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IIT-JEE-NEET-PHYSICS



KINETIC THEORY OF GASES

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KINETIC THEORY OF GASES

Welcome to the Kinetic Theory of Gases Study Module tailored for IIT-JEE aspirants. This module provides a deep dive into the fundamental principles of the Kinetic Theory of Gases, an essential topic in physics, particularly for mastering concepts in thermodynamics and statistical mechanics.

- **Comprehensive Coverage:** This module covers all essential concepts of the Kinetic Theory of Gases, providing a strong foundation for tackling advanced topics in thermodynamics and statistical mechanics.
- **Clear Explanation:** Concepts are explained clearly and concisely, supported by illustrative examples and diagrams to facilitate understanding.
- **Practice-oriented Approach:** Ample practice questions, problem-solving exercises, and conceptual quizzes enable students to apply their knowledge and reinforce their understanding of the Kinetic Theory of Gases.



KEY - FEATURES

- **Particle Model:** The Kinetic Theory of Gases proposes that gases consist of a large number of tiny particles (atoms or molecules) in constant motion. These particles move randomly and collide with each other and the walls of the container.
- **Assumptions:** The theory is based on several assumptions, including that gas particles are point masses, their volume is negligible compared to the volume of the container, and there are no intermolecular forces except during collisions.
- **Kinetic Energy and Temperature:** Gas particles possess kinetic energy due to their motion, and the average kinetic energy is directly proportional to the absolute temperature of the gas (as described by the equation $KE = (3/2)kT$, where KE is the kinetic energy, k is Boltzmann's constant, and T is the temperature in Kelvin).
- **Pressure and Collision Model:** The pressure exerted by a gas on the walls of its container arises from the numerous collisions between gas particles and the container walls. The force exerted by each collision contributes to the overall pressure exerted by the gas.
- **Ideal Gas Law:** The behavior of gases under various conditions is described by the Ideal Gas Law, which relates the pressure, volume, temperature, and number of moles of a gas through the equation $PV = nRT$ (where P is pressure, V is volume, n is the number of moles, R is the gas constant, and T is the temperature in Kelvin).
- **Deviation from Ideal Behavior:** While the behavior of most gases can be described reasonably well by the Ideal Gas Law, real gases may deviate from ideal behavior under certain conditions, particularly at high pressures or low temperatures.

KINETIC THEORY OF GASES

02

IN THIS CHAPTER ...

- Ideal Gas or Perfect Gas
- Equation of State of a Perfect Gas
- Kinetic Theory of Gases
- Degree of Freedom
- Internal Energy of an Ideal Gas
- Law of Equipartition of Energy
- Specific Heat Capacity of Monoatomic, Diatomic and Polyatomic Gases
- Mean Free Path

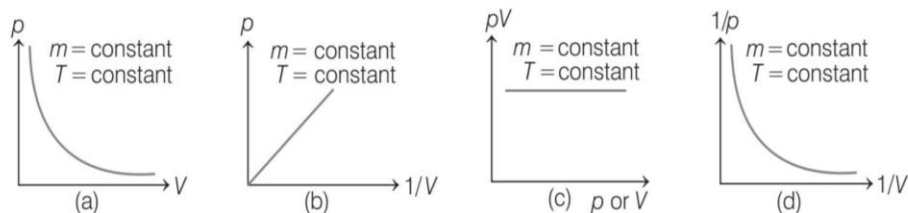
Ideal Gas or Perfect Gas

A gas which strictly obeys the gas laws such as Boyle's law, Charles' law, Gay-Lussac's law, Avogadro's law, etc., is known as an *ideal gas* or *perfect gas*.

These law (gaseous laws) are given below

Boyle's Law According to this law, "for a given mass of an ideal gas at constant temperature (called *isothermal process*), the volume of a gas is inversely proportional to its pressure." i.e. $V \propto \frac{1}{p}$ (if m and $T = \text{constant}$)

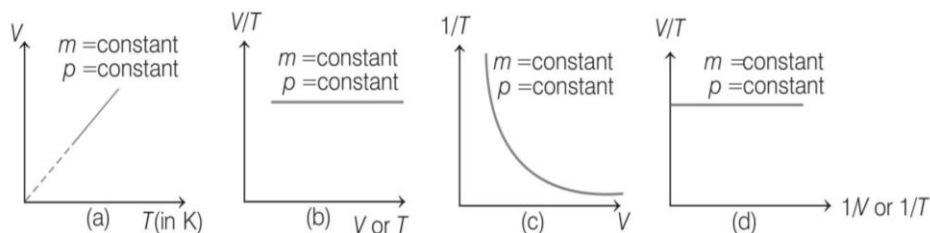
Graphical forms of such law are shown in following figure



Charles' Law According to this law, "for a given mass of an ideal gas at constant pressure (called **isobaric process**), volume of a gas is directly proportional to its absolute temperature."

i.e. $V \propto T$ (if m and $p = \text{constant}$)

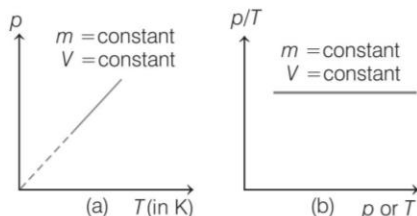
Graphical forms of such law are shown in following figure



Gay-Lussac's Law or Pressure Law According to this law, "for a given mass of an ideal gas at constant volume (called *isochoric process*), pressure of a gas is directly proportional to its absolute temperature."

i.e. $p \propto T$ (if m and $V = \text{constant}$)

Graphical forms of such law are shown in following figure



Avogadro's Law According to this law, "at same temperature and pressure, equal volumes of all the gases contain equal number of molecules."

i.e. $N_1 = N_2$

where, N_1 and N_2 are number at molecules at first and second, gas respectively.

Equation of State of a Perfect Gas

In practice, the gases do not obey the gas laws at all values of temperature and pressure. It is because of the intermolecular forces between the gas molecules.

Ideal gas equation is a form of combined effect of above first four laws. Thus, the equation is given by

$$pV = nRT = \frac{m}{M} RT$$

Here, $n = \text{number of moles of the gas} = \frac{m}{M}$,

$m = \text{total mass of the gas,}$

$M = \text{molecular mass}$

and $R = \text{universal gas constant}$

$$= 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

$$= 2.0 \text{ cal mol}^{-1} \text{ K}^{-1}.$$

Above equation is also known as equation of state of an ideal gas.

- In terms of density, the ideal gas equation may be expressed as

$$\frac{p}{\rho T} = \text{constant}$$

- In terms of number of molecules (n) per unit volume of a gas, the ideal gas equation may be expressed as

$$p = nkT$$

- If n_1 mole of a non-reactive gas in thermodynamical state (p_1, V_1, T_1) be mixed with n_2 mole of another non-reactive gas at (p_2, V_2, T_2) and the resultant gas mixture is at a state (p, V, T), then

$$\frac{p_1 V_1}{T_1} + \frac{p_2 V_2}{T_2} = \frac{pV}{T}$$

Work Done on Compressing a Gas

Work done on compressing a gas can be written as

$$W = \frac{mR(T_2 - T_1)}{1 - n}$$

where, $m = \text{total mass of the gas,}$

$R = \text{universal gas constant,}$

$n = \text{number of moles of the gas,}$

T_1 and $T_2 = \text{initial and final temperature of the gas.}$

Example 1. During an experiment, an ideal gas is found to obey an additional law $Vp^2 = \text{constant}$. The gas is initially at temperature T and volume V . What will be the temperature of the gas when it expands to a volume $2V$?

- (a) $\sqrt{3} T$ (b) $\sqrt{1/2} T$
 (c) $\sqrt{2} T$ (d) $\sqrt{3} T$

Sol. (c) Here it is given that $Vp^2 = \text{constant } K$ (say). Hence, we may write the gas equation as, $pV = nRT$

or $\sqrt{\frac{K}{V}} \cdot V = nRT$ or $\sqrt{V} = \frac{nR}{\sqrt{K}} T$

$$\Rightarrow \frac{\sqrt{V_1}}{\sqrt{V_2}} = \frac{T_1}{T_2}$$

$$\therefore T_2 = T_1 \sqrt{\frac{V_2}{V_1}} = T \sqrt{\frac{2V}{V}} = \sqrt{2} T$$

Example 2. A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases, at a temperature of 27°C and pressure of $1 \times 10^5 \text{ Nm}^{-2}$. The total mass of the mixture is 28 g . If the gram molecular weights of neon and argon are 20 and 40 respectively, find the masses of the individual gases in the container, assuming them to be ideal.

(Given, $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$)

- (a) $4 \text{ g, } 24 \text{ g}$ (b) $5 \text{ g, } 25 \text{ g}$
 (c) $6 \text{ g, } 26 \text{ g}$ (d) $7 \text{ g, } 27 \text{ g}$

Sol. (a) Let in the given container mass of neon be m and mass of argon be $(28 - m) \text{ g}$, so that

$$n_{\text{Ne}} = \frac{m}{20}$$

and $n_{\text{Ar}} = \frac{28 - m}{40}$

$$n = n_{\text{Ne}} + n_{\text{Ar}} = \frac{m}{20} + \frac{(28 - m)}{40} = \frac{28 + m}{40} \quad \dots(i)$$

and using ideal gas equation for the mixture, we have

$$n = \frac{pV}{RT} = \frac{1 \times 10^5 \times 0.02}{8.314 \times 300} = 0.8 \quad \dots(ii)$$

Comparing Eqs.(i) and (ii), we get

$$\frac{28 + m}{40} = 0.8$$

$$\Rightarrow m = 4 \text{ g}$$

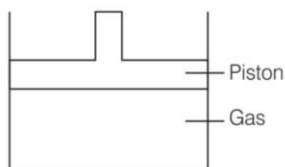
$$\therefore n_{\text{Ne}} = 4 \text{ g}$$

and $n_{\text{Ar}} = 28 - 4 = 24 \text{ g}$

Example 3. Two moles of an ideal gas is contained in a cylinder fitted with a frictionless movable piston, exposed to the atmosphere, at an initial temperature T_0 . The gas is slowly heated so that its volume becomes four times the initial value. The work done by the gas is

- (a) zero (b) $2RT_0$ (c) $4RT_0$ (d) $6RT_0$

Sol. (d) Given that gas is slowly heated, which means it remains in equilibrium with the atmosphere, i.e. the process takes place at constant pressure.



From the equation of ideal gas law,
 $pV = nRT$

For infinitesimal change,

$$pdV = nRdT$$

or

$$p\Delta V = nR\Delta T$$

Also, $p\Delta V =$ work done by the gas $= \Delta W$

$$\therefore \Delta W = nR\Delta T$$

Also, $\Delta V \propto \Delta T$

$$\therefore \Delta T \propto \Delta V \propto V_2 - V_1$$

Given, $V_2 = 4V_1$

$$\therefore \Delta T \propto 4V_1 - V_1 \propto 3V_1 \propto 3T_0$$

Also, it is given that, $n = 2$ moles

The expression for work done becomes

$$\Delta W = nR\Delta T$$

$$\Delta W = 2R \cdot 3T_0 = 6RT_0$$

Example 4. One mole of an ideal gas passes through a process, where pressure and volume obey the relation

$$p = p_0 \left[1 - \frac{1}{2} \left(\frac{V_0}{V} \right)^2 \right]. \text{ Here, } p_0 \text{ and } V_0 \text{ are constants. Calculate}$$

the change in the temperature of the gas if its volume changes from V_0 to $2V_0$. [JEE Main 2019]

- (a) $\frac{1}{2} \frac{p_0 V_0}{R}$ (b) $\frac{1}{4} \frac{p_0 V_0}{R}$ (c) $\frac{3}{4} \frac{p_0 V_0}{R}$ (d) $\frac{5}{4} \frac{p_0 V_0}{R}$

Sol. (d) Given process equation for 1 mole of an ideal gas is

$$p = p_0 \left[1 - \frac{1}{2} \left(\frac{V_0}{V} \right)^2 \right] \quad \dots(i)$$

Also, for 1 mole of ideal gas,

$$pV = RT$$

$$\therefore p = \frac{RT}{V} \quad \dots(ii)$$

So, from Eqs. (i) and (ii), we have

$$\frac{RT}{V} = p_0 \left[1 - \frac{1}{2} \left(\frac{V_0}{V} \right)^2 \right]$$

$$\therefore T = \frac{p_0 V}{R} \left[1 - \frac{1}{2} \left(\frac{V_0}{V} \right)^2 \right] \quad \dots(iii)$$

When volume of gas is V_0 , then by substituting $V = V_0$ in Eq. (iii), we get

Temperature of gas is

$$T_1 = \frac{p_0 V_0}{R} \left[1 - \frac{1}{2} \left(\frac{V_0}{V_0} \right)^2 \right] = \frac{p_0 V_0}{2R}$$

Similarly, at volume, $V = 2V_0$

Temperature of gas is

$$T_2 = \frac{p_0 (2V_0)}{R} \left[1 - \frac{1}{2} \left(\frac{V_0}{2V_0} \right)^2 \right] = \frac{7}{4} \frac{p_0 V_0}{R}$$

So, change in temperature as volume changes from V_0 to $2V_0$ is

$$\Delta T = T_2 - T_1 = \left(\frac{7}{4} - \frac{1}{2} \right) \frac{p_0 V_0}{R} = \frac{5}{4} \frac{p_0 V_0}{R}$$

Example 5. A vessel contains two non-reactive gases neon (monoatomic) and oxygen (diatomic). The ratio of their partial pressure is 3 : 2. The ratio of number of molecules is [Given atomic mass of Ne = 20.2 u, molecular mass of $O_2 = 32$ u] [NCERT]

- (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

Sol. (b) Each gas (assumed ideal) obey the gas laws. Since, V and T are common to the two gases, we have

$$p_1 V = \mu_1 RT \text{ and } p_2 V = \mu_2 RT$$

$$\frac{p_1}{p_2} = \frac{\mu_1}{\mu_2}$$

$$\frac{3}{2} = \frac{\mu_1}{\mu_2}$$

Here, 1 and 2 refer to neon and oxygen gas respectively.

$$\text{Given, } \frac{p_1}{p_2} = \frac{3}{2}$$

$$\frac{\mu_1}{\mu_2} = \frac{3}{2}$$

$$\text{By definition, } \mu_1 = \frac{N_1}{N_A} \text{ and } \mu_2 = \frac{N_2}{N_A}$$

where, N_1 and N_2 are the number of molecules of two gases and N_A is Avogadro's number.

$$\frac{N_1}{N_2} = \frac{\mu_1}{\mu_2} = \frac{3}{2}$$

Kinetic Theory of Gases

The kinetic theory of gases correlates the macroscopic properties of gases (e.g. pressure, temperature, etc) to the microscopic properties of gas molecules (e.g. speed, momentum, kinetic energy of molecules, etc). This theory attempts to develop a model of the molecular behaviour of gas, which results in the observed behaviour of an ideal gas. The kinetic theory of gases is based on the following assumptions

- (i) A gas consists of a large number of tiny, identical, spherical and electrically neutral, stable elastic particles called *molecules*.
- (ii) The space occupied by the molecules of a gas is extremely small as compared to the volume of the gas.

Critical Temperature, Pressure and Volume

Gases cannot be liquified above a temperature called *critical temperature* (T_c) however large the pressure may be. The pressure required to liquify the gas at critical temperature is called *critical pressure* (p_c) and the volume of the gas at critical temperature and pressure is called *critical volume* (V_c).

Value of critical constants in terms of Van der Waals' constants a and b are as under

$$V_c = 3b,$$

$$p_c = \frac{a}{27b^2}$$

and

$$T_c = \frac{8a}{27Rb}$$

Further, $\frac{RT_c}{p_c V_c} = \frac{8}{3}$ is called **critical coefficient** and is same for all gases.

Different Speeds of Gas Molecules

• **rms Speed** The square root of the mean of squares of the speeds of different molecules is known as root mean square (rms) speed of gas molecules.

$$i.e. \quad v_{rms} = \sqrt{\bar{v}^2}$$

$$= \sqrt{(v_1^2 + v_2^2 + \dots + v_N^2)/N}$$

According to kinetic theory of gases, it is observed that

$$v_{rms} = \sqrt{\bar{v}^2} = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3pV}{M}}$$

$$= \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{\frac{3kT}{m}}$$

where, M is the molar mass of gas, while m is the mass of a single gas molecule.

• **Average Speed** It is the arithmetic mean of the speeds of molecules in a gas. Thus,

$$v_{av} = \bar{v} = \frac{v_1 + v_2 + \dots + v_N}{N}$$

On the basis of kinetic theory, it is observed that

$$v_{av} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8p}{\pi\rho}} = \sqrt{\frac{8kT}{\pi m}}$$

• **Most Probable Speed** It is the speed possessed by maximum number of gas molecules in a given gas. Basis on kinetic theory, it is found that

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2p}{\rho}}$$

Thus, we find that for a given gas, $v_{mp} < v_{av} < v_{rms}$ and

$$v_{mp} : v_{av} : v_{rms} = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}.$$

Example 7. A flask contains argon and chlorine in the ratio of 2 : 1 by mass. The temperature of the mixture is 27°C. Ratio of root mean square speed v_{rms} of the molecules of the two gases is [Given, atomic mass of argon = 39.9 u, molecular mass of chlorine = 70.9 u] [NCERT]

- (a) 0.22 (b) 2.2 (c) 1.33 (d) 3.3

Sol. (c) The average kinetic energy (per molecule) of any (ideal) gas is always equal to $\frac{3}{2}k_B T$. It depends only on temperature and is independent of the nature of the gas. Since, argon and chlorine both have the same temperature in the flask, therefore the ratio of average kinetic energy (per molecule) of the two gases is 1 : 1.

$$\text{Now,} \quad \frac{1}{2} m v_{rms}^2 = \text{average kinetic energy per molecule}$$

$$= \frac{3}{2} k_B T$$

$$\frac{(v_{rms}^2)_{Ar}}{(v_{rms}^2)_{Cl}} = \frac{(m)_{Cl}}{(m)_{Ar}} = \frac{(M)_{Cl}}{(M)_{Ar}} = \frac{70.9}{39.9} = 1.77$$

where, M denotes the molecular mass of the gas. Taking square root

$$\frac{(v_{rms})_{Ar}}{(v_{rms})_{Cl}} = 1.33$$

Example 8. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be

- (a) 100√5 m/s (b) 80 m/s [JEE Main 2019]
(c) 100 m/s (d) 80√5 m/s

Sol. (a) For a gas molecule, $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$\therefore v_{rms} \propto \sqrt{T}$$

Let unknown rms speed be $v_{rms,2}$.

At $T_2 = 227^\circ\text{C}$ (or 500 K)

and at $T_1 = 127^\circ\text{C}$ (or 400 K)

$$v_{rms,1} = 200 \text{ m/s}$$

∴ Using the relation $v_{rms} \propto \sqrt{T}$, we can write

$$\frac{v_{rms,2}}{v_{rms,1}} = \sqrt{\frac{T_2}{T_1}} \quad \dots (i)$$

Substituting these given values in Eq. (i), we get

$$\therefore v_{rms,2} = \sqrt{\frac{500}{400}} \times 200 \text{ m/s}$$

$$= \frac{1}{2} \sqrt{5} \times 200 \text{ m/s} = 100\sqrt{5} \text{ m/s}$$

Example 9. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to (Given, Boltzmann constant, $k_B = 1.38 \times 10^{-23}$ J/K, Avogadro number $N_A = 6.02 \times 10^{26}$ /kg, radius of earth = 6.4×10^6 m and gravitational acceleration on earth = 10 ms^{-2})

- (a) 10^4 K (b) 650 K (c) 3×10^5 K (d) 800 K

Sol. (a) Root mean square velocity of hydrogen molecule is given as

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$$

Escape velocity of hydrogen molecule from the earth is given as

$$v_e = \sqrt{2gR_e}$$

Given, $v_{\text{rms}} = v_e$

or
$$\sqrt{\frac{3k_B T}{m}} = \sqrt{2gR_e}$$

$$\Rightarrow T = \frac{2gR_e m}{3 \times k_B}$$

Substituting the given values, we get

$$T = \frac{2 \times 10 \times 6.4 \times 10^6 \times 2}{3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{26}} \approx 10^4 \text{ K}$$

Note Question gives Avogadro number, $N_A = 6.02 \times 10^{26}/\text{kg}$ but we take $N_A = 6.02 \times 10^{26}/\text{kmol}$.

Van der Waals' Gas Equation

The gases actually found in nature are called *real gases*. They do not obey the gas laws. A real gas behaves as ideal gas most closely at *low pressure and high temperature*.

Equation of state for real gases is given by Van der Waals' equation with two corrections in ideal gas

(i) volume correction (ii) pressure correction.

Van der Waals' gas equation for 1 mole of gas is given by

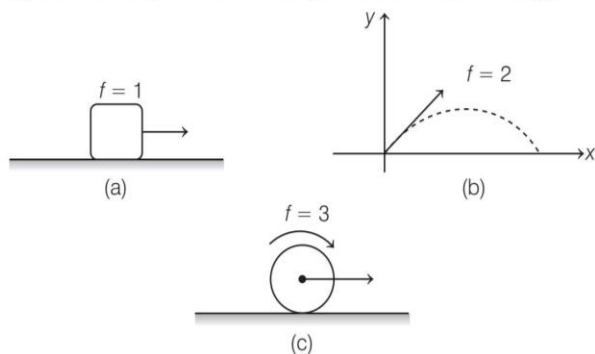
$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

For n moles,
$$\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

Here, a and b are constants called **Van der Waals' constants**.

Degree of Freedom (f)

The term degree of freedom refers to the number of possible independent ways in which a system can have energy.



For example In Fig. (a), block has one degree of freedom, because it is confined to move in a straight line and has only one translational degree of freedom.

In Fig. (b), the projectile has two degrees of freedom because it is confined to move in a plane and so it has two translational degrees of freedom.

In Fig. (c), the sphere has two degrees of freedom, *i.e.* one rotational and another translational.

Similarly, a particle free to move in space will have three translational degrees of freedom.

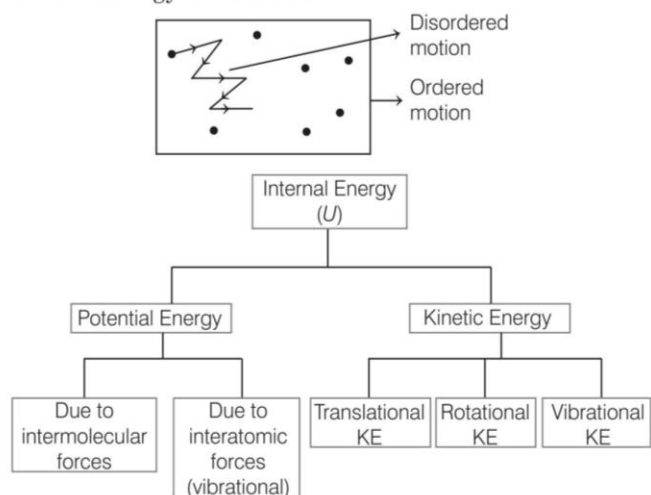
- Degrees of freedom for different gases according to atomicity of gas at low temperature is given below.

| Atomicity of Gas | Translational | Rotational | Total | |
|---|---------------|------------|-------|--|
| Monoatomic, e.g. Ar, Ne, ideal gas, etc. | 3 | 0 | 3 | |
| Diatomic, e.g. O ₂ , Cl ₂ , N ₂ , etc. | 3 | 2 | 5 | |
| Triatomic (linear), e.g. CO ₂ , C ₂ H ₂ , etc. | 3 | 2 | 5 | |
| Triatomic (non-linear) or Polyatomic, e.g. H ₂ O, NH ₃ , CH ₄ , etc. | 3 | 3 | 6 | |

- Degrees of freedom of a diatomic and polyatomic gas depends on temperature and since there is no clear cut demarcation line above which vibrational energy become significant. Moreover, this temperature varies from gas to gas. On the other hand, for a monoatomic gas there is no such confusion.
- Degree of freedom here is 3 at all temperatures. Unless and until stated in the question you can take $f = 3$ for monoatomic gas, $f = 5$ for a diatomic gas and $f = 6$ for a non-linear polyatomic gas.
- When a diatomic or polyatomic gas dissociates into atoms it behaves as a monoatomic gas, whose degrees of freedom are changed accordingly.

Internal Energy of an Ideal Gas

The internal energy (U) of the gas is concerned only with its disordered motion, i.e. it is not concerned with its ordered motion. When the temperature of the gas is increased, its disordered motion increases and hence its internal energy is increased.



Internal energy of an ideal gas depends only on its absolute temperature (T) and is directly proportional to T .

or $U \propto T$

Law of Equipartition of Energy

According to the law of equipartition of energy, an ideal gas distributes its internal energy equally in all degrees of freedom. In each degree of freedom, energy of one mole of an ideal gas is $\frac{1}{2}RT$, where T is the absolute

temperature of the gas. Thus, if f be the number of degrees of freedom, then the internal energy of 1 mole of the gas will be $\frac{f}{2}RT$ or internal energy of n moles of the gas will be $\frac{n}{2}fRT$. Thus,

$$U = \frac{n}{2}fRT \quad \dots(i)$$

For a monoatomic gas, $f = 3$.

Therefore, $U = \frac{3}{2}RT$

(for 1 mole of a monoatomic gas)

For a diatomic and linear polyatomic gas at low temperatures, $f = 5$, so,

$$U = \frac{5}{2}RT \quad \text{(for 1 mole)}$$

and for non-linear polyatomic gas at low temperatures, $f = 6$, so

$$U = \frac{6}{2}RT = 3RT \quad \text{(for 1 mole)}$$

Example 10. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T . Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is

- (a) 15 (b) 13
 (c) 11 (d) 20

Sol. (a) Internal energy of a gas mixture = Sum of internal energies of its components.

Also, internal energy of a gas is given by

$$U = \frac{f}{2}(nRT)$$

Here, gases are oxygen (diatomic, $f = 5$) and argon (monoatomic, $f = 3$).

So, total internal energy of the mixture is

$$U = \left(\frac{f}{2}nRT\right)_{O_2} + \left(\frac{f}{2}nRT\right)_{Ar}$$

$$= \frac{5}{2} \times 3 \times RT + \frac{3}{2} \times 5 \times RT = 15RT$$

Molar Specific Heat of the Gases

Consider a container containing m gram of gas of molecular mass M . If n is the number of moles of gas in container, ΔQ is the heat supplied and rise in temperature is ΔT , then

$$c = \frac{1}{m} \frac{\Delta Q}{\Delta T}$$

Further, $n = \frac{m}{M}$ or $m = nM$

so $c = \frac{1}{nM} \frac{\Delta Q}{\Delta T}$

Thus, molar specific heat,

$$C = Mc = \frac{1}{n} \left(\frac{\Delta Q}{\Delta T}\right)$$

where, C = molar specific heat of the mass and c = gram specific heat of the gas.

Molar specific heat is of two types

(i) **Specific heat at constant volume (C_V)** When heat is supplied to the gas at constant volume, the entire heat supplied just increases the internal energy of gas and does nothing else.

$$C_V = \frac{1}{n} \left(\frac{\Delta Q}{\Delta T}\right)_V$$

Thus, $C_V = \frac{1}{n} \left(\frac{\Delta U}{\Delta T}\right)$

(ii) **Specific heat at constant pressure (C_p)** When heat is supplied to the gas at constant pressure, a part of it increases the internal energy of the gas and remaining does an external work.

So, specific heat at constant pressure,

$$C_p = \frac{1}{n} \left(\frac{\Delta Q}{\Delta T}\right)_p$$

- At constant pressure, to increase the internal energy of the gas by the same amount (as in case of heat supplied at constant volume), more amount of heat is supplied. Hence, we conclude that $C_p > C_v$
- The relation between C_p and C_v is given by Mayer's relation which is $C_p - C_v = R$
 Here, R is gas constant.
- C_p and C_v in terms of degrees of freedom f can be written as $C_v = \frac{f}{2}R$
 and $C_p = C_v + R = \frac{f}{2}R + R = \left(\frac{f}{2} + 1\right)R$
- Ratio of specific heats C_p and C_v is $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

Specific Heat Capacity of Monoatomic, Diatomic and Polyatomic Gases

Monoatomic Gases The molar specific heat at constant volume C_v is

$$C_v (\text{monoatomic gas}) = \frac{dU}{dT} = \frac{3}{2}R$$

From Mayer's formula, $C_p - C_v = R$

where, C_p is molar specific heat at constant pressure.

Thus, $C_p = \frac{5}{2}R$

Ratio of specific heat, $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$

Diatomic Gases A diatomic molecule has 5 degrees of freedom, i.e. 3 translational and 2 rotational. Using the law of equipartition of energy, the total internal energy of a mole of such a gas is

$$U = \frac{5}{2}k_B T \times N_A = \frac{5}{2}RT$$

The molar specific heats are then given by

$$C_v (\text{rigid diatomic}) = \frac{5}{2}R, C_p = \frac{7}{2}R$$

$$\gamma (\text{rigid diatomic}) = \frac{7}{5}$$

If the diatomic molecule is not rigid but has an additional vibrational mode, then

$$U = \left(\frac{5}{2}k_B T + k_B T\right) N_A = \frac{7}{2}RT$$

$$C_v = \frac{7}{2}R, C_p = \frac{9}{2}R, \gamma = \frac{9}{7}R$$

Polyatomic Gases A polyatomic molecule has 3 translational, 3 rotational degrees of freedom and a certain number (f) of vibrational modes.

From law of equipartition of energy, one mole of such a gas has

$$U = \left(\frac{3}{2}k_B T + \frac{3}{2}k_B T + f k_B T\right) N_A$$

$$C_v = (3 + f)R$$

$$C_p = (4 + f)R$$

$$\gamma = \frac{(4 + f)}{(3 + f)}$$

Note Specific heat of lighter elements is higher than that of heavier elements and vice versa. Specific heat of the same substance in different states (solid, liquid and vapour) is different. For example, specific heat of water is $1 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$ and that of ice is $0.5 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$.

Example 11. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T . The total internal energy U of a mole of this gas and the value of $\gamma \left(= \frac{C_p}{C_v} \right)$ are given respectively, by

[JEE Main 2020]

- (a) $U = \frac{5}{2}RT$ and $\gamma = \frac{6}{5}$ (b) $U = 5RT$ and $\gamma = \frac{7}{5}$
 (c) $U = \frac{5}{2}RT$ and $\gamma = \frac{7}{5}$ (d) $U = 5RT$ and $\gamma = \frac{6}{5}$

Sol. (c) Given that, translational degree of freedom, $f_{\text{trans}} = 3$

Rotational degree of freedom, $f_{\text{rot}} = 2$

Total degree of freedom,

$$f = f_{\text{trans}} + f_{\text{rot}} = 3 + 2 = 5$$

At T temperature, total internal energy of an ideal gas is given by

$$U = \frac{f}{2} \mu RT$$

$$U = \frac{5}{2}RT \quad (\because \mu = 1)$$

$$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f} = 1 + \frac{2}{5}$$

$$\gamma = \frac{7}{5}$$

Example 12. A cylinder of fixed capacity 44.8 L contains helium gas at standard temperature and pressure. The amount of heat needed to raise the temperature of the gas in the cylinder by 15°C is (Given, $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$)

- (a) 45 J (b) 374 J
 (c) 273 J (d) 432 J

Sol. (b) From ideal gas law, $pV = \mu RT$

1 mole of any (ideal) gas at standard temperature (273 K) and pressure ($1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$) occupies a volume of 22.4 L. This universal volume is called molar volume. Thus, the cylinder in this example contains 2 mole of helium. Further, since helium is monoatomic, its predicted (and observed) molar specific heat at constant volume $C_v = \frac{3}{2}R$ and molar specific heat at constant pressure.

$$C_p = \frac{3}{2}R + R = \frac{5}{2}R$$

Since, volume of the cylinder is fixed, the heat required is determined by C_V .

$$\begin{aligned} \therefore \text{Heat required} &= \text{Number of moles} \times \text{Molar specific heat} \\ &\quad \text{at constant volume} \times \text{Rise in temperature} \\ &= 2 \times 1.5R \times 15 = 45R \\ &= 45 \times 8.31 = 374 \text{ J} \end{aligned}$$

Mean Free Path

Every gas consists of a very large number of molecules. These molecules are in a state of continuous rapid and random motion. They undergo perfectly elastic collision against one another. Therefore, path of a single gas molecule consists of a series of short zig-zag paths of different lengths. The mean free path of a gas molecule is the average distance between two successive collisions.

Mathematically, it is expressed as $\lambda = \frac{1}{\sqrt{2} \pi d^2 n_V}$

where, d = diameter of molecules,
 n_V = number of molecules per unit volume.

The number of molecules per unit volume can be determined from Avogadro's number and the ideal gas law leading to

$$n_V = \frac{nN_A}{V} = \frac{nN_A}{\frac{nRT}{p}} = \frac{N_A p}{RT}$$

$$\therefore \lambda = \frac{RT}{\sqrt{2} \pi d^2 N_A p}$$

One mole of an ideal gas will occupy a volume of 22.4L at STP.

Avogadro's number, $N_A = 6.0221367 \times 10^{23}$ mol

Note Standard temperature $0^\circ\text{C} = 273.15$ K

Standard pressure = 1 atmosphere = 760 mm of Hg = 101.3 kPa

Standard volume of 1 mole of an ideal gas at STP = 22.4 L.

Example 13. In a dilute gas at pressure p and temperature T , the mean time between successive collisions of a molecule varies with T as

- (a) T (b) $\frac{1}{\sqrt{T}}$ (c) $\frac{1}{T}$ (d) \sqrt{T}

Sol. (b) The expression of mean time (τ) between successive collisions of a molecule at pressure p and temperature T is given by

$$\tau = \frac{\lambda}{v_{\text{avg}}}$$

As, mean free path of gas molecule, $\lambda = \frac{1}{\sqrt{2} \pi d^2 n_V}$

where, n_V = number of molecules volume.

$$\Rightarrow \lambda \propto (T)^0$$

Also, average speed, $v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}}$

$$\Rightarrow v_{\text{avg}} \propto \sqrt{T} \therefore \tau \propto \frac{1}{\sqrt{T}}$$

Example 14. A vessel contains a mixture of 7 g of nitrogen and 11g of carbon dioxide at temperature $T = 300$ K. If the pressure of the mixture is 1 atm ($1 \times 10^5 \text{ N/m}^2$), its density is (Given, gas constant, $R = 2513 \text{ J/mol K}$)

- (a) 0.72 kg/m^3 (b) 1.44 kg/m^3
(c) 2.88 kg/m^3 (d) 5.16 kg/m^3

Sol. (b) The expression for density of a mixture of gases is given by,

$$\rho_{\text{mix}} = \frac{pM_{\text{mix}}}{RT}$$

where, M_{mix} is the mass of mixture of non-reactive gases, p is pressure, R is gas constant and T is temperature.

$$M_{\text{mix}} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$$

Given, Mass of nitrogen gas, $m_{\text{N}_2} = 7 \text{ g} = 7 \times 10^{-3} \text{ kg}$

Mass of carbon dioxide, $m_{\text{CO}_2} = 11 \text{ g} = 11 \times 10^{-3} \text{ kg}$

Molecular weight of nitrogen gas, $m_{\text{N}_2} = 28 \times 10^{-3} \text{ kg}$

Molecular weight of carbon dioxide, $M_{\text{CO}_2} = 44 \times 10^{-3} \text{ kg}$

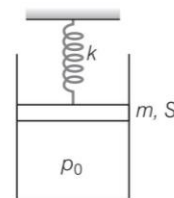
$$M_{\text{mix}} = \frac{n_{\text{N}_2} M_{\text{N}_2} + n_{\text{CO}_2} M_{\text{CO}_2}}{n_{\text{N}_2} + n_{\text{CO}_2}} = \frac{\frac{m_{\text{N}_2}}{M_{\text{N}_2}} M_{\text{N}_2} + \frac{m_{\text{CO}_2}}{M_{\text{CO}_2}} M_{\text{CO}_2}}{\frac{m_{\text{N}_2}}{M_{\text{N}_2}} + \frac{m_{\text{CO}_2}}{M_{\text{CO}_2}}}$$

$$M_{\text{mix}} = \frac{m_{\text{N}_2} + m_{\text{CO}_2}}{\frac{m_{\text{N}_2}}{M_{\text{N}_2}} + \frac{m_{\text{CO}_2}}{M_{\text{CO}_2}}} = \frac{(7 + 11) \times 10^{-3}}{\left(\frac{7 \times 10^{-3}}{28 \times 10^{-3}} + \frac{11 \times 10^{-3}}{44 \times 10^{-3}}\right)}$$

$$M_{\text{mix}} = \frac{18 \times 10^{-3}}{\frac{1}{4} + \frac{1}{4}} = 36 \times 10^{-3} \text{ kg}$$

$$\therefore \rho = \frac{(1 \times 10^5) (36 \times 10^{-3})}{\frac{25}{3} \times 300} = 1.44 \text{ kg/m}^3$$

Example 15. In the arrangement shown in the figure gas is thermally insulated. An ideal gas is filled in the cylinder having pressure p_0 ($>$ atmospheric pressure p_a). The spring of force constant k is initially unstretched. The piston of mass m and area S is frictionless. In equilibrium, the piston rises up a distance x_0 , then the decrease in internal energy of the gas is given by

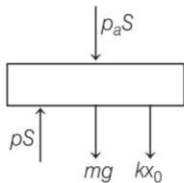


- (a) $p_a S x_0 + \frac{1}{2} k x_0^2 + m g x_0$ (b) $p_a S x_0 + \frac{4}{5} x_0^2 + 2 m g x_0$
(c) $2 p_a S x_0 + \frac{4}{5} x_0^2 + \frac{2}{3} m g x_0$ (d) $p_a S x_0 + 2 k x_0^2 + 2 m g x_0$

Sol. (a) Equilibrium of piston gives

$$pS = p_a S + mg + kx_0$$

$$p = p_a + \frac{mg}{S} + \frac{kx_0}{S} \quad (p = \text{final pressure of gas})$$



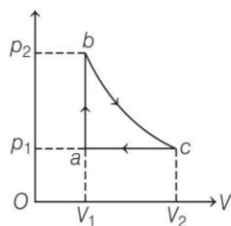
Work done by the gas is equal to work done against atmospheric pressure + elastic potential energy stored in the spring + increase in gravitational potential energy of the piston.

$$= p_a \Delta V + \frac{1}{2} kx_0^2 + mgx_0$$

$$= p_a Sx_0 + \frac{1}{2} kx_0^2 + mgx_0$$

This is also the decrease in internal energy of the gas, because the gas is thermally insulated and this work is done at the expense of internal energy of the gas.

Example 16. Carbon monoxide is carried around a closed cycle abc , in which bc is an isothermal process, as shown.



The gas absorbs 7000 J of heat as its temperature is increased from 300 K to 1000 K is going from a to b . The quantity of heat ejected by the gas. process ca is

- (a) 4200 J (b) 5000 J
 (c) 9800 J (d) 3500 J

Sol. (c) Given that gas absorbs 7000 J of energy, hence

$$(\Delta Q)_{ab} = \mu C_V \Delta T$$

$$\therefore +7000 = \mu C_V (1000 - 300)$$

$$\text{For the process } ca, \quad T_a = 300 \text{ K}$$

$$T_c = T_b = 1000 \text{ K}$$

$$(\Delta Q)_{ca} = \mu C_p \Delta T$$

$$= \mu C_p (300 - 1000)$$

$$= -\mu C_p \times 700$$

$$\text{Also,} \quad C_p - C_V = R$$

$$\therefore \quad C_p = R + C_V$$

$$(\Delta Q)_{ca} = -\mu (C_V + R) 700$$

$$\text{For carbon monoxide, } \gamma = 1 + \frac{2}{n} = 1 + \frac{2}{5} = \frac{7}{5}$$

$$C_V = \frac{R}{\gamma - 1} = \frac{R}{\frac{7}{5} - 1} = \frac{5R}{2}$$

$$\text{Hence, we have } (\Delta Q)_{ab} = \mu C_V 700$$

$$(\Delta Q)_{ab} = \mu \frac{5R}{2} \times 700 = 7000$$

$$\text{or} \quad \mu R = \frac{20}{5} = 4$$

$$\therefore \quad (\Delta Q)_{ca} = -(7000 + 4 \times 700) = -9800 \text{ J}$$

Negative sign shows that heat is ejected.

Practice Exercise

ROUND I Topically Divided Problems

Ideal Gas, Kinetic Theory of Gases and Degree of Freedom

- An oxygen cylinder of volume 30 L has an initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17°C. Estimate the mass of oxygen taken out of the cylinder. (Take, $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$ and molecular mass of $\text{O}_2 = 32 \text{ u}$) [NCERT]

(a) 0.14 g (b) 0.02 g (c) 0.14 kg (d) 0.014 kg
- At room temperature (27°C), the rms speed of the molecules of a certain diatomic gas is found to be 1920 ms^{-1} . The gas is

(a) Cl_2 (b) O_2 (c) N_2 (d) H_2
- The rms velocity of gas molecules is 300 ms^{-1} . The rms velocity of molecules of gas with twice the molecular mass and half the absolute temperature is

(a) 300 ms^{-1} (b) 600 ms^{-1}
 (c) 75 ms^{-1} (d) 150 ms^{-1}
- 8 g of O_2 , 14 g of N_2 and 22 g of CO_2 is mixed in a container of 10 L capacity at 27°C. The pressure exerted by the mixture in terms of atmospheric pressure is ($R = 0.082 \text{ L atm K}^{-1} \text{ mol}^{-1}$)

(a) 1.4 atm (b) 2.5 atm
 (c) 3.7 atm (d) 8.7 atm
- At a certain temperature, the ratio of the rms velocity of H_2 molecules to O_2 molecule is

(a) 1 : 1 (b) 1 : 4
 (c) 4 : 1 (d) 16 : 1
- Two containers of equal volume contain the same gas at the pressures p_1 and p_2 and absolute temperatures T_1 and T_2 respectively. On joining the vessels, the gas reaches a common pressure p and a common temperature T . The ratio p/T is equal to

(a) $\frac{p_1 T_2 + p_2 T_1}{T_1 \times T_2}$ (b) $\frac{p_1 T_2 + p_2 T_1}{T_1 + T_2}$
 (c) $\frac{1}{2} \left(\frac{p_1 T_2 + p_2 T_1}{T_1 T_2} \right)$ (d) $\frac{p_1 T_2 - p_2 T_1}{T_1 \times T_2}$
- The volume V of an enclosure contains a mixture of three gases : 16g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature T . Consider R as universal gas constant. The pressure of the mixture of gases is [JEE Main 2021]

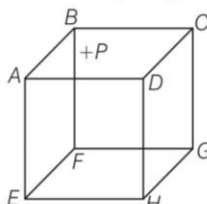
(a) $\frac{88 RT}{V}$ (b) $\frac{3 RT}{V}$ (c) $\frac{5}{2} \cdot \frac{RT}{V}$ (d) $\frac{4 RT}{V}$
- rms velocity of a particle is c at pressure p . If pressure is increased two times, then rms velocity becomes

(a) 0.5 c (b) c (c) 2 c (d) 3 c
- Calculate the rms speed of smoke particles each of mass $5 \times 10^{-17} \text{ kg}$ in their Brownian motion in air at NTP ($k = 1.38 \times 10^{-23} \text{ JK}^{-1}$)

(a) 1.5 mm s^{-1} (b) 1.5 ms^{-1}
 (c) 1.5 cms^{-1} (d) 1.5 kms^{-1}
- If c_s is the velocity of sound in air and c is the rms velocity, then

(a) $c_s < c$ (b) $c_s = c$
 (c) $c_s = c \left(\frac{\gamma}{3} \right)^{1/2}$ (d) None of these
- In a diatomic molecule, the rotational energy at a given temperature [NCERT Exemplar]

(a) do not obey Maxwell's distribution
 (b) have the same value for all molecules
 (c) equals the translational kinetic energy for each molecule
 (d) is (2/3)rd the translational kinetic energy for each molecule
- $ABCDEFGH$ is hollow cube made of an insulator. Face $ABCD$ has positive charge on it. Inside the cube, we have ionized hydrogen.



The usual kinetic theory expression for pressure [NCERT Exemplar]

- (a) will be valid
 (b) will be valid since the ions would experience forces larger than due to collisions with the walls
 (c) will not be valid since collisions with walls would not be elastic
 (d) will not be valid because isotropy is lost
- 13.** Consider a sample of oxygen behaving like an ideal gas. At 300 K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be (Take, molecular weight of oxygen = 32 g/mol and $R = 8.3 \text{ JK}^{-1} \text{ mol}^{-1}$) [JEE Main 2021]
- (a) $\sqrt{\frac{3}{8}}$ (b) $\sqrt{\frac{8}{3}}$
 (c) $\sqrt{\frac{3\pi}{8}}$ (d) $\sqrt{\frac{8\pi}{3}}$
- 14.** N molecules, each of mass m , of gas A and $2N$ molecules, each of mass $2m$, of gas B are contained in the same vessel which is maintained at a temperature T . The mean square velocity of molecules of B type is denoted by V_2 and the mean square velocity of A type is denoted by V_1 , then $\frac{V_1}{V_2}$ is
- (a) 2 (b) 1
 (c) 1/3 (d) 2/3
- 15.** If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of [JEE Main 2019]
- (a) 10^4 N/m^2 (b) 10^8 N/m^2
 (c) 10^0 N/m^2 (d) 10^{16} N/m^2
- 16.** To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 cal of heat is required. When the same mass of gas is cooled by 100°C at constant volume, 240 cal of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)? [JEE Main 2020]
- (a) 5 (b) 7
 (c) 6 (d) 3
- 17.** The specific heats, C_p and C_v of a gas of diatomic molecules, A are given (in units of $\text{J mol}^{-1} \text{K}^{-1}$) by 29 and 22, respectively. Another gas of diatomic molecules B , has the corresponding values of C_p and C_v given as 30 and 21, respectively. If they are treated as ideal gases, then respectively. [JEE Main 2019]
- (a) A has a vibrational mode but B has none
 (b) both A and B have a vibrational mode each
 (c) A has one vibrational mode and B has two
 (d) A is rigid but B has a vibrational mode

- 18.** A $25 \times 10^{-3} \text{ m}^3$ volume cylinder is filled with 1 mole of O_2 gas at room temperature (300 K). The molecular diameter of O_2 and its root mean square speed are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule? [JEE Main 2019]
- (a) $\sim 10^{10}$ (b) $\sim 10^{12}$
 (c) $\sim 10^{11}$ (d) $\sim 10^8$
- 19.** Consider two ideal diatomic gases A and B at some temperature T . Molecules of the gas A are rigid and have a mass m . Molecules of the gas B have an additional vibrational mode and have a mass $\frac{m}{4}$. The ratio of the specific heats (C_v^A and C_v^B) of gas A and B respectively is [JEE Main 2020]
- (a) 5 : 9 (b) 7 : 9
 (c) 3 : 5 (d) 5 : 7
- 20.** The average kinetic energy of a gas molecule at 27°C is 6.21×10^{-21} J. Its average kinetic energy at 127°C will be
- (a) 12.2×10^{-21} J
 (b) 8.28×10^{-21} J
 (c) 10.35×10^{-21} J
 (d) 11.35×10^{-21} J
- 21.** The root mean square velocity of the molecules in a sample of helium is 5/7th that of the molecules in a sample of hydrogen. If the temperature of the hydrogen as is 0°C , that of helium sample is about
- (a) 0°C (b) 4 K
 (c) 273°C (d) 100°C

Internal Energy, Specific Heat Capacity and Mean Free Path

- 22.** The value of molar specific heat at constant volume for 1 mole of polyatomic gas having n number of degrees of freedom at temperature T K is (R = universal gas constant)
- (a) $\frac{nR}{2T}$ (b) $\frac{nR}{2}$
 (c) $\frac{nRT}{2}$ (d) $2nRT$
- 23.** For a gas, if the ratio of specific heats at constant pressure and constant volume is γ , then the value of degrees of freedom is
- (a) $\frac{\gamma+1}{\gamma-1}$ (b) $\frac{\gamma-1}{\gamma+1}$
 (c) $\frac{(\gamma-1)}{2}$ (d) $\frac{2}{\gamma-1}$
- 24.** Two moles of monoatomic gas is mixed with three moles of a diatomic gas. The molar specific heat of the mixture at constant volume is
- (a) $1.55 R$ (b) $2.10 R$
 (c) $1.63 R$ (d) $2.20 R$

25. An ideal gas occupies a volume of 2 m^3 at a pressure of $3 \times 10^6 \text{ Pa}$. The energy of the gas is [JEE Main 2019]

- (a) $6 \times 10^4 \text{ J}$ (b) 10^8 J
(c) $9 \times 10^6 \text{ J}$ (d) $3 \times 10^2 \text{ J}$

26. Calculate the value of mean free path λ for oxygen molecules at temperature 27°C and pressure $1.01 \times 10^5 \text{ Pa}$. Assume the molecular diameter 0.3 nm and the gas is ideal. (Take, $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$) [JEE Main 2021]

- (a) 58 nm (b) 32 nm
(c) 86 nm (d) 102 nm

27. The value of molar specific heat at constant pressure for one mole of triatomic gas (triangular arrangement) at temperature $T \text{ K}$ is ($R =$ universal gas constant)

- (a) $3R$ (b) $\frac{2}{7}R$
(c) $\frac{5}{2}R$ (d) $4R$

28. The diameter of a gas molecule is $2.4 \times 10^{-10} \text{ m}$. The mean free path of gas molecule at NTP is ($k = 1.38 \times 10^{-23} \text{ JK}^{-1}$)

- (a) $1.46 \times 10^{-7} \text{ m}$ (b) $2.46 \times 10^{-6} \text{ m}$
(c) $1.46 \times 10^{-6} \text{ m}$ (d) $2.46 \times 10^{-7} \text{ m}$

29. Match the C_p/C_v ratio for ideal gases with different type of molecules [JEE Main 2020]

| Molecule type | C_p/C_v |
|----------------------------------|-----------|
| (A) Monatomic molecules | I. 7/5 |
| (B) Diatomic rigid molecules | II. 9/7 |
| (C) Diatomic non-rigid molecules | III. 4/3 |
| (D) Triatomic rigid molecules | IV. 5/3 |

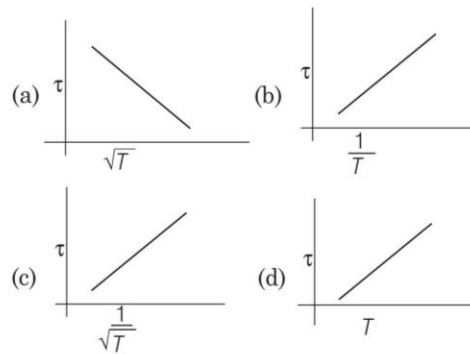
- | | |
|-----------------|-----------------|
| A B C D | A B C D |
| (a) IV I II III | (b) III IV II I |
| (c) II III I IV | (d) IV II I III |

30. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true? [JEE Main 2020]

- A. The mean free path of the molecules decreases.
B. The mean collision time between the molecules decreases.
C. The mean free path remains unchanged.
D. The mean collision time remains unchanged.

- (a) B and C (b) A and B
(c) C and D (d) A and D

31. The plot that depicts the behaviour of the mean free time τ (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively is (graphs are schematic and not drawn to scale)



32. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently, the mean collision time between the gas molecule changes from τ_1 to τ_2 . If $\frac{C_p}{C_v} = \gamma$ for this gas, then a good estimate for

$\frac{\tau_2}{\tau_1}$ is given by [JEE Main 2020]

- (a) $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$ (b) $\frac{1}{2}$
(c) 2 (d) $\left(\frac{1}{2}\right)^\gamma$

33. Two gases-argon (atomic radius 0.07 nm , atomic weight 40) and xenon (atomic radius 0.1 nm , atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closest to [JEE Main 2020]

- (a) 4.67 (b) 2.3
(c) 3.67 (d) 1.09

34. Two moles of an ideal gas with $\frac{C_p}{C_v} = \frac{5}{3}$ are mixed with 3 mol of another ideal gas with $\frac{C_p}{C_v} = \frac{4}{3}$. The value of $\frac{C_p}{C_v}$ for the mixture is [JEE Main 2020]

- (a) 1.42 (b) 1.47
(c) 1.50 (d) 1.45

35. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to (Given, mean kinetic energy of a molecule at T is $4 \times 10^{-14} \text{ erg}$, $g = 980 \text{ cm/s}^2$, density of mercury = 13.6 g/cm^3) [JEE Main 2020]

- (a) 5.8×10^{16} (b) 4.0×10^{16}
(c) 4.0×10^{18} (d) 5.8×10^{18}

36. Consider a mixture of n moles of helium gas and $2n$ moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_p / C_v value will be [JEE Main 2020]

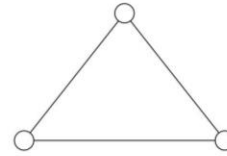
- (a) 40/27 (b) 23/15
(c) 19/13 (d) 67/45

37. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is \bar{v} , m is its mass and k_B is Boltzmann constant, then its temperature will be [JEE Main 2019]

- (a) $\frac{m\bar{v}^2}{3k_B}$ (b) $\frac{m\bar{v}^2}{7k_B}$ (c) $\frac{m\bar{v}^2}{5k_B}$ (d) $\frac{m\bar{v}^2}{6k_B}$

38. Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied

by atoms. The internal energy of a mole of the gas at temperature T is [JEE Main 2020]



- (a) $\frac{5}{2} RT$ (b) $\frac{3}{2} RT$
 (c) $\frac{9}{2} RT$ (d) $3 RT$

ROUND II Mixed Bag

Only One Correct Option

1. Three perfect gases at absolute temperatures T_1 , T_2 and T_3 are mixed. The masses of molecules are m_1 , m_2 and m_3 and the number of molecules are n_1 , n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is [AIIEE 2011]

- (a) $\frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$ (b) $\frac{n_1 T_1^2 + n_2 T_2^2 + n_3 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$
 (c) $\frac{n_1^2 T_1^2 + n_2^2 T_2^2 + n_3^2 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$ (d) $\frac{(T_1 + T_2 + T_3)}{3}$

2. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains $1 \times 10^5 \text{ Pa}$. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be [JEE Main 2017]

- (a) 1.38×10^{23} (b) 2.5×10^{25}
 (c) -2.5×10^{25} (d) -1.61×10^{23}

3. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure p_1 and temperature T_1 . The other chamber has volume V_2 and contains ideal gas at pressure p_2 and temperature T_2 . If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be [AIIEE 2008, 04]

- (a) $\frac{T_1 T_2 (p_1 V_1 + p_2 V_2)}{p_1 V_1 T_2 + p_2 V_2 T_1}$
 (b) $\frac{p_1 V_1 T_1 + p_2 V_2 T_2}{p_1 V_1 + p_2 V_2}$
 (c) $\frac{p_1 V_1 T_2 + p_2 V_2 T_1}{p_1 V_1 + p_2 V_2}$
 (d) $\frac{T_1 T_2 (p_1 V_1 + p_2 V_2)}{p_1 V_1 T_1 + p_2 V_2 T_2}$

4. 22 g of carbon dioxide at 27°C is mixed in a closed container with 16 g of oxygen at 37°C . If both gases are considered as ideal gases, then the temperature of the mixture is

- (a) 24.2°C (b) 28.5°C
 (c) 31.5°C (d) 33.5°C

5. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is $TV^x = \text{constant}$, then x is [JEE Main 2019]

- (a) $\frac{2}{5}$ (b) $\frac{2}{3}$
 (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

6. A cylinder is containing nitrogen at 2 atm and temperature 17°C . The radius of a nitrogen molecule is roughly 1.0A. Molecular mass of nitrogen = 28.0 u, Boltzmann constant, $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$. The mean free path of nitrogen molecule is

- (a) $1.1 \times 10^{-7} \text{ m}$ (b) $2.1 \times 10^{-7} \text{ m}$
 (c) $3.1 \times 10^{-7} \text{ m}$ (d) $0.8 \times 10^{-7} \text{ m}$

7. A polyatomic ideal gas has 24 vibrational modes. What is the value of γ ? [JEE main 2021]

- (a) 1.03 (b) 1.30
 (c) 1.37 (d) 10.3

8. Inside a cylinder closed at both ends is a movable piston. On one side of the piston is a mass m of a gas, and on the other side a mass $2m$ of the same gas.

What fraction of the volume of the cylinder will be occupied by the larger mass of the gas when the piston is in equilibrium? The temperature is the same throughout.

- (a) $\frac{2}{3}$ (b) $\frac{1}{3}$
 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

9. Half-mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20° C to 90° C .
 Work done by gas is close to
 (Take, gas constant, $R = 8.31 \text{ J/mol-K}$)
 (a) 291 J (b) 581 J [JEE Main 2019]
 (c) 146 J (d) 73 J
10. One kg of a diatomic gas is at a pressure of $8 \times 10^4 \text{ Nm}^{-2}$. The density of the gas is 4 kgm^{-3} .
 What is the energy of the gas due to its thermal motion? [AIEEE 2009]
 (a) $3 \times 10^4 \text{ J}$ (b) $5 \times 10^4 \text{ J}$
 (c) $6 \times 10^4 \text{ J}$ (d) $7 \times 10^4 \text{ J}$
11. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is [AIEEE 2010]
 (a) $\frac{b^2}{2a}$ (b) $\frac{b^2}{12a}$
 (c) $\frac{b^2}{4a}$ (d) $\frac{b^2}{6a}$
12. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled is about (Take, $R = 8.3 \text{ J/mol K}$) [JEE Main 2019]
 (a) 10 kJ (b) 0.9 kJ
 (c) 14 kJ (d) 6 kJ
13. Two chambers containing m_1 and m_2 gram of a gas at pressures p_1 and p_2 respectively are put in communication with each other, temperature remaining constant. The common pressure reached will be
 (a) $\frac{p_1 p_2 (m_1 + m_2)}{p_2 m_1 + p_1 m_2}$ (b) $\frac{p_1 p_2 m_1}{p_2 m_1 + p_1 m_2}$
 (c) $\frac{m_1 m_2 (p_1 + p_2)}{p_2 m_1 + p_1 m_2}$ (d) $\frac{m_1 m_2 p_2}{p_2 m_1 + p_1 m_2}$
14. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is $6 \times 10^{-8} \text{ s}$. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to [JEE Main 2019]
 (a) $4 \times 10^{-8} \text{ s}$ (b) $3 \times 10^{-6} \text{ s}$
 (c) $2 \times 10^{-7} \text{ s}$ (d) $0.5 \times 10^{-8} \text{ s}$
15. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume? (Take, $R = 8.3 \text{ J/mol-K}$) [JEE Main 2019]
 (a) 19.7 J/mol-K (b) 15.7 J/mol-K
 (c) 17.4 J/mol-K (d) 21.6 J/mol-K
16. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q , where V is the volume of the gas. The value of q is $\left(\gamma = \frac{C_p}{C_v}\right)$ [JEE Main 2015]
 (a) $\frac{3\gamma + 5}{6}$ (b) $\frac{3\gamma - 5}{6}$
 (c) $\frac{\gamma + 1}{2}$ (d) $\frac{\gamma - 1}{2}$
17. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio $\frac{C_p}{C_v}$ of the mixture is [AIEEE 2005]
 (a) 1.59 (b) 1.62 (c) 1.4 (d) 1.54
18. A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is l_1 and that below the piston is l_2 , such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T . If the piston is stationary, its mass m , will be given by (where, R is universal gas constant and g is the acceleration due to gravity) [JEE Main 2019]
 (a) $\frac{nRT}{g} \left[\frac{l_1 - l_2}{l_1 l_2} \right]$ (b) $\frac{nRT}{g} \left[\frac{1}{l_2} + \frac{1}{l_1} \right]$
 (c) $\frac{RT}{g} \left[\frac{2l_1 + l_2}{l_1 l_2} \right]$ (d) $\frac{RT}{ng} \left[\frac{l_1 - 3l_2}{l_1 l_2} \right]$
19. One mole of ideal monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of diatomic gas $\left(\gamma = \frac{7}{5}\right)$. What is γ for the mixture? γ denotes the ratio of specific heat at constant pressure, to that at constant volume. [AIEEE 2004]
 (a) $\frac{3}{2}$ (b) $\frac{23}{15}$ (c) $\frac{35}{23}$ (d) $\frac{4}{3}$
20. Two rigid boxes $\frac{1}{2}$ containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0 , while box B contains one mole of helium at temperature $(7/3) T_0$. The boxes are then put into thermal contact with each other and heat flows between them until the gases reach a common final temperature (ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f , in terms of T_0 is [AIEEE 2006]
 (a) $T_f = \frac{3}{7} T_0$ (b) $T_f = \frac{7}{3} T_0$
 (c) $T_f = \frac{3}{2} T_0$ (d) $T_f = \frac{5}{2} T_0$

21. The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-ar^4}$. Then, the total number of molecules is proportional to [JEE Main 2019]
 (a) $n_0 \alpha^{-3/4}$ (b) $\sqrt{n_0} \alpha^{1/2}$
 (c) $n_0 \alpha^{1/4}$ (d) $n_0 \alpha^{-3}$

Numerical Value Questions

22. The total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27°C and 1 atm pressure, is found to be 6.1×10^{13x} , then the value of x is
 (Take, Boltzmann constant = $1.38 \times 10^{-23} \text{ JK}^{-1}$)
23. A closed vessel contains 0.1 mole of a monatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to [JEE Main 2020]
24. If four molecules of a gas have speed 2, 4, 6, 8 kms^{-1} respectively, then their average speed (in kms^{-1}) and root mean square speed (in kms^{-1}) will be and, respectively.
25. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure p_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by p_2 . The ratio p_2/p_1 is [JEE Main 2020]
26. From the following data, find the magnitude of Joule's mechanical equivalent of heat. If C_p for hydrogen = $3.409 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$, C_v for

hydrogen = $2.409 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$ and molecular weight of hydrogen = 2, then the magnitude of joule's mechanical equivalent of heat (in Jcal^{-1}) will be

27. Nitrogen gas is at 300°C temperature. The temperature (in K) at which the rms speed of a H_2 molecule would be equal to the rms speed of a N_2 molecule is
 (Molar mass of N_2 gas is 28 g.) [JEE Main 2020]
28. In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cc of water at 27°C . The final temperature is 40°C . If heat losses to the surroundings is not negligible, then the value of specific heat (in $\text{J/g}^\circ\text{C}$) of the metal will be
29. A geyser heats water glowing at the rate of 3.0 Lmin^{-1} from 27°C to 77°C . If the geyser operates on a gas burner, then the rate of combustion (in gmin^{-1}) of fuel, when its heat of combustion is $4.0 \times 10^4 \text{ Jg}^{-1}$, will be
30. The molecules of a given mass of a gas have root mean square speeds of 100 ms^{-1} at 27°C and 1.00 atm pressure. If the root mean square speed of the molecules of the gas at 127°C and 2 atm pressure is found to be $\frac{200}{\sqrt{n}}$ m/s, then find the value of n . [JEE Main 2021]
31. An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12°C . The volume of bubble when it reaches the surface, which is at temperature of 3°C , is found to be $5.275 \times 10^{-x} \text{ m}^3$, then the value of x is
 (Given, 1 atm = $1.01 \times 10^5 \text{ Pa}$)

Answers

Round I

| | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (c) | 2. (d) | 3. (d) | 4. (c) | 5. (c) | 6. (c) | 7. (c) | 8. (b) | 9. (c) | 10. (c) |
| 11. (d) | 12. (d) | 13. (c) | 14. (b) | 15. (c) | 16. (c) | 17. (a) | 18. (d) | 19. (d) | 20. (b) |
| 21. (c) | 22. (b) | 23. (d) | 24. (b) | 25. (c) | 26. (d) | 27. (d) | 28. (a) | 29. (a) | 30. (a) |
| 31. (c) | 32. (a) | 33. (d) | 34. (a) | 35. (c) | 36. (c) | 37. (b) | 38. (d) | | |

Round II

| | | | | | | | | | |
|-----------|---------|---------|-------------|---------|----------|---------|---------|---------|---------|
| 1. (a) | 2. (c) | 3. (a) | 4. (c) | 5. (a) | 6. (a) | 7. (a) | 8. (a) | 9. (a) | 10. (b) |
| 11. (c) | 12. (a) | 13. (a) | 14. (a) | 15. (c) | 16. (c) | 17. (b) | 18. (a) | 19. (a) | 20. (c) |
| 21. (a) | 22. 2 | 23. 266 | 24. 5, 5.48 | 25. 5 | 26. 4.11 | 27. 41 | 28. 0.1 | | |
| 29. 15.75 | 30. 3 | 31. 6 | | | | | | | |

Solutions

Round I

1. Absolute pressure, $p_1 = (15 + 1)$ atm

$$[\because \text{Absolute pressure} = \text{Gauge pressure} + 1 \text{ atm}]$$

$$= 16 \times 1.013 \times 10^5 \text{ Pa}$$

$$V_1 = 30 \text{ L} = 30 \times 10^{-3} \text{ m}^3$$

$$T_1 = 273.15 + 27 = 300.15 \text{ K}$$

Using ideal gas equation, $pV = nRT$

$$\text{or } n = \frac{pV}{RT}$$

$$= \frac{p_1 V_1}{RT_1} = \frac{16 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.314 \times 300.15}$$

$$= 19.48$$

Final pressure, $p_2 = (11 + 1) = 12$ atm Pa

$$V_2 = 30 \text{ L} = 30 \times 10^{-3} \text{ m}^3$$

$$T_2 = 273.15 + 17 = 290.15 \text{ K}$$

$$\text{Number of moles} = \frac{p_2 V_2}{RT_2} = \frac{12 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.314 \times 290.15}$$

$$= 15.12$$

Hence, moles removed = $19.48 - 15.12 = 4.36$

Mass removed = $4.36 \times 32 \text{ g} = 0.1396 \text{ kg} \approx 0.14 \text{ kg}$

2. As, $c_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\text{or } M = \frac{3RT}{c_{\text{rms}}^2}$$

$$= \frac{3 \times 8.31 \times 300}{(1920)^2}$$

$$= 2 \times 10^{-3} \text{ kg} = 2 \text{ g}$$

Since, $M = 2$ for the hydrogen molecule. Hence, the gas is hydrogen.

3. As, $c = \sqrt{\frac{3pV}{M}} = \sqrt{\frac{3RT}{M}}$

and the new rms speed,

$$c_1 = \sqrt{\frac{3R(T/2)}{(2M)}} = \frac{1}{2} \sqrt{\frac{3RT}{M}}$$

$$= \frac{c}{2} = \frac{300}{2} = 150 \text{ ms}^{-1}$$

4. As, $p = \frac{n_1 RT + n_2 RT + n_3 RT}{V} = (n_1 + n_2 + n_3) \frac{RT}{V}$

$$= \left(\frac{8}{16} + \frac{14}{28} + \frac{22}{44} \right) \times \frac{0.082 \times 300}{10} = 3.69 \text{ atm} \approx 3.7 \text{ atm}$$

5. As, $c = \sqrt{\frac{3RT}{M}}$

$$\text{So, } \frac{c_H}{c_O} = \sqrt{\frac{M_O}{M_H}} = \sqrt{\frac{16}{1}} = 4$$

6. For a closed system, the total number of moles remains constant.

$$\text{So, } p_1 V = n_1 RT_1 \Rightarrow \frac{n_1 R}{V} = \frac{p_1}{T_1}$$

$$\text{and } p_2 V = n_2 RT_2 \Rightarrow \frac{n_2 R}{V} = \frac{p_2}{T_2}$$

$$\therefore p(2V) = (n_1 + n_2) RT$$

where, V is the volume of each vessel. When the vessels are joined.

$$\therefore \frac{p}{T} = \frac{(n_1 + n_2) R}{2V} = \frac{1}{2} \left(\frac{p_1}{T_1} + \frac{p_2}{T_2} \right) = \frac{1}{2} \left(\frac{p_1 T_2 + p_2 T_1}{T_1 T_2} \right)$$

7. $pV = (n_1 + n_2 + n_3) RT$

$$= \left(\frac{16}{32} + \frac{28}{28} + \frac{44}{44} \right) RT$$

$$= \left(\frac{1}{2} + 1 + 1 \right) RT$$

$$\Rightarrow pV = \frac{5}{2} RT$$

$$\Rightarrow p = \frac{5}{2} \cdot \frac{RT}{V}$$

8. rms velocity does not change with pressure, till temperature remains constant.

9. As, $c = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 273}{5 \times 10^{-17}}}$

$$= 15 \times 10^{-3} \text{ ms}^{-1} = 1.5 \text{ cms}^{-1}$$

10. As, $c_s = \sqrt{\frac{\gamma p}{\rho}}$

$$\text{and } c = \sqrt{\frac{3pV}{M}} = \sqrt{\frac{3p}{\rho}}$$

$$\frac{c_s}{c} = \sqrt{\frac{\gamma}{3}} \text{ or } c_s = c \sqrt{\frac{\gamma}{3}}$$

11. In a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's energy distribution law. As each such atom has three translational degrees of freedom and two rotational degrees of freedom, therefore, at a given temperature, rotational energy is $\frac{2}{3}$ -rd the translational KE of each molecule. Energy associated with each molecule per degree of freedom is fixed $\left(= \frac{1}{2} k_B T \right)$.

12. As face $ABCD$ has positive charge on it and the gas consists of ionized hydrogen, therefore, isotropy is lost. The usual expression for pressure on the basis of kinetic theory will not be valid as ions would also experience forces, other than the forces due to collisions with the walls of the container.

13.
$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$v_{\text{avg}} = \sqrt{\frac{8}{\pi} \cdot \frac{RT}{M}}$$

$$\therefore \frac{v_{\text{rms}}}{v_{\text{avg}}} = \sqrt{\frac{3\pi}{8}}$$

14. As, $p = \frac{1}{3} \frac{M}{V} c^2$
 or $c^2 = \frac{3pV}{M} = \frac{3RT}{M}$
 For gas A, $V_1^2 = \frac{3 \times RT}{M} = \frac{3RT}{M}$
 For gas B, $V_2^2 = \frac{3RT}{M}$;
 So, $\frac{V_1^2}{V_2^2} = 1 \Rightarrow \frac{V_1}{V_2} = 1$

15. Momentum imparted to the surface in one collision, $\Delta p = (p_i - p_f) = mv - (-mv) = 2mv$... (i)
 Force on the surface due to n collision per second,

$$F = \frac{n}{t} (\Delta p) = n \Delta p \quad (\because t = 1 \text{ s})$$

$$= 2mnv \quad [\text{from Eq. (i)}]$$

So, pressure on the surface, $p = \frac{F}{A} = \frac{2mnv}{A}$

Here, $m = 10^{-26}$ kg, $n = 10^{22}$ s⁻¹,
 $v = 10^4$ ms⁻¹, $A = 1$ m²

\therefore Pressure, $p = \frac{2 \times 10^{-26} \times 10^{22} \times 10^4}{1} = 2 \text{ N/m}^2$

So, pressure exerted is of order of 10^0 .

16. Given, in constant pressure process,

$$\Delta Q = 160 \text{ cal}$$

$$\Delta T = 50^\circ \text{C}$$

So, $\Delta Q = nC_p \Delta T$
 $\Rightarrow 160 = nC_p (50)$... (i)

Also, in constant volume process,

$$\Delta Q = 240 \text{ cal}$$

$$\Delta T = 100^\circ \text{C} \Rightarrow \Delta Q = nC_v \Delta T$$

$\Rightarrow 240 = nC_v (100)$... (ii)

Dividing Eq. (i) by Eq. (ii), we have

$$\frac{160}{240} = \frac{nC_p (50)}{nC_v (100)}$$

$\Rightarrow \frac{C_p}{C_v} = \frac{2 \times 160}{240} = \frac{4}{3}$

Now, $\frac{C_p}{C_v} = \gamma$ and degrees of freedom of a gas,

$$f = \frac{2}{\gamma - 1}$$

$\Rightarrow f = \frac{2}{\gamma - 1} = \frac{2}{\frac{4}{3} - 1} \Rightarrow f = 6$

17. A diatomic gas molecule has 5 degrees of freedom, i.e. 3 translational and 2 rotational, at low temperature ranges (~ 250 K to 750 K). At temperatures above 750 K, molecular vibrations occurs and this causes two extra degrees of freedom.

Now, in given case,

For gas A, $C_p = 29$, $C_v = 22$

For gas B, $C_p = 30$, $C_v = 21$

By using $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$, we have

For gas A,

$$1 + \frac{2}{f} = \frac{29}{22} \approx 1.3 \Rightarrow f = 6.67 \approx 7$$

So, gas A has vibrational mode of degree of freedom.

For gas B,

$$1 + \frac{2}{f} = \frac{30}{21} \approx 1.4 \Rightarrow f = 5$$

Hence, gas B does not have any vibrational mode of degree of freedom.

18. Given, volume, $V = 25 \times 10^{-3}$ m³

$$N = 1 \text{ mole of O}_2 = 6.023 \times 10^{23} \text{ atoms of O}_2$$

$$T = 300 \text{ K}$$

Root mean square velocity of a gas molecule of

$$\text{O}_2, v_{\text{rms}} = 200 \text{ m/s}$$

$$\text{Radius, } r = \frac{0.3}{2} \text{ nm} = \frac{0.3}{2} \times 10^{-9} \text{ m}$$

Now, average time, $\frac{1}{\tau} = \frac{v_{\text{av}}}{\lambda}$

where, $\lambda = \frac{RT}{\sqrt{2} N \pi r^2 p}$

As, $p = \frac{RT}{V}$

$\Rightarrow \lambda = \frac{V}{\sqrt{2} N \pi r^2}$

\therefore Average collision per second,

$$\frac{1}{\tau} = \frac{v_{\text{av}}}{\lambda} = \frac{\sqrt{\frac{8}{3\pi}} \times v_{\text{rms}}}{\lambda}$$

$$\frac{200 \times \sqrt{2} \times 6.023 \times 10^{23} \times \pi \times \frac{0.09}{4} \times 10^{-18}}{25 \times 10^{-3}}$$

$\Rightarrow \frac{1}{\tau} = 4.4 \times 10^8 \text{ per second} \approx 10^8$

19. For a gas, value of specific heat at constant volume,

$$C_v = \frac{1}{2} fR$$

where, f = degrees of freedom

and R = gas constant.

For a diatomic gas molecule, degrees of freedom is in general 5 but when vibration occurs, then degrees of freedom increases to 7.

So, for gas A, specific heat is $C_V^A = \frac{1}{2} 5R$

and for gas B, specific heat is $C_V^B = \frac{1}{2} 7R$

∴ Ratio of specific heats of gas A and B is

$$\frac{C_V^A}{C_V^B} = \frac{5}{7}$$

20. Given, $T_1 = 27^\circ \text{C} = 300 \text{K}$,

$$k_1 = 6.21 \times 10^{-21} \text{J}$$

and $T_2 = 127^\circ \text{C} = 400 \text{K}$.

We know that, $\frac{k_1}{k_2} = \frac{T_1}{T_2}$

$$k_2 = \frac{T_2}{T_1} \times k_1$$

$$= 6.21 \times 10^{-21} \times 400/300$$

$$= 8.28 \times 10^{-21} \text{J}$$

21. As, $\left(\frac{c_{\text{He}}}{c_{\text{H}}}\right) = \sqrt{\frac{\rho_{\text{H}}}{\rho_{\text{He}}}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$$(c_{\text{He}})_t = (c_{\text{He}})_0 \sqrt{\frac{T}{T_0}}$$

$$\therefore \frac{(c_{\text{He}})_t}{(c_{\text{H}})_0} = \frac{(c_{\text{He}})_0}{(c_{\text{H}})_0} \sqrt{\frac{T}{T_0}} = \frac{5}{7}$$

$$\text{or } T \approx 2T_0 = 2 \times 273 = 546^\circ \text{C} \\ = 273^\circ \text{C}$$

22. According to law of equipartition of energy, average kinetic energy per molecule per degree of freedom at temperature T is $\frac{1}{2} kT$. The average kinetic energy per

$$\text{molecule of polyatomic gas molecule} = \frac{n}{2} kT$$

(n = number of mole)

The average kinetic energy per mol of polyatomic gas

$$E = \frac{n}{2} kT \times N = \frac{n}{2} RT$$

$$C_V = \frac{d}{dT} \left(\frac{n}{2} RT \right) = \frac{n}{2} R$$

23. As, $\gamma = 1 + \frac{2}{n}$ or $\frac{2}{n} = \gamma - 1$ or $n = \frac{2}{\gamma - 1}$

24. Molar specific heat of the mixture at constant volume is

$$C_V = \frac{n_1 C_{V1} + n_2 C_{V2}}{(n_1 + n_2)} \\ = \frac{2 \left(\frac{3}{2} R \right) + 3 \left(\frac{5}{2} R \right)}{2 + 3} = 2.1 R$$

25. Internal energy of ' n ' moles of a gas with degree of freedom f ($= 3$ for an ideal gas), at temperature T is

$$E = \frac{f}{2} \cdot n RT = \frac{3}{2} nRT$$

For an ideal gas, internal energy, $E = \frac{3}{2} nRT$

$$= \frac{3}{2} p \cdot V \quad (\because \text{from } pV = nRT)$$

Substituting the given values, we get

$$= \frac{3}{2} \times 3 \times 10^6 \times 2 = 9 \times 10^6 \text{ J}$$

26. $\lambda = \frac{RT}{\sqrt{2\pi d^2 N_A P}}$

$$\lambda = 102 \text{ nm}$$

27. Here, $n = 6$,

$$C_p = \left(1 + \frac{n}{2} \right) R = \left(1 + \frac{6}{2} \right) R = 4R$$

28. At NTP, $T = 273 \text{K}$, $p = 1.01 \times 10^5 \text{ Nm}^{-2}$

Here, $d = 2.4 \times 10^{-10} \text{ m}$

$$\lambda = \frac{kT}{\sqrt{2\pi d^2 p}}$$

$$= \frac{(1.38 \times 10^{-23}) \times 273}{1.414 \times 3.14 \times (2.4 \times 10^{-10})^2 \times 1.01 \times 10^5}$$

$$= 1.46 \times 10^{-7} \text{ m}$$

29. $\therefore \gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$

where, f = degree of freedom.

Now,

(A) for monatomic gas, $f = 3$

$$\text{so, } \gamma = 1 + \frac{2}{3} = \frac{5}{3}$$

(B) for diatomic rigid gas, $f = 5$

$$\text{so, } \gamma = 1 + \frac{2}{5} = \frac{7}{5}$$

(C) for diatomic non-rigid gas, $f = 7$

$$\text{so, } \gamma = 1 + \frac{2}{7} = \frac{9}{7}$$

(D) for triatomic rigid gas, $f = 6$

$$\text{so, } \gamma = 1 + \frac{2}{6} = \frac{8}{6} = \frac{4}{3}$$

Therefore,

A-IV, B-I, C-II, D-III.

Hence, correct option is (a).

30. Mean free path, $\lambda = \frac{V}{\sqrt{2\pi d^2 N}} = \frac{kT}{\sqrt{2\pi d^2 p}}$

where, V = volume of conductor,

N = number of molecules,

d = diameter of molecule,

k = Boltzmann's constant,

T = temperature

and p = pressure.

But from gas equation, $\frac{p}{T} = \frac{nR}{V}$

For a constant n and V , $\frac{p}{T} = \text{constant}$.

So, mean free path does not depend on $\frac{p}{T}$ or p and T .

So, in given condition (closed container), mean free path remains constant or unchanged.

∴ Statement A is incorrect, but C is correct.

Also, mean time between two collisions is

$$\tau = \frac{\text{mean free path}}{v_{\text{avg}}} = \frac{\lambda}{v_{\text{avg}}}$$

As, $v_{\text{avg}} \propto \sqrt{T}$
 \Rightarrow Average time, $\tau \propto \frac{1}{\sqrt{T}}$

As, the temperature is increasing, so mean collision time (τ) decreases.

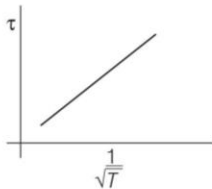
∴ Statement B is correct.
 Hence, option (a) is correct.

- 31.** Mean free time (τ) for a gas molecule is time elapsed between two successive collisions.

$$\begin{aligned} \therefore \tau &= \frac{\text{Mean free path length } (\lambda)}{\text{Average speed } (v_{\text{av}})} \\ &= \frac{1}{\left(\frac{\sqrt{2} \cdot \pi d^2 \cdot n}{\sqrt{\frac{8}{\pi} \frac{R}{M} \cdot T}\right)} \end{aligned}$$

So, for an ideal gas, $\tau \propto \frac{1}{\sqrt{T}}$

Thus, graph of τ versus $\frac{1}{\sqrt{T}}$ is a straight line as shown below



- 32.** Mean free path (λ) of a gas molecule and mean relaxation time (or collision time) τ are related as

$$\tau = \frac{\lambda}{v_{\text{mean}}} = \frac{\frac{1}{\sqrt{2}} \frac{k_B T}{\pi d^2 p}}{\sqrt{\frac{8}{\pi} \frac{k_B T}{m}}} = \frac{1}{4d^2} \left(\frac{k_B m}{\pi}\right)^{1/2} \cdot \left(\frac{T^{1/2}}{p}\right)$$

Using $pV = nRT$, we have

$$\begin{aligned} \tau &= \frac{1}{4d^2} \cdot \left(\frac{k_B m}{\pi}\right)^{1/2} \cdot \frac{VT^{1/2}}{nRT} \\ &= \left\{ \frac{1}{4d^2} \cdot \left(\frac{k_B m}{\pi}\right)^{1/2} \cdot \frac{1}{nR} \right\} \cdot \frac{V}{\sqrt{T}} \end{aligned}$$

$\Rightarrow \tau \propto \frac{V}{\sqrt{T}}$, as other quantities remain constant during the process.

So,
$$\frac{\tau_1}{\tau_2} = \frac{V_1}{V_2} \cdot \left(\frac{T_2}{T_1}\right)^{1/2} \quad \dots (i)$$

As the process is adiabatic,

$$\begin{aligned} T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\ \Rightarrow \frac{T_2}{T_1} &= \left(\frac{V_1}{V_2}\right)^{\gamma-1} \end{aligned}$$

So, Eq. (i) becomes

$$\begin{aligned} \frac{\tau_1}{\tau_2} &= \frac{V_1}{V_2} \cdot \left(\frac{V_1}{V_2}\right)^{\frac{\gamma-1}{2}} \\ \Rightarrow \frac{\tau_1}{\tau_2} &= \left(\frac{V_1}{V_2}\right)^{\frac{\gamma+1}{2}} \end{aligned}$$

Now, given $V_2 = 2V_1$, so

$$\frac{\tau_1}{\tau_2} = \left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$$

- 33.** Mean free path for a gas,

$$\lambda = \frac{1}{\sqrt{2} \pi n d^2}$$

where, n = number density
 and d = diameter of gas atom.

Mean speed of a gas atom,

$$v = \sqrt{\frac{8}{\pi} \frac{kT}{m}}$$

where, k = Boltzmann's constant,
 T = temperature of sample
 and m = atomic mass.

Hence, mean relaxation time,

$$T = \frac{\lambda}{v} = \frac{\left(\frac{1}{\sqrt{2} \pi n d^2}\right)}{\sqrt{\frac{8}{\pi} \frac{kT}{m}}}$$

$\tau \propto \frac{\sqrt{m}}{d^2}$, when T and n are same.

Hence,
$$\frac{\tau_{\text{Ar}}}{\tau_{\text{Xe}}} = \frac{\sqrt{m_{\text{Ar}}}}{\sqrt{m_{\text{Xe}}}} \cdot \frac{d_{\text{Xe}}^2}{d_{\text{Ar}}^2} = \frac{\sqrt{m_{\text{Ar}}}}{\sqrt{m_{\text{Xe}}}} \cdot \frac{r_{\text{Xe}}^2}{r_{\text{Ar}}^2}$$

Here, $m_{\text{Ar}} = 40$, $m_{\text{Xe}} = 140$,

$$r_{\text{Ar}} = 0.07 \text{ nm}, r_{\text{Xe}} = 0.1 \text{ nm}$$

$$\therefore \frac{\tau_{\text{Ar}}}{\tau_{\text{Xe}}} = \sqrt{\frac{40}{140}} \times \left(\frac{0.1}{0.07}\right)^2 = 1.09$$

- 34.** When n_1 moles of an ideal gas having specific heats C_{p_1} and C_{v_1} is mixed with n_2 moles of another ideal gas with specific heats C_{p_2} and C_{v_2} , specific heats of mixture are given by

$$C_{p(\text{mix})} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$$

and
$$C_{v(\text{mix})} = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$$

So, ratio of specific heats of mixture will be

$$\frac{C_{p(\text{mix})}}{C_{V(\text{mix})}} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}} \quad \dots(i)$$

Now, using $C_p = \left(\frac{\gamma}{\gamma-1}\right)R$ and $C_V = \frac{R}{\gamma-1}$

Note As, $\frac{C_p}{C_V} = \gamma$ and $C_p - C_V = R$

$$\text{So, } C_p - \frac{C_p}{\gamma} = R \text{ or } C_p = \frac{\gamma}{\gamma-1}R \text{ and } C_V = \frac{R}{\gamma-1}$$

We can write Eq. (i) as

$$\begin{aligned} \frac{C_{p(\text{mix})}}{C_{V(\text{mix})}} &= \frac{n_1 \left(\frac{\gamma_1}{\gamma_1-1}\right)R + n_2 \left(\frac{\gamma_2}{\gamma_2-1}\right)R}{n_1 \left(\frac{R}{\gamma_1-1}\right) + n_2 \left(\frac{R}{\gamma_2-1}\right)} \\ &= \frac{n_1 \gamma_1 (\gamma_2 - 1) + n_2 \gamma_2 (\gamma_1 - 1)}{n_1 (\gamma_2 - 1) + n_2 (\gamma_1 - 1)} \quad \dots(ii) \end{aligned}$$

Here, we are given

$$n_1 = 2, \gamma_1 = \frac{5}{3}, n_2 = 3 \text{ and } \gamma_2 = \frac{4}{3}$$

Substituting the above values in Eq.(ii), we have

$$\begin{aligned} \frac{C_{p(\text{mix})}}{C_{V(\text{mix})}} &= \frac{2 \times \frac{5}{3} \left(\frac{4}{3} - 1\right) + 3 \times \frac{4}{3} \left(\frac{5}{3} - 1\right)}{2 \times \left(\frac{4}{3} - 1\right) + 3 \times \left(\frac{5}{3} - 1\right)} \\ &= \frac{\frac{10}{3} + \frac{24}{3}}{\frac{2}{3} + \frac{6}{3}} \\ &= \frac{34 \times 3}{9 \times 3} = \frac{17}{12} = 1.42 \end{aligned}$$

35. Given, height, $h = 2$ cm

Kinetic energy, $U = 4 \times 10^{-14}$ erg

Density of mercury, $\rho = 13.6$ g/cm³

Volume, $V = 4$ cm³ $g = 980$ cm/s²

According to gas equation,

$$pV = NKT \Rightarrow N = \frac{pV}{KT}$$

For monoatomic gas,

$$\text{Energy, } U = \frac{3}{2}KT \Rightarrow KT = \frac{2}{3}U$$

$$\text{Now, } N = \frac{3pV}{2U}$$

$$\text{So, } N = \frac{3gphV}{2U} \quad (\because \text{pressure, } p = \rho gh) \quad \dots(i)$$

Substituting the given values in eq. (i), we get

$$\begin{aligned} N &= \frac{3 \times 980 \times 13.6 \times 2 \times 4}{2 \times 4 \times 10^{-14}} \\ &= 3.99 \times 10^{18} \\ &\approx 4 \times 10^{18} \end{aligned}$$

36. For a mixture of two gases, ratio of specific heats is given by

$$\begin{aligned} \gamma_{\text{mixture}} &= \frac{C_{p(\text{mixture})}}{C_{V(\text{mixture})}} \\ &= \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}} \quad \dots(i) \end{aligned}$$

Here, gas 1 is helium which is monoatomic, so

$$n_1 = n, C_{p_1} = \frac{5}{2}R, C_{V_1} = \frac{3}{2}R$$

and gas 2 is oxygen which is diatomic, so

$$n_2 = 2n, C_{p_2} = \frac{7}{2}R, C_{V_2} = \frac{5}{2}R$$

Hence, from Eq. (i), we get

$$\gamma_{\text{mix}} = \frac{n \times \frac{5R}{2} + 2n \times \frac{7R}{2}}{n \times \frac{3R}{2} + 2n \times \frac{5R}{2}} = \frac{19}{13}$$

37. According to the law of equipartition of energy,

$$\frac{1}{2}mv_{\text{rms}}^2 = \frac{n}{2}k_B T \text{ where, } n \text{ is the degree of freedom.}$$

Since, HCl is a diatomic molecule that has rotational, translational and vibrational motion.

So, $n = 7$

$$\Rightarrow \frac{1}{2}mv_{\text{rms}}^2 = \frac{7}{2}k_B T$$

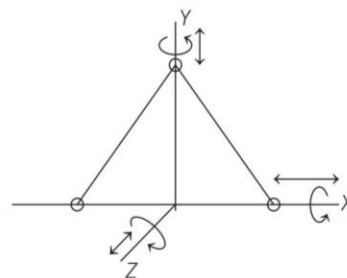
$$\begin{aligned} \text{Here, } v_{\text{rms}}^2 &= \bar{v} \\ \Rightarrow T &= \frac{m\bar{v}^2}{7k_B} \end{aligned}$$

38. Internal energy U of a mole of the gas at temperature T is given by

$$U = \frac{f}{2}RT$$

where, f = degree of freedom of gas molecule

and R = gas constant.



A triatomic triangular gas molecules can have three-translational motions (translation along X, Y or Z -axis) and three-rotational motions (rotation along X, Y or Z -axis). So, it has total 6 degrees of freedom.

$$\text{So, internal energy, } U = \frac{f}{2}RT = \frac{6}{2}RT = 3RT$$

Round II

$$1. \frac{F}{2} n_1 k T_1 + \frac{F}{2} n_2 k T_2 + \frac{F}{2} n_3 k T_3 = \frac{F}{2} (n_1 + n_2 + n_3) k T$$

[∵ F is degree of freedom]

$$\text{Temperature of mixture, } T = \frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$$

2. From $pV = nRT = \frac{N}{N_A} RT$, we have

$$n_f - n_i = \frac{pV N_A}{RT_f} - \frac{pV N_A}{RT_i}$$

$$\Rightarrow n_f - n_i = \frac{10^5 \times 30}{8.3} \times 6.02 \times 10^{23} \cdot \left(\frac{1}{300} - \frac{1}{290} \right)$$

$$= -2.5 \times 10^{25}$$

$$\therefore \Delta n = -2.5 \times 10^{25}$$

3. As no work is done and system is thermally insulated from surrounding, it means sum of internal energy of gas in two partitions is constant, i.e. $U = U_1 + U_2$.

Assuming both gases have same degree of freedom, then

$$\text{Internal energy, } U = \frac{f(n_1 + n_2)RT}{2}$$

and $U_1 = \frac{fn_1 RT_1}{2}, U_2 = \frac{fn_2 RT_2}{2}$

Solving, we get

$$\text{Common temperature, } T = \frac{(p_1 V_1 + p_2 V_2) T_1 T_2}{p_1 V_1 T_2 + p_2 V_2 T_1}$$

4. For carbon dioxide, number of mole (n_1) = $\frac{22}{44} = \frac{1}{2}$;

Molar specific heat of CO_2 at constant volume, $C_{V_1} = 3R$

For oxygen, number of moles (n_2) = $\frac{16}{32} = \frac{1}{2}$;

molar specific heat of O_2 at constant volume, $C_{V_2} = \frac{5R}{2}$.

Let T K be the temperature of mixture.

Heat lost by $\text{O}_2 = \text{Heat gained by } \text{CO}_2$.

$$n_2 C_{V_2} \Delta T_2 = n_1 C_{V_1} \Delta T_1$$

$$\frac{1}{2} \left(\frac{5}{2} R \right) (310 - T) = \frac{1}{2} \times (3R) (T - 300)$$

or $1550 - 5T = 6T - 1800$ or $T = 304.54 \text{ K} = 31.5^\circ \text{C}$

5. For an ideal gas undergoing an adiabatic process at room temperature,

$$pV^\gamma = \text{constant or } TV^{\gamma-1} = \text{constant}$$

For a diatomic gas, degree of freedom, $f = 5$

$$\therefore \gamma = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$$

As for adiabatic process, $TV^{\gamma-1} = \text{constant}$... (i)

and it is given that, here $TV^x = \text{constant}$... (ii)

Comparing Eqs. (i) and (ii), we get

$$\gamma - 1 = x \Rightarrow \frac{7}{5} - 1 = x \text{ or } x = 2/5$$

6. Here, $p = 2 \text{ atm} = 2 \times 1.013 \times 10^5 \text{ Nm}^{-2}$

$$T = 17 + 273 = 290 \text{ K}$$

$$\sigma = 2r = 2 \times 1 \text{ \AA} = 2 \times 10^{-10} \text{ m}$$

$$\lambda = \frac{kT}{\sqrt{2} m \sigma^2 p}$$

$$= \frac{(1.38 \times 10^{-23}) \times 290}{1.414 \times 3.14 \times (2 \times 10^{-10})^2 \times 2.026 \times 10^5}$$

$$= 1.11 \times 10^{-7} \text{ m}$$

7. Since, each vibrational mode has 2 degrees of freedom, hence total vibrational degrees of freedom = $24 \times 2 = 48$

∴ Total number of degree of freedom

$$= 3 + 3 + 48 = 54$$

$$\therefore \gamma = 1 + \frac{2}{f}$$

$$= 1 + \frac{2}{54}$$

$$= \frac{28}{27} = 1.03$$

8. When the piston is in equilibrium, the pressure is same on both the sides of the piston. It is given that temperature and weight of gas on the two sides of piston does not change. From ideal gas equation, $pV = nRT$, we have $V \propto \text{mass of the gas}$.

So, $\frac{V_1}{V_2} = \frac{m_1}{m_2}$

or $\frac{V_1}{V_2} + 1 = \frac{m_1}{m_2} + 1$

or $\frac{V_1 + V_2}{V_2} = \frac{m_1 + m_2}{m_2}$

or $\frac{V_2}{V_1 + V_2} = \frac{m_2}{m_1 + m_2}$

$$= \frac{2m}{m + 2m} = \frac{2}{3}$$

9. Work done by gas during heating process at constant pressure is given by

$$\Delta W = p\Delta V$$

Using ideal gas equation,

$$pV = nRT$$

$$\Rightarrow p\Delta V = nR\Delta T$$

So, $\Delta W = nR\Delta T$... (i)

Now, it is given that, $n = \frac{1}{2}$ and

$$\Delta T = 90^\circ \text{C} - 20^\circ \text{C}$$

$$= 363 \text{ K} - 293 \text{ K} = 70 \text{ K}$$

and R (gas constant) = 8.31 J/mol-K

Substituting these values in Eq. (i), we get

$$\Delta W = \frac{1}{2} \times 8.31 \times 70 = 290.85 \text{ J}$$

$$\Delta W \approx 291 \text{ J}$$

10. Thermal energy corresponds to internal energy

$$\text{Mass} = 1 \text{ kg}$$

$$\text{Density} = 4 \text{ kgm}^{-3}$$

$$\Rightarrow \text{Volume} = \frac{\text{Mass}}{\text{Density}} = \frac{1}{4} \text{ m}^3$$

$$\text{Pressure} = 8 \times 10^4 \text{ Nm}^{-2}$$

$$\text{As internal energy} = \frac{f}{2} nRT = \frac{f}{2} pV$$

$$\text{Degree of freedom for diatomic} = 5$$

$$\therefore \text{Internal energy} = \frac{5}{2} p \times V$$

$$\text{Putting the values of } p = 8 \times 10^4 \text{ Nm}^{-2} \text{ and } V = \frac{1}{4} \text{ m}^3,$$

we get

$$\text{Internal energy} = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4 \text{ J}$$

11. $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

$$U(x = \infty) = 0$$

$$\text{As, } F = -\frac{dU}{dx} = -\left[\frac{12a}{x^{13}} + \frac{6b}{x^7}\right]$$

$$\text{At equilibrium, } F = 0$$

$$\therefore x^6 = \frac{2a}{b}$$

$$\therefore U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = \frac{-b^2}{4a}$$

$$\therefore D = [U(x = \infty) - U_{\text{at equilibrium}}] = \frac{b^2}{4a}$$

12. We know that, from kinetic theory of gases,

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$\text{or } v_{\text{rms}} \propto \sqrt{T}$$

where, R is gas constant, T is temperature and M is molecular mass of a gas.

Here, to double the v_{rms} , temperature must be 4 times of the initial temperature.

$$\therefore \text{New temperature, } T_2 = 4 \times T_1$$

$$\text{As, } T_1 = 27^\circ \text{C} = 300 \text{ K}$$

$$\Rightarrow T_2 = 4 \times 300 = 1200 \text{ K}$$

As, gas is kept inside a closed vessel.

$$\therefore Q = nC_V \cdot dT = \frac{15}{28} \times \frac{5R}{2} (1200 - 300)$$

$$\left[\text{As, } n = \frac{m}{M} = \frac{15}{28} \text{ for nitrogen} \right]$$

$$= \frac{15}{28} \times \frac{5}{2} \times 8.3 \times 900$$

$$[\text{Given, } R = 8.3 \text{ J/K}\cdot\text{mol}]$$

$$\text{or } Q = 10004.4 \text{ J} \approx 10 \text{ kJ}$$

13. According to Boyle's law, $pV = k$ (a constant)

$$\text{or } p \frac{m}{\rho} = k \text{ or } \rho = \frac{pm}{k}$$

$$\text{or } \rho = \frac{p}{k} \quad \left(\text{where, } \frac{k}{m} = k = \text{constant} \right)$$

$$\text{So, } \rho_1 = \frac{p_1}{k}$$

$$\text{and } V_1 \frac{p_1}{k} = \frac{m_1}{\rho_1} = \frac{m_1}{p_1/k} = \frac{km_1}{p_1}$$

$$\text{Similarly, } V_2 = \frac{km_2}{p_2}$$

$$\text{Total volume} = V_1 + V_2 = k \left(\frac{m_1}{p_1} + \frac{m_2}{p_1} \right)$$

Let p be the common pressure and ρ be the common density of mixture. Then

$$\rho = \frac{m_1 + m_2}{V_1 + V_2} = \frac{m_1 + m_2}{k \left(\frac{m_1}{p_1} + \frac{m_2}{p_2} \right)}$$

$$\therefore p = k\rho = \frac{m_1 + m_2}{\frac{m_1}{p_1} + \frac{m_2}{p_2}} = \frac{p_1 p_2 (m_1 + m_2)}{(m_1 p_2 + m_2 p_1)}$$

14. Mean time elapsed between two successive, collisions

$$\text{is } t = \frac{\lambda}{v}$$

where, λ = mean free path length and

v = mean speed of gas molecule

$$\therefore t = \frac{\left(\frac{k_B t}{\sqrt{2} \pi d^2 p} \right)}{\sqrt{\frac{8}{\pi} \cdot \frac{k_B T}{M}}} \Rightarrow t = \frac{C\sqrt{T}}{p}$$

where, $C = \frac{1}{4d^2} \sqrt{\frac{k_B M}{\pi}}$ = a constant for a gas.

$$\text{So, } \frac{t_2}{t_1} = \frac{\sqrt{T_2}}{\sqrt{T_1}} \cdot \left(\frac{p_1}{p_2} \right) \quad \dots (i)$$

$$\text{Here given, } \frac{p_1}{p_2} = \frac{1}{2}, \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{500}{300}} = \sqrt{\frac{5}{3}}$$

$$\text{and } t_1 = 6 \times 10^{-8} \text{ s}$$

Substituting there values in Eq. (i), we get

$$t_2 = 6 \times 10^{-8} \times \sqrt{\frac{5}{3}} \times \frac{1}{2} = 3.86 \times 10^{-8} \text{ s} \approx 4 \times 10^{-8} \text{ s}$$

15. Let molar specific heat of the mixture is C_V .

Total number of molecules in the mixture = 3 + 2 = 5

$\therefore C_V$ can be determined using

$$n C_V dT = n_1 C_{V1} dT + n_2 C_{V2} dT$$

$$\text{or } (n_1 + n_2) (C_V)_{\text{mix}} = n_1 C_{V1} + n_2 C_{V2} \quad [\text{here, } n = n_1 + n_2]$$

Here, $C_{V_1} = \frac{3R}{2}$ (for helium); $n_1 = 2$

$C_{V_2} = \frac{5R}{2}$ (for hydrogen); $n_2 = 3$

[Because for monoatomic gases, $C_V = \frac{3}{2}R$ and for diatomic gases, $C_V = \frac{5}{2}R$]

$\therefore 5 \times C_V = \left(2 \times \frac{3R}{2}\right) + \left(3 \times \frac{5R}{2}\right) \Rightarrow 5C_V = \frac{21R}{2}$

or $C_V = \frac{21R}{10} = \frac{21 \times 8.3}{10} = \frac{174.3}{10}$

or $C_V = 17.4 \text{ J/mol-K}$

- 16.** For an adiabatic process $TV^{\gamma-1} = \text{constant}$.

We know that average time of collision between molecules

$$\tau = \frac{1}{n\pi\sqrt{2}v_{\text{rms}}d^2}$$

where, n = number of molecules per unit volume

v_{rms} = rms velocity of molecules

As $n \propto \frac{1}{V}$ and $v_{\text{rms}} \propto \sqrt{T}$

$$\tau \propto \frac{V}{\sqrt{T}}$$

Thus, we can write $n = K_1V^{-1}$ and $v_{\text{rms}} = K_2T^{1/2}$ where, K_1 and K_2 are constants.

For adiabatic process, $TV^{\gamma-1} = \text{constant}$. Thus, we can write

$$\tau \propto VT^{-1/2} \propto V(V^{1-\gamma})^{-1/2}$$

or $\tau \propto V^{\frac{\gamma+1}{2}}$

- 17.** For mixture, $C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$

For helium,

Number of moles, $n_1 = \frac{16}{4} = 4$ and $\gamma_1 = \frac{5}{3}$

For oxygen,

Number of moles, $n_2 = \frac{16}{32} = \frac{1}{2}$ and $\gamma_2 = \frac{7}{5}$

Then, $C_{V_1} = \frac{R}{\gamma_1 - 1} = \frac{R}{\frac{5}{3} - 1} = \frac{3}{2}R$

and $C_{V_2} = \frac{R}{\gamma_2 - 1} = \frac{R}{\frac{7}{5} - 1} = \frac{5}{2}R$

\therefore For mixture,

$$C_V = \frac{4 \times \frac{3}{2}R + \frac{1}{2} \times \frac{5}{2}R}{4 + \frac{1}{2}} = \frac{6R + \frac{5}{4}R}{\frac{9}{2}} = \frac{29R \times 2}{9 \times 4} = \frac{29R}{18}$$

Now, $C_V = \frac{R}{\gamma - 1}$

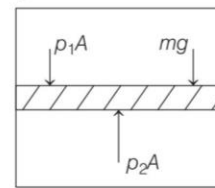
$\Rightarrow \gamma - 1 = \frac{R}{C_V}$

or $\gamma = \frac{R}{C_V} + 1 = \frac{R}{\frac{29}{18}R} + 1$

$\Rightarrow \frac{C_p}{C_V} = \frac{18}{29} + 1 = \frac{18 + 29}{29} = 1.62$

- 18.** As piston is in equilibrium, so net force on piston is zero.

When the piston is stationary, i.e. on equilibrium as shown in the figure below



then $p_1A + mg = p_2A$

$\Rightarrow mg = p_2A - p_1A$

or $mg = \left(\frac{nRTA}{V_2} - \frac{nRTA}{V_1}\right)$

[$\because pV = nRT$ (ideal gas equation)]

$$= nRT \left(\frac{A}{Al_2} - \frac{A}{Al_1}\right) = nRT \left(\frac{l_1 - l_2}{l_1 l_2}\right)$$

or $m = \frac{nRT}{g} \left(\frac{l_1 - l_2}{l_1 l_2}\right)$

- 19.** Mayer's formula is given by, $C_p - C_V = R$ and $\gamma = \frac{C_p}{C_V}$

Therefore, using above two relations, we find

$$C_V = \frac{R}{\gamma - 1}$$

For a mole of monatomic gas, $\gamma = \frac{5}{3}$

$\therefore C_V = \frac{R}{\left(\frac{5}{3}\right) - 1} = \frac{3}{2}R$

For a mole of diatomic gas, $\gamma = \frac{7}{5}$

$\therefore C_V = \frac{R}{\left(\frac{7}{5}\right) - 1} = \frac{5}{2}R$

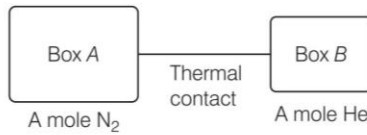
When these two moles are mixed, then heat required to raise the temperature to 1°C is $C_V = \frac{3}{2}R + \frac{5}{2}R = 4R$

Hence, for one mole, heat required $= \frac{4R}{2} = 2R$

$\therefore C_V = 2R$

$\Rightarrow \frac{R}{\gamma - 1} = 2R$ or $\gamma = \frac{3}{2}$

20. Here, change in internal energy of the system is zero, i.e. increase in internal energy of one is equal to decrease in internal energy of other.



Change in internal energy in box A,

$$\Delta U_A = 1 \times \frac{5}{2} R (T_f - T_0)$$

Change in internal energy in box B

$$\Delta U_B = 1 \times \frac{3}{2} R \left(T_f - \frac{7}{3} T_0 \right)$$

Now, $\Delta U_A + \Delta U_B = 0$

$$\Rightarrow \frac{5R}{2} (T_f - T_0) + \frac{3R}{2} \left(T_f - \frac{7T_0}{3} \right) = 0$$

$$\text{or } 5T_f - 5T_0 + 3T_f - 7T_0 = 0$$

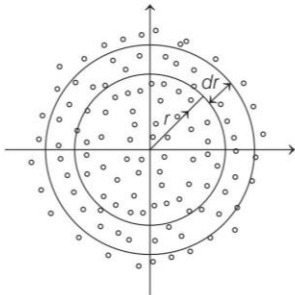
$$\text{or } 8T_f = 12T_0$$

$$\text{or } T_f = \frac{12}{8} T_0 = \frac{3}{2} T_0$$

21. Number density of gas molecules,

$$n = \frac{\text{Number of molecules}}{\text{Volume of gas}}$$

\Rightarrow Number of molecules, $N = n \times \text{Volume of gas}$



Now, consider a shell of radius r and thickness dr with centre at $r = 0$.

Volume of shell of differentiable thickness (dr),

$$dV = \text{surface area} \times \text{thickness} = 4\pi r^2 dr$$

Now, number of molecules in this shell is

$$dN = n(r) \cdot dV = n_0 e^{-\alpha r^4} \cdot 4\pi r^2 dr$$

So, total number of molecules present in given volume (extending from $r = 0$ to $r = \infty$) is

$$N = \int_0^\infty n(r) \cdot dV = \int_0^\infty n_0 e^{-\alpha r^4} \cdot 4\pi r^2 dr$$

$$= \int_0^\infty 4\pi n_0 e^{-\alpha r^4} \cdot r^2 dr \quad \dots(i)$$

Here, we take $\alpha r^4 = t$

$$\Rightarrow r = t^{\frac{1}{4}} \cdot \alpha^{-\frac{1}{4}} \Rightarrow 4\alpha r^3 dr = dt$$

$$\Rightarrow r^2 dr = \frac{dt}{4\alpha r} = \frac{dt}{4\alpha t^{\frac{1}{4}} \cdot \alpha^{-\frac{1}{4}}} = \frac{dt}{4\alpha^{\frac{3}{4}} t^{\frac{1}{4}}}$$

Also, when $r = 0, t = 0$ and when $r = \infty, t = \infty$ substituting in Eq. (i), we get

$$N = \int_0^\infty 4\pi n_0 e^{-t} \cdot \frac{dt}{4\alpha^{\frac{3}{4}} t^{\frac{1}{4}}}$$

$$N = \pi \alpha^{-3/4} \cdot n_0 \cdot \int_0^\infty e^{-t} \cdot t^{-\frac{1}{4}} dt$$

As value of definite integral $\int_0^\infty e^{-t} \cdot t^{-\frac{1}{4}} dt$ is a constant ($= k$ let), we have

$$N = \pi k n_0 \alpha^{-3/4}$$

$$\Rightarrow N \propto n_0 \alpha^{-3/4}$$

22. Here, $V = 25.0 \text{ m}^3$

$$T = 27 + 273 = 300 \text{ K}$$

$$k = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Now, $pV = nRT = n(Nk)T$

$$= (nN)kT = N'kT$$

where, $nN = N'$ = total number of air molecules in the given gas

$$N' = \frac{pV}{kT} = \frac{(1.01 \times 10^5) \times 25}{(1.38 \times 10^{-23}) \times 300}$$

$$= 6.10 \times 10^{26} = 6.10 \times 10^{13 \times 2}$$

$$\therefore x = 2$$

23. If T be the final equilibrium temperature, then applying conservation of internal energy,

$$U_1 + U_2 = U_1' + U_2'$$

$$n_1 C_V T_1 + n_2 C_V T_2 = (n_1 + n_2) C_V T$$

$$n_1 T_1 + n_2 T_2 = (n_1 + n_2) T$$

$$(0.1)(200) + (0.05)(400) = (0.1 + 0.05) T$$

$$20 + 20 = (0.15) T$$

$$T = \frac{40}{0.15} = 266.66 \text{ K} \approx 266 \text{ K}$$

24. Here, $c_1 = 2 \text{ kms}^{-1}$; $c_2 = 4 \text{ kms}^{-1}$

$$c_3 = 6 \text{ kms}^{-1} \text{ and } c_4 = 8 \text{ kms}^{-1}$$

(i) Average speed,

$$c_{av} = \frac{c_1 + c_2 + c_3 + c_4}{4} = \frac{2 + 4 + 6 + 8}{4} = 5 \text{ kms}^{-1}$$

(ii) Root mean square speed,

$$c = \sqrt{\frac{c_1^2 + c_2^2 + c_3^2 + c_4^2}{4}} = \sqrt{\frac{2^2 + 4^2 + 6^2 + 8^2}{4}}$$

$$= 5.48 \text{ kms}^{-1}$$

- 25.

$$\rho_1, V_1, \mu_1$$

At 250 K

$$\rho_2, V_2, \mu_2$$

At 2000 K

Let

$$\mu_1 = \mu_0, V_2 = 2V_1 \quad (\text{given})$$

Total number of moles at 2000 K = Number of moles of diatomic molecules + Number of moles of monoatomic

$$\mu_2 = 0.75\mu_0 + 2 \times 0.25\mu_0 = 1.25\mu_0$$

Ideal gas equation for situation 1,

$$p_1 V_1 = \mu_0 R T_1$$

$$p_1 V_1 = \mu_0 R (250) \quad \dots (i)$$

For situation 2,

$$p_2 V_2 = (1.25 \mu_0) R (2000)$$

$$p_2 (2V_1) = (1.25 \mu_0) R (2000) \quad \dots (ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{p_1 V_1}{p_2 (2V_1)} = \frac{\mu_0 R (250)}{(1.25 \mu_0) R (2000)}$$

$$\frac{p_1}{p_2} = \frac{1}{5} \text{ or } \frac{p_2}{p_1} = 5$$

26. Here, $J = ?$

$$C_p = 3.409 \text{ cal g}^{-1} \text{ } ^\circ\text{C}^{-1}$$

$$C_v = 2.409 \text{ cal g}^{-1} \text{ } ^\circ\text{C}^{-1}, M = 2$$

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

$$\text{As, } C_p - C_v = \frac{r}{J} = \frac{R}{MJ}$$

$$\therefore 3.409 - 2.409 = \frac{8.31}{2J}$$

$$1 = \frac{4.155}{J}$$

$$J = 4.11 \text{ J cal}^{-1}$$

27. The rms speed is given by $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\text{For nitrogen molecule, } (v_{\text{rms}})_{\text{N}_2} = \sqrt{\frac{3RT_1}{M_1}}$$

$$\text{and for hydrogen molecule, } (v_{\text{rms}})_{\text{H}_2} = \sqrt{\frac{3RT_2}{M_2}}$$

According to question, $(v_{\text{rms}})_{\text{N}_2} = (v_{\text{rms}})_{\text{H}_2}$

$$\Rightarrow \sqrt{\frac{3RT_1}{M_1}} = \sqrt{\frac{3RT_2}{M_2}} \Rightarrow \frac{T_1}{M_1} = \frac{T_2}{M_2}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{M_1}{M_2} \Rightarrow T_2 = \frac{T_1 M_2}{M_1} \quad \dots (i)$$

$$\text{Given, } T_1 = 300^\circ\text{C} = 300 + 273 = 573 \text{ K}$$

$$M_1 = 28 \text{ g and } M_2 = 2 \text{ g}$$

Substituting all these values in Eq. (i), we get

$$T_2 = 573 \times \frac{2}{28} = 40.93 \text{ K} \approx 41 \text{ K}$$

Hence, the required temperature is 41 K.

28. Here, mass of metal, $m = 0.20 \text{ kg} = 200 \text{ g}$

Fall in temperature of metal

$$\Delta T = 150^\circ\text{C} - 40^\circ\text{C} = 110^\circ\text{C}$$

If c is specific heat of the metal, then heat lost by the metal,

$$\Delta Q = mc\Delta T = 200 s \times 110 \quad \dots (i)$$

Volume of water = 150 cc

Mass of water, $m' = 150 \text{ g}$

Water equivalent of calorimeter

$$w = 0.025 \text{ kg} = 25 \text{ g}$$

Rise in temperature of water in calorimeter

$$\Delta T' = 40^\circ\text{C} - 27^\circ\text{C} = 13^\circ\text{C}$$

Heat gained by water and calorimeter

$$\Delta Q' = (m' + w) \Delta T' = (150 + 25) \times 13$$

$$\Delta Q' = 175 \times 13 \quad \dots (ii)$$

As $\Delta Q = \Delta Q'$

\therefore From Eqs. (i) and (ii), we get

$$200 \times s \times 100 = 175 \times 13$$

$$s = \frac{175 \times 13}{200 \times 110} \approx 0.1 \text{ J/g}^\circ\text{C}$$

29. Here, volume of water heated = 3.0 Lmin⁻¹

Mass of water heated, $m = 3000 \text{ gmin}^{-1}$

Rise of temperature, $\Delta T = 77^\circ\text{C} - 27^\circ\text{C} = 50^\circ\text{C}$

Specific heat of water, $s = 4.2 \text{ Jg}^{-1} \text{ } ^\circ\text{C}^{-1}$

$$\text{Amount of heat used, } \Delta Q = ms\Delta T = 3000 \times 4.2 \times 50$$

$$= 63 \times 10^4 \text{ Jmin}^{-1}$$

Heat of combustion = $4 \times 10^4 \text{ Jg}^{-1}$

$$\text{Rate of combustion of fuel} = \frac{63 \times 10^4}{4 \times 10^4} = 15.75 \text{ gmin}^{-1}$$

30. Given, at 27°C, root mean square speed,

$$(v_{\text{rms}})_1 = 100 \text{ ms}^{-1}$$

$$(v_{\text{rms}})_1 = \sqrt{\frac{3p_1}{d_1}} = \sqrt{\frac{3p_1 V_1}{M}} \quad \dots (i)$$

According to ideal gas equation,

$$\Rightarrow \frac{V_1}{V_2} = \frac{p_2 T_1}{p_1 T_2} = \frac{2 \times 300}{400} = \frac{3}{2}$$

\therefore At 127°C, root mean square speed

$$(v_{\text{rms}})_2 = \sqrt{\frac{3p_2}{d_2}} = \sqrt{\frac{3p_2 V_2}{M}} \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$\therefore (v_{\text{rms}})_2^2 = (v_{\text{rms}})_1^2 \times \frac{V_2}{V_1} \times \frac{p_2}{p_1} = (100)^2 \times \frac{2}{3} \times 2$$

$$\text{or } (v_{\text{rms}})_2 = \frac{200}{\sqrt{3}} \text{ ms}^{-1}$$

$$\therefore n = 3$$

31. When the air bubble is at 40 m depth, then

$$V_1 = 1 \text{ cm}^3 = 1.0 \times 10^{-6} \text{ m}^3$$

$$T_1 = 12^\circ\text{C} = 12 + 273 = 285 \text{ K}$$

$$P_1 = 1 \text{ atm} + h_1 \rho g = 1.01 \times 10^5 + 40 \times 10^3 \times 9.8$$

$$= 493000 \text{ Pa}$$

When the air bubble reaches at the surface of lake,

then, $V_2 = ?$, $T_2 = 35^\circ\text{C} = 35 + 273 = 308 \text{ K}$

$$P_2 = 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$$

$$\text{Now, } \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \text{ or } V_2 = \frac{p_1 V_1 T_2}{T_2 p_2}$$

$$\therefore V_2 = \frac{(493000) \times 1.0 \times 10^{-6} \times 308}{285 \times 1.01 \times 10^5}$$

$$= 5.275 \times 10^{-6} \text{ m}^3$$