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TRIGONOMETRY

CBSE-X

CBSE-MATHEMATICS
TOPICWISE Revision Module

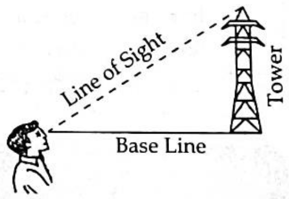
The word 'trigonometry' is derived from three Greek words 'Tri' means three, 'gon' means sides, 'metron' means measure. The literal meaning of trigonometry is measurement of triangle. In fact, it is the study of relations between the angles and sides of a triangle.

TOPICS

- Trigonometric Ratios
- Trigonometric Ratios of Some Specific Angles
- Trigonometric Identities

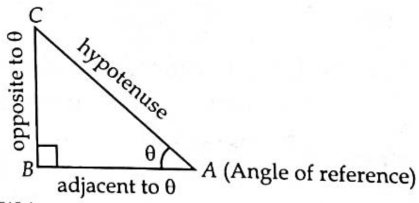
Use of Trigonometry

Sometimes we observe imaginary right triangle in nature, *e.g.*, if we look at the top of a tower then a right triangle can be imagined, as shown in figure.
In this situation, if we need to find the distances or heights then this can be done using trigonometry. Trigonometry also play an important role in the study of Physics, Engineering, Surveying, Navigation, etc.



TOPIC 1 : TRIGONOMETRIC RATIOS (T-RATIOS)

Here, we shall study about the ratios of the sides of a right triangle with respect to its acute angle, called **trigonometric ratios of the angle.**
Consider a right-angled ΔABC with $\angle B = 90°$.
Now, with respect to its acute $\angle A = \theta$ (say),
side BC = side opposite to θ is called perpendicular (P),
side AB = side adjacent to θ is called base (B),
and side AC = side opposite to 90° is called hypotenuse (H).



The trigonometric ratios of $\angle A$ *i.e.*, θ in the right-angled ΔABC are defined as below :

$\frac{BC}{AC} = \frac{\text{side opposite to } \theta}{\text{hypotenuse}}$ is called the sine of θ and written as $\sin\theta$.

$\frac{AB}{AC} = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}}$ is called the cosine of θ and written as $\cos\theta$.

$\frac{BC}{AB} = \frac{\text{side opposite to } \theta}{\text{side adjacent to } \theta}$ is called the tangent of θ and written as $\tan\theta$.

$\frac{AB}{BC} = \frac{\text{side adjacent to } \theta}{\text{side opposite to } \theta}$ is called the cotangent of θ and written as $\cot\theta$.

$\frac{AC}{AB} = \frac{\text{hypotenuse}}{\text{side adjacent to }\theta}$ is called the secant of θ and written as sec θ.

$\frac{AC}{BC} = \frac{\text{hypotenuse}}{\text{side opposite to }\theta}$ is called the cosecant of θ and written as cosec θ.

Thus, we have

$\sin\theta = P/H$	$\cos\theta = B/H$	$\tan\theta = P/B$
$\text{cosec}\,\theta = H/P$	$\sec\theta = H/B$	$\cot\theta = B/P$

Note : The ratios in row 2 are reciprocals of the corresponding ratios in row 1.

Similarly, if we consider $\angle C$ as angle of reference, we get

$\sin C = \frac{P}{H} = \frac{AB}{AC}$, $\cos C = \frac{B}{H} = \frac{BC}{AC}$, $\tan C = \frac{P}{B} = \frac{AB}{BC}$, $\text{cosec}\, C = \frac{H}{P} = \frac{AC}{AB}$, $\sec C = \frac{H}{B} = \frac{AC}{BC}$ and $\cot C = \frac{B}{P} = \frac{BC}{AB}$

POINTS TO REMEMBER

- Every trigonometric ratio is a real number and has no unit.
- $\sin A$ never means $\sin \times A$ *i.e.*, $\sin A \neq \sin \times A$. Similarly, in case of other trigonometric ratios.
- $\sin^k A = (\sin A)^k$ but $\sin (A^k) \neq (\sin A)^k$ where k is any natural number.
- The values of trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.
- If one of the trigonometric ratios of an acute angle is known, then the remaining trigonometric ratios of that angle can be easily determined.
- **Technique to Remember T-ratios**

  Some (sinθ) ⟶ People (P) ÷ Have (H)

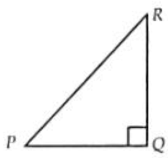
  Curly (cosθ) ⟶ Brown (B) ÷ Hair (H)

  To (tanθ) ⟶ Present (P) ÷ Beauty (B),

  where P is perpendicular, B is base and H is hypotenuse.
- **Pythagoras Theorem**

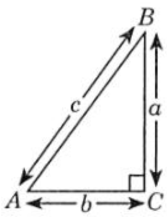
  In right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

  In ΔPQR, $(PR)^2 = (PQ)^2 + (QR)^2$



ILLUSTRATIONS

Q In the given triangle, indicate the lengths of the perpendicular (P), the hypotenuse (H) and the base (B) with respect to the angles A and B.



Sol. With respect to angle A,

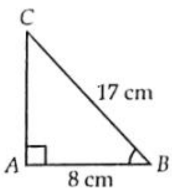
perpendicular = a, hypotenuse = c and base = b

With respect to angle B,

perpendicular = b, hypotenuse = c and base = a

Q In a ΔABC, right angled at A, if $AB = 8$ cm, $BC = 17$ cm, find all the trigonometric ratios of angle B.

Sol. We have $\angle A = 90°$, $AB = 8$ cm and $BC = 17$ cm. By Pythagoras theorem, we have



$BC^2 = AB^2 + AC^2$

$\Rightarrow \quad 17^2 = 8^2 + AC^2$

$\Rightarrow \quad AC^2 = 289 - 64 = 225$

$\Rightarrow \quad AC = 15$ cm

Now, with respect to $\angle B$,

$\sin B = \frac{AC}{BC} = \frac{15}{17}$, $\cos B = \frac{AB}{BC} = \frac{8}{17}$

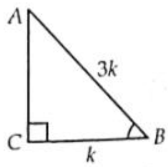
$\tan B = \frac{AC}{AB} = \frac{15}{8}$, $\cot B = \frac{AB}{AC} = \frac{8}{15}$

$\sec B = \frac{BC}{AB} = \frac{17}{8}$ and $\text{cosec}\, B = \frac{BC}{AC} = \frac{17}{15}$

Q If $\cos B = \frac{1}{3}$, find the other five trigonometric ratios.

Sol. We have, $\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{1}{3}$

Let us draw a triangle ABC, right-angled at C such that base $= BC = k$ units and hypotenuse $= AB = 3k$ units, where k is a positive number.



By Pythagoras theorem, we have $AB^2 = BC^2 + AC^2$

$\Rightarrow \quad (3k)^2 = k^2 + AC^2 \Rightarrow AC^2 = 9k^2 - k^2 = 8k^2$

$\Rightarrow \quad AC = \sqrt{8}k = 2\sqrt{2}\, k$ units

Now, we have

Base $(B) = BC = k$, perpendicular $(P) = AC = 2\sqrt{2}\,k$ and hypotenuse $(H) = AB = 3k$

$\therefore \quad \sin B = \dfrac{P}{H} = \dfrac{2\sqrt{2}k}{3k} = \dfrac{2\sqrt{2}}{3}$,

$\tan B = \dfrac{P}{B} = \dfrac{2\sqrt{2}k}{k} = \dfrac{2\sqrt{2}}{1} = 2\sqrt{2}$,

$\text{cosec } B = \dfrac{H}{P} = \dfrac{3k}{2\sqrt{2}\,k} = \dfrac{3}{2\sqrt{2}}$,

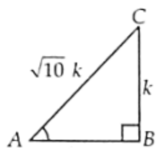
$\sec B = \dfrac{H}{B} = \dfrac{3k}{k} = \dfrac{3}{1} = 3$ and $\cot B = \dfrac{B}{P} = \dfrac{k}{2\sqrt{2}\,k} = \dfrac{1}{2\sqrt{2}}$

Q If $\sin A = 1/\sqrt{10}$, then find value of

(i) $\sin^2 A + \cos^2 A$, (ii) $\dfrac{\tan A + \cot A}{3\sin A + 2\cos A}$.

Sol. We have, $\sin A = \dfrac{1}{\sqrt{10}} = \dfrac{\text{perpendicular}}{\text{hypotenuse}}$

Let us draw a ΔABC, right-angled at B such that perpendicular $= BC = k$ units and hypotenuse $= AC = \sqrt{10}\,k$ units, where k is a positive number.



By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2 \Rightarrow 10k^2 = AB^2 + k^2$

$\Rightarrow \quad AB^2 = 10k^2 - k^2 = 9k^2 \Rightarrow AB = 3k$ units

Now, $\cos A = \dfrac{AB}{AC} = \dfrac{3k}{\sqrt{10}k} = \dfrac{3}{\sqrt{10}}$,

$\tan A = \dfrac{BC}{AB} = \dfrac{k}{3k} = \dfrac{1}{3}$ and $\cot A = \dfrac{AB}{BC} = \dfrac{3k}{k} = 3$

(i) $\sin^2 A + \cos^2 A = (\sin A)^2 + (\cos A)^2$

$= \left(\dfrac{1}{\sqrt{10}}\right)^2 + \left(\dfrac{3}{\sqrt{10}}\right)^2 = \dfrac{1}{10} + \dfrac{9}{10} = \dfrac{10}{10} = 1$

(ii) $\dfrac{\tan A + \cot A}{3\sin A + 2\cos A} = \dfrac{\frac{1}{3} + 3}{3\left(\frac{1}{\sqrt{10}}\right) + 2\left(\frac{3}{\sqrt{10}}\right)}$

$= \dfrac{10/3}{9/\sqrt{10}} = \dfrac{10\sqrt{10}}{27}$

Q If $\sin\alpha = 1/2$, then find the value of $3\sin\alpha - 4\sin^3\alpha$. *(CBSE 2017)*

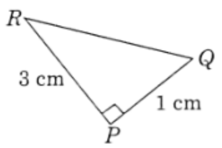
Sol. We have, $\sin\alpha = 1/2$

Now, $3\sin\alpha - 4\sin^3\alpha$

$= 3\left(\dfrac{1}{2}\right) - 4\left(\dfrac{1}{2}\right)^3 = \dfrac{3}{2} - \dfrac{4}{8} = \dfrac{3}{2} - \dfrac{1}{2} = 1$

TRY YOURSELF

1. In the given triangle PQR, find $\cos R$, $\cos Q$, $\cot Q$, $\sec R$ and $\tan R$.



$\left(\textbf{Ans. } \cos R = \dfrac{3}{\sqrt{10}}, \cos Q = \dfrac{1}{\sqrt{10}}, \cot Q = \dfrac{1}{3}, \sec R = \dfrac{\sqrt{10}}{3} \text{ and } \tan R = \dfrac{1}{3}\right)$

2. In a ΔABC, right angled at A; $AB = AC = 1$ unit, write down all the T-ratios of angle B.

(**Ans.** $\sin B = \dfrac{1}{\sqrt{2}}$, $\cos B = \dfrac{1}{\sqrt{2}}$, $\tan B = 1$, $\sec B = \sqrt{2}$, $\text{cosec } B = \sqrt{2}$ and $\cot B = 1$)

3. In ΔABC, $\tan A = 4/3$, find the other trigonometric ratios of angle A.

(**Ans.** $\sin A = 4/5$, $\cos A = 3/5$, $\cot A = 3/4$, $\text{cosec } A = 5/4$ and $\sec A = 5/3$)

4. Consider ΔACB, right-angled at C, in which $AB = 29$ units, $BC = 21$ units and $\angle ABC = \theta$. Determine the values of

(i) $\cos^2\theta + \sin^2\theta$ (ii) $\cos^2\theta - \sin^2\theta$ (**Ans.** (i) 1 (ii) 41/841)

5. If $\sec\alpha = \dfrac{5}{4}$, then evaluate $\dfrac{1 - \tan\alpha}{1 + \tan\alpha}$.

(**Ans.** 1/7)

Fundamental Relation between Trigonometric Ratios

(i) $\text{cosec }\theta = \dfrac{1}{\sin\theta}$ or $\sin\theta = \dfrac{1}{\text{cosec}\theta}$

(ii) $\sec\theta = \dfrac{1}{\cos\theta}$ or $\cos\theta = \dfrac{1}{\sec\theta}$

(iii) $\cot\theta = \dfrac{1}{\tan\theta}$ or $\tan\theta = \dfrac{1}{\cot\theta}$

(iv) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

ILLUSTRATIONS

Q If $\sin\theta = 3/5$ and $\cos\theta = 4/5$, then evaluate all other trigonometric ratios of θ.

Sol. We have, $\sin\theta = 3/5$ and $\cos\theta = 4/5$

We know that, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$$\Rightarrow \quad \tan\theta = \frac{3/5}{4/5} = \frac{3}{5}\times\frac{5}{4} = \frac{3}{4},$$

Also, $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{\left(\frac{3}{4}\right)} = \dfrac{4}{3},$

Similarly, $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\left(\frac{4}{5}\right)} = \dfrac{5}{4}$ and

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

Q Find the value of $\dfrac{\sin A + \cos A}{\sin A - \cos A}$, if $16\cot A = 12$.

Sol. We have, $16\cot A = 12 \Rightarrow \cot A = \dfrac{12}{16} = \dfrac{3}{4}$

Now, $\dfrac{\sin A + \cos A}{\sin A - \cos A} = \dfrac{\dfrac{\sin A}{\sin A} + \dfrac{\cos A}{\sin A}}{\dfrac{\sin A}{\sin A} - \dfrac{\cos A}{\sin A}}$

(Dividing both numerator and denominator by sin A)

$$= \frac{1+\cot A}{1-\cot A} = \frac{1+\frac{3}{4}}{1-\frac{3}{4}} = \frac{\frac{7}{4}}{\frac{1}{4}} = 7$$

Q Show that $\dfrac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \dfrac{3}{4}$, if $\tan\theta = \dfrac{1}{\sqrt{7}}$.

Sol. L.H.S. $= \dfrac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \dfrac{\left(\dfrac{1}{\sin^2\theta} - \dfrac{1}{\cos^2\theta}\right)}{\dfrac{1}{\sin^2\theta} + \dfrac{1}{\cos^2\theta}}$

$$= \frac{\left(\dfrac{\cos^2\theta - \sin^2\theta}{\sin^2\theta\cos^2\theta}\right)}{\left(\dfrac{\cos^2\theta + \sin^2\theta}{\sin^2\theta\cos^2\theta}\right)}$$

$$= \left(\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta\cos^2\theta}\right)\times\left(\frac{\sin^2\theta\cos^2\theta}{\cos^2\theta + \sin^2\theta}\right)$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \frac{\dfrac{\cos^2\theta}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}}$$

[Dividing both numerator and denominator by $\cos^2\theta$]

$$= \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-1/7}{1+1/7}$$

$$\left[\because \tan\theta = \frac{1}{\sqrt{7}} \Rightarrow (\tan\theta)^2 = \tan^2\theta = \frac{1}{7}\right]$$

$$= \frac{6/7}{8/7} = \frac{6}{8} = \frac{3}{4} = \text{R.H.S.}$$

Q Prove that $\dfrac{\sec^2\theta - \sin^2\theta}{\tan^2\theta} = \operatorname{cosec}^2\theta - \cos^2\theta$.

Sol. L.H.S. $= \dfrac{\sec^2\theta - \sin^2\theta}{\tan^2\theta} = \dfrac{\dfrac{1}{\cos^2\theta} - \sin^2\theta}{\dfrac{\sin^2\theta}{\cos^2\theta}}$

$$= \frac{\dfrac{1-\cos^2\theta\sin^2\theta}{\cos^2\theta}}{\dfrac{\sin^2\theta}{\cos^2\theta}} = \frac{1-\cos^2\theta\sin^2\theta}{\cos^2\theta}\times\frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{1-\cos^2\theta\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} - \frac{\cos^2\theta\sin^2\theta}{\sin^2\theta}$$

$= \operatorname{cosec}^2\theta - \cos^2\theta =$ R.H.S.

TRY YOURSELF

6. If $m\cot A = n$, then find the value of $\dfrac{m\sin A - n\cos A}{n\cos A + m\sin A}$. $\left(\textbf{Ans.}\ \dfrac{m^2-n^2}{m^2+n^2}\right)$

7. If $3\tan A = 4$, then prove that $\sqrt{\dfrac{\sec A - \operatorname{cosec} A}{\sec A + \operatorname{cosec} A}} = \dfrac{1}{\sqrt{7}}$.

8. Prove that $\dfrac{\operatorname{cosec}^2\theta - \cos^2\theta}{\cot^2\theta} = \sec^2\theta - \sin^2\theta$.

9. If $5\cos\theta = 7\sin\theta$, find the value of

$\dfrac{7\sin\theta + 5\cos\theta}{5\sin\theta + 7\cos\theta}$ **(Ans. 35/37)**

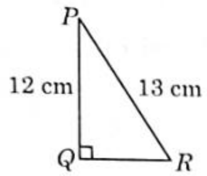
NCERT FOCUS

EXERCISE - 8.1

1. In ΔABC, right-angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine :

(i) $\sin A, \cos A$ (ii) $\sin C, \cos C$

2. In the figure, find $\tan P - \cot R$.



3. If $\sin A = \frac{3}{4}$, then calculate $\cos A$ and $\tan A$.

4. Given $15 \cot A = 8$, find $\sin A$ and $\sec A$.

5. Given $\sec \theta = 13/12$, calculate all other trigonometric ratios.

6. If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

7. If $\cot \theta = 7/8$; evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$

8. If $3 \cot A = 4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

9. In triangle ABC, right angled at B, if $\tan A = 1/\sqrt{3}$, find the value of :

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

10. In ΔPQR, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

11. State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = 12/5$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

(iv) $\cot A$ is the product of cot and A.

(v) $\sin \theta = 4/3$ for some angle θ.

CBSE FOCUS

Objective Type Questions

MCQs (1 Mark)

1. If $2 \tan A = 3$, then the value of $\frac{4\sin A + 3\cos A}{4\sin A - 3\cos A}$ is

(a) $\frac{7}{\sqrt{13}}$ (b) $\frac{1}{\sqrt{13}}$

(c) 3 (d) does not exit

(CBSE 2023)

2. Given that $\cos \theta = \frac{\sqrt{3}}{2}$, then the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$ is

(a) -1 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(CBSE Term I, Standard 2021-22)

3. $\frac{1}{\text{cosec}\,\theta(1-\cot\theta)} + \frac{1}{\sec\theta(1-\tan\theta)}$ is equal to

(a) 0 (b) 1

(c) $\sin\theta + \cos\theta$ (d) $\sin\theta - \cos\theta$

(CBSE Term I, Standard 2021-22)

Fill in the Blanks (1 Mark)

4. If $\tan A = 4/3$, then $\cot^2 A$ = _______.

5. If $\cos A = 3/4$, then $\text{cosec } A$ = _______.

VSA Type Questions (1 Mark)

6. In a ΔABC, right-angled at A, if $AB = 5$ cm, $AC = 12$ cm and $BC = 13$ cm, find $\sin B$ and $\cos C$.

7. If $6\cos\theta - 5\sin\theta = 4\sin\theta + \cos\theta$, then find the value of $\cot\theta$.

Short Answer Type Questions

SA Type I Questions (2 Marks)

8. If 15 cot A = 8, then find the value of $\frac{8\operatorname{cosec} A}{5}$.

9. If sin θ = a/b, then find the values of tan θ and sec θ.

10. If $\sin\alpha = \frac{1}{2}$, then show that

$(3\cos\alpha - 4\cos^3\alpha) = 0$.

11. If $\tan\theta = \frac{1}{\sqrt{3}}$, then prove that

$7\sin^2\theta + 3\cos^2\theta = 4$.

SA Type II Questions (3 Marks)

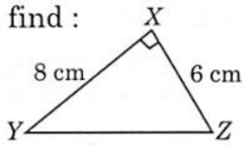
12. For the given triangle XYZ, find :

(i) sin Y

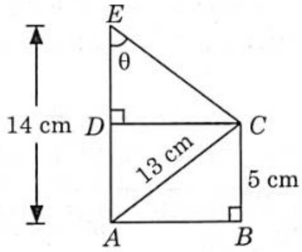
(ii) cos Y

(iii) tan Y

(iv) sec Z



13. In figure, if AE = 14 cm, AC = 13 cm, BC = 5 cm,then find the value of tanθ.



14. If $\operatorname{cosec}\theta = \frac{29}{20}$, then find the value of $\frac{42\tan\theta + 58\sin\theta}{10\cot\theta}$.

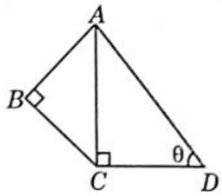
15. If θ is an acute angle and $\sec\theta = \sqrt{10}$, then evaluate : (i) cosecθ – cotθ (ii) tanθ – cotθ.

16. If $\cot\theta = \frac{8}{15}$, then evaluate $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta}$.

Long Answer Type Questions

LA Type Questions (5 Marks)

17. In the adjoining figure AB = 12 cm, BC = 16 cm and CD = 21 cm, then find the value of all trigonometric ratios.



18. In a ΔABC, right angled at A, if $\tan B = \sqrt{3}$, then find the value of sin$B$ cos$C$ + cos$B$ sin$C$.

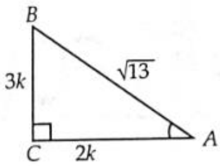
19. In ΔABC, ∠B = 90°, AB = 8 cm and AC = 10 cm. If CD bisects AB and ∠DCB = θ, then find : (i) sinθ (ii) cosθ (iii) tanθ (iv) $\sin^2\theta + \cos^2\theta$.

20. In ΔPQR, ∠Q = 90°, PQ = 8 cm, PR – QR = 2 cm. Find the lengths of PR and QR. Also, evaluate $\frac{1+\sin P}{1+\cos P}$.

SOLUTIONS

1. **(c)** : We have, 2 tan A = 3

$\Rightarrow \tan A = \frac{3}{2} = \frac{P}{B}$



Let $P = 3k$ and $B = 2k$

$AB = \sqrt{2^2 + 3^2}$

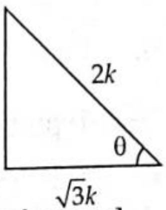
(By Pythagoras theorem)

$\Rightarrow H = \sqrt{13}$

$\therefore \sin A = \frac{P}{H} = \frac{3}{\sqrt{13}}, \cos A = \frac{B}{H} = \frac{2}{\sqrt{13}}$

Now, $\frac{4\sin A + 3\cos A}{4\sin A - 3\cos A} = \frac{4\left(\frac{3}{\sqrt{13}}\right) + 3\left(\frac{2}{\sqrt{13}}\right)}{4\left(\frac{3}{\sqrt{13}}\right) - 3\left(\frac{2}{\sqrt{13}}\right)} = 3$

2. **(c)** : Given, $\cos\theta = \frac{\sqrt{3}}{2} = \frac{B}{H}$



Let $B = \sqrt{3}k$ and $H = 2k$

$\therefore P = \sqrt{(2k)^2 - (\sqrt{3}k)^2}$ [By Pythagoras theorem]

$\Rightarrow P = \sqrt{k^2} = k$

$\therefore \operatorname{cosec}\theta = \frac{H}{P} = \frac{2k}{k} = 2, \quad \sec\theta = \frac{H}{B} = \frac{2k}{\sqrt{3}k} = \frac{2}{\sqrt{3}}$

$\therefore \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{(2)^2 - \left(\frac{2}{\sqrt{3}}\right)^2}{(2)^2 + \left(\frac{2}{\sqrt{3}}\right)^2} = \frac{4 - \frac{4}{3}}{4 + \frac{4}{3}} = \frac{8}{16} = \frac{1}{2}$

3. **(c)** : We have, $\frac{1}{\operatorname{cosec}\theta(1-\cot\theta)} + \frac{1}{\sec\theta(1-\tan\theta)}$

$= \frac{\sin\theta}{1 - \frac{\cos\theta}{\sin\theta}} + \frac{\cos\theta}{1 - \frac{\sin\theta}{\cos\theta}}$

$\left[\because \frac{1}{\operatorname{cosec}\theta} = \sin\theta, \frac{1}{\sec\theta} = \cos\theta, \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

$= \frac{\sin^2\theta}{\sin\theta - \cos\theta} + \frac{\cos^2\theta}{\cos\theta - \sin\theta}$

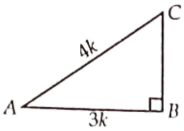
$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta} = \sin\theta + \cos\theta$

4. We have, $\tan A = 4/3$

Now, $\cot A = \dfrac{1}{\tan A} = \dfrac{1}{4/3} = \dfrac{3}{4}$ $\quad\therefore\ \cot^2 A = \left(\dfrac{3}{4}\right)^2 = \dfrac{9}{16}$

5. We have, $\cos A = 3/4$

Consider a ΔABC, right angled at B such that $AB = 3k$ units and $AC = 4k$ units.



$\therefore$ By Pythagoras theorem, we have

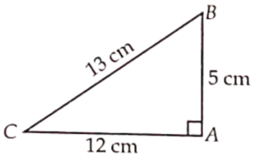
$AC^2 = BC^2 + AB^2 \Rightarrow (4k)^2 = (3k)^2 + BC^2$

$\Rightarrow BC^2 = 16k^2 - 9k^2 = 7k^2 \Rightarrow BC = \sqrt{7}k$ units.

$\therefore\ \operatorname{cosec} A = \dfrac{AC}{BC} = \dfrac{4k}{\sqrt{7}k} = \dfrac{4}{\sqrt{7}}$

6. In ΔABC, $\angle A = 90°$

$\therefore\ \sin B = \dfrac{AC}{BC} = \dfrac{12}{13}$



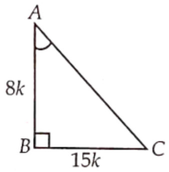
Also, $\cos C = \dfrac{AC}{BC} = \dfrac{12}{13}$

7. Given, $6\cos\theta - 5\sin\theta = 4\sin\theta + \cos\theta$

$\Rightarrow\ 5\cos\theta = 9\sin\theta \Rightarrow \dfrac{\cos\theta}{\sin\theta} = \dfrac{9}{5} \Rightarrow \cot\theta = \dfrac{9}{5}$

8. Consider a right triangle ABC, such that $\angle B = 90°$

Now, $15 \cot A = 8$ (Given)



$\Rightarrow\ \cot A = \dfrac{8}{15} \Rightarrow \dfrac{AB}{BC} = \dfrac{8}{15}$

Let $AB = 8k$ units and $BC = 15k$ units

Using Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2 = (8k)^2 + (15k)^2$

$= 64k^2 + 225k^2 = 289k^2 = (17k)^2$

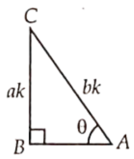
$\Rightarrow\ AC = \sqrt{(17k)^2} = 17k$ units

$\therefore\ \operatorname{cosec} A = \dfrac{AC}{BC} = \dfrac{17k}{15k} = \dfrac{17}{15}$

Thus, $\dfrac{8\operatorname{cosec} A}{5} = \dfrac{8}{5} \times \dfrac{17}{15} = \dfrac{136}{75}$

9. Consider a ΔABC in which $\angle B = 90°$ and $\angle BAC = \theta$.

Given, $\sin\theta = \dfrac{a}{b} = \dfrac{BC}{AC}$



Let $BC = ak$ units and $AC = bk$ units

Using Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2 \Rightarrow b^2k^2 = AB^2 + a^2k^2$

$\Rightarrow\ AB^2 = b^2k^2 - a^2k^2$

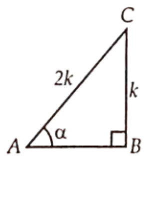
$\Rightarrow\ AB = \sqrt{b^2 - a^2}\,k$ units

$\therefore\ \tan\theta = \dfrac{BC}{AB} = \dfrac{a}{\sqrt{b^2 - a^2}}$ and $\sec\theta = \dfrac{AC}{AB} = \dfrac{b}{\sqrt{b^2 - a^2}}$

10. Consider a ΔABC in which $\angle B = 90°$ and $\angle BAC = \alpha$.

$\therefore\ \sin\alpha = \dfrac{BC}{AC} = \dfrac{1}{2}$

Let $BC = k$ units and $AC = 2k$ units, where k is a positive number.



By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow\ AB^2 = AC^2 - BC^2 = 4k^2 - k^2 = 3k^2$

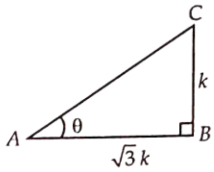
$\Rightarrow\ AB = \sqrt{3}k$ units

$\therefore\ \cos\alpha = \dfrac{AB}{AC} = \dfrac{\sqrt{3}k}{2k} = \dfrac{\sqrt{3}}{2}$

Now, L.H.S. $= (3\cos\alpha - 4\cos^3\alpha) = \dfrac{3\sqrt{3}}{2} - 4 \times \dfrac{3\sqrt{3}}{8} = 0 =$ R.H.S.

11. Consider a ΔABC in which $\angle B = 90°$ and $\angle BAC = \theta$.

We have, $\tan\theta = \dfrac{1}{\sqrt{3}}$ (Given)



Also, $\tan\theta = \dfrac{BC}{AB}$

$\therefore\ \dfrac{1}{\sqrt{3}} = \dfrac{BC}{AB}$

Let $BC = k$ units and $AB = \sqrt{3}k$ units

Using Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow\ AC^2 = (\sqrt{3}k)^2 + k^2 = 3k^2 + k^2 = 4k^2 \Rightarrow AC = 2k$

$\therefore\ \sin\theta = \dfrac{BC}{AC} = \dfrac{k}{2k} = \dfrac{1}{2}$, $\cos\theta = \dfrac{AB}{AC} = \dfrac{\sqrt{3}k}{2k} = \dfrac{\sqrt{3}}{2}$

Now, $7\sin^2\theta + 3\cos^2\theta = 7\left(\dfrac{1}{2}\right)^2 + 3\left(\dfrac{\sqrt{3}}{2}\right)^2 = \dfrac{7}{4} + \dfrac{9}{4} = \dfrac{16}{4} = 4$

Hence Proved.

12. Given, $\angle YXZ = 90°$, $XY = 8$ cm and $XZ = 6$ cm

Now, in ΔXYZ, $YZ^2 = XY^2 + XZ^2$ [Using Pythagoras theorem]

$\Rightarrow\ YZ^2 = 8^2 + 6^2 = 64 + 36 = 100$

$\Rightarrow\ YZ = \sqrt{100} = 10$ cm

(i) $\sin Y = \dfrac{XZ}{YZ} = \dfrac{6}{10} = \dfrac{3}{5}$ $\quad$ (ii) $\cos Y = \dfrac{XY}{YZ} = \dfrac{8}{10} = \dfrac{4}{5}$

(iii) $\tan Y = \dfrac{XZ}{XY} = \dfrac{6}{8} = \dfrac{3}{4}$ $\quad$ (iv) $\sec Z = \dfrac{YZ}{XZ} = \dfrac{10}{6} = \dfrac{5}{3}$

13. In right angle ΔABC, we have

$AC^2 = AB^2 + BC^2$ [Using Pythagoras theorem]

$\Rightarrow\ 13^2 = AB^2 + (5)^2 \Rightarrow AB^2 = 169 - 25 = 144$

$\Rightarrow\ AB = 12$ cm $= DC$

Also, $EA = 14$ cm

$\Rightarrow\ ED + DA = 14 \Rightarrow ED = 14 - 5 = 9$ cm ($\because\ DA = BC$)

Now, in right angle ΔCDE, we have

$\tan\theta = \dfrac{DC}{ED} \Rightarrow \tan\theta = \dfrac{12}{9} = \dfrac{4}{3}$

14. Consider a right ΔABC in which $\angle B = 90°$ and $\angle A = \theta$.

Given, $\operatorname{cosec}\theta = \dfrac{29}{20} = \dfrac{AC}{BC}$

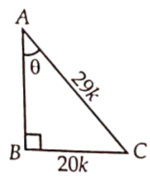
Let $AC = 29k$ units and $BC = 20k$ units

Using Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow\ AB^2 = AC^2 - BC^2 = (29k)^2 - (20k)^2$

$= 841k^2 - 400k^2 = 441k^2$



$\Rightarrow\ AB = \sqrt{441k^2} = 21k$ units

Now, $\sin\theta = \dfrac{BC}{AC} = \dfrac{20k}{29k} = \dfrac{20}{29}$,

$\tan\theta = \dfrac{BC}{AB} = \dfrac{20k}{21k} = \dfrac{20}{21}$ and $\cot\theta = \dfrac{AB}{BC} = \dfrac{21k}{20k} = \dfrac{21}{20}$

Now, $\dfrac{42\tan\theta + 58\sin\theta}{10\cot\theta} = \dfrac{42 \times \dfrac{20}{21} + 58 \times \dfrac{20}{29}}{10 \times \dfrac{21}{20}}$

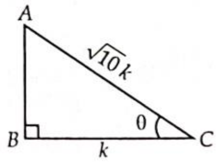
$= \dfrac{2\times 20+2\times 20}{\frac{21}{2}} = \dfrac{2(40+40)}{21} = \dfrac{160}{21}$

15. Consider a ΔABC in which $\angle B = 90°$ and $\angle C = \theta$.

Given, $\sec\theta = \sqrt{10}$

Also, $\sec\theta = \dfrac{AC}{BC}$

$\therefore \quad \dfrac{\sqrt{10}}{1} = \dfrac{AC}{BC}$



Let $AC = \sqrt{10}\,k$ units and $BC = k$ units

Using Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2 \Rightarrow \left(\sqrt{10}\,k\right)^2 = AB^2 + k^2$

$\Rightarrow \quad AB^2 = 10k^2 - k^2 = 9k^2 = (3k)^2 \Rightarrow AB = 3k$ units

$\therefore \quad \operatorname{cosec}\theta = \dfrac{AC}{AB} = \dfrac{\sqrt{10}k}{3k} = \dfrac{\sqrt{10}}{3},$

$\cot\theta = \dfrac{BC}{AB} = \dfrac{k}{3k} = \dfrac{1}{3}$ and $\tan\theta = \dfrac{AB}{BC} = \dfrac{3k}{k} = 3$

(i) $\quad \operatorname{cosec}\theta - \cot\theta = \dfrac{\sqrt{10}}{3} - \dfrac{1}{3} = \dfrac{\sqrt{10}-1}{3}$

(ii) $\quad \tan\theta - \cot\theta = 3 - \dfrac{1}{3} = \dfrac{9-1}{3} = \dfrac{8}{3}$

16. Given, $\cot\theta = 8/15$

Now, $\dfrac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} = \dfrac{\dfrac{\sin^2\theta}{\sin^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta}}{\dfrac{2\sin\theta\cos\theta}{\sin^2\theta}}$

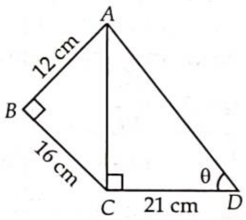
(Dividing both numerator and denominator by $\sin^2\theta$)

$= \dfrac{1-\cot^2\theta}{2\cot\theta} \quad \left(\because \dfrac{\cos\theta}{\sin\theta} = \cot\theta\right)$

$= \dfrac{1-(8/15)^2}{2(8/15)} = \left(1 - \dfrac{64}{225}\right)\times\dfrac{15}{16} = \dfrac{161}{225}\times\dfrac{15}{16} = \dfrac{161}{240}$

17. Given, $AB = 12$ cm, $BC = 16$ cm and $CD = 21$ cm.

Using Pythagoras theorem in ΔABC, we have



$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC^2 = (12)^2 + (16)^2$

$= 144 + 256 = 400 = 20^2$

$\Rightarrow \quad AC = 20$ cm $\quad$ ... (i)

Again using Pythagoras theorem in ΔACD, we have

$AD^2 = AC^2 + CD^2 \Rightarrow AD^2 = (20)^2 + (21)^2$ $\quad$ [Using (i)]

$\Rightarrow \quad AD^2 = 400 + 441 = 841 = 29^2 \Rightarrow AD = 29$ cm

$\therefore \quad \sin\theta = \dfrac{AC}{AD} = \dfrac{20}{29}, \ \cos\theta = \dfrac{CD}{AD} = \dfrac{21}{29}$

Now, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{20/29}{21/29} = \dfrac{20}{21}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{21}{20}; \ \sec\theta = \dfrac{1}{\cos\theta} = \dfrac{29}{21};$

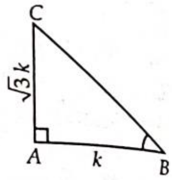
$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{29}{20}$

18. In right ΔABC, $\angle A = 90°$ and $\tan B = \sqrt{3}$

$\therefore \quad \tan B = \dfrac{\sqrt{3}}{1} = \dfrac{AC}{AB}$

Let $AC = \sqrt{3}k$ units and $AB = k$ units

Using Pythagoras theorem, we get



$BC^2 = AC^2 + AB^2$

$\Rightarrow \quad BC^2 = 3k^2 + k^2 = 4k^2 = (2k)^2$

$\Rightarrow \quad BC = 2k$ units

$\therefore \quad \sin B = \dfrac{AC}{BC} = \dfrac{\sqrt{3}k}{2k} = \dfrac{\sqrt{3}}{2},$

$\cos B = \dfrac{AB}{BC} = \dfrac{k}{2k} = \dfrac{1}{2}$

$\sin C = \dfrac{AB}{BC} = \dfrac{k}{2k} = \dfrac{1}{2}, \cos C = \dfrac{AC}{BC} = \dfrac{\sqrt{3}k}{2k} = \dfrac{\sqrt{3}}{2}$

$\sin B\cos C + \cos B\sin C$

$= \dfrac{\sqrt{3}}{2}\times\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\times\dfrac{1}{2} = \dfrac{3}{4}+\dfrac{1}{4} = \dfrac{4}{4} = 1$

19. Given, $AB = 8$ cm, $AC = 10$ cm and $\angle B = 90°$.

In right ΔABC, $AC^2 = AB^2 + BC^2$

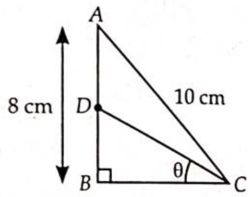
[Using Pythagoras theorem]

$\Rightarrow \quad BC^2 = 10^2 - 8^2 = 100 - 64 = 36 = 6^2$

$\Rightarrow \quad BC = 6$ cm

Also, CD bisects AB

$\therefore \quad BD = \dfrac{AB}{2} = \dfrac{8}{2} = 4\,\text{cm}$



In right ΔDBC, $DC^2 = BD^2 + BC^2$

$\Rightarrow \quad DC^2 = 4^2 + 6^2 = 16 + 36$

$= 52 = \left(2\sqrt{13}\right)^2$

$\Rightarrow \quad DC = 2\sqrt{13}$ cm

(i) $\quad \sin\theta = \dfrac{BD}{DC} = \dfrac{4}{2\sqrt{13}} = \dfrac{2}{\sqrt{13}} = \dfrac{2\sqrt{13}}{13}$

(ii) $\quad \cos\theta = \dfrac{BC}{DC} = \dfrac{6}{2\sqrt{13}} = \dfrac{3}{\sqrt{13}} = \dfrac{3\sqrt{13}}{13}$

(iii) $\quad \tan\theta = \dfrac{BD}{BC} = \dfrac{4}{6} = \dfrac{2}{3}$

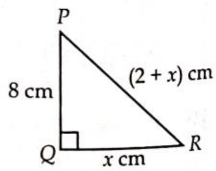
(iv) $\quad \sin^2\theta+\cos^2\theta = \left(\dfrac{2}{\sqrt{13}}\right)^2 + \left(\dfrac{3}{\sqrt{13}}\right)^2 = \dfrac{4}{13}+\dfrac{9}{13} = \dfrac{13}{13} = 1$

20. In ΔPQR, $PQ = 8$ cm, $\angle Q = 90°$ and $PR - QR = 2$ cm.

Let $QR = x$ cm

$\therefore \quad PR = (2+x)$ cm

Using Pythagoras theorem, we have



$PR^2 = PQ^2 + QR^2$

$\Rightarrow \quad (2+x)^2 = 8^2 + x^2$

$\Rightarrow \quad 4 + x^2 + 4x = 64 + x^2$

$\Rightarrow \quad 4x = 60$

$\Rightarrow \quad x = 15$

i.e., $\quad QR = 15$ cm

So, $\quad PR = 2 + 15 = 17$ cm

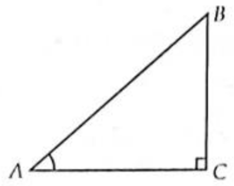
$\therefore \quad \sin P = \dfrac{QR}{PR} = \dfrac{15}{17}, \cos P = \dfrac{PQ}{PR} = \dfrac{8}{17}$

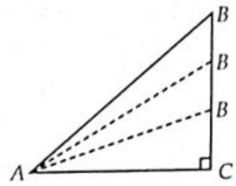
$\therefore \quad \dfrac{1+\sin P}{1+\cos P} = \dfrac{1+\dfrac{15}{17}}{1+\dfrac{8}{17}} = \dfrac{32}{25}$

TOPIC 2 : TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES

Trigonometric Ratios of 0°

Consider a right angle ΔABC, right-angled at C. Now, if $\angle A$ is made smaller and smaller in the right ΔABC, till it becomes zero, then the side BC also decreases and point B gets closer and closer to the point C. Finally when $\angle A$ becomes very close to 0°, the point B will almost coincide to point C and AB becomes almost equal to AC.





So, when $\angle A = 0°$, then

$$\sin A = \sin 0° = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AB} = \frac{0}{AC} = 0 \qquad [\because AB = AC]$$

Similarly, $\cos 0° = \frac{AC}{AB} = \frac{AC}{AC} = 1\ [\because AB = AC]$, $\tan 0° = \frac{BC}{AC} = \frac{0}{AC} = 0$,

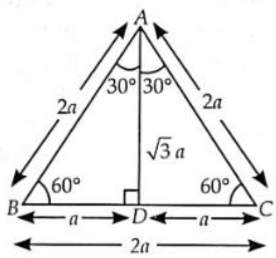
$\cot 0° = \frac{1}{\tan 0°} = \frac{1}{0} = \text{Not defined}$, $\sec 0° = \frac{1}{\cos 0°} = \frac{1}{1} = 1$ and $\text{cosec } 0° = \frac{1}{\sin 0°} = \frac{1}{0} = \text{Not defined}$.

Trigonometric Ratios of 30°, 45° and 60°

Consider an equilateral ΔABC in which each side is of length $2a$ units. Note that each angle of ΔABC will be of 60°.
Draw AD perpendicular to BC.
Since, ABC is an equilateral triangle.

$\therefore$ AD is the bisector of $\angle A$ and D is the mid-point of BC. $[\because \Delta ADC \cong \Delta ADB$ by RHS congruence rule]

$\Rightarrow$ $\angle BAD = \angle CAD = \frac{\angle A}{2} = \frac{60°}{2} = 30°$ and $BD = CD = \frac{1}{2} BC = a$ units



Now, as ΔABD is a right angled triangle, in which
$\angle D = 90°$, $AB = 2a$ units and $BD = a$ units
$\therefore$ Using Pythagoras theorem, we get
$AB^2 = AD^2 + BD^2$
$\Rightarrow (2a)^2 = AD^2 + (a)^2$
$\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2 \Rightarrow AD = \sqrt{3}a$ units

Trigonometric ratios of 30°

In ΔABD, $\angle A = 30°$,

$\therefore$ $\sin 30° = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$, $\cos 30° = \frac{AD}{AB} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{BD}{AD} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$,

$\cot 30° = \frac{1}{\tan 30°} = \frac{1}{1/\sqrt{3}} = \sqrt{3}$, $\sec 30° = \frac{1}{\cos 30°} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$ and $\text{cosec } 30° = \frac{1}{\sin 30°} = \frac{1}{1/2} = 2$.

Trigonometric ratios of 60°

In ΔABD, $\angle B = 60°$

$\therefore$ $\sin 60° = \frac{AD}{AB} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$, $\tan 60° = \frac{AD}{BD} = \frac{\sqrt{3}a}{a} = \sqrt{3}$,

$\cot 60° = \frac{1}{\tan 60°} = \frac{1}{\sqrt{3}}$, $\sec 60° = \frac{1}{\cos 60°} = \frac{1}{1/2} = 2$ and $\text{cosec } 60° = \frac{1}{\sin 60°} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$.

Trigonometric ratios of 45°

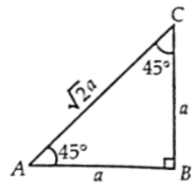
Consider a right angle ΔABC, right-angled at B, such that $\angle A = 45°$.
Then, $\angle A + \angle C = 90°$ $[\because$ Sum of interior angles of a triangle is 180° and $\angle B = 90°]$
$\Rightarrow \angle C = 45°$ $[\because \angle A = 45°]$

Now, as sides opposite to equal angles are equal

$\therefore \quad \angle A = \angle C = 45° \Rightarrow BC = AB = a$ units (say)

Now, by using Pythagoras theorem, we get $AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC^2 = a^2 + a^2 = 2a^2 \Rightarrow AC = \sqrt{2}\,a$ units



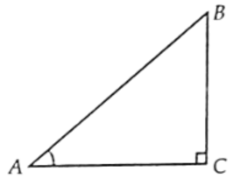
In ΔABC, $\angle A = 45°$

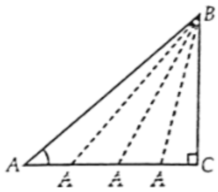
$\therefore \quad \sin 45° = \frac{BC}{AC} = \frac{a}{\sqrt{2}\,a} = \frac{1}{\sqrt{2}}$, $\cos 45° = \frac{AB}{AC} = \frac{a}{\sqrt{2}\,a} = \frac{1}{\sqrt{2}}$, $\tan 45° = \frac{BC}{AB} = \frac{a}{a} = 1$,

$\cot 45° = \frac{1}{\tan 45°} = \frac{1}{1} = 1$, $\sec 45° = \frac{1}{\cos 45°} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$ and $\operatorname{cosec} 45° = \frac{1}{\sin 45°} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$

Trigonometric ratios of 90°

Consider a right triangle ABC, right-angled at C. Now, if $\angle A$ is made larger and larger till it becomes 90°, then the side AC decreases and point A gets closer and closer to the point C. Finally when $\angle A$ becomes very close to 90°, the point A will almost coincide to point C and AB becomes almost equal to BC.





So, when $\angle A = 90°$, then

$\sin A = \sin 90° = \frac{BC}{AB} = \frac{BC}{BC} = 1 \quad [\because BC = AB]$

Similarly, $\cos 90° = \frac{AC}{AB} = \frac{0}{AB} = 0$, $\tan 90° = \frac{BC}{AC} = \frac{BC}{0} =$ Not defined,

$\cot 90° = \frac{AC}{BC} = \frac{0}{BC} = 0$, $\sec 90° = \frac{AB}{AC} = \frac{AB}{0} =$ Not defined and $\operatorname{cosec} 90° = \frac{AB}{BC} = \frac{BC}{BC} = 1$.

Note : (i) As we know that the values of trigonometric ratios do not vary, if the angle remains same, therefore above values are standard values.

(ii) Since the hypotenuse is the longest side in a right triangle, so the value of sin A or cos A is always less than 1 (or in particular equal to 1), whereas the value of sec A or cosec A is always greater than or equal to 1.

The following table gives the values of various trigonometric ratios of 0°, 30°, 45°, 60° and 90° for ready reference.

Table for T-ratios of 0°, 30°, 45°, 60° and 90°

T-Ratios / θ	sinθ	cosθ	tanθ	cotθ	secθ	cosecθ
0°	0	1	0	Not defined	1	Not defined
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
90°	1	0	Not defined	0	Not defined	1

From the above table, it can be seen that the value of sin θ increases from 0 to 1 and that of cosθ decreases from 1 to 0 as θ increases from 0° to 90°.

ILLUSTRATIONS

Q Write the value of $\sin^2 30° + \cos^2 60°$. *(CBSE 2020)*

Sol. We have, $\sin^2 30° + \cos^2 60°$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$

Q Evaluate :

$\sin^2 30° + \cos^2 45° + 4\cot^2 60° - 2\sin^2 90°$

Sol. We have, $\sin^2 30° + \cos^2 45° + 4\cot^2 60° - 2\sin^2 90°$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 - 2(1)^2$

$= \frac{1}{4} + \frac{1}{2} + \frac{4}{3} - 2 = \frac{25}{12} - 2 = \frac{25-24}{12} = \frac{1}{12}$

Q For $A = 30°$ and $B = 60°$, verify that $\sin A \cos B + \cos A \sin B = \sin (A+B)$.

Sol. L.H.S. = $\sin A \cos B + \cos A \sin B$

$= \sin 30° \cdot \cos 60° + \cos 30° \cdot \sin 60°$

$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$

R.H.S. = $\sin(A + B) = \sin (30° + 60°) = \sin 90° = 1$

Thus, L.H.S. = R.H.S.

Q By what number cos 45° must be multiplied to get tan 60°?

Sol. We know that, $\cos 45° = 1/\sqrt{2}$ and $\tan 60° = \sqrt{3}$.

Let x be the required number.

Then, $x \cdot \frac{1}{\sqrt{2}} = \sqrt{3} \Rightarrow x = \sqrt{3} \times \sqrt{2} = \sqrt{6}$.

Q If $\cos (A + B) = 1/2$ and $\tan (A - B) = 1/\sqrt{3}$; $0° < A + B \le 90°$, $A > B$, find A and B.

Sol. We have, $\cos (A + B) = 1/2$

and $\tan (A - B) = 1/\sqrt{3}$.

We know that $\cos 60° = 1/2$ and $\tan 30° = 1/\sqrt{3}$

$\therefore \quad A + B = 60° \quad ...(i)\ ; \ A - B = 30° \quad ...(ii)$

On adding (i) and (ii), we get $2A = 90° \Rightarrow A = 45°$

On substituting the value of A in (i), we get

$45° + B = 60° \quad \Rightarrow \quad B = 60° - 45° = 15°$

Hence, $A = 45°$ and $B = 15°$.

TRY YOURSELF

10. Evaluate : (i) $\frac{4\cos^2 30° - 5\sin 90°}{6\sec^2 30° + 2\cos 90°}$ (ii) $\frac{\cos 30°}{\sin^2 45°} - \tan 60° + 5\sin 0°$ **(Ans.** (i) $\frac{-1}{4}$ (ii) 0)

11. For $A = 60°$, verify that $\frac{1+\sin A}{\cos A} = \frac{\cos A}{1 - \sin A}$.

12. If $\sin (A - B) = \frac{1}{2}$, $\cos (A + B) = \frac{1}{2}$, $0° < A + B \le 90°$, $A > B$, find A and B. **(Ans.** $A = 45°$ and $B = 15°$)

13. Evaluate : $\frac{3\sin 3A + 2\cos(5A+10)°}{\sqrt{3}\tan 3A + \mathrm{cosec}\,(5A - 20)°}$, when $A = 10°$. **(Ans.** 5/6)

14. Evaluate : $\frac{\sin 90°}{\tan 45°} + \frac{1}{\sec 30°}$. **(Ans.** $1 + (\sqrt{3}/2)$)

Some Special Type of Problems

Here, we shall see, if one of the sides and any other part (either an acute angle or any side) of a right angled triangle is known, then the remaining sides and angles of the triangle can be determined.

ILLUSTRATIONS

Q ABC is a right angle triangle, right-angled at C. If $\angle A = 30°$ and $AB = 40$ units, then find the remaining two sides and $\angle ABC$.

Sol. We have, $\angle A + \angle B + \angle C = 180°$

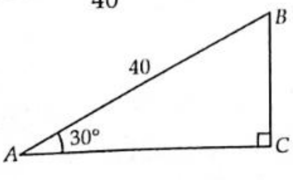
$\Rightarrow \quad 30° + \angle B + 90° = 180°$

[$\because \angle A = 30°$ and $\angle C = 90°$]

$\Rightarrow \quad \angle B = 180° - 120° = 60°$

Now, $\cos A = \frac{AC}{AB} \Rightarrow \cos 30° = \frac{AC}{40}$

$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AC}{40}$



$\Rightarrow \quad AC = \frac{\sqrt{3}}{2} \times 40$

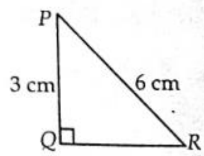
$\Rightarrow \quad AC = 20\sqrt{3}$ units

Also, $\sin A = \frac{BC}{AB} \Rightarrow \sin 30° = \frac{BC}{40}$

$\Rightarrow \quad \frac{1}{2} = \frac{BC}{40} \Rightarrow BC = 40 \times \frac{1}{2} = 20$ units

Hence, $AC = 20\sqrt{3}$ units, $BC = 20$ units and $\angle B = 60°$.

Q In the given figure, ΔPQR is right-angled at Q, $PQ = 3$ cm and $PR = 6$ cm. Determine $\angle QPR$ and $\angle PRQ$. *(NCERT)*



Sol. Given, $PQ = 3$ cm and $PR = 6$ cm

Clearly, $\sin R = \frac{PQ}{PR} \Rightarrow \sin R = \frac{3}{6} \Rightarrow \sin R = \frac{1}{2}$

$\Rightarrow \quad \sin R = \sin 30° \quad \left[\because \sin 30° = \frac{1}{2}\right]$

$\therefore \quad \angle PRQ = 30°$

Now, in ΔPQR, $\angle P + \angle Q + \angle R = 180°$

[By angle sum property of triangle]

$\Rightarrow \quad \angle P + 90° + 30° = 180° \quad \Rightarrow \angle P = 180° - 120°$

$\therefore \quad \angle P = 60°$ or $\angle QPR = 60°$

TRY YOURSELF

15. In a ΔABC, right-angled at B, $AB = 5$ cm and $\angle ACB = 30°$. Determine the lengths of the sides BC and AC. **(Ans.** $BC = 5\sqrt{3}$ cm and $AC = 10$ cm**)**

16. In ΔPQR, right angled at Q, if $PQ = \sqrt{2}$ cm and $PR = 2\sqrt{2}$ cm, then determine $\angle P$ and $\angle R$. **(Ans.** $\angle P = 60°$ and $\angle R = 30°$**)**

NCERT FOCUS

EXERCISE - 8.2

1. Evaluate the following:

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

2. Choose the correct option and justify your choice:

(i) $\dfrac{2\tan 30°}{1+\tan^2 30°} =$

(a) $\sin 60°$ (b) $\cos 60°$
(c) $\tan 60°$ (d) $\sin 30°$

(ii) $\dfrac{1-\tan^2 45°}{1+\tan^2 45°} =$

(a) $\tan 90°$ (b) 1 (c) $\sin 45°$ (d) 0

(iii) $\sin 2A = 2\sin A$ is true when $A =$

(a) 0° (b) 30° (c) 45° (d) 60°

(iv) $\dfrac{2\tan 30°}{1-\tan^2 30°} =$

(a) $\cos 60°$ (b) $\sin 60°$
(c) $\tan 60°$ (d) $\sin 30°$

3. If $\tan (A + B) = \sqrt{3}$ and $\tan (A - B) = 1/\sqrt{3}$; $0° < A + B \le 90°$; $A > B$, find A and B.

4. State whether the following are true or false. Justify your answer.

(i) $\sin (A + B) = \sin A + \sin B$.

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as θ increases.

(iv) $\sin \theta = \cos \theta$ for all values of θ.

(v) $\cot A$ is not defined for $A = 0°$.

CBSE FOCUS

Objective Type Questions

MCQs (1 Mark)

1. The value of $(\sin 45° + \cos 45°)$ is

(a) $1/\sqrt{2}$ (b) $\sqrt{2}$ (c) $\sqrt{3}/2$ (d) 1

(NCERT Exemplar)

2. Given that $\sin\alpha = \dfrac{\sqrt{3}}{2}$ and $\tan\beta = \dfrac{1}{\sqrt{3}}$, then the value of $\cos(\alpha - \beta)$ is

(a) $\dfrac{\sqrt{3}}{2}$ (b) $\dfrac{1}{2}$ (c) 0 (d) $\dfrac{1}{\sqrt{2}}$

(CBSE Term I, Standard 2021-22)

3. If $\sin(A + B) = \cos(A - B) = 1$, then
(a) $A = B = 0°$ (b) $A = B = 45°$
(c) $A = 60°, B = 30°$ (d) $A = 90°, B = 60°$
(CBSE Term I, Basic 2021-22)

Fill in the Blanks (1 Mark)

4. If $\cot 3\theta = 1/\sqrt{3}$, $0° < \theta \le 20°$, then the value of θ is ________.

5. If $\tan(3x + 30°) = 1$, then the value of x is ________.

VSA Type Questions (1 Mark)

6. Find the maximum value of $\sin\theta$ and $\cos\theta$.

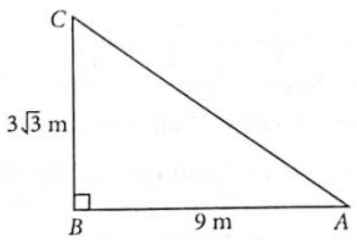
7. If $0° < \theta < 45°$, then find the value of θ when $\sqrt{3}\sin 2\theta = 3/2$.

8. If $\sin\theta = 1/\sqrt{2}$ and $\sec\phi = 2/\sqrt{3}$, then find the value of $(\theta + \phi)$.

Case-Study Based Questions (5 × 1 Mark)

9. **Hide and Seek**

Three friends – Anshu, Vijay and Vishal are playing hide and seek in a park. Anshu and Vijay hide in the shrubs and Vishal have to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure and forms a right angled triangle such that $AB = 9$ m, $BC = 3\sqrt{3}$ m and $\angle B = 90°$, then answer the following questions.



(i) The measure of $\angle A$ is
(a) 30° (b) 45°
(c) 60° (d) None of these

(ii) The measure of $\angle C$ is
(a) 30° (b) 45°
(c) 60° (d) None of these

(iii) The length of AC is
(a) $2\sqrt{3}$ m (b) $\sqrt{3}$ m
(c) $4\sqrt{3}$ m (d) $6\sqrt{3}$ m

(iv) $\cos 2A =$
(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

(v) $\sin\left(\frac{C}{2}\right) =$
(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

Short Answer Type Questions

SA Type I Questions (2 Marks)

10. Evaluate $2\sec^2\theta + 3\operatorname{cosec}^2\theta - 2\sin\theta\cos\theta$ if $\theta = 45°$. *(CBSE 2023)*

11. Prove that
$(\sqrt{3}+1)(3-\cot 30°) = \tan^3 60° - 2\sin 60°$.
(NCERT Exemplar)

12. For what value of A, $\frac{\cos A + \sin A}{\cos A - \sin A} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$?

13. If $\frac{1+\tan A}{1-\tan A} = 1$, then find the value of $\sin A + \cos A$.

SA Type II Questions (3 Marks)

14. If $\sqrt{3}\sec(3x - 21)° = 2$, then find the value of $\sin^2(x + 13)° + \cot^2(x + 13)°$.

15. If $\sin(A - B) = 1/2$ and $\tan(A + B) = \sqrt{3}$, then find the values of A and B.

16. In a rectangle $XYZW$, $XY = 30$ cm, $\angle YXZ = 30°$, Find the lengths of YW and YZ.

Long Answer Type Questions

LA Type Questions (5 Marks)

17. Find cosec30° and cos60° geometrically.

18. An equilateral triangle is inscribed in a circle of diameter 24 cm. Find its side.

SOLUTIONS

1. (b) : Clearly, $\sin 45° + \cos 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

2. (a) : Given, $\sin\alpha = \frac{\sqrt{3}}{2}$

$\Rightarrow \alpha = 60°$ $\quad (\because \sin 60° = \frac{\sqrt{3}}{2})$

and $\tan\beta = \frac{1}{\sqrt{3}}$

$\Rightarrow \beta = 30°$ $\quad (\because \tan 30° = \frac{1}{\sqrt{3}})$

Now, $\cos(\alpha - \beta) = \cos(60° - 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

3. (b) : We have, $\sin(A + B) = 1$

$\Rightarrow A + B = 90°$ ...(i)

and $\cos(A - B) = 1$

$\Rightarrow A - B = 0°$ ...(ii)

From (i) and (ii), we have, $A = B = 45°$.

4. Given, $\cot 3\theta = \frac{1}{\sqrt{3}} \Rightarrow \cot 3\theta = \cot 60°$

$\Rightarrow 3\theta = 60° \Rightarrow \theta = 20°$

5. We have, $\tan(3x + 30°) = 1$

$\Rightarrow \tan(3x + 30°) = \tan 45°$ $\quad [\because \tan 45° = 1]$

$\Rightarrow 3x + 30° = 45° \Rightarrow 3x = 45° - 30° = 15° \Rightarrow x = 5°$

6. Maximum value of $\sin\theta = 1$ when $\theta = 90°$ and maximum value of $\cos\theta = 1$, when $\theta = 0°$.

7. Given, $\sqrt{3}\sin 2\theta = \frac{3}{2} \Rightarrow \sin 2\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin 2\theta = \sin 60° \Rightarrow 2\theta = 60° \Rightarrow \theta = 30°$

8. Given, $\sin\theta = \frac{1}{\sqrt{2}} = \sin 45° \Rightarrow \theta = 45°$ ...(i)

Also, $\sec\phi = \frac{2}{\sqrt{3}} = \sec 30° \Rightarrow \phi = 30°$ ...(ii)

$\therefore \theta + \phi = 45° + 30° = 75°$ [Using (i) and (ii)]

9. (i) (a) : We have, $AB = 9$ m, $BC = 3\sqrt{3}$ m

In ΔABC, we have

$\tan A = \frac{BC}{AB} = \frac{3\sqrt{3}}{9} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan A = \tan 30° \Rightarrow \angle A = 30°$ ...(1)

(ii) (c) : Similarly, $\tan C = \frac{AB}{BC} = \frac{9}{3\sqrt{3}} = \sqrt{3}$

$\Rightarrow \tan C = \tan 60° \Rightarrow \angle C = 60°$ ...(2)

(iii) (d) : Since, $\sin A = \frac{BC}{AC} \Rightarrow \sin 30° = \frac{BC}{AC}$ [Using (1)]

$\Rightarrow \frac{1}{2} = \frac{3\sqrt{3}}{AC} \Rightarrow AC = 6\sqrt{3}$ m

(iv) (b) : $\because \angle A = 30°$ [From (1)]

$\therefore \cos 2A = \cos(2 \times 30°) = \cos 60° = \frac{1}{2}$

(v) (b) : $\because \angle C = 60°$ [Using (2)]

$\therefore \sin\left(\frac{C}{2}\right) = \sin\left(\frac{60°}{2}\right) = \sin 30° = \frac{1}{2}$

10. Put $\theta = 45°$ in $2\sec^2\theta + 3\,\text{cosec}^2\,\theta - 2\sin\theta\cos\theta$

$= 2\sec^2(45°) + 3\,\text{cosec}^2(45°) - 2\sin(45°)\cos(45°)$

$= 2(\sqrt{2})^2 + 3(\sqrt{2})^2 - 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 4 + 6 - 1 = 9$

11. L.H.S. $= (\sqrt{3}+1)(3-\cot 30°) = (\sqrt{3}+1)(3-\sqrt{3})$

$= (\sqrt{3}+1)\sqrt{3}(\sqrt{3}-1) = \sqrt{3}\,[(\sqrt{3})^2 - 1] = \sqrt{3}(3-1) = 2\sqrt{3}$

R.H.S. $= \tan^3 60° - 2\sin 60°$

$= (\sqrt{3})^3 - 2 \times \frac{\sqrt{3}}{2} = 3\sqrt{3} - \sqrt{3} = 2\sqrt{3}$

$\therefore$ L.H.S. = R.H.S.

12. Given, $\frac{\cos A + \sin A}{\cos A - \sin A} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\Rightarrow \sqrt{3}\cos A - \cos A + \sqrt{3}\sin A - \sin A$

$= \sqrt{3}\cos A - \sqrt{3}\sin A + \cos A - \sin A$

$\Rightarrow 2\sqrt{3}\sin A = 2\cos A$

$\Rightarrow \tan A = \frac{1}{\sqrt{3}} = \tan 30°$

$\therefore A = 30°$

13. We have, $\frac{1+\tan A}{1-\tan A} = 1 \Rightarrow 1 + \tan A = 1 - \tan A$

$\Rightarrow 2\tan A = 0 \Rightarrow \tan A = 0 \Rightarrow A = 0°$

$\therefore \sin A + \cos A = \sin 0° + \cos 0° = 0 + 1 = 1$

14. Given, $\sqrt{3}\sec(3x - 21)° = 2$

$\Rightarrow \sec(3x - 21)° = \frac{2}{\sqrt{3}} \Rightarrow \sec(3x-21)° = \sec 30°$

$\Rightarrow 3x - 21 = 30 \Rightarrow 3x = 51 \Rightarrow x = 17$

Now, $\sin^2(x + 13)° + \cot^2(x + 13)°$

$= \sin^2(17 + 13)° + \cot^2(17 + 13)° = \sin^2 30° + \cot^2 30°$

$= \left(\frac{1}{2}\right)^2 + (\sqrt{3})^2 = \frac{1}{4} + 3 = \frac{1+12}{4} = \frac{13}{4}$

15. Given, $\sin(A - B) = \frac{1}{2}$ and $\tan(A + B) = \sqrt{3}$

$\Rightarrow \sin(A - B) = \sin 30°$ and $\tan(A + B) = \tan 60°$

$\Rightarrow A - B = 30°$ ...(i) and $A + B = 60°$ ...(ii)

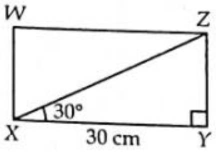
Adding (i) and (ii), we get, $2A = 90°$

$\Rightarrow A = 45°$

Substituting the value of A in (i), we get $45° - B = 30°$

$\Rightarrow B = 15°$.

16. In a rectangle $XYZW$, it is given that $XY = 30$ cm and $\angle YXZ = 30°$.



Now, $\tan 30° = \dfrac{ZY}{XY}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{ZY}{30} \Rightarrow ZY = \dfrac{30}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow \quad ZY = \dfrac{30\sqrt{3}}{3} = 10\sqrt{3}$ cm

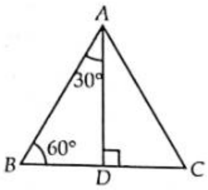
Also, $\sin 30° = \dfrac{ZY}{XZ} \Rightarrow \dfrac{1}{2} = \dfrac{10\sqrt{3}}{XZ} \Rightarrow XZ = 20\sqrt{3}$ cm $= YW$

($\because$ Diagonals of a rectangle are equal)

17. Consider an equilateral triangle ABC with each side of length $2a$ units, and $\angle A = \angle B = \angle C = 60°$

$\Rightarrow \quad AB = BC = CA = 2a$

Now, draw $AD \perp BC$



Now, in ΔADB and ΔADC

$\angle ADB = \angle ADC$ (Each 90°)

$AB = AC$ (Given)

$AD = AD$ (Common)

$\therefore \quad \Delta ADB \cong \Delta ADC$ (By RHS congruency criterion)

$\therefore \quad BD = DC$ and $\angle BAD = \angle CAD$ (By C.P.C.T.)

$\therefore \quad BD = DC = a$ units and $\angle BAD = 30°$

In ΔABD, by Pythagoras theorem, we have

$AB^2 = AD^2 + BD^2$

$\Rightarrow \quad (2a)^2 = AD^2 + a^2 \Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$

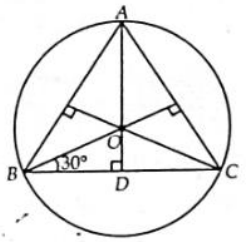
$\Rightarrow \quad AD = \sqrt{3}a$ units

$\therefore \quad$ In ΔABD, $\operatorname{cosec} 30° = \dfrac{AB}{BD} = \dfrac{2a}{a} = 2$

Again in ΔADB, we have $\angle ABD = 60°$

$\therefore \quad \cos 60° = \dfrac{BD}{AB} = \dfrac{a}{2a} = \dfrac{1}{2}$

18. It is given that an equilateral triangle ABC is inscribed in a circle of diameter 24 cm.



Since, ABC is an equilateral triangle, so $\angle A = \angle B = \angle C = 60°$

Now, draw the perpendicular from each vertex to the corresponding opposite side such that it bisects the vertex angle as well as the opposite side.

$\therefore \quad \angle OBD = 30°$

Radius $= OA = OB = OC = 24/2 = 12$ cm

Now, in right angled ΔOBD

$\cos 30° = \dfrac{BD}{OB} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{BD}{12} \Rightarrow BD = 6\sqrt{3}$ cm

Similarly, $CD = 6\sqrt{3}$ cm

$\therefore \quad BC = BD + DC = 6\sqrt{3} + 6\sqrt{3} = 12\sqrt{3}$ cm

TOPIC 3 : TRIGONOMETRIC IDENTITIES

As we know that an equation is called an identity, if it is true for all values of the variables involved. Therefore, an equation involving trigonometric ratios of an angle (say θ) is said to be a trigonometric identity, if it is true for all values of θ for which the trigonometric ratios are defined.

For example, $\tan\theta = 1/\cot\theta$, is a trigonometric identity, as it holds for all values of θ except $\theta = 0°$ and $90°$.

[$\because$ tan 90° and cot 0° are not defined.]

Thus, $\tan\theta = 1/\cot\theta$ for all $0° < \theta < 90°$.

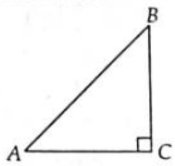
Some Simple Trigonometric Identities and their Proof

We have three simple trigonometric identities which are given below :

(i) $\sin^2\theta + \cos^2\theta = 1$ (ii) $1 + \tan^2\theta = \sec^2\theta$ (iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

Now, we will prove one trigonometric identity and use it further to prove other useful trigonometric identities.

Consider a ΔABC, right angled at C. Then, by Pythagoras theorem, we have



$AB^2 = AC^2 + BC^2$ ...(*)

On dividing each term by AB^2, we get

$$\frac{AB^2}{AB^2} = \frac{AC^2}{AB^2} + \frac{BC^2}{AB^2} \Rightarrow 1 = \left(\frac{AC}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2$$

$\Rightarrow \quad 1 = (\cos A)^2 + (\sin A)^2$ $\left[\because \sin\theta = \dfrac{P}{H} \text{ and } \cos\theta = \dfrac{B}{H}\right]$

$\Rightarrow \quad 1 = \cos^2 A + \sin^2 A \Rightarrow \cos^2 A + \sin^2 A = 1$

This is true for all A such that $0° \le A \le 90°$. So, this is a trigonometric identity.

Now, let us divide (*) by AC^2, we get $\dfrac{AB^2}{AC^2} = \dfrac{AC^2}{AC^2} + \dfrac{BC^2}{AC^2} \Rightarrow \left(\dfrac{AB}{AC}\right)^2 = 1 + \left(\dfrac{BC}{AC}\right)^2$

$\Rightarrow \quad \sec^2 A = 1 + \tan^2 A$, which is true for all A such that $0° \le A < 90°$ [$\because$ tan A and sec A are not defined for $A = 90°$]

Similarly, when (*) is divided by BC^2, we get $\dfrac{AB^2}{BC^2} = \dfrac{AC^2}{BC^2} + \dfrac{BC^2}{BC^2} \Rightarrow \left(\dfrac{AB}{BC}\right)^2 = \left(\dfrac{AC}{BC}\right)^2 + 1$

$\Rightarrow$ $\text{cosec}^2 A = \cot^2 A + 1$, which is true for all A such that $0° < A \le 90°$ [$\because$ cosec A and cot A are not defined for $A = 0°$]

Now, from the above discussion, we have

(i) $\sin^2\theta + \cos^2\theta = 1 \ \forall\ 0° \le \theta \le 90°$

(ii) $\sec^2\theta = 1 + \tan^2\theta \ \forall\ 0° \le \theta < 90°$

(iii) $\text{cosec}^2\theta = 1 + \cot^2\theta \ \forall\ 0° < \theta \le 90°$

Note :

- $\sin^2\theta = 1 - \cos^2\theta$
- $\cos^2\theta = 1 - \sin^2\theta$
- $\sec^2\theta - \tan^2\theta = 1$
- $\sec^2\theta - 1 = \tan^2\theta$
- $\text{cosec}^2\theta - 1 = \cot^2\theta$
- $\text{cosec}^2\theta - \cot^2\theta = 1$

ILLUSTRATIONS

Q Check whether the following equations are identities or not :

(i) $\tan^2\theta + \cos\theta = \sin^2\theta$ (ii) $\cos^2\theta + \cos\theta = 1$

Sol. (i) For $\theta = 45°$,

L.H.S. $= \tan^2 45° + \cos 45° = 1 + 1/\sqrt{2}$

R.H.S. $= \sin^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

$\because$ L.H.S. $\ne$ R.H.S. for $\theta = 45°$

$\therefore$ Equation given in (i) is not an identity.

(ii) For $\theta = 60°$,

L.H.S. $= \cos^2 60° + \cos 60°$

$= \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \ne$ R.H.S for $\theta = 60°$

$\therefore$ Equation given in (ii) is not an identity.

Q Evaluate : (i) $(\sec^2\theta - 1)\cot^2\theta$

(ii) $\text{cosec}\,\phi\,(1 - \cos\phi)(\text{cosec}\,\phi + \cot\phi)$

Sol. (i) $(\sec^2\theta - 1)\cot^2\theta = \tan^2\theta \cdot \cot^2\theta$

$= \tan^2\theta \cdot \frac{1}{\tan^2\theta} = 1$

(ii) $\text{cosec}\,\phi\,(1 - \cos\phi)(\text{cosec}\,\phi + \cot\phi)$

$= \left(\text{cosec}\,\phi - \frac{\cos\phi}{\sin\phi}\right)(\text{cosec}\,\phi + \cot\phi)$

$\left(\because \text{cosec}\,\phi = \frac{1}{\sin\phi}\right)$

$= (\text{cosec}\,\phi - \cot\phi)(\text{cosec}\,\phi + \cot\phi)$

$= \text{cosec}^2\phi - \cot^2\phi = 1$

Q Find the value of $(\sin^2 33° + \sin^2 57°)$. *(Delhi 2019)*

Sol. We have, $\sin^2 33° + \sin^2 57°$

$= \sin^2(90° - 57°) + \sin^2 57°$

$= \cos^2 57° + \sin^2 57°$ [$\because \sin(90° - \theta) = \cos\theta$]

$= 1$ [$\because \sin^2\theta + \cos^2\theta = 1$]

Q Prove that $\frac{1}{1+\cos A} + \frac{1}{1-\cos A} = 2\,\text{cosec}^2 A$.

Sol. L.H.S. $= \frac{1}{1+\cos A} + \frac{1}{1-\cos A}$

$= \frac{(1-\cos A)+(1+\cos A)}{1^2 - \cos^2 A} = \frac{2}{1-\cos^2 A} = \frac{2}{\sin^2 A}$ [$\because 1 - \cos^2\theta = \sin^2\theta$]

$= 2\,\text{cosec}^2 A =$ R.H.S.

Q Simplify : $\frac{\cos\theta - 2\cos^3\theta}{2\sin^3\theta - \sin\theta}$.

Sol. We have, $\frac{\cos\theta - 2\cos^3\theta}{2\sin^3\theta - \sin\theta} = \frac{\cos\theta(1 - 2\cos^2\theta)}{\sin\theta(2\sin^2\theta - 1)}$

$= \frac{\cos\theta(1 - 2(1-\sin^2\theta))}{\sin\theta(2\sin^2\theta - 1)} = \frac{\cos\theta(1 - 2 + 2\sin^2\theta)}{\sin\theta(2\sin^2\theta - 1)}$

$= \frac{\cos\theta(2\sin^2\theta - 1)}{\sin\theta(2\sin^2\theta - 1)} = \frac{\cos\theta}{\sin\theta} = \cot\theta$

TRY YOURSELF

17. Evaluate : $\sec^2\theta - \frac{1}{\text{cosec}^2\theta - 1}$. **(Ans. 1)**
18. Prove that : $\sec A\,(1 - \sin A)(\sec A + \tan A) = 1$.
19. Prove that : $\cot A + \tan A = \sec A\,\text{cosec}\,A$.
20. Prove that : $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\,\text{cosec}\,\theta$.
21. Evaluate : $\sin^2\theta\,(1 + \cot^2\theta)$. **(Ans. 1)**

Transformation of Trigonometric Ratios in Terms of other Trigonometric Ratios

Using the identities, $\sin^2\theta + \cos^2\theta = 1$, $\sec^2\theta = 1 + \tan^2\theta$ and $\text{cosec}^2\theta = 1 + \cot^2\theta$, we can express each trigonometric ratio in terms of other trigonometric ratios as shown in table.

Transformation of sinθ in terms of cosθ, tanθ, cotθ, secθ and cosecθ.	Transformation of cosθ, tanθ, cotθ, secθ and cosecθ in terms of sinθ.
We know, $\sin^2\theta + \cos^2\theta = 1$ $\Rightarrow \sin^2\theta = 1 - \cos^2\theta \Rightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$ But $\sin\theta \geq 0$ for all $0° \leq \theta \leq 90°$ $\therefore \sin\theta = \sqrt{1-\cos^2\theta}$ [Transformation of sin θ in cos θ] $= \sqrt{1-\frac{1}{\sec^2\theta}}$ $\left[\because \cos\theta = \frac{1}{\sec\theta}\right]$ $= \frac{\sqrt{\sec^2\theta - 1}}{\sec\theta}$ [Transformation of sinθ in secθ] $= \sqrt{\frac{\tan^2\theta}{1+\tan^2\theta}}$ $[\because \sec^2\theta = 1 + \tan^2\theta]$ $= \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$ [Transformation of sinθ in tanθ] $= \frac{1/\cot\theta}{\sqrt{1+\frac{1}{\cot^2\theta}}}$ $\left[\because \tan\theta = \frac{1}{\cot\theta}\right]$ $= \frac{1/\cot\theta}{\left(\frac{\sqrt{\cot^2\theta+1}}{\cot\theta}\right)} = \frac{1}{\cot\theta} \times \frac{\cot\theta}{\sqrt{\cot^2\theta+1}}$ $= \frac{1}{\sqrt{1+\cot^2\theta}}$ [Transformation of sinθ in cotθ] Also we know, $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$ [Transformation of sinθ in cosecθ]	We know, $\sin^2\theta + \cos^2\theta = 1$ $\Rightarrow \cos^2\theta = 1 - \sin^2\theta \Rightarrow \cos\theta = \pm\sqrt{1-\sin^2\theta}$ But $\cos\theta \geq 0$ for all $0° \leq \theta \leq 90°$ $\therefore \cos\theta = \sqrt{1-\sin^2\theta}$ [Transformation of cos θ in sin θ] Now, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $= \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$ [Transformation of tan θ in sin θ] Also, $\cot\theta = \frac{1}{\tan\theta} = \frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$ [Transformation of cotθ in sinθ] And $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{1-\sin^2\theta}}$ [Transformation of secθ in sinθ] Also, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ [Transformation of cosecθ in sinθ]
Similarly, we can express other trigonometric ratios.	Similarly, we can do other cases, shown in next table.

Transformation of Trigonometric Ratios in Terms of other Trigonometric Ratios

	sin θ	cos θ	tan θ	cot θ	sec θ	cosec θ
sin θ	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\operatorname{cosec}\theta}$
cos θ	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$
tan θ	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$
cot θ	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\sqrt{\operatorname{cosec}^2\theta-1}$
sec θ	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sec\theta$	$\frac{\operatorname{cosec}\theta}{\sqrt{\operatorname{cosec}^2\theta-1}}$
cosec θ	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\operatorname{cosec}\theta$

ILLUSTRATIONS

Q Express the ratios tan A and cosec A in terms of cos A.

Sol. We know that, $\tan A = \dfrac{\sin A}{\cos A} = \dfrac{\sqrt{1-\cos^2 A}}{\cos A}$

$[\because \sin^2\theta = 1 - \cos^2\theta \Rightarrow \sin\theta = \sqrt{1-\cos^2\theta}]$

Now, $\operatorname{cosec} A = \dfrac{1}{\sin A} = \dfrac{1}{\sqrt{1-\cos^2 A}}$

Q If sin A = 3/4, find cos A and tan A.

Sol. We have, $\sin A = 3/4$

Now, $\cos A = \sqrt{1-\sin^2 A}$

$= \sqrt{1-\left(\dfrac{3}{4}\right)^2} = \sqrt{1-\dfrac{9}{16}} = \sqrt{\dfrac{7}{16}} \Rightarrow \cos A = \sqrt{7}/4$

Now, $\tan A = \dfrac{\sin A}{\cos A} = \dfrac{3/4}{\sqrt{7}/4} = \dfrac{3}{\sqrt{7}}$

(**Note** : We can also do this by the method discussed in Topic 1)

Q Prove that: $\dfrac{2\cos^3\theta - \cos\theta}{\sin\theta - 2\sin^3\theta} = \cot\theta$ *(CBSE 2020)*

Sol. L.H.S. $= \dfrac{2\cos^3\theta - \cos\theta}{\sin\theta - 2\sin^3\theta}$

$= \dfrac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(1-2\sin^2\theta)} = \dfrac{\cot\theta(2(1-\sin^2\theta)-1)}{(1-2\sin^2\theta)}$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= \dfrac{\cot\theta(2-2\sin^2\theta-1)}{(1-2\sin^2\theta)} = \dfrac{\cot\theta(1-2\sin^2\theta)}{(1-2\sin^2\theta)}$

$= \cot\theta$ = R.H.S.

TRY YOURSELF

22. Express the ratios cosec A, tan A and cot A in terms of cos A.

$\left(\textbf{Ans. } \operatorname{cosec} A = \dfrac{1}{\sqrt{1-\cos^2 A}}, \tan A = \dfrac{\sqrt{1-\cos^2 A}}{\cos A} \text{ and } \cot A = \dfrac{\cos A}{\sqrt{1-\cos^2 A}}\right)$

23. If $\tan\theta = \dfrac{12}{5}$, then find the value of $\dfrac{1+\sin\theta}{1-\sin\theta}$. (**Ans.** 25)

24. If $\tan\theta = 1/\sqrt{3}$, then find the value of $3\sin^2\theta + 7\cos^2\theta$. (**Ans.** 6)

NCERT FOCUS

EXERCISE - 8.3

1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

2. Write all the other trigonometric ratios of $\angle A$ in terms of sec A.

3. Choose the correct option. Justify your choice.

(i) $9\sec^2 A - 9\tan^2 A =$

(a) 1 (b) 9
(c) 8 (d) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) =$

(a) 0 (b) 1
(c) 2 (d) –1

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(a) sec A (b) sin A
(c) cosec A (d) cos A

(iv) $\dfrac{1+\tan^2 A}{1+\cot^2 A} =$

(a) $\sec^2 A$ (b) -1
(c) $\cot^2 A$ (d) $\tan^2 A$

4. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\text{cosec } \theta - \cot \theta)^2 = 1 - \cos \theta / 1 + \cos \theta$

(ii) $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2 \sec A$

(iii) $\dfrac{\tan \theta}{1-\cot\theta} + \dfrac{\cot \theta}{1-\tan\theta} = 1 + \sec \theta \text{ cosec } \theta$

(iv) $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$

(v) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \text{cosec } A + \cot A$, using the identity $\text{cosec}^2 A = 1 + \cot^2 A$.

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(viii) $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $(\text{cosec } A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

CBSE FOCUS

Objective Type Questions

MCQs (1 Mark)

1. If $\text{cosec } \theta - \cot \theta = \dfrac{1}{3}$, then the value of $\text{cosec } \theta + \cot \theta$ is

(a) 1 (b) 2 (c) 3 (d) 4

(CBSE Term I, Basic 2021-22)

2. Given that $\sin\theta = \dfrac{p}{q}$, $\tan \theta$ is equal to

(a) $\dfrac{p}{\sqrt{p^2-q^2}}$ (b) $\dfrac{q}{\sqrt{p^2-q^2}}$

(c) $\dfrac{p}{\sqrt{q^2-p^2}}$ (d) $\dfrac{q}{\sqrt{q^2-p^2}}$

(CBSE Term I, Standard 2021-22)

3. If $\sin A + \sin^2 A = 1$, then $\cos^2 A + \cos^4 A =$

(a) 1/2 (b) 1 (c) 2 (d) 4

(NCERT Exemplar)

4. If $\sin A = 1/2$, then the value of $\cot A$ is

(a) $\sqrt{3}$ (b) $1/\sqrt{3}$ (c) $\sqrt{3}/2$ (d) 1

(NCERT Exemplar)

5. If $\sec\theta = \dfrac{17}{8}$, then $\sin^2\theta - \cos^2\theta =$

(a) 1 (b) $\dfrac{15}{17}$ (c) $\dfrac{8}{17}$ (d) $\dfrac{161}{289}$

Fill in the Blanks (1 Mark)

6. The value of $\left(\dfrac{-101}{\cos^2 A} + \dfrac{101}{\cot^2 A}\right)$ is ______.

7. If $\cot\theta - \cot^2\theta = 1$, then the value of $\text{cosec}^2\theta - \text{cosec}^4\theta$ is ______.

VSA Type Questions (1 Mark)

8. Prove that $1 + \dfrac{\cot^2 x}{1+\text{cosec } x} = \text{cosec } x$.

(NCERT Exemplar)

9. Simplify : $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

(NCERT Exemplar)

10. Simplify : $\sqrt{\dfrac{1-\sin^2 A}{\tan^2 A + 1}}$

Short Answer Type Questions

SA Type I Questions (2 Marks)

11. If $2\sin^2\theta - \cos^2\theta = 2$, then find the value of θ. *(NCERT Exemplar)*

12. Simplify : $\dfrac{\cot(90° - A)}{\cos A \cos(90° - A)} - 1$

13. If $\cot^2\theta(\sec\theta - 1)(1 + \cos\theta) = k\cos\theta$, then find the value of k.

14. Evaluate:

$$\frac{\cos 70°}{3\sin 20°} + \frac{4(\sec^2 59° - \cot^2 31°)}{3} - \frac{2}{3}\sin 90°$$

15. Prove that $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = 2\operatorname{cosec}\theta$

SA Type II Questions (3 Marks)

16. If $\dfrac{(\cos^2 31° + \cos^2 59°)}{5(\sin^2 25° + \sin^2 65°)} = 5P$, then find the value of $\sqrt{P}$.

17. Express $\sec\theta$ in terms of $\operatorname{cosec}\theta$.

18. Prove that : $(\tan A + \operatorname{cosec} B)^2 - (\cot B - \sec A)^2 = 2\sec A \operatorname{cosec} B(\sin A + \cos B)$.

19. If $4\tan\theta = 3$, evaluate $\left(\dfrac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$. *(CBSE 2018)*

20. Prove that $\sin^2\theta\cdot\tan\theta + \cos^2\theta\cdot\cot\theta + 2\sin\theta\cdot\cos\theta = \tan\theta + \cot\theta$. *(CBSE 2017)*

Long Answer Type Questions

LA Type Questions (5 Marks)

21. Prove that $\dfrac{1+\sec\theta - \tan\theta}{1+\sec\theta+\tan\theta} = \dfrac{1-\sin\theta}{\cos\theta}$. *(NCERT Exemplar)*

22. Prove that

$$\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}.$$

(Delhi 2019)

23. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$; prove that : $m^2 - n^2 = 4\sqrt{mn}$.

24. If $\cot\theta = \sqrt{1-k^2}$, then find the value of $\operatorname{cosec}\theta + \cot^3\theta\sec\theta$.

25. If $\operatorname{cosec}\theta + \cot\theta = p$, then prove that $\cos\theta = \dfrac{p^2-1}{p^2+1}$.

SOLUTIONS

1. (c) : We have, $\operatorname{cosec}\theta - \cot\theta = \dfrac{1}{3}$

$\because \operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

$\Rightarrow (\operatorname{cosec}\theta + \cot\theta) = \dfrac{1}{\operatorname{cosec}\theta - \cot\theta} = 3.$

2. (c) : Given, $\sin\theta = \dfrac{p}{q}$

$\therefore \cos\theta = \sqrt{1 - \left(\dfrac{p}{q}\right)^2}$ $\left[\because \cos\theta = \sqrt{1-\sin^2\theta}\right]$

$\Rightarrow \cos\theta = \dfrac{\sqrt{q^2-p^2}}{q}$

Now, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{p}{q}}{\frac{\sqrt{q^2-p^2}}{q}} = \dfrac{p}{\sqrt{q^2-p^2}}$

3. (b) : Given, $\sin A + \sin^2 A = 1$

$\Rightarrow \sin A = 1 - \sin^2 A$

$\Rightarrow \sin A = \cos^2 A$ $[\because \sin^2\theta + \cos^2\theta = 1]$

On squaring both sides, we get,

$\sin^2 A = \cos^4 A$

$\Rightarrow 1 - \cos^2 A = \cos^4 A \Rightarrow \cos^2 A + \cos^4 A = 1$

4. (a) : Given, $\sin A = \dfrac{1}{2}$

$\therefore \cos A = \sqrt{1-\sin^2 A} = \sqrt{1 - \left(\dfrac{1}{2}\right)^2} = \sqrt{1 - \dfrac{1}{4}} = \sqrt{\dfrac{3}{4}} = \dfrac{\sqrt{3}}{2}$

Now, $\cot A = \dfrac{\cos A}{\sin A} = \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

5. (d) : Given, $\sec\theta = \dfrac{17}{8} \Rightarrow \cos\theta = \dfrac{8}{17}$

Now, $\sin^2\theta = 1 - \cos^2\theta = 1 - \dfrac{(8)^2}{(17)^2} = \dfrac{225}{289}$

$\therefore \sin^2\theta - \cos^2\theta = \dfrac{225}{289} - \dfrac{64}{289} = \dfrac{161}{289}$

6. We have, $\dfrac{-101}{\cos^2 A} + \dfrac{101}{\cot^2 A}$

$= \left(\dfrac{-101}{\cos^2 A} + \dfrac{101 \times \sin^2 A}{\cos^2 A}\right) = \dfrac{-101(1-\sin^2 A)}{\cos^2 A}$

$= \dfrac{-101\cos^2 A}{\cos^2 A} = -101$ $[\because \sin^2 A + \cos^2 A = 1]$

7. Given, $\cot\theta - \cot^2\theta = 1$

$\Rightarrow$ $\cot\theta = 1 + \cot^2\theta$ ...(i)

$\Rightarrow$ $\cot\theta = \text{cosec}^2\theta$ ...(ii) $[\because 1 + \cot^2 A = \text{cosec}^2 A]$

Now, $\text{cosec}^2\theta - \text{cosec}^4\theta = \cot\theta - \cot^2\theta$ [Using (ii)]

$= 1$ [Using (i)]

8. L.H.S. $= 1 + \dfrac{\cot^2 x}{1+\text{cosec}\,x}$

$= 1 + \dfrac{\text{cosec}^2 x - 1}{1+\text{cosec}\,x} = 1 + \dfrac{(\text{cosec}\,x - 1)(\text{cosec}\,x + 1)}{1+\text{cosec}\,x}$

$= 1 + \text{cosec}\,x - 1 = \text{cosec}\,x = $ R.H.S.

9. $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$= (1 + \tan^2\theta)(1 - \sin^2\theta) = \sec^2\theta \cdot \cos^2\theta$

$[\because 1 + \tan^2\theta = \sec^2\theta$ and $1 - \sin^2\theta = \cos^2\theta]$

$= \dfrac{1}{\cos^2\theta}\cdot\cos^2\theta = 1$ $\left[\because \sec\theta = \dfrac{1}{\cos\theta}\right]$

10. Consider, $\sqrt{\dfrac{1-\sin^2 A}{\tan^2 A + 1}} = \sqrt{\dfrac{\cos^2 A}{\sec^2 A}}$

$[\because \sin^2\theta + \cos^2\theta = 1$ and $1 + \tan^2\theta = \sec^2\theta]$

$= \sqrt{\cos^2 A \times \cos^2 A} = \sqrt{\cos^4 A} = \cos^2 A$

11. Given, $2\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow$ $2\sin^2\theta - (1 - \sin^2\theta) = 2$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow$ $2\sin^2\theta + \sin^2\theta - 1 = 2$

$\Rightarrow$ $3\sin^2\theta = 3 \Rightarrow \sin^2\theta = 1$

$\Rightarrow$ $\sin\theta = 1 = \sin 90^\circ$ $[\because \sin 90^\circ = 1]$

$\therefore$ $\theta = 90^\circ$

12. Consider, $\dfrac{\cot(90^\circ - A)}{\cos A\cos(90^\circ - A)} - 1 = \dfrac{\tan A}{\cos A\cdot\sin A} - 1$

$[\because \cot(90^\circ - A) = \tan A, \cos(90^\circ - A) = \sin A]$

$= \dfrac{\sin A}{\cos^2 A\cdot\sin A} - 1 = \dfrac{1}{\cos^2 A} - 1 = \sec^2 A - 1$

$= \tan^2 A$ $[\because 1 + \tan^2 A = \sec^2 A]$

13. Given, $\cot^2\theta(\sec\theta - 1)(1 + \cos\theta) = k\cos\theta$

$\Rightarrow$ $\dfrac{\cos^2\theta}{\sin^2\theta}\left(\dfrac{1}{\cos\theta} - 1\right)(1+\cos\theta) = k\cos\theta$

$\Rightarrow$ $\dfrac{\cos^2\theta}{\sin^2\theta}\dfrac{(1-\cos\theta)}{\cos\theta}(1+\cos\theta) = k\cos\theta$

$\Rightarrow$ $\dfrac{1-\cos^2\theta}{\sin^2\theta} = k$

$\Rightarrow$ $\dfrac{\sin^2\theta}{\sin^2\theta} = k \Rightarrow k = 1$ $[\because \sin^2\theta + \cos^2\theta = 1]$

14. We have, $\dfrac{\cos 70^\circ}{3\sin 20^\circ} + \dfrac{4(\sec^2 59^\circ - \cot^2 31^\circ)}{3} - \dfrac{2}{3}\sin 90^\circ$

$= \dfrac{\cos(90^\circ - 20^\circ)}{3\sin 20^\circ} + \dfrac{4[\sec^2 59^\circ - \cot^2(90^\circ - 59^\circ)]}{3} - \dfrac{2}{3}\times 1$

$= \dfrac{\sin 20^\circ}{3\sin 20^\circ} + \dfrac{4(\sec^2 59^\circ - \tan^2 59^\circ)}{3} - \dfrac{2}{3} = \dfrac{1}{3} + \dfrac{4}{3} - \dfrac{2}{3} = \dfrac{3}{3} = 1$

$[\because \sec^2\theta - \tan^2\theta = 1]$

15. L.H.S. $= \sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$= \dfrac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}} + \dfrac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}} = \dfrac{1+\cos\theta+1-\cos\theta}{\sqrt{1-\cos\theta}\sqrt{1+\cos\theta}}$

$= \dfrac{2}{\sqrt{1-\cos^2\theta}} = \dfrac{2}{\sqrt{\sin^2\theta}} = \dfrac{2}{\sin\theta} = 2\,\text{cosec}\,\theta = $ R.H.S.

16. Given, $\dfrac{\cos^2 31^\circ + \cos^2 59^\circ}{5(\sin^2 25^\circ + \sin^2 65^\circ)} = 5P$

$\Rightarrow$ $\dfrac{\cos^2 31^\circ + \sin^2(90^\circ - 59^\circ)}{5(\sin^2 25^\circ + \cos^2(90^\circ - 65^\circ))} = 5P$

$[\because \sin(90^\circ - \theta) = \cos\theta$ and $\cos(90^\circ - \theta) = \sin\theta]$

$\Rightarrow$ $\dfrac{\cos^2 31^\circ + \sin^2 31^\circ}{5(\sin^2 25^\circ + \cos^2 25^\circ)} = 5P$

$\Rightarrow$ $\dfrac{1}{5\times 1} = 5P$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow$ $P = \dfrac{1}{25}$

$\Rightarrow$ $\sqrt{P} = \sqrt{\dfrac{1}{25}} = \dfrac{1}{5}$

17. We know that, $\sec\theta = \dfrac{1}{\cos\theta} \Rightarrow \sec^2\theta = \dfrac{1}{\cos^2\theta}$

$\Rightarrow$ $\sec^2\theta = \dfrac{1}{1-\sin^2\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \dfrac{1}{1 - \dfrac{1}{\text{cosec}^2\theta}} = \dfrac{1}{\dfrac{\text{cosec}^2\theta - 1}{\text{cosec}^2\theta}} = \dfrac{\text{cosec}^2\theta}{\text{cosec}^2\theta - 1}$

$\therefore$ $\sec\theta = \dfrac{\text{cosec}\theta}{\sqrt{\text{cosec}^2\theta - 1}}$

18. L.H.S. $= (\tan A + \text{cosec}\,B)^2 - (\cot B - \sec A)^2$

$= \tan^2 A + \text{cosec}^2 B + 2\tan A\,\text{cosec}\,B - [\cot^2 B + \sec^2 A - 2\cot B\sec A]$

$= \tan^2 A + \text{cosec}^2 B + 2\tan A\,\text{cosec}\,B - \cot^2 B - \sec^2 A + 2\cot B\sec A$

$= -1 + 1 + 2\tan A\,\text{cosec}\,B + 2\cot B\sec A$

$[\because 1 + \tan^2 A = \sec^2 A, \cot^2 A + 1 = \text{cosec}^2 A]$

$= 2[\tan A\,\text{cosec}\,B + \cot B\sec A]$

$= 2\left[\dfrac{\sin A}{\cos A}\times\dfrac{1}{\sin B} + \dfrac{\cos B}{\sin B}\times\dfrac{1}{\cos A}\right]$

$= 2\sec A\,\text{cosec}\,B\,[\sin A + \cos B] = $ R.H.S.

19. Given, $4\tan\theta = 3 \Rightarrow \tan\theta = 3/4$

Also, $\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1 + \left(\dfrac{3}{4}\right)^2} = \sqrt{\dfrac{16+9}{16}} = \dfrac{5}{4}$

We have, $\dfrac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}=\dfrac{4\dfrac{\sin\theta}{\cos\theta}-\dfrac{\cos\theta}{\cos\theta}+\dfrac{1}{\cos\theta}}{4\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\cos\theta}-\dfrac{1}{\cos\theta}}$

$=\dfrac{4\tan\theta-1+\sec\theta}{4\tan\theta+1-\sec\theta}$ $\left[\because \tan\theta=\dfrac{\sin\theta}{\cos\theta} \text{ and } \sec\theta=\dfrac{1}{\cos\theta}\right]$

$=\dfrac{4\left(\dfrac{3}{4}\right)-1+\dfrac{5}{4}}{4\left(\dfrac{3}{4}\right)+1-\dfrac{5}{4}}=\dfrac{\dfrac{12}{4}-1+\dfrac{5}{4}}{\dfrac{12}{4}+1-\dfrac{5}{4}}=\dfrac{\dfrac{12-4+5}{4}}{\dfrac{12+4-5}{4}}=\dfrac{13}{11}$

20. L.H.S. = $\sin^2\theta\tan\theta+\cos^2\theta\cot\theta+2\sin\theta\cos\theta$

$=\sin^2\theta\dfrac{\sin\theta}{\cos\theta}+\cos^2\theta\dfrac{\cos\theta}{\sin\theta}+2\sin\theta\cos\theta$

$=\dfrac{\sin^3\theta}{\cos\theta}+\dfrac{\cos^3\theta}{\sin\theta}+2\sin\theta\cos\theta$

$=\dfrac{\sin^4\theta+\cos^4\theta}{\cos\theta\sin\theta}+2\sin\theta\cos\theta$

$=\dfrac{\sin^4\theta+\cos^4\theta+2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$

$=\dfrac{(\sin^2\theta+\cos^2\theta)^2}{\sin\theta\cos\theta}=\dfrac{1}{\sin\theta\cos\theta}$ ...(i)

Now, R.H.S. = $\tan\theta+\cot\theta$

$=\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}=\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$

$=\dfrac{1}{\sin\theta\cos\theta}$ ...(ii)

From (i) and (ii), L.H.S. = R.H.S.

21. L.H.S. $=\dfrac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$

On multiplying numerator and denominator by $(\sec\theta-\tan\theta)$, we get

$\dfrac{(1+\sec\theta-\tan\theta)(\sec\theta-\tan\theta)}{(1+\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}$

$=\dfrac{(1+\sec\theta-\tan\theta)(\sec\theta-\tan\theta)}{(\sec\theta-\tan\theta)+(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}$

$=\dfrac{(1+\sec\theta-\tan\theta)(\sec\theta-\tan\theta)}{(\sec\theta-\tan\theta)+(\sec^2\theta-\tan^2\theta)}$

$=\dfrac{(1+\sec\theta-\tan\theta)(\sec\theta-\tan\theta)}{(\sec\theta-\tan\theta+1)}$ $[\because \sec^2\theta-\tan^2\theta=1]$

$=\sec\theta-\tan\theta$

$=\dfrac{1}{\cos\theta}-\dfrac{\sin\theta}{\cos\theta}$ $\left[\because \sec\theta=\dfrac{1}{\cos\theta} \text{ and } \tan\theta=\dfrac{\sin\theta}{\cos\theta}\right]$

$=\dfrac{1-\sin\theta}{\cos\theta}=$ R.H.S.

22. L.H.S. $=\dfrac{\tan^2 A}{\tan^2 A-1}+\dfrac{\operatorname{cosec}^2 A}{\sec^2 A-\operatorname{cosec}^2 A}$

$=\dfrac{\dfrac{\sin^2 A}{\cos^2 A}}{\dfrac{\sin^2 A}{\cos^2 A}-1}+\dfrac{\dfrac{1}{\sin^2 A}}{\dfrac{1}{\cos^2 A}-\dfrac{1}{\sin^2 A}}$

$=\dfrac{\sin^2 A}{\sin^2 A-\cos^2 A}+\dfrac{\cos^2 A}{\sin^2 A-\cos^2 A}$

$=\dfrac{\sin^2 A+\cos^2 A}{\sin^2 A-\cos^2 A}$

$=\dfrac{1}{\sin^2 A-\cos^2 A}$ $[\because \sin^2 A+\cos^2 A=1]$

$=\dfrac{1}{1-\cos^2 A-\cos^2 A}=\dfrac{1}{1-2\cos^2 A}=$ R.H.S.

23. We have, $\tan\theta+\sin\theta=m$ ...(i)

and $\tan\theta-\sin\theta=n$ ...(ii)

Squaring (i) and (ii) and then subtracting, we get

$m^2-n^2=\tan^2\theta+\sin^2\theta+2\tan\theta\sin\theta-\tan^2\theta-\sin^2\theta+2\tan\theta\sin\theta$

$\Rightarrow m^2-n^2=4\tan\theta\sin\theta$ ...(iii)

Multiplying (i) and (ii), we get

$\tan^2\theta-\sin^2\theta=mn$

$\Rightarrow \dfrac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta=mn$ $\left[\because \tan\theta=\dfrac{\sin\theta}{\cos\theta}\right]$

$\Rightarrow \dfrac{\sin^2\theta}{\cos^2\theta}(1-\cos^2\theta)=mn$

$\Rightarrow \tan^2\theta\sin^2\theta=mn$

$\Rightarrow \tan\theta\sin\theta=\sqrt{mn}$

Using above value in (iii), we get

$m^2-n^2=4\sqrt{mn}$

24. Given, $\cot\theta=\sqrt{1-k^2}\Rightarrow\cot^2\theta=1-k^2$

$\Rightarrow \dfrac{1}{\tan^2\theta}=1-k^2\Rightarrow\tan^2\theta=\dfrac{1}{1-k^2}$

Now, $\sec^2\theta=1+\tan^2\theta=1+\dfrac{1}{1-k^2}$

$\Rightarrow \sec^2\theta=\dfrac{1-k^2+1}{1-k^2}=\dfrac{2-k^2}{1-k^2}$

$\Rightarrow \sec\theta=\sqrt{\dfrac{2-k^2}{1-k^2}}$

Again, $\operatorname{cosec}^2\theta=\cot^2\theta+1=1+(1-k^2)$

$\Rightarrow \operatorname{cosec}\theta=\sqrt{2-k^2}$

$\therefore \operatorname{cosec}\theta+\cot^3\theta\sec\theta=\sqrt{2-k^2}+(1-k^2)^{3/2}\times\sqrt{\dfrac{2-k^2}{1-k^2}}$

$$= \sqrt{2-k^2} + \frac{\sqrt{1-k^2} \times (1-k^2)\sqrt{2-k^2}}{\sqrt{1-k^2}}$$

$$= \sqrt{2-k^2}(1+1-k^2) = \sqrt{2-k^2}(2-k^2) = (2-k^2)^{3/2}$$

25. Given, $\text{cosec}\theta + \cot\theta = p$

$$\Rightarrow \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = p$$

$$\left[\because \text{cosec}\theta = \frac{1}{\sin\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$\Rightarrow \frac{1+\cos\theta}{\sin\theta} = p$$

$$\Rightarrow \frac{(1+\cos\theta)^2}{\sin^2\theta} = p^2 \quad \text{[Taking square on both sides]}$$

$$\Rightarrow \frac{1+\cos^2\theta + 2\cos\theta}{\sin^2\theta} = p^2$$

Now, $\text{R.H.S.} = \frac{p^2-1}{p^2+1} = \frac{\frac{1+\cos^2\theta+2\cos\theta}{\sin^2\theta} - 1}{\frac{1+\cos^2\theta+2\cos\theta}{\sin^2\theta} + 1}$

$$= \frac{1+\cos^2\theta + 2\cos\theta - \sin^2\theta}{1+\cos^2\theta + 2\cos\theta + \sin^2\theta}$$

$$= \frac{\cos^2\theta + 2\cos\theta + \cos^2\theta}{1+1+2\cos\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2\cos^2\theta + 2\cos\theta}{2+2\cos\theta} = \frac{2\cos\theta(\cos\theta+1)}{2(1+\cos\theta)}$$

$= \cos\theta = \text{L.H.S.}$