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SIMILAR TRIANGLES

Objective Section (1 mark each)

Fill in the Blanks

Q. 1. **In Fig. 1, MN || BC and AM : MB = 1 : 2, then** **[CBSE OD, Set 1, 2020]**

$$\frac{ar(\Delta AMN)}{ar(\Delta ABC)} = \text{.........}$$

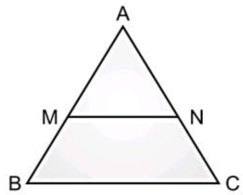


Fig. - 1

Ans. $\frac{1}{9}$

Explanation :

Given : MN || BC and AM : MB = 1 : 2

In ΔAMN and ΔABC,

$\angle MAN = \angle BAC$ [Common angle]

$\angle AMN = \angle ABC$ [Corresponding angles]

$\therefore \Delta AMN \sim \Delta ABC$

$$\therefore \frac{ar(\Delta AMN)}{ar(\Delta ABC)} = \left(\frac{AM}{AB}\right)^2 = \left(\frac{AM}{AM+MB}\right)^2 = \left(\frac{1}{1+2}\right)^2 = \frac{1}{9}$$ **Ans.**

Q. 2. **In ΔABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm, then ∠B = ...** **[CBSE OD, Set 1, 2020]**

Ans. 90°

Explanation :

$$\because (12)^2 = (6\sqrt{3})^2 + (6)^2$$

$$\Rightarrow AC^2 = AB^2 + BC^2$$

∴ By the converse of Pythagoras theorem,

$\therefore \angle B = 90°.$ **Ans.**

Q. 3. **Two triangles are similar if their corresponding sides are ....** **[CBSE OD, Set 1, 2020]**

Ans. Proportional.

Q. 4. **Given ΔABC ~ ΔPQR, if $\frac{AB}{PQ} = \frac{1}{3}$, then**

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \text{..............}$$

[CBSE Delhi, Set 1, 2020]

Ans. $\frac{1}{9}$.

Explanation : As ΔABC ~ ΔPQR

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

Q. 5. **ABC is an equilateral triangle of side 2*a*, then length of one of its altitude is ....** **[CBSE Delhi, Set 1, 2020]**

Ans. $\sqrt{3}\,a$

Explanation : We know that for an equilateral triangle,

$$\Rightarrow \text{Length of altitude} = \frac{\sqrt{3}}{2} \times \text{side}$$

$$= \frac{\sqrt{3}}{2} \times 2a = \sqrt{3}a$$ **Ans.**

Q. 6. **ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is ....** **[CBSE Delhi, Set 2, 2020]**

Ans. 4 : 1

Explanation :

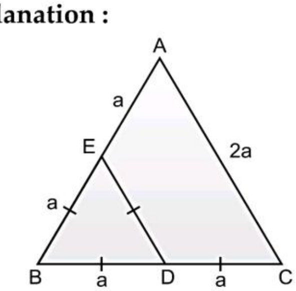


Figure – 1

$\Delta ABC \sim \Delta BDE$

($\because$ both are equilateral)

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \left(\frac{2a}{a}\right)^2 = \frac{4}{1} = 4:1$$

Very Short Answer Type Questions (1 mark each)

Q. 1. In Figure 1, *ABC* is an isosceles triangle right angled at *C* with *AC* = 4 cm. Find the length of *AB*.

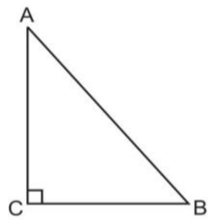
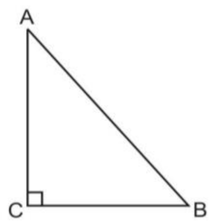


Figure 1

[CBSE OD, Set 1, 2019]

Ans. Given, $\angle C = 90°$ and $AC = 4$ cm

$AB = ?$



$\because$ ΔABC is an isosceles triangle so,

$$BC = AC = 4 \text{ cm}$$

On applying Pythagoras theorem, we have

$$AB^2 = AC^2 + BC^2$$
$$= 4^2 + 4^2$$
$$= 16 + 16 = 32$$

$$\Rightarrow \quad AB = \sqrt{32}$$
$$= 4\sqrt{2} \text{ cm}$$

Q. 2. In Figure 2, *DE* || *BC*. Find the length of side *AD*, given that *AE* = 1.8 cm, *BD* = 7.2 cm and *CE* = 5.4 cm.

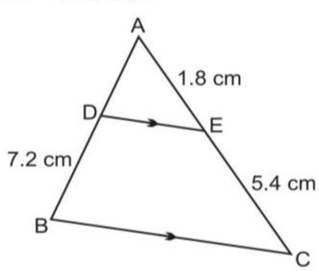


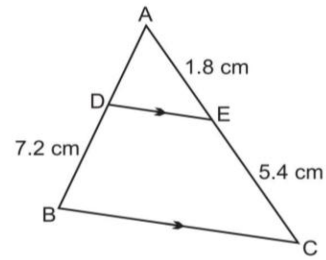
Figure 2

[CBSE OD, Set 1, 2019]

Ans. Given, $DE \parallel BC$

On applying Thales theorem, we have

$$\frac{AD}{AB} = \frac{AE}{AC}$$



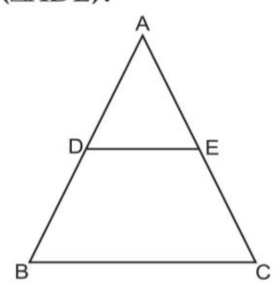
$$\frac{AD}{AD+7.2} = \frac{1.8}{1.8+5.4}$$

$$\frac{AD}{AD+7.2} = \frac{1.8}{7.2}$$

$$\frac{AD}{AD+7.2} = \frac{1}{4}$$

$$4AD = AD + 7.2$$
$$3AD = 7.2$$
$$AD = 2.4 \text{ cm}$$

Q. 3. In Fig., *DE* || *BC*, *AD* = 1 cm and BD = 2 cm. what is the ratio of the ar (ΔABC) to the ar (Δ*ADE*)?



[CBSE Delhi, Set 1, 2019]

Ans. Given,

$AD = 1$ cm, $BD = 2$ cm

$\therefore$ $AB = 1 + 2 = 3$ cm

Also, $DE \parallel BC$ (Given)

$\therefore$ $\angle ADE = \angle ABC$ ...(i) (corresponding angles)

In ΔABC and ΔADE

$\angle A = \angle A$ (common)

$\angle ABC = \angle ADE$ [by equation (i)]

$\therefore$ $\Delta ABC \sim \Delta ADE$ (by AA rule)

$$\therefore \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta ADE)} = \left(\frac{AB}{AD}\right)^2$$

or

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta ADE)} = \left(\frac{3}{1}\right)^2 = \frac{9}{1}$$

$\therefore$ ar (ΔABC) : ar (ΔADE) = 9 : 1

Q. 4. **Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then find $\frac{ar\Delta ABC}{ar\Delta PQR}$.** **[CBSE, 2018]**

Ans. Given, $\Delta ABC \sim \Delta PQR$

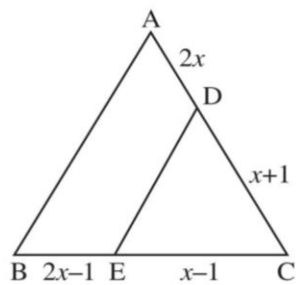
and $\frac{AB}{PQ} = \frac{1}{3}$

Now, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2}$

$= \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

Q. 5. **In ΔABC, D and E are points AC and BC respectively such that $DE \parallel AB$. If $AD = 2x$, $BE = 2x - 1$, $CD = x + 1$ and $CE = x - 1$, then find the value of x.** **[CBSE Term 1, 2016]**

Ans.



$DE \parallel AB$

So, $\frac{AD}{CD} = \frac{BE}{EC}$ [By B.P.T.]

$\Rightarrow \frac{2x}{x+1} = \frac{2x-1}{x-1}$

$\Rightarrow 2x(x-1) = (x+1)(2x-1)$

$\Rightarrow 2x^2 - 2x = 2x^2 + 2x - x - 1$

$\Rightarrow -2x = x - 1$

$\Rightarrow 1 = 3x$

$\Rightarrow x = \frac{1}{3}$

Q. 6. **In ΔDEW, $AB \parallel EW$. If $AD = 4$ cm, $DE = 12$ cm and $DW = 24$ cm, then find the value of DB.** **[CBSE Term 1, Set 1, 2015]**

Ans. Let $BD = x$ cm.

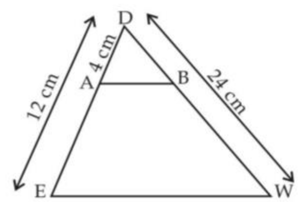
$\because DW = 24$ cm.

Then, $BW = (24 - x)$ cm, $AE = 12 - 4 = 8$ cm

In ΔDEW, $AB \parallel EW$

$\therefore \frac{AD}{AE} = \frac{BD}{BW}$ [Thales' Theorem]

$\Rightarrow \frac{4}{8} = \frac{x}{24-x}$



$\Rightarrow 8x = 96 - 4x$

$\Rightarrow 12x = 96$

$\Rightarrow x = \frac{96}{12} = 8$ cm

$\therefore DB = 8$ cm

Short Answer Type Questions-I (2 marks each)

Q. 1. **In fig. 5, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$**

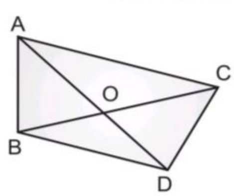


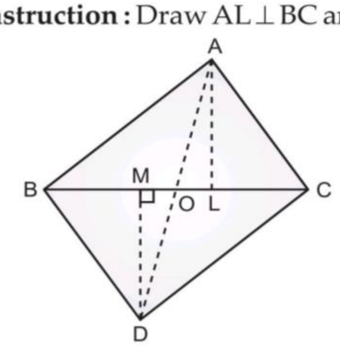
Fig. - 5

[CBSE OD, Set-I, 2020]

Ans. Given : ΔABC and ΔDBC are on the same base BC and AD intersect BC at O.

To prove : $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.

Construction : Draw $AL \perp BC$ and $DM \perp BC$.



Proof : In ΔALO and ΔDMO, we have

$\angle ALO = \angle DMO = 90°$

and $\angle AOL = \angle DOM$

[Vertically opposite angles]

$\therefore \angle ALO \sim \angle DMO$ [AA-similarity]

$\therefore \quad \dfrac{AL}{DM} = \dfrac{AO}{DO}$

[corresponding part of similar triangles] ...(i)

$$\text{Now, } \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times BC \times DM} = \frac{AL}{DM} = \frac{AO}{DO}$$

[using (i)]

$\Rightarrow \dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{AO}{DO}$. **Hence Proved.**

Q. 2. In fig. 6, if AD ⊥ BC, then prove that

$AB^2 + CD^2 = BD^2 + AC^2$

[CBSE OD, Set-I, 2020]

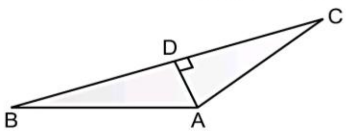


Fig - 6

Ans. Given : ΔABC in which AD ⊥ BC.

To prove : $AB^2 + CD^2 = BD^2 + AC^2$

Proof : From right ΔADB, we have

$AB^2 = AD^2 + BD^2$

[By Pythagoras theorem]

$\Rightarrow AB^2 - BD^2 = AD^2$. ...(i)

From right ΔADC, we have

$AC^2 = AD^2 + CD^2$

$\Rightarrow AC^2 - CD^2 = AD^2$ ...(ii)

From (i) and (ii), we get

$AB^2 - BD^2 = AC^2 - CD^2$

$\Rightarrow AB^2 + CD^2 = BD^2 + AC^2$ **Hence Proved.**

Q. 3. In Fig. 2, DE || AC and DC || AP. Power that $\dfrac{BE}{EC} = \dfrac{BC}{CP}$ [CBSE Delhi, Set-I, 2020]

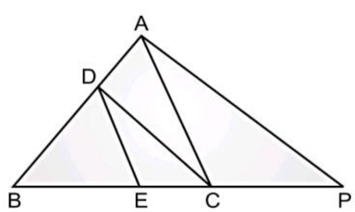
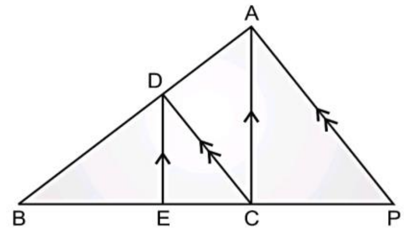


Fig. 2

Ans. Given : DE || AC and DC || AP.

To Prove : $\dfrac{BE}{EC} = \dfrac{BC}{CP}$



Proof : In ΔABC,

DE || AC

$\therefore \quad \dfrac{BE}{EC} = \dfrac{BD}{AD}$ ...(i)

(By Basic Proportionality Theorem)

Similarly, In ΔABP,

DC || AP

$\therefore \quad \dfrac{BC}{CP} = \dfrac{BD}{AD}$ ...(ii)

From (i) and (ii), we get

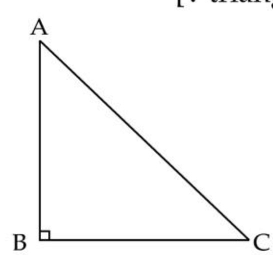
$\dfrac{BE}{EC} = \dfrac{BC}{CP}$ **Hence Proved.**

Q. 4. In an isosceles Δ *ABC* right angled at *B*, prove that $AC^2 = 2AB^2$.

[CBSE Term 1, 2016]

Ans. In Δ*ABC*, *AB* = *BC* ...(i)

[∵ triangle is isosceles]



In Δ*ABC* by pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC^2 = AB^2 + AB^2$ [From (i)]

$\Rightarrow \quad AC^2 = 2AB^2$ **Hence Proved.**

Q. 5. *X* and *Y* are points on the sides *AB* and *AC*, respectively of a triangle *ABC* such that $\dfrac{AX}{AB}$, *AY* = 2 cm and *YC* = 6 cm. Find whether *XY* || *BC* or not.

[CBSE Term 1, Set 1, 2015]

Ans. $\dfrac{AX}{AB} = \dfrac{1}{4}$

i.e., $AX = 1K, AB = 4K$

(K– constant)

$\therefore \quad BX = AB - AX$

$= 4K - 1K = 3K$

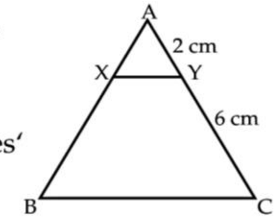
Now, $\quad \frac{AX}{XB} = \frac{1K}{3K} = \frac{1}{3}$

and, $\quad \frac{AY}{YC} = \frac{2}{6} = \frac{1}{3}$

$\frac{AX}{XB} = \frac{AY}{YC}$

$\therefore \quad XY \parallel BC$

(By converse of Thales' theorem)



Short Answer Type Questions-II (3 marks each)

Q. 1. In Fig. 7, if ΔABC ~ ΔDEF and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle. [CBSE OD, Set 1, 2020]

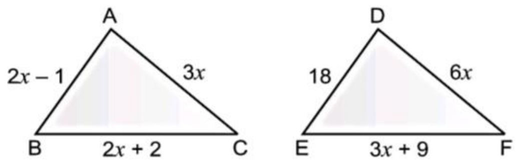


Fig. – 7

Ans. Given : ΔABC ~ ΔDEF

$\therefore \quad \frac{AB}{DE} = \frac{AC}{DF}$

[Corresponding parts of similar triangles]

$\Rightarrow \quad \frac{2x-1}{18} = \frac{3x}{6x}$

$\Rightarrow \quad \frac{2x-1}{18} = \frac{1}{2}$

$\Rightarrow \quad 4x - 2 = 18$

$\Rightarrow \quad 4x = 20$

$\Rightarrow \quad x = 5$

Now, lengths of sides of triangle ABC are,

$AB = 2x - 1 = 9$ cm

$BC = 2x + 2 = 12$ cm

$AC = 3x = 15$ cm

And, lengths of sides of triangle DEF are,

$DE = 18$ cm

$EF = 3x + 9 = 24$ cm

$DF = 6x = 30$ cm **Ans.**

Q. 2. In Fig. 5, ∠D = ∠E and $\frac{AD}{DB} = \frac{AE}{EC}$, prove that BAC is an isosceles triangle.

[CBSE Delhi, Set 1, 2020]

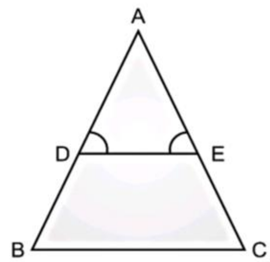


Fig. 5

Ans. Given : $\quad \angle D = \angle E$

and, $\quad \frac{AD}{DB} = \frac{AE}{EC}$

To prove : ΔBAC is an isosceles triangle

Proof : In ΔABC, $\frac{AD}{DB} = \frac{AE}{EC}$ (given)

$\Rightarrow \quad DE \parallel BC$

{By converse of Basic Proportionality theorem}

$\therefore \quad \angle ADE = \angle ABC$ ...(i)

{∵ Corresponding angles are equal as DE || BC}

and $\quad \angle AED = \angle ACB$ ...(ii)

But $\quad \angle ADE = \angle AED$ (Given)...(iii)

$\therefore \quad \angle ABC = \angle ACB$

(From eq. (i), (ii) and (ii))

$\Rightarrow \quad AB = AC$

∴ ΔABC is an isosceles triangle as two of its sides are equal. **Hence Proved.**

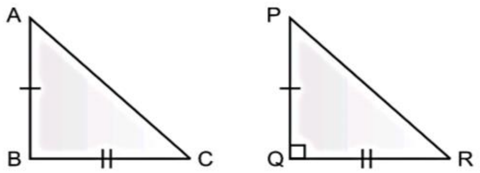
Q. 3. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite of the first side is a right angle.

[CBSE Delhi, Set 1, 2020]

Ans. Given : ΔABC in which

$AC^2 = AB^2 + BC^2$

To prove : $\angle B = 90°$



Construction : Draw ΔPQR in which PQ = AB, QR = BC and ∠Q = 90°.

Proof : In ΔABC,

$\Rightarrow \quad AC^2 = AB^2 + BC^2$ (Given)

$\Rightarrow \quad AC^2 = PQ^2 + QR^2$ (Given) ...(i)

Now, $\quad PR^2 = PQ^2 + PR^2$ ...(ii)

(By Pythagoras Theorem)

From equations (i) and (ii), we get

$AC^2 = PR^2$

$\Rightarrow \quad AC = PR$

∴ In ΔABC and ΔPQR,

$AB = PQ$

$BC = QR$

$AC = PR$

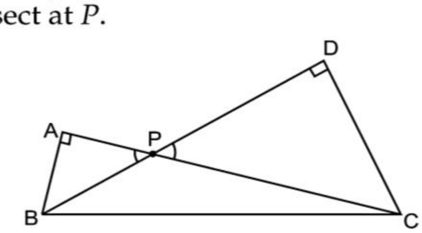
$\therefore \quad \Delta ABC \cong \Delta PQR$

$\Rightarrow \quad \angle ABC = \angle PQR$ (C.P.C.T.)

$\therefore \quad \angle ABC = 90^\circ$ **Hence Proved.**

Q. 4. Two right triangles *ABC* and *DBC* are drawn on the same hypotenuse *BC* and on the same side of *BC*. If *AC* and *BD* intersect at *P*, prove that $AP \times PC = BP \times DP$. [CBSE OD, Set 1, 2019]

Ans. Given, ΔABC and ΔDBC are right angle triangles, right angled at A and D respectively, on same side of BC. AC & BD intersect at P.



In ΔAPB and ΔPDC,

$\angle A = \angle D = 90^\circ$

$\angle APB = \angle DPC$ (Vertically opposite)

$\therefore \quad \Delta APB \sim \Delta DPC$ (By AA Similarity)

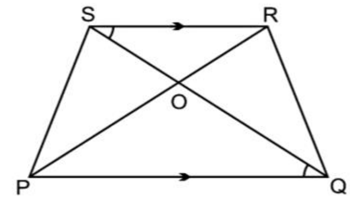
$\therefore \quad \frac{AP}{BP} = \frac{PD}{PC}$ (by c.s.s.t.)

$\Rightarrow AP \times PC = BP \times PD.$ **Hence Proved.**

Q. 5. Diagonals of a trapezium *PQRS* intersect each other at the point *O*, $PQ \parallel RS$ and $PQ = 3RS$. Find the ratio of the areas of traingles *POQ* and *ROS*.

[CBSE OD, Set 1, 2019]

Ans. Given, *PQRS* is a trapezium where $PQ \parallel RS$ and diagonals intersect at *O* and $PQ = 3RS$



In ΔPOQ and ΔROS, we have

$\angle ROS = \angle POQ$

(vertically opposite angles)

$\angle OQP = \angle OSR$ (alternate angles)

Hence, $\Delta POQ \sim \Delta ROS$ by *AA* similarity then, If two triangles are similar, then ratio of areas is equal to the ratio of square of its corresponding sides.

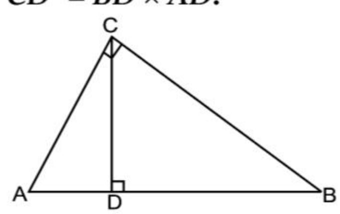
Then,

$$\frac{\text{area of } \Delta POQ}{\text{area of } \Delta ROS} = \frac{(PQ)^2}{(RS)^2}$$

$$= \frac{(3RS)^2}{(RS)^2} = \frac{9}{1}$$

$$= 9 : 1$$

Q. 6. In Fig. $\angle ACB = 90^\circ$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.



[CBSE Delhi, Set 1, 2019]

Ans. Given, ΔACB in which $\angle ACB = 90^\circ$ and $CD \perp AB$

To prove: $CD^2 = BD \times AD$

Proof: In ΔADC and ΔACB

$\angle A = \angle A$ (common)

$\angle ADC = \angle ACB$ (90° each)

$\therefore \quad \Delta ADC \sim \Delta ACB$ (By AA rule)

$\Rightarrow \quad \frac{AD}{CD} = \frac{AC}{BC}$ ...(i)

Similarly,

$\Delta CDB \sim \Delta ACB$ (By AA rule)

$\Rightarrow \qquad \dfrac{AD}{CD} = \dfrac{AC}{BC}$ ...(ii)

From equation (i) and (ii)

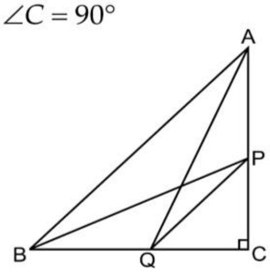
$$\frac{AD}{CD} = \frac{CD}{DB}$$

$\Rightarrow \qquad CD^2 = AD \cdot BD$

$\Rightarrow \qquad CD^2 = BD \times AD$ **Hence Proved.**

Q. 7. If P and Q are the points on side CA and CB respectively of ΔABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$. [CBSE Delhi, Set 1, 2019]

Ans. Given, ABC is a right angled triangle in which $\angle C = 90°$



To prove: $AQ^2 + BP^2 = AB^2 + PQ^2$

construction: Join AQ, PB and PQ

Proof: In ΔAQC, $\angle C = 90°$

$\therefore \qquad AQ^2 = AC^2 + CQ^2$ ...(i)

(Using Pythagoras theorem)

In ΔPBC, $\qquad \angle C = 90°$

$\therefore \qquad BP^2 = BC^2 + CP^2$ ...(ii)

(Using Pythagoras theorem)

Adding equation (i) and (ii)

$$AQ^2 + BP^2 = AC^2 + CQ^2 + BC^2 + CP^2$$
$$= AC^2 + BC^2 + CQ^2 + CP^2$$

or $\qquad AQ^2 + BP^2 = AB^2 + PQ^2$

Hence Proved.

Q. 8. Prove that the area of an equilateral triangle described on one side of the square is equal to half the area of the equilateral triangle described on one of its diagonal. [CBSE, 2018]

Ans.

Topper's Answers

17) (Choice 1) Given. Square ABCD. ΔAED and ΔAFC are equilateral.

To prove: Area $\Delta AFC = 2\times$ Area ΔAED.

Construction: Draw $EP \perp AD$ and $FQ \perp AC$.

Proof: Let side of square be x.

$\Rightarrow$ sides of $\Delta AED = x$.

In ΔABC, $\angle B = 90°$.

$\Rightarrow$ By Pythagoras Theorem,

$AB^2 + BC^2 = AC^2$

$x^2 + x^2 = AC^2 \Rightarrow AC = \sqrt{2}x. \Rightarrow$ sides of $\Delta AFC = \sqrt{2}x$.

We know, altitude of equilateral Δ bisects the base.

$\rightarrow PD = \frac{x}{2}$, $AQ = \frac{x}{\sqrt{2}}$.

In ΔAEP, $\angle P = 90°$.

By Pythagoras theorem, $AE^2 = EP^2 + AP^2$.

$x^2 = EP^2 + \left(\frac{x}{2}\right)^2$.

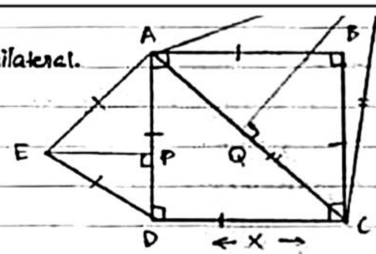
$EP^2 = \frac{3x^2}{4} \Rightarrow EP = \frac{\sqrt{3}}{2}x$.

In ΔAFQ, $\angle Q = 90°$.

By Pythagoras theorem, $AF^2 = FQ^2 + AQ^2$

$2x^2 = FQ^2 + \frac{x^2}{2}$.

$FQ^2 = \frac{3x^2}{2} \Rightarrow FQ = \frac{\sqrt{3}x}{\sqrt{2}}$.



We know, Area of triangle $= \frac{1}{2} \times$ Base $\times$ height sq. units.

$\Rightarrow$ Area of $\triangle AFC = \frac{1}{2} \times \sqrt{2}x \times FQ$

$= \frac{1}{2} \cdot \sqrt{2}x \cdot \frac{\sqrt{3}}{\sqrt{2}} x$

$= \frac{\sqrt{3}}{2} x^2.$

Area of $\triangle AED = \frac{1}{2} \times x \times EP.$

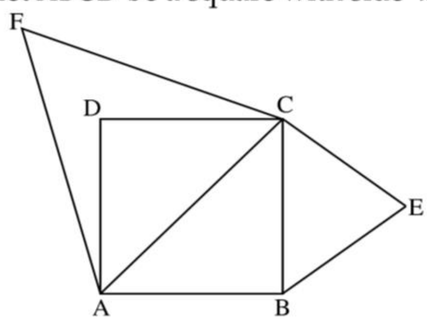
$= \frac{1}{2} \cdot x \cdot \frac{\sqrt{3}}{3} x$

$= \frac{\sqrt{3}}{4} x^2.$

$2 \cdot$ Area of $\triangle AED = \frac{\sqrt{3}}{2} x^2 =$ Area of $\triangle AFC$.

hence proved.

Let $ABCD$ be a square with side 'a'.



In ΔABC,

$$AC^2 = AB^2 + BC^2$$
$$= a^2 + a^2 = 2a^2$$
$$AC = \sqrt{2a^2} = \sqrt{2}a.$$

Area of equilateral ΔBEC (formed on side BC of square $ABCD$)

$$= \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} a^2 \quad \text{...(i)}$$

Area of equilateral ΔACF (formed on diagonal AC of square $ABCD$)

$$= \frac{\sqrt{3}}{4} (\sqrt{2}a)^2 = \frac{\sqrt{3}}{4}(2a^2)$$

$$= 2\frac{\sqrt{3}}{4} a^2 \quad \text{...(ii)}$$

From eq. (i) and (ii),

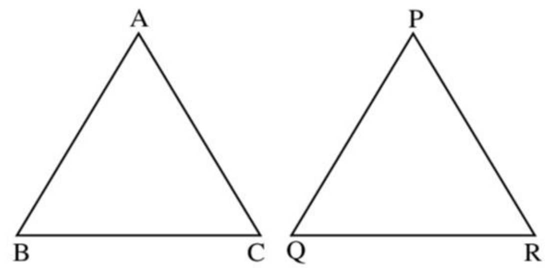
$$\text{ar } \Delta ACF = 2 \times \text{ar } \Delta BCF$$

or $\text{ar }(\Delta BCF) = \frac{1}{2} \text{ ar }(\Delta ACF)$

i.e., area of triangle described on one side of square is half the area of triangle described on its diagonal. **Hence Proved.**

Q. 9. If the area of two similar triangles are equal, prove that they are congruent. [CBSE, 2018]

Ans. Given, $\Delta ABC \sim \Delta PQR$



And $\text{ar }(\Delta ABC) = \text{ar }(\Delta PQR)$

To prove:

$$\Delta ABC \cong \Delta PQR$$

Proof:

Given, $\Delta ABC \sim \Delta PQR$

$$\therefore \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

(Ratio of area of similar triangles is equal to the square of corresponding sides)

But $\frac{\text{ar }(\Delta ABC)}{\text{ar }(\Delta PQR)} = 1$ (Given)

$$\therefore \quad \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} = 1$$

So, $AB^2 = PQ^2$ or $AB = PQ$

$BC^2 = QR^2$ or $BC = QR$

$AC^2 = PR^2$ or $AC = PR$

By SSS congruency axiom

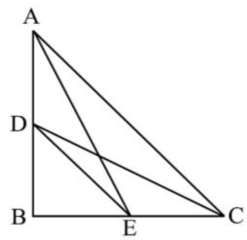
$\Delta ABC \cong \Delta PQR$ **Hence Proved.**

Q. 10. $\Delta\, ABC$ **is a right angled triangle in which** $\angle B = 90°$**.** D **and** E **are any point on** AB **and** BC **respectively. Prove that**

$$AE^2 + CD^2 = AC^2 + DE^2.$$

[CBSE, Term 1, 2016]

Ans. In $\Delta\, ABC$, $\angle B = 60°$ and D, E are points on AB, BC respectively.



To prove:

$$AC^2 + DE^2 = AE^2 + CD^2$$

In $\Delta\, ABC$ by using Pythagoras theorem,

$$AC^2 = AB^2 + BC^2 \quad \text{...(i)}$$

In $\Delta\, ABE$ by using Pythagoras theorem

$$AE^2 = AB^2 + BE^2 \quad \text{...(ii)}$$

In $\Delta\, BCD$ by using Pythagoras theorem

$$CD^2 = BD^2 + BC^2 \quad \text{...(iii)}$$

In $\Delta\, DBE$ by using Pythagoras theorem

$$DE^2 = DB^2 + BE^2 \quad \text{...(iv)}$$

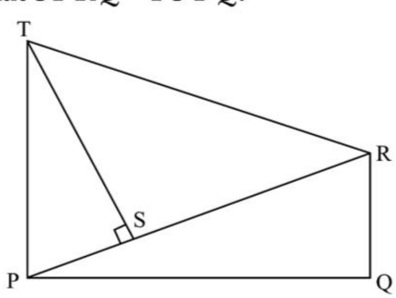
Adding eq. (i) and eq. (iv),

$$AC^2 + DE^2 = AB^2 + BC^2 + BD^2 + BE^2$$
$$= AB^2 + BE^2 + BC^2 + BD^2$$
$$AC^2 + DE^2 = AE^2 + CD^2$$

[From eq. (ii) and eq. (iii)]

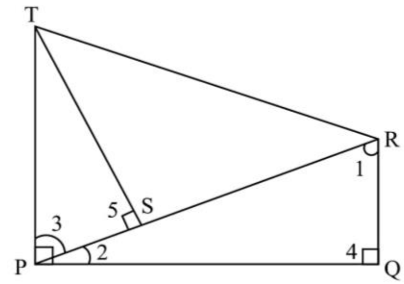
Hence Proved.

Q. 11. In the given figure, RQ **and** TP **are perpendicular to** PQ**, also** $TS \perp PR$ **prove that** $ST \cdot RQ = PS \cdot PQ$**.**



[CBSE, Term 1, 2016]

Ans.



In ΔRPQ,

$$\angle 1 + \angle 2 + \angle 4 = 180°$$
$$\Rightarrow \angle 1 + \angle 2 + 90° = 180°$$
$$\Rightarrow \quad \angle 1 + \angle 2 = 180° - 90°$$
$$\Rightarrow \quad \angle 1 = 90° - \angle 2 \quad \text{...(i)}$$
$$\because \quad TP \perp PQ$$
$$\therefore \quad \angle TPQ = 90°$$
$$\Rightarrow \quad \angle 2 + \angle 3 = 90°$$
$$\Rightarrow \quad \angle 3 = 90° - \angle 2 \quad \text{...(ii)}$$

From eq. (i) and eq. (ii),

$$\angle 1 = \angle 3$$

Now in ΔRQP and ΔPST,

$$\angle 1 = \angle 3 \quad \text{[Proved above]}$$
$$\angle 4 = \angle 5 \quad \text{[Each 90°]}$$

So by AA similarity

$$\Delta RQP \sim \Delta PST$$
$$\frac{ST}{QP} = \frac{PS}{RQ}. \quad \text{[By c.p.c.t.]}$$
$$\Rightarrow \quad ST \cdot RQ = PS \cdot PQ$$

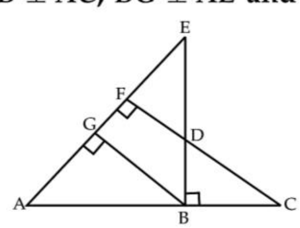
Hence Proved.

Q. 12. In given figure, $EB \perp AC$**,** $BG \perp AE$ **and** $CF \perp AE$**.**

Prove that:

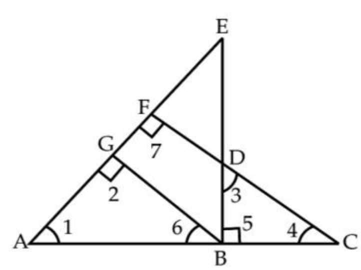
(i) $\Delta ABG \sim \Delta DCB$

(ii) $\dfrac{BC}{BD} = \dfrac{BE}{BA}$



[CBSE, Term 1, 2015]

Ans.



Given: $EB \perp AC$, $BG \perp AE$ and $CF \perp AE$

To prove: (i) $\Delta ABG \sim \Delta DCB$

(ii) $\dfrac{BC}{BD} = \dfrac{BE}{BA}$

Proof: (i) In ΔABG and ΔDCB, $BG \parallel CF$ as corresponding angles 2 and 7 are equal.

$\angle 2 = \angle 5$ [Each 90°]

$\angle 6 = \angle 4$ [Corresponding angles]

$\therefore$ $\Delta ABG \sim \Delta DCB$ Hence Proved. [By AA similarity]

$\therefore$ $\angle 1 = \angle 3$ [c.p.c.t]

(ii) In ΔABE and ΔDBC,

$\angle 1 = \angle 3$ [Proved above]

$\angle ABE = \angle 5$ [Each is 90°, $EB \perp AC$ (Given)]

$\therefore$ $\Delta ABE \sim \Delta DBC$ [By AA similarity]

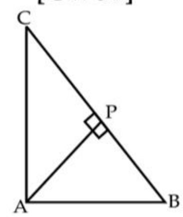
In similar triangles, corresponding sides are proportional

$\therefore$ $\dfrac{BC}{BD} = \dfrac{BE}{BA}$ **Hence Proved.**

Q. 13. In triangle ABC, if $AP \perp BC$ and $AC^2 = BC^2 - AB^2$, then prove that $PA^2 = PB \times CP$.

[CBSE, Term 1, Set 1, 2015]

Ans. $AC^2 = BC^2 - AB^2$ [Given]



$\Rightarrow$ $AC^2 + AB^2 = BC^2$

$\therefore$ $\angle BAC = 90°$ [By converse of Pythagoras' theorem]

$\therefore$ $\Delta APB \sim \Delta CPA$

[If a perpendicular is drawn from the vertex of the right angle of a triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other]

$\Rightarrow$ $\dfrac{AP}{CP} = \dfrac{PB}{PA}$

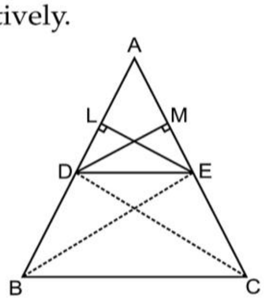
[In similar triangles, corresponding sides are proportional]

$\Rightarrow$ $PA^2 = PB \,.\, CP$ **Hence Proved**

Long Answer Type Questions (4 marks each)

Q. 1. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio. [CBSE OD, Set 1, 2019] [CBSE Term 1, Set 1, 2015]

Ans. Given, a ΔABC in which $DE \parallel BC$ and DE intersect AB and AC at D and E respectively.



To prove: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Construction: Join BE and CD

Draw $EL \perp AB$ and $DM \perp AC$

Proof: we have

area $(\Delta ADE) = \dfrac{1}{2} \times AD \times EL$

and area $(\Delta DBE) = \dfrac{1}{2} \times DB \times EL$

$\left(\because \Delta = \dfrac{1}{2} \times b \times h\right)$

$\therefore$ $\dfrac{\text{area}(\Delta ADE)}{\text{area}(\Delta DBE)} = \dfrac{\frac{1}{2} \times AD \times EL}{\frac{1}{2} \times DB \times EL}$

$= \dfrac{AD}{DB}$ ...(i)

Again, area $(\Delta ADE) =$ area (ΔAED)

$= \dfrac{1}{2} \times AE \times DM$

and area $(\Delta ECD) = \dfrac{1}{2} \times EC \times DM$

$$\therefore \frac{\text{area}(\Delta ADE)}{\text{area}(\Delta ECD)} = \frac{\frac{1}{2}\times AE\times DM}{\frac{1}{2}\times EC\times DM} = \frac{AE}{EC} \quad ...(ii)$$

Now, ΔDBE and ΔECD, being on same base DE and between the same parallels DE and BC, we have

$$\text{area}(\Delta DBE) = \text{area}(\Delta ECD) \quad ...(iii)$$

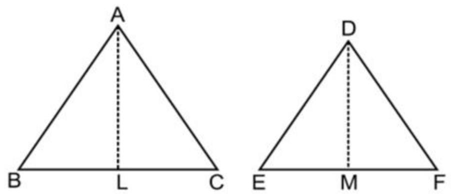
From equations (i), (ii) and (iii), we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$ **Hence Proved.**

Q. 2. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides.
[CBSE OD, Set 2, 2019]
[CBSE Delhi, Set 3, 2019]

Ans. Given, $\Delta ABC \sim \Delta DEF$

To prove: $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$



Construction: Draw $AL \perp BC$ and $DM \perp EF$.

Proof: Since $\Delta ABC \sim \Delta DEF$ it follows that they are equiangular and their sides are proportional.

$\therefore \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$ and

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \quad ...(i)$$

Now, $\text{area}(\Delta ABC) = \frac{1}{2}\times BC\times AL$

$\text{area}(\Delta DEF) = \frac{1}{2}\times EF\times DM$

$$\therefore \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{\frac{1}{2}\times BC\times AL}{\frac{1}{2}\times EF\times DM} = \frac{BC}{EF}\times\frac{AL}{DM}$$

Also, $$\frac{AL}{DM} = \frac{BC}{EF} \quad ...(ii)$$

($\because$ In similar triangles, the ratio of the corresponding sides is the same as the ratio of corresponding altitudes)

Using equations (i) and (ii), we get

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \left(\frac{BC}{EF}\times\frac{BC}{EF}\right) = \frac{BC^2}{EF^2}$$

Similarly, $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{AB^2}{DE^2}$

and $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{AC^2}{DF^2}$

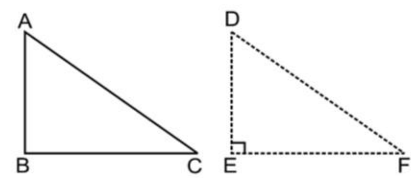
Hence, $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$

Hence Proved.

Q. 3. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite the first side is a right angle.
[CBSE OD, Set 3, 2019]

Ans. Given, ΔABC in which

$$AC^2 = AB^2 + BC^2$$



To prove: $\angle B = 90°$

Construction: Draw a ΔDEF such that

$DE = AB$, $EF = BC$ and $\angle E = 90°$.

Proof: In ΔDEF we have $\angle E = 90°$

So, by Pythagoras theorem, we have

$$DF^2 = DE^2 + EF^2$$

$$\Rightarrow DF^2 = AB^2 + BC^2 \quad ...(i)$$

($\because DE = AB$ and $EF = BC$)

But $AC^2 = AB^2 + BC^2$ ...(ii) (Given)

From equations (i) and (ii), we get

$$AC^2 = DF^2 \Rightarrow AC = DF.$$

Now, in ΔABC and ΔDEF, we have

$AB = DE$, $BC = EF$ and $AC = DF$.

$\therefore \Delta ABC \cong \Delta DEF$.

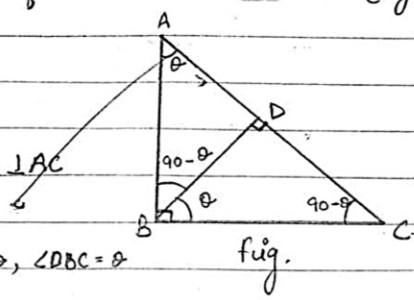
Hence, $\angle B = \angle E = 90°$. **Hence Proved.**

Q. 4. Prove that in a right angle triangle, the square of the hypotenuse is equal the sum of squares of the other two sides. **[CBSE Delhi, Set 1, 2019] [CBSE 2018]**

Topper's Answers

24. To prove: Square of hypotenuse, in a right triangle, is equal to the sum of squares of other two sides. (Pythagoras theorem)

That is, $AC^2 = AB^2 + BC^2$.



Construction: Construct $BD \perp AC$

Name $\angle BAC = \theta$.

Then, $\angle BCA = 90 - \theta$, $\angle ABD = 90 - \theta$, $\angle DBC = \theta$

It is clear that,

$\Delta ABD \sim \Delta ACB \sim \Delta BCD$.

Using $\Delta ABD \sim \Delta ACB$, we get:

$\frac{AB}{AC} = \frac{AD}{AB} \Rightarrow AB^2 = AC \times AD$ —— ①

Similarly, using $\Delta BCD \sim \Delta ACB$, we get:

$\frac{BC}{AC} = \frac{CD}{CB} \Rightarrow \frac{BC}{AC} = \frac{CD}{BC} \Rightarrow BC^2 = AC \times CD$ —— ②

Adding ① and ②, gives:

$AB^2 + BC^2 = AC \times AD + AC \times CD$

$\Rightarrow AB^2 + BC^2 = AC(AD + CD)$ [AD+CD = AC Figure]

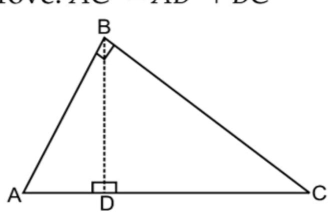
$\Rightarrow AB^2 + BC^2 = AC \times AC$

$\Rightarrow \boxed{AB^2 + BC^2 = AC^2}$

Hence, proved that in a right triangle, sum of square of the other 2 sides is equal to the square of hypotenuse.

Ans. Given, A ΔABC right angled at B.

To prove: $AC^2 = AB^2 + BC^2$



Construction: Draw $BD \perp AC$

Proof: In ΔADB and ΔABC

$\angle A = \angle A$ (common)

$\angle ADB = \angle ABC$ (90° each)

$\therefore \Delta ADB \sim \Delta ABC$ (By AA rule)

So, $\frac{AD}{AB} = \frac{AB}{AC}$ (sides are proportional)

or $AB^2 = AD \cdot AC$ ...(i)

Also, In ΔBDC and ΔABC

$\angle C = \angle C$ (common)

$\angle BDC = \angle ABC$ (90° each)

$\therefore \quad \Delta BDC \sim \Delta ABC$

So, $\quad \dfrac{CD}{BC} = \dfrac{BC}{AC}$

or $\quad BC^2 = CD \cdot AC \quad$ ...(ii)

Adding equation (i) and (ii), we get

$AB^2 + BC^2 = AD \cdot AC + CD \cdot AC$

$= AC\,(AD + CD)$

$= AC \times AC$

$= AC^2$

or $\quad AC^2 = AB^2 + BC^2$

Hence Proved.

Q. 5. In an equilateral ΔABC, D is a point on side BC such that $BD = \dfrac{1}{3}BC$. Prove that $9(AD)^2 = 7(AB)^2$. **[CBSE, 2018]**

Ans.

Topper's Answers

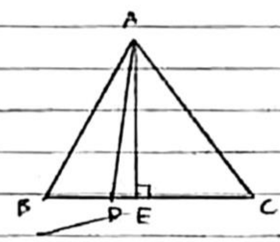
25) Given: ΔABC is equilateral.

$\rightarrow AB = BC = CA$, $\angle A = \angle B = \angle C = 60°$

D is a point on BC such that $BD = \frac{1}{3} BC$.

To prove: $9(AD)^2 = 7(AB)^2$.

Construction: Draw $AE \perp BC$.



Proof: Let $BD = x$.

$\Rightarrow BC = 3x = AB = AC$ [$\because \Delta ABC$ is equilateral] [Given $BD = \frac{1}{3}BC$].

Also, we know that $BE = \frac{1}{2}BC$ [Altitude in equilateral Δ bisects base].

As $\angle AEB = 90°$,

In ~~triangle~~ ΔABE, by Pythagoras Theorem,

$BE^2 + AE^2 = AB^2 \quad \rightarrow AB^2 = 9x^2 \rightarrow$ ⓐ.

$\left(\frac{3x}{2}\right)^2 + AE^2 = (3x)^2$.

$\frac{9x^2}{4} + AE^2 = 9x^2 \quad \rightarrow AE^2 = \frac{3 \times 9x^2}{4} \rightarrow$ ①

Now, in ΔADE, $\angle E = 90°$. $\quad DE = BE - BD$

By Pythagoras Theorem, $\quad = \frac{3x}{2} - x$.

$DE^2 + AE^2 = AD^2$

$\left(\frac{3x}{2} - x\right)^2 + \frac{27x^2}{4} = AD^2$ [From ①].

$\left(\frac{x}{2}\right)^2 + \frac{27x^2}{4} = AD^2$.

$\frac{x^2 + 27x^2}{4} = AD^2$

$\rightarrow AD^2 = \frac{28x^2}{4} = 7x^2 \rightarrow$ ⓑ.

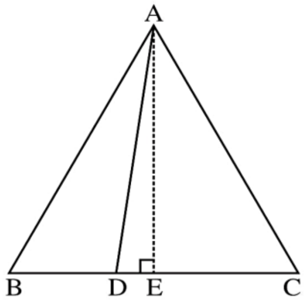
From ⓐ and ⓑ,

$AB^2 = 9x^2, \quad AD^2 = 7x^2$.

$7AB^2 = 63x^2, \quad 9AD^2 = 63x^2$.

$\Rightarrow 7AB^2 = 9AD^2$.

hence proved.



Given, ABC is an equilateral triangle and D is a point on BC such that $BD = \frac{1}{3}BC$.

To prove:

$$9AD^2 = 7AB^2$$

Construction: Draw $AE \perp BC$

Proof: $BD = \frac{1}{3}BC$ ...(i) (Given)

and $AE \perp BC$

We know that perpendicular from a vertex of equilateral triangle to the base divides base in two equal parts.

$\therefore$ $BE = EC = \frac{1}{2}BC$ ...(ii)

In ΔAED,

$$AD^2 = AE^2 + DE^2$$

(Pythagoras theorem)

or $AE^2 = AD^2 - DE^2$ ...(iii)

Similarly, In ΔAEB,

$$AB^2 = AE^2 + BE^2$$

$$= AD^2 - DE^2 + \left(\frac{1}{2}BC\right)^2$$

[from equations (ii) and (iii)]

$$= AD^2 - (BE - BD)^2 + \frac{1}{4}BC^2$$

$$= AD^2 - BE^2 - BD^2 + 2\cdot BE\cdot BD + \frac{1}{4}BC^2$$

$$= AD^2 - \left(\frac{1}{2}BC\right)^2 - \left(\frac{1}{3}BC\right)^2 + 2.\frac{1}{2}BC.\frac{1}{3}BC + \frac{1}{4}BC^2$$

[From equations (i) and (ii)]

$\Rightarrow$ $AB^2 = AD^2 - \frac{1}{9}BC^2 + \frac{1}{3}BC^2$

$\Rightarrow$ $AB^2 = AD^2 + \frac{2}{9}BC^2$

$\Rightarrow$ $AB^2 = AD^2 + \frac{2}{9}AB^2$ ($\because BC = AB$)

$\Rightarrow AB^2 - \frac{2}{9}AB^2 = AD^2$

$\Rightarrow$ $\frac{7}{9}AB^2 = AD^2$

$\Rightarrow$ $7AB^2 = 9AD^2$

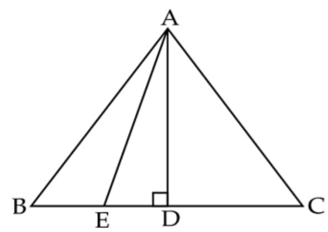
or $9(AD)^2 = 7(AB)^2$ **Hence Proved.**

Q. 6. In an equilateral ΔABC, E is any point on BC such that $BE = \frac{1}{4}BC$. Prove that $16\,AE^2 = 13\,AB^2$.

[CBSE Term 1, 2016]

Ans. Given $BE = \frac{1}{4}BC$

Draw $AD \perp BC$.



In ΔAED by pythagoras theorem,

$$AE^2 = AD^2 + DE^2 \quad ...(i)$$

In ΔADB,

$$AB^2 = AD^2 + BD^2$$

$\Rightarrow$ $AB^2 = AE^2 - DE^2 + BD^2$ [From (i)]

$$= AE^2 - DE^2 + (BE + DE)^2$$

$\Rightarrow AB^2 = AE^2 - DE^2 + BE^2 + DE^2 + 2BE.DE$

$\Rightarrow AB^2 = AE^2 + BE^2 + 2BE.DE$

$\Rightarrow AB^2 = AE^2 + \left(\frac{BC}{4}\right)^2 + 2\frac{BC}{4}.(BD - BE)$

$\Rightarrow AB^2 = AE^2 + \frac{BC^2}{16} + \frac{BC}{2}\left(\frac{BC}{2} - \frac{BC}{4}\right)$

$\Rightarrow AB^2 = AE^2 + \frac{AB^2}{16} + \frac{AB}{2}\left[\frac{2AB - AB}{4}\right]$

$\Rightarrow AB^2 = AE^2 + \frac{AB^2}{16} + \frac{AB}{2} \times \frac{AB}{4}$

$\Rightarrow$ $AB^2 - \frac{AB^2}{16} - \frac{AB^2}{8} = AE^2$

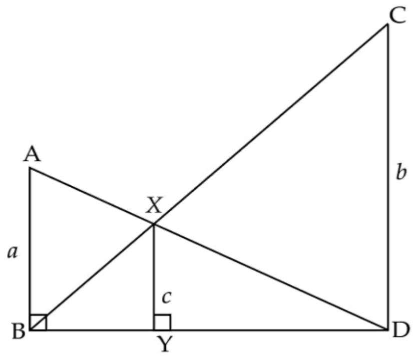
$\Rightarrow$ $\frac{16AB^2 - AB^2 - 2AB^2}{16} = AE^2$

$\Rightarrow \quad 16AB^2 - 3AB^2 = 16AE^2$

$\Rightarrow \quad 13AB^2 = 16AE^2$

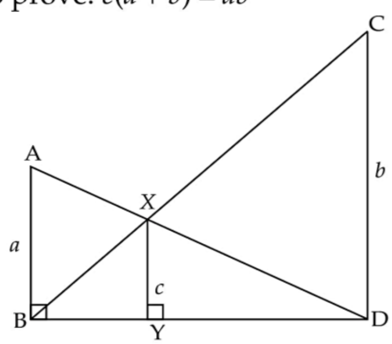
Hence Proved.

Q. 7. In the figure, if $\angle ABD = \angle XYD = \angle CDB = 90°$. $AB = a$, $XY = c$ and $CD = b$, then prove that $c(a + b) = ab$.



[CBSE Term 1, 2016]

Ans. To prove: $c(a + b) = ab$



In ΔABD & ΔDXY,

$\angle B = \angle XYD$ [Each 90°]

$\angle XDY = \angle ADB$ [Common]

So by AA similarity,

$$\Delta DAB \sim \Delta DXY$$

$$\therefore \quad \frac{DY}{DB} = \frac{XY}{AB}$$

$$\Rightarrow \quad DY = \frac{c}{a}(BD) \quad \text{...(i)}$$

In ΔBCD & ΔBYX,

$\angle XYB = \angle D$ [Each 90°]

$\angle CBD = \angle XBY$ [Common]

So by AA similarity,

$$\Delta BYX \sim \Delta BDC$$

$$\therefore \quad \frac{BY}{BD} = \frac{XY}{CD}$$

$$\Rightarrow \quad BY = \frac{c}{b}(BD) \quad \text{...(ii)}$$

Adding equation (i) and equation (ii),

$$DY + BY = \frac{c}{a}(BD) + \frac{c}{b}(BD)$$

$$\Rightarrow \quad BD = BD\left[\frac{c}{a} + \frac{c}{b}\right]$$

$$\Rightarrow \quad \frac{BD}{BD} = \left[\frac{cb + ca}{ab}\right]$$

$$\Rightarrow \quad 1 = \frac{c(a+b)}{ab}$$

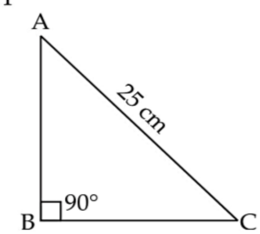
$\Rightarrow \quad c(a + b) = ab$ **Hence Proved.**

Q. 8. The perimeter of a right triangle is 60 cm. Its hypotenuse is 25 cm. Find the area of the triangle.

[CBSE Delhi, Term 2, Set 2, 2016]

Ans. Given, the perimeter of right triangle $= 60$ cm

and hypotenuse $= 25$ cm



$\therefore \quad AB + BC + CA = 60$ cm

$\Rightarrow \quad AB + BC + 25 = 60$

$\therefore \quad AB + BC = 35 \quad$ ...(i)

Now, by pythagoras theorem,

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow \quad (25)^2 = (AB)^2 + (BC)^2$$

$$\therefore \quad AB^2 + BC^2 = 625 \quad \text{...(ii)}$$

We know that, $(a + b)^2 = a^2 + b^2 + 2ab$

then, $\quad (AB + BC)^2 = (AB)^2 + (BC)^2 + 2AB \cdot BC$

$$\Rightarrow \quad (35)^2 = 625 + 2\,AB \cdot BC$$

$$\Rightarrow \quad 2AB \cdot BC = 1225 - 625$$

$$\Rightarrow \quad 2AB \cdot BC = 600$$

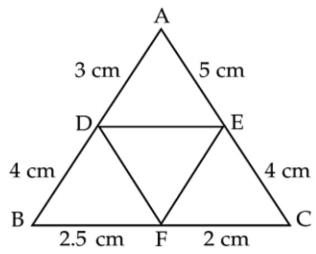
$$\therefore \quad AB \cdot BC = 300$$

$$\therefore \quad \text{Area of } \Delta\, ABC = \frac{1}{2} \times AB \times BC$$

$$= \frac{1}{2} \times 300$$

$$= 150 \text{ cm}^2$$

Q. 9. In the given figure, $AD = 3$ cm, $AE = 5$ cm, $BD = 4$ cm, $CE = 4$ cm, $CF = 2$ cm, $BF = 2.5$ cm, then find the pair of parallel lines and hence their lengths.



[CBSE Term 1, Set 1, 2015]

Ans. $\frac{EC}{EA} = \frac{4}{5}$ and $\frac{CF}{FB} = \frac{2}{2.5} = \frac{4}{5}$

$\Rightarrow \quad \frac{EC}{EA} = \frac{CF}{FB}$

In ΔABC, $\quad EF \parallel AB$

[Converse of Thales' theorem]

Also, $\quad \frac{CE}{CA} = \frac{4}{4+5} = \frac{4}{9} \quad$ ...(i)

$\frac{CF}{CB} = \frac{2}{2+2.5} = \frac{2}{4.5} = \frac{4}{9}$

$\Rightarrow \quad \frac{EC}{EA} = \frac{CF}{CB}$

$\angle ECF = \angle ACB$ [Common]

$\therefore \quad \Delta CFE \sim \Delta CBA$ [SAS similarity]

$\Rightarrow \quad \frac{EF}{AB} = \frac{CE}{CA}$

[In similar Δ's, corresponding sides are proportional]

$\Rightarrow \quad \frac{EF}{7} = \frac{4}{9} \quad [\because \; AB = 3 + 4 = 7 \text{ cm}]$

$\therefore \quad EF = \frac{28}{9}$ cm and $AB = 7$ cm