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TRIANGLE AND ITS ANGLES

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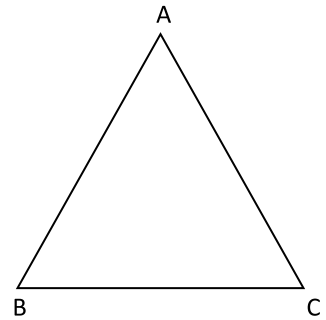
TRIANGLE AND ITS ANGLES

TRIANGLE

DEFINITION: A plane figure bounded by three lines in a plane is called a triangle.

Let A, B, C be three points such that all are not in a line. Then, the line segments AB, BC and CA form a triangle with vertices A, B and C. The segments AB, BC and CA are called the sides and the angles BAC, ABC and ACB are called the angles of the triangle ABC.



For the sake of convenience we shall denote ∠BAC, ∠ABC and ∠ACB by ∠A, ∠B and ∠C respectively. We shall also use the symbol 'Δ' (read as 'delta') in place of the word "triangle". Thus, triangle ABC will be denoted by the Δ ABC.

TYPES OF TRIANGLES

Triangles are classified into various types on the basis of the lengths their sides as well as on the basis of the measures of their angles. Following are the types of triangles on the basis of sides:

¶ **SCALENE TRIANGLE:** A triangle, no two of whose sides are equal is called a scalene triangle.

¶ **ISOSCELES TRIANGLE:** A triangle, two of whose sides are equal in length is called an isosceles triangle.

¶ **EQUILATERAL TRIANGLE:** A triangle, all of whose sides are equal is called an equilateral triangle. Following are the types of triangles on the basis of angles.

¶ **ACUTE TRIANGLE:** A triangle, each of whose angles is acute, is called on an acute triangle or an acute angled triangle.

¶ **RIGHT TRIANGLE:** A Triangle with one angle a right angle is called a right triangle or a right angled triangle.

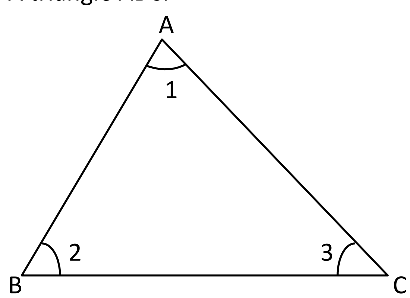
¶ **OBTUSE TRIANGLE:** A triangle with one angle an obtuse angle, is known as an obtuse triangle or obtuse angled triangle.

◘◘ It should be noted that an equilateral triangle is an isosceles triangle but the converse is not true.

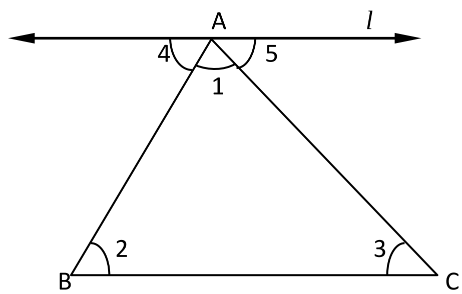
ANGLE SUM PROPERTY OF A TRIANGLE

◘ THEOREM 1: **The sum of the three angles of a triangle is 180°.**

GIVEN: A triangle ABC.



(i)



(ii)

To prove: ∠A + ∠B + ∠C = 180° i.e., ∠1 + ∠2 + ∠3 = 180°

Construction: Through A, draw a line *l* parallel to BC.

Proof: Since *l* ∥ BC. Therefore,

	∠2 = ∠4	[Alternate interior angles]
and,	∠3 = ∠5	[Alternate interior angles]
∴	∠2 + ∠3 = ∠4 + ∠5	
=>	∠1 + ∠2 + ∠3 = ∠1 + ∠4 + ∠5	[Adding ∠1 on both sides]
=>	∠1 + ∠2 + ∠3 = ∠4 + ∠1 + ∠5	
=>	∠1 + ∠2 + ∠3 = 180°	[∵ Sum of angles at a point on a line is 180°] ∴ ∠4 + ∠2 + ∠5 = 180°
=>	∠A + ∠B + ∠C = 180°	

Thus, the sum of the three angles of a triangle is 180°.

COROLLARY: If the bisector of angles ∠ABC and ∠ACB of a triangle ABC meet at a point O, then ∠BOC = 90° + ½ ∠A

GIVEN: A ΔABC such that the bisector of ∠ABC and ∠ACB meet at a point O.

To prove: ∠BOC = 90° + ½ ∠A

Proof: In BOC, we have

∠1 + ∠2 + ∠BOC = 180°

In ΔABC, we have

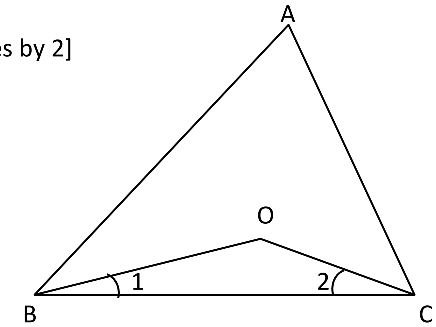
∠A + ∠B + ∠C = 180°

=> ∠A + 2(∠1) + 2 (∠2) = 180° [∵ BO and CO are bisectors of ∠ABC and ∠ACB respectively ∴ ∠B = 2 ∠ 1 and ∠C = 2∠2]

=> ∠A + 2 (∠1) + 2 (∠2) = 180°

=> $\frac{\angle A}{2}$ + ∠1 + ∠2 = 90° [Dividing both sides by 2]

=> ∠1 + ∠2 = 90° – ∠A ... (ii)



Substituting this value of ∠1 + ∠2 in (i), we get

90° – ∠A + ∠BOC = 180°

=> ∠BOC = 180° – 90° + $\frac{\angle A}{2}$

=> ∠BOC = 90° + $\frac{\angle A}{2}$

THEOREM 2: If two parallel lines are intersected by a transversal, prove that the bisectors of the two pairs of interior angles enclose a rectangle.

Given: Two parallel lines AB and CD and a transversal EF intersecting them at G and H respectively. GM, HM, GL and HL are the bisectors of the two pairs of interior angles.

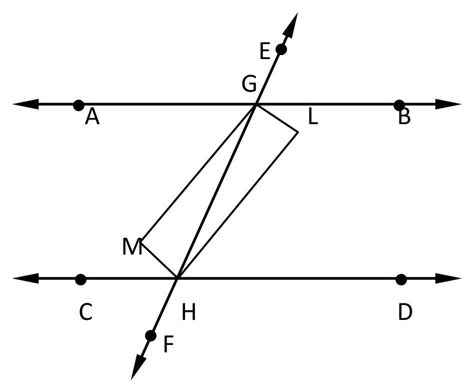
To Prove: GMHL is a rectangle.

Proof: We have,

∠AGH = ∠DHG [Alternate interior angles]

=> ½ ∠AGH = ½ ∠DHG

=> ∠HGM = ∠GHL



Thus, lines GM and HL are intersected by a transversal GH at G and H respectively such that pair of alternate angles are equal i.e., ∠HGM = ∠GHL.

∵ GM ∥ HL

Similarly, we can prove that GL ∥ HM. So, GMHL is a parallelogram

Since AB ∥ CD and EF is a transversal.

∴ ∠BGH + ∠DHG = 180° [∵ Sum of interior angles on the same side of a transversal = 180°]

=> ½ ∠BGH + ½ ∠DHG = 90°

=> ∠LHG + ∠LHG = 90° [∵ ½ ∠BGH = ∠LGH and, ½ DHG = ∠LHG]

But, ∠LGH + ∠LHG + ∠GLH = 180° [Sum of the angles of a triangle is 180°]

∴ 90° + ∠GLH = 180° [∵ ∠LGH + ∠LHG = 90°]

=> ∠GLH = 180° – 90°

=> ∠GLH = 90°

Thus, in the parallelogram GMHL, we have ∠GLH = 90°

Hence, GMHL is a rectangle.

Illustrative Examples...

Q. 1. In a ΔABC, ∠B = 105°, ∠C = 50°. Find ∠A.

Sol. We have,

∠A + ∠B + ∠C = 180°

=> ∠A + 105° + 50° = 180° => ∠A = 180° – 155° = 25°

Q. 2. The sum of two angles of a triangle is equal to its third angle. Determine the measure of the third angle.

Sol. Let ABC be a triangle such that

∠A + ∠B = ∠C ... (i)

We know that ∠A + ∠B + ∠C = 180° ... (ii)

We know that ∠A + ∠B + ∠C = 180°

Putting ∠A + ∠B = ∠C in (ii), we get

∠C + ∠C = 180° => 2∠C = 180° => ∠C = 90°

Thus, measure of the third angle is of 90°.

Q. 3. Of the three angles of a triangle, one is twice the smallest and another is three times the smallest. Find the angles.

Sol. Let the smallest angle of the given triangle be of x°. Then, the other two angles are of 2x° and 3x°.

So, x + 2x + 3x = 180

=> $6x = 180 \Rightarrow x = \frac{180}{6} = 30$

Hence, measures of the angles of the triangle are 30°, 60° and 90°.

Q. 4. If the angles of a triangle are in the ratio 2 : 3 : 4, determine three angles.

Sol. Let the angles of the triangle be 2x°, 3x° and 4x°. Then,

∴ 2x + 3x + 4x = 180 => 9x = 180 => x = 20

Hence, the angles of the triangle are 40°, 60°and 80°.

Q. 5. The sum of two angles of a triangle is 80° and their difference is 20°. Find all the angles.

Sol. Let ABC be a triangle such that

∠A + ∠B = 80° and ∠A – ∠B = 20°

Adding and subtracting these two, we get

(∠A + ∠B) + (∠A – ∠B) = 80° + 20°

and, (∠A + ∠B) – (∠A – ∠B) = 80° – 20°

=> 2 (∠A) = 100° and 2(∠B) = 60°

=> ∠A = 50° and ∠B = 30°

Putting the values of ∠A and ∠B in ∠A + ∠B + ∠C = 180°, we get

50° + 30° + ∠C = 180°

=> ∠C = 180° – (50° + 30°) = 100°

Hence, ∠A = 50°, ∠B = 30° and ∠C = 100°

Q. 6. In a Δ ABC, if 2 ∠A = 3 ∠B = 6 ∠C, determine ∠A, ∠B and ∠C.

Sol. We have,

2 ∠A = 3 ∠B = 6 ∠C

=> $\frac{\angle A}{3} = \frac{\angle B}{2} = \frac{\angle C}{1}$ [Dividing throughout by 6 i.e., the l.c.m of 2, 3 and 6]

=> ∠A : ∠B : ∠C = 3 : 2 : 1

Let ∠A = 3x, ∠B = 2x and ∠C = x. Then,

∠A + ∠B + ∠C = 180°

=> 3x + 2x + x = 180° => 6x = 180° => x = 30°

Q. 7. A, B, C are the three angles of a triangle. If A – B = 15°, B – C = 30°, find ∠A, ∠B and ∠C.

Sol. We have,

A – B = 15° and B – C = 30°

=> A = 15° + B and C = B – 30° ... (i)

Now, $A + B + C = 180°$

=> $15° + B + B + B - 30° = 180°$ => $3B - 15° = 180°$ => $3B = 195°$ => $B = 65°$

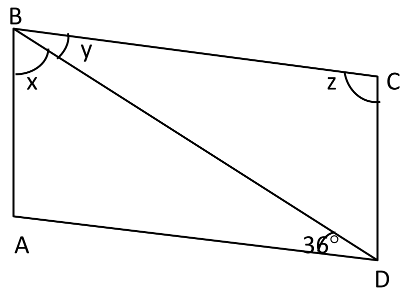
Putting $B = 65°$ in (i), We get , $A = 15° + 65° = 80°$ and $C = 65° - 30° = 35°$

Q. 8. In fig. AB ∥ DC. If $x = \frac{4y}{3}$ and $y = \frac{3z}{8}$, find ∠BCD, ∠ABC and ∠BAD.

Sol. Since AB ∥ DC and transversal BC intersects them at B and D respectively.

∴ ∠ABC = ∠BDC and, ∠CBD = ∠ADB [Alternate angles]

=> ∠BDC = x and y = 36 [∵ ∠ABD = x° and ∠ADB = 36° (Given)]



But, it it given that:

$x = \frac{4y}{3}$ and $y = \frac{3z}{8}$

∴ $x = \frac{4}{3} \times 36$ and $36 = \frac{3z}{8}$

=> $x = 48$ and $z = \frac{36 \times 8}{3} = 96$

IN Δ BAD, we have

∠BAD + ∠ADB + ∠ABD = 180°

=> ∠BAD + 36 + x = 180

=> ∠BAD + 36 + 48 = 180

=> ∠BAD = 96

Thus, ∠BCD = z° = 96°, ∠ABC = x° + y° = 48° + 36° = 84° and ∠BAD = 96°

Q. 9. A triangle ABC is right angled at A. AL is drawn perpendicular to BC. Prove that ∠BAL = ∠ACB.

Sol. IN Δ ABL, we have

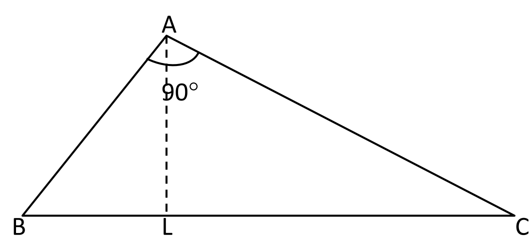
∠BAL + ∠ALB + ∠B = 180°

=> ∠BAL + 90° + ∠B = 180° [∵ AL ⊥ BC ∴ ∠ALB = 90°]

=> ∠BAL + ∠B = 90°

=> ∠BAL = 90° – ∠B ... (i)

In Δ ABC, we have



∠A + ∠B + ∠C = 180°

=> 90° + ∠B + ∠C = 180°

=> ∠B + ∠C = 180° – 90°

=> ∠B + ∠C = 90°

=> ∠C = 90° – ∠B

=> ∠ACB = 90° – ∠B ... (ii)

From (i) and (ii), we get

∠BAL = ∠ACB

In Δ ABC, we have

∠A + ∠B + ∠C = 180°

=> 90° + ∠B + ∠C = 180°

=> ∠B + ∠C = 180° – 90°

=> ∠B + ∠C = 90°

=> ∠C = 90° – ∠B

=> ∠ACB = 90° – ∠B

From (i) and (ii), we get

∠BAL = ∠ACB

Q. 10. In Fig. PS is the bisector of ∠QPR and PT ⊥ QR. Shown that ∠TPS = ½ (∠Q – ∠R).

Sol. Since PS is the bisector of ∠QPR.

∴ ∠QPS = ∠SPR ... (i)

In Δ PQT, we have

∠PQT + ∠PTQ + ∠QPT = 180°

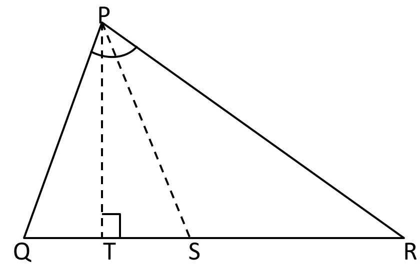
Q. 10. In Fig. PS is the bisector of ∠QPR and PT ⊥ QR. Shown that ∠TPS = ½ (∠Q – ∠R).

=> $\angle PQT + 90^\circ + \angle QPT = 180^\circ$

=> $\angle PQT + \angle QPT = 90^\circ$

=> $\angle PQT = 90^\circ - \angle QPT$

=> $\angle Q = 90^\circ - QPT$ ... (ii)



In Δ PTR, we have

$\angle PRT + \angle TPR + \angle PTR = 180^\circ$

=> $\angle PRT + \angle TPR = 90^\circ$

=> $\angle PRT + \angle TPR = 90^\circ$

=> $\angle PRT = 90^\circ - \angle TPR$

=> $\angle R = 90^\circ - \angle TPR$ ... (iii)

Subtracting (iii) from (ii), we get

$\angle Q - \angle R = (90^\circ - \angle QPT) - (90^\circ - \angle TPR)$

=> $\angle Q - \angle R = \angle TPR - \angle QPT$

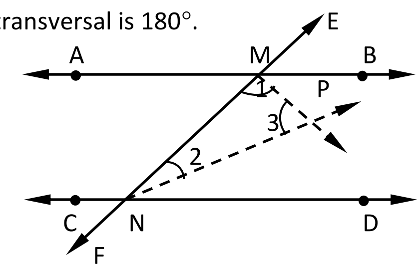
=> $\angle Q - \angle R = (\angle TPS + \angle SPR) - (\angle QPS - \angle TPS)$

=> $\angle Q - \angle R = 2\angle TPS$ [Using (i)]

=> $\angle TPS = \frac{1}{2}(\angle Q - \angle R)$

Q. 11. If two parallel lines are intersected by a transversal, prove that the bisectors of the interior angles on the same side of transversal intersect each other at right angles.

Sol. We know that the sum of the interior angles on the same side of the transversal is 180°.



∴ $\angle BMN + \angle DNM = 180^\circ$

=> $\frac{1}{2}\angle BMN + \frac{1}{2}\angle DNM = 90^\circ$

=> $\angle PMN + \angle PNM = 90^\circ$

=> $\angle 1 + \angle 2 = 90^\circ$ ... (i)

In Δ PMN, we have

$\angle 1 + \angle 2 + \angle 3 = 180^\circ$ ... (ii)

From (i) and (ii), we get

$90^\circ + \angle 3 = 180^\circ$

=> $\angle 3 = 90^\circ$

=> PM and PN intersect at right angles.

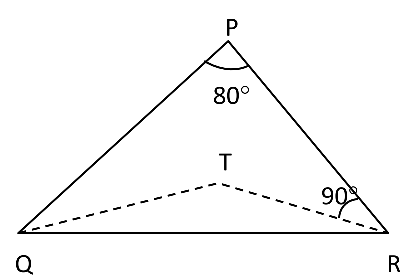
Q. 12. In Fig. TQ and TR are the bisectors of ∠Q and ∠R respectively. If ∠QPR = 80° and ∠PRT = 30°, determine ∠TQR and ∠QTR.

Sol. Since the bisector of ∠Q and ∠R meet at T.

∴ $\angle QRT = 90^\circ + \frac{1}{2}\angle QPR$ [By corollary on page 2]

=> $\angle QTR = 90^\circ + \frac{1}{2}(80^\circ)$

=> $\angle QTR = 90^\circ + 40^\circ = 130^\circ$



In Δ QTR, we have

$\angle TQR + \angle QTR + \angle TRQ = 180^\circ$

=> $\angle TQR + 130^\circ + 30^\circ = 180^\circ$ [∵ $\angle TRQ = \angle PRT = 30^\circ$]

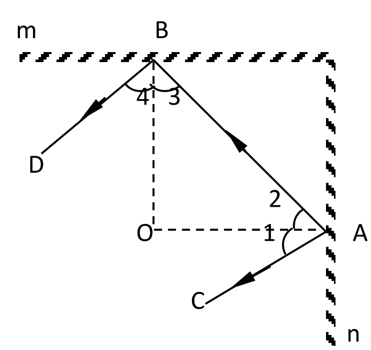
=> $\angle TQR = 20^\circ$

Thus, $\angle TQR = 20^\circ$ and $\angle QTR = 130^\circ$

Q. 13. In Fig. m and n are twp plane mirrors perpendicular to each other. Show that the incident ray CA is parallel to the reflected ray BD.

Sol. In order to prove that CA ∥ BD. It is sufficient to show that

$\angle CAB + \angle ABD = 180^\circ$



In Δ BOA, we have

=> $\angle 2 + \angle 3 + \angle BOA = 180^\circ$

=> $\angle 2 + \angle 3 + 90^\circ = 180^\circ$ [∵ $\angle BOA = 90^\circ$]

=> $\angle 2 + \angle 3 = 90^\circ$

=> $2(\angle 2 + \angle 3) = 180^\circ$ [Multiply both sides by 2]

=> $2(\angle 2) + 2(\angle 3) = 180^\circ$

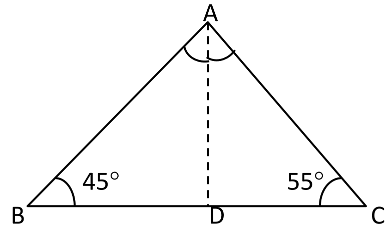
=> $\angle CAB + \angle ABD = 180^\circ$

(∠ of incidence = ∠ of reflection
∴ $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$
=> $2\angle 2 = \angle CAB$ and $2\angle 3 = \angle BAD$)

Thus, CA and BD are two lines intersected by a transversal AB such that ∠CAB + ∠ABD = 180° i.e., the sum of the interior angles on the same side of AB is 180°. Hence, CA ∥ BD.

Q. 14. In Δ ABC, ∠B = 45°, ∠C = 55° and bisector of ∠A meets BC at a point D. Find ∠ADB and ∠ADC.

Sol. In ΔABC, we have



∠A + ∠B + ∠C = 180°

=> ∠A + 45° + 55° = 180°

=> ∠A = 180° – 100°

=> ∠A = 80°

Since AD is the bisector of ∠A.

∴ ∠BAD = ∠CAD = ½ ∠A

=> ∠BAD = ∠CAD = 40°

In Δ ADB, we have

∠BAD + ∠ABD + ∠ADB = 180°

=> 40° + 45° + ∠ADB = 180°

=> ∠ADB = 180° – 85° = 95°

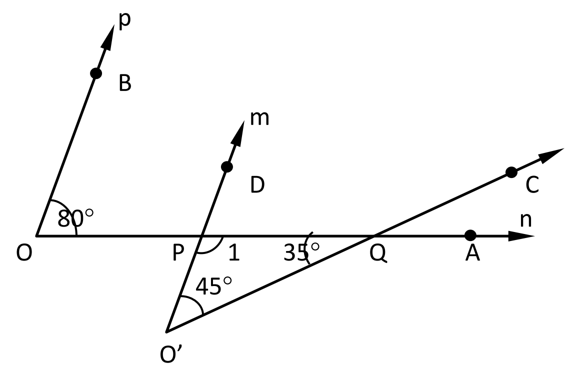
Since ∠ADB and ∠ADC from a linear pair.

∴ ∠ADB + ∠ADC = 180°

=> 95° + ∠ADC = 180°

=> ∠ADB = 95° and ∠ ADC = 85°

Q. 15. In Fig. prove that p ‖ m.



Sol. In Δ PO'Q, we have

∠O'PQ + ∠PQ'O + ∠PQO' = 180°

=> ∠1 + 45° + 35° = 180°

=> ∠1 = 180° – 80°

=> ∠1 = 100°

Since ∠QPD and ∠QPD' from a linear pair.

∴ ∠QPD + ∠QPD' = 180°

=> ∠QPD + 100° = 180°

=> ∠QPD = 80°

Now, p and m are two lines such that a transversal n intersects them at 0 and P respectively such that the corresponding angles on the same side are equal i.e.,

∠AOB = ∠QPD = 80°. Hence, p ‖ m.

EXERCISE 1.1

1. In a Δ ABC, if ∠A = 55°, ∠B = 40°, find ∠C.
2. If the angles of a triangle are in the ratio 1:2:3, determine three angles.

3. The angles of the triangle are (x – 40)°, (x – 20)° and [½ x – 10]°. Find the value of x.
4. The angles of a triangle are arranged in ascending order of magnitude. If the difference between two consecutive angles is 10°, find the three angles.
5. Two angles of a triangle are equal and the third angle is greater than each of those angles by 30°. Determine all the angles of the triangle.
6. If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right triangle.
7. ABC is a triangle in which ∠A = 72°, the internal bisectors of angles B and C meet in O. Find the magnitude of ∠BOC.
8. The bisectors of base angles of a triangle cannot enclose a right angle in any case.
9. If the bisectors of the base angles of a triangle enclose an angle of 135°, prove that the triangle is a right triangle.
10. In a Δ ABC, ∠ABC = ∠ ACB and the bisectors of ∠ABC and ∠ACB intersect at O such that ∠BOC = 120°. Show that ∠A = ∠B = ∠C = 60°.
11. Can a triangle have:
    (i) Two right angles? (ii) Two obtuse angles?
    (iii) Two acute angles? (iv) All angles more than 60°?
    (v) All angles less than 60°? (vi) All angles equal to 60°?
    Justify your answer in each case.
12. If each angle of a triangle is less than the sum of the other two, show that the triangle is acute angled.

ANSWERS

1. 85° 2. 30°, 60°, 90° 3. 100° 4. 50°, 60°, 70°
5. 50°, 50°, 80° 7. 126° 11. (i) No (ii) No (iii) Yes (iv) No (v) No (vi) Yes

EXTERIOR ANGLES OF A TRIANGLE

EXTERIOR ANGLES: If the side BC of a triangle ABC is produced to form ray BD, then ∠ACD is called an exterior angle of Δ ABC at C and is denoted by ext. ∠ACD.

With respect to ext ∠ACD of ΔABC at C, the angles A and B are called remote interior angles or interior opposite angles. Now, if we produce side AC to form ray AE, then ∠BCE is also an exterior angle of Δ ABC at C. Clearly, these two angles viz. ext ∠ACD and ext. ∠BCE are vertically opposite angles.

∴ ext. ∠ACD = ext. ∠BCE

Also, angles A and B are the interior opposite angles with respect to ext. ∠BCE.

It follows from the above discussion that at each vertex of a triangle, there are two exterior angles of the triangle and these two angles are equal.

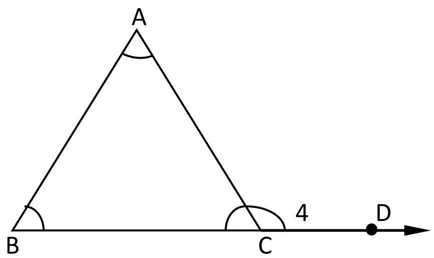
An exterior angle of a triangle is closely related to the interior opposite angles as proved in the following theorem.

■ <u>**THEOREM 1**</u> **(Exterior Angle Theorem): If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles**.

GIVEN: A triangle ABC. D is a point on BC produced, forming exterior angle ∠A.

PROOF In triangle ABC, we have

$\angle 1 + \angle 2 + \angle 3 = 180^\circ$



Also, $\angle 3 + \angle 4 = 180^\circ$ [∵ ∠3 and ∠4 form a linear pair] ... (ii)

From (i) and (ii), we have

$\angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$

=> $\angle 1 + \angle 2 = \angle 4$

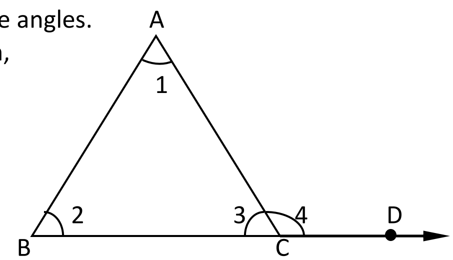
Hence, $\angle 4 = \angle 1 + \angle 2$ i.e., $\angle ACD = \angle CAB + \angle CBA$

COROLLARY An exterior angle of a triangle is greater than either of the interior opposite angles.

PROOF Let ABC be a triangle whose side BC is produced to form exterior angle ∠A. Then,

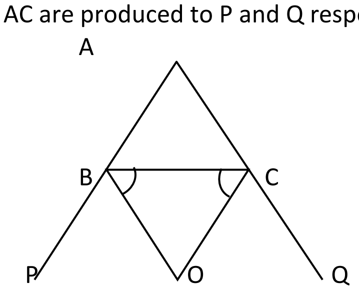
$\angle 1 + \angle 2 = \angle 4$

=> $\angle 4 > \angle 1$ and $\angle 4 > \angle 2$ i.e., $\angle ACD > \angle CAB$ and $\angle ACD > \angle CBA$



■<u>**THEOREM 2**</u> **The sides AB and AC of a Δ ABC are produced to P and Q respectively. If the bisectors of ∠PBC and ∠QCB intersect at O, then $\angle BOC = 90^\circ - \frac{1}{2}\angle A$**

GIVEN A Δ ABC in which sides AB and AC are produced to P and Q respectively. The bisectors of ∠PBC and ∠QCB intersect at O.



PROVE $\angle BOC = 90^\circ - \frac{1}{2}\angle A$

PROOF Since ∠ABC and ∠CBP form a linear pair.

∴ $\angle ABC + \angle CBP = 180^\circ$

=> $\angle B + 2\angle 1 = 180^\circ$ [BO is the bisector of ∠CBP ∴ ∠CBP = 2∠1]

=> $2\angle 1 = 180^\circ - \angle B$ => $\angle 1 = 90^\circ - \frac{1}{2}\angle B$ ... (i)

Again, ∠ACB and ∠QCB form a linear pair.

∴ $\angle ACB + \angle QCB = 180^\circ$

=> $\angle C + 2\angle 2 = 180°$ [∵ DC is the bisector of $\angle QCB$ ∴ $\angle QCB = 2\angle 2$]

=> $2\angle 2 = 180° - \angle C$ => $\angle 2 = 90° - ½ \angle C$ … (ii)

In Δ BOC, we have

$\angle 1 + \angle 2 + \angle BOC = 180°$

=> $90° - ½ \angle B + 90° - ½ \angle C + \angle BOC = 180°$ [Using (i) and (ii)]

=> $180° - ½ (\angle B + \angle C) + \angle BOC = 180°$

=> $\angle BOC = ½ (\angle B + \angle C)$

=> $\angle BOC = ½ (180° - \angle A)$ $\left[\begin{array}{l} ∵ \angle A + \angle B + \angle C = 180° \\ ∴ \angle B + \angle C = 180° - \angle A \end{array}\right]$

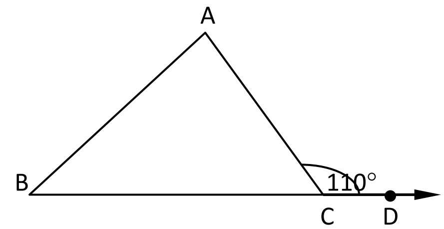
=> $\angle BOC = 90° - ½ \angle A$.

Illustrative Examples……………………………………………………………………………

Q. 1. An exterior angle of a triangle is 110°, and one of the interior opposite angles is 30°. Find the other two angles of the triangle.

Sol. Let ABC be a triangle whose side BC is produced to form an interior and ∠ACD such that ext. ∠ACD = 110°

Let ∠B = 30°. By exterior angle theorem, we have



ext. $\angle ACD = \angle B + \angle A$

=> $110° = 30° + \angle A$

=> $\angle A = 110° - 30° = 80°$

In Δ ABC, we have

$\angle A + \angle B + \angle C = 180°$

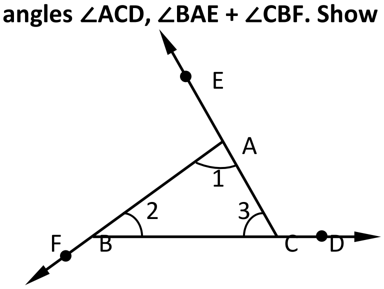
=> $80° + 30° + \angle C = 180°$ => $\angle C = 180° - (80° + 30°) = 70°$

Hence, the other two angles of the triangle are 80° and 70°.

Q. 2. The sides BC, CA and AB of a Δ ABC, are produced in order, forming exterior angles ∠ACD, ∠BAE + ∠CBF. Show that

$\angle ACD + \angle BAE + \angle CBF = 360°$

Sol. In Δ ABC, by using exterior angle theorem, we have,



$\angle ACD = \angle 1 + \angle 2$

$\angle BAE = \angle 2 + \angle 3$

and, $\angle CBF = \angle 1 + \angle 3$

By adding these three, we get

$\angle ACD + \angle BAE + \angle CBF = (\angle 1 + \angle 2) + (\angle 2 + \angle 3) + (\angle 1 + \angle 3)$

=> $\angle ACD + \angle BAE + \angle CBF = 2 (\angle 1 + \angle 2 + \angle 3)$

=> $\angle ACD + \angle BAE + \angle CBF = 2 \times 180° = 360°$ [∵ $\angle 1 + \angle 2 + \angle 3 = 180°$]

Q. 3. In Fig. if QT ⊥ PR, ∠TQR = 40° and ∠SPR = 30°, find x and y.

Sol. In triangle TQR, we know two angles ∠TQR and ∠QTR. Therefore, we apply the angle sum property in ΔTQR.

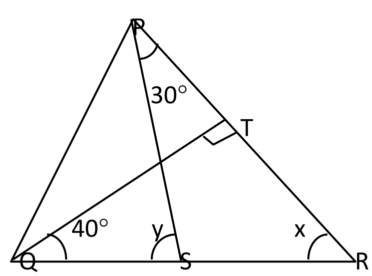
We have,

$\angle TQR + \angle QTR + \angle TRQ = 180°$

=> $40° + 90° + \angle TRQ = 180°$

=> $\angle TRQ = 180° - 130° = 50°$

=> $x = 50°$



In Δ PSR, using exterior angle property, we have

$\angle PSQ = \angle PRS + \angle RPS$

=> $y = x + 30°$

=> $y = 50° + 30° = 80°$

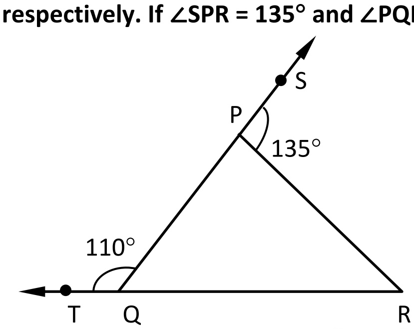
Q. 4. In Fig. sides QP and RQ of Δ PQR are produced to point S and T respectively. If ∠SPR = 135° and ∠PQR = 110°, Find ∠PRQ.

Sol. Since QPS is a straight line.

∴ $\angle QPR + \angle SPR = 180°$

=> $\angle QPR + 135° = 180°$

=> $\angle QPR = 45°$



Using exterior angle property is ΔPQR, we have

$\angle PQT = \angle QPR + \angle PRQ$

=> $110° + 45° + \angle PRQ$

=> $\angle PRQ = 110° - 45° = 65°$

Q. 5. In Fig. ∠x = 62°, ∠XYZ = 54°. If YO and ZO are bisectors of ∠XYZ and ∠XZY respectively of ΔXYZ, find ∠OZY and ∠YOZ.

Sol. In ΔXYZ, we have

62° + 54° + ∠XZY = 180°

=> ∠XZY = 180° – 116° = 64°

=> 2 ∠OZY = 64° [∵ ∠XZY = 2 ∠OZY]

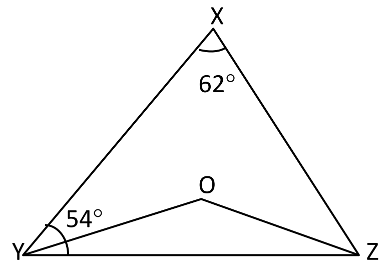
=> ∠OZY = 32°

In Δ OYZ, we have

∠OYZ + ∠OZY + ∠YOZ = 180°

=> 27° + 32° + ∠YOZ = 180°

=> ∠YOZ = 180° – (27° + 32°) = 180° – 59° = 121°



Alter From corollary of theorem 1 on page 2, we have

∠YOZ = 90° + ∠8 = 90° + 31° = 121°

Q. 6. In Fig. if AB ∥ DE, ∠BAC = 35° and ∠CDE = 53°, find ∠DCE.

Sol. Since AB ∥ DE and AE is the transversal.

∴ ∠AED = ∠BAE

=> ∠AED = 35° [∠BAE = ∠BAC = 35°]

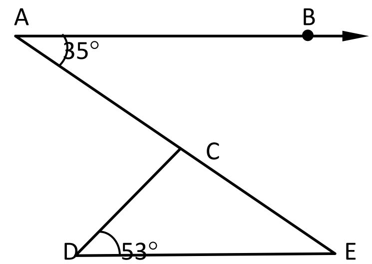
=> ∠CED = 35°

In Δ DCE, we have

∠CDE + ∠DCE + ∠CED = 180°

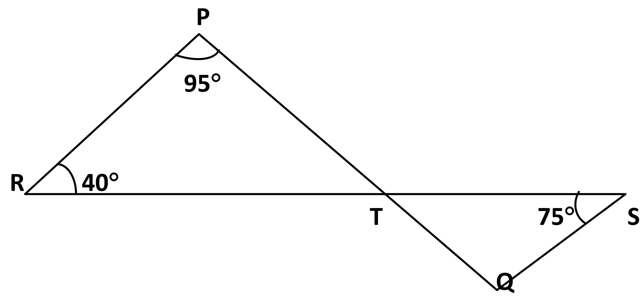
=> 53° + ∠DCE + 35° = 180°

=> ∠DCE = 180° – 88° = 92°



Q. 7. In fig. if lines PQ and RS intersect at a pint T such that ∠PRT = 40°, ∠RPT = 95° and ∠TSQ = 75°, find ∠SQT.

Sol.



Sol. In Δ PRT, we have

∠P + ∠R + ∠T = 180°

=> 95° + 40° + ∠T = 180°

=> ∠T = 180° – 95° – 40° = 45°

=> ∠PTR = 45° {∵ ∠QTS and ∠PTR are vertically opposite angles ∴ ∠QTS = ∠PTR}

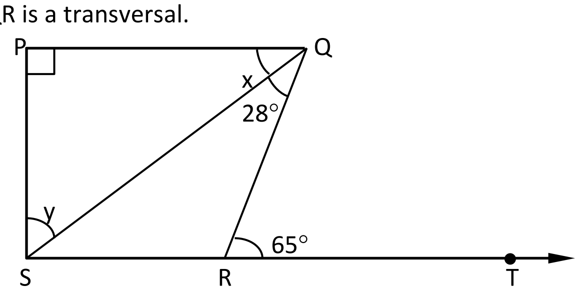
In Δ SQT, we have

∠QTS + ∠SQT + ∠TSQ = 180°

=> 45° + ∠SQT + 75° = 180° => ∠SQT = 180° – 120° = 60°

Q. 8. In Fig. If PQ ⊥ PS, PQ ∥ SR, ∠SQR = 28° and ∠QRT = 65°, then find the values of x and y.

Sol. Since PQ ∥ SR and QR is a transversal.



∴ ∠PQR = ∠TRQ [∵ Alternate angles]

=> x + 28° = 65°

=> x = 37°

In Δ PQS, we have,

$\angle QPS + \angle PQS + \angle PSQ = 180^\circ$

=> $90^\circ + 37^\circ + y = 180^\circ \Rightarrow y = 180^\circ - 127^\circ = 53^\circ$

Q. 9. The side BC of a Δ ABC is produced on both sides. Show that the sum of the exterior angles so formed is greater than ∠A by two right angles.

Sol. By exterior angle theorem, we have

$\angle 4 = \angle 1 + \angle 3$ and $\angle 5 = \angle 1 + \angle 2$

Adding these two, we get

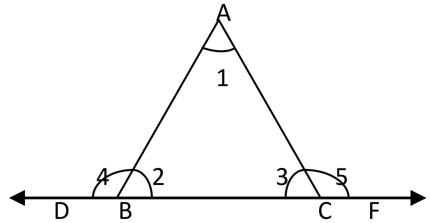
$\angle 4 + \angle 5 = (\angle 1 + \angle 3) + (\angle 1 + \angle 2)$

=> $\angle 4 + \angle 5 = \angle 1 + (\angle 1 + \angle 2 + \angle 3)$

=> $\angle 4 + \angle 5 + \angle 1 + 180^\circ$ [∵ $\angle 1 + \angle 2 + \angle 3 = 180^\circ$]

=> $\angle 4 + \angle 5 = \angle A + 2 \times 90^\circ$

=> $\angle 4 + \angle 5$ exceeds $\angle A$ by two right angles.



Q. 10. Sides BC, CA and BA of a triangle ABC are produced to D, Q ,P respectively as shown in Fig. If ∠ACD = 100° and ∠QAP = .35°, find all the angles of the triangle.

Sol. Since ∠QAP and ∠BAC are vertically opposite angles.

∴ $\angle BAC = \angle QAP \Rightarrow \angle BAC = 35^\circ$ [∵ $\angle QAP = 35^\circ$]

By exterior angle theorem, we have

∴ $\angle ACD = \angle BAC + \angle CBA$

=> $100^\circ = 35^\circ + \angle CBA$

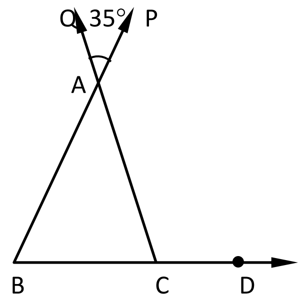
=> $\angle CBA = 100^\circ - 35^\circ = 65^\circ$

Since ∠ACB and ∠ACD form linear pairs,

∴ $\angle ACB + \angle ACD = 180^\circ$

=> $\angle ACB + 100^\circ = 180^\circ = 180^\circ - 100^\circ = 80^\circ$

Hence, the angles of the Δ ABC are ∠A = 35°, ∠B = 65° and ∠C = 80°.



Q. 11. In Fig. the side BC of Δ ABC is produced to form ray BD as shown. Ray CE is drawn parallel to BA. Show directly, without using the angle sum property of a triangle that ∠ACD = ∠A + ∠B and deduced that ∠A + ∠B + ∠C = 180°.

Sol. Since AB ∥ CE and transversal AC cuts them at A and C respectively.

∴ $\angle 1 = \angle 4$ [Alt int ∠s] ... (i)

Again, AB ∥ CE and transversal BD cuts them

∴ $\angle 2 = \angle 5$ [Corres. ∠s axiom] ... (ii)

Adding (i) and (ii), we get

$\angle 1 + \angle 2 = \angle 4 + \angle 5$

=> $\angle A + \angle B = \angle ACD$ [∵ $\angle 4 + \angle 5 = \angle ACD$]

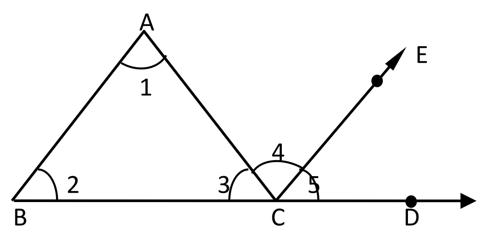
This proves the first part,

Now,

$\angle A + \angle B = \angle ACD$

=> $\angle A + \angle B + \angle C = \angle ACD + \angle C$ [Adding ∠C on both sides]

=> $\angle A + \angle B + \angle C = 180^\circ$ [∵ ∠ACD and ∠C form a linear pair ∴ $\angle ACD + \angle C = 180^\circ$]



Q. 12. Prove that the angle between internal bisector of one base angle and the external bisector of the other base angle of a triangle is equal to one half of the vertical angle.

Given A Δ ABC with base BC. The internal bisector of ∠B and the external bisector of ext. ∠ACD meet at E.

TO PROVE $\angle E = \frac{1}{2} \angle A$

PROOF We have,

ext. $\angle ACD = \angle A + \angle B$

=> $\frac{1}{2}$ ext. $\angle ACD = \frac{1}{2} \angle A + \frac{1}{2} \angle B$

=> $\angle 2 = \angle 1 + \frac{1}{2} \angle A$ ... (i)

In Δ BCE, we have

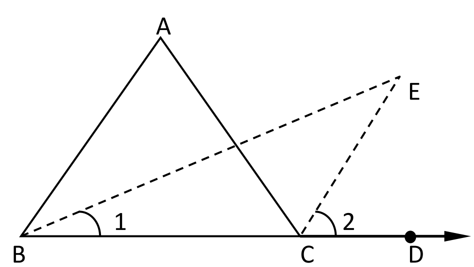
ext. $\angle ECD = \angle 1 + \angle E$

=> $\angle 2 = \angle 1 + \angle E$ ... (ii)

From (i) and (ii), we get

=> $\angle 1 + \frac{1}{2} \angle A = \angle 1 + \angle E$

=> $\frac{1}{2} \angle A = \angle E \Rightarrow \frac{1}{2} \angle A$



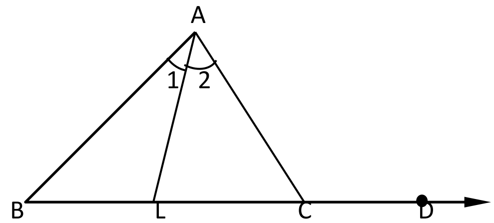
Q. 13. **The side BC of a Δ ABC is produced, such that D is on ray BC. The bisector of ∠A meets BC in L as shown in Fig. Prove that ∠ABC + ACD = 2 ∠ALC** **11**

Sol. In Δ ABC, we have

ext. ∠ACD = ∠B + ∠A

=> ext. ∠ACD = ∠B + 2 ∠1 [∵ AL is the bisector of ∠A ∴ ∠A = 2 ∠ 1]

=> ∠ACD = ∠B + 2 ∠1 ... (i)



In Δ ABL, we have

ext. ∠ALC = ∠B + ∠BAL

=> ext. ∠ALC = ∠B + ∠1

=> 2 ∠ALC = 2 ∠B + 2 ∠1 ... (ii)

[Multiplying both sides by 2]

Subtracting (i) from (ii), we get

2 ∠ALC – ∠ACD = ∠B

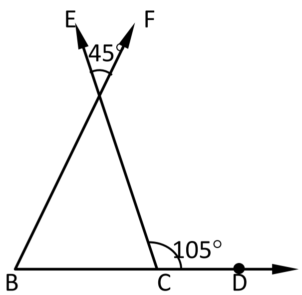
=> ∠ACD + ∠B = 2 ∠ALC => ∠ACD + ∠ABC = 2 ∠ALC

EXERCISE 1.2

1.The exterior angles, obtained on producing the base of a triangle both ways are 104° and 136°. Find all the angles of the triangle.

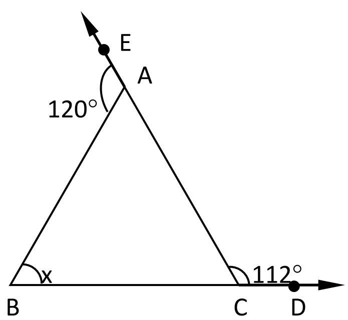
2.In a Δ ABC, the internal bisectors of ∠B and ∠C meet at P and the external bisector of ∠B and ∠C meet at Q. Prove that ∠BPC + ∠BQC = 180°

3. In Fig. the sides BC, CA and AB of a Δ ABC have been produced to D, E and F respectively. If ∠ACD = 105° and ∠EAF = 45°, find all the angles of the Δ ABC.

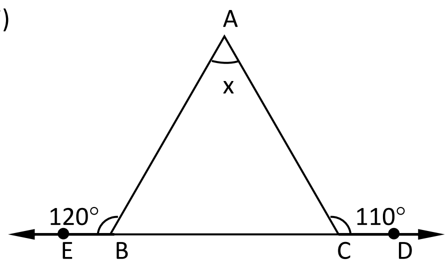


4. Compute the value of x in each of the following figures:

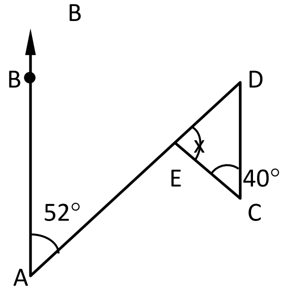
(i)

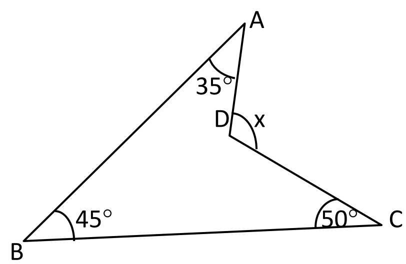


(ii)

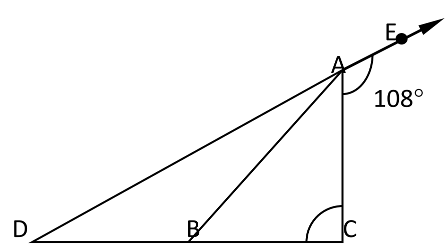


(iii)



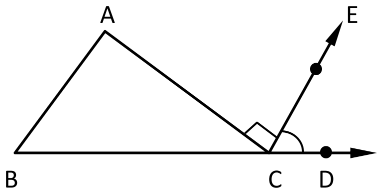


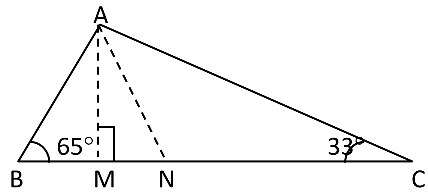
5. In Fig., AB divides ∠DAC in the ratio 1:3 and AB = DB. Determine the value of x.



6. ABC is a triangle. The bisector of the exterior angle at B and the bisector of ∠C intersect each other at D. Prove that ∠D = ½ ∠A.

7. In Fig. AC ⊥ CE and ∠A : ∠B : ∠C = 3 : 2 : 1, find the value of ∠ECD.



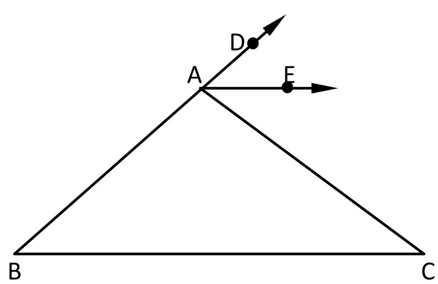


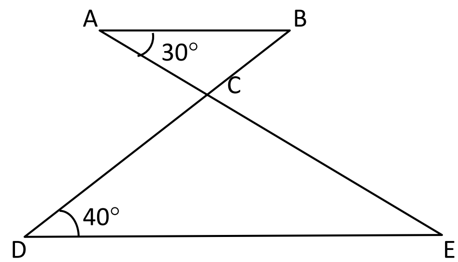
8. In Fig. AM ⊥ BC and AN is the bisector of ∠A. If ∠B = 65° and ∠C = 33°, find ∠MAN.

9. In a Δ ABC, AD bisects ∠A and ∠C > ∠B. Prove that ∠ADB > ∠ADC.

10. In Δ ABC, BD ⊥ AC and CE ⊥ AB. If BD and CE intersect at O, prove that ∠BOC = 180° – A.

11. In Fig. AE bisects ∠CAD and ∠B = ∠C. Prove that AE ∥ BC.





12. In Fig. AB ∥ DE. Fin ∠ACD.

13. Which of the following statements are true (T) and which are false (F):
(i) Sum of the three angles of a triangle is 180°.
(ii) A triangle can have two right angles.
(iii) All the angles of a triangle can be less than 60°.
(iv) All the angles of a triangle can be greater than 60°
(v) All the angles of a triangle can be equal to 60°.
(vi) A triangle can have two obtuse angles.
(vii) A triangle can have at most one obtuse angles.
(viii) If one angle of the triangle is obtuse, then it cannot be a right angled triangle.
(ix) An exterior angle of a triangle is less than either of its interior opposite angles.
(x) An exterior angle of a triangle is equal to the sum of the two interior opposite angles.
(xi) An exterior angle of a triangle is greater than the opposite interior angles.

14. Fill in the blanks to make the following statements true:
(i) Sum of the angles of a triangle is …………………. .
(ii) An exterior angle of a triangle is equal to the two ………. opposite angles.
(iii) An exterior angle of a triangle is always …………. than either of the interior opposite angle.
(iv) A triangle cannot have more than ………. right angles.
(v) A triangles cannot have more than ………. obtuse angles.

ANSWERS

1. 60°, 76°, 44° 3. ∠A = 45°; ∠C = 75°; ∠B = 60° 4. 52°, 50°, 88°, 130°
5. 90° 7. 60° 8. 16°
12. 70° 13. (i) T (ii) F (iii) F (iv) F (v) T (vi) F (vii) T (viii) T (ix) F (x) T (xi) T
14. (i) 180° (ii) interior (iii) Greater (iv) one (v) one

CONGRUENT TRIANGLES

CONGRUENCE OF LINE SEGMENTS

We know that two congruent line segments have the same length and conversely two line segments of equal length are congruent. Thus, a simple criterion for the congruence of two-line segments is:

Two-line segments are congruent if and only if their lengths are equal.

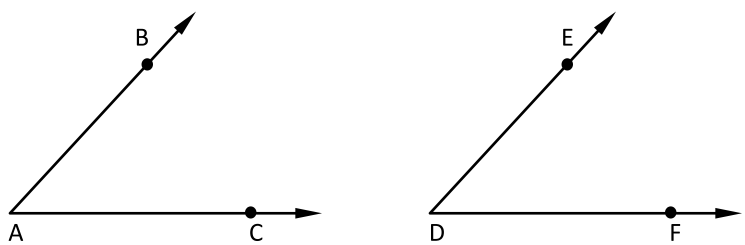
OR

Two-line segments AB and CD are congruent if and only if AB = CD.

(i) (ii)

CONGRUENCE OF ANGLES

- Two angles are congruent if any only if their measures are equal.
- A sufficient condition for the congruence of two angles is as follows:
- Two angles BAC and EDF are congruent if m ∠BAC = m ∠EDF



(i) (ii)

CONGRUENCE OF TRIANGLES

Let ΔABC and ΔDEF be two congruent triangles. Then, we can superpose ΔABC on ΔDEF, so as to cover it exactly. In such a superposition the vertices of Δ ABC will fall on the vertices of Δ DEF, in some order, Let us assume that the vertex A falls on vertex D, vertex B on vertex E and vertex C on vertex F.

Then, side AB falls on DE, BC on EF and CA on FD. Also ∠A superposes on the corresponding angle ∠D, ∠B on ∠E and ∠C on ∠F. Thus, the order in which the vertices match, automatically determines a correspondence between the sides and angles of the two triangles. And, if the superposition is exact ***i.e. the triangles are congruent, the corresponding sides and angles are congruent. Consequently, we get six equalities three of the corresponding sides and three of the corresponding angles.***

i.e., if Δ ABC superposes on Δ DEF exactly such that the vertices of ΔABC fall on the vertices of Δ DEF in the following order

AD$\leftrightarrow$ D, B $\leftrightarrow$ E, C $\leftrightarrow$ F

Then, we have the following six equalities

AB = DE, BC = EF, CA = FD (i.e., corresponding sides are congruent)

∠A = ∠D, ∠B = ∠E, ∠C = ∠F (i.e., corresponding angles are congruent)

In the above discussion we have considered one correspondence between the vertices of triangles ABC and DEF viz. A $\rightarrow$ D, B $\rightarrow$ E and C $\rightarrow$ F. But there can be many other matchings between the vertices of two triangles as discussed below.

In two triangles ABC and DEF, we have the following six matchings or correspondence between their vertices:

A $\leftrightarrow$ D, B $\leftrightarrow$ E and C $\leftrightarrow$ F written as ABC $\leftrightarrow$ DEF
A $\leftrightarrow$ E, B $\leftrightarrow$ F and C $\leftrightarrow$ D written as ABC $\leftrightarrow$ EFD
A $\leftrightarrow$ F, B $\leftrightarrow$ D and C $\leftrightarrow$ E written as ABC $\leftrightarrow$ FDE
A $\leftrightarrow$ D, B $\leftrightarrow$ F and C $\leftrightarrow$ E written as ABC $\leftrightarrow$ DFE
A $\leftrightarrow$ E, B $\leftrightarrow$ D and C $\leftrightarrow$ F written as ABC $\leftrightarrow$ EDF
A $\leftrightarrow$ F, B $\leftrightarrow$ E and C $\leftrightarrow$ D written as ABD $\leftrightarrow$ FED

If Δ ABC is congruent to ΔDEF, then in one of these six matchings Δ ABC superpose on ΔDEF exactly and in that particular matching corresponding sides and angles will be congruent. Consequently, we will have three equalities of corresponding sides and three equalities of the measures of corresponding angles.

If Δ ABC is not congruent to ΔDEF, then ΔABC will not superpose exactly ΔDEF in none of the above the six possible matchings. Infact, in each mapping at least one part (a side or an angle) of ΔABC will not be equal to the corresponding part of ΔDEF.

GENERAL CONDITION FOR THE CONGRUENCE OF TWO TRIANGLES:

► ***Two triangles are congruent of and only if there exists a correspondence between their vertices such that the corresponding sides and the corresponding angles of the two triangles are equal or congruent.***

► **ΔABC is congruent to ΔDEF and the correspondence ABC ↔ DEF makes the six pairs of corresponding parts of the two triangles congruent, then we write**

$$\Delta ABC \cong \Delta DEF$$

Thus, Δ ABC ≅ ΔDEF if and only if AB = DE, BC = EF, CA = FD, ∠A = ∠D, ∠B = ∠E and ∠C = ∠F.

- The letters in the names of two triangles will indicate the correspondence between the vertices of two triangles. For example, Δ ABC ≅ Δ DEF will indicate the correspondence ABC ↔ DEF and Δ ABC ≅ Δ DFE will indicate the correspondence ABC ↔ DFE. Thus, we can easily infer the six equalities between the corresponding parts of two triangles from the notation Δ ABC ↔ Δ DEF. We shall use the abbreviation "c.p.c.t" to indicate corresponding parts of congruent triangles.

- **Δ PQR ≅ ΔUVW will mean that**
  ∠P = ∠U, ∠Q = ∠V, ∠R = ∠W, PQ = UV, QR = VW and PR = UW

CONGRUENCE RELATION

From the definition of congruence of two triangles, we obtain the following results:

(i) Every triangle is congruent to itself i.e., Δ ABC ≅ Δ ABC

(ii) If Δ ABC ≅ Δ DEF, then Δ DEF ≅ Δ ABC

(iii) If Δ DEF ≅ Δ ABC, and Δ DEF ≅ Δ PQR, then Δ ABC ≅ Δ PQR

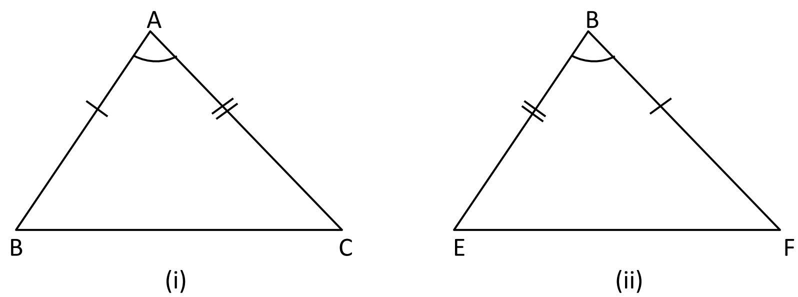
SUFFICIENT CONDITIONS (CRITERIA) FOR CONGRUENCE OF TRIANGLES

In this section we shall prove that *if three properly chosen conditions out of the six conditions are satisfied, then the other three are automatically satisfied.*

SIDE-ANGLE-SIDE (SAS) CONGRUENCE CRITERION

THEOREM 1: Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angle of the other triangle.

GIVEN: Two triangles ABC and DEF such that AB = DE, AC = DF and ∠A = ∠D



TO PROVE: Δ ABC ≅ Δ DEF

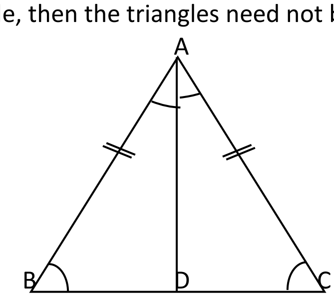
PROOF: Place Δ ABC over Δ DEF such that the side AB falls on side DF, vertex A falls on vertex D and B on E. Since ∠A = ∠D. Therefore, AC will fall on DF. But AC = DF and A falls on D. Therefore, C will fall on F. Thus, AC coincides with DF.

Now, B falls on E and C falls on F. Therefore, BC coincides with EF.

Thus, ΔABC when superposed on ΔDEF, covers it exactly, Hence, by definition of congruence, ΔABC ≅ Δ DEF.

It shall be noted that in ***SAS criterion the equality of included angles is very essential***. If two sides and one angle (not included between the two sides) of one triangle are equal to two sides and one angle of the other triangle, then the triangles need not be congruent. So, the equal angle should be the angle included between the sides.

THEOREM 2: Angles opposite to two equal sides of a triangle are equal.



GIVEN: ΔABC in which AB = AC

TO PROVE: ∠C = ∠B

CONSTRUCTION: Draw the bisector AD of ∠A which meets BC in D.

PROOF: In Δs ABD and ACD, we have

AB = AC	[Given]
∠BAD = ∠CAD	[By construction]
AD = AD	[Common side]

Therefore, by SAS criterion of congruence, we have

$\Delta ABD \cong \Delta ACD$

=> $\angle B = \angle C$ [Corresponding parts of congruent triangles are equal]

✱ The ***converse of the above theorem is also true*** i.e., if two angles of a triangle are equal, then the sides opposite to them are also equal. To prove this, we shall take the help of another criterion of congruence.

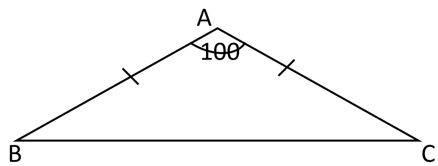
Examples.......

Q. 1. In Δ ABC, $\angle A = 100°$ and AB = AC. Find $\angle B$ and $\angle C$

Sol. We have,

AB = AC

=> $\angle B = \angle C$ [∵ Angles opp. to equal sides are equal]



In Δ ABC, we have

$\angle A + \angle B + \angle C = 180°$

=> $\angle A + \angle B + \angle B = 180°$ [∵ $\angle B = \angle C$]

=> $100° + 2\angle B = 180°$

=> $2\angle B = 80°$

=> $\angle B = 40°$

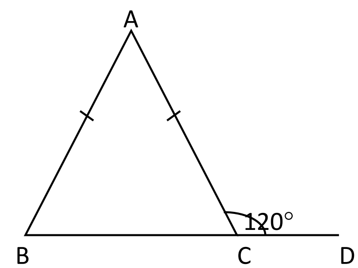
=> $\angle B = \angle C = 40°$

Q. 2. In Fig. AB = AC and $\angle$ ACD = 120°. Find $\angle A$.

Sol. We have,

AB = AC

=> $\angle B = \angle C$ [∵ Angles opposite to equal sides are equal]



Now, $\angle ACB + \angle ACD = 180°$ [Angles of a linear pair]

=> $\angle C + 120° = 180°$

=> $\angle C = 60°$

∴ $\angle B = 60°$ [∵ $\angle B = \angle C$]

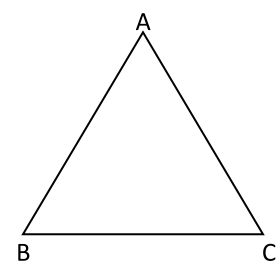
Now, $\angle A + \angle B + \angle C = 180°$

=> $\angle A + 60° + 60° = 180°$

=> $\angle A = 60°$

Q. 3. Prove that measure of each angle of an equilateral triangle is 60°.

Sol. Let Δ ABC be an equilateral triangle. Then, AB = BC = CA



Since angles opposite to equal sides of a triangle are equal.

∴ AB = BC and BC = CA

=> $\angle C = \angle A$ and $\angle A = \angle B$

=> $\angle A = \angle B = \angle C$

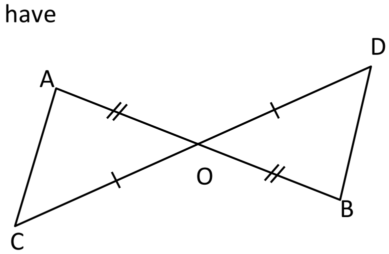
But, $\angle A + \angle A$ and $\angle A = \angle B$

∴ $\angle A + \angle A + \angle A = 180°$ => $3\angle A = 180°$ => $\angle A = 60°$

Hence, $\angle A = \angle B = \angle C = 60°$

Q. 4. In Fig. O is the mid-point of AB and CD. Prove that
(i) Δ AOC $\cong$ Δ BOD (ii) AC = BD and (iii) AC ∥ BD

Sol. In Δ s AOC and BOD, we have



AO = OB [∵ O is the mid-point of AB]

$\angle AOC = \angle BOD$ [Vertically opposite angles]

and, CO = OD [∵ O is the mid-point of CD]

So, by SAS congruence criterion, we have

$\Delta AOC \cong \Delta BOD$

=> AC = BD and, $\angle CAO = \angle DBO$ [Corresponding parts of congruent triangles are equal]

Now, AC and BD are two lines intersected by a transversal AB such that $\angle CAO = \angle DBO$, i.e., alternate angles are equal. Therefore, AC ∥ BD

Q. 5. In Fig. it is given that AB = CF, EF = BD and ∠AFE = ∠ CBD. Prove that Δ AFE ≅ Δ CBD.

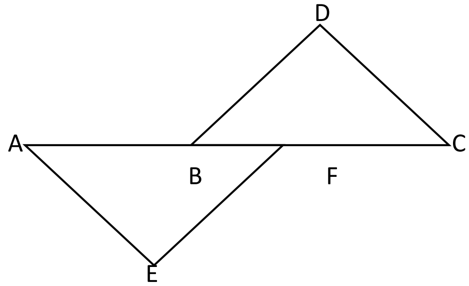
Sol. We have,

=> AB + BF = CF + BF [Adding BF on both sides]

=> AF = CB ... (i)

In Δs AFE and CBD, we have

AF = CB [From (i)]



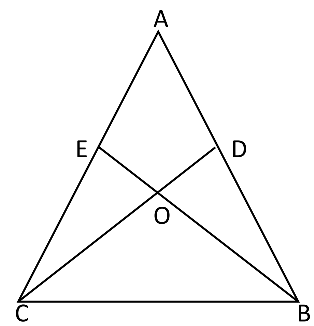
∠AFE = ∠DBC

and, EF = BD

So, by SAS criterion of congruence, we have

Δ AFE ≅ Δ CBD

Q. 6. In Fig., it is given that AE = AD and BD = CE. Prove that
Δ AEB ≅ Δ ADC



Sol. We have,

AE = AD and CE = BD

=> AE + CE = AD + BD

=> AC = AB ... (i)

Now, in Δs AEB and ADC, we have

AE = AD [Given]

∠EAB = ∠DAC [Common]

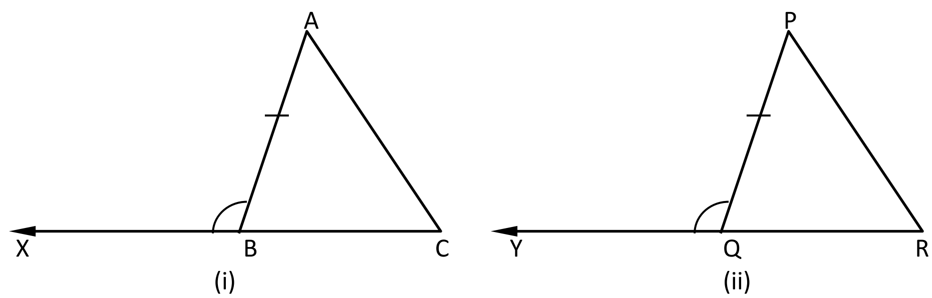
and, AC = AB [From (iii)]

So, by SAS criterion of congruence, we have

ΔAEB ≅ Δ ADC

Q. 7. In Δ ABC and Δ PQR, AB = PQ, BC = QR and CB and RQ are extended to X and Y respectively and ∠ABX = ∠PQY. Prove that Δ ABC ≅ Δ PQR.

Sol. We have,



∠ABX = ∠PQY

=> 180° – ∠ABC = 180° – ∠PQR [∵ ∠ABX + ∠ABC = 180 linear pair
∴ ∠ABX = 180° – ∠ABC
Similarly, ∠POY = 180° – ∠PQR]

=> ∠ABC = ∠PQR ... (i)

In Δ ABC and Δ PQR, we have

AB = PQ [Given]

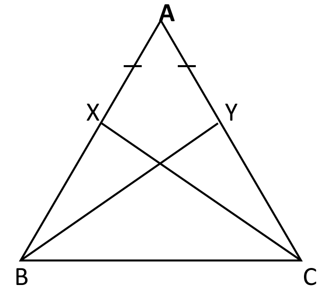
∠ABC = ∠PQR [From (i)]

and, BC = QR [Given]

So, by SAS criterion of congruence, we have

ΔABC ≅ Δ PQR

Q. 8. In Fig., X and Y are two points on equal sides AB and AC of a Δ ABC such that AX = AY. Prove that XC = YB.



Sol. In Δs AXC and AYB, we have

AX = AY [Given]

∠A = ∠A [Common angle]

and, AC = AB

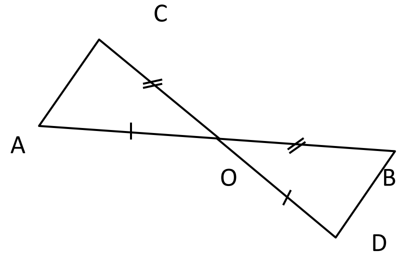
So, by SAS criterion of congruence, we have

Δ AXC ≅ Δ AYB

=> XC = YB [∵ Corresponding parts of congruent triangles are equal]

Q. 9. Suppose line segments AB and CD intersect at O in such a way that AO = OD and OB = OC. Prove that AC = BD but AC may not be parallel to BD.

Sol. In Δs AOC and DOB, we have



AO = OD [Given]

∠AOC = ∠DOB [Vertically opposite angles]

and, OC = OB [Given]

So, by SAS criterion of congruence, we have

ΔAOC ≅ ΔDOB

=> ∠OAC = ∠ODB

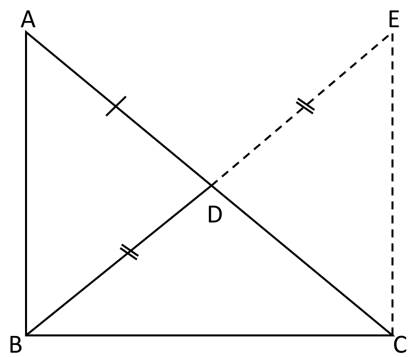
and, ∠OCA = ∠OBD [∵ c.p.ct]

Clearly, AC will be parallel to BD only when ∠OAC = ∠OBD. But ∠OAC may not be equal to ∠OBD, So, AC may not be parallel to BD.

Q. 10. If D is the mid-point of the hypotenuse AC of a right triangle ABC, prove that BD = ½ AC

GIVEN: A Δ ABC in which ∠B = 90° and D is the mid-point of AC.

TO PROVE: BD = ½ AC



CONSTRUCTION: Produce BD to E so that BD = DE. Join EC.

PROOF: In Δs ADB and CDE, we have

AD = DC [Given]

BD = DE [By construction]

and, ∠ADB = ∠CDE [Ver. opp. angles]

So, by SAS criterion of congruence, we have,

ΔADB ≅ Δ CDE

=> EC = AB and ∠CED = ∠ABD

Thus, transversal BE cuts AB and CE such that the alternate angles ∠CED and ∠ABD are equal. So,

CE ∥ AB

=> ∠ABC + ∠ECB = 180° [Sum of the interior angles on the same side of transversal BC intersecting parallel lines AB and CE]

=> 90° + ∠ECB = 180° [∵ ∠ABC = 90°]

=> ∠ECB = 90°

Thus, in Δs ABC and ECB, we have

AB = EC [From (i)]

BC = CB [Common]

and, ∠ABC = ∠ECB [Each equal to 90°]

So, by SAS criterion of congruence, we have

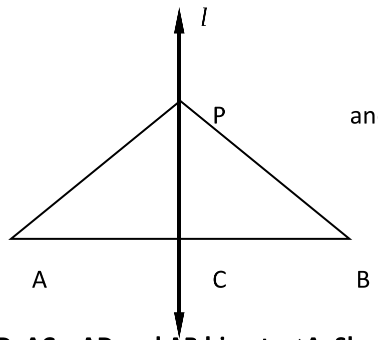
Δ ABC ≅ Δ ECB

=> AC = BE [c. p. c. t.]

=> ½ AC = ½ BE = ½ AC = BD

Q. 11. AB is a line segment and line l is its perpendicular bisector. If a point P lies on l, shown that P is equidistant from A and B.

Sol. Let C be the mid-point of AB. Clearly, line l passes through C and its perpendicular to AB.



In Δ PCA and Δ PCB, we have

AC = BC [∵ C is the mid-point of AB]

∠PCA = ∠PCB [Each equal to 90° as l ⊥ AB]

and, PC = PC

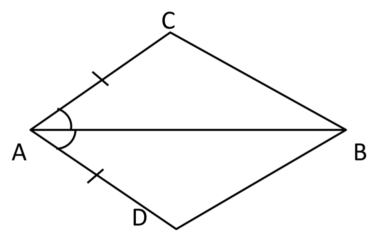
So, by SAS congruence rule, we have

ΔPCA ≅ Δ PCB

=> PA = PB [c. p. c. t.]

Q. 12. In quadrilateral ACBD, AC = AD and AB bisects ∠A. Show that Δ ABC ≅ Δ ABD. What can you say about BC and BD?

Sol.



In Δ ABC and Δ ABD, AC = AD [Given]

∠CAB = ∠DAB [∵ AB is the bisector of ∠ DAC]

and, AB = AB [Common]

So, by SAS congruence rule, we have

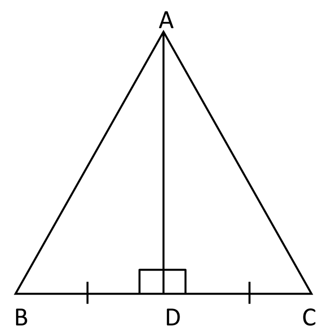
Δ ABC ≅ Δ ABD

=> BC = BD [c.p.ct]

Q. 13. Prove that Δ ABC is isosceles if any one of the following holds:

(i) Altitude AD bisects BC (ii) Median AD is perpendicular to the base BC.

Sol. (i) Let ABC be a triangle and let AD be altitude from A on BC. Suppose D bisects BC i.e., BD = CD. We have to prove that the Δ ABC isosceles.



In Δ ADB and ADC, we have,

AD = AD [Common side]

∠ADB = ∠ADC [Each equal to 90°]

and, BD = DC [Given]

So, by SAS criterion of congruence, we have

ΔADB ≅ Δ ADC

=> AB = AC [c. p. c. t.]

Hence, Δ ABC is isosceles.

(ii) Let ABC be a triangle such that the median AD is perpendicular to the base BC.

Then, we have to prove that the triangle ABC is isosceles.

In Δs ADB and ADC, we have

AD = AD [Common side]

∠ADB = ∠ADC [∵ AD ⊥ BC ∴ ∠ADB = ∠ADC = 90°]

and, BD = DC [∴ AD is the median ∴ D is the mid-point of BC]

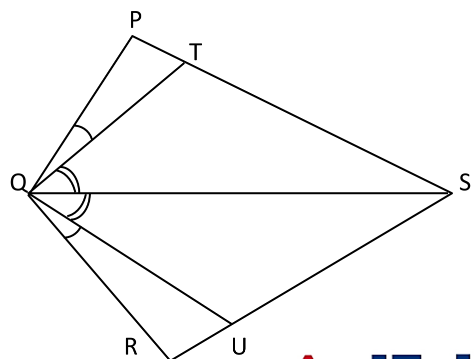
So, by SAS criterion of congruence, we have

Δ ADB ≅ Δ ADC

=> AB = AC [c. p. c. t.] Hence, Δ ABC is isosceles.

Q. 14. In Fig. PQRS is a quadrilateral and T and U are respectively points on PS and RS such that PQ = RQ, ∠RQU and ∠TQS = ∠UQS. Prove that QT = QU.

Sol. We have,∠PQT = ∠RQU



and, ∠TQS = ∠UQS

∴ ∠PQT + ∠TQS = ∠RQU + ∠UQS

=> ∠PQS = ∠RQS ... (i)

Thus, in triangles PQS and RQS, we have

PQ = RQ [Given]

∠PQS = ∠RQS [From (i)]

and, QS = QS [Common side]

Therefore, by SAS congruence criterion, we have

ΔPQS ≅ Δ RQS

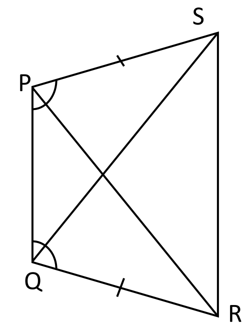
=> ∠QPS = ∠QRS [c. p. c. t.]

=> ∠QPT = ∠QRU ... (ii)

Now, consider triangles QPT and QRS. In these two triangles, we have

QP = QR [Given]
∠PQT = ∠RQU [given]
and, ∠QPT = ∠QRT [From (ii)]
Therefore, by ASA congruence criterion, we have
ΔQPT ≅ Δ QRU => QT = QU [c.p.c.t]

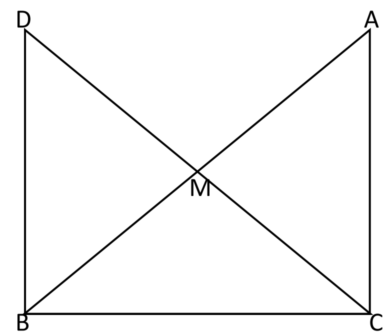
Q. 15. In Fig. PS = QR and ∠SPQ = ∠RQP. Prove that ΔPQS ≅ Δ QPR, PR = QS and ∠QPR = ∠PQS.



Sol. In Δ PQS and Δ QPR, we have
PS = QR [Given]
∠SPQ = ∠ RQP [Given]
PQ = PQ [Common]
Therefore, by SAS criterion of congruence, we have
Δ PQS ≅ Δ QPR
=> QS = PR and ∠QPR = ∠PQS [c. p. c. t]

Q. 16. In right triangle ABC, right angle at C, M is the mid-point of the hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that
(i) Δ AMC ≅ Δ BMD (ii) ∠DBC = ∠ACB (iii) Δ DBC ≅ Δ ACB (iv) CM = ½ AB

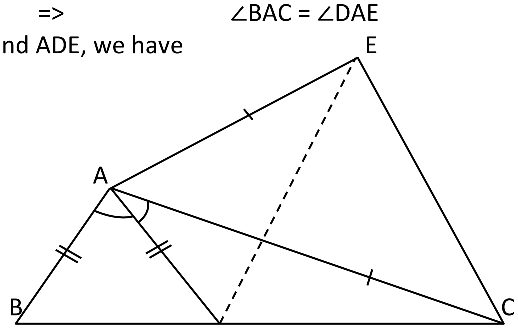
Sol. (i) In Δ AMC and Δ BMD, we have
AM = BM [Given]
∠AMC = ∠BMD [Vertically opposite angles]
MC = MD [Given]
So, by SAS congruence criterion, we have
ΔAMC ≅ Δ BMD , this proves (i).
(ii) We have, Δ AMC ≅ Δ BMD => ∠ ABD = ∠ CAB
Now, AC and DB are two lines and AB is a transversal such that ∠ ABD = ∠CAB i.e., alternate angles are equal.
∴ AC ∥ BD
Now, AC ∥ BD and BC is a transversal
∴ ∠DBC + 90° = 180°
=> ∠DBC + 90° = 180° [∵ ∠ACB = 90° (given)]
=> ∠DBC = 90°
Hence, ∠DBC = ∠ACB = 90°
This proves (ii)



(iii) In Δ DBC and Δ ACB, we have
DB = AC [∵ Δ AMC ≅ Δ BMD ∴ AC = BD]
∠DBC = ∠ACB = 90° [As proved in (ii)]
and, BC = CB [Common]
So, by SAS congruence criterion, we have
Δ DBC ≅ Δ ACB
(iv) We have, Δ DBC ≅ Δ ACB
=> DC = AB [c. p. c. t.]
=> 2 CM = AD [∵ M is the mid-point of DC]
=> CM = ½ AB

Q. 17. In Fig., AC = AE, AB = AD and ∠BAD = ∠EAC. Prove that BC = DE.

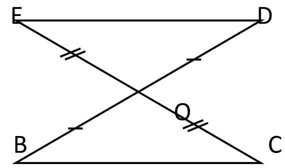
Sol. Join DE . We have, ∠BAD = ∠EAC
=> ∠BAD + ∠DAC = ∠EAC + ∠DAC [Adding ∠DAC to both sides]
=> ∠BAC = ∠DAE ... (i)
Now, in triangles ABC and ADE, we have



AB = AD [Given]
∠BAC = ∠DAE [From (i)]
and, AC = AE [Given]
So, by SAS congruence criterion, we have
Δ ABC ≅ Δ ADE
=> BC = DE [c. p. c. t.]

Self Evalution test paper ....

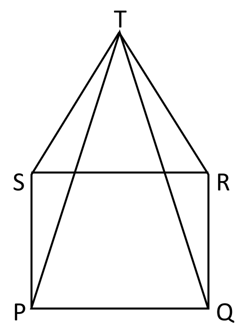
Q. 1. In Fig., the sides BA and CA have been produced such that BA = AD and CA = AE. Prove that segment DE ∥ BC.



Q. 2. In a Δ PQR, if PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that LN = MN.

Q. 3. In Fig. PQRS is a square and SRT is an equilateral triangle. Prove that

(i) PT = QT (ii) ∠ TQR = 15°



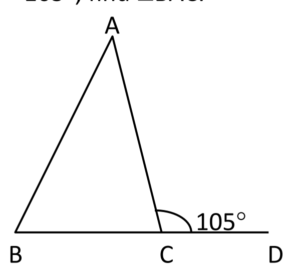
Q. 4. Prove that the medians of an equilateral triangle are equal.

Q. 5. In a ΔABC, if ∠A = 120° and AB = AC. Find ∠B and ∠C.

Q. 6. In a ΔABC, if AB = AC and ∠B = 70°, find ∠A.

Q. 7. The vertical angle of an isosceles triangle is 100°. Find the base angles.

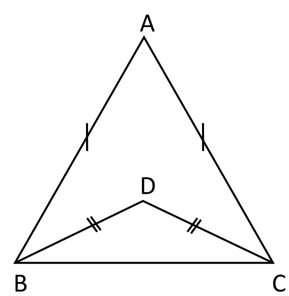
Q. 8. In Fig., AB = AC and ∠ACD = 105°, find ∠BAC.



Q. 9. Find the measure of each exterior angle of an equilateral triangle.

Q. 10. If the base of an isosceles triangle is produced on both sides, prove that the exterior angles so formed are equal to each other.

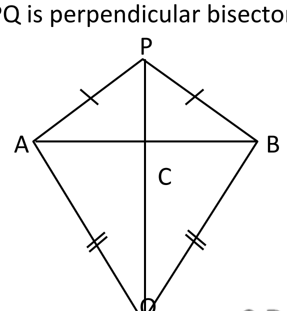
Q. 11. In Fig., AB = AC and DB = DC, find the ratio ∠ABD : ∠ACD.



Q. 12. Determine the measure of each of the equal angles of a right-angled isosceles triangle.

OR

Q. 13. AB is a line segment. P and Q are points on opposite sides of AB such that each of them is equidistant from he points A and B (see fig.). Show that the line PQ is perpendicular bisector of AB.



Answers.........

5. ∠B = ∠C = 30° **6.** 40° **7.** 40° **8.** 30° **9.** 120° **11.** 1 : 1 **12.** 45°

HINTS TO SELECTED PROBLEMS

1. **In order to prove that DE ∥ BC it** is sufficient to prove that either ∠CED = ∠ECB or ∠BDE = ∠DBC. For this, let us consider the triangles EAD and CAB. In these two triangles, we have

EA = AC [Given]
AD = AB [Given]
∠EAD = ∠BAC [Vertically opposite angles]

So, by SAS congruence criterion, we have

ΔEAD ≅ Δ CAB
=> ∠AED = ∠ACB and ∠ADE = ∠ABC
=> ∠CED = ∠ECB and ∠BDE = ∠ABC

Hence, DE ∥ BC

2. **In Δ PQR, we have**

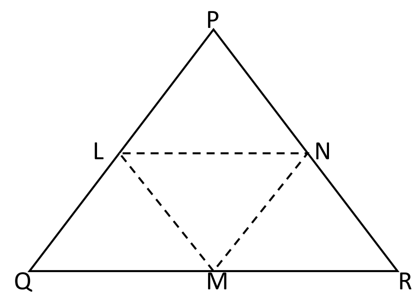
PQ = QR and ∠R = ∠P
=> ½ PQ = ½ QR and ∠P = ∠R
=> PL = MR and ∠P = ∠R

Thus, in Δ's MRN and LPN, we have

PL = MR
∠R = ∠R
and, PN = NR [∵ N is the mid-point of PR]

So, by SAS congruence criterion, we have

Δ MRN ≅ ΔLPN => MN = LN



3. **(i) Since PQRS is a square and ΔSRT is an equilateral triangle.** 21

∴ ∠ PSR = 90° and ∠TSR = 60°
=> ∠PSR + ∠TSR = 90° + 60°
=> ∠PST = 150°

Thus, in triangles PST and QRT, we have

PS = QR
∠PST = ∠QRT = 150°
and, ST = RT

So, by SAS congruence criterion, we have

Δ PST ≅ Δ QRT => PT = QT

(ii) In Δ TQR, we have

QR = RT
=> ∠TQR = ∠QTR = x (say)

Now, ∠TQR + ∠QTR + ∠QRT = 180° => 2x + 150° = 180° => x = 15°

4. **Let Δ ABC be an equilateral triangle with AD, BE and CF as its medians.**

Let AB = AC = BC = x units

In triangles BFC and CEB, we have

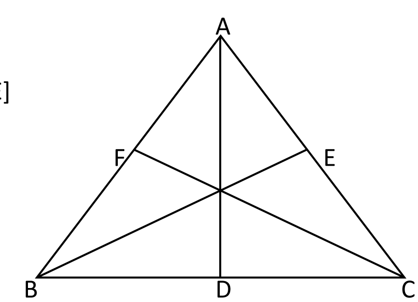
BF = CE [∵ AB = AC => ½ AB = ½ AC => BF = CE]
∠ABC = ∠ACB [Each equal to 60°]
and, BC = BC [Common]

So, by SAS congruence criterion, we have

ΔBFC ≅ ΔCEB => BE = CF

Similarly, we have AB = BE

Hence, AD = BE = CF



5. **We have,**

∠A = 120° and AB = AC => ∠B = ∠C

But, ∠A + ∠B + ∠C = 180°

∴ 120° + ∠B + ∠B = 180° => 2 ∠B = 60° => ∠B = 30°

6. **We have,**

AB = AC => ∠C = ∠B => ∠B = ∠C = 70° [∵ ∠B = 70 (Given)]

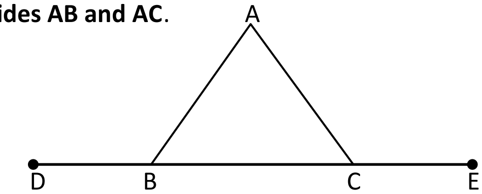
Now, ∠A + ∠B + ∠C = 180° => ∠A + 70° + 70° = 180° => ∠A = 40°

7. **Let Δ ABC be an isosceles triangle with vertical angle ∠A = 100° and**
AB = AC. Then,
AB = AC => ∠B = ∠C
But, ∠A + ∠B + ∠C = 180°
∴ 100° + 2∠B = 180° => ∠B = 40°

8. **We have,**
AB = AC => ∠C = ∠B => ∠B = 180° – 105° = 75° [∵ ∠C = 180° – 105° = 75°]
Now, ∠A + ∠B + ∠C = 180° => ∠A = 180° – 75° – 75° = 30°

9. ***Each angle of an equilateral triangle is of 60°. Therefore, each exterior angle is equal to 180° – 60° = 120°***

10. **Let ABC be an isosceles triangle with base BC and equal sides AB and AC.**



Then, ∠ABC = ∠ACB i.e., ∠B = ∠C
Now, ∠ADB + ∠ABC = 180° and ∠ACB + ∠ACE = 180°
=> ∠ADB = 180° – ∠B and ∠ACE = 180° – ∠C
=> ∠ADB = 180° – ∠B and ∠ACE = 180° – ∠B
=> ∠ADB = ∠ACE

11. **We have,**
AB = AC and DB = DC
=> ∠ABC = ∠ACB and ∠DBC = ∠DCB
=> ∠ABC – ∠DBC = ∠ACB – ∠DCB
=> ∠ABD – ∠ACD => $\frac{\angle ABD}{\angle ACD}$ = 1 i.e., ∠ABD : ∠ACD = 1 : 1

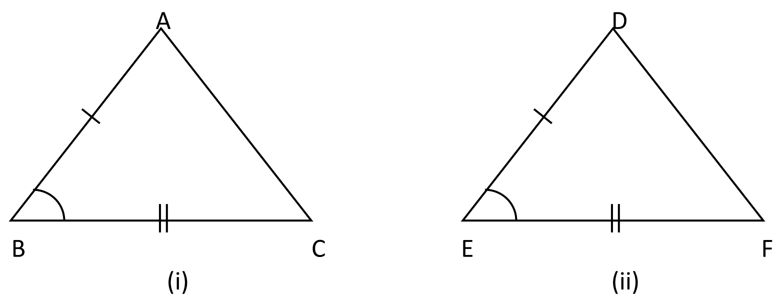
12. **We have,**
AB = AC and ∠A = 90° => ∠B = ∠C and ∠A = 90°
Now, ∠A + ∠B + ∠C = 180° => 90° + 2∠B = 180° => ∠B = 45°
Hence, ∠B = ∠C = 45°

ANGLE-SIDE-ANGLE (ASA) CONGRUENCE CRITERION

THEOREM: Two triangles are congruent if two angles and the included side of one triangle are equal to the corresponding two angles and the inclined side of the other triangle.

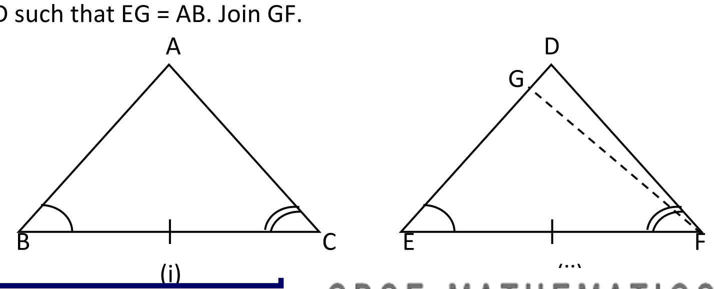
Given: Two Δs ABC and DEF such that ∠B = ∠E, ∠C = ∠F and BC = EF
To prove: Δ ABC ≅ Δ DEF
Proof: There are three possibilities.

☑**CASE I** When AB = DE:



In this case, we have
AB = DE
∠B = ∠E [Given] and, BC = EF [Given]
So, by SAS criterion of congruence, we have, ΔABC ≅ Δ DEF

☑**CASE II** **When AB < ED**
In this case take a point G on ED such that EG = AB. Join GF.



Now, in Δs ABC and GEF, we have

AB = GE [By supposition]

∠B = ∠E [Given]

and, BC = EF [Given]

So, by SAS criterion of congruence, we have

ΔABC ≅ Δ GEF

=> ∠ACB = ∠GFE [c. p. c. t.]

But, ∠ACB = ∠DFE [Given]

∴ ∠GFE = ∠DFE

This is possible only when ray FG coincides with ray FD or G coincides with D. Therefore, AB must be equal to DE.

Thus, in Δs ABC and DEF, we have

AB = DE [As proved above]

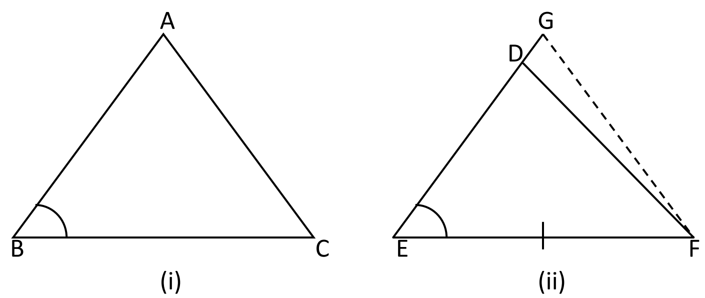
∠B = ∠E [Given]

and, BC = EF [Given]

ΔABC ≅ ΔDEF [by SAS criterion of congruent]

☑**CASE III** **When AB > ED.**

In this case take a point G on ED produced such that EG = AB. Join GF. Now, proceeding exactly on the same lines as in case II, we can prove that



ΔABC ≅ Δ DEF Hence, in all the three cases, we have Δ ABC ≅ Δ DEF

Examples

Q. 1. In Fig., diagonal AC of quadrilateral ABCD bisects the angles A and C. Prove that AB = AD and CB = CD.

Sol. Since diagonal AC bisects the angle A and C.

∴ ∠BAC = ∠DAC and ∠BCA = ∠DCA

In triangles ABC and ADC, we have

∠BAC = ∠DAC [Given]

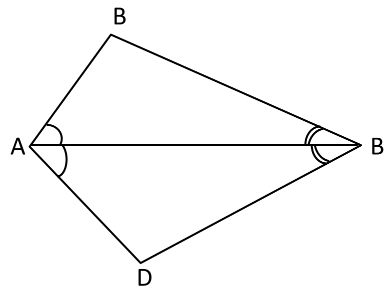
∠BCA = ∠DCA [Given]

and, AC = AC [Common]

So by ASA congruence criterion, we have

Δ BAC ≅ Δ DAC

=> BA = DA and CB = CD



Q. 2. AB is a line segment, AX and BY are two equal line segments drawn on opposite sides of line AB such that AX ∥ BY. If AB and XY intersect each other at P, prove that

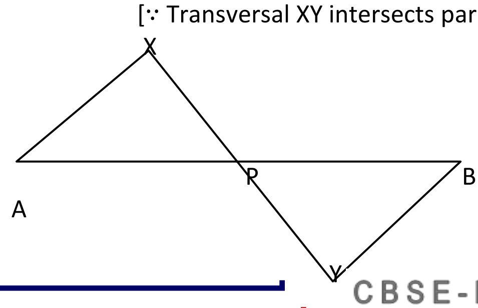
(i) Δ APX ≅ Δ BPY (ii) AB and XY bisect each other.

Sol. Since AX ∥ BY and transversal AB intersects them at A and B respectively. Therefore,

∠BAX = ∠ABY [Alternate angles]

Similarly, we have

∠AXY = ∠BYX [∵ Transversal XY intersects parallel lines AX and BY at X and Y respectively]



Thus, in triangles PAX and PBY, we have

∠PAX = ∠PBY

∠AXP = ∠BPY

and, AX = BY [Given]

So, by ASA congruence criterion, we have

Δ APX ≅ Δ BPY

=> AP = BP and PX = PY

Hence, Δ APX ≅ Δ BPY and AB and XY bisect each other.

Q. 3. ***l*** **and** ***m*** **are two parallel lines intersected by another pair of parallel lines p and q as shown in Fig. Show that Δ ABC ≅ Δ CDA.**

Sol. Since *l* and *m* are parallel lines and AC is a transversal. Therefore,

∠1 = ∠4 ... (i)

Similarly, transversal AC cuts parallel lines p and q.

∴ ∠2 = ∠3 ... (ii)

In triangles ABC and CDA, we have

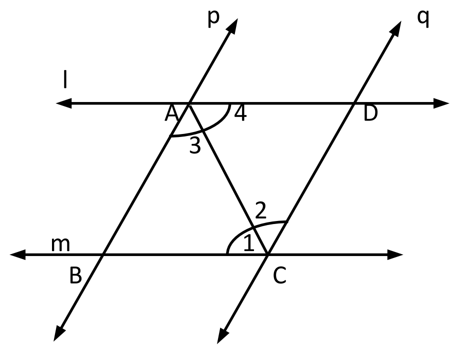
∠2 = ∠3 [From (ii)]

AC = AC [Common]

and, ∠1 = ∠4 [From (i)]

So, by ASA congruence criterion, we have

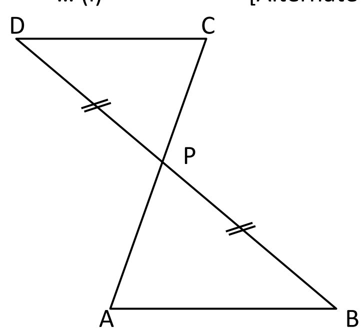
Δ ABC ≅ Δ CDA



Q. 4. **In Fig., if AB || DC and P is the mid-point of BD, prove that P is also the mid-point of AC.**

Sol. Since AB || DC and transversal AC cuts them at A and C respectively.

∴ PAB = ∠PCD ... (i) [Alternate angles]



Similarly, AB || DC and transversal BD cuts them at B and D respectively.

∴ ∠ABP = ∠CDP ... (ii) [Alternate angles]

Since AC and BD intersect at P. Therefore,

∠APB = ∠CPD ... (iii) [Vertically opposite angles]

Thus, in triangles APB and CPD, we have

∠ABP = ∠CDP [From (i)]

BP = DP [∵ P is the mid-point of BD (Given)]

and, ∠APB = ∠CPD [From (iii)]

So, by ASA congruence criterion, we have

Δ APB ≅ Δ CPD

=> AP = PC [∵ Corresponding parts of congruent triangles are equal]

Hence, P is the mid-point of AC.

Q. 5. **In Fig. ∠BCD = ∠ADC and ∠ACB = ∠BDA. Prove that AD = BC and ∠A = ∠B.**

Sol. We have,

∠1 = ∠2 and ∠3 = ∠4

=> ∠1 + ∠3 = ∠2 + ∠4

=> ∠ACD = ∠BDC ... (i)

Thus, in triangle ACD and BDC, we have

∠ ADC = ∠ BCD [Given]

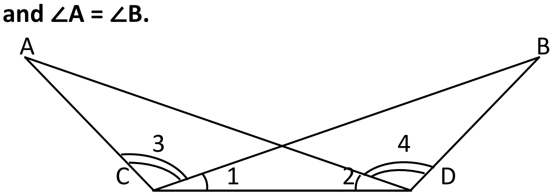
CD = CD [Common]

and, ∠ACD = ∠BDC [From (i)]

So, by ASA criterion of congruence, we have

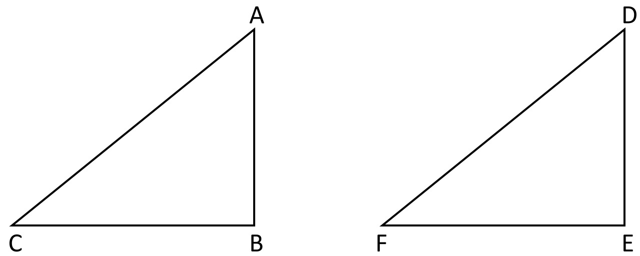
Δ ACD ≅ Δ BDC

=> AD = BC and ∠ A = ∠ B [c. p. c. t.]



Q. 6. In two right triangles, one side and an acute angle of one triangle are equal to one side and the corresponding acute angle of the other triangle. Prove that the two triangles are congruent.

Sol. Let ABC and DEF be two right triangles such that BC = EF and ∠ABC = ∠DEF. Then, we have to prove that Δ ABC ≅ Δ DEF.



In Δ ABC ≅ Δ DEF, we have

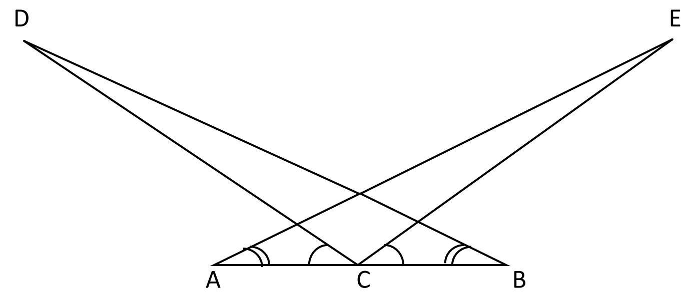
∠ACB = ∠DEF [Given]

BC = EF [Given]

and, ∠ABC = ∠DEF [Each equal to 90°]

So, by ASA congruence criterion, we have Δ ABC ≅ Δ DEF

Q. 7. In Fig. AC = BC, ∠DCA = ∠ECB and ∠DBC = ∠EAC. Prove that triangles DBC and EAC are congruent, and hence DC = EC and BD = AE.



Sol. We have,

∠DCA = ∠ECB

=> ∠DCA + ∠ECD = ∠ECB + ∠ECD

=> ∠ECA = ∠DCB ... (i)

Now, in Δs DBC and EAC, we have

∠DCB = ∠ECA [From (i)]

BC = AC [Given]

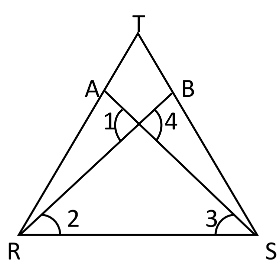
and, ∠DBC = ∠EAC [Given]

So, by ASA criterion of congruence, we have

Δ DBC ≅ Δ EAC => DC = EC and BD = AE [c. p. c. t.]

Self Evalution test paper..........

Q. 1. In Fig. it is given that RT = TS, ∠1 = 2∠2 and ∠4 = 2 ∠3. Prove that Δ RBT ≅ Δ SAT.



Q. 2. Two lines AB and CD intersect at O such that BC is equal and parallel to AD. Prove that the lines AB and CD bisect at O.

Q. 3. BD and CE are bisectors of ∠B and ∠C of an isosceles Δ ABC with AB = AC. Prove that BD = CE.

HINTS TO SELECTED PROBLEMS

1. In Δ RTS, we have

RT = ST => ∠ TSR = ∠TRS ... (i)

We have,

∠1 = ∠4 [Vertically opposite angles]

=> 2 ∠2 = 2 ∠3 [∵ 2 ∠2 and ∠4 = 2 ∠3 (given)]

=> ∠2 =∠3 ... (ii)

Subtracting (ii) from (i), we get

∠TRS – ∠2 = ∠TSR – ∠3 => ∠TRB = ∠TSA ... (iii)

Thus, in triangles RBT and SAT, we have

∠RTB = ∠STA [Common]

RT = ST [Given]

and, ∠TRB = ∠TSA [From (iii)]

So, by ASA congruence criterion, we have

Δ RBT ≅ Δ SAT

2. In order to prove that AB and CD bisect each other at O, it is sufficient to prove that Δ AOD ≅ Δ BOC.

In these two triangles, we have

AD = BC

∠OBC = ∠OAD [∵ AD ∥ BC and AB is transversal]

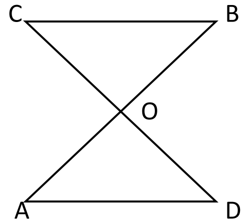
and, ∠OCB = ∠ODA [∵ AD ∥ BC and CD is transversal]

So, by ASA congruence criterion, we have

Δ AOD ≅ Δ BOC

=> OA = OB and OD = OC

=> AB and CD bisect each other at O.



3. In order to prove that BD = CE, we will prove that

Δ BEC ≅ Δ CDB

In these two triangles, we have

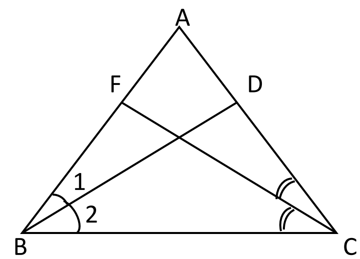
∠B = ∠C

AB = AC

and, ∠2 = ∠3 [∵ AB = AC => ∠B = ∠C
=> 2∠2 = 2 ∠3 => ∠2 = ∠3]

So, by ASA congruence criterion, we have

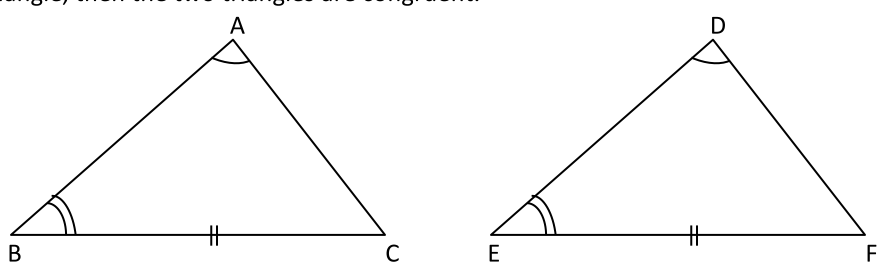
Δ BEC ≅ Δ CDB => EC = BD



ANGLE-ANGLE-SIDE (AAS) CONGRUENCE CRITERION

If two angles and even a non-included side of one triangle are equal to the corresponding angles and side of another triangle, then also, the triangles are congruent as proved in the following theorem.

◘THEOREM 1 If any two angles and a non-included side of one triangle are equal to the corresponding angles and side of another triangle, then the two triangles are congruent.



GIVEN: Two Δs ABC and DEF such that

∠A = ∠D, ∠B = ∠E, BC = EF

TO PROVE: Δ ABC ≅ Δ DEF

PROOF: We have,∠A = ∠D and ∠B = ∠E

=> ∠A + ∠B = ∠D + ∠E

=> 180° – ∠C = 180° – ∠F [∵ ∠A + ∠B + ∠C = 180° ∴ ∠A + ∠B = 180° – ∠C
Similarly, ∠D + ∠E = 180° – ∠F]

=> ∠C = ∠F ... (i)

Thus, in Δs ABC and DEF, we have ∠A = ∠D, ∠B = ∠E and ∠C = ∠F

Now, in Δ ABC and Δ DEF, we have

∠B = ∠E [Given]

BC = EF [Given]

and, ∠C = ∠F [From (i)] So, by ASA criterion of congruence, Δ ABC ≅ Δ DEF.

◘THEOREM 2 If two angles of a triangle are equal, then sides opposite to them are also equal.

GIVEN: A Δ ABC in which ∠B = ∠C

TO PROVE: AB = AC

CONSTRUCTION: Draw the bisector of ∠A and let it meet BD and D.

PROOF: In Δs ABD and ACD, we have

∠B = ∠C [Given]

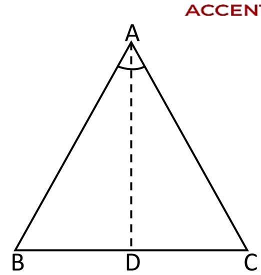
∠BAD = ∠CAD [By construction]

AD = AD [Common]

So, by AAS criterion of congruence, we have

Δ ABD ≅ Δ ACD

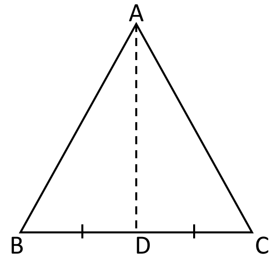
=> AB = AC [c. p. c. t.]



THEOREM 3: If the altitude from one vertex of a triangle bisects the opposite side, then the triangle is isosceles.

GIVEN: A Δ ABC such that the altitude AD from A on the opposite side BC bisects BC i.e., BD = DC.

TO PROVE: AB = AC i.e., the triangle ABC is isosceles.



PROOF: In Δs ADB and ADC, we have

BD = DC [Given]

∠ADB = ∠ADC = 90° [∵ AD ⊥ BC]

AD = AD [Common]

So, by SAS criterion of congruence, we have

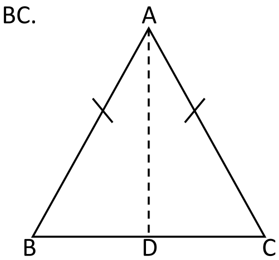
Δ ADB ≅ Δ ADC

=> AB = AC

THEOREM 4: In an isosceles triangle altitude from the vertex bisects the base.

GIVEN: An isosceles triangle ABC such that AB = AC and an altitude AD from A on side BC.

TO PROVE: D bisects BC i.e., BD = DC.



PROOF: In Δs ADB and ADC, we have

∠ADB = ∠ADC [Each equal to 90°]

AD = AD [Common]

∠B = ∠C [∵ AB = AC ∴ ∠B = ∠C]

So, by AAS criterion of congruence, we have

Δ ADB ≅ Δ ADC

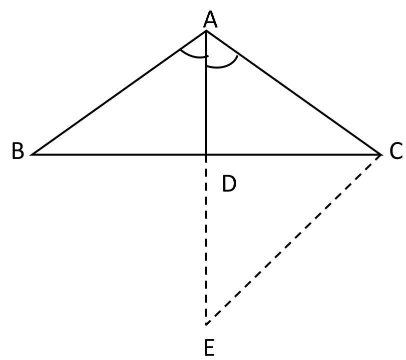
=> BD = DC

THEOREM 5: If the bisector of the vertical angle of a triangle bisects the base of the triangle, then the triangle is isosceles.

GIVEN: A Δ ABC in which AD is the bisector of ∠A meeting BC in D such that BD = DC.

TO PROVE: Δ ABC is an isosceles triangle.

CONSTRUCTION: Produce AD to E such that AD = DE. Join EC.



PROOF: In Δs ADB and EDC, we have

BD = DC [Given]

AD = DE [By construction]

and, ∠ADB = ∠EDC

So, by SAS criterion of congruence, we have

ΔADB ≅ Δ EDC

=> AB = EC

and, ∠BAD = ∠CED [c. p. c. t] ... (i)

But, ∠BAD = ∠CAD [Given]

∴ ∠CAD = ∠CED

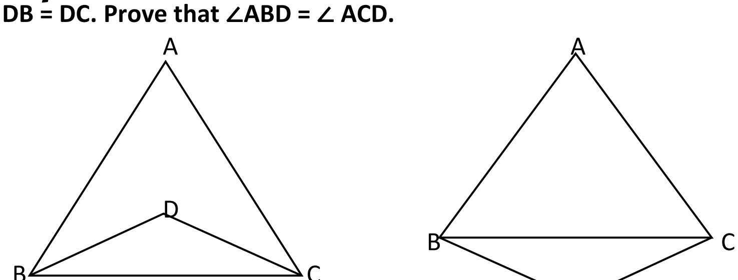
=> AC = EC [∵ Sides opposite to equal angles are equal]

=> AC = AB [∵ AB = EC (From (i)]

Hence, Δ ABC is an isosceles triangle.

Illustrative Examples.......

Q. 1. In Fig. AB = AC and DB = DC. Prove that ∠ABD = ∠ ACD.



(i) (ii)

Sol. In Δ ABC, we have

AB = AC [Given]

=> ∠ABC = ∠ACB ... (i) [∵ Angles opposite to equal sides are equal]

Again, in Δ DBC, we have

DB = DC [Given]

=> ∠DBC = ∠DCB ... (ii) [∵ Angles opposite to equal sides are equal]

Subtracting (ii) from (i), we get

$\angle ABC - \angle DBC = \angle ACB - \angle DCB$

$\angle ABD = \angle ACB$ [From Fig. (i)]

Adding (i) and (ii), we get

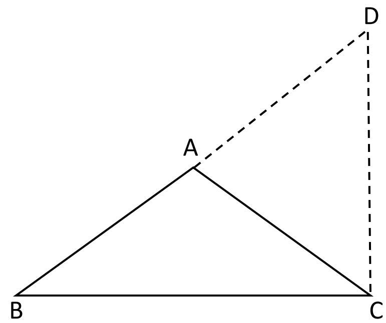
$\angle ABC + \angle DBC = \angle ACB + \angle DCB$ => $\angle ABD = \angle ACD$ [From Fig. (ii)]

***Q. 2. Δ ABC is an isosceles triangle with AB = AC. Side BA is produced to D such that AB = AD. Prove that ∠BCD is a right angle.**

GIVEN: A Δ ABC such that AB = AC. Side BA is produced to D such that AB = AD.

CONSTRUCTION: Join CD.

TO PROVE: $\angle BCD = 90^\circ$



PROOF: In Δ ABC, we have

AB = AC

=> $\angle ACB = \angle ABC$ ... (i) [∵ Angles opp. to equal sides are equal]

Now, AB = AD [Given]

∴ AD = AC [∵ AB = AC]

Thus, in Δ ADC, we have AD = AC

=> $\angle ACD = \angle ADC$ ... (ii) [∵ Angles opp. to equal sides are equal]

Adding (i) and (ii), we get

$\angle ACB + \angle ACD = \angle ABC + \angle ADC$

=> $\angle BCD = \angle ABC + \angle BDC$ [∵ $\angle ADC = \angle BDC$]

=> $\angle BCD = \angle BCD = \angle ABC + \angle BDC + \angle BCD$ [Adding ∠BCD on both sides]

=> $2\angle BCD = 180^\circ$ [∵ Sum of the angles of a Δ is 180°]

=> $\angle BCD = 90^\circ$ Hence, ∠BCD is a right angle.

Q. 3. In Fig. AB = AC. BE and CF are respectively the bisectors of ∠B and ∠C. Prove that Δ EBC ≅ Δ FCB.

Sol. In Δ ABC, we have

AB = AC [Given]

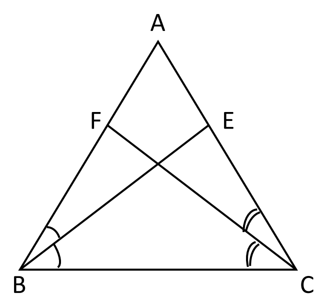
=> $\angle ACB = \angle ABC$

=> $\angle ECB = \angle FBC$ ... (i) [∵ $\angle ACB = \angle FCB$ and $\angle ABC = \angle FBC$]

Again, $\angle ACB = \angle ABC$

=> $\frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$

=> $\angle FCB = \angle EBC$ ... (ii) [∵ CF and BE are bisectors of ∠ACB and ∠ABC respectively]



Now, in Δs EBC and FCB, we have

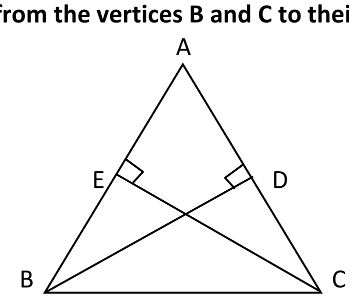
$\angle ECB = \angle FBC$ [From (i)]

BC = BC [Common]

and, $\angle FCB = \angle EBC$

So, by ASA criterion of congruence, we have , Δ EBC ≅ Δ FCB

Q. 4. If Δ ABC is an isosceles triangle with AB = AC. Prove that the perpendiculars from the vertices B and C to their opposite sides are equal.



Sol. In Δ ABC, we have

AB = AC [Given]

=> $\angle B = \angle C$ ... (i) [∵ Angles opp. to equal sides are equal]

Now, in Δs BCE and BCD, we have

$\angle B = \angle C$ [From (i)]

$\angle CEB = \angle BDC$ [Each equal to 90°]

and, BC = BC [Common]

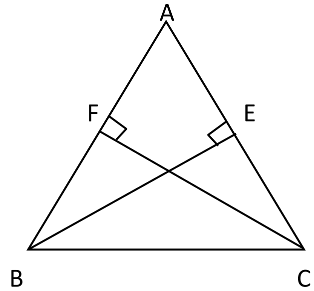
So, by AAS criterion of congruence, we have

Δ BCE = Δ BCD

=> BD = CE [∵ Corresponding parts of congruent triangles are equal]

Hence, BD = CE

Q. 5. If the altitudes from two vertices of a triangle to the opposite sides are equal, prove that the triangle is isosceles.



GIVEN: A Δ ABC is isosceles i.e., AB = AC

PROOF: In Δs ABE and ACF, we have

∠AEB = ∠AFC [Each equal to 90°]

∠BAE = ∠CAF [Common angle]

and, BE = CF [Given]

So, by AAS criterion of congruence, we have

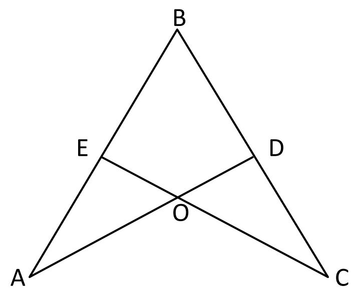
Δ ABE ≅ Δ ACF

=> AB = AC

Hence, Δ ABC is isosceles.

Q. 6. In Fig. It is given that ∠A = ∠C and AB = BC. Prove that Δ ABD ≅ Δ CBE.

Sol. In Δs AOE and COD, we have



∠A = ∠C

and, ∠AOE = ∠COD [Vertically opp. angles]

∴ ∠A + ∠AOE = ∠C + ∠COD

=> 180° – ∠AEO = 180° – ∠CDO [∵ ∠A + ∠AOE + ∠AEO = 180° and ∠C + ∠COD + ∠CDO = 180°]

=> ∠AEO = ∠CDO ... (i)

Now, ∠AEO + ∠OEB = 180° [Angles of a linear pair]

and, ∠CDO + ∠ODB = 180° [Angles of a linear pair]

∴ ∠AEO + ∠OEB = ∠CDO + ∠ODB

=> ∠OEB = ∠ODB [Using (i)]

=> ∠CEB = ∠ADB [∠OEB = ∠CEB and ∠ODB = ∠ADB] ... (ii)

Now, in Δ ABD and Δ CBE, we have

∠A = ∠C [Given]

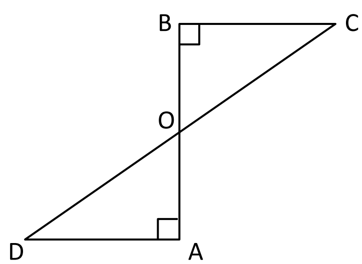
∠ADB = ∠CEB [From (ii)]

and, AB = BC [Given]

So, by AAS criterion of congruence, we have , Δ ABD ≅ Δ CBE

Q. 7. AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB.

Sol. In triangles OAD and OBD, we have



∠AOD = ∠BOC

∠OAD = ∠OBC

and, AD = BC

So, by AAS congruence criterion, we have

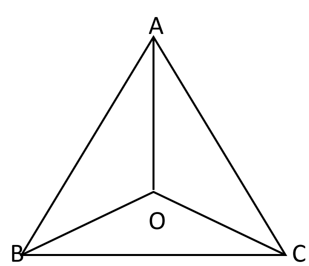
ΔAOD ≅ Δ BOC

=> OA = OB

=> CD bisects AB

Q. 8. In Δ ABC, AB = AC, and the bisectors of angles B and C intersect at point O. Prove that BO = CO and the ray AO is the bisector of angle BAC.

Sol. In Δ ABC, we have



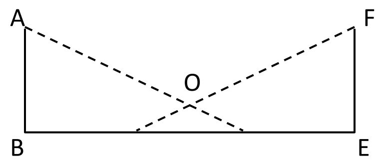
AB = AC
=> ∠B = ∠C [∵ Angles opp. to equal sides are equal]
=> ½ ∠B = ½ ∠C
=> ∠OBC = ∠OCB [∵ OB and OC are bisector of ∠B and ∠C respectively ∴ ∠OBC = ½ ∠B & ∠OCB = ½ ∠C] ... (i)
=> OB = OC [∵ Sides opposite to equal angles are equal] ... (ii)

Now, in ∆ ABO and ∆ ACO, we have
AB = AC [Given]
∠OBC = ∠OCB [From (i)]
and, OB = OC [From (ii)]

So, by SAS criterion of congruence
∆ ABO ≅ ∆ ACO
=> ∠BAO = ∠CAO => AO is the bisector of ∠BAC.

Q. 9. In Fig. it is shown that AB = EF, BC = DE, AB ⊥ BD and FE ⊥ CE. Prove that ∆ ABD ≅ ∆ FEC.

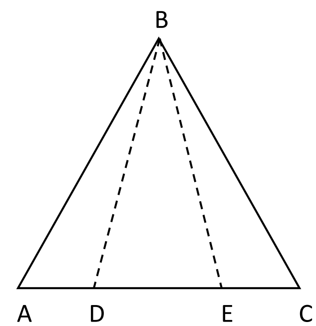
Sol. We have,



BC = DE
=> BC + CD = DE + CD [Adding CD on both sides]
Thus, in ∆s ABC and FEC, we have ... (i)
AB = EF [Given]
∠ABD = FEC [∵ AB ⊥ BD and FE ⊥ CE (Given) ∴ ∠ABD = 90° and ∠FEC = 90°]
and, BD = CE [From (i)]
So, by ASA criterion of congruence, we have, ∆ ABD ≅ ∆ FEC

Q. 10. In Fig. it is given that AB = BC, and AD = EC. Prove that ∆ ABE ≅ ∆ CBD

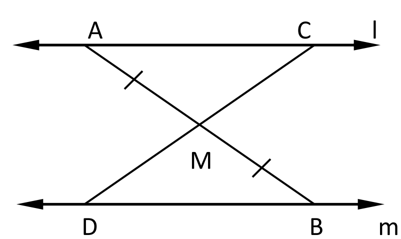
Sol. In ∆ ABC, we have



BA = BC [Given]
=> ∠BCA = ∠BAC [∵ Angles opp. to equal sides are equal]
=> ∠BAE = ∠BCD ... (i)
We have,
=> AD + DE = DE + EC [Adding DE on both sides]
=> AE = CD ... (ii)
Thus, in ∆ ABE and CBD, we have
AB = BC [Given]
∠BAE = ∠BCD [From (i)]
and, AE = CD [From (ii)]
So, by SAS criterion of congruence, we have
∆ ABE ≅ ∆ CBD

Q. 11. In Fig. *l* ∥ *m* is the mid-point of the line segment AB. Prove that M is also the mid-point of any line segment CD having its end-points on *l* and *m* respectively.

Sol. In ∆s AMC and BMD, we have

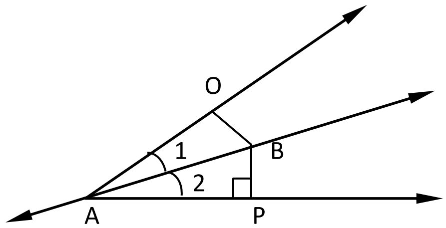


∠BAC = ∠ABD [Alternative angles]
∠AMC = ∠BMC [Vertically opp. angles]
and, AM = BM [Given]
So, by AAS criterion of congruence, we have,
∆ AMC ≅ ∆ BMD
=> CM = DM [∵ Corresponding parts of congruent triangles are equal]
Hence, M is the mid-point of CD also.

Q. 12. In Fig. line l is the bisector of angle A and B is any point on l. BP and PQ are perpendiculars from B to the arms of A. Show that: (i) ∆ APB ≅ ∆ BMD
(ii) BP = BQ or B is equidistant from the arms of ∠A

Sol. In triangles APB and AQB, we have

∠1 = ∠2 [line l is the bisector s of ∠A]

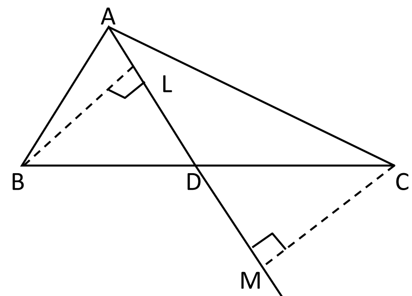


∠APB = ∠AQB [Each equal to 90°]

AB = AB [Common]

So, by AAS congruence criterion, we have, Δ APB ≅ Δ AQB => BP = BQ

Q. 13. In Fig., AD is a median and BL, CM are perpendicular drawn from B and C respectively on AD and AD produced. Prove that BL = CM.



Sol. In Δs BDL and CDM, we have

∠BLD = ∠CMD [Each equal to 90°]

∠BDL = ∠CDM [Vert. opp. ∠s]

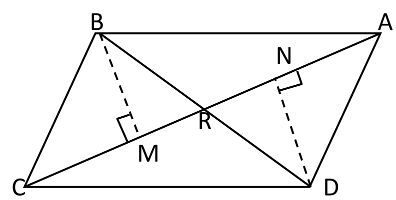
and, BD = DC [∵ D is the mid-point of BC]

So, by AAS criterion of congruence, we have

Δ BDL ≅ Δ CDM

=> BL = CM [c. p. c. t.]

Q. 14. In Fig. BM and DN are both perpendicular to the segments AC and BM = DN. Prove that AC bisects BD.



Sol. In Δs BMR and DNR, we have

∠BMR = ∠DNR [Each equal to 90° ∵ BM ⊥ AC and DN ⊥ AC]

∠BRM = ∠DRN [Vert. opp. angles]

and, BM = DN [Given]

So, by AAS criterion of congruence, we have

Δ BMR ≅ Δ DNR

=> BR = DR [∵ Corresponding parts of congruent triangles are equal]

=> R is the mid-point of BD.

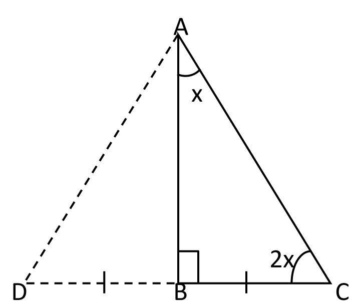
Hence, AC bisects BD.

***Q. 15. In a right angled triangle, one acute angle is double the other. Prove that the hypotenuse is double the smallest side.**

GIVEN: A Δ ABC in which ∠B = 90° and ∠ACB = 2 ∠CAB.

TO PROVE: AC = 2 BC

CONSTRUCTION: Produce CB and D such that BD = CB. Join AD.



PROOF: In Δs ABD and ABC, we have

BD = BC [By construction]

AB = AB [Common side]

and, ∠ABD = ∠ABC [Each equal to 90°]

So, by SAS criterion of congruence we have

Δ ABD ≅ Δ ABC

=> AD = AC and ∠DAB = ∠CAB [c. p. c. t.]

=> AD = AC and ∠DAB = x [∵ ∠CAB = x]

Now, ∠DAC = ∠DAB + ∠CAB

=> ∠DAC = x + x = 2x

=> ∠DAC = ∠ACB [∵ ∠ ACB = 2x]

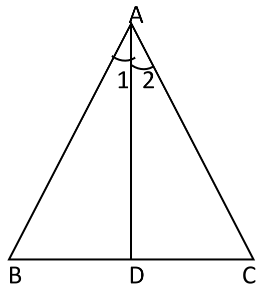
=> DC = AD [∵ BC = DB ∴ DC = 2 BC]

=> 2 BC = AC [∵ AD = AC (proved above)]

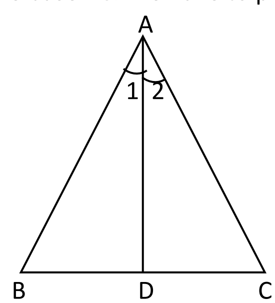
Hence, the hypotenuse AC is double the smallest side BC.

Q. 16. A triangle ABC is an isosceles triangle of any one of the following conditions hold:
(i) Altitude AD bisects ∠BAC. (ii) Bisector of ∠BAC is perpendicular to the base BC.

Sol. (i) Let ABC be a triangle such that the altitude AD bisects ∠BAC. Then, we have to prove that the triangle is isosceles.
In triangle ADB and ADC, we have
∠1 = ∠2 [∵ AD is bisector of ∠BAC ∴ ∠1 = ∠2]
AD = AD [Common side]

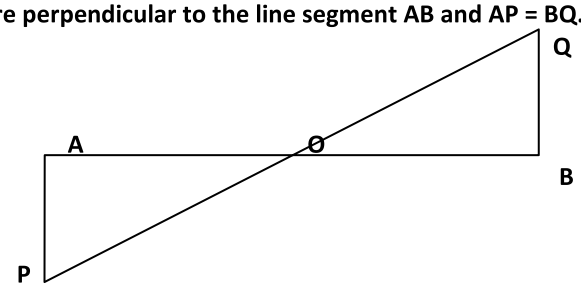


∠ADB = ∠ADC [Each equal to 90°]
So, by ASA criterion of congruence, we have Δ ADB ≅ Δ ADC => AB = AC [c. p. c. t.]
Hence, Δ ABC is an isosceles triangle.
(ii) Let ABC be a triangle such that the bisector AD of ∠BAC is perpendicular to the base BC. We have to prove that the triangle is isosceles.



In Δs ADB and ADC, we have
∠1 = ∠2 [∵ AD is the bisector of ∠ BAC]
=> ∠ADB = ∠ADC [∵ AD ⊥ BC ∴ ∠ADB = ∠ADC = 90°]
AD = AD [Common side]
So, by ASA criterion of congruence, we have
Δ ADB ≅ Δ ADC
=> AB = AC [c. p. c. t.]
Hence, Δ ABC is an isosceles triangle.

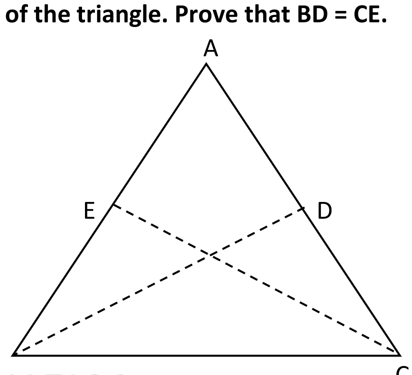
Q. 17. In Fig. AP and BQ are perpendicular to the line segment AB and AP = BQ. Prove that O is the mid-point of line segment AB and PQ.



Sol. Since AB and PQ intersect at O.
∴ ∠AOP = ∠BOQ ... (i) [Vertically opposite angles]
In triangles AOP and BOQ, we have
∠AOP = ∠BOQ [From (i)]
∠OAP = ∠OBP [Each equal to 90°]
and, AP = BQ [Given]
So, by AAS congruence criterion, we have Δ AOP ≅ Δ BOQ
=> OA = OB and OP = OQ [∵ Corresponding parts of congruent triangles are equal]
Hence, O is the mid-point of the line segments AB and PQ.

Q. 18. In Fig. ABC is an isosceles triangle with AB = AC. BD and CE are two medians of the triangle. Prove that BD = CE.

Sol. In Δ ABC, it is given that
AB = AC
=> ∠B = ∠C
Again AB = AC ... (i)
=> ½ AB = ½ AC
=> BE = CD ... (ii) [∵ D and E are the mid-points of AC and AB respectively]

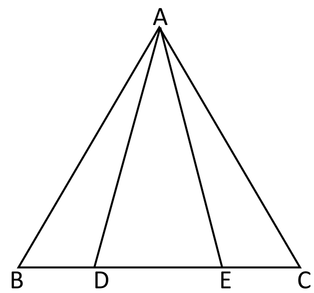


Thus, in Δ DCB and Δ EBC, we have
DC = EB [From (ii)]
CB = BC [Common side]
and, ∠DCB = ∠EBC [From (i)]

Δ DCB $\cong$ Δ EBC
=> BD = CE [∵ Corresponding parts of congruent triangles are equal]

Q. 19. In Fig. AD = AE and D and E are points on BC such that BD = EC. Prove that AB = AC.

Sol. In Δ ADE, we have



AD = AE
=> ∠ADE = ∠AED
=> 180° – ADE = (180° – ∠AED)
=> ∠ADB = ∠AEC ... (i)

Now, in Δ ABD and Δ ACE, we have
AD = AE [Given]
∠ADB = ∠AEC [From (i)]
and, BD = EC
So, by SAS congruence criterion, we have
Δ ABD $\cong$ Δ ACE
=> AB = AC [∵ Corresponding parts of cong. triangles are equal]

Q. 20. In Fig. if AB = AC and BE = CD, prove that AD = AE.

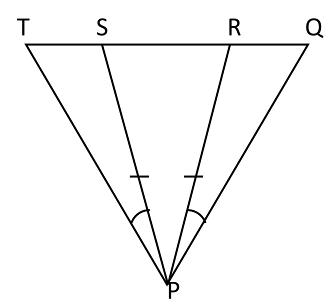
Sol. We have,
BE = CD
=> BE + DE = CE + DE
=> BD = CE ... (i)

In Δ ABC, it is given that
AB = AC
=> ∠B = ∠C ... (ii)

Thus, in Δ ABD and Δ ACE, we have
AB = AC [Given]
∠ABD = ∠ACD [From (ii)]
and, BD = CE [From (i)]
So, by SAS congruence criterion, we have
Δ ABD $\cong$ Δ ACE
=> AD = AE [∵ Corresponding parts of cong. triangles are equal]

Q. 21. In Fig. PS = PR, ∠TPS = ∠QPR. Prove that PT = PQ

Sol. In Δ PRS, we have



PS = PR
=> ∠PRS = ∠PSR [∵ Angles opposite to equal sides are equal]
=> 180° – ∠PRS = 180° – ∠PSR
=> ∠PRQ = ∠PST ... (i)

Thus, in Δ PST and Δ PRQ we have
∠TPS = ∠QPR [Given]
PS = PR [Given]
∠PST = ∠PRQ [From (i)]
So, by ASA congruence criterion, we have
ΔPST $\cong$ Δ PRQ
=> PT = PQ

Q. 22. In Fig. if PQ = PT and ∠TPS = ∠QPR, prove that Δ PRS is isosceles.

Sol. In Δ PQT, it is given that
PQ = PT
=> ∠PTQ = ∠PQT ... (i) [Angles opposite to equal sides are equal]

Thus, in Δ PQR and Δ PTS, we have
PQ = PT [Given]
∠QPR = ∠TPS [Given]
and, ∠PQR = Δ PTS [From (i)]
So, by ASA congruence criterion, we have
Δ PQR $\cong$ Δ PTS
=> PR = PS [∵ Corresponding parts of cong. triangles are equal]
Hence, Δ PRS is an isosceles triangle.

Q. 23. **In Fig. ABC and DBC are two isosceles triangles on the same base BC such that AB = AC and DB = DC. Prove that ∠ABD = ∠ACD.**

Sol. In Δ ABC, we have

AB = AC

=> ∠ABC = ∠ACB [∵ Angles opposite to equal sides are equal] ... (i)

In Δ BCD, we have

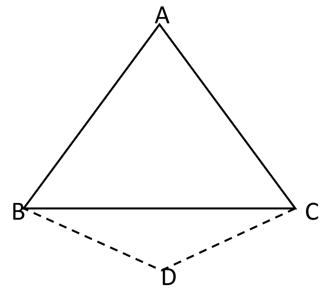
BD = CD

=> ∠DBC = ∠DCB [∵ Angles opposite to equal sides are equal] ... (ii)

From (i) and (ii), we have

∠ABC + ∠DBC = ∠ACB + ∠DCB

=> ∠ABD = ∠ACD



<u>ALTER</u> Join AD.

In Δ's ABD and ACD, we have

AB = AC [Given]

BD = CD [Given]

AD = AD [Common]

So, by SSS criterion of congruence, we have

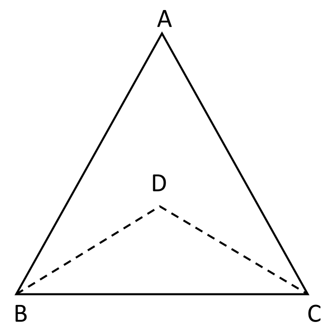
Δ ABD ≅ Δ ACD => ∠ABD = ∠ACD [c. p. c. t.]

Q. 24. **In Fig. Δ ABC and Δ DBC are two triangles on the same base BC such that AB = AC and DB = DC. Prove that ∠ABD = ∠ACD.**

Sol. In Δ ABC, it is given that

AB = AC

=> ∠ ABC = ∠ACB [∵ Angles opposite to equal sides are equal] ... (i)



In Δ DBC, it is given that

DB = DC

=> ∠DBC = ∠DCB ... (ii)

Subtracting (ii) from (i), we get

∠ABC – ∠DBC = ∠ACB – ∠DCB

=> ∠ABD = ∠ACD

Q. 25. **In Fig. BD and CE are two altitudes of a ΔABC such that BD = CE. Prove that Δ ABC is isosceles.**

Sol. In Δ ABD and Δ ACE, we have

∠ADB = ∠AEC = 90° [Given]

∠BAD = ∠CAE [Common]

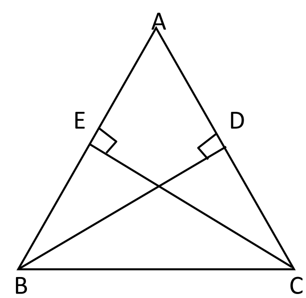
and, BD = CE [Given]

So, by AAS congruence criterion, we have

Δ ABD ≅ Δ ACE

=> AB = AC [∵ Corresponding parts of congruent triangles are equal]

Hence, Δ ABC is isosceles.



Q. 26. **In Fig. line segment AB is parallel to another line segment CD. O is the mid-point of AD. Show that:**

(i) Δ AOD ≅ Δ DOC **(ii) O is also the mid-point of BC**

Sol. (i) Since AB ∥ CD and BC is the transversal.

∴ ∠ABO = ∠DCO ... (i)

In triangles AOB and DOC, we have

∠ABO = ∠DCO [From (i)]

∠ABO = ∠DOC [Vertically opposite angles]

OA = OD [Given]

So, by AAS congruence criterion, we have

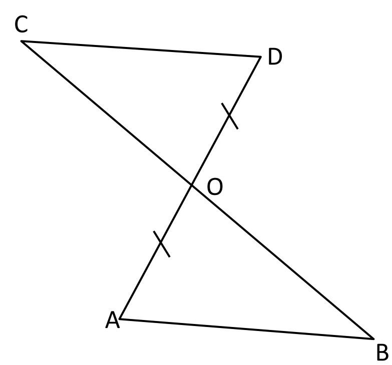
Δ AOB ≅ Δ DOC

(ii) We have,

Δ AOB ≅ Δ DOC [As proved above]

=> OB = OC

=> O is the mid-point of BC.



Exercise 1.3...........

Q. 1. In two right triangles one side an acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are concurrent.

Q. 2. If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.

Q. 3. In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Q. 4. PQR is a triangle in which PQ = PR and S is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that PS = PT. 34

Q. 5. In a Δ ABC, it is given that AB = AC and the bisectors of ∠B and ∠C intersect at O. If M is a point of BO produced, prove that ∠MOC = ∠ABC.

Q. 6. P is a point on the bisector of an angle ∠ABC. If the line through P parallel to AB meets BC at Q, prove that triangle BPQ is isosceles.

Q. 7. Prove that each angle of an equilateral triangle is 60°.

Q. 8. Angles A, B, C of a triangle ABC are equal to each other. Prove that Δ ABC is equilateral.

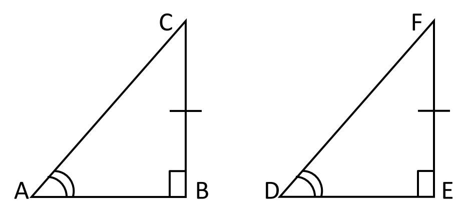
Q. 9. ABC is a triangle in which ∠B = 2∠C. D is a point on BC such that AD bisects ∠BAC and AB = CD. Prove that ∠BAC = 72°.

Answers...........

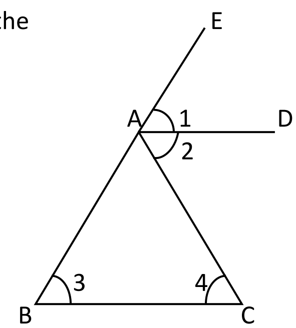
3. 30°, 20°, 120°

HINTS TO SELECTED PROBLEMS

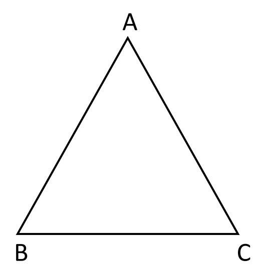
1. Let ABC and DEF be two right triangles such that ∠A = ∠D, BC = EF and ∠B = ∠E = 90°.
Thus, in Δ's ABC and DEF, we have
∠A = ∠D, ∠B = ∠E = 90° and BC = EF
So, by AAS congruence criterion, we have
Δ ABC ≅ Δ DEF



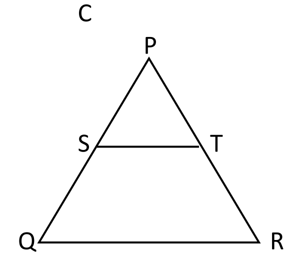
2. Let ABC be a triangle such that the bisector AD of ∠CAE is parallel to the base BC as shown in Fig.
We have,
∠1 = ∠3 [Corresponding angles]
and, ∠2 = ∠4 [Alternate angles]
=> ∠3 = ∠4 [∵ ∠1 = ∠2]
=> AB = AC
Hence, Δ ABC is isosceles.



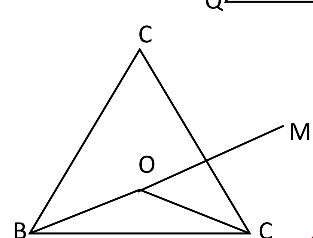
3. Let ABC be an isosceles triangle such that AB = AC. Then,
AB = AC => ∠C = ∠B = x (say)
we have,
∠A = 2 (∠B + ∠C) [Given]
=> ∠A = 2 (∠B + ∠B) [∵ ∠B = ∠C]
=> ∠A = 4 x
Now, ∠A + ∠B + ∠C = 180°



4. In Δ PQR, we have
PQ = PR => ∠R = ∠Q
Now, ST || QR
=> ∠PST = ∠PQR and ∠PTS = ∠PRQ [∵ Corresponding angles are equal]
=> ∠PST = ∠Q and ∠PTS = ∠R
=> ∠PST = ∠PTS
=> PT = PS



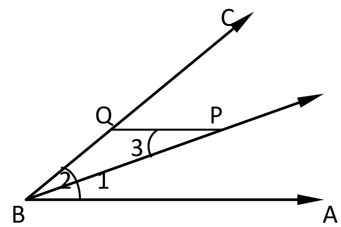
5. In Δ ABC, we have
AB = AC
=> ∠C = ∠B
=> ½ ∠C = ½ ∠B
=> ∠OCB = ∠OBC ... (i)
In Δ OBC, we have
∠MOC = ∠OBC + ∠OCB



=> ∠MOC = ∠OBC + ∠OBC [Using (i)]

=> ∠MOC = 2 ∠OBC = ∠ABC

6. We have,



∠1 = ∠2 [∵ BP is the bisector of ∠ABC]

∠1 = ∠3 [∵ PQ ∥ BA]

∴ ∠2 = ∠3

=> PQ = BQ

=> Δ PBQ is isosceles.

7. Let ABC be an equilateral triangle. Then,

AB = AC => ∠C = ∠B and, BC = AC => ∠A = ∠ ∴ ∠A = ∠B = ∠C But, ∠A + ∠B + ∠C = 180°

Hence, ∠A = ∠B = ∠C = 60°

8. We have, ∠A = ∠B => BC = AC and, ∠B = ∠C => CA = AB

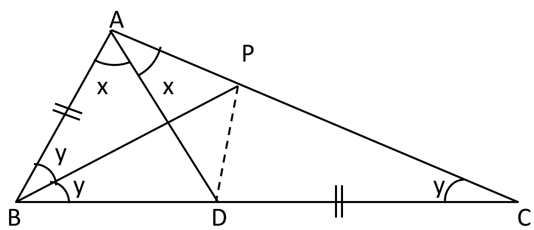
∴ AB = BC = CA => Δ ABC is equilateral.

9. In Δ ABC, we have

∠B = 2 ∠C or, ∠B = 2y, where ∠C = y

AD is the bisector of ∠BAC. So, let ∠BAD = ∠CAD = x

Let BP be the bisector of ∠ABC. Join PD.



In Δ BPC, we have

∠CBP = ∠BCP = y => BP = PC

In Δ's ABP and DCP, we have

∠ABP = ∠DCP = y

AB = DC [Given]

and, BP = PC [As proved above]

So, by SAS congruence criterion, we have

Δ ABP ≅ Δ DCP

=> ∠BAP = ∠CDP and AP = DP

=> ∠CDP = 2x and ∠ADP = DAP = x [∵ ∠A = 2x)

In Δ ABD, we have

∠ADC = ∠ABD + ∠BAD => x + 2x = 2y + x => x = y

In Δ ABC, we have

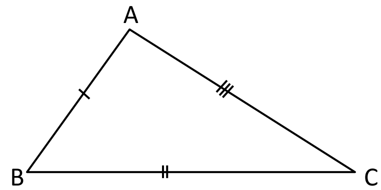
∠A + ∠B + ∠C = 180° => 2x + 2y + y = 180° => 5x = 180° [∵ x = y]

=> x = 36°

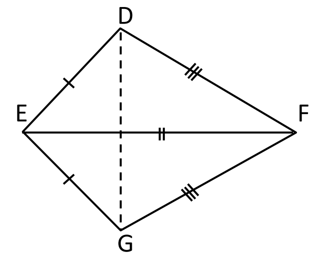
Hence, ∠BAC = 2x = 72°

SIDE-SIDE-SIDE (SSS) CONGRUENCE CRITERION

THEOREM 9 Two triangles are congruent if the three sides of one triangle are equal to the corresponding three sides of the other triangle.



(i)



(ii)

GIVEN: Two ∆s ABC and DEF such that AB = DE, BC = EF and AC = DF.

TO PROVE: ∆ ABC ≅ ∆ DEF

CONSTRUCTION: Suppose BC is the longest side. Draw EG such that ∠FEG = ∠ABC and EG = AB. Join GF and GD.

PROOF: In ∆s ABC and GEF, we have

BC = EF [Given]
AB = GE [By construction]
and, ∠ABC = ∠FEG [By construction]
So, by SAS criterion of congruence, we have
∆ ABC ≅ ∆ GEF
=> ∠A = ∠G and AC = GF [c.p.c.t.]
Now, AB = DE and AB = GE
=> DE = GE … (i)
Similarly, AC = DF and AC = GF
=> DF = GF … (ii)
In ∆ EGD, we have
DE = GE [From (i)]
=> ∠EDG = ∠EGD … (iii)
In ∆ FGD, we have
DF = GF [From (ii)]
=> ∠FDG = ∠FGD … (iv)
From (iii) and (iv), we have
∠EDG + ∠FDG = ∠EGD + ∠FGD
=> ∠D = ∠G
∴ ∠A = ∠D … (v)
Thus, in ∆s ABC and DEF, we have
But, ∠G = ∠A [Proved above]
AB = DE [Given]
∠A = ∠D [From (v)]
and, AC = DF [Given]
So, by SAS criterion of congruence, we have
∆ ABC ≅ ∆ DEF

Illustrative Examples

Q. 1. In Fig. it is given that AB = CD and AD = BC. Prove that ∆ ADC ≅ ∆ CBA

Sol. In ∆s ADC and CBA, we have
AB = CD [Given]
AD = BC [Given]
and, AC = AC [Common side]
So, by SSS criterion of congruence, we have
∆ ADC ≅ ∆ CBA

Q. 2. ABCD is a parallelogram, if the two diagonals are equal, find the measure of ∠ABC.

Sol. Since ABCD is a parallelogram. Therefore,
AB = CD and AD = BC [∵ Opposite sides of a parallelogram are equal]
Thus, in ∆s ABD and ACB, we have
AD = BC [As proved above]
BD = AC [Given]
and, AB = AB [Common]
So, by SSS criterion of congruence, we have
AD = BC [As proved above]
BD = AC [Given]
and, AB = AB [Common]

So, by SSS criterion of congruence, we have

$\Delta ABD \cong \Delta ACB$

=> $\angle BAD = \angle ABC$ [c. p. c. t.] ... (i)

Now, AD || BC and transversal AB intersects them at A and B respectively.

∴ $\angle BAD + \angle ABC = 180^\circ$ [Sum of the interior angles on the same side of a transversal is 180°]

=> $\angle ABC + \angle ABC = 180^\circ$

=> $2\angle ABC = 180^\circ$

=> $\angle ABC = 90^\circ$, Hence, the measure of ∠ABC is 90°.

Q. 3. If two isosceles triangles have a common base, prove that the line joining their vertices bisects them at right angles.

GIVEN: Two isosceles triangles ABC and DBC having the common base BC such that AB = AC and DB = DC.

TO PROVE: AD (or AD produced) bisects BC at right angle.

PROOF: In Δs ABD and ACD, we have

AB = AD [Given]

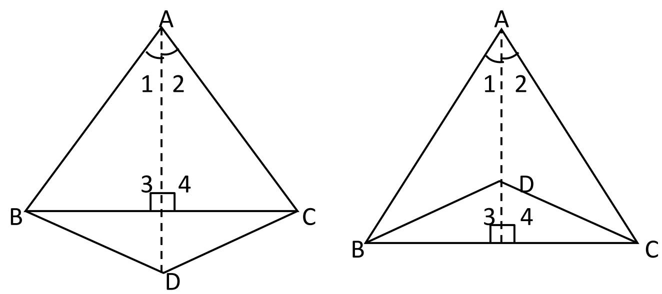
BD = CD [Given]

and, AD = AD [Common side]

So, by SSS criterion of congruence, we have

$\Delta ABD \cong \Delta ACD$

=> $\angle 1 = \angle 2$ [c. p. c. t.] ... (i)



Thus, in Δ ABE and Δ ACE, we have

AB = AC [Given]

$\angle 1 = \angle 2$ [From (i)]

and, AE = AE [Common side]

So, by SAS criterion of congruence, we have

$\Delta ABE \cong \Delta ACE$

=> BE = CE [∵ Corresponding parts of congruent triangles are equal]

and, $\angle 3 = \angle 4$

But, $\angle 3 + \angle 4 = 180^\circ$ [∵ Sum of the angles of a linear pair is 180°]

=> $2\angle 3 = 180^\circ$ [∵ $\angle 3 = \angle 4$]

=> $\angle 3 = 90^\circ$

∴ $\angle 3 = \angle 4 = 90^\circ$ Hence, AD bisects BC at right angles.

Q. 4. Δ ABC and Δ DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that

(i) Δ ABD ≅ Δ ACD **(ii) Δ ABP ≅ Δ ACP**

(iii) AP bisects ∠A as well as ∠D. **(iv) AP is the perpendicular bisector of BC.**

Sol. (i) In triangles ABD and ACD, we have

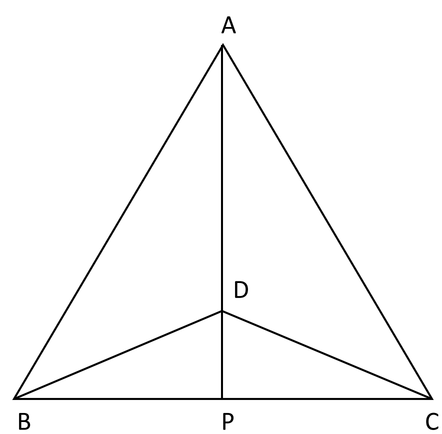
AB = AC [Given]

BD = CD [Given]

and, AD = DA [Common]

So, by SSS criterion of congruence, we have

$\Delta ABD \cong \Delta ACD$



(ii) In triangles ABP and ACP, we have

AB = AC

$\angle BAP = \angle CAP$ [∵ ΔABD ≅ Δ ACD ∴ ∠BAD = ∠CAD => ∠BAP = ∠CAP]

and, AP = AP

So, by SAS congruence criterion, we have

$\Delta ABP \cong \Delta ACP$

(iii) We have proved in (i) that

$\Delta ABD \cong \Delta ACD$

=> $\angle BAD = \angle CAD$

=> $\angle BAP = \angle CAP$

=> AP is the bisector of ∠A

In triangles BDP and CDP, we have

BD = CD [Given]

BP = CP [∵ Δ ABP ≅ Δ ACP ∴ BP = CP]

and, DP = DP [Common]

So, by SSS congruence criterion, we have

Δ BDP ≅ Δ CDP

=> ∠BDP = ∠CDP

=> DP is the bisector of ∠D

Hence, AP is the bisector of ∠A as well as ∠D.

(iv) In (iii) We have proved that

Δ BDP ≅ Δ CDP => BP = CP and ∠BPD = ∠CPD

=> BP = CP and ∠BPD = ∠CPD = 90° [∵ ∠BPD and ∠CPD form a linear pair]

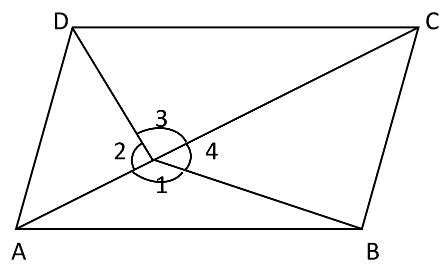
=> DP is the perpendicular bisector of BC, Hence, AP is the perpendicular bisector of BC.

Q. 5. A point O is taken inside an equilateral four sides figure ABCD such that its distance from the angular points D and B are equal. Show that AO and OC are in one and the same straight line.

GIVEN: A point O inside an equilateral quadrilateral four sided figure ABCD such that BO = OD.

TO PROVE: AO and OC are in one and the same straight line.

PROOF: In Δs AOD and AOB, we have



AD = AB [Given]

AO = AO [Common side]

and, OD = OB [Given]

So, by SSS criterion of congruence, we have

Δ AOD ≅ Δ AOB

=> ∠1 = ∠2 [c. p. c. t.] ... (i)

Similarly, Δ DOC ≅ Δ BOC

=> ∠3 = ∠4 [c. p. c. t.] ... (ii)

But, ∠1 + ∠2 + ∠3 + ∠4 = 4 right angles [Sum of the angles at a point is 4 right angles]

=> 2∠2 + 2∠3 = 4 right angles [Using (i) and (ii)]

=> ∠2 + ∠3 = 2 right angles

=> ∠2 + ∠3 = 180° => ∠2 and ∠3 form a linear pair.

=> AO and OC are in the same straight line => AC is a straight line.

Q. 6. In Fig. two sides AB and BC and the median AD of Δ ABC are equal respectively to the two sides PQ and QR and the median PM of the other triangle PQR. Prove that (i) Δ ABD ≅ Δ PQM (ii) Δ ABC ≅ Δ PQR

GIVEN: Two Δs ABC and PQR in which AB = PQ, BC = QR and AD = PM.

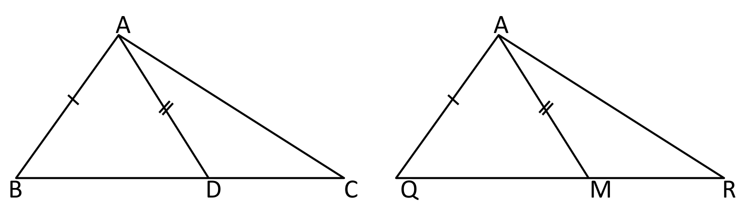
TO PROVE: Δ ABC ≅ Δ PQR

PROOF: Since AD and PM are medians of triangles ABC and PQR respectively. Therefore D and M are Mid-points of BC and QR respectively.

Now, BC = QR [Given]

=> ½ BC = ½ QR

=> BD = QM ... (i)



Now, in Δs ABD and PQM, we have

AB = PQ [Given]

BD = QM [From (i)]

and, AD = PM [Given]

So, by SSS criterion of congruence, we have

ΔABD ≅ Δ PQM

=> ∠B = ∠Q [c. p. c. t.] ... (ii)

Now, In Δ ABC and Δ PQR, we have

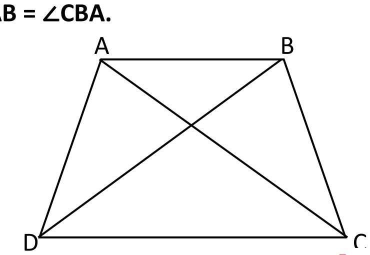
AB = PQ [Given]

∠B = ∠Q [From (ii)]

and, BC = QR [Given] So, by SAS criterion of congruence, we haveΔ ABC ≅ Δ PQR

Q. 7. In Fig. AD = BC and BD = CA. Prove that ∠ADB = ∠BCA and ∠DAB = ∠CBA.

Sol. In triangles ABD and ABC, we have



AD = BC [Given]

BD = CA [Given]

and, AB = AB [Common]

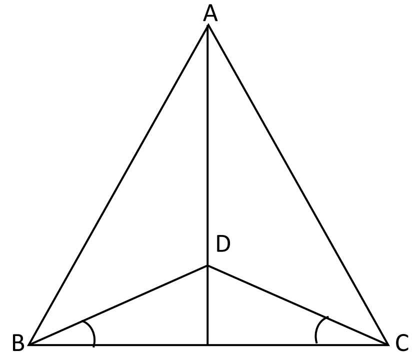
So, by SSS congruence criterion, we have

Δ ABD ≅ Δ CBA

=> ∠DAB = ∠ABC [c. p. c. t.]

=> ∠DAB = ∠CBA

Q. 8. In Fig. AB = AC, D is the point in the interior of Δ ABC such that ∠DBC = ∠DCB. Prove that AD bisects ∠BAC of Δ ABC.



Sol. In Δ BDC, we have

∠DBC = ∠DCB [Given]

=> DC = DB ... (i) [∵ Sides opposite to equal angles of Δ DBC are equal]

Now, in Δ ABD and Δ ACD, we have

AB = AC [Given]

BD = CD [From (i)]

and, AD = AD [Common side]

So, by SSS congruence criterion, we have

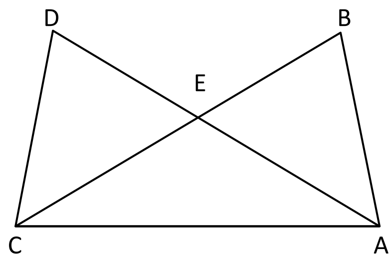
Δ ABD ≅ Δ ACD

=> ∠BAD = ∠CAD [c. p. c. t.]

Hence, AD is bisector of ∠BAC.

Exercise 1.4............

Q. 1. In Fig., it is given that AB = CD and AD = BC. Prove that Δ ADC ≅ Δ CBA.



Q. 2. In A Δ PQR, if PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that LN = MN.

HINTS TO SELECTED PROBLEMS

1. In Δ's ADC and CBA, we have

AB = CD [Given]

AD = BC [Given]

and, AC = AC [Common]

So, by SSS congruence criterion, we have

Δ ADC ≅ Δ Δ ACBA

2. Using mid-point theorem, we have MN ∥ PQ and MN

Similarly, we have

LM = PN

In triangles NML and LPN, we have

MN = PL

LM = ON

and, LN = NL

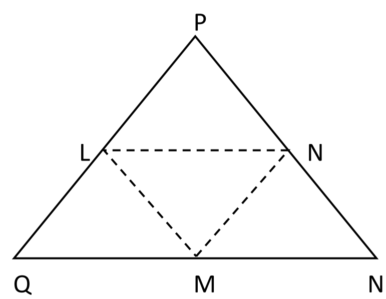
So, by SSS congruence criterion, we have

Δ NML ≅ Δ LPN

=> ∠ MNL = ∠PLN and ∠MLN = ∠LNP

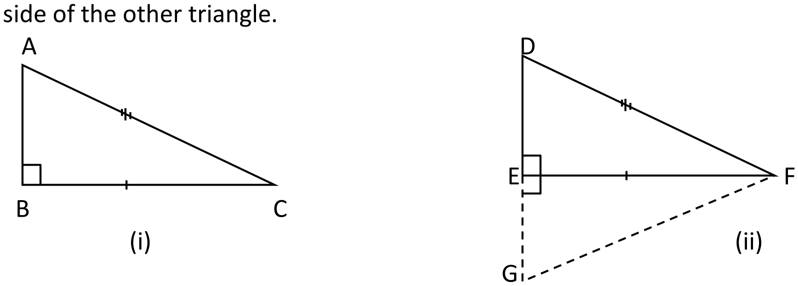
=> ∠MNL = ∠LNP = ∠PLM = ∠MLN

=> ∠PNM = ∠PLM



RIGHT ANGLE-HYPOTENUSE-SIDE (RHS) CONGRUENCE CRITERION

***THEOREM** Two right triangles are congruence if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other triangle.



(i) (ii)

GIVEN: Two right triangles ABC and DEF in which ∠B = ∠E = 90°, AC = DF, BC = EF

TO PROVE: Δ ABC ≅ Δ DEF

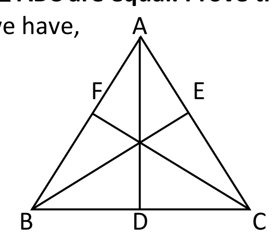
CONSTRUCTION: produce DE to G so that EG = AB. Join GF.
PROOF: In Δs ABC and FEF, we have

AB = GE [By construction]
$\angle B = \angle FEG = 90^\circ$
and, BC = EF [Given]
So, by SAS criterion of congruence, we have $\Delta ABC \cong \Delta GEF$
=> $\angle A = \angle G$ ... (i)
AC = GF [c. p. c. t.] ... (ii)
Now, AC = DF [From (ii)]
and, AC = DF [Given]
$\therefore$ DF = GF
=> $\angle D = \angle G$ [Angles opposite to equal sides in Δ DGF are equal] ... (iii)
From (i) and (iii), we get
$\angle A = \angle D$ ... (iv)
Thus, in Δs ABC and DEF, we have
$\angle A = \angle D$ [From (iv)]
$\angle B = \angle E$ [Given]
=> $\angle A + \angle B = \angle D + \angle E$
=> $180^\circ - \angle C = 180^\circ - F$ [$\because \angle A + \angle B + \angle C = 180^\circ$ and $\angle D + \angle E + \angle F = 180^\circ$]
=> $\angle C = \angle F$ ... (v)
Now, in Δs ABC and DEF, we have
BC = EF [Given]
$\angle C = \angle F$ [From (v)]
and, AC = DF
So, by SAS criterion of congruence, we have
$\Delta ABC \cong \Delta DEF$

Illustrative Examples........

Q. 1. AD, BE and CF, the altitudes of Δ ABC are equal. Prove that Δ ABC is an equilateral triangle.

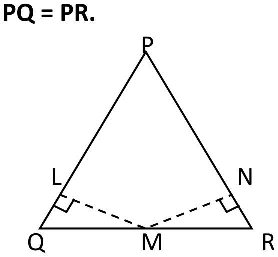
Sol. In right triangles BCE and BFC, we have,



Hyp, BC = Hyp. BC
BE = CF
So, by RHS criterion of congruence, we have
$\Delta BCE \cong \Delta BFC$
=> $\angle B = \angle C$ [$\because$ Corresponding parts of congruent triangles are equal]
=> AC = AB [$\because$ Sides opposite to equal angles are equal] ... (i)
Similarly, $\Delta ABD \cong \Delta ABE$
=> $\angle B = \angle A$ [Corresponding parts of congruent triangles are equal]
=> AC = BC [$\because$ Sides opposite to equal angles are equal] ... (ii)
From (i) and (ii), we get
AB = BC = AC Hence, Δ ABC is an equilateral triangle.

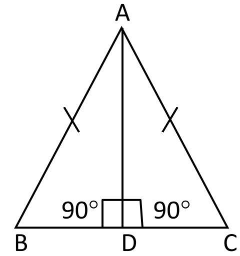
Q. 2. In Fig. it is given that LM = MN, QM = MR, ML $\perp$ PQ and MN $\perp$ PR. Prove that PQ = PR.

Sol. In right triangles QLM and MNR, we have



Hyp. QM = Hyp. MR [Given]
LM = MN [Given]
So, by RHS criterion of congruence, we have
$\Delta QLM \cong \Delta RNM$
=> $\angle Q = \angle R$ [c. p. c. t]
=> PR = PQ [$\because$ Sides opposite to equal angles are equal]

Q. 3. **If Δ ABC is an isosceles triangle such that AB = AC and AD is an altitude from A on BC. Prove that (i) ∠B = ∠C (ii) AD bisects BC (iii) AD bisects ∠A.**



Sol. In right triangles ADB and ADC, we have

Hyp. AB = Hyp. AC [Given]

AD = AD [Common side]

So, by RHS criterion of congruence, we have

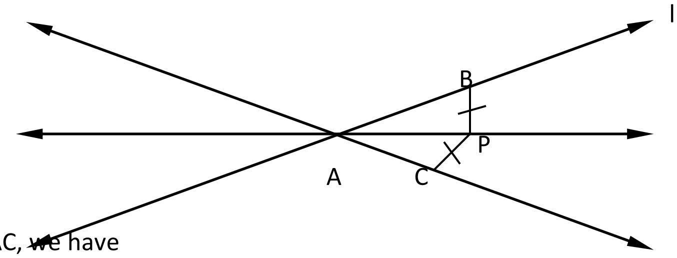
Δ ABD ≅ Δ ACD

=> ∠B = ∠C, BD = DC and ∠BAD = ∠CAD [c. p. c. t.]

=> ∠B = ∠C, AD bisects BC and ∠A

Q. 4. **P is a point equidistant from two lines l and m intersecting at a point A (see fig.). Show that AP bisects the angle between them.**

Sol. Let PB and PC be perpendiculars form P on lines l and m respectively. Since P is equidistant from lines l and m. Therefore,



PB = PC

In triangles PAB and PAC, we have

PB = PC [Given]

∠PBA = ∠PCA [Each equal to 90°]

and, PA = PA [Common]

So, by RHS congruence criterion, we have

Δ PAB ≅ Δ PAC

=> ∠PAB = ∠PAC

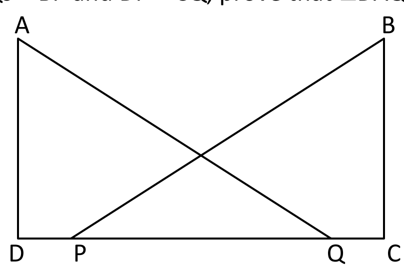
Exercise 1.4...........

Q. 1. ABC is a triangle and D is the mid-point of BC. The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.

Q. 2. ABC is a triangle in which BE and CF are, respectively, the perpendiculars to the sides AC and AB. If BE = CF, prove that Δ ABC is isosceles.

Q. 3. If the perpendiculars from any point within an angle on its arms are congruent, prove that it lies on the bisector of that angle.

Q. 4. In Fig., AD ⊥ CD and CB ⊥ CD. If AQs = BP and DP = CQ, prove that ∠DAQ = ∠CBP.



Q. 5. ABCD is a square, X and Y are points on sides AD and BC respectively such that AY = BX. Prove that BY = AX and ∠BAY = ∠ABX.

Q. 6. Which of the following statements are true (T) and which are false (F):

(i) Sides opposite to equal angles of a triangle may be unequal.

(ii) Angles opposite to equal sides of a triangle are equal.

(iii) The measure of each angle of an equilateral triangle is 60°.

(iv) If the altitude from one vertex of a triangle bisects the opposite side, then the triangle may be isosceles.

(v) The bisectors of two equal angles of a triangle are equal.

(vi) If the bisector of the vertical angle of a triangle bisects the base, then the triangle may be isosceles.

(vii) The two altitudes corresponding to two equal sides of a triangle need not be equal.

(viii) If any two sides of a right triangle are respectively equal to two sides of other right triangle, then the two triangles are congruent.

(ix) Two right triangles are congruent if hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.

Q. 7. Fill in the blanks in the following so that each of the following statements is true.

(i) Sides opposite to equal angles of a triangle are ……………..

(ii) Angle opposite to equal sides of a triangle are …….

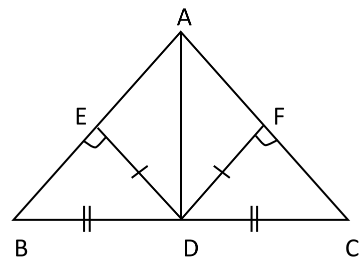
(iii) In an equilateral triangle all angles are ………………
(iv) In a Δ ABC if ∠A = ∠C, then AB = ………
(v) If altitudes CE and BF of a triangle ABC are equal, then AB = ………….
(vi) In an isosceles triangle ABC with AB = AC. If BD and CE are its altitudes, then BD is ……… CE.
(vii) In right triangles ABC and DEF, if hypotenuse AB = EF and side AC = DE, then Δ ABC ≅ Δ ………

Answers…………….

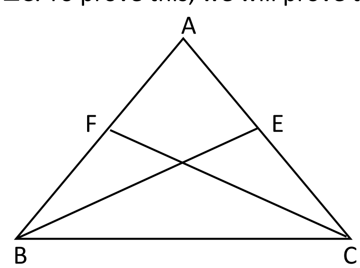
6.	(i) F	(ii) T	(iii) T	(iv) F	
	(v) T	(vi) F	(vii) F	(viii) F	(ix) T
7.	(i) equal	(ii) equal	(iii) equal	(iv) BC	(v) AC
	(vi) equal to	(vii) EFD			

HINTS TO SELECTED PROBLEMS

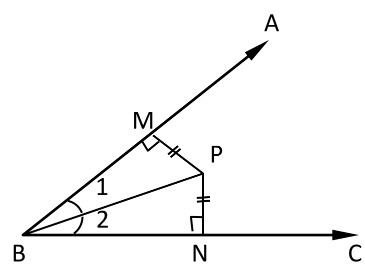
1. Let DE and DF be perpendiculars from D on AB and AC respectively.
In order to prove that AB = AC, we will prove that Δ BDE ≅ Δ CDF.
In these two triangles, we have
∠BEF = ∠CFD = 90°
BD = CD [∵ D is the mid-point of BC]
DE = DF [Given]
So, by RHS congruence criterion, we have
Δ BDE ≅ Δ CDF
=> ∠B = ∠C
=> AC = AB
=> Δ ABC is isosceles.



2. In order to prove that the triangle ABC is isosceles, we will prove that ∠B = ∠C. To prove this, we will prove that Δ BFC ≅ Δ CEB.
In these two triangles, we have
BE = CF [Given]
BC = BC [Common]
So, by RHS congruence criterion, we have
Δ BFC ≅ Δ CEB
=> ∠FBC = ∠ECB
=> ∠ABC = ∠ACB
=> AB = AC [∵ Sides opposite to equal angles are equal]

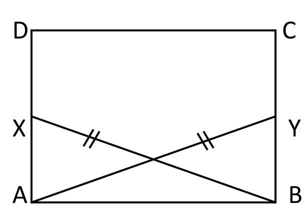


3. Let P be a point within ∠ABC such that PM = PN. We have to prove that P lies on the bisector of ∠ABC i.e., ∠1 = ∠2
In Δ's PMB and PNB, we have
PM = PN [Given]
BP = BP [Common]
So, by RHS congruence criterion, we have
Δ PMB ≅ Δ PBN
=> ∠1 = ∠2 [∵ ∠B = ∠C]
=> P lies on the bisector of ∠ABC



4. In Δ's ADQ and BCP, we have
DQ = PC [∵ DP = QC (Given) ∴ DP + PQ = PQ + QC => DQ = PC]
and, AQ = BP [Given]
So, by RHS congruence criterion, we have
Δ ADQ ≅ Δ BCP
=> ∠A = ∠B i.e., ∠DAQ = ∠CBP

5. In right triangles BAY and ABX, we have
AY = BX [Given]
and, BA = AB [Common]
So, by RHS congruence criterion we have
Δ BAY ≅ Δ ABX
=> BY = AX and ∠BAY = ∠ABX



SOME INEQUALITY RELATIONS IN A TRIANGLE

THEOREM 1 If two sides of a triangle are unequal, the longer side has greater angle opposite to it.

GIVEN: A Δ ABC in which AC > AB.

TO PROVE: ∠ABC > ∠ ACB

CONSTRUCTION: Mark a point D on AC such that AB = AD. Join BD.

PROOF: In Δ ABD, we have

AB = AD [By construction]

=> ∠1 = ∠2 [∵ Angle opp. to equal sides are equal] ... (i)

Now, consider Δ BCD. We find ∠2, is the exterior angle of Δs BCD and an exterior angle is always greater than interior opposite angle. Therefore,

∠2 > ∠DCB

=> ∠2 > ∠ACB [∵ ∠ACB = ∠DCB] ... (ii)

From (i) and (ii), we have

∠1 = ∠2 and ∠2 > ACB

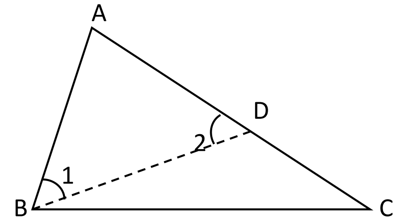
=> ∠1 > ∠ ACB ... (iii)

But, ∠1 is a part of ∠ ABC.

∴ ∠ABC > ∠1 ... (iv)

From (iii) and (iv), we get

∠ABC > ∠ ACB



THEOREM 2 (Converse of Theorem 1) In a triangle the greater angle has the longer side opposite to it.

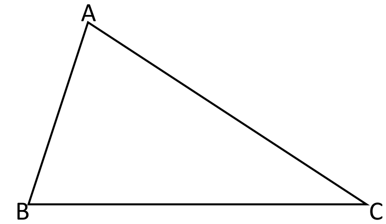
GIVEN: A Δ ABC in which ∠ ABC > ∠ACB.

TO PROVE: AC > AB.

PROOF: In Δ ABC, we have the following three possibilities.

(i) AC = AB (ii) AC < AB (iii) AC > AB.

Out of these three possibilities exactly one must be true.



☑ **CASE I** When AC = AB

AC = AB

=> ∠ABC = ∠ACB [Angles opp. to equal sides are equal]

This is contradiction,

Since, ∠ ABC > ∠ ACB [Given]

∴ AC ≠ AB

☑ **CASE II** When AC < AB

AC < AB

=> ∠ACB > ABC [∵ Longer side has the greater angle opposite to it]

This also contradicts the given hypothesis.

Thus, we are left with the only possibility, AC > AB, which must be true. Hence, AC > AB

THEOREM 3 The sum of any two sides of a triangle is greater than the third side.

GIVEN: A Δ ABC

TO PROVE: AB + AC > BC, AB + BC > AC and BC + AC > AB

CONSTRUCTION: Produce side BA to D such that AD = AC. Join CD.

PROOF: In Δ ACD, we have

AC = AD [By construction]

=> ∠ADC = ∠ACD [Angles opp. to equal sides are equal]

=> ∠ACD = ∠ADC

=> ∠BCA + ∠ACD > ∠ADC [∵ ∠BCA + ∠ACD > ∠ACD]

=> ∠BCD > ∠ADC

=> ∠BCD > ∠BDC [∵ ∠ADC = ∠BDC]

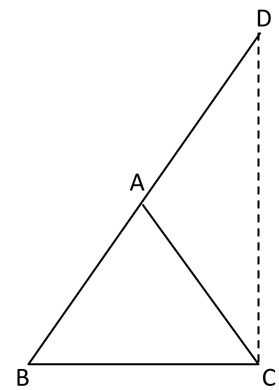
=> BD > BC [∵ Side opp. to greater angle is larger]

=> BA + AD > BC

=> BA + AC > BC [∵ AC = AD (By Construction)]

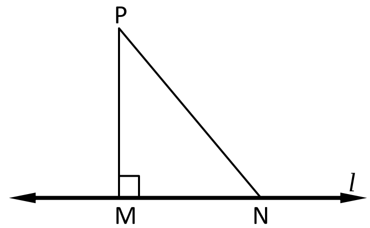
=> AB + AC > BC

=> Thus, AB + AC > BC Similarly, AB + BC > AC and BC + AC > AB



***THEOREM 4** Of all the line segments that can be drawn to a given line, from a point, not lying on it, the perpendicular line segment is the shortest.

GIVEN: A straight line *l* and a point P not lying on *l*. PM ⊥ *l* and N is any point on 1 other than M.



TO PROVE: PM < PN

PROOF: In Δ PMN, we have

$\angle M = 90°$

=> $\angle N < 90°$ $\left(\because \angle M = 90° \Rightarrow \angle MPN + \angle PNM = 90° \Rightarrow \angle P + \angle N = 90° \Rightarrow \angle N < 90°\right)$

=> $\angle N < \angle M$

=> PM < ON [Side opp. to greater angle is larger]

Thus, PM < PN

Hence, PM is the shortest of all line segments from P to AB.

DISTANCE BETWEEN A LINE AND A POINT: The distance between a line and a point, not on it, is the length of the perpendicular line segment from the point to the line.

Illustrative Examples.........

Q. 1. In a Δ ABC, if $\angle A = 45°$ and $\angle B = 70°$. Determine the shortest and largest sides of the triangle.

Sol. We have, $\angle A = 45°$ and $\angle B = 70°$

$\therefore$ $\angle A + \angle B + \angle C = 180°$

=> $45° + 70° + \angle C = 180°$ => $\angle C = 180° - 115°$ => $\angle C = 65°$

Since the side opposite to the greatest angle is largest. Therefore, side AC is largest. The side opposite to the least angle is the smallest. So, side opposite to $\angle A$ i.e., side BC is the smallest.

Q. 2. In a Δ ABC, if $\angle A = 50°$ and $\angle B = 60°$, determine the shortest and largest sides of the triangle.

Sol. We have,

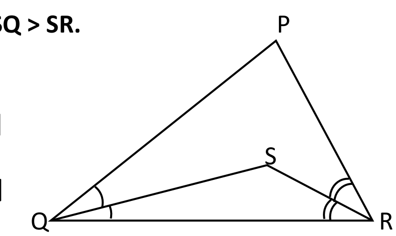
$\angle A = 50°$ and $\angle B = 60°$

$\therefore$ $\angle A + \angle B + \angle C = 180°$

=> $50° + 60° + \angle C = 180°$ => $\angle C = 70°$

Since $\angle$ A and $\angle C$ are the smallest and largest angles respectively. Therefore, sides BC and AB are the smallest and largest sides respectively of the triangle.

Q. 3. In Fig. PQ > PR. QS and RS are the bisector of $\angle Q$ and $\angle R$ respectively. Prove that SQ > SR.



Sol. In Δ PQR, we have

PQ > PR [Given]

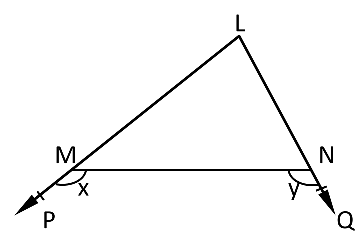
=> $\angle PRQ > PQR$ [Angle opp. to larger side of a triangle is greater]

=> $\frac{1}{2}\angle PRQ > \frac{1}{2}\angle PQR$

=> $\angle SRQ > \angle SQR$ [RS and QS are bisectors of $\angle PRQ$ and $\angle PQR$ respectively]

=> SQ > SR [$\because$ Side opp. to greater angle is larger]

Q. 4. In Fig. sides LM and LN of Δ LMN are extended to P and Q respectively. If x > y, show that LM > LN.



Sol. We have,

$\angle LMN + x = 180°$ [Angles of a linear pair] ... (i)

=> $\angle LMN + y = 180°$ [Angles of a linear pair] ... (ii)

$\therefore$ $\angle LMN + x = \angle LNM + y$

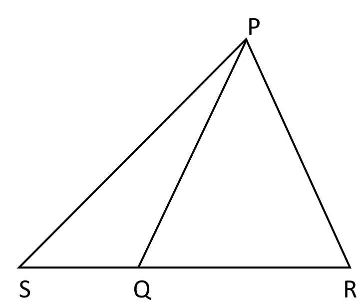
But, $x > y$

$\therefore$ $\angle LMN < \angle LNM$

=> $\angle LNM > \angle LMN$

=> LM > LN [$\because$ Side opp. to greater angle is larger]

Q. 5. In Fig. PQ = PR. Show that PS > PQ



Sol. In Δ PQR, we have

PQ = PR [Given]

=> $\angle PRQ = \angle PQR$ [Angles opp. to equal sides are equal] ... (i)

In Δ PSQ, SQ is produced to R and exterior angle of triangle is greater than each of interior opposite angle.

$\therefore$ Ext. $\angle PQR > \angle PSQ$ ... (ii)

From (i) and (ii), we have

$\angle PSQ > \angle PSQ$

=> $\angle PRS > \angle PSR$ [$\because \angle PRQ = \angle PRS$ and $\angle PSQ = \angle PSR$]

Thus, in Δ PSR, we have

$\angle PRS > \angle PSR$

=> PS > PR [∵ Side opposite to greater angle is larger]
But, PR = PQ [Given]
∴ PS > PQ

Q. 6. In Fig. AB > AC. Show that AB > AD

Sol. In Δ ABC, we have

AB > AC [Given]

=> $\angle ACB > \angle ABC$ ... (i)

Now, in Δ ACD, CD is produced to B, forming and ext.

$\angle ADB$ and exterior angle of a triangle is greater than each of interior opposite angle

∴ $\angle ADB > \angle ACD$

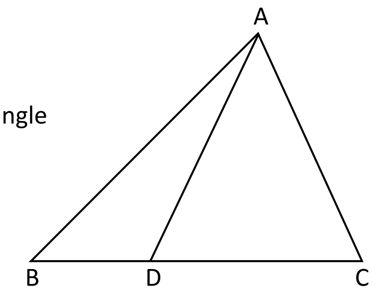
=> $\angle ADB > \angle ACB$ [∵ $\angle ACD = \angle ACB$] ... (ii)

From (i) and (ii), we get

$\angle ADB > \angle ABC$

=> $\angle ADB > \angle ABD$ [∵ $\angle ABC = \angle ABD$]

=> AB > AD [∵ Side opp. to greater angle is larger]



Q. 7. If D is any point on the base BC produced, of an isosceles triangle ABC, prove that AD > AB.

Sol. In Δ ABC, we have

AB =AC

=> $\angle ABC = \angle ACB$ [∵ Angles opp. to equal sides are equal] ... (i)

In Δ ABD, we have

Ext. $\angle ABC > \angle ADB$ [∵ Exterior angle of a Δ is greater than each of interior opp. angle]

=> $\angle ABC > \angle ADB$ ... (ii)

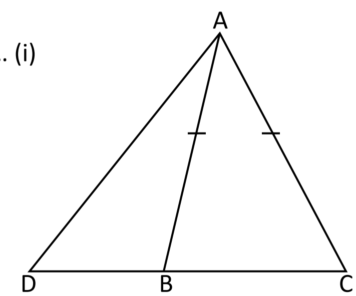
From (i) and (ii), we get

$\angle ACB > \angle ADB$

=> $\angle ACD > \angle ADC$ [∵ $\angle ACB = \angle ACD$, $\angle ADB = \angle ADC$]

=> AD > AC [∵ Side opp. to greater angle is larger]

=> AD > AB [∵ AB = AC]



Q. 8. In Fig. if AD is the bisector of $\angle A$, show that:

(i) AB > BD **(ii) AC > CD**

Sol. In Δ ABC, AD is the bisector of $\angle A$

∴ $\angle 1 = \angle 2$ ... (i)

Since exterior angle of a triangle is greater than each of interior opposite angle.

Therefore, in Δ ADC, we have

Ext. $\angle ADC > \angle 2$

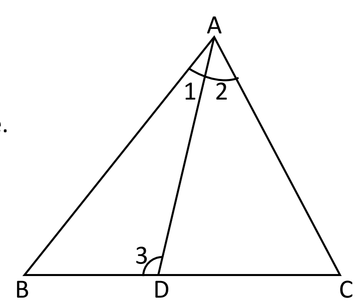
=> $\angle 3 > \angle 2$

=> $\angle 3 > \angle 1$ [Using (i)]

Thus, in Δ ABD, we have

$\angle 3 > \angle 1$ => AB > BD [Side opp. to greater angle is larger]

Hence, AB > BD,Similarly, we can prove that AC > CD.



Q. 9. Show that in a right triangle the hypotenuse is the longest side.

GIVEN: A right triangle ABC in which $\angle ABC = 90°$

TO PROVE: Hypotenuse AC is the longest side, i.e.,

(i) AC > AB **(ii) AC > BC**

PROOF: In Δ ABC, we have

$\angle ABC = 90°$

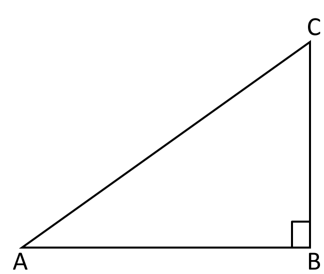
But, $\angle ABC + \angle BCA + \angle CAB = 180°$

∴ $90° + \angle BCA + \angle CAB = 180°$

=> $\angle BCA + \angle CAB = 90°$

=> $\angle BCA$ and $\angle CAB$ are acute angles

=> $\angle BCA < 90°$ and $\angle CAB < 90°$ => AC > AB and AC > BC [∵ Side opp. to greater angle is larger]



Q. 10. In Fig. AC > AB and AD is the bisector of $\angle A$. Show that $\angle ADC > \angle ADB$.

Sol. In Δ ABC, we have

AC > AB [Given]

=> $\angle ABC > \angle ACB$ [Angle opp. to larger side is greater]

=> $\angle ABC + \angle 1 > \angle ACB + \angle 1$ [Adding $\angle 1$ on both sides]

=> $\angle ABC + \angle 1 > \angle ACB + \angle 2$ ... (i) [$\because$ AD is the bisector of $\angle A$ $\therefore$ $\angle 1 = \angle 2$]

Now, in triangles ABD and ADC, we have

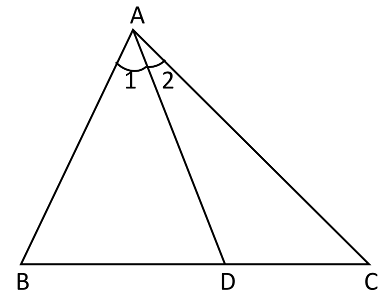
$\angle ABC + \angle 1 + \angle ADB = 180^\circ$ and $\angle ACB + \angle 2 + \angle ADC = 180^\circ$

=> $\angle ABC + \angle 1 = 180^\circ - \angle ADB$ and $\angle ACB + \angle 2 = 180^\circ - \angle ADC$

$\therefore$ $180^\circ - \angle ADB > 180^\circ - \angle ADC$ [From (i)]

=> $180^\circ - \angle ADB - 180^\circ - \angle ADC > 0$

=> $\angle ADC - \angle ADB > 0$ => $\angle ADC > \angle ADB$



***Q. 11. Show that the sum of the three altitudes of a triangle is less than the sum of three sides of the triangle.**

GIVEN: A Δ ABC in which AD $\perp$ BC, BE $\perp$ AC and CF $\perp$ AB.

TO PROVE: AD + BE + CF < AB + BC + AC

PROOF: We know that of all the segments that can be drawn to a given line, from a point not lying on it, the perpendicular line segment is the shortest. Therefore, AD $\perp$ BC

=> AB > AD and AC > AD

=> AB + AC > AD + AD

=> AB + AC > 2 AD ... (i)

BE $\perp$ AC

=> BC > BE and BA > BE

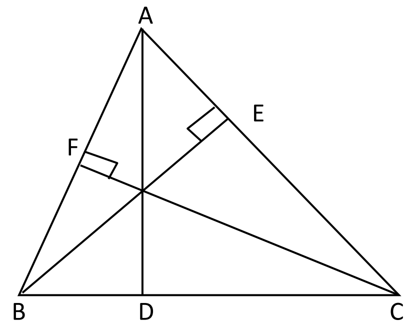
=> BC + BA > BE + BE ... (ii)

=> BA + BC > 2 BE

and, CF $\perp$ AB

=> AC > CF and BC > CF

=> AC + BC > 2 CF ... (iii)



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Adding (i), (ii) and (iii), we get

(AB + AC) + (AB + BC) + (AC + BC) > 2 AD + 2 BE + 2 CF

=> 2 (AB + BC + AC) > 2 (AD + BE + CF) => AD + BE + CF < AB + BC + AC

***Q. 12. Prove that any two sides of a triangle are together greater than twice the median drawn to the third side.**

GIVEN: Δ ABC in which AD is a median

TO PROVE: AB + AC > 2 AD

CONSTRUCTION: Produce AD to E such that AD = DE. Join EC.

PROOF: In Δs ADB and EDC, we have

AD = DE [By construction]

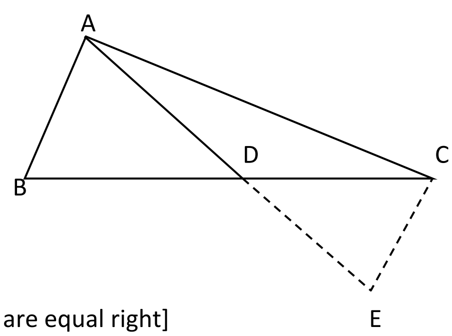
BD = DC [$\because$ D is the mid point of BC]

and, $\angle ADB = \angle EDC$ [Ver. opp. $\angle$s right]

So, by SAS criterion of congruence, we have

Δ ADB $\cong$ Δ EDC

=> AB = EC [Corresponding parts of congruent triangle are equal right]



Thus, in Δ AEC, we have

AC + EC > AE [$\because$ Sum of any two sides of a Δ is greater than the third]

=> AC + AB > 2 AD [$\because$ AD = DE $\therefore$ AE = AD + DE = 2 AD and EC = AB]

***Q. 13. Prove that the perimeter of a triangle is greater than the sum of its three medians.**

GIVEN: A Δ ABC in which AD, BE and CF are its medians.

TO PROVE: AB + BC + AC > AD + BE + CF

PROOF: We know that the sum of any two sides of a triangle is greater than twice the median bisecting the third side. Therefore,

AD is the median bisecting BC

=> AB + AC > 2 AD ... (i)

BE is the median bisecting AC

=> AB + BC > 2 BE ... (ii)

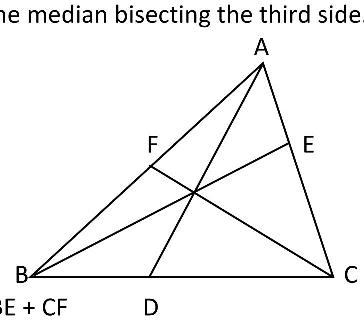
And, CF is the median bisecting AB

=> BC + AC > 2 CF ...(iii)

Adding (i), (ii) and (iii), we get

(AB + AC) + (AB + BC) + (BC + AC) > 2 . AD + 2 . BE + 2 . CF

=> 2 (AB + BC + AC) > 2 (AD + BE + CF) => AB + BC + AC > AD + BE + CF



***Q. 14. Show that the difference of any two sides of a triangle is less than the third side.**

GIVEN: A Δ ABC

TO PROVE: (i) AC – AB < BC (ii) BC – AC < AB (iii) BC – AB < AC

CONSTRUCTION: Take a point D on AC such that AD = AB. Join BD.

PROOF: In Δ ABD, side AD has been produced to C.

∴ ∠3 > ∠1 ... (i) [∵ Exterior angle of a Δ is greater than each of interior opp. angle]

In Δ ACD, side CD has been produced to A.

∴ ∠2 > ∠4 ... (ii) [∵ Exterior angle of a Δ is greater than each of interior opp. angle]

In Δ ABD, we have

AB = AD

=> ∠2 = ∠1 ... (iii) [Angles opp. to equal sides are equal]

From (i) and (iii), we get

∠3 > ∠2 ... (iv)

From (ii) and (iv), we get

∠3 > ∠ 2 and ∠2 > ∠4

=> ∠3 > ∠4

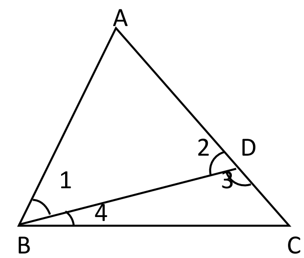
=> BC > CD [Side opp. to greater angle is larger]

=> CD < BC

=> AC – AD < BC

=> AC – AB < BC [∵ AD = AB]

Similarly, BC – AC < AB and BC – AB < AC



***Q. 15. In Fig. PQR is a triangle and S is any point in its interior, show that SQ + SR < PQ + PR.**

GIVEN: S is any point in the interior of Δ PQR.

TO PROVE: SQ + SR < PQ + PR

CONSTRUCTION: Produce QS to meet PR in T.

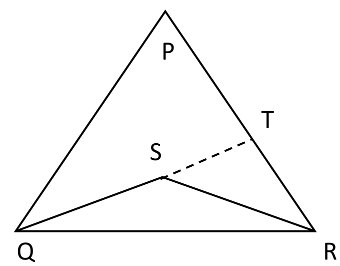
PROOF: In Δ PQT, we have,

PQ + PT > QT [∵ Sum of the two sides of a Δ is a greater than the third side]

=> PQ + PT > QS + ST [∵ QT = QS = ST] ... (i)

In Δ RST, we have

ST + TR > SR ... (ii)



Adding (i) and (ii), we get

PQ + PT + ST + TR > SQ + ST + SR

=> PQ + (PT + TR) > SQ + SR

=> PQ + PR > SQ + SR => SQ + SR < PQ + PR

Q. 16. In Δ PQR, S is any point on the side QR. Show that PQ + QR + RP > 2PS

Sol. In Δ PQS, we have

PQ + QS > PS [∵ Sum of the two sides of a Δ is greater than the third side] ... (i)

Similarly, in Δ PRS, we have

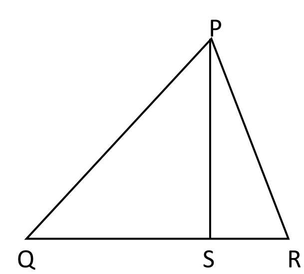
RP + RS > PS ... (ii)

Adding (i) and (ii), we get

(PQ + QS) + (RP + RS) > PS + PS

=> PQ + (QS + RS) + RP > 2 PS

=> PQ + QR + RP > 2 PS [∵ QS + RS = QR]



***Q. 17. In Fig., AP ⊥ *l* and PR > PQ. Show that AR > AQ.**

GIVEN: AP ⊥ *l* and PR > PQ

TO PROVE: AR > AQ

CONSTRUCTION: Mark a point S on PR such that PS = PQ. Join AS.

PROOF: In Δs APQ and APS, we have

AP = AP [Common side]

∠APQ = ∠APS [Each equal to 90°]

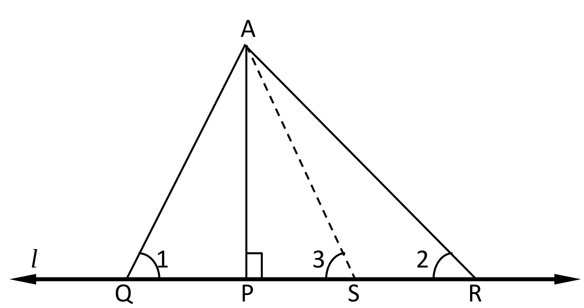
and, PQ = PS [By construction]

So, by SAS criterion of congruence

Δ APQ ≅ Δ APS

=> AQ = AS [∵ Corresponding parts of similar triangles are equal]

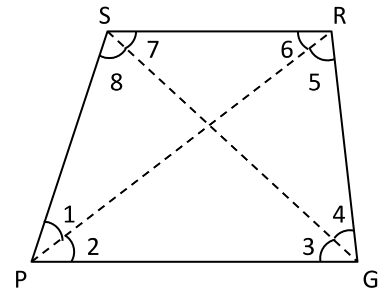
Thus, in Δ AQS, we have



AQ = AS
=> ∠1 = ∠3 ... (i) [∵ Angles opposite to equal sides are equal]
In Δ ARS, we have
∠3 > ∠2 ... (ii) [∵ Exterior angle of a Δ is greater than each of interior opp. angle]
From (i) and (ii), we get
∠1 < ∠2
=> AR > AQ [∵ Side opp. to greater angle is larger]
Hence, AR > AQ

***Q. 18. In Fig. PQRS is a quadrilateral. PQ is its longest side and RS is its shortest side. Prove that ∠R > ∠P and ∠S > ∠Q.**

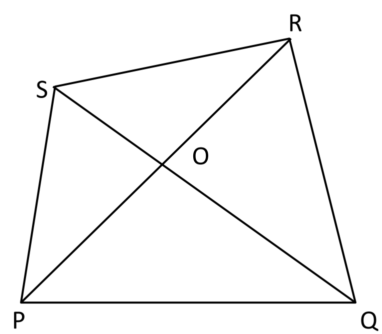
GIVEN: PQRS is a quadrilateral. PQ is its longest side and RS is its shortest side.
TO PROVE: (i) ∠R > ∠P (ii) ∠S > ∠Q
CONSTRUCTION: Join PR and QS.



PROOF: (i) Since PQ is the longest side of quadrilateral PQRS.
Therefore, in Δ PQR, we have
PQ > QR
=> ∠5 > ∠2 ... (i) [∵ Angle opp. to longer side is greater]
Since RS is the smallest side of quadrilateral PQRS.
Therefore, in Δ OSR, we have
PS = RS
=> ∠6 > ∠1 ... (ii) [∵ Angle opp. to longer side is greater]
Adding (i) and (ii), we get
∠5 + ∠6 > ∠2 + ∠1
=> ∠R > ∠P
(ii) In Δ PQS, we have
PQ > PS [∵ PQ is the longest side]
=> ∠8 > ∠3 ... (iii)
In Δ SRQ, we have
RQ > RS [∵ RS is the shortest side]
Adding (iii) and (iv), we get
∠8 + ∠7 > ∠3 + ∠4
=> ∠S > ∠Q Hence, ∠R > ∠P and ∠S > ∠Q.

Q. 19. In Fig. PQRS is a quadrilateral in which diagonals PR and QS intersect in O. Show that
(i) PQ + QR + RS + SP > PR + QS
(ii) PQ + QR + RS + SP < 2 (PR + QS)

Sol. (i) Since the sum of any two sides of a triangle is greater than the third side. Therefore,
In Δ PQR, we have , PQ + QR > PR ... (i)
In Δ RSP, we have ,RS + SP > PR ... (ii)
In Δ PQS, we have
PQ + SP > QS ... (iii)
In Δ QRS, we have
QR + RS > QS ... (iv)
Adding (i), (ii), (iii) and (iv), we get
2 (PQ + QR + RS + SP) > 2 (PR + QS)
=> PQ + QR + RS + SP > PR + QS

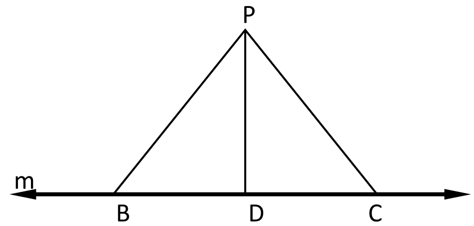


(ii) In Δ OPQ, we have
OP + OQ > PQ ... (v)
In Δ OQR, we have , OQ + OR > QR ... (vi)
In Δ ORS, we have , OR + OS > RS ... (vii)
In Δ OSP, we have , OS + OP > SP ... (viii)
Adding (v), (vi), (vii) and (viii), we get
2 (OP + OQ + OR + OS) > PQ + QR + RS + SP
=> 2 [(OP + OR) + (OQ + OS)] > PQ + QR + RS + SP
=> 2 (PR + QS) > PQ + QR + RS + SP [∵ OP + OR = PR and OQ + OS = QS]
=> PQ + QR + RS + SP < 2 (PR + QS)

Q. 20. Of all the line segments drawn from a point P to a line m not containing P, let PD be the shortest. If B and C are points on m such that D is the mid-point of BC, prove that PB = PC.

Sol. It is given that PD is the shortest line segment among all the line segments drawn from P to a line m not containing P.
Therefore, PD ⊥ m.

=> ∠PDB = ∠PDC = 90° ... (i)



It is also given that D is the mid-point of BC.

∴ BD = DC ... (ii)

Now, in Δ PBD and Δ PCD, we have

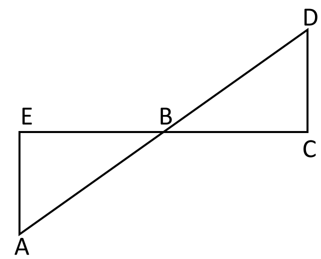
BD = DC [From (ii)]
∠PDB = ∠PDC = 90° [From (i)]
and, PD = PD [Common]

So, by SAS congruence criterion, we have

Δ PBD ≅ Δ PCD **Hence, PB = PC**

Q. 21. In Fig. ∠E > ∠A and ∠C > ∠D. Prove that AD > EC.

Sol. In Δ ABE, it is given that



∠E > ∠A
=> AB > BE ... (i)

In Δ BCD, it is given that

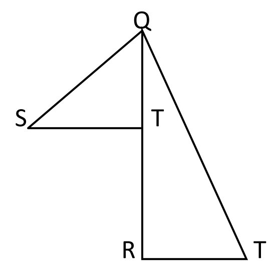
∠C > ∠D
=> BD > BC ... (ii)

Adding (i) and (ii), we get

AB + BD > BE + BC => AD > EC

Q. 22. In Fig. T is a point on side QR of Δ PQR and S is a point such that RT = ST. Prove that PQ + PR > QS.

Sol. In Δ PQR, we have



PQ + PR > QR
=> PQ + PR > QT + RT [∵ QR = QT + RT]
=> PQ + PR > QT + ST ... (i) [∵ RT = ST (Given)]

In Δ QST, we have

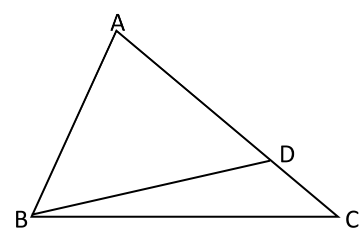
QT + ST > QS ... (ii)

From (i) and (ii), we have

PQ + PR > QS

Q. 23. In Fig. AC > AB and D is the point on AC such that AB = AD. Prove that BC > CD.

Sol. In Δ ABD, we have



AB = AD ... (i)

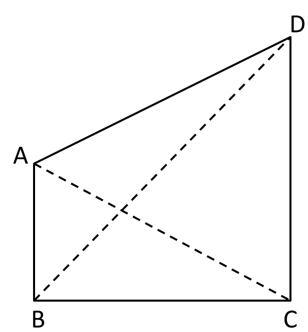
In Δ ABC, we have

AB + BC > AC
=> AB + BC > AD + CD
=> AB + BC > AB + CD [∵ AD = AB {from (i)}]
=> BC > CD

Q. 24. In Fig. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD. Show that ∠A > ∠C and ∠B > ∠D.

Sol. In Δ ABC, we have

BC > AB [∵ AB is the smallest side]
=> ∠BAC > ∠BCA ... (i)



In Δ ACD we gave

CD > AD [∵ CD is the largest side]
=> ∠CAD > ∠ACD ... (ii)

Adding (i) and (ii), we get

∠BAC + ∠CAD > ∠BCA + ∠ACD
=> ∠BAD > ∠BCD
=> ∠A > ∠C

In Δ ABD, we have

AD > AB [∵ AB is the smallest side]
=> ∠ABD > ∠ADB ... (iii)

In Δ BCD, we have

CD > BC [∵ CD is the largest side]
=> ∠DBC > ∠BDC ... (iv)

Adding (iii) and (iv), we get

$\angle ABD + \angle DBC > \angle ADB + \angle BDC$

=> $\angle ABC > \angle ADC$

=> $\angle B > \angle D$

Hence, $\angle A > \angle C$ and $\angle B > \angle D$.

Exercise 1.4..........

Q. 1. In Δ ABC, if $\angle A = 40°$ and $\angle B = 60°$. Determine the longest and shortest sides of the triangle.

Q. 2. In a Δ ABC, if $\angle B = \angle C = 45°$, which is the longest side?

Q. 3. In Δ ABC, side AB is produced to D so that BD = BC. If $\angle B = 60°$ and $\angle = 70°$, prove that: (i) AD > CD (ii) AD > AC

Q. 4. Is it possible to draw a triangle with sides of length 2 cm, 3 cm and 7 cm?

Q. 5. In Δ ABC, $\angle B = 35°$, $\angle C = 65°$ and the bisector of $\angle BAC$ meets BC in P. Arrange AP, BP and CP in descending order.

Q. 6. O is any point in the interior of Δ ABC, Prove that

(i) AB + AC > OB + OC (ii) AB + BC + CA > OA + OB + OC

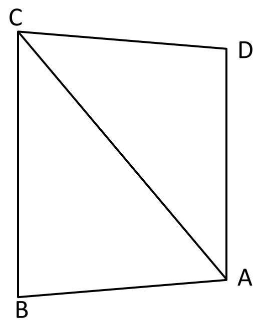
(iii) OA + OB + OC > ½ (AB + BC + CA)

Q. 7. Prove that the perimeter of a triangle is greater than the sum of its altitudes.

Q. 8. Prove that in a quadrilateral the sum of all the sides is greater than the sum of its diagonals.

Q. 9. In Fig., prove that:

(i) CD + DA + AB + BC > 2 AC (ii) CD + DA + AB > BC



Q. 10. Which of the following statements are true (T) and which are false (F)?

(i) Sum of the three sides of a triangle is less than the sum of its three altitudes.

(ii) Sum of any two sides of a triangle is greater than twice the median drawn to the third side.

(iii) Sum of any two sides of a triangle is greater than the third side.

(iv) Difference of any two sides of a triangle is equal to the third side.

(v) If two angles of a triangle are unequal, then the greater angle has the larger side opposite to it.

(vi) Of all the line segments that can be drawn from a point to a line not containing it, the perpendicular line segment is the shortest one.

Q. 11. Fill in the blanks to make the following statements true.

(i) In a right triangle the hypotenuse is the ……….. side.

(ii) The sum of three altitudes of a triangle is ……….. than its perimeter.

(iii) The sum of any two sides of a triangle is ………… than the third side.

(iv) If two angles of a triangle are unequal, then the smaller angle has the ………. side opposite to it.

(v) Difference of any two sides of a triangle is ………. than the third side.

(vi) If two sides of a triangle are unequal, then the larger side has ……… angle opposite to it.

Answers...............

1. Longest = AB, shortest = BC			2. BC		5. BP, AP, CP
10. (i) F	(ii) T	(iii) T	(iv) F	(v) T	(vi) T
11. (i) largest	(ii) less	(iii)greater	(iv) smaller	(v) less	(vi) greater

HINTS TO SELECTED PROBLEMS

3. We have, $\angle A = 70°$ and $\angle B = 60°$

So, $\angle C = 50°$; $\angle CBD = 120°$ and $\angle BDC = \angle DCB = 30°$

Now, $\angle ACD = 50° + 30° = 80°$, $\angle CAD = 70°$ and $\angle ADC = 30°$

∴ $\angle ACD > \angle CAD$ and $\angle ACD > \angle CDA$

=> AD > CD and AD > AC

4. A triangle can be drawn only when the sum of any two sides is greater than the third side.
Here, 2 + 3 $\not>$ 7. so, the triangle does not exist.

5. In Δ ACP, we have
∠ACP > ∠CAP
=> AP > CP ... (i)
In Δ ABP, we have
∠BAP > ∠ABP => BP > AP ... (ii)
from (i) and (ii), we have
BP > AP > CP

6. Produce BO to meet AC at D.
In Δ ABD, we have
AB + AD > BD
=> AB + AD > OB + OD ... (i)
In Δ ODC, we have
OD + DC > OC ... (ii)
Adding (i) and (ii), we get
AB + AD + OD + DC > OB + OD + OC
=> AB + AC > OB + OC
This proves (i)
Similarly, we have
BC + BA > OA + OC
and, CA + CB > OA + OB
Adding these three in equalities, we get
2 (AB + BC + CA) > 2 (OA + OB + OC)
=> AB + BC + CA > OA + OB + OC
This proves (ii)
In Δ's OAB, OBC and OCA, we have
OA + OB > AB, OB + OC > BC and OC + OA > AC
=> 2 (OA + OB + OC) > AB + BC + CA
=> OA + OB + OC > ½ (AB + BC + CA)

9. (i) in Δ ABC, we have
AB + BC > AC ... (i)
In Δ ACD, we have
AD + CD > AC ... (ii)
Adding (i) and (ii), we get
AB + BC + AD + CD > 2 AC

(ii) In Δ ACD, we have
CD + DA > CA
=> CD + DA + AB > CA + AB
=> CD + DA + AB > BC [∵ AB + AC > BC]

... END.