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PHYSICS

ELECTRIC POTENTIAL

CH:03

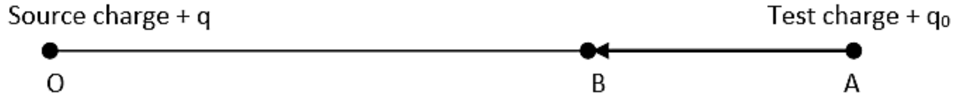
ELECTRIC – POTENTIAL

- Electric potential energy
- Electrostatic potential difference
- Electrostatic potential
- Electric potential due to a point charge
- Electric potential due to group of charges.
- Electric potential at any point due to an electric dipole
- Conservative nature of electric field
- Relation between electric field intensity and electric potential
- Electric potential due to a uniformly charged thin spherical shell.
- Electric potential due to a continuous charge distribution
- Equipotential surfaces
- Properties of equipotential surfaces
- Electric potential energy
- Potential energy in an external field
- Potential energy of dipole in uniform electric field

Concepts of Potential difference and Electric potential:

Potential difference: Consider a point charge + q located at a point O. Let A and B be two points in its electric field. When a test charge q_0 is moved from A to B, a work W_{AB} has to be done in moving against the repulsive force exerted by the charge +q. We then calculate the potential difference between points A and B by the equation:

$$V = V_B - V_A = \frac{W_{AB}}{q_0} \quad \text{... (i)}$$



[To define potential difference]

So, the **potential difference between two points in an electric field may be defined as the amount of work done in moving a unit positive charge from one point to the other against the electrostatic forces.**

► In the above definition, **we have assumed that the test charge is so small that it does not disturb the distribution of the source charge. Secondly, we just apply so much external force on the test charge that it just balances the repulsive electric force on it and hence does not produce any acceleration in it.**

◙ **SI unit of potential difference is Volt (V):** It has been named after the Italian scientist Alessandro Volta.

$$1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

or $\quad 1\ V = 1\ Nm\ C^{-1} = 1\ JC^{-1}$

Hence **the potential difference between two points in an electric field is said to be 1 volt if 1 joule of work has to be done in moving a positive charge of 1 coulomb from one point to the other against the electrostatic forces.**

ELECTRIC POTENTIAL: The electric potential at a point located far away from a charge is taken to be zero. In Fig, if the point A lies at infinity, then $V_A = 0$, so that

$$V = V_B = \frac{W}{q_0}$$

where W is the amount of work done in moving the test charge q_0 from infinity to the point B and V_B refers to the potential at point B.∞

So, the **electric potential at a point in an electric field is the amount of work done in moving a unit positive charge from infinity to that point against the electrostatic forces.**

$$\text{Electric potential} = \frac{\text{Work done}}{\text{Charge}}$$

Thus, **"Electric potential at any point in the electric field is the work done per unit charge in bringing a unit positive charge from infinity to that point along any path".**

◙ **SI unit of electric potential is volt (V):** The *electric potential* at a point in an electric field **is said to be 1 volt if one joule of work has to be done in moving a positive charge of 1 coulomb from infinity to that point against the electrostatic forces.**

◙ **Dimension Formula:** $[M L^2 T^{-3} A^{-1}]$

- Electrostatic potential of a body represents the degree of electrification of the body.
- Electrostatic potential of a body also determines the direction of flow of charge.
- The charge always flows from a body at higher potential to another body at lower potential. (The flow of charge stops as soon as the potential of the two bodies become equal). Thus,

Electrostatic force is conservative in nature as it obeys inverse square law. And, therefore, work done by electrostatic force between two points is same for all the paths between the points.

- Work done by electrostatic force along a closed path is ZERO.

Cgs unit: **'Stat volt'**; Since , V = W / q , therefore, 1 Stat volt = 1 erg / 1stat coulomb
thus, **"Electric potential at a point is said to be '1 Stat volt', when '1 erg' of work is done in moving 1 Stat coulomb charge from infinity to that point".**

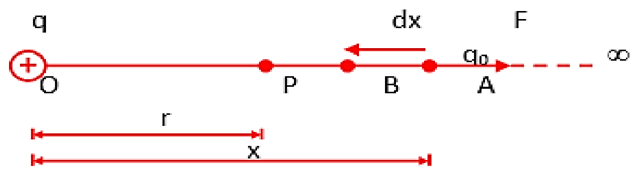
Relationship Between 'Volt' and 'Stat volt':

$$1\ \text{Volt} = \frac{1\ \text{Joule}}{1\ \text{coulomb}} = \frac{10^7\ \text{erg}}{3\times10^9\ \text{esu}} = \frac{1\ \text{erg}}{300\ \text{esu}} = \frac{1}{300}\ \text{stat volt}$$

- Electric potential is ***Characteristic property*** of the field. It does not matter, whether a charge is placed in the electric field or not.
- **Electric potential and potential difference are scalar quantities.**
- The electric potential due to an isolated positive charge is positive and due to isolate negative charge **is negative**.

ELECTRIC POTENTIAL DUE TO A POINT CHARGE

Consider a positive point charge q placed at the origin O. We wish to calculate its electric potential at a point P at distance r from it, as shown in Fig. By definition, the electric potential at point P will be equal to the amount of work done in bringing a unit positive charge from infinity to the point P.



Suppose a test charge q_0 is placed at point A at distance x from O. By Coulomb's law, the electrostatic force acting on charge q_0 is

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{x^2}$$

The force F acts away from the charge q. The small work done in moving the test charge q_0 from A to B through small displacement dx against the electrostatic force is

$$dW = \vec{F}\cdot\vec{dx} = F dx \cos 180° = -F dx$$

The total work done in moving the charge q_0 form infinity to the point P will be

$$W = \int dW = -\int_{\infty}^{r} F dx = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{x^2}dx$$

$$= -\frac{qq_0}{4\pi\varepsilon_0}\int_{\infty}^{r} x^{-2}dx = -\frac{qq_0}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{r}$$

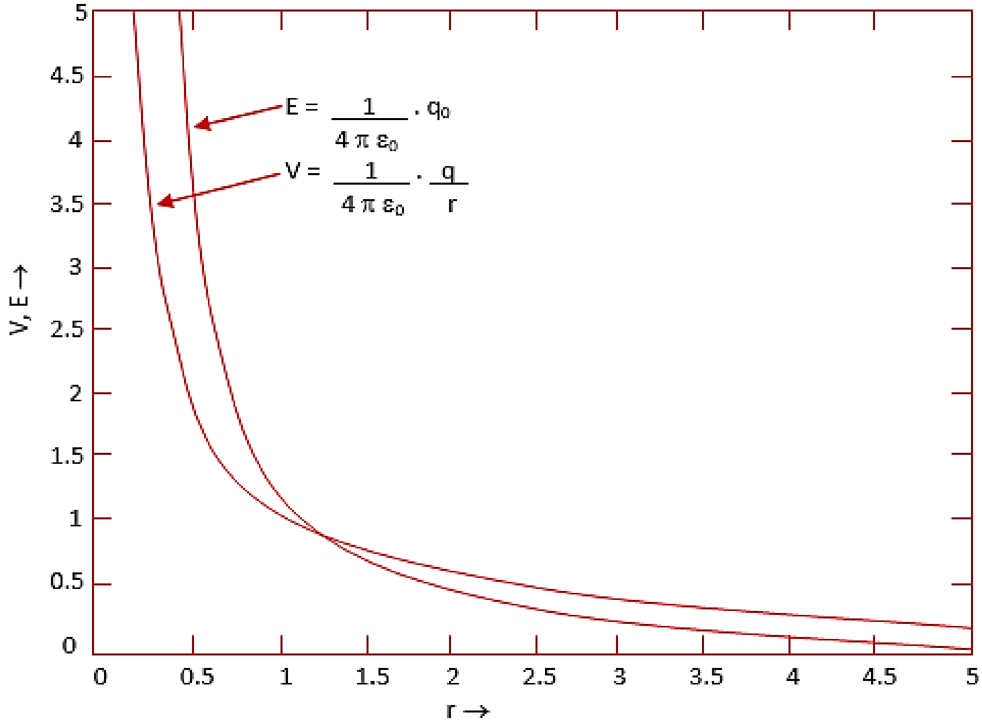
$$= \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{r}$$

Hence the work done in moving a unit test charge from infinity to the point P, or the electric potential at point P is

$$V = \frac{W}{q_0} \quad \text{or} \quad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} \text{-------------------------[a]}$$

Clearly, $V \propto 1/r$. Thus the electric potential due to a point charge is spherically symmetric as it depends only on the distance of the observation point from the charge and not on the direction of that point with respect to the point charge. Moreover, we not that the potential at infinity is zero.

Fig. shows the variation of electrostatic potential ($V \propto 1/r$) and the electrostatic field ($E \propto 1/r^2$) with distance r from a charge q.



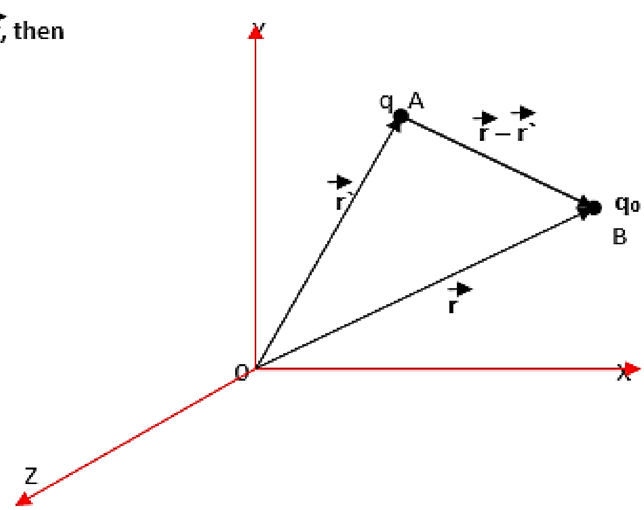
== From [a] , It is clear that at equal distance from a point charge , say 'r' , the value of V is the same .Hence electric potential due to single charge is **spherical symmetry.**

== **From [a],** It is clear that , when q is positive , V is also positive and when q is negative , V is also negative.

●● **Due to single charge ,** $F \ \alpha \ \frac{1}{r^2}$; $E \ \alpha \ \frac{1}{r^2}$ **But** $V \ \alpha \ \frac{1}{r}$

●● Although 'V' is called the potential at a point but actually V is equal to the potential difference between points **'r' & '∞'.**

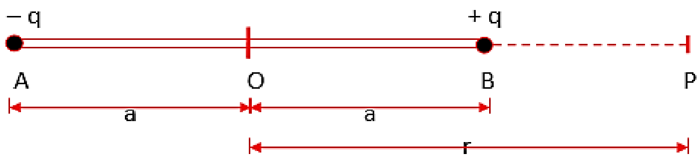
●●If the source charge located at a point, whose position vector is $\vec{r}$**, then**



$$V(r) = \frac{q}{4\pi\varepsilon_0} \frac{1}{|r - r`|}$$

ELECTRIC POTENTIAL DUE TO A DIPOLE

◙ **Electric potential at an axial point of a dipole**: Consider an electric dipole consisting of two-point charges – q and + q and separated by distance 2a. Let P be a point on the axis of the dipole at a distance r from its centre O.



[Potential at an axial point of a dipole]

Electric potential at point P due to the dipole is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{AP} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{BP}$$

$$= -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r+a} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r-a}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r-a} - \frac{1}{r+a}\right)$$

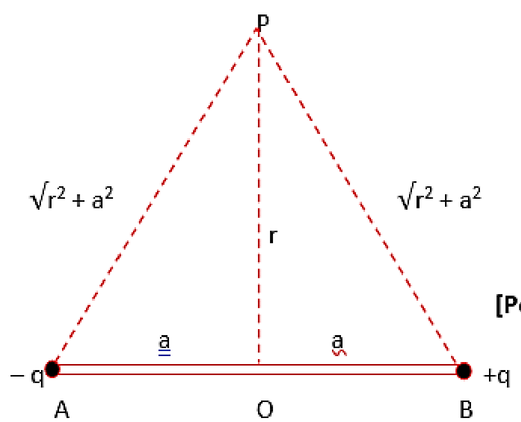
$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{(r+a)-(r-a)}{r^2-a^2}\right) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times 2a}{r^2-a^2}$$

or $$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^2-a^2} \quad [\because p = q \times 2a]$$

For a short dipole, $a^2 << r^2$, so

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^2}$$

◙ **Electric potential at an Equatorial point of a dipole**: consider an electric dipole consisting of charges – q and + q and separated by distance 2a. Let P be a point on the perpendicular bisector of the dipole at distance r from its centre O.



[Potential at an equatorial point of a dipole]

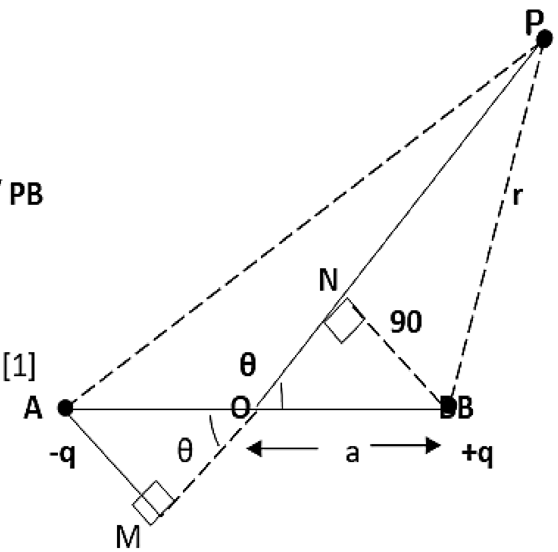
$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{AP} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{BP}$$

$$= -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{r^2+a^2}} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{r^2+a^2}} = 0$$

▶ ***Thus, the potential at a point on the equatorial line of an electric dipole is zero.***

ELECTRIC POTENTIAL At any point due to an ELECTRIC DIPOLE:

Consider an electric dipole AB having charge +q (at B) and –q (at A) separated by a small distance AB = **2a**, with the centre O. Let P be any point at a distance 'r' from the centre O (where, **Electric field due to dipole is to be determined.** Let ∟**POB = θ**



The potential at a point **'P'** due to charge –q (at A), $V_1 = -kq / PA$

IIy, The potential at a point **'P'** due to charge +q (at B), $V_2 = +kq / PB$

Net Potential at P due to dipole, $V = V_1 + V_2$

$$= -kq/PA + kq/PB$$

$$= kq\left\{\frac{1}{PB} - \frac{1}{PA}\right\} \text{ ----------- [1]}$$

Now, Draw BN and AM perpendiculars to OP and OP produced

In rt. ∟^{d} Δ AMO , Cos θ = OM / OA => Cos θ = OM / a

∴ OM = a Cos θ

also, PA ≃ PM {since dipole is small as compared to 'r'}

= PO + OM = r + a Cos θ

Again, In rt. ∟^{d} Δ BNO , Cos θ = ON / OB => ON = OB Cos θ

∴ ON = a Cos θ

PB ≃ PN {since dipole is small as compared to 'r'}

= PO -- ON = r -- a Cos θ

From [1],

$$V = kq\left\{\frac{1}{r - a\cos\theta} - \frac{1}{r + a\cos\theta}\right\} = kq\left[\frac{\cancel{r} + a\cos\theta - \cancel{r} + a\cos\theta}{(r - a\cos\theta)(r + a\cos\theta)}\right]$$

$$= kq\,\frac{2a\cos\theta}{r^2 - a^2\cos^2\theta} = \frac{K\,(q \times 2a)\cos\theta}{r^2 - a^2\cos^2\theta}$$

{Since |p| = q × 2a ; dipole moment}

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\,\frac{p\cos\theta}{r^2 - a^2\cos^2\theta}}$$

special cases : [I] When 'P' lies on the axial line of the dipole i.e, θ = 0

then $V_{AXIAL} = k\,\dfrac{p \times \cos 0}{r^2 - a^2\cos^2 0} = k\,\dfrac{p}{r^2 - a^2}$ [since cos0 = 1]

[II] When 'P' lies on the equatorial line of the dipole i.e, θ = 90 , then

$V_{EQUATORIAL} = k\,\dfrac{p \times \cos 90}{r^2 - a^2\cos^2 90} = 0$ [since cos 90 = 0]

☞ **If r >> a , then**

$$\boxed{V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}}$$

☞ ***Potential due to a point charge , $V \propto 1/r$***
☞ ***Potential due to a point charge , $V \propto 1/r^2$***

Differences between electric potentials of a dipole and a single charge:

1. The potential due to a dipole depends not only on distance r but also on the angle between the position vector r of the observation point and the dipole moment vector p. The potential due to a single charge depends only on r.

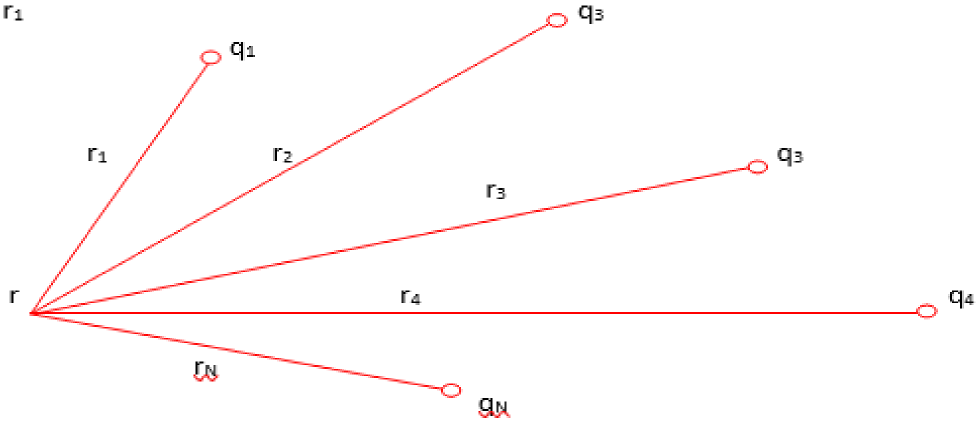
2. The potential due to a dipole is cylindrically symmetric about the dipole axis. If we rotate the observation point P about the dipole axis (keeping r and θ fixed), the potential V does not change. The potential due to a single charge is spherically symmetric.

3. At large distance, the dipole potential falls off as $1/r^2$ while the potential due to a single charge falls off as $1/r$.

ELECTRIC POTENTIAL DUE TO A SYSTEM OF CHARGES

suppose N point charges $q_1, q_2, q_3, \ldots, q_N$ lie at distances $r_1, r_2, r_3, \ldots r_N$ from a point P.
Electric potential at point P due to charge q_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1}$$



Similarly, electric potentials at point P due to other charges will be

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2},\ V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3},\ \ldots,\ V_N = \frac{1}{4\pi\varepsilon_0}\frac{q_N}{r_N}$$

As electric potential is a scalar quantity, so the total potential at point P will be equal to the algebraic sum of all the individual potentials, i.e.,

$$V = V_1 + V_2 + V_3 + \ldots + V_N$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} \ldots + \frac{q_N}{r_N}\right)$$

or

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i}$$

If $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots \vec{r}_N$ are the position vectors of the N point charges, the electric potential at a point whose position vector is $\vec{r}$, would be

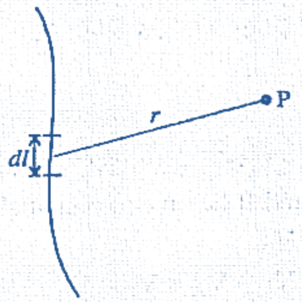
$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{|\vec{r} - \vec{r}_i|}$$

POTENTIAL due to CONTINUOS CHARGE DISTRIBUTION:

***(i)* Potential due to linear charge distribution :** Consider a long line charge L having linear charge density λ. Let dl be the small element of the line charge L.

Charge on elementary length dl

$$dq = \lambda dl$$



Linear Charge Distribution

Fig. 24*(a)*

Electric potential at point P at distance r from small element.

$$dV_l = \frac{1}{4\pi\epsilon_0}\frac{dq}{r}$$

If the charge is distributed uniformly along the line of length l, then potential due to the linear charge distribution at point P is

$$V_l = \frac{1}{4\pi\epsilon_0}\int_l \frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\int_l \frac{\lambda dl}{r} \quad ...(1)$$

***(ii)* Potential due to surface charge distribution :** Consider a surface (S) having surface charge density σ. Let dS be the small area element. Then charge on elementary length

$$dq = \sigma dS$$

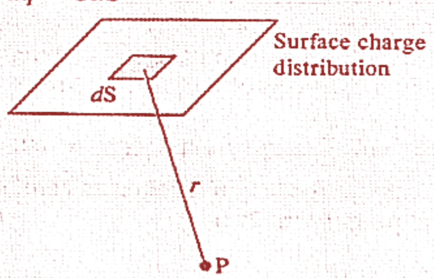


Fig. 24*(b)*

The electric potential at point P at distance r from small element.

$$dV_S = \frac{1}{4\pi\epsilon_0}\frac{dq}{r}$$

If the charge is distributed uniformly over the surface of area S, then potential at point P due to this surface charge distribution is

$$V_S = \frac{1}{4\pi\epsilon_0}\int_S \frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\int_S \frac{\sigma dS}{r} \quad ...(2)$$

***(iii)* Potential due to volume charge distribution :** Consider the volume V having volume charge density ρ, then charge on the small element.

$$dv = dq = \rho dv$$

Electric potential at point P at distance r from the small element.

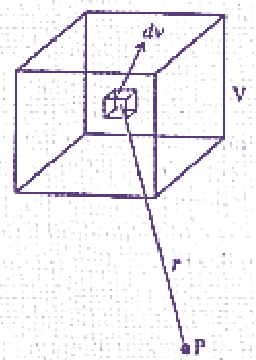


Fig. 24*(c)*

$$dV_v = \frac{1}{4\pi\epsilon_0}\frac{dq}{r}$$

If the charge is distributed uniformly over volume V then potential due to this volume charge at point P is

$$V_v = \frac{1}{4\pi\epsilon_0}\int_V \frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho dV}{r} \quad ...(3)$$

- If all the three types of charge distributions are present simultaneously then total potential at P.

$$V = V_l + V_s + V_v$$

$$= \frac{1}{4\pi\,\epsilon_0}\int_l \frac{\lambda dl}{r} + \frac{1}{4\pi\,\epsilon_0}\int_s \frac{\sigma dS}{r} + \frac{1}{4\pi\,\epsilon_0}\int_v \frac{\rho dV}{r}$$

$$V = \frac{1}{4\pi\epsilon_0}\left[\int_l \frac{\lambda dl}{r} + \int_s \frac{\sigma dS}{r} + \int_v \frac{\rho dV}{r}\right] \quad ...(4)$$

ELECTRIC POTENTIAL DUE TO A UNIFORMLY CHARGED THIN SPHERICAL SHELL

Consider a uniformly charged thin spherical shell of radius R carrying charge q. Wish to calculate the electric potential at any point P at a distance r from the centre (O) of shell.

***(i)* When point P lies outside the spherical shell :** Let the point P lies at a distance r from the centre of shell and lies outside the shell. We know that for a uniformly charged spherical shell the electric field outside shell is such that the entire charge is concentrated at the centre of shell and is given by

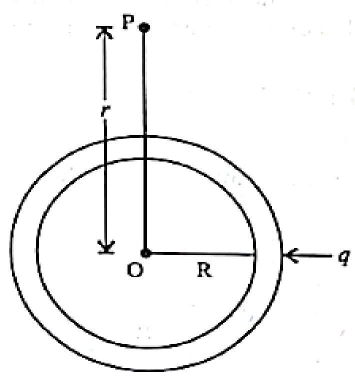


Fig. 22

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$E = -\frac{dV}{dr} \quad \therefore dV = -E\,dr = -\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}dr \qquad ...(1)$$

Integrating equation (1) both sides

$$\int dV = \int -\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}dr$$

$$V = -\frac{q}{4\pi\epsilon_0}\int r^{-2}dr$$

$$= -\frac{q}{4\pi\epsilon_0}\left(\frac{r^{-1}}{-1}\right) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r}\right)$$

$$\therefore \quad V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad ...(2)$$

So, the electric potential at any point outside the charged spherical shell is such as if whole the charge is concentrated at centre of the spherical shell.

***(ii)* When point P lies on surface of spherical shell :** When the point lies on the surface of shell then $r = R$. Hence from equation (2)

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{R} \qquad ...(3)$$

***(iii)* When point P lies inside the spherical shell :** Electric field at any point on the surface of shell is zero. Hence electric potential due to a uniformly charged spherical shell is constant everywhere inside the spherical shell.

$$E = -\frac{dV}{dr} \quad \therefore dV = -E\,dr$$

as $E = 0,\ dV = 0$

$\therefore$ V = constant = electric potential on the surface of spherical shell

$$\therefore \quad V = \frac{1}{4\pi\epsilon_0}\frac{q}{R} \qquad ...(4)$$

The variation of the electric potential due to uniformly charged spherical shell with distance measured from the centre of shell. Note that V = constant $\left(\frac{q}{4\pi\epsilon_0 R}\right)$ from $r = 0$ to $r = R$ along a horizontal line and thereafter $V \propto \frac{1}{r}$ for points outside the shell.

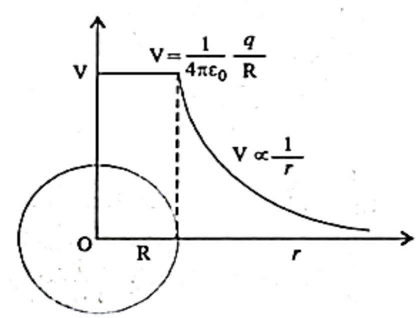


Fig. 23

CONCEPTS...

☑ Electric potential is a scalar quantity while potential gradient is a vector quantity.

☑ The electric potential near an isolated positive charge is positive because work has to be done by an external agent to push a positive charge in, from infinity.

☑ The electric potential near an isolated negative charge is negative because the positive test charge is attracted by the negative charge.

☑ The electric potential due to a charge q at its own location is not defined – it is infinite.

☑ Because of arbitrary choice of the reference point, the electric potential at a point is arbitrary to within an additive constant, but it is immaterial because it is the potential difference between two points which is physically significant.

☑ For defining electric potential at any point, generally a point far away from the source charges is taken as the reference point. Such a point is assumed to be at infinity.

☑ As the electrostatic force is a conservative force, so the work done in moving a unit positive charge from one point to another or the potential difference between two points does not depend on the path along which the test charge is moved.

Examples based on Quantization of Electric Charge

❖ **Formulae Used**

1. Potential difference = $\frac{\text{Work done}}{\text{Charge}}$ or $V = \frac{W}{q}$

2. Electric potential due to a point charge q at distance r form it,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

3. Electric potential at a point due to N point charges,

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1} \frac{q_i}{r_i}$$

4. Electric potential at a point due to a dipole,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}\cdot\vec{r}}{r^3}$$

❖ **Units Used**

Charge q is in coulomb, distance r in metre, work done W in joule and potential difference V in volt.

Q. 1. ***If 100 J of work has to be done in moving an electric charge of 4C from a place, where potential is – 10 V to another place, where potential is V volt, find the value of V.***

Sol. Here W_{AB} = 100 J, q_0 = 4C, V_A = – 10 V, V_B = V

As $V_B - V_A = \frac{W_{AB}}{q_0}$ $\quad V - (-10) = \frac{100}{4} = 25$

or V = 25 – 10 = 15 V

Q. 2. ***Determine the electric potential at the surface of a gold nucleus. The radius is 6.6 × 10^{-15} m and the atomic number Z = 79. Given charge on a proton = 1.6 × 10^{-19} C.***

Sol. As nucleus is spherical, it behaves like a point charge for external points.

Here $q = ne = 79 \times 1.6 \times 10^{-19}$ C,

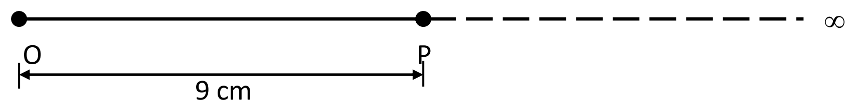
$r = 6.6 \times 10^{-15}$ m

$\therefore$ $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = \frac{9 \times 10^9 \times 79 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-15}}$ V $= 1.7 \times 10^7$ V

Q. 3. ***(i) Calculate the potential at a point P due to a charge of 4 × 10^{-7} C located 9 cm away. (ii) Hence obtain the work done in bringing a charge of 2 × 10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought?***

Sol. (i) Here $q = 4 \times 10^{-7}$ C, r = 9 cm = 0.09 m

$q = 4 \times 10^{-7}$ C



Electric potential at point P is

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = 9 \times 10^9 \times \frac{4 \times 10^{-7}}{0.09}$$

$$= 4 \times 10^4 \text{ V}$$

(ii) By definition, electric potential at point P is equal to the work done in bringing a unit positive charge from infinity to the point P. Hence the work-done in bringing a charge of 2×10^{-9} C from infinity to the point P is

$$W = q_0 V = 2 \times 10^{-9} \times 4 \times 10^4$$

$$= 8 \times 10^{-5} \text{ J}$$

No, the answer does not depend on the path along which the charge is brought.

Q. 4. ***A metal wire is bend in a circle of radius 10 cm. It is given a charge of 200 μC which spreads on it uniformly. Calculate the electric potential at its centre.***

Sol. Here $q = 200\ \mu C = 2 \times 10^{-4}$ C, r = 10 cm = 0.10 m

We can consider the circular wire to be made of a large number of elementary charges dq. Potential due to one such elementary charge dq at the centre,

$$dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r}$$

Total potential at the centre of the circular wire,

$$V = \sum dV = \sum \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0 r} \sum dq$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = \frac{9 \times 10^9 \times 2 \times 10^{-4}}{0.10} = 18 \times 10^6 \text{ V}$$

Q. 5. ***Electric field intensity at point 'B' due to a point charge 'Q' kept at point 'A' is 24 NC^{-1} and the electric potential at point 'B' due to same charge is 12 JC^{-1}. Calculate the distance AB and also the magnitude of charge Q.***

Sol. Electric field of a point charge,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} = 24\ NC^{-1}$$

Electric potential of a point charge,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r} = 12\ JC^{-1}$$

The distance AB is given by

$$r = \frac{V}{E} = \frac{12}{24} = 0.5 \text{ m}$$

The magnitude of the charge,

$$Q = 4\pi\varepsilon_0 Vr = \frac{1}{9 \times 10^9} \times 12 \times 0.5 = 0.667 \times 10^{-9} \text{ C.}$$

Q. 6. ***To what potential we must charge an insulated sphere of radius 14 cm so that the surface charge density is equal to 1 μ Cm^{-2}?***

Sol. Here $r = 14$ cm $= 14 \times 10^{-2}$ m, $\sigma = 1\ \mu\ Cm^{-2} = 10^{-6}\ Cm^{-2}$

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4\pi r^2 \sigma}{r} = \frac{1}{4\pi\varepsilon_0} \cdot 4\pi r\sigma$$

$$= 9 \times 10^9 \times 4 \times \frac{22}{7} \times 14 \times 10^{-2} \times 10^{-6} \text{ V} \qquad = 15840 \text{ V}$$

Q. 7. ***A charge of 24 μC is given to a hollow metallic sphere of radius 0.2 m. Find the potential***
(i) At the surface of the sphere, and
(ii) At the distance of 0.1 cm from the centre of the sphere.

Sol. (i) $q = 24\ \mu C = 24 \times 10^{-6}$ C, r = 0.2 m

Potential at the surface of the sphere is

$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = \frac{9\times10^9 \times 24\times10^{-6}}{0.2}$ V $= 1.08\times10^6$ V

(ii) As potential at any point inside the sphere = Potential on the surface

∴ Potential at a distance of 0.1 cm from the centre $= 1.08\times10^6$ V

Q. 8. ***Twenty-seven drops of same size are charged at 220 V each. They coalesce to form a bigger drop. Calculate the potential of the bigger drop.***

Sol. Let radius of each small drop = r

Radius of large drop = R

Then $\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$

or $R = 3r$

Potential of each small drop,

$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$

∴ Total charge on 27 drops,

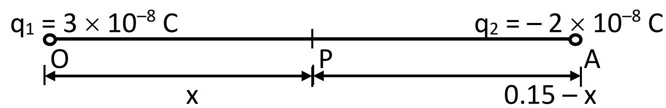
$Q = 27\,q = 27 \times 4\pi\varepsilon_0\, r\, V$

Potential of large drop,

$V' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{27 \times 4\pi\varepsilon_0\, r\, V}{3r} = 9\,V = 9 \times 220 = 1980$ V

Q. 9. ***Two charges 3×10^{-8} C and -2×10^{-8} C are located 15 cm apart. At what point on the line joining the two charges are the electric potential zero? Take the potential at infinity to be zero.***

Sol. As shown in Fig. suppose the two-point charges are placed on X-axis with the positive charge located on the origin O.



[Zero of electric potential for two charges]

Let the potential be zero at the point P and OP = x. For x < 0 (i.e., to the left of O), the potentials of the two charges cannot add up to zero. Clearly, x must be positive. If x lies between O and A, then

$V_1 + V_2 = 0$

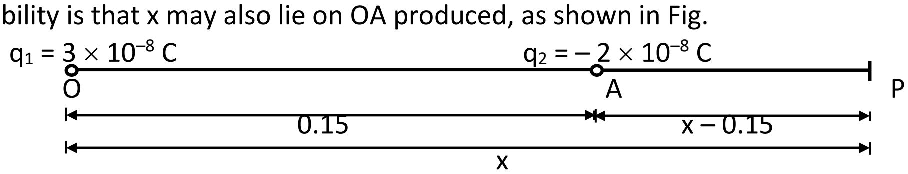
$\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{x} + \frac{q_2}{0.15 - x}\right) = 0$

or $9\times10^9\left(\frac{3\times10^{-8}}{x} - \frac{2\times10^{-8}}{0.15 - x}\right) = 0$

or $\frac{3}{x} - \frac{2}{0.15 - x} = 0$

which gives $x = 0.09$ m = 9 cm

The other possibility is that x may also lie on OA produced, as shown in Fig.



As $V_1 + V_2 =$

∴ $\frac{1}{4\pi\varepsilon_0}\left(\frac{3\times10^{-8}}{x} - \frac{2\times10^{-8}}{x - 0.15}\right) = 0$

which gives $x = 0.45$ m = 45 m

Thus, the electric potential is zero at 9 cm and 45 cm away from the positive charge on the side of the negative charge.

Q. 10. ***Calculate the electric potential at the center of a square of side $\sqrt{2}$ m, having charges 100 μC, – 50 μC, 20 μC, and – 60 μC at the four corners of the square.***

Sol. Diagonal of the square

$= \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2$ m

Distance of each charge from the centre of the square is

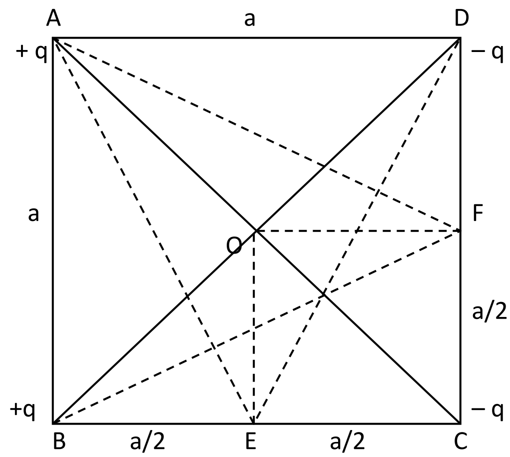
r = Half diagonal = 1 m

∴ Potential at the centre of the square is

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right)$$

$$V = 9 \times 10^9 \left(\frac{100 \times 10^{-6}}{1} - \frac{50 \times 10^{-6}}{1} + \frac{20 \times 10^{-6}}{1} - \frac{60 \times 10^{-6}}{1}\right) = 9 \times 10^9 \times 10^{-6} \times 10 = 9 \times 10^4 \text{ V}.$$

Q. 11. Four charges +q, +q, – q and – q are placed respectively at the corners A, B, C and D of a square of side 'a' arranged in the given order. Calculate the electric potential at the center O. If E and F are the midpoints of sides BC and CD respectively, what will be the work done in carrying a charge 'e' from O to E and from O to F.



Sol. Let OA = OB = OC = OD = r

Then the potential at the centre O is

$$V_0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{q}{r} + \frac{q}{r} + \frac{q}{r}\right) = 0$$

Again, the potential at point E is

$$V_E = \left(\frac{1}{4\pi\varepsilon_0}\ \frac{q}{AE} + \frac{q}{BE} + \frac{q}{CE} + \frac{q}{DE}\right) = 0 \qquad [\because AE = DE, BE = CE]$$

Now, $AF = BF = \sqrt{a^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{5}a}{2}$

∴ The potential at point F is

$$V_F = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{AF} + \frac{q}{BF} - \frac{q}{CF} - \frac{q}{DF}\right)$$

$$= \frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{AF} - \frac{1}{CF}\right) \qquad [\because AF = BF, CF = DF]$$

$$= \frac{2q}{4\pi\varepsilon_0}\left(\frac{2}{\sqrt{5}} - \frac{2}{a}\right) = \frac{q}{\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{5}} - 1\right)$$

Work done in moving the charge 'e' from O to E is

$$W = e\,[V_E - V_0] = e \times 0 = 0$$

Work done in moving the charge 'e' from O to F is

$$W = e\,[V_F - V_0] = e\left(\frac{q}{\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{5}} - 1\right) - 0\right)$$

$$= \frac{q_e}{\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{5}} - 1\right)$$

Q. 12. An infinite number of charges, each equal to q, are placed along the x-axis at x = 1, x = 2, x = 4…. And so on. (i) Find the potential at the point x = 0 due to this set up the consecutive charges have opposite signs?

Sol. (i) The potential at x = 0 due to given set of charges is

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{1}+\frac{q}{2}+\frac{q}{4}+\frac{q}{8}+......\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+......\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{1-½} = \frac{q}{2\pi\varepsilon_0} \quad \left(\because \text{Sum of an infinite G.P.} = \frac{b}{1-r}\right)$$

(ii) When the consecutive charges have opposite signs, potential at x = 0 is

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{1}-\frac{q}{2}+\frac{q}{4}-\frac{q}{8}+.......\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+.......\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{1-[-½]} = \frac{q}{6\pi\varepsilon_0}$$

Q. 13. ***A charge Q is distributed over two concentric hollow spheres of radii r and R, where R > r, such that the surface charge densities are equal. Find the potential at the common centre.***

Sol. Let Q_r and Q_R be the charges distributed over the smaller and the larger spheres, respectively.

Then $Q = Q_r + Q_R$

The surface charge densities will be

$$\sigma = \frac{Q_R}{4\pi R^2} = \frac{Q_r}{4\pi r^2}$$

or $\frac{Q_r}{Q_R} = \frac{r^2}{R^2}$ or $\frac{Q_r + Q_R}{Q_R} = \frac{r^2 + R^2}{R^2}$

or $\frac{Q}{Q_R} = \frac{r^2 + R^2}{R^2}$ or $Q_R = \left(\frac{R^2}{R^2 + r^2}\right)Q$

Similarly, $Q_r = \left(\frac{r^2}{R^2 + r^2}\right)Q$

Potential due to the charge on the smaller sphere is

$$V_r = \frac{Q_r}{4\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0 r}\cdot\left(\frac{R^2}{R^2 + r^2}\right)Q = \frac{Q.R}{4\pi\varepsilon_0 (R^2 + r^2)}$$

Potential due to the charge on the larger sphere is

$$V_r = \frac{Q_r}{4\pi\varepsilon_0 R} = \frac{1}{4\pi\varepsilon_0 R}\cdot\left(\frac{R^2}{R^2 + r^2}\right)Q = \frac{Q.R}{4\pi\varepsilon_0 (R^2 + r^2)}$$

Total potential at the Centre,

$$V = V_r + V_R = \frac{Q(r + R)}{4\pi\varepsilon_0(R^2 + r^2)}$$

Q. 14. ***A short electric dipole has dipole moment of 4×10^{-9} Cm. Determine the electric potential due to the dipole at a point distant 0.3 m from the Centre of the dipole situated (a) on the axial line (b) on equatorial line and (c) on a line making an angle of 60° with the dipole axis.***

Sol. Here $p = 4 \times 10^{-9}$ Cm, r = 0.3 m.

(a) Potential at a point on the axial line is

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2} = \frac{9\times 10^9 \times 4\times 10^{-9}}{(0.3)^2}$$

= 400 V

(b) Potential at a point on the equatorial line = 0

(c) Potential at a point on a line that makes an angle of 60° with dipole axis is

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{r^2}$$

$$= \frac{9\times 10^9 \times 4 \times 10^{-9}\cos 60°}{(0.3)^2}$$

= 200 V.

Q. 15. *Two-point charges of + 3 μC and – 3 μC are placed 2 × 10^{-3} m apart from each other. Calculate (i) electric field and electric potential at a distance of 0.6 m from the dipole in broad-side-on position (ii) electric field and electric potential at the same point after rotating the dipole through 90°.*

Sol. Dipole moment, $P = q \times 2l = 3 \times 10^{-6} \times 2 \times 10^{-3} = 6 \times 10^{-9}$ Cm

(i) Electric field in broad-side-on position is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3} = \frac{9 \times 10^9 \times 6 \times 10^{-9}}{(0.6)^3} = 250 \text{ NC}^{-1}$$

Electric potential in broad-side-on position, V = 0.

(ii) When the dipole is rotated through 90°, the same point is now in end-on-position with respect to the dipole.

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3} = 500 \text{ NC}^{-1}$$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^2} = \frac{9 \times 10^9 \times 6 \times 10^{-9}}{(0.6)^2} = 150 \text{ V}$$

Example 16. ***n*** **small drops of same size are charged to V volt each. They coalesce to form a bigger drop. Calculate potential of bigger drop.** ***(Punjab S.B. 2001)***

Solution : Let r be the radius of small drop and R be the radius of bigger drop.

When n small drops coalesce to form a bigger drop then volume remains the same

Volume of bigger drop = n × volume of smaller drop

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$$

$$R = n^{1/3} r$$

Charge on bigger drop, $Q = nq$ (q is charge on small drop)

$$\therefore \text{Potential of smaller drop } (V_s) = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad ...(1)$$

$$\text{Potential of bigger drop} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

$$V_b = \frac{1}{4\pi\epsilon_0}\frac{nq}{n^{1/3}r} = n^{2/3}\left(\frac{1}{4\pi\epsilon_0}\frac{q}{r}\right) \quad ...(2)$$

From equations (1) and (2)

Potential of bigger drop $= n^{2/3}$ (Potential of smaller drop)

$$V_b = n^{2/3} V_s$$

Example 17. Three concentric metallic shells A, B and C of radii ***a, b*** **and** ***c*** **(*a* < *b* < *c*) have surface charge densities + σ, – σ and + σ respectively.**

(a) **Find the potentials of three shells A, B and C.**

(b) **If the shells A and C are at the same potential, obtain the relation between** ***a, b, c.***

(I.I.T., C.B.S.E. Sample Paper 2003)

Solution : Three given concentric shells A, B and C are as shown in Fig. 17.

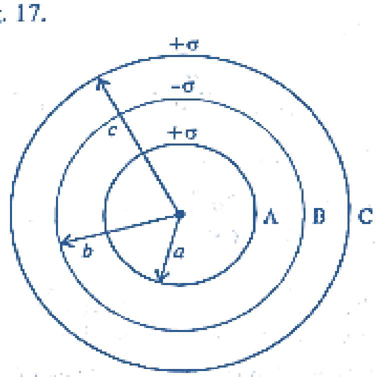


Fig. 17

Example 14. Eight charged water droplets, each with a radius of 1 mm and charge 10^{-9} C coalesce to form a single drop. Calculate potential of bigger drop.

Solution : Let r be the radius of each small drop and R be the radius of large (big) drop

Volume of 8 small drops = Volume of big drop

$$8 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$\therefore \quad R = 2r = 2 \times 1 \times 10^{-3} \text{ m} = 2 \times 10^{-3} \text{ m}$$

Charge on each drop = 1×10^{-9} C

Charge on 8 drops = 8×10^{-9} C

$$\therefore \text{Potential of large drop } V' = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

$$= \frac{9 \times 10^9 \times 8 \times 10^{-9}}{2 \times 10^{-3}} = 3{\cdot}6 \times 10^4 \text{ V}$$

Charge on A = $+4\pi a^2\sigma$

Charge on B = $-4\pi b^2\sigma$

Charge on C = $+4\sigma c^2\sigma$

(a) Potential of A due to its own charge

$$= +\frac{4\pi a^2\sigma}{4\pi\epsilon_0 a} = \frac{\sigma a}{\epsilon_0}$$

$$\text{Potential of A due to charge on B} = -\frac{4\pi b^2\sigma}{4\pi\epsilon_0 b} = -\frac{\sigma b}{\epsilon_0}$$

$$\text{Potential of A due to charge on C} = +\frac{4\pi c^2\sigma}{4\pi\epsilon_0 c} = \frac{\sigma c}{\epsilon_0}$$

Total potential of A *i.e.*,

$$V_A = \frac{\sigma}{\epsilon_0}(a - b + c)$$

$$\text{Similarly, } V_B = \frac{4\pi a^2\sigma}{4\pi\epsilon_0 b} - \frac{4\pi b^2\sigma}{4\pi\epsilon_0 b} + \frac{4\pi c^2\sigma}{4\pi\epsilon_0 c}$$

$$V_B = \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{b} - b + c\right)$$

$$\text{and} \quad V_C = +\frac{4\pi a^2\sigma}{4\pi\epsilon_0 c} - \frac{4\pi b^2\sigma}{4\pi\epsilon_0 c} + \frac{4\pi c^2\sigma}{4\pi\epsilon_0 c}$$

$$V_C = \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$$

(b) If $V_A = V_C$

$$\frac{\sigma}{\epsilon_0}(a - b + c) = \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$$

$$a - b = \frac{a^2 - b^2}{c} = \frac{(a+b)(a-b)}{c}$$

$$c(a - b) = (a + b)(a - b)$$

$$c = a + b$$

▶ Relation Between ELECTRIC FIELD INTENSITY (E) And POTENTIAL (V)

{Electric field as a potential Gradient}

Consider two points A and B in the electric field E due to a point charge +q placed at O. Points A & B are so close to each other that electric field intensity between A and B is uniform.

Let q_0 be the test charge placed at A. The force acting on q_0 is $\vec{F} = q_0 \vec{E}$ (along the direction of E)

Work done to move the test charge from A to B is

$$dW = \vec{F}.\vec{dr} = q_0 \vec{E}.\vec{dr} = q_0 E\, dr \cos 180^0$$
$$= - q_0 E\, dr$$

or, $\dfrac{dW}{q_0} = - E\, dr$

But, $dV = \dfrac{dW}{q_0}$ = potential difference (By definition)

∴ $dV = \dfrac{- q_0 E\, dr}{q_0}$

$dV = - E\, dr$

or, $$E = \frac{- dV}{dr}$$

Thus, ***"Electric field intensity at a point is equal to the negative gradient of the electric potential that point".***

☞ The negative sign shows that electric field intensity is in the direction of decreasing potential.

Conclusion: The component of E in any direction is negative of the rate of change of electric potential with distance in that direction

☞ **In vector form: E = - dV / dr can be written as**

$$E_x = - \frac{dV_x}{dr} ; \quad E_y = - \frac{dV_y}{dr} ; \quad E_z = - \frac{dV_z}{dr}$$

- Electric potential is a **scalar quantity**, while electric gradient is **vector quantity** (numerically equal to electric field intensity
- The SI unit of electric field intensity is same as that of potential gradient i.e., **V / m**
- The relation $E = - dV / dr$ is valid for non-uniform field also.

The quantity dV is the rate of change of potential with distance and is called potential gradient. Thus, the electric field at any point is equal to the negative of the potential gradient at that point. The negative sign shows that the direction of the electric field is in the direction of decreasing potential. Moreover, the field is in the direction where this decrease is steepest.

Conclusions:

(i) Electric field is in that direction in which the potential decreases is steepest.

(ii) The magnitude of electric field is equal to the change in the magnitude of potential per unit displacement (called potential gradient) normal to the equipotential surface at the given point.

Computing electric potential from electric field:

$$\vec{E} = -\frac{dV}{d\vec{r}}$$

or $dV = -\vec{E}.d\vec{r}$

Integrating the above equation between points r_1 and r_2, we get

$$\int_{V_1}^{V_2} dV = -\int_{\vec{r_1}}^{\vec{r_2}} \vec{E}.\vec{d}r$$

or $$V_2 - V_1 = -\int_{\vec{r_1}}^{r_2} \vec{E}.\vec{d}r$$

where V_1 and V_2 are the potentials at $\vec{r_1}$ and r_2 respectively. If we taken $\vec{r_1}$ at infinity, then $\vec{V_1} = 0$ and put $r_2 = r$, we get

$$V(r) = -\int_{\infty}^{\vec{r}} \vec{E}.\vec{d}r$$

Hence by knowing electric potential at any point, we can evaluate the electric field at that point.

units: ***volt/metre and newton/coulomb are equivalent.***

► **SI units of electric field**: Electric field at any points is equal to the negative of the potential gradient. It suggests that the SI unit of electric field is volt per metre. But electric field is also defined as the force experienced by a unit positive charge, so SI unit of electric field is newton per metre. Both of these units are equivalent as shown below.

$$\frac{\text{Volt}}{\text{metre}} = \frac{\text{joule/coulomb}}{\text{metre}}$$

$$= \frac{\text{newton-metre}}{\text{Coulomb-metre}} = \frac{\text{newton}}{\text{coulomb}}$$

or $1\ Vm^{-1} = 1\ NC^{-1}$

Examples based on Relation between Electric Field and Potential

❖ **Formulae Used**

1. Electric field in a region can be determined from the electric potential by using relation,

$$E = -\frac{dV}{dr}$$

or $$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = \frac{\partial V}{\partial z}$$

2. Electric field between two parallel conductors,

$$E = \frac{V}{d}$$

3. Electric potential in a region can be determined from the electric field by using the relation,

$$V = -\int_{\infty}^{r} \vec{E}.\vec{d}r$$

❖ **Units Used**

E is in NC^{-1} or Vm^{-1}, V in volt, r in metre.

Q. 1. Find the electric field between two metal plates 3 mm apart, connected to 12 V battery.

Sol. Electric field,

$$E = \frac{V}{d} = \frac{12\text{ V}}{3 \times 10^{-3}\text{ m}} = 4 \times 10^{3}\text{ Vm}^{-1}$$

Q. 2. Calculate the voltage needed to balance an oil drop carrying 10 electrons when located between the plates of a capacitor which are 5 mm apart. ($g = 10\text{ m s}^{-2}$). The mass of oil drop is 3×10^{-16} kg.

Sol. $q = ne = 10 \times 1.6 \times 10^{-19}$ C, $m = 3 \times 10^{-16}$ kg, $d = 5\text{ mm} = 5 \times 10^{-3}$ m

$$E = \frac{V}{d} = \frac{V}{5 \times 10^{-3}}\text{ Vm}^{-1}$$

For the charged oil drop to remain stationary in electric field,

$$qE = mg$$

$$\therefore \quad 10 \times 1.6 \times 10^{-19} \times \frac{V}{5 \times 10^{-3}} = 3 \times 10^{-16} \times 10$$

or $$V = \frac{3 \times 10^{-16} \times 10 \times 5 \times 10^{-3}}{10 \times 1.6 \times 10^{-19}} = 9.47\text{ V}$$

Q. 3. An infinite plane sheet of charge density 10^{-8} Cm^{-2} is held in air. In this situation how far apart are two equipotential surfaces, whose p.d. is 5 V?

Sol. Electric field of an infinite plane sheet of charge, $E = \frac{\sigma}{2\varepsilon_0}$

If Δr is the separation between two equipotential surfaces having potential difference ΔV, then

$$E = \frac{\Delta V}{\Delta r}$$

$$\therefore \quad \frac{\sigma}{2\varepsilon_0} = \frac{\Delta V}{\Delta r}$$

or $$\Delta r = \frac{2\varepsilon_0 \Delta V}{\sigma} = \frac{2 \times 8.85 \times 10^{-12} \times 5}{10^{-8}} = 8.85 \times 10^{-3}\text{ m} = 8.85\text{ mm}$$

Q. 4. A spark passes in air when the potential gradient at the surface of a charged conductor is 3×10^{6} Vm^{-1}. What must be the radius of an insulated metal sphere which can be charged to a potential of 3×10^{6} V before sparking into air?

Sol. Potential gradient,

$$\frac{dV}{dr} = 3 \times 10^{6}\text{ Vm}^{-1}$$

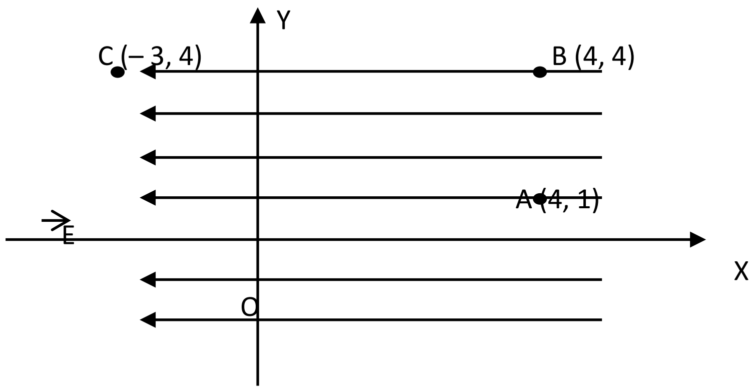
or $dV = 3 \times 10^{6}\, dr$

or $V = 3 \times 10^{6}$ V

But $V = 3 \times 10^{6}$ V

$\therefore \quad 3 \times 10^{6}\, r = 3 \times 10^{6}$ or $r = 1$ m

Q. 5. A uniform electric field $\vec{E}$ of 300 NC^{-1} is directed along negative X-axis. A, B and C are three points in the field, having x and y coordinates (in meter), as shown in Fig. Find the potential difference ΔV_{BA}, ΔV_{CB} and ΔV_{CA}.



Sol. (i) No work is done in moving a unit positive charge from A to B because the displacement of the charge is perpendicular to the electric field. Thus, the points A and B are at the same potential.

∴ $\Delta V_{BA} = 0$

(ii) Work is done by the electric field as the positive charge moves from B to C (i.e., in the direction of E). Thus, the point C is at a lower potential than the point B.

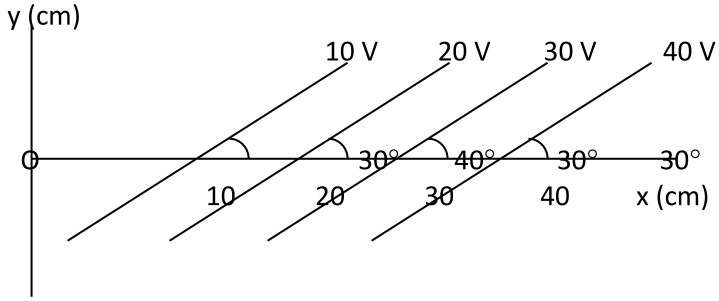
As $E = -\dfrac{\Delta V}{\Delta x}$

∴ $\Delta V_{CB} = -E\,\Delta x = -300\ NC^{-1} \times 7\ m = -2100\ V$

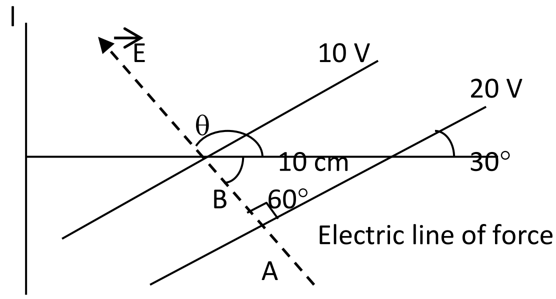
(iii) Points A and B lie on an equipotential surface. So $V_B = V_A$.

$\Delta V_{CA} = V_C - V_A = V_C - V_B = \Delta V_{CB} = -2100\ V$

Q. 6. Fig. shows some equipotential surfaces. What can you say about the magnitude and direction of the electric field?



Sol. As shown in Fig., consider two consecutive equipotential surfaces. Electric field is normal to the equipotential surfaces and always directed from higher potential to lower potential.



The normal distance between two consecutive equipotential surfaces is

$dr = AB = 10\ cm \times \cos 60° = 5\ cm$ $\left(\dfrac{AB}{10\ cm} = \cos 60°\right)$

Also, $dV = 10 - 20 = -10\ V$

∴ $E = -\dfrac{dV}{dr} = -\dfrac{-10\ V}{5 \times 10^{-2}\ m} = 200\ Vm^{-1}$

Angle made by $\vec{E}$ with positive X-axis is $\theta = 180 - 60° = 120°$

Q. 7. If the potential in the region of space around the point (– 1m, 2m, 3m) is given by $V = (10x^2 + 5y^2 - 3z^2)$ volt, calculate the three components of electric field at this point.

Sol. Here $x = -1m$, $y = 2\ m$, $z = 3\ m$

As $V = 10x^2 + 5y^2 - 3z^2$

∴ $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\partial}{\partial x}(10x^2 + 5y^2 - 3z^2)$

$= -20x = -20 \times (-1) = 20\ Vm^{-1}$

$E_y = \dfrac{\partial V}{\partial y} = -\dfrac{\partial}{dy}(10x^2 + 5y^2 - 3z^2) = -10y = -10 \times 2 = -20\ Vm^{-1}$

$E_z = -\frac{\partial V}{\partial y} = -\frac{\partial}{dy}(10x^2 + 5y^2 - 3z^2) = 6y = 6 \times 3 = 18\ Vm^{-1}$

Q. 8. ***The electric potential V at any point x, y, z (all in meters) in space is given by $V = 4x^2$ volts. Calculate the electric field at the point (1 m, 0, 2 m).***

Sol. Here $V = 4x^2$

$E_x = -\frac{\partial V}{\partial x} = -8x,\quad E_y = -\frac{\partial V}{\partial y} = 0,\quad E_z = -\frac{\partial V}{\partial z} = 0$

$E = E_x\hat{i} + E_y\hat{j} + E_z k = -8x\,\hat{i}$

At point (1m, 0, 2 m),

$E = -8 \times 1\,\hat{i} = -8\,\hat{i}\ Vm^{-1}$

Q. 9. ***The electric field outside a charged long straight wire is given by $E = \frac{1000}{r}\ Vm^{-1}$, and is directed outwards. What is the sign of the charge on the wire? If two points A and B are situated such that $r_A = 0.2$ m and $r_B = 0.4$ m, find the value of $(V_B - V_A)$.***

Sol. As the electric field is directed outwards, the charge on the wire must be positive.

$$V_B - V_A = -\int_A^B \vec{E}\cdot\vec{dl} = -\int_{r=0.2\,m}^{r=0.4\,m} \frac{1000}{r}\,dr$$

$= -1000\,[\log_e r]_{0.2}^{0.4} = -1000\,[\log_e 0.4 - \log_e 0.2]$

$= -1000 \log_e \frac{0.4}{0.2} = -1000 \times \log_e 2$

$= -1000 \times 0.6931 = -693.1$ V

Example 24. Two identical plane metallic surfaces A and B are kept parallel to each other in air separated by a distance of 1·0 cm as shown in Fig. 20. Surface A is given a positive potential of 10 V and the outer surface of B is earthed. *(i)* What is the magnitude and direction of uniform electric field between points X and Z ? *(ii)* What is the workdone in moving a charge of 20 μC from point X to point Y, where X is situated on surface A.

(C.B.S.E. Sample Paper 2010)

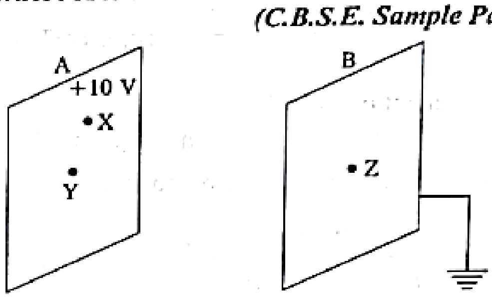


Fig. 20

Solution : Here $dV = 10V$, $dr = 1{\cdot}0$ cm $= 1 \times 10^{-2}$ m

(i) Magnitude of electric field $E = \left|-\frac{dV}{dr}\right| = \frac{dV}{dr}$

$$E = \frac{10V}{1 \times 10^{-2}\,m} = 10^3\ Vm^{-1}$$

Direction of electric field is from Y to Z

(ii) Since surface A is an equipotential surface, ∴ potential difference between X and Y is $\Delta Y = 0$

∴ Workdone $W = q\Delta V = 0$

∴ No work is done in moving charge from X to Y.

Example 20. Calculate the voltage needed to balance an oil drop carrying 10 electrons when located between the plates of capacitor which are 5 mm apart ($g = 10\ ms^{-2}$). The mass of oil drop is 3×10^{-16} kg.

Solution : Here mass $(m) = 3 \times 10^{-16}$ kg,

$d = 5$ mm $= 5 \times 10^{-3}$ m

∴ $q = ne = 10 \times 1{\cdot}6 \times 10^{-19}$ C

☐ **Step 1.** $E = \frac{V}{d} = \frac{V}{5 \times 10^{-3}}\ Vm^{-1}$

Since the oil drop remain stationary in electric field

∴ $qE = mg$

$$10 \times 1{\cdot}6 \times 10^{-19} \times \frac{V}{5 \times 10^{-3}} = 3 \times 10^{-16} \times 10$$

$$V = \frac{3 \times 10^{-16} \times 10 \times 5 \times 10^{-3}}{10 \times 1{\cdot}6 \times 10^{-19}} = 9{\cdot}47\ V$$

EQUIPOTENTIAL SURFACE:

"Any surface which has same electrostatic potential at every point, is called an equipotential surface".

In other words, **"An equipotential surface is that surface on which, at every point, electric potential is same".**

► The surface may be surface of a charged conductor is an equipotential surface. By joining points of constant potential, we can draw equipotential surfaces through-out the region in which an electric field exists.

Explanation (PROPERTIES):

(i) We know that, potential difference between two-point A & B = Work done in carrying unit positive charge from A to B.

$$V_B - V_A = \frac{W_{AB}}{q_0}$$

But, $V_A = V_B$ (as A & B lie on an equipotential surface)

$\therefore \quad W_{AB} = 0$

☞☞ **Hence no work is done in moving a test charge between two point on an equipotential surface.**

(ii) The electric field is always at right angles to the equipotential surface (In other words, electric lines of force are always perpendicular to the equipotential surface).

As, $dW = \vec{E} \cdot \vec{dl} = E\,dl \cos\theta$

$0 = E\,dl \cos\theta$

$\cos\theta = 0$

$\cos\theta = \cos 90$

$\theta = 90$

i.e., $E \perp dl$

☞☞ **Electric field intensity is always at 90° to the equipotential surface.**

(iii) The equipotential surface help to distinguish region of strong field from those of weak field .

We Know that,

$$E = -\frac{dV}{dr} \quad \text{or,} \quad dr = -\frac{dV}{E}$$

or, $dr \propto \frac{1}{E}$ (if dV = constant)

i.e., *The spacing of equipotential surface are closer together, where the electric field in the region, where the electric field is stronger and vice versa.*

☞☞ **The equipotential surface are closer together, where the electric field is stronger and farther apart where the field is weaker.**

(iv) The equipotential surface tells us the direction of the electric field.

We know that $E = -\,dV / dr$

-- The negative sign tells that electric field is directed in direction of decrease in electric potential with distance.

(v) No two equipotential surface can intersect each other.

Explanation: If two equipotential surfaces intersect each other then at their point of intersection, there will be two value of electric potential. This is because, two equipotential surfaces cannot intersect each other.

STUDY OF EQUIPOTENTIAL SURFACE:

EQUIPOTENTIAL SURFACES OF VARIOUS CHARGE SYSTEMS

The equipotential surfaces for:

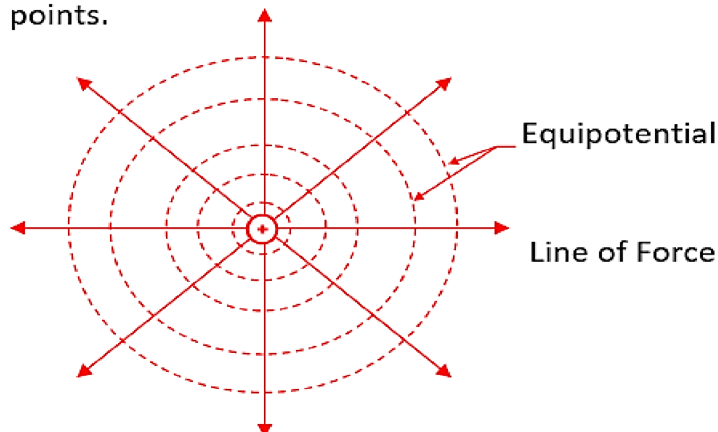
(i) a point charge,
(ii) two-point charges + q and – q, separated by a small distance,
(iii) two-point charges + q and + q separated by a small distance and
(iv) a uniform electric field.

For the various charge systems, we represent equipotential surfaces by dashed curved and lines of force by full line curves. Between any two adjacent equipotential surfaces, we assume a constant potential difference.

(i) Equipotential surfaces of a positive point charge: The electric potential due to a point charge q at distance r from it is given by

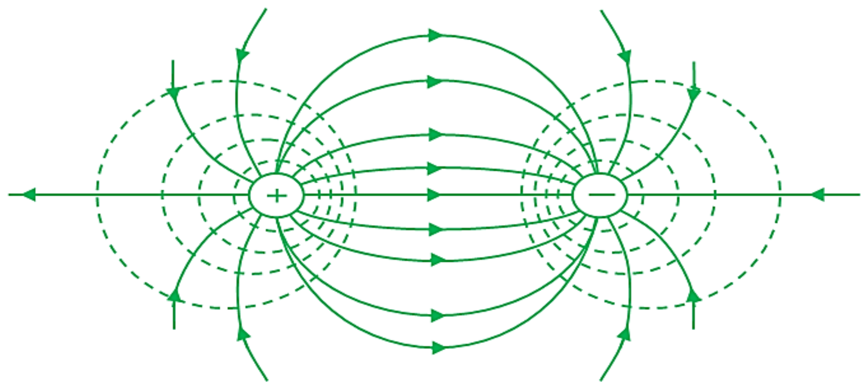
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

This shows that V is constant if r is constant. Thus the equipotential surfaces of a single point charge are concentric spherical shells with their centres at the point charge, as shown in Fig. As the lines of force point radially outwards, so they are perpendicular to the equipotential surfaces at all points.



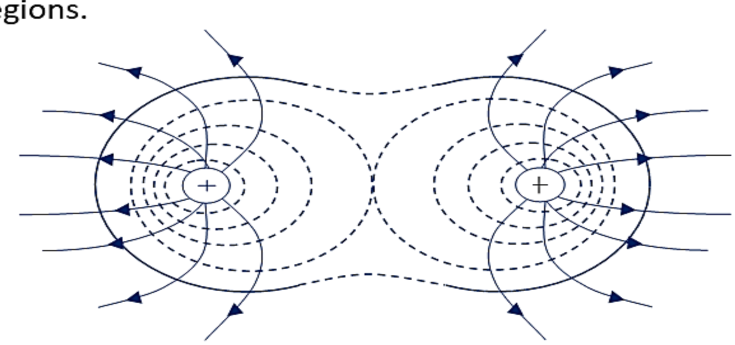
[Equipotential surfaces of a +ve point charge]

(ii) Equipotential surfaces of two equal and opposite point charge: Electric dipole. Fig. shows the equipotential surfaces of two equal and opposite charges, + q and –q, separated by a small distance. They are close together in the region in between the two charges.



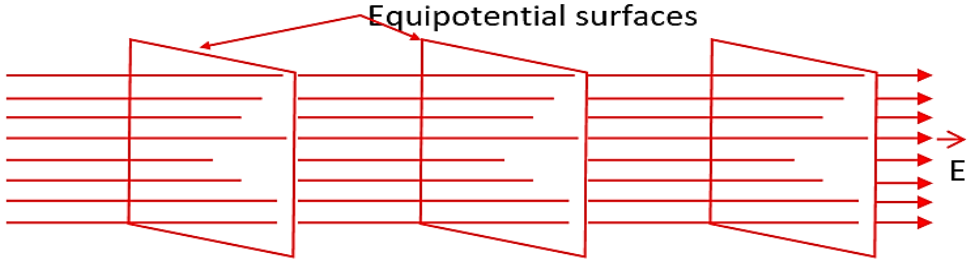
[Equipotential surfaces for two equal and opposite charges]

(iii) Equipotential surfaces of two equal positive charges: Fig. shows the equipotential surfaces of two equal and positive charges, each equal to + q, separated by a small distance. The equipotential surfaces are far apart in the regions in between the two charges, indicating a weak field in such regions.



[Equipotential surfaces of two equal positive charges]

(iv) Equipotential surfaces for a uniform electric field: Fig. show the equipotential surfaces for a uniform electric field. The lines of force are parallel straight lines and equipotential surfaces are equidistant parallel planes perpendicular to the lines of force.



[Equipotential surfaces for a uniform electric field]

► **Importance of equipotential surfaces:** Like the lines of force, the equipotential surfaces give a visual picture of both the direction and the magnitude of field $\vec{E}$ in a region of space. If we draw equipotential surfaces at regular intervals of V, we find that equipotential surfaces are closer together in the regions of strong field and farther apart in the regions of weak field. Moreover, E is normal to the equipotential surface at every point.

POTENTIAL ENERGY IN AN EXTERNAL FIELD

Potential energy of a single charge: We wish to determine the potential energy of a charge q in an external electric field E at a point P where the corresponding external potential is V. By definition, V at a point P is the amount of work done in bringing a unit positive charge from infinity to the point P. Thus, the work done in bringing a charge q from infinity to the point P will be qV, i.e.,

$$W = qV$$

This work done is stored as the potential energy of the charge q. If r is the position vector of P relative to some origin, then

$$U(\vec{r}) = qV(\vec{r})$$

P.E. of a charge an external field = Charge × external electric potential

As $V = \frac{U}{q}$

So we can define electric potential at a given point in an external field as the potential energy of a unit positive charge at that point.

Electron volt. [in joule].

Units of electrostatic potential energy: Suppose an electron ($q = 1.6 \times 10^{-19}$ C) is moved through a potential difference of 1 volt, then the change in its P.E. would be

$$\Delta U = q\,\Delta V = 1.6 \times 10^{-19}\ C \times 1\ V = 1.6 \times 10^{-19}\ J$$

This is a commonly used unit of energy in atomic physics and we call it electron volt (eV).
Thus, electron volt is the potential energy gained or lost by an electron in moving through a potential difference of 1 volt.

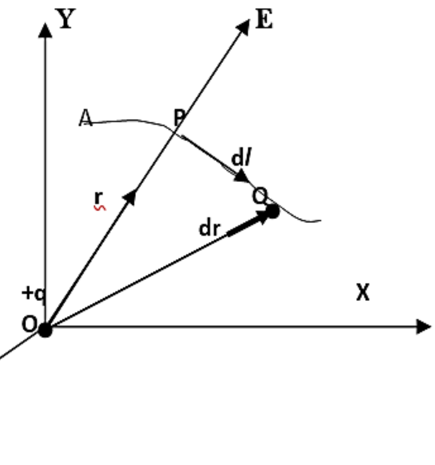
$$1\ eV = 1.6 \times 10^{-19}\ J$$

Multiples and submultiples of eV:

- **1 meV (milli electron volt) = 10^{-3} eV = 1.6×10^{-22} J**
- **1 keV (kilo electron volt) = 10^{3} eV = 1.6×10^{-16} J**
- **1 MeV (million electron volt) = 10^{6} eV = 1.6×10^{-13} J**
- **1 GeV (giga electron volt) = 10^{9} eV = 1.6×10^{-10} J**
- **1 TeV (tera electron volt) = 10^{12} eV = 1.6×10^{-7} J**

LINE integral of an ELECTROSTATIC FIELD: (Work done by electrostatic field)

"Line integral of electric field intensity $\vec{E}$ represents the work done on a unit positive charge in moving it from one point to another in the field".



Consider a point charge **'+q'** at origin. Let AB be any path of unit positive charge in electric field of +q (at O).

∴ Force acting on the unit positive charge at P due to +q

$$\vec{F} = \frac{k\,q \times 1}{r^2}\,\hat{r}$$

Also, Electric field intensity $\vec{E}$ at point P due to +q (charge at O)

$$\vec{E} = \frac{k\,q}{r^2}\,\hat{r}$$

The unit positive charge moves from P to Q in the electric field (such that PQ = d*l*).

∴ Work done by the electric field intensity E in moving the unit positive charge from P to Q is

$$dW = \vec{F}\cdot d\vec{l}$$

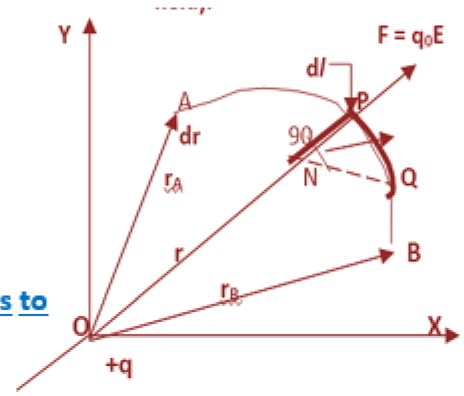
$$dW = \vec{E}\cdot d\vec{l} \qquad \{\text{since } \vec{F} = \vec{E}\}$$

Total work done by the electric field E to move the unit positive charge from A to B is

$$W_{AB} = \int dW = \int_A^B \vec{E}\cdot d\vec{l}$$

Called **'line integral of electric field '**between the point A & B.

❒ PATH INDEPENDENCE of line integral of electrostatic field (Conservative nature of electrostatic field):



Suppose a point charge +q at origin of rectangular co-ordinate system
A & B are two points in the electric field. Let OA = r_A , OB = r_B
Suppose a small positive test charge q_0 is moved from A to B
At any point P on the path AB, OP = r, let the electric field intensity is $\vec{E}$
∴Force on small test charge q_0 at P , F = q_0E (directed radially outward away from +q, at O)

To prevent acceleration of test charge under this force , external agent has to apply a force i.e., F = - q_0E

Consider a small segment of path AB, i.e., $\vec{PQ} = d\vec{l}$
Now, work done to move the test charge from P to Q is,

$$dW = \vec{F} \cdot d\vec{l} = -q_0 \vec{E} \cdot d\vec{l}$$

$$= -q_0E\, dl \cos\theta \quad \text{-------------------------------- [1]}$$

In Δ PNQ, QN ⊥ NP, Let PN = dr

$$\therefore \quad \cos\theta = \frac{PN}{PQ} = \frac{dr}{dl} \qquad \therefore \quad dr = dl \cos\theta$$

From [1] , $dW = -q_0E\, dr$

Total amount of work done by the external force in carrying the charge from A to B

$$W_{AB} = \int dW = \int_{r_A}^{r_B} -q_0E\, dr \qquad \{\text{Since } r_A \text{ and } r_B \text{ are the distance of A and B from O}\}$$

$$= -q_0 \int_{r_A}^{r_B} E \cdot dr = -q_0 \int_{r_A}^{r_B} \frac{kq}{r^2} dr = -q_0kq \int_{r_A}^{r_B} r^{-2} dr = -kq_0q \left| \frac{r^{-1}}{-1} \right|_{r_A}^{r_B}$$

$$= kq_0q \left| \frac{1}{r} \right|_{r_A}^{r_B} = \frac{q_0q}{4\pi\varepsilon_0} \left| \frac{1}{r_B} - \frac{1}{r_A} \right|$$

$$\boxed{W_{AB} = -q_0 \int_A^B \vec{E} \cdot d\vec{l} = \frac{q_0q}{4\pi\varepsilon_0} \left| \frac{1}{r_B} - \frac{1}{r_A} \right|}$$

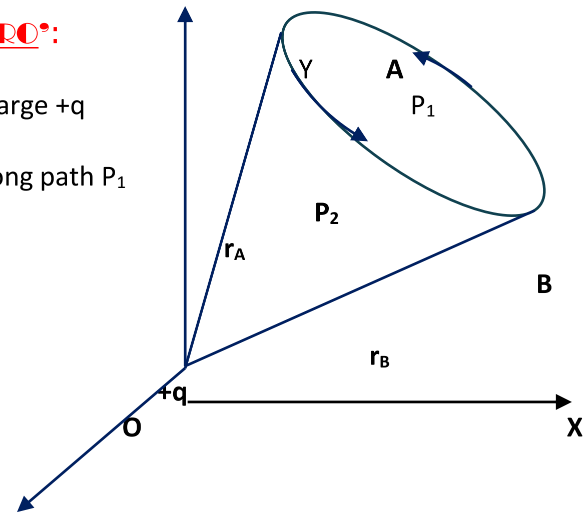
☞**Hence,** ***this work done is independent of the actual path followed between A and B but depends only end position (i.e., r_A and r_B) of the path AB.***

Also,
$$\boxed{\frac{W_{AB}}{q_0} = -\int_A^B \vec{E} \cdot d\vec{l} = \frac{q}{4\pi\varepsilon_0} \left| \frac{1}{r_B} - \frac{1}{r_A} \right|}$$

☞Mathematically, $-\int \vec{E} \cdot d\vec{l}$ = same for all paths between points A and B .(This is because if A and B are joined by other curve r_A & r_B will not change)

Hence,
$$\boxed{\int \vec{E} \cdot d\vec{l} = \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{r_A} - \frac{1}{r_B} \right]}$$

❒ Line integral of electric field along closed path is 'ZERO':



PROOF: Consider a closed path AP_1BP_2A in the electric field of point charge +q placed at origin.

The line integral of electric field between two-point A and B along path P_1

$$\int_{A\,(\text{along } P1)}^{B} \vec{E}\cdot d\vec{l} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right] \quad \text{------ -- -- -- - - - - -[1]}$$

lly,

$$\int_{B\,(\text{along } P2)}^{A} \vec{E}\cdot d\vec{l} = \frac{q}{4\pi\varepsilon_0}\left|\frac{1}{r_B} - \frac{1}{r_A}\right| \quad \text{------ -- -- -- - - - - - [2]}$$

Adding [1] and [2] , we get

$$\int_{A\,(\text{along } P1)}^{B} \vec{E}\cdot d\vec{l} + \int_{B\,(\text{along } P2)}^{A} \vec{E}\cdot d\vec{l} = \frac{q}{4\pi\varepsilon_0}\left|\frac{1}{r_A} - \frac{1}{r_B} + \frac{1}{r_B} - \frac{1}{r_A}\right| = 0 \text{ (zero)}$$

But, $\int_{(\text{along } P1)} \vec{E}\cdot d\vec{l} + \int_{(\text{along } P2)} \vec{E}\cdot d\vec{l} = \oint \vec{E}\cdot d\vec{l}$

∴ $\boxed{\oint \vec{E}\cdot d\vec{l} = \text{Zero}}$ → Read as **'integration over a closed path.**

Examples based on Relation between Electric Field , Dipole and Potential Energy

❖ **Formulae Used**

1. Electric potential energy of a system of two-point charges,

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{12}}$$

2. Electric potential energy of a system of N point charges,

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_j q_k}{r_{jk}}$$

3. Potential energy of an electric dipole in a uniform electric field,

$$U = -pE(\cos\theta_2 - \cos\theta_1)$$

If initially the dipole is perpendicular to the field E, $\theta_1 = 90°$ and $\theta_2 = \theta$ (say), then

$$U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

If initially the dipole is parallel to the field E, $\theta_1 = 0°$ and $\theta_2 = \theta$ (say), then

$$U = -pE(\cos\theta - 1) = pE(1 - \cos\theta)$$

❖ **Units Used**

Charges are in coulomb, distances in metre, energy in joule or in electron volt (eV) and dipole moment in coulomb metre (Cm).

1 eV = 1.6×10^{-19} C, 1 meV = 1.6×10^{-13} C

Q. 1. ***(a) Determine the electrostatic potential energy of a system consisting of two charges 7 μC and – 2 μC (and with no external field) placed at (– 9 cm, 0, 0) and (9 cm, 0, 0) respectively.***

(b) How much work is required to separate the two charges infinite away from each other?

(c) Suppose the same system of charges is new placed in an external electric field $E = A(1/r^2)$; $A = 9 \times 10^9$ cm^{-2}. What would the electrostatic energy of the configuration be?

Sol. (a) $q_1 = 7\ \mu C = 7 \times 10^{-6}$ C, $q_2 = -2 \times 10^{-6}$ C r = 18 cm = 0.18 m

Electrostatic potential energy of the two charges is

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r}$$

$$= \frac{9 \times 10^9 \times 7 \times 10^{-6} \times (-2) \times 10^{-6}}{0.18} = -0.7\text{ J}$$

(b) Work required to separate two charges infinitely away from each other,

$$W = U_2 - U_1 = 0 - U = -(-0.7) = 0.7\text{ J}$$

(c) Energy the two charge in the external electric field = Energy of interaction of two charges with the external electric field+ Mutual interaction energy of the two charges

$$= q_1 V(r_1) + q_2 V(r_2) + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

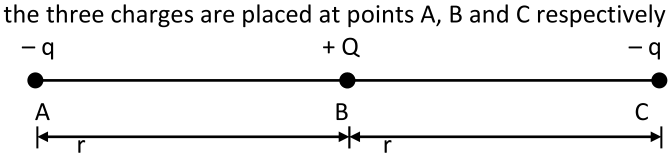
$$= q_1 \frac{A}{r_1} + q_2 \frac{A}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad \left(V = Er = \frac{A}{r}\right)$$

$$= \left(\frac{7\,\mu C}{0.09\text{ m}} + \frac{-2\mu C}{0.09\text{ m}}\right) \times 9 \times 10^5\text{ cm}^{-2} - 0.7\text{ J}$$

$$= 50 - 0.7 = 49.3\text{ J}$$

Q. 2. ***Three charges – q, + Q and – q are placed at equal distances on a straight line. If the potential energy of the system of three charges is zero, find the ration Q/q.***

Sol. As shown in Fig., suppose the three charges are placed at points A, B and C respectively on a straight line, such that AB = BC = r



As the total P.E. of the system is zero, so

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{-qQ}{r} + \frac{(-q)(-q)}{2r} + \frac{Q(-q)}{r}\right) = 0$$

or $\quad -Q + \frac{q}{2} - Q = 0 \quad$ or $\quad 2Q = \frac{q}{2} \quad$ or $\quad \frac{Q}{q} = \frac{1}{4} = 1:4$

Q. 3. ***Two positive point charges of 0.2 μC and 0.01 μC are placed 10 cm apart. Calculate the work done in reducing the distance to 5 cm.***

Sol. Here q_1 0.2×10^{-6} C, $q_2 = 0.01 \times 10^{-6}$ C

Initial separation (r_i) = 10 cm = 0.10 m

Final separation (r_f) = 5 cm = 0.05 m

Work done = Change in potential energy = Final P.E. – Initial P.E.

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_f} - \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_i} = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

$$= 0.2 \times 10^{-6} \times 0.01 \times 10^{-6} \times 9 \times 10^9 \left(\frac{1}{0.05} - \frac{1}{0.10}\right) = 1.8 \times 10^{-4}\text{ J}$$

Q. 4. ***Two electrons, each moving with a velocity of 10^6 ms^{-1}, are released towards each other. What will be the closest distance of approach between them?***

Sol. Let r_0 be the distance of closest approach of the two electrons. At this distance, the entire K.E. of the electron's changes into their P.E. Therefore,

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{e\,e}{r_0}$$

$$r_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{mv^2} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{9.1 \times 10^{-31} \times (10^6)^2} = 2.53 \times 10^{-10}\text{ m}$$

Q. 5. ***Two particles have equal masses of 5.0 g each and opposite charges of + 4 × 10^{-5} C and – 4.0 × 10^{-5} C. They are released from rest with a separation of 1.0 m between them. Find the speeds of the particles when the separation is reduced to 50 cm.***

Sol. Here m = 5.0 g = 5×10^{-3} kg, q = $\pm 4 \times 10^{-5}$ C, r_1 = 1.0 m, r_2 = 50 cm = 0.50 m

Let v = speed of each particle at the separation of 50 cm

From energy conservation principle,

K.E. of the two particles at 50 cm separation + P.E. of the two particles at 50 cm separation
= P.E. of the two particles of 1.0 m separation

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_1}$$

$$mv^2 = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad \text{or} \quad v^2 = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

$\therefore \quad v^2 = \dfrac{4 \times 10^{-5} \times (-4 \times 10^{-5}) \times 9 \times 10^9}{5 \times 10^{-3}} \left(\dfrac{0.50 - 1.0}{1.0 \times 0.50}\right) = 2880 \quad$ or $\quad v = 53.67\ ms^{-1}$

Q. 6. ***Four + q charges are arranged at the corners of a square ABCD of side d as shown in Fig. (i) Find the work required to put together this arrangement. (ii) A charge q_0 is brought to the centre E of the square, to the centre E of the square, the four charges being held fixed at its corners. How much extra work is needed to do this?***

Sol. (i) Given AB = BC = Cd = AD = d

$\therefore \quad AC = BD = \sqrt{d^2 + d^2} = \sqrt{2}\, d$

Work required to put the four charges together = Total electrostatic P.E. of the four charges

$= \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q_A q_B}{AB} + \dfrac{q_A q_C}{AC} + \dfrac{q_A q_D}{AD} + \dfrac{q_B q_C}{BC} + \dfrac{q_B q_D}{BD} + \dfrac{q_C q_D}{CD}\right)$

$= \dfrac{1}{4\pi\varepsilon_0}\left(-\dfrac{q^2}{d} + \dfrac{q^2}{\sqrt{2}d} - \dfrac{q^2}{d} - \dfrac{q^2}{d} + \dfrac{q^2}{\sqrt{2}d} - \dfrac{q^2}{d}\right)$

$= -\dfrac{q^2}{4\pi\varepsilon_0}(4 - \sqrt{2})$

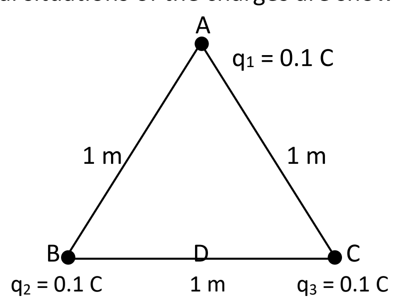
(ii) Extra work needed to bring charge q_0 to centre E.

W = q_0 × Electrostatic potential at E due to the four charges

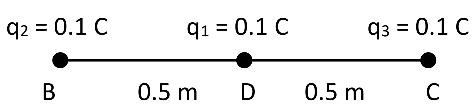
$= q_0\left(\dfrac{q}{4\pi\varepsilon_0 (d/\sqrt{2})} + \dfrac{-q}{4\pi\varepsilon_0 (d/\sqrt{2})} + \dfrac{q}{4\pi\varepsilon_0 (d/\sqrt{2})} + \dfrac{-q}{4\pi\varepsilon_0 (d/\sqrt{2})}\right) = 0$

Q. 7. ***Three charges of 0.1 C each are placed on the corners of an equilateral triangle of side 1.0 m. If the energy is supplied to the system at the rate of 1.0 kW, how much time would be required to move one of the charges onto the midpoint of the line joining the other two charges?***

Sol. The initial and final situations of the charges are shown in Fig. (a) & (b) respectively.



(a)



(b)

The initial electrostatic P.E. of the three charges is $U_i = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q_1 q_2}{AB} + \dfrac{q_2 q_3}{BC} + \dfrac{q_3 q_1}{CA}\right)$

$= 9 \times 10^9 \left(\dfrac{0.1 \times 0.1}{1} + \dfrac{0.1 \times 0.1}{1} + \dfrac{0.1 \times 0.1}{1}\right) = 0.27 \times 10^9$ J

The final electrostatic P.E. of the system of charges is

$U_f = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q_1 q_2}{BD} + \dfrac{q_1 q_3}{DC} + \dfrac{q_2 q_3}{BC}\right)$

$= 9 \times 10^9 \left(\dfrac{0.1 \times 0.1}{0.5} + \dfrac{0.1 \times 0.1}{0.5} + \dfrac{0.1 \times 0.1}{1}\right) = 0.45 \times 10^9$ J

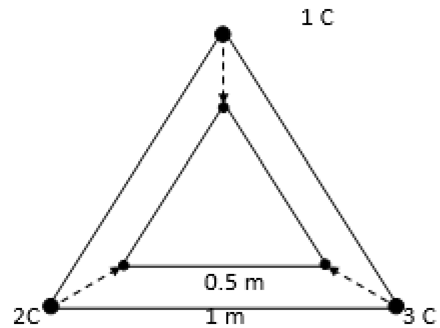
Work done = $U_f - U_i = (0.454 - 0.27) \times 10^9$

$= 0.18 \times 10^9$ J

Required time = $\dfrac{\text{Work done}}{\text{Power}} = \dfrac{0.18 \times 10^9}{1.0 \times 10^3} = 1.8 \times 10^5$ s

Q. 8. Three-point charges of 1C, 2C and 3 C are placed at the corners of an equilateral triangle of side 1 m. Calculate the work required to move these charges to the corners of a smaller equilateral triangle of sides 0.5 m as shown in Fig.

Sol. Initial P.E. of the system is



$$U_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{13}}\right)$$

$$= 9 \times 10^9 \left(\frac{1 \times 2}{1} + \frac{2 \times 3}{1} + \frac{3 \times 1}{1}\right)$$

$$= 9 \times 10^9 \times 11 = 9.9 \times 10^{10}\ \text{J}$$

Final P.E. Of the system is

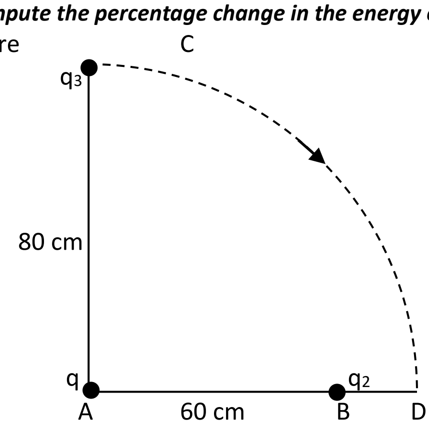
$$U_f = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r'_{12}} + \frac{q_2 q_3}{r'_{23}} + \frac{q_3 q_1}{r'_{13}}\right)$$

$$= 9 \times 10^9 \left(\frac{1 \times 2}{0.5} + \frac{2 \times 3}{0.5} + \frac{3 \times 1}{0.5}\right) = 9 \times 10^9 \times 22 = 19.8 \times 10^{10}\ \text{J}$$

Work done $= U_f - U_i = (19.8 - 9.9) \times 10^{10} = 9.9 \times 10^{10}$ J

Q. 9. Two charges $q_1 = +2 \times 10^{-8}$ C and $q_2 = -0.4 \times 10^{-8}$ C are placed 60 cm apart, as shown in Fig. A third charge $q_3 = +0.2 \times 10^{-8}$ C is moved along the arc of a circle of radius 80 cm from C to D. Compute the percentage change in the energy of the system.

Sol. Initially the charge q_2 is at C. Its distances from q_1 and q_2 are



$r_{13} = 80$ cm $= 0.80$ m

and $r_{23} = \sqrt{80^2 + 60^2} = 100$ cm $= 1.0$ m

Hence the initially energy of the system is

$$U_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$= \frac{q_3}{4\pi\varepsilon_0}\left(\frac{2 \times 10^{-8}}{0.8} - \frac{0.4 \times 10^{-8}}{1.0}\right)$$

$$= \frac{q_3}{4\pi\varepsilon_0}\left(\frac{2 \times 10^{-8}}{0.8} - \frac{0.4 \times 10^{-8}}{1.0}\right)$$

$$= \frac{q_3}{4\pi\varepsilon_0} \times 2.1 \times 10^{-8}\ \text{J}$$

When the charge q_3 is moved to D, its distances from q_2 and q_3 are

$r_{13} = 80$ cm $= 0.80$ m and $r_{23} = 20$ cm $= 0.20$ m

Hence the final potential energy of the system is

$$U_f = \frac{q_3}{4\pi\varepsilon_0}\left(\frac{2 \times 10^{-8}}{0.8} - \frac{0.4 \times 10^{-8}}{0.2}\right)$$

$$= \frac{q_3}{4\pi\varepsilon_0} \times 0.5 \times 10^{-8}\ \text{J}$$

Decrease in energy of the system

$$= U_i - U_f = \frac{q_0}{4\pi\varepsilon_0} \times 1.6 \times 10^{-8}\ \text{J}$$

Percentage decrease $= \dfrac{1.6 \times 10^{-8}}{2.1 \times 10^{-8}} \times 100 = 76.2\%$

Q. 10. An electric dipole of length 2 cm is placed with its axis making an angle of 60° to a uniform electric field of 10^5 NC^{-1}. If it experiences a torque of $8\sqrt{3}$ Nm, calculate the
(i) Magnitude of the charge on the dipole, and (ii) Potential energy of the dipole.

Sol. Here $2a = 2$ cm $= 0.02$ m, $\theta = 60°$m, $E = 10^5$ NC^{-1}, $\tau = 8\sqrt{3}$ Nm

(i) $\tau = pE \sin\theta = q \times 2a \times E \sin\theta$

∴ $8\sqrt{3} = q \times 0.02 \times 10^5 \times \sin 60°$

or $q = \dfrac{8\sqrt{3} \times 2}{0.02 \times 10^5 \times \sqrt{3}} = 8 \times 10^{-3}$ C

(ii) P.E. of the dipole is $U = -pE\cos\theta = -q \times 2a \times E\cos\theta = -8 \times 10^{-3} \times 0.02 \times 10^5 \times \cos 60° = -8$ J

Q. 11. ***An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field experiences a torque of $4\sqrt{3}$ Nm. Calculate the (i) magnitude of the electric field, (ii) potential energy of the dipole, if the dipole has charge of ±8 nC.***

Sol. Here 2a = 4 cm = 0.04 m, $\theta = 60°$, $\tau = 4\sqrt{3}$ Nm, q = 8 nC = 8×10^{-9} C

Dipole moment, $p = q \times 2a = 8 \times 10^{-9} \times 0.04 = 0.32 \times 10^{-9}$ Cm

(i) As $\tau = pE \sin\theta$

$$\therefore \quad E = \frac{\tau}{p \sin\theta} = \frac{4\sqrt{3}}{0.32 \times 10^{-9} \times \sin 60°}$$

$$= \frac{4\sqrt{3} \times 10^{9} \times 2}{0.32 \times \sqrt{3}} = 2.5 \times 10^{10}\ NC^{-1}$$

(ii) $U = -pE\cos\theta = -0.32 \times 10^{-9} \times 2.5 \times 10^{10} \times \cos 60° = -4J$

Q. 12. ***An electric dipole consists of two opposite charges each of magnitude 1 μC separated by 2 cm. The dipole is placed in an external electric field of 10^5 NC^{-1}. Find (i) the maximum torque exerted by the field on the dipole (ii) the work which the external agent will have to do in turning the dipole through 180° starting from the position θ = 0°.***

Sol. Here q = 1 μC = 10^{-6} C,

2a = 2 cm = 0.02 m, E = 10^5 NC^{-1}

$\therefore$ Dipole moment, $p = q \times 2a = 10^{-6} \times 0.02 = 2 \times 10^{-8}$ Cm

(i) Maximum torque,

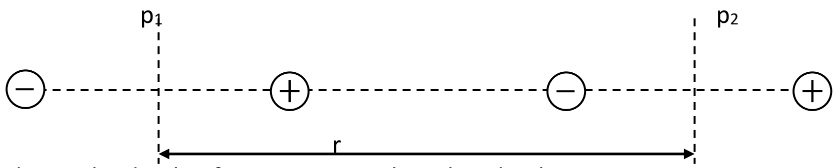
$\tau_{max} = pE \sin 90° = 2 \times 10^{-8} \times 10^{5} \times 1$

$= 2 \times 10^{-3}$ Nm.

(ii) Here $\theta_1 = 0°$ and $\theta_2 = 180°$

$\therefore$ $W = pE(\cos\theta_1 - \cos\theta_2) = 2 \times 10^{-8} \times 10^{5}(\cos 0° - \cos 180°) = 2 \times 10^{-3}(1 + 1) = 4 \times 10^{-3}$ J

Q. 13. ***Two electric dipoles of moments p_1 and p_2 are in a straight line. Show that the potential energy of each in the presence of the other is $-\frac{1}{2\pi\varepsilon_0} \cdot \frac{p_1 p_2}{r^3}$, where r is the distance between the dipoles. (Assume r to be much greater than the length of the dipole).***



Sol. Electric field dye to the dipole of moment p_1 at the other dipole is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p_1}{r^3}$$

The potential energy of the other dipole of moment p_2 in the electric field E is

$$U = -p_2 E = -p_2 \times \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p_1}{r^3} = -\frac{1}{2\pi\varepsilon_0} \frac{p_1 p_2}{r^3}$$

Q. 14. ***A molecule of a substance has permanent electric dipole moment equal to 10^{-29} Cm. A mole of this substance is polarized (at low temperature) by applying a strong electrostatic field of magnitude (10^6 Vm^{-1}). The direction of the field of magnitude (10^6 Vm^{-1}). The direction of the field is suddenly changed by an angle of 60°. Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity assume 100% polarization of the sample.***

Sol. Here p = 10^{-29} Cm, E = 10^6 Vm^{-1}, $\theta = 60°$, N = 6×10^{23}

Work required to bring one dipole from position $\theta = 0°$ to position θ is

$W = pE - pE\cos\theta = pE(1 - \cos\theta)$

$= 10^{-29} \times 10^{6}(1 - \cos 60°)$ J $= 0.5 \times 10^{-23}$ J

Work required for one mole of dipoles

$= W \times N = 0.5 \times 120^{-23} \times 6 \times 10^{23} = 3.0$ J

Heat released = Loss in P.E. = Work done = 3.0 J

Knowledge Plus

1. Potential due to an isolated positive charge is positive and due to an isolated negative charge is negative.
2. Potential is scalar quantity.
3. Potential gradient is a vector quantity.
4. The electric potential due to a charge q at its own location is not defined, (it is infinite).
5. To define electric potential at any point generally a point far away from the source charge is taken as the reference point. Such a point is taken at infinity.

Example 38. A molecule of a substance has permament dipole moment of 10^{-29} Cm. A mole of this substance is polarised by applying a strong electrostatic field of 10^6 Vm^{-1}. The direction of the field is suddenly changed by an angle of 60°. Calculate heat released by the substance.

Solution : A mole contains molecules or dipoles equal to Avogadro number

$$N = 6{\cdot}023 \times 10^{23}$$

Dipole moment of each molecule $(p) = 10^{-29}$ Cm

Total dipole moment $= Np = 6{\cdot}023 \times 10^{23} \times 10^{-29}$

$$= \mathbf{6{\cdot}023 \times 10^{-6}\ Cm}$$

Now, when the substance is completely polarised $\theta = 0$

i.e., dipole moment is along the directions of field

$\therefore$ Initial P.E. of system $U_i = -NpE\cos\theta_1$

$$= -6{\cdot}023 \times 10^{-6} \times 10^{6}\cos 0$$

$$= -(6{\cdot}023) = -\mathbf{6{\cdot}023\ J}$$

Secondly when the dipole moment makes an angle of 60° with E

$$\therefore \quad U_f = -Np\,E\cos 60^\circ$$

$$= -6{\cdot}023 \times 10^{-6} \times 10^{6} \times \cos 60^\circ$$

$$= -6{\cdot}023 \times \frac{1}{2} = -\mathbf{3{\cdot}012\ J}$$

$\therefore$ Heat released $= U_f - U_i$

$$= -3{\cdot}012 - (-6{\cdot}023)$$

$$= 6{\cdot}023 - 3{\cdot}012 = \mathbf{3{\cdot}011\ J}$$

Knowledge Plus

1. Electric potential energy is a scalar quantity. During calculation of its value, the value of various charges must be substituted with proper signs.
2. Potential energy of two like charges $(q_1q_2 > 0)$ is positive. Since the nature of electrostatic force is repulsive, so a positive amount of work has to be done against this force in bringing charge from infinity to finite separation.
3. Potential energy of two unlike charges is negative $(q_1q_2 < 0)$. Since electrostatic force is attractive here, so a positive amount of work has to be done against this force to take charge from a specific position to infinity. Conversely a negative amount of work is to be done so as to bring the charge from infinity to the present locations.
4. Positive potential energy implies that the work can be obtained by releasing the charges whereas negative potential energy indicates that an external agency will have to do the work to separate charges infinite distance apart.

Example 31. Two charges of magnitudes $5nC$ and $-2nC$ are placed at points (2 cm, 0, 0) and (x cm, 0, 0) in a region of space where there is no other external field. If the electrostatic potential energy of the system is $-0{\cdot}5\ \mu$J, what is the value of x ? *(C.B.S.E. 2008)*

Solution : Given $q_1 = 5nC = 5 \times 10^{-9}$ C,

$$q_2 = -2nC = -2 \times 10^{-9}\ C$$

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{(x-2)^2 + 0 + 0} = (x-2)\ cm$$

$$U = -5{\cdot}0\ \mu J = -5{\cdot}0 \times 10^{-6}\ J$$

But $$U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

$$r = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{U} = \frac{9\times10^9 \times 5\times10^{-9} \times -2\times10^{-9}}{-5\times10^{-6}}$$

$$= 18 \times 10^{-3}\ m = 1{\cdot}8\ cm$$

$$\therefore \quad x - 2 = 1{\cdot}8 \ \therefore\ x = \mathbf{3{\cdot}8\ cm}$$