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CBSE-XII

02

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MATHEMATICS
MOCK TEST 02
CBSE-XII

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. tan 150° = ? **[1]**

a) $\frac{-1}{\sqrt{3}}$ b) $\frac{1}{\sqrt{3}}$

c) $-\sqrt{3}$ d) $\sqrt{3}$

2. Let f(x) = (x - 1) Then, **[1]**

a) f (|x|) = f(x) b) $f(x^2) = (f(x)]^2$

c) None of these d) f (x + y) = f(x) f(y)

3. Two dice are thrown simultaneously. The probability of obtaining total score of seven is **[1]**

a) $\frac{6}{36}$ b) $\frac{8}{36}$

c) $\frac{7}{36}$ d) $\frac{5}{36}$

4. $\lim\limits_{x\to3} \frac{\sqrt{x^2+10}-\sqrt{19}}{x-3}$ is equal to **[1]**

a) 1 b) $\frac{6}{\sqrt{19}}$

c) $\frac{3}{\sqrt{19}}$ d) 0

5. The two lines ax + by = c and a′x + b′y = c′ are perpendicular if **[1]**

a) ab′ = ba′ b) aa′ + bb′ = 0

c) ab + a′b′ = 0 d) ab′ + ba′ = 0

6. The number of non-empty subsets of the set {1, 2, 3, 4} is: **[1]**

a) 14 b) 16

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c) 17 d) 15

7. Mark the correct answer for $\left(\frac{1-i}{1+i}\right)^2 = ?$ **[1]**

a) $\frac{1}{\sqrt{2}}$ b) -1

c) $\frac{-1}{2}$ d) 1

8. The range of the function f(x) = |x - 1| is **[1]**

a) R b) $(-\infty, 0)$

c) $(0, \infty)$ d) $[0, \infty)$

9. If x belongs to set of integers, A is the solution set of 2(x - 1) < 3x - 1 and B is the solution set of 4x - 3 $\leq$ 8 + x, find A $\cap$ B **[1]**

a) {0, 2, 4} b) {1, 2, 3}

c) {0, 1, 2} d) {0, 1, 2, 3}

10. At 3 : 40, the hour and minute hands of a clock are inclined at **[1]**

a) $\frac{7\pi^c}{18}$ b) $\frac{2\pi^c}{3}$

c) $\frac{3\pi^c}{18}$ d) $\frac{13\pi^c}{18}$

11. Let A = {x : x $\in$ R, x > 4} and B = {x $\in$ R : x < 5}. Then, A $\cap$ B = **[1]**

a) [4, 5) b) [4, 5]

c) (4, 5] d) (4, 5)

12. If in an infinite G.P., first term is equal to 10 times the sum of all successive terms, then its common ratio is **[1]**

a) $\frac{1}{9}$ b) $\frac{1}{11}$

c) $\frac{1}{10}$ d) $\frac{1}{20}$

13. $(\sqrt{5}+1)^4 + (\sqrt{5}-1)^4$ is **[1]**

a) an irrational number b) a negative real number

c) a rational number d) a negative integer

14. Solve the system of inequalities -2 $\leq$ 6x - 1 < 2 **[1]**

a) $-\frac{1}{6} \leq x < \frac{1}{2}$ b) $-\frac{1}{6} < x < \frac{3}{2}$

c) none of these d) $-\frac{1}{7} \leq x > \frac{1}{2}$

15. If A = {1, 3, 5, B} and B = {2, 4}, then **[1]**

a) {4} $\subset$ A b) None of these

c) B $\subset$ A d) 4 $\in$ A

16. The value of sec θ can **[1]**

a) can't lie between -1 and 1 b) can't be less than 1

c) can't be greater than 1 d) can't be equal to 1

17. Mark the correct answer for: $i^{326} = ?$ **[1]**

a) -i b) i

c) -1 d) 1

18. If ${}^{n}C_{18} = {}^{n}C_{12}$, then ${}^{32}C_{n}$ = ? **[1]**

a) None of these b) 248

c) 992 d) 496

19. **Assertion (A):** The expansion of $(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$. **[1]**

Reason (R): If x = -1, then the above expansion is zero.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** The mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is 3. **[1]**

Reason (R): The mean deviation about the mean for the data 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 is 8.5.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

Section B

21. If A = (1, 2, 3), B = {4}, C = {5}, then verify that $A \times (B - C) = (A \times B) - (A \times C)$. **[2]**

OR

Let A = {-2, -1, 0, 1, 2} and f: A $\rightarrow Z$ be given by f(x) = x^2 - 2x - 3 find pre image of 6. -3 and 5.

22. Evaluate: $\lim\limits_{x \rightarrow 0} \left(\frac{e^{3x} - e^{2x}}{x}\right)$. **[2]**

23. Find the eccentricity of an ellipse whose latus rectum is one half of its major axis. **[2]**

OR

Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum: $x^2 = -16y$

24. Write the set in roster form: C = {x : x is a two-digit number such that the sum of its digits is 9}. **[2]**

25. Find the angles between the pairs of straight lines x - 4y = 3 and 6x - y = 11. **[2]**

Section C

26. Let A = {1, 2} and B = {2, 4, 6}. Let f = {(x, y) : x $\in$ A, y $\in$ B and y > 2x +1}. Write f as a set of ordered pairs. Show that f is a relation but not a function from A to B. **[3]**

27. Solve systems of linear inequation: $\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}, x > 0$ **[3]**

28. Find the equation of the set of points P, the sum of whose distances from A(4, 0, 0) and B(-4, 0, 0) is equal to 10. **[3]**

OR

Show that the points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of a right angled isosceles triangle.

29. Find a, b and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729, 7290 and 30375 respectively. **[3]**

OR

Using g binomial theorem, expand $\left\{(x+y)^5 + (x-y)^5\right\}$ and hence find the value of $\{(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5\}$

30. If (a + ib) $= \frac{c+i}{c-i}$, where c is real, prove that $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2-1}$.. **[3]**

OR

Evaluate: $\sqrt{5+12i}$.

31. Using the properties of sets and their complements prove that (A ∪ B) - C = (A - C) ∪ (B - C) **[3]**

Section D

32. A fair coin is tossed four times, and a person win Rs. 1 for each head and lose Rs. 1.50 for each tail that turns up. Form the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts. **[5]**

33. Differentiate $\frac{\sin x}{x}$ from first principle. **[5]**

OR

Differentiate log sin x from first principles.

34. In an increasing GP, the sum of the first and last terms is 66, the product of the second and the last but one is 128 and the sum of the terms is 126. How many terms are there in this GP? **[5]**

35. $0 \leq x \leq \pi$ and x lies in the IInd quadrant such that sin x = $\frac{1}{4}$. Find the values of $\cos\frac{x}{2}$, $\sin\frac{x}{2}$ and $\tan\frac{x}{2}$. **[5]**

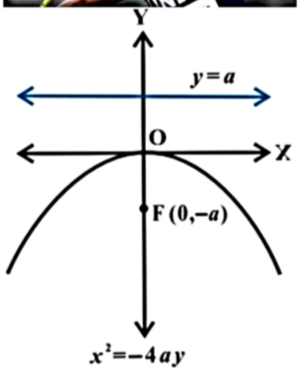
OR

Prove that: $\sin 20^o \sin 40^o \sin 80^o = \frac{\sqrt{3}}{8}$

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal for India at the Olympics.



(i) Name the shape of path followed by a javelin. If equation of such a curve is given by $x^2 = -16y$, then find the coordinates of foci.

(ii) Find the equation of directrix and length of latus rectum of parabola $x^2 = -16y$.

(iii) Find the equation of parabola with Vertex (0,0), passing through (5,2) and symmetric with respect to y-axis and also find equation of directrix.

OR

Find the equation of the parabola with focus (2, 0) and directrix x = -2 and also length of latus rectum.

37. **Read the text carefully and answer the questions:** **[4]**

Consider the data.

Class	Frequency
0-10	6
10-20	7

20-30	15
30-40	16
40-50	4
50-60	2

(i) Find the mean deviation about median.

(ii) Find the Median.

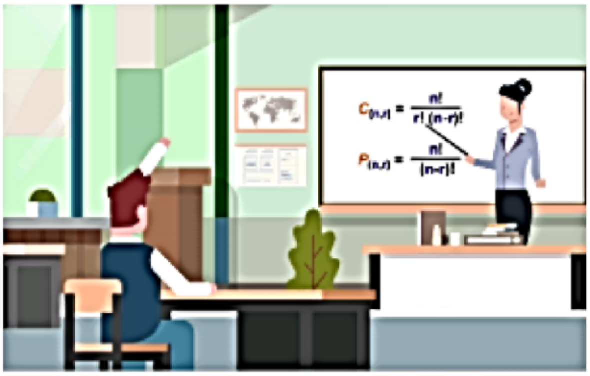
(iii) Write the formula to calculate the Mean deviation about median?

OR

Write the formula to calculate median?

38. **Read the text carefully and answer the questions:** **[4]**

During the math class, a teacher clears the concept of permutation and combination to the 11th standard students. After the class was over she asks the students some questions, one of the question was: how many numbers between 99 and 1000 (both excluding) can be formed such that:



(i) How many numbers between 99 and 1000 (both excluding) can be formed such that every digit is either 3 or 7.

(ii) How many numbers between 99 and 1000 (both excluding) can be formed such that without any restriction?

Solution

Section A

1. **(a)** $\frac{-1}{\sqrt{3}}$
   Explanation: tan 150° = tan (180° - 30°) = - tan 30° $= \frac{-1}{\sqrt{3}}$

2.
   (c) None of these
   Explanation: f(x) = x-1
   $f(x^2) = x^2-1$
   $[f(x)]^2 = (x-1)^2$
   $= x^2 + 1 - 2x$
   So, $f(x^2) \neq [f(x)]^2$
   f(x + y) = x + y - 1
   f(x)f(y) = (x - 1)(y - 1)
   So, f(x + y) ≠ f(x) f(y)
   f(|x|) = |x|-1 $\neq$ f(x)

3. **(a)** $\frac{6}{36}$
   Explanation: When two dices are thrown, there are (6 $\times$ 6) = 36 outcomes.
   The set of all these outcomes is the sample space given by
   S = (1, 1) , (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
   (2, 1) , (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
   (3, 1) , (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
   (4, 1) , (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
   (5, 1) , (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
   (6, 1) , (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)
   ∴ n(S) = 36
   Let E be the event of getting a total score of 7.
   Then E = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}
   ∴ n(E) = 6
   Hence, required probability = nEnS = $\frac{6}{36}$

4.
   (c) $\frac{3}{\sqrt{19}}$
   Explanation: Using L'Hospital,
   $\lim_{x \to 3} \frac{\frac{2x}{\sqrt[2]{x^2+10}}}{1}$
   Substituting x = 3 in $\frac{\frac{2x}{2\sqrt{x^2+10}}}{1}$
   We get $\frac{3}{\sqrt{19}}$

5.
   (b) aa′ + bb′ = 0
   Explanation: We know that Slope of the line ax + by = c is $\frac{-a}{b}$, and the slope of the line a′x + b′y = c′ is $\frac{-a'}{b'}$ The lines are perpendicular if $\tan\theta = \frac{3}{5-x}$ (1)
   $\frac{-a}{b}\frac{-a'}{b'} = -1$ or $aa' + bb' = 0$

6.
   (d) 15

Explanation: Total no. of subset including empty set = 2^n

So total subset = $2^4 = 16$

The no. of non empty set = 16 - 1 = 15

7.

(b) -1

Explanation: $\frac{(1-i)}{(1+i)} = \frac{(1-i)}{(1+i)} \times \frac{(1-i)}{(1-i)} = \frac{(1-i)^2}{(1-i^2)} = \frac{1+i^2-2i}{(1+1)} = \frac{1-1-2i}{2} = \frac{-2i}{2}$ = -i

$\Rightarrow \left(\frac{1-i}{1+i}\right)^2 = (-i)^2 = i^2 = -1$

8.

(d) $[0, \infty)$

Explanation: A modulus function always gives a positive value

$R(f) = [0, \infty)$

9.

(d) {0, 1, 2, 3}

Explanation: Given $2(x - 1) < 3x - 1$

$\Rightarrow 2x - 2 < 3x - 1$

$\Rightarrow 2x - 2 + 2 < 3x - 1 + 2$

$\Rightarrow 2x < 3x + 1$

$\Rightarrow 2x - 3x < 3x + 1 - 3x$

$\Rightarrow -x < +1$

$\Rightarrow x > -1$ but $x \in Z$

Hence A = {0, 1, 2, 3, 4, ...}

Now $4x - 3 \leq 8 + x$

$\Rightarrow 4x - 3 + 3 \leq 8 + x + 3$

$\Rightarrow 4x \leq 11 + x$

$\Rightarrow 4x - x \leq 11 + x - x$

$\Rightarrow 3x \leq 11$

$\Rightarrow \frac{3x}{3} \leq \frac{11}{3}$

$\Rightarrow x \leq \frac{11}{3}$

$\Rightarrow x \leq 3\frac{2}{3}$, but $x \in Z$

Therefore B = {....., -2, -1, 0, 1, 2, 3}

Hence $A \cap B$ = {0, 1, 2, 3}

10.

(d) $\frac{13\pi^c}{18}$

Explanation: We know, in clock 1 rotation gives 360^o

i.e. 60 minutes = 360^o and 12 hours = 360^o

So,1 minute = 6^o and 1 hour = 30^o

Now, For hour hand:

3 hours = $3 \times 30^o = 90^o$ and for another 40 minute = $(\frac{30^\circ}{60}) \times 40 = 20^o$

i.e. angle traced by hour hand is $90^o + 20^o = 110^o$

Now, for minute hand:

40 minute = $40 \times 6^o = 240^o$

i.e. angle traced by minute hand is 240^o.

So, the angle between hour hand and minute hand = $240^o - 110^o$

$= 130^o$

$= 130^\circ \times \frac{\pi^c}{180}$

$= \frac{13\pi^c}{18}$

11.

(d) (4, 5)

Explanation: We have, A = {x : x ∈ R, x > 4} and B = {x ∈ R : x < 5}

A ∩ B = (4, 5)

12\.

(b) $\frac{1}{11}$

Explanation: Let the first term of the G.P. be a

Let its common ratio be r.

We are given that,

First term = 10 [Sum of all successive terms]

a = $10\left(\frac{ar}{1-r}\right)$

⇒ a - ar = 10ar

⇒ 11ar = a

$\Rightarrow r = \frac{a}{11a} = \frac{1}{11}$

13\.

(c) a rational number

Explanation: We have $(a + b)^n + (a - b)^n$

= $\left[{}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + {}^nC_3 \quad a^{n-3}b^3 + \ldots.. + {}^nC_nb^n\right] +$
$\left[{}^nC_0a^n - {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 - {}^nC_3a^{n-3}b^3 + \ldots.. + (-1)^n \cdot {}^nC_n \quad b^n\right]$

= $2[{}^nC_0 \quad a^n + {}^nC_2 \quad a^{n-2}b^2 + \ldots]$

Let a = $\sqrt{5}$ and b = 1 and n = 4

Now we get $(\sqrt{5}+1)^4 + (\sqrt{5}-1)^4 = 2\left[{}^4C_0(\sqrt{5})^4 + {}^4C_2(\sqrt{5})^2 1^2 + {}^4C_4(\sqrt{5})^0 1^4\right]$

= 2[25 + 30 + 1] = 112

14\. **(a)** $-\frac{1}{6} \le x < \frac{1}{2}$

Explanation: $-2 \le 6x - 1 < 2$

$\Rightarrow -2 + 1 \le 6x - 1 + 1 < 2 + 1$

$\Rightarrow -1 \le 6x < 3$

$\Rightarrow \frac{-1}{6} \le \frac{6x}{6} < \frac{3}{6}$

$\Rightarrow \frac{-1}{6} \le x < \frac{1}{2}$

15\.

(b) None of these

Explanation: 4 ∉ A

{4} ⊄ A

B ⊄ A

Therefore, we can say that none of these options satisfy the given relation.

16\. **(a)** can't lie between -1 and 1

Explanation: $|\sec\theta| \ge 1 \Rightarrow (\sec\theta \le -1)$ or $(\sec\theta \ge 1)$

∴ value of sec θ can never lie between - 1 and 1

17\.

(c) -1

Explanation: $i^{326} = (i^4)^{81} \times i^2 = 1^{81} \times (-1) = 1 \times (-1) = -1$

18\.

(d) 496

Explanation: ${}^nC_{18} = {}^nC_{12}$

⇒ n = (18 + 12) = 30

$\therefore {}^{32}C_n = {}^{32}C_{30} = {}^{32}C_2 = \frac{32 \times 31}{2} = 496$

19\.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion:

$(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$

Reason:

$(1 + (-1))^n = n_{c_0}1^n + n_{c_1}(1)^{n-1}(-1)^1 + n_{c_2}(1)^{n-2}(-1)^2 + ... + {}^nc_n(1)^{n-n}(-1)^n$

$= n_{c_8} - n_{c_1} + n_{c_2} - n_{c_3} + ... (-1)^n n_{c_n}$

Each term will cancel each other

$\therefore (1 + (-1))^n = 0$

Reason is also the but not the correct explanation of Assertion.

20.

(c) A is true but R is false.

Explanation: Assertion Mean of the given series

$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$

$= \frac{4+7+8+9+10+12+13+17}{8} = 10$

xi	$\|xi - \bar{x}\|$
4	$\|4 - 10\| = 6$
7	$\|7 - 10\| = 3$
8	$\|8 - 10\| = 2$
9	$\|9 - 10\| = 1$
10	$\|10 - 10\| = 0$
12	$\|12 - 10\| = 2$
13	$\|13 - 10\| = 3$
17	$\|17 - 10\| = 7$
$\sum x_i = 80$	$\sum \|x_i - \bar{x}\| = 24$

$\therefore$ Mean deviation about mean

$= \frac{\Sigma|x_i - \bar{x}|}{n} = \frac{24}{8} = 3$

Reason Mean of the given series

$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$

$= \frac{38+70+48+40+42+55+63+46+54+44}{} = 50$

$\therefore$ Mean deviation about mean

$= \frac{\Sigma|x_i - \bar{x}|}{n}$

$= \frac{84}{10} = 8.4$

Hence, Assertion is true and Reason is false.

Section B

21. As given in the question we have, A = {1, 2, 3}, B = {4} and C = {5}

From set theory, (B - C) = {4}

$\therefore \quad A \times (B - C) = \{1, 2, 3\} \times \{4\}$ = {(1, 4), (2, 4), (3, 4)}........(i)

Now,

$A \times B = \{1, 2, 3\} \times \{4\}$ = {(1, 4), (2, 4), (3, 4)}

and, $A \times C = \{1, 2, 3\} \times \{5\}$ = {(1,5), (2, 5), (3, 5)}

$\therefore \quad (A \times B) - (A \times C)$ = {(1, 4), (2, 4), (3, 4)}.......(ii)

From equation (i) and equation (ii), we get

$A \times (B - C) = (A \times B) - (A \times C)$

We can see the equations (i) and (ii) have same ordered pairs.

Hence verified.

OR

From the given we can assume,

Let x be a pre-image of 6 Then

f(x) = 6 = x^2 - 2x - 3 = 6 = x^2 - 2x - 9 = 0 = x = $1 \pm \sqrt{10}$

Since x = $1 \pm \sqrt{10} \notin A$ so there is nor pre image of 6 f(x) = -3 = x^2 - 2x - 3 = -3 = x^2 - 2x = 0 = x = 0.2

Clearly, 0.2 $\in A$ So 0 and 2 are pre image of -3

Let x be a pre image of 5 then

f(x) = 5 = x^2 - 2x - 3 = 5 = x^2 - 2x - 8 = 0 = (x - 4) (x + 2) = 0 = x = 4,

Since, -2A be 4A so, -2 is a pre image of 5

22. To evaluate: $\lim\limits_{x\to0}\left(\frac{e^{3x}-e^{2x}}{x}\right)$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = 0$ or $\pm\infty$ then

$\lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$

As x $\to$ 0, we have

$\lim\limits_{x\to0}\left(\frac{e^{3x}-e^{2x}}{x}\right) = \frac{0}{0}$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$\lim\limits_{x\to0}\left(\frac{e^{3x}-e^{2x}}{x}\right) = \lim\limits_{x\to0}\frac{\frac{d}{dx}(e^{3x}-e^{2x})}{\frac{d}{dx}(x)}$

$\lim\limits_{x\to0}\left(\frac{e^{3x}-e^{2x}}{x}\right) = \lim_{x\to0}\frac{3e^{3x}-2e^{2x}}{1}$

$\lim\limits_{x\to0}\left(\frac{e^{3x}-e^{2x}}{x}\right)$ = 3 - 2

$\lim\limits_{x\to0}\left(\frac{e^{3x}-e^{2x}}{x}\right)$ = 1

Thus, the value of $\lim\limits_{x\to0}\left(\frac{e^{3x}-e^{2x}}{x}\right)$ is 1

23. Given that, Length of Latus Rectum $= \frac{1}{2}$ major Axis

Let the equation of the required ellipse is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ... (i)

As we know that,

Length of Latus Rectum $= \frac{2b^2}{a}$ and Length of Major Axis = 2a

So, according to the question,

$\frac{2b^2}{a} = \frac{1}{2}\times 2a \Rightarrow \frac{2b^2}{a} = a \Rightarrow$ $2b^2 = a^2$ ... (ii)

$\Rightarrow a = \sqrt{2b^2} \Rightarrow a = b\sqrt{2}$

Eccentricity $= \frac{c}{a}$ ... (iii)

where, $c^2 = a^2 - b^2$

So, $c^2 = 2b^2 - b^2$ [from (ii)]

$\Rightarrow c^2 = b^2$

Putting the value of c and a in eq. (iii), we get

Eccentricity $= \frac{c}{a} = \frac{b}{\sqrt{2}b} \Rightarrow e = \frac{1}{\sqrt{2}}$

OR

The given equation of parabola is $x^2 = 16y$ which is of the form $x^2 = -4ay$

$\therefore$ 4a = 16 $\Rightarrow$ a = 4

$\therefore$ Coordinates of focus are (0, -4)

Axis of parabola is x = 0

Equation of the directrix is y = 4 $\Rightarrow$ y - 4 = 0

Length of latus rectum = 4 $\times$ 4 = 16

24. We have,

9 = 0 + 9, Numbers can be 09, 90

9 = 1 + 8, Numbers can be 18, 81

9 = 2 + 7, Numbers can be 27, 72

9 = 3 + 6, Numbers can be 36, 63

9 = 4 + 5, Numbers can be 45, 54

9 = 5 + 4, Numbers can be 54, 45

The elements of this set are 18, 27, 36, 45, 54, 63, 72, 81 and 90 and

Therefore, C = {18, 27, 36, 45, 54, 63, 72, 81, 90}

25. Given that equations of the lines are,

x - 4y = 3 .... (i)

6x - y = 11 .... (ii)

Let m_1 and m_2 be the slopes of these lines.

Here, $m_1 = \frac{1}{4}$, $m_2 = 6$

Let θ be the angle between the lines.

Then,

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{\frac{1}{4} - 6}{1 + \frac{3}{2}}\right|$

$= \frac{23}{10}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{23}{10}\right)$

Therefore, the acute angle between the lines is $\tan^{-1}\left(\frac{23}{10}\right)$

Section C

26. We have, A = f{1, 2} and B = {2, 4, 6}

Also it is given that, f = {(x, y) : x ∈ A, y ∈ B and y > 2x + 1}.

Put x = 1 in y > 2x + 1, we obtain

y > 2(1) + 1

⇒ y > 3

and y ∈ B

This means y = 4,6 if x = 1 because it satisfies the condition y > 3.

Put x = 2 in y > 2x + 1, we get

y > 2(2) + 1

⇒ y > 5

This means y = 6 if x = 2 because, it satisfies the condition y > 5.

∴ f = {(1, 4),(1, 6),(2, 6)}

(1,2),(2,2),(2,4) are not the members of 'f' because they do not satisfy the given condition y > 2x + 1

Firstly, we have to show that f is a relation from A to B.

First elements in F = 1, 2

All the first elements are in Set A. So, the first element is from set A

Second elements in F = 4, 6

All the second elements are in Set B

So, the second element is from set B

Since the first element is from set A and second element is from set B

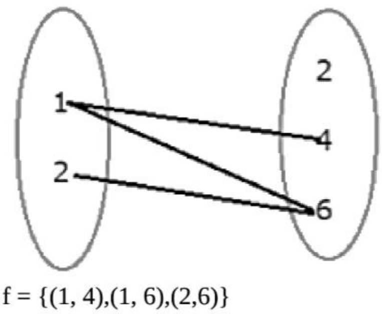
Hence, F is a relation from A to B.

All elements of the first set are associated with the elements of the second set.

i. An element of the first set has a unique image in the second set.

   Now, we have to show that f is not a function from A to B

   f = {(1, 4),(1, 6),(2, 6)}



   f = {(1, 4),(1, 6),(2,6)}

   Here, 1 is coming twice.

Hence, it does not have a unique (one) image.

So, it is not a function.

27. Given that,

$\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}, x > 0$

==> 4 $\leq$ 3(x+1) < 6 [multiply by (x+1)]

==> 4 $\leq$ 3x + 3 < 6

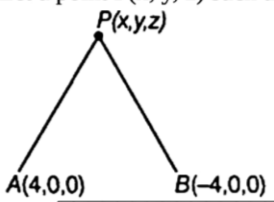
now, 3x + 3 $\geq$ 4 and 3x + 3 < 6

==> 3x $\geq$ 1 and 3x < 3

==> x $\geq \frac{1}{3}$ and x < 1

==> $\frac{1}{3} \leq$ x < 1

28. Let a point P(x, y, z) such that PA + PB = 10



$\Rightarrow \sqrt{(x-4)^2+(y-0)^2+(z-0)^2}+\sqrt{(x+4)^2+(y-0)^2+(z-0)^2}=10$

[$\because$ distance $= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$]

$\Rightarrow \sqrt{x^2-8x+16+y^2+z^2}+\sqrt{x^2+8x+16+y^2+z^2}=10$

$\Rightarrow \sqrt{x^2+y^2+z^2-8x+16}= 10-\sqrt{x^2+y^2+z^2+8x+16}$

On squaring sides, we get

$x^2 + y^2 + z^2 - 8x +16 = 100 + x^2 + y^2 + z^2 + 8x +16$

$-20\sqrt{x^2+y^2+z^2+8x+16}$

$\Rightarrow -16x-100=-20\sqrt{x^2+y^2+z^2+8x+16}$

$\Rightarrow \quad 4x+25=5\sqrt{x^2+y^2+z^2+8x+16}$ [dividing both sides by -4]

Again squaring on both sides, we get

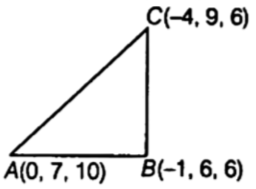
$16x^2 + 200x + 625 = 25(x^2 + y^2 + z^2 + 8x + 16)$

$\Rightarrow 16x^2 + 200x + 625 = 25x^2 + 25y^2 + 25z^2 + 200x + 400$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 =0$

OR

Let A (0, 7, 10), B (-1, 6,6) and C (-4, 9, 6) be the given points. We have,



Now, $AB=\sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}$ [$\because$ distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$]

$=\sqrt{1+1+16}=\sqrt{18}=3\sqrt{2}$

$BC=\sqrt{(-4+1)^2+(9-6)^2+(6-6)^2}$

$=\sqrt{9+9+0}=\sqrt{18}=3\sqrt{2}$

and $AC=\sqrt{(-4-0)^2+(9-7)^2+(6-10)^2}$

$=\sqrt{16+4+16}$

$\therefore \quad AC=\sqrt{36}=6$ ...... (i)

Now, $AB^2 + BC^2 = (3\sqrt{2})^2+(3\sqrt{2})^2 = 18 + 18 = 36$

$\therefore AB^2 + BC^2 = AC^2$ [from Eq. (i)]

Also, AB = BC = $3\sqrt{2}$

Hence, ABC is a right isosceles triangle.

29. We have $T_1={}^nC_0a^nb^0=729$ ... (i)

$T_2={}^nC_1a^{n-1}b=7290$ ... (ii)

$T_3={}^nC_2a^{n-2}b^2=30375$ ... (iii)

From (i) $a^n = 729$ . . . (iv)

From (ii) $na^{n-1} b = 7290$ . . . (v)

From (iii) $\frac{n(n-1)}{2}a^{n-2}b^2 = 30375$ . . . (vi)

Multiplying (iv) and (vi), we get

$\frac{n(n-1)}{2}a^{2n-2}b^2 = 729 \times 30375$ . . . (vii)

Squaring both sides of (v) we get

$n^2a^{2n-2}b^2 = (7290)(7290)$(viii)

Dividing (vii) by (viii), we get

$\frac{n(n-1)a^{2n-2}b^2}{2n^2a^{2n-2}b^2} = \frac{729\times30375}{7290\times7290}$

$\Rightarrow \frac{(n-1)}{2n} = \frac{30375}{72900} \Rightarrow \frac{n-1}{2n} = \frac{5}{12} \Rightarrow 12n - 12 = 10n$

$\Rightarrow 2n = 12 \Rightarrow n = 6$

From (iv) $a^6 = 729 \Rightarrow a^6 = (3)^6 \Rightarrow a = 3$

From (v) $6 \times 3^5 \times b = 7290 \Rightarrow b = 5$

Thus a = 3, b = 5 and n = 6.

OR

We have

$(x+y)^5 + (x-y)^5 = 2\left[{}^5C_0\, x^5 + {}^5C_2\, x^3y^2 + {}^5C_4\, x^1y^4\right]$

$= 2\,(x^5 + 10x^3y^2 + 5xy^4$

Putting $x = \sqrt{2}$ *and* $y = 1$, *we get*

$(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5 = 2\left[(\sqrt{2})^5 + 10(\sqrt{2})^3 + 5\sqrt{2}\right]$

$= 2\left[4\sqrt{2} + 20\sqrt{2} + 5\sqrt{2}\right]$

$= 58\sqrt{2}$

30. Here $a + ib = \frac{c+i}{c-i}$

$= \frac{c+i}{c-i} \times \frac{c+i}{c+i} = \frac{(c+i)^2}{c^2-i^2}$

$= \frac{c^2+2ci+i^2}{c^2+1}$

$= \frac{c^2-1}{c^2+1} + \frac{2c}{c^2+1}i$

Comparing real and imaginary parts on both sides, we have

$a = \frac{c^2-1}{c^2+1}$ and $b = \frac{2c}{c^2+1}$

Now $a^2 + b^2 = \left(\frac{c^2-1}{c^2+1}\right)^2 + \left(\frac{2c}{c^2+1}\right)^2$

$= \frac{(c^2-1)^2+4c^2}{(c^2+1)^2} = \frac{(c^2+1)^2}{(c^2+1)^2} = 1$

Also $\frac{b}{a} = \frac{\frac{2c}{c^2+1}}{\frac{c^2-1}{c^2+1}} = \frac{2c}{c^2-1}$

OR

Let, $(a + ib)^2 = 5 + 12i$

$\Rightarrow a^2 + (bi)^2 + 2abi = 5 + 12i$ $[(a + b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow a^2 - b^2 + 2abi = 5 + 12i$ $[i^2 = -1]$

now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 5$…………..eq.1

$\Rightarrow 2ab = 12$

$\Rightarrow a = \frac{6}{b}$……..eq.2

now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{6}{b}\right)^2 - b^2 = 5$

$\Rightarrow 36 - b^4 = 5b^2$

$\Rightarrow b^4 + 5b^2 - 36 = 0$

=> $(b^2 + 9)(b^2 - 4) = 0$

$\Rightarrow b^2 = -9$ or $b^2 = 4$

As b is real no. so, $b^2 = 4$

b = 2 or b= -2

put value of b in equation (2) ===> a = 3 or a= -3

Hence the square root of the complex no. is 3 + 2i and -3 - 2i.

31. $(A \cup B) - C = (A - C) \cup (B - C)$

Let $x \in [(A \cup B) - C]$

$x \in (A \cup B)$ and $x \notin C$

$(x \in A$ or $x \in B)$ and $x \notin C)$

$(x \in A$ and $x \notin C)$ or $(x \in B$ and $x \notin C)$

$x \in \{(A - C)$ or $x \in (B - C)\}$

$x \in \{(A - C) \cup (B - C)\}$

$(A \cup B) - C \subseteq (A - C) \cup (B - C)$ ...(i)

Again, let $y \in [(A - C) \cup (B - C)]$

$y \in (A - C)$ or $y \in (B - C)$

$(y \in A$ and $y \notin C)$ or $(y \in B$ and $y \notin C)$

$(y \in A$ or $y \in B)$ and $y \notin C$

$y \in \{(A \cup B)$ and $y \notin C$

$y \in \{(A \cup B) - C\}$

$(A - C) \cup (B - C) \subseteq (A \cup B) - C$ ...(ii)

From eqs. (i) and (ii),

$(A \cup B) - C = (A - C) \cup (B - C)$ Hence proved

Section D

32. Here a coin is tossed four times. So number of elements in the sample space (S) will be $2^4 = 16$. n(S) = 16.

The sample space,

S = {HHHH, HHHT, HHTH, HTHH, HTTH, HTHT, HHTT, HTTT, THHH, THHT, THTH, TTHH, TTTH, TTHT, THTT, TTTT}

Amounts:

i. When 4 heads turns up = Rs(1 + 1+ 1 + 1)= Rs. 4. i.e., Person wins Rs. 4
ii. When 3 heads and 1 tail turns up = Rs(1+ 1+1 –1.50 =Rs. 1.50. i.e.,Person wins Rs. 1.50
iii. When2 heads and 2 tails turns up = Rs(1 + 1– 1.50– 1.50) =– Rs. 1. i.e., Person losesRs. 1
iv. When1 head and 3 tails turns up = Rs(1– 1.50– 1.50– 1.50)=– Rs 3.50. i.e., Person losesRs. 3.50
v. When4 tails turns up= Rs(– 1.50– 1.50– 1.50– 1.50) =– Rs 6. i.e., Person loses Rs. 6

Let the events for which the person wins Rs 4, wins Rs 1.50, loses Re1, loses Rs 3.50 and loses Rs 6 be denoted by E_1, E_2, E_3, E_4 and E_5.

i.e., E_1 = {HHHH}, E_2 = {HHHT, HHTH, HTHH, THHH} E_3 = {HHTT, HTHT, HTTH, THTH, THHT, TTHH}

E_4 = {HTTT, TTTH, THTT, TTHT}, E_5 = {TTTT}

Here, $n(E_1) = 1$, $n(E_2) = 4$, $n(E_3) = 6$, $n(E_4) = 4$ and $n(E_5) = 1$.

Hence, $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{16}$,

$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{16} = \frac{1}{4}$

$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{6}{16} = \frac{3}{8}$

$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{4}{16} = \frac{1}{4}$

and $P(E_5) = \frac{n(E_5)}{n(S)} = \frac{1}{16} = \frac{1}{16}$

33. Let $f(x) = \frac{\sin x}{x}$

By using first principle of derivative,

$f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\therefore f'(x)=\lim\limits_{h\to 0}\frac{\frac{\sin(x+h)}{x+h}-\frac{\sin x}{x}}{h}$

$=\lim\limits_{h\to 0}\frac{x\sin(x+h)-(x+h)\sin x}{x(x+h)\times h}$

$=\lim\limits_{h\to 0}\frac{x[\sin(x+h)-\sin x]-h\sin x}{h\cdot x(x+h)}$

$=\lim\limits_{h\to 0}\frac{x\left[2\cdot\cos\left(\frac{x+h+x}{2}\right)\cdot\sin\left(\frac{x+h-x}{2}\right)\right]-h\sin x}{h\cdot x(x+h)}$

$\left[\because \sin C-\sin D=2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$

$=\lim\limits_{h\to 0}\frac{x\left[2\cdot\sin\frac{h}{2}\cdot\cos\left(x+\frac{h}{2}\right)\right]-h\sin x}{h\cdot x(x+h)}$

$=\lim\limits_{\frac{h}{2}\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\cdot\lim\limits_{h\to 0}\frac{\cos\left(x+\frac{h}{2}\right)}{(x+h)}-\lim\limits_{h\to 0}\frac{\sin x}{x(x+h)}$

$=(1)\cdot\frac{\cos x}{x}-\frac{\sin x}{x^2}\left[\because \lim\limits_{x\to 0}\frac{\sin x}{x}=1\right]$

$=\frac{\cos x}{x}-\frac{\sin x}{x^2}$

OR

Let f (x) = log sin x. Then, f (x + h) = log sin (x + h)

$\therefore \frac{d}{dx}(f(x))=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow \frac{d}{dx}(f(x))=\lim\limits_{h\to 0}\frac{\log\sin(x+h)-\log\sin x}{h}$

$\Rightarrow \frac{d}{dx}(f(x))=\lim\limits_{h\to 0}\frac{\log\left\{\frac{\sin(x+h)}{\sin x}\right\}}{h}$

$\Rightarrow \frac{d}{dx}(f(x))=\lim\limits_{h\to 0}\frac{\log\left\{1+\frac{\sin(x+h)}{\sin x}-1\right\}}{h}$

$\Rightarrow \frac{d}{dx}(f(x))=\lim\limits_{h\to 0}\frac{\log\left\{1+\frac{\sin(x+h)-\sin x}{\sin x}\right\}}{h}$

$\Rightarrow \frac{d}{dx}(f(x))=\lim\limits_{h\to 0}\frac{\log\left\{1+\frac{\sin(x+h)-\sin x}{\sin x}\right\}}{h\left\{\frac{\sin(x+h)-\sin x}{\sin x}\right\}}\times\left\{\frac{\sin(x+h)-\sin x}{\sin x}\right\}$

$\Rightarrow \frac{d}{dx}(f(x))=\lim\limits_{h\to 0}\frac{\log\left\{1+\frac{\sin(x+h)-\sin x}{h}\right\}}{\left\{\frac{\sin(x+h)-\sin x}{h}\right\}}\times\frac{\sin(x+h)-\sin x}{h}\times\frac{1}{\sin x}$

$\Rightarrow \frac{d}{dx}(f(x))=\lim\limits_{h\to 0}\frac{\log\left\{1+\frac{\sin(x+h)-\sin x}{h}\right\}}{\left\{\frac{\sin(x+h)-\sin x}{h}\right\}}\times\lim\limits_{h\to 0}\frac{2\sin\frac{h}{2}\cos\left(x+\frac{h}{2}\right)}{h}\times\frac{1}{\sin x}$

$\Rightarrow \frac{d}{dx}(f(x))=\lim\limits_{h\to 0}\frac{\log\left\{1+\frac{\sin(x+h)-\sin x}{h}\right\}}{\left\{\frac{\sin(x+h)-\sin x}{h}\right\}}\times\lim\limits_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)\cos\left(x+\frac{h}{2}\right)}{\frac{h}{2}}\times\frac{1}{\sin x}$

$\Rightarrow \frac{d}{dx}(f(x))=1\times\cos x\times\frac{1}{\sin x}=\cot x.$

34. Let the given GP contain n terms. Let abe the first term and r be the common ratio of this GP.

Since the given GP is increasing, we have r > 1

Now, $T_1+T_n=66\Rightarrow a+ar^{(n-1)}=66$ ...(i)

And, $T_2\times T_{n-1}=128\Rightarrow ar\times ar^{(n-2)}=128$

$\Rightarrow a^2r^{(n-1)}=128\Rightarrow ar^{(n-1)}=\frac{128}{a}$ ...(ii)

Using (ii) and (i), we get

$a+\frac{128}{a}=66\Rightarrow a^2-66a+128=0$

$\Rightarrow a^2-2a-64a+128=0$

$\Rightarrow a(a-2)-64(a-2)=0$

$\Rightarrow (a-2)(a-64)=0$

$\Rightarrow a=2$ or $a=64$

Putting a = 2 in (ii), we get

$r^{(n-1)} = \frac{128}{a^2} = \frac{128}{4} = 32$ ...(iii)

Putting a = 64 in (ii), we get

$r^{(n-1)} = \frac{128}{a^2} = \frac{128}{64\times 64} = \frac{1}{32}$, which is rejected, since r > 1.

Thus, a = 2 and $r^{(n-1)} = 32$

Now, $S_n = 126 \Rightarrow \frac{a(r^n-1)}{(r-1)} = 126$

$\Rightarrow 2\left(\frac{r^n-1}{r-1}\right) = 126 \Rightarrow \frac{r^n-1}{r-1} = 63$

$\Rightarrow \frac{r^{(n-1)}\times r-1}{r-1} = 63 \Rightarrow \frac{32r-1}{r-1} = 63$

$\Rightarrow 32r - 1 = 63r - 63 \Rightarrow 31r = 62 \Rightarrow r = 2$

$\therefore r^{(n-1)} = 32 = 25 \Rightarrow n - 1 = 5 \Rightarrow n = 6$

Hence, there are 6 terms in the given GP

35. We know,

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cos^2 x = 1 - \left(\frac{1}{4}\right)^2$ ... $[\because \sin x = \frac{1}{4}]$

$\cos^2 x = 1 - \frac{1}{16} = \frac{16-1}{16} = \frac{15}{16}$

$\cos x = \pm\frac{\sqrt{15}}{4}$

Since, $x \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow$ cos x will be negative in second quadrant

So, $\cos x = -\frac{\sqrt{15}}{4}$

We know,

$\cos 2x = 2\cos^2 x - 1$

$\cos x = 2\cos^2\frac{x}{2} - 1$

$-\frac{\sqrt{15}}{4} = 2\cos^2\frac{x}{2} - 1$ ... $[\because \cos x = -\frac{\sqrt{15}}{4}]$

$2\cos^2\frac{x}{2} = -\frac{\sqrt{15}}{4} + 1 = \frac{-\sqrt{15}+4}{4}$

$\cos^2\frac{x}{2} = \frac{-\sqrt{15}+4}{8}$

$\cos\frac{x}{2} = \pm\sqrt{\frac{-\sqrt{15}+4}{8}}$

Since, $x \in \left(\frac{\pi}{2}, \pi\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\cos\frac{x}{2}$ will be positive in first quadrant

So, $\cos\frac{x}{2} = \sqrt{\frac{-\sqrt{15}+4}{8}}$

We know,

$\cos 2x = 1 - 2\sin^2 x$

$\cos x = 1 - 2\sin^2\frac{x}{2}$ ... $[\because \cos x = -\frac{\sqrt{15}}{4}]$

$-\frac{\sqrt{15}}{4} = 1 - 2\sin^2\frac{x}{2}$

$2\sin^2\frac{x}{2} = \frac{\sqrt{15}}{4} + 1 = \frac{\sqrt{15}+4}{4}$

$\sin^2\frac{x}{2} = \frac{\sqrt{15}+4}{8} = \pm\sqrt{\frac{\sqrt{15}+4}{8}}$

Since, $x \in \left(\frac{\pi}{2}, \pi\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\sin\frac{x}{2}$ will be positive in first quadrant

So, $\sin\frac{x}{2} = \sqrt{\frac{\sqrt{15}+4}{8}}$

We know,

$\tan\frac{x}{2} = \frac{\sqrt{\frac{\sqrt{15}+4}{8}}}{\sqrt{\frac{-\sqrt{15}+4}{8}}}$

$\tan\frac{x}{2} = \sqrt{\frac{\sqrt{15}+4}{8} \times \frac{8}{-\sqrt{15}+4}}$

$\tan\frac{x}{2} = \sqrt{\frac{\sqrt{15}+4}{-\sqrt{15}+4}}$

On rationalising:

$\tan\frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}} \times \frac{4+\sqrt{15}}{4+\sqrt{15}}}$

$\tan\frac{x}{2} = \sqrt{\frac{(4+\sqrt{15})^2}{4^2-(\sqrt{15})^2}}$ ... $\{\because (a + b)(a - b) = a^2 - b^2\}$

$\tan\frac{x}{2} = \sqrt{\frac{(4+\sqrt{15})^2}{16-15}} = \sqrt{\frac{(4+\sqrt{15})^2}{1}} = 4 + \sqrt{15}$

Hence, values of $\cos\frac{x}{2}$, $\sin\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\sqrt{\frac{-\sqrt{15}+4}{8}}$, $\sqrt{\frac{\sqrt{15}+4}{8}}$ and $4 + \sqrt{15}$ respectively

OR

Given, LHS $= sin20^o sin40^o sin80^o$

$= \frac{1}{2}$ [2 sin 20° · sin 40°] sin 80° [multiplying and dividing by 2]

$= \frac{1}{2}[cos(20^o - 40^o) - cos(20^o + 40^o)] \cdot \sin 80^\circ$ $[\because 2\sin x \cdot \sin y = \cos(x - y) - \cos(x + y)]$

$= \frac{1}{2}[cos(-20^o) - cos60^o]sin80^o$

$= \frac{1}{2}[\cos 20^o - \frac{1}{2}] \cdot \sin 80^\circ$ $[\because \cos(-\theta) = \cos\theta$ and $\cos 60^o = \frac{1}{2}]$

$= \frac{1}{2} \times \frac{1}{2}\left[2\left(\cos 20^\circ - \frac{1}{2}\right) \cdot \sin 80^\circ\right]$ [again multiplying and dividing by 2]

$= \frac{1}{4}[2\cos 20^\circ \cdot \sin 80^\circ - \sin 80^\circ]$

$= \frac{1}{4}[sin(20^o + 80^o) - sin(20^o - 80^o) - sin80^o]$ $[\because 2\cos x \cdot \sin y = sin(x + y) - sin(x - y)]$

$= \frac{1}{4}[sin100^o - sin(-60^o) - sin80^o]$

$= \frac{1}{4}[\sin 100^o + \sin 60^o - \sin 80^o]$ $[\because \sin(-\theta) = -\sin\theta]$

$= \frac{1}{4}[\sin(180^o - 80^o) + \sin 60^o - \sin 80^o]$ $[\because \sin 100^o = \sin(180^o - 80^o)]$

$= \frac{1}{4}[\sin 80^o + \sin 60^o - \sin 80^o]$ $[\because \sin(\pi - \theta) = \sin\theta]$

$= \frac{1}{4} \times \sin 60^o = \frac{1}{4} \times \frac{\sqrt{3}}{2}$ $[\because \sin 60^o = \frac{\sqrt{3}}{2}]$

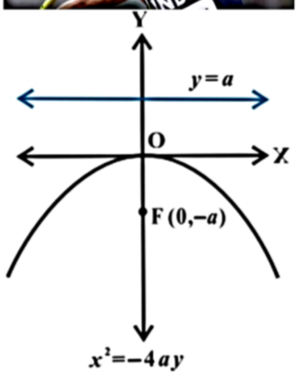
$= \frac{\sqrt{3}}{8}$ = RHS

Hence proved.

Section E

36. **Read the text carefully and answer the questions:**

Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal for India at the Olympics.



(i) The path traced by Javelin is parabola. A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.

compare $x^2 = -16y$ with $x^2 = -4ay$

$\Rightarrow -4a = -16$

$\Rightarrow a = 4$

coordinates of focus for parabola $x^2 = -4ay$ is (0, -a)

⇒ coordinates of focus for given parabola is (0, -4)

(ii) compare $x^2 = -16y$ with $x^2 = -4ay$

⇒ -4a = -16

⇒ a = 4

Equation of directrix for parabola $x^2 = -4ay$ is y = a

⇒ Equation of directrix for parabola $x^2 = -16y$ is y = 4

Length of latus rectum is 4a = 4 × 4 = 16

(iii)Equation of parabola with axis along y - axis

$x^2 = 4ay$

which passes through (5, 2)

⇒ 25 = 4a × 2

⇒ $4a = \frac{25}{2}$

hence required equation of parabola is

$x^2 = \frac{25}{2}y$

⇒ $2x^2 = 25y$

Equation of directrix is y= -a

Hence required equation of directrix is 8y + 25 = 0.

OR

Since the focus (2,0) lies on the x-axis, the x-axis itself is the axis of the parabola.

Hence the equation of the parabola is of the form either $y^2 = 4ax$ or $y^2 = -4ax$.

Since the directrix is x = -2 and the focus is (2,0), the parabola is to be of the form $y^2 = 4ax$ with a = 2.

Hence the required equation is $y^2 = 4(2)x = 8x$

length of latus rectum = 4a = 8

37. **Read the text carefully and answer the questions:**

Consider the data.

Class	**Frequency**
0-10	6
10-20	7
20-30	15
30-40	16
40-50	4
50-60	2

(i) We make the table from the given data.

Class	f_i	cf	Mid-point(x_i)	$\|x_i - M\|$	$f_i\|x_i - M\|$
0-10	6	6	5	23	138
10-20	7	13	15	13	91
20-30	15	28	25	3	45
30-40	16	44	35	7	112
40-50	4	48	45	17	68
50-60	2	50	55	27	54
	50				508

Here, $\frac{N}{2} = \frac{50}{2} = 25$

Here, 25th item lies in the class 20-30. Therefore, 20-30 is the median class.

Here, l = 20, cf = 13, f = 15, b = 10 and N = 50

$\because$ Median, M = $l + \frac{\frac{N}{2} - cf}{f} \times b$

$\Rightarrow$ M = 20 + $\frac{25-13}{15}$ $\times$ 10 = 20 + 8 = 28

Thus, mean deviation about median is given by

MD (M) = $\frac{1}{N}\sum_{i=1}^{6} f_i |x_i - M|$ = $\frac{1}{50}$ $\times$ 508 = 10.16

Hence, mean deviation about median is 10.16.

(ii) Here, l = 20, cf = 13, f = 15, b = 10 and N = 50

$\because$ Median, M = $l + \frac{\frac{N}{2} - cf}{f} \times b$

$\Rightarrow$ M = 20 + $\frac{25-13}{15}$ $\times$ 10 = 20 + 8 = 28

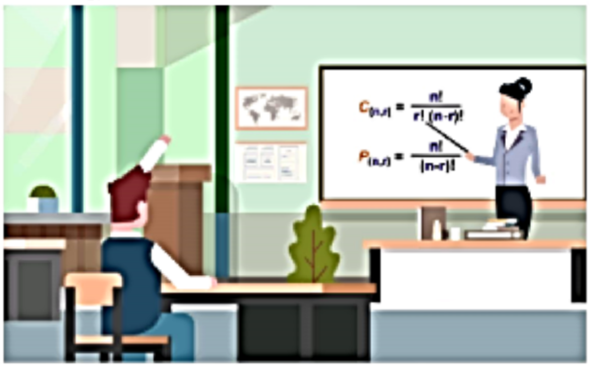
(iii) MD = $\frac{\Sigma f_i |x_i - M|}{N}$

OR

M = 1 + $\frac{\frac{N}{2} - cf}{f} \times h$

38. **Read the text carefully and answer the questions:**

During the math class, a teacher clears the concept of permutation and combination to the 11th standard students. After the class was over she asks the students some questions, one of the question was: how many numbers between 99 and 1000 (both excluding) can be formed such that:



(i) Here we need to get a 3-digit number

Three vacant paces are fixed with 3 or 7. Therefore, by the multiplication principle, the required number of three-digit numbers with every digit 3 or 7 3 or 7 is 2 $\times$ 2 $\times$ 2 =8

(ii) Three vacant paces are fixed with all 10 digits, but first place is fixed with 9 digits excluding 0.

Therefore, by the multiplication principle, the required number of three digits numbers without any restriction = 9 $\times$ 10 $\times$ 10 = 900