ESTD: 2001

AEP

STUDY CIRCLE

UNIT OF ACCENTS EDUCATIONAL PROMOTERS

CBSE-XII

E - POTENTIAL & CAPACITANCE

IIT-JEE

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NEET

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E - POTENTIAL & CAPACITANCE

Q AND A (STRICTLY BASED ON CBSE PATTERN)

HELPLINE:
+91-9939586130 ; +91-773965050

email:
aepstudycircle@gmail.com

Q AND ANSWERS

basic concepts

E - POTENTIAL & CAPACITANCE

1. Electric Potential

The electric potential is the physical quantity which determines the direction of charge flow between two bodies when brought in contact. The positive charge always flows from a body at higher potential to that at lower potential.

Definition: The electric potential at any point in an electric field is defined as the work done in bringing a unit positive test charge from infinity to that point without acceleration.

If W is the work done in bringing infinitesimal positive test charge q_0 from infinity to given point, then electric potential

$$V = \frac{W}{q_0}$$

Electric potential at any point is also defined as the negative line integral of electric field from infinity to given point (independent of path followed).

i.e., $$V = -\int_{\infty}^{r} \vec{E}.\vec{dl}$$

The unit of electric potential is joule/coulomb or volt and its dimensional formula is $[ML^2 T^{-3} A^{-1}]$.

2. Potential Difference

The potential difference between two points in an electric field is defined as the work done in bringing unit positive charge from one point to another.

3. Formulae for Electric Potential

(*a*) Due to a point charge q at a point distant r is $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

(*b*) Due to a short electric dipole at a distance r from its centre

(*i*) at its axis is $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$

(*ii*) at its equatorial position, $V = 0$

(*iii*) at a general point having polar coordinates (r, θ) with respect to centre of dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

(*c*) due to a system of charges is

$$V = V_1 + V_2 + ... + V_N = \sum_{i=1}^{N} \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + ... + \frac{q_N}{r_N}\right]$$

4. Equipotential Surface

An equipotential surface is the surface having the same potential at each point. The surface of a charged conductor in equilibrium is a equipotential surface.

5. Electric Potential Energy of a System of Point Charges

If q_1 and q_2 are point charges at separation r_{12}, then electric potential energy $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$.

If there are n point charges $q_1, q_2,.... q_n$ in system at separation r_{ij} between i^{th} and j^{th} charge $(i=1, 2,..., n, j=2, 3,...n)$ then potential energy of system

$$U = \frac{1}{4\pi\varepsilon_0}\sum_i \sum_{j>i} \frac{q_i q_j}{r_{ij}} \qquad (i=1, 2,..., n, j=2, 3,...n)$$

6. Electric Potential Energy of a Dipole in Uniform Electric Field

Potential energy of dipole in uniform electric field is

$$U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

Work done in rotating the dipole in uniform electric field from inclination θ_1 to θ_2

$$W = U_2 - U_1 = pE(\cos\theta_1 - \cos\theta_2)$$

If dipole is initially in stable equilibrium position $(\theta_1 = 0)$ and finally its inclination is θ, then

$$W = pE(1 - \cos\theta)$$

7. Conductors and Insulators

Conductors are those substances which contain free charge carriers and so allow easy flow of current.

Insulators are those substances which contain practically no free charge carriers and do not allow the flow of current.

8. Free and Bound Charges Inside a Conductor

The electrons are free charge carriers inside a metallic conductor while positive ions fixed in lattice are bound charge carriers.

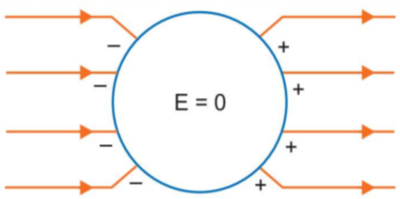
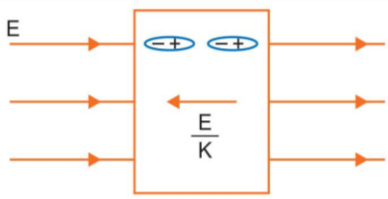
9. Dielectrics and Electric Polarisation

The insulators are often referred as dielectrics. Each dielectric is formed of atoms/molecules. In some dielectrics the positive and negative charge centres coincide, such dielectrics are said to be non-polar dielectrics. While in some other dielectrics the centres of positive and negative charges do not coincide, such dielectrics have permanent electric dipole moment and said to be polar dielectrics. The example of polar dielectric is water, while example of non-polar dielectric is carbon dioxide (CO_2).

When a dielectric is placed in an external electric field, the centres of positive and negative dipoles get separated (in non-polar dielectrics) or get farther away (in polar dielectrics), so that molecules of dielectric gain a permanent electric dipole moment; this process is called polarisation and the dipole is said to be polarised.

The induced dipole moment developed per unit volume in an electric field is called polarisation density. Numerically it is equal to surface charge density induced at the faces which are perpendicular to the direction of applied electric field.

10. The Behaviour of a Conductor and Dielectric in the Presence of External Electric Field.

Conductor	Dielectric
	 where K is dielectric constant
1. No electric field lines travel inside conductor.	1. Alignment of atoms takes place due to electric field.

2. Electric field inside a conductor is zero.	2. This results in a small electric field inside dielectric in opposite direction. Net field inside the dielectric is $\frac{E}{K}$.

11. Capacitor and Capacitance

A capacitor contains two oppositely charged metallic conductors at a finite separation. It is a device by which capacity of storing charge may be varied simply by changing separation and/or medium between the conductors.

The capacitance of a capacitor is defined as the ratio of magnitude of charge (Q) on either plate and potential difference (V) across the plate, *i.e.*,

$$C = \frac{Q}{V}$$

The unit of capacitance is coulomb/volt or farad (F).

12. Combination of Capacitors in Series and Parallel

(a) Series Combination: When capacitors are connected in series, then net capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Net charge $Q = q_1 = q_2 = q_3$ (remain same)

Net potential difference $V = V_1 + V_2 + V_3$

(b) Parallel Combination: When capacitors are connected in parallel, then the net capacitance

$$C = C_1 + C_2 + C_3$$

In parallel combination net charge $Q = q_1 + q_2 + q_3$

Net potential difference $V = V_1 = V_2 = V_3$ (remain same)

13. Capacitance of Parallel Plate Capacitor

A parallel plate capacitor consists of two parallel metallic plates separated by a dielectric. The capacitance is given by

$$C = \frac{K\varepsilon_0 A}{d},$$

where K is dielectric constant, A = area of each plate and d = separation between the plates.

Special Cases:

(*i*) When there is no medium between the plates, then $K=1$, so

$$C_{vacuum} = \frac{\varepsilon_0 A}{d} = C_0$$

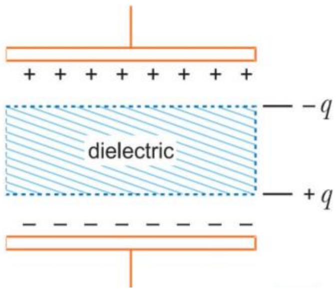
(*ii*) When space between the plates is partly filled with a medium of thickness t and dielectric constant K, then capacitance

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

Clearly, $C > C_0$, *i.e.*, on introduction of a dielectric slab between the plates of a parallel plate capacitor, its capacitance increases.

14. Charge Induced on a Dielectric

$q' = -q\left(1 - \frac{1}{K}\right)$ where q is free charge on the capacitor plates.



15. Energy stored in a Charged Capacitor

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$$

This energy resides in the medium between the plates.

The unit is joule (J) .The energy stored per unit volume of a charged capacitor is given by

$$u = \frac{U}{V} = \frac{1}{2}\varepsilon_0 E^2$$

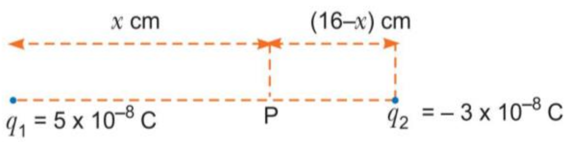
where E is electric field strength. The unit is joule/m^3(J/m^3)

Selected NCERT Textbook Questions

Electric Potential and Potential Energy

Q. 1. Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Ans. Let P be a point on the line joining charges $q_1 = 5 \times 10^{-8}$ C and $q_2 = -3 \times 10^{-8}$ C at a distance x cm from charge q_1.



Its distance from charge q_2 will be $(16 - x)$ cm.

For potential at P

$$V_1 + V_2 = 0 \Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} = 0 \Rightarrow \frac{q_1}{r_1} + \frac{q_2}{r_2} = 0$$

Given, $r_1 = x \text{ cm} = x \times 10^{-2}$ m, $r_2 = (16 - x) \text{ cm} = (16 - x) \times 10^{-2}$ m

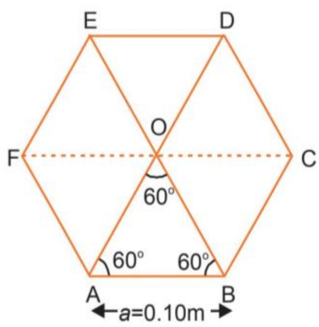
$$\therefore \quad \left[\frac{5 \times 10^{-8}}{x \times 10^{-2}} + \frac{(-3 \times 10^{-8})}{(16 - x) \times 10^{-2}}\right] = 0$$

$$\Rightarrow \frac{5}{x} - \frac{3}{(16 - x)} = 0 \Rightarrow \frac{5}{x} = \frac{3}{(16 - x)}$$

$$\Rightarrow \quad 5(16 - x) = 3x \text{ or } \quad 8x = 80 \quad \text{or} \quad x = \mathbf{10\ cm}$$

Q. 2. A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.

Ans. **Key idea:** The potential due to similar charges is additive.



Let O be the centre of the hexagon.

In triangle OAB all angles are 60°, so

$$OA = OB = AB = a$$

So, in a regular hexagon distance of each corner from centre is equal to the side of the hexagon

$r = OA = OB = OC = OD = OE = OF = a = 10 \text{ cm} = 0.10$ m

The net potential at $O, V = 6 \times \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$.

Here $q = 5\,\mu\text{C} = 5 \times 10^{-6}\text{C}, \; a = 0.10$ m

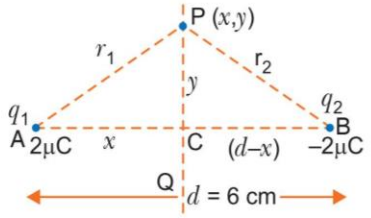
$$\therefore \quad V = 6 \times 9 \times 10^9 \times 5 \times \frac{10^{-6}}{0.10} = \mathbf{2.7 \times 10^6 \text{ volt}}$$

Q. 3. Two charges 2 μC and – 2 μC are placed at points A and B 6 cm apart.

(*a*) Identify an equipotential surface of the system.

(*b*) What is the direction of the electric field at every point on this surface?

Ans. (*a*) Let $P(x, y)$ be a point on zero potential surface. Let A (location of charge $q = 2\ \mu C$) be origin of coordinate system.



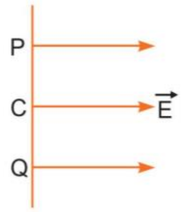
Distance $r_1 = \sqrt{x^2 + y^2}$, Distance $r_2 = \sqrt{(d-x)^2 + y^2}$

where $d = 6$ cm $= 6 \times 10^{-2}$ m.

Potential at P due to charges $q_1 = +2\ \mu C$ and $q_2 = -2\ \mu C$ is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} = 0 \Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{2\times10^{-6}}{\sqrt{x^2+y^2}} + \frac{1}{4\pi\varepsilon_0}\frac{(-2\times10^{-6})}{\sqrt{(d-x)^2+y^2}} = 0$$

or $$\frac{1}{\sqrt{x^2+y^2}} = \frac{1}{\sqrt{(d-x)^2+y^2}} \Rightarrow x^2+y^2 = (d-x)^2+y^2 \Rightarrow x = \frac{d}{2} = \mathbf{3\ cm}$$



So, plane passing through mid point of line joining A and B has zero potential everywhere.

(*b*) The direction of electric field is normal to surface PCQ everywhere as shown in figure.

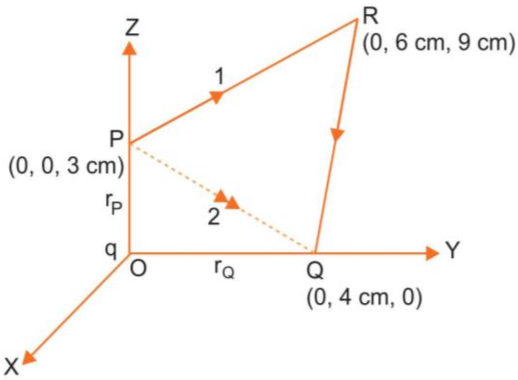
Q. 4. A charge 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point *P*(0, 0, 3 cm) to a point Q(0, 4 cm, 0) via a point *R*(0, 6 cm, 9 cm).

Ans. In electric field the work done in carrying a charge depends only on initial and final points and is independent of path.

The points P, Q, R are shown in figure. Charge $q = 8$ mC$=8\times10^{-3}$ C is located at the origin O. Clearly,

$OP = r_P = 3$ cm $= 3\times10^{-2}$ m

$OQ = r_Q = 4$ cm $= 4\times10^{-2}$ m



As electrostatic field is conservative; so the work done is independent of path. Hence, work done along path PRQ (path 1) is same as work done along path PQ directly (path 2). By work-energy theorem, the work done is simply the change in electrostatic potential energy at two positions of charge q_0(say)$=-2\times10^{-9}$ C

Work, W=Potential energy of system when charge q_0 is at Q –Potential energy of system when charge q_0 is at P

$$= \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r_Q} - \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r_P} = \frac{1}{4\pi\varepsilon_0}qq_0\left(\frac{1}{r_Q} - \frac{1}{r_P}\right)$$

Substituting given values, we get

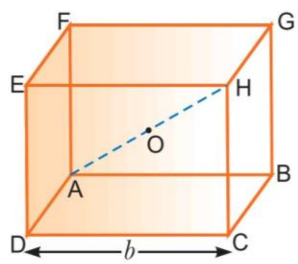
$$W = 9\times10^9\times(8\times10^{-3})\times(-2\times10^{-9})\left[\frac{1}{4\times10^{-2}} - \frac{1}{3\times10^{-2}}\right] = -144\times10^{-1}\left(\frac{3-4}{12}\right) = \mathbf{1.2\ joule.}$$

Q. 5. A cube of side *b* has a charge *q* at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Ans. O is the centre of cube $ABCDEFGH$. Charge q is placed at each of eight corners of the cube.

Electric Potential: Side of cube $= b$

Length of each diagonal $= \sqrt{b^2 + b^2 + b^2} = \sqrt{3}\,b$



Distance of each corner from centre O = half the diagonal $= \dfrac{\sqrt{3}\,b}{2}$

Potential at O due to charge at each corner $= \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{(\sqrt{3}\,b/2)} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2q}{\sqrt{3}\,b}$

$\therefore$ Net potential at O due to all 8 charges at corners of the cube

$$V = 8\times\frac{1}{4\pi\varepsilon_0}\frac{2q}{\sqrt{3}\,b} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{16q}{\sqrt{3}\,b}$$

Electric Field: The electric field at O due to charges at all corners of the cube is zero, since, electric fields due to charges at opposite corners such as A and H, G and D, B and E, F and C are equal and opposite.

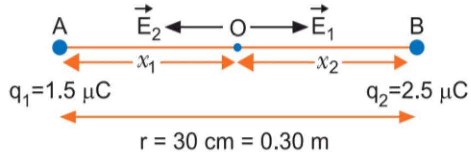
Q. 6. Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field

(a) at the mid-point of the line joining the two charges, and

(b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.

Ans. The potential due to similar charges is additive while electric field at a point due to individual charges are added vectorially.

(a) The electric potential at mid point O,



$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{x_1} + \frac{q_2}{x_2}\right)$$

Here, $x_1 = x_2 = \dfrac{0{\cdot}30}{2} = 0{\cdot}15\,\text{m}$

$$V = 9\times10^9\left[\frac{1{\cdot}5\times10^{-6}}{0{\cdot}15} + \frac{2{\cdot}5\times10^{-6}}{0{\cdot}15}\right] = 9\times10^9\left[10\times10^{-6} + \frac{50}{3}\times10^{-6}\right]$$

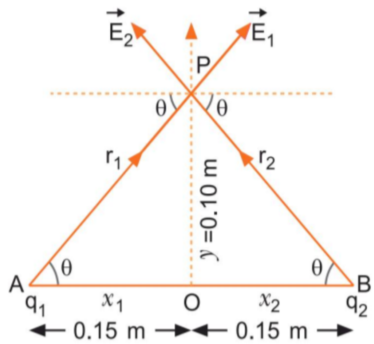
$$= 9\times10^9\times\frac{80}{3}\times10^{-6} = \mathbf{2.4\times10^5\ V}$$

Electric field at O due to q_1 is towards $\overrightarrow{AB}$ and that due to q_2 is towards $\overrightarrow{BO}$. The net electric field at mid point O is

$$E = E_2 - E_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{x_2^2} - \frac{q_1}{x_1^2}\right) = 9\times10^9\left[\frac{2{\cdot}5\times10^{-6}}{(0{\cdot}15)^2} - \frac{1{\cdot}5\times10^{-6}}{(0{\cdot}15)^2}\right]$$

$= \mathbf{4.0 \times 10^5\ N/C}$ directed from q_2 to q_1.

(b) Let P be a point at distance 10 cm = 0.10 m from O, in a plane normal to line AB.



$$AP = BP = \sqrt{(0{\cdot}15)^2 + (0{\cdot}10)^2} = 0{\cdot}18\,\text{m}$$

Electric potential at P.

$$V_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{(AP)} + \frac{q_2}{(BP)}\right]$$

$$= 9\times10^9\left[\frac{1{\cdot}5\times10^{-6}}{0{\cdot}18} + \frac{2{\cdot}5\times10^{-6}}{0{\cdot}18}\right]$$

$$= \frac{9\times10^9\times4.0\times10^{-6}}{0.18} = \mathbf{2.0\times10^5\ V}$$

Electric field at P due to q_1,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\text{ along }\overrightarrow{AP} = 9\times10^9\times\frac{1.5\times10^{-6}}{(0.18)^2}\text{ along }\overrightarrow{AP}$$

Electric field at P due to q_2

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\text{ along }\overrightarrow{BP} = 9\times10^9\times\frac{2.5\times10^{-6}}{(0.18)^2}\text{ along }\overrightarrow{BP}$$

Resolving E_1 and E_2 along and normal to AB.

Net electric field along $\overrightarrow{BA}$, $E_x = E_2\cos\theta - E_1\cos\theta$

$$= (E_2 - E_1)\cos\theta = (E_2 - E_1)\frac{x_1}{r_1}$$

$$= 9\times10^9\left[\frac{2\cdot5\times10^{-6} - 1\cdot5\times10^{-6}}{(0\cdot18)^2}\right]\times\left(\frac{0\cdot15}{0\cdot18}\right)$$

$$= \frac{9\times10^9\times1.0\times10^{-6}}{(0.18)^2}\times\left(\frac{0.15}{0.18}\right) = 2.3\times10^5 \text{ N/C}$$

Net electric field normal to AB, $E_y = (E_2 + E_1)\sin\theta$

$$= 9\times10^9\left[\frac{2\cdot5\times10^{-6} + 1\cdot5\times10^{-6}}{(0\cdot18)^2}\right]\times\frac{0\cdot10}{0\cdot18}$$

$$= 9\times10^9\times\frac{4\cdot0\times10^{-6}}{(0\cdot18)^2}\times\frac{10}{18} = 6.2\times10^5 \text{ N/C}$$

Net electric field $E = \sqrt{E_x^2 + E_y^2} = \sqrt{(2\cdot3\times10^5)^2 + (6\cdot2\times10^5)^2} = \mathbf{6.6\times10^5\ N/C}$

If α is the angle made by resultant field with AB then

$$\tan\alpha = \frac{E_y}{E_x} = \frac{6.2\times10^5}{2.3\times10^5} = 2.69$$

$\Rightarrow$ $\alpha = \tan^{-1}(2.69) = \mathbf{69.6°}$

That resultant electric field at point P is 6.6×10^5 N/C making an angle 69.6° to the line joining the charge 2.5 μC to 1.5 μC.

Q. 7. In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å.

(*a*) Estimate the potential energy of the system in eV, taking the zero of potential energy at infinite separation of electron from proton.

(*b*) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (*a*)?

(*c*) What are the answers to (*a*) and (*b*) above if the zero of potential energy is taken at 1.06 Å separation? **[*HOTS*]**

Ans. (*a*) Charge on proton $q_1 = +1.6\times10^{-19}$ C

Charge on electron $q_2 = -1.6\times10^{-19}$ C

Separation $r = 0.53$ Å $= 0.53\times10^{-10}$ m

Potential energy of system $U = U_{at\ r} - U_{at\ \infty}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} - 0$$

$$= 9\times10^9\times\frac{(1.6\times10^{-19})(-1.6\times10^{-19})}{0.53\times10^{-10}}$$

$$= -43.47\times10^{-19}\text{ J}$$

As 1 eV $= 1.6\times10^{-19}$ J, we have

$$U = -\frac{43.47\times10^{-19}}{1.6\times10^{-19}}\text{eV} \approx \mathbf{-27.2\ eV}$$

(*b*) Kinetic energy is always positive, so kinetic energy of electron $= \frac{27.2}{2} = 13.6$ eV

Total energy of electron $= -27.2 + 13.6 = -13.6$ eV

Minimum work required to free the electron $= -$ Total energy of bound electron $=$ **13.6 eV**

(*c*) Potential energy at separation, $r_0 = 1.06$ Å is

$$U_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

$$= 9 \times 10^9 \times \frac{(1.6 \times 10^{-19})(-1.6 \times 10^{-19})}{1.06 \times 10^{-10}}$$

$$= -21.73 \times 10^{-19}\text{ J} = -13.6\text{ eV}$$

$\therefore$ Potential energy of system when zero of potential energy is taken at $r_0 = 1.06$ Å

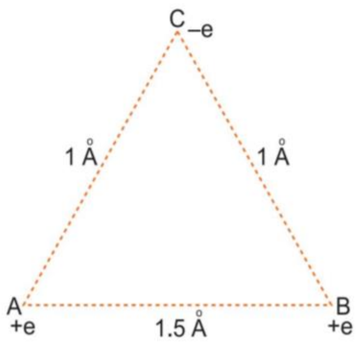
$$U = U(r) - U_0 = -27.2 + 13.6 = \mathbf{-13.6\ eV}$$

Now total energy of hydrogen atom is zero

$\therefore$ Minimum work $= E - U = 0 - (-13.6)$ eV $=$ **13.6 eV**

Q. 8. If one of the two electrons of a H_2 molecule is removed, we get a hydrogen-molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy. ***[HOTS]***

Ans. The choice of zero potential energy is when all charges are initially at infinite distance apart.



The system of charges: 2 protons (each of charge $+e$) and an electron (of charge $-e$) is shown in figure.

The potential energy of system

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{(e.e)}{r_{AB}} + \frac{e(-e)}{r_{AC}} + \frac{e(-e)}{r_{BC}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}e^2\left[\frac{1}{r_{AB}} - \frac{1}{r_{AC}} - \frac{1}{r_{BC}}\right]$$

Given: $r_{AB} = 1.5$ Å $= 1.5 \times 10^{-10}$ m, $r_{AC} = r_{BC} = 1$ Å $= 10^{-10}$ m, $e = 1.6 \times 10^{-19}$C

$$\therefore \quad U = 9 \times 10^9 \times (1.6 \times 10^{-19})^2\left[\frac{1}{1.5 \times 10^{-10}} - \frac{1}{10^{-10}} - \frac{1}{10^{-10}}\right]$$

$$= 9 \times 2.56 \times 10^{-19} \times \left(-\frac{4}{3}\right)$$

$$= \mathbf{-30.72 \times 10^{-19}\ J}$$

Converting it into eV (keeping in mind 1 eV$=1.6\times10^{-19}$J)

$$U = \frac{-30.72 \times 10^{-19}}{1.6 \times 10^{-19}}\text{eV} = \mathbf{-19.2\ eV}$$

Thus, electrostatic potential energy of system

$$U = -30.72 \times 10^{-19}\text{ joule or } -19.2\text{ eV}$$

Q. 9. Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions. ***[HOTS]***

Ans. When conducting spheres are connected by a wire, the potential of each sphere will be the same.

$$i.e., \ V_1 = V_2$$

If q_1 and q_2 are charges on them after connection, then

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{a} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{b}$$

Ratio of charges $\dfrac{q_1}{q_2} = \dfrac{a}{b}$ ...(*i*)

That is, the ratio of charges on two spheres after their electrical contact is the same as the ratio of their radii.

Electric field strengths on the surfaces of two spheres

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a^2}, \qquad E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{b^2}$$

$\therefore$ $$\frac{E_1}{E_2} = \frac{q_1}{q_2}\frac{b^2}{a^2} = \left(\frac{a}{b}\right)\left(\frac{b}{a}\right)^2 \quad \text{[using (}i\text{)]}$$

or $$\boldsymbol{\frac{E_1}{E_2} = \frac{b}{a}}$$

Thus, the ratio of electric field strengths on their surfaces is equal to the inverse ratio of their radii.

If σ_1 and σ_2 are the surface charge densities of two spheres, then $q_1 = 4\pi a^2\sigma_1$ and $q_2 = 4\pi b^2\sigma_2$

From (*i*), $\frac{4\pi a^2\sigma_1}{4\pi b^2\sigma_2} = \frac{a}{b} \qquad \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{b}{a}$

A flat portion is equivalent to a spherical surface of large radius and a pointed portion that of small radius.

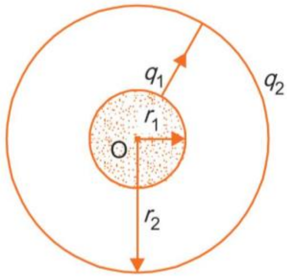
$\therefore$ $$\frac{\sigma_{\text{flat}}}{\sigma_{\text{pointed}}} = \frac{\text{small}}{\text{large}}$$

Obviously, charge density on flatter parts is very small and on sharp and pointed ends it is very large.

Q. 10. A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire), no matter, what the charge q_2 on the shell is.

Ans. The potential of inner sphere (due to its own charge and due to charge on shell) is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$



Potential of shell, $$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2 + q_1}{r_2}$$

$\therefore$ Potential difference, $$V = V_1 - V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} - \frac{q_1}{r_2}\right)$$

This is independent of q_2. If q_1 is positive, the potential of inner sphere is always greater than the potential of shell; so if both inner sphere and shell are connected by a wire, the charge will necessarily flow from sphere to shell.

Capacitors

Q. 11. A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1pF = 10^{-12} F). What will be the capacitance if the distance between the plates is reduced by half and the space between them is filled with a substance of dielectric constant 6?

Ans. Capacitance of parallel plate air capacitor,

$$C = \frac{\varepsilon_0 A}{d} = 8\text{ pF} \qquad \ldots(1)$$

When separation between the plates becomes $\frac{d}{2}$ and the space between the plates is filled with dielectric ($K = 6$), then new capacitance

$$C' = \frac{K\varepsilon_0 A}{d/2} = \frac{2K\varepsilon_0 A}{d} \qquad \ldots(2)$$

$\Rightarrow \quad \frac{C'}{C} = 2K$

or $\quad C' = 2KC = 2\times6\times8$ pF = **96 pF**

Q. 12. Three capacitors each of capacitance 9 pF are connected in series:
(a) What is the total capacitance of the combination?
(b) What is the potential difference across each capacitor if the combination is connected to 120 V supply?

Ans. (a) Given $C_1 = C_2 = C_3 = 9$ pF

When capacitors are connected in series, the equivalent capacitance C_S is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$$

$C_S = \mathbf{3\ pF}$

(b) In series, charge on each capacitor remains the same, so charge on each capacitor

$$q = C_S V = (3 \times 10^{-12} \text{ F}) \times (120 \text{ V}) = 3.6 \times 10^{-10} \text{ coulomb}$$

Potential difference across each capacitor, $V = \frac{q}{C_1} = \frac{3.6\times10^{-10}}{9\times10^{-12}} = \mathbf{40\ V}$

Q. 13. Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.
(a) What is the total capacitance of the combination?
(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Ans. $C_1 = 2$ pF, $C_2 = 3$ pF, $C_3 = 4$ pF

(a) Total capacitance when connected in parallel, $C_p = C_1 + C_2 + C_3 = 2 + 3 + 4 = \mathbf{9\ pF}$

(b) In parallel, the potential difference across each capacitor remains the same, *i.e.*, $V = 100$ V.

Charge on $C_1 = 2$ pF is $q_1 = C_1V = 2 \times 10^{-12} \times 100 = \mathbf{2 \times 10^{-10}\ C}$

Charge on $C_2 = 3$ pF, $q_2 = C_2V = 3 \times 10^{-12} \times 100 = \mathbf{3 \times 10^{-10}\ C}$

Charge on $C_3 = 4$ pF, $q_3 = C_3V = 4 \times 10^{-12} \times 100 = \mathbf{4 \times 10^{-10}\ C}$

Q. 14. In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m² and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor? [*HOTS*]

Ans. Capacitance of parallel plate air capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

Given $A = 6 \times 10^{-3}$ m², $d = 3$ mm $= 3 \times 10^{-3}$ m, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m.

$$\therefore \quad C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}} = \mathbf{17.7\times10^{-12}\ F}$$

Charge on each plate of capacitor,

$$Q = CV = 17.7 \times 10^{-12} \times 100 = 1.77 \times 10^{-9} \text{ coulomb} = \mathbf{1.77\ nC}$$

Q. 15. Explain what would happen if in the capacitor a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates given in Q 14 above.
(a) While the voltage supply remained connected.
(b) After the supply was disconnected.

Ans. Capacitance of parallel plate air capacitor,

$$C = \frac{\varepsilon_0 A}{d} = 17.7 \times 10^{-12} \text{ F} = 17.7 \text{ pF}$$

When dielectric is introduced between the plates, the new capacitance

$$C' = \frac{K\varepsilon_0 A}{d} = 6 \times 17.7 \text{ pF} = 106.2 \text{ pF}.$$

(*a*) When voltage supply remains connected, voltage across plates remains 100 V and so charge becomes 6-times = 6 × 1.77 nC = **10.62 nC.**

(*b*) When voltage supply was disconnected, the charge on each plate remains the same $q = 1.77$ nC.

As capacitance is increased to K times, the potential difference $V = \frac{q}{C}$ must decrease to $\frac{1}{K}$ times.

New potential difference $V' = \frac{V}{K} = \frac{100}{6} = \mathbf{16.6 \text{ volt}}$

Q. 16. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?

Ans. Electrostatic energy stored in capacitor, $U = \frac{1}{2}CV^2$

Here $C = 12 \text{ pF} = 12 \times 10^{-12}$ F, $V = 50$ V

$$\therefore \quad U = \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2 = \mathbf{1.5 \times 10^{-8} \text{ J}}$$

Q. 17. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to the another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Ans. Given, $C_1 = 600 \text{ pF} = 600 \times 10^{-12}$ F, $V_1 = 200$ V

Initial energy stored, $U_{initial} = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times 600 \times 10^{-12} \times (200)^2 = 12 \times 10^{-6}$ J

When another uncharged capacitor C_2=600 pF is connected across capacitor C_1 then common potential difference

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + 0}{C_1 + C_2} = \frac{C_1V_1}{C_1 + C_2}$$

$$= \frac{600 \times 10^{-12} \times 200}{(600 + 600) \times 10^{-12}} = 100 \text{ V}$$

$\therefore$ Final electrostatic energy, $U_{\text{final}} = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(600 + 600) \times 10^{-12} \times (100)^2 = 6 \times 10^{-6}$ J

$\therefore$ Energy lost, $\Delta U = U_{\text{initial}} - U_{\text{final}} = 12 \times 10^{-6} - 6 \times 10^{-6} = \mathbf{6 \times 10^{-6} \text{ J}}$

Q. 18. An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him, each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires a minimum number of capacitors. [*HOTS*]

Ans. The potential difference can only be increased by connecting capacitors in series, while capacitance can only be increased by connecting capacitances in parallel.

To acquire the required arrangement let there be m rows, connected in parallel, each row containing n capacitors in series. Then total number of capacitors $N=mn$.

If V is the net potential difference and V_0 the potential difference across each capacitor, then

$$V = nV_0, \text{i.e.}, n = \frac{V}{V_0} = \frac{1 \text{ kV}}{400 \text{ V}} = \frac{1000 \text{ V}}{400 \text{ V}} = 2.5$$

As n cannot be a fraction, we must take $n = 3$. If C_0 is capacitance of each capacitor, the capacitance of a row $= \frac{C_0}{n}$

As m rows are connected in parallel, net capacitance

$$C = \frac{mC_0}{n}$$

Given, C=2 μF and C_0=1μF, n=3

$$\therefore \quad 2\,\mu F = \frac{m \times (1\mu F)}{3} \quad \text{or } m = \frac{2\times 3}{1} = 6$$

Minimum number of capacitors, $N = mn = 3\times 6 = \mathbf{18}$

Q. 19. What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realise from your answer why ordinary capacitors are in the range of μF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

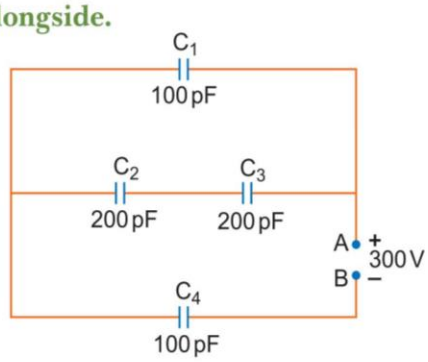
Ans. Capacitance of a parallel plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

$$\text{Area } A = \frac{Cd}{\varepsilon_0} = \frac{2\times(0.5\times 10^{-2})}{8.85\times 10^{-12}} = \mathbf{1.13\times 10^9\,m^2}$$

This is too large. That is why ordinary capacitors are in the range of μF or even less. However, in electrolytic capacitors the separation (d) is very small, so they have capacitances of the order of 0.1 F.

Q. 20. Obtain the equivalent capacitance of the network in figure alongside. For a 300 V supply, determine the charge and voltage across each capacitor.



Ans. Given, $C_1 = C_4 = 100$ pF, $C_2 = C_3 = 200$ pF.

The capacitors C_2 and C_3 are connected in series. Their equivalent capacitance

$$C' = \frac{C_2C_3}{C_2 + C_3} = \frac{200\times 200}{200+200} = 100 \text{ pF}$$

The combination of C_2 and C_3 (*i.e.*, C') is connected in parallel with C_1, therefore, equivalent capacitance of C_1 and C',

$$C'' = C_1 + C' = 100 + 100 = 200 \text{ pF}$$

The capacitance C'' is in series with C_4 hence equivalent capacitance between A and B.

$$C = \frac{C''C_4}{C''+C_4} = \frac{200\times 100}{200+100} = \frac{200}{3}\text{pF} = \mathbf{66{\cdot}7\ pF}$$

$$\text{Total charge, } Q = CV = \left(\frac{200}{3}\times 10^{-12}\text{F}\right)\times(300\text{ V}) = 2\times 10^{-8} \text{ coulomb}$$

As C_4 is connected in series with battery, charge on C_4 is, $Q_4 = \mathbf{2\times 10^{-8}\ C}$

$$\text{Potential difference across } C_4 \text{ is } V_4 = \frac{Q_4}{C_4} = \frac{2\times 10^{-8}C}{100\times 10^{-12}\text{F}} = 200 \text{ V}$$

As C_2 and C_3 have resultant capacitance C' equal to C_1 =100 pF, so the charge Q is equally divided among two branches; charge on C_1 is $Q_1 = \frac{Q}{2} = 1\times 10^{-8}\text{ C} = \mathbf{10^{-8}\ C}$

Charge in branch C_2 and C_3 is also 1×10^{-8} C. As charge in series remains same, so charges on C_2 and C_3 are equal to 1×10^{-8} C.

$$Q_2 = Q_3 = \mathbf{10^{-8}\ C}$$

$$\text{Potential across } C_1 = V_1 = \frac{Q_1}{C_1} = \frac{10^{-8}}{100\times 10^{-12}} = \mathbf{100\ V}$$

$$\text{Potential across, } C_2 = \frac{Q_2}{C_2} = \frac{10^{-8}}{200\times 10^{-12}} = \mathbf{50\ V}$$

$$\text{Potential across, } C_3 = \frac{Q_3}{C_3} = \frac{10^{-8}}{200\times 10^{-12}} = \mathbf{50\ V}$$

Q. 21. The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.

(a) How much electrostatic energy is stored by the capacitor?

(b) View this energy as stored in the electrostatic field between the plates and obtain the energy per unit volume *u*. Hence arrive at a relation between *u* and the magnitude of electric field *E* between the plates. [*HOTS*]

Ans. (a) Given area, $A = 90\ cm^2 = 90 \times 10^{-4}\ m^2$

Separation, $d = 2.5\ mm = 2.5 \times 10^{-3}$ m

Capacitance, $C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 90 \times 10^{-4}}{2.5 \times 10^{-3}} = 31.9 \times 10^{-12} F = 31.9\ pF$

Energy stored,

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 31.9 \times 10^{-12} \times (400)^2 = \mathbf{2.55 \times 10^{-6}\ J}$$

(b) Volume of space between the plates

$$V = Ad = 90 \times 10^{-4} \times 2.5 \times 10^{-3} = 22.5 \times 10^{-6} m^3$$

∴ Energy density or energy per unit volume

$$u = \frac{U}{V} = \frac{2.55 \times 10^{-6}}{22.5 \times 10^{-6}} = \mathbf{0.113\ Jm^{-3}}$$

Expression for energy stored per unit volume

$$u = \frac{U}{V} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2}{Ad} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$

If E is electric field strength between the plates, then $E = \frac{V}{d}$.

∴ Energy density, $u = \frac{1}{2}\varepsilon_0 E^2$

Q. 22. A 4 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation? [*CBSE (F) 2012*]

Ans. Given, $C_1 = 4\ \mu F = 4 \times 10^{-6}$ F, $V_1 = 200$ V

Initial energy of first capacitor

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times (4 \times 10^{-6}) \times (200)^2 = 8 \times 10^{-2}\ J$$

When another uncharged capacitor $C_2 = 2\ \mu F$, is connected across first capacitor

Common potential,

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + 0}{C_1 + C_2} = \frac{4 \times 10^{-6} \times 200}{(4+2) \times 10^{-6}} = \frac{400}{3} \text{ volt}$$

Final energy, $U_2 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2} \times (4+2) \times 10^{-6} \times \left(\frac{400}{3}\right)^2$

$$= \frac{16}{3} \times 10^{-2}\ J = 5{\cdot}33 \times 10^{-2}\ J$$

Energy loss, $\Delta U = U_1 - U_2 = 8 \times 10^{-2} - 5{\cdot}33 \times 10^{-2} = \mathbf{2.67 \times 10^{-2}\ J}$

Multiple Choice Questions [1 mark]

Choose and write the correct option(s) in the following questions.

1. **The ratio of charge to potential of a body is known as**
   (*a*) capacitance (*b*) inductance (*c*) conductance (*d*) resistance

2. **On moving a charge of 20 C by 2 cm, 2 J of work is done. Then the potential difference between the points is**
   (*a*) 0.1 V (*b*) 8 V (*c*) 2 V (*d*) 0.5 V

3. **In brining an electron towards another electron, the electrostatic potential energy of the system**
   (*a*) increases (*b*) decreases
   (*c*) remains unchanged (*d*) becomes zero

4. **Electric potential of earth is taken to be zero, because earth is a good**
   (*a*) insulator (*b*) conductor (*c*) semi-conductor (*d*) dielectric

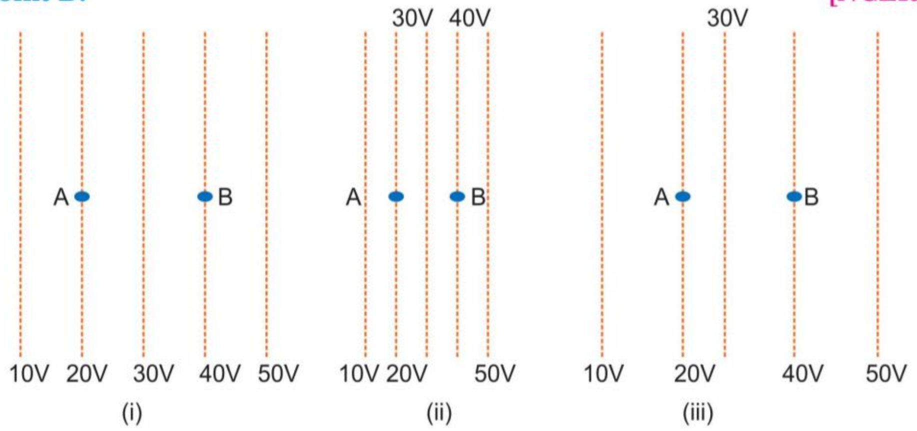
5. **Some charge is being given to a conductor. Then, its potential**
   (*a*) is maximum at surface.
   (*b*) is maximum at centre.
   (*c*) remains the same throughout the conductor.
   (*d*) is maximum somewhere between surface and centre.

6. **Equipotential surface associated with an electric field, which is increasing in magnitude along the X-direction, are**
   (*a*) planes parallel to YZ-plane.
   (*b*) planes parallel to XZ-plane.
   (*c*) planes parallel to XY-plane.
   (*d*) coaxial cylinder of increasing radii around the X-axis.

7. **What is angle between electric field and equipotential surface?**
   (*a*) 90° always (*b*) 0° always (*c*) 0° to 90° (*d*) 0° to 180°

8. **A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge** [*NCERT Exemplar*]
   (*a*) remains a constant because the electric field is uniform.
   (*b*) increases because the charge moves along the electric field.
   (*c*) decreases because the charge moves along the electric field.
   (*d*) decreases because the charge moves opposite to the electric field.

9. **Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B.** [*NCERT Exemplar*]



   (*a*) The work done in Fig. (*i*) is the greatest.
   (*b*) The work done in Fig. (*ii*) is least.
   (*c*) The work done is the same in Fig. (*i*), Fig. (*ii*) and Fig. (*iii*).
   (*d*) The work done in Fig. (*iii*) is greater than Fig. (*ii*) but equal to that in Fig. (*i*).

10. The electrostatic potential on the surface of a charged conducting sphere is 100 V. Two statements are made in this regard: *[NCERT Exemplar]*

S_1 : At any point inside the sphere, electric intensity is zero.

S_2 : At any point inside the sphere, the electrostatic potential is 100 V.

Which of the following is a correct statement?

(*a*) S_1 is true but S_2 is false.

(*b*) Both S_1 and S_2 are false.

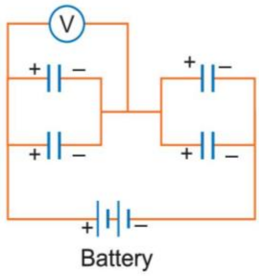
(*c*) S_1 is true, S_2 is also true and S_1 is the cause of S_2.

(*d*) S_1 is true, S_2 is also true but the statements are independent.

11. Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately *[NCERT Exemplar]*

(*a*) spheres (*b*) planes (*c*) paraboloids (*d*) ellipsoids

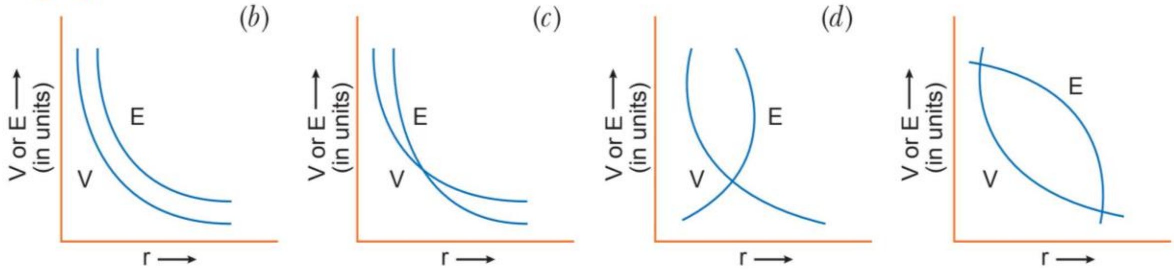
12. Four capacitors, each 50 μF are connected as shown. The DC voltmeter V reads 100 V. The charge on each plate of each capacitor is



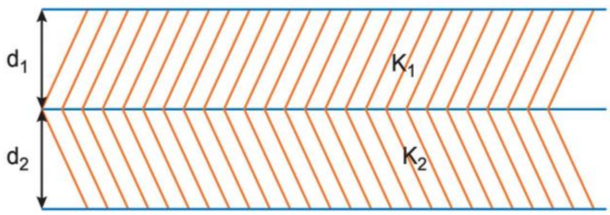
(*a*) 2×10^{-3} C (*b*) 5×10^{-3} C (*c*) 0.2 C (*d*) 0.5 C

13. The variation potential *V* with *r* and electric field *E* with *r* for a point charge is correctly shown in the graphs.

(*a*) (*b*) (*c*) (*d*)



14. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant k_1 and the other has thickness d_2 and dielectric constant k_2 as shown in figure. This arrangement can be thought as a dielectric slab of thickness $d\ (= d_1 + d_2)$ and effective dielectric constant *k*. The *k* is *[NCERT Exemplar]*



(*a*) $\dfrac{k_1 d_1 + k_2 d_2}{d_1 + d_2}$ (*b*) $\dfrac{k_1 d_1 + k_2 d_2}{k_1 + k_2}$ (*c*) $\dfrac{k_1 k_2 (d_1 + d_2)}{k_1 d_1 + k_2 d_2}$ (*d*) $\dfrac{2 k_1 k_2}{k_1 + k_2}$

15. Equipotential surfaces *[NCERT Exemplar]*

(*a*) are closer in regions of large electric fields compared to regions of lower electric fields.

(*b*) will be more crowded near sharp edges of a conductor.

(*c*) will be more crowded near regions of large charge densities.

(*d*) will always be equally spaced.

16. A 2 μF capacitor is charged to 200 volt and then the battery is disconnected. When it is connected in parallel to another uncharged capacitor, the potential difference between the plates of both is 40 volt. The capacitance of the other capacitor is

(*a*) 2 μF (*b*) 4 μF (*c*) 8 μF (*d*) 16 μF

17. Two identical metal plates, separated by a distance *d* form a parallel-plate capacitor. A metal sheet of thickness *d*/2 is inserted between the plates. The ratio of the capacitance after the insertion of the sheet to that before insertion is

(*a*) $\sqrt{2}:1$ (*b*) 2 : 1 (*c*) 1 : 1 (*d*) 1 : 2

18. *n* identical capacitors joined in parallel are charged to a common potential *V*. The battery is disconnected. Now, the capacitors are separated and joined in series. For the new combination:

(*a*) energy and potential difference both will remain unchanged

(*b*) energy will remain same, potential difference will become nV

(*c*) energy and potential both will become n times

(*d*) energy will become n times, potential difference will remain V.

19. The capacitance of a capacitor becomes $\frac{7}{6}$ times its original value if a dielectric slab of thickness $t = \frac{2}{3}d$ is introduced in between the plates, where *d* is the separation between the plates. The dielectric constant of the slab is

(*a*) $\frac{14}{11}$ (*b*) $\frac{11}{14}$ (*c*) $\frac{7}{11}$ (*d*) $\frac{11}{7}$

20. Two capacitors of capacitances 3 μF and 6 μF are charged to a potential of 12 V each. They are now connected to each other, with the positive plate of each joined to the negative plate of the other. The potential difference across 3 μF will be

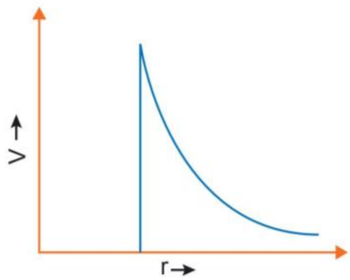
(*a*) 3 V (*b*) zero (*c*) 6 V (*d*) 4 V

21. The plates of a parallel plate capacitor are 4 cm apart, the first plate is at 300 V and the second plate at – 100 V. The voltage at 3 cm from the second plate is

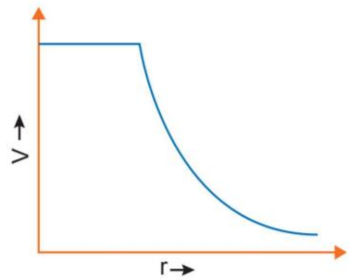
(*a*) 200 V (*b*) 400 V (*c*) 250 V (*d*) 500 V

22. In the case of a charged metallic sphere, potential (*V*) changes with respect to distance (*r*) from the centre as

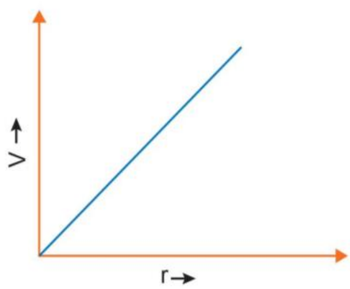
(*a*)



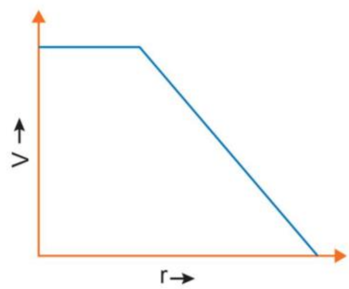
(*b*)



(*c*)



(*d*)



23. Three capacitors of capacitance 1μF, 2 μF and 3 μF are connected in series and a p.d. of 11 V is applied across the combination. Then, the p.d. across the plates of 1 μF capacitor is

(*a*) 2 V (*b*) 4 V (*c*) 1 V (*d*) 6 V

24. A conducting sphere of radius R is given a charge Q. The electric potential and the electric field at the centre of the sphere respectively are

(*a*) zero and $\frac{Q}{4\pi\varepsilon_0 R^2}$

(*b*) $\frac{Q}{4\pi\varepsilon_0 R}$ and zero

(*c*) $\frac{Q}{4\pi\varepsilon_0 R}$ and $\frac{Q}{4\pi\varepsilon_0 R^2}$

(*d*) both are zero

25. Four point charges $-Q$, $-q$, $2q$ and $2Q$ are placed, one at each corner of the square. The relation between Q and q for which the potential at the centre of the square is zero is

(*a*) $Q = \frac{1}{2}q$

(*b*) $Q = -q$

(*c*) $Q = -\frac{1}{2}q$

(*d*) $Q = q$

Answers

1. (*a*)	**2.** (*a*)	**3.** (*a*)	**4.** (*b*)	**5.** (*c*)	**6.** (*a*)
7. (*a*)	**8.** (*c*)	**9.** (*c*)	**10.** (*c*)	**11.** (*a*)	**12.** (*b*)
13. (*b*)	**14.** (*c*)	**15.** (*a*), (*b*), (*c*)	**16.** (*c*)	**17.** (*b*)	**18.** (*b*)
19. (*a*)	**20.** (*d*)	**21.** (*a*)	**22.** (*b*)	**23.** (*d*)	**24.** (*b*)
25. (*b*)					

Fill in the Blanks [1 mark]

1. The magnitude of electric field is given by the change in the magnitude of potential per unit ______________ normal to the equipotential surface at the point.
2. For linear isotropic dielectrics, $\vec{P} = \chi_e \vec{E}$ who χ_e is a constant characteristic of the dielectric and is known as the ______________ of the dielectric medium.
3. The potential energy of two like charged ($q_1q_2 > 0$) is ______________.
4. The potential energy of two unlike charges ($q_1q_2 < 0$) is ______________.
5. The maximum electric field that a dielectric medium can withstand without break-down of its insulting property is called its ______________.
6. The dielectric constant of a substance is a factor (>1) by which the capacitance ______________ from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor.
7. It is safer to be inside the car rather than standing outside under a tree during lightening is based on ______________ concept.
8. Equipotential surfaces due to long linear change distribution will be ______________ in shape.
9. Two capacitors each of capacitance 2 μF are connected in series. Equivalent capacitance will be ______________.
10. Electric field is in the direction in which the potential ______________ steepest.

Answers

1. displacement
2. susceptibility
3. positive
4. negative
5. dielectric strength
6. increases
7. electrostatic shielding
8. cylindrical
9. 1 μF
10. decreases

Very Short Answer Questions [1 mark]

Q. 1. Name the physical quantity whose SI unit is JC^{-1}. Is it a scalar or a vector quantity? *[CBSE Delhi 2010]*

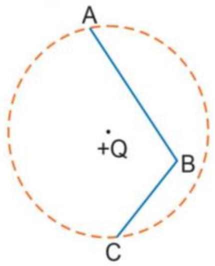
Ans. Electric potential. It is a scalar quantity.

Q. 2. Why is the electrostatic potential inside a charged conducting shell constant throughout the volume of the conductor? *[CBSE 2019 (55/5/1)]*

Ans. $\because E = 0$ inside the conductor & has no tangential component on the surface.

$\because$ No work is done in moving charge inside or on the surface of the conductor and potential is constant.

Q. 3. In the given figure, charge $+Q$ is placed at the centre of a dotted circle. Work done in taking another charge $+q$ from A to B is W_1 and from B to C is W_2. Which one of the following is correct: $W_1 > W_2$, $W_1 = W_2$ and $W_1 < W_2$? *[CBSE Sample Paper 2018]*



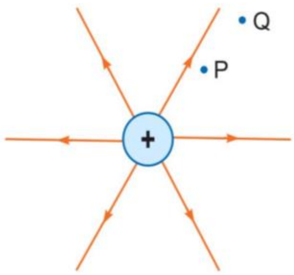
Ans. The points A and C are at same distance from the charge +Q at the centre, so

$V_A = V_C$

Therefore, $V_A - V_B = V_C - V_B$

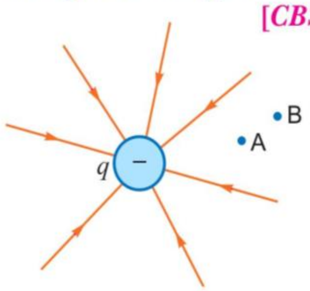
Hence, the magnitude of work done in taking charge $+q$ from A to B or from B to C will be the same *i.e.*, $W_1 = W_2$.

Q. 4. Figure shows the field lines on a positive charge. Is the work done by the field in moving a small positive charge from Q to P positive or negative? Give reason. *[CBSE (F) 2014]*



Ans. The work done by the field is negative. This is because the charge is moved against the force exerted by the field.

Q. 5. The field lines of a negative point charge are as shown in the figure. Does the kinetic energy of a small negative charge increase or decrease in going from B to A? *[CBSE Patna 2015]*



Ans. The kinetic energy of a negative charge decreases while going from point B to point A, against the movement of force of repulsion.

Q. 6. A point charge $+Q$ is placed at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero? *[CBSE Delhi 2016]*

Ans. The potential due to a point charge decreases with increase of distance. So, $V_A - V_B$ is positive.

Explanation: Let the distance of point A and B from charge Q be r_A and r_B respectively.

$$V_A = \frac{+Q}{4\pi\varepsilon_0 r_A} \text{ and } V_B = \frac{+Q}{4\pi\varepsilon_0 r_B}$$

$$V_A - V_B = \frac{+Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

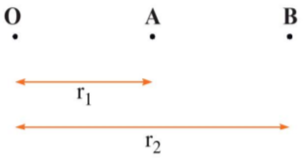
Also $r_A < r_B$

$$\Rightarrow \frac{1}{r_A} > \frac{1}{r_B} \quad \Rightarrow \frac{1}{r_A} - \frac{1}{r_B} > 0 \quad \Rightarrow \quad \frac{1}{r_A} - \frac{1}{r_B} \text{ has positive value}$$

Also Q is positive.

Hence $V_A - V_B$ is positive.

Q. 7. A point charge Q is placed at point 'O' as shown in figure. Is the potential at point A, *i.e.*, V_A, greater, smaller or equal to potential, V_B, at point B, when Q is (*i*) positive, and (*ii*) negative charge? *[CBSE (F) 2017]*



Ans. (*i*) If Q is positive,

$$V_A = \frac{KQ}{r_1} \quad \text{and} \quad V_B = \frac{KQ}{r_2}$$

Clearly, $V_A > V_B$

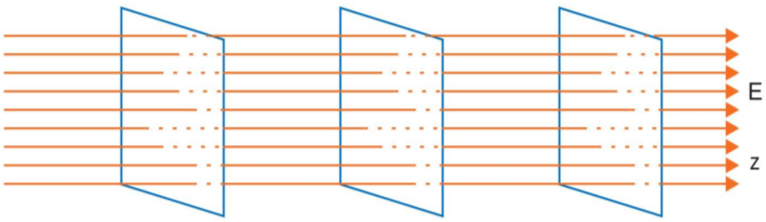
(*ii*) If Q is negative,

$$V_A = -\frac{KQ}{r_1} \quad \text{and} \quad V_B = -\frac{KQ}{r_2}$$

Clearly, $V_A < V_B$

Q. 8. Draw the equipotential surfaces corresponding to a uniform electric field in the *z*-direction. *[CBSE 2019 (55/1/1)]*

Ans. The equipotential surfaces are the equidistant planes normal to the *z*-axis, *i.e.*, planes parallel to the X–Y plane.



Q. 9. A point charge Q is placed at point O as shown in the figure. The potential difference $V_A - V_B$ is positive. Is the charge Q negative or positive? *[CBSE (F) 2016]*

Ans. We know that, $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

$$\Rightarrow \qquad V \propto \frac{1}{r}$$

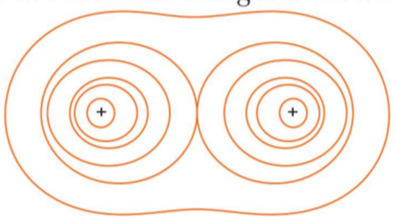
The potential due to a point charge decreases with increase of distance.

$$V_A - V_B > 0 \qquad \Rightarrow \qquad V_A > V_B$$

Hence, the charge Q is positive.

Q. 10. Depict the equipotential surfaces for a system of two identical positive point charges placed a distance '*d*' apart. *[CBSE Delhi 2010]*

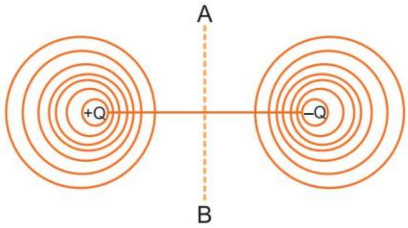
Ans. Equipotential surfaces due to two identical charges is shown in figure.



Q. 11. Draw an equipotential surface for a system consisting of two charges Q, – Q separated by a distance *r* in air. Locate the points where the potential due to the dipole is zero.

[CBSE Delhi 2017, (AI) 2008, 2013, 2019 (55/2/1)]

Ans. The equipotential surface for the system is as shown. Electric potential is zero at all points in the plane passing through the dipole equator *AB*.



Q. 12. Why do the equipotential surfaces due to a uniform electric field not intersect each other?

CBSE (F) 2012]

Ans. This is because at the point of intersection there will be two values of electric potential, which is not possible.

Q. 13. "For any charge configuration, equipotential surface through a point is normal to the electric field." Justify. *[CBSE Delhi 2014]*

Ans. The work done in moving a charge from one point to another on an equipotential surface is zero. If electric field is not normal to the equipotential surface, it would have non-zero component along the surface. In that case work would be done in moving a charge on an equipotential surface.

Q. 14. Why is the potential inside a hollow spherical charged conductor constant and has the same value as on its surface? *[CBSE (F) 2012]*

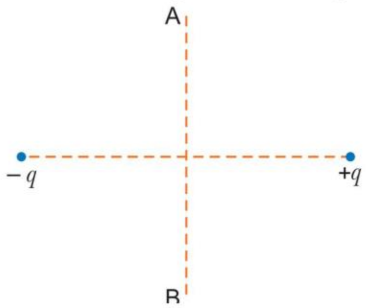
Ans. Electric field intensity is zero inside the hollow spherical charged conductor. So, no work is done in moving a test charge inside the conductor and on its surface. Therefore, there is no potential difference between any two points inside or on the surface of the conductor.

$V_A - V_B = -\int \vec{E}.\vec{dl} = 0 \quad \Rightarrow V_A = V_B = \text{Constant}$

Q. 15. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. What is the potential at the centre of the sphere? *[CBSE (AI) 2011]*

Ans. Potential at centre of sphere = 10 V. Potential at all points inside the hollow metal sphere (or any surface) is always equal to the potential at its surface.

Q. 16. A charge '*q*' is moved from a point *A* above a dipole of dipole moment '*p*' to a point *B* below the dipole in equatorial plane without acceleration. Find the work done in the process. *[CBSE Central 2016]*



Ans. Work done in the process is zero. Because, equatorial plane of a dipole is equipotential surface and work done in moving charge on equipotential surface is zero.

$$W = qV_{AB} = q \times 0 = 0$$

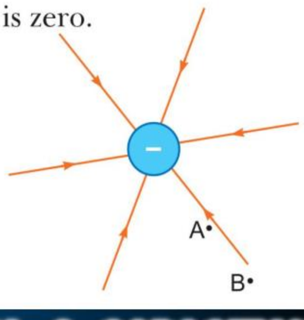
Q. 17. Why is there no work done in moving a charge from one point to another on an equipotential surface? *[CBSE (F) 2012]*

Ans. The potential difference between any two points of equipotential surface is zero. We have

$$V_1 - V_2 = \frac{W}{q} = 0 \qquad \Rightarrow \qquad W = 0$$

therefore, the work done in moving a charge on an equipotential surface is zero.

Q. 18. Figure shows the field lines due to a negative point charge. Give the sign of the potential energy difference of a small negative charge between the points *A* and *B*. *[CBSE (F) 2014]*



Ans.
$$U = \frac{1}{4\pi\varepsilon_0} . \frac{q_1 q_2}{r}$$

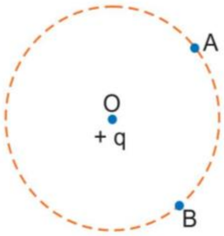
Since $\quad r_A < r_B$

$\therefore \qquad \dfrac{kq_1q_2}{r_A} > \dfrac{kq_1q_2}{r_B}$

$\therefore \qquad U_A > U_B$

Therefore, $U_A - U_B$ is positive.

Q. 19. What is the amount of work done in moving a point charge *Q* around a circular arc of radius '*r*' at the centre of which another point charge '*q*' is located? *[CBSE North 2016]*



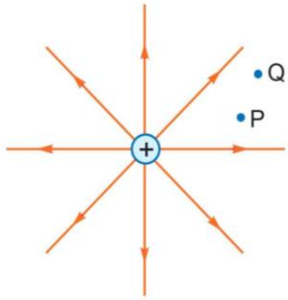
Ans. The potential of points *A* and *B* are same being equal to

$$V_A = V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

where R is the radius of the circle.

Work done $W = q\,(V_B - V_A) = q\,(V_A - V_A) = 0.$

Q. 20. The figure shows the field lines of a positive point charge. What will be the sign of the potential energy difference of a small negative charge between the points Q and P? Justify your answer. *[CBSE Guwahati 2015]*



Ans. The sign of the potential energy difference of a small negative charge will be positive. This is because negative charge moves from a point at a lower potential energy to a point at a higher potential energy.

Q. 21. Do free electrons travel to region of higher potential or lower potential? *[NCERT Exemplar]*

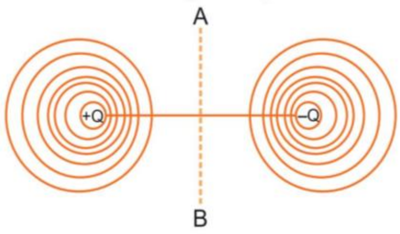
Ans. Free electrons would travel to regions of higher potentials as they are negatively charged.

Q. 22. Can there be a potential difference between two adjacent conductors carrying the same charge? *[NCERT Exemplar]*

Ans. Yes.

Q. 23. Show that the equipotential surfaces are closed together in the regions of strong field and far apart in the regions of weak field. Draw equipotential surfaces for an electric dipole. *[CBSE Sample Paper 2016]*

Ans. Equipotential surfaces are closer together in the regions of strong field and farther apart in the regions of weak field.

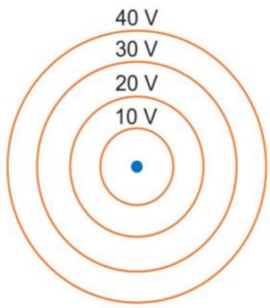


$$E = -\frac{dV}{dr}$$

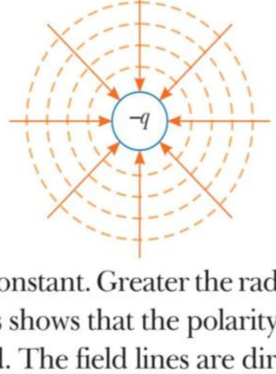
E = negative potential gradient

For same change in dV, $E \propto \dfrac{1}{dr}$ where 'dr' represents the distance between equipotential surfaces.

Q. 24. Concentric equipotential surfaces due to a charged body placed at the centre are shown. Identify the polarity of the charge and draw the electric field lines due to it. *[HOTS][CBSE Sample Paper 2016]*



Ans. For a single charge the potential is given by $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$

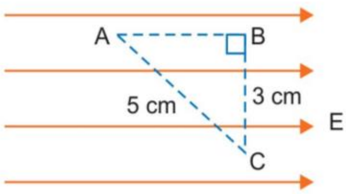


This shows that V is constant if r is constant. Greater the radius smaller will be the potential. In the given figure, potential is increasing. This shows that the polarity of charge is negative $(-q)$. The direction of electric field will be radially inward. The field lines are directed from higher to lower potential.

Short Answer Questions–I [2 marks]

Q. 1. Three points A, B and C lie in a uniform electric field (E) of 5×10^3 NC^{-1} as shown in the figure. Find the potential difference between A and C. **[CBSE (F) 2009]**

Ans. The line joining B to C is perpendicular to electric field, so potential of B = potential of C *i.e.*, $V_B = V_C$



Distance AB = 4 cm

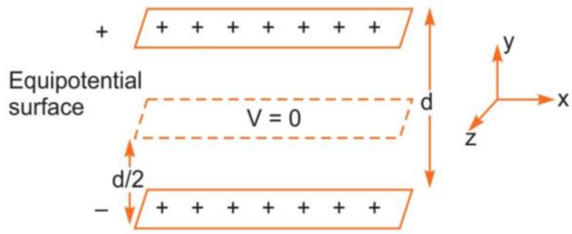
Potential difference between A and C = $E \times (AB)$

$$= 5 \times 10^3 \times (4 \times 10^{-2})$$

$$= 200 \text{ volt}$$

Q. 2. Two uniformly large parallel thin plates having charge densities +σ and – σ are kept in the X-Z plane at a distance 'd' apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m and charge '$-q$' remains stationary between the plates, what is the magnitude and direction of this field? **[CBSE Delhi 2011]**

Ans. The equipotential surface is at a distance $d/2$ from either plate in X-Z plane. For a particle of charge ($-q$) at rest between the plates, then



(*i*) weight mg acts vertically downward

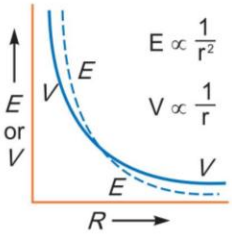
(*ii*) electric force qE acts vertically upward.

So, $mg = qE$

$$E = \frac{mg}{q}$$, vertically downward, *i.e.*, along (–)Y-axis.

Q. 3. Plot a graph comparing the variation of potential 'V' and electric field 'E' due to a point charge 'Q' as a function of distance 'R' from the point charge. **[CBSE Delhi 2012]**

Ans. The graph of variation of potential and electric field due to a point charge Q with distance R from the point charge is shown in figure.



Q. 4. What is electrostatic shielding? How is this property used in actual practice? Is the potential in the cavity of a charged conductor zero? **[CBSE South 2016]**

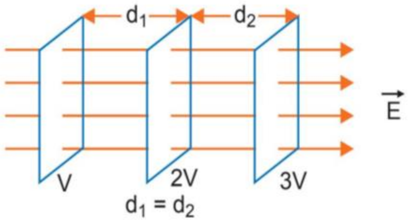
Ans. Whatever be the charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence. The field inside a conductor is zero. This is known as electrostatic shielding.

- Sensitive instruments are shielded from outside electrical influences by enclosing them in a hollow conductor.
- During lightning it is safest to sit inside a car, rather than near a tree. The metallic body of a car becomes an electrostatic shielding from lightening.

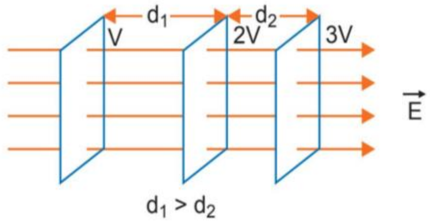
Potential inside the cavity is not zero. Potential is constant.

Q. 5. Draw 3 equipotential surfaces corresponding to a field that uniformly increases in magnitude but remains constant along Z-direction. How are these surfaces different from that of a constant electric field along Z-direction? **[CBSE (AI) 2009]**

Ans. For constant electric field $\vec{E}$

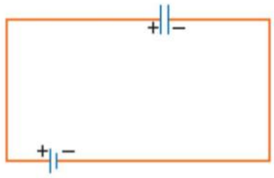


For increasing electric field



Difference: For constant electric field, the equipotential surfaces are equidistant for same potential difference between these surfaces; while for increasing electric field, the separation between these surfaces decreases, in the direction of increasing field, for the same potential difference between them.

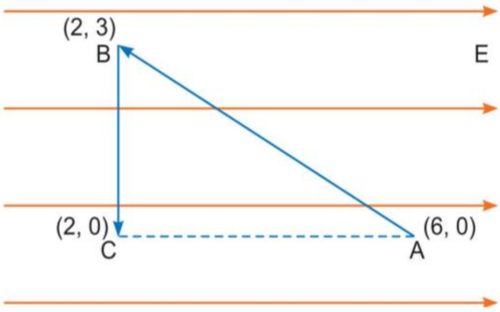
Q. 6. Why does current in a steady state not flow in a capacitor connected across a battery? However momentary current does flow during charging or discharging of the capacitor. Explain. *[CBSE (AI) 2017]*



Ans. (*i*) In the steady state no current flows through capacitor because, we have two sources (battery and fully charged capacitor) of equal potential connected in opposition.

(*ii*) During charging or discharging there is a momentary flow of current as the potentials of the two sources are not equal to each other.

Q. 7. A test charge '*q*' is moved without acceleration from *A* to *C* along the path from *A* to *B* and then from *B* to *C* in electric field *E* as shown in the figure. (*i*) Calculate the potential difference between *A* and *C*. (*ii*) At which point (of the two) is the electric potential more and why? *[CBSE (AI) 2012]*



Ans. (*i*) Since electric field is conservative in nature, the amount of work done will depend upon initial and final positions only.

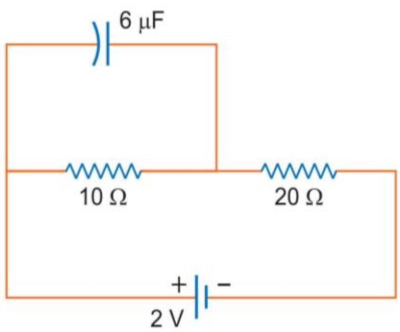
$\therefore$ Work done $W = \vec{F}\cdot\vec{d} = q\vec{E}\cdot\vec{d} = qE.4\cos 180°$

$$= -4\,qE$$

Hence $V_A - V_C = \dfrac{W}{q} = -4E$

(*ii*) $V_C > V_A$, because direction of electric field is in decreasing potential.

Q. 8 . Find the charge on the capacitor as shown in the circuit. *[CBSE (F) 2014]*



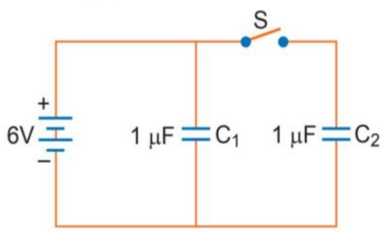
Ans. Total resistance, $R = 10\ \Omega + 20\ \Omega = 30\ \Omega$

The current, $I = \dfrac{V}{R} = \dfrac{2\text{V}}{30\ \Omega} = \dfrac{1}{15}\text{ A}$

Potential difference, $V = IR = \dfrac{1}{15}\times 10 = \dfrac{2}{3}\text{V}$

Charge, $q = CV = 6\times\dfrac{2}{3} = \mathbf{4\ \mu C}$

Q. 9. Figure shows two identical capacitors, C_1 and C_2, each of 1 μF capacitance connected to a battery of 6 V. Initially switch '*S*' is closed. After sometimes '*S*' is left open and dielectric slabs of dielectric constant $K = 3$ are inserted to fill completely the space between the plates of the two capacitors. How will the (*i*) charge and (*ii*) potential difference between the plates of the capacitors be affected after the slabs are inserted?

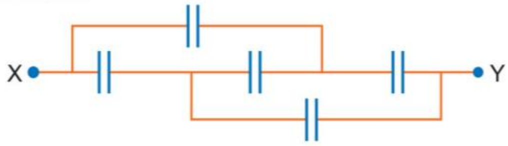


[CBSE Delhi 2011]

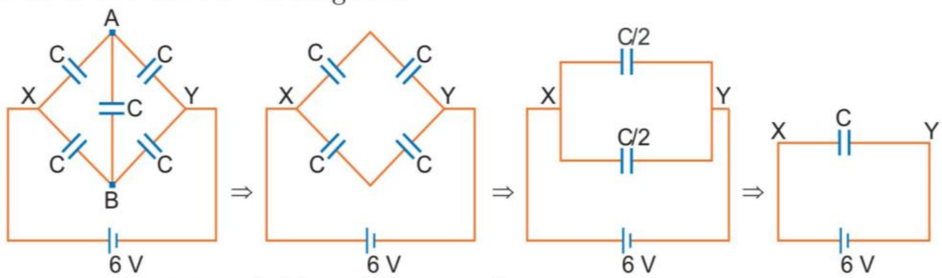
Ans. When switch S is closed, p.d. across each capacitor is 6V

$V_1 = V_2 = 6\text{ V}$

Q. 12. Find the equivalent capacitance of the network shown in the figure, when each capacitor is of 1 μF. When the ends X and Y are connected to a 6 V battery, find out (*i*) the charge and (*ii*) the energy stored in the network. [*CBSE Patna 2015*]



Ans. The given circuit can be rearranged as



It is known as wheatstone bridge of the capacitor.

Since $V_A = V_B$, so the bridge capacitor between points A and B can be removed.

(*i*) The equivalent capacitor of the network

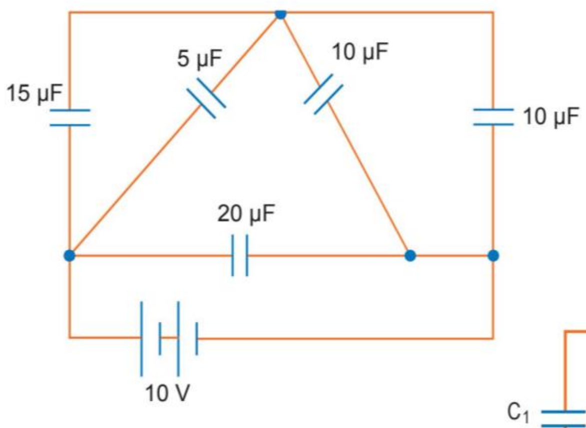
$$C_{eq} = \frac{C \times C}{C + C} + \frac{C \times C}{C + C}$$
$$= \frac{C}{2} + \frac{C}{2}$$
$$= C = 1 \mu F$$

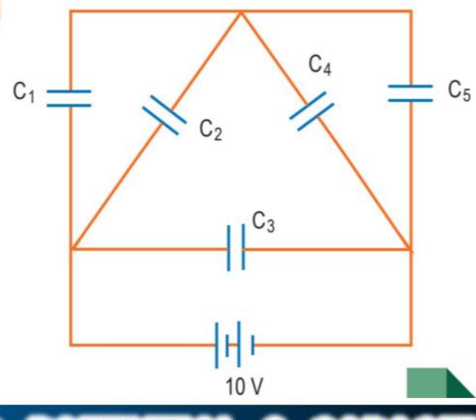
Charge in the network, $Q = C_{eq} V$
$$= C \times V$$
$$= 1 \mu F \times 6 V = \mathbf{6 \mu C}$$

(*ii*) Energy stored in the capacitor,

$$U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} \times 1 \mu F \times (6)^2$$
$$= \mathbf{18 \mu J}$$

Q. 13. The figure shows a network of five capacitors connected to a 10 V battery. Calculate the charge acquired by the 5 μF capacitor. [*CBSE 2019 (55/3/3)*]





Ans. Net capacitance of parallel C_1 & $C_2 = C_1 + C_2$

$$C_{12} = 15 + 5 = 20 \mu F$$

Net capacitance of parallel C_4 & $C_5 = C_4 + C_5$

$$C_{45} = 10 + 10 = 20 \mu F$$

C_{12}, C_{45} in series, $C_{1245} = \frac{C_{12} C_{45}}{C_{12} + C_{45}} = \frac{20 \times 20}{20 + 20} = 10 \mu F$

C_3 in parallel with $C_{1245} = C_{1245} + C_3 = 10 + 20 = 30\,\mu F$

P.D. across $C_{1245} = 10$ V

P.D. across $C_{12} = C_{45} = 5$ V

Charge on $5\,\mu F$, $Q = CV$

$= 5 \times 10^{-6} \times 5$ C

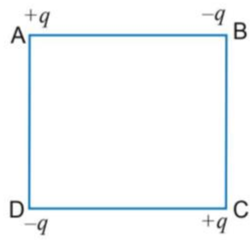
$= \mathbf{25 \times 10^{-6}}$ **C**

Q. 14. Four charges $+q, -q, +q$ and $-q$ are to be arranged respectively at the four corners of a square *ABCD* of side '*a*'.

(*a*) Find the work required to put together this arrangement.

(*b*) A charge q_0 is brought to the centre of the square, the four charges being held fixed. How much extra work is needed to do this? **[*HOTS*][*CBSE (F) 2015*]**

Ans. (*a*) Work done in bringing charge $+q$ at point *A*



$$W_A = 0$$

Work done in bringing charge $-q$ to the point *B*

$$W_B = W_{AB} = -q \times \frac{1}{4\pi\varepsilon_0}\frac{q}{a} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}$$

Work done in bring the charge $+q$ to the point *C*

$$W_C = W_{AC} + W_{BC}$$

$$= q \times \frac{1}{4\pi\varepsilon_0}\frac{q}{a\sqrt{2}} + q \times \left(-\frac{1}{4\pi\varepsilon_0}\frac{q}{a}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a\sqrt{2}} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}$$

Work done in bringing a charge $-q$ to the point *D*

$$W_D = W_{AD} + W_{BD} + W_{CD}$$

$$= -q \times \frac{1}{4\pi\varepsilon_0}\frac{q}{a} + (-q)\left(\frac{1}{4\pi\varepsilon_0}\frac{-q}{a\sqrt{2}}\right) + (-q) \times \frac{1}{4\pi\varepsilon_0}.\frac{q}{a}$$

Total work done $W = W_A + W_B + W_C + W_D$

$$= 2 \times \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a\sqrt{2}} - 4 \times \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}(\sqrt{2} - 4)$$

(*b*) Work done in bringing a charge from infinity to a point is given by

$$W = q_0 V_p \quad (V_p = \text{Electric potential at the point})$$

Electric potential at the centre of the square is

$$V_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{+q}{s}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{s}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{+q}{s}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{s}\right) = 0$$

and electric potential at infinity is always zero.

Hence, work done $W = 0$.

Q. 15. Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If the two are at the same potential, the larger sphere has more charge than the smaller sphere. State whether the charge density of the smaller sphere is more or less than that of the larger one. **[*HOTS*][*NCERT Exemplar*]**

Ans. Since two spheres are at the same potential, therefore

$$V_1 = V_2$$

$$\frac{Q_1}{4\pi\varepsilon_0 R_1} = \frac{Q_2}{4\pi\varepsilon_0 R_2}$$

$$\Rightarrow \quad \frac{Q_1}{Q_2} = \frac{R_1}{R_2} \qquad \ldots(i)$$

Given, $R_1 > R_2$, $\therefore$ $Q_1 > Q_2$

$\Rightarrow$ Larger sphere has more charge

Now, $\sigma_1 = \dfrac{Q_1}{4\pi R_1^2}$ and $\sigma_2 = \dfrac{Q_2}{4\pi R_2^2}$

$$\frac{\sigma_2}{\sigma_1} = \frac{Q_2}{Q_1} \cdot \frac{R_1^2}{R_2^2}$$

$$\Rightarrow \frac{\sigma_2}{\sigma_1} = \frac{R_2}{R_1} \cdot \frac{R_1^2}{R_2^2} \quad \text{[From equation (i)]}$$

Since $R_1 > R_2$, therefore $\sigma_2 > \sigma_1$.

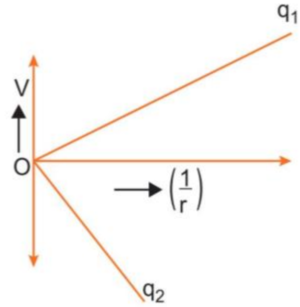
Charge density of smaller sphere is more than that of larger one.

Q. 16. The two graphs are drawn below, show the variations of electrostatic potential (V) with $\frac{1}{r}$ (r being the distance of field point from the point charge) for two point charges q_1 and q_2.

(i) What are the signs of the two charges?

(ii) Which of the two charges has the larger magnitude and why?

[HOTS]



Ans. (i) The potential due to positive charge is positive and due to negative charge, it is negative, so, q_1 is **positive** and q_2 is **negative**.

(ii) $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$

The graph between V and $\frac{1}{r}$ is a straight line passing through the origin with slope $\dfrac{q}{4\pi\varepsilon_0}$.

As the magnitude of slope of the line due to charge q_2 is greater than that due to q_1, q_2 has larger magnitude.

Q. 17. Two identical capacitors of 12 pF each are connected in series across a 50 V battery. Calculate the electrostatic energy stored in the combination. If these were connected in parallel across the same battery, find out the value of the energy stored in this combination.

[CBSE 2019 (55/5/1)]

Ans. Net capacitance in series combination is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \frac{1}{C_s} = \frac{1}{12} + \frac{1}{12}$$

$$\Rightarrow C_s = 6 \text{ pF}$$

$$E_s = \frac{1}{2}C_sV^2$$

$$E_s = \frac{1}{2} \times 6 \times 10^{-12} \times 50 \times 50$$

$$= 7500 \times 10^{-12} \text{ J}$$

$$= \mathbf{7.5 \times 10^{-9}\ J}$$

Net capacitance in parallel combination is given by

$$C_p = 12 \text{ pF} + 12 \text{ pF}$$

$$= 24 \text{ pF}$$

$$E_p = \frac{1}{2}C_pV^2$$

$$E_p = \frac{1}{2} \times 24 \times 10^{-12} \times 50 \times 50$$

$$= \mathbf{3 \times 10^{-8}\ J}$$

Short Answer Questions–II [3 marks]

Q. 1. Define an equipotential surface. Draw equipotential surfaces [*CBSE Central 2016*]

(*i*) in the case of a single point charge and

(*ii*) in a constant electric field in Z-direction.

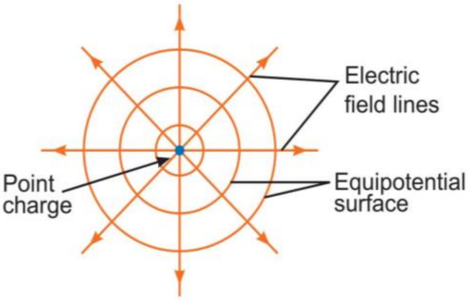
Why the equipotential surfaces about a single charge are not equidistant?

(*iii*) Can electric field exist tangential to an equipotential surface? Give reason.

Ans. An equipotential surface is the surface with a constant value of potential at all points on the surface.

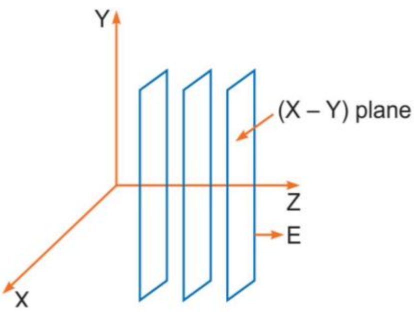
Equipotential surface :

(*i*) In case of a single point charge



Here point charge is positive, if it is negative then electric field will be radially inward but equipotential surfaces are same and are concentric spheres with centres at the charge.

(*ii*) In case of electric field in Z-direction



Potential of a point charge at a distance $r = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

$$\therefore \qquad V \propto \frac{1}{r}$$

Hence equipotential surfaces about a single charge are not equidistant.

(*iii*) No if the field lines are tangential, work will be done in moving a charge on the surface which goes against the definition of equipotential surface.

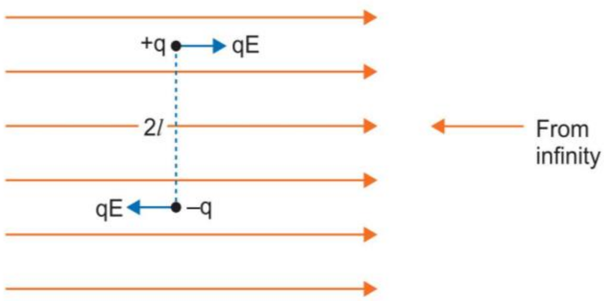
Q. 2. Show that the potential energy of a dipole making angle θ with the direction of the field is given by $U(\theta) = -\vec{P}\cdot\vec{E}$. Hence find out the amount of work done in rotating it from the position of unstable equilibrium to the stable equilibrium. [*CBSE East 2016*]

Ans. *The potential energy of an electric dipole in an electric field is defined as the work done in bringing the dipole from infinity to its present position in the electric field.*

Suppose the dipole is brought from infinity and placed at orientation θ with the direction of electric field. The work done in this process may be supposed to be done in two parts.

(*i*) The work done (W_1) in bringing the dipole perpendicular to electric field from infinity.

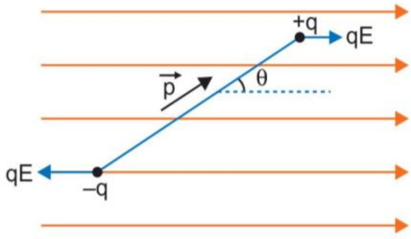
(*ii*) Work done (W_2) in rotating the dipole such that it finally makes an angle θ from the direction of electric field.



Let us suppose that the electric dipole is brought from infinity in the region of a uniform electric field such that its dipole moment $\vec{P}$ always remains perpendicular to electric field. The electric forces on charges $+q$ and $-q$ are qE and $-qE$, along the field direction and opposite to field direction respectively.

As charges $+q$ and $-q$ traverse equal distance under equal and opposite forces; therefore, net work done in bringing the dipole in the region of electric field perpendicular to field-direction will be zero, *i.e.*, $W_1 = 0$.

Now the dipole is rotated and brought to orientation making an angle θ with the field direction (*i.e.*, $\theta_0 = 90°$ and $\theta_1 = \theta$), therefore, work done



$$W_2 = pE\ (\cos\theta - \cos\theta_1)$$
$$= pE\ (\cos 90° - \cos\theta) = -pE\cos\theta$$

∴ Total work done in bringing the electric dipole from infinity, *i.e.*,

Electric potential energy of electric dipole

$$U = W_1 + W_2 = 0 - pE\cos\theta = -pE\cos\theta$$

In vector form $U = -\vec{p}\,.\,\vec{E}$

For rotating dipole from position of unstable equilibrium ($\theta_0 = 180°$) to the stable equilibrium ($\theta = 0°$)

∴ $W_{req} = pE(\cos 180° - \cos 0°)$

$pE(-1 - 1) = -2pE$

Q. 3. Three concentric metallic shells *A*, *B* and *C* of radii *a*, *b* and *c* (*a* < *b* < *c*) have surface charge densities +σ, –σ and +σ respectively as shown in the figure.

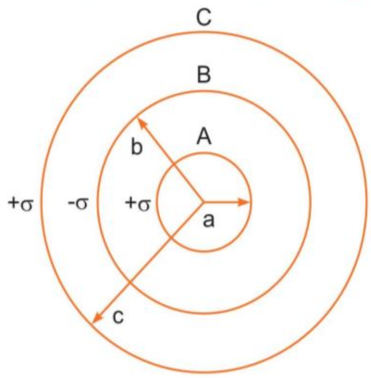
If shells *A* and *C* are at the same potential, then obtain the relation between the radii *a*, *b* and *c*.

[CBSE (F) 2014, 2019 (55/5/1)]

Ans. Charge on shell *A*, $q_A = 4\pi a^2\sigma$

Charge on shell *B*, $q_B = -4\pi b^2\sigma$

Charge of shell *C*, $q_C = 4\pi c^2\sigma$



Potential of shell *A*: Any point on the shell *A* lies inside the shells *B* and *C*.

$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{a} + \frac{q_B}{b} + \frac{q_C}{c}\right]$$
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{4\pi a^2\sigma}{a} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right]$$
$$= \frac{\sigma}{\varepsilon_0}(a - b + c)$$

Any point on *B* lies outside the shell *A* and inside the shell *C*. Potential of shell *B*,

$$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{b} + \frac{q_B}{b} + \frac{q_C}{c}\right]$$
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{4\pi a^2\sigma}{b} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right] = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right]$$

Any point on shell *C* lies outside the shells *A* and *B*. Therefore, potential of shell *C*.

$$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{c} + \frac{q_B}{c} + \frac{q_C}{c}\right]$$
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{4\pi a^2\sigma}{c} - \frac{4\pi b^2\sigma}{c} + \frac{4\pi c^2\sigma}{c}\right]$$
$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{c} - \frac{b^2}{c} + c\right]$$

Now, we have

$$V_A = V_C$$
$$\frac{\sigma}{\varepsilon_0}(a - b + c) = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$$
$$a - b = \frac{(a-b)(a+b)}{c}$$

or $\boldsymbol{a + b = c}$

Q. 4. A parallel plate capacitor each with plate area A and separation 'd' is charged to a potential difference V. The battery used to charge it is then disconnected. A dielectric slab of thickness d and dielectric constant K is now placed between the plates. What change if any, will take place in ***[CBSE (F) 2010]***

(*i*) charge on the plates,

(*ii*) electric field intensity between the plates,

(*iii*) capacitance of the capacitor?

Justify your answer in each case.

Ans. Initial capacitance $C_0 = \frac{\varepsilon_0 A}{d}$, Potential difference $= V$

(*i*) Initial charge, $q_0 = C_0 V = \frac{\varepsilon_0 A}{d} V$

$\therefore$ When battery is disconnected the charge on the capacitor remains unchanged and equal to

$$q = q_0 = \frac{\varepsilon_0 A}{d} V.$$

(*ii*) Initial electric field between the plates, $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q/A}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$

After introduction of dielectric; the permittivity of medium becomes $K\varepsilon_0$;

so final electric field between the plates, $E = \frac{q}{AK\varepsilon_0} = \frac{E_0}{K}$ *i.e.*, electric field reduces to $\frac{1}{K}$ times.

(*iii*) After introduction of dielectric, the capacitance becomes KC_0.

Q. 5. A parallel plate capacitor is charged by a battery, which is then disconnected. A dielectric slab is then inserted in the space between the plates. Explain what changes, if any, occur in the values of

(*i*) capacitance

(*ii*) potential difference between the plates

(*iii*) electric field between the plates, and

(*iv*) the energy stored in the capacitor. ***[CBSE Delhi 2010, (AI) 2009, 2012]***

Ans. (*i*) The capacitance of capacitor increases to K times (since $C = \frac{K\varepsilon_0 A}{d} \propto K$)

(*ii*) The potential difference between the plates becomes $\frac{1}{K}$ times.

Reason: $V = \frac{Q}{C}$; Q same, C increases to K times; $V' = \frac{V}{K}$

(*iii*) As $E = \frac{V}{d}$ and V is decreased; therefore, electric field decreases to $\frac{1}{K}$ times.

(*iv*) Energy stored will be decreased. The energy becomes, $U = \frac{Q_0^2}{2C} = \frac{Q_0^2}{2KC_0} = \frac{U_0}{K}$

Thus, energy is reduced to $\frac{1}{K}$ times the initial energy.

Q. 6. A parallel plate is charged by a battery. When the battery remains connected, a dielectric slab is inserted in the space between the plates. Explain what changes if any, occur in the values of

(*i*) potential difference between the plates

(*ii*) electric field strength between the plates

(*iii*) capacitance

(*iv*) charge on the plates

(*v*) energy stored in the capacitor. ***[CBSE Delhi 2010]***

Ans. (*i*) When battery remains connected, the potential difference remains the same.

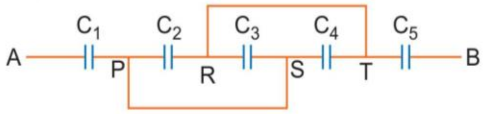
(*ii*) As electric field $E = \frac{V}{d}$, V = constant and d = constant; therefore, electric field strength remains the same.

(*iii*) The capacitance of capacitor increases as $K > 1$.

(*iv*) The charge $Q = CV$, V = same, C = increases; therefore, charge on plates increases.

(*v*) Energy stored by capacitor $U = \frac{1}{2}CV^2$, also increases.

Q. 7. (*i*) Find equivalent capacitance between A and B in the combination given below. Each capacitor is of 2 μF capacitance.



(*ii*) If a dc source of 7 V is connected across *AB*, how much charge is drawn from the source and what is the energy stored in the network? [*CBSE Delhi 2017*]

Ans. (*i*) Capacitors C_2, C_3 and C_4 are in parallel

$$C_{234} = C_2 + C_3 + C_4 = 2\ \mu F + 2\ \mu F + 2\ \mu F$$

$$\therefore \quad C_{234} = 6\ \mu F$$

Capacitors C_1, C_{234} and C_5 are in series,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{234}} + \frac{1}{C_5} = \frac{1}{2} + \frac{1}{6} + \frac{1}{2}$$

$$= \frac{7}{6}\ \mu F$$

$$C_{eq} = \mathbf{\frac{6}{7}\ \mu F}$$

(*ii*) Charge drawn from the source

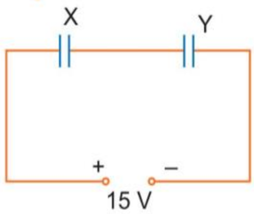
$$Q = C_{eq} V$$

$$= \frac{6}{7} \times 7\ \mu C = \mathbf{6\ \mu C}$$

Energy stored in the network, $U = \frac{Q^2}{2C}$

$$= \frac{6 \times 6 \times 10^{-12} \times 7}{2 \times 6 \times 10^{-6}} J = 21 \times 10^{-6}\ J = \mathbf{21\ \mu J}$$

Q. 8. Two parallel plate capacitors *X* and *Y* have the same area of plates and same separation between them. *X* has air between the plates while *Y* contains a dielectric medium $\varepsilon_r = 4$.



(*i*) Calculate the capacitance of each capacitor if equivalent capacitance of the combination is 4 μF.

(*ii*) Calculate the potential difference between the plates of *X* and *Y*.

(*iii*) Estimate the ratio of electrostatic energy stored in *X* and *Y*.

[*CBSE Delhi 2016*]

Ans. (*i*) Capacitance of X, $C_X = \frac{\varepsilon_0 A}{d}$

Capacitance of Y, $C_Y = \frac{\varepsilon_r \varepsilon_0 A}{d} = 4\frac{\varepsilon_0 A}{d}$

$$\therefore \quad \frac{C_Y}{C_X} = 4 \Rightarrow C_Y = 4C_X \qquad \ldots(i)$$

As X and Y are in series, so

$$C_{eq} = \frac{C_X C_Y}{C_X + C_Y} \Rightarrow 4\ \mu F = \frac{C_X . 4C_X}{C_X + 4C_X}$$

$$\Rightarrow \quad C_X = \mathbf{5\ \mu F} \text{ and } C_Y = 4C_X = \mathbf{20\ \mu F}$$

(*ii*) In series charge on each capacitor is same, so

$$\text{P.d. } V = \frac{Q}{C} \Rightarrow V \propto \frac{1}{C}$$

$$\therefore \quad \frac{V_X}{V_Y} = \frac{C_Y}{C_X} = 4 \quad \Rightarrow V_X = 4V_Y \qquad \ldots(ii)$$

Also $\quad V_X + V_Y = 15 \qquad \ldots(iii)$

From (*ii*) and (*iii*),

$$4V_Y + V_Y = 15 \quad \Rightarrow \quad V_Y = 3 \text{ V}$$

$$V_X = 15 - 3 = 12 \text{ V}$$

Thus potential difference across *X*, $\mathbf{V_X = 12\ V}$, P.d. across Y, $\mathbf{V_Y = 3\ V}$

(*iii*) $$\frac{\text{Energy stored in } X}{\text{Energy stored in } Y} = \frac{Q^2/2C_X}{Q^2/2C_Y} = \frac{C_Y}{C_X} = \frac{4}{1} \Rightarrow \frac{U_X}{U_Y} = \frac{\mathbf{4}}{\mathbf{1}}$$

Q. 9. In a parallel plate capacitor with air between the plates, each plate has an area of 5×10^{-3} m^2 and the separation between the plates is 2.5 mm.

(*i*) Calculate the capacitance of the capacitor.

(*ii*) If this capacitor is connected to 100 V supply, what would be the charge on each plate?

(*iii*) How would charge on the plates be affected, if a 2.5 mm thick mica sheet of $K = 8$ is inserted between the plates while the voltage supply remains connected? [*CBSE (F) 2014*]

Ans. (*i*) Capacitance, $C = \dfrac{\varepsilon_0 A}{d}$

$$= \frac{8.85 \times 10^{-12} \times 5 \times 10^{-3}}{2.5 \times 10^{-3}}$$

$$= \mathbf{17.7 \times 10^{-12}\ F}$$

(*ii*) Charge $Q = CV$

$$= 17.7 \times 10^{-12} \times 100$$

$$= \mathbf{17.7 \times 10^{-10}\ C}$$

(*iii*) New charge, $Q = KQ$

$$= 8 \times 17.7 \times 10^{-10}$$

$$= \mathbf{1.416 \times 10^{-8}\ C}$$

Q. 10. A 200 μF parallel plate capacitor having plate separation of 5 mm is charged by a 100 V *dc* source. It remains connected to the source. Using an insulated handle, the distance between the plates is doubled and a dielectric slab of thickness 5 mm and dielectric constant 10 is introduced between the plates. Explain with reason, how the (*i*) capacitance, (*ii*) electric field between the plates, (*iii*) energy density of the capacitor will change? [*CBSE 2019 (55/2/1)*]

Ans. Dielectric slab of thickness 5 mm is equivalent to an air capacitor of thickness $= \dfrac{5}{10}$ mm.

Effective separation between the plates with air in between is = (5 + 0.50) mm = 5.5 mm

(*i*) Effective new capacitance

$$C' = 200\,\mu\text{F} \times \frac{5\text{ mm}}{5.5\text{ mm}} = \frac{2000}{11}\,\mu\text{F}$$

$$\approx \mathbf{182\,\mu F}$$

(*ii*) Effective new electric field

$$E' = \frac{100\,V}{5.5 \times 10^{-3}\text{m}} = \frac{200000}{11}\text{ V/m, where } E = \frac{V}{d} = \frac{100}{5 \times 10^{-3}} = 20000 \text{ V/m}$$

$$\approx \mathbf{18182\ V/m}$$

(*iii*) $$\frac{\text{New energy density}}{\text{Original energy density}} = \frac{\frac{1}{2}\varepsilon_0 E'^2}{\frac{1}{2}\varepsilon_0 E^2} = \left(\frac{E'}{E}\right)^2 = \left(\frac{10}{11}\right)^2$$

New Energy density will be $\left(\frac{10}{11}\right)^2$ of the original energy density = $\mathbf{\frac{100}{121}}$ the original energy density.

Q. 11. A parallel plate capacitor of capacitance *C* is charged to a potential *V*. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor. [*CBSE (AI) 2014*]

Ans. Energy stored in the capacitor $= \frac{1}{2}CV^2$

$$= \frac{q^2}{2C}$$

Net capacitance of the parallel combination (when capacitors are connected together)

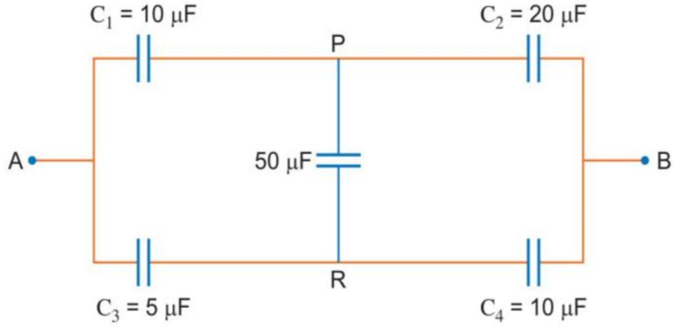
$$= C + C = 2C$$

Since the total charge Q remains same, initial energy $= \frac{q^2}{2C}$

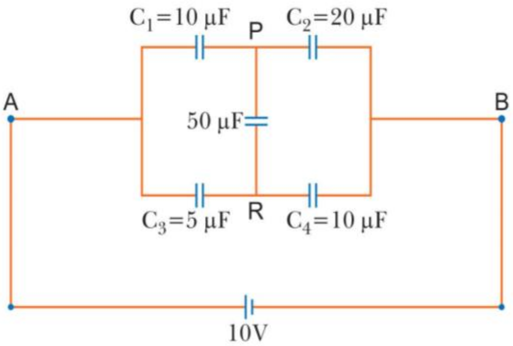
$$\text{Final energy} = \frac{q^2}{2(2C)}$$

$$\frac{U_f}{U_i} = \mathbf{1:2}$$

Q. 12. Calculate the equivalent capacitance between points *A* and *B* in the circuit below. If a battery of 10 V is connected across *A* and *B*, calculate the charge drawn from the battery by the circuit. [*CBSE East 2016*]



Ans. ∴ $$\frac{C_1}{C_2} = \frac{C_3}{C_4}$$



This is the condition of balance so there will be no current across PR (50 μF capacitor)

Now C_1 and C_2 are in series

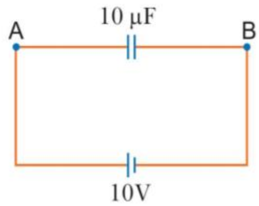
$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{10 \times 20}{10 + 20} = \frac{200}{30} = \frac{20}{3}\ \mu F$$

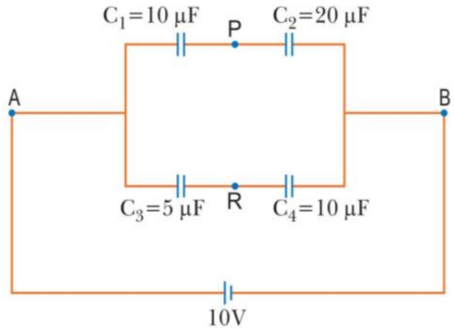
∵ C_3 and C_4 are in series

$$C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{5 \times 10}{5 + 10} = \frac{50}{15} = \frac{10}{3}\ \mu F$$

Equivalent capacitance between A and B is

$$C_{AB} = C_{12} + C_{34} = \frac{20}{3} + \frac{10}{3} = 10\ \mu F$$





Hence, charge drawn from battery (Q) = CV

$= 10 \times 10\ \mu C = 100\ \mu C = \mathbf{10^{-4}\ C}$

Q. 13. Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, determine the value of C_1 and C_2. Also calculate the charge on each capacitor in parallel combination. *[CBSE Delhi 2015]*

Ans. Energy stored in a capacitor, $E = \frac{1}{2}CV^2$

In parallel, $$0.25 = \frac{1}{2}(C_1 + C_2)(100)^2 \quad ...(i)$$

In series, $$0.045 = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(100)^2 \quad ...(ii)$$

From (*i*) $$C_1 + C_2 = 0.25 \times 2 \times 10^{-4}$$
$$C_1 + C_2 = 5 \times 10^{-5} \quad ...(iii)$$

From (*ii*) $$\frac{C_1C_2}{C_1 + C_2} = 0.045 \times 2 \times 10^{-4}$$
$$\frac{C_1C_2}{C_1 + C_2} = 0.09 \times 10^{-4} = 9 \times 10^{-6}$$

From (*iii*) $$C_1 C_2 = 9\times10^{-6}\times5\times10^{-5} = 4.5 \times 10^{-10}$$
$$C_1 - C_2 = \sqrt{(C_1 + C_2)^2 - 4C_1C_2}$$
$$C_1 - C_2 = 2.64 \times 10^{-5} \quad ...(iv)$$

Solving (*iii*) and (*iv*) $C_1 = \mathbf{38.2\ \mu F}$

$C_2 = \mathbf{11.8\ \mu F}$

In parallel $Q_1 = C_1 V$

$= 38.2 \times 10^{-6} \times 100 = \mathbf{38.2 \times 10^{-4}\ C}$

$Q_2 = C_2 V$

$= 11.8 \times 10^{-6} \times 100 = \mathbf{11.8 \times 10^{-4}\ C}$

Q. 14. Two capacitors of capacitance 10 μF and 20 μF are connected in series with a 6 V battery. After the capacitors are fully charged, a slab of dielectric constant (*K*) is inserted between the plates of the two capacitors. How will the following be affected after the slab is introduced:

(*a*) the electric field energy stored in the capacitors?

(*b*) the charges on the two capacitors?

(*c*) the potential difference between the plates of the capacitors?

Justify your answer. *[CBSE Bhubaneshwer 2015]*

Ans. Let Q be the charge on each capacitor. So, $Q = \frac{C_1C_2}{C_1 + C_2}V$.

Initial electric field energy in each capacitor becomes

$$U_1 = \frac{1}{2}\frac{Q^2}{C_1} \text{ and } U_2 = \frac{1}{2}\frac{Q^2}{C_2}$$

Initial charge on each capacitor

$$Q = C_1V_1, \quad Q = C_2V_2 \text{ and } Q = \frac{C_1C_2}{C_1 + C_2}.V$$

where V_1 and V_2 are *p.d* across the capacitors

On inserting the dielectric slab the capacitance of each capacitor becomes

$$C'_1 = KC_1 \text{ and } C'_2 = KC_2$$

and equivalent capacitance becomes

$$C'_{eq} = \frac{KC_1 \times KC_2}{KC_1 + KC_2} = K\frac{C_1C_2}{C_1 + C_2}$$

New charge on the capacitor becomes

$$Q' = C'_{eq}V' = K\left(\frac{C_1C_2}{C_1 + C_2}\right) \times V$$

$$Q' = \frac{C_1C_2}{C_1 + C_2}.V \times K$$

$$Q' = Q \times K$$

$$Q' = KQ$$

(*a*) New electric field energy becomes

$$U'_1 = \frac{Q'^2}{2KC_1} = \frac{KQ^2}{2C_1}$$

$$U'_2 = \frac{1}{2}\frac{Q'^2}{KC_2} = \frac{KQ^2}{2C_2}$$

i.e., electric field energy increases in each capacitor.

(*b*) $Q' = KQ$ (as stated above) *i.e.*, charges are increases on each capacitor.

(*c*) $$V'_1 = \frac{Q'}{C_1'} = \frac{KQ}{KC_1} = \frac{Q}{C_1}$$

and $$V'_2 = \frac{Q'}{C_2'} = \frac{KQ}{KC_2} = \frac{Q}{C_2}$$

i.e., *p.d* across each capacitor remains same.

Q. 15. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor? If another capacitor of 6 pF is connected in series with it with the same battery connected across the combination, find the charge stored and potential difference across each capacitor. [*CBSE Delhi 2017*]

Ans. Electrostatic energy stored, $U = \frac{1}{2}CV^2$

$$= \frac{1}{2} \times 12 \times 10^{-12} \times 50 \times 50 \text{ J} = 1.5 \times 10^{-8} \text{ J}$$

C = Equivalent capacitance of 12 pF and 6 pF, in series

$\therefore$ $$\frac{1}{C} = \frac{1}{12} + \frac{1}{6} = \frac{1+2}{12}$$

$\Rightarrow$ $$C = 4 \text{ pF}$$

Charge stored across each capacitor

$$Q = CV = 4 \times 10^{-12} \times 50 \text{ V}$$

$$= \mathbf{2 \times 10^{-10} \text{ C}}$$

In series combination, charge on each capacitor is same.

Charge on each capacitor, 12 pF as well as 6 pF is same.

$\therefore$ Potential difference across capacitor C_1 (12 pF capacitor)

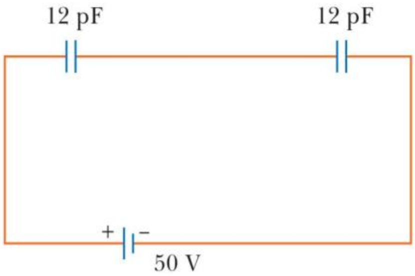
$\therefore$ $$V_1 = \frac{2 \times 10^{-10}}{12 \times 10^{-12}}\text{V} = \mathbf{\frac{50}{3}V} \qquad \left(V = \frac{Q}{C}\right)$$

Potential difference across capacitor C_2 (6 pF capacitor)

$$V_2 = \frac{2 \times 10^{-10}}{6 \times 10^{-12}}\text{V} = \mathbf{\frac{100}{3}V}$$

Q. 16. Two identical capacitors of 12 pF each are connected in series across a battery of 50 V. How much electrostatic energy is stored in the combination? If these were connected in parallel across the same battery, how much energy will be stored in the combination now? Also find the charge drawn from the battery in each case. *[CBSE Delhi 2017]*

Ans. In series combination: $\frac{1}{C_S} = \left(\frac{1}{12} + \frac{1}{12}\right) \Rightarrow \frac{1}{C_S} = \frac{1}{6}$



$\therefore \quad C_s = 6 \times 10^{-12}$ F

$U_s = \frac{1}{2}CV^2$

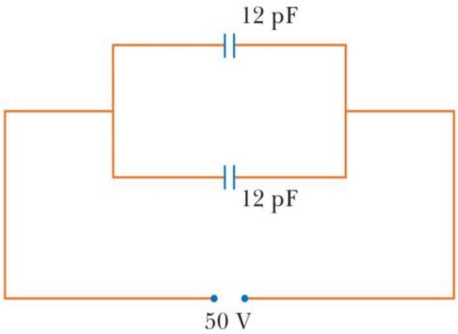
$U_s = \frac{1}{2} \times 6 \times 10^{-12} \times 50 \times 50$ J

$\therefore \quad U_s = \mathbf{75 \times 10^{-10}\ J}$

$Q_s = C_s V = 6 \times 10^{-12} \times 50$

$= 300 \times 10^{-12}$ C $= \mathbf{3 \times 10^{-10}\ C}$

In parallel combination: $C_p = (12 + 12)$ pF



$\therefore \quad C_p = 24 \times 10^{-12}$ F

$U_s = \frac{1}{2} \times 24 \times 10^{-12} \times 2500$ J

$= \mathbf{3 \times 10^{-8}\ J}$

$Q_p = C_p V$

$Q_p = 24 \times 10^{-12} \times 50$ C

$Q_p = \mathbf{1.2 \times 10^{-9}\ C}$

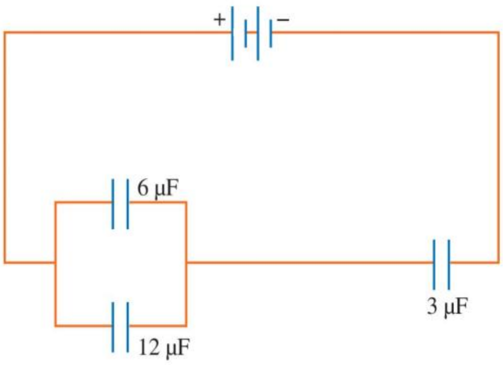
Q. 17. In the following arrangement of capacitors, the energy stored in the 6 μF capacitor is *E*. Find the value of the following:

(*i*) Energy stored in 12 μF capacitor.

(*ii*) Energy stored in 3 μF capacitor.

(*iii*) Total energy drawn from the battery.

[CBSE (F) 2016]



Ans. Given that energy stored in 6 μF is *E*.

(*i*) Let *V* be the voltage across 6 μF capacitor

Also, 6 μF and 12 μF capacitors are in parallel.

Therefore, voltage across 12 μF = Voltage across 6 μF capacitor

$$E = \frac{1}{2}CV^2 = \frac{1}{2} \times 6 \times V^2 \quad \Rightarrow \quad V = \sqrt{\frac{E}{3}}$$

Energy stored in $12\mu\text{F} = \frac{1}{2} \times 12 \times \left(\sqrt{\frac{E}{3}}\right)^2 = \mathbf{2E}$

(*ii*) Since charge remains constant in series. Sum of charge on 6 μF capacitor and 12 μF capacitor is equal to charge on 3 μF capacitor.

Using $Q = CV$,

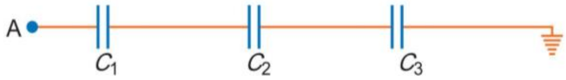
Charge on 3 μF capacitor = $(6 + 12) \times V = 18 \times V$

Energy stored in 3 F capacitor $= \frac{Q^2}{2C} = \frac{(18V)^2}{2 \times 3} = \frac{18 \times 18}{6}\left(\frac{\sqrt{E}}{3}\right)^2 = \mathbf{18E}$

(*iii*) Total energy drawn from battery = $E + 2E + 18E = \mathbf{21E}$

Q. 18. Calculate the potential difference and the energy stored in the capacitor C_2 in the circuit shown in the figure. Given potential at A is 90 V, $C_1 = 20\ \mu F$, $C_2 = 30\ \mu F$, $C_3 = 15\ \mu F$.

[CBSE Allahabad 2015]



Ans. Capacitors C_1, C_2 and C_3 are in series. So, its net capacitance is

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{20} + \frac{1}{30} + \frac{1}{15}$$

$$C_S = \frac{20}{3}\mu F$$

Net charge on the capacitors, C_1, C_2 and C_3 remain same.

$$q = C_S (V_A - V_E)$$

$$= \frac{20}{3}\mu F \times (90 - 0) = 600\,\mu C$$

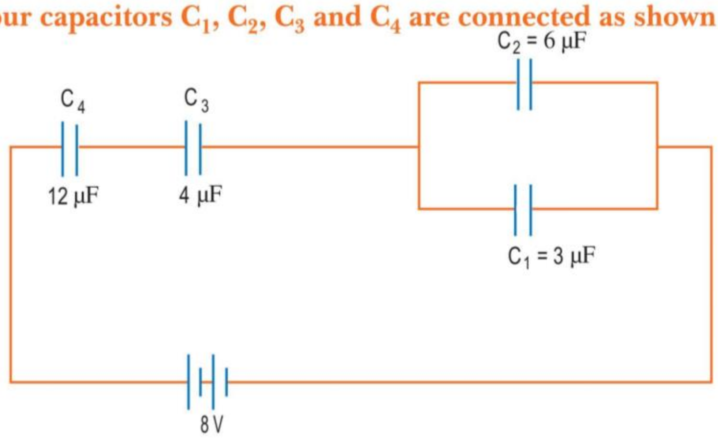
The $p.d$ across C_2 due to charge 600 μC is

$$V_2 = \frac{q}{C_2} = \frac{600}{30} = \mathbf{20\ V}$$

Energy stored in the capacitor C_2,

$$U_2 = \frac{1}{2}\frac{q^2}{C_2}\left(\text{or} \frac{1}{2}C_2 V_2^2\right) = \frac{1}{2} \times 30\,\mu F \times (20)^2 = 6000\,\mu J = \mathbf{6 \times 10^{-3}\ J}$$

Q. 19. In a network, four capacitors C_1, C_2, C_3 and C_4 are connected as shown in the figure.



(a) Calculate the net capacitance in the circuit.

(b) If the charge on the capacitor C_1 is 6 μC, (i) calculate the charge on the capacitors C_3 and C_4, and (ii) net energy stored in the capacitors C_3 and C_4 connected in series.

[CBSE 2019 (55/2/3)]

Ans. (a) Capacitance across C_3 & C_4

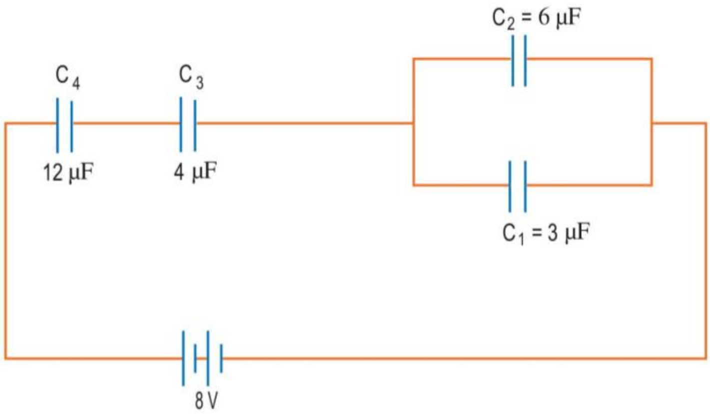
$$C_{34} = \frac{12 \times 4}{16} = 3\,\mu F$$

Capacitance across C_2 & C_1

$$C_{12} = 6 + 3 = 9\,\mu F$$

Equivalent capacitance

$$C_{eq} = \frac{9 \times 3}{12} = \mathbf{\frac{9\,\mu F}{4}}$$



(b) (i) $Q_1 = 6\,\mu C,\ V_1 = \frac{Q_1}{C_1}$

$$= \frac{6 \times 10^{-6}}{3 \times 10^{-6}} = 2\ V$$

$Q_2 = C_2 V_1 = 6 \times 10^{-6} \times 2 = 12\,\mu C$

As C_3 & C_4 are in series they carry a charge of $18\,\mu C$ each

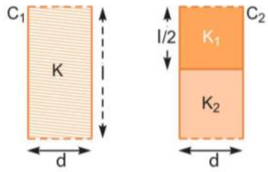
(*ii*) $Q = 18\,\mu C$

$C_{43} = 3\,\mu F$

$$E_{34} = \frac{1}{2}\frac{Q^2}{C_{34}} = \frac{1}{2} \times \frac{(18\times10^{-6})^2}{3\times10^{-6}}$$

$$E_{34} = \mathbf{54\times10^{-6}\,joule}$$

Q. 20. Two identical parallel plate (air) capacitors C_1 and C_2 have capacitances C each. The space between their plates is now filled with dielectrics as shown. If the two capacitors still have equal capacitance, obtain the relation between dielectric constants K, K_1 and K_2. *[HOTS] [CBSE (F) 2011]*



Ans. Let $A \to$ area of each plate.

Let initially $C_1 = C = \dfrac{\varepsilon_0 A}{d} = C_2$

After inserting respective dielectric slabs:

$$C'_1 = KC \qquad \ldots(i)$$

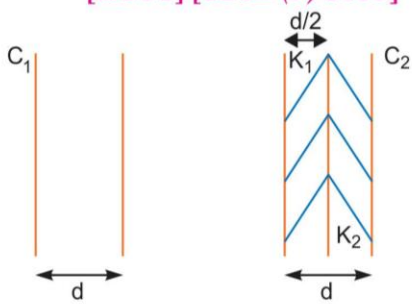
and $$C'_2 = K_1\frac{\varepsilon_0 (A/2)}{d} + \frac{K_2\varepsilon_0 (A/2)}{d} = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

$$C'_2 = \frac{C}{2}(K_1 + K_2) \qquad \ldots(ii)$$

From (*i*) and (*ii*)

$$C'_1 = C'_2 \Rightarrow KC = \frac{C}{2}(K_1 + K_2) \Rightarrow K = \frac{1}{2}(K_1 + K_2)$$

Q. 21. You are given an air filled parallel plate capacitor C_1. The space between its plates is now filled with slabs of dielectric constants K_1 and K_2 as shown in C_2. Find the capacitances of the capacitor C_2. if area of the plates is A and distance between the plates is d. *[HOTS] [CBSE (F) 2011]*



Ans. $C_1 = \dfrac{\varepsilon_0 A}{d}$

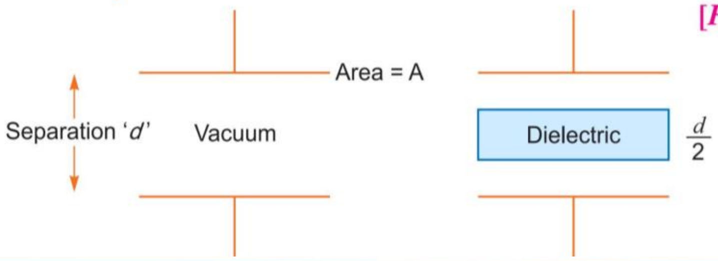
$$\frac{1}{C_2} = \frac{1}{K_1\frac{\varepsilon_0 A}{d/2}} + \frac{1}{K_2\frac{\varepsilon_0 A}{d/2}} = \frac{d}{2.K_1\varepsilon_0 A} + \frac{d}{2.K_2\varepsilon_0 A}$$

$$\frac{1}{C_2} = \frac{d}{2\varepsilon_0 A}\left[\frac{1}{K_1} + \frac{1}{K_2}\right] \Rightarrow C_2 = \frac{2.\varepsilon_0 A}{d}\left[\frac{K_1K_2}{K_1 + K_2}\right]$$

$$C_2 = 2C_1\left[\frac{K_1K_2}{K_1 + K_2}\right] \Rightarrow C_2 = C_1\left[\frac{2K_1K_2}{K_1 + K_2}\right]$$

Q. 22. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but has the thickness $d/2$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor. *[HOTS] [CBSE (AI) 2013]*

Ans.

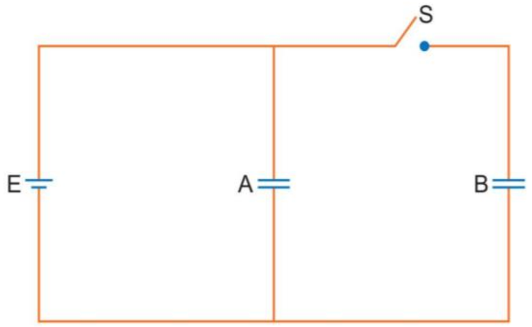


Capacitance with dielectric of thickness 't'

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} \qquad \text{Put } t = \frac{d}{2}$$

$$C = \frac{\varepsilon_0}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{\varepsilon_0 A}{\frac{d}{2} + \frac{d}{2K}} \qquad \Rightarrow \qquad \frac{\varepsilon_0 A}{\frac{d}{2}\left(1 + \frac{1}{K}\right)} = \frac{2\varepsilon_0 AK}{d(K+1)}$$

Q. 23. Two identical parallel plate capacitors A and B are connected to a battery of V volts with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. ***[CBSE (AI) 2017]***



Ans. Two capacitors are connected in parallel. Hence, the potential on each of them remains the same. So, the charge on each capacitor is

$$Q_A = Q_B = CV$$

Formula for energy stored $= \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

Net capacitance with switch S closed $= C + C = 2C$

$\therefore$ Energy stored $= \frac{1}{2} \times 2C \times V^2 = CV^2$

After the switch S is opened, capacitance of each capacitor $= KC$

In this case, voltage only across A remains the same.

The voltage across B changes to $V' = \frac{Q}{C'} = \frac{Q}{KC}$

$\therefore$ Energy stored in capacitor $A = \frac{1}{2}KCV^2$

Energy stored in capacitor $B = \frac{1}{2}\frac{Q^2}{KC} = \frac{1}{2}\frac{C^2V^2}{KC} = \frac{1}{2}\frac{CV^2}{K}$

$\therefore$ Total energy stored

$$= \frac{1}{2}KCV^2 + \frac{1}{2}\frac{CV^2}{K}$$

$$= \frac{1}{2}CV^2\left(K + \frac{1}{K}\right)$$

$$= \frac{1}{2}CV^2\left(\frac{K^2+1}{K}\right)$$

Required ratio $= \frac{2CV^2 . K}{CV^2(K^2+1)} = \frac{2K}{(K^2+1)}$

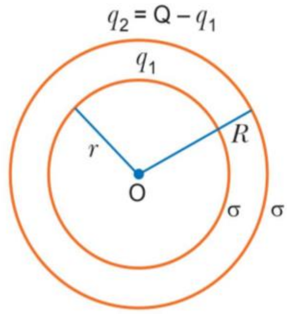
Q. 24. A charge Q is distributed over the surfaces of two concentric hollow spheres of radii r and R ($R >> r$), such that their surface charge densities are equal. Derive the expression for the potential at the common centre. ***[CBSE 2019 (55/5/1)]***

Ans. If charge q_1 is distributed over the smaller sphere and q_2 over the larger sphere, then

$$Q = q_1 + q_2 \qquad \ldots(i)$$

If σ is the surface charge density of the two spheres, then

$$\sigma = \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$$



or $\quad q_1 = 4\pi r^2\sigma \quad$ and $\quad q_2 = 4\pi R^2\sigma$

From (*i*), we have

$$Q = 4\pi r^2\sigma + 4\pi R^2\sigma$$
$$= 4\pi\sigma(r^2 + R^2)$$

or $\quad \sigma = \dfrac{Q}{4\pi(r^2 + R^2)}$

The potential at a point inside the charged sphere is equal to the potential at its surface.

So, the potential due to the smaller sphere at the common centre,

$$V_1 = \frac{1}{4\pi\varepsilon_0}.\frac{q_1}{r}$$

Also, the potential due to the larger sphere at the common centre,

$$V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{q_2}{R}$$

$\therefore$ Potential at common centre

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{R}\right)$$

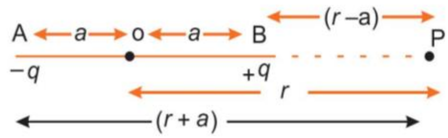
$$= \frac{1}{4\pi\varepsilon_0} \times \left[\frac{4\pi r^2\sigma}{r} + \frac{4\pi R^2\sigma}{R}\right]$$

$$= \frac{(r + R)\sigma}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q(r + R)}{r^2 + R^2}\right] \text{ (By putting the value of } \sigma)$$

Q. 25. (*a*) Derive an expression for the electric potential at any point along the axial line of an electric dipole.

(*b*) Find the electrostatic potential at a point on equatorial line of an electric dipole.

Ans. (*a*) Potential at point P



$$V_P = V_{-q} + V_{+q}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r + a)} + \frac{1}{4\pi\varepsilon_0}\frac{q}{(r - a)}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r - a)} - \frac{1}{(r + a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r + a - r + a}{(r - a)(r + a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0} \times \frac{2a}{(r^2 - a^2)} = \frac{q \times 2a}{4\pi\varepsilon_0(r^2 - a^2)}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{p}{(r^2 - a^2)} \text{ (where } p \text{ is the dipole moment)}$$

For a short dipole, $a^2 << r^2$, so $V = V = \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{p}{r^2}$

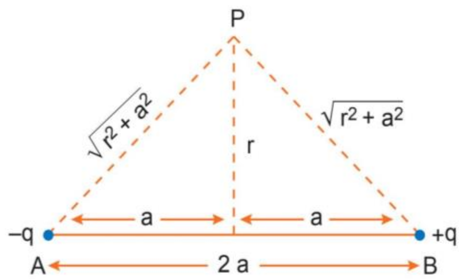
(*b*) Let *P* be a point on the equatorial line of an electric dipole due to charges $-q$ and $+q$ with separation $2a$

The distance of point P from centre of dipole = r

$$AP = BP = \sqrt{r^2 + a^2}$$

Electrostatic potential at P, $V_P = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{BP} - \frac{q}{AP}\right)$

$$\Rightarrow \quad V_P = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{r^2+a^2}} - \frac{q}{\sqrt{r^2+a^2}}\right] = \mathbf{0}$$



That is electrostatic potential at each equatorial point of an electric dipole is **zero**.

Q. 26. If N drops of same size each having the same charge, coalesce to form a bigger drop. How will the following vary with respect to single small drop? [*CBSE Sample Paper* 2017]

(*i*) Total charge on bigger drop

(*ii*) Potential on the bigger drop

(*iii*) Capacitance

Ans. Let r, q and v be the radius, charge and potential of the small drop.

The total charge on bigger drop is sum of all charge on small drops.

(*i*) $\therefore \quad Q = \boldsymbol{Nq}$ (where Q is charge on bigger drop)

(*ii*) The volume of N small drops $= N\frac{4}{3}\pi r^3$

Volume of the bigger drop $\frac{4}{3}\pi R^3$

Hence, $\quad N\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \quad \Rightarrow \quad R = N^{1/3}r$

Potential on bigger drop, $V = \frac{1}{4\pi\epsilon_0} \times \frac{Q}{R}$

$$= \frac{1}{4\pi\epsilon_0}\frac{Nq}{N^{1/3}r} = \frac{1}{4\pi\epsilon_0}\frac{N^{2/3}.q}{r}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r}.N^{2/3} = \boldsymbol{N^{2/3}v} \quad \left[\therefore v = \frac{1}{4\pi\epsilon_0}\frac{q}{r}\right]$$

(*iii*) Capacitance $= 4\pi\varepsilon_0 R$

$= 4\pi\varepsilon_0 N^{1/3} r$

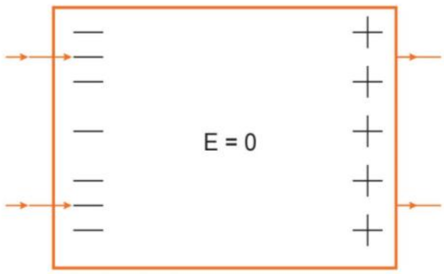
$= N^{1/3}\,(4\pi\varepsilon_0 r)$

$= \mathbf{N^{1/3}C}$ [where C is capacitance of the small drop]

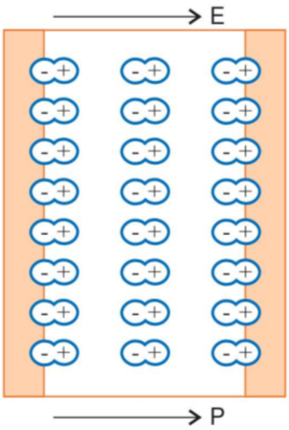
Q. 27. (*a*) Explain briefly, using a proper diagram, the difference in behaviour of a conductor and a dielectric in the presence of external electric field.

(*b*) Define the term polarization of a dielectric and write the expression for a linear isotropic dielectric in terms of electric field. [*CBSE 2019 (55/3/1)*]

Ans. (*a*) **For conductor:** Due to induction the free electrons collect on the left face of slab creating equal positive charge on the right face. Internal electric field is equal and opposite to external field; hence net electric field (inside the conductor) is zero.



For dielectric: Due to alignment of atomic dipoles along $\vec{E}$, the net electric field within the dielectric decreases.



(*b*) The net dipole moment developed per unit volume in the presence of external electric field is called polarization vector $\vec{P}$.

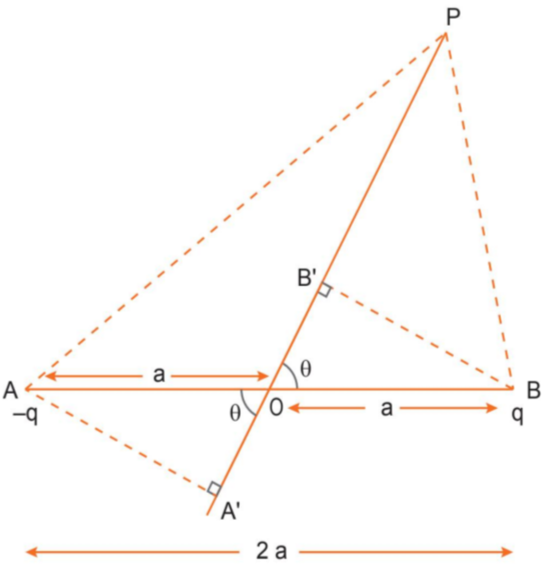
Expression: $\vec{P} = \chi_e \vec{E}$

Long Answer Questions [5 marks]

Q. 1. Derive an expression for the electric potential at a point due to an electric dipole. Mention the contrasting features of electric potential of a dipole at a point as compared to that due to a single charge. **[CBSE Delhi 2008, 2017]**

Ans. Potential at a point due to a dipole.

Suppose, the negative charge $-q$ is placed at a point A and the positive charge q is placed at a point B (fig.), the separation $AB = 2a$. The middle point of AB is O. The potential is to be evaluated at a point P where $OP = r$ and $\angle POB = \theta$. Also, let $r >> a$.



Let AA' be the perpendicular from A to PO and BB' be be the perpendicular from B to PO. Since a is very small compared to r,

$$AP = A'P = OP + OA'$$
$$= OP + AO\cos\theta$$
$$= r + a\cos\theta$$

Similarly, $BP = B'P = OP - OB'$

$$= r - a\cos\theta$$

The potential at P due to the charge $-q$ is

$$V_1 = -\frac{1}{4\pi\varepsilon_0}\frac{q}{AP} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r + a\cos\theta}$$

The potential at P due to the charge q is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r - a\cos\theta}$$

The net potential at P due to the dipole is

$$V = V_1 + V_2$$
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r - a\cos\theta} - \frac{q}{r + a\cos\theta}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q\,2a\cos\theta}{r^2 - a^2\cos^2\theta}$$

$$V = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2}$$

Special Cases:

(*i*) When point *P* lies on the axis of dipole, then $\theta = 0°$

$\therefore \quad \cos\theta = \cos 0^0 = 1$

$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$

(*ii*) When point *P* lies on the equatorial plane of the dipole, then

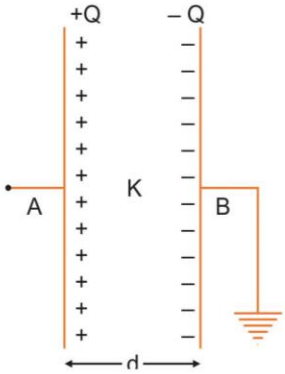
$\therefore$ cos θ =cos 90°= 0

$\therefore \quad V = 0$

It may be noted that the electric potential at any point on the equitorial line of a dipole is zero.

Q. 2. Briefly explain the principle of a capacitor. Derive an expression for the capacitance of a parallel plate capacitor, whose plates are separated by a dielectric medium.

Ans. **Principle of a Capacitor:** A capacitor works on the principle that the capacitance of a conductor increases appreciably when an earthed conductor is brought near it.



Parallel Plate Capacitor: Consider a parallel plate capacitor having two plane metallic plates A and B, placed parallel to each other (see fig.). The plates carry equal and opposite charges +Q and –Q respectively.

In general, the electric field between the plates due to charges +Q and –Q remains uniform, but at the edges, the electric field lines deviate outward. If the separation between the plates is much smaller than the size of plates, the electric field strength between the plates may be assumed uniform.

Let *A* be the area of each plate, '*d*' the separation between the plates, K the dielectric constant of medium between the plates. If σ is the magnitude of charge density of plates, then

$$\sigma = \frac{Q}{A}$$

The electric field strength between the plates

$$E = \frac{\sigma}{K\epsilon_0} \text{ where } \varepsilon_0 = \text{permittivity of free space.} \quad ...(i)$$

The potential difference between the plates, $V_{AB} = Ed = \frac{\sigma d}{K\varepsilon_0}$ ...(*ii*)

Putting the value of σ, we get

$$V_{AB} = \frac{(Q/A)d}{K\,\varepsilon_0} = \frac{Qd}{K\,\varepsilon_0 A}$$

$\therefore$ Capacitance of capacitor,

$$C = \frac{Q}{V_{AB}} = \frac{Q}{(Qd/K\varepsilon_0 A)} \text{ or } C = \frac{K\,\varepsilon_0 A}{d} \quad ...(iii)$$

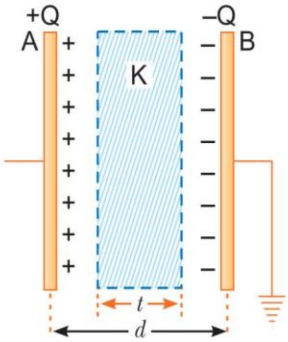
This is a general expression for capacitance of parallel plate capacitor. Obviously, the capacitance is directly proportional to the dielectric constant of medium between the plates.

For air capacitor (*K*=1); capacitance $C = \frac{\varepsilon_0 A}{d}$. This is expression for the capacitance of a parallel plate air capacitor. It can be seen that the capacitance of parallel plate (air) capacitor is

(*a*) directly proportional to the area of each plate.
(*b*) inversely proportional to the distance between the plates.
(*c*) independent of the material of the plates.

Q. 3. Derive an expression for the capacitance of a parallel plate capacitor when a dielectric slab of dielectric constant K and thickness $t = \frac{d}{2}$ but of same area as that of the plates is inserted between the capacitor plates. (d = separation between the plates). [*CBSE (F) 2010*]

Ans. Consider a parallel plate capacitor, area of each plate being A, the separation between the plates being d. Let a dielectric slab of dielectric constant K and thickness $t < d$ be placed between the plates. The thickness of air between the plates is $(d - t)$. If charges on plates are $+Q$ and $-Q$, then surface charge density



$$\sigma = \frac{Q}{A}$$

The electric field between the plates in air, $E_1 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

The electric field between the plates in slab, $E_2 = \frac{\sigma}{K\varepsilon_0} = \frac{Q}{K\varepsilon_0 A}$

∴ The potential difference between the plates

V_{AB} = work done in carrying unit positive charge from one plate to another

$=\Sigma Ex$ (as field between the plates is not constant).

$$= E_1(d-t) + E_2 t = \frac{Q}{\varepsilon_0 A}(d-t) + \frac{Q}{K\varepsilon_0 A}t$$

$$\therefore \quad V_{AB} = \frac{Q}{\varepsilon_0 A}\left[d - t + \frac{t}{K}\right]$$

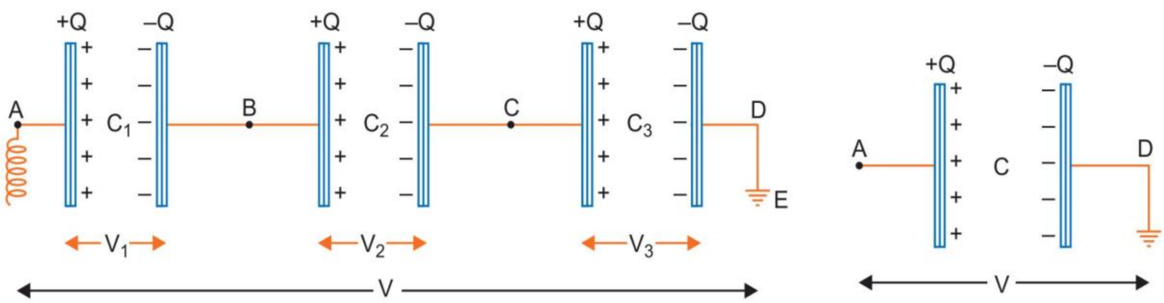
$$\therefore \quad \text{Capacitance of capacitor, } C = \frac{Q}{V_{AB}} = \frac{Q}{\frac{Q}{\varepsilon_0 A}\left(d - t + \frac{t}{K}\right)}$$

or,
$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

Here, $t = \frac{d}{2}$ ∴ $C = \frac{\varepsilon_0 A}{d - \frac{d}{2}\left(1 - \frac{1}{K}\right)} = \frac{\boldsymbol{\varepsilon_0 A}}{\boldsymbol{\frac{d}{2}\left(1 + \frac{1}{K}\right)}}$

Q. 4. Derive an expression for equivalent capacitance of three capacitors when connected (*i*) in series and (*ii*) in parallel.

Ans. (*i*) In fig. (*a*) three capacitors of capacitances C_1, C_2, C_3 are connected in series between points A and D.



(a) (b)

In series first plate of each capacitor has charge $+Q$ and second plate of each capacitor has charge $-Q$ *i.e.*, charge on each capacitor is Q.

Let the potential differences across the capacitors C_1, C_2, C_3 be V_1, V_2, V_3 respectively. As the second plate of first capacitor C_1 and first plate of second capacitor C_2 are connected together, their potentials are equal. Let this common potential be V_B. Similarly the common potential of second plate of C_2 and first plate of C_3 is V_C. The second plate of capacitor C_3 is connected to earth, therefore its potential $V_D=0$. As charge flows from higher potential to lower potential, therefore $V_A>V_B>V_C>V_D$.

For the first capacitor, $V_1 = V_A - V_B = \frac{Q}{C_1}$ ...(*i*)

For the second capacitor, $V_2 = V_B - V_C = \frac{Q}{C_2}$ ...(*ii*)

For the third capacitor, $V_3 = V_C - V_D = \frac{Q}{C_3}$ ...(*iii*)

Adding (*i*), (*ii*) and (*iii*), we get

$$V_1 + V_2 + V_3 = V_A - V_D = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad ...(iv)$$

If V be the potential difference between A and D, then

$$V_A - V_D = V$$

$\therefore$ From (*iv*), we get

$$V = (V_1 + V_2 + V_3) = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad ...(v)$$

If in place of all the three capacitors, only one capacitor is placed between A and D such that on giving it charge Q, the potential difference between its plates become V, then it will be called **equivalent capacitor**. If its capacitance is C, then

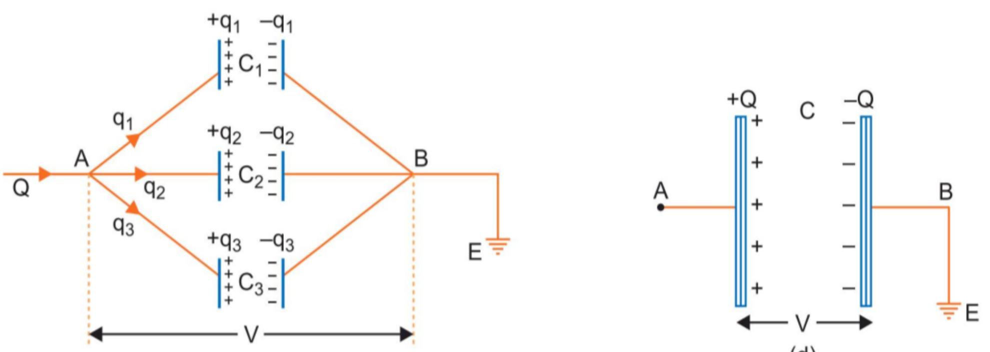
$$V = \frac{Q}{C} \quad ...(vi)$$

Comparing (*v*) and (*vi*), we get

$$\frac{Q}{C} = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad \text{or} \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad ...(vii)$$

Thus in series arrangement, *"The reciprocal of equivalent capacitance is equal to the sum of the reciprocals of the individual capacitors."*

(*ii*) Parallel Arrangement: In fig. (*c*) three capacitors of capacitance C_1, C_2, C_3 are connected in parallel.



(c) (d)

In parallel the potential difference across each capacitor is same V (say). Clearly the potential difference between plates of each capacitor

$$V_A - V_B = V \text{ (say)}$$

The charge Q given to capacitors is divided on capacitors C_1, C_2, C_3.

Let q_1, q_2, q_3 be the charges on capacitors C_1, C_2, C_3 respectively.

Then $Q = q_1 + q_2 + q_3$ ...(*i*)

and $q_1 = C_1V, q_2 = C_2V, q_3 = C_3V$

Substituting these values in (*i*), we get

$$Q = C_1V + C_2V + C_3V \quad or \quad Q = (C_1 + C_2 + C_3)V \qquad ...(ii)$$

If, in place of all the three capacitors, only one capacitor of capacitance C be connected between A and B; such that on giving it charge Q, the potential difference between its plates be V, then it will be called **equivalent capacitor.** If C be the capacitance of equivalent capacitor, then

$$Q = CV \qquad ...(iii)$$

Comparing equations (*ii*) and (*iii*), we get

$CV=(C_1+C_2+C_3)V$ or $C=(C_1+C_2+C_3)$ ...(*iv*)

Important Note: It may be noted carefully that the formula for the total capacitance in series and parallel combination of capacitors is the reverse of corresponding formula for combination of resistors in current electricity.

Q. 5. (*a*) Derive an expression for the energy stored in a parallel plate capacitor *C*, charged to a potential difference *V*. Hence derive an expression for the energy density of a capacitor.

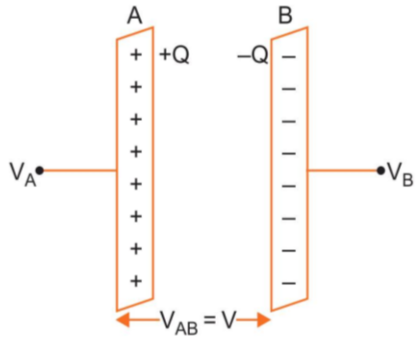
[*CBSE (AI) 2012, (F) 2013, Allahabad 2015, 2020(55/3/1)*]

OR

Obtain an expression for the energy stored per unit volume in a charged parallel plate capacitor.

(*b*) Find the ratio of the potential differences that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 2 so that the energy stored in the two cases becomes the same. [*CBSE Central 2016*]

Ans. (*a*) When a capacitor is charged by a battery, work is done by the charging battery at the expense of its chemical energy. This work is stored in the capacitor in the form of electrostatic potential energy.



Consider a capacitor of capacitance C. Initial charge on capacitor is zero. Initial potential difference between capacitor plates is zero. Let a charge Q be given to it in small steps. When charge is given to capacitor, the potential difference between its plates increases. Let at any instant when charge on capacitor be q, the potential difference between its plates $V = \frac{q}{C}$.

Now work done in giving an additional infinitesimal charge dq to capacitor.

$$dW = V\,dq = \frac{q}{C}dq$$

The total work done in giving charge from 0 to Q will be equal to the sum of all such infinitesimal works, which may be obtained by integration. Therefore total work

$$W = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{C}\left(\frac{Q^2}{2} - \frac{0}{2}\right) = \frac{Q^2}{2C}$$

If V is the final potential difference between capacitor plates, then $Q=CV$

$$\therefore \quad W = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

This work is stored as electrostatic potential energy of capacitor *i.e.*,

Electrostatic potential energy, $U = \dfrac{Q^2}{2C} = \dfrac{1}{2}CV^2 = \dfrac{1}{2}QV$

Energy density: Consider a parallel plate capacitor consisting of plates, each of area A, separated by a distance d. If space between the plates is filled with a medium of dielectric constant K, then

Capacitance of capacitor, $C = \dfrac{K\varepsilon_0 A}{d}$

If σ is the surface charge density of plates, then electric field strength between the plates

$E = \dfrac{\sigma}{K\varepsilon_0} \Rightarrow \sigma = K\varepsilon_0 E$

Charge on each plate of capacitor, $Q = \sigma A = K\varepsilon_0 EA$

Energy stored by capacitor, $U = \dfrac{Q^2}{2C} = \dfrac{(K\varepsilon_0 EA)^2}{2(K\varepsilon_0 A/d)} = \dfrac{1}{2}K\varepsilon_0 E^2 Ad$

But Ad = volume of space between capacitor plates

$\therefore$ Energy stored, $U = \dfrac{1}{2}K\varepsilon_0 E^2 Ad$

Electrostatic Energy stored per unit volume, $u_e = \dfrac{U}{Ad} = \dfrac{1}{2}K\varepsilon_0 E^2$

This is expression for electrostatic energy density in medium of dielectric constant K.

In air or free space (K=1) therefore energy density, $u_e = \dfrac{1}{2}\varepsilon_0 E^2$

(*b*) $U_S = \dfrac{1}{2}C_S V_S^2$, $U_P = \dfrac{1}{2}C_P V_P^2$

Also, $\dfrac{C_1}{C_2} = \dfrac{1}{2}$(given) $\Rightarrow C_2 = 2C_1$

$$U_S = U_P$$

$$\Rightarrow \quad \frac{V_{series}}{V_{parallel}} = \sqrt{\frac{C_{equivalent\ parallel}}{C_{equivalent\ series}}}$$

$$= \sqrt{\frac{C_1 + C_2}{\frac{C_1 C_2}{C_1 + C_2}}}$$

$$= \frac{C_1 + C_2}{\sqrt{C_1 C_2}} = \frac{3C_1}{\sqrt{2C_1^2}} = \frac{3}{\sqrt{2}}$$

Q. 6. Find the expression for the energy stored in the capacitor. Also find the energy lost when the charged capacitor is disconnected from the source and connected in parallel with the uncharged capacitor. Where does this loss of energy appear? *[CBSE Sample Paper 2017]*

Ans. Refer to Q. 5 (a), Page number 98.

Let a charged capacitor of capacitance C_1 is charged by a cell of emf V volt. When this capacitor is connected with uncharged capacitor C_2 and charge distributes between capacitors still they acquire common potential say V_0 volt.

Energy stored in C_1, $U_i = \dfrac{1}{2}\ C_1\ V^2$

Charge on other capacitor of capacitance C_2 is $q_2 = C_2\ V_0$

But total charge on pair of plates committed together remains constant equal to $Q = q_1 + q_2$

$$Q = C_1\ V = C_1\ V_0 + C_2\ V_0$$

where, V_0 = common potential

$$V_0 = \frac{C_1 V}{C_1 + C_2}$$

Energy stored in both capacitor, $U_2 = \frac{1}{2}(C_1 + C_2)\times\left(\frac{C_1 V}{C_1 + C_2}\right)^2$

$$= \frac{1}{2}\frac{C_1^2 V^2}{C_1 + C_2}$$

Loss of energy $H = U_1 - U_2 = \frac{1}{2}C_1 V^2 - \frac{1}{2}\frac{C_1^2 V^2}{C_1 + C_2}$

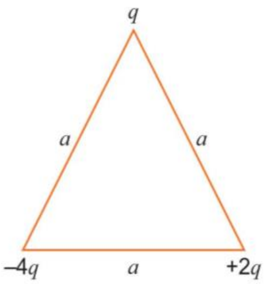
$$= \frac{1}{2}C_1 V^2\left(1 - \frac{C_1}{C_1 + C_2}\right) = \mathbf{\frac{C_1 C_2 V^2}{2C_1 + C_2}}$$

The lost energy appears in the form of heat.

Q. 7. (*a*) **Explain why, for any charge configuration, the equipotential surface through a point is normal to the electric field at that point.**

Draw a sketch of equipotential surfaces due to a single charge (– q), depicting the electric field lines due to the charge.

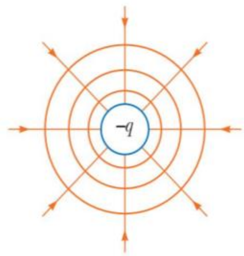
(*b*) **Obtain an expression for the work done to dissociate the system of three charges placed at the vertices of an equilateral triangle of side 'a' as shown below.** [*CBSE North 2016*]



Ans. (*a*) The work done in moving a charge from one point to another on an equipotential surface is zero. If the field is not normal to an equipotential surface, it would have a non zero component along the surface. This would imply that work would have to be done to move a charge on the surface which is contradictory to the definition of equipotential surface.

Mathematically

Work done to move a charge dq on a surface can be expressed as



$$dW = dq(\vec{E}.\vec{dr})$$

But $dW = 0$ on an equipotential surface

$\therefore \quad \vec{E} \perp \vec{dr}$

Equipotential surfaces for a charge $-q$ is shown alongside.

(*b*) Work done to dissociate the system = – Potential energy of the system

$$= \frac{-1}{4\pi\varepsilon_0}\left[\frac{(-4q)(q)}{a} + \frac{(2q)(q)}{a} + \frac{(-4q)(2q)}{a}\right]$$

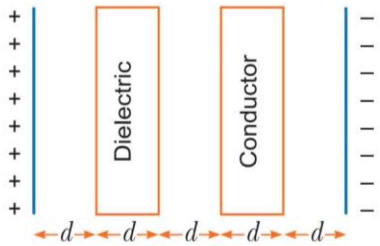
$$= -\frac{1}{4\pi\varepsilon_0}\left[-4q^2 + 2q^2 - 8q^2\right] = +\left[\mathbf{\frac{10q^2}{4\pi\varepsilon_0 a}}\right]$$

Q. 8. **(*i*) Compare the individual dipole moment and the specimen dipole moment for H_2O molecule and O_2 molecule when placed in**

(*a*) Absence of external electric field

(*b*) Presence of external eclectic field. Justify your answer.

(*ii*) Given two parallel conducting plates of area *A* and charge densities $+\sigma$ and $-\sigma$. A dielectric slab of constant *K* and a conducting slab of thickness *d* each are inserted in between them as shown.



(*a*) Find the potential difference between the plates.

(*b*) Plot *E* versus *x* graph, taking $x = 0$ at positive plate and $x = 5d$ at negative plate.

[CBSE Sample Paper 2016]

Ans. (*i*)

	Non-Polar (O_2)	Polar (H_2O)
(a) Absence of electric field		
Individual	No dipole moment exists	Dipole moment exists
Specimen	No dipole moment exists	Dipole are randomly oriented. Net P = 0
(b) Presence of electric field		
Individual	Dipole moment exists (molecules become polarised)	Torque acts on the molecules to align them parallel to $\vec{E}$
Specimen	Dipole moment exists	Net dipole moment exists parallel to $\vec{E}$

(*ii*) (*a*) The potential difference between the plates is given by

$$V = E_0 d + \frac{E_0}{K} d + E_0 d + 0 + E_0 d \quad \Rightarrow \quad V = 3E_0 d + \frac{E_0}{K} d$$

(*b*) *E* versus *x* graph