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IIT-JEE-NEET-PHYSICS



KINETIC THEORY OF GASES

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KINETIC THEORY OF GASES

Welcome to the Kinetic Theory of Gases Study Module tailored for IIT-JEE aspirants. This module provides a deep dive into the fundamental principles of the Kinetic Theory of Gases, an essential topic in physics, particularly for mastering concepts in thermodynamics and statistical mechanics.

- **Comprehensive Coverage:** This module covers all essential concepts of the Kinetic Theory of Gases, providing a strong foundation for tackling advanced topics in thermodynamics and statistical mechanics.
- **Clear Explanation:** Concepts are explained clearly and concisely, supported by illustrative examples and diagrams to facilitate understanding.
- **Practice-oriented Approach:** Ample practice questions, problem-solving exercises, and conceptual quizzes enable students to apply their knowledge and reinforce their understanding of the Kinetic Theory of Gases.



KEY - FEATURES

- **Particle Model:** The Kinetic Theory of Gases proposes that gases consist of a large number of tiny particles (atoms or molecules) in constant motion. These particles move randomly and collide with each other and the walls of the container.
- **Assumptions:** The theory is based on several assumptions, including that gas particles are point masses, their volume is negligible compared to the volume of the container, and there are no intermolecular forces except during collisions.
- **Kinetic Energy and Temperature:** Gas particles possess kinetic energy due to their motion, and the average kinetic energy is directly proportional to the absolute temperature of the gas (as described by the equation $KE = (3/2)kT$, where KE is the kinetic energy, k is Boltzmann's constant, and T is the temperature in Kelvin).
- **Pressure and Collision Model:** The pressure exerted by a gas on the walls of its container arises from the numerous collisions between gas particles and the container walls. The force exerted by each collision contributes to the overall pressure exerted by the gas.
- **Ideal Gas Law:** The behavior of gases under various conditions is described by the Ideal Gas Law, which relates the pressure, volume, temperature, and number of moles of a gas through the equation $PV = nRT$ (where P is pressure, V is volume, n is the number of moles, R is the gas constant, and T is the temperature in Kelvin).
- **Deviation from Ideal Behavior:** While the behavior of most gases can be described reasonably well by the Ideal Gas Law, real gases may deviate from ideal behavior under certain conditions, particularly at high pressures or low temperatures.



KINETIC THEORY OF GASES



REVIEW OF BASIC CONCEPTS

1. Assumptions Regarding the Model of an Ideal Gas

- A gas consists of very tiny particles called molecules which are identical in all respects.
- The molecules are in random motion and obey Newton's laws of motion.
- The total number of molecules is very large.
- The volume of molecules is negligibly small compared to the volume occupied by the gas.
- The molecules do not exert any appreciable force on each other except during collision.
- The collisions of molecules between themselves and with the walls of the container are perfectly elastic. The collisions are of negligible duration.

2. Pressure Exerted by an Ideal Gas

The pressure of a gas in a container is a result of the continuous bombardment of the gas molecules against the walls of the container and is given by

$$P = \frac{1}{3} \frac{mnv_{\text{rms}}^2}{V} = \frac{1}{3} \frac{Mv_{\text{rms}}^2}{V} = \frac{1}{3} \rho v_{\text{rms}}^2 = \frac{2}{3} \frac{U}{V}$$

where m = mass of each molecule, n = number of molecules in the container, v_{rms} = root mean square speed of molecules, V = volume of container, M = mass of gas in the container, ρ = density of the gas and U = internal energy of the gas.

3. Root Mean Square Speed

The root mean square (rms) speed is defined as

$$v_{\text{rms}} = \left[\frac{1}{n} (v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2) \right]^{1/2}$$

where $v_1, v_2, v_3, \dots, v_n$ are the speeds of the molecules 1, 2, 3, \dots, n respectively. In terms of P and ρ , v_{rms} is given by

$$v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$$

where m is the mass of each molecule, T is the absolute temperature of the gas and k is a constant called *Boltzmann's constant*. Its value is

$$k = 1.38 \times 10^{-23} \text{ J K}^{-1} \text{ per molecule}$$

R is the universal gas constant and its value is

$$R = 8.315 \text{ J K}^{-1} \text{ mol}^{-1}$$

4. Mean Translational Kinetic Energy

The mean translational kinetic energy of a molecule of a gas is given by

$$E = \frac{1}{2} mv_{\text{rms}}^2$$

In terms of E , the pressure of the gas is given by

$$P = \frac{1}{3} \rho v_{\text{rms}}^2 = \frac{2}{3} \frac{nE}{V} = \frac{2}{3} \frac{U}{V}$$

where $U = nE$ is the total translational kinetic energy of all the n molecules of the gas. It is also called the internal energy of the gas.

5. Avogadro's Law

Equal volumes of all gases at the same temperature and pressure contain an equal number of molecules. This is Avogadro's law. The number of molecules in one mole of any gas is 6.0255×10^{23} which is called the *Avogadro number*. The amount of a substance which contains as many elementary units as there are in 0.012 kg (or 12 g) or carbon-12 is called a *mole*. In other words, a mole of substance is the number of grams equal to its molecular mass in grams.

Thus, the mass of 1 mole of carbon is 12 g or 0.012 kg and that of 1 mole of oxygen is 32 g (0.032 kg).

6. Equation of State of an Ideal Gas

The relationship between pressure P , volume V and absolute temperature T of an ideal gas is called the equation of state. For n moles of a gas, this relation is

$$PV = nRT$$

where R is the molar gas constant.

From Avogadro's law, it follows that one mole of all gases, at the same temperature and pressure, occupies equal volume. Experiments confirm that one mole of any gas occupies 22.4 litres at STP. Consequently, for one mole the ratio $\frac{PV}{T}$ is constant for all gases. This constant is the molar gas constant and can be evaluated as follows:

At STP, $V = 22.4 \text{ litre} = 22.4 \times 10^{-3} \text{ m}^3$
 $P = 0.76 \text{ m Hg} = 1.013 \times 10^5 \text{ Nm}^{-2}$
 $T = 273 \text{ K}$

$$\therefore R = \frac{PV}{T} = \frac{1.013 \times 10^5 \times 22.4 \times 10^{-3}}{273}$$

$$= 8.315 \text{ J mol}^{-1} \text{ K}^{-1}$$

7. Van der Waal's Equation of State

According to Van der Waal, the true pressure exerted by a gas is greater than P by an amount a/V^2 (where a is a constant) due to attractive forces between molecules and the true volume of the gas is less than V by an amount b (where b is another constant) because molecules themselves occupy a finite space. The Van der Waal's equation of state is

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT$$

At high pressures, when the molecules are too many and too close together, the correction factors a and b both become important. But at low pressures, when they are not too many and not too close together, a gas behaves like an ideal gas and obeys the equation $PV = nRT$.

8. Degrees of Freedom and Equipartition of Energy

The total number of coordinates or independent quantities required to completely specify the position or configuration of a dynamical system is called the degrees of freedom of the system.

The molecules of a monoatomic gas consist of single atoms. Therefore, the molecules of a monoatomic gas have

three degrees of freedom corresponding to translational motion. The molecules of a diatomic gas have five degrees of freedom—three corresponding to translational motion and two for rotational motion. A polyatomic molecule has six degrees of freedom including one of vibrational motion.

The law of equipartition of energy is stated as follows. In any dynamical system with a uniform absolute temperature T , the total energy is distributed equally among all the degrees of freedom and the average energy per degree of freedom per molecule equals $\frac{1}{2} kT$, where $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

If the molecules of a gas have f degrees of freedom, then kinetic energy per molecule = $f \times \frac{1}{2} kT$. Therefore, kinetic energy per mole is

$$U = Nf \times \frac{1}{2} kT = \frac{f}{2} RT$$

Now $C_v = \frac{\Delta U}{\Delta T} = \frac{f}{2} R$

and $C_p = C_v + R = \frac{f}{2} R + R = \left(\frac{f}{2} + 1\right) R$

$$\therefore \gamma = \frac{C_p}{C_v} = \frac{\left(\frac{f}{2} + 1\right)R}{\frac{f}{2}R} = 1 + \frac{2}{f}$$

Thus $\gamma = 1 + \frac{2}{3} = \frac{5}{3}$ for a monoatomic gas = $1 + \frac{2}{5} = \frac{7}{5}$

for a diatomic gas = $1 + \frac{2}{6} = \frac{4}{3}$ for a triatomic or polyatomic gas

⊙ **EXAMPLE 1** Calculate the root mean square speed of the molecules of hydrogen gas at S.T.P. Density of hydrogen at S.T.P. is $9 \times 10^{-2} \text{ kg m}^{-3}$.

⊙ **SOLUTION** At S.T.P., pressure $P = 1.01 \times 10^5 \text{ Pa}$ and density $\rho = 9 \times 10^{-2} \text{ kg m}^{-3}$.

$$v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{9 \times 10^{-2}}}$$

$$= 1840 \text{ ms}^{-1}$$

⊙ **EXAMPLE 2** Calculate the temperature (in kelvin) at which the root mean square speed of a gas molecule is half its value at 0°C .

⊙ **SOLUTION** $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$

$$\therefore \frac{v'_{\text{rms}}}{v_{\text{rms}}} = \sqrt{\frac{T'}{T}} = \sqrt{\frac{T'}{273}}$$

$$\Rightarrow \frac{1}{2} = \sqrt{\frac{T'}{273}} \Rightarrow T' = 68.25 \text{ K}$$

⊙ **EXAMPLE 3** Find the mean translational kinetic energy of an oxygen molecule at 0°C. Given Avogadro number $N = 6.03 \times 10^{23}$ per mole and $R = 8.3 \text{ JK}^{-1} \text{ mol}^{-1}$.

⊙ **SOLUTION** $E = \frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} kT = \frac{3}{2} \frac{RT}{N}$

$$\therefore E = \frac{3 \times 8.3 \times 273}{2 \times 6.03 \times 10^{23}} = 5.64 \times 10^{-21} \text{ J}$$

⊙ **EXAMPLE 4** Calculate the mean translational kinetic energy of 1 mole of hydrogen at S.T.P. Density of hydrogen at S.T.P. is 0.09 kg m^{-3} .

⊙ **SOLUTION** $v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{0.09}}$
 $= 1.84 \times 10^3 \text{ ms}^{-1}$

Mass of 1 mole is

$$m = 22.4 \times 10^{-3} \text{ m}^3 \times 0.09 \text{ kg m}^{-3}$$

$$= 2.016 \times 10^{-3} \text{ kg}$$

$$\therefore \text{K.E.} = \frac{1}{2} m v_{\text{rms}}^2$$

$$= \frac{1}{2} \times (2.016 \times 10^{-3}) \times (1.84 \times 10^3)^2$$

$$= 3.41 \times 10^3 \text{ J}$$

⊙ **EXAMPLE 5** Calculate (a) the average translational kinetic energy of the molecules of an ideal gas at 0°C and at 100°C and (b) the energy per mole of the gas at 0°C and 100°C. Given Avogadro's number $N = 6.02 \times 10^{23}$ and Boltzmann's constant $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$.

⊙ **SOLUTION**

(a) Average translational K.E. of a molecule of an ideal gas is

$$E = \frac{3}{2} kT,$$

where $T =$ temperature in kelvin

At $T = 0^\circ\text{C} = 273 \text{ K}$,

$$E = \frac{3}{2} \times (1.38 \times 10^{-23}) \times 273$$

$$= 5.65 \times 10^{-21} \text{ J}$$

At $T = 100^\circ\text{C} = 373 \text{ K}$,

$$E = \frac{3}{2} \times (1.38 \times 10^{-23}) \times 373$$

$$= 7.72 \times 10^{-21} \text{ J}$$

(b) Number of molecules in 1 mole of a gas is $N = 6.02 \times 10^{23}$

$$\therefore \text{K.E. of 1 mole at } 273 \text{ K} = (5.65 \times 10^{-21}) \times (6.02 \times 10^{23}) = 3.40 \text{ J}$$

$$\text{K.E. of 1 mole at } 373 \text{ K} = (7.72 \times 10^{-21}) \times (6.02 \times 10^{23}) = 4.65 \text{ J}$$

⊙ **EXAMPLE 6** The speed of sound in a gas at S.T.P. is 330 ms^{-1} and the density of the gas is 1.3 kg m^{-3} . Find the number of degrees of freedom of a molecule of the gas.

⊙ **SOLUTION** $v = \sqrt{\frac{\gamma P}{\rho}}$

$$\therefore \gamma = \frac{v^2 \rho}{P} = \frac{(330)^2 \times 1.3}{1.01 \times 10^5} = 1.4$$

If f is the number of degrees of freedom, then

$$f = \frac{2}{\gamma - 1} = \frac{2}{(1.4 - 1)} = 5$$

⊙ **EXAMPLE 7** The volume of 2 moles of a diatomic ideal gas at 300 K is doubled keeping its pressure constant. Find the change in the internal energy of the gas. Given $R = 8.3 \text{ JK}^{-1} \text{ mol}^{-1}$.

⊙ **SOLUTION** According to kinetic theory, there are no internal forces of interaction between the molecules of an ideal gas. This implies that the potential energy is zero. Hence, for an ideal gas, the internal energy is only due to kinetic energy of the molecules.

For n moles of an ideal gas at absolute temperature T , the internal energy is

$$U = \frac{f}{2} nRT$$

where f is the number of degrees of freedom. For a diatomic gas $f = 5$. Therefore

$$U = \frac{5}{2} nRT$$

Since pressure is kept constant, $V \propto T$ (Charles' law), i.e.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\Rightarrow T_2 = \left(\frac{V_2}{V_1}\right) T_1 = 2T_1 = 600 \text{ K} \quad (\because V_2 = 2V_1)$$

$$\therefore \Delta U = U_2 - U_1 = \frac{5}{2} nR(T_2 - T_1)$$

$$= \frac{5}{2} \times 2 \times 8.3 \times (600 - 300)$$

$$= 1.245 \times 10^4 \text{ J}$$

$$\Rightarrow \frac{1}{2} = \sqrt{\frac{T'}{273}} \Rightarrow T' = 68.25 \text{ K}$$

⊙ **EXAMPLE 3** Find the mean translational kinetic energy of an oxygen molecule at 0°C. Given Avogadro number $N = 6.03 \times 10^{23}$ per mole and $R = 8.3 \text{ JK}^{-1} \text{ mol}^{-1}$.

⊙ **SOLUTION** $E = \frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} kT = \frac{3}{2} \frac{RT}{N}$

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⊙ **SOLUTION**

(a) Average translational K.E. of a molecule of an ideal gas is

$$E = \frac{3}{2} kT,$$

where $T =$ temperature in kelvin

At $T = 0^\circ\text{C} = 273 \text{ K}$,

$$E = \frac{3}{2} \times (1.38 \times 10^{-23}) \times 273$$

$$= 5.65 \times 10^{-21} \text{ J}$$

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$$E = \frac{3}{2} \times (1.38 \times 10^{-23}) \times 373$$

$$= 7.72 \times 10^{-21} \text{ J}$$

(b) Number of molecules in 1 mole of a gas is $N = 6.02 \times 10^{23}$

$$\therefore \text{K.E. of 1 mole at } 273 \text{ K} = (5.65 \times 10^{-21}) \times (6.02 \times 10^{23}) = 3.40 \text{ J}$$

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⊙ **SOLUTION** According to kinetic theory, there are no internal forces of interaction between the molecules of an ideal gas. This implies that the potential energy is zero. Hence, for an ideal gas, the internal energy is only due to kinetic energy of the molecules.

For n moles of an ideal gas at absolute temperature T , the internal energy is

$$U = \frac{f}{2} nRT$$

where f is the number of degrees of freedom. For a diatomic gas $f = 5$. Therefore

$$U = \frac{5}{2} nRT$$

Since pressure is kept constant, $V \propto T$ (Charles' law), i.e.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\Rightarrow T_2 = \left(\frac{V_2}{V_1}\right) T_1 = 2T_1 = 600 \text{ K} \quad (\because V_2 = 2V_1)$$

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$$= \frac{5}{2} \times 2 \times 8.3 \times (600 - 300)$$

$$= 1.245 \times 10^4 \text{ J}$$

Alternative Method

$$\Delta U = nC_v\Delta T$$

For diatomic gas, $C_v = \frac{5R}{2}$. Therefore

$$\begin{aligned}\Delta U &= 2 \times \frac{5R}{2} \times (600 - 300) \\ &= 1.245 \times 10^4 \text{ J}\end{aligned}$$

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SECTION

Multiple Choice Questions with One Correct Choice

- The root mean square speed of gas molecules
 - is the same for all gases at the same temperature
 - depends on the mass of the gas molecule and its temperature
 - is independent of the density and pressure of the gas
 - depends only on the temperature and volume of the gas
- The relation between volume V , pressure P and absolute temperature T of an ideal gas is $PV = xT$ where x is a constant. The value of x depends upon
 - the mass of the gas molecule
 - the average kinetic energy of the gas molecules
 - P , V and T
 - the number of gas molecules in volume V
- The equation of state for n moles of an ideal gas is $PV = nRT$, where R is a constant. The SI unit for R is
 - J K^{-1} per molecule
 - $\text{J kg}^{-1} \text{K}^{-1}$
 - $\text{J K}^{-1} \text{mol}^{-1}$
 - $\text{J K}^{-1} \text{g}^{-1}$
- Four molecules of a gas have speeds 1, 2, 3 and 4 km s^{-1} . The value of the root mean square speed of the gas molecule is
 - $\frac{1}{2}\sqrt{15} \text{ km s}^{-1}$
 - $\frac{1}{2}\sqrt{10} \text{ km s}^{-1}$
 - 2.5 km s^{-1}
 - $\sqrt{15/2} \text{ km s}^{-1}$
- The average kinetic energy of a molecule of a gas at absolute temperature T is proportional to
 - $1/T$
 - \sqrt{T}
 - T
 - T^2
- The root mean square speed of the molecules of a gas at absolute temperature T is proportional to
 - $1/T$
 - \sqrt{T}
 - T
 - T^2
- The following four gases are at the same temperature. In which gas do the molecules have the maximum root mean square speed?
 - Hydrogen
 - Oxygen
 - Nitrogen
 - Carbon dioxide
- If k is the Boltzmann constant, the average kinetic energy of a gas molecule at absolute temperature T is
 - $kT/2$
 - $3kT/4$
 - kT
 - $3kT/2$
- E_0 and E_h respectively represent the average kinetic energy of a molecule of oxygen and hydrogen. If the two gases are at the same temperature, which of the following statements is true?
 - $E_0 > E_h$
 - $E_0 = E_h$
 - $E_0 < E_h$
 - Nothing can be said about the magnitude of E_0 and E_h as the information given is insufficient.
- Choose the correct statement from the following:
 - The average kinetic energy of a molecule of any gas is the same at the same temperature.
 - The average kinetic energy of a molecule of a gas is independent of its temperature.
 - The average kinetic energy of 1 g of any gas is the same at the same temperature.
 - The average kinetic energy of 1 g of a gas is independent of its temperature.
- The root mean square speed of the molecules of an enclosed gas is v . What will be the root mean square speed if the pressure is doubled, the temperature remaining the same?
 - $v/2$
 - v
 - $2v$
 - $4v$
- The mass of an oxygen molecule is about 16 times that of a hydrogen molecule. At room temperature the rms speed of oxygen molecules is v . The rms speed

of the hydrogen molecules at the same temperature will be

- (a) $v/16$ (b) $v/4$
 (c) $4v$ (d) $16v$
13. The average kinetic energy of hydrogen molecules at 300 K is E . At the same temperature, the average kinetic energy of oxygen molecules will be
 (a) $E/16$ (b) $E/4$
 (c) E (d) $4E$
14. Choose the only correct statement from the following:
 (a) the pressure of a gas is equal to the total kinetic energy of the molecules in a unit volume of the gas.
 (b) the product of pressure and volume of a gas is always constant.
 (c) the average kinetic energy of molecules of a gas is proportional to its absolute temperature.
 (d) the average kinetic energy of molecules of a gas is proportional to the square root of its absolute temperature.
15. If a gas has f degrees of freedom, the ratio C_p/C_v of the gas is
 (a) $\frac{1+f}{2}$ (b) $1+\frac{f}{2}$
 (c) $\frac{1}{2}+f$ (d) $1+\frac{2}{f}$
16. At room temperature (27°C) the rms speed of the molecules of a certain diatomic gas is found to be 1920 ms^{-1} . The gas is
 (a) H_2 (b) F_2
 (c) O_2 (d) Cl_2
17. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K, the root mean square speed of the gas molecules is v , then at 480 K it will be
 (a) $4v$ (b) $2v$
 (c) $\frac{v}{2}$ (d) $\frac{v}{4}$
18. Three closed vessels A , B and C are at the same temperature. Vessel A contains only O_2 , B only N_2 and C a mixture of equal quantities of O_2 and N_2 . If the average speed of O_2 molecules in vessel A is v_1 , that of N_2 molecules in vessel B is v_2 , the average speed of O_2 molecules in vessel C is
 (a) $\frac{1}{2}(v_1+v_2)$ (b) v_1

(c) $\sqrt{v_1 v_2}$ (d) $\sqrt{\frac{3kT}{M}}$

19. The average translational energy and the rms speed of molecules of a sample of oxygen gas at 300 K are $6.21 \times 10^{-21}\text{ J}$ and 484 ms^{-1} respectively. The corresponding values at 600 K are nearly (assuming ideal gas behaviour)
 (a) $12.42 \times 10^{-21}\text{ J}$, 968 ms^{-1}
 (b) $8.78 \times 10^{-21}\text{ J}$, 684 ms^{-1}
 (c) $6.21 \times 10^{-21}\text{ J}$, 968 ms^{-1}
 (d) $12.42 \times 10^{-21}\text{ J}$, 684 ms^{-1}
20. A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T . The pressure of the gas is P . An identical vessel containing one mole of He gas (molar mass 4) at a temperature $2T$ has a pressure of
 (a) $\frac{P}{8}$ (b) P
 (c) $2P$ (d) $8P$
21. A vessel contains a mixture of 1 mole of oxygen and two moles of nitrogen at 300 K. The ratio of the rotational kinetic energy per O_2 molecule to that per N_2 molecule is
 (a) 1 : 1 (b) 1 : 2
 (c) 2 : 1 (d) depends on the moment of inertia of the two molecules.
22. Two thermally insulated vessels 1 and 2 are filled with air at temperature (T_1, T_2), volume (V_1, V_2) and pressure (P_1, P_2) respectively. If the valve joining the two vessels is opened, the temperature inside the vessel at equilibrium will be
 (a) $T_1 + T_2$ (b) $(T_1 + T_2)/2$
 (c) $T_1 T_2 (P_1 V_1 + P_2 V_2) / (P_1 V_1 T_2 + P_2 V_2 T_1)$
 (d) $T_1 T_2 (P_1 V_1 + P_2 V_2) / (P_1 V_1 T_1 + P_2 V_2 T_2)$
23. Which of the following phenomena gives evidence of the molecular structure of matter?
 (a) Brownian movement
 (b) Diffusion
 (c) Evaporation
 (d) All the above
24. Choose the correct statement. When the temperature of a gas is increased
 (a) the kinetic energy of its molecules increases
 (b) the potential energy of its molecules increases

- (c) the potential energy decreases and the kinetic energy increases; the total energy remaining unchanged
 (d) the potential energy increases and the kinetic energy decreases; the total energy remaining unchanged
25. If the temperature of a gas is increased from 27°C to 927°C, the root mean square speed of its molecules
 (a) becomes half (b) becomes doubled
 (c) becomes four times (d) remains unchanged
26. At what temperature will oxygen molecules have the same root mean square speed as hydrogen molecules at 200 K?
 (a) 527°C (b) 1327°C
 (c) 2127°C (d) 2927°C
27. At what temperature would the root mean square speed of a gas molecule have twice its value at 100°C?
 (a) 200°C (b) 273°C
 (c) 1219°C (d) 1492°C
28. An enclosure of volume V contains a mixture of 8 g of oxygen, 14 g of nitrogen and 22 g of carbon dioxide at absolute temperature T . The pressure of the mixture of gases is (R is universal gas constant)
 (a) $\frac{RT}{V}$ (b) $\frac{3RT}{2V}$
 (c) $\frac{5RT}{4V}$ (d) $\frac{7RT}{5V}$
29. At what absolute temperature T is the root mean square speed of a hydrogen molecule equal to its escape velocity from the surface of the moon? The radius of moon is R , g is the acceleration due to gravity on moon's surface, m is the mass of a hydrogen molecule and k is the Boltzmann constant
 (a) $\frac{mgR}{2k}$ (b) $\frac{2mgR}{k}$
 (c) $\frac{3mgR}{2k}$ (d) $\frac{2mgR}{3k}$
30. Two ideal gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy in this process. If n_1 and n_2 are the respective number of molecules of the gases, the temperature of the mixture will be
 (a) $\frac{n_1T_1 + n_2T_2}{n_1 + n_2}$ (b) $\frac{n_2T_1 + n_1T_2}{n_1 + n_2}$
 (c) $T_1 + \frac{n_2}{n_1}T_2$ (d) $T_2 + \frac{n_1}{n_2}T_1$
31. An insulated box containing a diatomic gas of molar mass M is moving with a velocity v . The box is suddenly stopped. The resulting change in temperature is (R is the gas constant)
 (a) $\frac{Mv^2}{2R}$ (b) $\frac{Mv^2}{3R}$
 (c) $\frac{Mv^2}{5R}$ (d) $\frac{2Mv^2}{5R}$
32. 0.014 kg of nitrogen is enclosed in a vessel at a temperature of 27 °C. The amount of heat energy to be supplied to the gas to double the rms speed of its molecules is approximately equal to
 (a) 6350 J (b) 7350 J
 (c) 8350 J (d) 9350 J
33. A vessel of volume V contains an ideal gas at absolute temperature T and pressure P . The gas is allowed to leak till its pressure falls to P' . Assuming that the temperature remains constant during leakage, the number of moles of the gas that have leaked is
 (a) $\frac{V}{RT} (P + P')$ (b) $\frac{V}{2RT} (P + P')$
 (c) $\frac{V}{RT} (P - P')$ (d) $\frac{V}{2RT} (P - P')$



Answers

- | | | | |
|---------|---------|---------|---------|
| 1. (b) | 2. (d) | 3. (c) | 4. (d) |
| 5. (c) | 6. (b) | 7. (a) | 8. (d) |
| 9. (b) | 10. (a) | 11. (b) | 12. (c) |
| 13. (c) | 14. (c) | 15. (d) | 16. (a) |
| 17. (b) | 18. (b) | 19. (d) | 20. (c) |
| 21. (a) | 22. (c) | 23. (d) | 24. (a) |
| 25. (b) | 26. (d) | 27. (c) | 28. (c) |
| 29. (d) | 30. (a) | 31. (c) | 32. (d) |
| 33. (c) | | | |



Solutions

1. The root mean square speed of the molecules of a gas is given by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

where m is the mass of the molecule of the gas, T its absolute temperature and k is the Boltzmann's constant. Hence the correct choice is (b).

2. The correct choice is (d).
3. The product PV has dimensions of work and is expressed in joules (J), n is the number of moles and T is expressed in kelvin (K). Thus, dimensions of
- $$R = \frac{\text{dimension of } PV}{\text{dimension of } (n \times T)} = \frac{\text{J}}{\text{mol K}} = \text{JK}^{-1} \text{ mol}^{-1}.$$

Hence the correct choice is (c).

4. The rms speed = $\left[\frac{1^2 + 2^2 + 3^2 + 4^2}{4} \right]^{1/2}$
 $= \sqrt{\frac{15}{2}} \text{ kms}^{-1}.$

Hence the correct choice is (d).

5. The average kinetic energy of a molecule of a gas at absolute temperature T is given by

$$E = \frac{3}{2} kT$$

Hence the correct choice is (c).

6. The root mean square speed of the molecules of a gas is given by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

Hence the correct choice is (b).

7. The rms speed is the maximum for the gas which is the lightest. Hence the correct choice is (a).
8. The correct choice is (d).
9. The average kinetic energy of molecule = $\frac{1}{2} kT$

which is independent of the molecular mass. Hence the correct choice is (b).

10. The correct choice is (a).
11. The rms speed = $\sqrt{3kT/m}$ which is independent of pressure. Hence the correct choice is (b).
12. $v = \sqrt{3kT/m}$. Therefore, $v_0 = \sqrt{3kT/m_0}$ and $v_h = \sqrt{3kT/m_h}$.

$$\text{Thus } \frac{v_0}{v_h} = \sqrt{\frac{m_h}{m_0}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

$$\text{or } v_h = 4 v_0.$$

Hence the correct choice is (c).

13. The correct choice is (c) as explained above.

14. The correct choice is (c)
15. We know that $C_v = \text{no. of degrees of freedom} \times \text{molar gas constant}/2$

$$\text{or } C_v = fR/2. \text{ Also } C_p - C_v = R.$$

Therefore

$$\frac{C_p}{C_v} = \frac{R}{C_v} + 1 = \frac{2R}{fR} + 1 = \frac{2}{f} + 1$$

Hence the correct choice is (d).

16. We know that $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ which gives

$$M = \frac{3RT}{v_{\text{rms}}^2} = \frac{3 \times 8.3 \times 300}{(1920)^2} = 2 \times 10^{-3} \text{ kg} = 2 \text{ g}$$

Since $M = 2$, the gas is hydrogen.

17. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ or $v_{\text{rms}} \propto \sqrt{T}$. Hence the correct choice is (b).

18. For a given M , $v_{\text{av}} \propto \sqrt{T}$ only. Therefore, the correct choice is v_1 as the temperatures of vessels A and C are the same.

19. $E \propto T$ and $v_{\text{rms}} \propto \sqrt{T}$. Hence at 600K,

$$E = 6.21 \times 10^{-21} \times \frac{600}{300}$$

$$= 12.42 \times 10^{-21} \text{ J}$$

$$\text{and } v_{\text{rms}} = 484 \times \sqrt{\frac{600}{300}} = 684 \text{ ms}^{-1}$$

which is choice (d).

20. For a gas, $PV = nRT$. Hence

$$(P)_{\text{O}_2} = \frac{(1 \text{ mole})RT}{V}$$

$$\text{and } (P)_{\text{He}} = \frac{(1 \text{ mole})R(2T)}{V}$$

$$\therefore \frac{(P)_{\text{He}}}{(P)_{\text{O}_2}} = 2 \text{ or } (P)_{\text{He}} = 2 (P)_{\text{O}_2}$$

21. Since both the gases are diatomic, each has two degrees of freedom associated with rotational motion. According to the law of equipartition of energy, the rotational kinetic energy per degree of freedom is $(1/2)kT$. Since the temperatures of the two gases are equal, their rotational kinetic energies will be equal. Hence the correct choice is (a).

22. According to the kinetic theory, the average kinetic energy (KE) per molecule of a gas = $\frac{3}{2} kT$. Let n_1 and n_2 be the number of moles of air in vessels 1 and 2 respectively.

Before mixing, the total KE of molecules in the two vessels is

$$E_1 = \frac{3}{2} n_1 k T_1 + \frac{3}{2} n_2 k T_2$$

$$= \frac{3}{2} k (n_1 T_1 + n_2 T_2)$$

After mixing, the total KE of molecules is

$$E_2 = \frac{3}{2} (n_1 + n_2) k T$$

where T is the temperature when equilibrium is established. Since there is no loss of energy (because the vessels are insulated), $E_2 = E_1$ or

$$\frac{3}{2} (n_1 + n_2) k T = \frac{3}{2} k (n_1 T_1 + n_2 T_2)$$

or
$$T = \frac{n_1 T_1 + n_2 T_2}{(n_1 + n_2)} \quad (1)$$

Now $P_1 V_1 = n_1 R T_1$ and $P_2 V_2 = n_2 R T_2$ which give

$$n_1 = \frac{P_1 V_1}{R T_1} \text{ and } n_2 = \frac{P_2 V_2}{R T_2}$$

Using these in Eq. (1) and simplifying, we get

$$T = \frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{(P_1 V_1 T_2 + P_2 V_2 T_1)}$$

23. The correct choice is (d)
 24. The correct choice is (a).
 25. The root mean square speed is given by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

i.e. $v_{\text{rms}} \propto T$. Initial temperature $T_1 = 27 + 273 = 300$ K. Final temperature $T_2 = 927 + 273 = 1200$ K. Since temperature is increased by 4 times, the speed is doubled. Hence the correct choice is (b).

26. For oxygen:

$$v_{\text{rms}} = \sqrt{\frac{3kT_0}{m_0}}$$

For hydrogen:

$$v_{\text{rms}} = \sqrt{\frac{3kT_h}{m_h}}$$

The value of v_{rms} will be the same if

$$\frac{T_0}{m_0} = \frac{T_h}{m_h}$$

or
$$T_0 = \frac{m_0}{m_h} \times T_h = 16 \times 200$$

$$= 3200 \text{ K} = 2927^\circ\text{C}$$

Hence the correct choice is (d).

27. Use $v_{\text{rms}} \propto T$. The correct choice is (c).
 28. The pressure exerted by a gas is given by

$$P = \frac{nRT}{V}$$

$$= \frac{\text{mass}}{\text{molecular weight}} \times \frac{RT}{V}$$

Pressure exerted by oxygen $P_1 = \frac{8}{32} \frac{RT}{V} = \frac{1}{4} \frac{RT}{V}$

Pressure exerted by oxygen $P_2 = \frac{14}{28} \frac{RT}{V} = \frac{1}{2} \frac{RT}{V}$

Pressure exerted by carbon dioxide

$$P_3 = \frac{22}{44} \frac{RT}{V} = \frac{1}{2} \frac{RT}{V}$$

From Dalton's law of partial pressures, the total pressure exerted by the mixture is given by

$$P = P_1 + P_2 + P_3$$

$$= \frac{1}{4} \frac{RT}{V} + \frac{1}{2} \frac{RT}{V} + \frac{1}{2} \frac{RT}{V}$$

$$= \frac{5RT}{4V}, \text{ which is choice (c).}$$

29. The root mean square speed is given by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

The escape velocity is given by $v_e = \sqrt{2gR}$

For $v_{\text{rms}} = v_e$, we require

$$\sqrt{\frac{3kT}{m}} = \sqrt{2gR} \text{ or } T = \frac{2mgR}{3k}, \text{ which is choice (d).}$$

30. Average kinetic energy per molecule of a perfect gas = $\frac{3}{2} kT$.

∴ Average KE of molecules of the first gas

$$= \frac{3}{2} n_1 kT_1$$

Average KE of molecules of the second gas

$$= \frac{3}{2} n_2 kT_2$$

∴ Total KE of the molecules of the two gases before they are mixed is

$$\begin{aligned} K &= \frac{3}{2} n_1 kT_1 + \frac{3}{2} n_2 kT_2 \\ &= \frac{3}{2} (n_1 T_1 + n_2 T_2)k \end{aligned} \quad (1)$$

If T is the temperature of the mixture, the kinetic energy of the molecules $(n_1 + n_2)$ in the mixture is

$$K' = \frac{3}{2} (n_1 + n_2)kT \quad (2)$$

Since there is no loss of energy $K = K'$. Equating (1) and (2) we get

$$n_1 T_1 + n_2 T_2 = (n_1 + n_2) T \text{ or } T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$$

Hence the correct choice is (a).

31. Let n be the number of moles of the gas in the box.

The kinetic energy of the gas = $n\left(\frac{1}{2} Mv^2\right)$. When

the box is suddenly stopped, this energy is used up in changing the internal energy, as a result of which the temperature of the gas rises. The change in internal energy is given by

$$\Delta U = nC_v \Delta T$$

For a diatomic gas $C_v = \frac{5}{2} R$.

$$\text{Hence } n \times \frac{5R}{2} \times \Delta T = n \times \frac{1}{2} Mv^2$$

$$\text{or } \Delta T = \frac{Mv^2}{5R}, \text{ which is choice (c)}$$

32. The root mean square speed is related to absolute temperature T as

$$c_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

For a given gas, m is fixed. Therefore, $c_{\text{rms}} \propto \sqrt{T}$. Hence in order to double the root mean square speed, the absolute temperature must be increased to four times the initial value. Initial temperature $T_1 = 273 + 27 = 300$ K. Therefore, final temperature $T_2 = 4 T_1 = 1200$ K.

Since the volume of the vessel is fixed, $\Delta V = 0$. Hence the heat energy supplied to the gas does no work on the gas; it only increases the internal energy of the molecules.

The increase in internal energy is

$$\Delta U = n C_v \Delta T$$

Since nitrogen is diatomic $C_v = \frac{5R}{2}$. The number of moles of nitrogen in the vessel is

$$n = \frac{\text{mass in kg}}{\text{molecular mass}} = \frac{0.014 \times 10^3}{28} = 0.5$$

$$\begin{aligned} \therefore \Delta U &= 0.5 \times \frac{5R}{2} \times (1200 - 300) \\ &= \frac{0.5 \times 5 \times 8.314 \times 900}{2} = 9353 \text{ J.} \end{aligned}$$

So the correct choice is (d)

33. Number of moles present initially is $n = \frac{PV}{RT}$. Let

n' be the number of moles of the gas that leaked till the pressure falls to P' . Since volume V of the vessel cannot change and temperature T remains constant during leakage, we have

$$n' = \frac{P'V}{RT}$$

∴ Number of moles that leaked is

$$\Delta n = n - n' = \frac{PV}{RT} - \frac{P'V}{RT} = \frac{V}{RT} (P - P')$$

So the correct choice is (c).

2

SECTION

Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage.

Passage I

Kinetic Theory of Gases

The molecules of a gas move in all directions with various speeds. The speeds of the molecules of a gas increase with rise in temperature. During its random motion, a fast molecule often strikes against the walls of the container of the gas. The collisions are assumed to be perfectly elastic, i.e. the molecule bounces back with the same speed with which it strikes the wall. Since the number of molecules is very large, billions of molecules strike against the walls of the container every second. These molecules exert a sizeable force on the wall. The force exerted per unit area is the pressure exerted by the gas on the walls. According to the kinetic theory, the pressure of a gas of density ρ at absolute temperature T is given by

$$P = \frac{1}{3} \rho v_{rms}^2$$

where v_{rms} is the root mean square speed of the gas molecule and is given by

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

where m is the mass of a molecule and k is Boltzmann constant.

- From the relation $v_{rms} = \sqrt{\frac{3kT}{m}}$, it follows that the constant k should be expressed in units of
 - newton per metre per kelvin
 - newton per kelvin
 - joule per kelvin
 - joule per kilogram per kelvin
- Choose the only correct statement from the following.
 - The pressure of a gas is equal to the total kinetic energy of its molecules per unit volume of the gas.
 - The product of pressure and volume of a gas is always constant.
 - The average kinetic energy of the molecules of a gas is proportional to its absolute temperature.
 - The root mean square speed of a molecule is proportional to the absolute temperature of the gas.

- If the temperature of a gas is increased from 27°C to 927°C , the root mean square speed of its molecules
 - becomes half
 - becomes doubled
 - becomes four times
 - remains unchanged
- The root mean square speed of oxygen gas molecule at $T = 320\text{ K}$ is very nearly equal to (the molar mass of oxygen is $M = 0.0320\text{ kg per mole}$ and gas constant $R = 8.31\text{ J mol}^{-1}\text{ K}^{-1}$)
 - 300 ms^{-1}
 - 500 ms^{-1}
 - 700 ms^{-1}
 - 900 ms^{-1}



Solutions

- Squaring we have

$$k = \frac{mv_{rms}^2}{3T} = \frac{\text{unit of energy}}{\text{unit of temperature}}$$

$$= \text{joule per kelvin}$$
- The correct choice is (c)
- The correct choice is (b) since v_{rms} is proportional to the square root of absolute temperature
- $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 320}{0.032}}$

$$= 499.3\text{ ms}^{-1}$$

Hence the correct choice is (b)

Questions 5 to 7 are based on the following passage.

Passage II

Van der Waals Equation of State

The equation of state $PV = nRT$ holds for an ideal gas. The behaviour of real gases shows departures from an ideal gas behaviour especially at high pressures. The model of an ideal gas is based on a number of assumptions. Van der Waals modified the ideal gas equation $PV = nRT$ by taking into account two of those assumptions which may not be valid. He argued that (i) the volume of the molecules may not be negligible compared to the volume V occupied by the gas and (ii) the attractive forces between the molecules may not be negligible. He said that pressure P in equation $PV = nRT$ is less than the true pressure by an amount p because of attractive forces between the molecules. According to him, the pressure 'defect' p is inversely proportional to the square of volume, i.e.

$$p \propto \frac{1}{V^2}$$

or
$$p = \frac{a}{V^2}$$

where a is constant depending on the nature of the gas.

Thus the true pressure of the gas is $P' = P + p = P + \frac{a}{V^2}$.

He further argued that V is not the true volume of the gas because the molecules themselves occupy a finite volume. According to him, the true volume of the gas is $V' = (V - b)$ where b is a factor depending on the actual volume of the molecules themselves. Thus Van der Waals' equation for real gases is $P' V' = nRT$, i.e.

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT$$

At high pressures, when the molecules are too many and too close together, the correction factors a and b become important.

- The dimensional formula for ab is
 - ML^2T^{-2}
 - ML^4T^{-2}
 - ML^6T^{-2}
 - ML^8T^{-2}
- The correction factors a and b depend upon
 - the pressure of the gas
 - the volume of the gas
 - the temperature of the gas
 - the nature of the gas
- The equation of state $PV = nRT$ holds if the gas has
 - low pressure and low density
 - low pressure and high density
 - high pressure and low density
 - high pressure and high density.



Solutions

- Dimensions of ab = dimensions of $PV^2 \times$ dimensions of V
 = dimensions of (PV^3)
 = $ML^{-1}T^{-2} \times (L^3)^3 = ML^8T^{-2}$

Hence the correct choice is (d).

- The correct choice is (d).
- The correct choice is (a).

Questions 8 and 9 are based on the following passage.

Passage III

Mean Free Path

During their random motion, the molecules of a gas often come close to each other. When the distance between two

molecules is comparable with the diameter of a molecule, the forces between them become very strong. As a result, their individual momenta before and after the encounter are different. When this happens a 'collision' is said to have occurred. The average distance a molecule travels before it suffers a collision with another molecule is called the mean free path (l_c), which can be estimated as follows. Suppose the average speed of a molecule of diameter d is \bar{v} . In one second, this molecule sweeps out a volume $\pi d^2 \bar{v}$. If it finds any other molecules in this volume, it will suffer collisions with them. If n is the number density (i.e. number per unit volume) of the molecules, then the number of molecules in this volume = $\pi d^2 \bar{v} n$. The number of collisions per second = $v_c = \pi d^2 \bar{v} n$. Therefore, the average time between two collisions (called collision period T_c) is

$$T_c = \frac{1}{v_c} = \frac{1}{\pi d^2 \bar{v} n}$$

Hence the mean free path (i.e. the average distance the molecule travels between two successive collisions) is

$$l_c = \bar{v} T_c = \frac{1}{\pi d^2 n}$$

For air at S.T.P. the value of $l_c \approx 3 \times 10^{-7}$ m.

- The mean free path of a molecule of a gas depends
 - only on its diameter (d)
 - only on the number density (n) of the molecules
 - on both d and n
 - neither on d nor on n .
- The average collision period in a gas
 - increases if the pressure is increased
 - decreases if the pressure is increased
 - increases if the temperature of the gas is increased.
 - decreases if the temperature of the gas is increased.



Solutions

- The mean free path is given by

$$l_c = \frac{1}{\pi d^2 n}$$

Hence the correct choice is (c).

- The collision period is given by

$$T_c = \frac{1}{\pi d^2 \bar{v} n}$$

If the pressure is increased, the volume of the gas decreases. Hence number density n increases. Therefore, T_c will decrease. If the temperature of the gas is increased, the average speed \bar{v} of the molecules increases. Hence T_c will decrease. Thus the correct choices are (b) and (d).

3
SECTION

Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside the container
- increases.
 - decreases.
 - remains the same.
 - decreases for some, while increases for others.

[2002]

2. At what temperature is the rms velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C?

- 80 K
- 73 K
- 3 K
- 20 K

[2002]

3. A vessel contains 1 mol of O₂ gas (molar mass 32) at a temperature T . The pressure of the gas is P . An identical vessel containing 1 mol of He gas (molar mass 4) at temperature $2T$ has pressure of

- $P/8$
- $2P$
- P
- $8P$

[2002]

4. One kilogram of diatomic gas is at a pressure of 8×10^4 N/m². The density of the gas is 4 kg/m³. What is the energy of the gas due to its thermal motion?

- 3×10^4 J
- 5×10^4 J
- 6×10^4 J
- 7×10^4 J

[2009]

5. Three perfect gases at absolute temperatures T_1 , T_2 and T_3 are mixed. The masses of molecules are m_1 , m_2 and m_3 and the number of molecules are n_1 , n_2 , n_3 , respectively. Assuming no loss of energy, the final temperature of the mixture is

- $\frac{(T_1 + T_2 + T_3)}{3}$
- $\frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$
- $\frac{n_1 T_1^2 + n_2 T_2^2 + n_3 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$
- $\frac{n_1^2 T_1^2 + n_2^2 T_2^2 + n_3^2 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$

[2011]

6. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ . It is moving with speed v and is suddenly brought to

rest. Assuming no heat is lost to the surroundings, its temperature increases by

- $\frac{(\gamma - 1)}{2(\gamma + 1)R} Mv^2 K$
- $\frac{(\gamma - 1)}{2\gamma} Mv^2 K$
- $\frac{\gamma Mv^2}{2R} K$
- $\frac{(\gamma - 1)}{2R} Mv^2 K$

[2011]

7. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4:3. The ratio of their densities is :

- 1 : 4
- 1 : 2
- 6 : 9
- 8 : 9

[2013]



Answers

- | | | | |
|--------|--------|--------|--------|
| 1. (c) | 2. (d) | 3. (b) | 4. (b) |
| 5. (b) | 6. (d) | 7. (d) | |



Solutions

1. The speed of the centre of mass of gas molecules in a container has no effect on the temperature of the gas molecules.

$$2. v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

It is given that v_{rms} is the same for both gases. Since R is a constant, it follows that

$$v_{\text{rms}} \propto \sqrt{\frac{T}{M}}$$

Thus $\frac{T}{M} = \text{constant}$

$$\therefore \frac{T}{2} = \frac{273 + 47}{32} \Rightarrow T = 20 \text{ K}$$

3. $PV = nRT$. Therefore

$$P_1 V = n_1 R T_1 \text{ and } P_2 V = n_2 R T_2$$

$$\therefore \frac{P_2}{P_1} = \frac{n_2}{n_1} \times \frac{T_2}{T_1}$$

$$\Rightarrow \frac{P_2}{P} = \frac{1}{1} \times \frac{2T}{T} = 2$$

$$\Rightarrow P_2 = 2P$$

4. Volume $V = \frac{\text{mass}}{\text{density}}$

$$= \frac{1 \text{ kg}}{4 \text{ kg m}^{-3}} = \frac{1}{4} \text{ m}^3$$

Energy of the gas due to thermal motion is

$$U = \gamma PV$$

$$= \frac{5}{2} \times (8 \times 10^4) \times \frac{1}{4} = 5 \times 10^4 \text{ J}$$

5. Let $T_1 < T_2 < T_3$. If T is the final temperature of the mixture, we have

$$n_1 c(T - T_1) + n_2 c(T - T_2) = n_3 c(T_3 - T)$$

$$\Rightarrow T = \frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$$

7. From energy conservation,

$$\frac{1}{2} M v^2 = n \times C_v \times \Delta T$$

$$\frac{1}{2} M v^2 = 1 \times \frac{R}{\gamma - 1} \times \Delta T$$

$$\Rightarrow \Delta T = \frac{M v^2 (\gamma - 1)}{2R} \text{ kelvin}$$

$$7. PV = n RT \Rightarrow \frac{PM}{\rho} = n RT \Rightarrow PM = n \rho RT$$

Since R and T are constant, then for given n

$$\frac{PM}{\rho} = \text{constant}$$

or $\frac{P_1 M_1}{\rho_1} = \frac{P_2 M_2}{\rho_2}$

$$\Rightarrow \frac{\rho_1}{\rho_2} = \left(\frac{P_1}{P_2} \right) \times \left(\frac{M_1}{M_2} \right) = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$$