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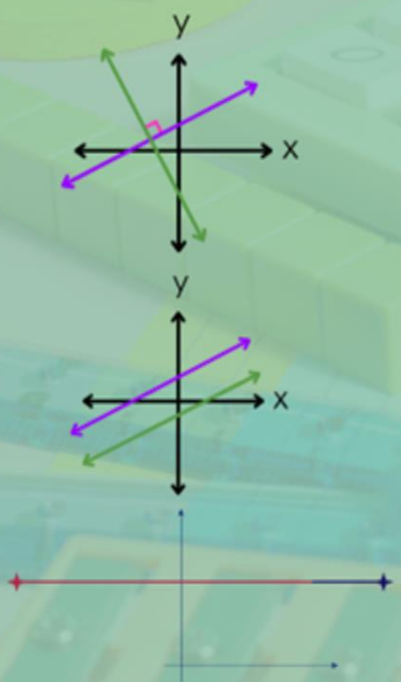
LINEAR EQUATIONS IN TWO VARIABLES



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02

Introduction

In mathematics, an equation is a statement that asserts the equality of two expressions which are connected by the equal sign "=".

Example: $2x = 10$, $5x + 5 = 40$ etc.

The variables for which the equation has to be solved are also called unknowns and the values of the unknowns that satisfy the equation are known as the solutions of the equation.

A linear equation is an algebraic equation in which all variables involved have the highest power 1.

For example: $3x - 4 = 0$.

Linear Equation

A linear equation is an algebraic equation in which all variables involved have the highest power as 1.

For example : $3x - 4 = 0$, $2x + y = 6$.

Linear Equations in one Variable

An equation which contains only one variable with degree 1 is known as a linear equation in one varibale.

Example: $4y = 20$, $15x - 10$, $13x = 52$ etc.

Linear Equations in two Variables

An equation which contains two unknown variables is known as a linear equation in two variables

Example: $2x - y = 10$, $10x + 4y = 3$ etc.

OR

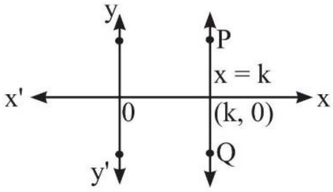
If an equation is written in the form $ax + by = c$, where a, b, c are real numbers and coefficients of x and y i.e., a and b are not equal to zero is known as a linear equation in two variables x and y.

- For a linear equation $ax + by = c$, the graph is a straight line.

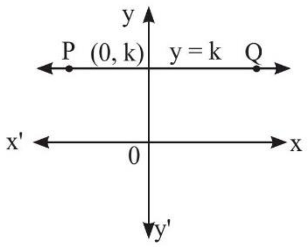
  $6x + 4y = 36$, is an example of a linear equation in two variables.

Note:

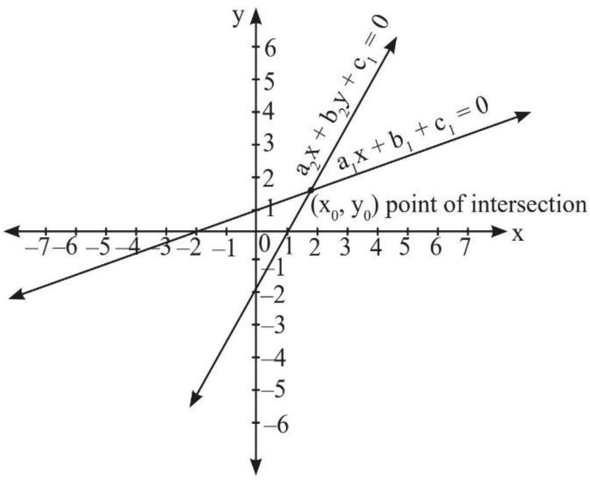
- A straight line parallel to the y-axis at a distance of k units from the y-axis is the graph of an equation of type $x = k$ (where k is a constant).



- A straight line parallel to the x-axis at a distance of k units from the x-axis is the graph of an equation of type y = k (where k is a constant).



- The solution of the two equations is given by the point of intersection of the two lines.



- There are infinite number of solutions for a single linear equation in two variables.

Pair of Linear Equations In Two Variables

Two or more linear equations in the same two variables are known as the pair of linear equation in two variables.

For example: $\left.\begin{array}{l} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{array}\right\}$ General form of pair of Linear equation in two variables.

Here a_1, a_2, b_1, b_2, c_1, c_2 are real numbers, such that $a_1^2 + b_1^2 \neq 0$ and $a_2^2 + b_2^2 \neq 0$.

It is clear that, a system of simultaneous linear equations is formed by a pair of linear equations in two variables.

For e.g.: 2x + 3y = 7 and 9x – 2y = –8 are pair of linear equations.

Pair of Linear Equations are of Two Types

- **Consistent Pair of Linear Equations:** A pair of linear equation in two variables which has atleast one solution is called a consistent pair of linear equations.
  - **Independent Pair of Linear Equations:** A pair of linear equation which has exactly one solution is known as independent pair of linear equations.
  - **Dependent Pair of Linear Equations:** A pair of linear equation which has infinitely many distinct common solutions. It needs to be noted that a dependent pair of linear equations is always consistent.
- **Inconsistent Pair of Linear Equations:** A pair of linear equation which has no solution is called an inconsistent pair of linear equation.

Note:

- Dependent equations are the equation of the type ax + by = c and kax + kby = kc.

  e.g., 2x + 3y = 9 and, ...(i)

  4x + 6y = 18 ...(ii)

As, we can see that equation (ii) is the multiple of equation (i). Hence, both equations are basically same that means equation (ii) is dependent on equation (i). Dependent pair of equations have infinitely many solutions.

- Inconsistent equations are the equation of the type $ax + by = c$ and $kax + kby = mc$.

  For e.g., $x + 2y = 4$ and $2x + 4y = 12$.

Solution of a Pair of Linear Equations

The pair of linear equations in two variables can be solved by the following two methods:

(i) Graphical Method

(ii) Algebraic Method

Graphical Method of Solution of a Pair of Linear Equations

The graph of a pair of linear equations in two variables is shown by two straight lines. The behaviour of lines representing a pair of linear equations in two variables and the existence of solutions can be summarized as follows:

(i) Lines are intersecting, if the system has a unique solution.

(ii) Lines are coincident, if the system has infinite number of solutions.

(iii) Lines are parallel, if the system has no solution.

Graphical representation of simultaneous equations:

(i) Intersecting lines (Unique solution)

$x + y = 4$ ...(i)

and $3x + 2y = 11$ ...(ii)

To obtain the equivalent geometric representation, we find two points on the line representing each equation. That is, we find two solutions of each equation.

These solutions are given below in the table.

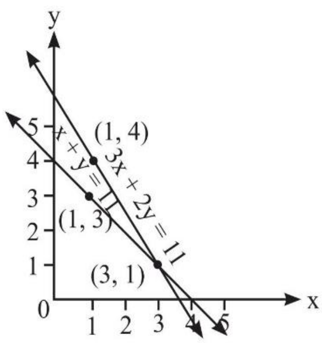
For $x + y = 4$

x	1	3
y	3	1

For $3x + 2y = 11$

x	3	1
y	1	4

The given equations representing two lines, intersect each other at a unique point (3, 1). Hence, the equations are consistent with a unique solution.



(ii) Coincident lines (Infinite number of solutions)

$x + y = 8$ ...(i)

and $2x + 4y = 16$ ...(ii)

To obtain the equivalent geometric representation, we find two points on the line representing each equation. That is, we find two solutions of each equation.

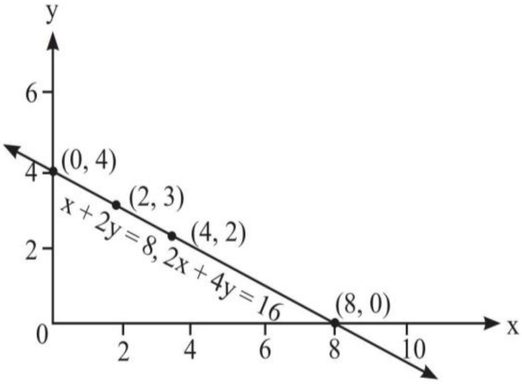
These solutions are given below in the table.

For x + 2y = 8

x	0	8
y	4	0

For 2x + 4y = 16

x	2	4
y	3	2



Clearly, the graph of given equations are coincident lines. Hence, given system of equations has infinite many solution.

(iii) Parallel lines (No solution)

x + y = 4 ...(i)

2x + 2y = 12 ...(ii)

To obtain the equivalent geometric representation, we find two points on the line representing each equation. That is, we find two solutions of each equation.

These solutions are given below in the table.

For x + y = 4

x	0	4
y	4	0

For 2x + 2y = 12

x	0	6
y	6	0

Clearly, the graphs of the given equations are parallel lines. As they have no common point, there is no common solution. Hence the given system of equations has no solutions.

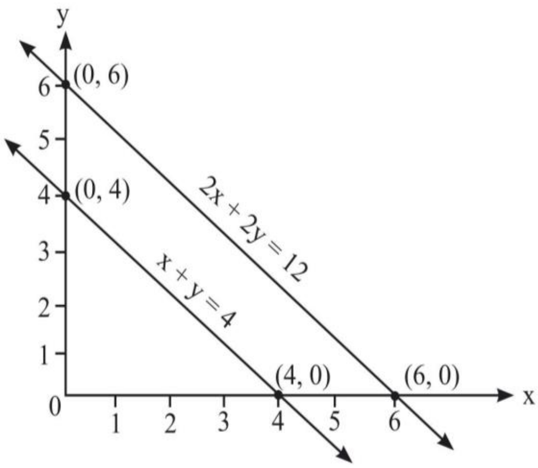


Table: Conditions for simultaneous equations

Pair of linear equations $a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	Algebraic conditions	Graphical representation	Algebraic interpretation
Consistent (Independent)	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique solution)
Consistent (Dependent)	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
In-consistent	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Pair of parallel lines	No solution

Example

1. Consider the following system of equations:

2x + 5y = 0 and 4x + 3y = 0

(a) How many solutions are possible, if the system is consistent?

(b) If the coefficient of y in second equation is replaced by 10, will there be any change in the number of solutions? Solve the following system of simultaneous linear equations graphically and explain your answer..

Ans. (a) Method-I: The given equations are:

$2x + 5y = 0$ ...(i)

$4x + 3y = 0$ ...(ii)

Here, $a_1 = 2$, $b_1 = 5$, $a_2 = 4$, $b_2 = 3$

Now, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{5}{3}$

i.e., $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given system is consistent and has exactly one solution.

Method-II:

$2x + 5y = 0 \Rightarrow y = \frac{-2x}{5}$

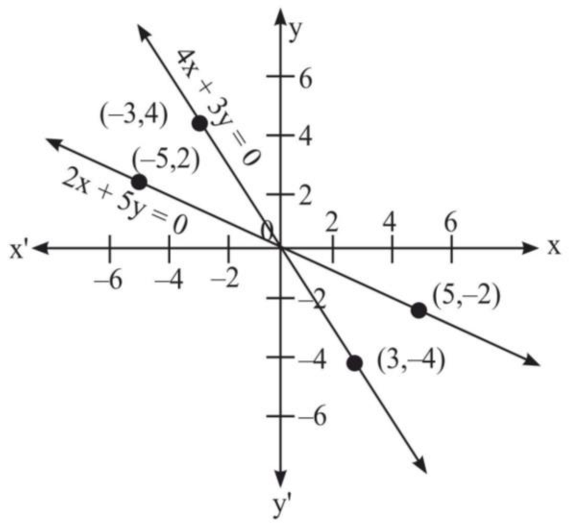
x	0	–5	5
y	0	2	–2

Hence, points are (0, 0), (–5, 2) and (5, –2)

$4x + 3y = 0 \Rightarrow y = \frac{-4x}{3}$

x	0	3	–3
y	0	–4	4

Hence, points are (0, 0), (–3, 4) and (3, –4)



As we can see from the graph that the two lines intersect at a point (0, 0). Hence, the solution of the pair of linear equations is x = 0, y = 0.

(b) If the coefficient of y in second equation is replaced by 10, then:

$2x + 5y = 0$ ...(iii)

$4x + 10y = 0$ ...(iv)

$2x + 5y = 0$

$\Rightarrow y = \frac{-2x}{5}$

x	0	5	–5
y	0	–2	2

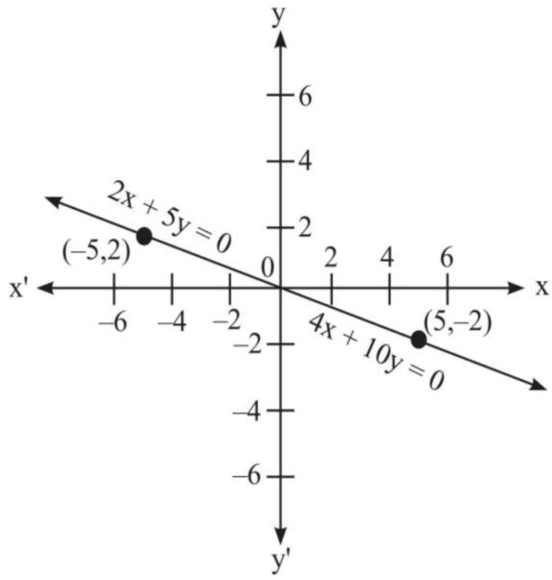
Hence, points are (0,0), (–5,2), (5, –2)

$4x + 10y = 0$

$\Rightarrow y = \frac{-2x}{5}$

x	0	–5	5
y	0	2	–2

Hence, points are (0,0), (5,–2), (–5,2)



As we can see from the graph that two lines are coincident. Hence, the system has infinitely many solutions.

2. A music store sold 7 violins in one week for a total of ₹1500. Two different types of violins were sold. One type cost ₹200 and the other type cost ₹300. Represent this situation algebraically and graphically.

Ans. Let us suppose, A and B are the two types of violins.

Hence,

Number of violins of type A + Number of violins of type B = Total number of violins sold

And,

(Price of type A violin) (Number of type A violins) + (Price of type B violin) (Number of type B violins) = Total sales

Let,

Number of type A violins = x

Number of type B violins = y

Total number of violins sold = 7

Price of type A violin= ₹ 200 (per violin)

Price of type B violin = ₹ 300 (per violin)

Total sales = ₹ 1500

Now, according to the question,

Algebraically, $x + y = 7$ ...(i)

$200x + 300y = 1500$ ...(ii)

Now, graphically

$x + y = 7$

$\Rightarrow y = 7 - x$

x	0	3	7
y	7	4	0

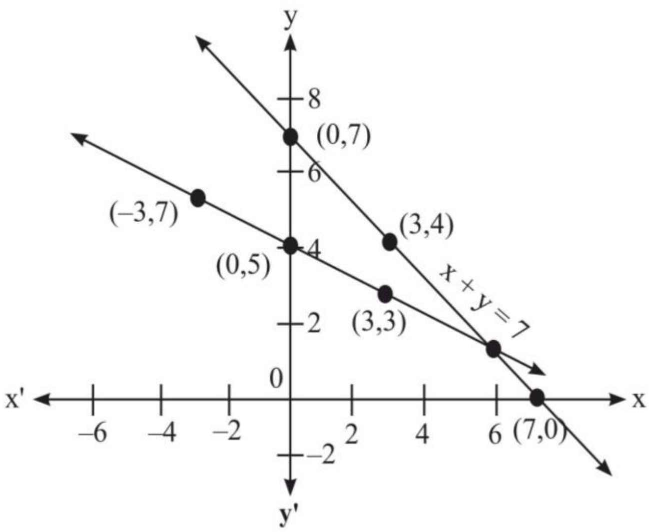
Hence, points are (0,7), (3,4), (7,0)

$200x + 300y = 1500$

$\Rightarrow 2x + 3y = 15 \Rightarrow y = \frac{15 - 2x}{3}$

x	0	–3	3
y	5	7	3

Hence, points are (0,5), (–3,7), (3,3)



Hence, we can see from the graph that two lines are intersecting in nature.

3. Two rails are represented by the equations x + 2y – 6 = 0 and x + 2y – 4 = 0. Represent this situation graphically.

Ans. The given equations are:

$x + 2y - 6 = 0$ ...(i)

$x + 2y - 4 = 0$ ...(ii)

$x + 2y - 6 = 0$

$\Rightarrow y = \frac{6 - x}{2}$

x	0	2	6
y	3	2	0

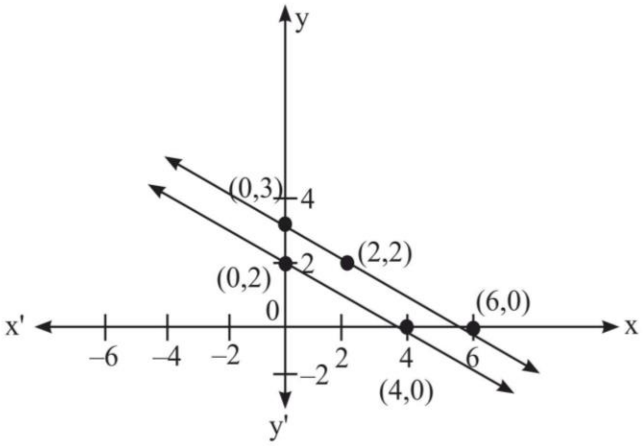
Hence, points are (0,3), (2,2), (6,0)

$x + 2y - 4 = 0$

$\Rightarrow y = \frac{4 - x}{2}$

x	0	2	4
y	2	1	0

Points are (0,2), (2,1), (4,0)



Hence, we can see from the graph that two lines are parallel to each other.

4. Solve graphically the following system of linear equations:

$3x + y + 1 = 0$

$2x - 3y + 8 = 0$

And, shade the area of the region bounded by the lines and x-axis.

Ans. The given equations are:

$3x + y + 1 = 0$ ...(i)

$\Rightarrow y = -(3x + 1)$

Hence, table of solutions for (i)

x	–2	–1	1
y = – (3x + 1)	5	2	–4

Hence, points are (–2, 5), (–1, 2) and (1, –4)

$2x - 3y + 8 = 0$ ...(ii)

$\Rightarrow y = \frac{(2x + 8)}{3}$

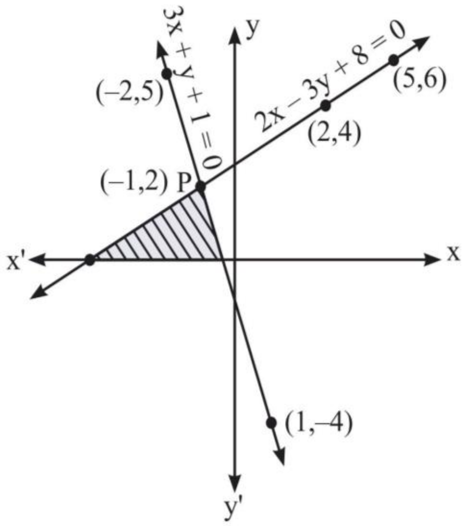
Table of solutions for (ii)

x	2	–1	5
$y=\frac{(2x+8)}{3}$	4	2	6

Hence, points are (2, 4), (–1, 2) and (5, 6)

As we can see from the graph that the two straight lines intersect each other at the point P(–1, 2)

Therefore, x = –1, y = 2 is the solution of the given system. The area of the region bounded by the two lines and the x-axis is shaded in the graph.



5. Solve graphically the following system of linear equations: 2x + y = 6, x – 2y + 2 = 0. Find the vertices of the triangle formed by the above two lines and the x-axis. Also, find the area of the triangle.

Ans. The given equations are:

2x + y = 6 ...(i)

⇒ y = 6 – 2x

Hence table of solutions for (i)

x	1	2	4
y = 6 – 2x	4	2	–2

Therefore, points are (1, 4), (2, 2) and (4, –2)

x – 2y + 2 = 0 ...(ii)

$\Rightarrow y=\frac{x+2}{2}$

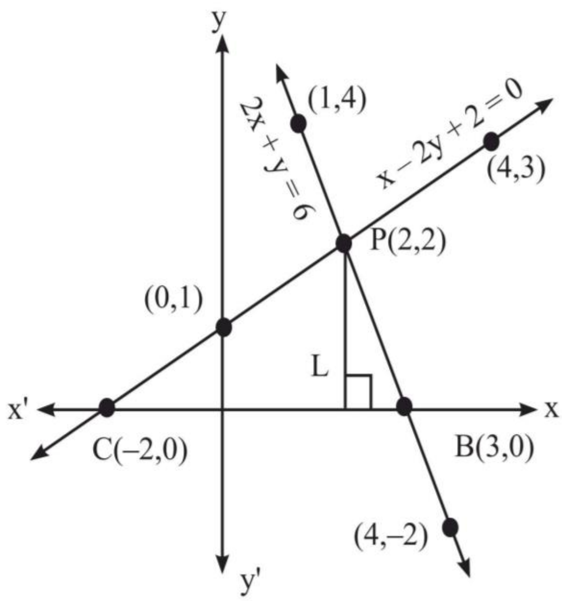
Table for solutions for (ii)

x	0	2	4
$y=\frac{x+2}{2}$	1	2	3

Hence, points are (0, 1), (2, 2) and (4, 3)

As we can see from the graph that the two straight lines intersect at P(2, 2).

Therefore, x = 2, y = 2 is the solution of the given system.



Vertices of the triangle are P(2,2), B(3,0) and C(–2,0)

PL ⊥ BC.

As, BC = 5 units, PL = 2 units

Therefore, Area of ΔPBC $=\frac{1}{2}(5\times2)=5$ sq. units.

Algebraic Methods of Solving a Pair of Linear Equations

For finding the solutions of a pair of linear equations, following methods are used:

1. Substitution method
2. Elimination method
3. Cross-multiplication method

❒ Substitution Method

In method, we first solve one equation to get the value of one variable (y) in terms of another variable (x). Then, in the second equation, this value of y is substituted. The second equation, becomes a linear equation in x only which may be solved for x. Then, we can find the value of y by putting the value of x in the first equation.

Solving a system of linear equations using this method is known as the method of **elimination by substitution**.

i.e., A pair of linear equations in two variables x and y can be solved by substitution method by using the following steps:

$2x - 3y = 7$ ...(i)

and $x + y = 1$ ...(ii)

Step-1: Express y in terms of x (or x in terms of y) by choosing one of the above two equations. That means one variable is expressed in terms of another variable.

Using the second equation, find x in terms of y.

i.e. $x + y = 1$

$\Rightarrow y = 1 - x$

Step-2: Value of y obtained in step-1 is then substituted in the other equation to get a linear equation in x.

$\therefore 2x - 3(1 - x) = 7$

Step-3: Then, value of x can be obtained by solving the linear equation obtained in step-2.

$2x - 3 + 3x = 7$

$\Rightarrow 5x = 7 + 3 = 10$

$\Rightarrow x = 2$

Step-4: The value of x obtained is then substituted in the relation obtained in step-1 and then find the value of y.

$y = 1 - x$

$\Rightarrow y = 1 - 2$

$\Rightarrow x = -1$

$\therefore$ The solution for the given pair of equations is x = 2 and y = –1.

Note :

This method is known as the method of elimination by substitution, the reason behind this is 'Elimination' because, y is eliminated from the second equation and 'substitution' because value of y is substituted in the second equation.

Example

1. Solve the given equations for x and y : 4x + 3y = 24, 3y –2x = 6.

Ans. The given equations are:

$4x + 3y = 24$ ...(i)

$3y - 2x = 6$ ...(ii)

By using equation (i), we get

$$y = \frac{24 - 4x}{3} \quad ...(iii)$$

On substituting the value of y in eq. (ii), we get

$$3\left(\frac{24 - 4x}{3}\right) - 2x = 6 \quad ...(iv)$$

$\Rightarrow 24 - 4x - 2x = 6$

$\Rightarrow -6x = -24 + 6$

$\Rightarrow 6x = 18$

$\Rightarrow x = 3$

On substituting the value of x in eq. (iii) we get

$$y = \frac{24 - 12}{3} \Rightarrow \frac{12}{3} = 4$$

Therefore, x = 3 and y = 4.

2. Solve the following pair of linear equations by the substitution method.

$\sqrt{2}x+\sqrt{3}y=0$ and $\sqrt{3}x-\sqrt{8}y=0$

Ans. The given equations are:

$\sqrt{2}x+\sqrt{3}y=0$ ...(i)

and $\sqrt{3}x-\sqrt{8}y=0$ ...(ii)

From equation (i), we get $y=\frac{-\sqrt{2}x}{\sqrt{3}}$ ...(iii)

On substituting the value of y in eq. (ii), we get

$$\sqrt{3}x-\sqrt{8}\left(\frac{-\sqrt{2}x}{\sqrt{3}}\right)=0$$

$\Rightarrow \sqrt{3}x+\frac{4x}{\sqrt{3}}=0$

$\Rightarrow 3x + 4x = 0$

$\Rightarrow 7x = 0$

$\Rightarrow x = 0$

On substituting the value of x in eq. (iii),

we get, $y=\frac{-\sqrt{2}\times 0}{\sqrt{3}}=0$

Therefore, the solution is x = 0 and y = 0.

3. Rajan combine 2 solutions to form a mixture that is 40% acid. One solution was 20% acid and the other was 50% acid. If Rajan had 90 milliliters of the mixture, how much of each solution was used to create the mixture.

Ans. Let us suppose, A and B be the two types of solutions.

Volume of solution A + Volume of solution B
= Volume of mixture

Acid in solution A + Acid in solution B
= Acid in mixture

Let volume of solution A = x mL

Let volume of solution B = y mL

Volume of mixture = 90 mL

Acid in solution A = 0.2 x mL

Acid in solution B = 0.5 y mL

Acid in mixture = 0.4 × 90 mL = 36 mL

According to the question,

$x + y = 90$ ...(i)

$0.2x + 0.5y = 36$ ...(ii)

Now, on solving eq. (i) for value of x and on multiplying eq. (ii) by 10, we get

$x = 90 – y$ ...(iii)

$2x + 5y = 360$ ...(iv)

On putting value of x from eq. (iii) in (iv), we get

$2(90 – y) + 5y = 360$

$180 – 2y + 5y = 360$

$180 + 3y = 360$

$3y = 180$

$y = 60$

Put value of y in eq. (i),

$x + 60 = 90$

$x = 30$

Hence, 30 mL of solution A and 60 mL of solution B were used to form the mixture.

4. One day, the National Museum of India, New Delhi admitted 321 adults and children and collected ₹1590. The price of admission is ₹6 for an adult and ₹4 for a child. How many adults and how many children were admitted to the museum on the day?

Ans. Let us suppose, x and y be the number of adults and number of children respectively.

Total number of people admitted = 321

Price of admission for an adult = ₹ 6

Price of admission for a child = ₹ 4

Total amount collected = ₹ 1590

According to the question,

$x + y = 321$ ...(i)

$6x + 4y = 1590$ ...(ii)

On solving eq. (i)

$x = –y + 321$ ...(iii)

On putting value of x in eq. (ii), we get

$6(–y + 321) + 4y = 1590$

$–6y + 1926 + 4y = 1590$

$–2y + 1926 = 1590$

$–2y = –336$

$y = 168$

On putting value of y in eq. (iii), we get

$x = –(168) + 321 = 153$

Hence, 153 adults and 168 children were admitted to the National Museum of India that day.

❒ Elimination Method

A pair of linear equation in two variables x and y can be solved by elimination method, by using the following steps:

Let the two equations be

$2x + 3y = 19$ ...(i)

and $5x + 4y = 37$ ...(ii)

Step-1: The given equations is multiplied by a suitable number so that the coefficient of one of the variables become numerically equal.

Here, let us eliminate the y-term and in order to eliminate the y term, we have to multiply the first equation by 4 and the second equation by 3.

$(2x + 3y = 19) \times 4 \Rightarrow 8x + 12y = 76$ ...(iii)

$(5x + 4y = 37) \times 3 \Rightarrow 15x + 12y = 111$ ...(iv)

Step-2: If the numerically equal coefficients are opposite in sign, then add the new equations otherwise subtract them.

Here, we subtract the equation (iii) from (iv) as the signs of numerically equal coefficients are same i.e., +ve.

$$\begin{array}{r} 15x + 12y = 111 \\ -\ \ 8x + 12y = 76 \\ \hline \end{array}$$

Step-3: Then, the linear equations in one variable obtained in step-2 is solved to give the value of one variable.

On solving, we get

$7x = 35$

$\Rightarrow x = 5$

Step-4: This value of the variable obtained in step-3 is then substituted in any of the two equations to find the value of another variable.

Substitute the value of x in the second equation, we get

$5(5) + 4(y) = 37$

$\Rightarrow \quad 4y = 37 - 25$

$\Rightarrow \quad 4y = 12$

$\Rightarrow \quad y = 3$

$\therefore$ The solution of the given pair of equaion is x = 5 and y = 3

Example

1. Use elimination method, to solve the following pair of equations.

x + y = 5 and 2x – 3y = 4

Ans. The given equations are:

$x + y = 5$ ...(1)

$2x - 3y = 4$ ...(2)

On multiplying eq. (1) by 3 and (2) by 1 and adding, we get

$$\begin{array}{l} 3x + 3y = 15 \\ 2x - 3y = 4 \\ \hline 5x \quad\quad = 19 \end{array}$$

$\Rightarrow x = \frac{19}{5}$

On putting value of x in eq. (1), we get

$\frac{19}{5} + y = 5$

$\Rightarrow y = 5 - \frac{19}{5}$

$\Rightarrow y = \frac{6}{5}$

Therefore, $x = \frac{19}{5}$ and $y = \frac{6}{5}$.

2. Solve the following pair of linear equations by elimination method 3x + 4y = 10 and 2x – 2y = 2.

Ans. The given equations are:

$3x + 4y = 10$ ...(1)

and $2x - 2y = 2$ ...(2)

On multiplying eq. (2) by 2, we get

$4x - 4y = 4$ ...(3)

Now on adding eqn (1) and (3), we get

$3x + 4y = 10$

$4x - 4y = 4$

$7x \quad = 14$

$\Rightarrow x = 2$

On putting x = 2 in equation (2), we get

$2 \times 2 - 2y = 2$

$\Rightarrow 2y = 4 - 2$

$\Rightarrow y = 1$

Hence, the solution is x = 2 and y = 1.

3. A gold crown, suspected of containing some silver, was found to have a mass of 714 grams and a volume of 46 cubic centimeters. The density of gold is about 19 grams per cubic centimeter. The density of silver is about 10.5 grams per cubic centimeter. What percent of the crown is silver?

Ans. Let volume of gold = G (in cm^3)

Let volume of silver = S (in cm^3)

Total volume = 46 (in cm^3)

Density of gold = 19 (g/cm^3)

Density of silver = 10.5 (g/cm^3)

Total mass = 714 (in grams)

According to the question,

$G + S = 46$ ...(i)

$19G + 10.5S = 714$ ...(ii)

On multiplying eq. (i) by-19, we get

$-19\,G - 19\,S = -874$ ...(iii)

Now, on adding eq. (ii) and iii, we get

$-8.5\,S = -160$

$\Rightarrow S \simeq 18.5$

Hence, the volume of silver is nearly about 19 cm^3. As, the crown has a volume of 46 cm^3, therefore the crown is $\frac{19}{46} \times 100 = 41\%$ silver by volume.

❒ Cross-multiplication method

Cross-multiplication is a method to find the solution of linear equations in two variables.

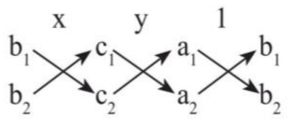
Consider the following system of linear equations:

$a_1x + b_1y + c_1 = 0$ ...(1) ;

$a_2x + b_2y + c_2 = 0$ ...(2)

A pair of linear equations can be solved by using the following steps:

Step-1: Firstly write the coefficients of variables as follows:



$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Now,

Case-I: If $a_1b_2 - a_2b_1 \neq 0 \Rightarrow$ system of equation will have unique solution and there will be some finite values for x and y.

Case-II: If $a_1b_2 - a_2b_1 = 0 \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

There are two more sub cases:

(a) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \lambda\ (\lambda \neq 0)$, then $a_1 = a_2\lambda$, $b_1 = b_2\lambda$ and $c_1 = c_2\lambda$

On substituting these values in equation $a_1x + b_1y + c_1 = 0$, we have ...(1)

$a_2\lambda x + b_2\lambda y + c_2\lambda = 0$

$\Rightarrow \lambda(a_2x + b_2y + c_2) = 0$ but $\lambda \neq 0$

$\Rightarrow a_2x + b_2y + c_2 = 0$ ...(2)

Therefore eq. (1) and (2) are dependent. Hence, there are infinite number of solutions.

(b) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow a_1b_2 - b_1a_2 = 0$

But $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \frac{\text{Finite value}}{0} =$ does not exist and $y = \frac{\text{Finite value}}{0} =$ does not exist

Hence, system of equations is inconsistent.

Example

1. Solve the given system of linear equations using cross-multiplication method:

3x + 2y = –25, –2x – y = 10

Ans. The given equations are:

$3x + 2y = -25 \Rightarrow 3x + 2y + 25 = 0$

$-2x - y = 10 \Rightarrow -2x - y - 10 = 0$

Here, $a_1 = 3$, $b_1 = 2$, $c_1 = 25$, $a_2 = -2$, $b_2 = -1$ and $c_2 = -10$.

By the method of cross multiplication, we have

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{b_2a_1 - b_1a_2}$$

On substituting the values in the above equation, we get

$$\frac{x}{[2\times(-10)]-[(-1)(25)]} = \frac{y}{[25\times(-2)]-[(-10)\times 3]} = \frac{1}{[3\times(-1)]-[(-2)\times(2)]}$$

$$\Rightarrow \frac{x}{-20-(-25)} = \frac{y}{-50-(-30)} = \frac{1}{-3-(-4)}$$

$$\Rightarrow \frac{x}{-20+25} = \frac{y}{-50+30} = \frac{1}{-3+4}$$

$$\Rightarrow \frac{x}{5} = \frac{y}{-20} = \frac{1}{1}$$

Hence, x = 5 and y = –20.

Mind it

- Equation of the form,

  $a_1x + b_1y + c_1z = d_1$

  $a_2x + b_2y + c_2z = d_2$

  $a_3x + b_3y + c_3z = d_3$

The following method may be used to solve the above type of equations.

Step 1: Consider anyone of the three given equations and find the value of one of the variables, say z, from it.

Step 2: Then substitute the value of z in the other two equations to obtain two linear equations in variables x, y.

Step 3: Solve the equations obtained in step-3 to get value of x and y by any method, we studied above.

Step 4: The values of x and y obtained in step-4 is then substituted in the equation obtained in step-2 to get value of z.

Example

1. From a bus stand in Bangalore, if we buy 2 tickets to Malleswarm and 3 tickets to Yeshwanthpur, the total cost is ₹46; but if we buy 3 tickets to Malleswarm, and 5 tickets to yeshwanthpur the total cost is ₹74. Find the fares from Bangalore to Malleswarm and to Yeshwanthpur.

Ans. Let fare from Bangalore bus stand to Malleswarm = ₹x

And fare from Bangalore bus stand to yeshwanthpur = ₹y

From the given information, we have

$2x + 3y = 46$ i.e., $2x + 3y - 46 = 0$ ...(i)

$3x + 5y = 74$ i.e., $3x + 5y - 74 = 0$ ...(ii)

Now, solving the equations by using cross multiplication method,

$$\Rightarrow \frac{x}{[3(-74)-(5)(-46)]} = \frac{y}{[(-46)(3)-(-74)(2)]} = \frac{1}{[(2)(5)-(3)(3)]}$$

$$\Rightarrow \frac{x}{-222+230} = \frac{y}{-138+148} = \frac{1}{10-9}$$

$$\Rightarrow \frac{x}{8} = \frac{y}{10} = \frac{1}{1}$$

$$\Rightarrow \frac{x}{8} = 1 \text{ and } \frac{y}{10} = 1$$

$\Rightarrow x = 8$ and $y = 10$

Hence, the fare from Bangalore to malleswarm = ₹8 and the fare from Bangalore to Yeshwanthpur = ₹10

2. Solve the following equations by cross-multiplication method

$(a - b)x + (a + b)y = 2(a^2 - b^2)$,

$(a + b)x - (a - b)y = 4ab$.

Ans. The given equations are:

$(a - b)x + (a + b)y - 2(a^2 - b^2) = 0$

$(a + b)x - (a - b)y - 4ab = 0$

Now, solving the above equations by cross-multiplication method, we get

$$\frac{x}{[(a+b)(-4ab)]-[-(a-b)-2(a^2-b^2)]}$$

$$= \frac{y}{\left[-2(a^2-b^2)(a+b)\right]-[(-4ab)(a-b)]}$$

$$= \frac{1}{[(a-b)-(a-b)]-[(a+b)(a+b)]}$$

On simplifying the expression under x, we get

$-4ab\ (a + b) - 2\ (a - b)\ (a^2 - b^2)$

$= -4ab(a + b) - 2(a - b)(a - b)(a + b)$

$[\because a^2 - b^2 = (a - b)(a + b)]$

$= -2(a + b)\ [2ab + (a - b)^2]$

$= -2(a + b)\ (2ab + a^2 + b^2 - 2ab)$

$= -2(a + b)(a^2 + b^2)$

On simplifying the expression under y, we get:

$-2(a^2 - b^2)\ (a + b) + 4ab\ (a - b)$

$= -2(a + b)\ (a - b)\ (a + b) + 4ab(a - b)$

$= -2(a - b)\ [(a + b)\ (a + b) - 2ab]$

$= -2(a - b)\ [a^2 + b^2 + 2ab - 2ab]$

$= -2(a - b)(a^2 + b^2)$

On simplifying of the expression under 1, we get

$-(a - b)^2 - (a + b)^2$

$= -(a^2 + b^2 - 2ab) - (a^2 + b^2 + 2ab)$

$= -2(a^2 + b^2)$

Therefore,

$$\frac{x}{-2(a+b)\left(a^2+b^2\right)} = \frac{y}{-2(a-b)\left(a^2+b^2\right)}$$

$$= \frac{1}{-2\left(a^2+b^2\right)}$$

$$\Rightarrow \frac{x}{a+b} = \frac{y}{a-b} = \frac{1}{1}$$

Hence, x = (a + b) and y = (a – b).

3. Solve the following system of equations.

$x - z = 5;\ y + z = 3;\ \ x - y = 2$

Ans. Given, $x - z = 5$ ....(i)

$y + z = 3$ ....(ii)

$x - y = 2$ .....(iii)

From equation (i), we can write,

$z = x - 5$

On putting value of z in eq. (ii), we get

$y + x - 5 = 3$

$\Rightarrow x + y = 8$ ....(iv)

On adding eq. (iii) and (iv), we get;

$2x = 10 \Rightarrow x = 5$

Now, on putting value of x in eq. (i), we get;

$5 - z = 5 \Rightarrow z = 0$

From eq. (ii), we get y = 3

Therefore, the solution of the given system of equation is x = 5, y = 3, z = 0.

Equation of the form ax + by = c and bx + ay = d, where a ≠ b.

For solving the equations of below form:

$ax + by = c$ ...(i) and $bx + ay = d$ ...(ii)

where $a \neq b$, the following steps need to be followed:

Step-1: First add eq. (i) and (ii) to get $(a + b)x + (b + a)y = c + d$.

Step-2: Then, subtract eq. (ii) from (i) to get $(a - b)x - (a - b)y = c - d$.

Step-3: Then, solve the equations obtained in step - 1 and step - 2 to get x and y.

Example

1. Solve the following equation for x and y : 47x + 31y = 63, 31x + 47y = 15.

Ans. The given equations are:

$47x + 31y = 63$ ...(i)

and $31x + 47y = 15$ ...(ii)

On adding eq. (i) and (ii), we get:

$78x + 78y = 78$

$\Rightarrow x + y = 1$ ...(iii)

On subtracting eq. (ii) from (i), we get:

$16x - 16y = 48 \Rightarrow x - y = 3$ ...(iv)

Now, on adding eq. (iii) and (iv), we get

$2x = 4 \Rightarrow x = 2$

On putting value of x in eq. (iii), we get:

$2 + y = 1 \Rightarrow y = -1$

Therefore, the solution is x = 2 and y = –1.

Equations Reducible to a Pair of Linear Equations in two Variables

Reciprocal equations are the equations which contain the variables, only in the denominators. These equations can be classified into three types and can be solved by the following methods,

Type-1: $\frac{a}{x}+\frac{b}{y}=c$ and $\frac{a'}{x}+\frac{b'}{y}=c'$ where, $a, b, c, a', b', c' \in R$

Example: $\frac{3}{x}+\frac{4}{y}=5$ and $\frac{7}{x}+\frac{9}{y}=63$

Put $\frac{1}{x}=p$ and $\frac{1}{y}=q$ and find the value of p and q by using any method explained earlier.

Thus $x=\frac{1}{p}$ and $y=\frac{1}{q}$.

Type-2: ax + by = cxy and a'x + b'y - c'xy where a, b, c, a', b', c' $\in$ R

Example: 3x + 4y = 5xy and 7x + 9y = 63xy

On dividing both equations by xy these equations can be converted in the form explained in type-1.

Type-3: $\frac{a}{lx+my}+\frac{b}{cx+dy}=k$ and $\frac{a'}{lx+my}+\frac{b'}{cx+by}=k$ where a, b, k, a', b', k' $\in$ R

Example: $\frac{1}{2x+3y}+\frac{3}{5x+7y}=7$ and $\frac{5}{2x+3y}+\frac{9}{5x+7y}=11$.

Put $\frac{1}{lx+my}=u$ and $\frac{1}{cx+dy}=v$

Hence, the above equations become, au + bv = k and a'u + b'v = k'

Then, find the values of u and v and put $lx+my=\frac{1}{u}$ and $cx+dy=\frac{1}{v}$

Then again solve the equations for x and y by using any method explained earlier.

Example

1. Solve for x and y: $\frac{3}{x}-\frac{2}{y}+5=0$ and $\frac{1}{x}+\frac{3}{y}-2=0$ $(x \neq 0, y \neq 0)$ [Type-1]

Ans. The given equations are: $\frac{3}{x}-\frac{2}{y}+5=0$ and

$\frac{1}{x}+\frac{3}{y}-2=0$

Put $\frac{1}{x}=u$ and $\frac{1}{y}=v$. Hence, the given equations become,

$3u - 2v = -5$ ...(i)

and $u + 3v = 2$ ...(ii)

On multiplying eq. (i) by 3 and eq. (ii) by 2, we obtain

$9u - 6v = -15$ ...(iii)

and $2u + 6v = 4$ ...(iv)

On adding eq. (iii) and (iv), we get

$11u = -11$

$u=\frac{-11}{11}$

$\Rightarrow u = -1$

Now, on putting value of u in eq. (ii), we get

$-1 + 3v = 2$

$\Rightarrow 3v = 3 \Rightarrow v = 1$

And as, $\frac{1}{x}=u$ and $\frac{1}{y}=v$

Hence, $\frac{1}{x}=-1 \Rightarrow x = -1$

and $\frac{1}{y}=1 \Rightarrow y=1$ [Since u = – 1, v = 1]

Therefore, the solution is x = –1 and y = 1.

2. Solve the equations for x and y : 7x – 2y = 5xy and 8x + 7y = 15xy. [Type-2]

Ans. The given equations are:

$7x - 2y = 5xy$

On dividing the above equation by xy, we get

$\Rightarrow \frac{7}{y}-\frac{2}{x}=5$ ...(i)

Also, $8x + 7y = 15xy$

On dividing the above equation by xy, we get

$\Rightarrow \frac{8}{y}+\frac{7}{x}=15$ ...(ii)

Let $\frac{1}{y}=u$ and $\frac{1}{x}=v$,

Hence, the given equation become,

$7u - 2v = 5$ ...(iii)

$8u + 7v = 15$ ...(iv)

On multiplying eq. (iii) by 7 and eq. (iv) by 2, we get,

$49u - 14v = 35$ ...(v)

and $16u + 14v = 30$ ...(vi)

Now, on adding eq. (v) and (vi), we get

$65u = 65 \Rightarrow u = 1 \Rightarrow \frac{1}{y}=1$ or $y = 1$

On putting value of u in eq (iii), we get

$7 - 2v = 5$

$\Rightarrow v = 1 \Rightarrow \frac{1}{x}=1$ or $x = 1$

Therefore, x = 1, y = 1.

3. Solve $\frac{57}{x+y}+\frac{6}{x-y}=5$ and $\frac{38}{x+y}+\frac{21}{x-y}=9.$ [Type-3]

Ans. The given equations are:

$\frac{57}{x+y}+\frac{6}{x-y}=5$

$\Rightarrow \frac{57}{x+y}+\frac{6}{x-y}-5=0.$

and $\frac{38}{x+y}+\frac{21}{x-y}=9$

$\Rightarrow \frac{38}{x+y}+\frac{21}{x-y}-9=0$

Put $\frac{1}{x+y}=p$ and $\frac{1}{x-y}=q$

Hence, the given equations become,

$57p + 6q - 5 = 0$ and $38p + 21q - 9 = 0$

On solving the equations, by using cross multiplication method:

$$\begin{matrix} & p & & q & & 1 & \\ 6 & \searrow\nearrow & -5 & \searrow\nearrow & 57 & \searrow\nearrow & 6 \\ 21 & & -9 & & 38 & & 21 \end{matrix}$$

$$\frac{p}{6\times(-9)-21\times(-5)} = \frac{q}{(-5)\times 38-(-9)\times 57}$$

$$= \frac{1}{57\times 21-38\times 6}$$

$$\Rightarrow \frac{p}{-54+105} = \frac{q}{-190+513} = \frac{1}{1197-228}$$

$$\Rightarrow \frac{p}{51} = \frac{q}{323} = \frac{1}{969}$$

$$\Rightarrow p = \frac{51}{969} = \frac{1}{19}$$

and $q = \frac{323}{969} = \frac{1}{3}$

And, as $\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$.

Hence, $\frac{1}{x+y} = \frac{1}{19} \Rightarrow x+y = 19$ ...(i)

$\frac{1}{x-y} = \frac{1}{3} \Rightarrow x-y = 3$ ...(ii)

On adding eq. (i) and (ii), we get $2x = 22 \Rightarrow x = 11$

On putting value of x in eq. (i), we get

$11 + y = 19 \Rightarrow y = 8$

Therefore, the solution is $x = 11$ and $y = 8$.

4. Solve the following system of equation in x and y: $\frac{a}{x} - \frac{b}{y} = 0$ and $\frac{ab^2}{x} + \frac{a^2b}{y} = a^2 + b^2$, where x, y ≠ 0. [Type-1]

Ans. Put $\frac{1}{x} = p$ and $\frac{1}{y} = q$

Hence, the equations become,

$ap - bq = 0$ ...(1)

$ab^2p + a^2bq = a^2 + b^2$ ...(2)

On multiplying eq. (1) by a^2 and eq. (2) by 1, we get

$a^3p - a^2bq = 0$ ...(3)

$ab^2p + a^2bq = a^2 + b^2$ ...(4)

Now on adding eq. (3) and 4, we get

$(a^3 + ab^2)p + (-a^2b + a^2b)q = a^2 + b^2$

$(a^3 + ab^2)p = a^2 + b^2$

$a(a^2 + b^2)p = a^2 + b^2$

$\Rightarrow p = \frac{1}{a}$

$\Rightarrow \frac{1}{x} = \frac{1}{a} \Rightarrow x = a$ $\left(\because p = \frac{1}{x}\right)$

On putting value of p in eq. 3, we get

$a^3\left(\frac{1}{a}\right) - a^2bq = 0$

$a^2 - a^2bq = 0$

$a^2(1 - bq) = 0$

$1 - bq = 0$

$\Rightarrow q = \frac{1}{b}$

$\Rightarrow \frac{1}{y} = \frac{1}{b}$

$\Rightarrow y = b$

Therefore, $x = a$ and $y = b$ is the required solution.

5. Solve the following system of equations for x and y: $\frac{x}{a} + \frac{y}{b} = 2$ and $ax - by = a^2 - b^2$.

Ans. The given equations are-

$\frac{x}{a} + \frac{y}{b} - 2 = 0$ ...(i)

$ax - by = a^2 - b^2 \Rightarrow ax - by - (a^2 - b^2) = 0$...(ii)

By using cross multiplication method, we get

$$\begin{matrix} & x & & y & & 1 & \\ \frac{1}{b} & \searrow\nearrow & -2 & \searrow\nearrow & \frac{1}{a} & \searrow\nearrow & \frac{1}{b} \\ -b & & -(a^2-b^2) & & a & & -b \end{matrix}$$

$$\frac{x}{-\frac{1}{b}(a^2-b^2)-2b} = \frac{y}{-2a+\frac{1}{a}(a^2-b^2)}$$

$$= \frac{1}{\frac{1}{a}(-b)-\frac{1}{b}(a)}$$

$$\Rightarrow x = \frac{ab}{b} = a \quad \text{and} \quad y = \frac{ab}{a} = b$$

Hence, the required solution is $x = a$ and $y = b$.

6. Solve the following equations $\frac{1}{x-1}+\frac{2}{y-1}=3$ and $\frac{4}{x-1}+\frac{5}{y-1}=9.$ [Type-3]

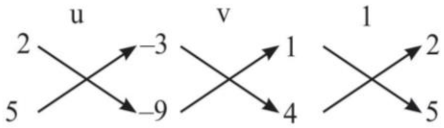
Ans. Put $\frac{1}{x-1}=u$ and $\frac{1}{y-1}=v$

Hence, the given equations become,

$u + 2v = 3 \Rightarrow u + 2v - 3 = 0$ ...(1)

$4u + 5v = 9 \Rightarrow 4u + 5v - 9 = 0$ ...(2)

Now, by using cross-multiplication method, we get



$$\frac{u}{2(-9)-5(-3)}=\frac{v}{-3(4)-(-9)(1)}=\frac{1}{5-8}$$

$$\Rightarrow \frac{u}{-18+15}=\frac{v}{-12+9}=\frac{1}{5-8}$$

$$\Rightarrow \frac{u}{-3}=\frac{v}{-3}=\frac{1}{-3}$$

$\Rightarrow u = 1$ & $v = 1$

Hence,

$$\Rightarrow \frac{1}{x-1}=\frac{1}{y-1}=1$$

$\Rightarrow x = y = 2$

Therefore, the required solution is $x = y = 2$.

7. Solve the following system of equations in x and y: $ax + by = 1$ and $bx+ay=\frac{(a+b)^2}{a^2+b^2}-1.$

Ans. The given equations are:

$ax + by = 1 \Rightarrow ax + by - 1 = 0$ ...(1)

$$bx + ay = \frac{(a+b)^2}{a^2+b^2}-1$$

$$bx + ay = \frac{a^2+b^2+2ab-(a^2+b^2)}{a^2+b^2}$$

$$bx + ay = \frac{2ab}{a^2+b^2}$$

$$bx + ay - \frac{2ab}{a^2+b^2} = 0 \quad ...(2)$$

Now, by using cross-multiplication method, we get

$$\frac{x}{b\left(\frac{-2ab}{a^2+b^2}\right)-(-a)}=\frac{y}{-b-a\left(-\frac{2ab}{a^2+b^2}\right)}=\frac{1}{a^2-b^2}$$

$$\frac{x}{\frac{-2ab^2+a(a^2+b^2)}{a^2+b^2}}=\frac{y}{\frac{-b(a^2+b^2)+2a^2b}{a^2+b^2}}=\frac{1}{a^2-b^2}$$

$$\frac{x}{\frac{-2ab^2+a^3+ab^2}{a^2+b^2}}=\frac{y}{\frac{-ba^2-b^3+2a^2b}{a^2+b^2}}=\frac{1}{a^2-b^2}$$

$$\frac{x}{\frac{a^3-ab^2}{a^2+b^2}}=\frac{y}{\frac{a^2b-b^3}{a^2+b^2}}=\frac{1}{a^2-b^2}$$

$$\frac{x}{\frac{a(a^2-b^2)}{a^2+b^2}}=\frac{y}{\frac{b(a^2-b^2)}{a^2+b^2}}=\frac{1}{a^2-b^2}$$

$$\Rightarrow x=\frac{a\left(a^2-b^2\right)}{\left(a^2+b^2\right)\left(a^2-b^2\right)}=\frac{a}{\left(a^2+b^2\right)} \text{ and}$$

$$y=\frac{b\left(a^2-b^2\right)}{\left(a^2+b^2\right)\left(a^2-b^2\right)}=\frac{b}{\left(a^2+b^2\right)}$$

Therefore, the required solution is $x=\frac{a}{a^2+b^2}$ and $y=\frac{b}{a^2+b^2}$.

8. It is given that, the denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6, then the denominator becomes 12 times the numerator. Determine the fraction.

Ans. Let us suppose, x and y be the numerator and denominator of the fraction respectively.

Hence, the fraction = x/y

Given,

Denominator = 2 (Numerator) + 4

i.e., $y = 2x + 4$

$\Rightarrow 2x - y + 4 = 0$

Now, according to the question,

$y - 6 = 12(x - 6)$

$\Rightarrow y - 6 = 12x - 72$

$\Rightarrow -6 + 72 = 12x - y$

$\Rightarrow 12x - y - 66 = 0$

Hence, now we have the following system of equations:t

$2x - y + 4 = 0$ ...(i)

$12x - y - 66 = 0$ ...(ii)

On subtracting eq. (i) from eq. (ii), we obtain,

$10x - 70 = 0$

$\Rightarrow 10x = 70$

$\Rightarrow x = 7$

On putting value of x in eq. (i), we get

$(2 \times 7) - y + 4 = 0$

$14 - y + 4 = 0$

$\Rightarrow y = 18$

Therefore, required fraction = 7/18

9. 37 pens and 53 pencils together cost Rs. 320, while 53 pens and 37 pencils together cost Rs. 400. Find the cost of a pen and that of a pencil.

Ans. Let the cost of a pen = ₹x

Let the cost of a pencil = ₹y

Now, according to question:

$37x + 53y = 320$ ...(i)

and $53x + 37y = 400$ ...(ii)

On adding eq. (i) and (ii), we obtain

$90x + 90y = 720$

$\Rightarrow x + y = 8$ ...(iii)

On subtracting equation (i) from (ii), we obtain

$16x - 16y = 80$

$\Rightarrow x - y = 5$ ...(iv)

Now, on adding eq. (iii) and (iv), we obtain

$2x = 13$

$\Rightarrow x = 6.5$

On putting value of x in eq. (iii), we get

$y = (8 - 6.5) = 1.5$

Therefore, cost of one pen = ₹6.50 and cost of one pencil = ₹1.50.

Problems based on time, distance and speed

Following formulae are used to solve the problems based on time, distance and speed.

Distance = Speed × Time

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} \text{ and Speed} = \frac{\text{Distance}}{\text{Time}}$$

Some basic definitions:

- **Stream:** The moving water in a river is called a stream.
- **Upstream:** It means that boat is moving in the opposite direction of the stream.
- **Downstream:** It means that boat is moving along in the direction of the flow of the stream.
- **Still water:** When the water is still and not moving and there's no flow like that in the case of ponds then it is called still water.

And, if Speed of a boat in still water = x km/hr

Speed of the stream or current = y km/hrt

Then, Speed in upstream = (x – y) km/hr

Speed in downstream = (x + y) km/hr

Example

1. A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

Ans. Let the speed of the boat in still water = x km/hr

Let the speed of the stream = y km/hr.

Then, Speed of boat in upstream = (x – y) km/hr

Speed of boat in downstream = (x + y) km/hr

Hence, time taken to cover 32 km upstream

$$=\frac{\text{Distance}}{\text{Speed}}=\frac{32}{x-y}\text{hrs}$$

And, time taken to cover 36 km downstream

$$=\frac{\text{Distance}}{\text{Speed}}=\frac{36}{x+y}\text{hrs}$$

But, the total time taken = 7 hours

Therefore, $\frac{32}{x-y}+\frac{36}{x+y}=7$ ...(i)

Similarly time taken to cover 40 km upstream

$$=\frac{40}{x-y}$$

And, time taken to cover 48 km downstream

$$=\frac{48}{x+y}$$

But in this case, total time taken = 9 hours

Therefore, $\frac{40}{x-y}+\frac{48}{x+y}=9$ ...(ii)

Put $\frac{1}{x-y}=u$ and $\frac{1}{x+y}=v$ in equations (i) and (ii), we get

$32u + 36v = 7 \Rightarrow 32u + 36v - 7 = 0$ ...(iii)

$40u + 48v = 9 \Rightarrow 40u + 48v - 9 = 0$ ...(iv)

Now, on solving these equations by cross-multiplication, we obtain

$$\frac{u}{36\times(-9)-(48\times(-7))}=\frac{v}{(-7)\times(40)-(-9)\times 32}=\frac{1}{32\times 48-40\times 36}$$

$$\Rightarrow \frac{u}{-324+336}=\frac{v}{-280+288}=\frac{1}{1536-1440}$$

$$\Rightarrow \frac{u}{12}=\frac{v}{8}=\frac{1}{96}$$

$$\Rightarrow u=\frac{12}{96} \text{ and } v=\frac{8}{96}$$

$$\Rightarrow u=\frac{1}{8} \text{ and } v=\frac{1}{12}$$

And, as $u=\frac{1}{8}\Rightarrow\frac{1}{x-y}=\frac{1}{8}\Rightarrow x-y=8$ ...(v)

and, $v=\frac{1}{12}\Rightarrow\frac{1}{x+y}=\frac{1}{12}\Rightarrow x+y=12$ ...(vi)

On solving equations (v) and (vi), we get x = 10 and y = 2.

Therefore, speed of the boat in still water = 10 km/hr and speed of the stream = 2 km/hr.

Summary

- An equation of the form ax + by + c = 0, where a, b, c are real numbers, a and b $\neq$ 0 is called a linear equation in two variables x and y.
- Two linear equations in the same two variables are called a **pair of linear equations in two variables.**
- The **general form** of pair of linear equations is:

$$\left.\begin{aligned} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{aligned}\right\} \text{ where } a_1, a_2, b_1, b_2, c_1, c_2 \text{ are real numbers, such that } a_1^2 + b_1^2 \neq 0,\ a_2^2 \neq 0$$

  - ☞ Intersecting lines, then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
  - ☞ Parallel lines, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
  - ☞ Coincident lines, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- Pair of Linear equations can be solved by the following method :
  - ☞ Graphical method
  - ☞ Algebraic method →
    - Elimination method
    - Substitution method
    - Cross - Multiplication method
- **Graphical method**

The graph of a linear equation in two variables is always a straight line.

Compare the ratios	Graphical representation	Algebraic representation
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution, Consistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions, Consistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution, Inconsistent

- **Substitution Method**

In this method, we express one variable in terms of the other in any one of the equations and substitute this in the other equation.

- **Elimination Method**

Elimination method involves removing one of the variables from the equations.

- **Method of cross - multiplication**

For equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ by cross - multiplication, we have

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{-1}{a_1b_2 - a_2b_1}$$

Quick Recall

Fill in the blanks

1. The pair of linear equations is ______________, if the lines intersect at a point which gives the unique solution of the two equations.
2. ______________ equation is an equation whose degree is 1.
3. If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is ______________
4. If a pair of linear equations is represented by the intersecting lines graphically, then the pair of linear equations has ______________ solution (s).
5. For the system of linear equations $5x + 4y + 6 = 0$ and $10x + 8y = 12$, the number of common solutions is ______________
6. If $\frac{1}{x}+\frac{1}{y}=k$ and $\frac{1}{x}-\frac{1}{y}=k$, then the value of y ______________
7. The graph of a pair of linear equations which has infinitely many solutions, is represented by a pair of ______________ lines.
8. If $p + q = k$, $p - q = n$ and $k > n$, then q is ______________ (positive/negative).
9. The pair of linear equations having no solution, is represented by a pair of ______________ lines.
10. If a pair of linear equations has a solution, either a unique solution or infinitely many solutions, then it is said to be ______________

True and False Statements

1. The pair of equations $x - 2y = 0$ and $3x + 4y = 20$ has a unique solution.
2. A pair of intersecting lines representing a pair of linear equations in two variables has infinitely many solutions.
3. $x + 2y = 4$, $2x + 4y = 12$ has no solution.
4. If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ and $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. In this case, the pair of linear equations is consistent.
5. If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ and $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. In this case, the pair of linear equations is inconsistent.
6. $2x + 3y = 9$, $4x + 6y = 18$ has infinitely many solutions.
7. If a pair of linear equation is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ and $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. In this case the pair of linear equations is consistent.
8. For all real values of c, the pair of equations $x - 2y = 8$, $5x + 10y = c$ have a unique solution.
9. $2x + 4y = 3$, $4x + 8y = 15$ are consistent pair of equations.
10. In a ΔPQR, $\angle R = 3\angle Q = 2(\angle P + \angle Q)$, then angles are 20°, 40° and 120°.

Match The Followings

1. Column-II gives value of x and y for pair of equations given in Column-I.

	Column I		Column II
(1)	$x + y = 10, x - y = 4$	(A)	(3, 4)
(2)	$\frac{1}{7x} + \frac{1}{6y} = 3, \frac{1}{2x} - \frac{1}{3y} = 5$	(B)	(4, 5)
(3)	$5x + 3y = 35, 2x + 4y = 28$	(C)	(1/14, 1/6)
(4)	$15x + 4y = 61, 4x + 15y = 72$	(D)	(7, 3)

a. 1-D 2-C 3-B 4-A
b. 1-B 2-D 3-A 4-C
c. 1-C 2-D 3-A 4-B
d. 1-C 2-A 3-D 4-B

2. Match the following:

	Column I		Column II
(1)	No solution	(A)	$2x + 3y = 9$ $4x + 6y = 18$
(2)	Infinitely many solutions	(B)	$x + 2y = 4$ $2x + 4y = 12$
(3)	Unique solution	(C)	$2x + y = 6$ $4x - 2y - 4 = 0$

a. 1-B 2-A 3-C
b. 1-A 2-B 3-C
c. 1-A 2-C 3-B
d. 1-C 2-B 3-A

Answers

Fill in the Blanks:

1. consistent
2. linear
3. inconsistent
4. unique
5. 0
6. does not exist
7. coincident
8. positive
9. parallel
10. consistent pair of linear equations

True and False:

1. True
2. False
3. True
4. True
5. False
6. True
7. False
8. False
9. False
10. True

Match the Followings

1. (a) **2.** (a)

NCERT Exercise

Exercise-I

1. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Exp: Let us suppose, the present age of Aftab = 'x' years

And, the present age of his daughter = 'y' years

Seven years ago,

Age of Aftab = x – 7

And, age of his daughter = y – 7

According to given condition,

$x - 7 = 7(y - 7)$

$\Rightarrow x - 7 = 7y - 49$

$\Rightarrow x - 7y = -49 + 7$

$\Rightarrow x - 7y = -42$ ...(1)

Also, three years from now or after three years,

Age of Aftab will become = x + 3.

Age of his daughter will become = y + 3

According to given condition,

$x + 3 = 3(y + 3)$

$\Rightarrow x + 3 = 3y + 9$

$\Rightarrow x - 3y = 9 - 3$

$\Rightarrow x - 3y = 6$

$\Rightarrow x - 3y - 6 = 0$ ..(ii)

On solving eq. (1), we obtain

$x - 7y + 42 = 0$

$\Rightarrow 7y = x + 42$

$\Rightarrow y = \frac{(x+42)}{7}$

Hence, solution table is,

x	0	7
y	6	7

i.e., when x = 0

$y = \frac{(0+42)}{7}$ $\left[\because y = \frac{(x+42)}{7}\right]$

$y = \frac{42}{7}$

$y = 6$

And, when x = 7, we obtain

$y = \frac{(7+42)}{7}$ $\left[\because y = \frac{(x+42)}{7}\right]$

$y = \frac{49}{7}$

$y = 7$

Therefore, when x = 0, y = 6, i.e., (0, 6)

And, when x = 7, y = 7 i.e., (7, 7)

Now, on solving eq. (2), we get

$x - 3y - 6 = 0$

$\Rightarrow 3y = x - 6$

$\Rightarrow y = \frac{(x-6)}{3}$

The solution table is,

x	0	6
y	–2	0

When, x = 0

$y = \frac{(0-6)}{3}$ $\left[\because y = \frac{(x-6)}{3}\right]$

$= \frac{-6}{3}$

$= -2$

And, when, x = 6

$y = \frac{(6-6)}{3}$ $\left[\because y = \frac{(x-6)}{3}\right]$

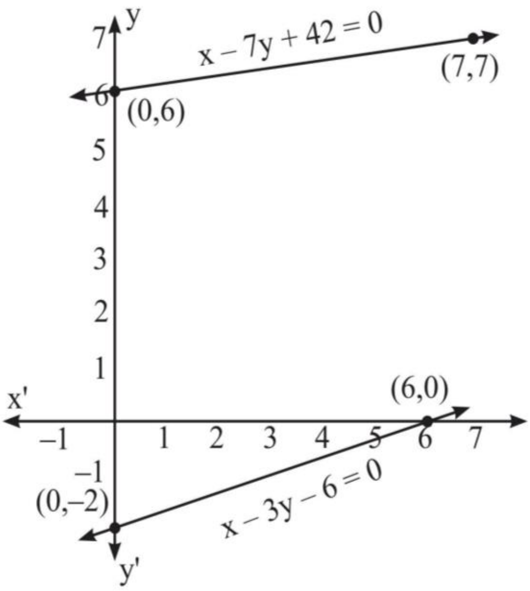
$= \frac{0}{3}$

$= 0$

Therefore, when x = 0, y = –2 i.e., (0, –2)

And, when x = 6, y = 0, i.e., (6, 0)

Now, the graphical representation is as follows:



2. The coach of a cricket team buys 3 bats and 6 balls for Rs.3900. Later, she buys another bat and 3 more balls of the same kind for Rs.1300. Represent this situation algebraically and geometrically.

Exp: Let the cost of a bat be ₹x

And, the cost of one ball = ₹y

According to the question, the algebraic representation is:

$3x + 6y = 3900$ ...(i)

and $x + 3y = 1300$ ...(ii)

For $3x + 6y = 3900$,

$$x = \frac{(3900 - 6y)}{3}$$

The solution table is

x	300	100	–100
y	500	600	700

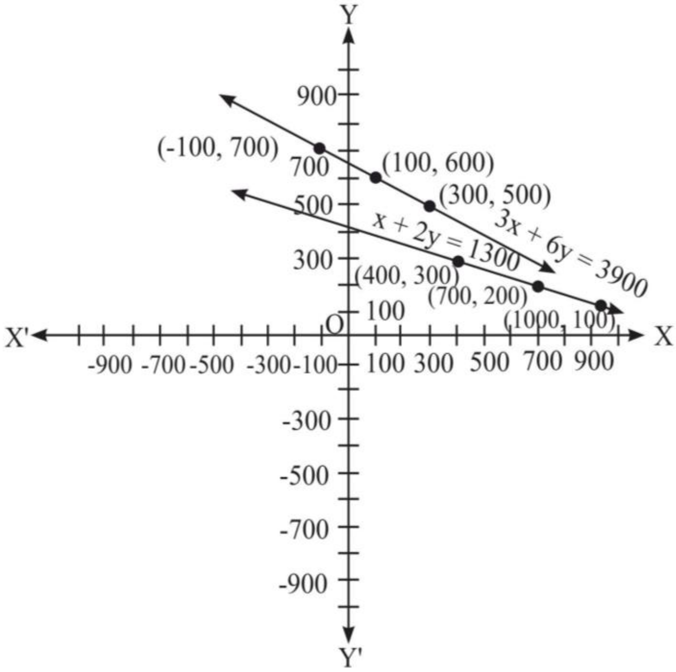
For, $x + 3y = 1300$

$x = 1300 - 3y$

The solution table is

x	1000	700	400
y	100	200	300

The graphical representation is as follows.



3. The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs.160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs.300. Represent the situation algebraically and geometrically.

Exp: Let us suppose, the cost of 1 kg of apples = ₹ x

And, the cost of 1 kg of grapes = ₹ y

Now, according to the given conditions,

$2x + y = 160$ ...(i)

And $4x + 2y = 300$ ...(ii)

For, $2x + y = 160$

$\Rightarrow y = 160 - 2x$

The solution table is

x	50	60	70
y = 160 – 2x	60	40	20

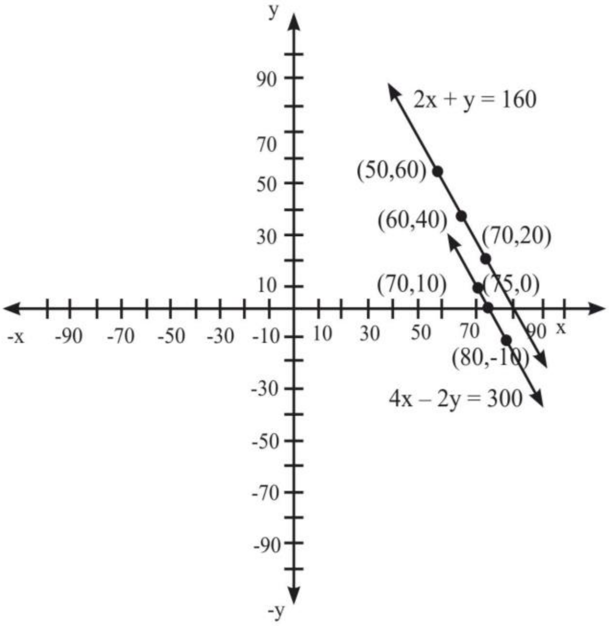
For, $4x+2y = 300$

$$\Rightarrow y = \frac{(300 - 4x)}{2}$$

The solution table is

x	70	80	75
$y = \frac{(300-4x)}{2}$	10	–10	0

Now, let us plot the graph,



Exercise-II

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost 50, whereas 7 pencils and 5 pens together cost 46. Find the cost of one pencil and that of one pen.

Exp: (i) Let us suppose, the number of girls = x

And, the number of boys = y

According to given conditions,

$x + y = 10$

And, $x - y = 4$

Now, for $x + y = 10$

$\Rightarrow y = 10 - x$

The solution table is

x	4	5	6
y	6	5	4

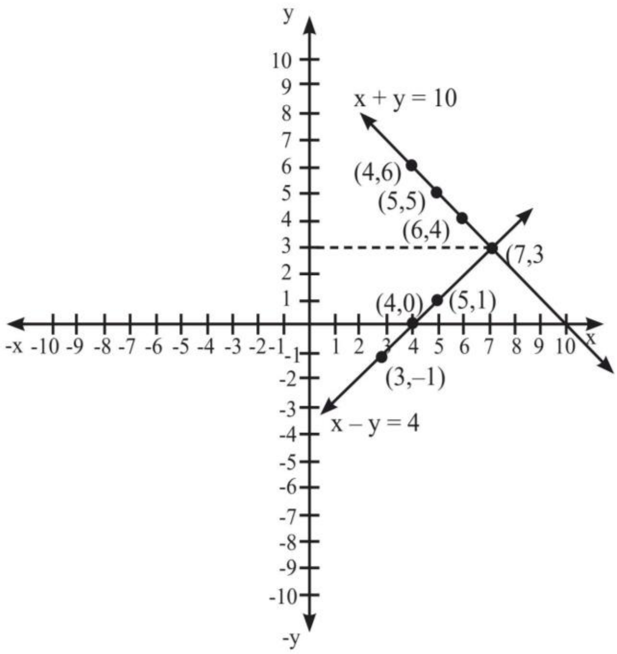
For $x - y = 4$

or $y = x - 4$

The solution table is

x	5	4	3
y	1	0	–1

Now, let us plot the graph



As, it is clear from the graph that the given lines cross each other at point (7, 3). Hence, there are 7 girls and 3 boys in the class.

(ii) Do it yourself.

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

a. $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

b. $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

c. $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Exp: (a) The given equations are:

$5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Now, comparing equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we obtain

$a_1 = 5, b_1 = -4, c_1 = 8$

$a_2 = 7, b_2 = 6, c_2 = -9$

$\therefore \frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = -\frac{4}{6} = -\frac{2}{3}, \frac{c_1}{c_2} = \frac{8}{-9}$

As, $\frac{a_1}{a_2} \neq \frac{b_1}{b}$

Hence, the given pair of equations have a unique solution and the pair of lines represented by given equations intersect each other at exactly one point.

(b) Do it yourself.

(c) The given equations are:

$6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Now, comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we obtain

$a_1 = 6, b_1 = -3, c_1 = 10$

$a_2 = 2, b_2 = -1, c_2 = 9$

$\therefore \frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}, \frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}, \frac{c_1}{c_2} = \frac{10}{9}$

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, pair of lines represented by given equations are parallel lines and there is no possible solution for the given pair of equations.

3. On comparing the ratio, $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

a. $3x + 2y = 5$; $2x - 3y = 7$

b. $2x - 3y = 8$; $4x - 6y = 9$

c. $\frac{3}{2}x + \frac{5}{3}y = 7; 9x - 10y = 14$

d. $5x - 3y = 11$; $-10x + 6y = -22$

e. $\frac{4}{3}x + 2y = 8; 2x + 3y = 12$

Exp: (a) The given equations are:

$3x + 2y = 5$ or $3x + 2y - 5 = 0$

and $2x - 3y = 7$ or $2x - 3y - 7 = 0$

Now, comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we obtain

$a_1 = 3, b_1 = 2, c_1 = -5$

$a_2 = 2, b_2 = -3, c_2 = -7$

$\therefore \frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3}, \frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7}$

As, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Pair of lines represented by the given equations intersect each other at one point.

Therefore, equations are consistent.

(ii) The given equations are:

$2x - 3y = 8$ and $4x - 6y = 9$

Hence,

$a_1 = 2, b_1 = -3, c_1 = -8$

$a_2 = 4, b_2 = -6, c_2 = -9$

$\therefore \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{8}{9}$

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

The equations are parallel to each other and there is no possible solution for the given pair of equations. Hence, the equations are inconsistent.

(c) Do it yourself.

(d) Do it yourself.

(e) Do it yourself.

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

a. $x + y = 5, 2x + 2y = 10$

b. $x - y = 8, 3x - 3y = 16$

c. $2x + y - 6 = 0, 4x - 2y - 4 = 0$

d. $2x - 2y - 2 = 0, 4x - 4y - 5 = 0$

Exp: (a) The given equations are:

$x + y = 5$ and $2x + 2y = 10$

Now,

$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{5}{10} = \frac{1}{2}$

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given equations are coincident and have infinitely many solutions.

Therefore, the given equations are consistent.

For, $x + y = 5$ or $y = 5 - x$

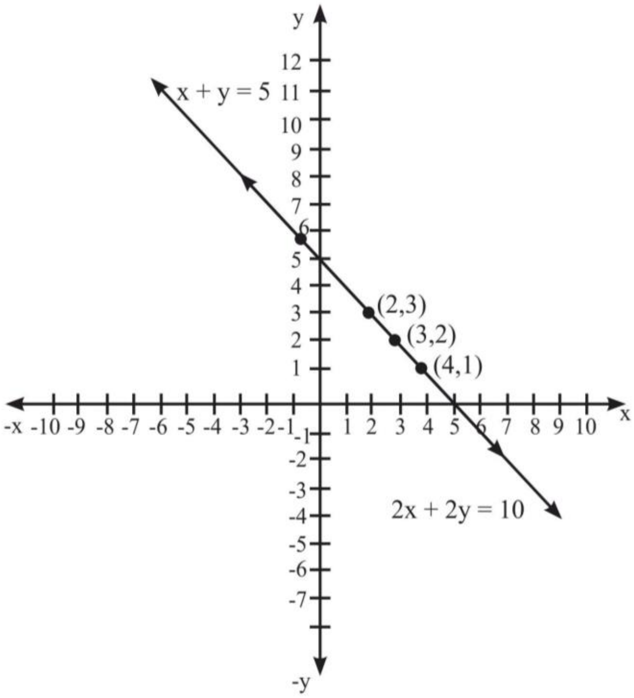
The solution table is

x	2	3	4
$y = 5 - x$	3	2	1

For $2x + 2y = 10$ or $y = \frac{10 - 2x}{2}$

x	2	3	4
$y = \frac{10-2x}{2}$	3	2	1

Hence, the graphical representation is as follows:



As we can see from the graph that the lines represented by given equations are coincident lines.

Hence, the equations have infinite possible solutions.

(b) The given equations are:

x – y = 8 and 3x – 3y = 16

Now,

$$\frac{a_1}{a_2}=\frac{1}{3},\frac{b_1}{b_2}=\frac{-1}{-3}=\frac{1}{3},\frac{c_1}{c_2}=\frac{8}{16}=\frac{1}{2}$$

As, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

Hence, the pair of lines represented by given equations are parallel lines and the given equations have no solution. Therefore, the given pair of linear equations is inconsistent.

(c) Do it yourself.

(d) Do it yourself.

5. **Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.**

Exp: Let us suppose, the width of the garden = x m

and the length of the garden = y m

It is given that length of rectangular garden is 4m more than its width.

∴ Length = 4 + Breadth

y = 4 + x

y – x = 4 ...(i)

Also, half of the perimeter of rectangular garden is 36 m.

$\therefore\ \frac{1}{2}[2(\text{length + Breadth})] = 36$

⇒ y + x = 36 ...(ii)

For, y – x = 4 or y = x + 4

The solution table is

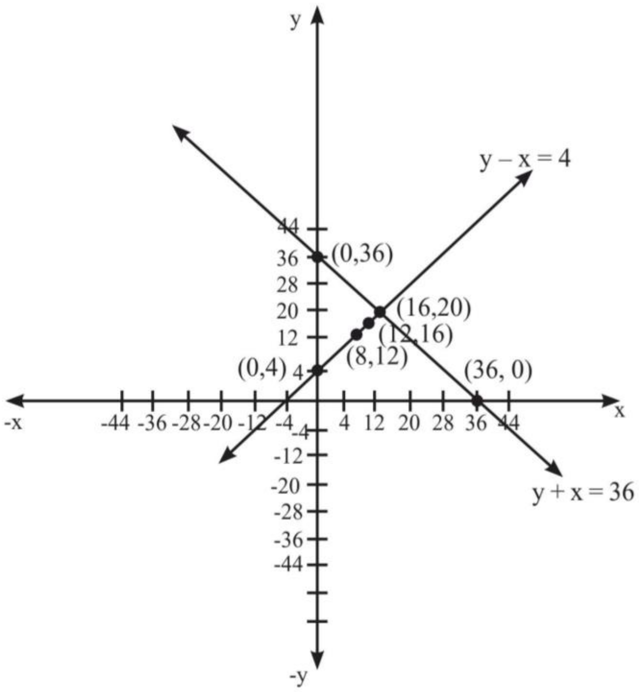
x	0	12	8
y = x + 4	4	16	12

For, y + x = 36 or y = 36 – x

The solution table is

x	0	16	36
y = 36 – x	36	20	0

Now, let us plot the graph



As we can see from the graph that the lines intersects each other at a point(16, 20). Therefore, the width of the garden is 16 m and length is 20 m.

6. **Given the linear equation 2x + 3y – 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is:**

a. Intersecting lines

b. Parallel lines

c. Coincident lines

Exp: (a) The given equation is 2x + 3y – 8 = 0.

i.e., 2x + 3y = 8

For intersecting lines, it should satisfy the following condition,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, let another equation be 2x – 7y + 9 = 0, Now,

$\frac{a_1}{a_2} = \frac{2}{2} = 1$ and $\frac{b_1}{b_2} = \frac{3}{-7}$

Therefore, these equation satisfies the condition for intersecting lines.

(b) The given linear equation is 2x + 3y – 8 = 0

i.e., 2x + 3y = 8

For parallel lines, it should satisfy the following condition

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, let another equation be 6x + 9y + 9 = 0,

Now,

$\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}, \frac{c_1}{c_2} = -\frac{8}{9}$

It is clear that these equation satisfies the condition for parallel lines.

(c) The given linear equation is 2x + 3y – 8 = 0.

i.e., 2x – 3y = 8

For coincident lines, it should satisfy the following condition

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, let another equation be 4x + 6y – 16 = 0

Now,

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$

It is clear that these equation satisfies the condition for coincident lines.

7. Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Exp: The given equations are:

x – y + 1 = 0 and,

3x + 2y – 12 = 0

For, x – y + 1 = 0 or y = x + 1

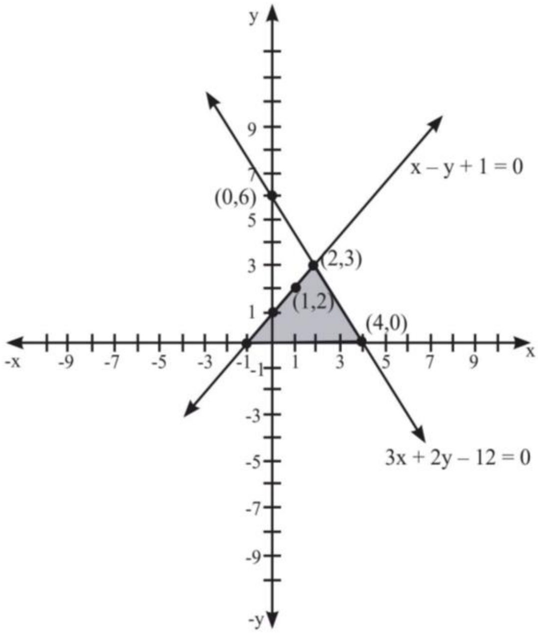
The solution table is

x	0	1	2
y = x + 1	1	2	3

For, 3x + 2y – 12 = 0 or $y = \frac{12 - 3x}{2}$

x	0	2	4
$y = \frac{12-3x}{2}$	6	3	0

Hence, the graphical representation of these equations is as follows;



As we can see that these lines intersect each other at point (2, 3) and x-axis at (−1, 0) and (4, 0). Hence, the vertices of the triangle are (2, 3), (−1, 0), and (4, 0).

Exercise-III

1. Solve the following pair of linear equations by the substitution method

a. x + y = 14

x – y = 4

b. s – t = 3

$\left(\frac{s}{3}\right) + \left(\frac{t}{2}\right) = 6$

c. 3x – y = 3

9x – 3y = 9

d. 0.2x + 0.3y = 1.3

0.4x + 0.5y = 2.3

e. $\sqrt{2}x + \sqrt{3}y = 0$

$\sqrt{3}x - \sqrt{8}y = 0$

f. $\left(\frac{3x}{2}\right)-\left(\frac{5y}{3}\right)=-2$

$\left(\frac{x}{3}\right)+\left(\frac{y}{2}\right)=\left(\frac{13}{6}\right)$

Exp: (a) The given equations are:

x + y = 14 and ...(i)

x – y = 4 ...(ii)

On substituting the value of x in eq. (ii), we get

(14 – y) – y = 4

14 – 2y = 4

–2y = 4 – 14

⇒ 2y = 10

⇒ y = 5

Now, on putting value of y in eq. (i), we get

x + 5 = 14

∴ x = 14 – 5

⇒ x = 9

Therefore, x = 9 and y = 5.

(b) Do it yourself.

(c) Do it yourself.

(d) The given equations are:

0.2x + 0.3y = 1.3 ...(i)

and, 0.4x + 0.5y = 2.3 ...(ii)

From eq. (i), we obtain

$x=\frac{(1.3-0.3y)}{0.2}$ ...(iii)

On putting the value of x in eq. (ii), we get

$0.4\left[\frac{1.3-0.3y}{0.2}\right]+0.5y=2.3$

⇒ 2(1.3 – 0.3y) + 0.5y = 2.3

⇒ 2.6 – 0.6y + 0.5y = 2.3

⇒ 2.6 – 0.1 y = 2.3

⇒ –0.1y = 2.3 – 2.6

⇒ 0.1 y = 0.3

⇒ y = 3

On putting the value of y in eq. (iii), we obtain

$x=\frac{1.3-0.3(3)}{0.2}=\frac{(1.3-0.9)}{0.2}=\frac{0.4}{0.2}=2$

Hence, x = 2 and y = 3.

(e) Do it yourself.

(f) The given equations are:

$\frac{3}{2}x-\frac{5}{3}y=-2$ and, ...(i)

$\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$ ...(ii)

From eq. (i), we obtain

$\left(\frac{3}{2}\right)x=-2+\left(\frac{5y}{3}\right)$

$\Rightarrow \frac{3}{2}x=\frac{-6+5y}{3}$

$\Rightarrow x=\frac{2(-6+5y)}{9}=\frac{(-12+10y)}{9}$ ...(iii)

On putting the value of x in eq. (ii), we obtain

$\frac{\left(\frac{-12+10y}{9}\right)}{3}+\frac{y}{2}=\frac{13}{6}$

$\Rightarrow\left(\frac{-12+10y}{27}\right)+\frac{y}{2}=\frac{13}{6}$

$\Rightarrow\frac{-12+10y}{27}+\frac{y}{2}=\frac{13}{6}$

$\Rightarrow\frac{-12+10y}{27}-\frac{13}{6}=\frac{-y}{2}$

$\Rightarrow\frac{-24+20y-117}{54}=\frac{-y}{2}$

⇒ –24 + 20y = –27y + 117

⇒–141 = –47y

⇒ y = 3

On substituting the value of y in equation (i), we get,

$\left(\frac{3x}{2}\right)-\frac{5(3)}{3}=-2$

$\Rightarrow\left(\frac{3x}{2}\right)-5=-2$

$\Rightarrow\frac{3}{2}x=3$

⇒ x = 2

Hence, x = 2 and y = 3.

2. Solve 2x + 3y = 11 and 2x – 4y = – 24 and hence find the value of 'm' for which y = mx + 3.

Exp: The given equations are:

2x + 3y = 11 and, ...(i)

2x – 4y = –24 ...(ii)

Now, from eq. (i), we obtain

$x=\frac{(11-3y)}{2}$ ...(iii)

On substituting the value of x in eq. (ii), we obtain

$$\frac{2(11-3y)}{2}-4y=-24$$

11 – 3y – 4y = –24

–7y = –24 – 11

–7y = –35

y = 5 (iv)

On substituting the value of y in eq. (iii), we get

$$x=\frac{(11-3\times5)}{2}=\frac{-4}{2}=-2$$

Therefore, x = –2, y = 5

It is also given that,

y = mx + 3

5 = –2m +3

5 – 3 = –2m

⇒ –2m = 2

⇒ m = –1

Hence, the value of m is –1.

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

a. The difference between two numbers is 26 and one number is three times the other. Find them.

Exp: Let us suppose x and y be the I[st] number and II[nd] number respectively, such that y > x

According to the given conditions,

y = 3x ...(i)

y – x = 26 ...(ii)

On putting the value of y from eq. (i) into eq. (ii), we get

3x – x = 26

2x = 26

x = 13 ...(iii)

On substituting the value of x from eq. (iii) in eq. (i), we get

y = 39

Therefore, the numbers are 13 and 39.

b. The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Exp: Do it yourself

c. The coach of a cricket team buys 7 bats and 6 balls for Rs.3800. Later, she buys 3 bats and 5 balls for Rs.1750. Find the cost of each bat and each ball.

Exp: Do it yourself.

d. The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

Exp: Let us suppose, fixed charges = ₹x

and, charge for per km = ₹y

According to given conditions,

x + 10y = 105 ...(i)

x + 15y = 155 ...(ii)

Now from eq. (i), we obtain

x = 105 – 10y ...(iii)

On puttting value of x in eq. (ii), we get

105 – 10y + 15y = 155

5y = 155 – 105

5y = 50

⇒ y = 10 ...(iv)

On substituting the value of y in eq. (iii), we get

x = 105 – 10 × 10 = 5

Therefore, fixed charge is ₹5 and charges per km is ₹10

Hence, charge for 25 km = x + 25y = 5 + 250 = ₹ 255.

e. A fraction becomes 9/11 , if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes 5/6. Find the fraction.

Exp: Let us suppose, x and y be the numerator and denominator of the fraction respectively.

Therefore, the fraction is $\frac{x}{y}$.

According to the given conditions,

$$\frac{(x+2)}{(y+2)}=\frac{9}{11}$$

11x + 22 = 9y + 18

11x – 9y = –4 ...(i)

And,

$$\frac{(x+3)}{(y+3)}=\frac{5}{6}$$

6x + 18 = 5y +15

6x – 5y = -3 ...(ii)

From equation (i), we obtain

$$x = \frac{-4+9y}{11} \quad ...(iii)$$

Now, on putting the value of x in eq. (ii), we get

$$6\left[\frac{9y-4}{11}\right] - 5y = -3$$

$$\frac{54y-24}{11} - 5y = -3$$

$-24 + 54y - 55y = -33$

$-y = -9$

$\Rightarrow y = 9 \quad ...(iv)$

On putting the value of y in eq. (iii), we obtain

$$x = \frac{(-4+9\times 9)}{11} = 7$$

Therefore, the fraction is 7/9.

f. Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Exp: Do it yourself.

Exercise-IV

1. Solve the following pair of linear equations by the elimination method and the substitution method:

a. $x + y = 5$ **and** $2x - 3y = 4$

b. $3x + 4y = 10$ **and** $2x - 2y = 2$

c. $3x - 5y - 4 = 0$ **and** $9x = 2y + 7$

d. $\frac{x}{2} + \frac{2y}{3} = -1$ **and** $x - \frac{y}{3} = 3$

Exp: (a) By using elimination method,

The given equations are:

$x + y = 5 \quad ...(1)$

$2x - 3y = 4 \quad ...(2)$

On multiplying eq. (1) by 2, we obtain

$2x + 2y = 10 \quad ...(3)$

Now, subtract eq. (2) from eq. (3),

$(2x + 2y) - (2x - 3y) = 10 - 4$

$5y = 6$

$$\Rightarrow y = \frac{6}{5} \quad (4)$$

On putting the value of y in eq. (1), we obtain

$$x + \frac{6}{5} = 5$$

$$\Rightarrow x = 5 - \frac{6}{5}$$

$$\Rightarrow x = \frac{25-6}{5} = \frac{19}{5}$$

Now, by using substitution method,

Using eq. (1), we get

$x = 5 - y \quad ...(5)$

On putting value of x in eq. (2), we get

$2(5 - y) - 3y = 4$

$10 - 2y - 3y = 4$

$-5y = -6$

$$\Rightarrow y = \frac{6}{5}$$

On putting value of y in eq. (5), we obtain:

$$x = 5 - \frac{6}{5} = \frac{19}{5}$$

Therefore, $x = \frac{19}{5}$ and $y = \frac{6}{5}$.

(b) Do it yourself.

(c) Do it yourself.

(d) The given equations are:

$$\frac{x}{2} + \frac{2y}{3} = -1 \text{ and } x - \frac{y}{3} = 3$$

By using elimination method,

Equations can be rewritten as,

$3x + 4y = -6 \quad ...(1)$

$3x - y = 9 \quad ...(2)$

On subtracting eq. (2) from eq. (1), we get

$5y = -15$

$y = -3 \quad ...(3)$

On putting the value of y in eq. (1) we obtain

$3x + 4(-3) = -6$

$3x - 12 = -6$

$3x = 6$

$\Rightarrow x = 2$

Therefore, $x = 2$, $y = -3$

By using substitution method,

From eq. (2), we obtain

$$x = \frac{(y+9)}{3} \quad ...(4)$$

On putting value of x in eq. (1), we obtain

$$\frac{3(y+9)}{3} + 4y = -6$$

$(y + 9) + 4y = -6$

$5y = -15$

$y = -3$

On putting value of y in eq. (4), we obtain

$$x = \frac{(-3+9)}{3} = 2$$

Hence, x = 2 and y = –3.

2. **Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:**

   a. **If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?**

Exp: Let us suppose, x and y be the numerator and denominator of the fraction respectively.

Hence, the fraction is $\frac{x}{y}$.

Now, according to the given conditions,

$$\frac{(x+1)}{(y-1)} = 1$$

$\Rightarrow x + 1 = y - 1$

$\Rightarrow x - y = -2$ ...(1)

And, $\frac{x}{y+1} = \frac{1}{2}$

$\Rightarrow 2x - y = 1$ ...(2)

On subtracting eq. (1) from eq. (2), we obtain

$x = 3$ ...(3)

On putting value of x in eq. (1), we get

$3 - y = -2$

$-y = -5$

$y = 5$

Hence, x = 3 and y = 5

Therefore, the fraction is $\frac{3}{5}$.

   b. **Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?**

Exp: "Do it yourself".

   c. **The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.**

Exp: Let us suppose, the digit at unit's place be x and the digit at ten's place by y.

Therefore, two digit number = 10y + x

After reversing the order of the digits,

the number becomes = 10x + y

Now, according to given conditions

$x + y = 9$ ...(i)

and, $9(10y + x) = 2(10x + y)$

$90y + 9x = 20x + 2y$

$88y - 11x = 0$

$-x + 8y = 0$ ...(ii)

On adding eq. (i) and (ii), we obtain

$9y = 9$

$\Rightarrow y = 1$ ...(iii)

On putting this value of y in eq. (i), we get

$x = 8$

As, y = 1 and x = 8

Therefore, the number is $10y + x = 10 \times 1 + 8 = 18$.

   d. **Meena went to a bank to withdraw Rs.2000. She asked the cashier to give her Rs.50 and Rs.100 notes only. Meena got 25 notes in all. Find how many notes of Rs.50 and Rs.100 she received.**

Exp: "Do it yourself".

   e. **A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs.27 for a book kept for seven days, while Susy paid Rs.21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.**

Exp: "Do it yourself"

Exercise-V

1. **Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method.**

   a. **$x - 3y - 3 = 0$ and $3x - 9y - 2 = 0$**

   b. **$2x + y = 5$ and $3x + 2y = 8$**

   c. **$3x - 5y = 20$ and $6x - 10y = 40$**

   d. **$x - 3y - 7 = 0$ and $3x - 3y - 15 = 0$**

Exp: **(a)** The given equations are:

$x - 3y - 3 = 0$ and $3x - 9y - 2 = 0$

Hence, $\frac{a_1}{a_2} = \frac{1}{3},\ \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3},\quad \frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$

Hence, $\left(\frac{a_1}{a_2}\right) = \left(\frac{b_1}{b_2}\right) \neq \left(\frac{c_1}{c_2}\right)$

Clearly, this is the condition for no solution. Thus, the lines will be parallel and hence no solution exists.

(b) The given equations are:

$2x + y = 5$ and $3x + 2y = 8$

Here, $\frac{a_1}{a_2} = \frac{2}{3},\ \frac{b_1}{b_2} = \frac{1}{2},\quad \frac{c_1}{c_2} = \frac{-5}{-8}$

Hence, $\left(\frac{a_1}{a_2}\right) \neq \left(\frac{b_1}{b_2}\right)$

Clearly, this is the condition for unique solution. Thus, the lines will be intersecting and hence unique solution exists.

By using cross-multiplication method,

$$\frac{x}{(b_1c_2 - c_1b_2)} = \frac{y}{(c_1a_2 - c_2a_1)} = \frac{1}{(a_1b_2 - a_2b_1)}$$

$$\frac{x}{-8-(-10)} = \frac{y}{(-15+16)} = \frac{1}{(4-3)}$$

$$\frac{x}{2} = \frac{y}{1} = 1$$

Therefore, $x = 2$ and $y = 1$

(c) "Do it yourself".

(d) "Do it yourself".

2. (i) For which values of a and b does the following pair of linear equations have infinite number of solutions?

$2x + 3y = 7$

$(a - b)\,x + (a + b)\,y = 3a + b - 2$

(ii) For which value of k will the following pair of linear equations have no solution?

$3x + y = 1$

$(2k - 1)\,x + (k - 1)\,y = 2k + 1$

Exp: (i) The given equations are:

$2x + 3y = 7$

$(a - b)\,x + (a + b)\,y = 3a + b - 2$

$\frac{a_1}{a_2} = \frac{2}{(a-b)},\ \frac{b_1}{b_2} = \frac{3}{(a+b)},\ \frac{c_1}{c_2} = \frac{-7}{-(3a+b-2)}$

It should satisfy the following condition, for infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, $\frac{2}{(a-b)} = \frac{7}{(3a+b-2)}$

$\Rightarrow 6a + 2b - 4 = 7a - 7b$

$\Rightarrow 2b + 7b - 4 = 7a - 6a$

$\Rightarrow a - 9b = -4$ ...(1)

And, $\frac{2}{(a-b)} = \frac{3}{(a+b)}$

$2(a + b) = 3(a - b)$

$2a + 2b = 3a - 3b$

$2b + 3b = 3a - 2a$

$\Rightarrow a - 5b = 0$ ...(2)

Subtracting (1) from (2), we get

$4b = 4$

$\Rightarrow b = 1$

On putting the value of b in eq. (2), we get

$a - 5 \times 1 = 0$

$\Rightarrow a = 5$

Hence, the given equations will have infinite solutions at $a = 5$ and $b = 1$.

(ii) The given equations are:

$3x + y - 1 = 0$

and $(2k - 1)\,x + (k - 1)\,y - (2k + 1) = 0$

Here,

$\frac{a_1}{a_2} = \frac{3}{(2k-1)},\ \frac{b_1}{b_2} = \frac{1}{(k-1)},\ \frac{c_1}{c_2} = \frac{-1}{(-2k-1)} = \frac{1}{(2k+1)}$

For no solution, it should satisfy the following condition,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, $\frac{3}{(2k-1)} = \frac{1}{(k-1)} \neq \frac{1}{(2k+1)}$

Taking, $\frac{3}{(2k-1)} = \frac{1}{(k-1)}$

$3(k - 1) = 1(2k - 1)$

$3k - 3 = 2k - 1$

$\Rightarrow k = 2$

Hence, the given equations will have no solution for $k = 2$.

3. Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$8x + 5y = 9$

$3x + 2y = 4$

Exp: The given equations are:

$8x + 5y = 9$ ...(i)

$3x + 2y = 4$ ...(ii)

Using eq. (ii), we have

$x = \frac{4-2y}{3}$ ...(iii)

On putting value of x in eq. (i)

8(4-2y)/3 + 5y = 9

32 – 16y +15y = 27

–y = 27 – 32

–y = –5

y = 5 ...(iv)

On putting this value of y in eq. (ii), we get

3x + 2 × 5 = 4

3x + 10 = 4

3x = –6

x = –2

Therefore, x = -2 and y = 5.

Now, by using Cross Multiplication method:

8x +5y – 9 = 0 and,

3x + 2y – 4 = 0

$$\frac{x}{-20+18} = \frac{y}{-27+32} = \frac{1}{16-15}$$

–x/2 = y/5 =1/1

Therefore, x = -2 and y = 5.

4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

a. A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs.1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs.1180 as hostel charges. Find the fixed charges and the cost of food per day.

b. A fraction becomes 1/3 when 1 is subtracted from the numerator and it becomes 1/4 when 8 is added to its denominator. Find the fraction.

c. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

d. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

e. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Exp: **(a)** Let, the fixed charge = ₹ x

and the charge of food per day = ₹ y

According to given conditions,

x + 20y = 1000 ...(1)

x + 26y = 1180 ...(2)

On subtracting eq. (1) from eq. (2), we get

6y = 180

⇒ y = 30

On putting the value of y in eq. (2), we get

x + (26 × 30) = 1180

x = 1180 – 26 × 30

x = 400

Hence, fixed charges is ₹400 and charges of food per day is ₹30.

(b) Do it yourself.

(c) Let us suppose, x and y be the number of right answers and number of wrong answers respectively.

According to given conditions,

3x − y = 40 and, ...(i)

4x − 2y = 50

⇒ 2x − y = 25 ...(ii)

On subtracting eq. (ii) from eq. (i),

x = 15 (iii)

On substituting the value of x in eq. (ii), we get

30 – y = 25

⇒ y = 5

Hence, number of right answers = 15 and number of wrong answers = 5

Therefore, total number of questions = 15 + 5 = 20

(d) Let the speed of the car from point A is x km/h.

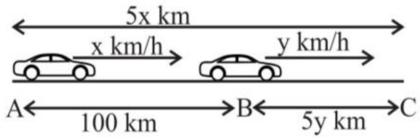
And the speed of the car from point B is y km/h. Distance travelled by the first car in 5 hours = AC = Speed * Time = 5x km

Distance travelled by the second car in 5 hours = BC = Speed * Time = 5y km

According to the question,

If the cars travel in the same direction, then they meet up in 5 hours.

Let they meet at point C.



$$AC = AB + BC$$

$$5x = 100 + 5y$$

$\Rightarrow \quad 5x - 5y = 100$

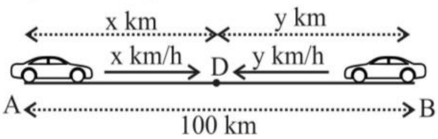
$\Rightarrow \quad x - y = 20$ ...(1)

When the cars are moving in the same direction then they meet up in 1 hour.

Distance travelled by the first car in 1 hour = AD = Speed * Time = x km

Distance travelled by the second car in 1 hour = BD = Speed * Time = y km

Let they meet at point D.



$$AD + DB = AB$$

$x + y = 100$ ...(2)

From eq. (1) and (2), we get

$x = 60$ km/h ...(3)

On putting the value of x in eq. (1), we get

$60 - y = 20$

$\Rightarrow y = 40$ km/h

Hence, the speed of car from point A = 60 km/h and speed of car from point B = 40 km/h.

(e) "Do it yourself".

Exercise-VI

1. Solve the following pair of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x} + \frac{1}{3y} = 2$

$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

Exp: Let us suppose $\frac{1}{x} = p$ and $\frac{1}{y} = q$, then the equations become,

$\frac{p}{2} + \frac{q}{3} = 2$

$\Rightarrow 3p + 2q - 12 = 0$ ...(i)

And, $\frac{p}{3} + \frac{q}{2} = \frac{13}{6}$

$\Rightarrow 2p + 3q - 13 = 0$ ...(ii)

Now, by using cross-multiplication method, we have,

$$\frac{p}{-26-(-36)} = \frac{q}{-24-(-39)} = \frac{1}{(9-4)}$$

$$\frac{p}{10} = \frac{q}{15} = \frac{1}{5}$$

$$\frac{p}{10} = \frac{1}{5} \text{ and } \frac{q}{15} = \frac{1}{5}$$

Therefore, $p = 2$ and $q = 3$

Hence, $\frac{1}{x} = 2$ and $\frac{1}{y} = 3$

i.e., $x = \frac{1}{2}$ and $y = \frac{1}{3}$

(ii) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

Exp: We have, $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$ ...(1)

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$ ...(2)

Substituting $\frac{1}{\sqrt{x}} = u$ and $\frac{1}{\sqrt{y}} = v$ in equations (1) and (2), we get:

$2u + 3v = 2$ ...(3)

$4u - 9v = -1$ ...(4)

Multiplying equation (3) by 3, we obtain

$6u + 9v = 6$ ...(5)

On adding equations (4) and (5), we obtain

$10u = 5$

$\Rightarrow \quad u = \frac{5}{10} = \frac{1}{2}$

Substituting $u = \frac{1}{2}$ in equation (3), we get:

$$2 \times \frac{1}{2} + 3v = 2$$

$\Rightarrow \quad 3v = 2 - 1$

$\Rightarrow \quad v = \frac{1}{3}$

But $\quad u = \frac{1}{\sqrt{x}} = \frac{1}{2}$

$\Rightarrow \quad \sqrt{x} = 2$

$\Rightarrow \quad x = 4$

and $\quad v = \frac{1}{\sqrt{y}} = \frac{1}{3}$

$\Rightarrow \quad \sqrt{y} = 3$

$\Rightarrow \quad y = 9$

Thus, the solution is x = 4 and y = 9.

(iii) $\mathbf{\frac{4}{x} + 3y = 14}$

$\mathbf{\frac{3}{x} - 4y = 23}$

Exp: "Do it yourself".

Hint: Take $\frac{1}{x} = p$

(iv) $\mathbf{\frac{5}{(x-1)} + \frac{1}{(y-2)} = 2}$

$\mathbf{\frac{6}{(x-1)} - \frac{3}{(y-2)} = 1}$

Exp: Put $\frac{1}{(x-1)} = u$ and $\frac{1}{(y-2)} = v$, hence equations become,

$5u + v = 2 \quad ...(1)$

$6u - 3v = 1 \quad ...(2)$

On multiplying eq. (1) by 3, we obtain

$15u + 3v = 6 \quad ...(3)$

On adding eq. (2) and (3), we obtain

$21u = 7$

$u = \frac{1}{3}$

On putting value of u in eq. (1), we obtain

$5 \times \frac{1}{3} + v = 2$

$v = 2 - \frac{5}{3} = \frac{1}{3}$

As, $u = \frac{1}{(x-1)}$

$\Rightarrow \frac{1}{3} = \frac{1}{(x-1)} \qquad \left(\because u = \frac{1}{3}\right)$

$\Rightarrow (x - 1) = 3$

$\Rightarrow x = 4$

And, $v = \frac{1}{(y-2)}$

$\Rightarrow \frac{1}{3} = \frac{1}{(y-2)} \qquad \left(\therefore v = \frac{1}{3}\right)$

$\Rightarrow (y - 2) = 3$

$\Rightarrow y = 5$

Therefore, x = 4 and y = 5

(v) $\mathbf{\frac{(7x-2y)}{xy} = 5}$

$\mathbf{\frac{(8x+7y)}{xy} = 15}$

Exp: "Do it yourself".

(vi) 6x + 3y = 6xy

2x + 4y = 5xy

Exp: 6x + 3y = 6xy

Dividing both sides by xy, we get

$\frac{6}{y} + \frac{3}{x} = 6$

Put, $\frac{1}{y} = u$ and $\frac{1}{x} = v$

Hence, equations become,

$\Rightarrow 6u + 3v = 6$

$\Rightarrow 6u + 3v - 6 = 0 \quad ...(1)$

And, 2x + 4y = 5xy

Dividing both sides by xy, we get

$\frac{2}{y} + \frac{4}{x} = 5$

Put, $\frac{1}{y} = u$ and $\frac{1}{x} = v$

$\Rightarrow 2u + 4v = 5$

$\Rightarrow 2u + 4v - 5 = 0 \quad ...(2)$

Now, by using cross-multiplication method, we get

$$\frac{u}{-15-(-24)}=\frac{v}{-12-(-30)}=\frac{1}{24-6}$$

$$\frac{u}{9}=\frac{v}{18}=\frac{1}{18}$$

Now, $\frac{u}{9}=\frac{1}{18}$

$\Rightarrow u=\frac{1}{2}$

And, $\frac{v}{18}=\frac{1}{8}$

$\Rightarrow v = 1$

$\therefore u=\frac{1}{2}$ and $v=1$

As, $u=\frac{1}{y}=\frac{1}{2}$

$\Rightarrow y=2$

And, $v=1=\frac{1}{x}$

$\Rightarrow x=1$

Therefore, x = 1 and y = 1.

(vii) $\frac{10}{(x+y)}+\frac{2}{(x-y)}=4$

$\frac{15}{(x+y)}-\frac{5}{(x-y)}=-2$

Exp: "Do it yourself"

(viii) $\frac{1}{(3x+y)}+\frac{1}{(3x-y)}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Exp: Put, $\frac{1}{(3x+y)}=u$ and $\frac{1}{(3x-y)}=v$

Hence, equations become

$u+v=\frac{3}{4}$ ...(i)

$\frac{u}{2}-\frac{v}{2}=-\frac{1}{8}$

$u-v=-\frac{1}{4}$ ...(ii)

On adding eq. (i) and (ii), we have

$2u=\frac{3}{4}-\frac{1}{4}$

$2u=\frac{2}{4}$

$2u=\frac{1}{2}$

$\Rightarrow u=\frac{1}{4}$

On putting value of u in eq. (ii), we get

$\frac{1}{4}-v=-\frac{1}{4}$

$v=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

As, $u=\frac{1}{(3x+y)}=\frac{1}{4}$

$\Rightarrow 3x + y = 4$ ...(iii)

And, $v=\frac{1}{(3x-y)}=\frac{1}{2}$

$\Rightarrow 3x - y = 2$ ...(iv)

On adding eq. (iii) and (iv), we obtain

$6x = 6$

$\Rightarrow x = 1$ ...(v)

On putting value of x in eq. (iii), we get

$3(1) + y = 4$

$\Rightarrow y = 1$

Therefore, x = 1 and y = 1.

2. Formulate the following problems as a pair of equations, and hence find their solutions.

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Exp: **(i)** Let us suppose,

Speed of Ritu in still water = x km/hr

Speed of current = y km/hr

Now, Speed of Ritu during, downstream = x + y km/h

Speed of Ritu during upstream = x – y km/h

According to the question, Ritu can row 20 km downstream in 2 hours.

As we know,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\therefore x + y = \frac{20}{2}$$

$\Rightarrow x + y = 10$ ...(i)

Also, Ritu can row 4 km upstream in 2 hours.

As we know,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\therefore x - y = \frac{4}{2}$$

$x - y = 2$ ...(ii)

On adding eq. (i) and (ii), we obtain

$2x = 12$

$\Rightarrow x = 6$

On substituting the value of x in eq. (i), we get

$6 + y = 10$

$\Rightarrow y = 4$

Hence,

Speed of Ritu rowing in still water = 6 km/hr

And, speed of current = 4 km/hr

(ii) Let,

Time taken by one woman to finish the work = x

And, time taken by one man to finish the work = y

Hence,

Work done by women in one day $= \frac{1}{x}$

And, Work done by man in one day $= \frac{1}{y}$

Now, according to given condition,

Time taken by 2 women and 5 men to finish the work = 4 days.

$\therefore$ Work finished by 2 women and 5 men in 1 day $= \frac{1}{4}$

$\Rightarrow$ 2 × (work finished by 1 women in 1 day) + 5 × (work finished by 1 men in 1 day) $= \frac{1}{4}$

$$\Rightarrow 2\left(\frac{1}{x}\right) + 5\left(\frac{1}{y}\right) = \frac{1}{4}$$

$$\Rightarrow \left(\frac{2}{x} + \frac{5}{y}\right) = \frac{1}{4} \quad \text{...(i)}$$

Similarly, by using the second condition, we get

$$\left(\frac{3}{x} + \frac{6}{y}\right) = \frac{1}{3} \quad \text{...(ii)}$$

Put $\frac{1}{x} = u$ and $\frac{1}{y} = v$,

Hence, equations become,

$$2u + 5v = \frac{1}{4}$$

$\Rightarrow 8u + 20v = 1$ ...(iii)

$$3u + 6v = \frac{1}{3}$$

$\Rightarrow 9u + 18v = 1$ ...(iv)

Now, by using cross multiplication method,

$$\frac{u}{(20-18)} = \frac{v}{(9-8)} = \frac{-1}{(144-180)}$$

$$\frac{u}{2} = \frac{v}{1} = \frac{1}{36}$$

Now,

$$\frac{u}{2} = \frac{1}{36}$$

$$\Rightarrow u = \frac{1}{18}$$

And,

$$u = \frac{1}{x} = \frac{1}{18}$$

$\Rightarrow x = 18$

Also,

$$v = \frac{1}{y} = \frac{1}{36}$$

$\Rightarrow y = 36$

Hence,

Time taken by one woman to finish the work = 18 days

Time taken by one man to finish the work = 36 days.

(iii) Let us suppose,

Speed of the train = x km/h

And, speed of the bus = y km/h

It is given that Roohi travels 60 km by train and the remaining 240 km by bus.

As we know, Time $= \frac{\text{Distance}}{\text{Speed}}$

Time taken to travel 60 km by train = $\frac{60}{x}$

And time taken to travel 240 km by bus = $\frac{240}{y}$

According to the question, Roohi takes 4 hours if she travels 60 km by train and remaining by bus.

∴ Total time taken = 4 hours

$\frac{60}{x} + \frac{240}{y} = 4$ ...(i)

If Roohi travels 100 km by train and the remaining 200 km by bus, then she takes 4 hours and 10 minutes.

As we know, Time = $\frac{\text{Distance}}{\text{Speed}}$

Time taken to cover 100 km by train = $\frac{100}{x}$

And time taken to cover 200 km by bus = $\frac{200}{y}$

Total time taken = 4 hours and 10 minutes

$\frac{100}{x} + \frac{200}{y} = 4 + \frac{10}{60}$

$\frac{100}{x} + \frac{200}{y} = \frac{240+10}{60}$

$\frac{100}{x} + \frac{200}{y} = \frac{25}{6}$ ...(ii)

Now, substitute $\frac{1}{x} = u$ and $\frac{1}{y} = v$. Hence, the equations become,

$60u + 240v = 4$ ...(iii)

$100u + 200v = \frac{25}{6} \Rightarrow 600u + 1200v = 25$ ...(iv)

On multiplying eq. (iii) by 10, we obtain

$600u + 2400v = 40$ ...(v)

Now, on subtracting eq. (iv) from eq. (v), we obtain

$1200v = 15$

$v = \frac{15}{1200} = \frac{1}{80}$

Now, on substituting the value of v in eq. (iii),

We get

$60u + 240 \times \frac{1}{80} = 4$

$60u + 3 = 4$

$u = \frac{1}{60}$

As, $u = \frac{1}{x} = \frac{1}{60}$

$\Rightarrow x = 60$

And $v = \frac{1}{y} = \frac{1}{80}$

$\Rightarrow y = 80$

Hence,

Speed of the train = 60 km/h

Speed of the bus = 80 km/h

Exercise-VII

1. **The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.**

Exp: Let the age of Ani = A years

and the age of Biju = B years.

It is given that Ani's father Dharam is twice as old as Ani

∴ Age of Dharam(D) = 2A

It is also given that Biju is twice as old as Cathy.

∴ 2 × Age of Cathy (C) = B

⇒ Age of Cathy (C) = $\frac{B}{2}$

Case I: Ani is older than Biju.

The ages of Ani and Biju differ by 3 years.

$A - B = 3$ ...(i)

The ages of Cathy and Dharam differs by 30 years.

∴ $D - C = 30$

$2A - B/2 = 30$ [∵ D = 2A and C = B/2]

$\Rightarrow 4A - B = 60$ ...(ii)

On subtracting equation (i) from (ii), we get

$3A = 57$

$A = \frac{57}{3} = 19$

On putting value of A in equation (i), we get

$19 - B = 3$

$B = 19 - 3$

$B = 16$

Hence, the age of Ani = 19 yrs

And the age of Biju = 16 yrs.

Case-II: Biju is older than Ani.

The ages of Ani and Biju differ by 3 years.

$\therefore \quad B - A = 3$ ...(i)

The ages of Cathy and Dharam differs by 30 years.

$D - C = 30$

$2A - B/2 = 30 \quad [\because D = 2A \text{ and } C = B/2]$

$4A - B = 60$ ...(ii)

On adding equations (i) and (ii), we get

$3A = 63$

$A = \frac{63}{3} = 21$

On putting value of A in equation (i), we get

$B - 21 = 3$

$B = 24$

Hence, the age of Ani = 21 yrs

And the age of Biju = 24 yrs.

2. One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [Hint : x + 100 = 2(y – 100), y + 10 = 6(x – 10)].

Exp: Do it yourself.

3. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Exp: Let us suppose, the speed of the train = x km/hr.

And, time taken by train = y hr.

Let, distance travelled by train = D km

As, $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

Therefore, $x = \frac{D(\text{distance})}{y(\text{time})}$

$\Rightarrow D = xy$ ...(i)

According to the given conditions,

When the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time.

$(x+10) = \frac{D}{(y-2)}$

$\Rightarrow (x + 10)(y - 2) = D$

Using eq. (i), we get

$(x + 10)(y - 2) = xy$

$\Rightarrow xy - 2x + 10y - 20 = xy$

$\Rightarrow -2x + 10y - 20 = 0$

$\Rightarrow -2x + 10y = 20$ ...(ii)

And when the train were slower by 10 km/h, it would have taken 2 hours more than the scheduled time.

$(x-10) = \frac{D}{(y+3)}$

$(x - 10)(y + 3) = xy$

$\Rightarrow xy + 3x - 10y - 30 = xy$

$\Rightarrow 3x - 10y - 30 = 0$

$\Rightarrow 3x - 10y = 30$ ...(iii)

On adding eq. (ii) and (iii),

we get

x = 50 km/hr

On putting value of x in eq. (ii), we get

$(-2) \times (50) + 10y = 20$

$-100 + 10y = 20$

$\Rightarrow 10y = 120$

y = 12 hours

On putting values of x and y in eq. (i), we get

Distance travelled by the train, D = xy

$= 50 \times 12$

= 600 km

Therefore, the distance travelled by the train = 600km.

4. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Exp: Let us suppose, number of rows = x

And, number of students in a row = y

Total students in class = Number of rows × Number of students in a row = xy

According to the question,

Condition-I: If 3 students are extra in a row, there would be 1 row less.

Total number of students = (x – 1) (y + 3)

$\Rightarrow xy = (x - 1)(y + 3)$

$\Rightarrow xy = xy - y + 3x - 3$

$\Rightarrow 3x - y = 3$ ...(i)

Condition-II: If 3 students are less in a row, there would be 2 rows more.

Total Number of students = (x + 2) (y – 3)

$\Rightarrow$ xy = xy + 2y – 3x – 6

$\Rightarrow$ 3x – 2y = –6 ...(ii)

You can solve the equations yourself now.

The number of rows, X = 4

5. In a ΔABC, ∠C = 3 ∠B = 2 (∠A + ∠B). Find the three angles.

Exp: It is given that,

∠C = 3 ∠B = 2(∠B + ∠A)

3∠B = 2 ∠A+2 ∠B

∠B = 2 ∠A

2∠A – ∠B= 0 ...(1)

As, sum of all the interior angles of a triangle = 180°.

Therefore, ∠A + ∠B + ∠C = 180°

∠A + ∠B + 3 ∠B = 180° (∵∠C = 3∠B)

∠A + 4 ∠B = 180° ...(2)

You can solve the equations by yourself now.

The value of ∠A = 20°, ∠B = 40° and ∠C = 120°.

6. Draw the graphs of the equations 5x – y = 5 and 3x – y = 3. Determine the co-ordinates of the vertices of the triangle formed by these lines and the y axis.

Exp: "Do it yourself".

7. Solve the following pair of linear equations:

a. px + qy = p – q

qx – py = p + q

b. ax + by = c

bx + ay = 1 + c

c. $\frac{x}{a}-\frac{y}{b}=0$

ax + by = $a^2 + b^2$

d. (a – b)x + (a + b) y = $a^2 – 2ab – b^2$

(a + b)(x + y) = a2 + b2

e. 152x – 378y = – 74

–378x + 152y = – 604

Exp: (a) The given equations are: px + qy = p – q ...(1)

qx – py = p + q ...(2)

On multiplying eq. (1) and eq. (2) by p and q respectively, we get

$p^2x + pqy = p^2 - pq$ ...(3)

$q^2x - pqy = pq + q^2$ ...(4)

On adding eq. (3) and (4), we obtain

$p^2x + q^2x = p^2 + q^2$

$(p^2 + q^2) x = p^2 + q^2$

$$x=\frac{\left(p^2+q^2\right)}{p^2+q^2}=1$$

On putting value of x in eq. (1), we get

p(1) + qy = p – q

qy = p – q – p

qy = –q

y = –1

Hence, x = 1 and y = –1

(b) ax + by = c ...(1)

bx + ay = 1 + c ...(2)

On multiplying eq. (1) and (2) by (a) and (b) respectively, we get

$a^2x + aby = ac$ ...(3)

$b^2x + aby = b + bc$ ...(4)

On subtracting eq. (4) from eq. (3), we get

$(a^2 - b^2)\,x = ac - bc - b$

$$x=\frac{(ac-bc-b)}{\left(a^2-b^2\right)}$$

Now, using eq. (1) we get

ax + by = c

$$\Rightarrow a\left(\frac{ac-bc-b}{a^2-b^2}\right)+by=c$$

$$\Rightarrow \frac{a^2c-abc-ab+by(a^2-b^2)}{a^2-b^2}=c$$

$\Rightarrow a^2c - abc - ab + by(a^2 - b^2) = c(a^2 - b^2)$

$= by(a^2 - b^2) = a^2c - b^2c - a^2c + abc + ab$

$$\Rightarrow y=\frac{abc+ab-b^2c}{b(a^2-b^2)}$$

$$\Rightarrow y=\frac{b(ac+a-bc)}{b(a^2-b^2)}$$

$$\Rightarrow y=\frac{ac+a-bc}{a^2-b^2}$$

(c) Do it yourself.

(d) Do it yourself.

(e) The given equations are:

152x − 378y = − 74 ...(1)

–378x + 152y = − 604 ...(2)

On adding eq (1) and (2), we get

–226x – 226y = –678

$\Rightarrow$ –226 (x + y) = –678

$\Rightarrow$ x + y = 3 ...(3)

On subtracting eq. (2) from (1), we get

$530x - 530y = 530$

$530(x - y) = 530$

$\Rightarrow x - y = 1$ ...(4)

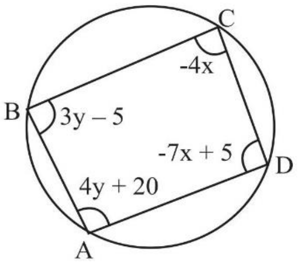
Now, on adding eq. (3) and (4), we obtain,

$2x = 4$

$\Rightarrow x = 2$

On putting value of x in eq. (3), we get

8. ABCD is a cyclic quadrilateral (see Fig). Find the angles of the cyclic quadrilateral.



Exp: As the sum of the opposite angles of a cyclic quadrilateral = 180°

Therefore, we have

$\angle C + \angle A = 180$

i.e., $(4y + 20) + (-4x) = 180°$

$-4x + 4y = 180° - 20°$

$-4x + 4y = 160°$

$-4(x - y) = 160°$

$x - y = -40°$ ...(i)

Also, $\angle B + \angle D = 180°$

$(3y - 5) + (-7x + 5) = 180°$

$-7x + 3y = 180°$ ...(ii)

On multiplying eq. (i) by (3), we have

$3(x - y) = -120°$

$3x - 3y = -120°$ ...(iii)

Now, on adding eq. (ii) and (iii), we have

$-7x + 3x = 180 - 120$

$-4x = 60$

$\Rightarrow x = -15$

On putting this value of x in eq (i), we get

$x - y = -40$

$-15 - y = -40$

$y = 40 - 15$

$\Rightarrow y = 25$

Now,

As, $\angle A = 4y + 20$

$\angle A = 4(25) + 20$

$\angle A = 120°$

Also, $\angle B = 3y - 5$

$\angle B = 3(25) - 5$

$\angle B = 70°$

Also, $\angle C = -4x$

$\angle C = -4(-15) = 60°$

And, $\angle D = -7x + 5$

$\angle D = -7(-15) + 5$

$\angle D = 105 + 5 = 110°$

Subjective Questions

Very Short Answer Type Questions

1. Using elimination by substitution method, solve the following system of linear equations:

   $2x + 3y = 7$ ...(1)

   $3x - y = 5$ ...(2)

2. Find the value of a, such that the equation $2ax + 4ay = 30$ may have $x = 1$, $y = 2$ as a solution.

3. Find the value of k for which the given system of equations

   $(k - 3)x + 3y = k$

   $kx + ky = 12$

   has infinitely many solutions.

4. Find the value of a such that $x = -1$, $y = 3$ is a solution of equation $9(x - a) + a(y + 7) = 8$.

5. If the ratio of incomes of P and Q is 5 : 6 and the ratio of their expenditure is 9 : 11. If each of them saves Rs 1000, then find their individual income.

6. A woman starts a job with a certain monthly salary and earned a fixed increment every year. If her salary was Rs 3625 after 5 years of service and Rs 4000 after 8 years of service, then find the amount of fixed annual increment?

7. Express x in terms of y, given that $5x - 3y = 8$. Check whether (–2, –6) is a point on the given line.

8. In a class there are certain number of benches, if 3 students sit on each bench then 9 students do not get any seat, but if 4 students sit on each bench then one bench remains unused. Express the statement in equation.

9. The following relation exist between centigrade (°C) and farenheit (°F) temperature, $\frac{C}{100} = \frac{F - 32}{180}$. Write this relation in ordinary form and draw its graph.

10. If cost of 5 books and 7 pens together is ₹79, whereas the cost of 7 books and 5 pens together is ₹77, then find the cost of 1 book and 2 pens.

Short Answer Type Questions

1. Solve the following system of linear equations graphically;

   $3x + y - 11 = 0$; $x - y - 1 = 0$

   Shade the region bounded by these lines and y-axis. Then, determine the areas of the region bounded by these lines and y-axis.

2. Solve, the following system of linear equations.

   $$\frac{1}{x+y} + \frac{2}{x-y} = 2$$

   $$\frac{2}{x+y} - \frac{1}{x-y} = 3$$

   where $x + y \neq 0$ and $x - y \neq 0$

3. Solve, the following system of linear equations:

   $x + 2y + z = 12$

   $2x - z = 4$

   $x - 2y = 4$

4. If the straight line $ax + by + c = 0$ passes through the point (1, 1), then solve the following system of equations:

   $bcx + cay = a^3 + b^3 + c^3$

   and $2x + 2y = 3(a + b)$

5. A man sold a chair and table together for ₹760 on a profit of 25% on chair and 10% on table. By selling them together for ₹767.50, he would have made a profit of 10% on chair and 25% on table. Find the cost price of each.

6. P and Q are standing on the circumference of a circle of radius 3 m subtending an angle of 60° at centre. Initially, Q is ahead of P. If they start moving at the same time, then find after what time they will meet if they are moving in clockwise direction with their speed 7m/s and 5m/s respectively.

7. Find the numbers, if the sum of 2 numbers is 1000 and the difference between their squares is 256000.

8. A boy walks a certain distance with certain speed. If he walks $\frac{1}{2}$ km an hour faster, he takes 1 hour less. But, if he walks 1 km an hour slower, he takes 3 more hours. Find the distance covered by the boy and his original rate of walking.

9. A person invested an amount at the rate of 12% simple interest and an amount at rate of 10% simple interest. She received yearly interest of ₹130%. But if he had interchanged the amount invested, she would have received ₹4 more as interest. How much amount did she invest at different rate?

10. An alloy of gold and silver weighing 100g is worth ₹36,400 but if the weight of gold and silvers be interchanged, it would be worth ₹24,600, if the price of gold be ₹600 per g, find the weight of silver and gold in the alloy and the price of silver per g.

Long Answer Type Questions

1. The difference of the digits of two digit number is 3. If this number is added to the number obtained by interchanging the digits, the sum is 99. Find the number?

2. Draw the graphs of $2x - y + 5 = 0$, $x + 2y = 6$ and $y = 3$. Find the area of the trapezium formed by these three graphs together with the x-axis.

3. By using substitution method, solve the following system of equations:

$2x - 3y - 1 = 0$

and $\frac{x+(3x-2y)}{x-(3x+2y)} = -3$

4. Priya travels 600 km to her home partly by train and partly by car. She takes 8 hours if she travels 120 km by train and the rest by car. She takes 20 minutes longer if she travels 200 km by train and the rest by car. Find the speed of the train and car.

5. In a cyclic quadrilateral PQRS, $\angle P = (2x + 4)^\circ$, $\angle Q = (y + 3)^\circ$, $\angle R = (2y + 10)^\circ$, $\angle S = (4x - 5)^\circ$. Find the four angles.

Integer Type Questions

1. The lateral surface area of the cylinder gets reduced by $75\pi cm^2$ when its diameter is reduced by 5 cm while keeping the height same. Its lateral surface area is increased by $100\pi cm^2$, if its height is increased by 5 cm, while keeping the diameter same. If diameter is D and height is H. Find (D – H).

2. A two digit number is equal to three times the sum of its digits. Also the unit's digit is one more than three times the ten's digit. Find the sum of digits of the number.

3. Out of 28 students, atleast one take Mathematics, English or History. Those students taking only one subject are all taking Mathematics. Of those students taking exactly two of the three subjects, it is known that the number taking Mathematics and English is identical to the number taking Mathematics only. Six students are taking Mathematics and History only and the number taking History and English is even non-zero and equal to 5 times then number taking all three subjects. Find the number of students taking mathematics and English only.

4. If the length of a rectangle is decreased by 5 metre and the breadth is increased by 3 metre, then its area is reduced by 8 m^2. If both length and breadth are increased by 3m and 2 m respectively, then its area is increased by 74 m^2, then find the difference of length and breadth.

5. If the cost of 7 audio cassettes and 3 video cassettes is ₹1110 and the cost of 5 audio cassettes and 4 video cassettes is ₹1350. If the cost of an audio cassette is 10P, then find the value of P.

6. If $kx + 6y = 27$ and $6x + ky = 28$ are inconsistent, then k can be ________.

7. A two-digit number is 18 more than the sum of its digits. The difference between the digits is z. Find the number of possible values of z.

8. Find the value of 15k if $3x + 2y = 11$ and $7x - 5ky = 23$ are inconsistent.

9. Find the value of k for the system of equations $3x + 8ky = 14$ and $9x + 24y = 42$ having infinite solutions.

10. Find the value of (x + y) if $5x + 6y = 21$ and $6x + 5y = 23$.

Multiple Choice Questions

Level-I

1. The given pair of equations

$6x - 3y + 10 = 0$

$2x - y + 9 = 0$

graphically represents two lines which are:

a. Intersecting at exactly one point.

b. Intersecting at exactly four points.

c. Coincident

d. Parallel

2. The given pair of linear equations $x + 2y - 5 = 0$ and $-3x - 6y + 15 = 0$ have:

a. Exactly one solution

b. Exactly two solutions

c. Infinitely many solutions

d. No solution

3. If a pair of linear equations is consistent, then the lines will be:

a. Intersecting or parallel

b. Always coincident

c. Intersecting or coincident

d. Always intersecting

4. The given pair of equations, $y = 0$ and $y = -7$ has

a. No solution

b. Two solutions

c. Infinitely many solutions

d. None of these

5. If the lines represented by given pair of equations,

$3x + 2ky = 2$

$2x + 5y + 1 = 0$

are parallel, then the value of k is

a. 3/5　　b. 2/5

c. 15/4　　d. 3/2

6. Find the value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions.

a. 3　　b. –5

c. –12　　d. None of these

7. If one equation of a pair of dependent linear equations is $-5x + 7y - 2 = 0$, then the second equation can be

a. $20x + 28y + 8 = 0$　　b. $-10x - 14y + 4 = 0$

c. $-10x + 14y + 4 = 0$　　d. $10x - 14y = -4$

8. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Then, find the numbers:

a. 36, 44　　b. 42, 48

c. 40, 48　　d. 44, 50

9. Find the solution of the equations $x - y = 2$ and $x + y = 4$.

a. 6 and 8　　b. 5 and 3

c. 3 and 1　　d. –1 and –3

10. For which values of a and b, will the following pair of linear equations have infinitely many solutions?

$x + 2y = 1$

$(a - b)x + (a + b)y = a + b - 2$

a. $a = 2$ and $b = 1$　　b. $a = 4$ and $b = 3$

c. $a = -3$ and $b = 1$　　d. $a = 3$ and $b = 1$

11. The father's age is six times his son's age. After four years, the age of the father will be four times his son's age. Find the present ages, of the son and the father, in years.

a. 8 and 32　　b. 5 and 30

c. 6 and 36　　d. 3 and 24

12. Rakhi has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins is, respectively

a. 35 and 15　　b. 35 and 20

c. 20 and 25　　d. 25 and 25

13. In a competitive examination, one mark is awarded for each correct answer while 1/2 mark is deducted for every wrong answer. Priya answered 120 questions and got 90 marks. How many questions did she answer correctly?

a. 100　　b. 74

c. 90　　d. 60

14. The angle of a cyclic quadrilateral PQRS are:

$\angle P = (6x + 10)^\circ$, $\angle Q = (5x)^\circ$

$\angle R = (x + y)^\circ$, $\angle S = (3y - 10)^\circ$

Then value of x and y are:

a. x = 20° and y = 30° b. x = 30° and y = 40°

c. x = 44° and y = 15° d. x = 15° and y = 15°

15. A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Preeti paid ₹ 22 for a book kept for six days, while Ritu paid ₹ 16 for the book kept for four days, then the charge for each extra day is:

a. ₹ 5 b. ₹ 8

c. ₹ 3 d. ₹ 2

16. The given pair of linear equations $x + 2y - 5 = 0$ and $-4x - 8y + 20 = 0$ have:

a. Exactly one solution

b. Exactly two solutions

c. Infinitely many solutions

d. No solution

17. The given pairs of linear equations $9x + 3y + 12 = 0$ and $18x + 6y + 26 = 0$ have

a. Exactly one solution

b. Exactly two solutions

c. Infinitely many solutions

d. No solution

18. Find the value of k, if the lines $3x + 2ky - 2 = 0$ and $2x + 5y + 1 = 0$ are parallel.

a. $\frac{3}{5}$ b. $\frac{15}{4}$

c. $\frac{4}{5}$ d. $\frac{5}{4}$

19. If one equation of a pair of dependent linear equations is $-3x + 5y - 2 = 0$, then the second equation will be:

a. $-6x + 10y - 4 = 0$ b. $6x - 10y - 4 = 0$

c. $6x + 10y - 4 = 0$ d. None of these

20. Find the solution of the equations $x - y = 2$ and $x + y = 4$.

a. 3 and 1 b. 3 and 5

c. 5 and 1 d. –1 and –3

21. A fraction becomes 1/3 when 1 is subtracted from the numerator and it becomes 1/4 when 8 is added to its denominator. The fraction obtained is:

a. $\frac{2}{12}$ b. $\frac{4}{12}$

c. $\frac{5}{12}$ d. $\frac{7}{12}$

22. The solution of $4/x + 3y = 14$ and $3/x - 4y = 23$ is:

a. $\frac{1}{5}$ and –2 b. $\frac{1}{2}$ and $\frac{1}{5}$

c. 3 and $\frac{1}{5}$ d. 2 and $\frac{1}{3}$

23. Kirti can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Her speed of rowing in still water and the speed of the current is:

a. 4km/hr and 2km/hr

b. 7km/hr and 4km/hr

c. 6km/hr and 4km/hr

d. 10km/hr and 6km/hr

24. The angles of cyclic quadrilaterals ABCD are: $A = (6x + 10)$, $B = (5x)°$, $C = (x + y)°$ and $D = (3y - 10)°$. The value of x and y is:

a. 10°, 20° b. 20°, 30°

c. 30°, 40° d. 40°, 50°

25. For what value of k will the equations $x + 2y + 7 = 0$, $2x + ky + 14 = 0$ represents coincident lines?

a. 6 b. 4

c. 3 d. 2

26. The equations $6x + 5y = 11$ and $9x + \frac{15}{2}y = 21$ have

a. Exactly one solution

b. No solution

c. Infinitely many solutions

d. Two solutions

27. Which among the following points lie on the line represented by $2x + 7y = 19$?

a. (–1, 3) b. (–2, 3)

c. (–3, 5) d. (5, 2)

28. If 2 is subtracted from the numerator and 1 is added to the denominator, a fraction becomes $\frac{1}{2}$ but when 4 is added to the numerator and 3 is subtracted from the denominator, it becomes $\frac{3}{2}$, then find the fraction:

a. $\frac{12}{11}$ b. $\frac{8}{11}$

c. $\frac{7}{11}$ d. $\frac{13}{13}$

29. Find the numbers, if the sum of two numbers is 69 and their difference is 25.

a. 47, 22 b. 45, 26

c. 27, 42 d. 46, 23

30. In class X of a certain school, the average marks secured in mathematics by boys is 76 and by girls is 60. If the average of marks secured in mathematics of class X be 70 and the number of girls be 30, then the number of boys is

a. 50 b. 25
c. 45 d. 55

31. 3 women and 4 girls can complete a piece of work in 6 days whereas the same work can be done by 5 women and 2 girls in 5 days. In how many days a women will complete that work?

a. 24 days b. 25 days
c. 15 days d. 30 days

32. Out of two numbers, if 3-times the bigger number is divided by smaller one, we get 5 as quotient and 1 as remainder, whereas if 6 times the smaller number is divided by bigger one, the quotient is 3 and remainder is 6. Then numbers are:

a. 12, 7 b. 8, 11
c. 16, 7 d. 12, 5

33. The lines $2(x + 3) = 3(x + 9)$ and $y(a + 1) = 2(y - 5)$ are $(a \neq 1)$

a. Parallel
b. Perpendicular
c. Both parallel to x-axis
d. None of these

34. Find the value of x, if $\frac{x}{a} + \frac{y}{b} = a + b$ and $\frac{x}{a^2} + \frac{y}{b^2} = 2$.

a. a^2 b. b^2
c. a^2b^2 d. ab

35. 2 tables and 3 chairs together cost Rs 2000 whereas 3 tables and 2 chairs together cost Rs 2500. Find the total cost of 1 table and 5 chairs.

a. ₹ 1700 b. ₹ 1500
c. ₹ 1300 d. ₹ 1450

Level-II

1. Find the numbers, if sum of two numbers is 15 and if the sum of their reciprocals is $\frac{3}{10}$.

a. 10 and 12 b. 7 and 9
c. 10 and 5 d. 12 and 5

2. If we increase the length of a rectangle by 2 units and breadth by 2 units, then the area of rectangle is increased by 54 square units. Find the perimeter of the rectangle.

a. 22 units b. 50 units
c. 56 units d. can't be determined

3. Find the values of a and b for which the following system of linear equations has infinite number of solutions:

$2x - 3y = 7$

$(a + b)x - (a + b - 3)y - (4a + b) = 0$

a. –3, –4 b. –5, –1
c. 2, 3 d. 5, 1

4. Shyam has some cows and some hens in his shed. The total number of legs is 92 and total number of heads is 29. Find the number of cows in shed?

a. 15 b. 14
c. 17 d. 19

5. At present the age of father is 4 times the age of his son. After 5 years, the age of father will be thrice the age of his son. Find the present age of father.

a. 38 years b. 35 years
c. 40 years d. 20 years

6. Ten years ago, the age of father was 4 times the age of his son whereas five years ago it was thrice the age of his son. Then present age of son in years is

a. 20 b. 30
c. 18 d. 10

7. The sum of a two-digit number and the number formed by interchanging the digits is 132. If the digits of the number differ by 4, then the number is

a. 87 or 78 b. 84 or 48
c. 64 or 48 d. 84 or 38

8. After six years, the age of father will be thrice the age of his son and three years ago the father was 9 times the age of his son. Find the present age of father in years.

a. 15 b. 25
c. 30 d. 40

9. The age of Ram and his father differ by 30 years. The difference between the square of their ages is 1560. Then, find the present age of son in years.

a. 15 b. 20
c. 10 d. 11

10. A man sold a chair and a table together for ₹1520 thereby making a profit of 25% on the chair and 10% on table. By selling them together for ₹1535, he would have made a profit of 10% on the chair and 25% on the table. Find the cost price of table.

a. ₹600 b. ₹700
c. ₹500 d. ₹400

11. The angles of a cyclic quadrilateral PQRS are given as $\angle P = x°$, $\angle Q = y°$, $\angle R = (x + 8)°$ and $\angle S = (2y - 3)°$. Find the largest angles

a. 86
b. 109°
c. 119°
d. 100°

12. A fraction becomes $\frac{11}{3}$, if the denominator of a fraction is added to its numerator and its numerator is subtracted from the denominator. Find the sum of numerator and denominator, if the numerator is less than the denominator by 3.

a. 11
b. 13
c. 15
d. 19

13. A train covered a certain distance at a uniform speed. If the train had been 10 km/hr faster, it would have taken 2 hour less than scheduled time and if the train was slower by 10 km/hr, train would have taken 4 hours more than scheduled time. Find the length of the journey.

a. 215 km
b. 230 km
c. 200 km
d. 240 km

14. Let a, b, c be the positive numbers, then the following system of equations in x, y and z

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1; \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1;$

$\frac{-x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ has

a. No solution
b. Exactly one solution
c. Infinitely many solutions
d. Finitely many solutions

Assertion & Reason Type Questions

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
c. Assertion (A) is true but reason (R) is false.
d. Assertion (A) is false but reason (R) is true.

1. Assertion: If the given system of linear equations $2x + 3y = 7$ and $2ax + (a + b)y = 28$ has infinitely many solutions then $2a - b = 0$

Reason: The system of equation $3x - 5y = 9$ and $6x - 10y = 8$ has a unique solution.

2. Assertion: If $x = 3$, $y = 1$ is the solution of the line $2x + y - q^2 - 3 = 0$, then the value of $q = \pm 2$.

Reason: The solutions of the line will satisfy the equation of the line.

3. Assertion: The value of k for which the system of equations $kx - y = 2$, $6x - 2y = 3$ has a unique solution is 3.

Reason: The system of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

4. Assertion: If $k = 2$, then $x + y - 4 = 0$ and $2x + ky - 3 = 0$ has no solution.

Reason: $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are consistent if $\frac{a_1}{a_2} \neq \frac{k_1}{k_2}$.

5. Assertion: Given pair of linear equations : $9x + 3y + 12 = 0$, $18x + 6y + 24 = 0$ have infinitely many solutions.

Reason: Pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ have infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Case-Based Type Questions

Case-Based-I: Seema bought 3 notebooks and 2 pens for ₹80. Her friend Preeti said that price of each notebook could be ₹25. also bought the same types of notebooks and pens. She paid ₹110 for 4 notebooks and 3 pens.

1. By taking cost of one notebook as ₹x and cost of one pen as ₹y, form the pair of linear equations in two variables.

a. $3x + 2y = 80$ and $4x + 3y = 110$
b. $3x - 2y = 80$ and $4x - 3y = 110$
c. $x + y = 80$ and $x + y = 110$
d. $3x + 2y = 110$ and $4x + 3y = 80$

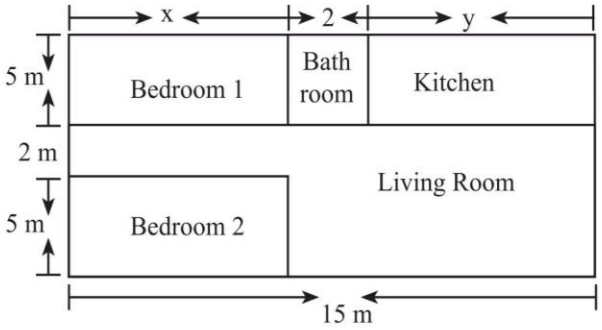
2. The cost of notebook is,

a. ₹20
b. ₹16
c. ₹5
d. ₹15

3. The cost of one pen is,

a. ₹2.5
b. ₹10
c. ₹5
d. ₹15

Case-Based-II: In the layout given below, the design and measurement has been made such that area of two bedroom and kitchen together is 95 m^2.



1. Find the area of two bedrooms and kitchen.

a. 5x, 5y b. 10x, 5y

c. 5x, 10y d. x, y

2. The length of the outer boundary of the layout is,

a. 25 m b. 15 m

c. 50 m d. 54 m

3. The area of each bedroom is,

a. 30 m^2 b. 35 m^2

c. 65 m^2 d. 42 m^2

4. The area of living room in the layout is,

a. 25 m^2 b. 35 m^2

c. 75 m^2 d. 65 m^2

5. Find the cost of laying tiles in Kitchen at the rate of ₹50 per m^2

a. ₹1500 b. ₹1000

c. ₹1750 d. ₹3000

Multi Correct MCQ's

1. Which among the following system of simultaneous linear equations are consistent.

a. $x + 2y = 7, 3x - y = 5$

b. $4x - 2y = 4, 2x - 4y = 1$

c. $x + y = 3, 2x + 2y = 6$

d. $2x + y = 0, 3x - 2y = 0$

2. For what value of k, the simultaneous linear equation $x + 3y = 2$ and $3x + ky = 5$ have one and only one solution.

a. 6 b. 8

c. 9 d. 12

3. Which of the following system of equations have no solution?

a. $x - y = 3, 2x - 2y = 1$

b. $x + y = 2, 2x + 2y = 1$

c. $y - 2x + 1 = 0, 2y - 4x + 2 = 0$

d. None of these

4. For what value of k, the system of equation $2x - 3y = 1$ and $kx + 2y = 3$ has unique solution

a. $k = \frac{-4}{3}$ b. $k = 4$

c. $k = \frac{2}{3}$ d. $k = 2$

Olympiad & NTSE Type Questions

1. The fare of a child is half the fare of an adult in railway but there is no concession in reservation fee. If the fare with reservation of an adult in second class sleeper from Patna to Delhi is ₹253 and that of a child be ₹134, then the fare of an adult is

a. ₹238 b. ₹230

c. ₹250 d. ₹220

2. Solution of the following system of equations:

$\frac{x-a}{c-a} + \frac{y-b}{c-b} = 1$ and $\frac{x+a}{c} + \frac{y-a}{a-b} = \frac{a}{c}$, is given by

a. $x = \frac{a^2}{b}, y = \frac{b^2}{a}$ b. $x = c, y = a$

c. $x = a, y = b$ d. $x = c, y = b$

3. If $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ and $u + v = s$, the expression for s in terms of u, v and f only is

a. $s = \frac{vf - u}{f}$ b. $s = \frac{uv}{f}$

c. $s = \frac{t^2}{v^2 - u^2}$ d. none of these

4. Riya travels 24 km to her home by covering half of it by walking through road and half of it by rowing a boat through river. Riya takes 5 hours to complete the journey of 12 km with the stream and remaining by walking. It takes 7 hours to complete the journey against the stream and remaining by walking. If there were no current, the journey would take $5\frac{2}{3}$ hours; the rate of the stream is

a. 2.5 km/hr b. 4 km/hr

c. 1.5 km/hr d. 3 km/hr

5. A part of salary of a salesman is fixed and the rest depends on the commission he gets on the sales. In one month he got an order of ₹31500 and got total monthly salary of ₹5575. In another month, he got an order of ₹45000 and got the total salary as ₹6250. If he gets the total salary of ₹7750 in next month, then order of sales he got

a. ₹60,000　　b. ₹75,000
c. ₹65,000　　d. ₹80,000

6. Find the solution of the following system of equations,

$$\frac{2x-3y}{3}=3+\frac{3y-4x}{4}, \frac{1}{3}(6y+7x)=\frac{1}{5}(7x+12y)+4$$

a. x = 8, y = 2　　b. x = 4, y = 6
c. x = 6, y = 4　　d. None of these

7. Find the solution of the following system of equations

$3(2u + v) = 7uv$, $u + 3v = \frac{11}{3}uv$ is

a. $u=-2, v=-\frac{3}{2}$　　b. $u=4, v=\frac{1}{3}$
c. $u=-1, v=-\frac{3}{2}$　　d. $u=1, v=\frac{3}{2}$

8. For the following system of equations

$$\frac{1}{2(2x+3y)}+\frac{12}{7(3x-2y)}=\frac{1}{2}$$

$$\frac{7}{2x+3y}+\frac{4}{3x-2y}=2, \text{ where } 2x+3y\neq 0, 3x-2y=0$$

find p, if y = px – 3.

a. 4　　b. –3
c. 2　　d. –2

9. A three digit number $\overline{abc}$ is 459 more than the sum of its digits. What is the sum of the 2-digit number $\overline{ab}$ and one-digit number a?

a. 71　　b. 81
c. 51　　d. None of these

10. On the national highway two points P and Q are situated at a distance of 160 km apart. A car starts from P and another car starts from Q at the same time. If they go in the same direction, they meet in 8 hours and if they go in opposite direction they meet in 2 hours, then speed of car starting from Q is

a. 20 km/hr　　b. 40 km/hr
c. 50 km/hr　　d. 55 km/hr

11. Acids of 77% purity and 99% purity are mixed to prepare 28 litres of acid of 88% purity, then find the quantity of each solution.

a. 12 litre each　　b. 11 litre each
c. 14 litre each　　d. 16 litre each

12. Two candles of equal length start burning at the same instant. One of the candles burn in 5 hrs and the other in 4 hrs. By the time one candle is 2 times the length of the other, the candles have already burnt for:

a. $2\frac{1}{2}$ hrs.　　b. $5\frac{1}{3}$ hrs.
c. $3\frac{1}{9}$ hrs.　　d. $3\frac{1}{3}$ hrs.

13. The solution of the equations $\frac{m}{x}+\frac{n}{y}=a, \frac{n}{x}+\frac{m}{y}=b$ is given by

a. $x=\frac{n^2+m^2}{am-bn}, y=\frac{m^2-n^2}{bm-an}$

b. $x=\frac{m^2+n^2}{am-bn}, y=\frac{n^2-m^2}{bm-an}$

c. $x=\frac{m^2-n^2}{am-bn}, y=\frac{m^2-n^2}{bm-an}$

d. None of these

14. A test has 50 questions. A student was awarded 1 mark for a correct answer, $-\frac{1}{3}$ for a wrong answer and $-\frac{1}{6}$ for not attempting a question. If the net score of a student is 32, the number of questions answered wrongly by the student can not be less than:

a. 5　　b. 4
c. 3　　d. 2

15. At a certain fast food restaurant, Priya can buy 3 burgers, 7 shakes and one order of fries for ₹120. At the same place, it would cost ₹146.50 for four burgers, 10 shakes and one order of fries. How much would it cost for an ordinary meal of one burger, one shake and one order of fries.

a. 67　　b. 41
c. 21　　d. Cannot be determined

Explanations

Subjective Questions

Very Short Answer Type Questions

1. Do it yourself. ($x = 2$ and $y = 1$)

2. As, it is given that, $x = 1$, $y = 2$ is a solution of the given equation $2ax + 4ay = 30$ and therefore, it should satisfy the given equation.

i.e., $2a(1) + 4a(2) = 30$

$\Rightarrow 2a + 8a = 30$

$\Rightarrow 10a = 30$

$\Rightarrow a = 3$

3. We can write the given equations as $(k - 3)x + 3y - k = 0$ and $kx + ky - 12 = 0$

It must satisfy the following condition, for infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

i.e., $\frac{k-3}{k} = \frac{3}{k} = \frac{-k}{-12}$

$\Rightarrow \frac{k-3}{k} = \frac{3}{k}$ and $\frac{3}{k} = \frac{k}{12}$

$\Rightarrow (k - 3) k = 3k$ and $36 = k^2$

$\Rightarrow k^2 - 3k = 3k$ and $k^2 = 36$

$\Rightarrow k^2 - 6k = 0$ and $k^2 = 36$

$\Rightarrow (k = 0$ or $k = 6)$ and $(k = \pm 6)$

$\Rightarrow k = 6$

Therefore, for $k = 6$, the given system has infinitely many solutions.

4. As, it is given that $x = -1$, $y = 3$ is a solution of the given equation, hence, these values should satisfy the given equation.

On putting $x = -1$ and $y = 3$ in the given equation, we obtain

$9(-1 - a) + a(3 + 7) = 8$

$\Rightarrow -9 - 9a + 10a = 8$

$\Rightarrow a = 8 + 9 = 17$

5. Let us suppose, income of P = ₹5x

and, income of Q = ₹6x

Let, expenditure of P = ₹9y

And, expenditure of Q = ₹11y

As we know,

Income – Expenditure = Saving

Now, according to given conditions,

$5x - 9y = 1000$ ...(i)

$6x - 11y = 1000$ ...(ii)

After solving above two equations, we get $x = 2000$ and $y = 1000$

Hence, income of P = ₹10,000

And, income of Q = ₹12,000

6. Let, starting salary of woman = ₹x

And, amount of fixed annual increment = ₹y

Salary after 5 years = ₹(x + 5y)

Salary after 8 years = ₹(x + 8y)

According to given conditions,

$x + 5y = 3625$ ...(1)

$x + 8y = 4000$ ...(2)

On solving eq. (1) and (2), we get

$y = 125$ and $x = 3000$

Hence, amount of fixed annual increment = ₹125

7. It is given that, $5x - 3y = 8$

$\therefore\ x = \frac{8+3y}{5}$ ...(1)

Now, put $x = -2$, $y = -6$ in eq. (1)

$-2 = \frac{8+3(-6)}{5}$

$-2 = -2$

As, LHS = RHS

Therefore point $(-2, -6)$ lie on the line because it satisfies the equation of the line.

8. Let the number of students in class = x

And, number of benches = y

It is given that if three students sit on each bench, nine students will be left. So we can deduce that the number of students is nine more than three times the number of benches.

$\therefore$ Total number of students = $3y + 9$

$\Rightarrow\ x = 3y + 9$ ...(1)

Also, it is given that if four students sit on each bench, one bench will be left. So we can deduce that the number of students is 4 less than four times the number of benches.

∴ Total number of students = 4y – 4

$\Rightarrow \quad x = 4y - 4 \quad$ ...(2)

Hence, equations (1) and (2) are the required equations.

On solving the equations, we get

Number of students = 48

Number of benches = 13

9. As, $\frac{C}{100} = \frac{F-32}{180}$

$\left(\frac{180}{100}\right)C = F - 32$

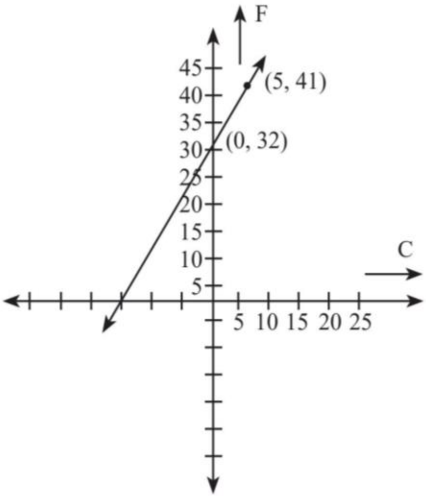
5F – 9C = 160

Hence, $F = \frac{9C+160}{5}$

The solution table is,

C	5	0
F	41	32

Let us plot the graph:



10. Let us suppose, cost of books = ₹x

Ans, cost of pen = ₹y

According to question,

$5x + 7y = 79 \quad$ ...(1)

$7x + 5y = 77 \quad$ ...(2)

On solving eq. (1) and (2), we get

x = 6, y = 7

Hence, total cost of 1 book = ₹6

Total cost of 2 pens = ₹14

Therefore, total cost of 1 book and 2 pens = ₹20

Short Answer Type Questions

1. The given equations are:

3x + y – 11= 0 and x – y – 1 = 0

(a) For equation, 3x + y – 11 = 0

i.e., 3x + y – 11 = 0

⇒ y = – 3x + 11

The solution table is

x	2	3
y = –3x + 11	5	2

(b) For equation, x – y – 1 = 0

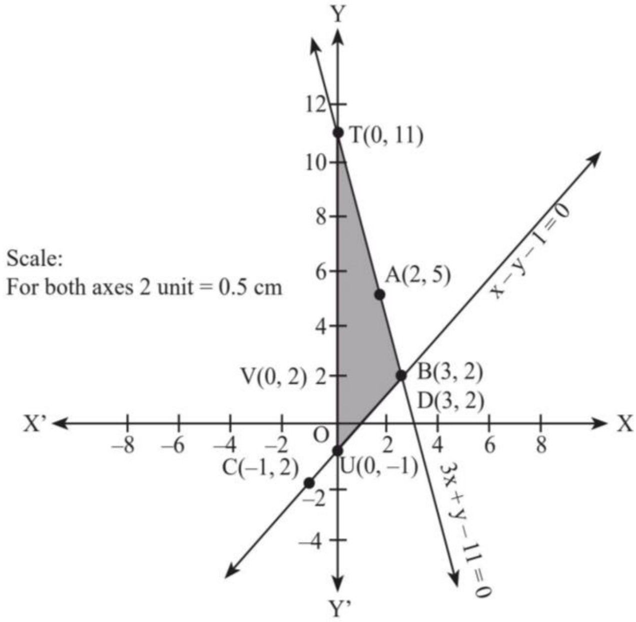
i.e., x – y – 1 = 0

⇒ y = x – 1

The solution table is:

x	–1	3
y = x – 1	–2	2

Now, let us plot the graph:



As it can be seen that the two lines intersect at point B(3, 2). Therefore, x = 3 and y = 2.

The area bounded by the lines represented by the given equations and y-axis is shaded, as shown in graph.

Area of the shaded region is,

⇒ Area of ΔBUT = $\frac{1}{2}$ × base × height

$= \frac{1}{2} \times (TU \times VB) = \frac{1}{2} \times (OT + OU) \times VB$

$= \frac{1}{2}(11 + 1) \times 3 = \frac{1}{2} \times 12 \times 3 = 18$ sq. units.

Therefore, required area = 18 sq. units.

2. Do it yourself.

Hint: Put $\frac{1}{x+y}=u$ and $\frac{1}{x-y}=v$ in the given equations.

On solving, we get

$x=\frac{45}{16}$ and $y=\frac{-35}{16}$.

3. The given equations are:

x + 2y + z = 12 ...(i)

2x – z = 4 ...(ii)

x – 2y = 4 ...(iii)

Using eq. (i), we have,

z = 12 – x – 2y.

On substituting value of z in eq (ii), we have

2x – (12 – x – 2y) = 4

⇒ 2x – 12 + x + 2y = 4

⇒ 3x + 2y = 16 ...(iv)

On adding equations (iii) and (iv), we obtain,

4x = 20

⇒ x = 5

Now, on substituting value of x in eq (ii), we have

(2 × 5) – z = 4

⇒ z = 10 – 4 = 6

Now, on again substituting the value of x in eq. (iii), we obtain

5 – 2y = 4 ⇒ y = 1/2

Therefore, the solution of the given system of equations is;

x = 5, y = 1/2 , z = 6

4. As, line ax + by + c = 0 passes through the point (1, 1).

Hence, replacing x, y by (1, 1)

⇒ a + b + c = 0

As we know,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

If the value of a + b + c = 0, then

⇒ $a^3 + b^3 + c^3 - 3abc = 0$

⇒ $a^3 + b^2 + c^3 = 3abc$

As, $bcx + cay = a^3 + b^3 + c^3$ (Given)

⇒ bcx + cay = 3abc

⇒ c (bx + ay) = 3abc

⇒ bx + ay = 3ab ...(i)

$x+y=\frac{3}{2}(a+b)$ ...(ii)

On solving eq. (i) and (ii), we get

$x=\frac{3}{2}a,\ y=\frac{3b}{2}$

5. Let us suppose, the cost price of 1 chair = ₹x

And, let the cost price of 1 table = ₹y

Hence, S.P. of 1 chair = C.P. + profit

$=x+\frac{25}{100}x$

$\Rightarrow\ x+\frac{x}{4}=\frac{5x}{4}$

And, S.P. of 1 table = C.P. + profit

$=y+\frac{10}{100}y=y+\frac{y}{10}=\frac{11y}{10}$

According to question,

$\frac{5x}{4}+\frac{11y}{10}=760\Rightarrow 25x+22y=15200$ ...(1)

When, the percentage of profit on chair and table are reversed and using eq. (1), we get

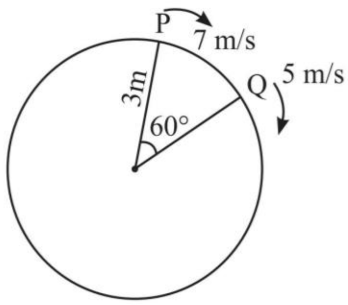
22x + 25y = 767.50 × 20

22x + 25y = 15350 ...(2)

On solving eq. (1) and (2), we have

x = 300, y = 350

6.



It is given that,

Speed of P = 7 m/sec and speed of Q = 5 m/sec

Length of arc PQ = $\frac{\theta}{360}(2\pi r)$

[You will study about this in detail in chapter "Areas related to circles'.]

$=\frac{60}{360}\times 2\pi r$ (As, θ = 60°)

$=\boxed{\frac{\pi r}{3}}$ (∵ r = 3m)

$= \frac{\pi}{3} \times 3m$

$= \pi m$

Let P and Q meet after t sec.

As we know,

Distance = Speed × Time

∴ Distance covered by P after t seconds = 7t m/s

Distance covered by Q after t seconds = 5t m/s

According to the question, P should cover the ditance of π m more than Q as Q is ahead of P by π m.

∴ $7t = 5t + \pi$

$7t - 5t = \pi$

$2t = \pi$

$t = \pi/2$

7. Let us suppose, one number be x

and another number be y

According to given conditions,

$x + y = 1000$ ...(1)

and, $x^2 - y^2 = 256000$ [x > y]

$(x - y)(x + y) = 256000$

$(x - y) \times 1000 = 256000$

$x - y = 256$ ...(2)

Now, on adding eq. (1) and (2), we get

$x + y = 1000$

$x - y = 256$

$2x = 1256$

$\Rightarrow x = 628$

On Putting value of x in eq. (1), we get

$x + y = 1000$

$628 + y = 1000$

$y = 1000 - 628$

$y = 372$

Therefore, one number is 628 and another number is 372

8. Let us suppose, v be the speed of the boy and t be the scheduled time of walking

As, distance = speed × time

∴ Distance = vt

Let, x be the distance covered

Hence,

$x = vt$ ...(i)

Now, according to the first condition that if the boy moves 1/2 km/h faster, then he takes 1 hour less to complete the journey.

$\left[v + \frac{1}{2}\right][t - 1] = x$

$vt - v + \frac{1}{2}t - \frac{1}{2} = x$

Now, using eq. (i)

$vt - v + \frac{1}{2}t - \frac{1}{2} = vt$

$\Rightarrow 2v - t = -1$ ...(ii)

According to the second condition that if the boy moves 1 km/h slower, then he takes 3 hour more to complete the journey.

$(v - 1)(t + 3) = x$

$\Rightarrow vt - t + 3v - 3 = vt$

$\Rightarrow 3v - t = 3$ ...(iii)

On solving eq. (ii) and (iii), we get

$v = 4$

Hence, speed of boy = 4 km/hr.

Time of walking = 9 hrs = t

The distance covered = 4 × 9

= 36 km.

9. Let us suppose, the amount invested at the rate of 12% be ₹x

Let amount invested at the rate of 10% be ₹y

Then, yearly interest on ₹x = $\frac{x \times 12}{100}$

And, yearly interest on ₹y = $\frac{y \times 10}{100}$

According to the given conditions,

$\frac{12x}{100} + \frac{10y}{100} = 130$

$12x + 10y = 13000$

$6x + 5y = 6500$ ...(1)

If the amount is interchanged,

$\frac{10x}{100} + \frac{12y}{100} = 134$

$5x + 6y = 6700$ ...(2)

On solving eq. (1) and (2), we get

$x = 500, y = 700.$

10. Let us suppose, weight of silver in alloy = x g

Then weight of gold in alloy = (100 – x) g

And price of silver per gram = ₹y

Price of gold per gram = ₹600

Condition-I says that the cost of 100 g alloy of gold and silver is ₹36,400.

∴ (price of silver) × (weight of silver) + (price of gold) × (weight of gold) = ₹36,400

$\Rightarrow y \times x + 600 \times (100 - x) = 36,400$

$\Rightarrow xy + 60,000 - 600x = 36,400$ ...(1)

Condition-II says that if the weight of gold and silver be interchanged i.e., weight of gold is xy and weight of silver is (100 – x)g, then its worth is ₹24,600

(price of silver) × (weight of silver) + (price of gold) × (weight of gold) = ₹24,600

$\Rightarrow y \times (100 - x) + 600x = 24,600$

$\Rightarrow 100y - xy + 600x = 24,600$ ...(2)

On adding eq. (1) and (2), we get

$y = 10$

On substituting value of y in eq. (2), we get

$x = 40$

Hence, required weight of silver in the alloy = 40 g,

required weight of gold = 100 – 40 = 60 g

and price of silver/gm = ₹10

Long Answer Type Questions

1. Let us suppose, 'x' be the unit's digit of a two digit number and let 'y' be the ten's digit of two digit number.

Hence, two digit number = 10y + x

Now, the number obtained by interchanging the digits = 10x + y

Acc. to question,

If $x > y$, then $x - y = 3$ ...(1)

and $(10y + x) + (10x + y) = 99$

$\Rightarrow 11x + 11y = 99$

$\Rightarrow 11(x + y) = 99$

$\Rightarrow x + y = 9$ ...(2)

and if $x < y$ then $y - x = 3$ ...(3)

On adding eq. (1) and (2), we obtian

$x - y = 3$

$x + y = 9$

$2x = 12$

$\Rightarrow x = 6$

Now, on substituting the value of x in eq. (1), we get

$x - y = 3$

$6 - y = 3$

$-y = 3 - 6$

$-y = -3$

$\Rightarrow y = 3$

Hence, two digit number = 10y + x = (10 × 3) + 6 = 36

On adding eq. (2) and (3), we get

$x + y = 9$

$-x + y = 3$

$2y = 12$

$\Rightarrow y = 6$

On substituting value of y in eq. (2), we get

$x + y = 9$

$x + 6 = 9$

$\Rightarrow x = 3$

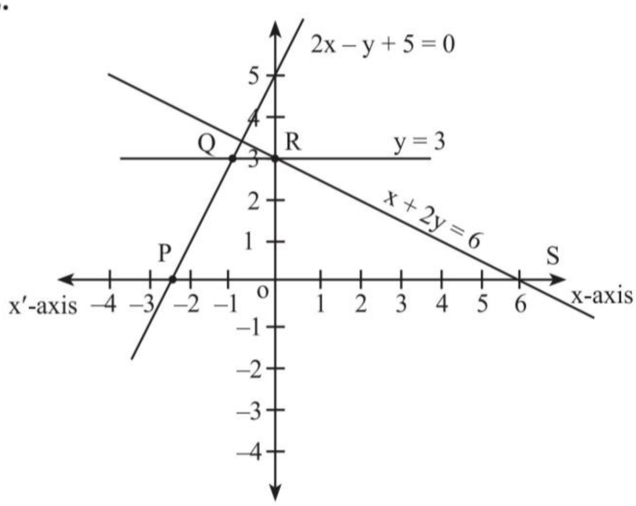
hence, the two digit number

= 10 × y + x

= (10 × 6) + 3

= 63

2.



You can draw the graph yourself. It will look somewhat like shown above.

$2x - y + 5 = 0$

$x + 2y = 6$

$P = \left(-\frac{5}{2}, 0\right)$

$Q = (-1, 3)$

$R = (0, 3)$

$S = (6, 0)$

Now, the area of the trapezium PQRS,

$= \frac{1}{2}(PS + QR) \times OR$

As, here PS, QR form the 2 bases and OR is the perpendicular height,

Here, $PS = 6 - \left(-\frac{5}{2}\right) = \frac{17}{2}$

$QR = 1$

$OR = 3$

Hence, the area of trapezium PQRS

$= \frac{1}{2} \times 3 \times \left(\frac{17}{2} + 1\right) = \frac{57}{4}$

3. $2x - 3y = 1$ ...(1)

and $\frac{x + (3x + 2y)}{x - (3x + 2y)} = -3$

> **Note:** Componendo and Dividendo is a theorem on proportions that allows for a quick way to perform calculations and reduce the number of expansions needed.
>
> $\frac{a}{b} = \frac{c}{d}$
>
> On applying Componendo and Dividendo, we get
>
> $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

On applying Componendo and Dividendo to both the sides, we get

$$\frac{x + (3x + 2y) + x - (3x + 2y)}{x + (3x + 2y) - x + (3x + 2y)} = \frac{-3+1}{-3-1}$$

$$\frac{2x}{6x + 4y} = \frac{-2}{-4}$$

$$\frac{2x}{2(3x + 2y)} = \frac{1}{2}$$

$$\frac{x}{3x + 2y} = \frac{1}{2}$$

$2x = 3x + 2y$

$2x - 3x - 2y = 0$

$-x - 2y = 0$

$x + 2y = 0$ ...(2)

On multiplying eq. (2) by 2 and then subtracting from eq. (1),

$2x - 3y = 1$

$2x + 4y = 0$

$-\quad -\quad -$

$-7y = 1$

$\Rightarrow \quad y = -\frac{1}{7}$

On substituting the value of y in (1), we get

$2x - 3\left(\frac{-1}{7}\right) = 1$

$2x + \frac{3}{7} = 1$

$2x = \frac{1}{1} - \frac{3}{7}$

$2x = \frac{7-3}{7}$

$2x = \frac{4}{7}$

$\boxed{x = \frac{2}{7}}$

Therefore, $x = \frac{2}{7}$ and $y = \frac{-1}{7}$

4. Let the speed of the car be x km/h.

And the speed of the train be y km/h

Total distance covered by priya = 600 km.

Condition I: It takes 8 hours to complete the journey if priya travels 120 km by train and travels the remaining journey i.e. 480 km by car.

As we know,

$\text{Time} = \frac{\text{distance}}{\text{speed}}$

$\therefore$ Time taken to cover 120 km by train $= \frac{120}{y}$ hours

Time taken to cover 480 km by car $= \frac{480}{x}$ hours

Total time taken = 8 hours

$\therefore \frac{120}{y} + \frac{480}{x} = 8$

Condition II: It takes 8 hours 20 minutes to complete the journey if she travels 200 km by train and the remaining 400 km by car.

As we know,

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$\therefore$ Time taken to cover 200 km by train = $\frac{200}{y}$ hours

Time taken to cover 400 km by car = $\frac{400}{x}$ hours

Total time taken = 8 hours and 20 minutes

$$\therefore \frac{200}{y} + \frac{400}{x} = 8 + \frac{20}{60}$$

$$\Rightarrow \frac{200}{y} + \frac{400}{x} = 8 + \frac{1}{3}$$

$$\Rightarrow \frac{200}{y} + \frac{400}{x} = \frac{25}{3} \quad \text{...(ii)}$$

Put $\frac{1}{y} = u$ and $\frac{1}{x} = v$

Hence, equations become:

$120u + 480v = 8$ ...(iii)

$200u + 400v = \frac{25}{3}$

$\Rightarrow 600u + 1200v = 25$ ...(iv)

On multiplying eq. (iii) by 5 and subtracting from eq. (iv), we get

$$\begin{array}{r} 600u + 1200v = 25 \\ 600u + 2400v = 40 \\ - \quad\quad - \quad\quad - \\ \hline -1200v = -15 \end{array}$$

$$v = \frac{15}{1200} = \frac{1}{80}$$

Now, on putting the value of v in eq. (iii), we get

$$120u + \left(480 \times \frac{1}{80}\right) = 8$$

$120u + 6 = 8$

$120u = 2$

$$u = \frac{2}{120}$$

$$u = \frac{1}{60}$$

As, $\frac{1}{y} = u$

$$\Rightarrow \frac{1}{y} = \frac{1}{60}$$

$\Rightarrow y = 60$

Also, $\frac{1}{x} = v$

$$\Rightarrow \frac{1}{x} = \frac{1}{80}$$

$\Rightarrow x = 80$

Therefore, speed of car = 80 km/h

and speed of train = 60 km/hr.

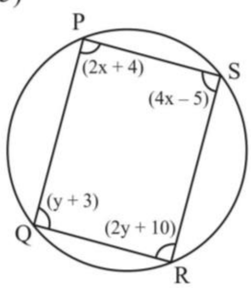
5. PQRS is a cyclic quadrilateral.

$\angle P = (2x + 4)°$

$\angle Q = (y + 3)°$

$\angle R = (2y + 10)°$

$\angle S = (4x - 5)°$



As, we know that the sum of the opposite angles of a cyclic quadrilateral is 180°

$\therefore \angle P + \angle R = 180°$

$(2x + 4) + (2y + 10) = 180°$

$2x + 2y + 14 = 180°$

$2x + 2y = 180 - 14$

$2x + 2y = 166°$

$\Rightarrow 2(x + y) = 166°$

$\Rightarrow x + y = 83$ ...(1)

Similarly, $\angle Q + \angle S = 180°$

$(y + 3) + (4x - 5) = 180°$

$4x + y - 2 = 180°$

$4x + y = 182°$ ...(2)

On subtracting eq. (1) and (2), we get

$$\begin{array}{r} x + y = 83 \\ 4x + y = 182 \\ - \quad - \quad - \\ \hline -3x \quad = -99 \end{array}$$

$\Rightarrow x = 33$

On substituting the value of x in eq. (1), we get

$x + y = 83$

$33 + y = 83$

$\Rightarrow y = 50$

Hence,

$\angle P = (2x + 4) = (2 \times 33 + 4) = 70°$

$\angle Q = y + 3 = (50 + 3) = 53°$

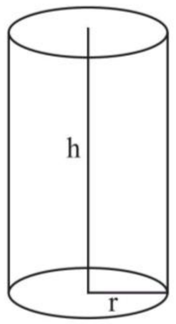
$\angle R = 2y + 10 = (100 + 10) = 110°$

$\angle S = 4 \times 33 - 5 = (132 - 5) = 127°$

Integer Type Questions

1. As, D is the diameter and H is height of the cylinder,

Therefore, lateral surface area of cylinder = $2\pi rh$ = πDH



As per the question

$A = (2\pi rH - 75\pi)$ if $D = (D - 5)$

According to question,

$\Rightarrow (\pi DH - 75\pi) = \pi(D - 5)H$ ...(i)

$\Rightarrow \pi DH + 100\pi = \pi D(H + 5)$ ...(ii)

On solving eq. (i) and (ii), we get

H = 15 cm, D = 20 cm

Hence, (D – H) = 5 cm.

2. Let us suppose, the number be $10x + y$

According to the question,

$10x + y = 3(x + y)$

$\Rightarrow 10x + y = 3x + 3y$

$\Rightarrow 7x = 2y$ ...(i)

According to the second condition,

$y = 3x + 1$

On substituting value of y in eq. (i), we get

$7x = 6x + 2$

$\Rightarrow x = 2 \Rightarrow y = 7$

Hence, the number is 27.

3. The total number of students are 28 which are divided into 5 possible groups with no students taking either English only or History only:

(1) Taking Math only i.e. x.

(2) Taking English and Math but not History i.e. y.

(3) Taking Math and History but not English i.e. z.

(4) Taking English and History but not Math i.e. u.

(5) Taking all the three subjects i.e. v.

The number of students taking Mathematics and History but not English is 6. Hence, the total number of students in group 1, 2, 4 and 5 is 28 – 6 = 22

$\therefore x + y + u + v = 22$ ...(i)

Also, the number of students taking Mathematics and English is identical to the number taking Mathematics only.

$\therefore x = y$ ...(ii)

Also, the number of students in group 4 is five times the number in group 5,

$\therefore u = 5v$ ...(iii)

On substituting eq. (ii) and (iii) in (i), we get

$\therefore y + y + (5v) + v = 22$

$\Rightarrow 6v + 2y = 22$

$\Rightarrow y = 11 - 3v$

As the number of students in group 5 are even and a non-negative integer.

Therefore, let v = 2

$y = 11 - 3(2) = 5$

Now, let v = 4

But, $y = 11 - 3(4)$

$= 11 - 12 = -1$ (negative not possible)

Therefore, y = 5 and v = 2

Using eq. (ii), we get

$x = y = 5$

Hence, students who took mathematics only = 5 student.

4. Let, x and y be the length and breadth of rectangle respectively.

Therefore, area = xy m^2

According to the 1st condition, if the length of a rectangle is decreased by 5 metre and the breadth is increased by 3 metre, then its area is reduced by 8 m^2.

$(x - 5)(y + 3) = xy - 8$

$(x - 5)(y + 3) + 8 = xy$

$3x - 5y = 7$ ...(1)

According to the second condition, if both length and breadth are increased by 3m and 2 m respectively, then its area is increased by 74 m^2.

$(x + 3)(x + 2) = xy + 74$

$2x + 3y = 68$ ...(2)

On solving eq. (1) and (2), we get

$x = 19$ and $y = 10$

Hence, length = 19 m and breadth = 10 m

$\therefore$ difference = 19 – 10 = 9 m.

5. Let us suppose, cost of audio cassettes = ₹x

And cost of video cassettes = ₹y

Now, according to question,

$7x + 3y = 1110$ ...(1)

$5x + 4y = 1350$ ...(2)

On multiplying eq. (1) by 4 and eq. (2) by 3 and subtracting eq. (2) from eq. (1), we get

$28x + 12y = 4440$

$15x + 12y = 4050$

$-\quad -\quad -$

$13x = 390$

$x = 30$

Therefore, cost of audio cassettes = ₹30

As, 10P = 30

Hence, P = 3

6. The given equations are:

$kx + 6y = 27 \Rightarrow kx + 6y - 27 = 0$ ...(i)

$6x + ky = 28 \Rightarrow 6x + ky - 28 = 0$ ...(ii)

Here, $a_1 = k, b_1 = 6, c_1 = -27, a_2 = 6, b_2 = k$ and $c_2 = -28$.

The given system of equations are inconsistent, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}.$$

$$\therefore \frac{k}{6} = \frac{6}{k}$$

$k^2 = 36$

$k = \sqrt{36}$

$k = \pm 36$

Therefore, the correct answer is 6.

7. Let us suppose, the two-digit number be 10a + b.

According to question, $10a + b = (a + b) + 18$

$\Rightarrow 10a - a + b - b = 18$

$\Rightarrow 9a = 18 \Rightarrow a = 2$

Therefore, the required numbers are 20, 21, 22, 23, ..., 29.

Difference between digits are 2, 1, 0, 1, 2, 3, 4, 5, 6, 7

Therefore, the number of different values of z is 8.

Hence, the correct answer is 8.

8. The given equations are:

$3x + 2y = 11 \Rightarrow 3x + 2y - 11 = 0$ ...(i)

$7x - 5ky = 23 \Rightarrow 7x - 5ky - 23 = 0$ ...(ii)

Here, $a_1 = 3, b_1 = 3, c_1 = -11, a_2 = 7, b_2 = -5k$ and $c_2 = -23$.

The given system of equations are inconsistent, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}.$$

$$\therefore \frac{3}{7} = -\frac{2}{5k}$$

$-15k = 14$

$$k = \frac{-14}{15}$$

$15k = -14$

Therefore, the correct answer is –14.

9. $3x + 8ky = 14$ ...(1)

$9x + 24y = 42$ ...(2)

Equations (1) and (2) have infinite common solutions.

$$\therefore \frac{3}{9} = \frac{8k}{24} = \frac{14}{42}$$

$$\Rightarrow \frac{k}{3} = \frac{1}{3} \Rightarrow k = 1$$

Therefore, the correct answer is 1.

10. The given equations are:

$5x + 6y = 21$ ...(1)

$6x + 5y = 23$ ...(2)

On adding equations (1) and (2), we obtain

$11x + 11y = 44$

$\Rightarrow 11(x + y) = 44$

$\Rightarrow x + y = 4$

Therefore, the correct answer is 4.

Multiple Choice Questions

Level-I

1. **(d)** The given equations are:

$6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Here, $\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}$

$\frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}$

$\frac{c_1}{c_2} = \frac{10}{9}$

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, lines are parallel.

2. **(c)** The given equations are:

$x + 2y - 5 = 0$

$-3x - 6y + 15 = 0$

Here, $\frac{a_1}{a_2} = \frac{1}{-3}$

$\frac{b_1}{b_2} = \frac{2}{-6} = \frac{1}{-3}$

$\frac{c_1}{c_2} = \frac{-5}{15} = \frac{-1}{3}$

As, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given pair of linear equations has infinitely many solutions.

3. **(c)** The two lines represented by the given pair of linear equations which is consistent, will definitely have a solution. Hence, the lines are either intersecting or coincident.

4. **(a)** The lines represented by the given pair of equations will be parallel lines. Therefore, the equations have no solution.

5. **(c)** As, we know that for parallel lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence,

$\Rightarrow \frac{3}{2} = \frac{2k}{5}$

$\Rightarrow 15 = 4k$

$\Rightarrow k = \frac{15}{4}$

6. **(a)** As, we know that for infinitely many solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{c}{6} = \frac{-1}{-2}$

$\Rightarrow -2c = -6$

$\Rightarrow c = 3$

Hence, value of c is 3.

7. **(d)** For dependent pair of linear equations,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Now, for option (d)

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{-1}{2}$

8. **(c)** According to given conditions,

$\frac{x}{y} = \frac{5}{6}$

$\Rightarrow 6x - 5y = 0$ ...(1)

and, $\frac{x-8}{y-8} = \frac{4}{5}$

$\Rightarrow 5(x - 8) = 4(y - 8)$

$\Rightarrow 5x - 40 = 4y - 32$

$\Rightarrow 5x - 4y = 8$ ...(2)

Using equation (1), we get

$6x - 5y = 0$

$\Rightarrow x = \frac{5y}{6}$ ...(3)

On putting value of x in eq. (2), we get

$5x - 4y = 8$

$\Rightarrow 5\left(\frac{5y}{6}\right) - 4y = 8$

$\Rightarrow \frac{25y}{6} - 4y = 8$

$\Rightarrow \frac{y}{6} = 8$

$\Rightarrow y = 48$

Now, using eq. (3), we get

$x = \frac{5y}{6}$

$\Rightarrow x = \frac{5 \times 48}{6}$ $(\because y = 48)$

$\Rightarrow x = 40$

Hence, $x = 40$ and $y = 48$.

9. **(c)** The given equations are:

$x - y = 2$ ...(i)

$x + y = 4$ ...(ii)

On adding eq. (i) and (ii), we get

x = 3

On putting value of x in eq. (i), we get,

y = 1

10. **(d)** As, we know that for infinitely many solutions:

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\Rightarrow \frac{1}{a-b}=\frac{2}{a+b}=\frac{-1}{-(a+b-2)}$$

$$\Rightarrow \frac{1}{a-b}=\frac{2}{a+b}$$

⇒ (a + b) = 2(a – b)

⇒ a + b = 2a – 2b

⇒ a – 2a = –2b – b

⇒ a – 3b = 0 ...(1)

And $\frac{2}{a+b}=\frac{-1}{-(a+b-2)}$

⇒ 2(a + b –2) = (a + b)

⇒ 2a + 2b – 4 = a + b

⇒ 2a – a + 2b – b = 4

⇒ a + b = 4 ...(2)

On solving eq. (1) and (2), we get

a = 3 and b = 1.

11. **(c)** Let us suppose, x and y be the age of father and his son respectively.

According to the given condition that father's age is 6 times of his son's age.

x = 6y …(1)

After four years, age of son will be y + 4 and age of father will be x + 4

According to the given condition that after four years, the age of the father will be four times his son's age.

x + 4 = 4 (y + 4)

x + 4 = 4y + 16

x – 4y = 12 …(2)

On solving equation (1) and (2), we obtain

y = 6 and x = 36.

12. **(d)** Let the number of ₹1 coins be x

Let the number of ₹2 coins be y

Now, according to given conditions,

x + y = 50 …(1)

and, (1 × x) + (2 × y) = 75

⇒ x + 2y = 75 …(2)

On subtracting eq. (1) from (2), we obtain

(x + 2y) – (x + y) = 75 – 50

⇒ y = 25

Hence, x = 50 – 25 = 25

So the number of coins are 25, 25 each.

13. **(a)** Let us suppose, the number of correct answers of the questions in a competitive exam be x.

Then, the number of wrong answers be 120 – x

According to given condition,

$$(1\times x)-\left[(120-x)\times\frac{1}{2}\right]=90$$

$$\Rightarrow x-60+\frac{x}{2}=90$$

$$\Rightarrow \frac{3x}{2}=150$$

$$\Rightarrow x=\frac{150\times 2}{3}$$

⇒ x = 100

Hence, Priya answered 100 questions correctly.

14. **(a)** As, sum of opposite angles of a cyclic quadrilateral is 180°

Hence,

6x + 10 + x + y = 180

⇒ 7x + y = 170 …(1)

And, 5x + 3y – 10 = 180

⇒ 5x + 3y = 190 …(2)

On solving eq. (1) and (2), we get

x = 20° and y = 30°.

15. **(c)** Let, fixed charges be ₹x and charges for each extra day be ₹y

According to given conditions,

x + 4y = 22 …(1)

x + 2y = 16 …(2)

On subtracting eq. (2) from (1), we obtain

y = ₹3

Hence charge for each extra day is ₹3.

16. **(c)** The given equations are: x + 2y – 5 =0 and –4x – 8y + 20 = 0

Here, $\frac{a_1}{a_2}=\frac{1}{-4}$

$\frac{b_1}{b_2} = \frac{2}{-8} = \frac{1}{-4}$

$\frac{c_1}{c_2} = \frac{-5}{20} = \frac{-1}{4}$

Therefore,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the pair of equations has infinitely many solutions.

17. **(d)** Do it yourself.

18. **(b)** For parallel lines:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, $\frac{3}{2} = \frac{2k}{5}$

$\Rightarrow 15 = 4k$

$\Rightarrow k = \frac{15}{4}$

19. **(a)** For dependent linear equations:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

For option a,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

20. **(a)** Do it yourself.

21. **(c)** Do it yourself.

22. **(a)** Put $\frac{1}{x} = m$

Hence, equations become

$4m + 3y = 14$

$3m - 4y = 23$

By using cross multiplication method we get;

$$\frac{m}{(-69-56)} = \frac{y}{\{-42-(-92)\}} = \frac{1}{(-16-9)}$$

$\frac{m}{-125} = \frac{y}{50} = \frac{-1}{25}$

$\frac{m}{-125} = \frac{-1}{25}$ and $\frac{y}{50} = \frac{-1}{25}$

$25m = 125$ and $25y = -50$

$\Rightarrow m = 5$ and $y = -2$

As, $m = \frac{1}{x}$ or $x = \frac{1}{m} = \frac{1}{5}$

Hence, $x = \frac{1}{5}$ and $y = -2$.

23. **(c)** Let us suppose, speed of Kirti in still water $= x$ km/hr

Speed of Stream $= y$ km/hr

Now, speed of Kirti, during downstream $= x + y$ km/h

Speed of Kriti during upstream $= x - y$ km/h

According to the question, Kirti can row 20 km downstream in 2 hours.

As we know,

Time * Speed = Distance

$\therefore 2(x + y) = 20$

$\Rightarrow x + y = 10$

Similarly, Kirti can row 4 km upstream in 2 hours.

As we know,

Time * Speed = Distance ...(i)

$\therefore 2(x - y) = 4$

$\Rightarrow x - y = 2$ ...(ii)

On adding equations (i) and (ii), we obtain

$2x = 12$

$\Rightarrow x = 6$

On substituting value of x in eq. (i), we get

$y = 4$

Hence speed of Kirti in still water = 6 km/hr

Speed of Stream = 4 km/hr.

24. **(b)** Do it yourself.

25. **(b)** Do it yourself.

26. **(b)** Do it yourself.

27. **(a)** **Hint:** To find out if a point (x, y) is on the graph of a line, we plug in the values and see if we get a true statement, such as 10 = 10.

28. **(b)** Let us suppose, the fraction be $\frac{x}{y}$

Now, according to given conditions,

$\frac{x-2}{y+1} = \frac{1}{2}$

$2(x - 2) = 1(y + 1)$

$2x - 4 = y + 1$

$2x - y = 5$ ...(1)

Again, according to question

$\frac{x+4}{y-3} = \frac{3}{2}$

$2(x + 4) = 3(y - 3)$

$2x + 8 = 3y - 9$

$2x - 3y = -17$ ...(2)

On solving eq. (1) and (2), we get

$x = 8, y = 11$

Hence, the fraction is $\frac{8}{11}$.

29. **(a)** Let us suppose, x and y be the two numbers

According to given conditions,

$x + y = 69$ ...(1)

$x - y = 25$ ...(2)

On solving eq. (1) and (2), we obtain

$x = 47, y = 22$

Therefore, numbers are 47 and 22.

30. **(a)** Let the number of boys = x

The number of girls = 30

$\therefore$ Total number of students in class X = 30 + x

Average marks scored by boys of class X = 76

Average marks scored by girls of class X = 60

Average mark scored by students of class X = 70

$\therefore$ Total marks scored by boys = Average marks obtained by boys × Total no. of boys = 76x

Similarly, total marks scored by girls = Average marks scored by girls × Total no. of girls

$= 60 \times 30 = 1800$

Similarly, total marks obtained by class = Average marks scored by class X students × Total no. of students = 70(30 + x)

Now, Total marks obtained by class = Total marks obtained by boys = Total marks obtained by girls.

$\therefore$ $70(30 + x) = 76x + 1800$

$70x + 2100 = 1800 + 76x$

$70x - 76x = 1800 - 2100$

$\Rightarrow x = 50$

Therefore, numbers of boys = 50

31. **(d)** Let the time taken by 1 women to finish the work = x days.

And the time taken by 1 girl to finish the work = y days

Hence, work done by 1 women in 1 day = 1/x

And work done by 1 girl in 1 day = 1/y

It is given that 3 women and 4 girls can complete the work in 6 days.

$\therefore$ Work done by 3 women and 4 girls in 1 days = $\frac{1}{6}$

$\Rightarrow \frac{3}{x} + \frac{4}{y} = \frac{1}{6}$ ...(i)

It is also given that 5 women and 2 girls can complete the work in 5 days.

$\therefore$ Work done by 5 women and 2 girls in 1 day = $\frac{1}{5}$

$\Rightarrow \frac{5}{x} + \frac{2}{y} = \frac{1}{5}$ ...(ii)

By putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in equation (i) and (ii), we get

$3u + 4v = 1/6$ ...(iii)

$5u + 2v = 1/5$ ...(iv)

You can solve these equations by yourself.

On solving these equations, we get

$u = \frac{1}{30}$ and $v = \frac{1}{60}$

As $u = \frac{1}{x} = \frac{1}{30}$

$\therefore x = 30$

Hence, a woman can complete the work in 30 days.

32. **(a)** Let us suppose, x and y be the larger and smaller number respectively.

We know that,

Dividend = Divisor × Quotient + Remainder ...(1)

When 3x is divided by y, we get 5 as quotient and 1 as remainder. Therefore, by using (1), we get

$3x = 5y + 1$

$3x - 5y = 1$ ...(2)

When 6y is divided by x, we get 3 as quotient and 6 as remainder. Therefore, by using (1), we get

$6y = 3x + 6$

$6y - 3x = 6$

$3(2y - x) = 6$

$2y - x = 2$ ...(3)

On solving eq. (2) and (3), we get

$y = 7$ and $x = 12$

Hence, the numbers are 12 and 7.

33. **(b)** The given equations of line are:

$2(x + 3) = 3(x + 9)$ ...(i)

$y(a + 1) = 2(y - 5)$ ...(ii)

On simplifying equation (i), we get

$2x + 6 = 3x + 27$

$3x - 2x = 6 - 27$

$x = -21$ (Line is ∥ to y-axis)

On simplifying equation (ii), we get

$ay + y = 2y - 10$

$ay + y - 2y = -10$

$ay - y = -10$

$y(a - 1) = -10$

$y = \frac{-10}{a-1}$ (Line is ∥ to x-axis)

So, the first line is ∥ to y-axis and the second line is ∥ to x-axis.

Since x and y axis are mutually perpendicular to one another.

So these 2 lines would also be perpendicular to one another.

34. **(a)** It is given that,

$\frac{x}{a}+\frac{y}{b} = a + b$ ...(i)

and $\frac{x}{a^2}+\frac{y}{b^2} = 2$ ...(ii)

On dividing eq. (i) by b, we get

$\frac{x}{ab}+\frac{y}{b^2} = \frac{a+b}{b}$ ...(iii)

On subtracting eq. (iii) from (ii), we get

$$\frac{x}{a^2}+\frac{y}{b^2}-\left(\frac{x}{ab}+\frac{y}{b^2}\right)=2-\frac{a+b}{b}$$

$$\frac{x}{a^2}+\frac{y}{b^2}-\frac{x}{ab}-\frac{y}{b^2}=\frac{2b-a-b}{b}$$

$$\frac{x}{a^2}-\frac{x}{ab}=\frac{b-a}{b}$$

$$\frac{bx-ax}{a^2b}=\frac{b-a}{b}$$

$$\frac{x(b-a)}{a^2b}=\frac{b-a}{b}$$

$$x=\frac{a^2b}{b}=a^2$$

35. **(a)** Do it yourself.

Level-II

1. **(c)** Let us suppose numbers be x and y

According to the question,

$x + y = 15 \Rightarrow y = 15 - x$ ...(1)

$\frac{1}{x}+\frac{1}{y}=\frac{3}{10}$ ...(2)

Now, using eq. (1), we get

$\Rightarrow \frac{1}{x}+\frac{1}{15-x}=\frac{3}{10}$

$\Rightarrow \frac{15-x+x}{x(15-x)}=\frac{3}{10}$

$\Rightarrow 15 \times 10 = 3x(15 - x)$

$\Rightarrow 150 = 45x - 3x^2$

$\Rightarrow 3x^2 - 45x + 150 = 0$

$\Rightarrow x^2 - 15x + 50 = 0$

$\Rightarrow x^2 - 10x - 5x + 50 = 0$

$\Rightarrow x(x - 10) - 5(x - 10) = 0$

$\Rightarrow x - 10 = 0$ or $x - 5 = 0$

$\Rightarrow x = 10$ or $x = 5$

Hence, the required numbers are 10 and 5.

2. **(b)** Let us suupose, x units and y units be the length and breadth of a rectangle respectively.

Area = xy

New length = x + 2

New breadth = y + 2

According to question,

$(x + 2)(y + 2) = xy + 54$

$xy + 2x + 2y + 4 = xy + 54$

$2(x + y) = 54 - 4$

$2(x + y) = 50$

Hence, perimeter = 2(x + y) = 50 units.

3. **(b)** The given system of linear equaitons is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where $a_1 = 2, b_1 = -3, c_1 = -7,$

$a_2 = (a + b), b_2 = -(a + b - 3), c_2 = -(4a + b)$

Now, $\frac{a_1}{a_2}=\frac{2}{a+b}, \frac{b_2}{b_2}=\frac{-3}{-(a+b-3)}, \frac{c_1}{c_2}=\frac{-7}{-(4a+b)}$

To have infinite number of solutions, it should satisfy the following condition:

$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$\Rightarrow \frac{2}{a+b} = \frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$

i.e., $\frac{2}{a+b} = \frac{3}{a+b-3}$ and $\frac{3}{a+b-3} = \frac{7}{4a+b}$

$\Rightarrow 2(a + b - 3) = 3(a + b)$ and $3(4a + b) = 7(a + b - 3)$

$\Rightarrow 2a + 2b - 6 = 3a + 3b$ and $12a + 3b = 7a + 7b - 21$

$\Rightarrow 2a - 3a = 3b - 2b + 6$ and $12a - 7a + 3b - 7b = -21$

$\Rightarrow -a = b + 6$ and $5a - 4b = -21$

$\Rightarrow a + b + 6 = 0$ ...(1)

and $5a - 4b + 21 = 0$ ...(2)

On multiplying eq. (1) by 5 and subtracting from (2), we obtain

$-9b - 9 = 0 \Rightarrow b = -1$

On substituting value of b in eq. (1), we get

$a = -5$

Therefore, the given system of linear equations has an infinite number of solutions when $a = -5$, $b = -1$.

4. **(c)** Let us suppose number of cows be x and number of hens be y.

As per fact that hens and cows each have one head and 2 legs and 4 legs respectively.

It is given that,

Numbers of legs = 92

According to question,

$4x + 2y = 92$ ...(1)

$x + y = 29$ ...(2)

on solving equations (1) and (2), we get

$x = 17$ and $y = 12$

Hence, number of cows = 17.

5. **(c)** Let us suppose, the age of son = x years

Age of father = 4x years

After five years,

Age of son = x + 5

Age of father = 4x + 5

According to given condition,

$4x + 5 = 3(x + 5)$

$4x + 5 = 3x + 15$

$4x - 3x = 15 - 5$

$x = 10$

Hence, present age of father = 4x = 40 years.

6. **(a)** Let us suppose, the present age of son = x years

And present age of father = y years

Ten years ago,

Age of son = x – 10

And, age of father = y – 10

According to given condition,

$4(x - 10) = (y - 10)$

$4x - 40 = y - 10$

$4x - y = 30$ ...(1)

Again 5 years ago

$3(x - 5) = y - 5$

$3x - 15 = y - 5$

$3x - y = 10$ ...(2)

On solving eq. (1) and (2), we obtain

$x = 20, y = 50$

Hence, present age of son = 20 years.

7. **(b)** Let us suppose, the two digit number be (10x + y)

According to given condition,

i.e., $(10x + y) + (10y + x) = 132$

$11x + 11y = 132$

$x + y = 12$ ...(1)

$x - y = 4$ ...(2)

On solving eq. (1) and (2), we get

$x = 8, y = 4$

Hence, number is 84 or 48.

8. **(c)** Let us suppose, present age of son be x years and present age of father be y years

After 6 years

Age of son = x + 6

Age of father = y + 6

According to given conditions,

$y + 6 = 3(x + 6)$

$y + 6 = 3x + 18$

$y - 3x = 12$ ...(1)

Again, according to ques,

3 years ago,

$y - 3 = 9(x - 3)$

$y - 3 = 9x - 27$

$y - 9x = -24$ ...(2)

On solving eq. (1) and (2), we get

$x = 6, y = 30$

Therefore, present age of father is 30 years.

9. **(d)** Let us suppose, age of Ram be x years

Age of his father be y years

According to given conditions

$y - x = 30$ ...(1)

and $y^2 - x^2 = 1560$

$(y + x)(y - x) = 1560$ $[\because a^2 - b^2 = (a + b)(a - b)]$

Now, using eq. (1), we get

$30(y + x) = 1560$

$y + x = 52$ ...(ii)

On solving eq. (1) and (2), we get

$y = 41$ and $x = 11$

Hence, age of son is 11 years.

10. **(b)** Let us suppose, x and y be the cost price (C.P) of 1 chair and 1 table respectively.

Profit on chair = 25%

Hence, SP of 1 chair = $x + \frac{25}{100}x = \frac{125}{100}x$

And, profit on table = 10%

Hence, SP of 1 table $y + \frac{10}{100}y = \frac{110}{100}y$

According to the question,

$$\frac{125}{100}x + \frac{110}{100}y = 1520$$

$$\frac{25}{20}x + \frac{22}{20}y = 1520$$

$\Rightarrow 25x + 22y = 30400$ ...(1)

If profit on a chair is 10% and on table is 25%

Then $\left(x + \frac{10}{100}x\right) + \left(y + \frac{25}{100}y\right) = 1535$

$\Rightarrow 22x + 25y = 30700$ ...(2)

On solving eq. (1) and (2), we get

$x = 600$ and $y = 700$

Hence, cost of table is ₹700.

11. **(c)** As, we know that the sum of opposite angles of cyclic quadrilateral is 180°.

Therefore, $\angle P + \angle R = 180°$

$\angle Q + \angle S = 180°$

Hence, so $x + x + 8 = 180°$

$2x = 180° - 8$

$x = 86°$

and $y + 2y - 3 = 180°$

$3y = 180 + 3$

$y = 61°$

Hence, the four angles are:

$\angle P = 86°$, $\angle Q = 61°$

$\angle R = 94°$, $S = \angle 119°$.

12. **(a)** Let us suppose, the fraction be $\frac{x}{y}$

According to given condition

$$\frac{x+y}{y-x} = \frac{11}{3}$$

$3(x + y) = 11(y - x)$

$3x + 3y = 11y - 11x$

$3x + 11x = 11y - 3y$

$14x = 8y$

$2(7x) = 2(4y)$

$7x = 4y$ ...(1)

It is also given that,

$y - x = 3$ ...(2)

On solving eq. (1) and (2), we get

$x = 4, y = 7$

Therefore, sum of numerator and denominator is (7 + 4) = 11.

13. **(d)** Let us suppose, speed of train = x km/h

And time taken = y hours

Therefore, distance = xy

According to given conditions,

$xy = (x + 10)(y - 2)$

$xy = xy - 2x + 10y - 20$

$2x - 10y = -20$

$x - 5y = -10$ ...(1)

Again,

$xy = (x - 10)(y + 4)$

$xy = xy + 4x - 10y - 40$

$4x - 10y = 40$ ...(2)

On solving eq. (1) and (2), we get

$x = 30$ and $y = 8$

As, distance = xy

$= 30 \times 8 = 240$ km.

14. **(c)** $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ ...(1)

$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ...(2)

$\frac{-x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ...(3)

On adding eq. (1) and (2), we get

$\Rightarrow \frac{2x^2}{a^2} = 2$

$\Rightarrow 2x^2 = 2a^2$

$\Rightarrow x^2 = a^2$

$\Rightarrow x = \pm a$

In the similar way, $y = \pm b$, $z = \pm c$

Hence, equation will have unique solution.

Assertion & Reason Type

1. **(c)** Assertion (A) is true but reason (R) is false.

The given equations are:

$2x + 3y = 7 \Rightarrow 2x + 3y - 7 = 0$ ...(i)

$2ax + (a + b)y = 28 \Rightarrow 2ax + (a + b)y - 28 = 0$...(ii)

Here $a_1 = 2$, $b_1 = 3$, $c_1 = -7$, $a_2 = 2a$, $b_2 = (a + b)$ and $c_2 = -28$.

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$$

$$\frac{1}{a} = \frac{3}{a+b} = \frac{1}{4}$$

$$\frac{1}{a} = \frac{3}{a+b}$$

$3a = a + b$

$3a - a = b$

$2a - b = 0$

Hence, assertion is true.

The given equations are

$3x - 5y = 9 \Rightarrow 3x - 5y - 9 = 0$ ...(i)

$6x - 10y = 8 \Rightarrow 6x - 10y - 8 = 0$ ...(ii)

Here, $a_1 = 3$, $b_1 = -5$, $c_1 = -9$, $a_2 = 6$, $b_2 = -10$ and $c_2 = -8$.

For unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Here, $\frac{3}{6} = \frac{-5}{-10}$

$$\frac{1}{2} = \frac{1}{2}$$

Hence, the given system of equation does not has a unique solution.

2. **(a)** Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

It is given that, $x = 3$ and $y = 1$ is the solution of line $2x + y - q^2 - 3 = 0$

Therefore,

$2 \times 3 + 1 - q^2 - 3 = 0$

$6 + 1 - 3 - q^2 = 0$

$4 - q^2 = 0$

$q = \pm\sqrt{4}$

$q = \pm 2$

Hence, both A and R are true and R explains A.

3. **(d)** Assertion (A) is false but reason (R) is true. If system of linear equations has a unique solution, then

$$\frac{k}{6} \neq \frac{-1}{-2}$$

$$\frac{k}{6} \neq \frac{1}{2}$$

$k \neq 3$

Hence, A is incorrect but R is correct.

4. **(b)** Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

For no solution,

$$\frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3} \text{ i.e., } \frac{4}{3}$$

$$k = 2 \left[\frac{1}{2} \neq \frac{4}{3} \text{ holds}\right]$$

Hence, assertion is true.

Reason does not explain result of assertion.

5. **(a)** Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

From the given equations, we have

$$\frac{9}{18} = \frac{3}{6} = \frac{12}{24}$$

$$\frac{1}{2} = \frac{1}{2} = \frac{1}{2} \text{ i.e., } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, both A and R are true and R explains A.

Case-Based Type Questions

Case-Based-I:

1. **(a)** The cost of one notebook be ₹x and that of pen be ₹y.

   According to the question,

   $3x + 2y = 80$ and $4x + 3y = 110$

2. **(a)** ₹20

3. **(b)** On solving $3x + 2y = 80$ and $4x + 3y = 110$, we obtain

   $x = 20$ and $y = 10$

   Therefore, cost of 1 notebook = ₹20 and cost of 1 pen = ₹10.

Case-Based-II:

1. **(b)** Area of one bedroom = $5x\ m^2$

   Area of two bedrooms = $10x\ m^2$

   Area of kitchen = $5y\ m^2$

2. **(d)** Length of outer boundary of the layout is

   $= 12 + 15 + 12 + 15 = 54$ m

3. **(a)** Area of two bedrooms = $10x\ m^2$

   Area of kitchen = $5y\ m^2$

   Therefore, $10x + 5y = 95 \Rightarrow 2x + y = 19$

   And, $x + 2 + y = 15 \Rightarrow x + y = 13$

   On solving $2x + y = 19$ and $x + y = 13$,

   we get $x = 6$ m and $y = 7$ m

   Therefore, area of bedroom = $5 \times 6 = 30\ m^2$.

4. **(c)** Area of living room = $(15 \times 7) - 30$

   $= 105 - 30 = 75m^2$

5. **(c)** Cost of $1m^2$ laying tiles in kitchen = ₹50

   Total cost of laying tiles in kitchen

   = ₹50 × 35 = ₹1750

Multi Correct MCQs

1. **(a, c)** For option (a), $x + 2y = 7$

   $3x - y = 5$

   As,

   $\frac{1}{3} \neq \frac{2}{-1}$ hence, equation is consistent.

   Now, for option (c),

   $x + y = 3$

   $2x + 2y = 6$

   $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$ Therefore, this equation is dependent and hence, will have infinitely many solutions.

2. **(a, b, d)** For unique solution, it should satisfy the following condition,

   $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

   The given equations are:

   $x + 3y = 2$ and $3x + ky = 5$

   $\therefore \frac{1}{3} \neq \frac{3}{k}$ i.e. $k \neq 9$

   Hence, k can have any value except 9.

   Therefore, a, b and d can be the value of k.

3. **(a, b)** For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

   For option (a),

   $x - y = 3$

   $2x - 2y = 1$

   $\Rightarrow \frac{1}{2} = \frac{1}{2} \neq \frac{3}{1}$ Hence, no solution.

   For option (b),

   $x + y = 2$

   $2x + 2y = 1$

   $\frac{1}{2} = \frac{1}{2} \neq \frac{2}{1}$ Hence, no solution.

4. **(b, c, d)**

   For unique solution, it should satisfy the following condition:

   $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

   $\therefore \frac{2}{k} \neq \frac{-3}{2}$

   $\Rightarrow k \neq \frac{-4}{3}$

   Hence, k can have any value except $\frac{-4}{3}$.

Olympiad & NTSE Type

1. **(a)** Let us suppose, the fare of child be ₹x

   Then, fare of adult be ₹2x

   Let amount of reservation fee = ₹y

   Then, the fare with reservation of an adult = ₹(2x + y)

   Then, the fare with reservation of a child = ₹(x + y)

   Now, according to given conditions,

   $2x + y = 253$ ...(1)

   $x + y = 134$ ...(2)

   On solving eq. (1) and (2), we obtain

   $x = 119$ and $y = 15$

   Hence, fare of an adult = $2x = 2 \times 119$ = ₹238

2. **(d)** The given equations are:

$$\frac{x-a}{c-a}+\frac{y-b}{c-b}=1 \quad \text{...(i)}$$

$$\frac{x+a}{c}+\frac{y-a}{a-b}=\frac{a}{c} \quad \text{(ii)}$$

Now, using eq. (i), we get

$(c - b)(x - a) + (c - a)(y - b) = (c - a)(c - b)$

$\Rightarrow x(c - b) + y(c - a) - a(c - b) - b(c - a)$

$= c^2 - bc - ac + ab$

$\Rightarrow x(c - b) + y(c - a) = ac - ab + bc - ab + c^2 - bc - ac + ab$

$\Rightarrow x(c - b) + y(c - a) = c^2 - ab$ ...(iii)

Now, from eq. (ii), we get

$(x + a)(a - b) + c(y - a) = a(a - b)$

$\Rightarrow x(a - b) + a(a - b) + cy - ac = a^2 - ab$

$\Rightarrow x(a - b) + cy = ac - a^2 + ab + a^2 - ab$

$\Rightarrow x(a - b) + cy = ac$ ...(iv)

On multiplying equation (iii) by c, we get

$c[x(c - b) + y(c - a)] = c(c^2 - ab)$

$\Rightarrow cx(c - b) + cy(c - a) = c^3 - abc$ ...(v)

On multiplying equation (iv) by (c – a), we get

$(c - a)[x(a - b) + cy] = (c - a)ac$

$\Rightarrow cx(a - b) + c^2y - ax(a - b) - acy = ac^2 - a^2c$

$\Rightarrow cx(a - b) - ax(a - b) + cy(c - a) = ac^2 - a^2c$

$\Rightarrow cx(a - b)[c - a] + cy(c - a) = ac^2 - a^2c$ ...(vi)

On subtracting equation (vi) from (v), we get

$cx(c - b) - cx(a - b)(c - a) = c^3 - abc - (ac^2 - a^2c)$

$x\{(c (c - b) - (c - a) (a - b))\} = c^3 - abc - ac(c - a)$

$\Rightarrow x(c^2 - ac + a^2 - ab) = c(c^2 - ab - ac + a^2)$

$\Rightarrow x = c$

On putting value of x in eq. (iv), we get

$c(a - b) + cy = ac$

$\Rightarrow y = b$

Therefore, the solution is x = c, y = b.

3. **(b)** The given equations are:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad \text{...(i)}$$

and $u + v = s$ ...(ii)

Also, form (i), $\frac{u+v}{uv}=\frac{1}{f}$

$\Rightarrow \frac{s}{uv}=\frac{1}{f}$ [Using eq (ii)]

Hence, $s=\frac{uv}{f}$

4. **(c)** Let, the speed of walking = x km/hr

And, the speed of rowing a boat = y km/hr

Let speed of current = z km/hr

With the current, the man rows (y + z) km/hr, and against the current (y – z) km/hr. Therefore, we have the following equations;

$$\frac{12}{x}+\frac{12}{y+z}=5 \quad \text{...(i)}$$

$$\frac{12}{x}+\frac{12}{y-z}=7 \quad \text{...(ii)}$$

$$\frac{12}{x}+\frac{12}{y}=5\frac{2}{3} \quad \text{...(iii)}$$

On subtracting (i) from (iii), we get

$$\frac{1}{y}-\frac{1}{y+z}=\frac{1}{18} \quad \text{...(iv)}$$

On subtracting (iii) from (ii), we get

$$\frac{1}{y-z}-\frac{1}{y}=\frac{1}{9} \quad \text{...(v)}$$

From eq. (iv), we get

$18z = y(y + z)$ ...(vi)

Now, from eq. (v), we get

$9z = y(y - z)$ ...(vii)

On dividing eq. (vi) and (vii), we get

$2=\frac{y+z}{y-z}$,

$\Rightarrow y = 3z.$

Now, from eq. (iv), $z = 1\frac{1}{2}$; and therefore, $y = 4\frac{1}{2}$, $x = 4$,

(using eq. (i))

Hence, the rates of walking and rowing are 4 km/hr and $4\frac{1}{2}$ km/hr respectively; and the speed of current is $1\frac{1}{2}$ km/hr.

5. **(b)** Salesman gets a fixed salary plus commission.
Income of salesman = Salary + Commission on sales
According to the question,
Income of sales in 1st month = Salary + Commission on ₹31500
$\Rightarrow$ 5575 = Salary + Commission on ₹31,500 ...(i)
Income of salesman in the 2nd month = Salary + Commission on ₹45,000
$\Rightarrow$ 6250 = Salary + Commission on ₹45,000 ...(ii)
On subtracting (i) from (ii), we get
6250 = Salary + Commission on ₹45,000
5575 = Salary + Commission on ₹31,500
– – –
675 = Commission on ₹13,500
$\therefore$ Commission on ₹13,500 = ₹675

$$\text{Commission on ₹1} = ₹\frac{675}{13500}$$

$$\text{Commission on } 45000 = ₹\frac{675}{13500}\times 45000$$

= ₹2250 ...(iii)

On subtracting equation (iii) in (ii), we get
6250 = Salary + 2250
Salary = 6250 – 2250 = ₹4000
In the third month, he got a salary of ₹7750 and an order of ₹x.
7750 = Fixed salary + Commission on ₹x
7750 = ₹4000 + Commission on ₹x
3750 = Commission on ₹x. ...(iv)
In the third month, he got a commission of ₹3750

$$\text{Commission on ₹x} = x \times \frac{675}{13500}$$

$$3750 = x \times \frac{675}{13500} \quad \text{(from iv)}$$

$$\frac{3750\times 13500}{675} = x$$

x = ₹75,000
Hence, the required order of sales = ₹75000.

6. **(c)** The given system of equations is

$$\frac{2x-3y}{3} = 3+\frac{3y-4x}{4} \quad ...(1)$$

$$\frac{1}{3}(6y+7x) = \frac{1}{5}(7x+12y)+4 \quad ...(2)$$

On multiplying eq. (1) by 12, we get
$4(2x - 3y) = 36 + 3(3y - 4x)$
$\Rightarrow 8x - 12y = 36 + 9y - 12x$
$\Rightarrow 8x + 12x - 12y - 9y = 36$
$\Rightarrow 20x - 21y = 36$ ...(3)
On multiplying eq. (2) by 15, we get
$5(6y + 7x) = 3(7x + 12y) + 60$
$\Rightarrow 30y + 35x = 21x + 36y + 60$
$\Rightarrow 30y - 36y + 35x - 21x = 60$
$\Rightarrow 14x - 6y = 60$
$\Rightarrow 2(7x - 3y) = 60$
$\Rightarrow 7x - 3y = 30$ ...(4)
On multiplying equation (3) by 7 and equation (4) by 20, we obtain
$140x - 147y = 252$ ...(5)
$140x - 60y = 600$ ...(6)
On subtracting equation (6) from equation (5), we obtain
$-87 = -348$
$\Rightarrow y = 4$
On putting value of y in eq. (3), we get
$20x - 84 = 36$
$\Rightarrow 20x = 120$
$\Rightarrow x = 6$
Therefore, x = 6 and y = 4 is the required solution.

7. **(d)** The given system of equations is
$6u + 3v = 7uv$ ...(1)
$3u + 9v = 11uv$ ...(2)
Equations (1) and (2) are not linear equations in u and v, but by an appropriate substitution, it can be reduced to linear equations.
If we put u = 0 in equation (1) or (2), we get v = 0
Therefore, u = 0, v = 0 is one set of solution. Such solution is known as a trivial solution.
To find the other solutions, we assume that $u \neq 0$, $v \neq 0$ Now, $u \neq 0$, $v \neq 0$
$\Rightarrow uv \neq 0$.
Now, dividing equations (1) and (2) by uv, we obtain

$$\frac{6}{v}+\frac{3}{u}=7 \quad ...(3)$$

$$\frac{3}{v}+\frac{9}{u}=11 \quad ...(4)$$

Substitute $\frac{1}{u}=x$ and $\frac{1}{v}=y$ in equations (3) and (4), we obtain 3x + 6y = 7 and, ...(5)

9x + 3y = 11 ...(6)

On multiplying equation (6) by 2, we obtain

18x + 6y = 22 ...(7)

On subtracting equation (7) from (5), we obtain

–15x = –15 ⇒ x = 1.

On substituting value of x in eq. (5), we get

$$3+6y=7 \Rightarrow y=\frac{4}{6}=\frac{2}{3}$$

As, $x=1 \Rightarrow \frac{1}{u}=1 \quad \Rightarrow u=1$

And $y=\frac{2}{3} \Rightarrow \frac{1}{v}=\frac{2}{3} \Rightarrow v=\frac{3}{2}$

Therefore, the given system of equations has two solutions:

(i) u = 0, v = 0 and (ii) u = 1, $v = \frac{3}{2}$

8. **(c)** Put $\frac{1}{2x+3y}=u$ and $\frac{1}{3x-2y}=v$ in the given equations.

Hence, the given equations become

$$\frac{1}{2}u+\frac{12}{7}v=\frac{1}{2}$$

⇒ 7u + 24v = 7 ...(1)

And 7u + 4v = 2 ...(2)

On subtracting eq. (2) from (1), we obtain

$20v = 5 \Rightarrow v = \frac{1}{4}$

On substituting value of v in eq. (1), we get

$7u + 6 = 7 \Rightarrow u = \frac{1}{7}$

As, $u = \frac{1}{7} \Rightarrow \frac{1}{2x+3y}=\frac{1}{7} \Rightarrow 2x+3y=7$ ...(3)

and $v=\frac{1}{4} \Rightarrow \frac{1}{3x-2y}=\frac{1}{4} \Rightarrow 3x-2y=4$ ...(4)

On multiplying equation (3) by 2 and equation (4) by 3, we get

4x + 6y = 14 ...(5)

9x – 6y = 12 ...(6)

On adding eq. (5) and (6), we obtain

13x = 26 ⇒ x = 2

On substituting value of x in eq. (5), we get

⇒ 8 + 6y = 14

⇒ 6y = 6

⇒ y = 1

Hence x = 2 and y = 1 is the required solution of the given system of equations.

Now, on substituting x = 2, y = 1 in y = px – 3, we obtain

1 = 2p – 3

⇒ p = 2

Therefore, value of p is 2.

9. **(c)** It is given that, $\overline{abc}$ = 459 + a + b + c

100a + 10b + c = 459 + a + b + c

100a – a + 10b – b + c = 459 + c

99a + 9b = 459

9(11a + b) = 459

11a + b = 51 ...(1)

We need to find the value of

$\overline{ab}$ + a = 10a + b + a

= 11a + b

Now, using eq. (1), we get

$\overline{ab}$ + a = 51

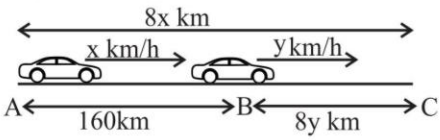
10. **(b)** Let the speed of the car from point P is x km/h. And the speed of the car from point Q is y km/h. Distance travelled by the first car in 8 hours = AC = Speed × Time = 8x km

Distance travelled by the second car in 8 hours = BC = Speed × Time = 8y km

According to the question,

If the cars travel in the same direction, then they meet up in 8 hours.

Let they meet at point C.



AC = AB + BC

⇒ 8x = 160 + 8y

⇒ 8x – 8y = 160

⇒ x – y = 20 ...(1)

When the cars are moving in the same direction then they meet up in 2 hours.

14. **(c)** Let the number of correct answers be 'x', number of wrong answers be 'y' and number of non-attempted questions be 'z'.

According to the question,

$x - \frac{1}{3}y - \frac{1}{6}z = 32 \Rightarrow 6x - 2y - z = 192$ ...(1)

$x + y + z = 50$ ...(2)

On adding eq. (1) and (2), we get

$7x - y = 242$

$y = 7x - 242$

242 is more than 7 × 34, so x has to be at least 35 or else y is negative.

So the smallest value of y occurs when x = 35 and in that case, 7x – 242 is 3.

So the smallest value of y i.e., the smallest possible number of wrong answers is 3.

15. **(a)** Let cost of burger = ₹B

Cost of shake = ₹S

Cost of fries = ₹F

According to the question,

$3B + 7S + F = 120$ ...(i)

$4B + 10S + F = 146.50$ ...(ii)

On subtracting equation (i) from (ii), we get

$4B + 10S + F - (3B + 7S + F) = 146.50 - 120$

$\Rightarrow B + 3S = 26.50$ ...(iii)

On multiplying equation (i) by 4, we get

$12B + 28S + 4F = 480$ ...(iv)

On multiplying equation (ii) by 3, we get

$12B + 30S + 3F = 439.50$ ...(v)

On subtracting equation (v) from (iv), we get

$12B + 28S + 4F - (12B + 30S + 3F) = 480 - 439.50$

$-2S + F = 40.50$ ...(vi)

On adding equations (iii) and (vi), we get

$B + 3S + (-2S + F) = 26.50 + 40.50$

$B + S + F = 67$

Hence, the cost of an ordinary meal that contains 1 Burger, 1 Shake and 1 order of fries is ₹67.