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PHYSICS MOTION

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MOTION

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MOTION - FN

Motion means the change in position of an object with respect to its surroundings in a given interval of time.

We see various things around us in motion, e.g. water flowing in a river, flowers nodding to blowing wind, birds flying in the sky, a player running in the playground and many more.

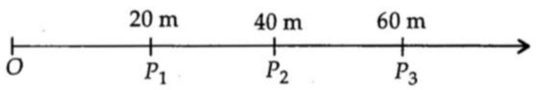
Motion is a relative phenomenon i.e. an object appearing to be in motion to one person can appear to be at rest to another person, e.g. to a person driving a car, trees on roadside might appear to move backward while same tree will appear to be at rest to a person standing on roadside. In this chapter we shall learn more about motion.

Describing Motion

Two different physical quantities the distance and the displacement, are used to describe the overall motion of an object and to locate its final position with reference to its initial position at a given time.

Since motion is a relative concept, we need a reference point to describe the position of an object. We call this reference point the origin.

Consider an object moving in a straight line. Let the object start from a point O. This starting point O is taken as the reference point. After 10 minutes it reaches the position P_3 through P_1 and P_2.



(time taken =10 min)
(time taken = 5 min)

In next 5 minutes it comes back to position P_2 (40 m away from O) and stops. So the total distance travelled by the object in these 15 minutes is $OP_3 + P_3P_2 = 60 + 20 = 80m.$

But the shortest distance between the initial and final positions of the object $= OP_2 = 40\,\text{m}.$

The shortest length of the path between initial and final positions is known as displacement.

So in the above example

Distance covered by the object = 80 m

Displacement of the object = 40 m right of 0.

Thus whereas distance is described with magnitude only, displacement requires both the magnitude and the direction. Physical quantities like distance, which can be described by magnitude only are called scalars, and the quantities are described using both magnitude and direction are called vectors. Hence distance is a scalar quantity and displacement is a vector quantity.

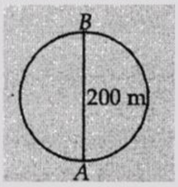
Do You Know

A vector is meaningful only if we know both magnitude and direction of vector. Without knowing direction, the description of a vector quantity is incomplete.

ILLUSTRATION

1. An athlete completes a round of a circular track of diameter 200 m in 20 s. Calculate
(i) the distance travelled by the athlete and
(ii) the magnitude of the displacement of the athlete at the end of 1 minute and 10 seconds.

Sol. Here, diameter of circular track, D=200 m



$\therefore$ Length of circular track = circumference of the circular track

$$= 2\pi r = \pi(2r) = \pi D = \frac{22}{7} \times 200 = 628.57\,\text{m}$$

(i) Distance travelled in 20 s = length of circular track = 628.57 m

Distance travelled in 1 $s=\frac{628.57}{20}m$

$\therefore$ Distance travelled in 1 minute and 10 s (or 70 s) $=\frac{628.57}{20}m\times 70=2199.99=2200\text{m}$

(ii) Number of rounds completed in 20 s = 1

Number of rounds completed in 70 s $=\frac{1}{20}\times 70=3\frac{1}{2}$

When athlete completes 3 rounds, his displacement = zero

The position of the athlete in next $\frac{1}{2}$ round is just opposite to his starting point.

So, displacement of the athlete at the end of 1 minute and 10 s = diameter of the circular track = 200 m.

2. Are the following statements true or false

(a) Displacement cannot be zero

(b) The magnitude of displacement is greater than the distance travelled by the object.

Sol. (a) False, consider a boy who starts walking from a corner of a park along its perimeter and finally comes back to the initial point. As both starting and final positions of the boy are same. So his displacement is zero.

(b) False, since displacement is the length of shortest path between initial and final positions of the moving object, its magnitude can never be greater than the distance travelled.

Uniform and Non-uniform Motion

- **Uniform Motion**

An object is said to be m uniform motion if it covers equal distances in equal intervals of time however big or small these time intervals may be.

For example, suppose a car covers 60 km in first hour, another 60 km in second hour, again 60 km in the third hour and so on. The motion of the car is uniform motion. Let us now understand the meaning of the words, ('howsoever small the time interval may be') used in the definition. In this example, the car travels a distance of 60 km in each hour. In the stricter sense, the car should travel 30 km in each half hour ; 15 km in every 15 minutes ; 10 km is every 10 minutes, 5 km is every 5 minutes and 1 km in every one minute. Only then, the motion of the car can be said to be uniform. However, in broader sense, we do not mind even when time interval is big. The motion of the car is taken as uniform when it covers a distance of 60 km in every one hour.

- **Non-uniform Motion**

Consider a bus starting from one stop. It proceeds slowly when it passes crowded area of the road. Suppose it manages to travel merely 100 m in 5 minutes due to heavy traffic. When it gets out and the road is clear, it speeds up and is able to travel about 2 km in 5 minutes. We say the motion of bus is non-uniform i.e. it travels unequal distances in equal intervals of time.

Example of non-uniform motion is

A speeding up or a slowing down vehicle.

Speed and Velocity

- **Speed**

Speed of a moving body gives us the idea about how fast or slow the body moves. In case of vehicles it is the quantity indicated by their speedometers.

Mathematically, it is the distance travelled by the object in unit time. We usually denote it by symbol v

$\therefore \upsilon=$ Distance travelled in unit time.

We also call it the instantaneous speed.

Its S.I. unit is meter per second represented by the symbol m/s or ms^{-1}. It is a scalar quantity as it requires magnitude only for its specification.

Many of the motions occurring around us are non - uniform motions. A train going from stop A to stop B, which are 100 km apart, may travel different intermediate intervals of path in different time spans. It will slow down or speed up at intermediate stops. Thus it does not move with a constant speed during its journey. We describe the rate of motion of such objects in terms of their average speed. Average speed is defined as the ratio of total distance travelled and total time taken.

$$\upsilon_{av}\frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Thus if the train discussed above travels first 25 km in 30 min, next 50 km in 40 min and the remaining 25 km in 30 min. The average speed of the train is

$$\upsilon_{av}=\frac{25\text{km}+50\text{km}+25\text{km}}{30\text{min}+40\text{min}+30\text{min}}$$

$$=\frac{100\,\text{km}}{100\,\text{min}}=\frac{1\text{km}}{\left(\frac{1}{60}\right)\text{h}}=60\,\text{km/h}$$

An object undergoing a uniform motion is said to move with uniform speed while the object which is in non - uniform motion is said to be moving with a non - uniform speed.

ILLUSTRATION

3. On a 60 km track, a train travels the first 30 km at a uniform speed of 30 km/h. How fast must the train travel the next 30 km so as to get an average of 40 km/h for the entire trip?

Sol. Total distance (d) = 60 km

Speed (υ_1) during the first half journey = 30 km/h

To calculate: Speed (υ_2) for the 2nd half =?

Now $\upsilon_{av} = \dfrac{\text{Total distance}}{\text{Total time}}$

or $\upsilon_{av} = \dfrac{\text{Total distance}}{t_1 + t_2}$ where $t_1 = \dfrac{30}{30} = 1hr$

According to question $40 = \dfrac{60}{1 + \dfrac{30}{\upsilon_2}}$ or

$40(\upsilon_2 + 30) = 60\upsilon_2 \Rightarrow \upsilon_2 = 60 \text{km/hr}$

Velocity (Speed with Direction)

The distance covered by a body per unit time in a specified direction is called the velocity.

Thus the quantity that specifies both the speed and direction of an object is called velocity.

It is denoted by symbol v S.I. unit of velocity is m/s. larger unit such as km/h is also used.

- **Uniform Velocity**
  If the velocity of an object does not change as time passes, it is said to move with a uniform velocity. In such a case, both its speed and direction remain constant. This means that the object is moving along a straight line, without turning back, with a fixed speed. The displacement of the particle is equal in equal time intervals, however small a time interval we choose. We also say in this situation that the object is in uniform motion. If the object undergoes unequal displacement in equal time intervals, the motion is non uniform.
- **Non-Uniform Velocity**
  If the velocity of a moving body does not remain constant in a given interval of time, we say the body has a non-uniform velocity in that time interval. A body is said to have non-uniform velocity if
  (i) its speed i.e. magnitude of velocity changes and direction remains constant
  (ii) its direction of motion changes and speed remains constant, e.g. A boy running on a circular track with a constant speed of 2 m/s.
  (iii) both speed and direction change.
  Speed and velocity share similar relation as distance and displacement do.

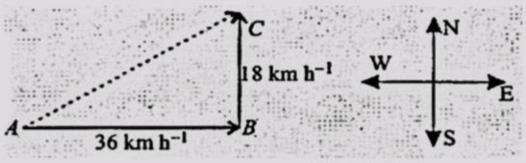
Thus speed $= \dfrac{\text{Distance travelled}}{\text{Time taken}}$ and velocity $= \dfrac{\text{Displacement}}{\text{Time taken}}$

There are many instances when displacement of an object is zero while distance travelled is non-zero. Thus velocity of an object can be zero or have some other non-zero value (depending on displacement) different from its speed.

ILLUSTRATION

4. A car starts at 36 km/h towards east. After 10 minutes, it turns toward north and travels at a rate of 18 km/h for next 10 minutes. What is its average speed and velocity?

Sol.



Average speed $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \dfrac{\text{AB+BC}}{20\,\text{min}}$

$= \dfrac{(36\text{km/h})\times 10\text{min} + (18\text{km/h})\times 10\text{min}}{20\,\text{min}}$

$= \dfrac{[36\text{km/h} + 18\text{km/h}]\times\left(\dfrac{10}{60}\text{h}\right)}{\left(\dfrac{20}{60}\right)\text{h}}$

$= \dfrac{[54\,\text{km/h}]}{2} = 27\,\text{km/h}$

Average velocity $= \dfrac{\text{Total displacement}}{\text{Total time taken}}$

$= \dfrac{\text{AC}}{20\text{min}} = \dfrac{(AB^2 + BC^2)^{\frac{1}{2}}}{20\text{min}}$

$= \dfrac{\left[(36)^2 + (18)^2\right]^{\frac{1}{2}} \times \left(\dfrac{10}{60}\right)\text{h}}{\dfrac{20}{60}\text{h}}$

$= \dfrac{1}{2}[40.25]\text{km/h}$ along AC.

= 20. 15 km/h along AC

Acceleration

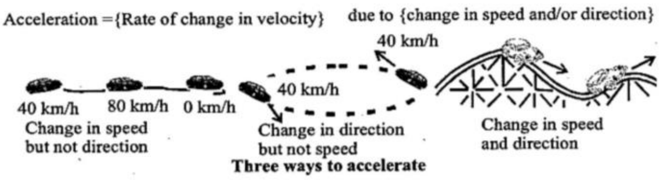
In non-uniform motion, velocity varies with time. The rate at which velocity varies is called acceleration. We can say that acceleration is the change in velocity per unit time.

If the velocity of an object from an initial value is u to the final value v in time t, then acceleration a is given by

$a = \frac{v-u}{t}$. This is also called **average acceleration.** As noted, velocity is a vector quantity, so is the acceleration. The S.I. unit of acceleration is s^{-2} or m/s^2.

Analysis of the equation, $a = \frac{v-u}{t}$

- When a body moves in a straight line without reversing its direction.
  (i) From the above equation if v > u, a is positive.
  $\Rightarrow$ If final velocity is greater than initial velocity, i.e. if the velocity increases with time, the value of acceleration is positive.
  (ii) If v < u, a is negative.
  $\Rightarrow$ If final velocity is less than initial velocity, i.e. if the velocity decreases with time, the value of acceleration is negative.
  Note: Negative acceleration is called retardation or deceleration.
  If the acceleration has a value of-2 m s-^{2}, then we say that the retardation is 2 m/s^2 or deceleration is 2 m/s^2.
  (iii) If v = u, 'a = 0. Acceleration is zero when the final velocity is equal to initial velocity.
  Note 1: An acceleration of 2ms^{-2} means that a change of velocity of 2 m/s is taking place in 1 s,
  i.e. $2\text{ms}^{-2} = \frac{2\text{ms}^{-2}}{1\text{s}}$
  Note 2: For constant acceleration, the average velocity for a given time interval can be calculated by the formula,
  Average velocity $v_{av} = \frac{u+v}{2}$



ILLUSTRATION

5. A boy starting from rest, starts running and attains a velocity of 6 m s^{-1} in 30 s. Then he slows down uniformly to 4 m s^{-1} in next 5 s. Calculate his acceleration in both cases.

Sol. In the first case
Initial velocity u = 0 ;
Final velocity, v = 6 ms^{-1};
Time t = 30 s. $a = \frac{v-u}{t}$
Substituting the given values of u, v and t in the above equation, we get
As $a = \frac{(6\text{ms}^{-1} - 0\text{ms}^{-1})}{30\text{s}} = 0.2\text{ms}^{-2}$
In the second case:
Initial velocity, $u = 6\text{ms}^{-1}$
Final velocity, $u = 4\text{ms}^{-1}$;
time, t = 5 s.
Then $a = \frac{(4\text{ms}^{-1} - 6\text{ms}^{-1})}{5\text{s}} = -0.4\text{s}^{-2}$

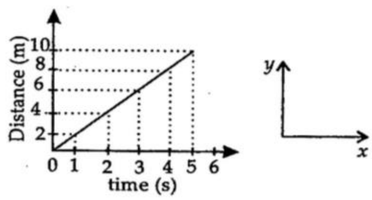
Study of Motion through Various Graphs

If information about various quantities related to motion is given in tabular form for various instants of time, it can be converted in graphical form. It makes easier for us to find out relation between various physical quantities.

Distance-Time Graphs

(i) Distance is taken along Y-axis and time along the X-axis. A convenient scale is chosen for both the axes.
Consider the following data taken by an observer for a bus in 5 sec. using this data we can obtain the distance - time graph for the bus as shown

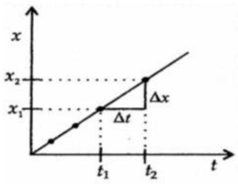
Distance (m)	Time (s)
0	0
2	1
4	2
6	3
8	4
10	5



We see, the graph is a straight line passing through the origin (intersection of two axes). It is clear that the bus is travelling 2 m in each second, or the body has a constant speed Let us find out speed of the bus from above graph. By definition speed of a body is given by

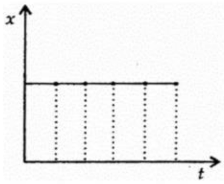
$v = \frac{\text{total distance travelled}}{\text{total time taken}}$

Let us take any two instances t_1 and t_2 on the distance-time graph when the distance travelled by the bus are x_1 and x_2 respectively.

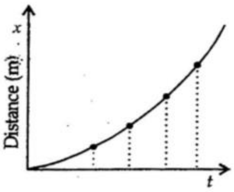


So the distance travelled between instances t_1 and t_2 is $(x_2 - x_1)$ and time taken is $t_2 - t_1$

$\therefore$ speed $= \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$ where $\frac{\Delta x}{\Delta t}$ is called the slope of distance time graph. In other words slope of the line obtained in a distance time graph gives the speed of the object.



(ii) If distance-time graph of an object is a straight line parallel to the time axis, as shown in graph, we say the position of the object does not change with time. Or the object remains at rest.



(iii) If we get a curved line in distance-time graph then, if we find slope for any two time intervals, it will not be same.

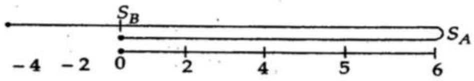
Or we say the object has a non-uniform speed i.e. the body is accelerated.

Displacement-Time Graph

As discussed earlier, we know that displacement of an object can be negative also while distance is always positive. Thus displacement time graph differs from distance-time graph in following manner.

First the objects moves with a constant velocity given by slope of line OA.

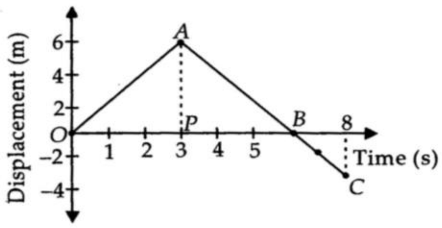
Velocity $= \frac{AP}{OP}$



After that object moves with a constant velocity given by slope of line AC.

Now slope of line $AC = \frac{S_A - S_B}{t_A - t_B}$

$= \frac{6-0}{3-6} = -\frac{6}{3} = -2$ It is negative.



We say the object moves with velocity -2 m/s i.e. with a velocity 2 m/s in opposite direction. It comes to original position in 6 sec (Zero Displacement) and then moves to the other side maintaining its velocity.

The distance-time and displacement-time graphs of a moving body are similar only when the body moves along a straight line in its positive direction without changing its direction.

Velocity-Time Graphs

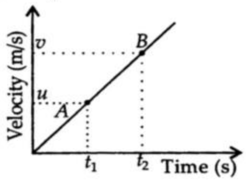
In these graph, variation of velocity of an object with time is shown. Time is taken along x-axis while velocity is taken along y-axis.

- **Velocity- time graph for uniformly accelerated motion:**

  Consider the velocity time graph shown.

  It is a straight line with a positive slope passing through the origin.

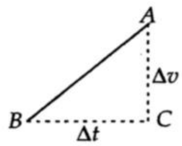
  Consider any two positions A and B of the object at instants t_1 and t_2 when the object has velocities u and v respectively.



  Now according to definition of acceleration

  $a = \frac{\Delta \upsilon}{\Delta t} = \frac{\upsilon - u}{t_2 - t_1}$ ....(i)

  $= \frac{BC}{AC} =$ slope of the straight line



  Thus slope of the graph obtained in a velocity time graph gives the acceleration of the object. Now for a straight line, the slope will be same for any two points considered. Or we can say acceleration will remain constant or uniform.

  Thus a uniform accelerated motion is shown by a straight line having a positive slope in a velocity time graph.

  Consider equation (i) above once more.

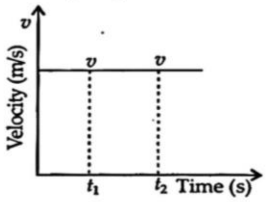
  $a = \frac{\upsilon - u}{t_2 - t_1}$ Let $t_2 - t_1 = t \Rightarrow a = \frac{\upsilon - u}{t}$ or $\upsilon = u + at$

  Which is one of the very important equation of motion.

- **Velocity time graph for a uniform motion:**

  When an object is in uniform motion, it will have a constant velocity at every instant, i.e. velocity won't change with time. Or we can say that velocity time graph is a straight line parallel to time-axis.

Now acceleration between any two instants t_1 and t_2 is $a = \frac{\Delta \upsilon}{\Delta t} = \frac{\upsilon - u}{t_2 - t_1} = 0$



Thus acceleration is zero for an object having uniform motion.

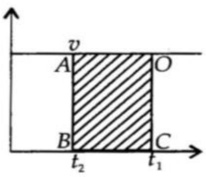
Also according to the definition of velocity

Average velocity $= \frac{\text{total displacment}}{\text{total time taken}}$

Again taking same time interval between two instants t_1 and t_2

$$\upsilon = \frac{s}{t_2 - t_1} \Rightarrow s = \upsilon \times (t_2 - t_1) = AB \times BC$$

= Area of the shaded region shown in adjacent graph.



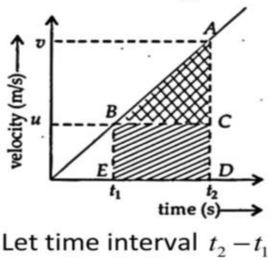
Thus distances of the object is given by the area enclosed by velocity time graph and the time axis. Distance can also be obtained for a uniformly accelerated motion using same method.

Total distance = Area of trapezium ABEDC

s = Area of rectangle BCDE + Area of triangle ABC.

$$= BE \times ED + \frac{1}{2} \times AC \times BC$$

$$= u \times (t_2 - t_1) + \frac{1}{2}(\upsilon - u) \times (t_2 - t_1)$$



Let time interval $t_2 - t_1 = t$

$$\Rightarrow \quad s = ut + \frac{1}{2}(v - u)t$$

Using equation, v = u + at, we get

$$s = ut + \frac{1}{2}at^2 \Rightarrow s = ut + \frac{1}{2}at^2$$

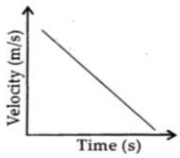
This is second equation of motion.

Consider the figure given above once again,

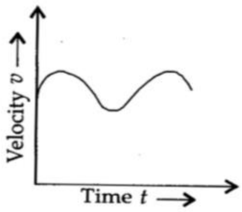
s = Area of trapezium ABEDC

$$= \frac{1}{2} \times ED(BE + AD) = \frac{1}{2}t(u + \upsilon)$$

$$= \frac{1}{2}\frac{(\upsilon - u)a}{}(\upsilon + u) = \frac{1}{2a}(\upsilon^2 - u^2)$$



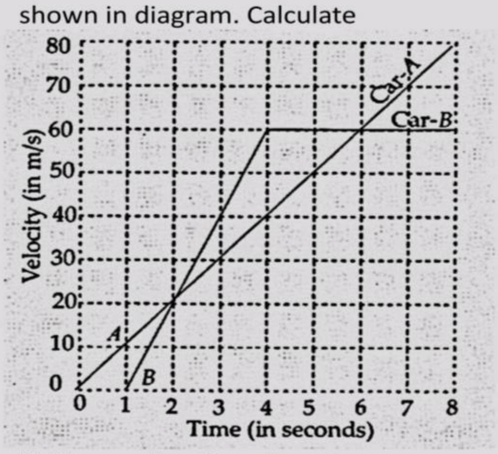
- **Uniformly retarded motion:** For this velocity-time graph is a straight line having a negative slope as shown
- **Non- uniformly acceleration motion:** For this velocity time graph can have any shape depending upon how the velocity varies. Here Time(s) acceleration will be different between different instants as the slope does not remain constant.



The graphs discussed above can also be used to make a comparative study of motion of two or more objects. Consider the illustration given.

ILLUSTRATION

6. The velocity-time graph of two cars A and B, which start from the same place and move along a straight road in the same direction, as shown in diagram. Calculate



(i) the acceleration of car A.

(ii) the acceleration of car B between 2 s to 4 s.

(iii) the points of time at which both the cars have the same velocity

(iv) which of the two cars is ahead after 8 seconds and by how much?

Sol. (i) Acceleration of car A

(ii) Acceleration of car B $a = \frac{80}{8} = 10\,\text{ms}^{-2}$

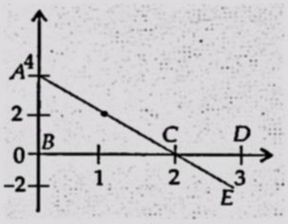
(iii) After 2 seconds and 6 seconds from start.

(iv) Distance travelled by car

$A=\frac{1}{2}\times 80\,\text{m/s}=320\text{m}$ and distance travelled by car $B=\frac{1}{2}\times(7+4)\text{s}\times 60\,\text{m/s}=330\,\text{m}$

$\therefore$ Car B is ahead by 330 – 320 = 10 m.

7. Find the displacement of a car between t = sec to t = 3 sec for the given velocity time graph.



Sol. As discussed earlier, the area between velocity-time graph and the time axis gives the displacement.

$\therefore$ Required displacement

= Area of ΔABC + Area of ΔDCE

$=\frac{1}{2}AB\times BC+\frac{1}{2}CD\times DE$

$s=\frac{1}{2}\times(4m/s)\times 2s+\frac{1}{2}(1s)\times(-2m/s)$

$=4m-1m$

S=3 m, So total displacement = 3 m.

If we study this graph carefully, it is a uniformly retarded motion. Initially at t = 0 sec, object has a velocity 4 m/s. It decreases uniformly till it becomes zero at t = 2 sec i.e. the object momentarily comes to rest at t =2 sec. Velocity still continues to decrease at the same rate i.e. the object now starts moving in opposite direction or we say the velocity now has a negative value. It is for this reason that displacement corresponding to ΔACB is taken as positive and for ΔDCE as negative.

Equations of Motion for Uniformly Accelerated Motion

The three important equations of motion, which we have derived in last section also, are given below:

(i) $v=u+at$

(ii) $s=ut+\frac{1}{2}at^2$

(iii) $2as=v^2-u^2$

Illustration based on these equations are as follows.

ILLUSTRATION

8. A bus starting from rest moves with a uniform acceleration of 0.1 ms^{-2} for 2 minutes. Find (a) the speed acquired, (b) the distance travelled.

Sol. Here, u = 0,

A = 0.1 ms^{-2}, t = 2 minutes = 120 s

(a) Using, $v=u+at, v=0+0.1\times 120=12\,\text{ms}^{-1}$

$\therefore$ speed acquired $=12\,\text{ms}^{-1}$

(b) Using, $s=ut+\frac{1}{2}at^2$, we get

$s=0+\frac{1}{2}\times 0.1\times 120\times 120=720\,\text{m}$

$\therefore$ Distance travelled = 720 m.

A bus decreases its speed from 80 km h^{-1} to 60 km h^{-1} in 5 s. Find the retardation of the bus.

Sol. Here, u = 80 km h^{-1} = $80\times\frac{5}{18}\text{ms}^{-1}$

$=22.22\,\text{ms}^{-1}$ $v=60\,\text{kmh}^{-1}=60\times\frac{5}{18}\text{ms}^{-1}$

$=16.67\,\text{ms}^{-1}$ t = 5 s

$\therefore$ retardation,

$a=\frac{v-u}{t}=\frac{(16.67-22.22)\text{ms}^{-1}}{5s}$

$a=\frac{-5.55\text{ms}^{-1}}{5\text{s}}=1.11\,\text{ms}^{-2}$

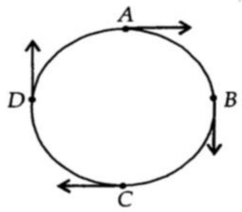
Uniform Circular Motion

If an object moves in a circular path with uniform speed, its motion is called uniform circular motion.

A circular path can be made up of an indefinite number of small sides, and a body moving along such a circular path changes its direction of motion continuously.

Therefore, if you run on a circular track, you change your direction infinite times in one round.

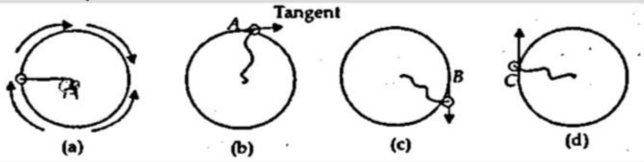
Four arbitrary points on the circular path and the direction of motion of the body at these points are shown. Since the direction of motion changes, uniform circular motion is a case of accelerated motion.



ACTIVITY CORNER

Take a small stone and tie it with one end of a strong thread. Now, move the stone in a circular path by holding the other end of the thread in your hand as shown in figure

(a). Leave the thread, when the stone is at position A on the circular path as shown figure (b). You will find that the stone moves in a straight line which the tangent to the position A on the circular path. Again, move the stone in the circular path and leave the thread, when stone is at position B as shown in figure (c). Once again, you will find that the stone moves in a straight line which is the tangent to the position B on the circular path. Repeat the activity and leave the thread, when the stone is at different position on the circular path.



From this simple activity, we conclude that the direction of motion of a body moving in a circular path is always along the tangent to a point on the circular path. Thus, the direction of motion of a body moving in a circular path is different at different positions of the circular path.

Note:
If a body moves around a circular path of radius r in the time t then the distance covered by the body is equal to the circumference of the circle, i.e., $2\pi r$. In such a case the velocity v is given by $\upsilon = \frac{2\pi r}{t}$

Examples for circular motion

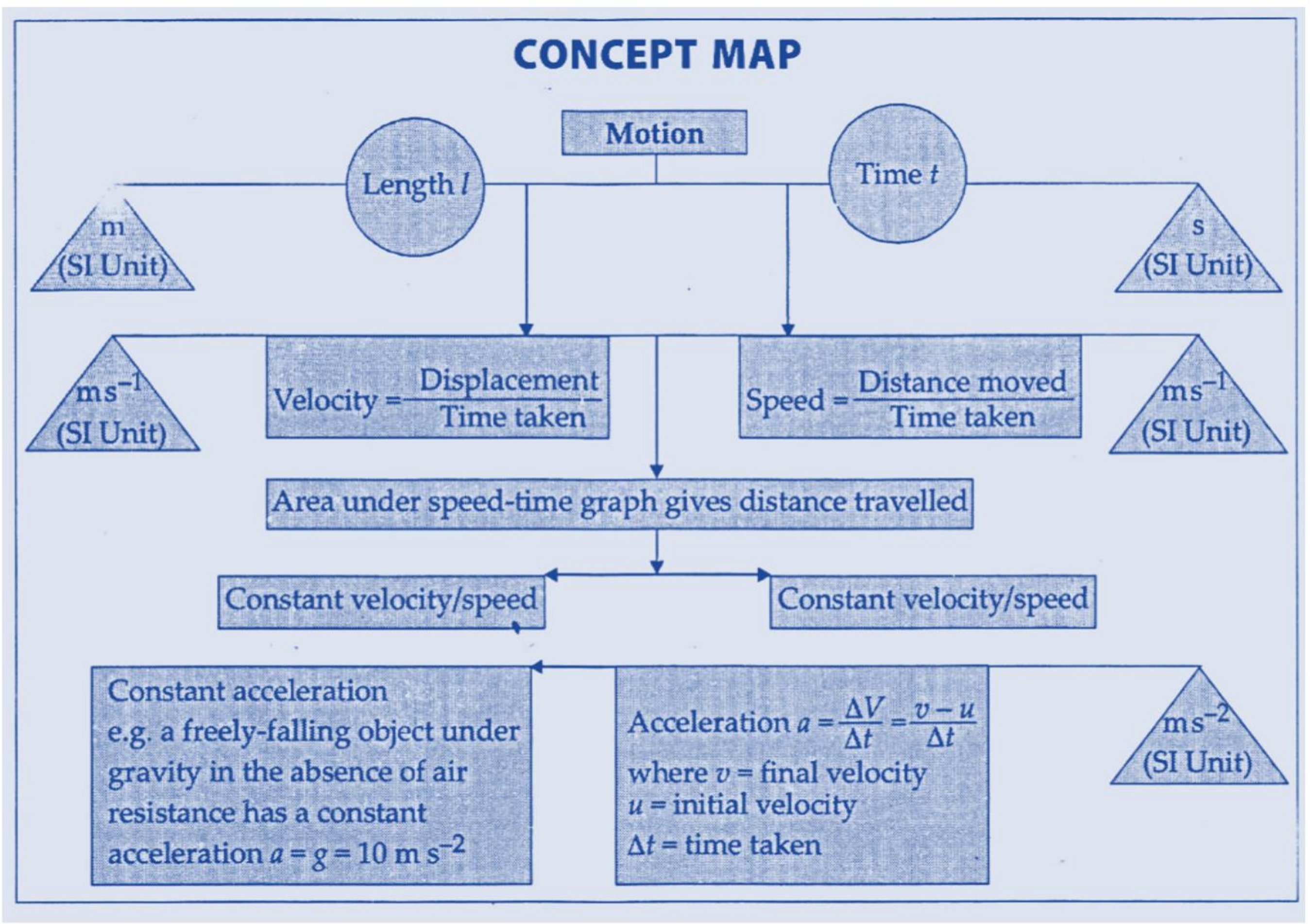
- **A stone tied to a thread and whirled in a circular path.**
- **Wheels of various vehicle rotating about their axles.**
- **A satellite revolving around the Earth in a circular path, at constant speed.**
- **The Moon revolving around the Earth in a circular path at constant speed.**
  In the above examples, the speed is uniform, but the velocity is variable due to continuous change in direction. Thus, the bodies have an accelerated motion.

ESSENTIAL POINTS For COMPETITIVE EXAMS

- **Rest:** When a body does not change its position with respect to time and its surroundings, the body is said to be at rest.
- **Motion:** When a body continuously changes its position with respect to time and its surroundings, the body is said to be in motion.
- **Characteristics (properties) of a moving body:**
  (i) There must be a reference point (a stationary object) to describe the position of a given body.
  (ii) The position of the given body must continuously change with time and with respect to reference point.
- **Distance:** It is the actual length of the path travelled by a moving body, irrespective of the direction of motion of the body.
- **Displacement:** The shortest distance of a moving body from the point of reference (initial position of the body) in a specified direction is called displacement.
- **Uniform motion:** When a body covers equal distances in equal intervals of time, however small may be time intervals, the body is said to be uniform motion.
- **Non-uniform motion:** When a body covers unequal distances in equal intervals of time, it is said to be moving with non-uniform motion.
- **Speed:** The rate of change of motion is called the speed.
- **Mathematical expression for speed:** Speed = Distance ÷ Time. S.I. unit of speed is metre per second (ms^{-1} or m/s)
- **Uniform speed:** When a body covers equal distances in equal intervals of time, however small may be the time intervals, the body is said to be moving with uniform speed.
- **Variable speed:** When a body covers unequal distances in equal intervals of time, the body is said to be moving with a variable speed.
- **Average speed:** The total distance covered by a body per unit time is called average speed.
- **Velocity:** The distance covered by a body per unit time in a specified direction is called velocity. It is a vector quantity and has same units as speed.
- **Uniform velocity:** When a body covers equal distances in equal intervals of time (however small may be the time intervals) in a specified direction, the body is said to be moving with uniform velocity.
- **Variable velocity or Non-uniform velocity:** When a body covers unequal distances in equal intervals of time in a specified direction or when a body covers equal distances in equal in- tervals of time, but its direction changes, then the body is said to be moving with a variable velocity.
- **Acceleration:** The rate of change of velocity of a moving body is called acceleration. It is a vector

quantity and its unit is metre per second square ($m\ ms^{-1}$ or m/s^2)

- **Positive acceleration:** The rate of change of velocity of a moving body, when the velocity is increasing is called positive acceleration.
- **Negative acceleration:** The rate of change of velocity of a moving body, when the velocity is decreasing is called negative acceleration or retardation.
- **Distance - time graph:**
  (a) The distance-time graph of an object moving with a uniform speed is a straight line. Conversely, if the distance-time graph of an object is a straight line, the object is moving with a uniform speed.
  (b) The slope of the distance-time graph of an object equals its speed.
  (c) If an object moves with no uniform speed, its distance-time graph is not a straight line.
- **Displacement-time graph:**
  (a) The displacement-time graph of an object moving with a uniform velocity is a straight line.
  (b) The slope of the displacement-time graph of an object equals its velocity.
- **Speed-time graph:**
  (a) If an object moves with a constant speed, its speed-time graph is a straight line parallel to the time-axis.
  (b) The area under the speed-time graph gives the distance traversed by the object in the corresponding time interval.
- **Velocity-time graph:**
  (a) If an object moves with a constant acceleration in a straight line, its velocity-time graph is a straight line.
  (b) The slope of the velocity-time graph gives the acceleration of the object.
  (c) The area under a velocity-time graph gives the displacement of the object.
- **Circular motion:**
  A particle moving in a circular path changes its direction continuously, and hence, is accelerated.
- **Mathematical equations :**
  $s = vt$ s = distance, v = speed (assumed constant), t = time $v = u + at$ u = velocity at t = 0, v = velocity at time t, a = acceleration (assumed constant). $s = ut + \frac{1}{2}at^2$ s = displacement during time 0 to t.
  u = velocity at t = 0, a = acceleration (assumed constant) $v^2 = u^2 + 2as$
  Symbols have the same meaning as in the above equations.



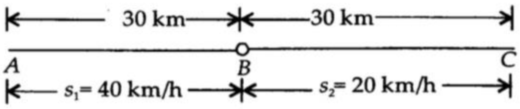
SOLVED EXAMPLES

1. A car travels 30 km at a uniform speed of 40 km/h and the next 30 km at a uniform speed of 20 km/h. find its average speed.

Sol. Total distance (d) = 30 + 30 = 60 km Speed for the first 30 km (u_1) = 40 km/h

Speed for the next 30 km (u_2) = 20 km/h

To calculate: Average speed (u_{av})=?



Formula to used $\frac{\text{Total distance(d)}}{\text{Total time (t)}}$

Now in this problem we are not given the total time. So our first problem is to find out the total time from the two speeds given to us by using the formula.

Time $(t) = \frac{\text{distance}}{\text{speed}}$ time (t_1) in going from A

$= \frac{30}{40} = \frac{3h}{4} = 45\,\text{min}$

(ii) time (t_2) in going from B to C

$= \frac{30}{20} = 1$ hr 30 min = 90 min

Total time $t = t_1 + t_2 = 45 + 90 = 135\,\text{min}$

or $t = \frac{135}{60}h$ Hence, $\upsilon_{av} = \frac{60 \times 60}{135} = \frac{80}{3}\text{km/h}$

2. A train travels at 60 km/h for 0.52 hour, 30 km/h for the next 0.24 hour and then 70 km/h for the next 0.71 hour. What is the average speed of the trip?

Sol. Given:

First speed (υ_1) = 60 km/h

Time for this part of the trip (t_1) = 0.52 h

Second speed (υ_2) = 30 km/h

Time for second part (t_2) = 0.24 h

Third speed (t_3) = 70 km/h

Time for third part (t_3) = 0.71 h

To calculate: Average speed (υ_{av}) =?

Formula to be used: $\frac{\text{Total distance (d)}}{\text{Total time taken (t)}}$

Now in this case we are not given the total distance travelled, so first of all we will find out the distance covered during the three given intervals by using the formula, distance = speed x time

(i) First part $(d_1) = \upsilon_1 \times t_1 = 60 \times 0.52 = \frac{156}{5}\text{km}$

(ii) Second $(d_2) = \upsilon_2 \times t_2 = 30 \times 0.24 = \frac{36}{5}\text{km}$

(iii) Third part

$(d_3) = \upsilon_3 \times t_3 = 70 \times 0.71 = \frac{497}{10}\text{km}$

Total distance $(d) = d_1 + d_2 + d_3$

$= \frac{156}{5} + \frac{36}{5} + \frac{497}{10} = \frac{881}{10}\text{km=88.1km}$

Total time taken $(t) = t_1 + t_2 + t_3$

$= 0.52 + 0.24 + 0.71 = 1.47\,\text{h}$

$\therefore\ \upsilon_{av} = \frac{88.1}{1.47} = 59.93\,\text{km/h}$

3. A ship is moving at a speed of 56 km/h. One second later it is moving at 58 km/h. What is its acceleration?

Sol. Given:

Initial speed {u} = 56 km/h

Final speed (v) = 58 km/h

Time taken $(t) = 1s = \frac{1}{60 \times 60}h$

To calculate : Acceleration (a) = ?

Formula to be used : $a = \frac{\upsilon - u}{t}$

$a = \frac{58-56}{\frac{1}{60 \times 60}} = 7200\,\text{km/h}^2$

4. The distance between the house and the school of a boy is 3.6 km. If he takes 6 minutes to reach his school by car, calculate his speed in m/s. Also express his speed in km/h.

Sol. Given:

Distance (d) = 3.6 km = 3.6 x 10^3 m

Time (t) = 6 min = 6 x 60 = 360 $s = \frac{6}{60}h$

To calculate: Speed (u) = ? in m/s and km/h

Formula to be used :

$\upsilon = \frac{d}{t} = \frac{3.6}{6/60} = \frac{3.6 \times 60}{6} = 36\text{km/h}$

Also, $\upsilon = \frac{3.6 \times 10^3}{360} = 10\,\text{m/s}$

5. In this figure represents a velocity-time graph of an object moving along a straight line.

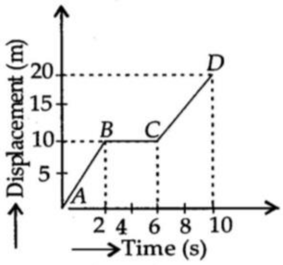
(a) Describe the motion of the body during the 10 s.

(b) How far did the object travel in the first

(c) What was the velocity of the object during the first 2s?

(d) What was the velocity during the next 4 s?

(e) What was it during the last 4s?



Sol. (a) For part AB, the object moved with a uniform velocity, for part BC, it was at rest and for part CD, it again moved with some other uniform velocity.

(b) In the first 2 s, it covered 10 m.

(c) Since velocity is given by the slope of the displacement-time graph, i.e.

Velocity $= \dfrac{\text{Displacement}}{\text{Time}}$

Velocity during the first 2 s (along AB)

$= \dfrac{(10-0)}{(2-0)} = 5\,\text{m/s}$

(d) Velocity during the next 4 s (along BC)

$= \dfrac{(10-10)}{(6-2)} = 0\,\text{m/s}$

(e) Velocity during the last 4 s (along CD)

$= \dfrac{(20-10)}{(10-6)} = 2.5\,\text{m/s}$

6. A body covers half of its journey with speed a m/s and the other half with speed b m/s. Calculate the average speed of the body during the whole journey.

Sol. Given:

Speed (u_1) = a m/s Speed (u_2) = b m/s

To calculate: Average speed $(u_{av}) = ?$

Formula to be used $\upsilon_{av} = \dfrac{\text{Total distance}}{\text{Total time taken}}$

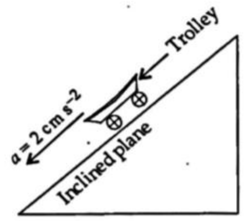
Suppose the total distance covered by the body is Id, out of which (d) is covered with a speed (a) and the other half (i.e., d) is covered with a speed (b). Let us suppose (t_1) and (t_2) be the times taken for the first and the second half respectively. $t_1 = \dfrac{d}{a}$ and $t_2 = \dfrac{d}{b}$

Total time taken

$(t) = t_1 + t_2 = \dfrac{d}{a} + \dfrac{d}{b} = d\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$

Also, $t_1 + t_2 \dfrac{2d}{\upsilon_{av}}$

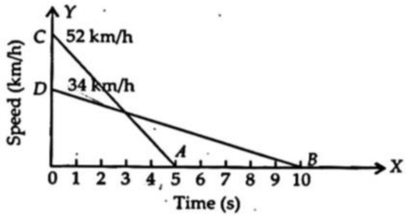
7. A trolley, while going down an inclined plane has an acceleration of 2cms^{-2}, What will be its velocity after 3 s of the start?



$\therefore\ \upsilon = u + at \quad \upsilon = 0 + 2\times 3 = 6\text{cm/s}$

8. A driver of a car travelling at 52 km h~1 applies the brakes and accelerates uniformly in the opposite direction. The car stops in 5 s. Another driver going to 34 km h^{-1} in another car applies his brakes slowly and stops in 10 s. On the same graph paper, plot the speed versus time graph for the two cars. Which of the two cars travelled farther after the brakes were applied?

Sol. For car X



$u = 52\text{km/h} = 52\times\dfrac{5}{18} = 14.4\,\text{m/s}\ \upsilon = 0, t = 5s$

For car Y $u = 34\text{km}\,\text{h}^{-1} = 34\times\dfrac{5}{18} = 9.4\,\text{m/s}$

$\upsilon = 0, t = 10s$

Both the graphs are straight line because the retardation is uniform.

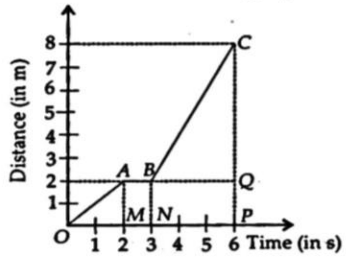
Distance travelled by X = Area of ΔOAN

$= \dfrac{1}{2}\times 10\times 9.4 = 47.2\,\text{m}$

9. If two objects move in circular path of radii in the ration of 1:3 and take same time to complete the circle, what is the ratio of their speeds?

Soln.: $\dfrac{\upsilon^1}{\upsilon_2} = \dfrac{2\pi r_1/t}{2\pi r_2/t} = \dfrac{r_1}{r_2} = \dfrac{1}{3}$

10. Interpret the distance-time graph



Sol. O to A

When we get a straight line inclined to time axis in a distance-time graph, it is case of constant speed. The speed is the slope of the line.

Speed $= \dfrac{AM}{OM} = \dfrac{2m}{2s} = 1\,\text{ms}^{-1}$ A to B

When we get a straight line parallel to time axis in a distance-time graph, the body is at rest.

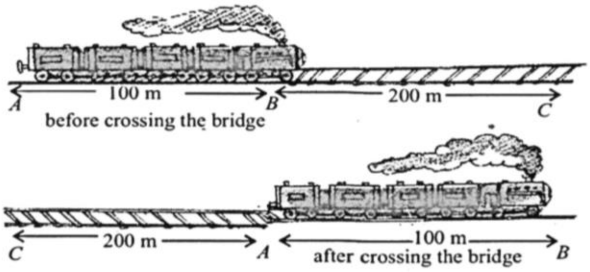
B to C It is a case of constant speed where

Speed $= \dfrac{CQ}{BQ} = \dfrac{6m}{3s} = 2\,\text{ms}^{-1}$

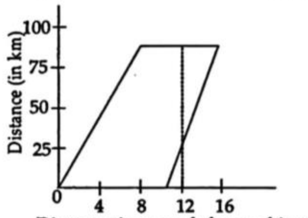
11. A 100 m long train crosses a bridge of length 200 m in 50 seconds with constant velocity. Find the velocity.

Sol. Distance travelled by the train=BC+AB=300 m.

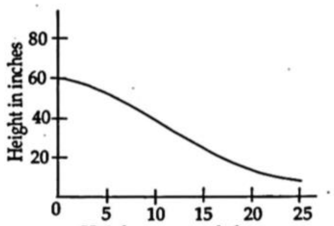
$\therefore$ Velocity $= \dfrac{300}{50} = 6\,\text{m/s}$



12. What is wrong with the following graphs?



Distance-time graph for an object



Height-age graph for a person

Sol. (a) If we draw a perpendicular on the time-axis at the point corresponding to 12 hours, it cuts the graph at two points. One corresponds to 25 km and the other corresponds to 75 km. Thus, according to the graph, the distance travelled in 12 hours is 25 km as well as 75 km, which is not possible.

(b) According to the graph, the height of a person gradually decreases as his age increases. Such a thing does not happen.

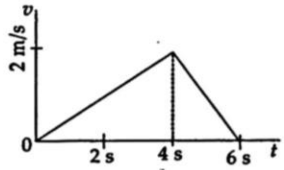
13. The velocity-time graph of a particle moving along a straight line is shown in the figure

(a) Is the motion uniform?

(b) Is the acceleration uniform?

(c) Does the particle change its direction of motion?

(d) Find the distances covered from 0 to 4 s and from 4 to 6 s.



Sol. (a) The velocity is changing with time. So the motion is not uniform

(b) The acceleration is given by the slope of the velocity-time graph. The slopes are different before and after t = 4 s. So the acceleration is not uniform for the entire time shown. It is uniform between 0 and 4 s and also between 4 and 6 s the slope does not change in these periods.

(c) The velocity always remains positive. It means that the particle keeps moving in the positive direction. In other words, it does not change direction.

(d) The displacement during the period 0-4 s is equal to the area under the velocity-time graph for this period. This area is in the shape of a triangle.

Area of the triangle $= \dfrac{1}{2} \times \text{base} \times \text{height}$

$= \dfrac{1}{2} \times 4s \times (2\text{m/s}) = 4\,\text{m}$

As the particle moves in the same direction, this is also the distance moved. For the period 4-6 s, the area is $\dfrac{1}{2}(2\text{s}) \times 2\left(\dfrac{m}{s}\right) = 2\,\text{m}$

So the particle moves 2 m in this period.

14. A particle moving with an initial velocity of 5.0 m/s is subjected to a uniform acceleration of -2.5 m/s^2. Find the displacement in the next 4.0 s.

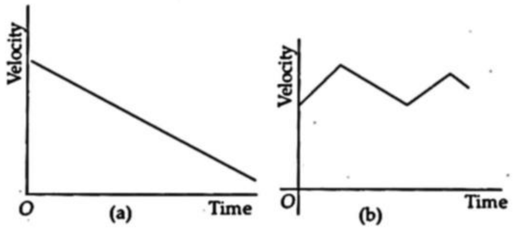
Soln.: The displacement is $s = ut + \dfrac{1}{2}at^2$

$= \left(5.0\dfrac{m}{s}\right) \times (4.0s) + \dfrac{1}{2}\left(-2.5\dfrac{m}{s^2}\right) \times (4.0s)^2$

$= 20m - 20m = 0$

So, after 4.0 s, the particle will be back at its initial position. Note that the distance traversed is not zero, as the particle moves in the forward direction and then comes back to the initial position.

15. What can you say about the nature of the motions of the particles for which the velocity-time graphs are given below



Sol. In case of figure (a), as time passes, the velocity decreases continuously. So the particle is slowing down continuously. We see this type of motion when we throw a ball up. The ball slows down continuously on its way up.

In case of figure (b), the velocity increases and decreases alternately. As the velocity remains positive throughout, the particle keeps moving in the same direction. You have this type of motion when a driver drives a car on a straight, busy road. He has to slow down (brake) and speed up (accelerate) alternately for a large part of the drive.

16. The second's hand of a clock is moving with uniform angular speed, calculate its angular speed. If the length of the second's hand is 2 cm, find out the speed of the tip of the second's hand.

Sol. In a clock, the second's hand makes one complete round in 1 minute. Therefore,

Time, t = 1 minute = 60 s

Angular displacement, $\theta = 2\pi$ radian

We know, angular velocity,

$$\omega \frac{\theta}{t} = \frac{2\pi}{60}\frac{\text{radian}}{s} = \frac{\pi}{30}\text{rad/s}$$

Now, linear speed = angular velocity x radius of the circular path Hence, the speed of the tip of the second's hand,

$$\upsilon = r\omega = 2\times\frac{\pi}{30}\text{cm/s}=\frac{\pi}{15}\text{cm/s}$$

$$=0.21\text{cm/s.}\left(\because \pi = \frac{22}{7}\right)$$

17. A truck running at 90 km/h, slows down to 54 km/h over a distance of 20 m. Calculate (i) the retardation produced by its brakes, and (ii) the time for which the brakes are applied.

Sol. Initial velocity of the truck (u) = 90 km/h = 25 m/s Final velocity of the truck (v) = 54 km/h =15 m/s

Distance covered (s) = 20 m

Retardation (-a) = ? (To be calculated)

Time (t) = ? (To be calculated)

Applying, $v^2 - u^2 = 2$ as

$\Rightarrow$

$$(15)^2 - (25)^2 = 2\times a \times 20 \Rightarrow 225 - 625 = 40a$$

$$\therefore \; a = \frac{-400}{40} = -10\,\text{m/s}^2$$

Retardation (-a) =-(-10 m/s^2) = 10 m/s^2

(ii) Applying, v = u + at

$\Rightarrow \; 15 = 25 - 10\times t \Rightarrow 10t = 10$ or Time, t = 1 s.

18. A sound is heard 5 seconds later than the lightning is seen in the sky on a rainy day. Find the distance of the location of lightning. Given speed of sound = 346 ms^{-1}.

Sol. Here, t = 5 s

Speed, v = 346 m ms^{-1}

Distance = ?

Using, distance = speed x time, we get distance = 346 m ms^{-1} x 5 s = 1730 m

Thus, the distance of the location of lightning = 1730 m.

19. The driver of train A travelling at a speed of 54 km h^{-1} applies brakes and retards the train uniformly. The train stops in 5 seconds. Another train B is travelling on the parallel track with a speed of 36 km h^{-1}. His driver applies the brakes and the train retards uniformly. The train B stops in 10 seconds. Plot speed-time graphs for both the trains on the same graph paper. Also calculate the distance travelled by each train after the brakes were applied.

Sol. For train A $u = 54\times\frac{5}{18} = 15\text{ms}^{-1}$

$\upsilon = 0; t = 5s$

For train B u = 36 km $\text{h}^{-1} = 36\times\frac{5}{18} = 10\,\text{ms}^{-1}$

$\upsilon = 0; t = 10s$ Speed-time graphs for trains A and B are shown in figure.

Distance travelled by train A = Area under curve EF = Area of ΔOEF

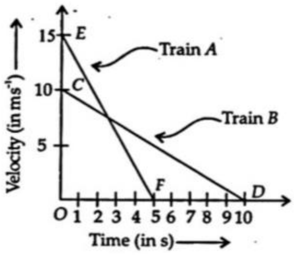
$$=\frac{1}{2}\times\text{Base}\times\text{Height} = \frac{1}{2}\times OF\times OE$$

$$=\frac{1}{2}\times 5s = 37.5\text{m}\times 15\text{ms}^{-1}$$

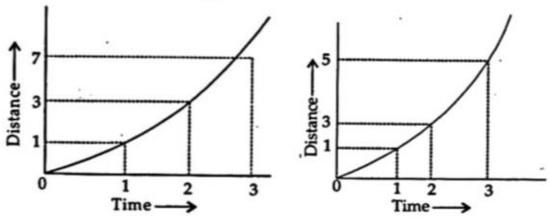
Distance travelled by train B = Area under curve CD = Area of ΔOCD

$$=\frac{1}{2}\times\text{Base}\times\text{Height} = \frac{1}{2}\times OD\times OC$$

$$=\frac{1}{2}\times 10\,\text{ms}^{-1}\times 10\text{s} = 50\,\text{m}$$



20. **Distance-time graph for the motion of a truck and bus are shown in figure A and figure B respectively. What can you say about the motion of these vehicles and which of these vehicles is moving fast?**



Sol. Since both truck and bus are travelling unequal distances in equal intervals of time, therefore, the motion of both the vehicles is non-uniform motion.

Slope of distance-time graph = speed of an object Since slope of distance-time graph for the motion of the bus is greater than the slope of distance- time graph for the motion of the truck, therefore, bus is moving faster than the truck.

21. **An artificial satellite is moving in a circular orbit of radius 42250 km. Calculate its speed if it takes 24 hours to revolve around the earth.**

Sol. Radius, R= 42250 km

Length of orbit = Circumference of the orbit

$$= 2\pi R = 2 \times \frac{22}{7} \times 42250 \text{km} = 265571.43 \text{km}$$

Time, t = 24 h

$\therefore$ Speed

$$= \frac{\text{Length of orbit}}{\text{Time}} = \frac{2\pi R}{t} = \frac{265571.43 \text{km}}{24 \text{h}}$$

$$= 11065.48 \text{km h}^{-1}$$

$$= \frac{11065.48}{3600} \text{km s}^{-1} = 3.07 \text{km s}^{-1}$$

NCERT SECTION

1. An object has moved through a distance. Can it have zero displacement? If yes support your answer with an example.

Ans. Yes, the object instead of moving through a distance can have zero displacement. Example If an object travels from point A and reaches to the same point *A,* then its displacement is zero.

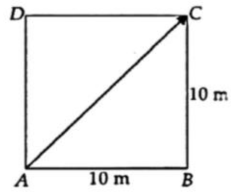
2. A farmer moves along the boundary of a square field of side 10 m in 40 s. What will be the magnitude of displacement of the farmer at the end of 2 minutes 20 seconds?

Ans. Figure *ABCD* is a square field of side 10 m.

Time for one round = 40 s /

Total time = 2 min 20 s /

= (2 x 60 + 20) s = 140 s / 10 m



Number of round completed $= \frac{140}{40} = 3.5$ If farmer starts from *A,* it will complete 3 rounds *A* 10m B

$(A \rightarrow B \rightarrow C \rightarrow D \rightarrow A)$ at A. In the last 0.5 round starting from A, he will finish at C.

3. Which of the following is true for displacement?

(a) It cannot be zero.

(b) Its magnitude is greater than the distance travelled by the object.

Ans. (a) False (b) False

4. Distinguish between speed and velocity?

Speed	Velocity
1. The distance travelled by a moving body per unit time is called its speed.	**1.** The distance travelled by moving body in a particular direction per unit time is called its velocity.
2. It is a scalar quantity.	**2.** It is a vector quantity.

5. Under what condition(s) is the magnitude of average velocity of an object equal to its average

Ans. The magnitude of average velocity of an object is equal to its average speed if the object moves in a straight line in a particular direction.

6. What does the odometer of an automobile measure?

Ans. The odometer of an automobile measure the distance travelled by a vehicle,

7. What does the path of an object look like when it is in uniform motion?

Ans. It is a straight line.

8. During an experiment, a signal from a spaceship reached the ground station in five minute?

What was the distance of the spaceship from the ground station? The signal travels at the speed of light, that is $3\times10^8 ms^{-1}$?

Ans. Time taken = 5 minutes = 5 x 60 s = 300 s Speed of signal $u=3\times10^8 ms^{-1}$ Distance =?

$$speed=\frac{distance}{time}\therefore distance = speed\times time$$

$$\therefore distance=3\times10^8\times300=9\times10^{10}m$$

9. When will you say a body is in

(i) uniform acceleration?

(ii) non-uniform acceleration

Ans. (i) Uniform acceleration: When a body travels with the same velocity in the given time, then the acceleration is said to be uniform.

(ii) Non-uniform acceleration: When a body moves with unequal velocity in the equal interval of time, the body is said to be moving with non-uniform acceleration.

10. A bus decreases its speed from 80 km h^{-1} to 60 km h^{-1} in 5 s. Find the acceleration of the bus.

Ans. Initial velocity u = 80 km h^{-1}

$$=\frac{80\times1000}{60\times60}=ms^{-1}=22.22ms^{-1}$$

Final velocity $u=60kmhr^{-1}$

$$=\frac{60\times1000}{60\times60}=16.66ms^{-1}$$

$\therefore$ The acceleration of bus is -1.11 m s^{-2}

11. A racing car has a uniform acceleration of 4 m s*2. What distance will it cover in 10 s after start?

Ans. $a=4ms^{-2}$

$t=10s$

$s=?$

$u=0$

$\therefore$

$$s=ut+\frac{1}{2}at^2=0\times10+\frac{1}{2}\times4\times(10)^2=0+\frac{1}{2}\times4\times100$$

$s=200m$

The distance covered in 10 s by the car is 200 m.

12. A stone is thrown in a vertically upward direction with a velocity of 5 m s^{-1}. If the acceleration of the stone during its motion is 10 m s^{-2} in the downward direction, what will be the height attained by the stone and how much time will it take to reach there?

Ans. $u=5ms^{-1}$

$\upsilon=0$

$a=-10ms^{-2}$

$s=?$

$t=?$

$\upsilon=u+at$

(i) $0=5+(-10)t$

$-5=-10t$

$\upsilon^2-u^2=2as$

(ii) $(0)2-(5)^2=2(-10)\times s$

$-25=-20\times s$

$\therefore t=\frac{5}{10}=0.5\sec$ $\qquad s=\frac{25}{20}=1.25m$

$t=0.5s$ $\qquad s=1.25m$

13. An athelete completes one round of a circular track of diameter 200 m in 40 s. What will be the distance covered and the displacement at the end of 2 minutes 20 s?

Ans. Diameter $=200m,\ r=\frac{d}{2}=100m$

Time for one round $=40s$

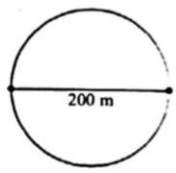
Distance travelled in 2 minutes and 20 s.

$=2\times60+20=140s$

$\therefore \frac{140}{40}=3.5$ rounds

Distance travelled = Circumference of the circle $\times3.5=2\pi r\times3.5$

$$=2\times\frac{22}{7}\times100\times3.5=2200m$$



(ii) Displacement after 3.5 revolution

= diameter of the track

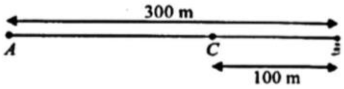
= 200 m.

14. Joseph jogs from one end A to the other end *B of a* straight 300 m road in 2 minutes 30 seconds and then turns around and jogs 100 m back to point C in another 1 minute. What are Joseph's average speeds and velocities in jogging, (a) from A to B and (b) from A to C?

Ans. (a) From A to B.

Time for A to B = 2 min 30

$=2\times 60+30=150s$



Average speed

$$=\frac{total\ distance}{time\ interval}=\frac{300}{150}=2ms^{-1}$$

Average velocity

$$=\frac{displacement}{time\ interval}=\frac{300}{150}=2ms^{-1}$$

(b) From A to C.

Time taken = A to B + B to C, total distance = 300 + 100 = 400

150 + 60 = 210 sec

$\therefore$ Average speed

$$=\frac{total\ distnace}{time\ interval}=\frac{400}{210}=1.9ms^{-1}$$

$\therefore$ Average velocity

$$=\frac{displacement}{time\ interval}=\frac{200}{210}=0.95ms^{-1}$$

15. Abdul, while driving to school, computes the average speed for his trip to be 20 km h^{-1}. On his return trip along the same route, there is less traffic and the average speed is 40 km h^{-1}. what is the average speed for Abdul's trip?

Ans. Let the school be at a distance of *x* km. If t_1 is time taken to reach the school, then $t_1=\frac{distance}{average\ speed}=\frac{x}{20}$ If t_2 is time taken to reach back, then $t_2=\frac{distance}{average\ speed}=\frac{x}{30}$

Total time,

$$t=t_1+t_2=\frac{x}{20}+\frac{x}{40}=x\left[\frac{1}{20}+\frac{1}{30}\right]=\frac{5x}{60}=\frac{x}{12}$$

Total time, $x+x=2x$

Average speed

$$=\frac{total\ distance}{total\ time}=\frac{2x}{x/12}=24kmh^{-1}$$

16. A motorboat starting from rest on a lake accelerates in a straight line at a constant rate of $3.0ms^{-2}$ for 8.0 s. How far does the boat travel during this time?

Ans. u = 0 $u=0$ $a=3.0ms^{-2}$

$$s=ut+\frac{1}{2}at^2=0\times t+\frac{1}{2}(3)(8)^2$$

$$s=\frac{1}{2}\times 3\times 64=96m$$

$\therefore$ Boat travelled a distance of 96 m.

17. A driver of a car travelling at 52 km h^{-1} applies the brakes and accelerates uniformly in the opposite direction. The car stops in 5 s. Another driver going at 3 km h^{-1} in another car applies his brakes slowly and stops in 10 s. On the same graph paper plot the speed versus time graphs for the two cars. Which of the two cars travelled farther after the brakes were applied?

Ans. The data given in this numerical problem are in different units. So, we should first convert km h^{-1} unit into m s^{-1} unit.

For first car:

Initial velocity

$$u=52kmh^{-1}=\frac{52km}{1h}=\frac{52\times 1000m}{1\times 3600s}=14.4ms^{-1}$$

Final velocity, $\upsilon=0kmh^{-1}=0.0ms^{-1}$

Time taken, $t=10s$

For second car:

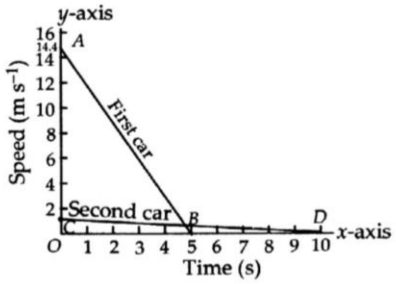
The distance travelled by a moving body is given by the area under its speed-time graph.

So, Distance travelled by the first car = Area of the triangle AOB

$$=\frac{1}{2}\times OB\times AO=\frac{1}{2}\times 144ms^{-1}\times 5s$$

$$=\frac{1}{2}\times 14.4\times 5m=36m$$

Similarly,



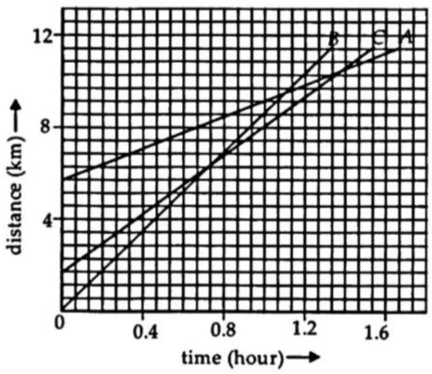
Distance travelled by the second car = Area of triangle COD

$= \frac{1}{2} \times OD \times CO = \frac{1}{2} \times 0.83 m s^{-1} \times 10 s$

$= \frac{1}{2} \times 0.83 \times 10 m = 4.1 m$

Thus, the second car travels 4.1 m and the first car travels 36 m before coming to rest. So, the second car travelled farther after the brakes were applied.

18. Figure given below shows the distance-time graph of three objects A, *B* and C, study the graph and answer the following questions?



(a) Which of the three is travelling the fastest?

(b) Are all three ever at the same point on the road?

(c) How far has C travelled when B passes A?

(d) How far has B travelled by the time it passes C?

Ans. (a) B is travelling fastest.

(b) As three lines do not meet at any point, the three objects never meet on the road.

(c)B passes A at D. At this time, C is at *E,* which corresponds to 7 km. Hence when B crosses A, then C is at 7 km from the origin.

(d) By the time B passes C, it has travelled 4.5 km.

19. A ball is gently dropped from a height of 20 m. If its velocity increases uniformly at the rate of 10 m s'1, with what velocity will it strike the ground? After what time will it strike the ground?

Ans. $s = 20m, u = 0, a = 10 m s^{-2}$

We have, $s = ut + \frac{1}{2}at^2$ $\therefore$

$(20) = 0 \times t + \frac{1}{2}(10)t^2$

$20 = \frac{1}{2} \times 10t^2$

$t = ? \frac{20 \times 2}{10} = t^2$

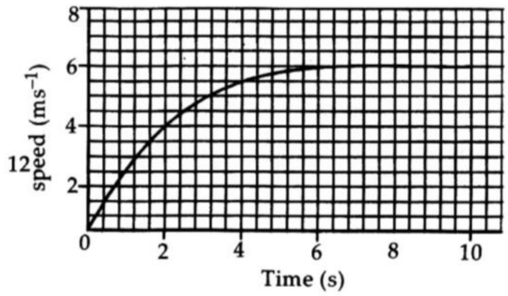
$t^2 = 4 \therefore t = 2s$

$\upsilon = u + at = 0 + 10 \times 2$

$\upsilon = 20 m s^{-1}$

The ball strike the ground after 2 s with the velocity of 20 m s^{-1}.

20. The speed-time graph for a car is shown in the figure.



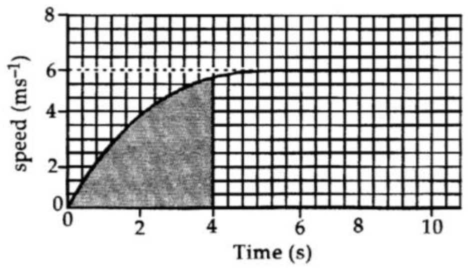
(a) Find how far does the car travel in the first 4 seconds. Shade the area on the graph that represents the distance travelled by the car during the period.

(b) Which part of the graph represents uniform motion of the car?

Ans. The motion during first 4 seconds is not uniformly accelerated. So, distance travelled by car in first 4 seconds is calculated by graphical method.

(a) Area of a small division on the graph $= \frac{2}{15} m s^{-1} \times \frac{2}{25} s = \frac{4}{375} m$ Total number of small divisions under the curve up to4s= 1615

Area under the curve up to $4s = \frac{1615 \times 4}{375} = 17.2 m$



Therefore, the car has covered a distance of 17.2 m in first 4 s.

(b) The limiting flat portion of the curve describes the constant speed of the car, *i.e.,* a speed of 6.7 m s^{-1}. At this stage, the acceleration of the car is zero.

Therefore, portion of the graph beyond 6.2 s describes the uniform motion of the car.

21. State which of the following situations are possible and give an example for each of these :

(a) an object with a constant acceleration but with zero velocity

(b) an object moving in a certain direction with an acceleration in the perpendicular direction

Ans. (a) free fall due to gravity

(b) object moving in a circular path

22. An artificial satellite is moving in a circular orbit of radius 42250 km. Calculate its speed if it takes 24 hours to revolve around the earth.

Ans. Radius of the orbit = 42250 km = 42250 x 1000 m

Time taken for one revolution = 24 hours

= 24 x 60 x 60 sec

Speed =?

$\therefore$

$$Speed = \frac{distance}{time} = \frac{2\pi r}{time} = 2\times\frac{22}{7}\times\frac{42250}{24\times60\times60}$$

$$Speed = 3073.74\,m s^{-1}$$

$$= 3.07\,km s^{-1}.$$

NCERT EXEMPLAR PROBLEMS-SOLUTIONS

Multiple Choice Questions (MCQs)

1. A particle is moving in a circular path of radius r. The displacement after half a circle would be

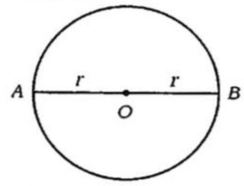
(a) zero (b) π r

(c) 2r (c) 2 π r

Ans. **(c)** Given, after half the circle, the particle will reach the diametrically opposite point i.e., from point A to point B. And we know displacement is shortest path between initial and final point.

$\therefore$ Displacement after half circle =AB=OA+OB

[$\because$ Given, OA and OB = r]



Hence, the displacement after half circle is 2r.

2. A body is thrown vertically upward with velocity u, the greatest height h to which it will rise is

(a) u/g (b) $u^2/2g$

(c) u^2/g (d) $u/2g$

Ans. **(b)** Given, initial velocity = u, height = h and a = g (acceleration due to gravity) At the highest point, final velocity becomes zero i.e., v = 0 From, third equation of motion, $v^2 = u^2 - 2gh$

$$0 = u^2 - 2gh$$

$$2gh = u^2$$

$$\Rightarrow \quad h = \frac{u^2}{2g}$$

Here, we have used negative sign because the body is moving against the gravity.

3. The numerical ratio of displacement and distance for a moving object is

(a) always less than 1

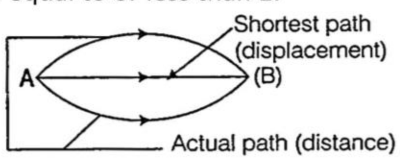
(b) always equal to 1

(c) always more than 1

(d) equal or less than 1

Ans. **(d)** Displacement of an object can be less than or equal to the distance covered by the object, because the magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without any change in direction.

So, the ratio of displacement to distance is always equal to or less than 1.



4. If the displacement of an object is proportional to square of time, then the object moves with
(a) uniform velocity
(b) uniform acceleration
(c) increasing acceleration
(d) decreasing acceleration

Ans. **(b)** From second equation of motion, $s = ut + \frac{1}{2}at^2$ If object starts from rest i.e., initial velocity (u) = 0 and aconite an acceleration (a) in time (t)

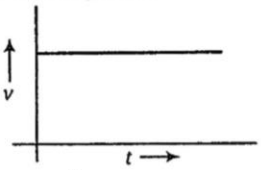
Then, $s = 0 \times t + \frac{1}{2}at^2$

$s = \frac{1}{2}at^2$

$s \propto t^2$, if a = constant

So, the object moves with constant or uniform acceleration.

5. From the given v-t graph (see figure), it can be inferred that the object is

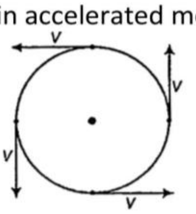


(a) in uniform motion
(b) at rest
(c) in non-uniform motion
(d) moving with uniform acceleration

Ans. **(a)** From the given v-t graph, it is clear that the velocity of the object is not changing with time i.e., the object is in uniform motion.

6. Suppose a boy is enjoying a ride on a merry-go-round which is moving with a constant speed of $10 ms^{-1}$.It implies that the boy is
(a) at rest
(b) moving with no acceleration
(c) in accelerated motion
(d) moving with uniform velocity

Ans. **(c)** In merry-go-round, the speed is constant but velocity is not constant, because its direction goes on changing i.e., there is acceleration in the motion. So, we can say that the boy is in accelerated motion.



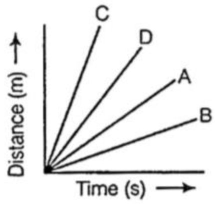
7. Area under v-t graph represents a physical quantity which has the unit
(a) m^2 (b) m
(c) m^3 (d) ms^{-1}

Ans. **(b)** Area under v-t graph represent displacement whose unit is metre or (m). Because, unit of velocity v = m/s and unit of time (T) = s.

$\therefore$ Unit of (v-t) graph $= \frac{m}{s} \times s = m$.

Hence, the unit of (v-t) graph is metre (m).

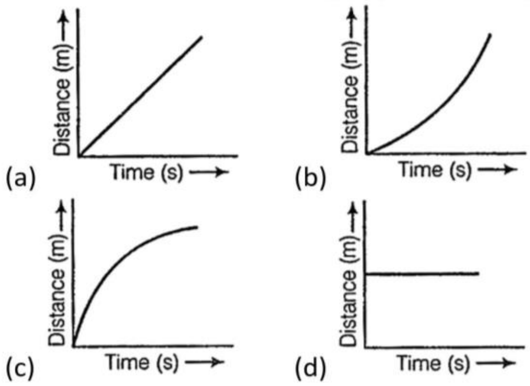
8. Four cars A, B, C and D are moving on a levelled road.
Their distance versus time graphs are shown in figure. Choose the correct statement.



(a) Car A is faster than car D
(b) Car B is the slowest
(c) Car D is faster than car C
(d) Car C is the slowest

Ans. **(b)** The slope of distance-time graph represents the speed. From the graph, it is clear that the slope of distance-time graph for car B is less than all other cars. So, the slope is minimum for car 6. Hence, car 6 is the slowest.

9. Which of the following figures represents uniform motion of a moving object correctly?



Ans. **(a)** For uniform motion, the distance-time graph is a straight line (because in uniform motion object covers equal distance in equal interval of time).

10. Slope of a velocity-time graph gives
(a) the distance (b) the displacement
(c) the acceleration (d) the speed

Ans. (c) Slope of velocity-time graph gives acceleration.

Because slope of the curve $= \frac{v}{t}$, where $\frac{v}{t} =$ acceleration.

11. In which of the following cases of motions, the distance moved and the magnitude of displacement are equal?
(a) If the car is moving on straight road
(b) If the car is moving in circular path
(c) The pendulum is moving to and fro
(d) The earth is revolving around the sun

Ans. (a) The distance moved and magnitude of displacement are equal only in the case of motion along a straight line. Because displacement is the shortest path between initial and find path.
So, for car moving on straight road, distance moved and magnitude of displacement are equal.

Short Answer Type Questions

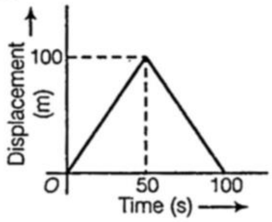
12. The displacement of a moving object in a given interval of a time is zero. Would the distance travelled by the object also be zero? Justify your answer?

Ans. The displacement of a moving object in a given interval is zero i.e., the object comes back to its initial position in the given time (displacement is the shortest distance between the initial and final position of an object).
The distance in this case will not be zero because distance is the total length of the path travelled by the body. If the object comes back to its initial position, then length of path travelled is not zero.

13. How will the equations of motion for an object moving with a uniform velocity change?

Ans. We know that, the equations of uniformly accelerated motion are
(i) $v = u + at$
(ii) $s = ut + \frac{1}{2}at^2$
(iii) $v^2 = u^2 + 2as$
where,
u = Initial velocity
v = Final velocity
a = Acceleration
t = Time and
s = Distance
For an object moving with uniform velocity (velocity which is not changing with time), then acceleration a = 0.
So, equations of motion will become (putting a = 0 in above equations)
(i) $v = u$
(ii) $s = ut$
(iii) $v^2 = u^2$

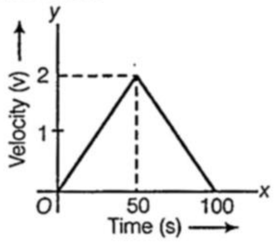
14. A girl walks along a straight path to drop a letter in the letter box and comes back to her initial position. Her displacement-time graph is shown in figure. Plot a velocity-time graph for the same.



Ans. From the graph,
(i) Initial velocity, u = 0
[Since, displacement and time is zero]
(ii) Velocity after 50s, $v = \frac{\text{Displacement}}{\text{Time}}$
[∵ Given, displacement = 100m]
$\Rightarrow \quad v = \frac{100}{50} = 2ms^{-1}$
(iii) Velocity after $100s, v = \frac{\text{Displacement}}{\text{Time}}$
[Here, displacement = zero and time = 100s]
$\Rightarrow v = \frac{0}{100} = 0$
Therefore,

v	0	2	0
t	0	50	100

Velocity-time graph plotted from the above data is shown below



15. A car starts from rest and moves along the X-axis with constant acceleration $5ms^{-2}$ for 8 s. If it then continues with constant velocity. What distance will the car cover in 12 s, since it started from rest?

Ans. Given, the car starts from rest, so its initial velocity u = 0
Acceleration, (a) $= 5ms^{-2}$ and time (t) = 8 s
From first equation of motion, v = u + at on putting a $a = 5ms^{-2}$ and t = 8 s in above equation, we get $v = 0 + 5 \times 8 = 40ms^{-1}$
So, final velocity v is $40ms^{-1}$

Again, from second equation of motion, $s = ut + \frac{1}{2}at^2$ on putting t = 8 s and a = 5 ms"2 in above equation, we get

$s = 0 \times 8 + \frac{1}{2} \times 5 \times (8)^2 = \frac{1}{2} \times 5 \times 64 = 5 \times 32 = 160m$

So, the distance covered in 8 s is 160 m.
Given, total time t = 12 s.
After 8 s, the car continues with constant velocity i.e., the car will move with a velocity of $40 ms^{-1}$.
So, remaining time t'= 12 s - 8 s = 4 s
The distance covered in the last 4s (s') = Velocity x Time [$\because$ Distance = Velocity x Time]
= 40 x 4 = 160 m [We have used the direct formula because after 8 s, car is moving with constant velocity i.e., zero acceleration].
$\therefore$ Total distance travelled in 12 s from the start D = s + s' = 160 + 160 = 320m

16. A motorcyclist drives from A to B with a uniform speed of $30 kmh^{-1}$ and returns back with a speed of $20 kmh^{-1}$, Find its average speed.

Thinking Process

To find the t_1 and t_2 by using formula, time $= \frac{\text{distance}}{\text{speed}}$ and then put this values in formula, average speed $= \frac{\text{total distance}}{\text{total time}}$ and further simplify it, to get the required average speed.

Ans. Let the distance between A and 6 be -x- km.
Time taken in driving from A to B

$t_1 = \frac{\text{Distance}}{\text{Speed}} = \frac{x}{30} h \quad \left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$

Similarly, time taken in returning from 6 to A.

$t_2 = \frac{\text{Distance}}{\text{Speed}} = \frac{x}{20} h$

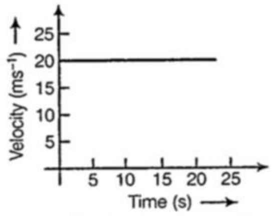
Average speed $= \frac{\textit{Total distnace}}{\textit{Total time}} = \frac{x + x}{t_1 + t_2}$

$x \frac{x + x}{\frac{x}{30} + \frac{x}{20}} = \frac{2x}{\frac{2x + 3x}{60}}$

$\frac{2x \times 60}{5x} = 2 \times 12 = 24 \text{kmh}^{-1}$

Hence, average speed of a motorcyclist is 24kmh^{-1}

17. The velocity-time graph (see figure) shows the motion of a cyclist. Find (i) its acceleration (ii) its velocity and (iii) the distance covered by the cyclist in 15 s.



Ans. (i) From the graph, it is clear that velocity is not changing with time i.e., acceleration is zero. $(as\, a = \frac{dv}{dt} + dv = 0)$

(ii) Again from the graph, we can see that there is no change in the velocity with time, so velocity after 15 s will remain same as 20ms^{-1}

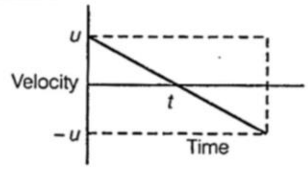
(iii) Distance covered in 15s= Velocity x Time
=20 x 15= 300m $\left[\because \text{Time} = \frac{\text{Distnace}}{\text{Speed}}\right]$

18. Draw a velocity versus time graph of a stone thrown vertically upwards and then coming downwards after attaining the maximum height.

Ans. When a stone is thrown vertically upwards, it has some initial velocity (let u). As the stone goes its velocity goes on decreasing ($\because$ it is moving against the gravity) and at the highest point i.e., maximum height) its velocity become zero. Let the stone takes time 'f second to reach at the highest point.
After that stone begins to fall (with zero initial velocity) and its velocity goes on increasing (since it is moving with the gravity) and it reaches its initial point of projection with the velocity v in the same time (with which it was thrown), So,

Velocity	u	0	-u
Time	0	t	2t

Here, we have taken -u because in the upward motion velocity of stone is in upward direction and in the downward motion, the velocity is in downward direction.
The velocity-time graph for the whole journey is shown below



Long Answer Type Questions

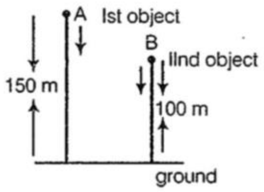
19. An object is dropped from rest at a height of 150 m and simultaneously another object is dropped from rest at height 100 m. What is the difference in their heights after 2 s, if both the objects drop with same accelerations? How does the difference in heights vary with time?

Thinking Process

Use second equation of motion i.e., h = ut $+\frac{1}{2}at^2$ and then find the difference between h_1 and h_2

Ans. For first object given, u = 0 (because object dropped from rest) and time (t) = 2s.

From second equation of motion, the distance covered by first object in 2s is



$h = ut + \frac{1}{2}gt^2 \quad h = 0 \times 2 + \frac{1}{2} \times 10 \times (2)^2$

$[\because g = 10 m/s^2]$

$h = 0 + \frac{1}{2} \times 10 \times 4 = 20m$

Height of first object from the ground after 2 s (h_1) = 150m -20m = 130m

For second object given, u=0 and time (t) = 2 s

From second equation of motion, the distance covered by second object in 2 s is

$h = ut + \frac{1}{2}gt^2 = 0 \times 2 + \frac{1}{2} + 10 \times (2)^2$

$[\because g = 10m/s^2]$

$= 0 + \frac{1}{2} \times 10 \times 4 = 20m$

Height of second object from the ground after 2s then h_2 = 100m - 20m = 80m

Now, difference in the height after 2 $s = h_1 - h_2$ = 130 - 80= 50 m

The difference in heights of the objects will remain same with time- as both the objects have been dropped from rest and are falling with same acceleration i.e., (acceleration due to gravity).

20. An object starting from rest travels 20 m in first 2 s and 160 m in next 4 s. What will be the velocity after 7s from the start?

Thinking Process

Firstly we find the acceleration by using the second equation of motion i.e., $h = ut + \frac{1}{2}at^2$ and then put this value in Newton's first equation of motion i.e., v = u + at to get required final velocity.

Ans. Given, object starts from rest, u = 0, t = 2s and s = 20 m From second equation of motion, $s = ut + \frac{1}{2}at^2$ On putting u = 0 in above equation, $20 = 0 \times 2 + \frac{1}{2} \times a(2)^2 = 0 + \frac{1}{2} \times a \times 4$

$20 = 2a \Rightarrow a = \frac{20}{2} \Rightarrow a = 10m/s^2$ Now, from first equation of motion, velocity after 7 s from the start v = u + at $[put\, a = 10m/s^2]$

= 0 + 10 x 7 = 70 m/s

21. Using following data, draw time-displacement graph for a moving object.

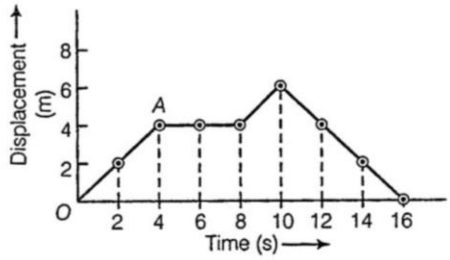
Time (s)	0	2	4	6	8	10	12	14	16
Displacement (m)	0	2	4	4	4	6	4	2	0

Use this graph to find average velocity for first 4 s, for next 4 s and for last 6 s.

Thinking Process

Firstly, consider the time is on x -axis and displacement is on y-axis. Now, plot all the points (0,0), (2,2), (4, 4), (6,4), (8,4), (10,6), (12,4), (14,2) and (16,0) on a graph paper and join by line segment.

Ans. Therefore, displacement-time graph is shown in figure.



Average velocity for first

$4s = \frac{\text{Change in displacement}}{\text{Total time taken}}$

$v = \frac{4-0}{4-0} = \frac{4m}{4s} = 1ms^{-1}$

Average velocity for next 4 s (i.e., in the interval of 4 s to 8 s), $v = \frac{4-4}{8-4} = \frac{0}{4} =$

Average velocity for last $6s = \frac{(0-6)m}{(16-10)s} = \left(\frac{-6}{6}\right) = -1ms^{-1}$

22. An electron moving with a velocity of $5\times10^4\ ms^{-1}$ enters into a uniform electric field and acquires a uniform acceleration of $10^4\ ms^{-2}$ in the direction of its initial motion.

(i) Calculate the time in which the electron would acquire a velocity double of its initial velocity.

(ii) How much distance the electron would cover in this time?

Thinking Process

Firstly, we find the time by using Newton's first equation of motion i.e., v =u+ at. And then put this value in Newton's second equation of motion i.e., $s = ut + \frac{1}{2}at^2$ to get required distance.

Ans. Given, initial velocity, $u = 5\times10^4 ms^{-1}$

and acceleration, $a = 10^4 ms^{-2}$

(i) According to the question, final velocity, v = 2u, t = ?

From first equation of motion, v = u + at

$2u = u + 10^4 \times t$ [$\because$ put $v = 2u, a = 10^4 ms^{-2}$]

$2u - u = 10^4 \times t$

$10^4 \times t = u$

$\Rightarrow t = \frac{(u)}{10^4} = \frac{5\times10^4}{10^4} = 5s$ [$\because u = 5\times10^4\ m/s$]

i.e., after 5 s electron will acquire a velocity double of its initial velocity.

(ii) From second equation of motion,

Distance covered in t second, $s = ut + \frac{1}{2}at^2$

$= 5\times10^4\times5 + \frac{1}{2}\times10^4(5)^2$

[$put, u = 5\times10^4 m/s, t = 5s$ and $a = 10^4 m/s^2$]

$= 25\times10^4 + \frac{1}{2}\times10^4\times25$

$= 25\times10^4 + 12.5\times10^4$

$= 10^4(25+12.5)$

$= 37.5\times10^4 m = 37.5\times10\times10^3 m = 375\times10^3$ m

$= 375$km [$\because 1000\text{m} = 1\text{km}$]

23. Obtain a relation for the distance travelled by an object moving with a uniform acceleration in the interval between 4th and 5th second.

Thinking Process

Firstly, we use the Newton's second equation of motion i.e., $s = ut + \frac{1}{2}at^2$ and then find the difference between s_5 and s_4.

Ans. From second equation of motion,

Distance travelled in t see $s = ut + \frac{1}{2}at^2$

Distance travelled in 4 s $s_4 = u\times4 + \frac{1}{2}a(4)^2$

[$\because$ put $t = 4s$]

$= 4u + \frac{1}{2}\times a\times16 = 4u + 8a$ ($S_5 =$ distance travelled in 4th sec) Again, distance travelled in 5 s $s_5 = ut + \frac{1}{2}at^2$ [$\because$ put $t = 5s$]

$= u\times5 + \frac{1}{2}a(5)^2 = 5u + \frac{25}{2}a$ ($S_5 =$ distance travelled in 5th sec) So, distance travelled in the interval between 4th and 5th second.

$s = s_5 - s_4 = \left(5u + \frac{25}{2}a\right) - (4u + 8a)$

$= 5u + \frac{25}{2}a - 4u - 8a$

$= 5u - 4u + \frac{25}{2}a - 8a$

$= u + \frac{25a - 15a}{2} = u + \frac{9}{2}a$

So, the relation will be [u + 9/2a)

24. Two stones are thrown vertically upwards simultaneously with their initial velocities u_1 and u_2 respectively. Prove that the height reached by them would be in the ratio of $u_1^2 : u_2^2$ (assume upward acceleration is – g and downward acceleration to be+ g).

Thinking Process

Firstly, we find the h_1 and h_2 by using Newton's third equation of motion i.e., $v^2 = u^2 + 2gh$ and then find the ratio of h_1 and h_2.

Ans. For 1st stone, given initial velocity $u = u_1$

Let height attained by it be h_1.

From third equation of motion, $v^2 = u^2 - 2gh$ for upward motion [Here, we have used negative sign, because as given upward acceleration is taken to be –q At the highest point the velocity becomes zero i.e., v = 0

$$0 = u_1^2 - 2gh_1$$

$$\Rightarrow \quad 2gh_1 = u_1^2 \Rightarrow h_1 = \frac{u_1^2}{2g}$$

Similarly,

For 2nd stone, given initial velocity, $u = u_2$

Let the height attained by it be h_2.

From third equation of motion, $v^2 = u^2 - 2gh$

[at the highest point the velocity becomes zero i.e., v = 0] $0 = u_2^2 - 2gh_2$

$\therefore \quad 2gh_2 = u_2^2$

$\Rightarrow \quad h_2 = \dfrac{u_2^2}{2g}$

Now, ratio of heights reached by the two stones $h_1 : h_2 = \dfrac{u_1^2}{2g} : \dfrac{u_2^2}{2g}$

$\therefore \quad h_1 : h_2 = u_1^2 : u_2^2$ Hence proved