AEP STUDY CIRCLE

Since 2001...

IIT-JEE | NEET | CBSE

ACCENTS EDUCATIONAL PROMOTERS

OFFLINE-ONLINE LEARNING ACADEMY

PHYSICS

CBSE-IX

GRAVITATION

FOUNDATION

CBSE-IX-PHYSICS-GRAVITATION

IIT MAINS & ADV

NEET

CBSE

The Success Destination...

CBSE-FND

GRAVITATION

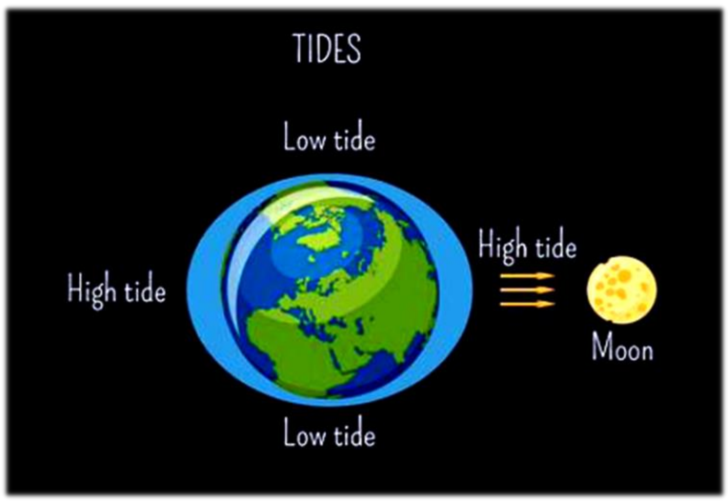
Call Us:
+91 99395 86130, +91 7739650505

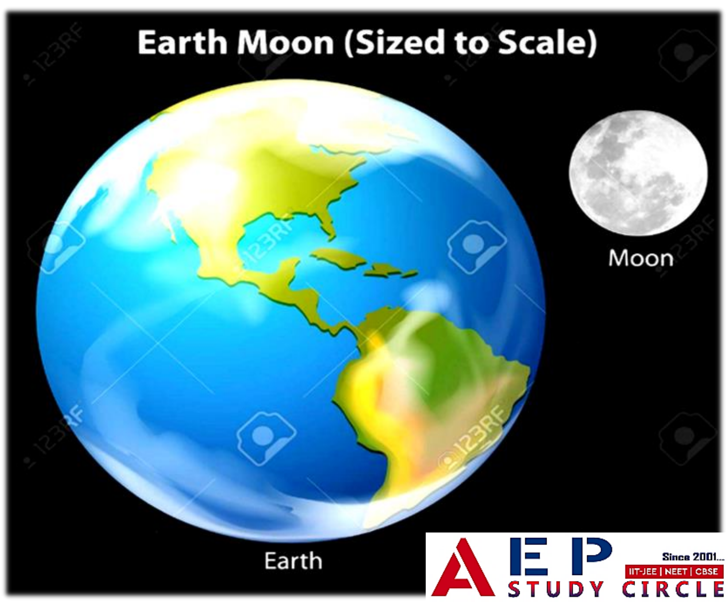
Visit Us:
www.aepstudycircle.com

2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH, JHARKAHAND - 829122

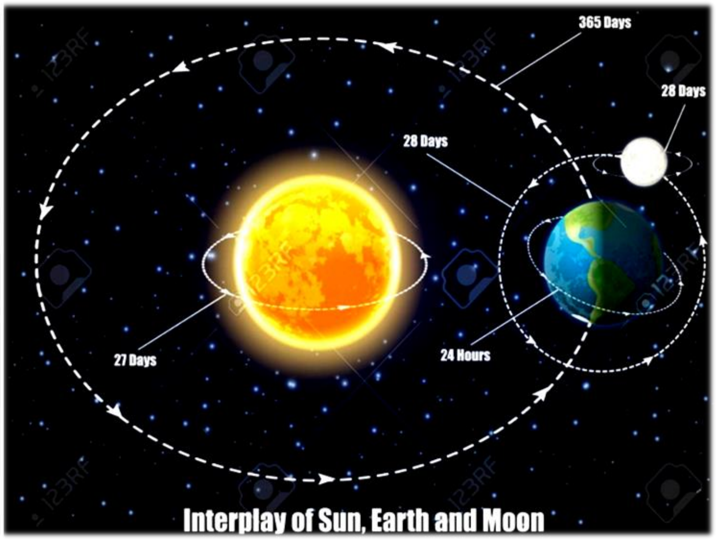
GRAVITATION

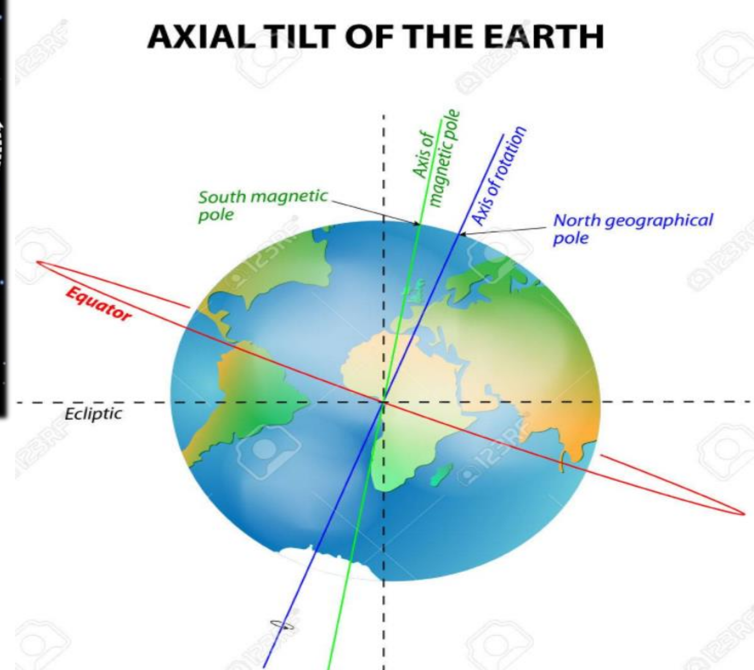
In this chapter we shall study about gravitation and universal law of gravitation. We will discuss about the motion of the body under the influence of gravitation. How the weight of the body varies from one place to another? It was **Sir Isaac** Newton who first gave the term gravitation when he saw an apple falling from a tree and thought that **'why an apple fall towards the earth and not going otherwise'**. Later he realized that it is because earth attracts every object towards itself with a force called **the gravitational force**. Later on the generalized the idea and said that not only the earth, but every other object in this universe attract each other with a force. Our knowledge about gravitation did not develop only as a result of Newton's observation of a falling apple. Before Newton there were many scholars who had been using the term **Gurutva akarshan** in India. Before Newton many scientist such as **Aryabhatta and Kepler** studied about the motion of the planet. It was Kepler who described 'how the planet moves around the sun?'.
Newton's laws of motion helps us to understand about the force which cause a planet's motion.

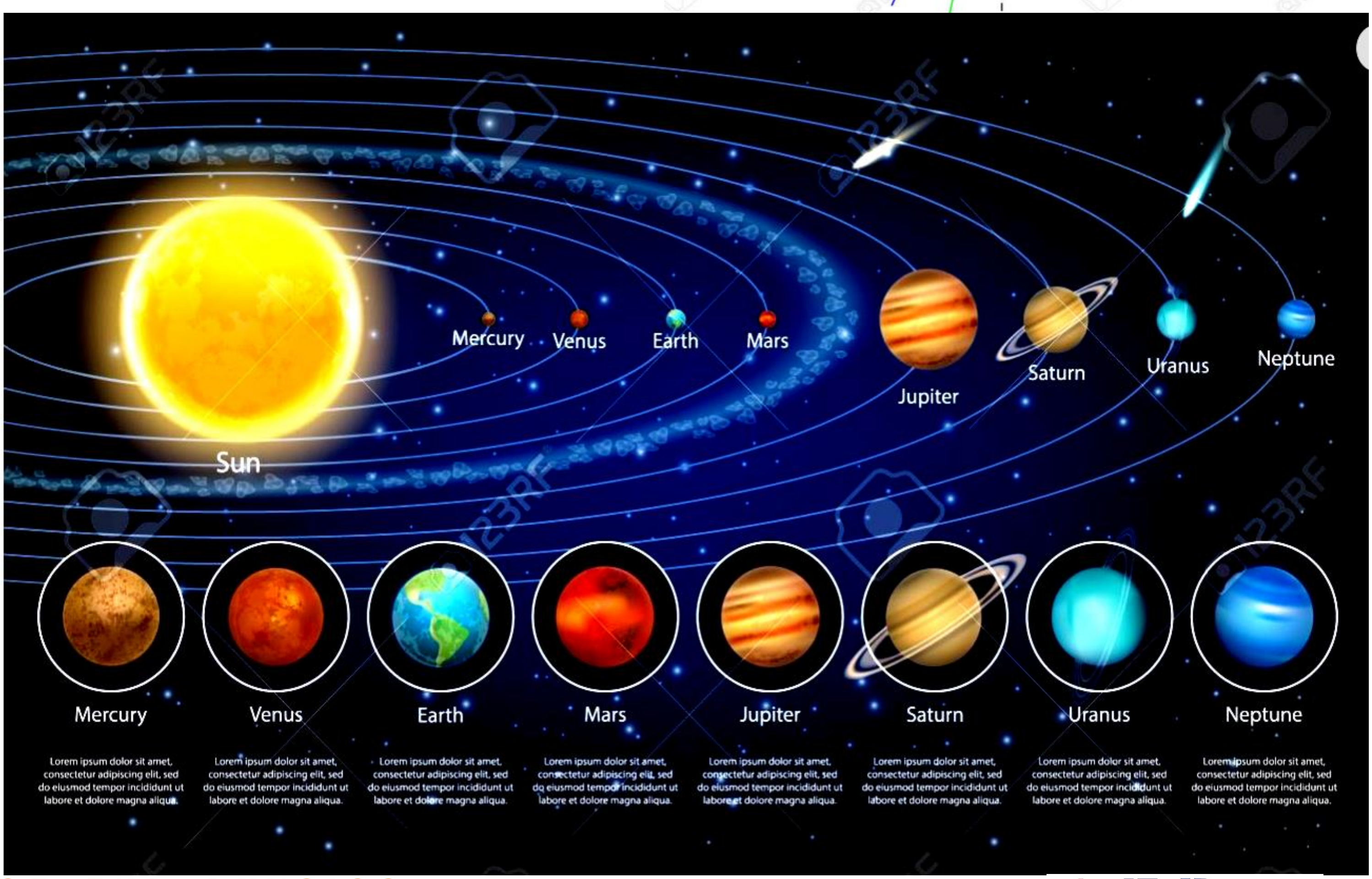


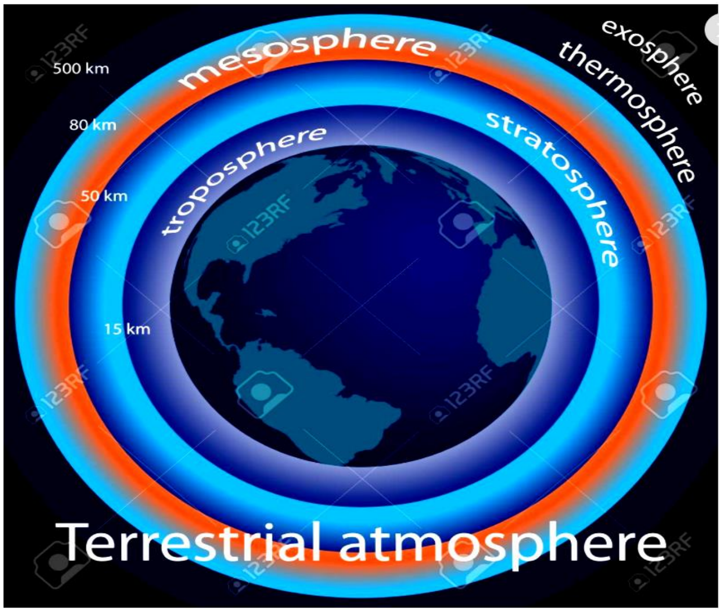


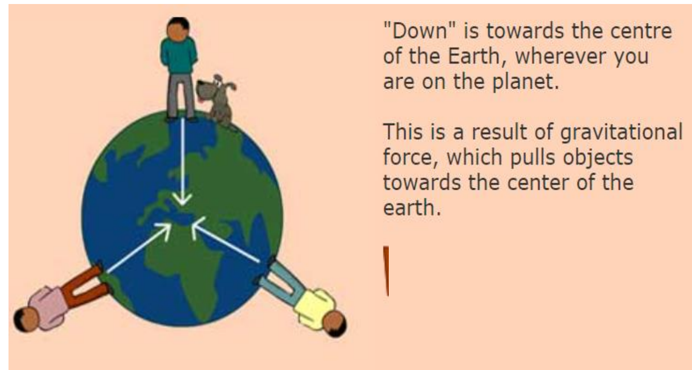
AEP STUDY CIRCLE
Since 2001...
IIT-JEE | NEET | CBSE
ACCENTS EDUCATIONAL PROMOTERS

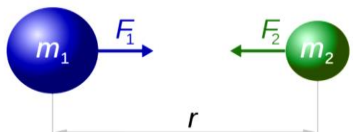

365 Days
28 Days
28 Days
27 Days
24 Hours
Interplay of Sun, Earth and Moon


AXIAL TILT OF THE EARTH
Axis of magnetic pole
Axis of rotation
South magnetic pole
North geographical pole
Equator
Ecliptic

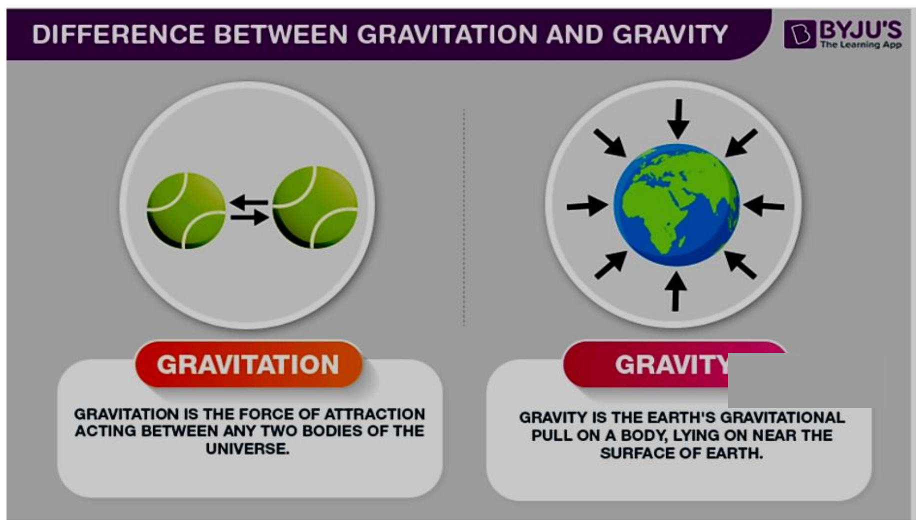

Mercury
Venus
Earth
Mars
Jupiter
Saturn
Uranus
Neptune
Sun
Mercury
Venus
Earth
Mars
Jupiter
Saturn
Uranus
Neptune







$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

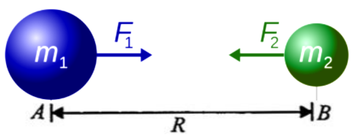


- Since, the gravitational force of the earth pulls the object in downward direction therefore, a force has to applied by us to raise an object above the surface of the earth to overcome the force of gravity.
- NEWTON generalised that, not only the earth, but any object in the universe attracts any other object.
- NEWTON also state that all object attracts each other along the line joining their centre with a force called gravitation force.
- Gravitational always acts between two bodies even there is nothing connecting the two bodies
- . If, the masses of the object are small, the force of gravitation between them is also small and cannot be detected easily.

Universal Law Of Gravitation

According to this law, the force of attraction between two particles or bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between these particles or bodies.

Consider two bodies A and B having masses m_1 and m_2 respectively. Let the distance between these bodies be R.



According to the law of gravitation, the force of attraction (F) or force of gravitation between these bodies is given by

$f \propto m_1 m_2$ ...(i)

and $F \propto \frac{1}{R^2}$ ...(ii)

Combining equations (i) and (ii), we get

$F \propto \frac{m_1 m_2}{R^2}$ or $F = G\frac{m_1 m_2}{R^2}$ ...(iii)

where G is constant and is known as Universal Gravitational Constant.

Equation (iii) is the mathematical form of Newton's law of gravitation. Here the magnitude of the force varies inversely with the square of the distance between the two particles. So the force law is also called Inverse Square Law.

According to the **Inverse Square Law,**

Force of gravitation $\propto \frac{1}{R^2}$

If the distance between the two particles is doubled, then the gravitational force becomes one-fourth.

When the distance is R, then Force, $F \propto \frac{1}{R^2}$

When the distance is 2R (doubled), then $F' \propto \frac{1}{(2R)^2}$

From these two relationships, one can write $\frac{F'}{F} = \frac{1}{(2R)^2} \times \frac{R^2}{1} = \frac{1}{4}$ or $F' = \frac{F}{4}$

Thus, when the distance between the two particles is doubled, the gravitational force becomes one-fourth.

If the distance between the two particles is halved, then the gravitational force becomes four times.

When the distance is R, then $F \propto \frac{1}{R^2}$

When the distance is R/2 (halved), then $F' \propto \frac{1}{(R/2)^2}$

From these relationships,

$$\frac{F'}{F} = \frac{\frac{1}{(R/2)^2}}{1/R^2} = \frac{R^2 \times 4}{R^2} = 4 \quad F' = 4F$$

Thus, when the distance between the two particles is halved, the gravitational force becomes four times.

Unit and Value of Gravitational Constant

According to Newton's law of gravitation, force between two bodies of masses m_1 and m_2 separated by a distance R is given by

$F = \frac{Gm_1 m_2}{R^2}$ ...(i)

Rearranging, we get $Gm_1 m_2 = F \times R^2$

or $G = \frac{F \times R^2}{m_1 m_2}$ ...(ii)

The SI unit of G can be obtained by putting the units of force distance and mass we get,

$G = N\,m^2 kg^{-2}$ If $m_1 = m_2 = 1$ kg, $R = 1m$,

From equation (ii) $G = \frac{F \times (1)^2}{1 \times 1} = F$

Hence, we can say that Universal Gravitational Constant G is numerically equal to the gravitational force of attraction between two bodies. The value of G in SI is $6.67 \times 10^{-11}\,\mathrm{N\,m^2kg^{-2}}$ and in CGS

ILLUSTRATION

1. A sphere of mass 40 kg is attracted by a second sphere of mass 15 kg when their centers are 20 cm apart, with a force of 0.1 milligram weight. Calculate the value of gravitational constant.

Soln.: Here, $m_1 = 40kg, m_2 = 15kg$

$R = 20cm = \frac{20}{100}m = 2 \times 10^{-1} m$

$F = 0.1$ milligram weight $= 0.1 \times 10^{-3}$ gram weight $= 10^{-4} \times 10^{-3} kg\,wt = 10^{-7} \times 9.8N$

$(1kg\,wt = 9.8N)$ From $F = \frac{Gm_1 m_2}{R^2}$

$G = \frac{F \times R^2}{m_1 m_2} = \frac{10^{-7} \times 9.8 \times (2 \times 10^{-1})^2}{40 \times 15}$

$F = 6.53 \times 10^{-11} N\,m^2 kg^{-2}$

DO U KNOW. . .

If the value of G becomes 10 times its present value, then we would be crushed to the floor by the earth's attraction. If the value of G becomes 1/10th of its present value, then the earth's attract would become very weak and in that case we can jump over a building with ease.

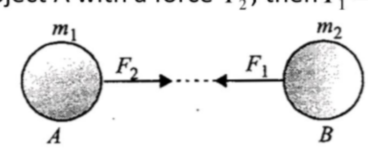
Importance of the Universal Law of Gravitation

The gravitational force is one of the fundamental forces in nature. The gravitational force is responsible, for the following:

- the existence of the solar system (motion of planets around the sun).
- holding the atmosphere near the surface of the earth.
- the flow of water in rivers.
- for rainfall and snowfall.
- motion of the moon around the earth.
- occurrence of tides.
- prediction about solar and lunar eclipses made on the basis of this law always come out to be true.

Characteristics of Gravitational Force

(i) Gravitational force between two bodies or objects does not need any contact between them.
It means, gravitational force is action at a distance.
(ii) Gravitational force between two bodies varies inversely proportional to the square of the distance between them. Hence, gravitational force is an inverse square force.
(iii) The gravitational forces between two bodies or objects form an action-reaction pair. If object A attracts object B with a force F_1 and the object B attracts object A with a force F_2, then $F_1 = -F_2$

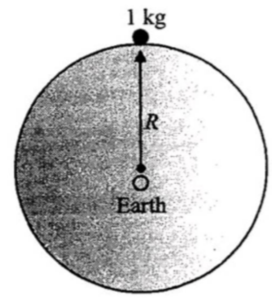


Estimation of Gravitational Force between Different Objects

The formula applied for calculating gravitational force between light objects and heavy objects is the same, i.e., $F = \frac{Gm_1m_2}{R^2}$ Let us take three different cases :

- **The gravitational force between a body of 1 kg and the earth [mass of earth $= 6\times10^{24}\,kg$, , radius of earth is $6.4\times10^6\,m$.]**

Let a body of mass 1 kg be placed on the surface of the earth as shown in figure. The distance between the centre of the earth and body is equal to the radius of the earth i.e., $R = 6.4\times10^6\,m$.



The magnitude of gravitational force between the earth and the body is $F = \frac{Gm_1m_2}{R^2}$

where, $m_1 = 1\,\text{kg}$
$m_2 = 6\times10^{24}\,\text{kg}$ (mass of the earth)
$R = 6.4\times10^6\,\text{m}$
$G = 6.67\times10^{-11}\,\text{N m}^2\text{kg}^{-2}$

$$\therefore \quad F = \frac{6.67\times10^{-11}\,N\,m^2kg^{-2}\times1kg\times6\times10^{24}\,kg}{(6.4\times10^6\,m)^2} = 9.8N$$

Thus, a body of mass 1 kg is attracted by the earth with a force of 9.8 N.

- **Gravitational force between the sun and the earth**

Mass of the earth, $m_1 = 6\times10^{24}\,\text{kg}$
Mass of the sun, $m_2 = 2\times10^{30}\,\text{kg}$
Distance between the sun and the earth, $R = 1.5\times10^{11}\,\text{m}$
Gravitational force between the sun and the earth,

$$F = \frac{Gm_1m_2}{R^2} = \frac{6.67\times10^{-11}\,\text{Nm}^2\text{kg}^{-2}\times6\times10^{24}\,\text{kg}\times2\times10^{30}\,\text{kg}}{(1.5\times10^{11}\,\text{m})^2}$$

$= 3.6\times10^{22}\,\text{N}$

The gravitational force between the sun and the earth is very large (i.e., $3.6\times10^{22}\,\text{N}$). This force keeps the earth bond to the sun.

- **Gravitational force between the moon and the earth.**

Mass of the earth, $m_1 = 6\times10^{24}\,\text{kg}$
Mass of the moon, $m_2 = 7.4\times10^{22}\,\text{kg}$
$\therefore$ Gravitational force between the earth and the moon.

$$F = \frac{Gm_1m_2}{R^2} = \frac{6.67\times10^{-11}\,\text{Nm}^2\text{kg}^{-2}\times6\times10^{24}\,\text{kg}\times7.4\times10^{22}\,\text{kg}}{(3.8\times10^8\,\text{m})^2}$$

$= 2.05\times10^{20}\,\text{N}$

This large gravitational force keeps the moon to move around the earth.
We find that,
(i) the gravitational force between two small bodies is very small.

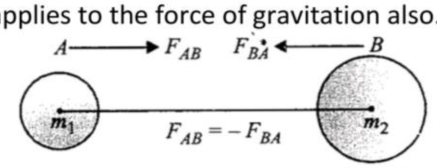
(ii) the gravitational force between a small body and a larger body (e.g. the earth) is large.
(iii) the gravitational force between two large bodies (e.g., the sun and the earth) is very large.

DO U KNOW. . .

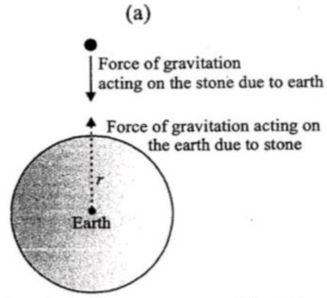
Newton was the first to realize that the moon is a satellite of the earth. It has just the right speed to keep moving round the earth in a nearly circular orbit. It completes one full circle in 27 days and 8 hours. It is constantly falling towards the earth but cannot hit the earth because the earth's surface curves down under it by the same amount.

Gravitational and Newton's Third Law of Motion

According to Newton's third law, to every action, there is always an equal and opposite reaction. It means if an object A exerts some force on another object B, then the object B exerts an equal and opposite force on the object A at the same instant. This law applies to the force of gravitation also.



Gravitational force follows Newton's third law of motion
(a)



Force of gravitation between a stone and the earth
(b)

According to Newton's second law of motion, force = mass x acceleration
i.e., for a given force, acceleration produced, varies inversely as the mass.
We know that acceleration produced in a body due to gravitational pull of earth on it is 9.8 m s^{-2}. As this acceleration is very large, we can see the body falling towards earth. We shall show that when gravitational pull of same magnitude acts on earth (where mass is 6 x 10^{24} kg), the acceleration produced in earth is $1.63\times10^{-24}\,s^{-2}$. As this value of acceleration is too small, we cannot see the earth moving towards the falling body.

ILLUSTRATION

2. Find the gravitationla force between two protons kept at a separation of 1 femtometre (1 femtometre $=10^{-15}\,m$). The mass of a proton is $1.67\times10^{-27}\,kg$.

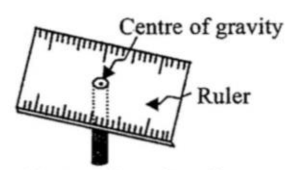
Soln.: The gravitational force between the two protons is $F=\dfrac{Gm_1m_2}{r^2}$

$$=\frac{(6.67\times10^{-11}\,N\,m^2\,kg^{-2})\times(1.67\times10^{-27}\,kg)^2}{(10^{-15}\,m)^2}$$

$=1.86\times10^{-34}\,N$

Centre of Mass and Centre of Gravity

The point in a body where its whole mass is assumed to be concentrated is called its centre of mass. The centre of mass of a homogeneous sphere or cube must lie at its geometric centre. A rigid extended body is a continuous distribution of mass. Each particle or portion of the body experiences the force of gravity. The net effect of all these forces of to the effect of a single force, mg acting through a point called centre of gravity of the body, or we can say that a point in any body at which the force of gravity on the whole of the body can be assumed to act is called its centre of gravity.



Centre of gravity of an object

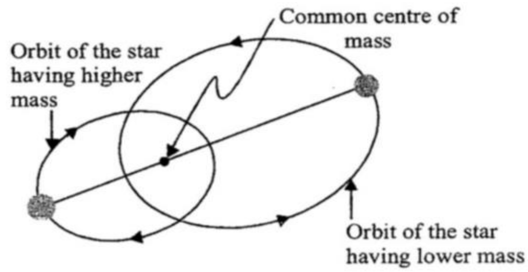
On the surface of the earth, or near it, where the force of gravity is constant, the centre of mass also becomes the centre of gravity.
If we assume the earth to be a sphere of uniform density, then its centre of mass lies at its centre.
The force of attraction of the earth on any body is, therefore, towards its centre.

Application Of Newton's Law Of Gravitation

- **Determination of the masses of planets and stars**

Knowing precise values of g, R and G, it is possible to determine accurately the mass of any planet or star by using the relationship,

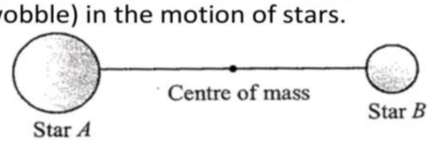
$$M=\frac{gR^2}{G}$$



Orbits of stars of a double-star system

1. Estimating the masses of double stars - A double star is a system consisting of two stars orbiting round their common centre of mass. From the extent of irregularity in the motion of a star due to the gravitational pull by some other star bound to it, can be used for estimating their masses. Such a small irregularity in motion is called wobble.

Many planets outside our solar system have been detected in recent years by measuring the irregularity (called wobble) in the motion of stars.



Free Fall

The motion of a body towards the earth when no other force except the force of gravity acts on it is called a free fall. Thus all the freely falling bodies, lighter or heavier, fall towards the earth with the same acceleration.

According to Galileo Galilee, if there were no air, all the bodies having different masses when dropped simultaneously from the same height would hit the ground at the same time.

Later Robert Boyle proved this experimentally. He placed a coin and a feather in a long glass tube and removed its air with the help of vacuum pump. When the tube was inverted both the coin and the feather fell to the bottom of the tube at the same time. Thus the acceleration produced in all the freely falling bodies is the same, and does not depend upon the mass of the falling body.

Acceleration due to Gravity of the Earth

The acceleration produced in a body due to the gravitational pull of the earth near its surface is called acceleration due to gravity of the earth. The gravitational force acting on a body of mass m near the surface of the earth is given by,

$$F = G\frac{m \times M}{R^2} \quad \text{...(i)}$$

where, m is the mass of the body, M is the mass of earth, and R is the radius of the earth.

If g is the acceleration produced in a body of mass m, then

$$F = m \times g \quad \text{...(ii)}$$

From equation (i) and (ii),

$$mg = G\frac{m \times M}{R^2} \text{ or } g = G\frac{M_e}{R_e^2} \quad \text{...(ii)}$$

From equations (iii), we see that acceleration produced in the body due to the earth does not depend upon its mass.

Gravity and Gravitation

According to Newton, the gravitational force of attraction between the moon and earth is responsible for providing the necessary centripetal force for moon to revolve around earth.

The terms gravity and gravitation are not the same. The force of attraction between any two objects by virtue of their masses is called gravitation (or gravitational force).

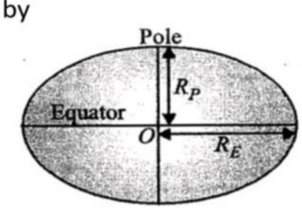
For example, force of attraction between any two objects such as books, tables, chairs, and between any two heavenly bodies are the examples of gravitation.

The force of gravitation exerted by a huge heavenly body such as, the earth, the moon, or the sun etc., on a smaller object near its surface is called its gravity (or force of gravity).

For example, earth pulls an object of mass 1 kg towards it with a force of 9.8 N. So, force of 9.8 N is the gravity (or, force of gravity) of the earth. Similarly, on the surface of moon, we can talk of the gravity of the moon. Thus, we see that the gravity is a particular case of gravitation.

Variation in the value of g

The value of g vary with the shape of the earth. The acceleration due to gravity g on the surface of the earth is given by



$$g = \frac{GM}{R^2} \quad \text{....(i)}$$

The expression for g is calculated by considering the earth as a spherical body, but earth is not spherical in shape. It is elliptical in shape as shown in the figure. Therefore, the radius of the earth (R) is not constant throughout. Hence, the value of g is different at different points on the earth. The equatorial radius (R_E) of the earth is .about 21 km longer than its polar-radius (R^P).

Now from equation (i) value of g at equator is given by

$$g_E = \frac{GM}{R_E^2} \quad \text{...(ii)}$$

Value of g at pole is given by $g_P = \frac{GM}{R_P^2}$ ...(iii)

Dividing (iii) by (ii) $\frac{g_P}{g_E} = \left(\frac{R_E}{R_P}\right)^2$

Since $R_E > R_p \therefore g_p > g_E$

Thus, value of g is more at equator than at poles.

Variation in the value of g with the altitude (or height) above the surface, of the earth.

We know, acceleration due to gravity on the surface of the earth is given by $g = \frac{GM}{R^2}$ ...(i)

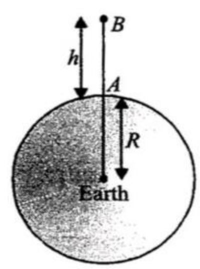
Let a body be at a height h above the surface of the earth as shown in figure. The distance of the body from the centre of the earth = (R + h).

Therefore, acceleration due to gravity at height h is given by $g_h = \frac{GM}{(R+h)^2}$ ...(ii)

Dividing (ii) by (i), we get

$$\frac{g_h}{g} = \frac{GM}{(R+h)^2} \times \frac{R_2}{GM} = \frac{R^2}{(R+h)^2}$$

or $\frac{g_h}{g} = \left(\frac{R}{R+h}\right)^2$ ...(iii)



Since $(R+h) > R \therefore \frac{g_h}{g} < 1$ or $g_h < g$

Thus, value of g decreases with the height from the surface of the earth.

Variation in the value of g with depth below the surface of the earth.

Let us consider a body of mass m at a depth h below the surface of the earth. Then,

Radius of the inner solid sphere of the earth = R - h

So,

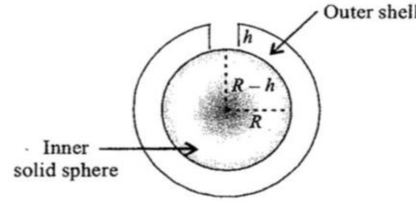
Volume of the inner solid sphere of the earth

$$= \frac{4}{3}\pi(R-h)^3$$

If d is the average density of the earth, then

Mass of the inner solid sphere of the earth

$$= \frac{4}{3}\pi(R-h)^3 d$$



According to the law of gravitational,

$$mg_d = \frac{G \times \frac{4}{3}\pi(R-h)^3 d \times m}{(R-h)^2}$$

This gives, $g_d = G \times \frac{4}{3}\pi(R-h)d$ ...(i)

On the surface of the earth,

$$g = \frac{GM}{R^2} = \frac{G \times \frac{4}{3}\pi R^3 d}{R^2} = G \times \frac{4}{3}\pi R d \quad \text{...(ii)}$$

From equation (i) and (ii), one gets

$$\frac{g_d}{g} = \frac{G \times \frac{4}{3}\pi(R-h)d}{G \times \frac{4}{3}\pi R d} = \frac{(R-h)}{R}$$

$$g_d = g\left(1 - \frac{h}{R}\right) \quad \frac{(R-h)}{R} < 1$$

So, $g_d < g$

Thus the value of g at a depth inside the earth is less than that on the surface of the earth. The value of $\frac{(R-h)}{R}$ decreases with the value of h, i.e., depth below the surface of the earth.

Therefore, the value of g decreases as we go down below the surface of the earth.

The difference between acceleration due to gravity (g) and gravitational constant (G) as shown in table below.

	Acceleration due to Gravity (g)		Universal Gravitational Constant (G)
1.	It is the acceleration acquired by a body due to the earth's gravitational pull on it.	1.	It is equal to the force of attraction between two masses of 1 kg each separated by a distance of 1 m.
2.	The value of g is different at different places on the surface of the earth. Its value varies from	2.	G is a universal constant, i.e., its value is the same every where in the universe $(G = 6.67 \times 10^{-11} \mathrm{N\,m^2\,kg^{-2}})$

	one celestial (heavenly) body to another.		
3.	It is a vector quantity.	3.	It is a scalar quantity.
4.	The value of g at the centre of the earth is zero.	4.	The value of G is not zero at the centre of the earth or any where else.

Equations of Motion for a Body Moving under Gravity

The value of g changes with height above the earth's surface. For smaller heights, the change in the acceleration due to gravity (g) is very small. So, the value of g can be taken as almost constant. The motion of a body under gravity near the earth's surface is a uniformly accelerated motion.

Equations of motion for a freely falling body.

For a freely falling body, acceleration a is equal to the acceleration due to gravity g and distance travelled s is equal to the height above the surface of the earth h. From the general equations of motion by replacing a by g and s by h as follows :

General equations of motion Equations of motion for freely

General equations of motion	Equations of motion for freely falling bodies
1. $v = u + at$	$v = u + gt$
2. $s = ut + \frac{1}{2}at^2$	$h = ut + \frac{1}{2}gt^2$
3. $v^2 = u^2 + 2as$	$v^2 = u^2 + 2gh$

Equations of motion for a body thrown vertically upwards.

For a body thrown vertically upwards the acceleration a is equal to minus of acceleration due to gravity $-g$. Distance travelled s is equal to height above the earth's surface h.

The general equations of motion as shown below.

General equations of motion	Equations of motion for bodies thrown vertically upwars
1. $v = u + at$	$v = u - gt$
2. $s = ut + \frac{1}{2}at^2$	$h = ut - \frac{1}{2}gt^2$
3. $v^2 = u^2 + 2as$	$v^2 = u^2 - 2gh$

ILLUSTRATION

3. A helicopter is ascending with a velocity of $2\,\text{ms}^{-1}$ at height of 24 m when it drops a mail packet. The packet contains material, which can be damaged if it hits ground with velocity greater than $72\,\text{kmh}^{-1}$. Was the packet damaged? $(g = 10\,\text{ms}^{-2})$

Soln.: Velocity during ascent $2 = \text{m s}^{-1}$

Height of the helicopter = 24 m

So, height from where the packet is dropped = 24 m

Initial velocity of the packet, $u = -2\,\text{ms}^{-1}$

Final velocity of the packet, $\upsilon = ?$

Using the equation,

$\upsilon^2 u^2 = 2as$

$\upsilon^2 - (-2\text{m s}^{-1})^2 = 2 \times 10\,\text{ms}^{-2} \times 24\,\text{m}$

$= 480\,\text{m}^2\text{s}^{-2}$

$\upsilon^2 = 480\,\text{m}^2\text{s}^{-2} + 4\,\text{m}^2\text{s}^{-2} = 484\ \text{m}^2\text{s}^{-2}$

This gives, $\upsilon = \sqrt{480\,\text{m}^2\text{s}^{-2}} = 22\,\text{ms}^{-1}$

Converting the units of velocity,

$\upsilon = \frac{22 \times 1\text{m}}{1\text{s}} = \frac{22 \times 10^{-3}\text{km}}{(1/60 \times 60)\text{h}}$

$= 22 \times 10^{-3} \times 60 \times 60\,\text{kmh}^{-1} = 79.2\,\text{kmh}^{-1}$

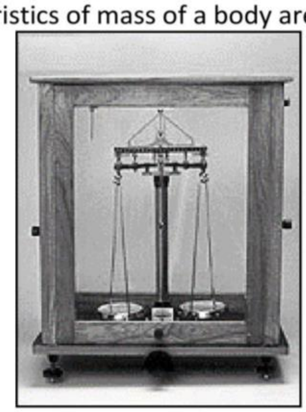
Thus, the packet hits the ground with a velocity of $79.2\,\text{kmh}^{-1}$.

Because the packet hits the ground with a velocity greater than the limit for safety $(72\,\text{kmh}^{-1})$, if will get damaged.

Mass and Weight

Mass: The quantity of matter in a body is called its mass. Mass is a measure of the number of atoms contained in any body. Since the number of atoms in any object remains constant, hence its mass remains constant. Mass is expressed in the mass units. The SI unit of mass is kilogram (kg). Mass

The characteristics of mass of a body are as follows:



Common Balance

Mass is a scalar quantity.

- Mass of a body does not depend on the shape, size and the state of the body.
- Mass of a body is proportional to the quantity of matter contained in it.
- Mass of a body can be measured with the help of a common balance.

Weight: The force with which a body is attracted by the earth is known as the weight of the body It varies from place to place. The weight of a body on earth is equal to the force of gravity exerted by the earth on that body.

We have, F = ma (From Newton's second law)

W = mg ...(i)

where g is the acceleration due to gravity.

Spring Balance

The S.I. unit of weight is same as that of the force, i.e., new ton (N).

From equation (i) it is clear that weight of a body depends upon the mass of the body and value of acceleration due to gravity g at a place.

The characteristics of weight of the body are as follows:

- Weight is a vector quantity.
- Weight is measured with a spring balance, or on a weighing machine.
- The weight of a body is directly proportional to its mass.
- The weight of a body changes with the value of g. So when g decreases, the weight of the body also decreases.
- The value of g at the poles is higher than that at the equator.
- The value of g on the surfaces of different planets of the solar system is different, therefore the weight of a body is different on the different planets.
- The value of g decreases with height from the surface of the earth. Therefore/ the weight of the body also decreases with height from the surface of the earth. That is why, the weight of a man is less on the peak of Mount Everest than the weight of the man at Delhi.
- The value of g decreases, with depth from the surface of the earth.
- Therefore, the weight of a body decreases with depth from the surface of the earth.

- The value of g at the centre of the earth is zero, hence weight of the body is zero at the centre of the earth.

Weight of an object on the surface of moon.

Consider an object of mass m on the surface of the earth. Let M be the mass of the earth and R be its radius.

According to universal law of gravitation, the force with which the earth attracts the object is given by

$$F = \frac{GMm}{R^2} \quad ...(i)$$

Since, force with which the earth attracts the object = weight of the object (W) i.e. F = W

$\therefore$ equation (i) becomes $W = \frac{GMm}{R^2}$ ...(ii)

Now let this object of mass m lies on the surface of the moon. Let M_m be the mass of the moon and R_m, be the radius of the moon. Therefore, according to universal law of gravitation, the force with which the moon attracts the object is given by $F' = \frac{GM_m m}{R_m^2}$ (ii)

But F' = weight of the object on the moon (W_m)

$$\therefore \quad W_m = \frac{GM_m m}{R_m^2} \quad ...(iii)$$

Dividing equation (iii) by equation (ii), we get

$$\frac{W_m}{W} = \frac{GM_m m}{R_m^2} \times \frac{R^2}{GMm} = \frac{M_m R^2}{MR_m^2} \quad ...(iv)$$

Now, M(mass of earth) $= 5.98 \times 10^{24}\,\text{kg}$

M_m (mass of moon) $= 7.36 \times 10^{22}\,\text{kg}$

R_m (radius of earth) $= 6400\,\text{km} = 6.4 \times 10^6\,\text{m}$

R_m (radius of moon) $= 1740\,\text{km} = 1.74 \times 10^6\,\text{m}$

Put these values in equation (iv), we get

$$\frac{W_m}{W} = \frac{7.36 \times 10^{22} \times (6.4 \times 10^6)^2}{5.98 \times 10^{24} \times (1.74 \times 10^6)^2} = 0.166$$

$= \frac{1}{6}$ or $W_m = \frac{1}{6} W$

object on the surface of moon

$= \frac{1}{6} \times$ Weight of the object on the surface of the earth.

- For this reason, moon exerts less force of attraction on objects.

ILLUSTRATION

4. The mass of an object is 60 kg on the surface of the earth. What would be its weight when measured on the surface of the moon? What would be its mass on the moon?

(take $g = 10\,\text{m/s}^2$)

Soln.: Mass of an object on the earth $m_e = 60\text{kg}$

Acceleration due to gravity on the earth

$$g_e = 10\text{m/s}^2$$

Weight of the object on the earth $W_e = ?$

(to be calculated)

We know, we $= m_e \times g_e$

By putting the values we get

$W_e = 60\text{kg} \times 10\text{ms}^{-2}$

$W_e = 600\text{N}$

Thus, the weight of the object on the earth is 600 N.

We know the weight of an object on the moon $= \left(\frac{1}{6}\right) \times$ its weight on the earth

$W_m = \frac{1}{6} \times 600\text{N}$

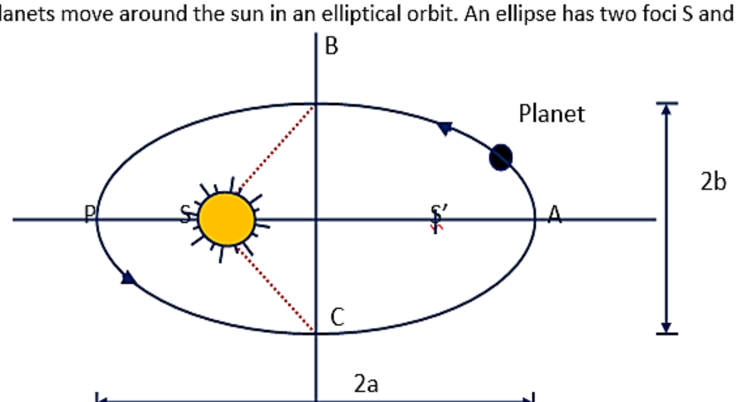
$W_m = 100\text{N}$

KEPLER'S LAWS OF PLANETARY MOTION

To explain the motion of the planets, Kepler formulated the following three laws:

1. Law of orbits (first law): Each planet revolves around the sun in an elliptical orbit with the sun situated at one of the two foci.

As shown in Fig. the planets move around the sun in an elliptical orbit. An ellipse has two foci S and S' the sun remains at one focus S.

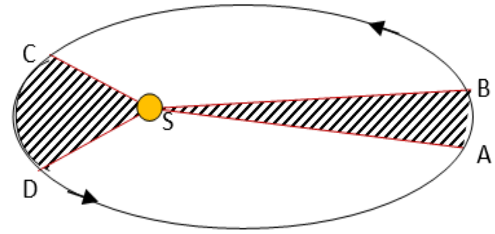


[Elliptical orbit of a planet, PA = 2a = major axis, BC = 2b = minor axis]

The point P and A on the orbit are called the **perihelion** and the **aphelion** and represent the closest and farthest distance from the sun respectively. The orbits of Pluto and Mercury are highly elliptical. The orbits of Neptune and Venus are circular. The orbits of other planets have slight ellipticity and may be taken as nearly circular.

2. Law of mass (second law): "The radius vector, drawn from the sun to a planet sweeps out equal areas in equal intervals of time i.e., the areal velocity (area covered per unit time) of a planet around the sun is constant".

Suppose a planet takes same time to go from position A to B as in going from C to D [Fig.]. From Kepler's second law, the areas ASB and CSD (covered in equal time) must be equal. Clearly, the planet covers a larger distance CD when it is near the sun than AB when it is farther away in the same interval of time. Hence the linear velocity of a planet is more when it is closer to the sun than its linear velocity when away from the sun.



[Kepler's second law or areas]

•3. Law of periods (Third law): The square of the period of the revolution of the planet around the sun is proportional to the cube of the semimajor axis of its elliptical orbit.

If T is the period of revolution of a planet and R is the length of semimajor axis of its elliptical orbit, then

$T^2 \propto R^3$ or $T^2 = KR^3$

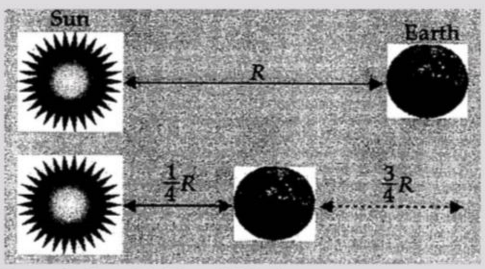
Where K is a proportionally constant. For two different planets, we can write

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

Thus, larger the distance of a planet from the sun, the larger will be its period of revolution around the sun. The period of revolution of the farthest planet Pluto around the sun is 247 years while that of the nearest planet mercury is 81 days.

ILLUSTRATION

5. If earth comes closer to sum by 3/4th of the present distance then the year of earth consists of how many days?



Soln.: Case-I $R_1 = R$ $T_1 = 365$ **days**

Cade II $R = R - \frac{3}{4}R = \frac{1}{4}R$ $T_2 = ?$

We know that $T^2 \propto R^3 \Rightarrow T^2 = KR^3$ ...(i)

Substituting the above values (i), we have,

$$\frac{T_1^2}{T_2^2} = \frac{KR_1^2}{KR_2^2} \Rightarrow T_2^2 = \left(\frac{R_2}{R_1}\right)^2 \times T_1^2$$

$$\Rightarrow T_2^2 = \left(\frac{1}{4}\right)^3 \times (365)^2$$

$$\Rightarrow T_2 = \frac{1}{8} \times 365 = 45.625 \text{ days}$$

∴ if the earth comes closer to the sun by 3/4th of the present distance, one year of earth consists of 45.625 days.

Derivation of Newton's Inverse Square Rule from Kepler's Third Law.

Newton derive mathematically his universal law of gravitation from Kepler's law of planetary motion.

Consider a planet of mass m revolving around the sun of mass M in a circular path of radius r. Let us take v as the orbital velocity of the planet and T is its time period to complete one revolution around the sun.

The distance travelled by the planet in one complete rotation is = 2πr.

We have/ velocity, $\upsilon = \frac{\text{Distance travelled}}{\text{Time taken}}$

i.e., $\upsilon = \frac{2\pi r}{T}$ ...(i)

or $\upsilon \propto \frac{r}{T}$ ($\because 2\pi$ constant) ...(ii)

Squaring both sides of equation (ii), we get

$\upsilon^2 \propto \frac{r^2}{T^2}$ or $\upsilon^2 \propto \frac{r^2}{T^2} \times \frac{r}{r}$ ...(iii)

According to Kepler's third law of planetary motion, $\frac{r^3}{T^2}$ is constant.

From equation (iii) $\upsilon^2 \propto \frac{1}{r}$ ...(iv)

Now, we know that the centripetal force, F required to keep the planet in a circular orbit is $F = \frac{m\upsilon^2}{r}$ ...(v)

or $F \propto \frac{\upsilon^2}{r}$ ($\because$ m is constant) ...(vi)

From equation (iv) and (vi), we get $F \propto \frac{1}{r^2}$

This is Newton's Inverse Square Rule

Gravitation

It is the phenomenon by which all objects with mass attract each other

Newton's Universal Law Of Gravitation

Every particle of matter in the universe exerts an attractive force on every other particle, the force being directly proportional to the product of their respective masses and inversely proportional to the square of their separation. The force of attraction always acts along the line joining the two particles.

Formula: $F = G\frac{m_1 m_2}{r^2}$

G is equal to the force of at $G = 6.67\times10^{-11}\ N\ m^2\ kg^2$ traction between unit masses placed at unit distance apart. SI unit is $N\,m^2 kg^{-2}$

$G = 6.67\times10^{-11}\ N\ m^2\ kg^2$

Kepler's law of motion

- All planets move around the sun, in all elliptical orbit with the sun located at one of its foci.
- The line joining the sun to any planet, known as radius vector sweeps out equal areas in equal time intervals.
- The square of the period of revolution of a planet around the sun varies directly as the cube of the semi-major axis of its elliptical path.

Acceleration due to gravity (g)

The uniform acceleration produced by a freely falling body under the gravitational pull of the earth. $g = 9.8\,m\ s^{-2} = 980\,cm\,s^{-2}$

- Mass is the quantity of matter contained in it. S.I. unit of mass is kilogram
- Weight of a body is the force with which it is attracted towards the centre of the earth $\therefore W = mg$ S.I. unit is newton
- Factors affecting
  - Altitude : $gh = \frac{GM}{(R+h)^2}$ $gh = g\left(1-\frac{2h}{R}\right) h << R$
  - Depth : $g_d = \left(1-\frac{d}{R}\right)g$
  - g is maximum at poles and minimum at equator.

Q AND A . . .

NCERT CORNER

Intext Questions

Q. 1. State the universal law of gravitation.
[NCERT Q. 1, Page 134]
[KVS, Patna Region, 2017-18, SA-I, 2015-16]

Ans. The universal law of gravitation states that, "Any two objects exert a gravitational force of attraction on each other. The direction of the force is along the line joining the objects. The magnitude of the force is proportional to the product of the masses of the objects, and inversely proportional to the square of the distance between them."

For two objects of masses m_1 and m_2 and the distance between them r, the force (F) of attraction acting between them is given by the universal law of gravitation as :

$$F = G\, m_1 m_2 / r^2$$

where, G is the universal gravitation constant and its value is 6.67×10^{-11} N m^2 kg^{-2}.

Q. 2. Write the formula to find the magnitude of the gravitational force between the Earth and an object on the surface of the Earth.
[NCERT Q. 2, Page 134]

Ans. Let 'M_E' be the mass of the Earth and 'm' be the mass of an object on its surface. If 'R' is the radius of the Earth, then according to the universal law of gravitation, the gravitational force F acting between the Earth and the object is given by the relation :

$$F = GM_E \frac{m}{R^2}$$

Q. 3. What do you mean by free fall ?
[NCERT Q. 1, Page 136]

Ans. Gravity of the Earth attracts every object towards its centre.

When an object is released from a height, it falls towards the surface of the Earth under the influence of gravitational force. The motion of the object is said to be free fall.

Q. 4. What do you mean by acceleration due to gravity ?
[NCERT Q. 2, Page 136]

Ans. When an object falls towards the ground from a height, its velocity changes during the fall.

This changing velocity produces acceleration in the object. This acceleration is known as **acceleration due to gravity** (g). Its value is given by 9.8 m/s^2.

Q. 5. What are the differences between mass of an object and its weight? **[NCERT Q. 1, Page 138]**

Ans. Differences between Mass and Weight :

S. No.	Mass	Weight
(i)	Mass is the quantity of matter contained in the body.	Weight is the gravitational force acting on the body.
(ii)	It is the measure of inertia of the body.	It is the measure of gravity.
(iii)	It is a constant quantity for any object.	Weight of an object is not a constant quantity. It is different at different places.
(iv)	It is a scalar quantity and only has magnitude.	It is a vector quantity as it has both magnitude and direction.
(v)	The S.I. unit of mass is kg.	Its S.I. unit is N (newton) same as the S.I. unit of force.

Q. 6. Why is the weight of an object on the moon 1/6th its weight on the Earth ? **[NCERT Q. 2, Page 138]**

Ans. Let 'M_E' be the mass of the Earth and 'm' be an object on the surface of the Earth and 'R_E' be the radius of the Earth.

According to the Universal law of gravitation, weight 'W_E' of the object on the surface of the Earth is given by,

$$W_E = \frac{G.M_E.m}{R^2_E}$$

Now again, let M_M and R_M be the mass and radius of the moon respectively.

Then, according to the universal law of gravitation, weight W_M of the object on the surface of the moon is given by,

$$W_M = \frac{G.M_M.m}{R^2_M}$$

Now, take the ratio of W_M to W_E

$$\Rightarrow \frac{W_M}{W_E} = \frac{M_M R_E^2}{M_E R_M^2}$$

where,

$M_E = 5.98 \times 10^{24}$ kg
$M_M = 7.36 \times 10^{22}$ kg
$R_E = 6.4 \times 10^6$ m
$R_M = 1.74 \times 10^6$ m

Now, put the values,

$$\Rightarrow \frac{W_M}{W_E} = \frac{[7.36 \times 10^{22} \times (6.4 \times 10^6)^2]}{[5.98 \times 10^{24} \times (1.74 \times 10^6)^2]}$$

$$= 0.165 \approx 1/6$$

Therefore, weight of an object on the Moon is $1/6^{th}$ of its weight on the Earth.

Exercise Questions

Q. 1. How does the force of gravitation between two objects change when the distance between them is reduced to half ?

Ans. According to the universal law of gravitation, gravitational force (F) acting between two objects is inversely proportional to the square of the distance (r) between them, that is,

$$F \propto \frac{1}{r^2}$$

If distance r becomes $r/2$, then the gravitational force, F will be proportional to

$$F \propto \frac{1}{(r/2)^2}$$

$$F \propto \frac{1}{(r^2/4)}$$

$$F \propto \frac{4}{r^2}$$

Hence, if the distance is reduced to half, then the gravitational force becomes four times larger than the previous value.

Q. 2. Gravitational force acts on all objects in proportion to their masses. Why then, a heavy object does not fall faster than a light object ?

Ans. All objects fall on ground with constant acceleration, called acceleration due to gravity (in the absence of air resistance). It is constant ($g = 9.8$ m/s^2) and it does not depend upon the mass of the object. Hence, heavy objects do not fall faster than light objects.

Let F be the gravitational force acting on a body of mass m, then it can be given by,

$$F = G\frac{Mm}{r^2} = mg$$

$$g = \frac{GM}{r^2}$$

From the above equations, we can see that

$$F \alpha m$$

Although 'g' (acceleration due to gravity) does not depend on mass 'm'. Hence, all bodies fall with the same acceleration provided there is no air or other resistance.

Q. 3. What is the magnitude of the gravitational force between the Earth and a 1 kg object on its surface? (Mass of the Earth = 6 × 10^{24} kg and radius of the Earth is 6.4 × 10^6 m)

Ans. Given that,

Mass of the body (m) = 1 kg
Mass of the Earth (M) = 6 × 10^{24} kg
Radius of the Earth (R) = 6.4 × 10^6 m

where,

G = 6.67 × 10^{-11} Nm^{2}kg^{-2}

Now, magnitude of the gravitational force (F) between the Earth and the body can be given as,

$$F = G\frac{Mm}{R^2}$$

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 1}{(6.4 \times 10^6)^2} N$$

$$= \frac{6.67 \times 6 \times 10}{6.4 \times 6.4} = 9.8 N (approx.)$$

So, the magnitude of the gravitational force between the Earth and a 1 kg object on its surface is 9.8 N.

Q. 4. The Earth and the moon are attracted to each other by gravitational force. Does the Earth attract the Moon with a force that is greater or smaller or the same as the force with which the Moon attracts the Earth? Why ?

Ans. According to the universal law of gravitation, two objects attract each other with equal force, but in opposite directions. The Earth attracts the moon with an equal force with which the moon attracts the Earth.

The universal law of gravitation, the force acting between Earth and moon will be :

$$F = G\frac{M_1 M_2}{d^2}$$

where, M_1 = Mass (Earth),
M_2 = Mass (Moon),
d = Distance between Earth and Moon

So, the magnitude of force (F) is same for Earth and Moon.

Q. 5. If the Moon attracts the Earth, why does the Earth not move towards the Moon ?

Ans. According to the Newton's third law of motion, the Moon also attracts the Earth with a force equal to that with which the Earth attracts the moon. However, the mass of the Earth is much larger than the mass of the moon.

Hence, it accelerates at a rate less than the acceleration rate of the moon towards the Earth. For this reason, the Earth does not move towards the Moon.

Q. 6. What happens to the force between two objects, if:

(i) the mass of one object is doubled?

(ii) the distance between the objects is doubled and tripled?

(iii) the masses of both objects are doubled ?

Ans. According to the universal law of gravitation, the force of gravitation between two objects, is given by,

$$F = \frac{GMm}{r^2}$$

(i) When the mass of either body is doubled, the force is also doubled as per the following equation, F' being the new force :

$$F' = G\frac{(2m_1)m_2}{r^2} = 2G\frac{m_1m_2}{r^2} = 2F$$

F is directly proportional to the masses of the objects. If the mass of one object is doubled, then the gravitational force will also get doubled.

(ii) When the distance between the objects is doubled, the force becomes one-fourth of the original force as per the following equation :

$$F' = G\frac{m_1m_2}{(2r)^2} = \frac{1}{4}G\frac{m_1m_2}{r^2} = \frac{1}{4}F$$

F is inversely proportional to the square of the distances between the objects. If the distance is doubled, then the gravitational force becomes one-fourth of its original value. Similarly, if the distance is tripled, then the gravitational force becomes one-ninth of its original value.

(iii) When the masses of both bodies are doubled, the force becomes four times the original force because F is directly proportional to the product of mass of both objects. If the masses of both the objects are doubled, then the gravitational force becomes four times the original value.

Q. 7. What is the importance of universal law of gravitation ?

Ans. Universal law of gravitation is important because with the help of this law we can explain natural phenomena such as,

1. the force of attraction that binds us to the Earth.
2. the motion of planets moving around the Sun.
3. the motion of Moon around the Earth.
4. the occurring of tides due to the gravitational forces of the Sun and Moon.

Q. 8. What is the acceleration of free fall ?

Ans. When objects fall towards the Earth under the effect of gravitational force alone, they are said to be in free fall. The acceleration of free fall is produced when a body falls under the influence of the force of gravitation of the Earth alone.

Acceleration of free fall is 9.8 m s^{-2}, which is constant for all objects (irrespective of their masses).

Q. 9. What do we call the gravitational force between the Earth and an object?

Ans. Gravitational force between the Earth and an object is known as the weight of the object.

The weight of an object is defined as the force of gravity on the object and it is equal to the product of mass and the acceleration due to gravity.

$$w = mg$$

Since the weight is a force, so its S.I. unit is Newton.

Q. 10. Amit buys few grams of gold at the poles as per the instruction of one of his friends. He hands over the same when he meets him at the equator. Will the friend agree with the weight of gold bought? If not, why? [Hint : The value of *g* is greater at the poles than at the equator].

Ans. Weight of a body on the Earth is given by,

$$W = mg$$

where,

m = Mass of the body

g = Acceleration due to gravity

The value of g is greater at poles than at the equator. Therefore, gold at the equator weighs less than at the poles. Hence, Amit's friend will not agree with the weight of the gold bought.

Q. 11. Why will a sheet of paper fall slower than one that is crumpled into a ball ?

Ans. When a sheet of paper is crumpled into a ball, then its surface area in contact with the air decreases.

Hence, resistance to its motion through the air decreases and it falls faster than the sheet of paper.

Q. 12. Gravitational force on the surface of the Moon is only 1/6 as strong as gravitational force on the Earth. What is the weight in newton of a 10 kg object on the moon and on the Earth ?

Ans. Given,

Weight of an object on the Moon $= 1/6 \times$ Weight of an object on the Earth

We know that,

Weight = Mass × Acceleration

Acceleration due to gravity, $g = 9.8\ m/s^2$

Therefore, weight of a 10 kg object on the Earth $= 10 \times 9.8 = 98$ N

And, weight of the same object on the moon $= 1/6 \times 98 = 16.3$ N

Q. 13. A ball is thrown vertically upwards with a velocity of 49 m/s. Calculate :

(i) the maximum height to which it rises.

(ii) the total time it takes to return to the surface of the Earth.

Ans. **(i)** According to the equation of motion under gravity, $v^2 - u^2 = 2gh$

where, u = Initial velocity of the ball

v = Final velocity of the ball

h = Height achieved by the ball

g = Acceleration due to gravity

At maximum height, final velocity of the ball is zero, that is, $v = 0$ m/s and $u = 49$ m/s

During upwards motion, $g = -9.8$ m s^{-2}

Let h be the maximum height attained by the ball. Hence, using $v^2 - u^2 = 2gs$

And we have, $0^2 - 49^2 = 2(-9.8)$ h

$$\Rightarrow h = \frac{[49 \times 49]}{[2 \times 9.8]}$$

$$= 122.5 \text{ m}$$

Let 't' be the time taken by the ball to reach the height 122.5 m, then according to the equation of motion, $v = u + gt$

We get, $0 = 49 + (-9.8)\,t \Rightarrow 9.8\,t$

$$= 49 \Rightarrow t = \frac{49}{9.8} = 5 \text{ s}$$

But, Time of ascent = Time of descent

Therefore, total time taken by the ball to return = 5 + 5 = 10 s

Q. 14. A stone is released from the top of a tower of height 19.6 m. Calculate its final velocity just before touching the ground.

Ans. According to the equation of motion under gravity,

$$v^2 - u^2 = 2gh$$

where,

u = Initial velocity of the stone = 0 m/s

v = Final velocity of the stone

h = Height of the stone = 19.6 m

g = Acceleration due to gravity = 9.8 m s^{-2}

Put the values,

$\therefore v^2 - 0^2 = 2 \times 9.8 \times 19.6 \Rightarrow v^2 = 2 \times 9.8 \times 19.6 = (19.6)^2$

$\Rightarrow v = 19.6$ m s^{-1}

Hence, the velocity of the stone just before touching the ground is 19.6 m s^{-1}.

Q. 15. A stone is thrown vertically upward with an initial velocity of 40 m/s. Taking g = 10 m/s^2, find the maximum height reached by the stone. What is the net displacement and the total distance covered by the stone ?

Ans. According to the equation of motion under gravity $v^2 - u^2 = 2gh$

where, u = Initial velocity of the stone = 40 m/s

v = Final velocity of the stone = 0 m/s

h = Height of the stone

g = Acceleration due to gravity = -10 ms^{-2}

If 'h' be the maximum height attained by the stone. Then, $v^2 - u^2 = 2gh$

Therefore, $0^2 - 40^2 = 2(-10)\,h$

$$\Rightarrow h = \frac{[40 \times 40]}{20} = 80\ m$$

Therefore, total distance covered by the stone during its upwards and downwards journey

= 80 + 80 = 160 m

Net displacement during its upwards and downwards journey = 80 + (−80) = 0 m

Q. 16. Calculate the force of gravitation between the Earth and the Sun, given that the mass of the Earth = 6 × 10^{24} kg and of the Sun = 2 × 10^{30} kg. The average distance between the two is 1.5 × 10^{11} m.

Ans. According to the universal law of gravitation, the force of attraction between the Earth and the Sun is given by

$$F = G \times M_{Sun} \times M_{Earth}/R^2$$

where,

M_{Sun} = Mass of the Sun = 2 × 10^{30} kg

M_{Earth} = Mass of the Earth = 6 × 10^{24} kg

R = Average distance between the Earth and the Sun = 1.5 × 10^{11} m

G = Universal gravitational constant = 6.7 × 10^{-11} Nm^2 kg^{-2}

Put the values-

$$F = \frac{G \times M_{Sun} \times M_{Earth}}{R^2}$$

$$F = \frac{[6.7 \times 10^{-11} \times 2 \times 10^{30} \times 6 \times 10^{24}]}{(1.5 \times 10^{11})^2}$$

$$\mathbf{= 3.57 \times 10^{22}\ N}$$

Hence, the force of gravitation between the Earth and the Sun is 3.57 × 10^{22} N.

Q. 17. A stone is allowed to fall from the top of a tower 100 m high and at the same time another stone is projected vertically upwards from the ground with a velocity of 25 m/s. Calculate when and where the two stones will meet.

Ans. Let the two stones meet after a time t.

When the stone dropped from the tower,

Initial velocity, u = 0 m/s

Let the displacement of the stone in time t from the top of the tower be h.

Acceleration due to gravity, g = 9.8 ms^{-2}

From the equation of motion,

$$h = ut + 1/2\,gt^2$$

$$h = 0 \times t + ½ \times 9.8 \times t^2$$

$$\Rightarrow \quad h = 4.9\,t^2 \qquad \text{...(i)}$$

When the stone is thrown upwards,

Initial velocity, u = 25 ms^{-1}

Let the displacement of the stone from the ground in time, t be h'.

Acceleration due to gravity, $g = -9.8$ ms^{-2}

Equation of motion, $h = ut + 1/2\,gt^2$

$$h' = 25 \times t - 1/2 \times 9.8 \times t^2$$

$$h' = 25t - 4.9\,t^2 \qquad \text{...(ii)}$$

The combined displacement of both the stones at the meeting point is equal to the height of the tower 100 m.

$h' + h = 100$

$\Rightarrow 25t - 4.9t^2 + 4.9t^2 = 100$

$\Rightarrow t = \frac{100}{25} \text{ s} = 4 \text{ s}$

In 4 s, the falling stone has covered a distance given by (i) as $h = 4.9 \times 4^2 = 78.4$ m. Therefore, the stones will meet after 4 s at a height (100 – 78.4) = 21.6 m from the ground.

Q. 18. A ball thrown up vertically returns to the thrower after 6 s. Find :

(a) the velocity with which it was thrown up,

(b) the maximum height it reaches, and

(c) its position after 4 s.

Ans. **(a)** Time of ascent is equal to the time of descent. The ball takes a total of 6 s for its upwards and downwards journey.

Hence, it has taken 3 s to attain the maximum height.

Final velocity of the ball at the maximum height, $v = 0$ m/s

Acceleration due to gravity, $g = -9.8 \text{ m s}^{-2}$

Using equation of motion, $v = u + gt$, [½]

Put the values $0 = u + (-9.8 \times 3)$

$\Rightarrow u = 9.8 \times 3 = 29.4$ m/s

Hence, the ball was thrown upwards with a velocity of 29.4 m/s.

(b) Let the maximum height attained by the ball = h

Initial velocity during the upward journey, $u = 29.4$ m/s

Final velocity, $v = 0$ m/s

Acceleration due to gravity, $g = -9.8 \text{ m s}^{-2}$

Using the equation of motion,

$$h = ut + \frac{1}{2}gt^2$$

$$h = 29.4 \times 3 - \frac{1}{2} \times 9.8 \times 3^2$$

$\Rightarrow h = 44.1$ m

Hence, the maximum height is 44.1 m.

(c) Ball attains the maximum height after 3 s. After attaining this height, it will start falling downwards.

In this case, Initial velocity, $u = 0$ m/s

Position of the ball after 4 s of the throw is given by the distance travelled by it during its downwards journey in 4 s − 3 s = 1 s.

Using the equation of motion,

$$h = ut + \frac{1}{2}gt^2$$

Put the values,

$$h = 0 \times 1 + \frac{1}{2} \times 9.8 \times 12 \Rightarrow s = 4.9 \text{ m}$$

Now, total height = 44.1 m

This means, the ball is 39.2 m (44.1 m − 4.9 m) above the ground after 4 seconds.

NCERT EXEMPLAR

Multiple Choice Questions

Q. 1. Two objects of different masses falling freely near the surface of moon would :

(a) have same velocities at any instant.

(b) have different accelerations.

(c) experience forces of same magnitude.

(d) undergo a change in their inertia.

Ans. Correct option : (a)

Explanation : Because they will have same acceleration due to gravity.

Q. 2. The value of acceleration due to gravity :

(a) is same on equator and poles.

(b) is least on poles.

(c) is least on equator.

(d) increases from pole to equator.

Ans. Correct option : (c)

Explanation : It is least at the equator and maximum at the poles due to rotation of Earth.

Q. 3. The gravitational force between two objects is F. If masses of both objects are halved without changing distance between them, then the gravitational force would become.

(a) F/4. **(b)** F/2.

(c) F. **(d)** 2 F.

Ans. Correct option : (a)

Explanation : We know that, according to law of gravitation

$$F = \frac{Gm_1m_2}{r^2} \quad (G = \text{Gravitational constant})$$

where m_1 and m_2 are the masses of two objects, respectively. And r is the distance between these two masses.

Now, according to the question, if masses of both objects are halved,

New force, $$F' = \frac{G\left(\frac{m_1}{2}\right)\left(\frac{m_2}{2}\right)}{r^2}$$

$$= \frac{1}{4}\frac{Gm_1m_2}{r^2} = \frac{F}{4}$$

where, $$\frac{Gm_1m_2}{r^2} = F$$

So, new force $$F' = \frac{F}{4}$$

Thus, the new gravitational force will become $\frac{1}{4}$ times its original gravitational force.

Q. 4. A boy is whirling a stone tied with a string in a horizontal circular path. If the string breaks, the stone :

(a) will continue to move in the circular path.

(b) will move along a straight line towards the centre of the circular path.

(c) will move along a straight line tangential to the circular path.

(d) will move along a straight line perpendicular to the circular path away from the boy.

Ans. Correct option : (c)

Explanation : In circular motion, the direction of velocity at a point is always along the tangent at that point. If string breaks, then the centripetal force acting on the stone becomes zero and it will move along a straight line tangential to the circular path.

Q. 5. An object is put one by one in three liquids having different densities. The object floats 1/9, 2/11 and 3/7 parts of their volumes outside the liquid surface in liquids of densities d_1, d_2 and d_3 respectively. Which of the following statement is correct?

(a) $d_1 > d_2 > d_3$ (b) $d_1 > d_2 < d_3$

(c) $d_1 < d_2 > d_3$ (d) $d_1 < d_2 < d_3$

Ans. Correct option : (d)

Explanation : In a liquid of higher density more part of the object remains outside the liquid. As buoyant force is directly proportional to density of the liquid and More buoyant force means more the floating.

$$\frac{1}{9} < \frac{2}{11} < \frac{3}{7}$$

Thus, the order of densities in increasing order is $d_1 < d_2 < d_3$.

Q. 6. In the relation F = G M m/d², the quantity G :

(a) depends on the value of g at the place of observation.

(b) is used only when the Earth is one of the two masses.

(c) is greatest at the surface of the Earth.

(d) is universal constant of nature.

Ans. Correct option : (d)

Explanation : The quantity G is universal constant of nature. It is applied to all objects present in the universe. It is constant of proportionality in Newton's universal law of gravitation.

Q. 7. Law of gravitation gives the gravitational force between :

(a) the Earth and *a* point mass only.

(b) the Earth and sun only.

(c) any two bodies having some mass.

(d) two charged bodies only.

Ans. Correct option : (c)

Explanation : Law of gravitation is applicable to all bodies having some mass. And is given by.

$$F = \frac{Gm_1m_2}{r^2}$$

where,

F = Force of attraction between the two bodies

m_1, m_2 = Mass of two bodies

G = Gravitational constant

r = Distance between the two bodies

Q. 8. The value of quantity G in the law of gravitation :

(a) depends on mass of Earth only.

(b) depends on radius of Earth only.

(c) depends on both mass and radius of Earth.

(d) is independent of mass and radius of the Earth.

Ans. Correct option : (d)

Explanation : G is the constant of proportionality and it is called the universal gravitational constant. It is independent of mass and radius of the Earth.

Q. 9. Two particles are placed at some distance. If the mass of each of the two particles is doubled, keeping the distance between them unchanged, the value of gravitational force between them will be :

(a) ¼ times. (b) 4 times.

(c) ½ times. (d) unchanged.

Ans. Correct option : (b)

Explanation : We know that, according to gravitational force

$$F = G\frac{Mm}{r^2} \quad \text{... (i)}$$

Where, F = Force between two masses

M = First mass

m = Second mass

G = Gravitational constant

r = Distance between the two masses

According to the question,

F' = New force when mass is doubled

If mass of each particle is doubled

That is, M ⇒ 2M and m ⇒ $2m$

put these values in eq. (i), we get

$$F' = G\frac{(2M)(2m)}{r^2}$$

$$F' = 4G\frac{Mm}{r^2}$$

From eq. (i) F' = 4F

Q. 10. The atmosphere is held to the Earth by :

(a) gravity. (b) wind.

(c) clouds. (d) Earth's magnetic field.

Ans. Correct option : (a)

Explanation : The atmosphere is held to the Earth by gravity due to Newton's law of gravitation.

Q. 11. The force of attraction between two-unit point masses separated by a unit distance is called :

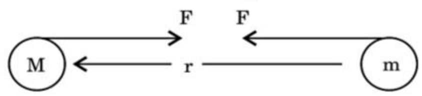
(a) gravitational potential.

(b) acceleration due to gravity.
(c) gravitational field.
(d) universal gravitational constant.

Ans. Correct option : (d)

Explanation : As we know that

$$F = G\frac{Mm}{r^2}$$



where,

F = Force between two masses
m = Mass of first body
M = Mass of second body
G = Gravitational Constant
r = Distance between the two masses

Given,
M = 1 unit, m = 1 unit, and r = 1 unit, we get

$$F = G\frac{1 \times 1}{(1)^2}$$

$$\Rightarrow \quad F = G$$

Therefore, the force of attraction between two unit point masses separated by a unit distance is called universal gravitational constant.

Q. 12. The weight of an object at the centre of the Earth of radius R is :

(a) zero.
(b) infinite.
(c) R times the weight at the surface of the Earth.
(d) $1/R^2$ times the weight at surface of the Earth.

Ans. Correct option : (a)

Explanation : The weight of an object is given by

$$w = mg$$

At the centre of the Earth, acceleration due to gravity g is zero.
And is given by

$$g' = g\left(1 - \frac{h}{R}\right)$$

Here, h = distance from surface of the Earth to centre of the Earth
And at centre $h = R$

$$g' = 0$$

$$\text{So, } w = m \times 0 = 0$$

Q. 13. An object weighs 10 N in air. When immersed fully in water, it weighs only 8 N. The weight of the liquid displaced by the object will be

(a) 2 N **(b)** 8 N
(c) 10 N **(d)** 12 N

Ans. Correct option : (a)

Explanation : Given that,
Weight of an object in air = 10 N
Weight of an object in water = 8 N
So, the weight of the liquid displaced by the object
F = 10 – 8 = 2N
According to Archimedes' Principle
Buoyancy force = Weight of the liquid displaced by the body

Q.14. A girl stands on a box having 60 cm length, 40 cm breadth and 20 cm width in three ways. In which of the following cases, pressure exerted by the brick will be

(a) maximum when length and breadth form the base.
(b) maximum when breadth and width form the base.
(c) maximum when width and length form the base.
(d) the same in all the above three cases.

Ans. Correct option : (b)

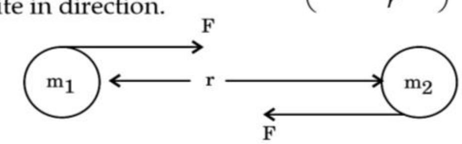
Explanation : When base is formed by breadth and width. Area will be minimum. And so, pressure will be maximum.

Q. 15. An apple falls from a tree because of gravitational attraction between the Earth and apple. If F_1 is the magnitude of force exerted by the Earth on the apple and F_2 is the magnitude of force exerted by apple on Earth, then :

(a) F_1 is very much greater than F_2.
(b) F_2 is very much greater than F_1.
(c) F_1 is only a little greater than F_2.
(d) F_1 and F_2 are equal.

Ans. Correct option : (d)

Explanation : According to Newton's universal law of gravitation, forces exerted on each other are equal in magnitude and given by $\left(F = \frac{Gm_1m_2}{r^2}\right)$ and opposite in direction.



Short Answer Type Questions

Q. 16. What is the source of centripetal force that a planet requires to revolve around the Sun? On what factors does that force depend?

Ans. The motion of the planet around the sun is due to the centripetal force. Gravitational force is the source of centripetal force that a planet requires to revolve around the Sun.
It depends upon the following factors

(i) Mass of the Earth and the Sun, that is, depends on the product of the masses of the planet and the Sun.
(ii) Distance between the Earth and the Sun, that is, depends on the square of distance between the Earth and the Sun.

Q. 17. On the Earth, a stone is thrown from a height in a direction parallel to the Earth's surface while another stone is simultaneously dropped from the same height. Which stone would reach the ground first and why ?

Ans. Both the stones will take the same time to reach the ground because the two stones fall from the same height.
Time taken by both stones is given by second equation of motion $s = ut + \frac{1}{2}at^2$
for both cases initial velocity in the vertical direction
$u = 0$
so,

$$s = \frac{1}{2}at^2$$

And here $s = h$, distance covered in vertical direction
where, $a = g$, acceleration due to gravity
t = time displacement to time

So,

$$h = \frac{1}{2}gt^2$$

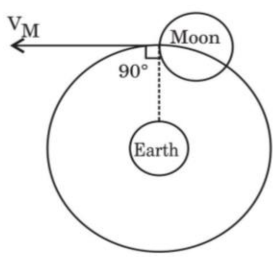
$$t = \sqrt{\frac{2h}{g}}$$

Therefore, if we drop down a stone from a height (h) in vertical direction and at the same time we throw another stone in horizontal direction, then both the stones strike the Earth simultaneously but at different places.

Q. 18. Suppose gravity of Earth suddenly becomes zero, then in which direction will the Moon begin to move if no other celestial body affects it ?

Ans. In the absence of gravity of Earth, the Moon flies off along a straight line.

This straight line will be a tangent to the circular path.



Q. 19. Identical packets are dropped from two aeroplanes, one above the equator and the other above the north pole, both at height *h*. Assuming all conditions are identical, will those packets take same time to reach the surface of Earth. Justify your answer.

Ans. At a given place, the value of acceleration due to gravity is constant but it varies from one place to another place on the Earth surface.

It is due to the fact that Earth is not a perfect sphere. It is flattened at the poles and bulges out at the equator (ellipsoidal shaped). Thus, the value of 'g' is minimum at the equator and maximum at the poles.

It means, 'g' increases as we go from equator to pole. Therefore, the packet falls slowly at equator compared to the poles. Thus, the packet will remain in air for a longer time, when it is dropped at the equator.

Q. 20. The weight of any person on the Moon is about 1/6 times that on the Earth. He can lift a mass of 15 kg on the Earth. What will be the maximum mass, which can be lifted by the same force applied by the person on the Moon ?

Ans. On Earth, force (F) applied by the man on Earth

$$F = m \times g \qquad (m = \text{mass})$$

At Moon, acceleration due to gravity is 1/6th to that of Earth. So, the maximum mass M is which can be lifted on Moon

$$M' = \frac{F}{(g/6)} = \frac{m \times g}{(g/6)}$$

$$= 6\text{ m}$$
$$= 6 \times 15\text{ kg}$$
$$= 90\text{ kg}$$

So, 90 kg will be maximum mass which can be lifted on the moon by the same force applied by the person.

Q. 21. Calculate the average density of the Earth in terms of *g*, *G* and *R*.

Ans. We know that, $\left(g = \frac{GM}{R^2}\right)$

where,
M = Mass of the Earth,
R = Radius of the Earth.

So, mass of the earth $M = \frac{gR^2}{G}$ ...(i)

If the Earth is considered as a sphere of radius R and of material of density ρ then

Mass of the earth = volume of the earth × density

[Volume of sphere $= \frac{4}{3}\pi r^3$]

$$M = \frac{4}{3}\pi R^3 \rho$$

From eq. (i), we get

$$\frac{4}{3}\pi R^3 \rho = \frac{gR^2}{G}$$

$$\rho = \frac{3g}{4\pi GR}$$

Q. 22. The Earth is acted upon by gravitation of Sun, even though it does not fall into the Sun. Why ?

Ans. According to Newton's first law of motion, an object in motion tends to move in straight-line motion at a constant speed unless acted upon by an external, unbalanced force.

When the Earth comes close to the Sun that has a large gravitational force, the path of the Earth is altered due to the unbalanced force of gravity on it. The Sun exerts an attractive force on the Earth, accelerating the Earth directly towards the Sun.

This force on the Earth, directed towards the centre of the orbit, is also known as the centripetal force. At the same time, due to the Earth's inertia and its curvilinear motion, another force called the centrifugal force is acting on the Earth. This force acts away from the centre of the orbit, thus away from the Sun. The magnitude of this force is exactly equal to the Sun's gravitational force on the Earth, thus holiness the centripetal force of the Sun's gravity. Thus the earth remains in orbit to the Sun.

Short Answer Questions

Q. 23. How does the weight of an object vary with respect to mass and radius of the Earth? In a hypothetical case, if the diameter of the Earth becomes half of its present value and its mass becomes four times of its present value, then how would the weight of any object on the surface of the Earth be affected ?

Ans. Let the radius of the Earth = R

The mass of the Earth = M

Mass of the object = m

The weight of the object will be = w

Then, $w = mg = \text{G M}m/\text{R}^2$

According to the question, the diameter of the Earth becomes half of its present value. So, radius also becomes half of its initial value.

So, $$\text{R}' = \frac{\text{R}}{2}$$

Again, mass of Earth becomes four times its present value.

So, $\text{M}' = 4\,\text{M}$

Now, weight of the object

$$w' = \frac{\text{GM}'\,m}{\text{R}^2}$$

Put $\text{R}' = \frac{\text{R}}{2}$ and $\text{M}' = 4\,\text{M}$ in above formula, we get

$$w' = \frac{\text{G}\,(4\,\text{M})m}{(\text{R}/2)^2}$$

$$= \frac{4\text{GM}m}{\text{R}^2} \times 4$$

$$= \frac{16\text{GM}m}{\text{R}^2}$$

From eq. (i), we get $w' = 16w$

Thus, weight of the object will become 16 times of its original weight.

Q. 24. How does the force of attraction between the two bodies depend upon their masses and distance between them ? A student thought that two bricks tied together would fall faster than a single one under the action of gravity. Do you agree with his hypothesis or not ? Comment.

Ans. The force of attraction between two bodies of masses m_1 and m_2 and separated by distance r is given by Newton's universal law of gravitation.

The universal law of gravitation states that, "Any two objects exert a gravitational force of attraction on each other. The direction of the force is along the line joining the objects. The magnitude of the force is proportional to the product of the masses of the objects, and inversely proportional to the square of the distance between them."

For two objects of masses m_1 and m_2 and the distance between them r, the force (F) of attraction acting between them is given by the universal law of gravitation as,

$$\text{F} = \frac{\text{G}m_1m_2}{r^2}$$

where, G is the universal gravitation constant and its value is $6.67 \times 10^{-11}\,\text{Nm}^2\,\text{kg}^{-2}$.

According to the universal law of gravitation, gravitational force (F) acting between two objects is inversely proportional to the square of the distance (r) between them, that is,

$$\text{F} \propto 1/r^2$$

and directly proportional to the product of the masses of both objects.

$$F \propto m_1m_2$$

All the objects fall on the ground with constant acceleration, called acceleration due to gravity (in the absence of air resistance). It is constant (g = 9.8 m/s^2) and it does not depend upon the mass of the object. Hence, heavy objects do not fall faster than light objects.

$$\text{g} = \frac{\text{GM}}{\text{R}^2}$$

where,

R = radius of the Earth

M= mass of the Earth

G = universal gravitation constant

And from the equation, it is clear that acceleration due to gravity (*i.e.*, g) depends only on mass of the Earth, the radius of the Earth. So, two bricks tied together will not fall faster than a single brick under the action of gravity. Hence, the hypothesis is not correct.

This is because acceleration due to gravity, with which brick/bricks fall does/do not depend upon the mass of the body falling body. So, in case of free fall, the two bricks tied together would not fall faster than a single one.

Q. 25. Two objects of masses m_1 and m_2 having the same size are dropped simultaneously from heights h_1 and h_2 respectively. Find out the ratio of time they would take in reaching the ground. Will this ratio remain the same if :

(i) one of the objects is hollow and the other one is solid and

(ii) both of them are hollow, size remaining the same in each case. Give reason.

Ans. Given,

Mass of the two objects m_1 and m_2 dropped simultaneously from heights h_1 and h_2.

Initial velocities u_1 and $u_2 = 0$

Using Newton's second equation of motion,

$$h = ut + 1/2\,gt^2$$

Then put the values,

$$h = 1/2\,gt^2 \qquad [\because u = 0]$$

$$2h = gt^2$$

$$t = \sqrt{\frac{2h}{g}}$$

Time taken by first object of mass, m_1

$$t_1 = \sqrt{\frac{2h_1}{g}} \qquad \text{[for second object } u = 0]$$

Similarly, time taken by object of mass m_2

$$t_2 = \sqrt{\frac{2h_2}{g}} \Rightarrow \frac{t_1}{t_2} = \sqrt{\frac{h_1}{h_2}}$$

(i) Acceleration due to gravity is independent of mass of falling body. So, the ratio remains the same.

(ii) If bodies are hollow, then also ratio remains the same, that is, $t_1 : t_2 = \sqrt{h_1} : \sqrt{h_2}$

So, the ratio will not change in either case because acceleration remains the same. In case of free-fall acceleration does not depend upon mass and size.

BOARD CORNER

Short Answer Type Questions

Q. 1. What happens to the force between two objects. if :

(i) the mass of one objects is doubled ?

(ii) the distance between objects is doubled and tripled ?

(iii) the masses of both objects are doubled ?

[KVS, 2018, Agra Region]

Ans. According to law of gravitation, the force between two objects (m_1 and m_2) is proportional to their masses and inversely proportional to the square of the distance (R) between them.

$F = \frac{Gm_1m_2}{R^2}$ (G = gravitational constant)

(i) If the mass of one object is doubled, let the new force be F

Then, $F' = \frac{G \times 2m_1 \times m_2}{R^2} F$

$F' = 2F$, hence force will be doubled.

(ii) If the distance between the objects is doubled and tripled

Then, $F' = \frac{G \times 2m_1 \times m_2}{(2R)^2} F = \frac{1}{4} F$

E hence force become one-fourth of its initial force.

and $F' = \frac{G \times 2m_1 \times m_2}{(3R)^2}$

$F' = \frac{1}{9} F$

E hence force become one-ninth of its initial force.

(iii) If the masses of both objects are doubled.

Then, $F' = \frac{G \times 2m_1 \times m_2}{R^2}$

$F' = 4F$, hence force will be four times more 3 its original value. **1+1+1**

Q. 2. A 8000 kg engine pulls a train of 5 wagons, each of 2000 kg, alone a horizontal track. It the engine exerts a force of 40000 N and the track offers a friction force 5000 N, then calculate :

(a) The net accelerating force ?

(b) The acceleration of the train.

(c) The force of wagon 1 on wagon 2.

[KVS, 2019, Agra Region]

Ans. (a) Given, $f = 40000$ N, $F_{track} = 5000$ N

Net accelerating force, $F_a = F - F_{track}$

$= 40000 - 5000$ N

$= 350000$ N

Total mass of train, $m = 80000 + 5 \times 2000$

$= 18000$ kg

(b) Acceleration of train, $a = \frac{F_a}{m}$

$= \frac{35000}{18000}$

$= 1.944$ m/s^2

(c) Force of wagon 1 on wagon 2

$= m_{wagon} \times 4 \times a$, as 4 wagons are pulled by wagon.

$= 4 \times 2000 \times 1.944$

$= 15552$ N **1+1+1**

Commonly Made Error

- Calculation error is commonly seen. In many answers, formula is missing and no unit is written for the final answer.

Answering Tip

- While solving numerical it is advisable that the formula need to be written in the beginning. Essential steps need to be shown and final answer needs to be expressed along with a proper unit.