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SETS

SECTION A NCERT EXERCISES

Exercise 1.1

1. Which of the following are sets ? Justify your answer.

(i) The collection of all the months of a year beginning with the letter J.

(ii) The collection of ten most talented writers of India.

(iii) A team of eleven best-cricket batsmen of the world.

(iv) The collection of all boys in your class.

(v) The collection of all natural numbers less than 100.

(vi) A collection of novels written by the writer Munshi Prem Chand.

(vii) The collection of all even integers.

(viii) The collection of questions in this chapter.

(ix) A collection of most dangerous animals of the world.

Sol. (i) It is a set. The set of months in a year beginning with J is {January, June, July}.

(ii) The collection of ten most talented writers of India is not a set, because the term 'most talented' is not well defined.

(iii) A team of eleven best-cricket batsmen of the world is not a set, because the term 'best-cricket batsman' is not well defined.

(iv) The collection of all boys in your class is a set, because the boys in your class are well-defined.

(v) The collection of all natural numbers less than 100 is a set, because natural numbers less than 100 are $\{1, 2, 3, \ldots, 99\}$.

(vi) The collection of novels written by the writer Munshi Prem Chand is a set, because this collection is well-defined.

(vii) The collection of all even integers is a set, because even integers are: $\{\ldots, -4, -2, 0, 2, 4, \ldots\}$.

(viii) The collection of questions in this chapter is a set, because the questions in this chapter are well-defined.

(ix) A collection of most dangerous animals of the world is not a set, because the term 'most dangerous animal' is not well-defined.

2. Let $A = \{1, 2, 3, 4, 5, 6\}$. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces:

(i) 5 ... A (ii) 8 ... A (iii) 0... A

(iv) 4 ... A (v) 2... A (vi) 10... A

Sol. (i) $5 \in A$ (ii) $8 \notin A$ (iii) $0 \notin A$

(iv) $4 \in A$ (v) $2 \in A$ (vi) $10 \notin A$

3. Write the following sets in roster form :

(i) $A = \{x : x$ is an integer and $-3 < x < 7\}$

(ii) $B = \{x : x$ is a natural number less than $6\}$

(iii) $C = \{x : x$ is a two-digit natural number such that the sum of its digits is $8\}$

(iv) $D = \{x : x$ is a prime number which is divisor of $60\}$

(v) E = The set of all letters in the word 'TRIGONOMETRY'

(vi) F = The set of all letters in the word 'BETTER'

Sol. (i) $A = \{-2, 1, 0, 1, 2, 3, 4, 5, 6\}$

(ii) $B = \{1, 2, 3, 4, 5\}$

(iii) $C = \{17, 26, 35, 44, 53, 62, 71, 80\}$

(iv) $D = \{2, 3, 5\}$

(v) $E = \{T, R, I, G, O, N, M, E, Y\}$

(vi) $F = \{B, E, T, R\}$.

4. Write the following sets in the set-builder form:

(i) {3, 6, 9, 12} **(ii) {2, 4, 8, 16, 32}**

(iii) {5, 25, 125, 625} **(iv) {2, 4, 6, ...}**

(v) {1, 4, 9, ... 100}

Sol. (i) $A = \{x : x = 3n, n \in N \text{ and } 1 \le n \le 4\}$

(ii) $B = \{x : x = 2^n,\ n \in N \text{ and } 1 \le n \le 5\}$

(iii) $C = \{x : x = 5^n,\ n \in N \text{ and } 1 \le n \le 4\}$

(iv) $D = \{x : x \text{ is an even natural number}\}$

(v) $E = \{x : x = n^2, n \in N \text{ and } 1 \le n \le 10\}$

5. List all the elements of the following sets :

(i) $A = \{x : x \text{ is an odd natural number}\}$

(ii) $B = \left\{x : x \text{ is an integer}, -\frac{1}{2} < x < \frac{9}{2}\right\}$

(iii) $C = \{x : x \text{ is an integer}, x^2 \le 4\}$

(iv) $D = \{x : x$ **is a letter in the word "LOYAL"}**

(v) $E = \{x : x$ **is a month of a year not having 31 days}**

(vi) $F = \{x : x$ **is a consonant in the English alphabet which precedes** $k\}$.

Sol. (i) $A = \{1, 3, 5, ...\}$

(ii) $B = \{0, 1, 2, 3, 4\}$

(iii) $C = \{-2, -1, 0, 1, 2\}$

(iv) $D = \{L, O, Y, A\}$

(v) $E = \{$February, April, June, September, November$\}$

(vi) $F = \{b, c, d, f, g, h, j\}$.

6. Match each of the set on the left in the roster form with the same set on the right described in set-builder form:

(i) {1, 2, 3, 6}	**(a) {x : x is a prime number and a divisor of 6}**
(ii) {2, 3}	**(b) {x : x is an odd natural number less than 10}**
(iii) {M,A,T,H,E,I,C,S}	**(c) {x : x is a natural number and divisor of 6}**
(iv) {1, 3, 5, 7, 9}	**(d) {x : x is a letter of the word 'MATHEMATICS'}**

Sol.

(i) {1, 2, 3, 6}	(c) {x : x is a natural number and divisor of 6}
(ii) {2, 3}	(a) {x : x is a prime number and divisor of 6}
(iii) {M,A,T,H,E,I,C,S}	(d) {x : x is a letter of the word MATHEMATICS'}
(iv) {1, 3, 5, 7, 9}	(b) {x : x is an odd natural number less than 10}

EXERCISE 1.2

1. Which of the following are examples of the null set :

(i) Set of odd natural numbers divisible by 2.

(ii) Set of even prime numbers.

(iii) $\{x : x$ **is a natural numbers,** $x < 5$ **and** $x > 7\}$

(iv) $\{y : y$ **is a point common to any two parallel lines}**

Sol. (i) Set of odd natural numbers divisible by 2 is a null set, because no odd natural number is divisible by 2.

(ii) We know that 2 is the only even prime number. Therefore, set of even prime numbers is not a null set .

(iii) $\{x : x \in N, x < 5 \text{ and } x > 7\}$ is a null set, because there is no natural number which is less than 5 and greater than 7 simultaneously.

(iv) $\{y : y$ is a point common to any two parallel lines$\}$ is a null set, because there is no point common to any two parallel lines.

2. Which of the following sets are finite or infinite :

(i) The set of months of a year

(ii) {1, 2, 3, ...} **(iii) {1, 2, 3, ..., 99, 100}**

(iv) The set of positive integers greater than 100.

(v) The set of prime numbers less than 99.

Sol. (i) Finite (ii) Infinite (iii) Finite (iv) Infinite (v) Finite.

3. State whether each of the following set is finite or infinite :

(i) The set of lines which are parallel to the x**-axis.**

(ii) The set of letters in the English alphabet.

(iii) The set of numbers which are multiple of 5.

(iv) The set of animals living on the earth.

(v) The set of circles passing through the origin (0, 0).

Sol. (i) Infinite (ii) Finite (iii) Infinite (iv) Finite (v) Infinite

4. In the following, state whether A = B or not :

(i) $A = \{a, b, c, d\}$ $B = \{d, c, b, a\}$

(ii) $A = \{4, 8, 12, 16\}$ $B = \{8, 4, 16, 18\}$

(iii) $A = \{2, 4, 6, 8, 10\}$

$B = \{x : x \text{ is positive even integer and } x \le 10\}$

(iv) $A = \{x : x \text{ is a multiple of } 10\}$

$B = \{10, 15, 20, 25, 30, ...\}$

Sol. (i) Yes, $A = B$ (ii) No, $A \neq B$ (iii) Yes, $A = B$ (iv) No, $A \neq B$.

5. Are the following pairs of sets equal? Gives reasons.

(i) $A = \{2, 3\}$, $B = \{x : x \text{ is solution of } x^2 + 5x + 6 = 0\}$

$A = \{x : x \text{ is a letter in the word 'FOLLOW'}\}$

(ii) $B = \{y : y \text{ is a letter in the word 'WOLF'}\}$

Sol. (i) No, $A = \{2, 3\}$

But solution of $x^2 + 5x + 6 = 0$

$\Rightarrow x^2 + 3x + 2x + 6 = 0$

$\Rightarrow x(x+3) + 2(x+3) = 0$

$\Rightarrow (x+3)(x+2) = 0$

$\Rightarrow x + 3 = 0$ or $x + 2 = 0$

$\Rightarrow x = -3$ or $x = -2$

$\Rightarrow B = \{-2, -3\}$

Hence, $A \neq B$

(ii) Yes, $A = \{F, O, L, W\}$

and $B = \{W, O, L, F\}$

Hence, $A = B$

6. From the sets given below, select equal sets :

$A = \{2, 4, 8, 12\}$, $B = \{1, 2, 3, 4\}$,

$C = \{4, 8, 12, 14\}$, $D = \{3, 1, 4, 2\}$,

$E = \{-1, 1\}$, $F = \{0, a\}$,

$G = \{1, -1\}$, $H = \{0, 1\}$

Sol. (i) $B = D$, (ii) $E = G$

EXERCISE 1.3

1. Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces :

(i) $\{2, 3, 4\}$ ... $\{1, 2, 3, 4, 5\}$

(ii) $\{a, b, c\}$ ... $\{b, c, d\}$

(iii) $\{x : x$ is a student of class XI of your school$\}$ ... $\{x : x$ is a student of your school$\}$

(iv) $\{x : x$ is a circle in the plane$\}$ ... $\{x : x$ is a circle in the same plane with radius 1 unit$\}$

(v) $\{x : x$ is a triangle in a plane$\}$ ... $\{x : x$ is a rectangle in the plane$\}$

(vi) $\{x : x$ is an equilateral triangle in a plane$\}$ ... $\{x : x$ is a triangle in the same plane$\}$

(vii) $\{x : x$ is an even natural number$\}$ ... $\{x : x$ is an integer$\}$.

Sol. (i) $\{2, 3, 4\} \subset \{1, 2, 3, 4, 5\}$

(ii) $\{a, b, c\} \subset \{b, c, d\}$

(iii) $\{x : x$ is a student of class XI of your school$\} \subset \{x : x$ is a student of your school$\}$

(iv) $\{x : x$ is a circle in the plane$\} \subset \{x : x$ is a circle in the same plane with radius 1 unit$\}$

(v) $\{x : x$ is a triangle in a plane$\} \subset \{x : x$ is a rectangle in the same plane$\}$

(vi) $\{x : x$ is an equilateral triangle in a plane$\} \subset \{x : x$ is a triangle in the same plane$\}$

(vii) $\{x : x$ is an even natural number$\} \subset \{x : x$ is an integer$\}$

2. Examine whether the following statements are true or false :

(i) $\{a, b\} \not\subset \{b, c, a\}$

(ii) $\{a, e\} \subset \{x : x$ is a vowel in the English alphabet$\}$

(iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$

(iv) $\{a\} \subset \{a, b, c\}$ **(v)** $\{a\} \in \{a, b, c\}$

(vi) $\{x : x$ is an even natural number less than $6\} \subset \{x : x$ is a natural number which divides $36\}$

Sol. (i) Since the element of the set $\{a, b\}$ are also present in the set $\{b, c, a\}$, therefore $\{a, b\} \subset \{b, c, a\}$ is **false**.

(ii) Vowels in the English alphabets are $\{a, e, i, o, u\}$.

$\{a, e\} \subset \{x : x$ is a vowel in the English alphabet$\}$ is **true**.

(iii) Since, all the elements of the set $\{1, 2, 3\}$ are not present in the set $\{1, 3, 5\}$, therefore $\{1, 2, 3\} \subset \{1, 3, 5\}$ is **false**.

(iv) Since, the element of the set $\{a\}$ is present in the set $\{a, b, c\}$, therefore $\{a\} \subset \{a, b, c\}$ is **true**.

(v) Since, $a \in \{a, b, c\}$ is but not $\{a\} \in \{a, b, c\}$, therefore $\{a\} \in \{a, b, c\}$ **false**.

(vi) $\{x : x$ is an even natural number less than $6\}$ $= \{2, 4\}$ and $\{x : x$ is a natural number which divides $36\}$.
$= \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$
Since, every element of the set $\{2, 4\}$ is contained in the set
$\{1, 2, 3, 4, 6, 9, 12, 18, 36\}$
Hence, the given statement is **true**.

3. Let $A = \{1, 2 \{3, 4\}, 5\}$. Which of the following statements are incorrect and why ?

(i) $\{3, 4\} \subset A$ **(ii)** $\{3, 4\} \in A$
(iii) $\{\{3, 4\}\} \subset A$ **(iv)** $1 \in A$
(v) $1 \subset A$ **(vi)** $\{1, 2, 5\} \subset A$
(vii) $\{1, 2, 5\} \in A$ **(viii)** $\{1, 2, 3\} \subset A$
(ix) $\phi \in A$ **(x)** $\phi \subset A$
(xi) $\{\phi\} \subset A$

Sol. (i) **Incorrect** as $\{3, 4\}$ is an element of the set A.
(ii) **Correct** as $\{3, 4\}$ is an element of the set A.
(iii) **Correct** as $\{3, 4\}$ is contained in the set A.
(iv) **Correct** as 1 is an element of the set A.
(v) **Incorrect** as 1 belongs to the set A, but not contained in A.
(vi) **Correct** as all the elements of the set $\{1, 2, 5\}$ are present in the set A.
(vii) **Incorrect** as $\{1, 2, 5\}$ is not the element of the set A.
(viii) **Incorrect** as 3 is not an element of the set A.
(ix) **Incorrect** as ϕ is not an element of the set A.
(x) **Correct** as empty set is a subset of every set.
(xi) **Incorrect** as $\{\phi\}$ is not contained in the set A.

4. Write down all the subsets of the following sets :

(i) $\{a\}$ **(ii)** $\{a, b\}$
(iii) $\{1, 2, 3\}$ **(iv)** ϕ

Sol. (i) $\{a\}$, it has two possible subsets ϕ, $\{a\}$.
(ii) $\{a, b\}$, it has four possible subsets ϕ, $\{a\}$, $\{b\}$, $\{a, b\}$.
(iii) $\{1, 2, 3\}$, it has eight possible subsets : ϕ, $\{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$.
(iv) ϕ, it has only one subset which is $\{\phi\}$.

5. How many elements has $P(A)$, if $A = \phi$?

Sol. If $A = \phi$, then it has only one subset which is $\{\phi\}$. Hence, $P(A)$ has only one element.

6. Write the following as intervals :

(i) $\{x : x \in R, -4 < x \le 6\}$
(ii) $\{x : x \in R, -12 < x < -10\}$
(iii) $\{x : x \in R, 0 \le x < 7\}$
(iv) $\{x : x \in R, 3 \le x \le 4\}$

Sol. (i) $(-4, 6]$ (ii) $(-12, -10)$ (iii) $[0, 7)$ (iv) $[3, 4]$

7. Write the following intervals in set-builder form :

(i) $(-3, 0)$ **(ii)** $[6, 12]$
(iii) $(6, 12]$ **(iv)** $[-23, 5)$

Sol. (i) $(-3, 0) = \{x : x \in R, -3 < x < 0\}$
(ii) $[6, 12] = \{x , x \in R, 6 \le x \le 12\}$
(iii) $(6, 12] = \{x : x \in R, 6 < x \le 12\}$
(iv) $[-23, 5] = \{x : x \in R, -23 \le x < 5\}$

8. What universal set(s) would you propose for each of the following :

(i) The set of right triangles.
(ii) The set of isosceles triangles.

Sol. (i) The set of triangles.
(ii) The set of triangles.

9. Given the sets $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$, which of the following may be considered as universal set (s) for all the three sets A, B and C.

(i) $\{0, 1, 2, 3, 4, 5, 6\}$
(ii) $\{\phi\}$
(iii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
(iv) $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Sol. (iii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ may be considered as universal set for all the three sets A, B and C.

EXERCISE 1.4

1. Find the union of each of the following pairs of sets:

(i) $X = \{1, 3, 5\}$, $Y = \{1, 2, 3\}$
(ii) $A = \{a, e, i, o, u\}$, $B = \{a, b, c\}$
(iii) $A = \{x : x$ is a natural number and multiple of $3\}$
$B = \{x : x$ is a natural number less than $6\}$.
(iv) $A = \{x : x$ is a natural number and $1 < x \le 6\}$
$B = \{x : x$ is a natural number and $6 < x < 10\}$
(v) $A = \{1, 2, 3\}$, $B = \phi$

Sol. (i) $X \cup Y = \{1, 3, 5\} \cup \{1, 2, 3\}$
$= \{1, 2, 3, 5\}$
(ii) $A \cup B = \{a, e, i, o, u\} \cup \{a, b, c\}$
$= \{a, b, c, e, i, o, u\}$
(iii) $A = \{3, 6, 9, ...\}$
$B = \{1, 2, 3, 4, 5\}$

Hence,

$A \cup B = \{3, 6, 9, 12, ...\} \cup \{1, 2, 3, 4, 5\}$

$= \{1, 2, 3, 4, 5, 6, 9, 12, ...\}$

$= \{x : x = 1 = 1, 2, 3, 4, 5 \text{ or a multiple of } 3\}$

(iv) $A = \{2, 3, 4, 5, 6\}$ $B = \{7, 8, 9\}$

$\therefore$ $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$

$= \{x : 1 < x < 10, x \in N\}$

(v) $A \cup B = \{1, 2, 3\} \cup \{\} = \{1, 2, 3\}$.

2. Let $A = \{a, b\}$, $B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Sol. Yes, $A \subset B$. Because, every element of A is also an element of B, therefore, A is a subset of B.

$A \cup B = \{a, b\} \cup (a, b, c) = \{a, b, c\}$

3. If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Sol. Since, A is a subset of B, therefore, every element of set A is contained in the set B.

Hence, $A \cup B = B$.

4. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$; find

(i) $A \cup B$ (ii) $A \cup C$ (iii) $B \cup C$

(iv) $B \cup D$ (v) $A \cup B \cup C$

(vi) $A \cup B \cup D$ (vii) $B \cup C \cup D$.

Sol. (i) $A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6\}$

(ii) $A \cup C = \{1, 2, 3, 4\} \cup \{5, 6, 7, 8\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(iii) $B \cup C = \{3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$

$= \{3, 4, 5, 6, 7, 8\}$

(iv) $B \cup D = \{3, 4, 5, 6\} \cup \{7, 8, 9, 10\}$

$= \{3, 4, 5, 6, 7, 8, 9, 10\}$

(v) $A \cup B \cup C$

$= \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$

(vi) $A \cup B \cup D$

$= \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\} \cup \{7, 8, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(vii) $B \cup C \cup D$

$= \{3, 4, 5, 6\} \cup \{5, 6, 7, 8\} \cup \{7, 8, 9, 10\}$

$= \{3, 4, 5, 6, 7, 8, 9, 10\}$.

5. Find the intersection of each pair of sets of Q. 1, above.

Sol. (i) $X \cap Y = \{1, 3, 5\} \cap \{1, 2, 3\} = \{1, 3\}$

(ii) $A \cap B = \{a, e, i, o, u\} \cap \{a, b, c\} = \{a\}$

(iii) $A \cap B = \{3, 6, 9, ...\} \cap \{1, 2, 3, 4, 5\} = \{3\}$

(iv) $A \cap B = \{2, 3, 4, 5, 6\} \cap \{7, 8, 9\} = \phi$

(v) $A \cap B = \{1, 2, 3\} \cap \phi = \phi$.

6. If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$; find

(i) $A \cap B$ (ii) $B \cap C$

(iii) $A \cap C \cap D$ (iv) $A \cap C$

(v) $B \cap D$ (vi) $A \cap (B \cup C)$

(vii) $A \cap D$ (viii) $A \cap (B \cup D)$

(ix) $(A \cap B) \cap (B \cup C)$

(x) $(A \cup D) \cap (B \cup C)$.

Sol. (i) $A \cap B = \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13\}$

$= \{7, 9, 11\}$

(ii) $B \cap C = \{7, 9, 11, 13\} \cap \{11, 13, 15\}$

$= \{11, 13\}$

(iii) $A \cap C \cap D$

$= \{3, 5, 7, 9, 11\} \cap \{11, 13, 15\} \cap \{15, 17\}$

$= \{11\} \cap \{15, 17\} = \phi$

(iv) $A \cap C = \{3, 5, 7, 9, 11\} \cap \{11, 13, 15\} = \{11\}$

(v) $B \cap D = \{7, 9, 11, 13\} \cap \{15, 17\} = \phi$

(vi) $A \cap (B \cup C)$

$= \{3, 5, 7, 9, 11\} \cap [\{7, 9, 11, 13\} \cup \{11, 13, 15\}]$

$= \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13, 15\}$

$= \{7, 9, 11\}$

(vii) $A \cap D = \{3, 5, 7, 9, 11\} \cap \{15, 17\} = \phi$

(viii) $A \cap (B \cup D)$

$- \{3, 5, 7, 9, 11\} \cap [\{7, 9, 11, 13\} \cup \{15, 17\}]$

$= \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13, 15, 17\}$

$= \{7, 9, 11\}$

(ix) $(A \cap B) \cap (B \cup C)$

$= [\{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13\}] \cap [\{7, 9, 11, 13\} \cup \{11, 13, 15\}]$

$= \{7, 9, 11,\} \cap \{7, 9, 11, 13, 15\}$

$= \{7, 9, 11\}$.

(x) $(A \cup D) \cap (B \cup C)$

$= [\{3, 5, 7, 9, 11\} \cup \{15, 17\}] \cap [\{7, 9, 11, 13\} \cup \{11, 13, 15\}]$

$= \{3, 5, 7, 9, 11, 15, 17\} \cap \{7, 9, 11, 13, 15\}$

$= \{7, 9, 11, 15\}$.

7. If $A = \{x : x$ is a natural number$\}$,
$B = \{x : x$ is an even natural number$\}$
$C = \{x : x$ is an odd natural number$\}$ and
$D = \{x : x$ is a prime number$\}$, find

(i) $A \cap B$ (ii) $A \cap C$

(iii) $A \cap D$ (iv) $B \cap C$

(v) $B \cap D$ (vi) $C \cap D$

Sol. Given, $A = \{1, 2, 3, 4, ...\}$

$B = \{2, 4, 6, 8, ...\}$

$C = \{1, 3, 5, 7, ...\}$

and $D = \{2, 3, 5, 7, 11, 13, ...\}$

(i) $A \cap B = \{1, 2, 3, 4, ...\} \cap \{2, 4, 6, 8, ...\}$
$= \{2, 4, 6, 8, ...\} = B$

(ii) $A \cap C = \{1, 2, 3, 4, ...\} \cap \{1, 3, 5, 7\}$
$= \{1, 3, 5, 7, ...\} = C$

(iii) $A \cap D = \{1, 2, 3, 4, ...\} \cap \{2, 3, 5, 7, 11, 13, ...\}$
$= \{2, 3, 5, 7, 11, 13, ...\} = D$

(iv) $B \cap C = \{2, 4, 6, 8, ...\} \cap \{1, 3, 5, 7, ...\} = \phi$

(v) $B \cap D = \{2, 4, 6, 8, ...\} \cap \{2, 3, 5, 7, 11, 13, ...\} = \{2\}$

(vi) $C \cap D = \{1, 3, 5, 7, ...\} \cap \{2, 3, 5, 7, 11, 13, ...\}$
$= \{3, 5, 7, 11, 13, ...\}$
$= \{x : x \text{ is an odd prime number}\}$.

8. Which of the following pairs of sets are disjoint:

(i) $\{1, 2, 3, 4\}$ and $\{x : x$ is a natural number and $4 \le x \le 6\}$.

(ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$

(iii) $\{x : x$ is an even integer$\}$ and $\{x : x$ is an odd integer$\}$

Sol. (i) $\{1, 2, 3, 4\}$ and $\{x : x$ is a natural number and $4 \le x \le 6\}$ i.e. $\{4, 5, 6\}$ are not disjoint sets as they have 4 as a common element.

(ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$ are not disjoint sets, because they have common element e.

(iii) $\{x : x$ is an even integer$\} = \{+2, +4, +6, ...\}$ and $\{x : x$ is an odd integer$\}$
$= \{\pm 1, \pm 3, \pm 5, ...\}$
are disjoint sets, because they have no common element.

9. If $A = \{3, 6, 9, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$, $D = \{5, 10, 15, 20\}$, find

(i) $A - B$ (ii) $A - C$
(iii) $A - D$ (iv) $B - A$
(v) $C - A$ (vi) $D - A$
(vii) $B - C$ (viii) $B - D$
(ix) $C - B$ (x) $D - B$
(xi) $C - D$ (xii) $D - C$

Sol. (i) $A - B = \{3, 6, 9, 12, 15, 18, 21\} - \{4, 8, 12, 16, 20\} = \{3, 6, 9, 15, 18, 21\}$

(ii) $A - C = \{3, 6, 9, 12, 15, 18, 21\} - \{2, 4, 6, 8, 10, 12, 14, 16\}$
$= \{3, 9, 15, 18, 21\}$

(iii) $A - D = \{\{3, 6, 9, 12, 15, 18, 21\} - \{5, 10, 15, 20\}$
$= \{3, 6, 9, 12, 18, 21\}$.

(iv) $B - A = \{4, 8, 12, 16, 20\} - \{3, 6, 9, 12, 15, 18, 21\}$
$= \{4, 8, 16, 20\}$.

(v) $C - A = \{2, 4, 6, 8, 10, 12, 14, 16\} - \{3, 6, 9, 12, 15, 18, 21\}$
$= \{2, 4, 8, 10, 14, 16\}$

(vi) $D - A = \{5, 10, 15, 20\} - \{3, 6, 9, 12, 15, 18, 21\}$
$= \{5, 10, 20\}$

(vii) $B - C = \{4, 8, 12, 16, 20\} - \{2, 4, 6, 8, 10, 12, 14, 16\} = \{20\}$

(viii) $B - D = \{4, 8, 12, 16, 20\} - \{5, 10, 15, 20\}$
$= \{4, 8, 12, 16\}$.

(ix) $C - B = \{2, 4, 6, 8, 10, 12, 14, 16\} - \{4, 8, 12, 16, 20\}$
$= \{2, 6, 10, 14\}$.

(x) $D - B = \{5, 10, 15, 20\} - \{4, 8, 12, 16, 20\}$
$= \{5, 10, 15\}$

(xi) $C - D = \{2, 4, 6, 8, 12, 10, 14, 16\} - \{5, 10, 15, 20\}$
$= \{2, 4, 6, 8, 12, 14, 16\}$

(xii) $D - C = \{5, 10, 15, 20\} - \{2, 4, 6, 8, 10, 12, 14, 16\}$
$= \{5, 15, 20\}$

10. If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find
(i) $X - Y$ (ii) $Y - X$ (iii) $X \cap Y$

Sol. (i) $X - Y = \{a, b, c, d\} - \{f, b, d, g\} = \{a, c\}$

(ii) $Y - X = \{f, b, d, g\} - \{a, b, c, d\} = \{f, g\}$

(iii) $X \cap Y = \{a, b, c, d\} \cap \{f, b, d, g\} = \{b, d\}$

11. If R is the set of real numbers and Q is the set of rational numbers, then what is $R - Q$?

Sol. $R - Q = \{$set of real numbers$\} - \{$set of rational numbers$\}$
$= \{$set of rational numbers and set of irrational numbers$\} - \{$set of rational numbers$\}$
$=$ set of irrational numbers

12. State whether each of the following statement is true or false. Justify your answer:

(i) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets.

(ii) $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets.

(iii) $\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$ are disjoint sets

(iv) $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets

Sol. (i) False.
$\{2, 3, 4, 5\} \cap \{3, 6\} = \{3\} \ne \phi$
The given sets are not disjoint.

(ii) False.
$\{a, e, i, o, u\} \cap \{a, b, c, d\} = \{a\} \ne \phi$
The given sets are not disjoint.

(iii) True.
$\{2, 6, 10, 14\} \cap \{3, 7, 11, 15\} = \phi$
The given sets are disjoint.

(iv) True.
$\{2, 6, 10\} \cap \{3, 7, 11\} = \phi$
The given sets are disjoint.

EXERCISE 1.5

1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. Find, (i) A', (ii) B' (iii) $(A \cup C)'$ (iv) $(A \cup B)'$ (v) $(A')'$ (vi) $(B - C)'$

Sol. Here, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$A = \{1, 2, 3, 4\}$
$B = \{2, 4, 6, 8\}$
and $C = \{3, 4, 5, 6\}$

(i) $A' = U - A = \{5, 6, 7, 8, 9\}$

(ii) $B' = U - B = \{1, 3, 5, 7, 9\}$

(iii) $A \cup C = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$
$= \{1, 2, 3, 4, 5, 6\}$
Hence $(A \cup C)' = U - (A \cup C) = \{7, 8, 9\}$

(iv) $A \cup B = \{1, 2, 3, 4\} \cup \{2, 4, 6, 8\}$
$= \{1, 2, 3, 4, 6, 8\}$
Hence $(A \cup B)' = U - (A \cup B) = \{5, 7, 9\}$

(v) $A' = U - A = \{5, 6, 7, 8, 9\}$
Hence $(A')' = U - A' = \{1, 2, 3, 4\}$

(vi) $B - C = \{2, 4, 6, 8\} - \{3, 4, 5, 6\} = \{2, 8\}$
Hence $(B - C)' = U - (B - C) = \{1, 3, 4, 5, 6, 7, 9\}$

2. If $U = \{a, b, c, d, e, f, g, h\}$ find the complements of the following sets :

(i) $A = \{a, b, c\}$ (ii) $B = \{d, e, f, g\}$
(iii) $C = \{a, c, e, g\}$ (iv) $D = \{f, g, h, a\}$

Sol. Here, $U - \{a, b, c, d, e, f, g, h\}$

(i) $A' = U - A = \{a, b, c, d, e, f, g, h\} - \{a, b, c\}$
$= \{d, e, g, g, h\}$

(ii) $B' = U - B = \{a, b, c, d, e, f, g, h\} - \{d, e, f, g\}$
$= \{a, b, c, h\}$

(iii) $C' = U - C = \{a, b, c, d, e, f, g, h\} - \{a, c, e, g\}$
$= \{b, d, f, h\}$

(iv) $D' = U - D = \{a, b, c, d, e, f, g, h\} - \{f, g, h, a\}$
$= \{b, c, d, e\}$.

3. Taking the set of natural numbers as the universal set, write down the complements of the following sets :

(i) $\{x : x$ is an even natural number$\}$
(ii) $\{x : x$ is an odd natural number$\}$
(iii) $\{x : x$ is a positive multiple of $3\}$
(iv) $\{x : x$ is a prime number$\}$
(v) $\{x : x$ is a natural number divisible by 3 and $5\}$
(vi) $\{x : x$ is a perfect square$\}$
(vii) $\{x : x$ is a perfect cube$\}$
(viii) $\{x : x + 5 = 8\}$
(ix) $\{x : 2x + 5 = 9\}$
(x) $\{x : x \geq 7\}$
(xi) $\{x : x \in N$ and $2x + 1 > 10\}$

Sol. (i) $\{x : x$ is an odd natural number$\}$

(ii) $\{x : x$ is an even natural number$\}$

(iii) $\{x : x \in N$ and x is not a multiple of $3\}$

(iv) $\{x : x$ is a positive composite number and $x = 1\}$

(v) $\{x : x \in n\ N$ and x is neither a multiples of 3 nor of $5\}$.

(vi) $\{x : x \in N$ and x is not a perfect square$\}$

(vii) $\{x : x \in N$ and x is not a perfect cube$\}$

(viii) $\{x : x \in N$ and $x \neq 3\}$
$[x + 5 = 8 \Rightarrow x = 8 - 5 = 3]$

(ix) $\{x : x \in \mathrm{N}$ and $x \neq 2\}$
$[2x + 5 = 9 \Rightarrow 2x = 9 - 5 \Rightarrow 2x = 4 \Rightarrow x = 2]$

(x) $\{x : x \in N$ and $x < 7\} = \{1, 2, 3, 4, 5, 6\}$,

(xi) $\left\{x : x \in N \text{ and } x \leq \dfrac{9}{2}\right\}$

$[2x + 1 > 10 \Rightarrow 2x > 10 - 1 \Rightarrow 2x = 9 \Rightarrow x > 9/2]$

4. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$. Verify that

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Sol. (i) $A \cup B = \{2, 4, 6, 8\} \cup \{2, 3, 5, 7\}$
$= \{2, 3, 4, 5, 6, 7, 8\}$
L.H.S. $= (A \cup B)' = U - (A \cup B)$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 3, 4, 5, 6, 7, 8\}$
$= \{1, 9\}$
Now, $A' = U - A$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 4, 6, 8\}$
$= \{1, 3, 5, 7, 9\}$
and $B' = U - B$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 3, 5, 7\}$
$= \{1, 4, 6, 8, 9\}$
R.H.S. $= A' \cap B'$
$= \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\} = \{1, 9\}$
L.H.S. = R.H.S.
Hence $(A \cup B)' = A' \cap B'$ is verified.

(ii) $A \cap B = \{2, 4, 6, 8\} \cap \{2, 3, 5, 7\} = \{2\}$
L.H.S. $= (A \cap B)' = U - (A \cap B)$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2\}$
$= \{1, 3, 4, 5, 6, 7, 8, 9\}$

R.H.S. $= A' \cup B'$

$= \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\}$

$= \{1, 3, 4, 5, 6, 7, 8, 9\}$

L.H.S = R.H.S.

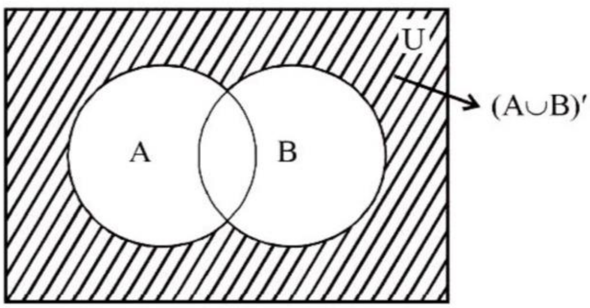
Hence, $(A \cap B) = A' \cup B'$ is verified.

5. Draw appropriate Venn diagram for each of the following :

(i) $(A \cup B)'$ **(ii) $A' \cap B'$**

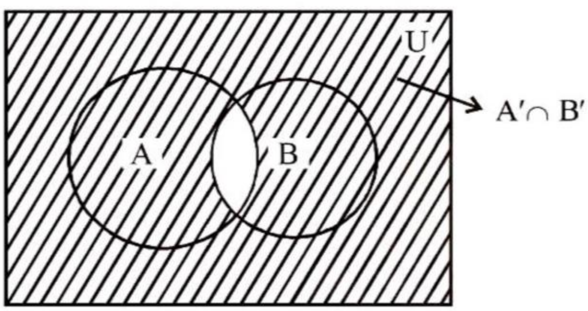
(iii) $(A \cap B)'$ **(iv) $A' \cup B'$.**

Sol. (i) $(A \cup B)'$



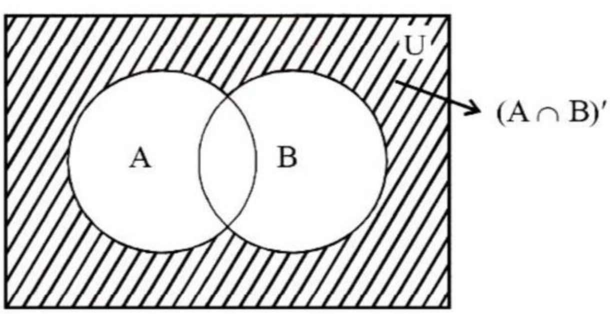
The shaded region indicates $(A \cup B)'$.

(ii) $A' \cap B'$



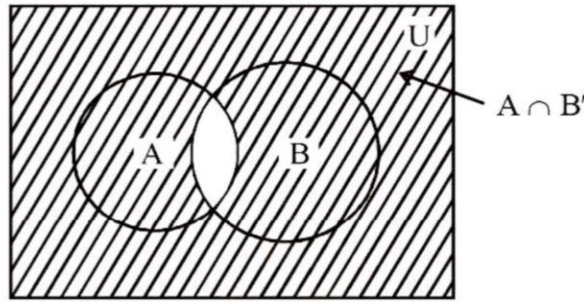
The shaded region indicates $A' \cap B'$.

(iii) $(A \cap B)'$



The shaded region indicates $(A \cap B)'$.

(iv) $A' \cup B'$



The shaded region indicates $A' \cup B'$.

All shaded region formed by all horizontal and vertical lines is $A' \cup B'$.

6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60°, what is A'?

Sol. A is the set of triangles in which no triangle is equilateral.

Hence A' is the set of all equilateral triangles.

7. Fill in the blanks to make each of the following a true statement :

(i) $A \cup A' =$ .... **(ii) $\phi' \cap A =$ ....**

(iii) $A \cap A' =$ .... **(iv) $U' \cap A =$ ....**

Sol. (i) $A \cup A' = U$ (ii) $\phi' \cap A = U \cap A = A$

(iii) $A \cap A' = \phi$ (iv) $U' \cap A = \phi \cap A = \phi$

EXERCISE 1.6

1. If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$, find $n(X \cap Y)$.

Sol. We know that

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$\Rightarrow 38 = 17 + 23 - n(X \cap Y)$.

$\Rightarrow 38 = 17 + 23 - n(X \cap Y)$

$\Rightarrow n(X \cap Y) = 40 - 38 = 2$.

2. If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements; how many elements does $X \cap Y$ have?

Sol. Here $n(X) = 8$ and $n(Y) = 15$

and $n(X \cup Y) = 18$

$\therefore n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$\Rightarrow 18 = 8 + 15 - n(X \cap Y)$

$\Rightarrow n(X \cap Y) = 23 - 18 = 5$.

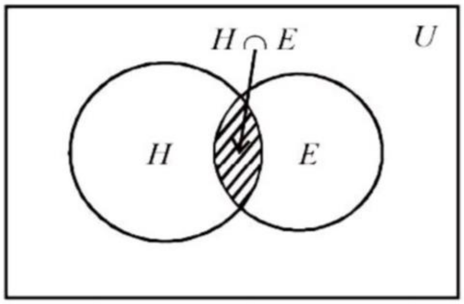
3. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

Sol. Let H = Set of people who can speak Hindi and E = Set of people who can speak English.

Then, $n(H) = 250$ $n(E) = 200$

and $n(H \cup E) = 400$

Now, $n(H \cup E) = n(H) + n(E) - n(H \cap E)$
$\Rightarrow 400 = 250 + 200 - n(H \cap E)$
$\Rightarrow n(H \cap E) = 450 - 400 = 50.$



Hence, 50 people can speak both Hindi and English.

4. If S and T are two sets such that S has 21 elements, T has 32 elements, and $S \cap T$ has 11 elements, how many elements does $S \cup T$ have?

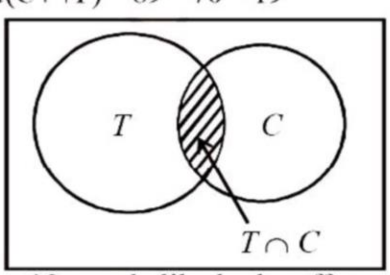
Sol. Here, $n(S) = 21$ and $n(T) = 32$
and $n(S \cap T) = 11$
Now, $n(S \cup T) = n(S) + n(T) - n(S \cap T)$
$= 21 + 32 - 11 = 53 - 11 = 42.$

5. If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?

Sol. Here, $n(X) = 40$
$n(X \cup Y) = 60$
$n(X \cap Y) = 10$
Now, $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$
$\Rightarrow 60 = 40 + n(Y) - 10$
$\Rightarrow n(Y) = 60 - 40 + 10 = 20 + 10 = 30.$

6. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?

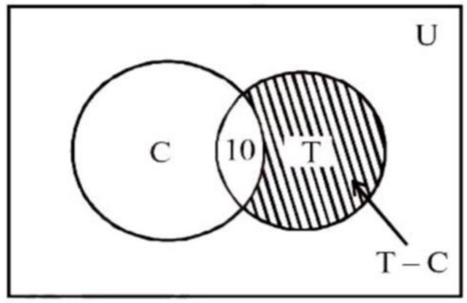
Sol. Let C = Set of people who like coffee and T = Set of people who like tea.
Then, $n(C \cup T) = 70$, $n(C) = 37$.
$n(T) = 52$, $n(C \cap T) = ?$
Now, we know that
$n(C \cup T) = n(C) + n(T) - n(C \cap T)$
$\Rightarrow 70 = 37 + 52 - n(C \cap T)$
$\Rightarrow n(C \cap T) = 89 - 70 = 19$



Hence, 19 people like both coffee and tea.

7. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

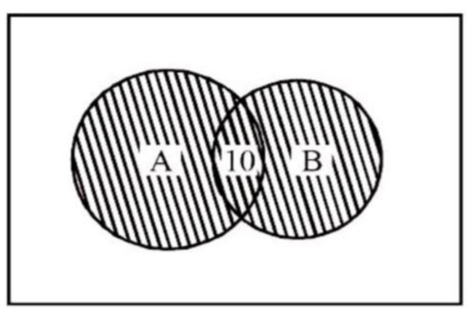
Sol. Let C = the set of people who like cricket and T = the set of people who like tennis.
Then, $n(C \cup T) = 65$, $n(C) = 40$
$n(C \cap T) = 10$



We know that
$$n(C \cup T) = n(C) + n(T) - n(C \cap T)$$
$\Rightarrow$ $65 = 40 + n(T) - 10$
$\Rightarrow$ $n(T) = 65 - 30 = 35$
Number of people who like only tennis
$= n(T) - n(C \cap T)$
$= 35 - 10 = 25$
Hence, number of people who like tennis is 35 and number of people who like tennis only is 25.

8. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Sol. Let F = the set of people who speak French and S = the set of people who speak Spanish
Then, $n(F) = 50$, $n(S) = 20$, $n(F \cap S) = 10$
As $n(F \cup S) = n(F) + n(S) - n(F \cap S)$
$= 50 + 20 - 10 = 60$



Hence, 60 people speak at least one of these two languages.

MISCELLANEOUS EXERCISE

1. Decide, among the following sets which sets are subsets of one and another
$A = \{x : x \in R$ and x satisfy $x^2 - 8x + 12 = 0\}$, $B = \{2, 4, 6\}$, $C = \{2, 4, 6, 8, ...\}$, $D = \{6\}$

Sol. We have $A = \{2, 6\}$,(since, only $x = 2$, and $x = 6$, satisfy the equation $x^2 - 8x + 12 = 0$)
$B = \{2, 4, 6\}$, $C = \{2, 4, 6, 8, ...\}$ and $D = \{6\}$
Every element of A is in B and C;
$\therefore$ $A \subset B$ and $A \subset C$
Again every element of B is in C; $\therefore$ $B \subset C$
Also, every element of D is in A, B and C;
$\therefore$ $D \subset A$, $D \subset B$ and $D \subset C$.

2. In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.
(i) If $x \in A$ and $A \in B$, then $x \in B$
(ii) If $A \subset B$ and $B \in C$, then $A \in C$
(iii) If $A \subset B$ and $B \subset C$, then $A \subset C$
(iv) If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$
(v) If $x \in A$ and $A \not\subset B$, then $x \in B$
(vi) If $A \subset B$ and $x \notin B$, then $x \notin A$

Sol. (i) **False.** Let $A = \{1\}$, $B = \{\{1\}, 2\}$
$\therefore$ $1 \in A$ and $A \in B$ but $1 \notin B$
So, $x \in A$ and $A \in B$ need not imply that $x \in B$

(ii) **False.** Let $A = \{1\}$, $B = \{1, 2\}$ and $C = \{\{1, 2\}, 3\}$
$\therefore$ $A \subset B$ and $B \in C$ but $A \notin C$
Thus, $A \subset B$ and $B \in C$ need not imply that $A \in C$.

(iii) **True.** Let $x \in A$, Then
$A \subset B \Rightarrow x \in B$, then $B \subset C \Rightarrow x \in C$
Thus, $A \subset B$ and $B \subset C \Rightarrow A \subset C$

(iv) **False.** Let $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{1, 2, 5\}$
Then $A \not\subset B$ and $B \not\subset C$ But $A \subset C$
Thus, $A \not\subset B$ and $B \not\subset C$ need not imply that $A \not\subset C$

(v) **False.** Let $A = \{1, 2\}$ and $B = \{2, 3, 4, 5\}$
Then $1 \in A$ and $A \not\subset B$ as $1 \notin B$
Thus, $x \in A$ and $A \not\subset B$ need not imply that $x \in B$

(vi) **True.** Let $A \subset B$, then
$x \in A \Rightarrow x \in B \Leftrightarrow x \notin B \Rightarrow x \notin A$

3. Let A, B, and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$

Sol. We have, $A \cup B = A \cup C$
$\Rightarrow (A \cup B) \cap C = (A \cup C) \cap C$
$\Rightarrow (A \cap C) \cup (B \cap C) = C$
$[\because (A \cup C) \cap C = C]$
$\Rightarrow (A \cap B) \cup (B \cap C) = C$ ...(i)
$[\because A \cap C = A \cap B]$
Again $A \cup B = A \cup C$
$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B$
$\Rightarrow B = (A \cap B) \cup (C \cap B)$
$[\because (A \cup B) \cap B = B]$
$(A \cap B) \cup (B \cap C) = B$ ...(ii)
$[\because C \cap B = B \cap C]$
From (i) and (ii) we get $B = C$

4. Show that the following four conditions are equivalent :
(i) $A \subset B$ (ii) $A - B = \phi$
(iii) $A \cup B = B$ (iv) $A \cap B = A$

Sol. (i) $\Leftrightarrow$ (ii) ; $A \subset B \Leftrightarrow$ All elements of A are in $B \Leftrightarrow A - B = \phi$
(ii) $\Leftrightarrow$ (iii) ; $A - B = \phi \Leftrightarrow$ All elements of A are in $B \Leftrightarrow A \cup B = B$
(iii) $\Leftrightarrow$ (iv) ; $A \cup B = B \Leftrightarrow$ All elements of A are in B
$\Leftrightarrow$ All the elements of A are common in A and $B \Leftrightarrow A \cap B = A$
Thus, all the four given conditions are equivalent.

5. Show that if $A \subset B$, then $C - B \subset C - A$.

Sol. $x \in C - B \Rightarrow x \in C$ but $x \notin B$
$\Rightarrow x \in C$ but $x \notin A \Rightarrow x \in C - A$
$\therefore C - B \subset C - A$

6. Assume that $P(A) = P(B)$. Show that $A = B$.

Sol. Let x be an arbitrary element of A. then, there exists a subset say X, of set A such that $x \in X$
Now, $X \subset A \Rightarrow X \in P(A)$
$\Rightarrow X \in P(B)$ $[\because P(A) = P(B)]$
$\Rightarrow X \subset B$ $\Rightarrow x \in B$
$[\because x \in X$ and $X \subset B \Rightarrow x \in B]$
Thus, $x \in A \Rightarrow x \in B$ $\therefore$ $A \subset B$ ...(i)
Now, let y be an arbitrary element of B
Then, there exists a subset say Y of set B such that $y \in Y$.
Now $Y \subset B \Rightarrow y \in P(B) \Rightarrow y \in P(A)$
$[\because P(A) = P(B)]$

$\Rightarrow Y \subset A \Rightarrow y \in A$. Thus $y \in B \Rightarrow y \in A$

$\therefore\ B \subset A$ ...(ii)

From (i) and (ii), we get $A = B$

7. Is it true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$? Justify your answer.

Sol. No, Let $A = \{a\}$, $B = \{b\}$ and $A \cup B = \{a, b\}$

$\therefore P(A) = \{\phi, \{a\}\}$, $P(B) = \{\phi, \{b\}\}$

and $P(A \cup B) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ ...(i)

and $P(A) \cup P(B) = \{\phi, \{a\}, \{b\}\}$ ...(ii)

From (i) and (ii), we have,

$P(A \cup B) \neq P(A) \cup P(B)$

8. Show that for any sets A and B,
$A = (A \cap B) \cup (A - B)$
and $A \cup (B - A) = (A \cup B)$

Sol. $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$

$[\because A - B = A \cap B']$

$= A \cap (B \cup B')$

[by distributive law]

$= A \cap X = A$

[X is a universal set]

$A \cup (B - A) = A \cup (B \cap A')$

$[\because B - A = B \cap A']$

$= (A \cup B) \cap (A \cup A')$

[by distributive law]

$= (A \cup B) \cap X$

$[\because X = A \cup A'$ is universal set$]$

$= A \cup B$

9. Using properties of sets, show that
(i) $A \cup (A \cap B) = A$
(ii) $A \cap (A \cup B) = A$

Sol. (i) $A \cup (A \cap B) = (A \cup A) \cap (A \cup B)$

[By distributive law]

$= A \cap (A \cup B)$ $[\because A \cup A = A]$

$= \text{A}$ $[\because A \subset A \cup B]$

(ii) $A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$

$= A \cup (A \cap B)$

$= A$ $[\because A \cap B \subset A]$

10. Show that $A \cap B = A \cap C$ need not imply $B = C$.

Sol. Let $A - \{1, 2\}$, $B - \{1, 3\}$ and $C - \{1, 4\}$

Now, $A \cap B = \{1, 2\} \cap \{1, 3\} = \{1\}$;

$A \cap C = \{1, 2\} \cap \{1, 4\} = \{1\}$

$\therefore A \cap B = A \cap C \Rightarrow B \neq C$

11. Let A and B be sets, If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that $A = B$.

Sol. We have $A \cup X = B \cup X$ for some set X

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow A = (A \cap B) \cup (A \cap X)$

$[\because A \cap (A \cup X) = A]$

$\Rightarrow A = (A \cap B) \cup \phi$

$[\because A \cap X = \phi$ (given)$]$

$\Rightarrow A = A \cap B \Rightarrow A \subset B$ ...(i)

Again, $A \cup X = B \cup X$

$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$

$[\because B \cap (B \cup X) = B]$

$\Rightarrow (B \cap A) \cup (B \cap X) = B$

$[\because B \cap X = \phi$ (given)$]$

$\Rightarrow (B \cap A) \cup \phi = B \Rightarrow B \cap A = B$

$\Rightarrow A \cap B = B \Rightarrow B \subset A$ ...(ii)

From (i) and (ii) we get, $\text{A} = \text{B}$

12. Find sets A, B, and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are not-empty sets and $A \cap B \cap C = \phi$.

Sol. Let $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{1, 3\}$,

$A \cap B = \{2\}$, $B \cap C = \{3\}$ and $A \cap C = \{1\}$

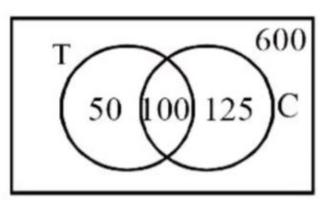
i.e., $A \cap B$, $B \cap C$ and $\text{A} \cap \text{C}$ are non-empty sets.

$\therefore\ (A \cap B) \cap C = \{2\} \cap \{1, 3\} = \phi$

13. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?

Sol. We have, $n(\text{T}) = 150, n(\text{C}) = 225$

and $n(\text{T} \cap \text{C}) = 100$



We know that

$n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$= 150 + 225 - 100 = 275$

Total no. of students = 600

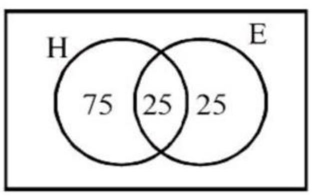
No. of students who neither take tea nor coffee

$= 600 - n(T \cup C) = 600 - 275 = 325$

14. In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

Sol. We have $n(H) = 100, n(E) = 50$ and

$$n(H \cap E) = 25$$



We know that

$$n(H \cup E) = n(H) + n(E) - n(H \cap E)$$

$$= 100 + 50 - 25 = 125$$

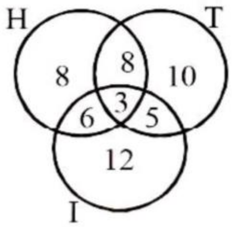
15. In a survey of 60 people, it was found that 25 people read newspaper *H*, 26 read Newspaper *T*, 26 read newspaper I, 9 read both *H* and *I*, 11 read both *H* and *T*, 8 read both *T* and I, 3 read all three newspapers. Find :

(i) the number of pepole who read at least one of the newspapers.

(ii) the number of people who read exactly one newspaper.

Sol. (i) No. of people who read atleast one newspaper

$$= 8 + 8 + 10 + 6 + 3 + 5 + 12 = 52$$



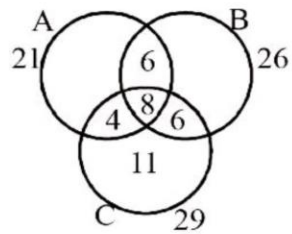
(ii) No. of people who read exactly one newspaper

$$= 52 - (8 + 3 + 6 + 5) = 52 - 22 = 30$$

16. In a survey it was found that 21 people liked product *A*, 26 liked product *B* and 29 liked product C. If 14 people liked products *A* and *B*, 12 people liked products *C* and *A*, 14 people liked products *B* and *C* and 8 liked all the three products. Find how many liked product *C* only.

Sol. $n(A \cap B) = 14$, $n(A \cap C) = 12$,

$n(B \cap C) = 14$ and $n(A \cap B \cap C) = 8$



$n(C \text{ only}) = 29 - 4 - 8 - 6 = 29 - 18 = 11$

SECTION B PRACTICE QUESTIONS

Short Answer Questions

1. Find the pairs of equal sets, if any give reason.
$A = \{0\}$, $B = \{x : x > 15 \text{ and } x < 5\}$
$C = \{x : x - 5 = 0\}$, $D = \{x : x^2 = 25\}$
$E = \{x : x$ is an integral positive root of the equation $x^2 - 2x - 15 = 0\}$
2. Let $V = \{a, e, i, o, u\}$ and $B = \{a, i, k, u\}$. Find $V - B$ and $B - V$
3. Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$ and $B = \{3, 4, 5\}$. Find $A', B', A' \cap B', A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$
4. Show that $(A \cup B) = A \cap B$ implies $A = B$
5. Write the following sets in roster form :
(i) $A = \{x : x$ is an integer and $-3 < x < 7\}$
(ii) $B - \{x : x$ is a natural number less than $6\}$
(iii) $C = \{x : x$ is a two-digit natural number such that the sum of its digits is $8\}$
(iv) $D = \{x : x$ is a prime number which is divisor of $60\}$
(v) $E =$ The set of all letters in the word TRIGONOMETRY
(vi) $F =$ The set of all letters in the word BETTER
6. Write the following sets in the set-builder form:
(i) $\{3, 6, 9, 12\}$ (ii) $\{2, 4, 8, 16, 32\}$
(iii) $\{5, 25, 125, 625\}$ (iv) $\{2, 4, 6, ...\}$
(v) $\{1, 4, 9, ..., 100\}$
7. How many elements has $P(A)$, if $A = \phi$?
8. Write the following intervals in set-builder form:
(i) $(-3, 0)$ (ii) $[6, 12]$
(iii) $(6, 12]$ (iv) $[-23, 5)$
9. What universal set(s) would you propose for each of the following
(i) The set of right triangles
(ii) The set of isosceles triangles.
10. Let $A = \{a, b\}$, $B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?
11. If R is the set of real numbers and Q is the set of rational numbers, then what is $R - Q$?
12. In a group of 400 people, each people speak in at least one of the language Hindi or English. 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?
13. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?
14. Which of the following pairs of sets are equal? Justify your answer
(i) $A =$ Set of letters in the word GOOD
$B =$ Set of letters in the word GOD
(ii) $A = \{x : x \in Z \text{ and } x^2 < 9\}$
$B = \{x : x$ is a multiple of 3 and $-3 \leq x \leq 3\}$
15. Given $U = \{x, x \in N, 11 < x < 22\}$ is the universal set and $A = \{x : x > 16\}$ and $B = \{x : x < 17\}$ are two subsets of U. Then, list the elements of A and B.
16. Prove that for a null set ϕ, the set $P(P(P(\phi)))$ contains 4 elements.
17. If A and B be any two sets, then find the value of $A \cap (B \cup C)$?
18. If $n(A) = 12$, $n(B) = 8$, $n(A \cap B) = 4$, then find the value of $n(A \cup B)$?
19. If $A = \{x \in C : x^2 = 1\}$ and $B = \{x \in C : x^4 = 1\}$, then evaluate $A \Delta B$.
20. If $A \times B = \{(5, 5), (5, 6), (5, 7), (8, 6), (8, 7), (8, 5)\}$, then find the value A.
21. Let X be the universal set for sets A and B. If $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$, Then $n(A' \cap B')$ is equal to 300. Determine the value of $n(X)$.
22. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n.
23. If $aN = \{ax : x \in N\}$ Describe the set $3N \cap 7N$.

Long Answer Questions

1. There are 200 individuals with a skin disorder, 120 had been exposed to the Chemical C_1, 50 to Chemical C_2 and 30 to both the Chemicals C_1 and C_2, Find the number of individuals exposed to
   (i) Chemical C_1 but not Chemical C_2
   (ii) Chemical C_2 but not Chemical C_1
   (iii) Chemical C_1 or Chemical C_2
2. A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only 3 men got medals in all the three sports, how many recieved medals in exactly two of the three sports?
3. Which of the following sets are finite or infinite?
   (i) The set of months of a year
   (ii) $\{1, 2, 3, ...\}$
   (iii) $\{1, 2, 3, ...99, 100\}$
   (iv) The set of positive integers greater than 100
   (v) The set of prime numbers less than 99
4. Are the following pair of sets equal? Give reasons.
   (i) $A = \{2, 3\}$,
   $B = \{x : x \text{ is solution of } x^2 + 5x + 6 = 0\}$
   (ii) $A = \{x : x \text{ is a letter in the word FOLLOW}\}$
   $B = \{y : y \text{ is a letter in the word WOLF}\}$
5. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$; find
   (i) $A \cup B$ (ii) $A \cup C$
   (iii) $B \cup C$ (iv) $B \cup D$
   (v) $A \cup B \cup C$ (vi) $A \cup B \cup D$
   (vii) $B \cup C \cup D$
6. If A = $\{3, 5, 7, 9, 11\}$, B = $\{7, 9, 11, 13\}$, C = $\{11, 13, 15\}$ and D = $\{15, 17\}$; find
   (i) $A \cap B$ (ii) $B \cap C$
   (iii) $A \cap C \cap D$ (iv) $A \cap C$
   (v) $B \cap D$ (vi) $A \cap (B \cup C)$
   (vii) $A \cap D$ (viii) $A \cap (B \cup D)$
   (ix) $(A \cap B) \cap (B \cup C)$
   (x) $(A \cup D) \cap (B \cup C)$
7. If U = $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, A = $\{2, 4, 6, 8\}$ and B = $\{2, 3, 5, 7\}$. Verify that
   (i) $(A \cup B)' = A' \cap B'$
   (ii) $(A \cap B)' = A' \cup B'$
8. Draw the appropriate Venn diagram for each of the following
   (i) $(A \cup B)'$ (ii) $A' \cap B'$
   (iii) $(A \cap B)'$ (iv) $A' \cup B'$
9. In a group of 65 people, each people like at least one of cricket and tennis, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket ? How many like tennis?
10. Describe the following sets in roster form :
   (i) $A = \{x : x \in N, 24 < x \leq 30\}$
   (ii) $B = \{x : x \in Z^+, x < 7\}$
   (iii) $C = \{x : x \in N, x \text{ is a factor of } 48\}$
   (iv) $D = \{x : x \in Z, -3 < x \leq 4\}$
   (v) $E = \{x : x$ is a letter in the word 'KOLKATA'$\}$
11. Describe each of the following sets in set-builder form .
   (i) $A = \{1, 2, 3, 4, 5, 6, 7\}$
   (ii) $B = \{-6, -5, -4, -3, -2, -1, 0, 1\}$
   (iii) $C = \left\{\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, ...\right\}$ (iv) $D = \{3, -3\}$
   (v) $E = \{\text{Mars, Mercury}\}$
12. Separate finite and infinite sets among the following
   (i) Set of all points on the circumference of a circle
   (ii) $\{x : x \in W, x < 5000\}$
   (iii) $\{x : x \in Z, x < 0\}$
   (iv) $\{x : x \in N, x > 100\}$
   (v) $\{x : x \in Q, 3 < x < 4\}$
   (vi) $\{1, 2, 3, 1, 2, 3, 1, 2, 3, ...\}$
13. Let A = $\{a, b, c, d, e\}$, B = $\{a, c, e, g\}$ and C = $\{b, e, f, a\}$ Verify the following identities:
   (i) $A \cap (B - C) = (A \cap B) - (A \cap C)$
   (ii) $A - (B \cup C) = (A - B) \cap (A - C)$
   (iii) $A - (B \cap C) = (A - B) \cup (A - C)$
14. If $A = \{1, 2, 4\}$, $B = \{2, 4, 5\}$, $C = \{2, 5\}$, then determine the value of $(A - C) \times (B - C)$
15. Evaluate $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$.
16. Suppose $A_1, A_2, ...A_{30}$ are thirty sets B each with five elements and $B_1, B_2, ... B_n$ are

n set each with three elements, let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j$. Assume that each element of S belong to exactly 10 of the A_i's and exactly 9 to B_j. Find the value of n.

17. If $A = \{\theta : 2\cos^2\theta + \sin\theta \le 2\}$ and

$B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right\}$ then evaluate $A \cap B$.

18. Find the value of $A \cap B$. If

$A = \{(x, y) : y = \frac{1}{x}, 0 \ne x \in R\}$ and

$B = \{(x, y) : y = -x, x \in R\}$

19. A survey shows that 63% of the Americans like cheese where as 76% like apples. If x % of the Americans like both cheese and apples then determine the value of x.

20. The set S and E are defined as given below :

S : $\{(x, y) : |x-3| < 1$ and $|y-3| < 1\}$;

E : $\{(x, y) : 4x^2 + 9y^2 - 32x - 54y + 109 \le 0\}$

Show that $S \subset E$

21. Given $A = \{2, 3\}$, $B = \{4, 5\}$, $C = \{5, 6\}$. Find the values of $A \times (B \cap C)$ and $A \times (B \cup C)$.

22. If $A = \{x : x = \frac{4}{y}, y \in N\}$, then A belongs to which values.

23. From 50 students taking examinations in subjects A, B and C, 37 passed A, 24 passed B and 43 passed C. At most 19 passed A and B at most 29 passed A and C and atmost 20 passed B and C. Find the largest possible number that could have passed all three examinations.

PRACTICE QUESTION'S SOLUTIONS

Short Answer Questions

1. We have $A \ne B$, $A \ne C$, $A \ne D$, $A \ne E$

$\because$ $0 \in A$ and $0 \notin B, C, D$ and E

$\because$ $B = \phi$ But none of the other set are empty.

$\therefore$ $B \ne A$, $B \ne C$, $B \ne D$ and $B \ne E$

Also $C = \{5\}$ but $-5 \in D$. Hence $C \ne D$.

$\because$ $E = \{5\}$, $C = E$, $D = \{-5, 5\}$ and $E = \{5\}$

Hence, $D \ne E$ $\therefore$ Equal sets are C and E.

2. We have, $V = \{a, e, i, o, u\}$ and $B = \{a, i, k, u\}$

$\therefore$ $V - B = \{e, o\}$

$\because$ the element e, o belong to V but not to B

$\therefore$ $B - V = \{k\}$

$\because$ the element k belong to $B - V$ but not to $V - B$. Hence, $V - B \ne B - V$.

3. We have $U = \{1, 2, 3, 4, 5, 6\}$,

$A = \{2, 3\}$, $A' = U - A = \{1, 4, 5, 6\}$

and $U = \{1, 2, 3, 4, 5, 6\}$, $B = \{3, 4, 5\}$

and $B' = U - B = \{1, 2, 6\}$

$\therefore$ $A' \cap B' = \{1, 6\}$ ...(i)

Also $A \cup B = \{2, 3, 4, 5\}$

$(A \cup B)' = U - (A \cup B) = \{1, 6\}$ ...(ii)

From (i) and (ii), $A' \cap B' = (A \cup B)'$. Hence, proved.

4. Let $a \in A$ then $a \in A \cup B$

$\because$ $A \cup B = A \cap B$, $a \in A \cap B$ $\therefore$ $A \subset B$

and let $b \in B$, since $A \cup B = A \cap B$,

$\therefore$ $b \in A \cap B$

So, $b \in A$, $\therefore$ $B \subset A$, Thus, $A = B$

5. (i) $A = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

(ii) $B = \{1, 2, 3, 4, 5\}$

(iii) $C = \{17, 26, 35, 44, 53, 62, 71, 80\}$

(iv) $D = \{2, 3, 5\}$

(v) $E = \{$T, R, I, G, O, N, M, E, Y$\}$

(vi) $F = \{$B, E, T, R$\}$

6. (i) $\{x : x$ is a natural number multiple of 3 and $x < 15\}$

(ii) $\{x : x = 2^n, n \in N$ and $n < 6\}$

(iii) $\{x : x = 5^n$, and $n \in N$ and $n \le 4\}$

(iv) $\{x : x$ is an even natural number $\}$

(v) $\{x : x = n^2, n \in N$ and $n < 11\}$

7. If $A = \phi$, then by the definition of power set, we have
$P(A) = P(\phi) = \{\phi\}$ $\quad [\because P(A) = 2^n]$
= a set containing one element.

8. (i) $(-3, 0) = \{x : x \in R,\ -3 < x < 0\}$
(ii) $[6, 12] = \{x : x \in R,\ 6 \le x \le 12\}$
(iii) $(6, 12] = \{x : x \in R,\ 6 < x \le 12\}$
(iv) $[-23, 5) = \{x : x \in R,\ -23 \le x < 5\}$

9. The set of all the possible triangles is the universal set for each of the given sets

10. Yes, $A \subset B$, because every element of A is also an elemetn of B, therefore. A is a subset of B i.e. $A \subset B$.
$A \cup B = \{a,b\} \cup \{a,b,c\} = \{a,b,c\}$

11. Since, set of real numbers contains a set of rational numbers and a set of irrational numbers
$\therefore R - Q$ = a set of irrational numbers.

12. Let H and E denote the sets of people who can speak Hindi and English respectively
Then, $n(H) = 250, n(E) = 200,\ n(H \cup E) = 400$
Now, $n(H \cap E) = n(H) + n(E) - n(H \cup E)$
$= 250 + 200 - 400 = 50$
Thus, 50 people can speak both Hindi and English

13. Let C be the set of people who like coffee and T be the set of people who like Tea. Then
$n(C \cup T) = 70$, $n(C) = 37, n(T) = 52$
We know that
$n(C \cup T) = n(C) + n(T) - n(C \cap T)$
$70 = 37 + 52 - n(C \cap T)$
$\therefore \quad n(C \cap T) = 89 - 70$
$\Rightarrow \quad n(C \cap T) = 19$
$\therefore$ 19 people like both coffee and Tea.

14. (i) Equal sets; (ii) $A \ne B$

15. $A = \{17, 18, 19, 20, 21\}$, $B = \{12, 13, 14, 15, 16\}$

16. Do it yourself.

17. **Ans :** ϕ
Hints : $A \cap (A \cup B)' = A \cap (A' \cap B')'$
$= (A \cap A') \cap B = \phi \cap B' = \phi$

18. 16

19. $\{-i, i\}$

20. $\{5, 8\}$

21. **Ans :** 700
Hints : Use
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ and
$n(A' \cap B') = n(A \cup B)' = n(X) - n(A \cup B)$

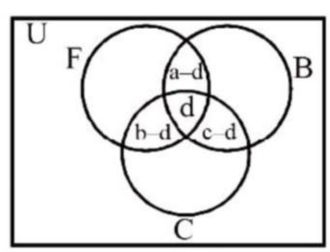
22. 6, 3

23. **Ans :** $21N$.
Hints : $aN = \{a, 2a, 3a, 4a, \ldots\ldots\ldots\}$
$\therefore 3N \cap 7N$
$= \{3, 6, 9, 12, 15, 18, 21, \ldots\ldots\ldots\} \cap \{7, 14, 21, 28, 35, \ldots\ldots\ldots\}$
$= \{21, 42, 63, \ldots\ldots\ldots\ldots\ldots\} = 21N$

Long Answer Questions

1. We have $n(U) = 200$, $n(A) = 120$, $n(B) = 50$, $n(A \cap B) = 30$
(i) We know that,
$n(A) = n(A - B) + n(A \cap B)$
$\Rightarrow$ $n(A - B) = n(A) - n(A \cap B)$
$= 120 - 30 = 90$
$n(A - B) = 90$
i.e., the number of individual exposed to Chemical C_1 but not to Chemical C_2 is 90
(ii) We know that,
$n(B) = n(B - A) + n(A \cap B)$
or $n(B - A) = n(B) - n(A \cap B) = 50 - 30 = 20$
Thus, the number of individuals exposed to chemical C_2 but not to Chemical C_1 is 20
(iii) The number of individuals exposed either to chemical C_1 or to chemical C_2
i.e. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 120 + 50 - 30 = 140$

2. We have



$n(F) = 38, n(B) = 15;\ n(C) = 20,$
$n(F \cup B \cup C) = 58;\ n(F \cap B \cap C) = 3$
$\therefore \quad n(F \cup B \cup C) = n(F) + n(B) + n(C)$
$-n(F \cap B) - n(F \cap C) - n(B \cap C) + n(F \cap B \cap C)$
$\Rightarrow \quad 58 = 38 + 15 + 20$
$-[n(F \cap B) + n(F \cap C) + n(B \cap C)] + 3$
$\therefore \quad n(F \cap B) + n(F \cap C) + n(B \cap C) = 18$

Let a denotes the no. of men who got medals in football and basket ball. b denotes the number of men who got medals in football and cricket, c denotes the number of men who got medals in basket ball and cricket and d denotes the number of men who got medal in all the three. i.e., $d = n(F \cap B \cap C) = 3$

$= a - d + b - d + c - d = a + b + c - 3d$

$= 18 - 3 \times 3 = 9$

Hence, no. of people who got medals in exactly two of the three sports.

3. (i) The set of months of a year.
It is a finite set as there are 12 members of the set which are the months of the year

(ii) $\{1, 2, 3, ...\}$. It is an inifinte set since there are infinite number of natural numbers

(iii) $\{1, 2, 3, ...99, 100\}$. It is a finite set it contains, first 100 natural numbers

(iv) The set of positive integers greater than 100. It is an infinite set since there are infinite number of positive integers viz : 101, 102, 103 ....>100

(v) The set of prime numbers less than 99. This is a finite set because the set is {2, 3, 5, 7, ...97}

4. (i) Since $x^2 + 5x + 6 = 0$

$\Rightarrow \ (x+2)(x+3) = 0$

$\Rightarrow \ x = -2 \text{ or } x = -3$

Solution set, $B = \{-2, -3\}$ $\therefore A \neq B$

$\therefore$ No.

(ii) $A = \{x : x \text{ is a letter in the word FOLLOW}\}$
$= \{F, O, L, W\}$

$B = \{y : y \text{ is a letter in the word WOLF}\}$

$B = \{W, O, L, F\}$

Since, every element of A is in B and every element of B is in A.

$\Rightarrow \ A = B$ $\quad \therefore$ Yes.

5. (i) $A \cup B = \{1,2,3,4\} \cup \{3,4,5,6\}$
$= \{1, 2, 3, 4, 5, 6\}$

(ii) $A \cup C = \{1,2,3,4\} \cup \{5,6,7,8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(iii) $B \cup C = \{3,4,5,6\} \cup \{5,6,7,8\}$
$= \{3, 4, 5, 6, 7, 8\}$

(iv) $B \cup D = \{3,4,5,6\} \cup \{7,8,9,10\}$
$= \{3, 4, 5, 6, 7, 8, 9, 10\}$

(v) $A \cup B \cup C = \{1,2,3,4\} \cup \{3,4,5,6\} \cup \{5,6,7,8\}$
$= \{1,2,3,4,5,6\} \cup \{5,6,7,8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(vi) $A \cup B \cup D = \{1,2,3,4\} \cup \{3,4,5,6\} \cup \{7,8,9,10\}$
$= \{1, 2, 3, 4, 5, 6\} \cup \{7, 8, 9, 10\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(vii) $B \cup C \cup D - \{3,4,5,6\} \cup \{5,6,7,8\} \cup \{7,8,9,10\}$
$= \{3, 4, 5, 6, 7, 8, 9, 10\}$

6. (i) $A \cap B = \{3,5,7,9,11\} \cap \{7,9,11,13\}$
$= \{7, 9, 11\}$

(ii) $B \cap C = \{7,9,11,13\} \cap \{11,13,15\}$
$= \{11, 13\}$

(iii) $A \cap C \cap D = \{3,5,7,9,11\}$
$\cap \{11, 13, 15\} \cap \{15, 17\}$
$= \{11\} \cap \{15,17\} = \phi$

(iv) $A \cap C = \{3,5,7,9,11\} \cap \{11, 13, 15\} = \{11\}$

(v) $B \cap D = \{7,9,11,13\} \cap \{15,17\} = \phi$

(vi) $A \cap (B \cup C)$
$= \{3,5,7,9,11\} \cap (\{7,9,11,13\} \cup \{11,13,15\})$
$= \{3,5,7,9,11\} \cap \{7,9,11,13,15\} = \{7,9,11\}$

(vii) $A \cap D = \{3,5,7,9,11\} \cap \{15,17\} = \phi$

(viii) $A \cap (B \cup D) = \{3,5,7,9,11\}$
$\cap (\{7,9,11,13\} \cup \{15,17\}$
$= \{3,5,7,9,11\} \cap \{7,9,11,13,15,17\}$
$= \{7,9,11\}$

(ix) $A \cap B = \{3,5,7,9,11\}$
$\cap \{7,9,11,13\} = \{7,9,11\}$
$B \cup C = \{7,9,11,13\} \cup \{11,13,15\}$
$= \{7,9,11,13,15\}$

$\therefore \ (A \cap B) \cap (B \cup C) = \{7,9,11\} \cap \{7,9,11,13,15\}$
$= \{7,9,11\}$

(x) $A \cup D = \{3,5,7,9,11\} \cup \{15,17\}$
$= \{3,5,7,9,11,15,17\}$
$B \cup C = \{7,9,11,13\} \cup \{11,13,15\}$
$= \{7,9,11,13,15\}$
$\therefore \ (A \cup D) \cap (B \cup C)$
$= \{3,5,7,9,11,15,17\} \cap \{7,9,11,13,15\}$
$= \{7,9,11,15\}$

7. (i) $A \cup B = \{2,4,6,8\} \cup \{2,3,5,7\}$
$= \{2,3,4,5,6,7,8\}$
$(A \cup B)' = \{1,2,3,4,5,6,7,8,9\}$
$- \{2,3,4,5,6,7,8\} = \{1,9\}$
$A' = \{1,2,3,4,5,6,7,8,9\} - \{2,4,6,8\}$
$= \{1,3,5,7,9\}$

$B' = \{1,2,3,4,5,6,7,8,9\} - \{2,3,5,7\}$
$= \{1,4,6,8,9\}$
$A' \cap B' = \{1,3,5,7,9\} \cap \{1,4,6,8,9\} = \{1,9\}$
$\therefore\ (A \cup B)' = A' \cap B'$

(ii) $(A \cap B) = \{2,4,6,8\} \cap \{2,3,5,7\} = \{2\}$

$(A \cap B)' = \{1,2,3,4,5,6,7,8,9\} - \{2\}$

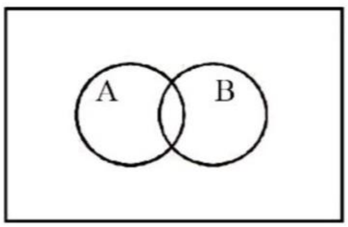
$= \{1,3,4,5,6,7,8,9\}$

$A' \cup B' = \{1,3,5,7,9\} \cup \{1,4,6,8,9\}$

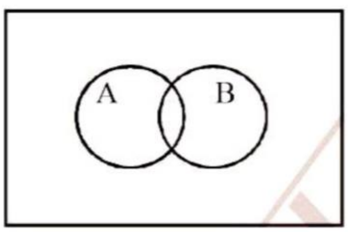
$= \{1,3,4,5,6,7,8,9\}$

Hence, $(A \cap B)' = A' \cup B'$

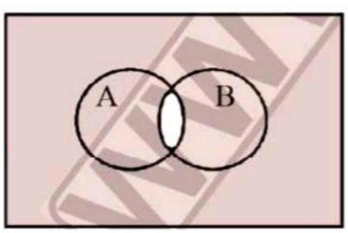
(i) Shaded region = $(A \cup B)'$



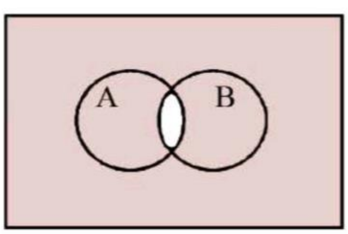
(ii) $A' \cap B'$ = common shaded region



(iii) $(A \cap B)'$ = shaded region



(iv) $A' \cup B'$ = shaded region



9. Let C be the set of people who like cricket and T be the set of people who like Tennis. Then

$n(C \cup T) = 65,\ n(C) = 40,\ \ n(C \cap T) = 10$

We know that

$$n(C \cup T) = n(C) + n(T) - n(C \cap T)$$
$$65 = 40 + n(T) - 10$$

$$n(T) = 65 - 40 + 10 = 35$$

No. of people like tennis only

$= n(T) - n(C \cap T)$
$= 35 - 10 = 25$

Number of people who like Tennis only and not cricket = 25 and number of people who like tennis is 35.

10. **Ans :** (i) $A = \{25, 26, 27, 28, 29, 30\}$

(ii) $B = \{0, 1, 2, 3, 4, 5, 6\}$

(iii) $C = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$

(iv) $D = \{-2, -1, 0, 1, 2, 3, 4\}$

(v) $E = \{$ k, O, L, A , T$\}$

Hints : In Roster form, the element of a set are actually written down separated by commas and enclosed within braces e.g., A, the set of odd natural numbers $< 10 = \{1, 3, 5, 7, 9\}$

11. **Ans :** (i) $A = \{x : x \in N,\ x < 8\}$

(ii) $B = \{x : x \in z, -6 \le x \le 1\}$

(iii) $C = \{x : x = \dfrac{n}{n+1},\ n \in N,\ n > 1\}$

(iv) $D = \{x : x \in z, x^2 = 9\}$

(v) $E = \{x : x$ is a planet whose name begins with M$\}$

Hints : Set Builder form is the property satisfied by the elements of the set.

e.g. $A = \{6, 7, 8, 9, 10, 11\} = \{x \mid x \in \text{N}, 5 < x < 12\}$

12. (i) Finite (ii) Finite
(iii) Infinite (iv) Infinite
(v) Infinite (vi) Finite

13. Do it yourself.

14. **Ans :** $\{(1, 4)(4, 4)\}$

Hints : $(A - C = \{1, 4\}$ and $B - C = \{4\}$

$\therefore (A - C) \times (B - C) = \{(1, 4), (4, 4)\}$

15. $B \cap C'$

16. **Ans :** 45

Hints : $n(S)=\frac{1}{10}\sum_{i=1}^{30}n(A_i)=\frac{1}{10}(30\times5)=15$;

$n(S)=\frac{1}{9}\sum_{j=1}^{n}n(B_j)\Rightarrow15=\frac{1}{9}(n\times3)=45$

17. **Ans :** $\left\{\theta:\frac{\pi}{2}\le\theta\frac{5\pi}{6}\text{ or }\pi\le\theta\le\frac{3\pi}{2}\right\}$

Hints :

$\sin\theta\ge0,\ \sin\theta\ge\frac{1}{2}\Rightarrow\sin\theta\ge\frac{1}{2}\Rightarrow\frac{\pi}{2}\le\theta\le\frac{5\pi}{6}$

and

$\sin\theta\le0,\ \sin\theta\le\frac{1}{2}\Rightarrow\sin\theta\le0\Rightarrow\pi\le\theta\le\frac{3\pi}{2}$

18. ϕ

19. $39\le x\le63$

20. Do it yourself.

21. {(2, 5), (3, 5) and {(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

22. **Ans :** $A\in1,\frac{1}{4}$

Hints : $\frac{4}{y}\ne0$ for any $y\in N$

$\therefore0\notin A$; $1=\frac{4}{4}\in A$, because, $4\in N$

$\frac{1}{4}=\frac{4}{16}\in A$, because, $16\in N$;

$\frac{10}{3}=\frac{4}{y}\Rightarrow y=\frac{12}{10}=\frac{6}{5}\notin N\quad\therefore\frac{10}{3}\notin A$

23. 14 students

SECTION C NCERT EXEMPLAR QUESTIONS

Fill in the Blanks

1. The set $\{x\in R:1\le1\ x<2\}$ can be written as __________.
2. When $A=\phi$, then number of elements in $P(A)$ is __________.
3. If A and B are finite sets such that $A\subset B$, then $n(A\cup B)=$ __________.
4. If A and B are any two sets, then $A-B$ is equal to __________.
5. Power set of the set $A=\{1,2\}$ is __________.
6. If the sets $A=\{1,3,5\}$. $B=\{2,4,6\}$ and $C=\{0,2,4,6,8)$. Then the universal set of all the three sets A, B and C can be __________.
7. If $U=\{1,2,3,4,5,6,7,8,9,10\}$, $A=\{1,2,3,5\}$, $B=\{2,4,6,7\}$ and $C=\{2,3,4,8\}$. Then
   (i) $(B\cup C)'$ is __________.
   (ii) $(C-A)'$ is __________.
8. For all sets A and B, $A-(A\cap B)$ is equal to __________.

True or False

1. If A is any set, then $A\subset A$.
2. If $M=\{1,2,3,4,5,6,7,8,9\}$ and $B=\{1,2,3,4,5,6,7,8,9\}$, then $B\not\subset M$.
3. The sets $\{1,2,3,4\}$ and $\{3,4,5,6\}$ are equal.
4. $Q\cup Z=Q$, where Q is the set of rational numbers and Z is the set of integers.
5. Let sets R and T be defined as
   $R=\{x\in Z\mid x\text{ is divisible by }2\}$
   $T=\{x\in Z\mid x\text{ is divisible by }6\}$. Then $T\subset R$
6. Given $A=\{0,1,2\}$, $B=\{x\in R\mid0\le x\le2\}$. Then $A=B$.

Short Answer Questions

1. Write the following sets in the roaster form.
   (i) $A=\{x:x\in R,2x+11=15\}$
   (ii) $B=\{x\mid x^2=x,x\in R\}$
   (iii) $C=\{x\mid x$ is a positive factor of a prime number $p\}$
2. Write the following sets in the roaster form.
   (i) $D=\{t\mid t^3=t,t\in R\}$
   (ii) $E=\left\{w\mid\frac{w-2}{w+3}=3,w\in R\right\}$
   (iii) $F=\{x\mid x^4-5x^2+6=0,x\in R\}$

3. If $Y = \{x \mid x$ is a positive factor of the number $2^{p-1}(2^p - 1)$, where $2^p - 1$ is a prime number$\}$. Write Y in the roaster form.
4. If $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6\}$ and $N = \{1, 3, 5\}$, then verify that $L - (M \cup N) = (L - M) \cap (L - N)$.
5. If A and B are subests of the universal set U, then show that
   (i) $A \subset A \cup B$
   (ii) $A \subset B \Leftrightarrow A \cup B = B$
   (iii) $(A \cap B) \subset A$
6. Given that $N = \{1, 2, 3, ..., 100\}$. Then, write
   (i) the subset of N whose elements are even numbers.
   (ii) the subset of N whose elements are perfect square numbers.
7. If $X = \{1, 2, 3\}$, if n represents any member of X, write the following sets containing all numbers represented by
   (i) $4n$ (ii) $n + 6$
   (iii) $\frac{n}{2}$ (iv) $n - 1$
8. If $Y = \{1, 2, 3, ... , 10\}$ and a represents any element of Y, write the following sets, containing all the elements satisfying the given conditions.
   (i) $a \in Y$ but $a^2 \notin Y$
   (ii) $a + 1 = 6, a \in Y$
   (iii) a is less than 6 and $a \in Y$
9. A, B and C are subsets of universal set U. If $A = \{2, 4, 6, 8, 12, 20\}$, $B = \{3, 6, 9, 12, 15\}$; $C = \{5, 10, 15, 20\}$ and U is the set of all whole numbers, draw a Venn diagram showing the relation of U, A, B and C.
10. Let U be the set of all boys and girls in a school, G be the set of all girls in the school, B be the set of all boys in the school and S be the set of all students in the school who take swimming. Some but not all, students in the school take swimming. Draw a Venn diagram showing one of the possible interrelationship among sets U, G, B and S.
11. For all sets A, B and C, show that $(A - B) \cap (A - C) = A - (B \cup C)$.
12. For all sets A and B, show that $(A - B) \cup (A \cap B) = A$.
13. For all sets A, B and C, find whether $A - (B - C)$ is equal to $(A - B) - C$ or not
14. For all sets A, B and C, if $A \subset B$, then show that $A \cap C \subset B \cap C$.
15. For all sets A, B and C, if $A \subset B$, then find whether $A \cup C \subset B \cup C$ is true or false
16. For all sets A, B and C, if $A \subset C$ and $B \subset C$, then show that $A \cup B \subset C$.
17. For all sets A and B prove that $A \cup (B - A) = A \cup B$.
18. For all sets A and B, show that $A - (A - B) = A \cap B$.
19. For all sets A and B, show that $A - (A \cap B) = A - B$.
20. Let $T = \left\{x \mid \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$. Is T an empty set? Justify your answer.

Long Answer Questions

1. If A, B and C be sets. Then show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
2. Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science, 4 in English and Science, 4 in all the three. Find how many passed
   (i) In English and Mathemtics but not in Science.
   (ii) In Mathematics and Science but not in English.
   (iii) In Mathematics only.
   (iv) In more than one subject only.
3. In a class of 60 students, 25 students play cricket and 20 students play tennis and 10 students play both the games. Find the number of students who play neither.
4. In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. Find the number of students who study all the three subjects.
5. In a town of 10000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspaper. Find
   (i) the number of families which buy newspaper A only.
   (ii) the number of families which buy none of A, B and C.

NCERT EXEMPLAR SOLUTIONS

Fill in the Blanks

1. $[1, 2]$
2. 1
3. $n(B)$
4. $A \cap B'$
5. $\{\phi, \{1\}, \{2\}, \{1, 2\}\}$
6. $\{0, 1, 2, 3, 4, 5, 6, 8\}$
7. (i) $\{1, 5, 9, 10\}$ (ii) $\{1, 2, 3, 5, 6, 7, 9, 10\}$
8. $A \cup B'$

True or False

1. True **2.** False **3.** False **4.** True
5. True **6.** False

Short Answer Questions

1. (i) Given $A = \{x : x \in R, 2x + 11 = 15\}$

Here, $2x + 11 = 15$

$\Rightarrow 2x = 15 - 11 \Rightarrow 2x = 4$

$\Rightarrow x = 2$

So, $A = \{2\}$

(ii) Given $B = \{x \mid x^2 = x, x \in R\}$

Here, $x^2 = x$

$\Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0$

$\Rightarrow x = 0, 1$

So, $B = \{0, 1\}$

(iii) Given $C = \{x \mid x$ is a positive factor of prime number $p\}$.

Since, positive factors of a prime number are 1 and the number itself.

So, $C = \{1, p\}$

2. (i) Given $D = \{t \mid t^3 = t, t \in R\}$

Here, $t^3 = t$

$\Rightarrow t^3 - t = 0$

$\Rightarrow t(t-1)(t+1) = 0 \Rightarrow t = 0, 1, -1$

So, $D = \{-1, 0, 1\}$

(ii) Given $E = \left\{w \mid \dfrac{w-2}{w+3} = 3, w \in R\right\}$

Here, $\dfrac{w-2}{w+3} = 3$

$\Rightarrow w - 2 = 3w + 9$

$\Rightarrow -2w = 11 \Rightarrow w = \dfrac{-11}{2}$

So, $E = \left\{\dfrac{-11}{2}\right\}$

(iii) Given $F = \{x \mid x^4 - 5x^2 + 6 = 0, x \in R\}$

$\Rightarrow x^4 - 5x^2 + 6 = 0$

$\Rightarrow x^4 - 3x^2 - 2x^2 + 6 = 0$

$\Rightarrow x^2(x^2 - 3) - 2(x^2 - 3) = 0$

$\Rightarrow (x^2 - 3)(x^2 - 2) = 0$

$\Rightarrow x = \pm\sqrt{3}, \pm\sqrt{2}$

So, $F = \{-\sqrt{3}, -\sqrt{2}, \sqrt{2}, \sqrt{3}\}$

3. Given $Y = \{x \mid x$ is a positive factor of the number $2^{p-1}(2^p - 1)$, where 2^{p-1} is a prime number$\}$.

So, the factors of 2^{p-1} are $1, 2, 2^2, 2^3, \ldots, 2^{p-1}$

$\therefore Y = \{1, 2, 2^2, 2^3, \ldots, 2^{p-1}, 2^p - 1\}$

4. We have $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6\}$ and $N = \{1, 3, 5\}$

$\therefore M \cup N = \{1, 3, 4, 5, 6\}$

$\Rightarrow L - (M \cup N) = \{2\}$

Now, $L - M = \{1, 2\}$, $L - N = \{2, 4\}$

$\therefore (L - M) \cap (L - N) = \{2\}$

Hence, $L - (M \cup N) = (L - M) \cap (L - N)$.

5. (i) Let $x \in A$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A \cup B$

Hence, $A \subset A \cup B$

(ii) If $A \subset B$

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ $[\because A \subset B]$

$\Rightarrow A \cup B \subset B$ ... (i)

But $A \subset B \cup B$ ... (ii)

From Eqs. (i) and (ii), we have

$A \cup B = B$

Next if $A \cup B = B$

Now let $y \in A$

$\Rightarrow y \in A \cup B \Rightarrow y \in B$ $[\because A \cup B = B]$

$\Rightarrow A \subset B$

Hence, $A \subset B \Leftrightarrow A \cup B = B$

(iii) Let $x \in A \cap B$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in A$

$\Rightarrow A \cap B \subset A$

6. We have, $N = \{1, 2, 3, 4, ..., 100\}$

(i) Required subset $= \{2, 4, 6, 8, ..., 100\}$

(ii) Required subset $= \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

7. We have $x = \{1, 2, 3\}$

(i) $\{4n \mid n \in X\} = \{4, 8, 12\}$

(ii) $\{n + 6 \mid n \in X\} = \{7, 8, 9\}$

(iii) $\left\{\frac{n}{2} \mid n \in X\right\} = \left\{\frac{1}{2}, 1, \frac{3}{2}\right\}$

(iv) $\{n - 1 \mid n \in X\} = \{0, 1, 2\}$

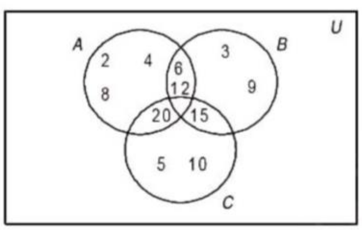
8. We have $Y = \{1, 2, 3, ..., 10\}$

(i) $\{a : a \in Y \text{ and } a^2 \notin Y\} = \{4, 5, 6, 7, 8, 9, 10\}$

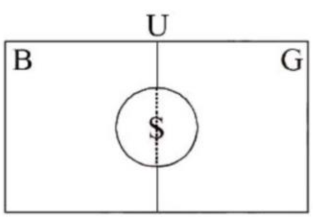
(ii) $\{a : a + 1 = 6, a \in Y\} = \{5\}$

(iii) $\{$is less than 6 and $a \in Y\} = \{1, 2, 3, 4, 5\}$

9.



10.



11. Let $x \in (A - B) \cap (A - C)$

$\Rightarrow x \in (A - B)$ and $x \in (A - C)$

$\Rightarrow (x \in A$ and $\notin B)$ and $(x \in A$ and $x \notin C)$

$\Rightarrow x \in A$ and $(x \notin B$ and $x \notin C)$

$\Rightarrow x \in A$ and $x \notin (B \cup C)$

$\Rightarrow x \in A - (B \cup C)$

$\Rightarrow (A - B) \cap (A - C) \subset A - (B \cup C)$ ... (1)

Now, let $y \in A - (B \cup C)$

$\Rightarrow y \in A$ and $y \notin (B \cup C)$

$\Rightarrow y \in A$ and $(y \notin B$ and $y \notin C)$

$\Rightarrow (y \in A$ and $y \notin B)$ and $(y \in A$ and $y \notin C)$

$\Rightarrow y \in (A - B)$ and $y \in (A - C)$

$\Rightarrow y \in (A - B) \cap (A - C)$

$\Rightarrow A - (B \cup C) \subset (A - B) \cap (A - C)$ ... (2)

From Eqs. (1) and (2)

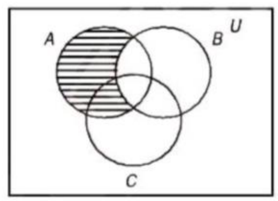
$$A - (B \cup C) = (A - B) \cap (A - C)$$

12. LHS $= (A - B) \cup (A \cap B)$

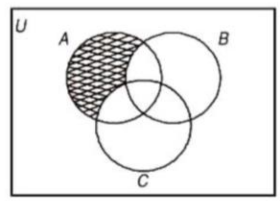
$= [(A - B) \cup A] \cap [(A - B) \cup B]$

$= A \cap (A \cup B) = A =$ RHS

13. The Venn diagrams given below, where shaded portions representing $A - (B - C)$ and $(A - B) - C$ respectively are



A − (B − C)



(A − B) − C

Clearly, $A - (B - C) \neq (A - B) - C$.

14. Let $x \in A \cap C$

$\Rightarrow x \in A$ and $x \in C$

$\Rightarrow x \in B$ and $x \in C$ $[\because A \subset B]$

$\Rightarrow x \in (B \cap C)$

$\Rightarrow (A \cap C) \subset (B \cap C)$

15. Let $x \in A \cup C$

$\Rightarrow x \in A$ or $x \in C$

$\Rightarrow x \in B$ or $x \in C$ $[\because A \subset B]$

$\Rightarrow x \in B \cup C \Rightarrow A \cup C \subset B \cup C$

Hence true.

16. Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in C$ or $x \in C$ $[\because A \subset C$ and $B \subset C]$

$\Rightarrow x \in C \Rightarrow A \cup B \subset C$

17. LHS$= A \cup (B - A) = A \cup (B \cap A')$

$[\because A - B = A \cap B']$

$= (A \cup B) \cap (A \cup A') = (A \cup B) \cap U$ $[\because A \cup A' = U]$

$= A \cup B =$ RHS $[\because A \cap U = A]$

Hence proved.

18. LHS$= A - (A - B) = A - (A \cap B')$

$[\because A - B = A \cap B']$

$= A \cap (A \cap B')' = A \cap [A' \cup (B')']$ $[\because (A \cap B)' = A' \cup B']$

$= A \cap (A' \cup B)$ $[\because (A')' = A]$

$= (A \cap A') \cup (A \cap B) = \phi \cup (A \cap B)$

$= A \cap B =$ RHS.

19. LHS $= A - (A \cap B) = A \cap (A \cap B)'$

$[\because A - B = A \cap B']$

$= A \cap (A' \cup B')$ $\quad [\because (A \cap B)' = A' \cup B']$

$= (A \cap A') \cup (A \cap B') = \phi \cup (A \cap B)$

$= A \cap B' [\because \phi \cup A = A]$

$= A - B =$ RHS

20. Given, $\quad T = \left\{ x \mid \dfrac{x+5}{x-7} - 5 = \dfrac{4x-40}{13-x} \right\}$

We have $\quad \dfrac{x+5}{x-7} - 5 = \dfrac{4x-40}{13-x}$

$\Rightarrow \quad \dfrac{x+5-5(x-7)}{x-7} = \dfrac{4x-40}{13-x}$

$\Rightarrow \quad \dfrac{-4x+40}{x-7} = \dfrac{4x-40}{13-x}$

$\Rightarrow \quad -(4x-40)(13-x) = (4x-40)(x-7)$

$\Rightarrow \quad (4x-40)(x-7+13-x) = 0$

$\Rightarrow \quad 24(x-10) = 0$

$\Rightarrow \quad x = 10$

$\therefore \quad T = \{10\}$

Hence, T is not an empty set.

Long Answer Questions

1. Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in (B \cup C)$

$\Rightarrow x \in A$ and $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

$\Rightarrow x \in A \cap B$ or $x \in A \cap C$

$x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow A \cap (B \cup C) \subset (A \cap B) \cup (A \cap C)$ ...(1)

Again, let $y \in (A \cap B) \cup (A \cap C)$

$\Rightarrow y \in (A \cap B)$ or $y \in (A \cap C)$

$\Rightarrow (y \in A$ and $y \in B)$ or $(y \in A$ and $y \in C)$

$\Rightarrow y \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow y \in A$ and $y \in (B \cup C)$

$\Rightarrow y \in A \cap (B \cap C)$

$\Rightarrow (A \cap B) \cup (A \cap C) \subset A \cap (B \cup C)$ ...(2)

From Eqs. (1) and (2),

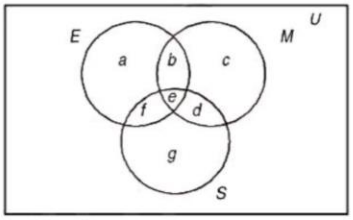
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

2. Let M be the set of students who passed in Mathematics, E be the set of students who passed in English and S be the set of students who passed in Science.

Then, $\quad n(U) = 100$

$n(E) = 15$, $n(M) = 12$, $n(S) = 8$, $n(E \cap M) = 6$, $n(M \cap S) = 7$, $n(E \cap S) = 4$, and $n(E \cap M \cap S) = 4$,

This situation can be shown by the venn Diagram as:



$\because \quad n(E) = 15$

$\Rightarrow a + b + e + f = 15$ ... (1)

$n(M) = 12$

$\Rightarrow b + c + e + d = 12$ ... (2)

$n(S) = 8$

$\Rightarrow d + e + f + g = 8$ ... (3)

$n(E \cap M) - 6$

$\Rightarrow b + e = 6$ ... (4)

$n(M \cap S) = 7$

$\Rightarrow e + d = 7$ ... (5)

$n(E \cap S) = 4$

$\Rightarrow e + f = 4$ ... (6)

$n(E \cap M \cap S) = 4$

$\Rightarrow e = 4$ ... (7)

From Eqns. (6) and (7) $\quad f = 0$

From Eqns. (5) and (7) $\quad d = 3$

From Eqns. (4) and (7) $\quad b = 2$

On substituting the values of d, e and f in Eqn. (3), we get

$$3 + 4 + 0 + g = 8$$

$\rightarrow \quad g = 1$

On substituting the value of b, e and d in Eqn. (2), we get

$$2 + c + 4 + 3 = 12$$

$\Rightarrow \quad c = 3$

On substituting b, e and f in Eqn. (1), we get

$$a + 2 + 4 + 0 = 15$$

$\Rightarrow \quad a = 9$

(i) Number of students who passed in English and Mathematics but not in Science = $b = 2$

(ii) Number of students who passed in Mathematics and Science but not in English $= d = 3$

(iii) Number of students who passed in Mathematics only $= c = 3$

(iv) Number of students who passed in more than one subject $= b + e + d + f$
$= 2 + 4 + 3 + 0 = 9$

3. Let C be the set of students who play cricket and T be the set of students who play tennis. Then, $n(U) = 60$, $n(C) = 25$, $n(T) = 20$, and $n(C \cap T) = 10$

We have $n(C \cup T) = n(C) + n(T) - n(C \cup T)$
$= 25 + 20 - 10 = 35$

So, Number of students who play neither
$= n(U) - n(C \cup T) = 60 - 35 = 25$

4. Let M be the set of students who study Mathematics, P be the set of students who study Physics and C be the set of students who study Chemistry.

Then, $n(U) = 200$, $n(M) = 120$, $n(P) = 90$,
$n(C) = 70$, $n(M \cap P) = 40$, $n(P \cap C) = 30$,
$n(C \cap M) = 50$ and $n(M' \cap P' \cap C') = 20$,

$\Rightarrow n(U) - n(M \cup P \cup C) = 20,$

$\Rightarrow n(M \cup P \cup C) = 200 - 20 = 180$

$\because n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(P \cap C) - n(C \cap M) + n(M \cap P \cap C)$

$\Rightarrow 180 = 120 + 90 + 70 - 40 - 30 - 50 + n(M \cap P \cap C)$

$\Rightarrow 180 = 160 + n(M \cap P \cap C)$

$\Rightarrow n(M \cap P \cap C) = 180 - 160 = 20$

So, the number of students who study all the three subjects is 20.

5. Let A be the set of families which buy newspaper A, B be the set of families which buy newspaper B and C be the set of families which buy newspaper C.

Then, $n(U) = 10000$, $n(A) = 40\%$ $n(B) = 20\%$ and $n(C) = 10\%$

$n(A \cap B) = 5\%$

$n(B \cap C) = 3\%$

$n(A \cap C) = 4\%$

$n(A \cap B \cap C) = 2\%$

(i) Percentage of families which buy newspaper A only $= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
$= (40 - 5 - 4 + 2)\% = 33\%$

$\therefore$ Number of families which buy newspaper A only - $10000 \times 33/100 = 3300$

(ii) Percentage of families which buy none of A, B and C $= n(U) - n(A \cup B \cup C)$

$= n(U) - [n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)]$

$= 100 - [40 + 20 + 10 - 5 - 3 - 4 + 2]$

$= (100 - 60)\% = 40\%$

$\therefore$ Number of families which buy none of A, B and C $= \dfrac{40}{100} \times 10000 = 4000$