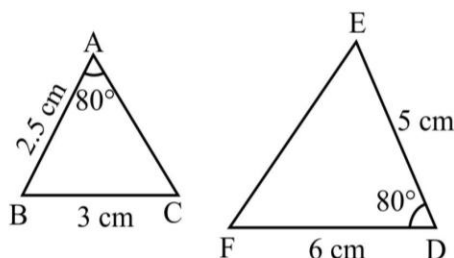


Figure (v)

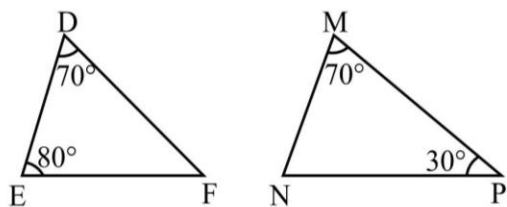


Figure (vi)

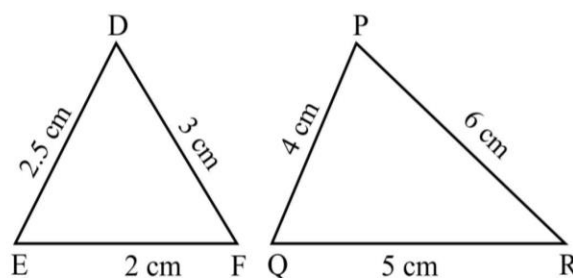


Figure (vii)

Sol. (i) $\angle A = \angle Q$, $\angle B = \angle P$ and $\angle C = \angle R$.

$\therefore \triangle ABC \sim \triangle QPR$ (AAA-similarity)

(ii) In triangle PQR and DEF, we observe that

$$\frac{PQ}{DE} = \frac{QR}{EF} = \frac{PR}{DF} = \frac{1}{2}$$

Therefore, by SSS-criterion of similarity, we have

$$\triangle PQR \sim \triangle DEF$$

(iii) SAS-similarity is not satisfied as included angles are not equal.

(iv) $\triangle CAB \sim \triangle QRP$ (SAS-similarity), as

$$\frac{CA}{QR} = \frac{CB}{QP} \text{ and } \angle C = \angle Q.$$

(v) In \triangle 's ABC and DEF, we have

$$\angle A = \angle D = 80^\circ$$

$$\text{But, } \frac{AB}{DE} \neq \frac{AC}{DF} \quad [\ominus AC \text{ is not given}]$$

So, by SAS-criterion of similarity these two triangles are not similar.

(vi) In \triangle 's DEF and MNP, we have

$$\angle D = \angle M = 70^\circ$$

$$\angle E = \angle N = 80^\circ \quad [\ominus \angle N = 180^\circ - \angle M - \angle P = 180^\circ - 70^\circ - 30^\circ = 80^\circ]$$

So, by AA-criterion of similarity

$$\triangle DEF \sim \triangle MNP.$$

(vii) FE = 2 cm, FD = 3 cm, ED = 2.5 cm

$$PQ = 4 \text{ cm, PR} = 6 \text{ cm, QR} = 5 \text{ cm}$$

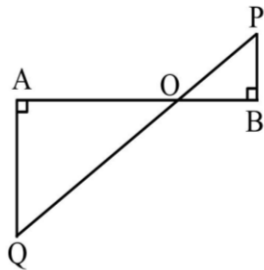
$\therefore \triangle FED \sim \triangle PQR$ (SSS-similarity)

Ex.28 In figure, QA and PB are perpendicular to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm. Find AQ.

Sol. In triangles AOQ and BOP, we have
 $\angle OAQ = \angle OBP$ [Each equal to 90°]
 $\angle AOQ = \angle BOP$
 [Vertically opposite angles]

Therefore, by AA-criterion of similarity

$$\triangle AOQ \sim \triangle BOP$$



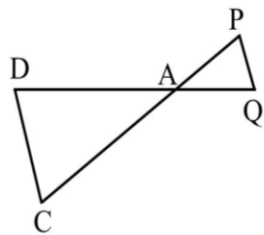
$$\Rightarrow \frac{AO}{BO} = \frac{OQ}{OP} = \frac{AQ}{BP}$$

$$\Rightarrow \frac{AO}{BO} = \frac{AQ}{BP} \Rightarrow \frac{10}{6} = \frac{AQ}{9}$$

$$\Rightarrow AQ = \frac{10 \times 9}{6} = 15 \text{ cm}$$

Ex.29 In figure, $\triangle ACB \sim \triangle APQ$. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm, AP = 2.8 cm, find CA and AQ.

Sol. We have, $\triangle ACB \sim \triangle APQ$



$$\Rightarrow \frac{AC}{AP} = \frac{CB}{PQ} = \frac{AB}{AQ}$$

$$\Rightarrow \frac{AC}{AP} = \frac{CB}{PQ} \text{ and } \frac{CB}{PQ} = \frac{AB}{AQ}$$

$$\Rightarrow \frac{AC}{2.8} = \frac{8}{4} \text{ and } \frac{8}{4} = \frac{6.5}{AQ}$$

$$\Rightarrow \frac{AC}{2.8} = 2 \text{ and } \frac{6.5}{AQ} = 2$$

$$\Rightarrow AC = (2 \times 2.8) \text{ cm} = 5.6 \text{ cm and}$$

$$AQ = \frac{6.5}{2} \text{ cm} = 3.25 \text{ cm}$$

Ex.30 The perimeters of two similar triangles ABC and PQR are respectively 36 cm and 24 cm. If PQ = 10 cm, find AB.

Sol. Since the ratio of the corresponding sides of similar triangles is same as the ratio of their perimeters.

$$\therefore \triangle ABC \sim \triangle PQR$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{36}{24}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{36}{24} \Rightarrow \frac{AB}{10} = \frac{36}{24}$$

$$\Rightarrow AB = \frac{36 \times 10}{24} \text{ cm} = 15 \text{ cm}$$

Ex.31 In figure, $\angle CAB = 90^\circ$ and $AD \perp BC$. If AC = 75 cm, AB = 1 m and BD = 1.25 m, find AD.

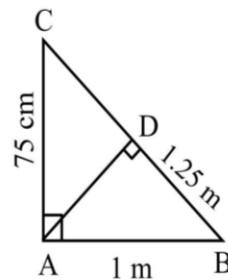
Sol. We have,

$$AB = 1 \text{ m} = 100 \text{ cm, } AC = 75 \text{ cm and } BD = 125 \text{ cm}$$

In $\triangle BAC$ and $\triangle BDA$, we have

$$\angle BAC = \angle BDA \text{ [Each equal to } 90^\circ]$$

and, $\angle B = \angle B$



So, by AA-criterion of similarity, we have

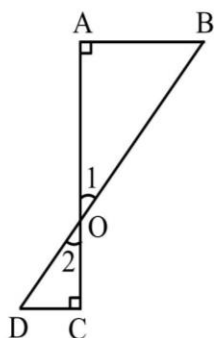
$$\triangle BAC \sim \triangle BDA$$

$$\Rightarrow \frac{BA}{BD} = \frac{AC}{AD}$$

$$\Rightarrow \frac{100}{125} = \frac{75}{AD}$$

$$\Rightarrow AD = \frac{125 \times 75}{100} \text{ cm} = 93.75 \text{ cm}$$

Ex.32 In figure, if $\angle A = \angle C$, then prove that $\triangle AOB \sim \triangle COD$.

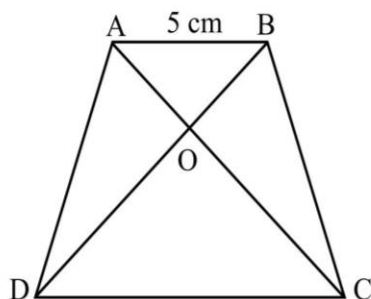


Sol. In triangles AOB and COD, we have
 $\angle A = \angle C$ [Given]
 and, $\angle 1 = \angle 2$ [Vertically opposite angles]
 Therefore, by AA-criterion of similarity, we have
 $\triangle AOB \sim \triangle COD$

Ex.33 In figure, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and $AB = 5$ cm.
 Find the value of DC.

Sol. In $\triangle AOB$ and $\triangle COD$, we have
 $\angle AOB = \angle COD$ [Vertically opposite angles]

$$\frac{AO}{OC} = \frac{BO}{OD} \quad [\text{Given}]$$



So, by SAS-criterion of similarity, we have

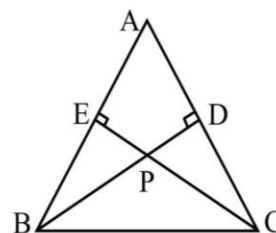
$$\begin{aligned} \triangle AOB &\sim \triangle COD \\ \Rightarrow \frac{AO}{OC} &= \frac{BO}{OD} = \frac{AB}{DC} \\ \Rightarrow \frac{1}{2} &= \frac{5}{DC} \quad [\because AB = 5 \text{ cm}] \\ \Rightarrow DC &= 10 \text{ cm} \end{aligned}$$

Ex.34 In figure, considering triangles BEP and CPD, prove that $BP \times PD = EP \times PC$.

Sol. **Given :** A $\triangle ABC$ in which $BD \perp AC$ and $CE \perp AB$ and BD and CE intersect at P .

To Prove : $BP \times PD = EP \times PC$

Proof : In $\triangle EPB$ and $\triangle DPC$, we have
 $\angle PEB = \angle PDC$ [Each equal to 90°]
 $\angle EPB = \angle DPC$ [Vertically opposite angles]



Thus, by AA-criterion of similarity, we have

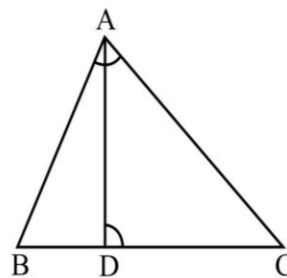
$$\triangle EPB \sim \triangle DPC$$

$$\frac{EP}{DP} = \frac{PB}{PC}$$

$$\Rightarrow BP \times PD = EP \times PC$$

Ex.35 D is a point on the side BC of $\triangle ABC$ such that $\angle ADC = \angle BAC$. Prove that $\frac{CA}{CD} = \frac{CB}{CA}$ or, $CA^2 = CB \times CD$.

Sol. In $\triangle ABC$ and $\triangle DAC$, we have
 $\angle ADC = \angle BAC$ and $\angle C = \angle C$



Therefore, by AA-criterion of similarity, we have

$$\begin{aligned} \triangle ABC &\sim \triangle DAC \\ \Rightarrow \frac{AB}{DA} &= \frac{BC}{AC} = \frac{AC}{DC} \\ \Rightarrow \frac{CB}{CA} &= \frac{CA}{CD} \end{aligned}$$

Ex.36 P and Q are points on sides AB and AC respectively of $\triangle ABC$. If $AP = 3$ cm, $PB = 6$ cm, $AQ = 5$ cm and $QC = 10$ cm, show that $BC = 3PQ$.

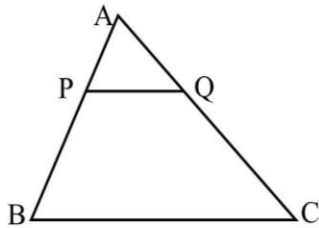
Sol. We have,

$$AB = AP + PB = (3 + 6) \text{ cm} = 9 \text{ cm}$$

$$\text{and, } AC = AQ + QC = (5 + 10) \text{ cm} = 15 \text{ cm.}$$

$$\therefore \frac{AP}{AB} = \frac{3}{9} = \frac{1}{3} \text{ and } \frac{AQ}{AC} = \frac{5}{15} = \frac{1}{3}$$

$$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$$



Thus, in triangles APQ and ABC, we have

$$\frac{AP}{AB} = \frac{AQ}{AC} \text{ and } \angle A = \angle A \text{ [Common]}$$

Therefore, by SAS-criterion of similarity, we have

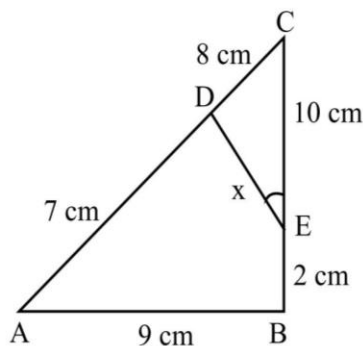
$$\triangle APQ \sim \triangle ABC$$

$$\Rightarrow \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$$

$$\Rightarrow \frac{PQ}{BC} = \frac{AQ}{AC} \Rightarrow \frac{PQ}{BC} = \frac{5}{15}$$

$$\Rightarrow \frac{PQ}{BC} = \frac{1}{3} \Rightarrow BC = 3PQ$$

Ex.37 In figure, $\angle A = \angle CED$, prove that $\triangle CAB \sim \triangle CED$. Also, find the value of x .



Sol. In $\triangle CAB$ and $\triangle CED$, we have

$$\angle A = \angle CED \text{ and } \angle C = \angle C \text{ [common]}$$

$$\therefore \triangle CAB \sim \triangle CED$$

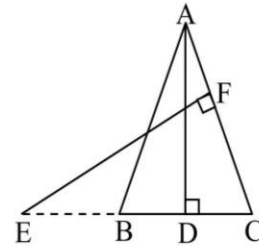
$$\Rightarrow \frac{CA}{CE} = \frac{AB}{DE} = \frac{CB}{CD}$$

$$\Rightarrow \frac{AB}{DE} = \frac{CB}{CD} \Rightarrow \frac{9}{x} = \frac{10+2}{8}$$

$$\Rightarrow x = 6 \text{ cm}$$

Ex.38 In the figure, E is a point on side CB produced of an isosceles $\triangle ABC$ with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that

$$\triangle ABD \sim \triangle ECF.$$



Sol. **Given :** A $\triangle ABC$ in which $AB = AC$ and $AD \perp BC$. Side CB is produced to E and $EF \perp AC$.

To prove : $\triangle ABD \sim \triangle ECF$.

Proof : we know that the angles opposite to equal sides of a triangle are equal.

$$\therefore \angle B = \angle C \quad [\because AB = AC]$$

Now, in $\triangle ABD$ and $\triangle ECF$, we have

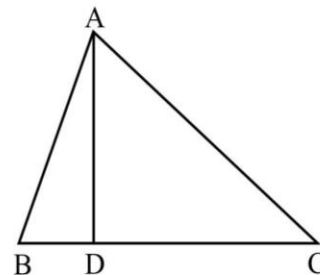
$$\angle B = \angle C \quad [\text{proved above}]$$

$$\angle ADB = \angle EFC = 90^\circ$$

$$\therefore \triangle ABD \sim \triangle ECF \text{ [By AA-similarity]}$$

Ex.39 In figure, $\angle BAC = 90^\circ$ and segment $AD \perp BC$. Prove that $AD^2 = BD \times DC$.

Sol. In $\triangle ABD$ and $\triangle ACD$, we have



$$\angle ADB = \angle ADC \quad [\text{Each equal to } 90^\circ]$$

and, $\angle DBA = \angle DAC$

$$[\text{Each equal to complement of } \angle BAD \text{ i.e., } 90^\circ - \angle BAD]$$

Therefore, by AA-criterion of similarity, we have

$$\triangle DBA \sim \triangle DAC$$

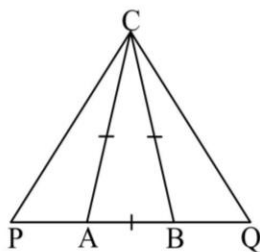
$$\left[\begin{array}{l} \therefore \angle D \leftrightarrow \angle D, \angle DBA \leftrightarrow \angle DAC \\ \text{and } \angle BAD \leftrightarrow \angle DCA \end{array} \right]$$

$$\Rightarrow \frac{DB}{DA} = \frac{DA}{DC}$$

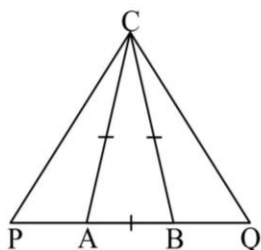
[In similar triangles corresponding sides are proportional]

$$\Rightarrow \frac{BD}{AD} = \frac{AD}{DC} \Rightarrow AD^2 = BD \times DC$$

Ex.40 In an isosceles $\triangle ABC$, the base AB is produced both ways in P and Q such that $AP \times BQ = AC^2$ and CE are the altitudes. Prove that $\triangle ACP \sim \triangle BCQ$.



Sol. $CA = CB \Rightarrow \angle CAB = \angle CBA$
 $\Rightarrow 180^\circ - \angle CAB = 180^\circ - \angle CBA$
 $\Rightarrow \angle CAP = \angle CBQ$
 Now, $AP \times BQ = AC^2$
 $\Rightarrow \frac{AP}{AC} = \frac{AC}{BQ} \Rightarrow \frac{AP}{AC} = \frac{BC}{BQ}$ [$\because AC = BC$]



Thus, $\angle CAP = \angle CBQ$ and $\frac{AP}{AC} = \frac{BC}{BQ}$.

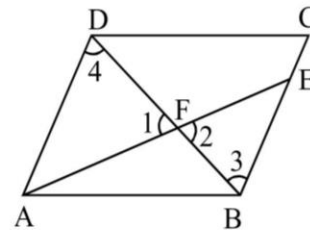
$\therefore \triangle ACP \sim \triangle BCQ$.

Ex.41 The diagonal BD of a parallelogram $ABCD$ intersects the segment AE at the point F , where E is any point on the side BC . Prove that $DF \times EF = FB \times FA$.

Sol. In $\triangle AFD$ and $\triangle BFE$, we have

$$\angle 1 = \angle 2 \quad [\text{Vertically opposite angles}]$$

$$\angle 3 = \angle 4 \quad [\text{Alternate angles}]$$



So, by AA-criterion of similarity, we have

$$\triangle FBE \sim \triangle FDA$$

$$\Rightarrow \frac{FB}{FD} = \frac{FE}{FA} \Rightarrow \frac{FB}{DF} = \frac{EF}{FA}$$

$$\Rightarrow DF \times EF = FB \times FA$$

Ex.42 Through the mid-point M of the side CD of a parallelogram $ABCD$, the line BM is drawn intersecting AC in L and AD produced in E . Prove that $EL = 2BL$.

Sol. In $\triangle BMC$ and $\triangle EMD$, we have

$$MC = MD \quad [\because M \text{ is the mid-point of } CD]$$

$$\angle CMB = \angle EMD \quad [\text{Vertically opposite angles}]$$

$$\text{and, } \angle MBC = \angle MED \quad [\text{Alternate angles}]$$

So, by AAS-criterion of congruence, we have

$$\therefore \triangle BMC \cong \triangle EMD$$

$$\Rightarrow BC = DE \quad \dots(i)$$

$$\text{Also, } AD = BC \quad \dots(ii)$$

[$\because ABCD$ is a parallelogram]

$$AD + DE = BC + BC$$

$$\Rightarrow AE = 2BC \quad \dots(iii)$$

Now, in $\triangle AEL$ and $\triangle CBL$, we have

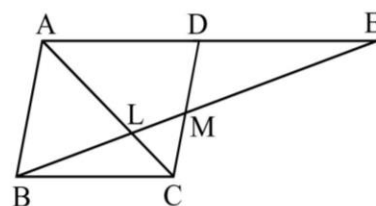
$$\angle ALE = \angle CLB$$

[Vertically opposite angles]

$$\angle EAL = \angle BCL$$

[Alternate angles]

So, by AA-criterion of similarity of triangles, we have



$$\triangle AEL \sim \triangle CBL$$

$$\Rightarrow \frac{EL}{BL} = \frac{AE}{CB} \Rightarrow \frac{EL}{BL} = \frac{2BC}{BC}$$

[Using equations (iii)]

$$\Rightarrow \frac{EL}{BL} = 2$$

$$\Rightarrow EL = 2BL$$

Ex.43 In figure, ABCD is a trapezium with $AB \parallel DC$. If $\triangle AED$ is similar to $\triangle BEC$, prove that $AD = BC$.

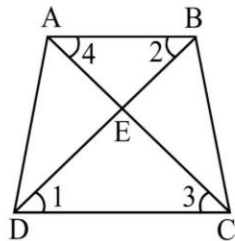
Sol. In $\triangle EDC$ and $\triangle EBA$, we have

$$\angle 1 = \angle 2 \quad [\text{Alternate angles}]$$

$$\angle 3 = \angle 4 \quad [\text{Alternate angles}]$$

and, $\angle CED = \angle AEB$ [Vertically opposite angles]

$$\therefore \triangle EDC \sim \triangle EBA$$



$$\Rightarrow \frac{ED}{EB} = \frac{EC}{EA}$$

$$\Rightarrow \frac{ED}{EC} = \frac{EB}{EA} \quad \dots(i)$$

It is given that $\triangle AED \sim \triangle BEC$

$$\therefore \frac{ED}{EC} = \frac{EA}{EB} = \frac{AD}{BC} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{EB}{EA} = \frac{EA}{EB}$$

$$\Rightarrow (EB)^2 = (EA)^2$$

$$\Rightarrow EB = EA$$

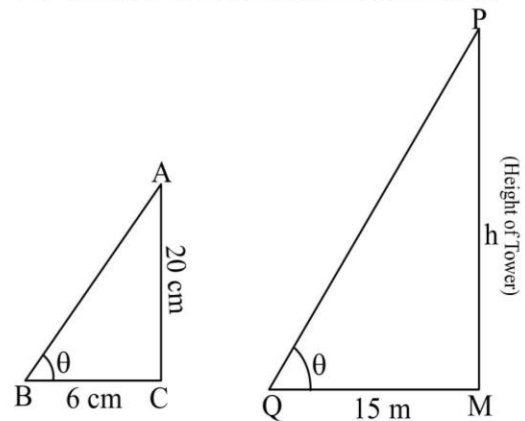
Substituting $EB = EA$ in (ii), we get

$$\frac{EA}{EA} = \frac{AD}{BC} \Rightarrow \frac{AD}{BC} = 1$$

$$\Rightarrow AD = BC$$

Ex.44 A vertical stick 20 cm long casts a shadow 6 cm long on the ground. At the same time, a tower casts a shadow 15 m long on the ground. Find the height of the tower.

Sol. Let the sun's altitude at that moment be θ .



$$\triangle PQM \sim \triangle ABC$$

$$\Rightarrow \frac{MP}{MQ} = \frac{AC}{CB}$$

$$\Rightarrow \frac{h}{15} = \frac{20}{6}$$

\therefore Height of the tower = 50 m.

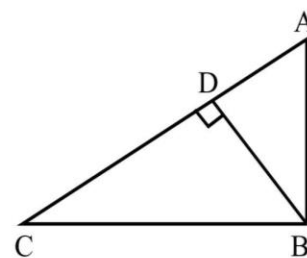
Ex.45 If a perpendicular is drawn from the vertex containing the right angle of a right triangle to the hypotenuse then prove that the triangle on each side of the perpendicular are similar to each other and to the original triangle. Also, prove that the square of the perpendicular is equal to the product of the lengths of the two parts of the hypotenuse. [NCERT]

Sol. **Given :** A right triangle ABC right angled at B, $BD \perp AC$.

To Prove :

- (i) $\triangle ADB \sim \triangle BDC$ (ii) $\triangle ADB \sim \triangle ABC$
- (iii) $\triangle BDC \sim \triangle ABC$ (iv) $BD^2 = AD \times DC$
- (v) $AB^2 = AD \times AC$ (vi) $BC^2 = CD \times AC$

Proof :



(i) We have,

$$\angle ABD + \angle DBC = 90^\circ$$

$$\text{Also, } \angle C + \angle DBC + \angle BDC = 180^\circ$$

$$\Rightarrow \angle C + \angle DBC + 90^\circ = 180^\circ$$

$$\Rightarrow \angle C + \angle DBC = 90^\circ$$

But, $\angle ABD + \angle DBC = 90^\circ$

$$\therefore \angle ABD + \angle DBC = \angle C + \angle DBC$$

$$\Rightarrow \angle ABD = \angle C \quad \dots(i)$$

Thus, in $\triangle ADB$ and $\triangle BDC$, we have

$$\angle ABD = \angle C \quad [\text{From (i)}]$$

and, $\angle ADB = \angle BDC$
 [Each equal to 90°]

So, by AA-similarity criterion, we have

$$\triangle ADB \sim \triangle BDC$$

(ii) In $\triangle ADB$ and $\triangle ABC$, we have

$$\angle ADB = \angle ABC \quad [\text{Each equal to } 90^\circ]$$

and, $\angle A = \angle A \quad [\text{Common}]$

So, by AA-similarity criterion, we have

$$\triangle ADB \sim \triangle ABC$$

(iii) In $\triangle BDC$ and $\triangle ABC$, we have

$$\angle BDC = \angle ABC$$

[Each equal to 90°]

$$\angle C = \angle C \quad [\text{Common}]$$

So, by AA-similarity criterion, we have

$$\triangle BDC \sim \triangle ABC$$

(iv) From (i), we have

$$\triangle ADB \sim \triangle BDC$$

$$\Rightarrow \frac{AD}{BD} = \frac{BD}{DC} \Rightarrow BD^2 = AD \times DC$$

(v) From (ii), we have

$$\triangle ADB \sim \triangle ABC$$

$$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC} \Rightarrow AB^2 = AD \times AC$$

(vi) From (iii), we have

$$\triangle BDC \sim \triangle ABC$$

$$\Rightarrow \frac{BC}{AC} = \frac{DC}{BC}$$

$$\Rightarrow BC^2 = CD \times AC$$

Ex.46 Prove that the line segments joining the mid-points of the sides of a triangle form four triangles, each of which is similar to the original triangle.

Sol. **Given :** $\triangle ABC$ in which D, E, F are the mid-points of sides BC, CA and AB respectively.

To Prove : Each of the triangles AFE, FBD, EDC and DEF is similar to $\triangle ABC$.

Proof : Consider triangles AFE and ABC.

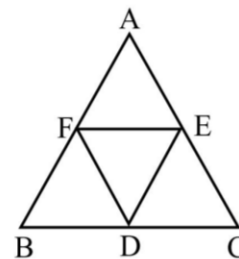
Since F and E are mid-points of AB and AC respectively.

$$\therefore FE \parallel BC$$

$$\Rightarrow \angle AEF = \angle B$$

[Corresponding angles]

Thus, in $\triangle AFE$ and $\triangle ABC$, we have



$$\angle AFE = \angle B$$

and, $\angle A = \angle A \quad [\text{Common}]$

$$\therefore \triangle AFE \sim \triangle ABC.$$

Similarly, we have

$$\triangle FBD \sim \triangle ABC \text{ and } \triangle EDC \sim \triangle ABC.$$

Now, we shall show that $\triangle DEF \sim \triangle ABC$.

Clearly, $ED \parallel AF$ and $DE \parallel EA$.

$$\therefore AFDE \text{ is a parallelogram.}$$

$$\Rightarrow \angle EDF = \angle A$$

[\ominus Opposite angles of a parallelogram are equal]

Similarly, BDEF is a parallelogram.

$$\therefore \angle DEF = \angle B$$

[\ominus Opposite angles of a parallelogram are equal]

Thus, in triangles DEF and ABC, we have

$$\angle EDF = \angle A \text{ and } \angle DEF = \angle B$$

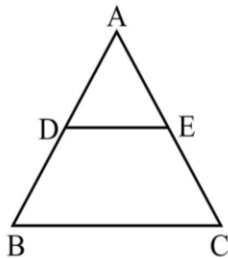
So, by AA-criterion of similarity, we have

$$\triangle DEF \sim \triangle ABC.$$

Thus, each one of the triangles AFE, FBD, EDC and DEF is similar to $\triangle ABC$.

Ex.47 In $\triangle ABC$, DE is parallel to base BC , with D on AB and E on AC . If $\frac{AD}{DB} = \frac{2}{3}$, find $\frac{BC}{DE}$.

Sol. In $\triangle ABC$, we have



$$DE \parallel BC \Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

Thus, in triangles ABC and ADE , we have

$$\frac{AB}{AD} = \frac{AC}{AE} \text{ and } \angle A = \angle A$$

Therefore, by SAS-criterion of similarity, we have

$$\triangle ABC \sim \triangle ADE$$

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC} \quad \dots(i)$$

It is given that

$$\frac{AD}{DB} = \frac{2}{3}$$

$$\Rightarrow \frac{DB}{AD} = \frac{3}{2}$$

$$\Rightarrow \frac{DB}{AD} + 1 = \frac{3}{2} + 1$$

$$\Rightarrow \frac{DB + AD}{AD} = \frac{5}{2}$$

$$\Rightarrow \frac{AB}{AD} = \frac{5}{2} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{BC}{DE} = \frac{5}{2}$$

Given : Two triangles ABC and DEF in which $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$, AP and DQ are their medians.

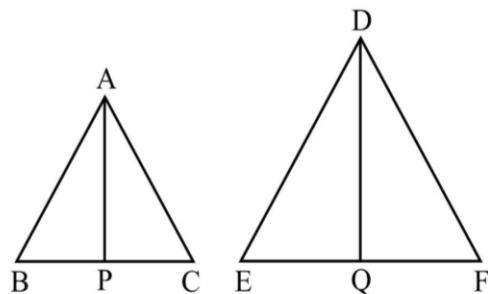


Figure (i)

Figure (ii)

To Prove : $\frac{BC}{EF} = \frac{AP}{DQ}$

Proof : Since equiangular triangles are similar.

$$\therefore \triangle ABC \sim \triangle DEF$$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} \Rightarrow \frac{AB}{DE} = \frac{2BP}{2EQ}$$

$$\left[\begin{array}{l} \text{P and Q are mid - points of BC} \\ \text{and EF respectively} \\ \therefore BC = 2BP \text{ and } EF = 2EQ \end{array} \right]$$

$$\Rightarrow \frac{AB}{DE} = \frac{BP}{EQ} \quad \dots(ii)$$

Now, in $\triangle ABP$ and $\triangle DEFQ$, we have

$$\frac{AB}{DE} = \frac{BP}{EQ} \quad [\text{From (ii)}]$$

$$\text{and, } \angle B = \angle E \quad [\text{Given}]$$

So, by SAS-criterion of similarity, we have

$$\triangle ABP \sim \triangle DEFQ$$

$$\Rightarrow \frac{AB}{DE} = \frac{AP}{DQ} \quad \dots(iii)$$

From (i) and (iii), we get

$$\frac{BC}{EF} = \frac{AP}{DQ}$$

Hence, the ratio of the corresponding sides is same as the ratio of corresponding medians.

Theorem 2 :

If two triangles are equiangular, prove that the ratio of the corresponding sides is same as the ratio of the corresponding angle bisector segments.

MORE ON CHARACTERISTIC PROPERTIES

Theorem 1 :

If two triangles are equiangular, prove that the ratio of the corresponding sides is same as the ratio of the corresponding medians.

Given : Two triangles ABC and DEF in which $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$; and AX, DY are the bisectors of $\angle A$ and $\angle D$ respectively.

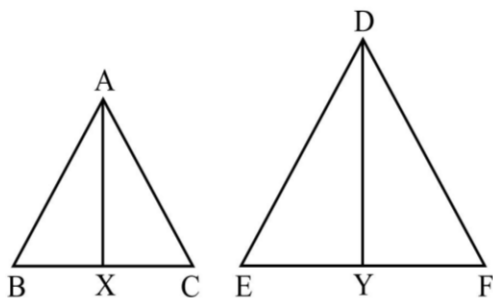


Figure (i)

Figure (ii)

To Prove : $\frac{BC}{EF} = \frac{AX}{DY}$

Proof : Since equiangular triangles are similar.

$$\triangle ABC \sim \triangle DEF$$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} \quad \dots(i)$$

In $\triangle ABX$ and $\triangle DEY$, we have

$$\angle B = \angle E \quad \text{[Given]}$$

and, $\angle BAX = \angle EDY$

$$\left[\begin{array}{l} \ominus \angle A = \angle D \Rightarrow \frac{1}{2} \angle A = \frac{1}{2} \angle D \\ \Rightarrow \angle BAX = \angle EDY \end{array} \right]$$

So, by AA-criterion of similarity, we have

$$\triangle ABX \sim \triangle DEY$$

$$\Rightarrow \frac{AB}{DE} = \frac{AX}{DY} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{BC}{EF} = \frac{AX}{DY}$$

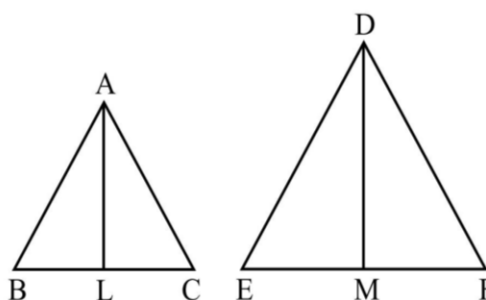
Theorem 3 :

If two triangles are equiangular, prove that the ratio of the corresponding sides is same as the ratio of the corresponding altitudes.

Given : Two triangles ABC and DEF in which

$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ and

$AL \perp BC$, $DM \perp EF$



To Prove : $\frac{BC}{EF} = \frac{AL}{DM}$

Proof : Since equiangular triangles are similar.

$$\therefore \triangle ABC \sim \triangle DEF$$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} \quad \dots(i)$$

In triangle ALB and DME, we have

$$\angle ALB = \angle DME \quad \text{[Each equal to } 90^\circ]$$

$$\angle B = \angle E \quad \text{[Given]}$$

So, by AA-criterion of similarity, we have

$$\triangle ALB \sim \triangle DME$$

$$\Rightarrow \frac{AB}{DE} = \frac{AL}{DM} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{BC}{EF} = \frac{AL}{DM}$$

Theorem 4 :

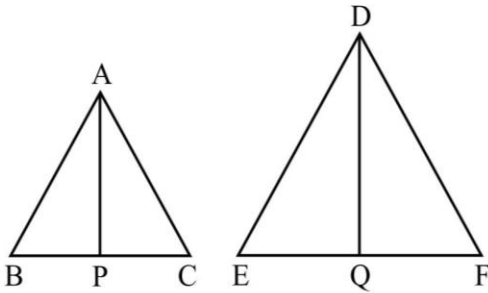
If one angle of a triangle is equal to one angle of another triangle and the bisectors of these equal angles divide the opposite side in the same ratio, prove that the triangles are similar.

Given : Two triangles ABC and DEF in which $\angle A = \angle D$. The bisectors AP and DQ of $\angle A$ and $\angle D$ intersect BC and EF in P and Q respectively

such that $\frac{BP}{PC} = \frac{EQ}{QF}$.

To Prove : $\triangle ABC \sim \triangle DEF$

Proof : We know that the bisectors of an angle of a triangle intersects the opposite side in the ratio of the sides containing the angle.



\therefore AP is the bisector of $\angle A$

$$\Rightarrow \frac{BP}{PC} = \frac{AB}{AC} \quad \dots(i)$$

DQ is the bisector of $\angle D$

$$\Rightarrow \frac{EQ}{QF} = \frac{DE}{DF} \quad \dots(ii)$$

But, $\frac{BP}{PC} = \frac{EQ}{QF}$ [Given]

Therefore, from (i) and (ii), we get

$$\frac{AB}{AC} = \frac{DE}{DF}$$

Thus, in triangles ABC and DEF, we have

$$\frac{AB}{AC} = \frac{DE}{DF}$$

and, $\angle A = \angle D$ [Given]

So, by SAS-criterion of similarity, we get

$$\triangle ABC \sim \triangle DEF$$

Theorem 5 :

If two sides and a median bisecting one of these sides of a triangle are respectively proportional to the two sides and the corresponding median of another triangle, then the triangles are similar.

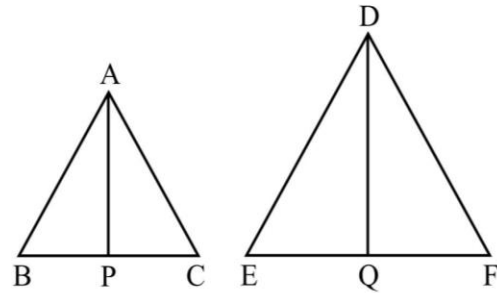
Given : $\triangle ABC$ and $\triangle DEF$ in which AP and DQ are the medians such that [NCERT]

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AP}{DQ}$$

To Prove : $\triangle ABC \sim \triangle DEF$

Proof : We have,

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AP}{DQ}$$



$$\Rightarrow \frac{AB}{DE} = \frac{\frac{1}{2}BC}{\frac{1}{2}EF} = \frac{AP}{DQ}$$

$$\Rightarrow \frac{AB}{DE} = \frac{BP}{EQ} = \frac{AP}{DQ}$$

$$\Rightarrow \triangle ABP \sim \triangle DEQ \quad [\text{By SSS-similarity}]$$

$$\Rightarrow \angle B = \angle E$$

Now, in $\triangle ABC$ and $\triangle DEF$, we have

$$\frac{AB}{DE} = \frac{BC}{EF} \quad [\text{Given}]$$

and, $\angle B = \angle E$

So, by SAS-criterion of similarity, we get

$$\triangle ABC \sim \triangle DEF$$

Theorem 6 :

If two sides and a median bisecting the third side of a triangle are respectively proportional to the corresponding sides and the median of another triangle, then the two triangles are similar.

[NCERT]

Given : Two triangle ABC and DEF in which AP and DQ are the medians such that

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{AP}{DQ}$$

To Prove : $\triangle ABC \sim \triangle DEF$

Construction : Produce AP to G so that $PG = AP$. Join CG. Also, produce DQ to H so that $QH = DQ$. Join FH.

Proof : In $\triangle APB$ and $\triangle GPC$, we have

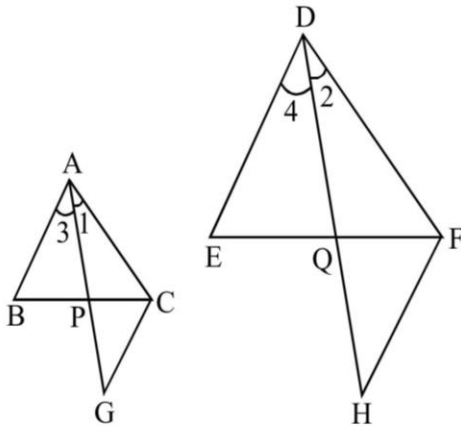
$$\begin{aligned} BP &= CP && [\ominus \text{ AP is the median}] \\ AP &= GP && [\text{By construction}] \end{aligned}$$

and, $\angle APB = \angle CPG$ [Vertically opposite angles]

So, by SAS-criterion of congruence, we have

$$\triangle APB \cong \triangle GPC$$

$\Rightarrow AG = GC$ (i)



Again, In ΔDQE and ΔHQF , we have

$EQ = FQ$ [∵ DQ is the median]

$DQ = HQ$ [By construction]

and, $\angle DQE = \angle HQF$ [Vertically opposite angles]

So, by SAS-criterion of congruence, we have

$\Delta DQE \cong \Delta HQF$

$\Rightarrow DE = HF$ (ii)

Now, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{AP}{DQ}$ [Given]

$\Rightarrow \frac{GC}{HF} = \frac{AC}{DF} = \frac{AP}{DQ}$

[∵ $AB = GC$ and $DE = HF$ (from (i) and (ii))]

$\Rightarrow \frac{GC}{HF} = \frac{AC}{DF} = \frac{2AP}{2DQ}$

$\Rightarrow \frac{GC}{HF} = \frac{AC}{DF} = \frac{AG}{DH}$
 [∵ $2AP = AG$ and $2DQ = DH$]

$\Rightarrow \Delta AGC \sim \Delta DHF$
 [By SSS-criterion of similarity]

$\Rightarrow \angle 1 = \angle 2$

Similarly, we have

$\angle 3 = \angle 4$

$\therefore \angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow \angle A = \angle D$ (iii)

Thus, in ΔABC and ΔDEF , we have

$\angle A = \angle D$ [From (iii)]

and, $\frac{AB}{DE} = \frac{AC}{DF}$ [Given]

So, by SAS-criterion of similarity, we have

$\Delta ABC \sim \Delta DEF$

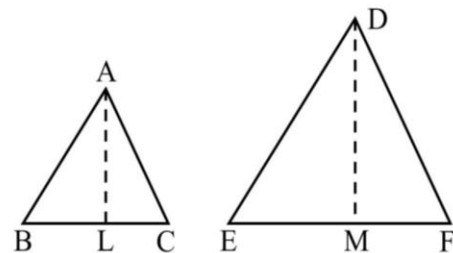
AREAS OF TWO SIMILAR TRIANGLES

Theorem 1 :

The ratio of the areas of two similar triangles are equal to the ratio of the squares of any two corresponding sides.

Given : Two triangles ABC and DEF such that $\Delta ABC \sim \Delta DEF$.

To Prove : $\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$



Construction : Draw $AL \perp BC$ and $DM \perp EF$.

Proof : Since similar triangles are equiangular and their corresponding sides are proportional. Therefore,

$\Delta ABC \sim \Delta DEF$

$\Rightarrow \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ (i)

Thus, in ΔALB and ΔDME , we have

$\Rightarrow \angle ALB = \angle DME$ [Each equal to 90°]

and, $\angle B = \angle E$ [From (i)]

So, by AA-criterion of similarity, we have

$\Delta ALB \sim \Delta DME$

$\Rightarrow \frac{AL}{DM} = \frac{AB}{DE}$ (ii)

From (i) and (ii), we get

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AL}{DM}$ (iii)

Now,

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{\frac{1}{2}(BC \times AL)}{\frac{1}{2}(EF \times DM)}$$

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{BC}{EF} \times \frac{AL}{DM}$$

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{BC}{EF} \times \frac{BC}{EF} \left[\text{From (iii), } \frac{BC}{EF} = \frac{AL}{DM} \right]$$

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{BC^2}{EF^2}$$

But, $\frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF}$

$$\Rightarrow \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$$

Hence, $\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Theorem 2 :

If the areas of two similar triangles are equal, then the triangles are congruent i.e. equal and similar triangles are congruent.

Given : Two triangles ABC and DEF such that $\Delta ABC \sim \Delta DEF$ and $\text{Area}(\Delta ABC) = \text{Area}(\Delta DEF)$.

To Prove : We have,

$$\Delta ABC \cong \Delta DEF$$

Proof : $\Delta ABC \sim \Delta DEF$

$$\Rightarrow \angle A = \angle D, \angle B = \angle E, \angle C = \angle F \text{ and}$$

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

In order to prove that $\Delta ABC \cong \Delta DEF$, it is sufficient to show that $AB = DE$, $BC = EF$ and $AC = DF$.

Now, $\text{Area}(\Delta ABC) = \text{Area}(\Delta DEF)$

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = 1$$

$$\Rightarrow \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1$$

$$\left[\ominus \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} \right]$$

$$\Rightarrow AB^2 = DE^2, BC^2 = EF^2 \text{ and } AC^2 = DF^2$$

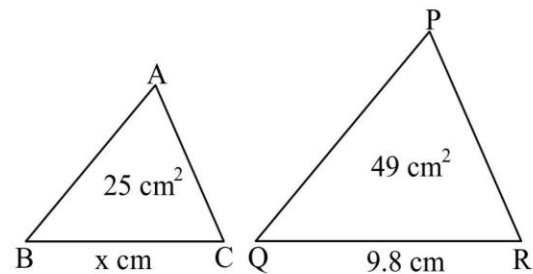
$$\Rightarrow AB = DE, BC = EF \text{ and } AC = DF$$

Hence, $\Delta ABC \cong \Delta DEF$.

❖ **EXAMPLES** ❖

Ex.48 The areas of two similar triangles ΔABC and ΔPQR are 25 cm^2 and 49 cm^2 respectively. If $QR = 9.8 \text{ cm}$, find BC .

Sol. It is being given that $\Delta ABC \sim \Delta PQR$, ar $(\Delta ABC) = 25 \text{ cm}^2$ and ar $(\Delta PQR) = 49 \text{ cm}^2$. We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.



$$\therefore \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \frac{25}{49} = \frac{x^2}{(9.8)^2}, \text{ where } BC = x \text{ cm}$$

$$\Rightarrow x^2 = \left(\frac{25}{49} \times 9.8 \times 9.8 \right)$$

$$\Rightarrow x = \sqrt{\frac{25}{49} \times 9.8 \times 9.8} = \left(\frac{5}{7} \times 9.8 \right) = (5 \times 1.4) = 7.$$

Hence $BC = 7 \text{ cm}$.

Ex.49 In two similar triangles ABC and PQR, if their corresponding altitudes AD and PS are in the ratio 4 : 9, find the ratio of the areas of ΔABC and ΔPQR .

Sol. Since the areas of two similar triangles are in the ratio of the squares of the corresponding altitudes.

$$\therefore \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AD^2}{PS^2}$$

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \left(\frac{4}{9} \right)^2 = \frac{16}{81}$$

$$[\ominus AD : PS = 4 : 9]$$

Hence, $\text{Area}(\Delta ABC) : \text{Area}(\Delta PQR) = 16 : 81$

Ex.50 If $\triangle ABC$ is similar to $\triangle DEF$ such that $\triangle DEF = 64 \text{ cm}^2$, $DE = 5.1 \text{ cm}$ and area of $\triangle ABC = 9 \text{ cm}^2$. Determine the area of AB.

Sol. Since the ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

$$\therefore \frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DEF)} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \frac{9}{64} = \frac{AB^2}{(5.1)^2}$$

$$\Rightarrow AB = \sqrt{3.65} \Rightarrow AB = 1.912 \text{ cm}$$

Ex.51 If $\triangle ABC \sim \triangle DEF$ such that area of $\triangle ABC$ is 16 cm^2 and the area of $\triangle DEF$ is 25 cm^2 and $BC = 2.3 \text{ cm}$. Find the length of EF.

Sol. We have,

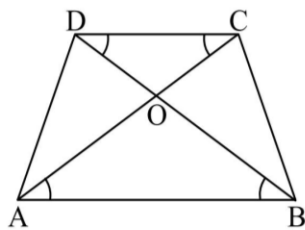
$$\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{16}{25} = \frac{(2.3)^2}{EF^2} \Rightarrow EF = \sqrt{8.265}$$

$$= 2.875 \text{ cm}$$

Ex.52 In a trapezium ABCD, O is the point of intersection of AC and BD, $AB \parallel CD$ and $AB = 2 \times CD$. If the area of $\triangle AOB = 84 \text{ cm}^2$. Find the area of $\triangle COD$.

Sol. In $\triangle AOB$ and $\triangle COD$, we have



$$\angle OAB = \angle OCD \text{ (alt. int. } \angle\text{s)}$$

$$\angle OBA = \angle ODC \text{ (alt. int. } \angle\text{s)}$$

$$\therefore \triangle AOB \sim \triangle COD \text{ [By AA-similarity]}$$

$$\Rightarrow \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \frac{AB^2}{CD^2} = \frac{(2CD)^2}{CD^2}$$

$$[\ominus AB = 2 \times CD]$$

$$= \frac{4 \times CD^2}{CD^2} = 4$$

$$\Rightarrow \text{ar}(\triangle COD) = 1/4 \times \text{ar}(\triangle AOB)$$

$$= \left(\frac{1}{4} \times 84\right) \text{ cm}^2 = 21 \text{ cm}^2$$

Hence, the area of $\triangle COD$ is 21 cm^2 .

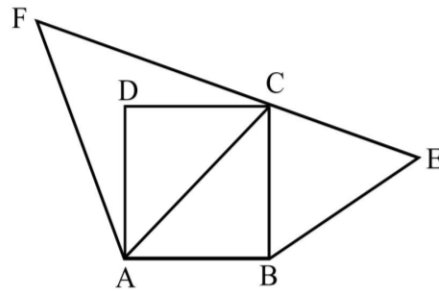
Ex.53 Prove that the area of the triangle BCE described on one side BC of a square ABCD as base is one half the area of the similar triangle ACF described on the diagonal AC as base.

Sol. ABCD is a square. $\triangle BCE$ is described on side BC is similar to $\triangle ACF$ described on diagonal AC.

Since ABCD is a square. Therefore,

$$AB = BC = CD = DA \text{ and } AC = \sqrt{2} BC$$

$$[\ominus \text{Diagonal} = \sqrt{2} (\text{Side})]$$



Now, $\triangle BCE \sim \triangle ACF$

$$\Rightarrow \frac{\text{Area}(\triangle BCE)}{\text{Area}(\triangle ACF)} = \frac{BC^2}{AC^2}$$

$$\Rightarrow \frac{\text{Area}(\triangle BCE)}{\text{Area}(\triangle ACF)} = \frac{BC^2}{(\sqrt{2}BC)^2} = \frac{1}{2}$$

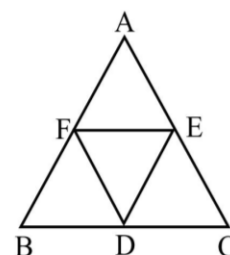
$$\Rightarrow \text{Area}(\triangle BCE) = \frac{1}{2} \text{Area}(\triangle ACF)$$

Ex.54 D, E, F are the mid-point of the sides BC, CA and AB respectively of a $\triangle ABC$. Determine the ratio of the areas of $\triangle DEF$ and $\triangle ABC$.

Sol. Since D and E are the mid-points of the sides BC and AB respectively of $\triangle ABC$. Therefore,

$$DE \parallel BA$$

$$\Rightarrow DE \parallel FA \quad \dots(i)$$



Since D and F are mid-points of the sides BC and AB respectively of $\triangle ABC$. Therefore,

$$DF \parallel CA \Rightarrow DF \parallel AE$$

From (i), and (ii), we conclude that AFDE is a parallelogram.

Similarly, BDEF is a parallelogram.

Now, in $\triangle DEF$ and $\triangle ABC$, we have

$$\angle FDE = \angle A$$

[Opposite angles of parallelogram AFDE]

and, $\angle DEF = \angle B$

[Opposite angles of parallelogram BDEF]

So, by AA-similarity criterion, we have

$$\triangle DEF \sim \triangle ABC$$

$$\Rightarrow \frac{\text{Area}(\triangle DEF)}{\text{Area}(\triangle ABC)} = \frac{DE^2}{AB^2} = \frac{(1/2AB)^2}{AB^2} = \frac{1}{4}$$

$$\left[\ominus DE = \frac{1}{2} AB \right]$$

Hence, $\text{Area}(\triangle DEF) : \text{Area}(\triangle ABC) = 1 : 4$.

Ex.55 D and E are points on the sides AB and AC respectively of a $\triangle ABC$ such that $DE \parallel BC$ and divides $\triangle ABC$ into two parts, equal in area. Find $\frac{BD}{AB}$.

Sol. We have,

$$\text{Area}(\triangle ADE) = \text{Area}(\text{trapezium BCED})$$

$$\Rightarrow \text{Area}(\triangle ADE) + \text{Area}(\triangle ADE)$$

$$= \text{Area}(\text{trapezium BCED}) + \text{Area}(\triangle ADE)$$

$$\Rightarrow 2 \text{Area}(\triangle ADE) = \text{Area}(\triangle ABC)$$

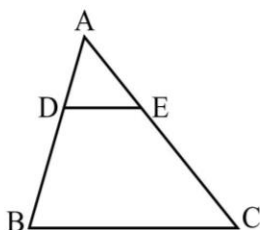
In $\triangle ADE$ and $\triangle ABC$, we have

$$\angle ADE = \angle B$$

[$\ominus DE \parallel BC \therefore \angle ADE = \angle B$ (Corresponding angles)]

and, $\angle A = \angle A$ [Common]

$$\therefore \triangle ADE \sim \triangle ABC$$



$$\Rightarrow \frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABC)} = \frac{AD^2}{AB^2}$$

$$\Rightarrow \frac{\text{Area}(\triangle ADE)}{2 \text{Area}(\triangle ADE)} = \frac{AD^2}{AB^2}$$

$$\Rightarrow \frac{1}{2} = \left(\frac{AD}{AB} \right)^2 \Rightarrow \frac{AD}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow AB = \sqrt{2} AD \Rightarrow AB = \sqrt{2} (AB - BD)$$

$$\Rightarrow (\sqrt{2} - 1) AB = \sqrt{2} BD$$

$$\Rightarrow \frac{BD}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}} = \frac{2 - \sqrt{2}}{2}$$

Ex.56 Two isosceles triangles have equal vertical angles and their areas are in the ratio 16 : 25. Find the ratio of their corresponding heights.

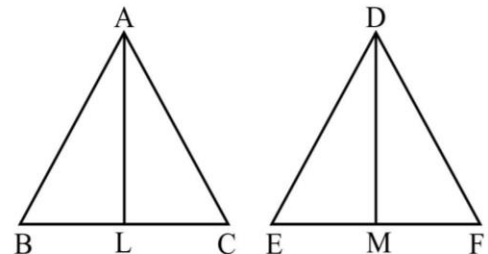
Sol. Let $\triangle ABC$ and $\triangle DEF$ be the given triangles such that $AB = AC$ and $DE = DF$, $\angle A = \angle D$.

$$\text{and, } \frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DEF)} = \frac{16}{25} \quad \dots(i)$$

Draw $AL \perp BC$ and $DM \perp EF$.

Now, $AB = AC$, $DE = DF$

$$\Rightarrow \frac{AB}{AC} = 1 \text{ and } \frac{DE}{DF} = 1$$



$$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF} \Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$$

Thus, in triangles ABC and DEF, we have

$$\frac{AB}{DE} = \frac{AC}{DF} \text{ and } \angle A = \angle D \quad [\text{Given}]$$

So, by SAS-similarity criterion, we have

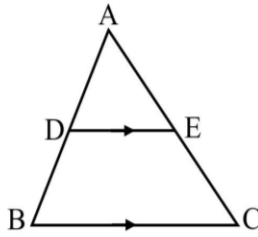
$$\triangle ABC \sim \triangle DEF$$

$$\Rightarrow \frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DEF)} = \frac{AL^2}{DM^2}$$

$$\Rightarrow \frac{16}{25} = \frac{AL^2}{DM^2} \quad [\text{Using (i)}]$$

$$\Rightarrow \frac{AL}{DM} = \frac{4}{5} \Rightarrow AL : DM = 4 : 5$$

Ex.57 In the given figure, $DE \parallel BC$ and $DE : BC = 3 : 5$. Calculate the ratio of the areas of $\triangle ADE$ and the trapezium $BCED$.



Sol. $\triangle ADE \sim \triangle ABC$.

$$\therefore \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle ABC)} = \frac{DE^2}{BC^2} = \left(\frac{DE}{BC}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

Let $\text{ar}(\triangle ADE) = 9x$ sq units

Then, $\text{ar}(\triangle ABC) = 25x$ sq units

$$\begin{aligned} \text{ar}(\text{trap. } BCED) &= \text{ar}(\triangle ABC) - \text{ar}(\triangle ADE) \\ &= (25x - 9x) = (16x) \text{ sq units} \end{aligned}$$

$$\therefore \frac{\text{ar}(\triangle ADE)}{\text{ar}(\text{trap. } BCED)} = \frac{9x}{16x} = \frac{9}{16}$$

PYTHAGORAS THEOREM

Theorem 1 :

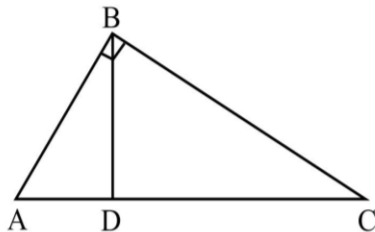
In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given : A right-angled triangle ABC in which $\angle B = 90^\circ$.

To Prove : $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$.

i.e., $AC^2 = AB^2 + BC^2$

Construction : From B draw $BD \perp AC$.



Proof : In triangle ADB and ABC , we have

$$\angle ADB = \angle ABC \quad [\text{Each equal to } 90^\circ]$$

$$\text{and, } \angle A = \angle A \quad [\text{Common}]$$

So, by AA-similarity criterion, we have

$$\triangle ADB \sim \triangle ABC$$

$$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$$

[\ominus In similar triangles corresponding sides are proportional]

$$\Rightarrow AB^2 = AD \times AC \quad \dots(i)$$

In triangles BDC and ABC , we have

$$\angle CDB = \angle ABC \quad [\text{Each equal to } 90^\circ]$$

$$\text{and, } \angle C = \angle C \quad [\text{Common}]$$

So, by AA-similarity criterion, we have

$$\triangle BDC \sim \triangle ABC$$

$$\Rightarrow \frac{DC}{BC} = \frac{BC}{AC}$$

[\ominus In similar triangles corresponding sides are proportional]

$$\Rightarrow BC^2 = AC \times DC \quad \dots(ii)$$

Adding equation (i) and (ii), we get

$$AB^2 + BC^2 = AD \times AC + AC \times DC$$

$$\Rightarrow AB^2 + BC^2 = AC (AD + DC)$$

$$\Rightarrow AB^2 + BC^2 = AC \times AC$$

$$\Rightarrow AB^2 + BC^2 = AC^2$$

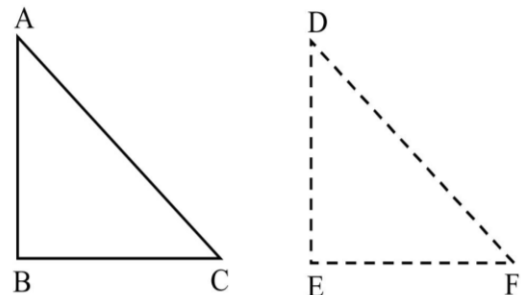
$$\text{Hence, } AC^2 = AB^2 + BC^2$$

The converse of the above theorem is also true as proved below.

Theorem 2 : (Converse of Pythagoras Theorem).

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the side is a right angle.

Given : A triangle ABC such that $AC^2 = AB^2 + BC^2$



Construction : Construct a triangle DEF such that $DE = AB$, $EF = BC$ and $\angle E = 90^\circ$,

Proof : In order to prove that $\angle B = 90^\circ$, it is sufficient to show that $\triangle ABC \sim \triangle DEF$.

For this we proceed as follows :

Since $\triangle DEF$ is a right angled triangle with right angle at E. Therefore, by Pythagoras theorem, we have

$$DF^2 = DE^2 + EF^2$$

$$\Rightarrow DF^2 = AB^2 + BC^2$$

$$[\ominus DE = AB \text{ and } EF = BC$$

(By construction)]

$$\Rightarrow DF^2 = AC^2 [\ominus AB^2 + BC^2 = AC^2 \text{ (Given)}]$$

$$\Rightarrow DF = AC \quad \dots(i)$$

Thus, in $\triangle ABC$ and $\triangle DEF$, we have

$$AB = DE, BC = EF \text{ [By construction]}$$

$$\text{and, } AC = DF \quad \text{[From equation (i)]}$$

$$\therefore \triangle ABC \cong \triangle DEF$$

$$\Rightarrow \angle B = \angle E = 90^\circ$$

Hence, $\triangle ABC$ is a right triangle right angled at B.

❖ EXAMPLES ❖

Ex.58 Side of a triangle is given, determine it is a right triangle.

$$(2a - 1) \text{ cm}, 2\sqrt{2a} \text{ cm and } (2a + 1) \text{ cm}$$

Sol. Let $p = (2a - 1) \text{ cm}$, $q = 2\sqrt{2a} \text{ cm}$ and $r = (2a + 1) \text{ cm}$.

Then,

$$(p^2 + q^2) = (2a - 1)^2 \text{ cm}^2 + (2\sqrt{2a})^2 \text{ cm}^2$$

$$= \{(4a^2 + 1 - 4a) + 8a\} \text{ cm}^2$$

$$= (4a^2 + 4a + 1) \text{ cm}^2$$

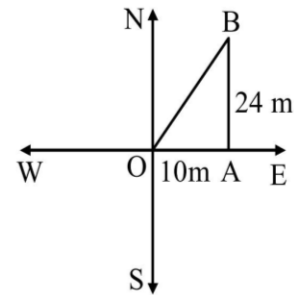
$$= (2a + 1)^2 \text{ cm}^2 = r^2.$$

$$\therefore (p^2 + q^2) = r^2.$$

Hence, the given triangle is right angled.

Ex.59 A man goes 10 m due east and then 24 m due north. Find the distance from the starting point.

Sol. Let the initial position of the man be O and his final position be B. Since the man goes 10 m due east and then 24 m due north. Therefore, $\triangle AOB$ is a right triangle right-angled at A such that $OA = 10 \text{ m}$ and $AB = 24 \text{ m}$.



By Pythagoras theorem, we have

$$OB^2 = OA^2 + AB^2$$

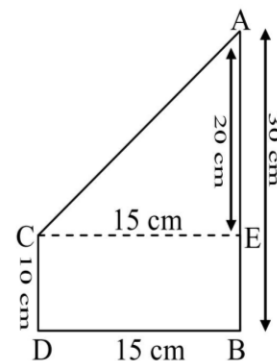
$$\Rightarrow OB^2 = 10^2 + 24^2 = 100 + 576 = 676$$

$$\Rightarrow OB = \sqrt{676} = 26 \text{ m}$$

Hence, the man is at a distance of 26 m from the starting point.

Ex.60 Two towers of heights 10 m and 30 m stand on a plane ground. If the distance between their feet is 15 m, find the distance between their tops.

Sol. $AC^2 = (15)^2 + (20)^2 = 625$
 $\Rightarrow AC = 25 \text{ m}$.



Ex.61 In Fig., $\triangle ABC$ is an obtuse triangle, obtuse angled at B. If $AD \perp CB$, prove that

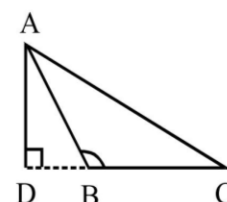
$$AC^2 = AB^2 + BC^2 + 2BC \times BD$$

Sol. **Given :** An obtuse triangle ABC, obtuse-angled at B and AD is perpendicular to CB produced.

To Prove : $AC^2 = AB^2 + BC^2 + 2BC \times BD$

Proof : Since $\triangle ADB$ is a right triangle right angled at D. Therefore, by Pythagoras theorem, we have

$$AB^2 = AD^2 + DB^2 \quad \dots(i)$$



Again $\triangle ADC$ is a right triangle right angled at D.

Therefore, by Pythagoras theorem, we have

$$\begin{aligned} AC^2 &= AD^2 + DC^2 \\ \Rightarrow AC^2 &= AD^2 + (DB + BC)^2 \\ \Rightarrow AC^2 &= AD^2 + DB^2 + BC^2 + 2BC \cdot BD \\ \Rightarrow AC^2 &= AB^2 + BC^2 + 2BC \cdot BD \end{aligned}$$

[Using (i)]

Hence, $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$

Ex.62 In figure, $\angle B$ of $\triangle ABC$ is an acute angle and $AD \perp BC$, prove that

$$AC^2 = AB^2 + BC^2 - 2BC \times BD$$

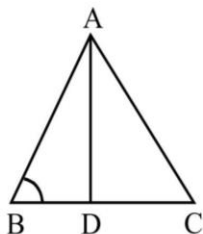
Sol. **Given :** A $\triangle ABC$ in which $\angle B$ is an acute angle and $AD \perp BC$.

To Prove : $AC^2 = AB^2 + BC^2 - 2BC \times BD$.

Proof : Since $\triangle ADB$ is a right triangle right-angled at D. So, by Pythagoras theorem, we have

$$AB^2 = AD^2 + BD^2 \quad \dots(i)$$

Again $\triangle ADC$ is a right triangle right angled at D.



So, by Pythagoras theorem, we have

$$\begin{aligned} AC^2 &= AD^2 + DC^2 \\ \Rightarrow AC^2 &= AD^2 + (BC - BD)^2 \\ \Rightarrow AC^2 &= AD^2 + (BC^2 + BD^2 - 2BC \cdot BD) \\ \Rightarrow AC^2 &= (AD^2 + BD^2) + BC^2 - 2BC \cdot BD \\ \Rightarrow AC^2 &= AB^2 + BC^2 - 2BC \cdot BD \end{aligned}$$

[Using (i)]

Hence, $AC^2 = AB^2 + BC^2 - 2BC \cdot BD$

Ex.63 If ABC is an equilateral triangle of side a, prove that its altitude = $\frac{\sqrt{3}}{2} a$.

Sol. $\triangle ABD$ is an equilateral triangle.

We are given that $AB = BC = CA = a$.

AD is the altitude, i.e., $AD \perp BC$.

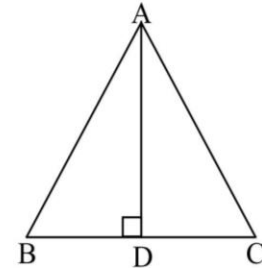
Now, in right angled triangles ABD and ACD, we have

$$AB = AC \quad (\text{Given})$$

$$\text{and } AD = AD \quad (\text{Common side})$$

$$\Rightarrow \triangle ABD \cong \triangle ACD \quad (\text{By RHS congruence})$$

$$\Rightarrow BD = CD \Rightarrow BD = DC = \frac{1}{2} BC = \frac{a}{2}$$



From right triangle ABD.

$$AB^2 = AD^2 + BD^2 \Rightarrow a^2 = AD^2 + \left(\frac{a}{2}\right)^2$$

$$\Rightarrow AD^2 = a^2 - \frac{a^2}{4} = \frac{3}{4} a^2 \Rightarrow AD = \frac{\sqrt{3}}{2} a.$$

Ex.64 ABC is a right-angled triangle, right-angled at A. A circle is inscribed in it. The lengths of the two sides containing the right angle are 5 cm and 12 cm. Find the radius of the circle.

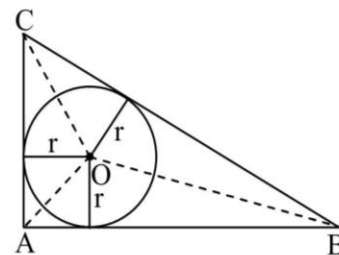
Sol. Given that $\triangle ABC$ is right angled at A.

$$AC = 5 \text{ cm and } AB = 12 \text{ cm}$$

$$BC^2 = AC^2 + AB^2 = 25 + 144 = 169$$

$$\Rightarrow BC = 13 \text{ cm}$$

Join OA, OB, OC



Let the radius of the inscribed circle be r

$$\text{Area of } \triangle ABC = \text{Area of } \triangle OAB$$

$$+ \text{Area of } \triangle OBC + \text{Area of } \triangle OCA$$

$$\Rightarrow \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2} (12 \times r) + \frac{1}{2} (13 \times r) + \frac{1}{2} (5 \times r)$$

$$\Rightarrow 12 \times 5 = r \times \{12 + 13 + 5\}$$

$$\Rightarrow 60 = r \times 30 \Rightarrow r = 2 \text{ cm}$$

Ex.65 ABCD is a rhombus. Prove that

$$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$$

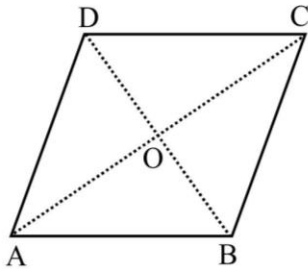
Sol. Let the diagonals AC and BD of rhombus ABCD intersect at O.

Since the diagonals of a rhombus bisect each other at right angles.

$$\therefore \angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$$

and $AO = CO, BO = OD$.

Since $\triangle AOB$ is a right triangle right-angle at O.



$$\therefore AB^2 = OA^2 + OB^2$$

$$\Rightarrow AB^2 = \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 \quad \left[\begin{array}{l} \ominus OA=OC \\ \text{and } OB=OD \end{array} \right]$$

$$\Rightarrow 4AB^2 = AC^2 + BD^2 \quad \dots(i)$$

Similarly, we have

$$4BC^2 = AC^2 + BD^2 \quad \dots(ii)$$

$$4CD^2 = AC^2 + BD^2 \quad \dots(iii)$$

$$\text{and, } 4AD^2 = AC^2 + BD^2 \quad \dots(iv)$$

Adding all these results, we get

$$4(AB^2 + BC^2 + AD^2) = 4(AC^2 + BD^2)$$

$$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$$

Ex.66 P and Q are the mid-points of the sides CA and CB respectively of a $\triangle ABC$, right angled at C. Prove that :

$$(i) \quad 4AQ^2 = 4AC^2 + BC^2$$

$$(ii) \quad 4BP^2 = 4BC^2 + AC^2$$

$$(iii) \quad (4AQ^2 + BP^2) = 5AB^2$$

Sol. (i) Since $\triangle AQC$ is a right triangle right-angled at C.

$$\therefore AQ^2 = AC^2 + QC^2$$

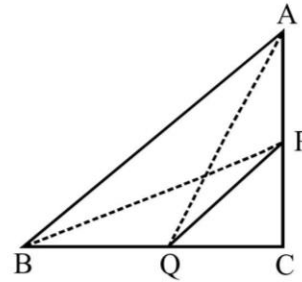
$$\Rightarrow 4AQ^2 = 4AC^2 + 4QC^2$$

[Multiplying both sides by 4]

$$\Rightarrow 4AQ^2 = 4AC^2 + (2QC)^2$$

$$\Rightarrow 4AQ^2 = 4AC^2 + BC^2 \quad [\ominus BC = 2QC]$$

(ii) Since $\triangle BPC$ is a right triangle right-angled at C.



$$\therefore BP^2 = BC^2 + CP^2$$

$$\Rightarrow 4BP^2 = 4BC^2 + 4CP^2$$

[Multiplying both sides by 4]

$$\Rightarrow 4BP^2 = 4BC^2 + (2CP)^2$$

$$\Rightarrow 4BP^2 = 4BC^2 + AC^2 \quad [\ominus AC = 2CP]$$

(iii) From (i) and (ii), we have

$$4AQ^2 = 4AC^2 + BC^2 \text{ and, } 4BC^2 = 4BC^2 + AC^2$$

$$\therefore 4AQ^2 + 4BP^2 = (4AC^2 + BC^2) + (4BC^2 + AC^2)$$

$$\Rightarrow 4(AQ^2 + BP^2) = 5(AC^2 + BC^2)$$

$$\Rightarrow 4(AQ^2 + BP^2) = 5AB^2$$

[In $\triangle ABC$, we have $AB^2 = AC^2 + BC^2$]

Ex.67 From a point O in the interior of a $\triangle ABC$, perpendicular OD, OE and OF are drawn to the sides BC, CA and AB respectively. Prove that : **[NCERT]**

$$(i) \quad AF^2 + BD^2 + CE^2 = OA^2 + OB^2$$

$$+ OC^2 - OD^2 - OE^2 - OF^2$$

$$(ii) \quad AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$$

Sol. Let O be a point in the interior of $\triangle ABC$ and let $OD \perp BC, OE \perp CA$ and $OF \perp AB$.

(i) In right triangles $\triangle OFA, \triangle ODB$ and $\triangle OEC$, we have

$$OA^2 = AF^2 + OF^2$$

$$OB^2 = BD^2 + OD^2$$

$$\text{and, } OC^2 = CE^2 + OE^2$$

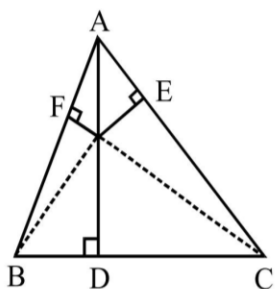
Adding all these results, we get

$$OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OF^2 + OD^2 + OE^2$$

$$\Rightarrow AF^2 + BD^2 + CE^2 = OA^2 + OB^2$$

$$+ OC^2 - OD^2 - OE^2 - OF^2$$

(ii) In right triangles $\triangle ODB$ and $\triangle ODC$, we have



$$OB^2 = OD^2 + BD^2$$

and, $OC^2 = OD^2 + CD^2$

$$\therefore OB^2 - OC^2 = (OD^2 + BD^2) - (OD^2 + CD^2)$$

$$\Rightarrow OB^2 - OC^2 = BD^2 - CD^2 \quad \dots(i)$$

Similarity, we have

$$OC^2 - OA^2 = CE^2 - AE^2 \quad \dots(ii)$$

and, $OA^2 - OB^2 = AF^2 - BF^2 \quad \dots(iii)$

Adding (i), (ii) and (iii), we get

$$(OB^2 - OC^2) + (OC^2 - OA^2) + (OA^2 - OB^2)$$

$$= (BD^2 - CD^2) + (CE^2 - AE^2) + (AF^2 - BF^2)$$

$$\Rightarrow (BD^2 + CE^2 + AF^2) - (AE^2 + CD^2 + BF^2) = 0$$

$$\Rightarrow AF^2 + BD^2 + CE^2 = AE^2 + BF^2 + CD^2$$

Ex.68 In a right triangle ABC right-angled at C, P and Q are the points on the sides CA and CB respectively, which divide these sides in the ratio 2 : 1. Prove that

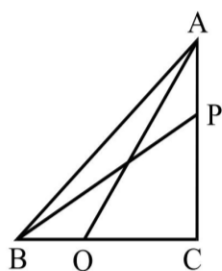
- (i) $9 AQ^2 = 9 AC^2 + 4 BC^2$
- (ii) $9 BP^2 = 9 BC^2 + 4 AC^2$
- (iii) $9 (AQ^2 + BP^2) = 13 AB^2$

Sol. It is given that P divides CA in the ratio 2 : 1. Therefore,

$$CP = \frac{2}{3} AC \quad \dots(i)$$

Also, Q divides CB in the ratio 2 : 1.

$$\therefore QC = \frac{2}{3} BC \quad \dots(ii)$$



(i) Applying pythagoras theorem in right-angled triangle ACQ, we have

$$AQ^2 = QC^2 + AC^2$$

$$\Rightarrow AQ^2 = \frac{4}{9} BC^2 + AC^2 \quad [\text{Using (ii)}]$$

$$\Rightarrow 9 AQ^2 = 4 BC^2 + 9 AC^2 \quad \dots(iii)$$

(ii) Applying pythagoras theorem in right triangle BCP, we have

$$BP^2 = BC^2 + CP^2$$

$$\Rightarrow BP^2 = BC^2 + \frac{4}{9} AC^2 \quad [\text{Using (i)}]$$

$$\Rightarrow 9 BP^2 = 9 BC^2 + 4 AC^2 \quad \dots(iv)$$

Adding (iii) and (iv), we get

$$9 (AQ^2 + BP^2) = 13 (BC^2 + AC^2)$$

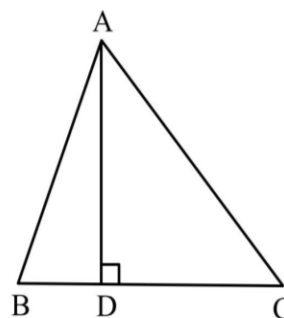
$$\Rightarrow 9 (AQ^2 + BP^2) = 13 AB^2$$

$$[\ominus BC^2 = AC^2 + AB^2]$$

Ex.69 In a ΔABC , $AD \perp BC$ and $AD^2 = BC \times CD$. Prove that ΔABC is a right triangle.

Sol. In right triangles ADB and ADC, we have

$$AB^2 = AD^2 + BD^2 \quad \dots(i)$$



$$\text{and, } AC^2 = AD^2 + DC^2 \quad \dots(ii)$$

Adding (i) and (ii), we get

$$AB^2 + AC^2 = 2 AD^2 + BD^2 + DC^2$$

$$\Rightarrow AB^2 + AC^2 = 2BD \times CD + BD^2 + DC^2$$

$$[\ominus AD^2 = BD \times CD \text{ (Given)}]$$

$$\Rightarrow AB^2 + AC^2 = (BD + CD)^2 = BC^2$$

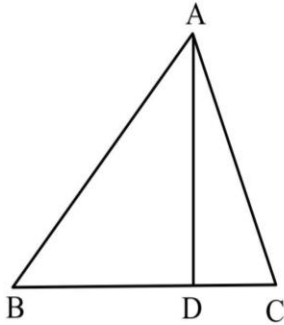
Thus, in ΔABC , we have

$$AB^2 = AC^2 + BC^2$$

Hence, ΔABC , is a right triangle right-angled at A.

Ex.70 The perpendicular AD on the base BC of a ΔABC intersects BC at D so that $DB = 3 CD$. Prove that $2AB^2 = 2AC^2 + BC^2$.

Sol. We have,



$$DB = 3CD$$

$$\therefore BC = BD + DC$$

$$\Rightarrow BC = 3CD + CD$$

$$\Rightarrow BD = 4CD \Rightarrow CD = \frac{1}{4}BC$$

$$\therefore CD = \frac{1}{4}BC \text{ and } BD = 3CD = \frac{3}{4}BC \dots(i)$$

Since ΔABD is a right triangle right-angled at D.

$$\therefore AB^2 = AD^2 + BD^2 \dots(ii)$$

Similarly, ΔACD is a right triangle right angled at D.

$$\therefore AC^2 = AD^2 + CD^2 \dots(iii)$$

Subtracting equation (iii) from equation (ii) we get

$$AB^2 - AC^2 = BD^2 - CD^2$$

$$\Rightarrow AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$$

$$\left[\text{From (i) } CD = \frac{1}{4}BC, BD = \frac{3}{4}BC \right]$$

$$\Rightarrow AB^2 - AC^2 = \frac{9}{16}BC^2 - \frac{1}{16}BC^2$$

$$\Rightarrow AB^2 - AC^2 = \frac{1}{2}BC^2$$

$$\Rightarrow 2(AB^2 - AC^2) = BC^2$$

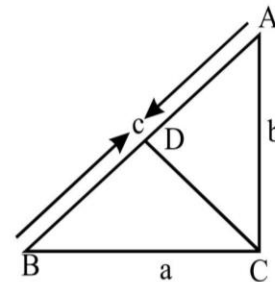
$$\Rightarrow 2AB^2 = 2AC^2 + BC^2$$

Ex.71 ABC is a right triangle right-angled at C. Let $BC = a$, $CA = b$, $AB = c$ and let p be the length of perpendicular from C on AB, prove that

(i) $cp = ab$

(ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Sol.(i) Let $CD \perp AB$. Then, $CD = p$.



$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}(\text{Base} \times \text{Height})$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}(AB \times CD) = \frac{1}{2}cp$$

Also,

$$\text{Area of } \Delta ABC = \frac{1}{2}(BC \times AC) = \frac{1}{2}ab$$

$$\therefore \frac{1}{2}cp = \frac{1}{2}ab \Rightarrow cp = ab$$

(ii) Since ΔABC is right triangle right-angled at C.

$$\therefore AB^2 = BC^2 + AC^2$$

$$\Rightarrow c^2 = a^2 + b^2$$

$$\Rightarrow \left(\frac{ab}{p}\right)^2 = a^2 + b^2 \left[\because cp = ab \therefore c = \frac{ab}{p} \right]$$

$$\Rightarrow \frac{a^2b^2}{p^2} = a^2 + b^2$$

$$\Rightarrow \frac{1}{p^2} = \frac{a^2 + b^2}{a^2b^2} \Rightarrow \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

IMPORTANT POINTS TO BE REMEMBERED

1. Two figures having the same shape but not necessarily the same size are called similar figures.
2. All congruent figures are similar but the converse is not true.
3. Two polygons having the same number of sides are similar, if
 - (a) Their corresponding angles are equal and
 - (b) Their corresponding sides are proportional
 (i.e., in the same ratio)
4. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.
5. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side of the triangle.
6. The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.
7. If a line through one vertex of a triangle divides the opposite side in the ratio of other two sides, then the line bisects the angle at the vertex.
8. The external bisector of an angle of a triangle divides the opposite side externally in the ratio of the sides containing the angle.
9. The line drawn from the mid-point of two sides of a triangle is parallel of another side bisects the third side.
10. The line joining the mid-points of two sides of a triangle is parallel to the third side.
11. The diagonals of a trapezium divide each other proportionally.
12. If a diagonals of a quadrilateral divide each other proportionally, then it is a trapezium.
13. Any line parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.
14. If three or more parallel lines are intersected by two transversals, then the intercepts made by them on the transversals are proportional.
15. **AAA similarity criterion** : If in two triangles, corresponding angles are equal, then the triangles are similar.
16. **AA Similarity criterion** : If in two triangles, two angles of one triangle are respectively equal the two angles of the other triangle, then the two triangles are similar.
17. **SSS Similarity criterion** : If in two triangles, corresponding sides are in the same ratio, then the two triangles are similar.
18. If one angle of a triangles is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the triangles are similar.
19. If two triangles are equiangular, then
 - (i) The ratio of the corresponding sides is same as the ratio of corresponding median.
 - (ii) The ratio of the corresponding sides is same as the ratio of the corresponding angle bisector segments.
 - (iii) The ratio of the corresponding sides is same as the ratio of the corresponding altitudes.
20. If one angle of a triangle is equal to one angle of another triangle and the bisectors of these equal angles divide the opposite side in the same ratio, then the triangles are similar.
21. If two sides and a median bisecting one of these sides of a triangle are respectively proportional to the two sides and the corresponding median of another triangle, then the triangles are similar.
22. If two sides and a median bisecting the third side of a triangle are respectively proportional to the two sides and the corresponding median of another triangle, then the triangles are similar.
23. The ratio of the areas of two similar triangles is equal to the ratio of
 - (i) The squares of any two corresponding sides
 - (ii) The squares of the corresponding altitudes.
 - (iii) The squares of the corresponding medians.
 - (iv) The squares of the corresponding angle bisector segments.

24. If the areas of two similar triangles are equal, then the triangles are congruent i.e., equal and similar triangles congruent.
25. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.
26. **Pythagoras Theorem** : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
27. **Converse of Pythagoras Theorem** : If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to first side is a right angle.
28. In any triangle, the sum of the squares of any two sides is equal to twice the square of half of the third side together with the twice of the square of the median which bisects the third side.
29. Three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.
30. Three times the square of any side of an equilateral triangle is equal to four times the square of the altitude.