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STUDY CIRCLE

UNIT OF ACCENT'S EDUCATIONAL PROMOTERS

OFFLINE ONLINE LEARNING ACADEMY

CBSE

PHYSICS ALTERNATING CURRENT

REVISION MODULE

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ALTERNATING CURRENT

Helpline
+91-9939586130
+91-7739650505

2nd FLOOR ,SATKOUDI COMPLEX
THANA CHOWK, RAMGARH
JHARKHAND - 829122

ALTERNATING CURRENT

Alternating Current

AC Current

Direction of current changes alternatively

$i = i_0 \sin\omega t$

$v = v_0 \sin\omega t$

RMS value or root mean square value

$i_{rms} = i_0/\sqrt{2}$

$v_{rms} = v_0/\sqrt{2}$

Mean value or Average value

$i_{mean} = 2i_0/\pi$

$v_{rms} = 2v_0/\pi$

Peak value or Maxm value

$i_0 = \sqrt{2}\, i_{rms}$

$v_0 = \sqrt{2}\, v_{rms}$

Contribution

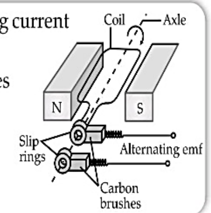
In 1883, Nikola Tesla inverted fixer AC motor, AC generator and AC transmission technology.

A C Generator

A device generating alternating current having elements magnet, Armature and graphite Brushes emf induced through coil

$E = NBA\,\omega \sin\omega t$

$i = E/R = NBA\,\omega \sin\omega t/R$



Transformer

It is used to obtain a high AC voltage from a Low–voltage AC source or vice–versa

Efficiency of a transformer

η = output power/Input power

99% efficiency can easily achieved

Turnsratio

$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = K$

Power in AC Circuit

$P = E_{rms} i_{rms} \cos\phi = \frac{E_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} \cos\phi = \frac{1}{2} E_0 i_0 \cos\phi$

Impedance

The ratio of emf applied and the current produced in an AC circuit

$Z = E_0/i_0$ Ohm

Reactance

The hindrance offered by inductor or capacitor or both to the flow of AC (c_1 or c_2)

A C Circuits

Inductive (L) circuit

$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

Phase difference b/w V&I, $\phi = \pi/2$

Power factor $\cos(\phi) = 0$

Voltage leads current by $\pi/2$

$P = 0$

Capacitive (C) circuit

$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$

$\phi = \pi/2$ or $-\pi/2$

Power factor $\cos\phi = 0$

Power $(P) = 0$

Current leads the voltage by $\pi/2$

Resistive (R) circuit

$i = i_0 \sin(\omega t)$

$\phi = 0°$

$\cos\phi = 1$

$P = V_0 I_0/2$

Current & voltage both are in same phase

R–C circuit

$i = i_0 \sin(\omega t + \phi)$

$Z = \sqrt{R^2 + X_c^2}$

$X_c = 1/\omega C$

$\tan\phi = 1/\omega CR$

$i_0 = E_0/Z$

- Power factor $\cos\phi = \frac{R^2}{\sqrt{R^2 + X_c^2}}$
- current leading.

L–C circuit

$i = i_0 \sin\left(\omega t \pm \frac{\pi}{2}\right)$

$X = X_L - X_C$

$\phi = \pi/2$

- Power factor = 0, Either voltage or current leading

L–R circuit

$i = i_0 \sin(\omega t + \phi)$

$Z = \sqrt{R^2 + X_L^2}$

$X_L = \omega L$

$\tan\phi = \omega L/R$

$i_0 = E_0/Z$

- power factor = R/Z
- voltage is leading

L–C–R circuit

$i = i_0 \sin(\omega t \pm \phi)$

$Z = \sqrt{R^2 + (X_c - X_L)^2}$

$X = X_c - X_L$

$= \left(\frac{1}{\omega c} - \omega L\right)$

$\tan\phi = \left(\frac{1}{\omega c} - \omega L\right)/R$, $i_0 = \frac{E_0}{Z}$

Trace the Mind Map
- First Level
- Second Level
- Third Level

Syllabus

- *Alternating currents, peak and RMS value of alternating current/voltage; reactance and impedance; LC oscillations (qualitative treatment only).*
- *LCR series circuit, resonance; power in A.C. circuits, power factor, wattless current.*
- *AC generator and transformer.*

Trend Analysis

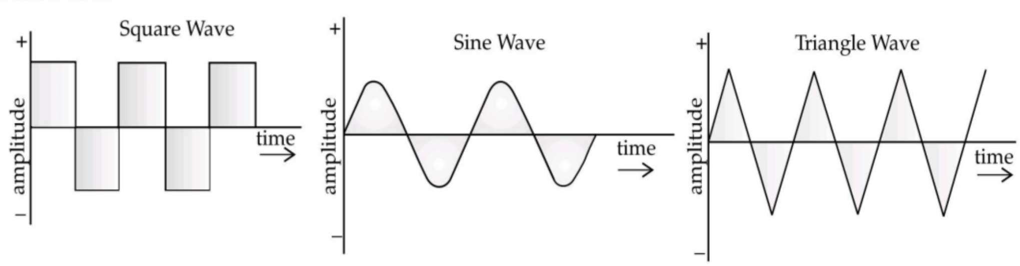
List of Concepts	2018		2019		2020	
	OD	D	OD	D	OD	D
Alternating Current	1Q (5M)	1Q (5M)				
LCR Circuit		2Q (5M)	2Q (3M)	1 Q (5M)	1 Q (2M)	4Q (1M) 1Q (5M)
AC Generator and Transformer		1Q (3M) 1Q (5 M)			1Q (3M)	

TOPIC-1
Alternating Current

Revision Notes

Alternating current

- Alternating current changes continuously in magnitude and periodically in direction.
- It is represented by sine curve or cosine curve as $I = I_0 \sin wt$ or $I = I_0 \cos wt$ where, I_0 is peak value of current and I is instantaneous value of current.
- Frequency of an alternating current supply f, is defined as the number of cycles completed per second. It is measured in Hertz (Hz). In India, the frequency is 50 Hz.
- The time period T, of an alternating supply, is time taken to complete one cycle.
- The behaviour of ohmic resistance R in ac circuit is the same as in dc circuit.
- Alternating current can be produced by using a device called as an alternator.
- AC waveforms are:



Peak and *rms* value of alternating current/voltage:

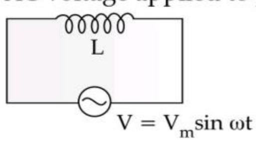
- Root mean square or *rms* is the root mean square of voltage or current in an *ac* circuit for one complete cycle denoted by V_{rms} or I_{rms}.
- *Rms* value is the standard way of measuring alternating current and voltage as it gives the *dc* equivalent values.
- *Rms* value of *ac* is also called effective value or virtual value of *ac* represented as I_{rms}, I_{eff} or I_v shown as

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

- *Rms* voltage value is the square root of averages of the squares of instantaneous voltages in a time varying waveform.

$$V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\ V_0$$

- AC voltage applied to pure inductive circuits:



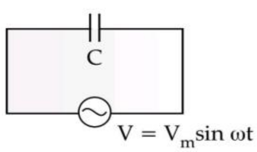
$$V = V_m \sin \omega t$$

$$i = i_m\left(\sin \omega t - \frac{\pi}{2}\right) \quad \text{[which shows current lags the voltage by } \frac{\pi}{2}\text{]}$$

$$\text{Average } P_L = \frac{i_m V_m}{2}[\sin(2\omega t)] = 0 \quad \text{[Since average of sin } 2\omega t \text{ over a complete cycle is zero]}$$

Thus the average power supplied to an inductor over one complete cycle is zero.

AC applied to pure capacitive circuit:



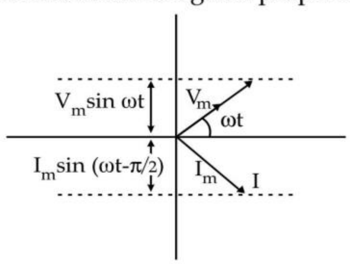
$$V = V_m \sin \omega t$$

$$I = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{[which shows current leads the voltage by } \frac{\pi}{2}\text{]}$$

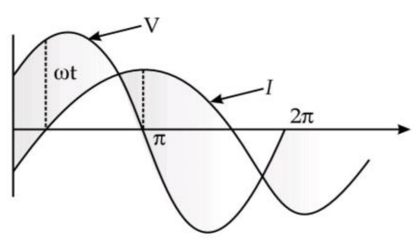
$$\text{Average } P_C = \frac{I_m V_m}{2}\sin(2\omega t) = 0 \quad \text{[Since average of sin } 2\omega t \text{ over a complete cycle is zero]}$$

Thus the average power supplied to an capacitor over one complete cycle is zero.

- **Phasor-diagram:** A phasor diagram represents sinusoidal *ac* current and sinusoidal voltage in a circuit along with the phase difference between current and voltage. The length of phasor is proportional to the instantaneous values of V and I and the maximum length is proportional to V_0 and I_0.



Phasor diagram of purely Inductive circuit



Graphical representation of ***V* and *i* versus ω*t*.**

Reactance and Impedance

- When an *ac* current is passed through a resistance, a voltage drop is produced which is in phase with the current and is measured in ohms (Ω).
- Reactance is the inertia against the motion of electrons where an alternating current after passing through it produces a voltage drop which is 90° out of phase with the current.
- Reactance is shown by "*X*" and is measured in ohms (Ω).
- Reactance is of two types: inductive and capacitive.
- Inductive reactance is linked with varying magnetic field that surrounds a wire or a coil carrying a current.
- Inductive reactance (X_L) is the resistance offered by an inductor and is given by $X_L = \omega L = 2\pi f L$
- Through a pure inductor, alternating current lags behind the alternating *emf* by phase angle of 90°.
- Capacitive reactance is linked with changing electric field between two conducting surfaces separated from each other by an insulating medium.
- Capacitive reactance (X_C) is the resistance offered by a capacitor and is given by

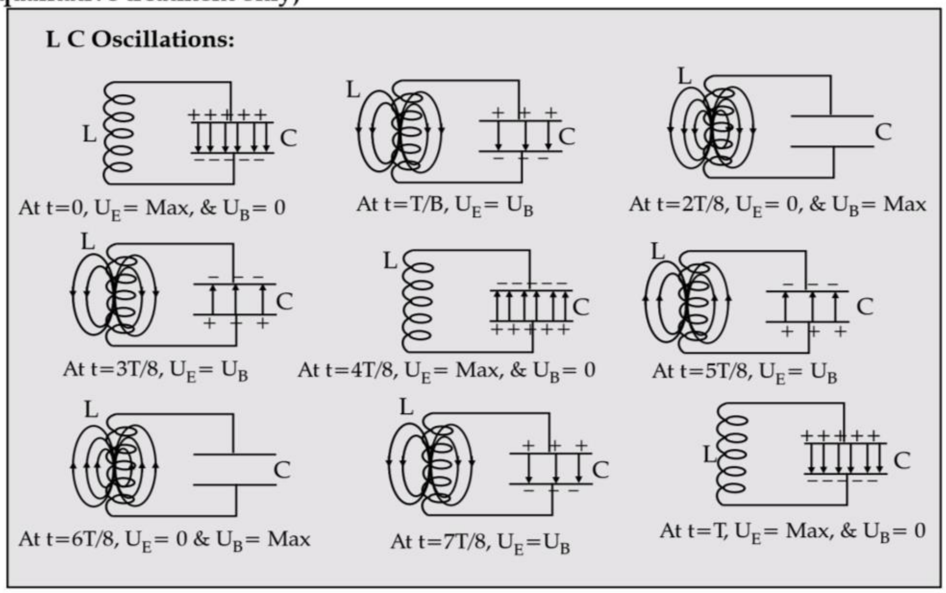
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

- Through a pure capacitor, alternating current leads the alternating *emf* by a phase angle of 90^o.
- Impedance is the comprehensive expression of all forms of opposition to electron flow, including resistance and reactance, where an alternating current after passing through it produces a voltage drop between 0^o and 90^o which will be out of phase with current given as,

$$Z = \sqrt{R^2 + X^2}$$

where, Z = Impedance of circuit, R = Resistance, X = Reactance

***LC* Oscillations (qualitative treatment only)**



- *LC* circuit comprises of inductor and capacitor connected in series where energy from the cell is given to capacitor which keeps on oscillating between inductor and capacitor.
- When *ac* voltage is applied to the capacitor, it keeps on charging and discharging continuously.
- When capacitor is fully charged, it starts discharging and charge gets transferred to the inductor which is connected to capacitor.
- Due to change in current, there is change in magnetic flux of the inductor in the circuit, which induces an *emf* in the inductor.
- The *emf* is given by $e = -L\frac{dI}{dt}$ which opposes the growth of the current.
- When capacitor gets completely discharged, all the energy stored in it, gets stored in the inductor as a result of which, inductor starts charging the capacitor and energy stored in the capacitor starts increasing.
- As there is no current in the circuit, energy in the inductor is zero, so total energy of *LC* circuit will be

$$U_E = \frac{1}{2}.\frac{q^2}{C}$$

- Band Width: It is the range of angular frequencies over which the average power is greater than ½ the maximum value of average power.
- **Impedance:** In an *ac*, the impedance is analogous to resistance in a *dc* circuit that measures the combined effect of resistance, capacitive reactance and inductive reactance.

Key Formulae

- *rms* value for current $I_{rms} = \frac{I_0}{\sqrt{2}}$
- *rms* value for voltage $V_{rms} = \frac{V_0}{\sqrt{2}}$
- Power $P = V_{rms}I_{rms}$

- In a purely inductive circuit if, $V = V_m \sin \omega t$

$$i = i_m \sin\left(\omega t - \frac{\pi}{2}\right), \quad \text{where } i_m = \frac{V_m}{X_L} \text{ and } X_L = \omega L$$

$$(P_{avg})_L = 0$$

- In a purely capacitive circuit if, $V = V_m \sin \omega t$

$$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \text{where, } i_m = \frac{V_m}{X_C} \text{ and } X_C = \frac{1}{\omega C}$$

- $$\text{Average Power} = \frac{1}{2}V_0 I_0 \cos\phi = V_{rms} I_{rms} \cos\phi \quad \left(\text{where, } \cos\phi = \frac{R}{Z} \text{ is power factor}\right)$$

- $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- $$\text{Induced emf} = e = -L\frac{dI}{dt}$$

- Energy in LC circuit, $$U_E = \frac{1}{2}\frac{q^2}{C}$$

How is it done on the GREENBOARD?

Q. 1. An electric lamp when connected to a 100 V dc supply, it draws 10 A current. The lamp is now connected to 200 V, 50 Hz supply.

(i) Calculate the inductance of choke required.

(ii) Find the value of the resistance which can replace the choke.

(iii) Which arrangement is better (i) or (ii)?

Solution:

Step I: Resistance of the lamp = R = V/I = 100/10 = 10 Ω

Step II: When connected to 200 V, 50 Hz ac supply, inductor is connected in series to keep the value of current same as before.

Impedance = $Z = V/I$ = 200/10 = 20 Ω

$$Z = \sqrt{R^2 + X_L^2}$$

Or, $$20 = \sqrt{10^2 + X_L^2}$$

Or, $$20^2 - 10^2 = X_L^2 \qquad 1$$

Or, $$X_L^2 = \sqrt{300}$$

Or, $$\omega L = \sqrt{300}$$

Or, $$2\pi f L = \sqrt{300}$$

Or, $$L = \frac{\sqrt{300}}{2\pi f}$$

Or, $$L = \frac{\sqrt{300}}{2\pi \times 50}$$

∴ Inductance = L = 0.055 H **1**

Step III. The resistance required to replace the choke = 20 – 10 = 10 Ω

Step IV. Use of choke will be better.

Total power supplied when choke is connected = VIcosϕ

$$= VI \times \frac{R}{Z}$$

$$= 200 \times 10 \times \frac{10}{20}$$

= 1000 W **½**

Step V. Total power supplied when a resistor is connected replacing the choke = VI = 200 × 10 = 2000 W

Since power supply is less when choke is connected then use of choke is the better option. **1**

Mnemonics

Concept: A.C. in pure inductive and capacitive circuit
Mnemonics: **C**hocolate **C**ookies are **V**ery **I**nteresting !
Interpretation: **C**hocolate: **C**urrent leads **C**ookies are: in **C**apacitive circuit **V**ery: **V**oltage leads **I**nteresting !: in **I**nductive circuit

Objective Type Questions

(1 mark each)

[A] Very Short Answer Type Questions

Q. 1. An alternating current from a source is given by i=10 sin 314 t. What is the effective value of current and frequency of source?

U [SQP 2020-21]

Ans. 7.07A, 50Hz ½ + ½

Detailed Answer:

$$I_{eff} = I_0/\sqrt{2}$$
$$= 10/\sqrt{2}$$
$$= 7.07\text{A} \qquad ½$$

Angular frequency = ω = 314

Or, $2\pi f = 314$

Or, $f = 314/2\pi$ ½

$\therefore$ f = 50 Hz

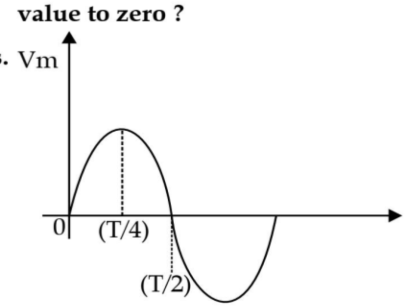
Q. 2. Why alternating current cannot be measured by a d.c. ammeter ? U

Ans. The full cycle of alternating current consists of two half cycles. For one half, current is positive and for second half, current is negative. Therefore, for an a.c. cycle, the net value of current average out to zero.

Hence, the alternating current cannot be measured by *DC* ammeter. 1

Q. 3. An a.c. source is rated at 220 V, 50 Hz. What is the time taken for the voltage to change from its peak value to zero ?

Ans.



Time to change from peak value to zero is

$$t = \left(\frac{T}{2}\right) - \left(\frac{T}{4}\right) = \left(\frac{2T}{8}\right) = \left(\frac{T}{4}\right)$$

Given f = 50 Hz, hence $T = \left(\frac{1}{50}\right)$sec

$$\therefore \qquad t = \left(\frac{T}{4}\right)$$
$$= \left[\frac{1}{\{50 \times 4\}}\right]$$
$$= \left(\frac{1}{200}\right)s \qquad 1$$

Commonly Made Error

- The students are confused about the basic equations of the alternating current variation with time.

Answering Tip

- Carefully observe the graph and then relate the given time periods to the equation of the alternating current.

Q. 4. The frequency of an alternating voltage is 50 cycles/sec and its amplitude is 120 V. Find rms value of current. A

Ans. The frequency of an alternating current voltage, f = 50 cycles /sec

Amplitude of the ac voltage (V_0) = 120V

$$V_{rms} = \frac{V_0}{\sqrt{2}} \qquad ½$$

where V_{rms} denotes root mean square value of voltage , V_0 is the peak value of voltage or amplitude. So,

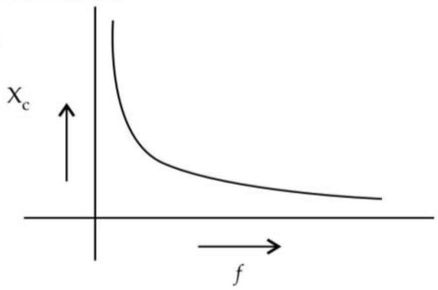
$$V_{rms} = 60\sqrt{2}$$
$$= 60 \times 1.414$$
$$= 84.84 \text{ V} \qquad ½$$

Q. 5. What is the average or mean value of current for half cycle of an a.c. ? R

Ans. $0.637I_m$ **1**

AI **Q. 6. Plot a graph showing variation of capacitive reactance with the change in the frequency of the *ac* source.** U

Ans.



1

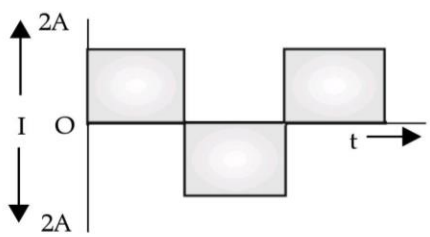
Q. 7. Can ever the *rms* value be equal to the peak value of an ac? U

Ans. Yes, the *rms* value of an ac is equal to the peak value in case of a square wave only but it is not possible in case of sinusoidal wave form. **1**

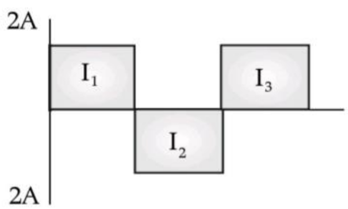
Q. 8. When an alternating current is passed through a moving coil galvanometer, it shows no deflection. Why? R

Ans. The moving coil galvanometer is designed so as to measure the average value of the current, which is zero for the complete cycle of ac. Hence, no deflection is shown by the galvanometer. **1**

Q. 9. Calculate the *rms* value of the alternating current as shown in the figure: A



Ans.



$$I_{rms} = \sqrt{\frac{(2)^2 + (-2)^2 + 2^2}{3}}$$

$$= \sqrt{\frac{12}{3}} = \sqrt{4} = 2\text{ A}$$ **1**

Q. 10. In an oscillating LC circuit the maximum charge on the capacitor is Q. Find the amount of charge on the capacitor when the energy is stored equally between the electric and magnetic field.

Ans. Let Q denote maximum charge on capacitor.

Let q denote charge when energy is equally shared

$$\therefore \quad \frac{1}{2}\left(\frac{1}{2}\frac{Q^2}{C}\right) = \frac{1}{2}\frac{q^2}{C} \Rightarrow Q^2 = 2q^2$$

$$\therefore \quad q = \frac{Q}{\sqrt{2}}.$$ **1**

Commonly Made Error

- The students are not able to apply the formula for the energy stored in the capacitor in terms of its charge and capacitance.

Answering Tip

- The factor of 1/2 should be carefully taken into account while performing the calculations.

Q. 11. What are the dimensions of the impedance?

(a) $[ML^2T^{-3}I^{-2}]$ **(b)** $[M^{-1}L^{-2}T^3I^2]$
(c) $[ML^3T^{-3}I^{-2}]$ **(d)** $[M^{-1}L^{-3}T^3I^2]$

Ans. Impedance has the same dimensions as that of the resistance:

$$R = \frac{V}{I}$$

$$= \frac{ML^2T^{-3}I^{-1}}{I}$$

$$= [ML^2T^{-3}I^{-2}]$$ **1**

[B] ASSERTION REASON TYPE QUESTIONS

For the following questions, two statements are gives one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is NOT the correct explanation of A

(c) A is true but R is false

(d) A is false and R is also false

Q. 1. Assertion (A): An alternating current does not show any magnetic effect.

Reason (R): Alternating current changes direction with time.

Ans. (d) **1**

Explanation: Current or moving charged particle creates magnetic field irrespective of direct current or alternating current. So assertion is false. Alternating current changes direction with time. So, the reason is true, but cannot explain the assertion.

Short Answer Type Questions-I

(2 marks each)

Q. 1. If a charged capacitor C is short circuited through an inductor L, the charge and current in the circuit oscillate simple harmonically.

(a) In what form the capacitor and the inductor store energy ?

(b) Write two reasons due to which the oscillations become damped. R

Ans. (a) The capacitor stores energy in the form of electric field and the inductor stores energy in the form of magnetic field. **1**

(b) Oscillations become damped due to:

(i) Resistance of the circuit ½

(ii) Radiation in the form of Em waves. ½

Q. 2. The current through a resistor is 2 A when connected to a 220 V, 50 Hz line. Find the value of capacitor which is to be connected to reduce the current to 1A. U

Ans. To reduce the current from 2 A to 1A, the impedance is to be doubled.

When only R is present, then $R = V/I = 220/2 = 110\ \Omega$

Now a capacitor C is connected in series and the impedance becomes 220 Ω.

$$Z = \sqrt{R^2 + X_C^2}$$

Or, $$220 = \sqrt{110^2 + X_C^2}$$ **1**

Or, $$220^2 = 110^2 + X_C^2$$

Or, $$X_C = 110\sqrt{3}$$

Or, $$\frac{1}{2rfC} = 110\sqrt{3}$$

Or, $$C = \frac{1}{2r110\sqrt{3}}$$

∴ $$C = 16.7\ \mu f$$ **1**

AI **Q. 3. An alternating voltage $V = 140 \sin 314t$ is connected across a 50 Ω resistor. Find**

(i) The frequency of the source

(ii) The r.m.s. current through the resistor. A

Ans. (i) Comparing the given equation of voltage with

$$V = V_0 \sin \omega t$$

$$\omega = 314$$

or, $$2\pi f = 314$$

∴ $$\text{Frequency} = f = \frac{314}{2r} = 50\text{ Hz}$$ **1**

(ii) $$I_{RMS} = \frac{V_{RMS}}{R}$$

Or, $$I_{RMS} = \frac{V_0}{\sqrt{2}}$$

Or, $$I_{RMS} = \frac{140}{\sqrt{2} \times 50}$$

∴ $$I_{RMS} = 1.98\text{ A}$$ **1**

Short Answer Type Questions-II

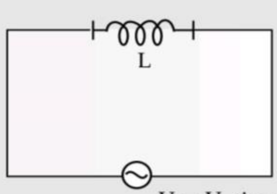
(3 marks each)

AI **Q. 1. A source of *ac* voltage $V = V_0 \sin \omega t$, is connected across a pure inductor of inductance L. Derive the expressions for the instantaneous current in the circuit. Show that average power dissipated in the circuit is zero.** U **[Foreign I, II, III 2017]**

Ans. Derivation of instantaneous current **2**
Derivation of average power dissipated **1**

Given: $V = V_0 \sin \omega t$

$$V = L\frac{di}{dt} \Rightarrow di = \frac{V}{L}dt$$



V = V_0 sin ωt

∴ $$di = \frac{V_0}{L}\sin \omega t dt$$ ½

Integrating, $$i = -\frac{V_0}{\omega L}\cos \omega t$$ ½

$$\therefore i = -\frac{V_0}{\omega L}\sin\left(\frac{\pi}{2} - \omega t\right) = I_0 \sin\left(\frac{\pi}{2} - \omega t\right)$$ ½

where, $$I_0 = \frac{V_0}{\omega L}$$ ½

Average power,

$$P_{av} = \int_0^T V i dt$$

$$= \frac{-V_0^2}{\omega L}\int_0^T \sin \omega t \cos \omega t\, dt$$

$$= \frac{-V_0^2}{2\omega L}\int_0^T \sin(2\omega t) dt$$

$$= 0$$ **1**

[CBSE Marking Scheme, 2017]

Long Answer Type Questions (5 marks each)

AI Q. 1. (a) Derive the expression for the current flowing in an ideal capacitor and its reactance when connected to an ac source of voltage $V=V_0 \sin\omega t$.

(b) Draw its phasor diagram.

(c) If resistance is added in series to capacitor what changes will occur in the current flowing in the circuit and phase angle between voltage and current. A [SQP 2020-21]

Ans. (a) Derivation of instantaneous current

$$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right) \quad 1$$

Reactance, $X_C = \dfrac{1}{\omega C}$ 1

(b) Phasor diagram showing v and i relation in pure C 1

(c) Explanation that adding R it will behave RC series ac circuit.

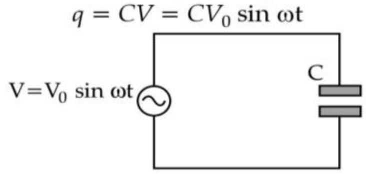
Calculation of current and phase angle. 1 + 1

Detailed Answer:

(a) An alternating voltage $V = V_0 \sin \omega t$ is supplied across the two ends of a capacitor of capacitance C.

At any instant the charge on capacitor $= q$

$$q = CV = CV_0 \sin \omega t$$



1

(b) Current $= I = dq/dt$

$$= \frac{d}{dt}(CV_0 \sin \omega t)$$

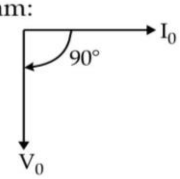
$$= \omega C V_0 \cos \omega t$$

$$= \frac{V_0}{1/\omega C}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \left[I_0 = \frac{V_0}{1/\omega C}\right]$$

Reactance $= X_C = 1/\omega C$ 1

Phasor diagram:



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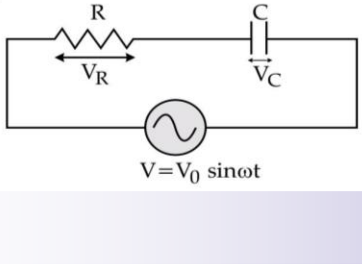
(c) A resistor is now connected with the capacitor in series:



Peak voltage drop across R is I_0R

Peak voltage drop across C is I_0X_C.

Voltage cross R is in phase with the current.

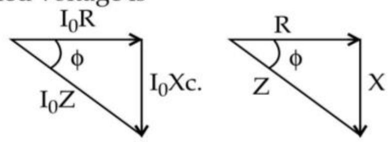
Voltage across C lags the current by 90°.

So, the voltage drops across R and across C are also not in phase. They are also out of phase by 90°.

So, $$V_0 = \sqrt{(I_0R)^2 + (I_m X_C)^2}$$

$$\therefore \quad I_0 = \frac{V_0}{\sqrt{R^2 + X_C^2}} \quad 1$$

The phase angle by which the current leads the applied voltage is



Phase Angle $= \phi = \tan^{-1}\dfrac{X_C}{R}$ 1

Q. 2. A device X is connected across an ac source of voltage $V = V_0 \sin \omega t$. The current through X is given as $I = I_0 \sin\left(\omega t + \dfrac{\pi}{2}\right)$.

(a) Identify the device X and write the expression for its reactance.

(b) Draw graphs showing variation of voltage and current with time over one cycle of ac, for X.

(c) How does the reactance of the device X vary with frequency of the ac ? Show this variation graphically.

(d) Draw the phasor diagram for the device X.

U [Delhi & OD, 2018]

Ans. (a) Identification of device and its reactance. ½+½

(b) Graphs of voltage and current with time 1+1

(c) Variation of reactance with frequency ½

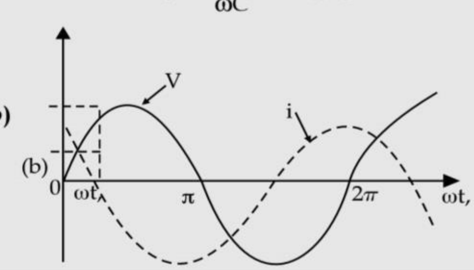
(Graphical variation) ½

(d) Phasor Diagram 1

(a) X: Capacitor ½

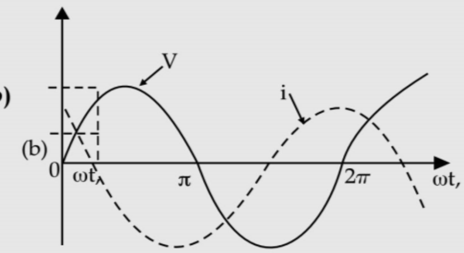
Reactance, $X_c = \dfrac{1}{\omega C} = \dfrac{1}{2\pi VC}$ ½

(b)



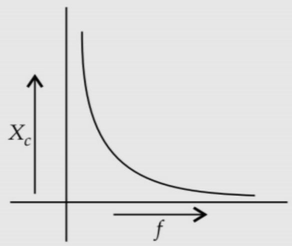
½+½

(b)



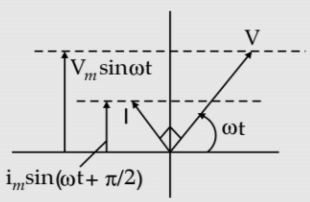
½+½

(c) Reactance of the capacitor varies in inverse proportion to the frequency *i.e.*, $X_c \propto \frac{1}{f}$ **1**



1

(d)



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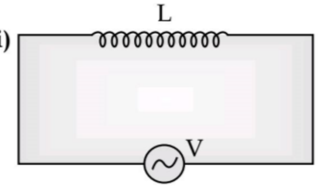
[CBSE Marking Scheme, 2018]

Q. 3. (i) Prove that current flowing through an ideal inductor connected across an ac source lags the voltage by π/2.

(ii) An inductor of self inductance 100 mH and a bulb are connected in series with an ac source of rms voltage 10 V, 50 Hz. It is found that effective voltage of the circuit leads the current by phase π/4.

Calculate the inductance of the inductor used and average power dissipated in the circuit, if a current of 1 A flows in the circuit. R & A

Ans. (i)



$$\text{Induced emf} = e = -L\frac{di}{dt}$$

$$\text{Net voltage} = V - L\frac{di}{dt}$$ **1**

Applying Kirchhoff's law:

$$V - L\frac{di}{dt} = 0$$

$$V_m \sin\omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin\omega t\,dt$$

$$i = -\frac{V_m}{\omega L}\cos\omega t$$ **1**

$$i = -\frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i = i_m \sin(\omega t - \frac{\pi}{2})$$

Hence current lags by $\frac{\pi}{2}$ **1**

(ii) $L = 100$ mH

Average power dissipation

$$= V_{RMS}\, i_{RMS} \cos\varphi$$ **1**

$$= 10 \times 1 \times \cos\frac{\pi}{4}$$

$$= \frac{10}{\sqrt{2}} \text{W}$$

$$= 5\sqrt{2}\ \text{W}$$ **1**

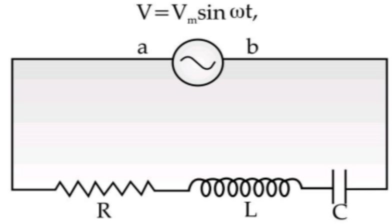
TOPIC-2
LCR Series Circuit

Revision Notes

***LCR* series circuit**

➤ In an LCR series circuit with resistor, inductor and capacitor, the expression for the instantaneous potential difference between the terminals a and b is given as

$$V = V_m \sin\omega t,$$



- The potential difference in this will be equal to the sum of the magnitudes of potential differences across R, L and C elements as

$$V = V_m \sin \omega t = RI + L\frac{dI}{dt} + \frac{1}{C}q$$

where, q is the charge on capacitor.

- The steady state situation will be

$$i = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t - \phi) \text{ and } i_m = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)}}$$

where, $\phi = \tan^{-1} \dfrac{\omega L - \frac{1}{\omega C}}{R}$

- From the equation, steady-state current varies sinusoidal with time, so steady-state current can be written as $I = I_m \sin (wt - f)$
- In an LCR circuit:

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$X = X_L - X_C = \omega L - \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + X^2}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_m}{\sqrt{R^2 + X^2}} = \frac{V_m}{Z}$$

Here, Z = Impedance of the circuit, X = Reactance of the circuit, X_L and X_C = Inductive and Capacitive reactance.

- For steady-state currents, maximum current I_m is related to maximum potential difference V_m by

$$I_m = \frac{V_m}{Z}$$

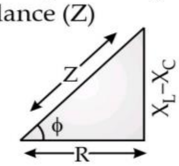
- Total effective resistance of LCR circuit is called Impedance (Z) of the circuit given as

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- The angle by which alternating voltage leads the alternating current in LCR circuit is given by

$$\tan \phi = \frac{X_L - X_C}{R}$$

- In an LCR circuit, impedance triangle is a right-angled triangle in which base is ohmic resistance R, perpendicular is reactance $(X_L - X_C)$ and hypotenuse is impedance (Z)



- When a condenser of capacity C charged to certain potential is connected to inductor L, energy stored in C oscillates between L and C where frequency of energy oscillations is given by

$$X_L = X_C \quad \text{or} \quad f = \frac{1}{2\pi\sqrt{LC}}$$

- In LCR circuit, if there is no loss of energy, then total energy in L and C at every instant will remain constant.
- Sign for phase difference (f) between I and E for a series LCR circuit:

ϕ is positive, when $X_L > X_C$.

ϕ is negative, when $X_L < X_C$.

ϕ is zero, when $X_L = X_C$.

$\phi = \pi/2$, when $\omega = \infty$.

$\phi = -\pi/2$, when $\omega = 0$.

Resonance

- Circuit in which inductance L, capacitance C and resistance R are connected in series and the circuit admits maximum current, such circuit is called as series resonant circuit.
- The necessary condition for resonance in LCR series circuit is: $V_C = V_L$

  $X_L = X_C$ which gives $\omega^2 = \frac{1}{LC}$ or $f = \frac{1}{2\pi\sqrt{LC}}$
- In this, frequency of ac fed to circuit will be equal to natural frequency of energy oscillations in the circuit under conditions,

$$Z = R$$

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{R}$$

- The sharpness of tuning at resonance is measured by Q factor or quality factor of the circuit given as

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

- At series LCR resonance or acceptor circuit, current is maximum.

$$I_{max} = \frac{E}{R}$$

Power in AC circuits

- When the current is out of phase with the voltage, the power indicated by the product of the applied voltage and the total current gives apparent power.
- If the instantaneous values of the voltage and current in an ac circuit are given by

$$E = E_0 \sin \omega t$$

$$i = i_0 \sin (\omega t - \phi)$$

  where ϕ is the phase difference between voltage and the current. Then, the instantaneous power

$$P_{in} = E \times i = E_0 i_0 \sin \omega t . \sin (\omega t - \phi)$$

  or average power

$$P_{avg} = \frac{1}{2} E_0 i_0 \cos\phi$$

$$= \frac{E_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} \cos\phi = V_{rms} \times I_{rms} \times \cos \phi$$

  where, $\cos \phi$ is known as power factor.
- Power factor ($\cos \phi$) is important in power systems as it shows how closely the effective power equals the apparent power which is given as:

$$\cos \phi = \frac{\text{Effective power}}{\text{Apparent power}}$$

- The value of power factor varies from 0 to 1.
- The instantaneous rate at which energy is supplied to an electrical device by ac circuit is

$$P = VI$$

- Average power in LCR where, $X_L = X_C$ over a complete cycle in a non-inductive circuit or pure resistive circuit is given as $P = V_0 I_0$ or $I_0^2 R$

Wattless Current

- The average power associated over a complete cycle with pure inductor or pure capacitor is zero which makes current through L and C as wattless or idle current.
- In LCR circuit at resonance, the power loss is maximum, so

  Wattless component of current $= I_{rms} \sin \phi$

  Power component of current $= I_{rms} \cos \phi$
- Phase angle: It is the amount by which the voltage and current are out of phase with each other in a circuit.
- Power factor: It is the amount by which the power delivered in the circuit which is less than the theoretical maximum value of the circuit due to voltage and current being out of phase.
- Quality factor: It is a dimensionless quantity that shows sharpness of the peak of bandwidth.
- Resonant frequency: It is the frequency at which the amplitude of the current is maximum where circuit oscillates when not driven by voltage source.

Know the Formulae

➢ Impedance for a series *LCR* circuit,

$$Z = \sqrt{R^2 + X^2} = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}.$$

➢ Average power, $$P = \frac{E_0 I_0}{2}\cos\phi = V_{rms} I_{rms}\cos\phi$$

➢ Power factor, $$\cos\phi = \frac{\text{Resistance}}{\text{Impedance}} = \frac{R}{Z} = \frac{\text{True power}}{\text{Apparent power}}$$

➢ Quality factor $$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Objective Type Questions

(1 mark each)

[A] Very Short Answer Type Questions

Q. 1. When the selectivity of a series LCR circuit will be large? U **[CBSE DEL SET 1, 2020 MODIFIED]**

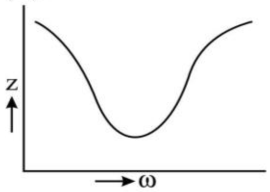
Ans. When L is large and R is small. 1

Q. 2. What is the power factor of a series LCR circuit at resonance? U **[CBSE DEL SET 1, 2020 MODIFIED]**

Ans. The power factor of a series LCR circuit at resonance is unity. 1

AI **Q. 3. A series combination of an inductor (L), capacitor (C) and a resistor (R) is connected across an a.c. source of emf of peak value E_0 and angular frequency (ω). Plot a graph to show variation of impedance of the circuit with angular frequency (ω).** U **[CBSE DEL SET, 2020]**

Ans.



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Q. 4. What is the phase difference between the current and voltage in a series LCR circuit at resonance? U **[CBSE DEL SET 2, 2020 MODIFIED]**

Ans. At resonance, phase difference is zero.

Q. 5. For an ideal conductor, connected across a sinusoidal ac voltage source. State which one of the following quantity is zero:

(i) Instantaneous power

(ii) Average power over full cycle of the ac voltage source. U **[Foreign 2016]**

Ans. Average power over full cycle of *ac* voltage source is zero. 1

[CBSE Marking Scheme, 2016]

Q. 6. The power factor of an *ac* circuit is 0.5. What is the phase difference between voltage and current in the circuit ? A **[O.D. I, 2016]**

Ans. Phase difference = $\cos^{-1}(0.5) = 60^0$

[B] ASSERTION REASON TYPE QUESTIONS

For the following questions, two statements are given-one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is NOT the correct explanation of A

(c) A is true but R is false

(d) A is false and R is also false

1. Assertion (A): The dimension of L/R is time.

Reason (R): Time constant (L/R) should be increased to reduce the rate of increase of current through a solenoid.

Ans. (b) 1

Explanation: For a solenoid, the magnitude of induced emf

$$e = L\frac{di}{dt}$$

$$i = \frac{e}{R} = \left(\frac{L}{R}\right)\left(\frac{di}{dt}\right)$$

$$\frac{di}{dt} = \frac{i}{\frac{L}{R}}$$

In left hand side of the above equation, denominator is time. So, in right hand side the denominator should be time. So, dimension of LR is time.

So, the assertion is true.

If *L/R* increases, *di/dt* decreases.

So, reason is also true.

But reason cannot properly explain the assertion.

Short Answer Type Questions-I

(2 marks each)

Q. 1. A resistor R and an inductor L are connected in series to a source of voltage $V = V_0 \sin \omega t$. The voltage is found to lead current in phase by $\frac{\pi}{4}$. If the inductor is replaced by a capacitor C, the voltage lags behind current in phase by $\frac{\pi}{4}$. When L, C and R are connected in series with the same source, Find the:

(i) average power dissipated and

(ii) instantaneous current in the circuit

Ans. For LR circuit — A [CBSE OD SET 1, 2020]

$$\frac{X_L}{R} = \tan 45^\circ = 1$$

So, $X_L = R$

For CR circuit

$$\frac{X_C}{R} = \tan 45^\circ = 1$$

So, $X_c = R$

For LCR circuit

When L, C and R are connected in series, then actually three R are connected in series. Equivalent impedance $= 3R$. Thus Circuit is resistive.

$$V = V_0 \sin \omega t$$

and $I = I_0 \sin \omega t$ **1**

(i) Average power dissipation, $P = VI$

$$P = V_0 \sin \omega t \times I_0 \sin \omega t$$

Over full cycle,

$$P_{AVG} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$ ½

(ii) Instantaneous current, $I = I_0 \sin \omega t$ ½

Q. 2. In a series LCR circuit, obtain the conditions under which

(i) impedance of the circuit is minimum.

(ii) wattless current flows in the circuit. A

Ans. (i) Impedance of LCR circuit $= Z = \sqrt{R^2 + (X_L - X_C)^2}$

Impedance will be minimum when $(X_L - X_C)^2 = 0$,

or, $X_L = X_C$

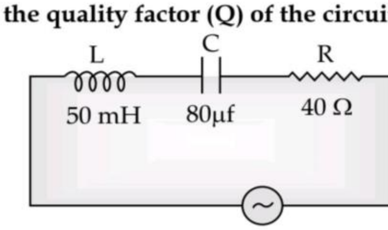
or, $\omega = \frac{1}{\sqrt{LC}}$ **1**

(ii) For wattless current to flow, the circuit should not have any Ohmic resistance *i.e.* $R = 0$ **1**

Q. 3. The figure shows a series LCR circuit connected to a variable frequency 200 V ac source.

Find (i) the source frequency which drives the circuit to resonance.

(ii) the quality factor (Q) of the circuit. A



Ans. (i) Resonance frequency $= \omega_0 = \frac{1}{\sqrt{LC}}$

or, $\omega_0 = \frac{1}{\sqrt{50 \times 10^{-3} \times 80 \times 10^{-6}}}$

or, $\omega_0 = 500$

or, $2\pi f = 500$

$\therefore$ $f = \frac{500}{2\pi} = 80 \text{ Hz}$ **1**

(ii) Quality factor $= Q = \frac{\omega_0 L}{R}$

or, $Q = \frac{500 \times 50 \times 10^{-3}}{40}$

$\therefore$ $Q = 0.625$ **1**

Short Answer Type Questions-II

(3 marks each)

Q. 1. A capacitor (C) and resistor (R) are connected in series with an ac source of voltage of frequency 50 Hz. The potential difference across C and R are respectively 120 V, 90 V, and the current in the circuit is 3 A. Calculate (i) the impedance of the circuit (ii) the value of the inductance, which when connected in series with C and R will make the power factor of the circuit unity.

R [CBSE OD SET 1, 2019]

Ans. Calculation of impedance 2

Calculation of inductance 1

(i) $Z = \sqrt{R^2 + X_C^2}$ ½

$R = \frac{V_R}{I_R} = 30\ \Omega$ ½

$X_C = \frac{V_C}{I_C} = \frac{120}{30} = 40\ \Omega$ ½

$$Z = \sqrt{(30)^2+(40)^2} = 50\ \Omega \quad ½$$

$$X_C = X_L$$

(ii) As power factor = 1

$$100\pi L = 40 \quad ½$$

$$L = \frac{2}{5\pi}\text{ henry} \quad ½$$

[CBSE Marking Scheme, 2019]

Detailed Answer:

Given: In *R-C* circuit,

Source frequency, $f = 50$ Hz

Potential difference across '*C*', $V_C = 120$ V

Potential difference across '*R*', $V_R = 90$ V

Circuit current $(i) = 3$ A

(i) Impedance of the circuit (Z)

Since, $V^2 = V_R^{\,2} + V_C^{\,2}$

$(iZ)^2 = V_R^{\,2} + V_C^{\,2}$

or, $Z = \dfrac{\sqrt{V_R^{\,2}+V_C^{\,2}}}{i}$

$= \dfrac{\sqrt{(90)^2+(120)^2}}{3}$

$Z = 50\ \Omega$ **2**

(ii) Let the inductance '*L*' be added in series with R and *C* to make power factor unity.

So, Power factor, $P = \dfrac{R}{Z}$

$1 = \dfrac{R}{\sqrt{R^2+(X_L-X_C)^2}}$

or, $R^2 = R^2 + (X_L - X_C)^2$

or, $X_L = X_C$ ...(i)

Since, $V_C = iX_C$

or, $X_C = \dfrac{V_C}{i} = \dfrac{120\text{ V}}{3\text{ A}} = 40$ W

Hence, from equation (i),

$X_L = 40$

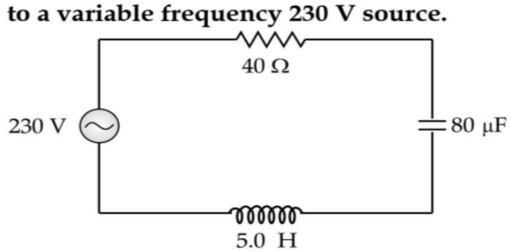
$2\pi fL = 40$

So, $L = \dfrac{40}{2\pi f}$

$= \dfrac{40}{2\times\pi\times 50}$

$L = 0.13$ H **1**

Q. 2. The figure shows a series LCR circuit connected to a variable frequency 230 V source.



(a) Determine the source frequency which drives the circuit in resonance.

(b) Calculate the impedance of the circuit and amplitude of current at resonance.

(c) Show that potential drop across LC combination is zero at resonating frequency.

A **[CBSE OD SET 1, 2019]**

Ans. Determining the source frequency **1**

Calculating impedance ½

For showing potential drop across LC 1 ½

(a) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{5\times 80\times 10^{-6}}} = \dfrac{1}{\sqrt{400\times 10^{-6}}}$

$\omega = \dfrac{1000}{20} = 50\,\text{Hz}$ **1**

(b) $Z = R = 40\ \Omega$

$\omega = \dfrac{1000}{20} = 50\,\text{Hz}$ ½

$I_m^{\max} = \dfrac{230\sqrt{2}}{R} = \dfrac{230\sqrt{2}}{40} = 8.1\,A$ ½

$V_c = I_m^{\max} X_c = \dfrac{230\sqrt{2}}{40}\times\dfrac{1}{\omega C} = 2033$ volt

$V_L = I_m^{\max} X_L = \dfrac{230\sqrt{2}}{40}\times 2\pi\nu\, L = 2033$ volt ½

(c) $V_C - V_L = 0$ ½

[CBSE Marking Scheme, 2019]

Detailed Answer:

Given: Source voltage, $V = 230$ V

$R = 40\ \Omega$, $C = 80$ mF, $L = 5.0$ H

(a) At resonance condition,

$V_C = V_L$

or, $iX_C = iX_L$

or, $\dfrac{1}{2\pi fC} = 2\pi fL$

or, $f = \dfrac{1}{2\pi\sqrt{LC}}$

$= \dfrac{1}{2\pi\sqrt{(5.0)\times(80\times 10^{-6})}}$ Hz

$= 7.96$ Hz **1**

(b) At resonance, $X_C = X_L$

So, the impedance of the circuit

$Z = \sqrt{R^2+(X_L-X_C)^2}$

or, $Z = R$ [as $X_L = X_C$]

or, $Z = 40$ W

Since, $I_{rms} = \dfrac{V_{in}}{Z}$

Amplitude of current, $I_0 = \sqrt{2}\ I_{rms} = \sqrt{2}\times\dfrac{V_{in}}{Z}$

$= \sqrt{2}\times\dfrac{230\text{ V}}{40\ \Omega} = 8.13$ A **1**

(c) At resonance condition,

$$X_L = X_C$$

or, $iX_L = iX_C$

or, $V_L = V_C$

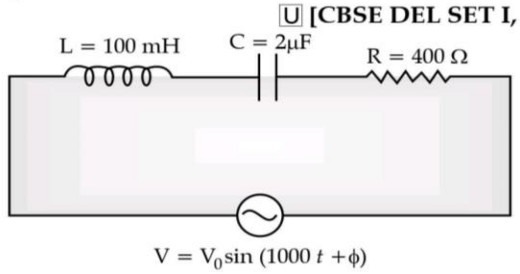
or, the potential drop across LC combination

$$V_L - V_C = 0. \qquad 1$$

Q. 3. (i) Find the value of the phase difference between the current and the voltage in the series *LCR* circuit shown below. Which one leads in phase: current or voltage ?

(ii) Without making any other change, find the value of the additional capacitor C_1, to be connected in parallel with the capacitor *C*, in order to make the power factor of the circuit unity.

U [CBSE DEL SET I, 2017]



Ans. (i) Calculation of phase difference between current and voltage **1**

Name of quantity which leads ½

(ii) Calculation of value of 'C_1', is to be connected in parallel **1½**

(i) $X_L = \omega L = (1000 \times 10^{-3})\ \Omega$

$= 100\ \Omega$

$$X_C = \frac{1}{\omega C} = \left(\frac{1}{1000 \times 2 \times 10^{-6}}\right)\Omega$$

$= 500\ \Omega$ ½

Phase angle

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\tan\phi = \frac{100 - 500}{400} = -1$$

$$\phi = -\frac{\pi}{4} \qquad ½$$

As $X_C > X_L$, ϕ phase angle is negative), hence current leads voltage ½

(ii) To make power factor unity,

$X_C' = X_L$

$$\frac{1}{\omega C'} = 100 \qquad ½$$

$C' = 10\ \mu F$ ½

$C' = C + C_1$

$10 = 2 + C_1$

$C_1 = 8\ \mu F$ ½

[CBSE Marking Scheme, 2017]

Q. 4. A capacitor of unknown capacitance, a resistor of 100Ω and an inductor of self inductance $L = (4/\pi^2)$ henry are connected in series to an *ac* source of 200 V and 50 Hz. Calculate the value of the capacitance and impedance of the circuit when the current is in phase with the voltage. Calculate the power dissipated in the circuit.

U [O.D. I, 2016]

Ans. $R = 100\ \Omega$, $L = 4/\pi^2$ H, $V_{RMS} = 200$ V, $f = 50$ Hz

When current and voltages are in phase,

$$2\pi f L = \frac{1}{2\pi f C} \qquad ½$$

or, $$2\pi \times 50 \times \frac{4}{\pi^2} = \frac{1}{2\pi \times 50\ C}$$

$\therefore$ $$C = \frac{1}{4} \times 10^{-4}\ \text{F} \qquad 1$$

When the current is in phase with the voltage then, $Z = R = 100\ \Omega$

$\therefore$ $$\text{Current} = i_{rms} = \frac{V_{rms}}{R} \qquad ½$$

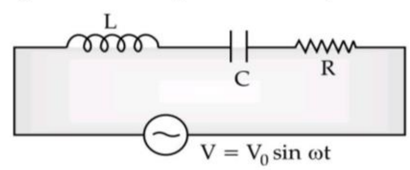
$$= \frac{200}{100} = 2\ \text{A} \qquad ½$$

$$\text{Power} = 200 \times 2 \times \cos 0°$$

$$= 400\ \text{W} \qquad ½$$

Q. 5. The current, in the *LCR* circuit shown in the figure is observed to lead the voltage in phase. Without making any other change in the circuit, a capacitor, of capacitance C_0, is (appropriately) joined to the capacitor *C*. This results in making the current, in the 'modified' circuit, flow in phase with the applied voltage.

Draw a diagram of the 'modified' circuit and obtain an expression for C_0 in terms of ω, *L* and *C*.



U [Foreign 2016]

Ans. The current leads the voltage in phase.

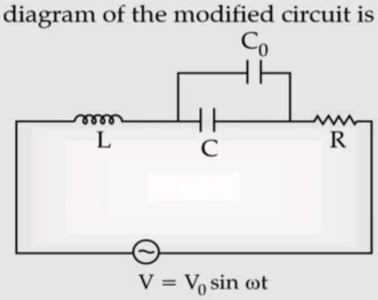
Hence, $X_c > X_L$

For resonance, we must have ½

New value of $X_c = X_L$

We, therefore, need to decrease $X_c = \left(\frac{1}{\omega C}\right)$. This requires an increase in the value of *C*. Hence, capacitor C_0 should be connected in parallel across *C*. ½

The diagram of the modified circuit is as shown.



1

For resonance, we have

$$\frac{1}{\omega(C+C_0)} = \omega L \qquad \frac{1}{2}$$

$$\therefore \quad C_0 = \left[\frac{1}{\omega^2 L} - C\right] \qquad \frac{1}{2}$$

[CBSE Marking Scheme, 2016]

Q. 6. A 200 mH (pure) inductor and a 5 μF (pure) capacitor are connected one by one, across a sinusoidal *ac* voltage source of $V = [70.7\sin(1000\,t)]$ voltage.
Obtain the expression for the current in each case. **A [Foreign, 2016]**

Ans. For the applied voltage

$V = 70.7\sin(1000\,t)$, we have

$$V_0 = 70.7 \text{ volts} \qquad \frac{1}{2}$$

$$\omega = 1000 \text{ rad/s} \qquad \frac{1}{2}$$

For the inductor

$$i_o = \frac{V_0}{\omega L} = \frac{70.7}{1000\times 200\times 10^{-3}} \text{ A}$$

$$= 35.35 \times 10^{-2} \text{ A}$$

$$= 0.3535 \text{ A} \qquad \frac{1}{2}$$

∴ Expression for current is

$$i = (0.3535)\sin\left(1000\,t - \frac{\pi}{2}\right) \qquad \frac{1}{2}$$

For the capacitor

$$i_0 = \frac{V_0}{\left(\frac{1}{\omega C}\right)} = V_0 \,.\, \omega C$$

$$= 70.7 \times 1000 \times 5 \times 10^{-6} \text{ A}$$

$$= 353.5 \times 10^{-3} \text{ A} = 0.3535 \text{ A} \qquad \frac{1}{2}$$

∴ Expression for current is

$$I = 0.3535\sin\left(1000t + \frac{\pi}{2}\right) \qquad \frac{1}{2}$$

[CBSE Marking Scheme, 2016]

Q. 7. Derive the expression for the average power dissipated in a series *LCR* circuit for an *ac* source of a voltage, $V = V_m \sin \omega t$, carrying a current, $i = i_m \sin(\omega t + \phi)$
Hence define the term "Wattless current". State under what condition it can be realized in a circuit. **U [Delhi Comptt. 2016]**

Ans.

$$V = V_m \sin \omega t$$

$$i = i_m \sin(\omega t + \phi)$$

Power at any instant,

$$P = Vi = V_m i_m \sin \omega t \sin(\omega t + \phi)$$

$$P = \frac{V_m i_m}{2}[\cos\phi - \cos(2\omega t + \phi)] \qquad \frac{1}{2}$$

The average of second term in the above expression is zero over a full cycle. ½

$$\therefore \text{ Average Power} = \overline{P} = \frac{V_m i_m}{2}\cos\phi$$

$$\overline{P} = \frac{V_m}{\sqrt{2}} \times \frac{i_m}{\sqrt{2}}\cos\phi \qquad \frac{1}{2}$$

$$\therefore \quad \overline{P} = V_{rms} I_{rms} \cos\phi \qquad \frac{1}{2}$$

Wattless current is the current which flows in the circuit but no power dissipation occurs. ½

It is realized only when circuit is purely inductive or capacitive, *i.e.*, when $\cos\phi = 0$ or $\phi = \frac{\pm\pi}{2}$ ½

[CBSE Marking Scheme, 2016]

Q. 8. A source of *ac* voltage $V = V_0 \sin \omega t$ is connected to a series combination of a resistor '*R*' and a capacitor '*C*'. Draw the phasor diagram and use it to obtain the expression for (i) impedance of the circuit and (ii) phase angle. **U [O.D. I, II, III 2015]**

Ans. The Pythagoras theorem gives

$$V_m^2 = V_{rm}^2 + V_{cm}^2$$

Substituting the values of V_{rm} and V_{cm} into this equation, gives

$$V_m^2 = (i_m R)^2 + (i_m X_C)^2$$

$$= i_m^2 (R^2 + X_C^2)$$

$$\therefore \quad i_m = \frac{V_m}{\sqrt{R^2 + X_C^2}} \qquad \frac{1}{2}$$

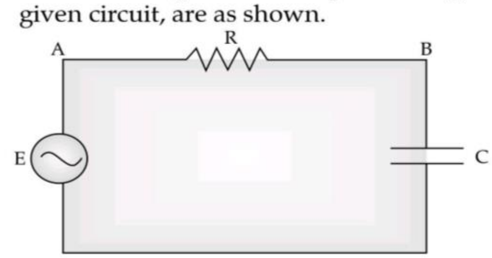
∴ The impedance of the circuit is given by:

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

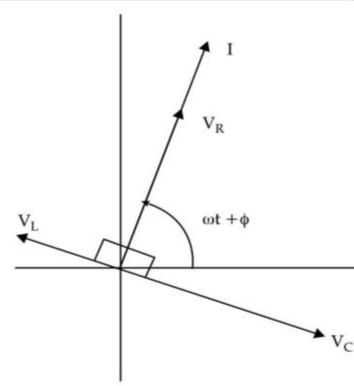
The phase angle is the angle between V_R and V. Hence

$$\tan\phi = \frac{X_C}{R} = \frac{1}{\omega CR} \qquad \frac{1}{2}$$

The circuit diagram and the phasor diagram, for the given circuit, are as shown.



½



½

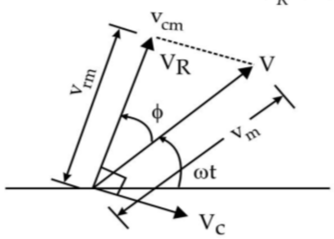
Derivation,

The voltage equation, for the circuit, can be written as:

$$V_R + V_C = V \quad ½$$

The phasor relation, whose vertical component gives the above equation, is

$$V_R + V_C = V$$



½

Q. 9. (i) When an *ac* source is connected to an ideal capacitor show that the average power supplied by the source over a complete cycle is zero.

(ii) A lamp is connected in series with a capacitor. Predict your observation when the system is connected first across a *dc* and then an *ac* source. What happens in each case if the capacitance of the capacitor is reduced ? A&E

Ans. (i) When an ideal capacitor is connected with *ac* source, the current flow continuously but due to dielectric in between the plates of capacitor, there is no current, *i.e.*,

$$I_{avg} = 0$$

$$\therefore \quad P_{avg} = V \times I_{avg}$$

$$P_{avg} = 0 \quad 1$$

(ii) For *dc*, lamp will not shine as capacitor blocks *dc* even if we reduce the capacitance, the lamp will not shine. 1

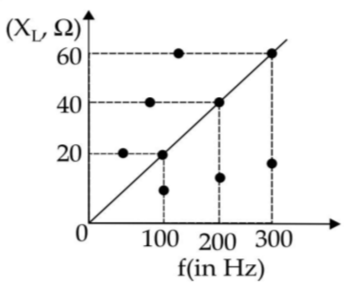
Lamp will shine, if *ac* is used on reducing *C*, with increase of impedance.

Hence, lamp will shine less brightly. 1

Long Answer Type Questions (5 marks each)

Q. 1. (a) Show that an ideal conductor does not dissipate power in an a.c. circuit.

(b) The variation of inductive reactance (X_L) of an inductor with the frequency (*f*) of the a.c. source of 100 V and variable frequency is shown in the figure.



(i) Calculate the self-inductance of the inductor.

(ii) When this inductor is used in series with a capacitor of unknown value and a resistor of 10 Ω at 300 s^{-1}, maximum power dissipation occurs in the circuit. Calculate the capacitance of the capacitor.

Ans. (a) Power dissipation $= P = V_{rms} I_{rms} \cos\phi$ ½

$$\cos\phi = \frac{R}{Z}. \quad ½$$

For ideal inductor $R = 0$

$$\therefore \quad \cos\phi = 0$$

$$\therefore \quad P = V_{rms} I_{rms} \cos\phi = 0$$

Thus, ideal inductor does not dissipate power in an ac circuit. 1

(b) (i) Inductive reactance $= X_L = 2\pi f L$

$$\therefore \quad L = \frac{X_L}{2\pi f}$$

From graph $f = 100$ Hz

$$X_L = 20\,\Omega$$

$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{20}{2\pi \times 100}$$

$$= 0.032\text{ H} = 32\text{ mH} \quad 1½$$

(ii) Power dissipation is maximum when

$$2\pi f L = \frac{1}{2\pi f C}$$

$$f = 300\text{ s}^{-1}$$

$$L = 0.032\text{ H}$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$\text{Or,} \quad 2\pi \times 300 \times 0.032 = \frac{1}{2\pi \times 300 \times C}$$

$$\therefore \quad C = 8.8 \times 10^{-6}\text{ F} = 8.8\,\mu\text{F} \quad 1½$$

Q. 2. (a) In a series *LCR* circuit connected across an ac source of variable frequency, obtain the expression for its impedance and draw a plot showing its variation with frequency of the *ac* source.

(b) What is the phase difference between the voltages across inductor and the capacitor at resonance in the *LCR* circuit ?

(c) When an inductor is connected to 200 V *dc* voltage, a current of 1A flows through it. When the same inductor is connected to a 200 V, 50 Hz *ac* source, only 0.5 A current flows. Explain why ? Also calculate the self inductance of the inductor.

R **[CBSE DEL SET I, 2019]**

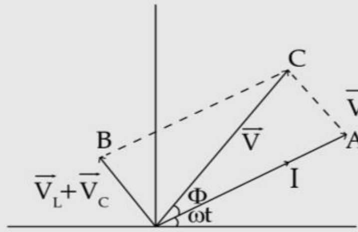
Ans. (a) Derivation of the expression for impedance 2

Plot of impedance with frequency ½

(b) Phase difference between voltage across inductor and capacitor ½

(c) Reason and calculation of self induction ½+ 1½

(a)



1

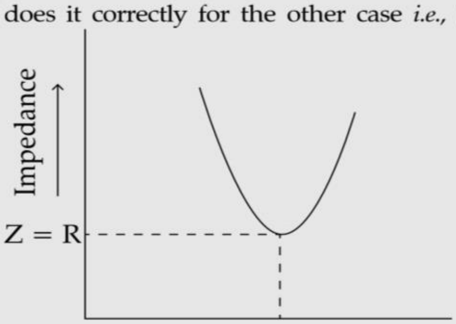
$$|\vec{V}| = V_m$$
$$|V_R| = V_{Rm}$$
$$|V_L| = V_{Lm}$$

1

From the figure, the pythagorus theorem gives

$$V_m^{\ 2} = V_{Rm}^{\ 2} + (V_{Lm} - V_{Cm})^2$$
$$V_{Rm} = i_m R, V_{Lm} = i_m X_L, V_{Cm} = i_m X_C$$
$$V_m = i_m Z$$ ½
$$(V_m)^2 = (i_m Z)^2 = (i_m R)^2 + (i_m X_L - i_m X_C)$$
or, $$Z^2 = R^2 + (X_L - X_C)^2$$
$\therefore$ $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$ ½

[**Note:** award these two marks, If a student does it correctly for the other case *i.e.*, ($V_C > V_L$)]



½

(b) Phase difference between voltage across inductor and the capacitor at resonance is 180° ½

(c) Inductor will offer an additional impedance to *ac* due to its self inductance. ½

$$R = \frac{V_{rms}}{I_{rms}} = \frac{200}{1} = 200\ \Omega$$

Impedance of the inductor,

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{200}{0.5} = 400\ \Omega$$ ½

Since, $$Z = \sqrt{R^2 + (X_L)^2}$$

$$\therefore (400)^2 - (200)^2 = (X_L)^2$$

$$X_L = \sqrt{600 \times 200} = 346.4$$ ½

$$\text{Inductance (L)} = \frac{X_L}{\omega} = \frac{364.4}{2 \times 3.14 \times 50} = 1.1\text{H}$$ ½

[CBSE Marking Scheme, 2019]

Q. 3. A device X is connected across an *ac* source of voltage $V = V_0 \sin \omega t$. The current through X is given as $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$.

(a) Identify the device X and write the expression for its reactance.
(b) Draw graphs showing variation of voltage and current with time over one cycle of *ac*, for *X*.
(c) How does the reactance of the device *X* vary with frequency of the *ac* ? Show this variation graphically.
(d) Draw the phasor diagram for the device *X*.

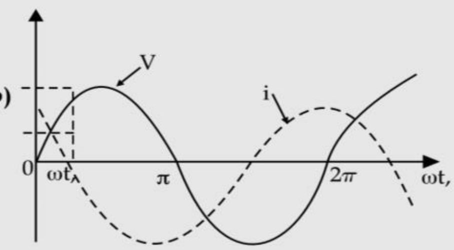
U [CBSE 2018]

Ans. (a) Identification of the device X Expression for reactance. 1
(b) Graphs of voltage and current with time 1+1
(c) Variation of reactance with frequency ½
(Graphical variation) ½
(d) Phasor Diagram 1

(a) *X*: Capacitor ½

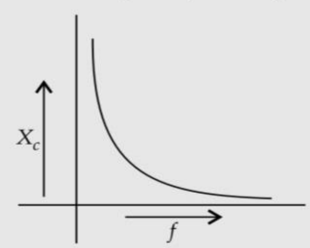
Reactance, $X_c = \frac{1}{\omega C} = \frac{1}{2\pi VC}$ ½

(b)



½+½

(c) Reactance of the capacitor varies in inverse proportion to the frequency *i.e.*, $X_c \propto \frac{1}{f}$ 1



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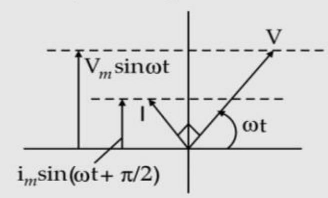
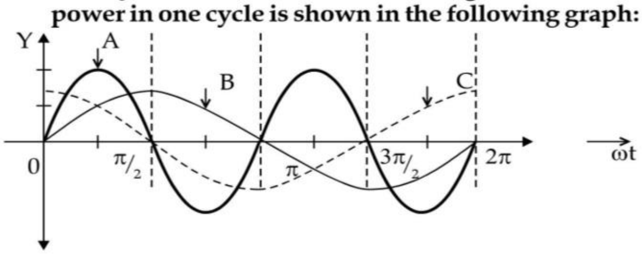


1

[CBSE Marking Scheme, 2018]

Q. 4. A device 'X' is connected to an *ac* source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is shown in the following graph:



(a) Identify the device 'X'. ½

(b) Which of the curves A, B and C represent the voltage, current and the power consumed in the circuit? Justify your answer. 1½

(c) How does its impedance vary with frequency of the *ac* source? Show graphically. 1

(d) Obtain an expression for the current in the circuit and its phase relation with *ac* voltage. 2

Ans. (a) Identification ½

(b) Identifying the curves 1

Justification ½

(c) Variation of Impedance with frequency ½

Graph ½

(d) Expression for current 1½

Phase relation ½

(a) The device X is a capacitor ½

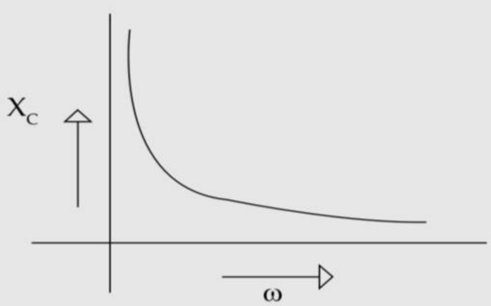
(b) Curve B ⟶ voltage

Curve C ⟶ current ½

Curve A ⟶ power ½

Reason: The current leads the voltage in phase, by $\pi/2$ for a capacitor. ½

(c) $$X_c = \frac{1}{\omega c} \quad \left(X_c \propto \frac{1}{\omega}\right)$$ ½



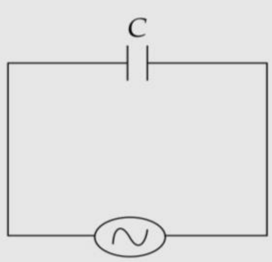
½

$$V = V_o \sin \omega t$$

$$q = CV = CV_o \sin \omega t$$ ½

$$I = \frac{dq}{dt} = \omega c V_o \cos \omega t$$ ½

$$= I_o \sin(\omega t + \pi/2)$$ ½



½

Current leads the voltage, in phase, by $\pi/2$

(Note: If the student identifies the device X as an Inductor but writes correct answers to parts (c) and (d) (in terms of an inductor), the student be given full marks for (only) these two parts)

[CBSE Marking Scheme, 2017]

Q. 5. (i) An a.c. source of voltage $V = V_0 \sin \omega t$ is connected to a series combination of L, C and R. Use the phasor diagram to obtain expressions for impedance of the circuit and phase angle between voltage and current. Find the condition when current will be in phase with the voltage. What is the circuit is this condition called?

(ii) In a series LR circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance C such that $X_L = X_C$ is put in series, the power factor becomes P_2. Calculate $\frac{P_1}{P_2}$.

U **[CBSE DEL SET 1, 2016]**

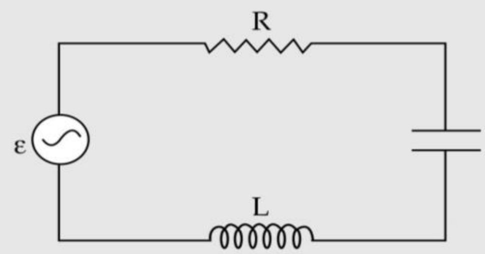
Ans. (i) Obtaining expression for impedance and phase angle 1½ + 1

Condition of current being in phase with voltage ½

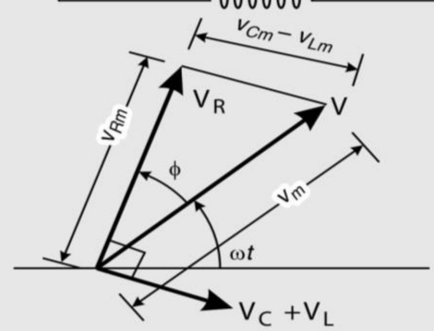
Naming of circuit condition ½

(ii) Calculation of $\frac{P_1}{P_2}$ 1½

(i)



½



From Figure

$$\vec{V} = \vec{V_L} + \vec{V_R} + \vec{V_C}$$ ½

where $$|\vec{V_R}| = i_m R$$

$$|\vec{V_L} + \vec{V_C}| = V_{Cm} - V_{Lm}$$ ½

$$= i_m (X_C - X_L)$$

$$\Rightarrow \quad V_m^2 = V_{Rm}^2 + (V_{Cm} - V_{Lm})^2$$

$$I_m^2 Z^2 = I_m^2 R^2 + I_m^2 (X_C - X_L)^2$$

$$\Rightarrow \quad Z = \sqrt{R^2 + (X_C - X_L)^2}$$

From Figure

$$\tan \phi = \frac{V_{Cm} - V_{Lm}}{V_{Rm}}$$ ½

$$= \frac{i_m(X_C - X_L)}{i_m R}$$ ½

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$ ½

Condition for current and voltage are in phase:

$$V_L = V_C \text{ or } X_L = X_C$$ ½

Circuit is called Resonant circuit. ½

(ii) Power factor, $P_1 = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + R^2}} = \frac{1}{\sqrt{2}}$

(as $X_L = R$) ½

Power factor when capacitor C of reactance $X_C = X_L$ is put in series in the circuit

$$P_2 = \frac{R}{Z} = \frac{R}{R} = 1$$

as $Z = R$ at resonance ½

$$\therefore \quad \frac{P_1}{P_2} = \frac{\frac{1}{\sqrt{2}}}{1} = \frac{1}{\sqrt{2}}$$ ½

[CBSE Marking Scheme, 2016]

Q. 6. (a) Draw graphs showing the variations of inductive reactance and capacitive reactance with frequency of the applied *ac* source.

(b) Draw the phasor diagram for a series *RC* circuit connected to an ac source.

(c) An alternating voltage of 220 V is applied across a device *X*, a current of 0.25 A flows, which lag behind the applied voltage in phase by $\frac{\pi}{2}$ radian.

If the same voltage is applied across another device *Y*, the same current flows but now it is in phase with the applied voltage.

(i) Name the devices *X* and *Y*.

(ii) Calculate the current flowing in the circuit when the same voltage is applied across the series combination of *X* and *Y*. U **[CBSE Comptt. 2018]**

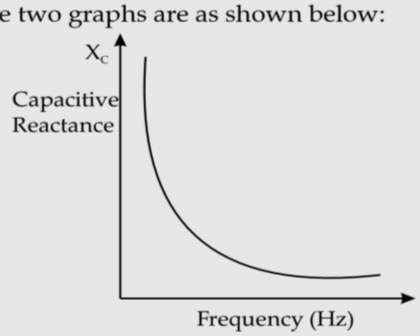
Ans. (a) Drawing the two graphs ½+½

(b) Drawing the phasor diagram 1

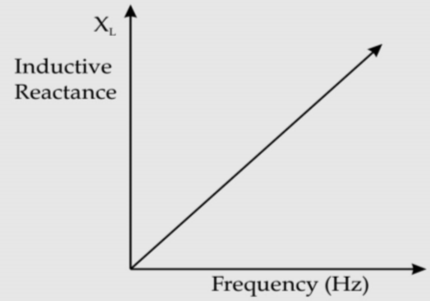
(c) (i) Naming the devices ½+½

(ii) Calculating the current flowing 2

(a) The two graphs are as shown below:

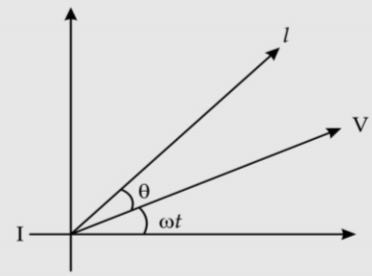


½



½

(b) (The current leads the voltage by an angle θ where $0 < \theta < \frac{\pi}{2}$). The required phasor diagram is as shown



1

[Here, $\theta = \tan^{-1}\left[\frac{1}{\omega CR}\right]$

(c) In device *X*:

Current lags behind the voltage by $\frac{\pi}{2}$

$\therefore$ *X* is an inductor.

In device *Y*: ½

Current in phase with the applied voltage.

$\therefore$ Y is resistor. ½

We are given that

$$0.25 = \frac{220}{X_L}$$

or $$X_L = \frac{220}{0.25}\,\Omega = 880\,\Omega$$ ½

Also $$0.25 = \frac{220}{R}$$

$$\therefore \quad X_R = \frac{220}{0.25}\,\Omega = 880\,\Omega$$ ½

For the series combination of *X* and *Y*,

Equivalent impedance $= \sqrt{X_L^2 + R^2} = \left(880\sqrt{2}\right)\Omega$ ½

$$\therefore \quad \text{Current flowing} = \frac{220}{880\sqrt{2}}\text{ A} = 0.177\text{ A}$$ ½

[CBSE Marking Scheme, 2018]

Q. 7. (i) Prove that an ideal capacitor in an *ac* circuit does not dissipate power.

(ii) An inductor of 200 mH, capacitor of 400 μF and a resistor of 10 Ω are connected in series to ac source of 50 V of variable frequency. Calculate the:

(a) angular frequency at which maximum power dissipation occurs in the circuit and the corresponding value of the effective current, and

(b) value of Q-factor in the circuit.

A [O.D. Comptt I, II, III 2017]

Ans. (i) Average Power dissipation is zero **2**

(ii) Numerical **3**

(i) Try yourself, Similar to Q. 1(a), Long Answer Type Questions-I **2**

(ii) (a)

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad ½$$

$$= \frac{1}{(200\times10^{-3}\times400\times40^{-6})^{1/2}}$$

$$= \frac{1}{\sqrt{8\times10^{-5}}}\text{rad/s}$$

$$= \frac{10^3}{\sqrt{80}}\text{rad/s}$$

$$\cong 111 \text{ rad/s.} \quad 1$$

$$I = \frac{V}{R} = \frac{50}{10} = 5\text{A} \quad ½$$

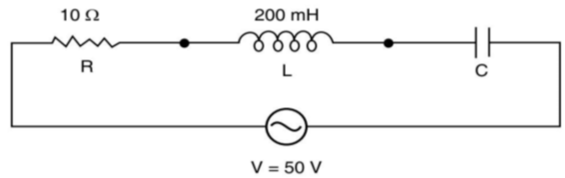
(b)

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{10}\sqrt{\frac{200\times10^{-3}}{400\times10^{-6}}} = \sqrt{5} \quad 1$$

[CBSE Marking Scheme, 2017]

Q. 8. In the following circuit, calculate (i) the capacitance of the capacitor, if the power factor of the circuit is unity, (ii) the Q-factor of this circuit. What is the significance of the Q-factor in *ac* circuit ? Given the angular frequency of the *ac* source to be 100 rad/s. Calculate the average power dissipated in the circuit. **A [O.D. Comptt I, II, III 2017]**



Ans. (i) Calculation of capacitance **1**

(ii) Q-factor of circuit and its importance **2**

Calculation of average power dissipated **2**

(i) As power factor is unity,

$$\therefore \quad X_L = X_C \quad ½$$

$$\Rightarrow \quad \omega = \frac{1}{\sqrt{LC}} \quad ½$$

$$100 = \frac{1}{\sqrt{200\times10^{-3}\times C}} \quad ½$$

$$10^4 \times 2 \times 10^2 \times 10^{-3} \times C = 1 \quad ½$$

$$C = \frac{1}{2\times10^3}\text{ F}$$

$$= 0.5 \times 10^{-3}\text{ F} \quad ½$$

$$= 0.5 \text{ mF} \quad ½$$

(ii) Quality factor,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \quad ½$$

$$= \frac{1}{10}\sqrt{\frac{200\times10^{-3}}{0.5\times10^{-3}}}$$

$$= \frac{1}{10}\times20 = 2 \quad ½$$

Significance: It measures the sharpness of resonance.

Average Power dissipated,

$$P = V_{rms}I_{rms}\cos\phi$$

$$= 50\times\frac{50}{10}\times1\text{ W}$$

$$= 250 \text{ watts} \quad 1$$

[CBSE Marking Scheme, 2017]

TOPIC-3
AC Generator and Transformer

Revision Notes

AC **generator**

- An alternator is an electrical machine which converts mechanical energy into alternating electrical energy.
- Alternator or a synchronous generator has a stator and rotor.
- It is similar to the basic working principle of a dc generator.
- It works on the principle of electromagnetic induction where a coil gets rotated in uniform magnetic field, sets an induced emf given as:

$$e = e_0 \sin\omega t = NBA\omega\sin\omega t$$

Transformer

- Transformer is an electrical device used for changing the alternating voltages.
  It is based on the phenomenon of mutual induction.

- The main use of transformer is in transmission of ac over long distances at extremely high voltages which reduces the energy losses in transmission.
- It comprises of two sets of coils which are insulated from each other and are wound on soft-iron core.
- In this, one of the coil is called as primary (input coil) having N_p turns while other coil is secondary (output coil) having N_s turns, so we have

$$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$$

- Transformer Ratio:

$$E_s = \left(\frac{N_s}{N_p}\right)E_p \text{ and } I_s = \left(\frac{N_p}{N_s}\right)I_p$$

$\frac{N_s}{N_p} = \frac{V_s}{V_p}$ is defined as the transformer ratio.

The value of turns ratio of a transformer $\qquad \frac{N_p}{N_s} = \frac{V_p}{V_s} = n$

- Step-up transformer: If secondary coil has more number of turns than primary ($N_s > N_p$), voltage gets stepped up ($V_s > V_p$).

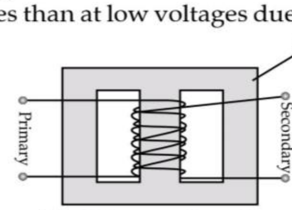
  In this, there is less current in secondary as compared to primary ($\frac{N_s}{N_p} > 1$ and $I_s < I_p$).

  The value of transformer ratio $\qquad k > 1$
- Step-down transformer: In this, the secondary coil has less number of turns than primary ($N_s < N_p$). In this, $V_s < V_p$ and $I_s > I_p$ as voltage gets stepped down or reduced with increase in current.

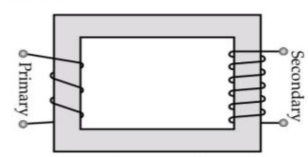
  In this, value of transformer ratio $\qquad k < 1$
- The main use of transformers is in stepping up voltage for power transmission.
- Electric power can be transmitted efficiently at high voltages than at low voltages due to less (I^2R) heat loss in a high voltage / low current transmission.
- Efficiency of transformer:

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\eta = \frac{E_sI_s}{E_pI_p}$$



Shell type coil and core arrangement



Core type coil and core arrengement

- Inspite of heavy power losses, the efficiency in a transformer is usually above 90%.
- An ideal transformer is 100% efficient as it delivers all energy it receives.
- Real transformer is not 100% efficient and at full load, its efficiency lies between 94% to 96%.
- A transformer operating with constant voltage and frequency with very high capacity, efficiency results as 98%.
- Energy losses in transformers:
  1. Flux Leakage
  2. Resistance of windings
  3. Eddy currents
  4. Hysteresis

Know the Formulae

For transformer:

- $$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$$
- $$V_s = \left(\frac{N_s}{N_p}\right)V_p \text{ and } I_s = \left(\frac{N_p}{N_s}\right)I_p$$
- The value of transformer ratio is greater than 1 for step up transformer and less than 1 for step-down transformer.
- $$\eta = \frac{E_sI_s}{E_pI_p}$$

➢ $\%\text{Efficiency} = \dfrac{\text{Output power}}{\text{Input power}} \times 100\%$

$$= \frac{\text{Input power} - \text{Losses}}{\text{Input power}} \times 100\%$$

For generator:

➢ $e = e_0 \sin \omega t = NBA\omega \sin \omega t$

➢ $I = \dfrac{e}{r} = \dfrac{NBA\omega \sin \omega t}{R}$

Very Short Answer Type Questions (1 mark each)

AI Q. 1. Mention the two characteristic properties of the material suitable for making core of a transformer. R

Ans. Any two of the following:

(i) Low coercivity / Low retentivity ½

(ii) Low hysteresis loss ½

OR

(i) High magnetic susceptibility / High Permeability ½

(ii) High resistivity ½

Q. 2. Does the step down transformer violate the principle of conservation of energy? U

Ans. No, it does not violate the principle of conservation of energy. If a voltage is increased, the current is decreased in the same ratio and the product VI (power) remains constant. 1

Q. 3. Why do we prefer carbon brushes than copper in an ac generator? R

Ans. The carbon brushes used in case of the generator are corrosion free. On small expansion on heating, it maintains the proper contact as well. 1

Q. 4. What is the relationship between the transformation ratio and the voltage? R

Ans. If all the magnetic flux due to any current in the primary is linked with the secondary, then

$$\frac{N_s}{N_p} = \frac{Vs}{Vp} = k$$

where k is the transformation ratio 1

Short Answer Type Questions-I (2 marks each)

AI Q. 1. State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distances done with the use of transformers ? R

Ans. A transformer is based on the principle of mutual induction which states that due to continuous change in the current in the primary coil, an emf gets induced across the secondary coil. 1

Electric power generated at the power station, is stepped-up to very high voltages by means of a step-up transformer and transmitted to a distant place. At receiving end, it is stepped down by a step-down transformer. 1

Q. 2. A power generating station produces electric power 600 kW at 4000 V which is to be transported to a distant village. The turns ratio of the step up transformer used at the generating station is 1:10. In the village if the voltage to be supplied at 200V, then what will be the turns ratio of the step-down transformer?

Ans. At generating station:

For step-up transformer, $\dfrac{V_P}{V_S} = \dfrac{N_P}{N_S}$

Or, $\dfrac{4000}{V_S} = \dfrac{1}{10}$

$\therefore \quad V_S = 40000\text{ V}$ 1

At destination village:

For step-up transformer, $\dfrac{V_P}{V_S} = \dfrac{N_P}{N_S}$

$$\frac{40000}{200} = \frac{N_P}{N_S}$$

$\dfrac{N_P}{N_S} = 200 : 1$ 1

AI Q. 3. An athlete peddles a stationary tricycle whose pedals are attached to a coil having 100 turns each of area 0.1 m^2. The coil, lying in XY plane, is rotated in this plane at the rate of 50 rpm, about the z-axis, in a region where a uniform magnetic field $\vec{B} = 0.01\hat{k}$ Tesla is present. Find the (i) maximum emf generated (ii) average emf generated in the coil over one complete rotation.

Ans. (i) Maximum emf generated $= \varepsilon_0 = NBA\omega$

$\varepsilon_0 = NBA \times 2\pi f$ ½

Or, $\varepsilon_0 = 100 \times 0.01 \times 0.1 \times 2\pi \times \frac{50}{60}$ ½

$\therefore$ $\varepsilon_0 = \frac{\pi}{6}$ V $= 0.52$ V ½

(ii) The average emf generated in the coil over complete rotation is 0. ½

Short Answer Type Questions-II

(3 marks each)

Q. 1. Explain with the help of a diagram, the working of a step-down transformer. Why is a laminated iron core used in a transformer ?

R & U **[CBSE OD SET 1, 2020]**

Ans. Working of a step-down transformer:

A step-down transformer converts a high voltage at the primary side to a low voltage at the secondary side.

Transformer works on the principle of "Faraday's law of electromagnetic induction".

The emf induced is determined by the number of turns in primary and secondary windings. This ratio is called as turn ratio.

The voltage reduction capability of step-down transformers depends on the turn ratio of the primary and secondary coils. As the number of turns in secondary coil is kept less as compared to the number of turns in primary coil, so the amount of flux linkage to the secondary coil of the transformer is less compared to the primary coil. Accordingly, the emf induced will be less in the secondary coil.

$$\frac{N_S}{N_p} = \frac{V_S}{V_p}$$ 1

Here,

N_s = Number of turns in secondary coil
N_p = Number of turns in primary coil
V_s = Voltage in secondary coil
V_p = Voltage in primary coil ½

The number of turns in secondary winding is always less than the number of turns in the primary winding of the transformer, *i.e.* $N_p > N_s$. ½

As the number of turns is less in secondary coil, so the induced emf (output voltage) in the secondary coil will be less than the primary input voltage.

Laminated iron core is used in transformers to reduce the Eddy current which prevents the efficient transfer of energy from the primary coil to the secondary coil and causes energy loss in form of heat generation in the core. 1

Q. 2. (a) Name the device used to change the alternating voltage to a higher or lower value. State one cause for power dissipation in this device.

(b) Explain with an example, how power loss is reduced if the energy is transmitted over long distances as an alternating current rather than a direct current. R & U **[CBSE OD SET 1, 2018]**

Ans. (a) Transformer is used to change the alternating voltage to a higher or lower value.

One cause of power dissipation in transformer is the resistance (I^2R loss) of the copper wire used for winding. **1½**

(b) At the generation end, the alternating voltage is stepped up by using a step-up transformer. So, the corresponding current reduces. It is transmitted through transmission lines. So, there will not be any appreciable I^2R loss. At the receiving point, the voltage is stepped down to the desired level by using a step-down transformer and then it is distributed to consumers.

This step-up and step down is not so easy process for direct voltage. **1½**

AI **Q. 3. (a) What is the principle of transformer?**

(b) Explain how laminating the core of a transformer helps to reduce eddy current losses in it.

(c) Why the primary and secondary coils of a transformer are preferably wound on the same core? R & U

Ans. (a) Try Yourself. See Q. No. 1 of 2 Marks Questions. **1**

(b) Each lamination being thin, its resistance is high. So, the eddy current is confined within thin lamination. Thus reduces the net eddy current. **1**

(c) For maximum sharing of magnetic flux, both the coils are preferably to be wound on the same core. **1**

Long Answer Type Questions

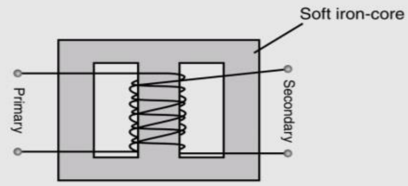
(5 marks each)

Q. 1. (a) Draw a labelled diagram of a step-up transformer. Obtain the ratio of secondary to primary voltage in terms of number of turns and currents in the two coils.

(b) A power transmission line feeds input power at 2200 V to a step-down transformer with its primary windings having 3000 turns. Find the number of turns in the secondary to get the power output at 220 V. A **[CBSE DEL SET 1, 2017]**

Ans. (a) Labelled diagram of a step-up transformer 1½
Derivation of ratio of secondary and primary voltage 2
(b) Calculation of number of turns in the secondary 1½

(a)



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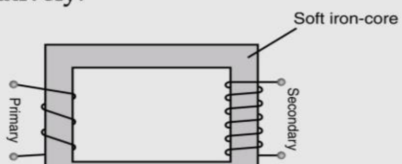


1

[Note: Deduct ½ mark, if labelling is not done]

When ac voltage is applied to primary coil, the resulting current produces an alternating magnetic flux, which also links the secondary coil.

The induced emf, in the secondary coil, having N_s turns, is

$$e_s = -N_s \frac{d\varphi}{dt} \quad ½$$

This flux, also induces an emf, called back emf, in the primary coil.

$$e_s = -N_s \frac{d\varphi}{dt} \quad ½$$

But $e_p = V_p$

and $e_s = V_s$ ½

$$\Rightarrow \frac{V_s}{V_p} = \frac{N_s}{N_p} \quad ½$$

For an ideal transformer

$$I_p V_p = i_s V_s \quad ½$$

$$\Rightarrow \frac{V_s}{V_p} = \frac{i_p}{i_s} \quad ½$$

(b)

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} \quad ½$$

$$\frac{N_s}{3000} = \frac{220}{2200}$$

$$\therefore N_s = 300 \quad ½$$

[CBSE Marking Scheme, 2017]

Q. 2. (i) Draw a labelled diagram of a step-down transformer. State the principle of its working.

(ii) Express the turn ratio in terms of voltages.

(iii) Find the ratio of primary and secondary currents in terms of turn ratio in an ideal transformer.

(iv) How much current is drawn by the primary of a transformer connected to 220 V supply when it delivers power to a 110 V – 550 W refrigerator?

A **[CBSE OD SET I, 2016]**

Ans.(i) Labelled diagram 1

Principle 1

(ii) Expression for the turn ratio in terms of voltage ½

(iii) Ratio of primary and secondary currents in terms of turns 1

(iv) Current drawn by primary

Formula: ½

Calculation and result ½ + ½

(i) and (ii) Try & Yourself, See Q. No. 1 of Short Answer Type Questions-II. **1 + 1**

(iii) For an ideal transformer,

$$i_p V_p = i_s V_s \quad ½$$

$$\therefore \frac{i_p}{i_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} \quad 1$$

(iv) We have

$$i_p V_p = i_s V_s = 550 \text{ W}$$

$$V_p = 220 \text{ V}$$

$$i_p = \frac{550}{220} = \frac{5}{2} = 2.5\text{A} \quad ½ + ½ + ½$$

[CBSE Marking Scheme, 2016]

AI **Q. 3. (i) Describe, the working principle of a step-up transformer with the help of a suitable diagram. Obtain the relation between input and output voltages in terms of the number of turns of primary and secondary windings and the currents in the input and output circuits.**

(ii) Given the input current 15 A and the input voltage of 100 V for a step-up transformer having 90% efficiency, find the output power and the voltage in the secondary if the output current is 3 A.

R & A **[Foreign I, II, III 2017]**

Ans. (i) Diagram ½

Principle ½

Relation between voltage, number of turns, and Currents 2½

(ii) Input power ½

Output power ½

Output voltage ½

(i) Try Yourself See Q. No. 1 of 5 marks Questions. **1+1**

$$\therefore \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} \quad 1$$

(ii) Input power, $P_i = I_i \times V_i = 15 \times 100 = 1500$ W ½

Power output, $P_0 = P_i \times \frac{90}{100} = 1350$ W ½

$$\Rightarrow I_0 V_0 = 1350 \text{ W} \quad ½$$

Output voltage, $V_0 = \frac{1350}{3}$ V $= 450$ V ½

[CBSE Marking Scheme, 2017]

Q. 4. (a) State the principle of working of a transformer.

(b) Define efficiency of a transformer.

(c) State any two factors that reduce the efficiency of a transformer.

(d) Calculate the current drawn by the primary of a 90% efficient transformer which steps down 220 V to 22 V, if the output resistance is 440 Ω.

U & A **[CBSE Comptt. I, II, III 2018]**

Ans. (a) Principle of working 1

(b) Defining efficiency 1

(c) Any two factors ½+½

(d) Calculating the current drawn 2

(a) Try it Yourself. See Q. No. 1 of 5 marks Questions. **1**

(b) The efficiency of a transformer equals the ratio of the output power to the input power. **1**

Alternatively:

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}}$$

$$\text{Efficiency} = \frac{V_s I_s}{V_p I_p}$$

(c) (i) Eddy current losses

(ii) joule heat losses

(iii) hysteresis losses ½+½

(iv) magnetic flux leakage losses **(Any two)**

(d) We have

$$\frac{V_s I_s}{V_p I_p} = 90\% = 0.9$$

$$\therefore \quad \frac{22 I_s}{220 I_p} = 0.9$$

$$\text{or,} \quad \frac{I_s}{I_p} = \frac{0.9}{0.1} = 9$$

$$\therefore \quad I_p = \frac{I_s}{9} = \frac{\left(\frac{22}{440}\right)}{9}\text{A}$$

$$= \frac{1}{180}\text{A}$$

$$= 0.0056 \text{ A}$$ **2**

[CBSE Marking Scheme, 2018]

Visual Case-based Questions **(1×4=4 marks)**

Attempt any 4 sub parts-from the given 5 Question. Each question carries 1 mark.

1. Tuning a radio set: In essence the simplest tuned radio frequency receiver is a simple crystal set. Desired frequency is tuned by a tuned coil / capacitor combination, and then the signal is presented to a simple crystal or diode detector where the amplitude modulated signal, is demodulated. This is then passed straight to the headphones or speaker. In radio set there is an LC oscillator comprising of a variable capacitor (or sometimes a variable coupling coil) , with a knob on the front panel to tune the receiver.

Capacitor used in old radio sets is gang capacitor. It consists of two sets of parallel circular plates one of which can rotate manually by means of a knob. The rotation causes overlapping areas of plates to change, thus changing its capacitance. Air gap between plates acts as dielectric.

The capacitor has to be tuned in tandem corresponding to the frequency of a station so that the LC combination of the radio set resonates at the frequency of the desired station.

When capacitive reactance (*XC*) is equal to the inductive reactance (*XL*), then the resonance occurs and the resonant frequency is given by $\omega_0 = \frac{1}{\sqrt{LC}}$

current amplitude becomes maximum at the resonant frequency. It is important to note that resonance phenomenon is exhibited by a circuit only if both *L* and *C* are present in the circuit. Only then do the voltages across *L* and *C* cancel each other (both being out of phase) and the current amplitude is $\frac{V_m}{R}$, the total source voltage appearing across *R*.

This means that we cannot have resonance in a RL or RC circuit.

(i) Name the phenomenon involved in tuning a radio set to a particular radio station.

(a) Stabilization **(b)** Rectification

(c) Resonance **(d)** Reflection

Ans. (c) **1**

Explanation: Phenomenon involved in tuning a radio set to a particular radio station is resonance.

The capacitor has to be tuned in tandem corresponding to the frequency of a station. So, that the LC combination of the radio set resonates at the frequency of the desired station.

(ii) Resonance may occur in:

(a) RL circuit

(b) RC circuit

(c) LC circuit

(d) Circuit having resistor only

Ans. (c) 1

Explanation: A simple radio receiver is a simple crystal set with a coil and capacitor combination. Desired frequency is tuned by tuning the coil - capacitor combination. Tuning means to make capacitive reactance (X_C) equal to the inductive reactance (X_L), so that the resonance occurs.

(iii) Resonance frequency is equal to:

(a) $\frac{1}{LC}$ **(b)** $\frac{1}{\sqrt{L_C}}$

(c) $\sqrt{\frac{L}{C}}$ **(d)** $\sqrt{\frac{C}{L}}$

Ans. (b) 1

Explanation: The resonant frequency is given by $\omega_0 = \frac{1}{\sqrt{L_C}}$

(iv) Resonance occurs only when:

(a) $X_C = R$ **(b)** $X_L = R$

(c) $X_L = X_C$ **(d)** $X_C > X_L$

Ans. (c) 1

Explanation: At resonance, capacitive reactance (X_C) is equal to the inductive reactance (X_L). Circuit is totally resistive and the current amplitude becomes maximum.

(v) Capacitor used in radio set for tuning is a:

(a) Parallel plate capacitor

(b) Spherical capacitor

(c) Paper capacitor

(d) Electrolytic capacitor

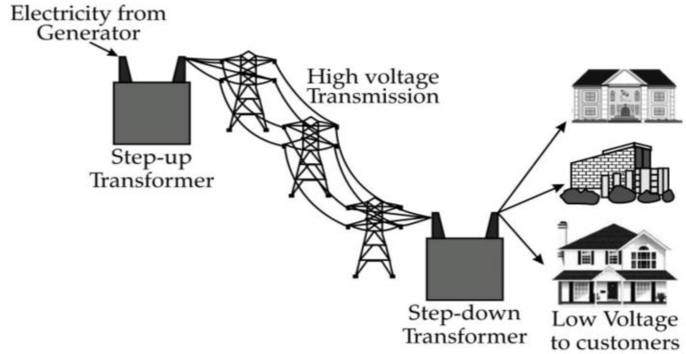
Ans. (a) 1

Explanation: Capacitor used in old radio is a parallel plate capacitor. It consists of two sets of parallel circular plates , one of which can rotate manually by means of a knob. The rotation causes overlapping areas of plates to change, thus changing its capacitance.

2. At power plant, a transformer increases the voltage of generated power by thousands of volts so that it can be sent of long distances through high-voltage transmission power lines. Transmission lines are bundles of wires that carry electric power from power plants to distant substations.

At substations, transformers lower the voltage of incoming power to make it acceptable for high-volume delivery to nearby end-users.

Electricity is sent at extremely high voltage because it limits so-called line losses. Very good conductors of electricity also offer some resistance and this resistance becomes considerable over long distances causing considerable loss.



At generating station, normally voltage is stepped up to around thousands of volts. Power losses increase with the square of current. Therefore, keeping voltage high current becomes low and the loss is minimized.

Another option of minimizing loss is the use of wires of super-conducting material. Super-conducting materials are capable of conducting without resistance, they must be kept extremely cold, nearly absolute zero, and this requirement makes standard super-conducting materials impractical to use. However, recent advances in super-conducting materials have decreased cooling requirement. In Germany recently 1 km super-conducting cable have been installed connecting the generating station and the destination. It has eliminated the line loss and the cable is capable of sending five times more electricity than conventional cable. Using super-conducting cables Germany has also get rid of the need of costly transformers.

Transformers generate waste heat when they are in operation and oil is the coolant of choice. It transfers the heat through convection to the transformer housing, which has cooling fins or radiators similar to heat exchangers on the outside.

Flush point is a very important parameter of transformer oil. Flashpoint of an oil is the temperature at which the oil ignites spontaneously. This must be as high as possible (not less than 160º C from the point of safety).

Fire point is the temperature at which the oil flashes and continuously burns. This must be very high for the chosen oil (not less than 200º C).

(i) Which of the following statement is true for long distance transmission of electricity?

(a) Step-down transformer is used at generating station and step-up transformer is used at destination substation

(b) Step-down transformers are used at generating station and destination substation

(c) Step-up transformers are used at generating station and destination substation

(d) None of the above

Ans. (d)

Explantion: At power plant, a step-up transformer increases the voltage of generated power by thousands of volts, so that it can be sent of long distances through high-voltage transmission power lines.

At substations, step-down transformers lower the voltage of incoming power to make it acceptable for high-volume delivery to nearby end-users.

(ii) Super-conducting transmission line has the following advantages:

(a) Resistance being zero, there is no I^2R loss

(b) There is no requirement of costly step-up and step-down transformers

(c) Cable is capable of sending more electricity

(d) All of the above

Ans. (d)

Explanation: Super-conducting materials are capable of conducting without resistance. So, this eliminates the line loss and the cable is capable of sending more electricity than conventional cable. Using super-conducting cables one can get rid of the need of costly transformers.

(iii) Why does stepping up voltages reduce power loss?

(a) Since resistance of conductor decreases with increase of voltage

(b) Since current decreases with increase of voltage

(c) Both of the above

(d) None of the above

Ans. (b)

Explanation: At generating station, normally voltage is stepped up to around thousands of volts. Power losses increase with the square of current. Therefore, keeping voltage high current becomes low and the loss is minimized.

(iv) Oil transfers heat from transformer winding by the process of:

(a) Convection **(b)** conduction

(c) Radiation **(d)** All of these

Ans. (a)

Explanation: Transformers generate waste heat when they are in operation and oil is the coolant of choice. It transfers the heat through convection to the transformer housing.

(v) Flush point of an oil is

(a) the temperature at which the oil flashes and continuously burns

(b) the temperature at which the oil ignites spontaneously

(c) the temperature at which the oil stars boiling

(d) The temperature at which the oil forms fumes

Ans. (b)

Explanation: Flush point is a very important parameter of transformer oil. Flashpoint of an oil is the temperature at which the oil ignites spontaneously. This must be as high as possible (not less than 160° C from the point of safety).

❑❑

SELF ASSESSMENT PAPER-4

Time : 1 Hours | **Maximum Marks : 25**

SECTION - A

1. (i) A square of side L meters lies in the x–y plane in a region where the magnetic field is given by $\vec{B} = B_0(2\hat{i} + 3\hat{j} + 4\hat{k})$ Tesla, where B_0 is constant. What will be the magnitude of flux passing through the square? **1**

(ii) If the number of turns per unit length of a coil of a solenoid is doubled, how will its self induction change? **1**

(iii) A metallic piece gets hot when surrounded by a coil carrying high frequency alternating current. Why? **1**

(iv) An iron-cored solenoid has self-inductance 2.8 H. When the core is removed, the self inductance becomes 2 mH. What is the relative permeability of the core used ? **1**

2. For question number 2 two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is NOT the correct explanation of A

(c) A is true but R is false

(d) A is false and R is also false

(i) Assertion (A) : At the timing of switching ON and OFF there is a high possibility of an electric bulb to fuse.

Reason (R) : A surge is produced due to inductive effect at the time of switching ON and OFF. **1**

(ii) Assertion (A) : Faraday's laws are consequence of conservation of energy.

Reason (R) : In a purely resistive *ac* circuit, the current lags behind the voltage. **1**

3. Read the Passage given below and answer any 4 Questions.

Tuning a radio set : In essence the simplest tuned radio frequency receiver is a simple crystal set. Desired frequency is tuned by a tuned coil / capacitor combination, and then the signal is presented to a simple crystal or diode detector where the amplitude modulated signal, is demodulated. This is then passed straight to the headphones or speaker. In radio set there is a LC oscillator comprising of a variable capacitor (or sometimes a variable coupling coil) , with a knob on the front panel to tune the receiver. **4**

Capacitor used in old radio sets is gang capacitor. It consists of two sets of parallel circular plates , one of which can rotate manually by means of a knob. The rotation causes overlapping areas of plates to change, thus changing its capacitance. Air gap between plates acts as dielectric.

The capacitor has to be tuned in tandem corresponding to the frequency of a station. so that the LC combination of the radio set resonates at the frequency of the desired station.

When Capacitive reactance (X_C) is equal to the inductive reactance (X_L), then the resonance occurs and the resonant frequency is given by $\omega_0 = \dfrac{1}{\sqrt{LC}}$

Current amplitude becomes maximum at the resonant frequency.

It is important to note that resonance phenomenon is exhibited by a circuit only if both L and C are present in the circuit. Only then do the voltages across L and C cancel each other (both being out of phase) and the current amplitude is V_m/R, the total source voltage appearing across R. This means that we cannot have resonance in a RL or RC circuit.

(i) Name the phenomenon involved in tuning a radio set to a particular radio station.

(a) Stabilization **(b)** Rectification
(c) Resonance **(d)** Reflection

(ii) Resonance may occur in

(a) RL circuit **(b)** RC circuit
(c) LC circuit **(d)** Circuit having resistor only

(iii) Resonance frequency is equal to :

(a) $\frac{1}{LC}$ **(b)** $\frac{1}{\sqrt{LC}}$

(c) $\sqrt{\frac{L}{C}}$ **(d)** $\sqrt{\frac{C}{L}}$

(iv) Resonance occurs only when :

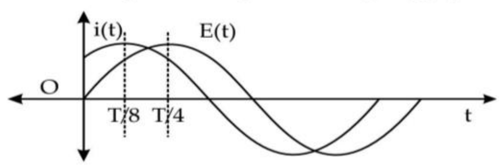
(a) $X_C = R$ **(b)** $X_L = R$
(c) $X_L = X_C$ **(d)** $X_C > X_L$

(v) Capacitor used in radio set for tuning is a :

(a) Parallel plate capacitor **(b)** Spherical capacitor
(c) Paper capacitor **(d)** Electrolytic capacitor

SECTION - B

4. The figure shows two sinusoidal curves representing oscillating supply voltage and current in an *ac* circuit.



Draw a phasor diagram to represent the current and supply voltage appropriately as phasors. State the phase difference between the two quantities. **2**

5. State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distances done with the use of transformers ? **2**

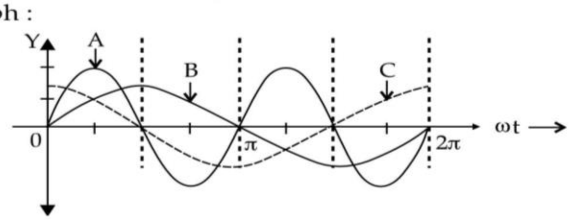
SECTION - C

6. State Lenz's law. Explain, by giving examples that Lenz's law is a consequence of conservation of energy. **3**

7. A source of *ac* voltage $V = V_0 \sin \omega t$, is connected across a pure inductor of inductance L. Derive the expressions for the instantaneous current in the circuit. Show that average power dissipated in the circuit is zero. **3**

SECTION - D

8. A device 'X' is connected to an *ac* source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is shown in the following graph :



(i) Identify the device 'X'.

(ii) Which of the curves A, B and C represent the voltage, current and the power consumed in the circuit ? Justify your answer.

(iii) How does its impedance vary with frequency of the *ac* source ? Show graphically.

(iv) Obtain an expression for the current in the circuit and its phase relation with *ac* voltage **5**