AEP
IIT-JEE | NEET | CBSE
STUDY CIRCLE
ACCENTS EDUCATIONAL PROMOTERS
OFFLINE-ONLINE LEARNING ACADEMY

XII
CBSE: PHYSICS
CAPACITANCE

THE SUCCESS DESTINATION

UNIT -1
ELECTROSTATICS
CH: 5
CAPACITANCE

HELPLINE:
+91-9939586130
+91-7739650505
www.aepstudycircle.com
2ND FLOOR SATKOUDI COMPLEX, THANA CHOWK, RAMGARH-829122

CAPACITANCE

CONDUCTORS, INSULATORS and DIELECTRICS

Conductors and insulators: On the basis of their behaviour in an external electric field, most of the materials can be broadly classified into two categories:

1. Conductors: These are the substances which allow large scale physical movement of electric charges through them when an external electric field is applied. For example, silver, copper, aluminium, graphite, human body, acids, alkalies etc.

✪**Conductors**: *"A substance which can be used to carry or conduct electric charge from one place to another is called a conductor".*

Examples: (i) Silver, gold (best conductors), Cu, Fe, Al, Zn
(ii) Liquid conductors: -- Acid, alkalis, salts etc

☑ In a metallic conductor, there are large no. of free electrons. Since an atom has a tendency of to have a filled valence shell, the valence shell electron in the atoms of a metal leave the atom and are free to move through the metal in a random manner. They have practically no affinity to their parent atom.

☑ The average velocity of free electrons in a metal is zero. But When external electric field is applied across the two ends of a metal , the free electrons experience force and gets accelerated . There is a net flow of electrons through the metal. As such, the metals are termed as conductors for electricity.

☑ The residual positive charge atoms in the metal cannot move and constitute the bound charge in the conductors.

2. Insulators: These are the substances which do not allow physical movement of electric charges through them when an external electric field is applied. For example, diamond, glass, wood, mica, wax, distilled water, ebonite, etc.

✪**Insulators**: *"The materials which do not have free electrons in them and are unable to conduct electricity, are termed as insulators".*

Examples: Glass, mica, plastics, wax, paper, wood etc.

☑ In an insulator, each electron is bound to a particular atom and is not free to move in the body of the insulators. Therefore an insulator does not possess freely movable charges to conduct electricity.

✪**Dielectrics**: *"Dielectrics are insulating material which transmit electric effects without conducting".*

☑ Dielectric cannot conduct electricity but when an external field is applied, induced charges appear on the surface of the dielectrics.

☑The rubbed insulators were able to retain charges placed on them, so they were called dielectrics. The rubbed conductors (metals) could not retain charges placed on them but immediately drained away the charges, so they were called non-electrics.

FREE AND BOUND CHARGES

Free and bound charges: The difference between the electrical behaviour of conductors and insulators can be understood on the basis of free and bound charges.

♦In metallic conductors, the electrons of the outer shells of the atoms are loosely bound to the nucleus. They get detached from the atoms and move almost freely inside the metal. In an external electric field, these free electrons drift in the opposite direction of the electric field. The positive ions which consist of nuclei and electrons of inner shells remain held in their fixed positions. These immobile charges constitute and bound charges.

♦In electrolytic conductors, both positive and negative ions act as charge carriers. However, their movements are restricted by the external electric field and the electrostatic forces between them.

♦In insulators, the electrons are tightly bound to the nuclei and cannot be detached from the atoms, i.e., charges in insulators and bound charges. Due to the absence of free charges, insulators are poor conductors of electricity.

CONCEPTUALS...

☑ **A third important category of materials is the semiconductors which we shall discuss in chapter 15.**

☑ **In metallic conductors, electrons of outer shells of at atoms are the free charges while the immobile positive ions are the bound charges.**

☑ **In electrolytic conductors, both positive and negative ions are the free charges.**

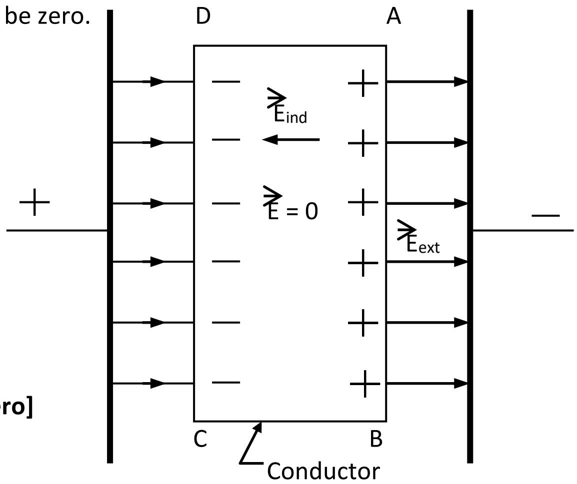
☑ **In insulators, both electrons and the positive ions are the bound charges.**

☑ **There is no clear-cut distinction between conductors and insulators – their electrical properties very continuously within a very large range. For example, the ratio of the electrical properties between a metal and glass may be as high 10^{20}.**

BEHAVIOUR OF CONDUCTORS IN ELECTROSTATIC FIELDS

Electrostatic properties of a conductor: When placed in electrostatic fields, the conductors show the following properties:

1. Net electrostatic field is zero in the interior of a conductor: when a conductor is placed in an electric field E_{ext}, its free electrons begin to move in the opposite direction of E_{ext}. Negative charges are induced on the left end and positive charges are induced on the left end and positive charges are induced on the right end of the conductor. The process continues till the electric field E_{ind} set up by the induced charges becomes equal and opposite to the field E_{ext}. The net field E ($= E_{ext} - E_{ind}$) inside the conductor will be zero.



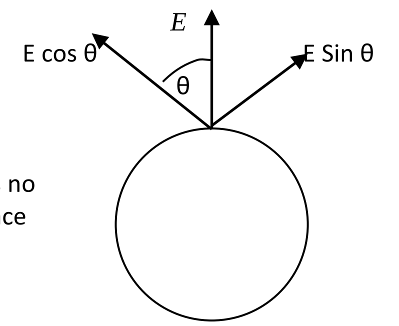
[Electric field inside a conductor is zero]

Explanation: A conductor contains large no. of free electrons. Suppose a conductor ABCD is held in an external electric field intensity E_0. Now, each free electron experiences a force ($F = -eE$) in a direction opposite to the direction of applied electric field E_0. So, the free electron will **move** and collects towards positive of the applied field on the surface of the conductor Therefore, side CD becomes **Negative** charge and side AB becomes positive charge due to transfer of free electrons. These are called induced charges and produced an electric field E_P which opposes the flow of free electrons from A B to CD. The flow, therefore stops as soon as E_P. becomes equal to E_0.

Net electric field inside the conductor $\vec{E}_{net} = \vec{E}_0 - \vec{E}_P = 0$

2. Just outside the surface of a charged conductor, electric field is normal to the surface: If the electric field is not normal to the surface, it will have a component tangential to the surface which will immediately cause the flow of charges, producing surface currents. But no such currents can exist under static conditions. Hence electric field is normal to the surface of the conductor at every point.

☑ Explanation: Electric field just outside the charged conductor is perpendicular to the surface of the conductor at every point:



Let E makes an angle θ to the surface of the conductor.

Resolve E:

(1) Tangential component --- E cos θ

(2) Radial component --- E Sin θ

Tangential component would lead to the flow of charges i.e., surface current. But there is no surface current (or flow of charge) in electrostatic. Thus, electric field just outside the surface can have only a normal component.

∴ Tangential component $= 0$

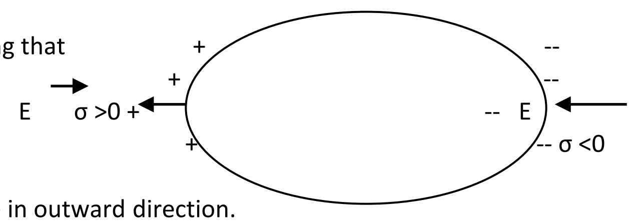
$E\cos\theta = 0$

$\cos\theta = 0 / E = 0$

$\cos\theta = \cos 90$

∴ $\boxed{\theta = 90^0}$ showing that

E just outside a charged conductor is perpendicular to the surface.



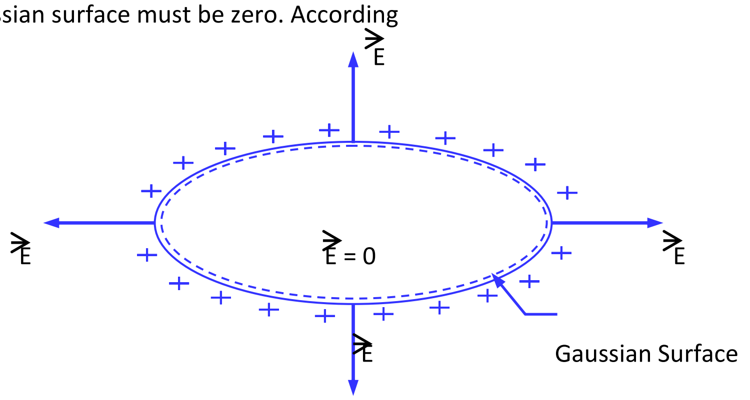
$E = \dfrac{\sigma \hat{n}}{\varepsilon_0}$ → Charge density; unit vector normal to the surface in outward direction.

$E = -\dfrac{\sigma \hat{n}}{\varepsilon_0}$ → Charge density; unit vector normal to the surface in inward direction.

3. Net Charge in the interior of a conductor is zero: consider a conductor carrying an excess charge q with no currents flowing in it. Choose a Gaussian surface inside the conductor just near its outer boundary. As the field E = 0 at all points inside the conductor, the flux ϕ_E through the Gaussian surface must be zero. According

to Gauss's theorem, $\phi_E = \oint \vec{E}.\,d\vec{S} = \dfrac{q}{\varepsilon_0}$

As $\phi_E = 0$, so q = 0



Hence there can be no charge in the interior of the conductor because the Gaussian surface lies just near the outer boundary. The entire excess charge q must reside at the surface of the conductor

4. Charge always resides on the surface of a conductor:

Explanation: when a conductor is charged, the excess charge given to it outside can stay only on the outer surface of the conductor, as the charge inside the conductor is zero.

5. Potential is constant within and on the surface of a conductor: Electric field at any point is equal to the negative of the potential gradient,

i.e., $E = -\dfrac{dV}{dr}$

But inside a conductor E = 0 and moreover, E has no tangential component on the surface, so

$\dfrac{dV}{dr} = 0$ or V = constant

Hence electric potential is constant throughout the volume of a conductor and has the same value (as inside) on its surface, Thus the surface of a conductor is an equipotential surface.

▣If a conductor is charged, there exists an electric field normal to its surface. This indicates that the potential on the surface will be different from the potential at a point just outside the surface

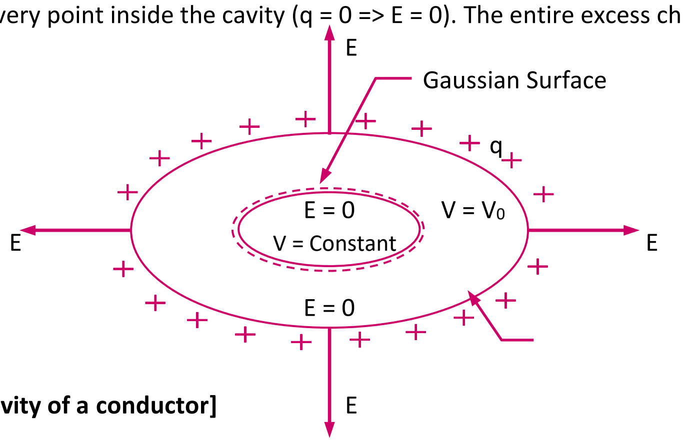
6. Surface charge density may be different at different points:

As, $\sigma = \dfrac{q}{\text{area}}$ may be different at different points on the surface of conductor. Portion of the surface having positive charge will have positive charge densities whereas those having negative charge have negative charge densities.

☞ ***Smaller surface area (say conical portion) of the surface of the conductor will have higher charge density.***

☞ ***Net volume charge density in the interior of a conductor is zero.***

7. Electric field is zero in the cavity of a hollow charged conductor: consider a charged conductor having a cavity, with no charges inside the cavity. Imagine a Gaussian surface inside the conductor quite close to the cavity. Everywhere inside the conductor, E = 0. By Gauss's theorem, charge closed by this Gaussian surface is zero (E = 0 => q = 0). Consequently, the electric field must be zero at every point inside the cavity (q = 0 => E = 0). The entire excess charge + q lies on its surface.



[Electric field vanishes in the cavity of a conductor]

ELECTROSTATIC SHIELDING: **" The method of protecting a certain region from the effect of electric field is called electrostatic shielding".**

Consider a conductor with a cavity, with no charges placed inside the cavity. Whatever be the size and shape of the cavity and whatever the charge on the conductor be and the external fields in which it might be placed, the electric field inside the cavity is zero, i.e., the cavity inside the conductor remains shielded from outside electric influence. This is known as electrostatic shielding. Such a field free region is called a **Faradays cage**.

The phenomenon of making a region free from any electric field is called electrostatic shielding. It is based on the fact that electric field vanishes inside the cavity of a hollow conductor.

♋ **Applications of electrostatic shielding:**

1. In a thunderstorm accompanied by lightning. It is safest to sit inside a car, rather than near a tree or on the open ground. The metallic body of the car becomes an electrostatic shielding from lightning.

2. Sensitive components of electronic devices are protected or shielded from external electric disturbances by placing metal shields around them.

3. In a coaxial cable, the outer conductor connected to ground provides an electrical shield to the signals carried by the central conductor.

Conceptual……………………………………….

☑ **In the interior of a conductor, the electric field and the volume charge density both vanish. Therefore, charges in a conductor can only be at the surface.**

☑ **Electric field at the surface of a charged conductor must be normal to the surface at every point.**

☑ **For a conductor without any surface charge, electric field is zero even at the surface.**

☑ **The entire body of each conductor, including its surface, is at a constant potential.**

☑ **If we have conductors of arbitrary size, shape and charge configuration, then each conductor will have a characteristics value of constant potential which may differ from one conductor to another.**

☑ **A cavity inside a conductor is shielded from outside electrical disturbances. However, the electrostatic shielding \ does not work the other way round, that is, if we place charges inside the cavity, the exterior of the conductor cannot be shielded from the electric fields of the inside charges.**

Ex: It is safer to sit in a car or a bus during thunderstorm accompanied by lightening rather than to stand under a tree or on the open ground.

ELECTRICAL CAPACITANCE OF A CONDUCTOR

"The measure of the ability of a conductor to store charge is known electrical capacitance (or capacity of a conductor)."

✪ i.e., Electrical capacitance of a conductor is related to its ability to store.

When an insulated conductor is given some charge, it acquires a certain potential. If we increase the charge on a conductor, its potential also increases. If a charge Q put on an insulated conductor increases its potential by V, then

$$q \propto V \qquad \text{or} \qquad q = CV$$

♦The proportionally constant C is called the capacitance of the conductor. Thus

$$\text{Capacitance} = \frac{\text{Charge}}{\text{Potential}}$$

Capacitance of a conductor may be defined as the charge required to increase the potential of the conductor by unit amount.

Or, *"Capacitance is the ratio of electric charge to the electric potential."*

✪ **--- Capacitor is a scalar quantity.**

◙ The capacitance of a conductor is the measure of its capacity to hold a large amount of charge without running a high potential. It depends upon the following factors:

●1. Size and shape of a conductor.
●2. Nature (permittivity) of the surrounding medium.
●3. Presence of the other conductors in its neighbourhood.

♦ Capacitance of a conductor does not depend on the nature of its material and the amount of charge existing on the conductor.

Units of capacitance: The SI unit of capacitance is farad (F), named in the honour of Michael Faraday.

"The capacitance of conductor is 1 farad if the addition of a charge of 1 coulomb to it, increases its potential by 1 volt".

$\therefore \quad 1 \text{ farad} = \dfrac{1 \text{ coulomb}}{1 \text{ Volt}} \quad$ or $\quad 1 \text{ F} = \dfrac{1 \text{ C}}{1 \text{ V}} = 1 \text{ CV}^{-1}$

♦One farad is a very large unit of capacitance. For practical purposes, we use its following submultiples:

◙1 millifarad = 1 mF = 10^{-3} F ◙1 microfarad = 1 μF = 10^{-6} F ◙1 picofarad = 1 pF = 10^{-12} F

Dimensions of capacitance: The unit of capacitance is

$$1 \text{ F} = \frac{1 \text{ C}}{1 \text{ V}} = \frac{1 \text{ C}}{1 \text{ J / C}} = \frac{1 \text{ C}^2}{1 \text{ J}} = \frac{1 \text{ (As)}^2}{1 \text{ Nm}}$$

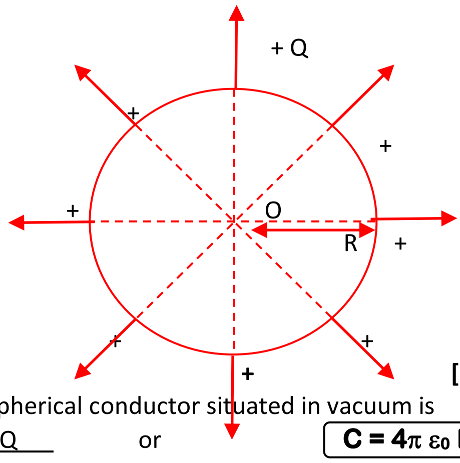
$\therefore$ Dimensions of capacitance $= \dfrac{A^2 T^2}{MLT^{-2} . L} = [M^{-1} L^{-2} T^4 A^2]$

☞ Electrostatic unit (esu) = stat farad.; 1 Farad = 9×10^{11} stat farad.

☞ Electromagnetic unit (emu) = ab farad; 1 Farad = 10^9 ab farad.

CAPACITANCE OF AN ISOLATED SPHERICAL CAPACITOR

Capacitance of an isolated spherical conductor: Consider an isolated spherical conductor of radius R. The charge Q is uniformly distributed over its entire surface. It can be assumed to be concentrated at the centre of the sphere. The potential at any point on the surface of the spherical conductor will be $V = \dfrac{1}{4 \pi \varepsilon_0} . \dfrac{Q}{R}$



[Capacitance of a spherical conductor]

$\therefore$ Capacitance of the spherical conductor situated in vacuum is

$$C = \frac{Q}{V} = \frac{Q}{\frac{1}{4 \pi\varepsilon_0} . \frac{Q}{R}} \quad \text{or} \quad \boxed{C = 4\pi \varepsilon_0 R} \qquad \boxed{C = \frac{r}{9 \times 10^9} \text{ F}}$$

Clearly, the capacitance of a spherical conductor is proportional to its radius.

Let us calculate the radius of the spherical conductor of capacitance 1 F.

$R = \dfrac{1}{4 \pi \varepsilon_0} . C = 9 \times 10^9 \text{ mF}^{-1}. 1 \text{ F} = 9 \times 10^9 \text{ m} = 9 \times 10^6 \text{ km}.$

This radius is about 1500 times the radius of the earth (~ 6×10^3 km). So we conclude:

1. One farad is a very large unit of capacitance.
2. It is not possible to have a single isolated conductor of very large capacitance.

Knowledge +......................................

☑ **The formula: $C = 4 \pi\varepsilon_0 R$ is valid for both hollow and solid spherical conductors.**

☑ **As $\varepsilon_0 = \dfrac{C}{4 \pi R}$**

So the SI unit of a ε_0 can be written as farad per metre (Fm^{-1}). From Coulomb's law, the SI unit of ε_0 comes out to be $C^2 N^{-1} m^{-2}$. Both of these units are equivalent.

☑ **The farad (1 F = 1 CV^{-1}) is an enormously large unit of capacitance because the coulomb is a very big unit of charge while the volt is the unit of potential having reasonable size.**

Examples based on Capacitance of Spherical Conductors

❖ **Formulae Used**

1. Capacitance of a spherical conductor of radius R,

$$C = 4\pi\varepsilon_0 R$$

2. Capacitance = $\frac{\text{Charge}}{\text{Potential}}$ or $C = \frac{q}{V}$

❖ **Units Used** Charge is in coulomb, potential in volt and capacitance in farad (F).

Q. 1. ***Assuming the earth to be a spherical conductor of radius 6400 km, calculate its capacitance.***

Sol. Here $R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{C}^{-2}$$

The capacitance of a spherical conductor,

$$C = 4\pi\varepsilon_0 R = \frac{1}{9 \times 10^9} \times 6.4 \times 10^6 = 0.711 \times 10^{-3} \text{ F} = 711 \mu\text{F}$$

Q. 2. ***Twenty seven spherical drops of radius 3 mm and carrying 10^{-12} C of charge are combined to form a single drop. Find the capacitance and the potential of the bigger drop.***

Sol. Let r and R be the radii of the small and bigger drops, respectively.

Volume of the bigger drop = 27 × Volume of a small drop

i.e., $\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$

or $R = 3r = 3 \times 3 \text{ mm} = 9 \times 10^{-3} \text{ m}$

∴ Capacitance of the bigger drop is

$$C = 4\pi\varepsilon_0 R = \frac{1}{\quad} . 9 \times 10^{-3} \text{ F} = 10^{-12} \text{ F} = 1 \text{ pF}$$

Charge on bigger drop

$q = 27 \times$ charge on a small drop $= 27 \times 10^{-12}$ C

∴ Potential of bigger drop is

$$V = \frac{q}{C} = \frac{27 \times 10^{-12}}{10^{-12}} = 217 \text{ V}$$

Q. 3. ***An isolated sphere has a capacitance 50 pF. (i) Calculate its radius. (ii) How much charge should be placed on it to raise its potential to 10^4 V?***

Sol. Here $C = 50 \text{ pF} = 50 \times 10^{-12}$ F, $V = 10^4$ V

(i) $R = \frac{1}{4\pi\varepsilon_0} . C = 9 \times 10^9 \text{ mF}^{-1} \times 50 \times 10^{-12} \text{ F} = 45 \times 10^{-2} \text{ m} = 45 \text{ cm}$

(ii) $q = CV = 50 \times 10^{-12} \times 10^4 = 5 \times 10^{-7} \text{ C} = 0.5 \mu\text{C}$.

Q. 4. ***A charged spherical conductor has a surface charge density of 0.07 C cm^{-2}. When the charge is increased by 4.4 C, the surface charge density changes by 0.084 C cm^{-2}. Find the initial charge and capacitance of the spherical conductor.***

Sol. Let q be the charge on the spherical conductor and r its radius. Its surface charge density is

$$\frac{q}{4\pi r^2} = 0.07 \text{ C cm}^{-2} \quad \ldots \text{(i)}$$

When the charge is increased by 4.4 C, the surface charge density becomes

$$\frac{q + 4.4}{4\pi r^2} = 0.084 \text{ C cm}^{-2} \quad \ldots \text{(ii)}$$

Dividing equation (ii) by (i), we get

$$\frac{q + 4.4}{q} = \frac{0.084}{0.07}$$

or $q = 22$ C

From equation (i), we get

$$r = \frac{q}{4\pi \times 0.07} = \frac{22 \times 7}{4 \times 22 \times 0.07}$$

$= 5 \text{ cm} = 0.05 \text{ m}$

Capacitance, $C = 4\pi\varepsilon_0 r = \frac{1}{9 \times 10^9} \times 0.05 = 5.56 \times 10^{-12}$ F

☑ Capacitance of Earth:

When earth is considered as spherical conductor of radius, $r = 6.4 \times 10^6$ m.

The capacitance of earth $C = \dfrac{r}{9\times10^9}$

$= \dfrac{6.4 \times 10^6}{9\times10^9} = 0.711 \times 10^{-3}$ F $= 711 \times 10^{-6}$ F $=$ **711μ F.**

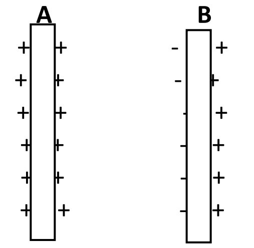
☞The capacitor of a spherical conductor is directly proportional to its radius $\boxed{C \propto r}$,

∴ **larger the sphere , larger is its capacity and vice versa** .

☑ Capacitor and its principle:

"A capacitor consists of two conducting bodies separated by non-conducting medium such that it can store large amount of electric charge in small space, and hence electric energy."

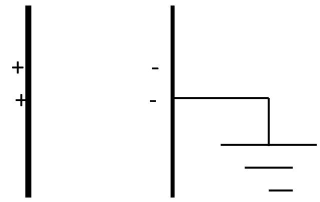
PRINCIPLE: Consider *an* insulated metal plate, A. Suppose that it is given positive charge, till its potential becomes maximum. The metal plate A will not hold any more charge over it. If charge is given to the metal plate A, it will leak to the surrounding. Now, consider another insulated metal plate B held near the plate A.



Due to induction, negative charge will be produced (induced) on the nearer face of B and positive charge on the farther face. The potential of A gets lowered due to induced negative charge on plate B and a bit raised due to induced positive charge. As the induced negative charge is closer to A , it is more effective . The overall potential of A reduces and hence more charge can be given to A to raise its potential to maximum.

✪It indicates that the capacity of a conductor increases by a small amount, when another uncharged conductor is placed near it.

Now connect the plate B to the earth



The induced positive charge on B will immediately flow to the earths as it is repelled by positive charge on plate A. However, the induced negative charge on plate B will stay on it as it is attracted by positive charge of A.

Due to induced negative charge on B, potential of A is greatly reduced.

Thus, a large amount of charge can be given to A to raise it to maximum potential.

✪**PRINCIPLE**: *Capacitance of an insulated conductor is increased considerably by bringing near it on uncharged earthed conductor.*

✪**Definition: (Capacitor)** --- ***"An arrangement of two metallic conductor, so that when one conductor is connected to the earth, the other conductor has the ability to store a large amount of charge on it, is called a capacitor".***

✪The two metallic conductors of the capacitor are called coating of the conductor (condenser).

------ If the coating is spherical, capacitor is called spherical capacitor.

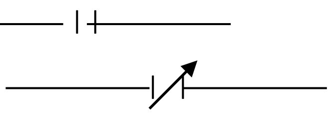
------ If the coating is plane, capacitor is called parallel plate capacitor.

------ If the coating is Cylindrical, capacitor is called cylindrical capacitor.

✪Pictorial symbol-----------------------------

→ Fixed capacitance

→ Variable capacitance



✪ **The** insulating medium between the two plates is called dielectric (air, paper, mica etc)

✪ If the insulating medium between two plates is air, it's called air capacitor.

✪If the dielectric has dielectric constant K, then capacitance of capacitor becomes K times its capacitance as air capacitor.

✠**USES:** used in electrical appliances like radio set, T.V fan, ignition system of engine, in electric motor, Computers etc.

✠ Types of Capacitors:

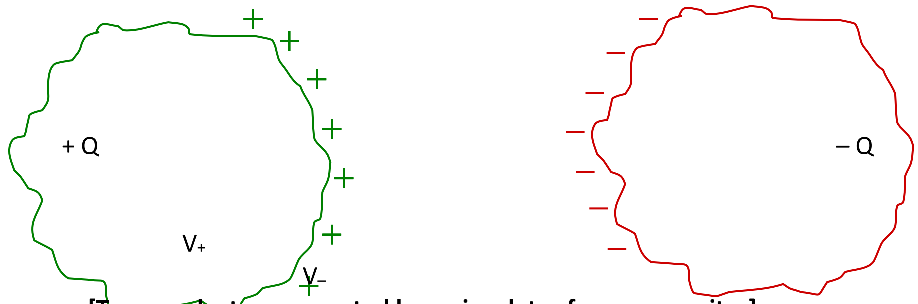
Capacitor are of many types. Some of the important types of capacitors are: --

[A] Parallel plate capacitor
[B] Spherical capacitor
[C] Cylindrical Capacitor.

◙***Capacitor:*** **A capacitor is an arrangement of two conductors separated by an insulating medium that is used to store electric charge and electric energy.**

●A capacitor, in general, consists of two conductors of any size and shape carrying different potentials and charges, and placed closed together in some definite positions relative to one another.

Capacitance of capacitor: As shown in Fig. usually a capacitor consists of two conductors having chargers + Q and – Q. The potential difference between them is $V = V_+ - V_-$. here Q is called the charge on the capacitor. Note that the charge on capacitor does not mean the total charge given to the capacitor which is + Q – Q =0



[Two conductors separated by an insulator form a capacitor]

For a given capacitor, the charge Q on the capacitor is proportional to the potential difference V between the two conductors. Thus, $Q \propto V$ or $Q = CV$

The proportionality constant C is called the capacitor of the capacitor. Clearly,

$$C = \frac{Q}{V}$$

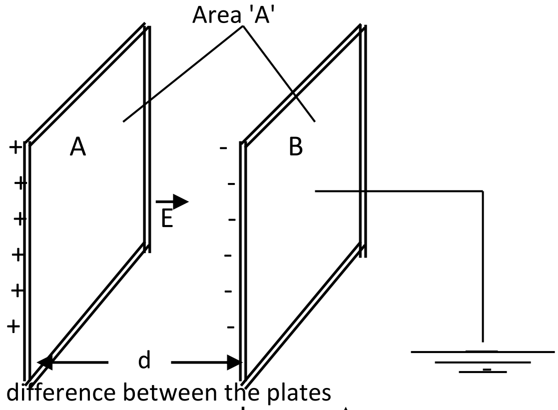
or Capacitance = $\dfrac{\text{Charge on either conductor}}{\text{P.D. between the two conductors}}$

The capacitance of a capacitor may be defined as the charge required to be supplied to either of the conductors of the capacitor so as to increase the potential difference between them by unit amount.

SI unit of capacitance is farad (F): A capacitor has a capacitance of farad if 1 coulomb of charge is transferred from its one conductor to another on applying a potential difference of 1 volt across the two conductors.

[A] Parallel plate capacitor

Parallel plate capacitor consists of two thin conducting plate each of area 'A' held parallel to each other. The separation 'd' between the plates is very small as compared to area 'A'. The plates are separated by an insulating medium like air, mica, glass, etc. One of the plates is insulated and the other is earth connected.



If charge +q is given to plate 'A', then charge –q is induced on the left face of B and charge +q on the right face. When B is earthed, the +q (of B) being free, flow to the earth. Due to +q on plate A and –q on plate B, Electric field is set up between the plate.

We know that electric field intensity between the plate is

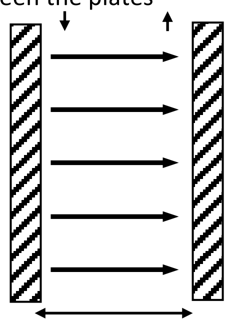
$$E = \frac{\sigma}{\xi_o}$$

An electric field intensity between is uniform, E = V /d

$$V = E\,d \quad \text{(For uniform electric field, } dV/dr = V/d)$$

or, $V = \dfrac{\sigma}{\xi_o} d$ Where , V = potential difference between the plates

$V = \dfrac{q\,d}{A\,\xi_o}$ [since q / A = σ]



If 'C' is the capacitance of parallel plate capacitor then $C = q/V$

or, $C = \dfrac{q}{\dfrac{q\,d}{A\,\xi_o}} = \dfrac{A\,\xi_o}{d}$

∴ capacitance of parallel plate capacitor (when its plate are held in air or vacuum)

$$\boxed{C = \frac{A\,\xi_o}{d}}$$

i.e., *'C' is directly proportional to the area of the plate and inversely proportional to their distance of separation.*

✪ **When plates of capacitor are separated by a dielectric medium of relative permittivity 'ξ_r' , its capacitance---**

$$C_m = \frac{A\,\xi}{d} = \frac{\xi_r\,\xi_o\,A}{d} = \xi_r \frac{\xi_o\,A}{d} = \xi_r C_o$$

◙Thus the capacitance of a parallel plate capacitor depends on the following factors:

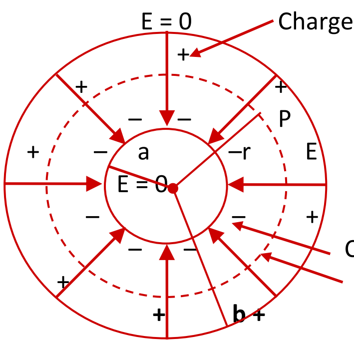
1. Area of the plates ($C \propto A$).
2. Distance between the plates ($C \propto 1/d$).
3. Permittivity of the medium between the plates ($C \propto \varepsilon$).

◙The direction of the electric field is from the positive to the negative plate and the field is uniform throughout. For plates with finite area, the field lines bend at the edges, this effect is called fringing of the field. But for larges plates separated by small distance ($A >> d^2$), the field is almost uniform in the regions far from the edges.

SPHERICAL CAPACITOR

Spherical capacitor: A spherical capacitor consists of two concentric spherical shells of inner and outer radii a and b. The two shells of inner and outer radii a and b. The two shells carry charges – Q and + Q respectively. Since the electric field inside a hollow conductor is zero, so $\vec{E} = 0$ for r < a. Also the field is zero outside the outer shell, i.e., $\vec{E} = 0$ for r > b. A radial field $\vec{E}$ exists in the region between the two shells due to the charge on the inner shell only.

To determine the electric field at any point P at distance r from the centre, consider a concentric sphere of radius r as the Gaussian surface. Using Gauss's theorem, $\phi = E.\,4\pi r^2 = \frac{Q}{\varepsilon_0}$ or $E = \frac{Q}{4\pi\varepsilon_0 r^2}$



[Spherical capacitor]

The potential difference (caused by the inner sphere alone) between the two shells will be

$$V = -\int_a^b \vec{E}\,.\,d\vec{r} = \int_a^b E dr = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$= \frac{Q}{4\pi\varepsilon_0}\int_a^b r^{-2}\,dr = \frac{Q}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)_a^b = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

[∵ $\vec{E}$ points radially inward and $d\vec{r}$ points outward so $\vec{E}.\,d\vec{r} = Edr\,180° = -Edr$]

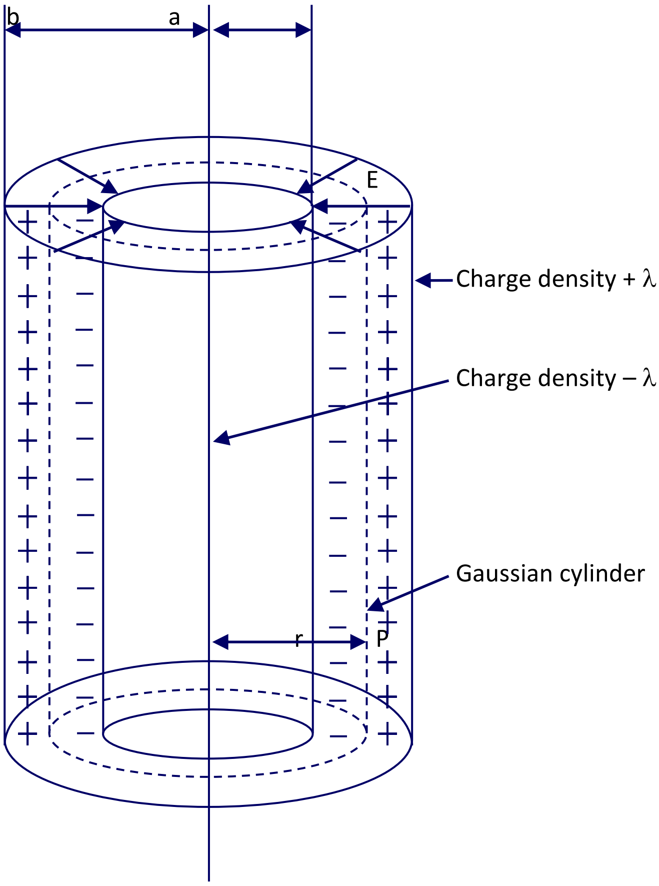
The capacitance of the spherical capacitor is

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)}$$

or $$C = \frac{4\pi\varepsilon_0 ab}{b-a}$$

CYLINDRICAL CAPACITOR.

Cylindrical capacitor: A cylindrical capacitor consists of two coaxial conducting cylinders of inner and outer radii a and b. Let the two cylinders have uniform linear charge densities of $\pm\lambda$ Cm^{-1}. The length L of the capacitor is so large (L >> radii a or b) that the edge effect can be neglected. The electric field in the region between the two cylinders comes only from the inner cylinder; the outer cylinder does not contribute due to shielding. To calculate the electric field E at any point P in between the two cylinders at a distance r from the central axis, we consider a coaxial Gaussian cylinder of radius r. Using Gauss's theorem, the flux through Gaussian surface must be



$$\phi_E = \frac{q}{\varepsilon_0}$$

or $$E.\,2\pi r\,L = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

∴ Potential difference between the two cylinder is

$$V = -\int_a^b E\,.\,dr$$

$$= \int_a^b E dr \qquad [\because \text{E and dr are in opposite directions}]$$

$$= \int_a^b \frac{\lambda}{2\pi\varepsilon_0 r} dr = \frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{1}{r} dr$$

$$= \frac{\lambda}{2\pi\varepsilon_0}[\ln r]_a^b = \frac{\lambda}{2\pi\varepsilon_0}[\ln b - \ln a]$$

or $$V = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}$$

Total charge on each cylinder is

$$Q = L\lambda$$

∴ Capacitance of cylindrical capacitor is

$$C = \frac{Q}{V} = \frac{L\lambda}{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}}$$

or $$C = \frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}}$$

Examples based on Capacitance of Air-Filled Capacitors

❖ **Formulae Used**

1. Capacitance, $C = \frac{q}{V}$

2. Capacitance of a parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$

3. P.D. between the two plates of a capacitor having charge q_1 and q_2,

$$V = \frac{q_1 - q_2}{2C}$$

4. Capacitance of a spherical capacitor,

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

Here a and b are the radii of inner and outer shells of the spherical capacitor.

5. Capacitance of a cylindrical capacitor,

$$C = 2\pi\varepsilon_0 \frac{L}{\log_e \frac{b}{a}} = 2\pi\varepsilon_0 \frac{L}{2.303 \log_{10} \frac{b}{a}}$$

Here a and b are the radii of inner and outer coaxial cylinders and L is the length of the capacitor.

❖ **Units Used** Capacitance C is in farad, charge q in coulomb, potential difference V in volt, thicknesses d and t in metre.

❖ **Constant Used** Permittivity constant, $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

Q. 1. ***When 1.0×10^{12} electrons are transferred from one conductor to another of a capacitor, a potential difference of 10 V develops between the two conductors. Calculate the capacitance of the capacitor.***

Sol. Here $q = ne = 1.0 \times 10^{12} \times 1.6 \times 10^{-19} = 1.6 \times 10^{-7}$ C ; V = 10 V

$\therefore$ $C = \frac{q}{V} = \frac{1.6 \times 10^{-7}}{10} = 1.6 \times 10^{-8}$ F

Q. 2. ***When the potential difference across a capacitor is reduced by 120 V μC. What is the capacitance of the capacitor?***

Sol. Let C be the capacitance of the capacitor and V the potential drop across its plates. Then

$$C = \frac{q}{V} = \frac{360 \times 10^{-6}}{V} \quad \dots (1)$$

When the potential difference is reduced by 120 V, the charge on capacitor plates reduces to 120 μC. therefore,

$$C = \frac{120 \times 10^{-6}}{V - 120} \quad \dots (2)$$

From equations (1) and (2), we get

$$\frac{360 \times 10^{-6}}{V} = \frac{120 \times 10^{-6}}{V - 120}$$

or $\frac{3}{V} = \frac{1}{V - 120}$

$\therefore$ V = 180 V

From (1), $C = \frac{360 \times 10^{-6}}{180} = 2 \times 10^{-6}$ F = 2 μF

Q. 3. ***Two identical metal plates are given charges q_1 and q_2 ($< q_1$) respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, what will be the potential difference between the plates?***

Sol. Let A be area of each plate. When the two plates are placed d distance apart, the capacitance of the parallel plate capacitor so formed is

$$C = \frac{\varepsilon_0 A}{d}$$

If E_1 and E_2 are the electric fields due to the two plates, then the net field between the two plates will be

$$E = E_1 - E_2 = \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}$$

$$= \frac{q_1/A}{2\varepsilon_0} - \frac{q_2/A}{2\varepsilon_0} = \frac{1}{2\varepsilon_0 A}(q_1 - q_2)$$

The potential difference between the two plates will be

$$V = Ed = \frac{1}{2\varepsilon_0 A}(q_1 - q_2) \times d = \frac{1}{2\varepsilon_0 A/d}(q_1 - q_2)$$

or $V = \frac{q_1 - q_2}{2C}$

Q. 4. ***A parallel plate capacitor has plate area of 25.0 cm² and a separation of 2.0 mm between its plates. The capacitor is connected to 12 V battery. (i) Find the charge on the capacitor. (ii) If the plate separation is decreased by 1.0 mm, what extra charge is given by the battery to the positive plate?***

Sol. $A = 25.0\ cm^2 = 25 \times 10^{-4}\ m^2$,

$d = 2.0\ mm = 2 \times 10^{-3}\ m$, $V = 12\ V$

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 25 \times 10^{-4}}{2 \times 10^{-3}}$$

$$= 1.1 \times 10^{-11}\ F$$

(i) $q = CV = 1.1 \times 10^{-11} \times 12 = 1.32 \times 10^{-10}\ C$

(ii) Here $d' = 2.0 - 1.0 = 1.0\ mm = 1 \times 10^{-3}\ m$

$$\therefore \quad C' = \frac{8.85 \times 10^{-12} \times 25 \times 10^{-4}}{1 \times 10^{-3}} = 2.2 \times 10^{-11}\ F$$

$q' = C'V = 2.2 \times 10 - 11 \times 12 = 2.64 \times 10^{-10}\ C$

Extra charge given by the battery to the positive plate is

$q' - q = (2.64 - 1.32) \times 10^{-10} = 1.32 \times 10^{-10}\ C$

Q. 5. ***Two parallel plate air capacitors have their plate areas 100 and 500 cm² respectively. If they have the same charge and potential and the distance between the plates of the first capacitor is 0.5 mm, what is the distance between the plates of the second capacitor?***

Sol. As capacitance, $C = q/V$ and the two capacitors have the same charge q and potential V, so they have the equal capacitances, i.e., $C_1 = C_2$

or $$\frac{\varepsilon_0 A_1}{d_1} = \frac{\varepsilon_0 A_2}{d_2} \quad \text{or} \quad d_2 = \frac{A_2}{A_1} d_1$$

But $A_1 = 100\ cm^2$, $A_2 = 500\ cm^2$,

$d_1 = 0.5\ mm = 0.05\ cm$

$$\therefore \quad d_2 = \frac{500 \times 0.05}{100} = 0.25\ cm = 2.5\ mm$$

Q. 6. ***A sphere of radius 0.03 m is suspended within a hollow sphere of radius 0.05 m. If the inner sphere is charged to a potential of 1500 volt and outer sphere is earthed, find the capacitance and the charge on the inner sphere.***

Sol. Here $a = 0.03\ m$, $b = 0.05\ m$, $V = 1500\ V$

The capacitance of the air-filled spherical capacitor is

$$C = \frac{4\pi\varepsilon_0 ab}{b - a} = \frac{0.03 \times 0.05}{9 \times 10^9 \times (0.05 - 0.03)}$$

$$= 8.33 \times 10^{-12}\ F = 8.33\ pF$$

Charge, $q = CV = 8.33 \times 10^{-12} \times 1500 = 1.25 \times 10^{-8}\ C$

Q. 7. ***The thickness of air layer between the two coatings of a spherical capacitor is 2 cm. The capacitor has the same capacitance as the sphere of 1.2 m diameter. Find the radii of its surfaces.***

Sol. Here $$\frac{4\pi\varepsilon_0 ab}{b - a} = 4\pi\varepsilon_0 R$$

or $$\frac{ab}{b - a} = R$$

Now $b - a = 2\ cm$ and $R = \frac{1.2}{2}\ m = 60\ cm$

$\therefore$ $ab/2 = 60$ or $ab = 120$

$(b + a) = (b - a)^2 + 4ab = 2^2 + 4 \times 120 = 484$

or $b + a = 22$ or $2 + a + a = 22$ $[\because b - a = 2\ cm]$

$\therefore$ $a = 10\ cm$ and $b = 12\ cm$

Q. 8. ***Assuming an expression for the potential of an isolated conductor, show that the capacitance of such a sphere will be increased by a factor by a factor n if it is enclosed within an earthed concentric sphere, the ratio of the radii of the spheres being n/(n – 1).***

Sol. The capacitance of an isolated conducting sphere of radius a is

$$C = 4\pi\varepsilon_0 a$$

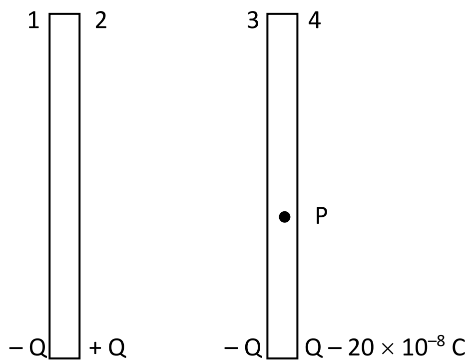
When surrounded by an earth sphere of radius b, its capacitance becomes

$$C' = 4\pi\varepsilon_0 \cdot \frac{ab}{b - a}$$

$$\therefore \quad \frac{C'}{C} = \frac{ab}{a(b - a)} = \frac{b}{b - a} = \frac{1}{1 - \frac{a}{b}} = \frac{1}{1 - \frac{n-1}{n}} = n$$

Q. 9. ***The negative plate of a parallel plate capacitor is given a charge of -20×10^{-8} C. Find the charges appearing on the four surfaces of the capacitor plates.***

Sol. As shown in Fig., let the charge appearing on the inner surface of the negative plate be $-Q$. Then the charge on its outer surface will be $Q - 20 \times 10^{-8}$ C.



The induced charge on the inner surface of the positive plate will be + Q and that on the outer surface will be – Q, as the positive plate is electrically neutral. To find Q, we consider the electric field at a point P inside the negative plate.

Field due to surface 1 = $\dfrac{Q}{2\varepsilon_0 A}$, toward left

Field due to surface 2 = $\dfrac{Q}{2\varepsilon_0 A}$, towards right

Field due to surface 3 = $\dfrac{Q}{2\varepsilon_0 A}$, towards left

Field due to surface 4 = $\dfrac{Q - 20 \times 10^{-8}\text{ C}}{2\varepsilon_0 A}$, towards left

As the point P lies inside the conductor, the field here must be zero.

$$\frac{Q}{2\varepsilon_0 A} - \frac{Q}{2\varepsilon_0 A} + \frac{Q}{2\varepsilon_0 A} + \frac{Q - 20 \times 10^{-8}}{2\varepsilon_0 A} = 0$$

or $Q = +10 \times 10^{-8}$ C

∴ Charge on surface 1 = -10×10^{-8} C

Charge on surface 2 = $+10 \times 10^{-8}$ C

Charge on surface 3 = -10×10^{-8} C

Charge on surface 4 = -10×10^{-8} C

COMBINATION OF CAPACITORS IN SERIES AND IN PARALLEL

The most common mode of capacitor is ----
- [1] Series grouping &
- [2] Parallel grouping

✪ In series, Net capacitance -- 'DECREASES'

✪ In parallel, Net capacitance -- ' INCREASES'

Capacitors in Series: ***When the negative plate of one capacitor is connected to the positive plate of the second and the negative of the second to the positive of third and so on, the capacitors are said to be connected in series.***

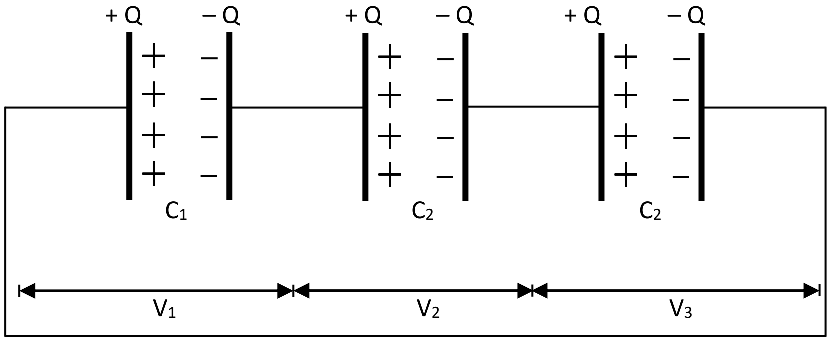
♦ The capacitor are said to connected in series between two points when proceed form one point to the other only through one path.

Consider three capacitors of capacitances C_1, C_2 and C_3 connected in series. A potential difference V is applied across the combination. This sets up charge ± Q on the two plates of each capacitor. What actually happens is, a charge + Q is given to the left plate of capacitor C_1 during the charging process. The charge + Q induces a charge – Q on the right plate of C_1 and a charge – Q on the left plate of C_2, etc.

The potential differences across the various capacitors are

$$V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2},\ V_3 = \frac{Q}{C_3}$$

For the series circuit, the sum of these potential differences must be equal to the applied potential differences.



[Capacitors in series]

∴ $V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

or $\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ ... (1)

Clearly, the combination can be regarded as an effective capacitor with charge Q and potential difference V. If C_S is the equivalent capacitance of the series combination, then

$C_S = \frac{Q}{V}$ or $\frac{1}{C_S} = \frac{V}{Q}$ ... (2)

From equations (1) and (2), we get

$$\boxed{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

For a series combination of n capacitors, we can write

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$$

☞ For 'n' capacitors connected in series, total capacitance would be

$$\boxed{\frac{1}{C_S} = \sum_{i=1}^{n} \frac{1}{C_i}}$$

Hence, Equivalent capacitance of any no. of capacitor joined in series is equal to sum of the reciprocal of the individual capacitance.

♋ For series combination of capacitors:

1. The reciprocal of equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances.

2. The equivalent capacitance is smaller than the smallest individual capacitance.

3. The charge on each capacitor is same.

4. The potential difference across any capacitor inversely proportional to its capacitance.

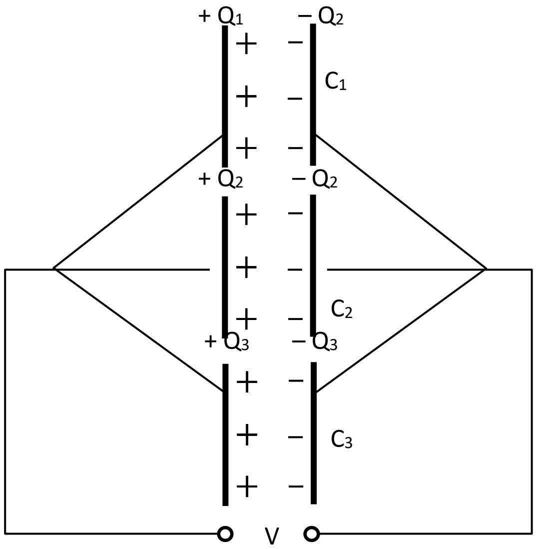
◙Capacitors in parallel:

When the positive plates of all capacitors are connected to one common point and the negative plates to another common point, the capacitors are said to be connected in parallel.

♦ The capacitors are said to be connected in parallel between any two points, if we can proceed from one point to the another along different path.

Consider three capacitors of capacitances C_1, C_2 and C_3 connected in parallel. A potential differences V is applied across the combination. All the capacitors have a common potential difference V but different charges given by

$Q_1 = C_1 V$, $Q_2 = C_2 V$, $Q_3 = C_3 V$



[Capacitors in parallel]

Total charge stored in the combination is

$$Q = Q_1 + Q_2 + Q_3 = (C_1 + C_2 + C_3)\,V \quad \text{... (1)}$$

If C_p is the equivalent capacitance of the parallel combination, then

$$Q = C_p V \quad \text{... (2)}$$

From equations (1) and (2), we get

$$C_p V = (C_1 + C_2 + C_3)\,V \quad \text{or} \quad C_p = C_1 + C_2 + C_3$$

$$C_P = (C_1 + C_2 + C_3)$$

☞ **When there are 'n' capacitor are connected in parallel , then** $\boxed{C_P = \sum_{i=1}^{n} C_i}$

Hence, Equivalent capacitance of any no. of capacitors are joined in series is equal to the sum of the individual capacitance

✪C_{SERIES} < least of the $C_1, C_2, C_3,$ - - - - - - - - C_n

✪$C_{PARALLEL}$ < largest of the $C_1, C_2, C_3,$ - - - - - - - - C_n.

✪ When there is a combination of capacitors in an electrical circuit, we replace that capacitor called equivalent capacitor and its capacitance is called equivalent capacitance.

♋ For parallel combination of capacitors:

1. The equivalent capacitance is equal to the sum of the individual capacitances.
2. The equivalent capacitance is larger than the largest individual capacitance.
3. The potential difference across each capacitor is same.
4. The charge on each capacitor is proportional to its capacitances.

Examples based on Grouping of Capacitors

❖ Formulae Used

1. In series combination, $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$
2. In parallel combination, $C_P = C_1 + C_2 + C_3 + ...$
3. In series combination, charge on each capacitor is same (equal to the charge supplied by battery) but potential difference across the capacitors may be different.
4. In parallel combination, potential difference on each capacitor is same but the charges on the capacitors may be different.

❖ Units Used

Capacitances are in farad, potential differences in volt and charges in coulomb.

Q. 1. ***Two capacitors of capacitance of 6 μF and 12 μF are connected in series with a battery. The voltage across the 6 μF capacitor is 2 V. Compute the total battery voltage.***

Sol. As the two capacitors are connected in series, the charge on each capacitor must be same.

∴ Charge on 6 μF capacitor = Charge on 12 μF capacitor

or $6\ \mu F \times 2$ volt = $12\ \mu F \times V$ volt

∴ P.D. across 12 μF capacitor = $\frac{6 \times 2}{12} = 1$ volt

Battery voltage = $V_1 + V_2 = 2V + 1\ V = 3\ V$

Q. 2. ***Two capacitors of capacitances 3 μF and 6 μF, are charged to potentials of 2V and 5V respectively. These two charged capacitors are connected in series. Find the potential across each of the two capacitors now.***

Sol. Total charge on the two capacitors

$= C_1 V_1 + C_2 V_2 = (3 \times 2 + 6 \times 5)\ \mu C = 36\ \mu C$

Potential on 3 μF capacitor = $\frac{q}{C_1} = \frac{36\ \mu C}{3\ \mu F} = 12\ V$

Potential on 6 μF capacitor = $\frac{q}{C_2} = \frac{36\ \mu C}{6\ \mu F} = 6\ V$

Q. 3. ***Two capacitors have a capacitance of 5 μF when connected in parallel and 1.2 μF when connected in series. Calculate their capacitances.***

Sol. Let the two capacitances be C_1 μF and C_2 μF.

In parallel, $C_p = C_1 + C_2 = 5\ \mu F$

In series, $C_S = \frac{C_1 C_2}{C_1 + C_2} = 1.2\ \mu F$

or $\frac{C_1 (5 - C_1)}{5} = 1.2$ or $C_1^2 - 5C_1 + 6 = 0$

Hence, $C_1 = 2$ or 3 μF

∴ The capacitances are of 2 μF and 3 μF.

Q. 4. ***Three capacitors of equal capacitance, when connected in series have net capacitance C_1, and when connected in parallel have net capacitance C_2. What is the value of C_1/C_2?***

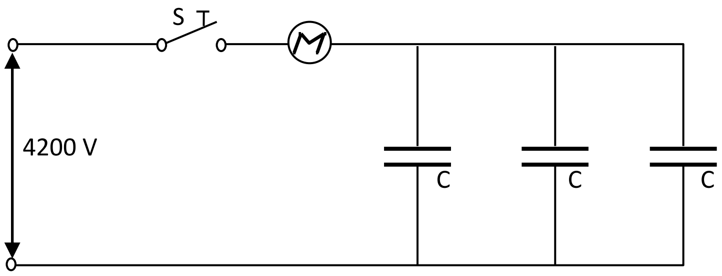
Sol. Let C = capacitance of each capacitor. For series combination,

$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$ or $C_1 = \frac{C}{3}$

For parallel combination,

$C_2 = C + C + C = 3\ C$ ∴ $\frac{C_1}{C_2} = \frac{C}{3} \cdot \frac{1}{3C} = \frac{1}{...}$

Q. 5. ***In Fig. each of the uncharged capacitors has a capacitance of 25 μF. What charge will flow through the metre M when the switch S is closed?***

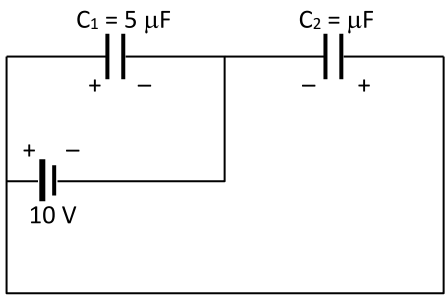


Sol. As the three capacitors are connected in parallel, their equivalent capacitance is

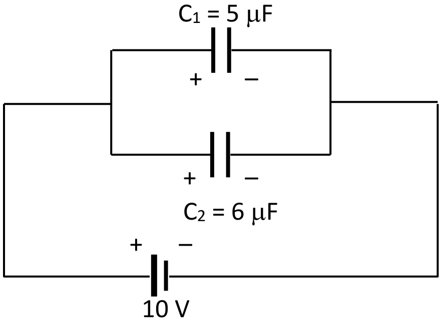
$C_p = C + C + C = 3C = 3 \times 25\ \mu F = 75\ \mu F$

∴ Charge, $q = C_p V = 75 \times 10^{-6} \times 4200 = 315 \times 10^{-3}\ C = 315$ mC

Q. 6. ***Calculate the charge supplied by the battery in the arrangement shown in Fig.***



Sol. The given arrangement is equivalent to the arrangement shown in Fig.



Clearly, the two capacitors are connected in parallel.

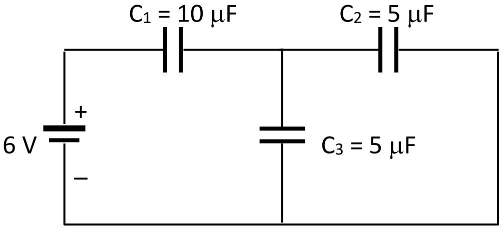
Their equivalent capacitance is

$C = C_1 + C_2 = 5 + 6 = 11\ \mu F$

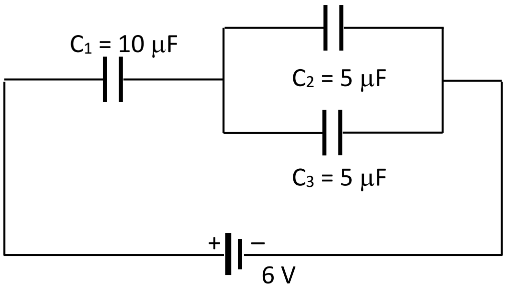
Charge supplied by the battery is

$q = CV = 11\ \mu F \times 10\ V = 110\ \mu C$

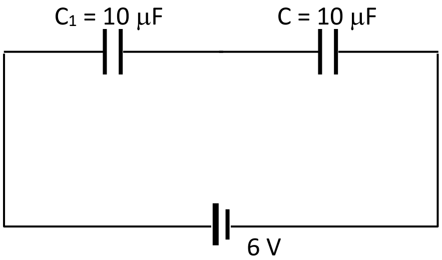
Q. 7. ***Three capacitors C_1, C_2 and C_3 are connected to a 6V battery, as shown in Fig. Find the charges on the three capacitors.***



Sol. The given arrangement is equivalent to the arrangement shown in Fig. (a)



(a)



(b)

Clearly, C_2 and C_3 are in parallel. Their equivalent capacitance is

$C' = C_2 + C_3 = 5 + 5 = 10\ \mu F$

Now C_1 and C' form a series combination, as shown in Fig. (b). Their equivalent capacitance is

$$C = \frac{C_1 C'}{C_1 + C'} = \frac{10 \times 10}{10 + 10} = 5\ \mu F$$

Charge drawn from the battery, $q = CV = 5\ \mu F \times 6\ V = 30\ \mu C$

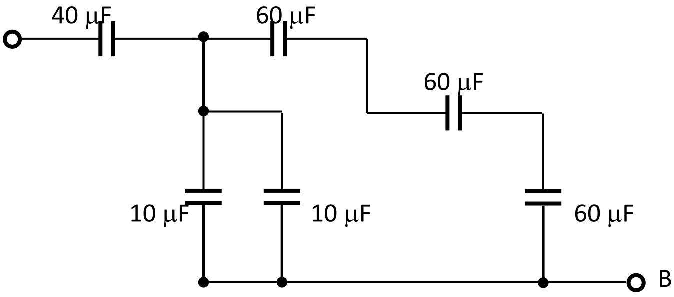
Charge on the capacitor $C_1 = q = 30\ \mu C$

Charge on the parallel combination of C_2 and $C_3 = q = 30\ \mu C$

As C_2 and C_3 are equal, so the charge is shared equally by the two capacitors.

Charge on C_2 = chare on $C_3 = \frac{30}{2} = 15\ \mu C$

Q. 8. ***Find the equivalent capacitance of the combination of capacitors between the points A and B as shown in Fig. Also calculate the total charge that flows in the circuits when a 100 V battery is connected between the point A and B.***



Sol. here three capacitors of 60 μF each are connected in series. Their equivalent capacitance C_1 is given by

$$\frac{1}{C_1} = \frac{1}{60} + \frac{1}{60} + \frac{1}{60} = \frac{3}{60} = \frac{1}{20}$$

or $C = 20\ \mu F$

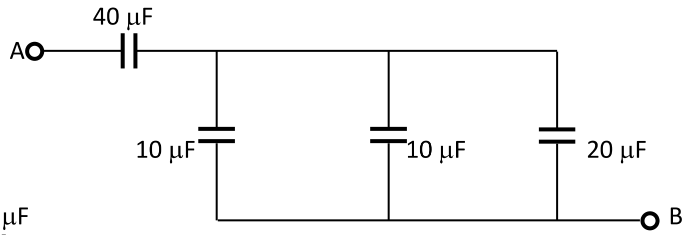
The given arrangement now reduces to the equivalent circuit shown n Fig. (a)

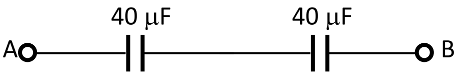
Clearly, the three capacitors of 10 μF, 10 μF and 20 μF are in parallel. Their equivalent capacitance is

$C_2 = 10 + 10 + 20 = 40\ \mu F$

Now the circuit reduces to the equivalent circuit shown in Fig. (b). We have two capacitors of 40 μF each connected in series. The equivalent capacitance between A and B is

$$C = \frac{40 \times 40}{40 + 40} = 20\ \mu F$$

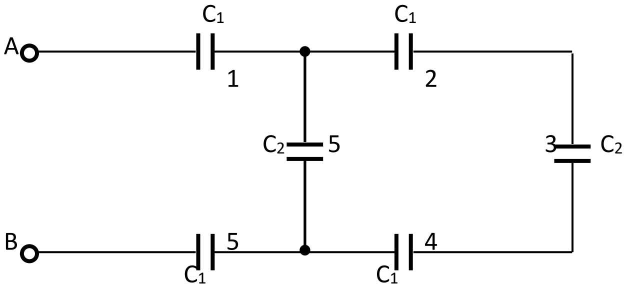




Given V = 100 V

∴ Charge, V = CV = 20 μF × 100 V= 2000 μC = 2 mC

Q. 9. ***If C_1 3 pF and C_2 = 2 pF, calculate the equivalent capacitance of the given network between points A and B.***



Sol. Clearly, capacitors 2, 3 and 4 form a series combination. Their total capacitance C' is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_1} = \frac{1}{3} + \frac{1}{2} + \frac{1}{3} = \frac{7}{6} \qquad \therefore \qquad C' = \frac{6}{7}\ pF$$

The capacitance C' forms a parallel combination with capacitor 5, so their equivalent capacitance is

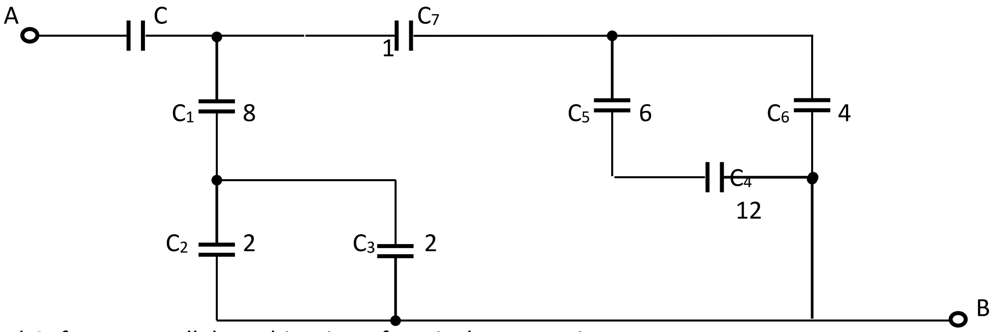
$$C'' = C' + C_2 = \frac{6}{7} + 2 = \frac{20}{7}\ pF$$

The capacitance C'' forms a parallel combination with capacitors 1 and 6. The equivalent capacitance C of the entire network is given by

$$\frac{1}{C} = \frac{1}{C''} + \frac{1}{C_1} + \frac{1}{C_1} = \frac{7}{20} + \frac{1}{3} + \frac{1}{3} = \frac{61}{60} \qquad \therefore \qquad C = \frac{60}{61}\ pF$$

Q. 10. ***From the network shown in Fig. find the value of the capacitance C if the equivalent capacitance between points A and B is to be 1 μF. All the capacitance are in μF.***

Sol.



Capacitors C_2 and C_3 from a parallel combination of equivalent capacitance,

$$C_8 = C_2 + C_3 = 2 + 2 = 4\ \mu F$$

Capacitors C_4 and C_5 form a series combination of capacitance C_9 is given by

$$\frac{1}{C_9} = \frac{1}{C_4} + \frac{1}{C_5} = \frac{1}{12} + \frac{1}{6} = \frac{3}{12} = \frac{1}{4} \qquad \therefore \qquad C_9 = 4\ \mu F$$

The equivalent circuit can be shown as in Fig. (a)

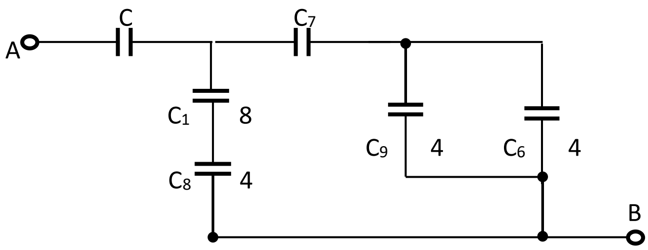


Fig. (a)

Capacitors C_1 and C_8 form a series combination of capacitance C_{10} given by

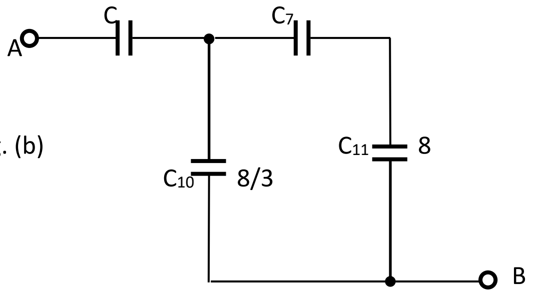
$$C_{10} = \frac{C_1 C_8}{C_1 + C_8} = \frac{8 \times 4}{8 + 4} = \frac{32}{12} = \frac{8}{3}\ \mu F$$

Capacitors C_6 and C_9 form a parallel combination of capacitance.

$$C_{11} = C_6 + C_9 = 4 + 4 = 8\ \mu F$$

The given network reduces to the equivalent circuit (b).

Fig. (b)

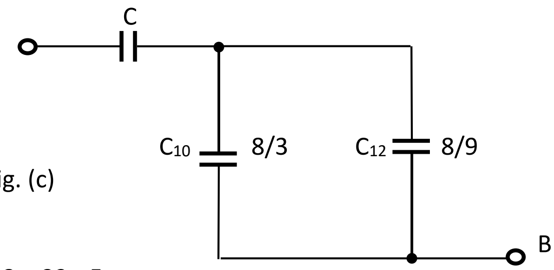


Again, capacitors C_7 and C_{11} form a series combination of capacitance C_{12} capacitance C_{12} given by

$$C_{12} = \frac{C_7 \times C_{11}}{C_7 + C_{11}} = \frac{1 \times 8}{1 + 8} = \frac{8}{9}\ \mu F$$

Now C_{10} and C_{12} form a parallel combination of capacitance C_{13} as shown in Fig. (c)

Fig. (c)



$$C_{13} = C_{10} + C_{12} = \frac{8}{3} + \frac{8}{9} = \frac{32}{9}\ \mu F$$

Finally, the capacitors C and C_{13} form a series combination of capacitance 1 μF as shown in Fig. (d).

$\therefore \quad \frac{1}{1} = \frac{1}{C} + \frac{9}{32} \qquad$ or $\qquad C = \frac{32}{23}\ \mu F$

Q. 11. ***Connect three capacitors of 3 μF, 3 μF and 6 μF such that their equivalent capacitance is 5 μF.***

Sol. Capacitors connected in parallel have maximum equivalent capacitance.

$$C_{max} = 3 + 3 + 6 = 12\ \mu F$$

Capacitors connected in series have minimum equivalent capacitance.

$\frac{1}{C_{min}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{6} = \frac{5}{6} \qquad$ or $\qquad C_{min} = \frac{6}{5} = 1.2\ \mu F$

The required equivalent capacitance of 5 μF lies between C_{max} and C_{min}. So

$$5\ \mu F = 3\ \mu F + 2\ \mu F = 3\ \mu F + \frac{1}{\frac{1}{3} + \frac{1}{6}}\ \mu F$$

So we should connect the series combination of 3 μF and 65 μF capacitors in parallel with the third capacitor of 3 μF.

Q. 12. ***Seven capacitors, each of capacitance 2 μF are to be connected in a configuration to obtain an effective capacitance of 10/11 μF. Suggest a suitable combination to achieve the desired result.***

Sol. Suppose a parallel combination of an capacitors is connected in series with a series combination of (7 – n) capacitors.

Capacitance of parallel combination, $C_1 = 2n\ \mu F$

Capacitance of series combination, $C_2 = \frac{2}{7 - n}\ \mu F$

As these two combinations are in series, so

$$C_s = \frac{10}{11}\ \mu F$$

But $\quad \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} \qquad \therefore \qquad \frac{11}{10} = \frac{1}{2n} + \frac{7 - n}{2}$

Multiplying both sides by 10 n, we get

$$11\ n = 5 + 35\ n - 5n^2$$

or $\quad 5n^2 - 24\ n - 5 = 0$

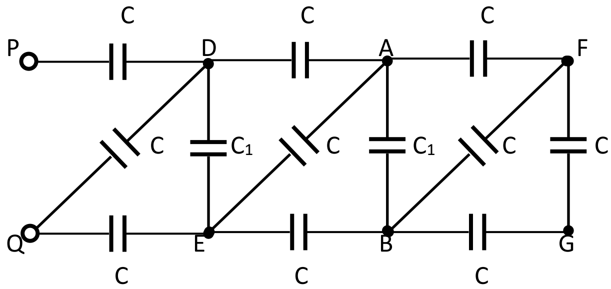
$(n - 5)(5n + 1) = 0$

or $n = 5$ [Rejecting –ve value]

Hence parallel combination of 5 capacitors must be connected in series with the other 2 capacitors.

Q. 13. ***Find the equivalent capacitance between the point s P and Q as shown in Fig. Given C = 18 μF and C_1 12 μF.***

Sol.



Equivalent capacitance between points F and B is

$$\frac{18 \times 18}{18 + 18} + 18 = 27\ \mu F$$

Equivalent capacitance between points A and B is

$$12 + \frac{18 \times 27}{18 + 27} = 12 + 10.8 = 22.8 \simeq 23\ \mu\ F$$

Equivalent capacitance between points A and E is

$$\frac{23 \times 18}{23 + 18} + 18 = 28\ \mu F$$

Equivalent capacitance between points D and E is

$$\frac{28 \times 18}{28 + 18} + 12 = 23\ \mu F$$

Equivalent capacitance between points D and Q is

$$\frac{23 \times 18}{23 + 18} + 185 = 28\ \mu F$$

Equivalent capacitance between points P and Q is

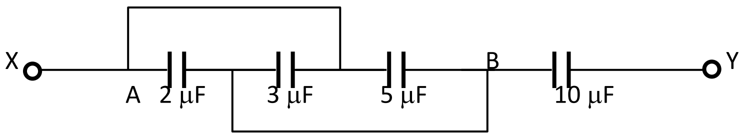
$$\frac{28 \times 18}{23 + 18} = 11\ \mu F$$

Equivalent capacitance between points P and Q is

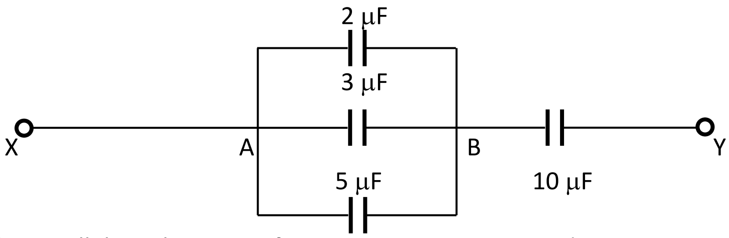
$$\frac{28 \times 18}{28 + 18} = 11\ \mu F$$

Q. 14. ***Four capacitors are connected as shown in Fig. Calculate the equivalent capacitance between the points X and Y.***

Sol.



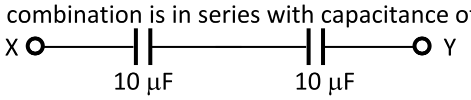
Clearly, the first plate of 2 μF capacitor, the second plate of 3 μF capacitor and the first plate of 5 μF capacitor are connected to the point A. On the other hand, the second plate of 2 μF capacitor, the first plate of 3 μF capacitor and the second plate of 5 μF capacitor are connected to the point B. Thus the capacitors of 2 μF, 3 μF and 5 μF are connected in parallel between points A and B, as shown in the equivalent circuit diagram of Fig.



Total capacitance of the parallel combination of capacitance 2 μF, 3 μF and 5 μF is

$C' = 2 + 3 + 5 = 10\ \mu F$

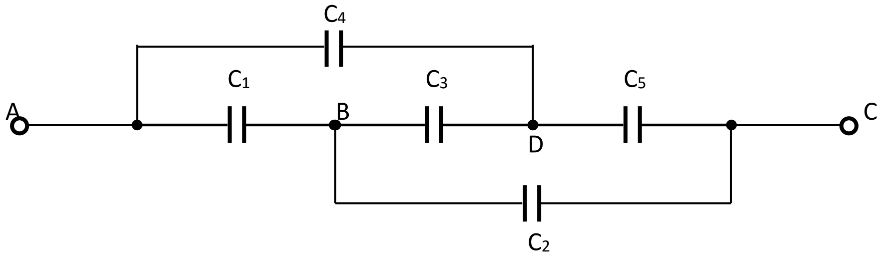
As shown in Fig. the parallel combination is in series with capacitance of 10 μF.



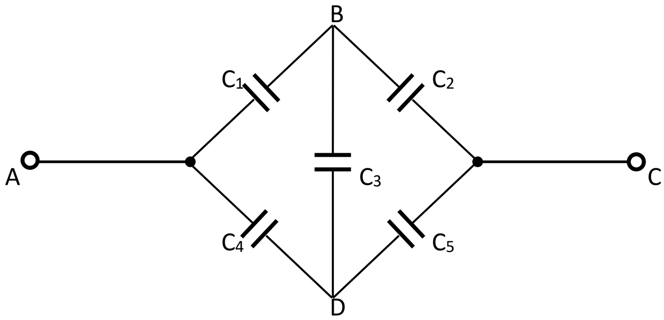
Equivalent capacitance between X and Y

$$= \frac{10 \times 10}{10 + 10} = 5\ \mu F$$

Q. 15. ***Five capacitors of capacitance 10 μF each are connected with each other, as shown in Fig. Calculate the total capacitance between the points A and C.***

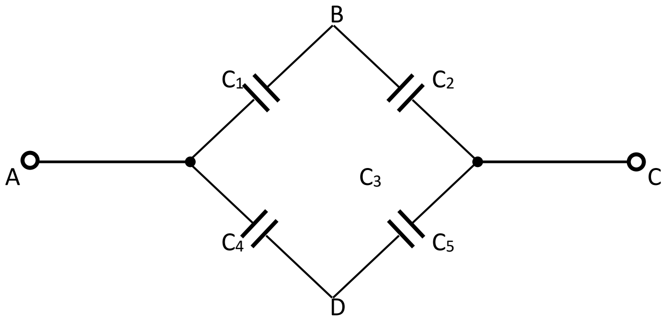


Sol. The given circuit can be redrawn in the form of a Wheatstone bridge as shown in Fig.



As $C_1 = C_2 = C_4 = C_5$, therefore, $\frac{C_1}{C_2} = \frac{C_4}{C_5}$

Thus, the given circuit is a balanced Wheatstone bridge. So the potential difference across the ends of capacitor C_3 is zero. Capacitance C_3 is ineffective. The given circuit reduces to the equivalent circuit shown in Fig. (a).



Capacitors C_1 and C_2 form a series combination of equivalent capacitance C_6 given by

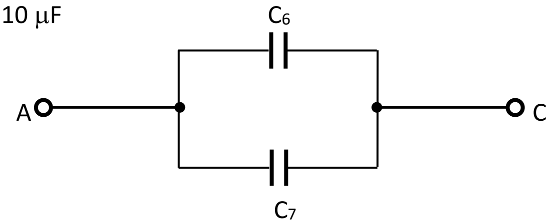
$$C_6 = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{10 \times 10}{10 + 10} = 5\ \mu F$$

Similarly, C_4 and C_5 form a series combination of equivalent capacitance C_7 given by

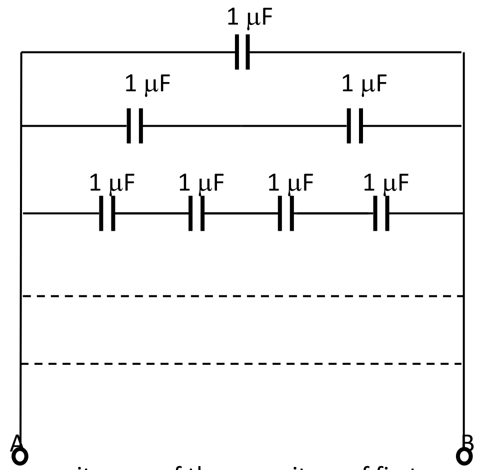
$$C_7 = \frac{C_4 \times C_5}{C_4 + C_5} = \frac{10 \times 10}{10 + 10} = 5\ \mu F$$

As shown in Fig. (b) C_6 and C_7 form a parallel combination. Hence the equivalent capacitance of the network is given by

$C = C_6 + C_7 = 5 + 5 = 10\ \mu F$



Q. 16. ***There are infinite number of capacitors, each of capacitance 1 μF. They are connected in rows, such that the number of capacitors in the first row, second row, third row, fourth row, are respectively 1, 2, 4, …. The rows these capacitors are then connected between points A and B, as shown in Fig. Determine the equivalent capacitance of the network between the points A and B.***



Sol. Let C_1, C_2, C_3, C_4, …. be the effective capacitances of the capacitors of first row, second, third row, fourth row, …. respectively. Then $C_1 = 1\ \mu F$

$$C_2 = \frac{1 \times 1}{1+1} = \frac{1}{2}\ \mu F$$

$$\frac{1}{C_3} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 4$$

∴ $C_3 = \frac{1}{4}\ \mu F$

Similarly, $C_4 = \frac{1}{8}\ \mu F$, and so on.

As these rows are connected in parallel between points A and B, so the equivalent capacitance between points A and B is

$$C = C_1 + C_2 + C_3 + C_4 + …. = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ….$$

This is an infinite geometric progression with first term a = 1 and common ratio r = ½. Hence

$$C = \frac{a}{1-r} = \frac{1}{1-½} = 2\ \mu F$$

☑ Energy stored in a capacitor

When a battery is connected across the plates capacitor, the work is done (or energy supplied) by the battery in charging the capacitor. As the capacitor charges, p.d across the plate increases, more and more work has to be done by the battery in delivering same amount of charge to the capacitor due to continuously increasing p.d across its plate.

"The work done charging a capacitor is stored in the capacitor in the form of electric energy."

EXPRESSION --

Consider a capacitor of capacitance 'C' , initially suppose the capacitor does not possess any charge .
On being connected to a battery, suppose that it charges to a potential V after some time. If 'q' is charge on the plate of capacitor at that time, then

$$q = CV \qquad ∴ \qquad V = \frac{q}{C}$$

Suppose, a battery supplies an infinitesimally small amount of charge 'dq' to the capacitor at a constant potential 'V'.

∴ Small amount of work done by the battery, $dW = V\,dq$ [since V = dW /dq]

$$dW = \frac{q}{C}\,dq$$

Amount of work in delivering charge 'q' to the capacitor is

$$\int_0^q dW = \int_0^q \frac{q}{C}\,dq = \frac{1}{C}\int_0^q q\,dq$$

$$∴ \qquad W = \frac{1}{\mathbf{C}}\left|\frac{q}{2}\right|_0^q = \frac{1}{\mathbf{2}}\frac{q^2}{\mathbf{C}}$$

∴ **Energy stored in the capacitor**

$$\boxed{U = W = \frac{1}{2}\frac{q^2}{C}}$$

But, $q = CV$ ∴ $U = 1/2\ \frac{C^2 V^2}{C}$

or, $\boxed{U = \frac{1}{2} C V^2}$

Putting the $CV = q$ $\therefore$ $\boxed{U = \frac{1}{2} CV.V = \frac{1}{2} qV}$

Hence, $\boxed{U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} CV^2 = \frac{1}{2} qV}$

The energy stored in a capacitor is in the form of electric field energy on the charged capacitor and it resides in the di electric medium between the field.

☑ Energy stored in combination of capacitor:

➢[1] In series combination of capacitor :

Consider 'n' capacitor each of capacitance C_1, C_2, C_3, --------C_n converted in series .

$\therefore$ equivalent capacitance $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \text{--------} + \frac{1}{C_n}$

We know that energy capacitor come the same charge 'q' (in series).

$\therefore$ Total energy $U = \frac{1}{2}\frac{q^2}{C_s} = \frac{1}{2} q^2 \times \frac{1}{C_s}$

$$U = \frac{q^2}{2}\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \text{--------} + \frac{1}{C_n}\right]$$

$$U = \frac{q^2}{2C_1} + \frac{q^2}{2C_2} + \frac{q^2}{2C_3} + \text{----------} + \frac{q^2}{2C_n}$$

$\therefore$ $\boxed{U = U_1 + U_2 + U_3 + \text{--------} U_n}$

Where U_1, U_2, U_3------U_n are energy stored in the individual capacitors respectively .

➢[2] In parallel combination of capacitor: --

Potential difference across each capacitor is same i.e. V.

$\therefore$ Total Energy, $U = \frac{1}{2} C_p V^2$

$$U = \frac{1}{2}(C_1 + C_2 + C_3 + \text{--------} + C_n) V^2$$

$$U = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 + \frac{1}{2} C_3 V^2 + \text{----------} \frac{1}{2} C_n V^2$$

$\therefore$ $\boxed{U = U_1 + U_2 + U_3 + \text{--------} U_n}$

Thus, Total energy stored in series or parallel combination of a capacitor is equal to the sum of energies stored in individual capacitor.

☑ Energy Density in a parallel plate capacito :

Energy density (U) is defined as the total energy store per unit volume of condenser.

i.e., $U = \frac{\text{Total energy}}{\text{Volume V}} = \frac{1/2\, C V^2}{A d \text{ (area)}} = \frac{1/2 \, \frac{\xi_0 A}{d} (E d)^2}{A d}$ (since $C = \frac{\xi_0 A}{d}$ and $V = E d$)

$$U = \frac{1}{2} \frac{\xi_0 A}{\cancel{d}} \frac{E^2 d^2}{\cancel{Ad}}$$

$\boxed{U = \frac{1}{2} \xi_0 E^2}$ Where E = electric field strength between the plates .

☑ **Common potential** : When two capacitors are charged to different potential & then connected by a conducting wire, Charge flows from, the one at higher potential to the other at lower potential. This flow continue till their potential becomes equal. This equal potential of two capacitor is called common potential.

EXPRESSION: Suppose C_1 and C_2 are capacitance of the two capacitors, charged to potential V_1 & V_2.

∴ Total charge before sharing $= C_1 V_1 + C_2 V_2$

If 'V' = common potential on sharing charges, then total charge after sharing $= C_1 V + C_2 V$

Since, no charge is lost in the process of sharing, therefore,

$$C_1 V_1 + C_2 V_2 = C_1 V + C_2 V$$

$$V (C_1 + C_2) = C_1 V_1 + C_2 V_2$$

$$\boxed{V = \frac{C_1 V_1 + C_2 V_2}{(C_1 + C_2)}} \quad \text{-----------------------} [1]$$

i.e., Common potential $= \dfrac{\text{Total charge}}{\text{Total capacity}}$

From [1] $C_1 V + C_2 V = C_1 V_1 + C_2 V_2$

$C_1 V_1 - C_1 V = C_2 V - C_2 V_2$

I.e., Charge loss by one condenser = Charge gain by the other condenser.

➢ **Loss of energy on sharing charges:** When charges are shared between two bodies, no charge is lost. But some energy is dissipated in the form of heat etc.

Consider two capacitors having Capacitance C_1 & C_2 and potential V_1 & V_2 respectively.

The charges flow from one capacitor at higher potential to the other at lower potential till their potential becomes equal

(i.e., Common potential, $V = \dfrac{C_1 V_1 + C_2 V_2}{(C_1 + C_2)}$)

Total potential energy of two capacitor before sharing, $U = U_1 + U_2$

$$= \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

Total potential energy of two capacitor after sharing, $U` = \dfrac{1}{2} C_1 V^2 + \dfrac{1}{2} C_2 V^2$ {Since both the capacitor acquires common potential}

$$U` = \frac{1}{2}(C_1 + C_2) V^2$$

$$U` = \frac{1}{2} \cancel{(C_1 + C_2)} \frac{(C_1 V_1 + C_2 V_2)^2}{\cancel{(C_1 + C_2)^2}}$$

$$\boxed{U` = \frac{1}{2} \frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)}}$$

Loss in energy $U - U` = \dfrac{1}{2} C_1 V_1^2 + \dfrac{1}{2} C_2 V_2^2 - \dfrac{1}{2}\dfrac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)}$

$$= \frac{1}{2(C_1 + C_2)} \left\{ C_1 V_1^2 + C_2 V_2^2 (C_1 + C_2) - (C_1 V_1 + C_2 V_2)^2 \right\}$$

$$= \frac{1}{2(C_1 + C_2)} \{C_1^2 V_1^2 + C_1 C_2 V_2^2 + C_1 C_2 V_1^2 + C_2^2 V_2^2 - (c_1^2 V_1^2 + C_2^2 V_2^2 + 2c_1c_2v_1v_2)\}$$ 21

$$= \frac{1}{2(C_1 + C_2)} \{\cancel{C_1^2 V_1^2} + C_1 C_2 V_2^2 + C_1 C_2 V_1^2 + \cancel{C_2^2 V_2^2} - \cancel{c_1^2 V_1^2} - \cancel{C_2^2 V_2^2} - 2c_1c_2v_1v_2\}$$

$$= \frac{1}{2(C_1 + C_2)} \{C_1 C_2 (V_1^2 + V_2^2 - 2v_1v_2)\}$$

$$\boxed{U - U` = \frac{C_1 C_2 (V_1^2 - V_2^2)}{2(C_1 + C_2)}}$$

i.e., U-**U` is positive quantity.**

i.e., U-**U` ≠ 0 or** U > U`, hence there occurs a loss of energy on sharing charges. This loss of energy appears in the form of sparking and heat produced in the connecting wire.

Examples based on Energy Stored in Capacitors

❖ **Formulae Used**

1. Energy stored in a capacitor,

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\cdot\frac{q^2}{C} = \frac{1}{2}qV$$

2. Energy stored per unit volume or the energy density of the electric field of a capacitor,

$$u = \frac{1}{2}\varepsilon_0 E^2$$

3. Electric field between capacitor plates, $E = \dfrac{\sigma}{\varepsilon_0}$

❖ **Units Used**

Capacitance is in farad, charge in coulomb, electric field in NC^{-1} or Vm^{-1}, energy in joule and energy density in Jm^{-3}.

Q. 1. ***How much work must be done to charge a 24 μF capacitor when the potential difference between the plates is 500 V?***

Sol. Here $C = 24\ \mu F = 24 \times 10^{-6}$ F, V = 500 V

Work done, $W = \frac{1}{2}CV^2 = \frac{1}{2} \times 24 \times 10^{-6} \times (500)^2 = 3$ J

Q. 2. ***A capacitor is charged through a potential difference of 200 V, when 0.1 C charge is stored in it. How much energy will it release, when it is discharged?***

Sol. Here V = 200 V, q = 0.1 C

Energy stored, $U = \frac{1}{2}qV = \frac{1}{2} \times 0.1 \times 200 = 10$ J

When the capacitor is discharged, it releases the same amount of energy i.e., 10 J

Q. 3. ***A parallel plate capacitor of 300 μF is charged to 200 V. If the distance between its plates is halved, what will be the potential difference between the plates and what will be the change in stored energy?***

Sol. Here $C = 300\ \mu F = 3 \times 10^{-4}$ F, V = 200 V

∴ $q = CV = 3 \times 10^{-4} \times 200 = 6 \times 10^{-2}$ C

When the distance between the plates is halved, capacitance is doubled.

∴ $C' = 2C = 2 \times 300 = 600\ \mu F = 6 \times 10^{-4}$ F

The potential difference between the plates becomes

$$V' = \frac{q}{C'} = \frac{6 \times 10^{-2}}{6 \times 10^{-4}} = 100 \text{ V}$$

Initial energy stored in the capacitor,

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 3 \times 10^{-4} \times (200)^2 = 6 \text{ J}$$

Final energy stored in the capacitor,

$$U' = \frac{1}{2}C' . V'^2 = \frac{1}{2} \times 6 \times 10^{-4} \times (100)^2 = 3 \text{ J}$$

Loss in energy = U – U′ = 6 – 3 = 3 J

Q. 4. ***Two capacitors of capacitances $C_1 = 3\ \mu F$ and $C_2 = 6\ \mu F$ arranged in series are connected in parallel with a third capacitor $C_3 = 4\ \mu F$. The arrangement is connected to a 6.0 V battery. Calculate the total energy stored in the capacitors.***

Sol. Equivalent capacitance of the series combination of C_1 and C_2 is given by

$$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{3 \times 6}{3 + 6} = 2\ \mu F$$

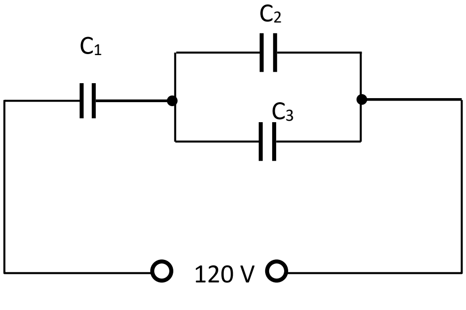
Combination C′ is in parallel with C_3,

∴ Total capacitance,

$$C = C' + C_3 = 2 + 4 = 6\ \mu F = 6 \times 10^{-6} \text{ F}$$

Energy stored, $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 6 \times 10^{-6} \times 6^2 = 1.08 \times 10^{-4}$ J

Q. 5. ***Three capacitors $C_1 = 15\ \mu F$, $C_2 = 25\ \mu F$ and $C_3 = 35\ \mu F$ are connected to a 120 V supply, as shown in Fig. Determine (i) the equivalent capacitance of the system and the energy stored in it (ii) charges and potential differences on C_1, C_2 and C_3.***



Sol. (i) Capacitors C_2 and C_3 are in parallel, Their equivalent capacitance

$$C' = C_2 + C_3 = 25 + 35 = 60\ \mu F$$

Now C' is in series with C_1. Hence the equivalent capacitance of the system is

$$C = \frac{C' C_1}{C' + C_1} = \frac{60 \times 15}{60 + 15} = 12\ \mu F$$

Energy stored, $U = \frac{1}{2} CV^2 = \frac{1}{2} \times 12 \times 10^{-6} \times (120)^2$

$= 8.64 \times 10^{-2}$ J

(ii) Total change $q = CV = 12 \times 10^{-6} \times 120 = 1.44 \times 10^{-3}$ C

$\therefore$ $q_1 = q = 1.44 \times 10^{-3}$ C

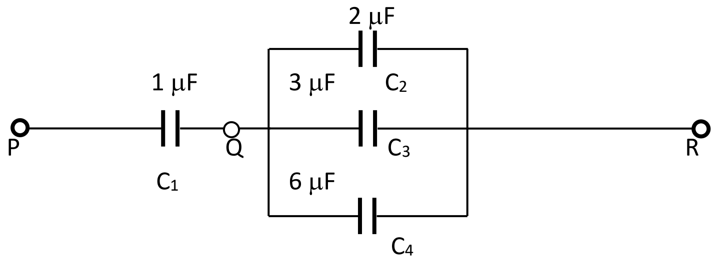
P.D. across C_2, or C_3,

$V_2 = V_3 = V - V_1 = 120 - 96 = 24$ V

$\therefore$ $q_2 = C_2 V_2 = 24\ \mu F \times 24\ V = 600\ \mu C$

$q_3 = C_3 V_3 = 35\ \mu F \times 24\ V = 840\ \mu C$

Q. 6. ***In Fig. the energy stored in C_4 is 27 J. Calculate the total energy stored in the system.***



Sol. Energy stored in C_4 is

$U_2 = \frac{1}{2} C_4 V^2 = 27$ J

or $\frac{1}{2} \times 6 \times 10^{-6} \times V^2 = 27$

or $V^2 = \frac{27 \times 2}{6 \times 10^{-6}} = 9 \times 10^6$

Energy stored in C_2, $U_2 = \frac{1}{2} \times 2 \times 10^{-6} \times 9 \times 10^6 = 9$ J

Energy stored in C_3, $U_3 = \frac{1}{2} \times 3 \times 10^{-6} \times 9 \times 10^6 = 13.5$ J

Energy stored in C_2, C_3 and $C_4 = U_2 + U_3 + U_4 = 9 + 13.5 + 27 = 49.5$ J

Equivalent capacitance of C_2, C_3 and C_4 connected in parallel

$= 2 + 3 + 5 = 11\ \mu F$

$\therefore$ $\frac{q^2}{2 \times 11 \times 10^{-6}} = 49.5$ J $\quad \left(U = \frac{q^2}{2C}\right)$

Energy stored in C_1, $U_1 = \frac{q^2}{2C_1} = \frac{49.5 \times 2 \times 11 \times 10^{-6}}{2 \times 1 \times 10^{-6}} = 544.5$ J

Total energy stored in the arrangement= 544.5 + 49.5 = 594.0 J

Q. 7. ***A 80 μF capacitor is charged by a 50 V battery. The capacitor is disconnected from the battery and then connected across another uncharged 320 μF capacitor. Calculate the charge on the second capacitor.***

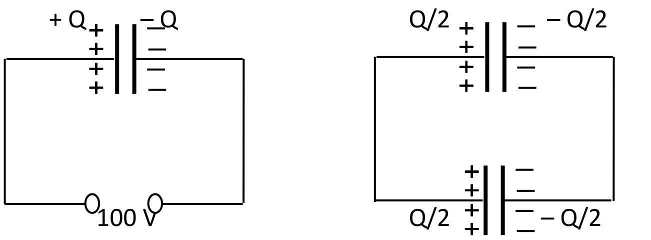
Sol. Here $C_1 = 80\ \mu F = 320 \times 10^{-6}$ F, $V_1 = 50$ V, $C_2 = 320\ \mu F = 320 \times 10^{-6}$ F, $V_2 = 0$

Common potential, $V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

$= \frac{80 \times 10^{-6} \times 50 + 0}{(80 + 320) \times 10^{-6}} = 10$ V

Charge on 320 μF capacitor= $C_2 V = 320 \times 10^{-6} \times 10 = 3.2 \times 10^{-3}$ C

Q. 8. ***(i) A 900 pF capacitor is charged by a 100 V battery. How much electrostatic energy is stored by the capacitor?***
(ii) The capacitor is disconnected from the battery and connected to another 900 pF capacitor. What is the electrostatic energy stored by the system?



(iii) Where has the remainder of the energy gone?

Sol. (i) The charge on the capacitor is

$q = CV = 900 \times 10^{-12}\ F \times 100\ V = 9 \times 10^{-8}$ C

The energy stored by the capacitor is

$U = \frac{1}{2} CV^2 = \frac{1}{2} qV = \frac{1}{2} \times 9 \times 10^{-8}\ C \times 100\ V$

$= 4.5 \times 10^{-6}$ J

(ii) In the steady situation, the two capacitors have their positive plates at the same potential, and them negative plates at the same potential. Let the common potential difference be V'. The charge on each capacitor is then q' = CV'. By charge conservation, q' = q/2.

∴ Total energy of the system

$= 2 \times \frac{1}{2} q' V' = q' V' = q' \cdot \frac{q'}{C}$

$= \frac{1}{4} \cdot \frac{q^2}{C} = \frac{1}{4} \cdot qV = \frac{1}{2} \times \frac{1}{2} qV \qquad \left(\because q' = \frac{q}{2} \text{ and } \frac{q}{C} = V\right)$

$= \frac{1}{2} \times 4.5 \times 10^{-6}\ J = 2.25 \times 10^{-6}\ J$

(iii) There is a transient period before the system settles to the situation (ii). During this period, a transient current flows from the first capacitor to the second. Energy is lost during this time in the form of heat and electromagnetic radiation.

Q. 9. ***A capacitor is charged to potential V_1. The power supply is disconnected and the capacitor is connected in parallel to another un charged capacitor.***
(i) Derive the expression for the common potential of the combination of capacitors.
(ii) Show that total energy of the combination is less than the sum of the energy stored in them before they are connected.

Sol. (i) Let C_1 and C_2 be the capacitances of the two capacitors and V be their common potential.

Then $V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

or $V = \frac{C_1 V_1}{C_1 + C_2} \qquad [\because V_2 = 0]$

(ii) Energy stored in the capacitors before connection,

$U_i = \frac{1}{2} C_1 V_1^2$

Total energy after connection,

$U_f = \frac{1}{2} (C_1 + C_2) V^2 = \frac{1}{2} (C_1 + C_2) \frac{C_1^2 V_1^2}{(C_1 + C_2)^2}$

$= \frac{1}{2} \frac{C_1^2 V_1^2}{C_1 + C_2}$

∴ $U_i - U_f = \frac{1}{2} C_1 V_1^2 \left(1 - \frac{C_1}{C_1 + C_2}\right)$

$= \frac{1}{2} C_1 V_1^2 \cdot \frac{C_2}{C_1 + C_2} = \frac{1}{2} \frac{C_1 C_2 V_1^2}{C_1 + C_2}$

Obviously, $U_i - U_f > 0$ or $U_i > U_f$.

Q. 10. ***Two capacitors are in parallel and the energy stored in 45 J. When the combination is raised to potential of 3000 V. With the same two capacitors in series, the energy stored in 4.05 J for the same potential. What are their individual capacitances?***

Sol. Let C_1 and C_2 be the capacitances of the two capacitors. For parallel combination, we have

$U = \frac{1}{2} (C_1 + C_2) V^2$

or $45 = \frac{1}{2} (C_1 + C_2) \times (3000)^2$

∴ $C_1 + C_2 = 10 \times 10^{-6}$ … (1)

For series combination, we have

$U = \frac{1}{2}$ … (1)

For series combination, we have

$U = \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} \cdot V^2$

or $4.05 = \frac{1}{2} \frac{C_1 C_2}{10 \times 10^{-6}} \times (3000)^2$

or $C_1 C_2 = 9 \times 10^{-12}$

Now $(C_1 - C_2)^2 = (C_1 + C_2)^2 - 4 C_1 C_2 = 100 \times 10^{-12} - 36 \times 10^{-12} = 64 \times 10^{-12}$

or $C_1 - C_2 = 8 \times 10^{-6}$ … (2)

On solving equations (1) and (2), we get

$C_1 = 9 \times 10^{-6}\ F = 9\ \mu F$ and $C_2 = 1 \times 10^{-6}\ F = 1\ \mu F$

Q. 11. ***A capacitor of capacitance 6 μF is charged to a potential of 150 V. Its potential falls to 90 V, when another capacitor is connected to it. Find the capacitance of the second capacitor and the amount of energy lost due to the connection.***

Sol. Here $C_1 = 6\ \mu F$, $V_1 = 150\ V$, $V_2 = 0$, $V = 90\ V$, $C_2 = ?$

Common potential,

$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

or $90\ V = \frac{6 \times 10^{-6} \times 150 + 0}{6 \times 10^{-6} + C_2}$

or $C_2 + 6 \times 10^{-6} = \dfrac{6 \times 10^{-6} \times 150}{90} = 10 \times 10^{-6}$

or $C_2 = 4 \times 10^{-6}\ F = 41\ \mu F$

Initial energy stored,

$U_i = U_1 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(6 + 4) \times 10^{-6} \times (90)^2 = 4.05 \times 10^{-2}\ J$

The loss of the energy on connecting the two capacitors

$\Delta U = U_i - U_f = (6.75 - 4.05) \times 10^{-2} = 2.7 \times 10^{-2}\ J = 0.027\ J$

Q. 12. ***A battery of 10 V is connected to a capacitor of capacity 0.1 F. The battery is now removed and this capacitor is connected to a second uncharged capacitor. If the charge distributes equally on these two capacitors, find the total energy stored in the two capacitors. Further, compare this energy with the initial energy stored in the first capacitor.***

Sol. Initial energy stored in the first capacitor is

$U_i = \frac{1}{2}CV^2 = \frac{1}{2} \times 0.1 \times (10)^2 = 5.0\ J$

When the first capacitor is connected to the second uncharged capacitor, the charge distributes equally. This implies that the capacitance of second capacitor is also C. The voltage across each capacitor is now V/2. The final total energy stored in the two capacitors is

$U_f = \frac{1}{2}C\left(\frac{V}{2}\right)^2 + \frac{1}{2}C\left(\frac{V}{2}\right)^2 = \frac{1}{4}CV^2 = 2.5\ J$

$\therefore \quad \dfrac{U_f}{U_i} = \dfrac{2.5}{5.0} = \frac{1}{2} = 1 : 2$

☑ BEHAVIOUR OF NON – CONDUCTING SUBSTANCE IN AN ELECTRIC FIELD :

There are two types of non – conducting substance

.... [1] POLAR
.... [2] NON – POLAR

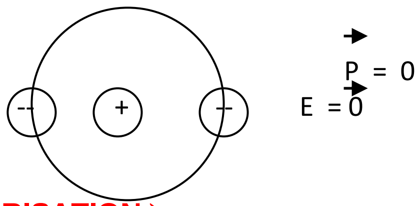
We know that dielectrics are insulating materials which transmit electric effect without conducting. Further, we know that in every atom, there is a positive charged nucleus and a negative charged electron cloud surrounding it. The two oppositely charged region have their own centre of mass.

.... The centre of positive charged is the centre of mass of positive charged protons in the nucleus.
.... The centre of negative charge is the centre of mass of negative charged electrons.

✠ NON-POLAR DIELECTRICS:**" A dielectric in the atom or the molecules in which, the center of mass of positive charge coincides with the center of mass of negative charge, is called non-polar dielectrics".**

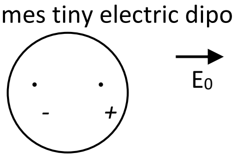
Ex: - H_2, N_2, CO_2, Benzene, CH_4 etc,

..... Each molecule has zero dipole moment in its normal state



✪ Effect of Electric field on the non- polar molecules (DIELECTRIC POLARISATION) :

When a non –polar dielectric is held in an external electric field E_0**,** the center of mass of positive charge (protons) in each molecule is pulled in the direction of E_0 (i.e., *Towards negative plate)* and the center of mass of negative charge (electron) is pulled in the direction opposite to $\mathbf{E_0}$ **(**i.e., *towards positive plate).* Therefore, the two center of positive and negative charges in the molecules are separated and the molecule is said to be polarized. Now, each molecule becomes tiny electric dipole moment parallel to the external electric field and proportional to it.



.... The separation between the charges continues till the force acting on them due to $\mathbf{E_0}$ **are** balanced by the internal forces.

.... Non – polar molecules, after polarization, called induced electric dipole & the dipole moment is called induced dipole moment.

.... Induced electric dipole moment disappears as soon as the external field is removed.

.... Force experience by positive charge in external Electric field, E, $\quad \vec{F} = q\vec{E}$
.... Force experience by negative charge in external Electric field, E, $\quad \vec{F} = -q\vec{E}$

✪ **POLARIZABILITY**: **Induced** electric dipole moment P is proportional to the applied electric field E_0 .

i.e.,

$$\boxed{\vec{P} = \alpha\ \xi_0 E_0}$$ where , α = Constant and is called atomic or molecular polarizability.

$$\boxed{\alpha = \frac{P}{\xi_0 E_0}}$$

➢ **S. I unit of polarizability :** $\alpha = \frac{p}{\xi_0 E_0} = \frac{\text{unit of P}}{\text{unit of } \xi_0 \times \text{unit of } E_0} = \frac{C - m}{C^2 N^{-1} m^{-2}\ (N / C)} = m^3$ **(unit of volume)**

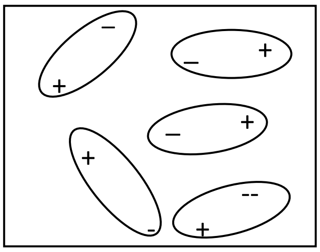
✪**Dimension of polarizability :** $\alpha = \frac{p}{\xi_0 E_0} = \frac{\text{Dim. of P}}{\text{Dim. of } \xi_0 \times \text{Dim. of } E_0} = [L^3]$

---- For most of atoms, the value of α is of the order of 10^{-20} to 10^{-30} m^3 (which is the order of atomic volume.

✠ **POLAR DIELECTRICS**: *" A dielectric in the atoms or molecule of which, the center of mass of positive charge and negative charge does not coincide is called a polar dielectric ".*

••Due to finite separation between the positive charge , polar molecules posses a finite electric dipole

Ex : NH_3 , HCl , H_2O , etc.

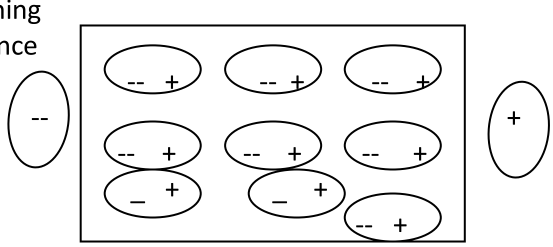


➢**Effect of Electric field on the polar molecules** :

A polar molecule has permanent electric dipole of its own in the absence of electric field. But these are randomly oriented due to this, Net dipole moment, $\vec{P} = 0$

When an electric field E is applied each dipole experience a torque. This torque tends to align the molecule in the direction of applied electric field giving surface layer of positive and negative charge. The dipole moment of all molecule now gets added and hence substance has finite net dipole moment in the presence of electric field.

** The effect of placing a polar substance in an electric field is simply aligning the existing $\vec{p}$ and it does not create any additional dipole moment. Hence **there importance is least in the capacitor**.



✪ **Polarisation o f a electric slab:**

"Polarisation of a dielectric slab is the process of inducing equal opposite charges on the two opposite faces of the dielectric on the application of electric field."

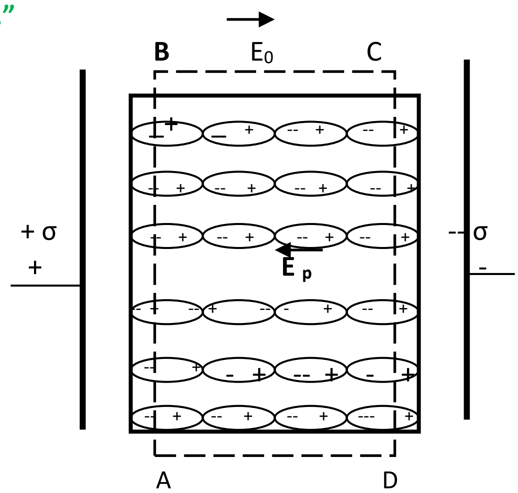
Let $\vec{E}_0$ = Electric field between the plate.

If $+\sigma$ = Surface charges density of positive plate

$-\sigma$ = Surface charges density of negative plate

Then magnitude of electric field

strength, $E = \frac{\sigma}{\xi_0}$



Now,

Introduce a non-polar dielectric slab ABCD between the plate. Each atom elongated due to displacement of its charges under the effect of electric field i.e., polarised.

Also, **Each atom/molecule** of the electric having dipole moment experience a torque in the direction of $\vec{E}_0$. (The interior charges in the dotted rectangle cancel the effect of one another, therefore the net effect is that opposite faces the electric slab will have equal opposite charges.

If $+\sigma$ and $-\sigma$ be the surface charge density of the opposite faces of the dielectric then the magnitude of electric field due to polarisation.

$$E_P = \frac{\sigma_P}{\xi_o} \text{ directed opposite to that of applied } \vec{E_o}$$

∴ Net electric field inside the dielectric is

$$E = E_o - E_P$$

or, $$E = \frac{\sigma}{\xi_0} - \frac{\sigma_P}{\xi_0}$$ $$\boxed{E = \frac{\sigma - \sigma_P}{\xi_0}} \quad \text{———— (i)}$$

Here E = reduced value of the electric field i.e., on placing a dielectric slab, the electric field intensity parallel plate is reduced.

☑ DIELECTRIC CONSTANT: --

"The ratio of applied electric field (Eo)to the reduced value of electric field intensity (E) on placing a dielectric between the plate of the capacitor is called dielectric constant."

i.e., $K = \frac{E_o}{E}$ Since $E_o > E$, therefore K is always more than 1. (Dielectric constant is also called relative permittivity.)

☑ POLARISATION DENSITY: -

The induced dipole moment developed per unit volume in a dielectric slab on placing it inside the electric field is called polarization density (P).

Suppose, all the atom of dielectric slab are uniformly polarized and N is the displacement between the centers of $\pm$ charges in the atom then moment of each atom P = q x.

Again, if N = No. of atoms per unit volume then dipole moment per unit vol. = Total (polarization density) dipole moment density.

i.e., $\boxed{P = N p = N q x}$

☞ **Unit of P** $= \frac{C - m}{m^3} = c\, m^{-2}$

Again, $P = \frac{\text{dipole moment}}{\text{volume}} = \frac{q\,d}{A\,d} = \frac{q}{A}$

☑ ELECTRIC SUSCEPTIBILITY:

It is found that electric polarisation P is directly proportional to the reduced value of electric field (E).

i.e., $P \propto E$

or $P = \xi_o \chi E$

where χ is called electric susceptibility and is read as 'chi' ξ_o is used to keep dimensionless.

χ describes the electric behaviour of dielectric,

or, $$\chi = \frac{P}{\xi_0 E}$$

☞ **Unit of χ** $= \frac{C\, m^{-2}}{C^2 N^{-1} m^2 \times N C^{-1}}$ = No unit

✪ Relation between χ and K :--

Reduced value of electric field in the dielectric is, $E = E_o - E_P$

$E_0 = E + E_P$

$E_0 = E + \sigma_p / \xi_0$

$E_0 = E + \frac{1}{\xi_0}\frac{q}{A}$ [Since, $\sigma_p = q / A$]

$E_0 = E + \frac{1}{\xi_0}(p)$ [Since, $P = q / A$]

$E_0 = E + \frac{P}{\xi_0} = E + \frac{\chi \cancel{\xi_0} E}{\cancel{\xi_0}}$

$E_0 = E (1 + \chi)$

$\frac{E_0}{E} = (1 + \chi)$

$$\boxed{K = (1 + \chi)}$$ { Since, dielectric Constant, $K = \frac{E_0}{E}$ }

(Relation between dielectric constant and electrical susceptibility.)

☞ **Dielectric Strength :** A dielectric is polarised under the action of an external field. As the strength of electric field applied are increased molecules of the dielectric under go more and more stretching and some strain is produced in the atom.

A stage may reach when the stretching is so much that the electrons break up from the molecule of the dielectric and the atom become positive ion. This is known as dielectric break down. The electron drift towards the positive plate of the capacitor and positive ion drift towards negative plate and the dielectric becomes conducting. Thus,

"The dielectric strength of dielectric is defined as the maximum value of electric field (or Potential gradient E = V / d) that can be applied to the dielectric without its electric breakdown".

☞ UNIT : V / m ☞ Practical unit :-- K V / mm

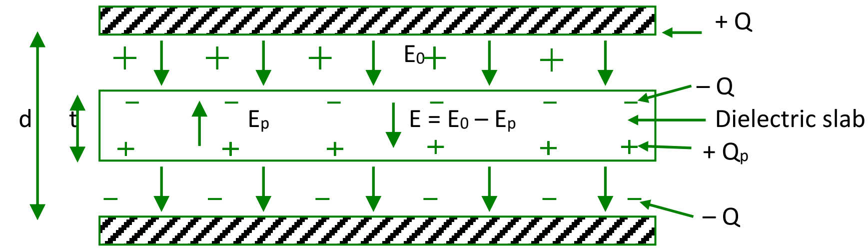
Dielectric constants and dielectric strengths of some common dielectrics.

Dielectric	Dielectric constant	Dielectric strength in kV mm^{-1}
Vacuum	1.00000	∞
Air	1.00054	0.8
Water	81	—
Paper	3.5	14
Pyrex glass	4.5	13
Mica	5.4	160
Porcelain	6.5	4

CAPACITANCE OF A PARALLEL PLATE CAPACITOR WITH A DIELECTRIC SLAB

The capacitance of a parallel plate capacitor of plate area A and plate separation d with vacuum between its plates is given by $C_0 = \dfrac{\varepsilon_0 A}{d}$

Suppose initially the charges on the capacitor plates are ± Q. Then the uniform electric field set up between the capacitor plates is $E_0 = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{A\,\varepsilon_0}$



[A dielectric slab placed in a parallel plate capacitor]

When a dielectric slab of thickness t < d is placed between the plates, the field E_0 polarises the dielectric. This induces charges – Q_p on the upper surface and + Q_p on the lower surface of the dielectric. These induced charges set up a field E_p inside the dielectric in the opposite direction of E_0. The induced field is given by

$$E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{P}{\varepsilon_0} \qquad [\sigma_p = Q/A = P, \text{ polarisation density}]$$

The net field inside the dielectric is

$$E = E_0 - E_p = \frac{E_0}{\kappa} \qquad \left(\because \frac{E_0}{E} = \kappa\right)$$

where κ is the dielectric constant of the slab. So, between the capacitor plates, the field E exists over a distance t and field E_0 exists over the remaining distance (d – t). Hence the potential difference between the capacitor plates is

$$V = E_0(d-t) + Et = E_0(d-t) + \frac{E_0}{\kappa}t \qquad \left(\because \frac{E_0}{E} = \kappa\right)$$

$$= E_0\left(d - t + \frac{t}{\kappa}\right) = \frac{Q}{\varepsilon_0 A}\left(d - t + \frac{t}{\kappa}\right)$$

The capacitance of the capacitor on introduction of dielectric slab becomes

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d - t + \dfrac{t}{\kappa}}$$

Special case: If the dielectric fills the entire space between the plates, then t = d, and we get

$$C = \frac{\varepsilon_0 A}{d} . \kappa = \kappa C_0$$

Thus, the capacitance of a parallel plate capacitor increases κ times when its entire space is filled with a dielectric material. Clearly, $\kappa = \frac{C}{C_0}$

Dielectric constant = $\dfrac{\text{Capacitance with dielectric between two plates}}{\text{Capacitance with vacuum between two plates}}$

Thus, **the dielectric constant of a dielectric material may be defined as the ratio of the capacitance of a capacitor completely filled with that material to the capacitance of the same capacitor with vacuum between its plates**.

CAPACITANCE OF A PARALLEL PLATE CAPACITOR WITH A CONDUCTING SLAB

Capacitance of a parallel plate capacitor with a conducting slab: Consider a parallel plate capacitor of plate area A and plate separation d. If the space between the plates is vacuum, its capacitance is given by

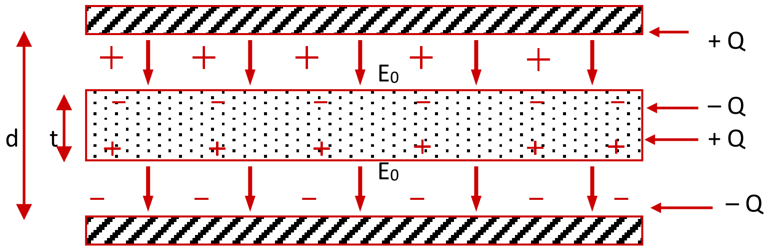
$$C_0 = \frac{\varepsilon_0 A}{d}$$

Suppose initially the charges on the capacitor plates are $\pm$ Q. Then the uniform electric field set up between the capacitor plates is $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\,\varepsilon_0}$

where σ is the surface charge density. The potential differences between the capacitor plates will be

$$V_0 = E_0 d = \frac{Qd}{A\varepsilon_0}$$

When a conducting slab of thickness t < d is placed between the capacitor plates, free electrons flow inside it so as to reduce the field to zero inside the slab, as shown in Fig. Charges – Q and + Q appear on the upper and lower faces of the slab. Now the electric field exists only in the vacuum regions between the plates of the capacitor on the either side of the slab, i.e.,



[A conducting slab placed in a parallel plate capacitor]

the field exists only in thickness d – t, therefore, potential difference between the plates of the capacitor is

$$V = E_0 (d - t) = \frac{Q}{A\,\varepsilon_0} (d - t)$$

$\therefore$ Capacitance of the capacitor in the presence of conducting slab becomes

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{(d - t)} = \frac{\varepsilon_0 A}{d} . \frac{d}{d - t}$$

or $$C = \left(\frac{d}{d - t}\right) . C_0$$

Clearly, $C > C_0$. Thus the introduction of a conducting slab of thickness t in a parallel plate capacitor increases its capacitance by a factor of $\frac{d}{d - t}$.

► ***Thus, Capacitance of a parallel plate capacitor increases on the introduction of a conducting slab between the slab of the capacitor.***

•• *If (t- d) = 0 i.e., conducting slab fits in completely in between the plates of the capacitor, then*

$$C = \frac{C_0}{(1 - t / d)} = \frac{C_0}{0} = \infty \text{ (infinity)}$$

i.e., Capacitance of the parallel plate capacitor becomes infinite. This means whatever the amount of charge may be given to this particular Capacitor; it will go on accepting it.

►**Capacitance of parallel plate capacitor increases on the introduction of a dielectric slab between the plates of the capacitor.**

- **Potential remains constant**
- **Electric field decreases.**
- **Energy 'E' increases.**

USES OF CAPACITORS

Capacitors are very useful circuit elements in any of the electric and electronic circuits. Some of their uses are:

1. To produce electric fields of desired patterns, e.g., for Millikan's experiment.
2. In radio circuits for tuning.
3. In power supplies for smoothing the rectified current.
4. In power supplies for smoothing the rectified current.
5. In the tank circuit of oscillators.
6. They store not only charge, but also energy in the electric field between their plates.

☑ VANDEGRAAF GENERATOR (Vande Graff accelerator) :--

- Designed by **Robort J. Vande Graff** in 1931.
- It is an electrostatic generator capable of generating very high potential of the order of 5×10^6 V.
- Vande graff generator was then made use in accelerating charged particles so as to carry out nuclear reaction. Now, a day it is used to accelerate charged particles like electrons ions etc, needed for various experiment of nuclear physics.

Principle

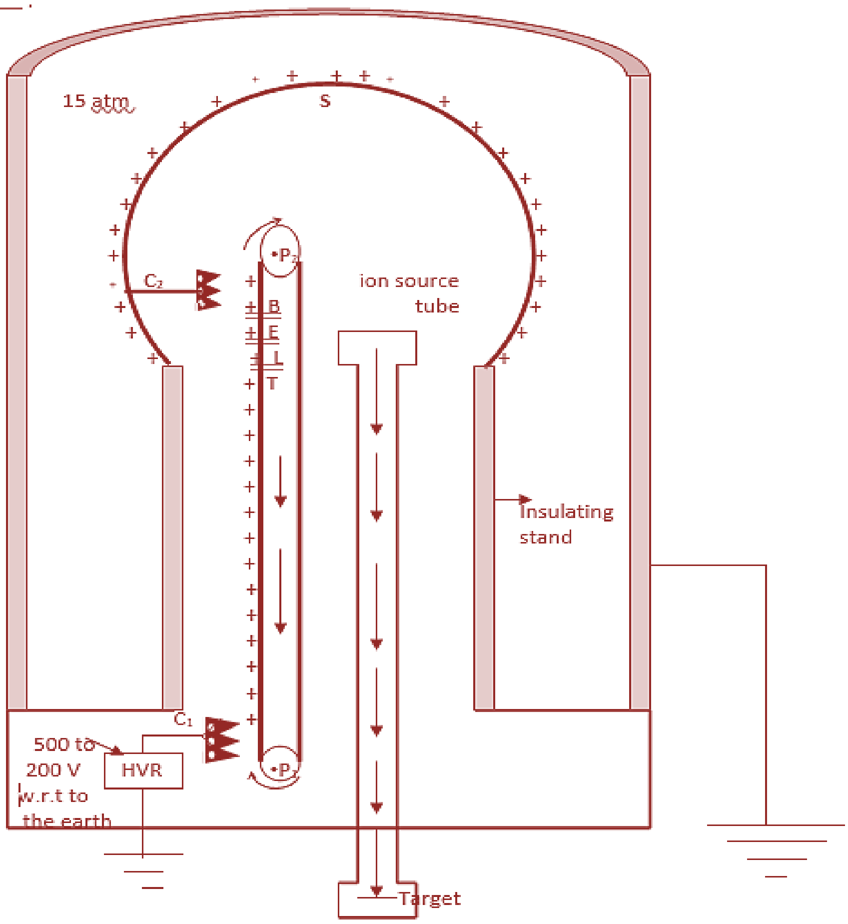
The generator is based on --- 1] The electrical discharge takes place in air or gases readily at pointed conductor. This is because, sharp point surfaces have large charge densities. (***corona discharge***).

2] Collecting action of hollow conductor – This means that if a charged conductor is brought in electrical contact with a hollow conductor (charged conductor located inside a large charged hollow conductor), the charge is transferred to the large hollow conductor no matter what is the potential of the hollow conductor? In this way, the potential of other large conductor increase considerably.

Construction

It Consists of a large spherical conducting shell (S) (of radius few meter) supported on the insulated stand. A long narrow belt of insulating material (like silk, rayon or rubber) is wound around two pulleys P_1 & P_2. The belt is kept moving continuously over the pulleys with the help of motor. A spray comb C_1 is connected to ***high voltage rectifier*** (HVR) which is earthed. A collecting comb C_2 is connected to the spherical shell 'S'.

The positive ions to be accelerated are produced in discharge tube D. The ion source lies at the head of the tube carrying target nucleus is earthed. The generator is enclosed in a steel chamber filled with **Methane** or **Nitrogen** or **Sulphur hexa floride** at a pressure of 15 **atm**.



- **WORKING**:

The spray comb is given a high potential, which spray +ive charge to the belt which is positively charged (sharp teeth of comb has high charge densities which set up corona discharging action, thus setting up a short of wind of positive charge). Since belt is moving up so it carries this positive charge upward.

Opposite charge appears on the teeth of collecting comb C_2 by induction from the belt. Due to this positive charge appears on the outer surface 'S'. As the belt is moving continuously so that the charge on the shell increases, consequently, the potential of 'S' rises to very high value.

Now, the charged particles at the top of tube (T)are at very high potential. With respect to the lower end of the tube which is earthed. Thus, these particles get accelerated downward and hit the target.

☞ Due to discharging action of sharp point of comb C_2, a negatively charged electric wind is set up and this neutralizes the positive charge on the belt. The uncharged belt returns down collects the positive charge from C_1 which in turn collected by C_2. This Process is repeated.

☞ As potential, $V = \frac{q}{C} = \frac{q}{4\pi\xi_0 R}$

Hence V of the spherical shell goes in increasing with increase in q.

The break down field of air = 3×10^6 V/m. The moment, the potential of spherical shell exceeds this value, air around is ionized and leakages of charge starts. The leakage is minimized by closing the generator inside a steel chamber filled with N_2 or CH_4 at high pressure. In addition to this the outer surface of spherical shell 's' is made extremely smooth so that there is no undue accumulation of charge at any point on the surface.

☞ If 'q' is the charge the ion to be accelerated and V is the p. d.
Then energy acquired by the ions = q V.

- At a particular stage of charging, the rate of loss of charge due to leakage becomes equal to the rate at which charge is transferred to the sphere. At this stage, there is no further rise in the potential of the sphere, thus *there is an upper limit to which we can rise the potential of the sphere.*

VERY SHORT ANSWER CONCEPTUAL PROBLEMS

Q. 1. ***Is the electrostatic potential necessarily zero at a point where the electric field strength is zero? Give an example to illustrate your answer.***

Sol. No, we know that the electric field is equal to the negative of potential gradient:

$$E = -\frac{dV}{dr}$$

The implies that even if the electric field at a point is zero, the potential may have some non-zero constant value at that point. Examples: (i) Electric field inside a charged conducting sphere is zero but potential at any inside point is the same as that on the surface of the sphere.

(ii) Electric field at the mid-point of the line joining two equal and similar charges is zero, but potential at this point is twice of that due to a single charge.

Q. 2. ***Can electrostatic potential at a point be zero, while electric field at that point is not zero?***

Sol. Yes, for example, the potential at any point on the perpendicular bisector of a dipole axis is zero, while electric field is not zero.

Q. 3. ***The electric potential is constant is in a given region. What can you say about the electric field there?***

Sol. We know that $E = -\frac{dV}{dr}$

As V is constant, so electric field E is zero.

Q. 4. ***In a certain 0.1 m³ of space, electric potential is found to be 5 V throughout. What is the electric field in this region?***

Sol. We know that $E = -\frac{dV}{dr}$

As electric potential is 5 V throughout i.e., constant so the electric field is zero in the given region.

Q. 5. ***Would electrons move away from higher potentials to lower potentials or vice-versa?***

Sol. Positive charges move from higher to lower potential regions. Electrons, being negatively charged, move from lower to higher potential regions because at a higher potential they have less potential energy.

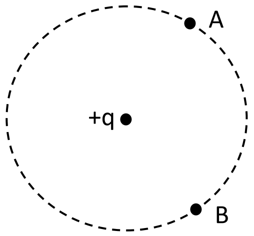
Q. 6. ***Can there be a potential difference between two neighbouring conductors carrying equal positive charges?***

Sol. Yes, the potential of a conductor depends not only on the net charge carried by it, but also on its geometrical shape and size. So two conductors of difference size and shape will have difference potentials even if they carry equal charges.

Q. 7. ***A positive charge + q is located at a point. What is the work done if a unit positive charge is carried once round this charge along a circle of radius r about this point?***

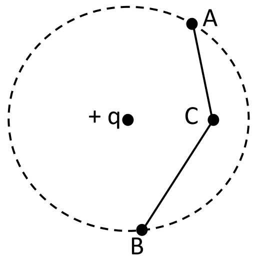
Sol. Work done is zero. Force on the unit positive charge is along the radius and direction of motion is perpendicular to it. $W = FS \cos 90^\circ = 0$

Q. 8. ***What would be the work done if a point charge + q, is taken from a point A to the point B on the circumference of a circle with another point charge + q at the same?***



Sol. The points A and B are at same distance from the charge +q at the centre, so $V_A = V_B$. Hence the work done in taking another charge from point A to B will be zero.

Q. 9. ***If a point charge + q is taken first from A to C and then from C to B of a circle drawn with path more work will be done?***

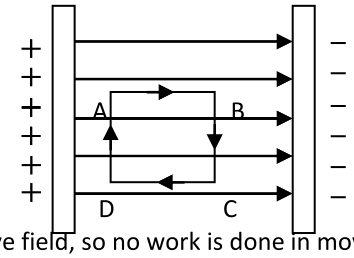


Sol. The points A and B are at same potential. Therefore,

$$V_C - V_A = V_C - V_B$$

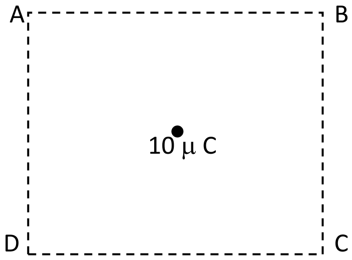
Hence the work done in taking a point charge from A to C or from C to B will be the same.

Q. 10. ***A uniform electric field E exists between two charged plates as shown in the figure. What would be the work done in moving a charge 'q' along the closed rectangular path ABCDA?***



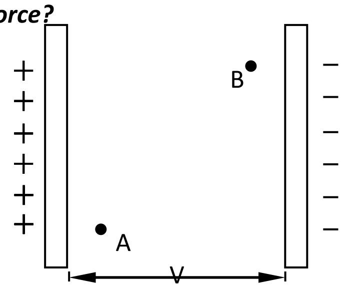
Sol. Zero, Electric field is a conservative field, so no work is done in moving a charge q along a closed path in a uniform electric field.

Q. 11. ***What is the work done in moving a 2 micro coulomb point charge from corner A to corner B of a square ABCD shown in Fig., when a 10 μC charge exists at the centre of the square?***



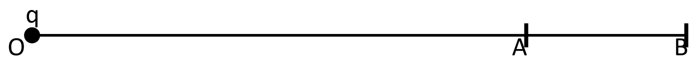
Sol. As the points A and B are at the same distance from the charge of 10 μC at the centre, so $V_A = V_B$. Hence the work done in moving a charge of 2 μC from A to B will be zero.

Q. 12. ***Two protons A and B are placed between two parallel plates having a potential difference V, as shown in Fig. Will these protons experience equal or unequal force?***



Sol. The electric field is uniform in the space between the two plates. Hence the protons A and B will experience equal force.

Q. 13. ***A point charge q is placed at O, as shown in Fig. Is $V_A - V_B$ positive, negative or zero, if q is a (i) positive, (ii) negative charge?***



Sol. Clearly,

$$V_A - V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{OA} - \frac{1}{4\pi\varepsilon_0}\frac{q}{OB} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{OA} - \frac{1}{OB}\right)$$

As OA < OB, so the quantity within brackets is positive.

(i) If q is a positive charge, then $V_A - V_B$ is positive.

(ii) If q is a negative charge, then $V_A - V_B$ Is negative.

Q. 14. ***Does the electric potential increase or decrease along the electric line of force?***

Sol. Electric potential decreases along the electric line of force.

Q. 15. ***Express the unit of electric potential in terms of the base units of SI.***

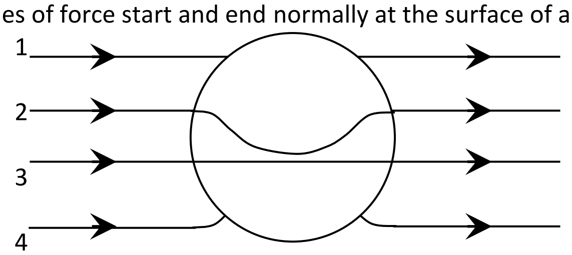
Sol. Electric potential

$$= \frac{\text{Work done}}{\text{Charge}} = \frac{ML^2\,T^{-2}}{AT} = ML^2T^{-3}\,A^{-1}$$

SI unit of electric potential = kg m^2 s^{-3} A^{-1}

Q. 16. ***A metallic sphere is placed in a uniform electric field. Which path is followed by lines of force shown in Fig.***

Sol. Path 4. This is because lines of force start and end normally at the surface of a conductor and do not exist inside it.



Q. 17. ***The work done in carrying a point charge from one point to another in an electric field does not depend on the path along which it is taken. Is it true or false? Give reason.***

Sol. False, Electric field is a conservation field, so the work done in carrying a point charge from one point to another does not depend on the path along which it is taken.

Q. 18. ***No work done is done taking a positive charge from one point to another inside a positively charged metallic shell, while outside the shell work has to be done in taking the charge from one point to the other towards the shell. Why?***

Sol. All points inside the metallic shell are at the same potential. So no work is done in moving any charge inside the charged shell. While outside the shell, there is a potential gradient and electric field. Work has to be done in moving the charge against this field.

Q. 19. ***A charge of + 1 C is placed at the centre of a spherical shell of radius 10 cm. What will be the work done in moving a charge of + 1 μC on its surface through a distance of 5 cm?***

Sol. Zero, this is because the surface of the spherical shell will be an equipotential surface.

Q. 20. ***Can two equipotential surfaces interest? Give reason.***

Sol. No, if two equipotential surfaces intersect, then there would be two values of electric potential at the point of intersection, which is not possible.

Q. 21. ***Equipotential surfaces are perpendicular to field lines, why?***

Sol. No work is done in moving a charge from one point to other on an equipotential surface. This indicates that the component of electric field along the equipotential surface is zero. Hence the equipotential surface is perpendicular to the field lines.

Q. 22. ***No work is done in moving a test charge over an equipotential surface. Why?***

Sol. The potential difference between any two points on a equipotential surface, $\Delta V = 0$

$\therefore$ Work done, $W = q_0 \, \Delta V = 0$

Q. 23. ***Why is a parallel plate capacitor named so?***

Sol. The plates of a parallel plates capacitors are plane and placed parallel to each other. Hence it is name so.

Q. 24. ***Two copper spheres of same radii, one hollow and other solid are charged to same potential. Which, if any of the two will have more charge?***

Sol. Capacitance of a spherical conductor, $C = 4\pi \varepsilon_0 R$ i.e., $C \propto R$

As both spheres have equal radii, they have the same capacitance. When charged to same potential, they will have the same charge.

Q. 25. ***Can a metal sphere of radius 1 cm hold a charge of 1 coulomb? Justify your answer?***

Sol. Here $R = 1$ cm $= 10^{-2}$ m, $q = 1$ C

Electric field on the surface of the sphere,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2} = 9 \times 10^9 \times \frac{1}{(10^{-2})^2} = 9 \times 10^{13} \text{ Vm}^{-1}$$

The field is much greater than the dielectric strength of air (3×10^6 Vm^{-1}). It causes ionisation of the surrounding air and the charge of the metal sphere leaks into the surrounding air. Thus, a metal sphere of 1 cm radius cannot hold a charge of 1 C.

Q. 26. ***What is the justification of choosing the earth as the zero of potential in practice?***

Sol. If we consider earth to be a conducting sphere surrounded by air, its capacitance will be equal to $4\pi\varepsilon_0 R$. Since the radius R of the earth is very large, so its capacitance is also very large. A charge q supplied to the earth will increase its potential by amount,

$$V = \frac{q}{C}$$

As C is very large, therefore $V = \frac{q}{C} \to 0$ for all finite charges.

Q. 27. ***Can we take the potential of the earth as + 100 V? What effect would such an assumption have on the measured values of (a) Potentials at various positions, and***
(b) Potential difference between two given points?

Sol. All potentials are measured relative to a reference position, to which we assign a zero value of potential. Earth is assigned zero potential value as it is generally taken as a reference.

However, earth can be assigned a value of + 100 V. This only increases the potential values by 100 V. But the potential difference between two points would still remain the same.

Q. 28. ***Can we give any desired charge to a capacitor?***

Sol. No, as we increase the charge on the plates of a capacitor, the potential difference between them also increases. A stage is reached when the electric field between the two plates attains the breakdown value of air. The surrounding air gets ionised and the charge begins to leak into air.

Q. 29. ***What happens if the plates of a charged capacitor are suddenly connected by a conducting wire?***

Sol. The capacitor plates will get discharged immediately. The energy stored in the capacitor changes into heat energy.

Q. 30. ***Why is the farad on inconveniently large unit of capacitance?***

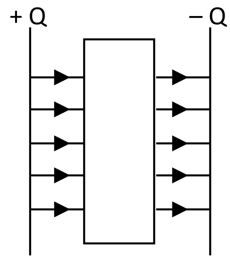
Sol. We know that

$$1\text{ F} = \frac{1\text{ C}}{1\text{ V}} = \frac{1\text{ C}^2}{1\text{ J}} \qquad \left(\because\ 1\text{ V} = \frac{1\text{ J}}{1\text{ C}}\right)$$

By daily standards, the SI unit of charge, the coulomb, is very large and the SI unit of energy, the joule, is a reasonable unit of energy. As a result, the SI unit of potential difference, the volt ($1\text{ V} = 1\text{ JC}^{-1}$) is a very small unit. So the farad, the ratio of the coulomb to the volt, is doubly large. In other words, the largeness of the farad goes as the square of the largeness of the coulomb.

Q. 31. ***A metal plate is introduced between the plates of a charged parallel plate capacitor. Sketch the electric lines of force between the plates.***

Sol. Inside the metal plate, electric field is zero, so no lines of force exist inside the metal plate, as shown in fig.



Q. 32. ***What is the dielectric constant of a metal?***

Sol. The dielectric constant of a metal is infinity. The electric field inside a conductor is zero so the dielectric constant, which is the ratio of applied electric field to the reduced electric field, will be infinite for the metallic conductor.

Q. 33. ***Is there any kind of material which when placed between the plates of a capacitor reduces its capacitance?***

Sol. No, the dielectric constant of a material is always greater than 1.

As $\kappa = C_d$ | $\kappa >$ | $\therefore$ $C_d > C_v$

i.e., the capacitance with dielectric between the plates is greater than that with vacuum between the plates. So there is no such material which when placed between the plates of a capacitor will reduced its capacitance.

Q. 34. ***For a given medium, the dielectric constant is unity. What is its permittivity?***

Sol. Permittivity, $\varepsilon = \varepsilon_0\,\kappa = 8.85 \times 10^{-12} \times 1 = 8.85 \times 10^{-12}\text{ C}^2\text{ N}^{-1}\text{ m}^{-2}$

Q. 35. ***Why do ordinary capacitors have capacities of the order of microfarads?***

Sol. Farad is a very large unit of capacitance. So, capacitors having smaller capacitances of the order of 1 μF (= 10^{-6} F) are more common.

Q. 36. ***Why does the electric field inside a dielectric decrease when it is placed in an external electric field?***

Sol. Due to polarisation of the dielectric, an electric field is induced inside the dielectric in the opposite direction of external electric field. Thus, the net electric field decreases inside the dielectric.

Q. 37. ***The introduction of dielectric slab between the capacitor plates increases the capacitance. Why?***

Sol. A dielectric slab of dielectric constant κ reduces the electric field from E to E/κ. This reduces the potential difference from V to V/κ. Hence the capacitance increases from C to κC.

Q. 38. ***A thin metal sheet is placed in the middle of a parallel plate capacitor. What will be the effect on the capacitance?***

Sol. No effect, when the metal sheet is placed in the middle, the new arrangement is equivalent to the series combination of two capacitors, each of plate separation d/2 and hence capacitance 2 C.

$$\therefore \qquad C_S = \frac{2\text{ C} \times 2\text{ C}}{2\text{ C} + 2\text{ C}} = \text{C}$$

Q. 39. ***Suppose a charge + Q_1 is given to the positive plate and a charge – Q_2 to the negative plate of a capacitor. What is the charge on the capacitor?***

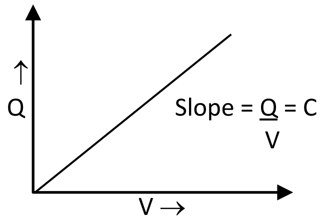
Sol. The charge on a capacitor is equal to the charge on its positive plate. So the charge on the given capacitor is + Q_1.

Q. 40. ***Is the capacitance C of a capacitor proportional to the charge Q?***

Sol. No, As the charge Q increases, the potential difference V also increases in the same proportion, so the capacitance C = Q/V, remains unaffected i.e., C is independent of Q.

Q. 41. ***Sketch of a graph to show how the charge Q acquired by a capacitor of capacitance C varies with increase in potential difference between its plates.***

Sol. As $Q \propto V$, the graph between Q and V is a straight line with slope Q/V = C, as shown in Fig.



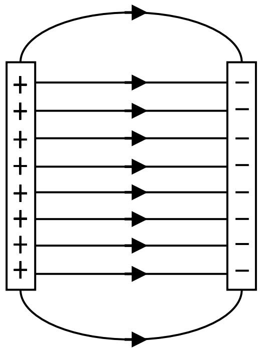
Q. 42. ***Sketch a graph to show how the capacitance C of a capacitor varies with the charge Q given to it.***

Sol. As capacitance C is independent of charge Q, so graph between C and Q is a strength line parallel to the charge axis, as shown in Fig.

Q. 43. ***Two plates are placed side by side. How many capacitors are formed?***

Sol. Three, first between distant earthed bodies and the first face of the first plate, the second between the two plates and the third between the second face of the second plate and the distant earthed objects. The capacitance of the first and third capacitors are negligibly small than capacitance of the second capacitor.

Q. 44. ***Draw the lines of force between the plates of a charged parallel plate capacitor.***



Sol. The electric lines of force between the plates of a charged parallel plate capacitor are shown in Fig.

Q. 45. ***The space between the plates of a parallel plate capacitor is filled consecutively with two dielectric layers 1 and 2 having the thicknesses d_1 and d_2 and the relative permittivity's ε_1 and ε_2 respectively. The area of each plate is equal to A. What is the capacitance of the capacitor?***

Sol. The effective separation between the plates is 1/ε times the geometrical separation. So, the capacitance of the given parallel plate capacitor is

$$C = \frac{\varepsilon_0 A}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}}$$

Q. 46. ***Two insulated charged spheres of radii 7 cm and 13 cm and having the same charge are connected by a conductor and then they are separated? Which of the two spheres will carry greater charge?***

Sol. The sphere of 13 cm radius has greater capacitance than the sphere of 7 cm radius ($C = 4\pi\varepsilon_0 R$). So, the sphere of 13 cm radius will carry more charge, Q = CV. Both spheres attain same potential V when connected by a conductor.

In fact, $V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

Q. 47. ***A spherical shell with radius a and charge Q is expanded to radius b. What is the work done by the electrostatic force in this process?***

Sol. Work done by the electrostatic force = Initial stored energy – Final stored energy

$$= \frac{1}{2} \cdot \frac{Q^2}{C_1} - \frac{1}{2}\frac{Q^2}{C_2} = \frac{1}{2} \cdot \frac{Q^2}{4\pi\varepsilon_0 a} - \frac{1}{2} \cdot \frac{Q^2}{4\pi\varepsilon_0 b}$$

$$= \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Q. 48. ***By what factor does the capacitance of a metal sphere increase if its volume is tripled?***

Sol. If V_1 and V_2 are the initial and final volumes, then

$$\frac{C_2}{C_1} = \frac{R_2}{R_1} = \left(\frac{V_2}{V_1}\right)^{1/3} = (3)^{1/3} = 1.44$$

or $C_2 = 1.44\, C_1$.

Thus, the capacitance increases 1.44 times.

Q. 49. ***How would you connect two capacitors across a battery, in series or parallel, so that they store greater (i) total charge and (ii) total energy?***

Sol. Total charge, $q = CV$

Total energy, $U = ½ CV^2$

As V is constant and $C_p > C_S$, so the capacitors must be connected in parallel for storing greater charge and greater energy.

Q. 50. ***Why should a circuit containing capacitors be handled cautiously, even when there is no current?***

Sol. Even if there is no current in the circuit, a capacitor may have charge. When such a circuit is touched, a discharge current is produced in the body and so the man touching it may receive a severe shock. This can be avoided by wearing shoes with ruber soles.

Q. 51. ***When moulded plastic parts are removed from metal dies, they develop a high voltage. Why?***

Sol. When the plastic part is removed, the capacitance of the metal die decreases but the charge (produced by friction) remains unchanged and so the voltage increases in accordance with the relation: $Q = CV$

Q. 52. ***What happens to the stored energy if after disconnecting the battery, the plates of the charged capacitor are drawn apart?***

Sol. As $C \propto 1/d$, so when the plates are drawn apart, the capacitance decreases. After disconnecting the battery, the charge on plates remains constant. Hence the energy stored in the capacitor, $U = q^2/2\,C$ increases.

Q. 53. ***What happens to the energy stored in a capacitor, if the plates of a charged capacitor are drawn apart, the battery remaining connected?***

Sol. When the plates are drawn apart, the capacitance decreases. As the battery remains connected, the potential difference remains constant, hence energy stored, $U = ½ CV^2$ decreases.

Q. 54. ***When a capacitor is charged and then discharged repeatedly, its dielectric gets heated. Why?***

Sol. The energy consumed during the polarisation of a dielectric is not completely recovered during the process of depolarization. Some energy is lost during the charging and discharging of the capacitor. This energy appears as heat.

Q. 55. ***n identical capacitors are joined in series and the combination is given a potential difference V. If these capacitors be disconnected and joined in parallel, what potential difference will be obtained across the combination?***

Sol. In series combination, each capacitor will have a potential difference V/n, which will remain same when capacitors are joined in parallel. Hence potential difference across the parallel combination will be V/n.

Q. 56. ***n identical capacitors are joined in parallel and the combination is given a potential difference V. if these capacitors be disconnected and joined in series, what potential difference will be obtained across the combination?***

Sol. In parallel combination, each capacitor will have a potential difference V. When the capacitors are disconnected and joined in series, the potential difference on n capacitors get added. Hence the p.d. across the series combination will be nV.

Q. 57. ***How can the whole charge of a conductor be transferred to another insulated conductor?***

Sol. When the charged conductor is placed in internal contact with the hollow insulated conductor, its whole charge is transferred to the hollow conductor.

Q. 58. ***Why is it that a man sitting in an insulated metal cage does not receive any shock when it is connected to a high voltage supply?***

Sol. The charge in the cage goes to its surface. The inside of the case is equipotential. There is no potential difference between the man and the cage. So the man does not receive any shock.

Q. 59. ***A large hollow metallic sphere A is charged positively to a potential of 100 volt and a small sphere B to a potential of 50 volt. Now B is placed inside A and they are connected by a wire. In which direction will the charge flow?***

Sol. The charge will flow B to A till no charge is left on B. Inside A, B will acquire potential of A and its own potential will become 150 V which is higher than that of A ($100\text{ V} < V_A < V_B$). Hence charge starts flowing from B to A.

Q. 60. ***Why should the radius of the sphere of a van de Graff generator be sufficiently large?***

Sol. Potential on a spherical shell is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

Larger the radius R of the spherical shell, the more charge it can collect without being raised to a high potential.

Q. 61. ***Why is van de Graff generator enclosed inside an earth connected steel tank filled with air under pressure?***

Sol. It prevents the leakage of charge due to ionisation. In air under pressure, as soon as free ions are formed, they recombine to form neutral air molecules.

Q. 62. ***Why is there loss of energy when two charged conductors at different potentials are connected by a conducting wire?***

Sol. As the charges flow in connecting wires, some potential energy of the charges changes into heat energy which is lost into the surroundings.

Q. 63. ***Why does the conductivity of earth's atmosphere increase with altitude?***

Sol. The conductivity increases as we go high in the atmosphere because (i) more ions are present at high altitudes due to ionisation caused by cosmic rays (ii) mean free path of ions increases at high attitude due to decrease of air density.

Q. 64. ***How does the atmospheric temperature change with altitude in the range 0 – 50 km?***

Sol. (i) In the altitude range from 0 to 12 km, the temperature falls uniformly from 290 K to 220 K.
(ii) In the altitude range from 12 to 50 km, the temperature rises uniformly form 220 K to 280 K.

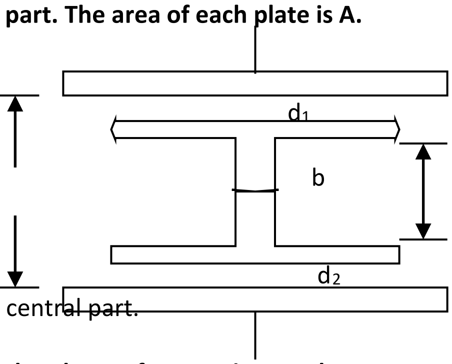
Q. 65. ***During lightning, it is safest to air inside a car rather than near a tree. Why?***

Sol. The metallic body of the car provides an electrostatic shielding from lightning. When we stand near a tree, our body provides an easy path.

Q. 66. ***Is there some way of producing high voltage on your body without getting a shock?***

Sol. Yes, if we stand on an insulating surface and touch the live wire of a high-power supply, a high potential is developed on our body, without causing any shock.

Q67. **Two capacitors are joined in series as shown in fig . The central part is movable. Prove that the equivalent capacitance of this combination is independent of the position of the central part. The area of each plate is A.**



Sol : $C_1 = \dfrac{\varepsilon_0 A}{d_1}$ & $C_2 = \dfrac{\varepsilon_0 A}{d_2}$

Equivalent capacitance in series, $C = \dfrac{C_1 C_2}{C_1 + C_2}$

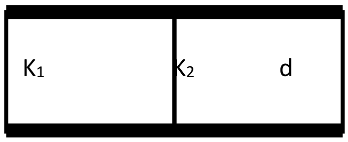
$= \dfrac{\varepsilon_0 A / d_1 \times \varepsilon_0 A / d_2}{\varepsilon_0 A / d_1 + \varepsilon_0 A / d_2}$

$= \dfrac{\varepsilon_0 A}{d_1 + d_2} = \dfrac{\varepsilon_0 A}{a - b}$

Clearly, the capacitance of the combination is independent of the position of the central part.

36. **Two equal dielectric slabs of dielectric constant K_1 & K_2 have been put between the plates of a capacitor as shown in the fig. If the area of each plate of the capacitor is A and separation between them is 'd' , find the capacitance of the capacitor.**

Area 'A'



Sol: The given arrangement is equivalent to parallel combination of two capacitors, each of plate area A / 2, plate separation 'd' such that one has medium of dielectric constant K_1 and other K_2

Let the capacitance of two capacitors are C_1 & C_2, then

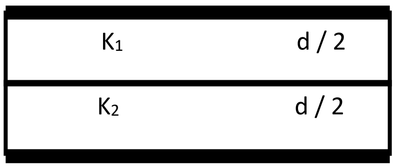
$C_1 = \dfrac{\varepsilon_0 K_1 A/2}{d}$ & $C_2 = \dfrac{\varepsilon_0 K_2 A/2}{d}$

Therefore, equivalent capacitance in parallel, $C = C_1 + C_2 = \dfrac{\varepsilon_0 K_1 A}{2d} + \dfrac{\varepsilon_0 K_2 A}{2d}$

$= \dfrac{\varepsilon_0 (K_1 + K_2) A}{d}$

37. **Two equal dielectric slabs of dielectric constant K_1 & K_2 have been put between the plates of a capacitor as shown in the fig. What should be the capacitance of the capacitor?**

Area 'A'



Sol: The given arrangement is equivalent to series combination of two capacitors, each of plate area A, plate separation 'd / 2' such that one has medium of dielectric constant K_1 and other K_2

Let the capacitance of two capacitors are C_1 & C_2, then

$C_1 = \dfrac{\varepsilon_0 K_1 A}{d/2}$ & $C_2 = \dfrac{\varepsilon_0 K_2 A}{d/2}$

$C_1 = \dfrac{2\varepsilon_0 K_1 A}{d}$ & $C_2 = \dfrac{2\varepsilon_0 K_2 A}{d}$

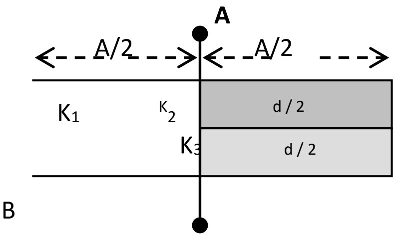
∴ equivalent capacitance in series, $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{d}{2\varepsilon_0 K_1 A} + \dfrac{d}{2\varepsilon_0 K_2 A}$

$= \dfrac{d}{2\varepsilon_0 A}\left(\dfrac{1}{K_1} + \dfrac{1}{K_2}\right)$

$= \dfrac{d}{2\varepsilon_0 A}\left(\dfrac{K_1 + K_2}{K_1 K_2}\right)$

∴ $C = \dfrac{2\varepsilon_0 A K_1 K_2}{d (K_1 + K_2)}$

38. **A parallel place capacitor is constructed using three different dielectric materials as shown in figure. The parallel plates across which a potential different is applied are separated by a distance d = 2 mm. If $K_1 = 4$, $K_2 = 6$ and $K_3 = 2$, find capacitance across points A and B .**



Sol : The arrangement is equivalent to three capacitors C_1 , C_2 and C_3 such that C_1 is parallel to the series combination of C_2 and C_3 .

Now, $C_1 = \dfrac{\xi_0 K_1 A/2}{d} = \dfrac{\xi_0 K_1 A}{2d}$

$C_2 = \dfrac{\xi_0 K_2 A/2}{d/2} = \dfrac{\xi_0 K_2 A}{d}$

$C_3 = \dfrac{\xi_0 K_3 A/2}{d/2} = \dfrac{\xi_0 K_3 A}{d}$

The equivalent capacitance is given by $C = C_1 + \dfrac{C_2C_3}{C_2+C_3}$

$= \dfrac{\xi_0 K_1 A}{2d} + \dfrac{(\xi_0 K_2 A/d) \times \xi_0 K_3 A/d}{\xi_0 K_2 A/d + \xi_0 K_3 A/d}$

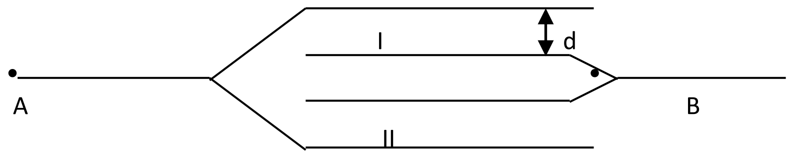
$= \dfrac{\xi_0 K_1 A}{2d} + \dfrac{\xi_0 K_2 K_3 A}{(K_2 + K_3)\, d} = \dfrac{\xi_0 A}{d}\left[\dfrac{K_1}{2} + \dfrac{K_2 K_3}{K_2 + K_3}\right]$

Here, $K_1 = 4$; $K_2 = 6$; $K_3 = 2$, $A = 1\ cm^2 = 10^{-4}\ m^2$ and $d = 2\ mm = 2 \times 10^{-3}\ m$.

$\therefore \quad C = \dfrac{8.854 \times 10^{-12} \times 10^{-4}}{2 \times 10^{-3}}\left[\dfrac{4}{2} + \dfrac{6 \times 2}{6 + 2}\right] = 1.55 \times 10^{-12}\ F = 1.55\ pF.$

39. **What is the capacitance of arrangement of 4 plates each of area A at a distance d in air in figure .**

Sol : Suppose the pair of plates A is connected to positive terminal of the battery and the pair of the plates B is connected to negative terminal of the battery. As is clear from the figure, we have two capacitors I and II. Positive plate of I is connected to positive plate of II and negative to negative therefore, they are in parallel.



As $C_p = C_1 + C_2$

therefore, $C_p = 2\,C = 2\,\xi_0 A/d$.

40. **What is the capacitance of 4 plates of area A at distance d in air in figure?**

Ans: Suppose the pair of plates A is connected to positive terminal of the battery and the pair of plates B is connected negative terminal of the battery, As is clear from the figure we have three capacitors, I , II and III. Their positive plates are connected to each other and so are the negative plates. Therefore, these are joined in parallel.

$C_p = C_1 + C_2 + C_3 = 3\,C = 3\,\xi_0 A/d$.

CONCEPTUALS:

1. **For a given potential difference, does a capacitor store more or less charge with a dielectric than it does without a dielectric?**
Ans: A capacitor with a dielectric would store more charge, as its capacity increases.

2. **An isolated conducting sphere is given a positive charge. Does its mass increases, decreases or remains the same?**
Ans: Its mass decreases slightly as it loses some electron.

3. **A capacitor is charged by using a battery, which is then slipped between the plates. Describe qualitatively what happens to the charge, the capacitance, the potential difference, electric field strength and stored energy?**
Ans: Charge remains the same; Capacitance increases; potential difference decreases; Electric field strength decreases and stored energy decreases.

4. **Is there any conductor which can be given almost unlimited charge?**
Ans: Yes, Earth can be almost unlimited charge, because its capacity is very large.

5. Two sphere of silver of same radii, one solid and the other hollow are charged to the same potential, which one has greater charge?

Ans: Both the sphere will have the same charge, as $q = CV = 4\pi \xi_0 r (V)$

6. Two insulated charged spheres of radii 10 cm and 20 cm having same charge are connected by a conductor and then they are separated. Which of the two spheres will carry more charge?

Ans: Bigger sphere will carry more charges as its capacity is larger (since $q = CV$) . The potential V becomes same on connecting them with a wire.

7. The distance between the plates of a parallel plate capacitor is 'd'. A metal plate of thickness d/2 is placed between the plates, what will be the new capacity?

Ans: As electric field inside the metal plate is zero, d becomes d / 2. Hence C becomes twice (from $C = \xi_0 A/d$)

8. Can there be a potential difference between two conductor of same volume carrying equal positive charges?

Ans: Yes, because two conductors of same volume may have different shapes and hence different capacitance.

9. What is the net charge on a charged capacitor?

Ans: Zero, because one plate has positive charge and the other carries an equal negative charge.

10. **If the parallel plate capacitor be suddenly connected to each other by a wire, what will happens ?**

Ans: The capacitor will be discharged immediately.

11. On what factor does the capacitance of capacitor depends?

Ans: (i) Geometry of the plates; (ii) distance between the plates and (iii) Nature of the dielectric medium separating the plates.

12. How will you obtain maximum capacitance from three given capacitor?

Ans: By connecting them in parallel.

13. What is the basic use of capacitor?

Ans: To store charge and energy.

14. What is the relation between dielectric constant and electric susceptibility?

Ans : $K = (1 + \chi)$

15. Why does the electric conductivity of earth's atmosphere increases with altitude?

Ans: This is because of ionization caused by highly energetic cosmic rays' particles, which are hitting the atmosphere of earth.

16. What do you mean by the capacity of single conductor?

Ans: A single conductor can be visualized as a capacitor whose second plate is far away at infinity.

17. A parallel plate capacitor has a capacity of a 6 µF in air and 60 µF when dielectric medium is introduced. What is dielectric constant of medium?

Ans: $K = \frac{C_m}{C_o} = \frac{60}{6} = 10.$

18. Where does the energy of a capacitor reside?

Ans: The energy resides in the dielectric medium separating the two plates.

19. What is the order of capacitances used in power supplies?

Ans: 1 **µF to** 10 **µF**

20. Why does the electric field inside a dielectric decrease when it is placed in an external electric field?

Ans: Because the dielectric gets polarized.

21. When a battery is connected across a capacitor, are the charges on the plates always equal and opposite, even for the plates of different sizes?

Ans: Yes, charges on the plates are always equal and opposite, irrespective of their areas. This is because charge is conserved.

22. What is the basic difference between a charged capacitor and an electric cell?

Ans: A capacitor supplies electrical energy stored in it. A cell supplies electrical energy by converting chemical energy into electrical energy at constant potential difference.

23. Given a battery, how would you connect two capacitors, in series or in parallel for them to store the greater (a) total charge (b) total energy?

Ans: Total charge, q = CV and total energy, E = ½ CV^2. Now, V is constant and $C_P > C_S$, therefore, Parallel combination is required for storing greater energy and greater Charge.

24.Why a space ship entering the ionosphere is not sufficiently heated, though temperature at the top o ionosphere is 700K?

Ans: Density of air in ionosphere is so low that very few molecules only bombard the spaceship.

25. A man fixes outside his house one evening a two-meter-high insulating slab carrying on its top a large aluminium sheet of area 1 m^2 . Will he get and electric shock if he touches the metal sheet next morning?

Ans : Yes , he get a shock . The steady discharging current in the atmosphere due to small conducting of air charges the aluminium sheet during night. On touching the metal sheet, charge flows to earth through his body and he gets shock .

26. Where is the knowledge of dielectric strength helpful?

Ans: Dielectric strength is the maximum strength of electric field that can be tolerated by the dielectric without electric breakdown . Its Knowledge helps us in designing a capacitor by determining the maximum potential that can be applied across the plates of the capacitor.

27. A sensitive instrument is to be shifted from the strong electrostatic field in its environment, suggest a possible way.

Ans: For this, the instrument must be enclosed fully in a metallic cover. This will provide electrostatic shielding to the instrument.

28. How can the whole charge of a conductor be transferred to another Isolated conductor?

Ans: This is done by placing the charged conductor inside the hollow insulated conductor and connecting the two way a wire . The whole charge will shift to outer surface of the conductor.

29. If two isolated conductors each having a definite capacity are far apart and are connected to each other by a fine wire how do you calculate the capacity of the combination? In joining them with the wire have you connected them in parallel or in series?

Ans: On connecting, both the spheres acquire a common potential V . If C is total capacity, then total charge

$q = CV = q_1 + q_2$, as charge is conserved .

Therefore, $C = \frac{q_1}{V} + \frac{q_2}{V} = C_1 + C_2$.

30. In what form is the energy stored in a charged capacitor?

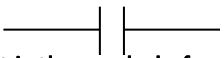
Ans: The energy is stored in the capacitor in the form of electric field.

31. What is the approximate value of electric field at the surface of the earth?

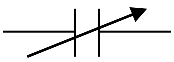
Ans: 100 V m^{-1} (directed vertically downward).

32. What is the symbol of a parallel plate capacitor of fixed capacitance?

Ans:



33. What is the symbol of a parallel plate capacitor of variable capacitance?



34. Define dielectric constant in terms of the capacitance of a capacitor.

Ans: Dielectric constant of a material is the ratio of the capacity of the capacitor with the material as dielectric to the capacity of the same capacitor without dielectric between the plates.

35. The capacitor C_1 = 3 μF and C_2 = 6 μF, are connected in the series and charged by connecting a battery of voltage V = 10 volt in series with them . They are then disconnected from the battery, and loose wires are connected together. What is the final charge on each?

Ans the capacitors are connected in series. So originally, they have equal charges. When the loose wires are reconnected, they neutralize each other giving zero final charge.

36. A metal foil is placed in the middle of a parallel pate capacitor. W hat is the effect on the capacitance of the system?

Ans: There will be practically no effect on the capacitance of the capacitor.

37. Distinguish between polar and non-polar dielectrics.

Ans: Polar dielectric is that dielectric whose molecules possess electric dipole moment even in the absence of external electric field. On the other hand , a non-polar dielectric is that dielectric whose molecules do not possess permanent electric dipole moment.

38. Guess a possible reason why water is much greater dielectric constant [= 80] then say mica [= 6] ?

Ans: This is because water has a permanent dipole moment. On the other hand, mica dose not possess permanent dipole moment.

39. What is the dielectric constant of a conductor?

Ans: The dielectric constant of a conductor is infinity. This is because the dielectric constant is the ratio of applied field and the reduced value of field. Since electric field inside a conductor is zero therefore dielectric constant of a conductor is infinity.

40. What is the role of evaporation in the atmospheric electricity?

Ans: Evaporation is the continuously taking place from the surface of water in seas end rivers. The water vapours moving up carry positive charges while an equivalent number of negative charges is left behind. Thus, evaporation contributes towards atmospheric electricity.

41. Discus the role of water vapours in the formation of clouds?

Ans: The water vapor condense on the ions present in the atmosphere. These condensed drops of water carry electric charge. Coalescence of these drops leads to the formation of clouds. The potential of the cloud is very large as compared to the potential of the cloud is very large as compared to the potential of an individual charged ion.

42. Name the physical quantity, whose SI unit is coulombs volt $^{-1}$.

Ans: It is electric capacitance.

43. Can we give any desired amount of charge to a capacitor?

Ans: No, the maximum charge that can be given to a capacitor is limited by the dielectric strength of the medium between the two plates of capacitor.

44. Why is the parallel plate capacitor named so?

Ans: The pales of a parallel plate capacitor are plane and are placed parallel to each other. For this reason, It is called a parallel plate capacitor.

45. The dielectric constant of a conductor can be taken to be infinitely large, infinitely small or optimum. Which of the three alternatives is correct?

Ans: When the conductor is placed inside in a electric field, the field inside the conductor becomes zero. The dielectric constant which is ratio of the strength of applied electric field to the reduced value of electric field will be infinite for the conductor.

46. On inserting a dielectric between the plates of capacitor, its capacitance is found to increase 5 times. What is the relative permittivity of dielectric?

Ans: When dielectric of relative permittivity ξ_r is inserted, the capacitance of the capacitor increases ξ_r times. Since on inserting dielectric, capacitance increases 5 times, the relative permittivity of dielectric is 5.

47. An air capacitor is given a charge of 2 µC raising its potential to 200V . If on a inserting a dielectric medium to 50 V, what is the dielectric constant of a medium?

Ans: When dielectric is introduced, the potential between the plates of the capacitor decreases by a factor equal to dielectric constant. Therefore, **K** = $\frac{V}{V'} = \frac{200}{50} = 4$.

48. Why the van de Graaff generator is enclosed inside an earth connected steel tank filled with air under pressure?

Ans: It prevents the leakage of charge due to ionization. The reason is that in air under pressure, a soon as free ions are produced they recombine to form neutral air molecules.

49. The atmosphere is not electrically neutral. Explain why?

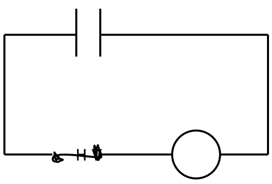
Ans: The atmosphere of earth is being continuously charged by lightning and thunderstorms all over the globe and maintains an equilibriums with the discharge during normal weather condition. Hence atmosphere cannot become electrically neutral.

50. Name the form of energy of the atmosphere and in what forms this energy appears during the thunder storms and lightning

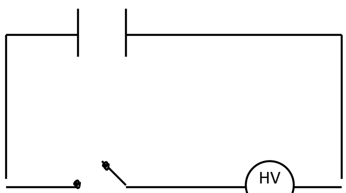
Ans: Electrical energy of the atmosphere appears as light , sound and heat energies during thunder storms and lightning .

51. Why should be capacitor circuit be carefully handled when the circuit is just switched off?

Ans: A capacitor when charged discharges slowly on the disconnection of circuit i.e., removal of external potential difference across it. If this capacitor is touched by someone, he may feel shock due to large charge still present on the capacitor.



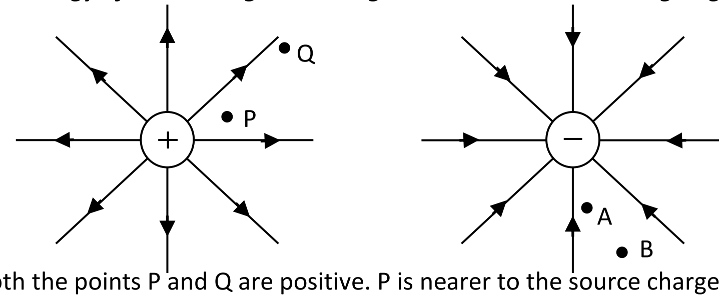
Charged capacitor



Remains charged for some time.

SHORT ANSWER TYPE CONCEPTUAL PROBLEMS

Q. 1. ***Fig. (a) and (b) show the field lines of a single positive and negative charge respectively.***
(i) Give the signals of the potential differences, $V_P - V_Q$; $V_B - V_A$.
(ii) Give the sign of the potential energy difference of a small negative charge between the points Q and P; A and B.
(iii) Give the sign of the work done by the field in moving a small positive charge from Q to P.
(iv) Give the sign of the work done by an external agency in moving a small negative charge from B to A.
(v) Does the kinetic energy of a small negative charge increase or decrease in going from B to A?

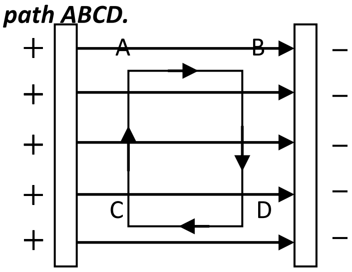


Sol. (i) potentials at both the points P and Q are positive. P is nearer to the source charge than Q. Also, we know that the electrostatic potential. At a point is inversely proportional to the distance of the point from the charge i.e., $V \propto 1/r$,
therefore , $V_P > V_Q$ or $V_P - V_Q > 0$
Potentials at both the points A and B are negative. Point B is farther from the charge than the point A. So, potential at B is less negative than at A.
$\therefore$ $V_B > V_A$ or $V_B - V_A > 0$
(ii) P.E. of two point charges, $U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$
The P.E. of a positive charge and a negative charge is negative. The potential energy of a negative charge will be more negative at P than at Q, so $(P.E)_Q - (P.E.)_P > 0$
The P.E. of two negative charge is positive. So the P.E. of a negative charge will be more positive at A than at B, hence $(P.E.)_A - (P.E.)_B > 0$
(iii) As $V_P > V_Q$, if a small positive charge is moved by an external agency from Q to P will be negative.
(iv) The negative charge will experience less repulsion at B than at A, so the work done by the external agency in moving a negative charge from B to A is positive.
(v) As the negative charge moves from B to A, it experiences more repulsion, its velocity decreases, and so its kinetic energy decreases.

Q. 2. ***Answer carefully:***
(i) A comb run through one's dry hair attracts small bits of paper. Why? What happens if the hair is wet or if it is a rainy day?
(ii) Ordinary rubber is an insulator. But the special rubber tyres of aircrafts are made slightly conducting. Why is this necessary?
(iii) Vehicles carrying inflammable materials usually have metallic ropes touching the ground during motion. Why>
(iv) A bird perches on a bare high power line and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why?

Sol. (i) When the comb runs through dry hair, it gets charged by friction. The molecules in the paper get polarized by the charged comb, resulting in a net force of attraction. If the hair is wet, or if it is rainy day, friction between hair and the comb reduces. The comb does not get charged and thus it will not attract small bits of paper.
(ii) During landing, the tyres of aircraft may get highly charged due to friction between tyres and the air strip. If the tyres are made slightly conducting, they will lose the charge to the earth otherwise too much of static electricity accumulated may produced spark and result in fire.
(iii) Moving vehicle gets charged due to friction. The inflammable material may catch fire due to the spark produced by charged vehicle. When metallic rope is used, the charge develops on the vehicle is transferred to the ground and so the fire is prevented.
(iv) Bird's whole body is at same potential. NO charge flows and no shock is produced. The man touching the ground maintains a potential difference between different parts of his body. A large current flow, which electrolysis blood and causes death.

Q. 3. ***A uniform electric field $\vec{E}$ exist between two charged plates, as shown in Fig. Calculate the work done in moving a charge q along the closed rectangular path ABCD.***



Sol. Force on charge q in field $\vec{E}$, $F = q\vec{E}$

Work done in moving the charge q along path ABCD,

$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$= q\vec{E}\,.\,\vec{AB} + q\vec{E}\,.\,\vec{BC} + q\vec{E}\,.\,\vec{CD} + q\,\vec{E} + \vec{DA}$

$= qE\,(AB)\cos 0^\circ + qE\,(BC)\cos 90^\circ + qE\,(CD \cos 180^\circ + qE\,(DA)\cos 90^\circ$

$= qE\,(AB) + 0 + qE\,(CD)\,(-1) + 0 \quad = 0$

Q. 4. ***Define electric potential at a point. When kept in an electric field, does a proton move from lower to higher or from higher to lower potential region?***

Sol. The electric potential at a point in an electric field is defined as the amount of work done in moving a unit positive charge from infinity to that point against the electrostatic force.

Proton is a positively charged particle. In an electric field, it will move from higher to lower potential region so as to reduce its potential energy.

Q. 5. ***Why electric potential in the field of a negative charge is lower at near points and higher at distant points?***

Sol. For a negative charge the electric potential is

$$V = -\frac{1}{4\pi\varepsilon_0}\,.\,\frac{q}{r}$$

For near points, r is less and V is more negative (lower).

For distant points, r is more and V is less negative (higher).

Q. 6. ***The electric field due to a point charge at a distance r depends according to inverse square law $\left(\propto \frac{1}{r^2}\right)$. State how are the following quantities depends upon r?***

(i) Potential due to a point charge

(ii) Potential at a distance r from the centre of a charged metallic sphere of radius R (r < R)?

Sol. (i) Potential due to a point charge,

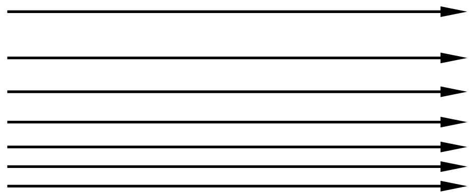
$$V = \frac{1}{4\pi r_0}\,.\,\frac{q}{r} \quad \text{i.e.,} \quad V \propto \frac{1}{r}$$

(ii) In case of a charged metallic sphere,

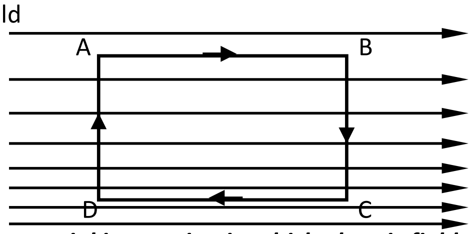
$$V_{\text{inside}} = V_{\text{surface}} = \frac{q}{4\pi\varepsilon_0 R} = \text{constant}$$

$\therefore$ Potential V does no depend on r.

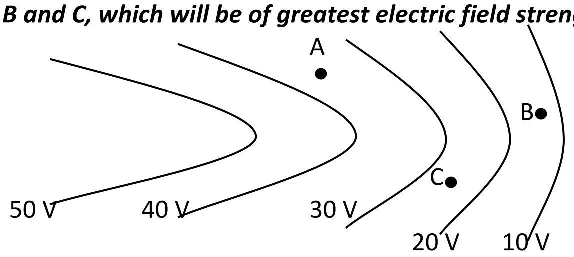
Q. 7. ***Is it possible to create an electric field in which all the lines of force are parallel lines and whose density increases gradually in a direction perpendicular to the lines of force, as shown in Fig. (a)?***



Sol. No, This is not possible because the work done in carrying a test charge along a closed ABCD, as shown in Fig. (b), will not be zero. More work is done along CD, less along AB, zero along BC and DA. But in an electric field, work done is essentially zero as it is a conservative field



Q. 8. ***Fig. shows lines of constant potential in a region in which electric field exists. The values of the potential are indicated. Out of the points A, B and C, which will be of greatest electric field strength? Give reason.***

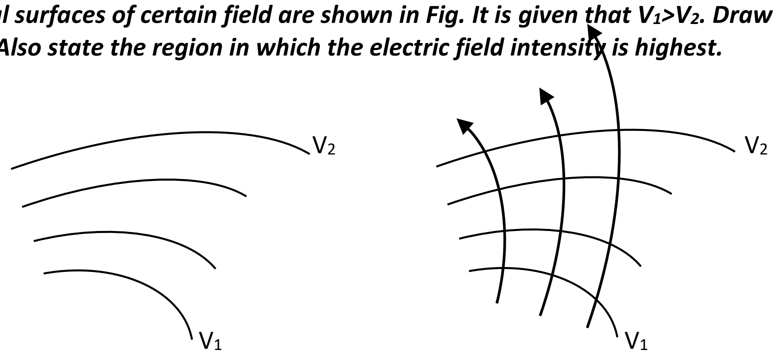


Sol. Electric field is the rate of fall of potential i.e.,

$$E = -\frac{dV}{dr}$$

For constant dV, $E \propto 1/dr$. The stronger the field, the closer the equipotential surfaces. As the equipotential surfaces are closed in the neighbourhood of B, so the field is greatest at B.

Q. 9. ***The equipotential surfaces of certain field are shown in Fig. It is given that $V_1 > V_2$. Draw the corresponding lines of force for this pattern. Also state the region in which the electric field intensity is highest.***

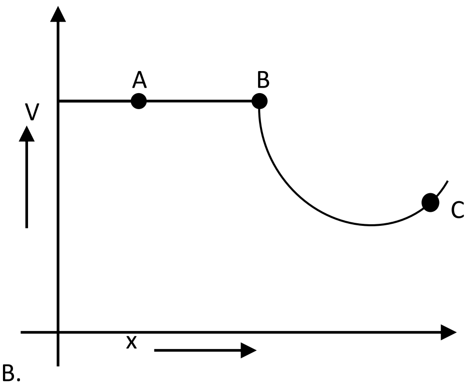


Sol. As shown in Fig., the lines of force are perpendicular to the equipotential surfaces and directed from higher potential to lower potential. The electric field intensity is highest in the lower left region where the equipotential surfaces are closest to each other.

Q. 10. ***The adjoining figure shows the variation of electrostatic potentials V with distance 'x' for a given charge distribution. For the points marked A, B and C, identify the point at which the electric field is:***

(i) zero ***(ii) maximum***

Explain your answer in each case.



Sol. At any point, we have

$$E = -\frac{dV}{dx} = \text{Negative slope of } V - x \text{ graph}$$

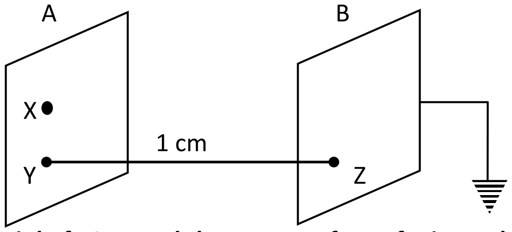
At point A, $\frac{dV}{dx} = 0$

At point B, $\frac{dV}{dx} < 0$

At point C, $\frac{dV}{dx} > 0$

Therefore, (i) E is zero at point A. (ii) E is maximum at point B.

Q. 11. ***Two identical plane metallic surfaces A and B are kept parallel to each other in air, separated by a distance of 1 cm, as shown in Fig.***



Surface A is given a positive potential of 10 V, and the outer surface of B is earthed.

(i) What is the magnitude and direction of the uniform electric field between points Y and Z?

(ii) What is the work done in moving a charge of 20 μC from point X to point Y.

Sol. (i) $E = -\frac{dV}{dr} = -\frac{10\ V}{1\ cm}$

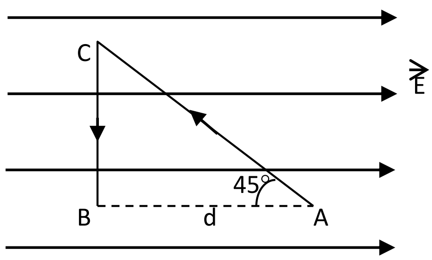
$$= -\frac{10\ V}{10^{-2}\ m} = -1000\ Vm^{-1}$$

Magnitude of the uniform electric field between X and Y = 1000 Vm^{-1}

The direction of the electric field is from plate A to plate B.

(ii) Zero, This is because the point X and Y are at the same potential.

Q. 12. ***A test charge q_0 is moved without acceleration from point A to B along the path $A \rightarrow C \rightarrow B$, as shown in Fig. Calculate the potential difference between A and B.***



Sol. P.D. does not depend on the path along which the test charged is moved. Therefore,

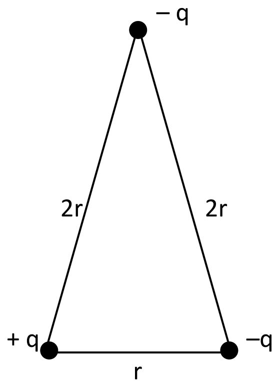
$$E = -\frac{dV}{dr} = -\frac{(V_B - V_A)}{d} = \frac{V_A - VB}{d}$$

or $V_A - V_B = Ed.$

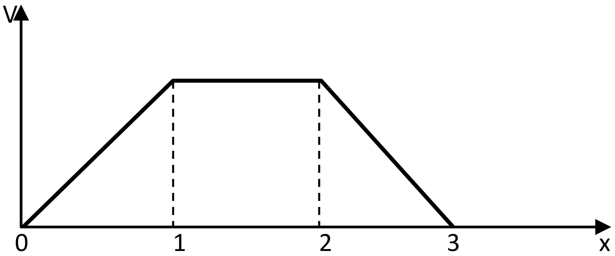
Q. 13. Suggest a configuration of three point charges separated by finite distances that has zero electric potential energy.

Sol. The configuration of three point charges separated by finite distance that has zero electric potential energy.

$$U = k\frac{q.\ q}{r} + k\frac{q\ (-q)}{2r} + k\frac{(-q)\ q}{2r} = 0$$



Q. 14. The electric potential as a function of distance x is shown in Fig. Construct a graph of the electric field strength E.



Sol. We know that

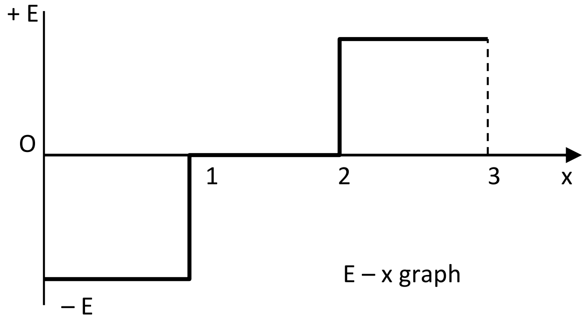
$$E = -\frac{dV}{dx} = \text{Negative slope of } V - x \text{ graph}$$

For $0 < x < 1$, $\frac{dV}{dx} = +\text{ve constant, so field} = -E$

For $1 < x < 2$, $\frac{dV}{dx} = 0$, so field = 0

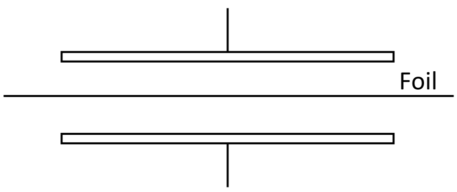
For $2 < x < 3$, $\frac{dV}{dx} = -\text{ve constant, so field} = +E$

Consequently, we get the E – x graph as shown below:



E – x graph

Q. 15. A sheet of aluminium foil of negligible thickness is placed between the plates of a capacitor, as shown in Fig. What effect has it on the capacitance of (i) The foil is electrically insulated, and
(ii) The foil is connected to the upper plate with a conducting wire.



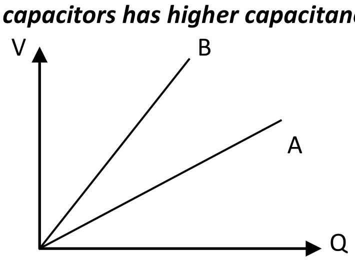
Sol. (i) The arrangement is equivalent to two capacitors connected in series. Each such capacitor has plate separation d/2 and hence capacitance 2 C. Total capacitance is

$$C_S = \frac{2\,C \times 2\,C}{2\,C + 2\,C} = C$$

i.e., the capacitance remains unaffected.

(ii) The capacitance becomes twice the original capacitance because d becomes d/2.

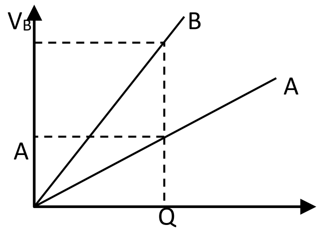
Q. 16. The graph shows the variation of voltage V across the plates of two capacitors A and B versus increase of charge Q stored on them. Which of the capacitors has higher capacitance? Give reason for your answer.



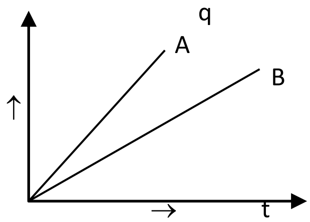
Sol. From Fig. $C_A = \frac{Q}{V_A}$ and $C_B = \frac{Q}{V_B}$

But $V_A < V_B$, therefore, $C_A > C_B$

Thus, capacitor A has a higher capacitance.



Q. 17. ***The given graph shows the variation of charge q versus potential difference V for two capacitors C_1 and C_2. The two capacitors have same plate separation but the plate area of C_2 is double than that of C_1. Which of the lines in the graph correspond to C_1 and C_2 and why?***

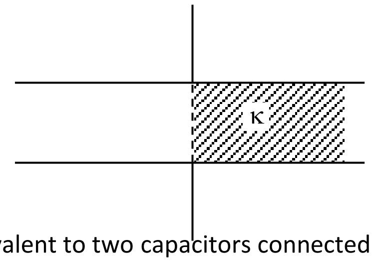


Sol. As C = q/V and graph A has a larger slop than B, so the graph A represents a capacitor of large capacitance.

Also, $C = \varepsilon_0 A$ i.e., $C \propto A$

As the plate area of C_2 is double of that of C_1, so C_2 has a larger capacitance. Hence the line A of graph corresponds to C_2.

Q. 18. ***As shown in Fig. a dielectric material of dielectric constant κ is inserted in half portion between the plates of a parallel-plate capacitor. If its initial capacitance is C, what is the new capacitance?***



Sol. The new arrangement is equivalent to two capacitors connected in parallel.

$$C_1 = \frac{\varepsilon_0 A/2}{d}, \quad C_2 = \frac{\kappa\,\varepsilon_0 A/2}{d} \quad \therefore \quad C_p = C_1 + C_2 = \frac{\varepsilon_0 A}{2d} + \frac{\kappa\,\varepsilon_0 A}{2d} = \frac{\varepsilon_0 A}{2d}(\kappa + 1) = \frac{C}{2}(\kappa + 1)$$

Q. 19. ***Find the capacitance of three parallel plates, each of area A metre^2 and separated by d_1 and d_2 metre. The in-between spaces are filled with dielectrics of relative permittivity ε_1 and ε_2. The permittivity of free space is ε_0.***

Sol. The given system is equivalent to two parallel-plate capacitors connected in series. Their capacitances are

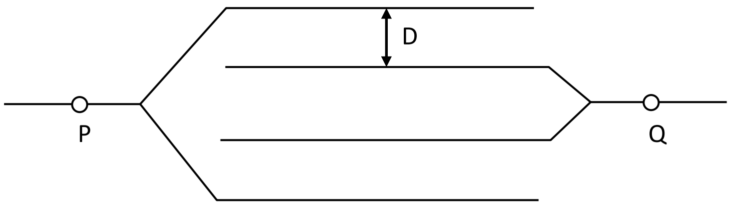
$$C_1 = \frac{\varepsilon_1\,\varepsilon_0\,A}{d_1} \quad \text{and} \quad C_2 = \frac{\varepsilon_2\,\varepsilon_0\,A}{d_2}$$

If C_S is the equivalent capacitance of the series combination, then

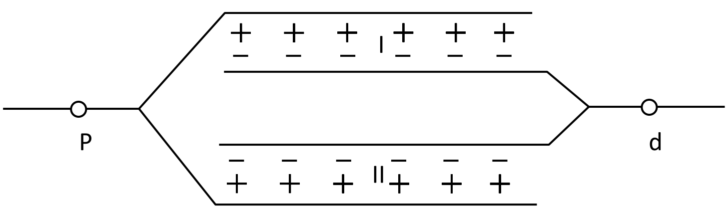
$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\varepsilon_0 A}\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right) = \frac{1}{\varepsilon_0 A}\,\frac{\varepsilon_2\,d_1 + \varepsilon_1\,d_2}{\varepsilon_1\,\varepsilon_2}$$

or $$C_S = \frac{\varepsilon_0 A}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}} = \frac{\varepsilon_1\,\varepsilon_2\,\varepsilon_0\,A}{\varepsilon_2\,d_1 + \varepsilon_1\,d_2}$$

Q. 20. ***What is the capacitance of arrangement of 4 plates of area A at distance d in air in Fig.***



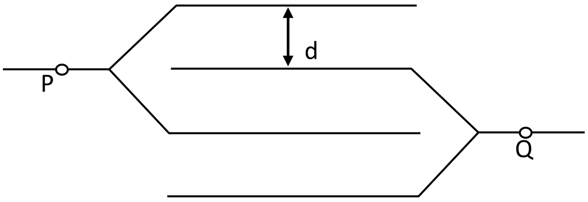
Sol. As shown in Fig. suppose the point P is connected to the positive terminal and point Q to the negative terminal of a battery. Clearly, we have two capacitors I and II with their positive plates connected together and their negative plates connected together. So the two capacitors are in parallel.



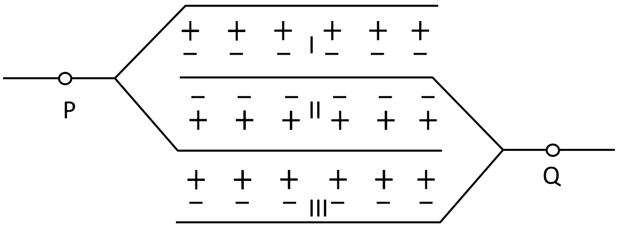
∴ Equivalent capacitance,

$$C_p = C_1 + C_2 = C + C = 2C = \frac{2\,\varepsilon_0 A}{d}$$

Q. 21. ***What is the capacitance of arrangement of 4 plates of area A at distance d in air in Fig.***



Sol. As shown in Fig. suppose the point P is connected to the positive terminal and point Q to the negative terminal of a battery. Clearly, we have three capacitors I, II and III. Their positive plates are connected to the same point P while the negative plates are connected to the same point Q. So the three capacitors are in parallel.



∴ Equivalent capacitance,

$$C_p = C_1 + C_2 + C_3 = 3C = \frac{3\,\varepsilon_0 A}{d}$$

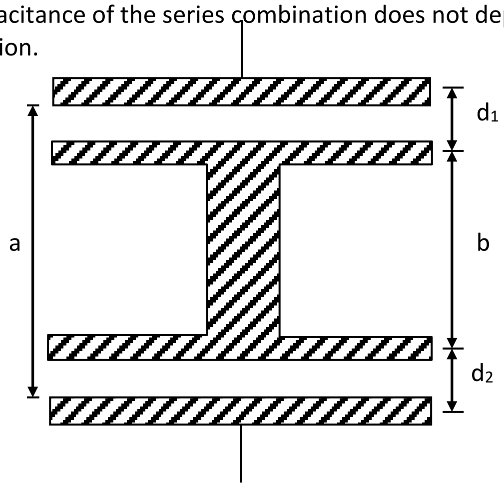
Q. 22. ***Fig. shows two capacitors joined in series, the rigid central section of length b being movable. Prove that the equivalent capacitance of the combination is independent of the position of the central section.***

Sol. Here $C_1 = \frac{\varepsilon_0 A}{d_1}$ and $C_2 = \frac{\varepsilon_0 A}{d_2}$

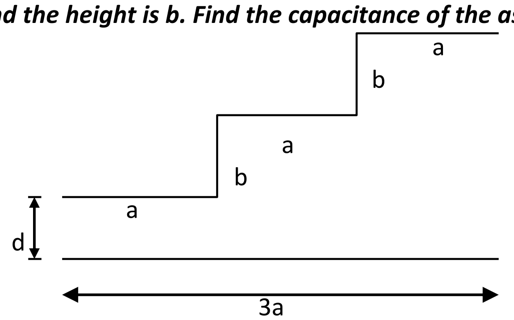
The equivalent capacitance of the series combination,

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_0 A/d_1 \times \varepsilon_0 A/d_2}{\frac{\varepsilon_0 A}{d_1} + \frac{\varepsilon_0 A}{d_2}} = \frac{\varepsilon_0 A}{d_1 + d_2} = \frac{\varepsilon_0 A}{a - b}$$

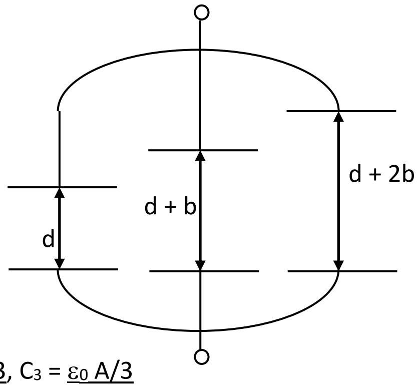
Clearly, the equivalent capacitance of the series combination does not depend on d_1 and d_2 i.e., it is independent of the position of the central section.



Q. 23. ***A capacitor is made of a flat plate of area A and a second plate having a stair-like structure, as shown in Fig. The width of each stair is a and the height is b. Find the capacitance of the assembly.***



Sol. As shown in Fig. the given arrangement is equivalent to a parallel combination of three capacitors of capacitances C_1, C_2, C_3. Here



$$C_1 = \frac{\varepsilon_0 A/3}{d},\quad C_2 = \frac{\varepsilon_0 A/3}{d+b},\quad C_3 = \frac{\varepsilon_0 A/3}{d+2b}$$

$$\therefore \quad C = C_1 + C_2 + C_3 = \frac{\varepsilon_0 A}{3}\left(\frac{1}{d} + \frac{1}{d+b} + \frac{1}{d+2b}\right)$$

$$= \frac{\varepsilon_0 A}{3}\left(\frac{(d+b)(d+2b) + d(d+2b) + d(d+b)}{d(d+b)(d+2b)}\right)$$

$$= \boldsymbol{\frac{\varepsilon_0 A (3d^2 + 6bd + 2b^2)}{3d(d+b)(d+2b)}}$$

Q. 24. ***A parallel plate capacitor, when there is vacuum between the plates, has capacitance C_0. What will be its capacitance, when (i) distance between the plates is doubled,***
(ii) A sheet of thickness t of a dielectric of relative permittivity κ is introduced between the plates?

Sol. (i) Capacitance of the capacitor with vacuum between its plate is

$$C_0 = \frac{\varepsilon_0 A}{d}$$

When the distance is doubled, capacitance becomes

$$C = \frac{\varepsilon_0 A}{2d} = \frac{1}{2} C_0$$

(ii) When a sheet of relative permittivity κ and thickness t is placed between the plates of a capacitor, its capacitance becomes

$$C = \frac{\kappa C_0}{\left[\frac{t}{d} + \kappa\left(1 - \frac{t}{d}\right)\right]}$$

As $t = d$, therefore, $C = \kappa C_0$

Q. 25. ***Two metal plates from a parallel plate capacitor. The distance between the plates is d. A metal sheet of thickness d/2 and of the same area is introduced between the plates. What is the ratio of the capacitances in the two cases?***

Sol. Capacitance of the air-filled parallel plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

When a metal plate of thickness t is introduced, t is introduced, its capacitance becomes

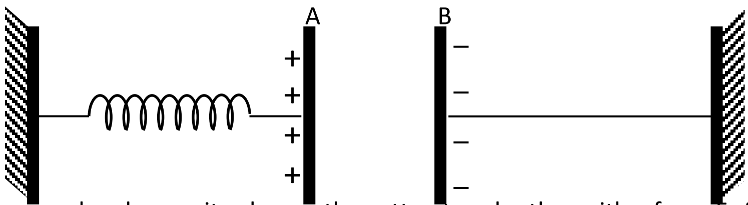
$$C' = \frac{\varepsilon_0 A}{d - t}$$

As $t = \frac{d}{2}$, therefore, $C' = \frac{\varepsilon_0 A}{d - d/2} = \frac{2\varepsilon_0 A}{d}$

Ratio of the capacitances,

$$\frac{C'}{C} = \frac{2\,\varepsilon_0 A/d}{\frac{\varepsilon_0 A}{d}} = \frac{2}{1} = 2 : 1$$

Q. 26. ***The plate A of a parallel plate capacitor is connected to a spring of force constant k and can move, while the plate B is fixed. The arrangement is held between two rigid supports as shown in Fig. If a charge + q is placed on plate A and – q on plate B, by how much does the spring elongate?***



Sol. As the two plates carry equal and opposite charge, they attract each other with a force F. As a result, the spring elongates by length l. Then

$$E = -kl$$

Where k is the force constant of the spring. The capacitance of the parallel plate capacitor with plate separation x is

$$C = \frac{\varepsilon_0 A}{x}$$

The energy stored in the capacitor is

$$U = \frac{1}{2} \cdot \frac{q^2}{C} = \frac{q^2}{2} \cdot \frac{x}{\varepsilon_0 A}$$

As the electrostatic force is a conservation force, we can write

$$F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{q^2 x}{3\,\varepsilon_0 A}\right) = -\frac{q^2}{3\,\varepsilon_0 A} \qquad \text{... (ii)}$$

The negative sign shows that the force is attractive. From equations (i) and (ii), we get

$$-kl = -\frac{q^2}{2\,\varepsilon_0 A} \quad \text{or} \quad l = \frac{q^2}{2\,\varepsilon_0 kA}$$

Q. 27. ***(i) Two circular metal plates, each of radius 10 cm, are kept parallel to each other at a distance of 1 mm. What kind of capacitor do they make? Mention one application of this capacitor.***
(ii) If the radius of each of the plates is increased by a factor of √2 and their distance of separation decreases to half of its initial value, calculate the ratio of the capacitance in the two cases.
(iii) Suggest any one possible method by which the capacitance in the second case be increased by n times.

Sol. (i) The two plates from a parallel plate capacitor.
Application: Along with an inductor, a capacitor is used in an oscillatory circuit.
(ii) Original capacitance, $C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \times \pi r^2}{d}$

New capacitance,

$$C' = \frac{\varepsilon_0 \times \pi (r\sqrt{2})^2}{d/2} = \frac{4\,\varepsilon_0 \times \pi r^2}{d} = 4\,C$$

$$\therefore \quad \frac{C}{C'} = \frac{1}{4} = 1 : 4$$

(iii) The capacitance of a capacitor can be increased n times by any of the following methods:
(a) By inserting a dielectric of dielectric constant n between the capacitor plates. Then C' = nC.
(b) By decreasing the distance d between the plates by a factor n, because $C \propto 1/d$.
(c) By increasing the area of the plates n times, because $C \propto A$.
(d) By increasing the area of the plates $\sqrt{n}$ times and simultaneously decreasing the distance between the plates by a factor of $\sqrt{n}$.

Q. 28. ***A parallel plate capacitor with air as dielectric is charged by a d.c. source to a potential 'V'. Without disconnecting the capacitor from the source, air is replaced by another dielectric medium of dielectric constant 10. State with reason, how does (i) electric field between the plates, and (ii) energy stored in the capacitor changes.***

Sol. (i) The potential difference V across the capacitor plates remains unchanged because the battery remains connected. Hence
the electric field, E = V/d remains unchanged.
(ii) With dielectric medium of dielectric constant 10, the capacitance becomes

$$C = 10\,C_0$$

$$\therefore \quad U = \frac{1}{2} CV^2 = \frac{1}{2}(10\,C_0)\,V_0^2$$

$$= 10\left[\frac{1}{2} C_0 V_0^2\right] = 10\,C_0$$

Thus the stored energy increases 10 times.

Q. 29. ***A parallel plate capacitor is charged to a potential difference 'V' by a d.c. source. The capacitor is then disconnected from the source. If the distance between the plates is doubled, state with reason how the following will change:***
(i) Electric field between the plates, (ii) capacitance and (iii) energy stored in the capacitor.

Sol. Here $C_0 = \frac{\varepsilon_0 A}{d}$, $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$

$$U_0 = \frac{q^2}{2\,C_0}$$

On disconnecting the battery, the charge q on the capacitor plates remains unchanged. If the distance d is doubled, then
(i) $E = \frac{q}{\varepsilon_0 A} = E_0$ i.e., the electric field remains unchanged.
(ii) $C = \frac{\varepsilon_0 A}{2d} = \frac{1}{2} C_0$ i.e., the capacitance is halved.
(iii) $U = \frac{q^2}{2\,C} = \frac{q^2}{C_0} = 2\,U_0$ i.e., the stored energy is doubled.

Q. 30. ***Capacitors P, Q and R have each a capacity C. A battery can charge the capacitor P to a potential difference V. If after charging P, the battery is disconnected from it and the charged capacitor P is connected in following separated instances to Q and R:(i) to Q in parallel, and (ii) to R in series, then what will be the potential differences between the plates of P in the two instances?***

Sol. (i) When P is connected to Q in parallel, the capacitors P and Q share charged equally.

∴ P.D. between the plate of P = $\frac{q/2}{C} = \frac{q}{2C} = \frac{V}{2}$

(ii) When P is connected to R is series, charges on both capacitors remain same. The capacitor P with the same charge will have the same potential difference V across it.

Q. 31. ***The plates of a parallel plate capacitor are drawn apart keeping them connected to a battery. Next the same plates are drawn apart from the same initial condition keeping the battery disconnected. In which case is more work done? Give reason.***

Sol. In the second case. In the first case, when the plates are drawn apart, the p.d. remains constant but the capacitance the hence the charge across the plates decreases. As a result, the force of attraction between the plated decreases gradually. In the second case, the charged on the plates remain constant. So when the plates are moved through equal distances, more work has to be done in the second case.

Q. 32. ***An uncharged capacitor is connected to a battery. Show that half the energy supplied by the battery is lost as heat while charging the capacitor.***

Sol. Let capacitance of the capacitor = C

emf of the battery = V

∴ Charge given to the capacitor, $q = CV$

Energy supplied by the battery = Work done by the battery = qV

Energy stored in the capacitor = $\frac{1}{2}CV^2 = \frac{1}{2}qV$

∴ Energy lost as heat = $qV - \frac{1}{2}qV = \frac{1}{2}qV$

Thus, half the energy supplied by the battery is lost as heat while charging the capacitor.

Q. 33. ***A capacitor is connected across a battery. (i) Why does each plate receive a charge of exactly the same magnitude? (ii) Is this true even if the plates are of different sizes?***

Sol. (i) This is because of conservation of charge. If q_1 and q_2 must be zero because the charge on the battery is simply redistributed and not created or destroyed.

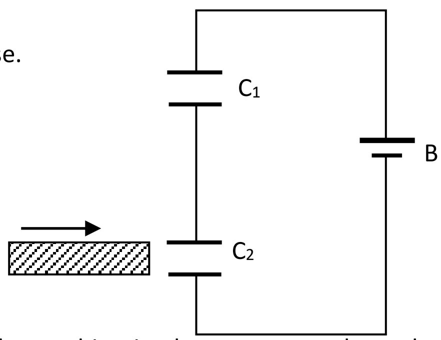
(ii) Yes, the charge will be of equal magnitude even if the two plates have different sizes.

Q. 34. ***Two identical capacitors C_1 and C_2 are connected to a battery B, as shown in Fig. A dielectric slab is slipped between the plates of C_2, the battery remaining connected. What happens to the charge, the capacitance, the potential difference and stored energy of each capacitor?***

Sol. On introducing the dielectric slab between the plates of capacitor C_2, its capacitance increases, hence the capacitance of the whole combination increases. As the battery remains connected, the p.d. across the combination remains constant, and so the charge on the combination will increase. As C_1 and C_2 are in series, charges on them will be equal. As charge on C_1 has increased, its capacitance is same and so p.d. across C_1 will increase. But $V_1 + V_2$ = constant, so the potential difference across C_2 will decrease.

Energy of C_1, $u_1 = \frac{1}{2}C_1 V_1^2$ will increase.

Energy of C_2, $U_2 = \frac{1}{2}(\kappa C_2)\left(\frac{V_2}{\kappa}\right)^2 = \frac{1}{2}\frac{C_2 V_2^2}{\kappa}$ will decrease.

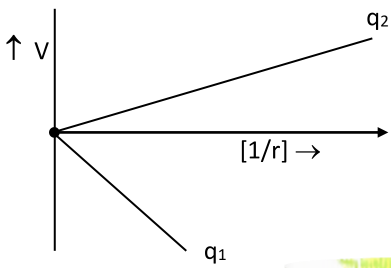


However, there will be a net increase in the energy of the combination because more charge has been drawn by the capacitors from the battery.

Q. 35. ***The two graphs drawn here, show the variation of electrostatic potential (V) with 1/r (r being distance of the field point from the point charge) for two point charges q_1 and q_2.***

(i) What are the signs of the two charges?

(ii) Which of the two charges has a larger magnitude and why?



Sol. (i) Charge q_1 is – ve while charge q_2 is +ve.

(ii) As $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r}$

∴ Slope of V vs. $\frac{1}{r}$ graph = $\frac{q}{4\pi\varepsilon_0}$

As the graph for q_1 has a slope of larger magnitude than that for q_2, so q_1 has a slope of larger magnitude than that for q_2, so q_1 has a larger magnitude than q_2.

Q. 38. ***Keeping the voltage of the charging source constant, what would be the percentage change in the energy stored in a parallel plate capacitor if the separation between its plates were to be decreased by 10 %?***

Sol. $U = \frac{1}{2} CV^2 = \frac{1}{2} \frac{\varepsilon_0 A}{d} V^2$ i.e., $U \propto 1/d$

When the separation between the plates is decreased by 10 %, the energy stored becomes U' such that,

$$\frac{U'}{U} = \frac{d}{d'} = \frac{100}{90} \quad \text{or} \quad U' = \frac{10}{9} U$$

Percentage change in the stored energy,

$$\frac{U' - U}{U} \times 100 = \left(\frac{U'}{U} - 1\right) \times 100 = \frac{10}{9} - 1 \times 100 = \frac{100}{9}$$

$= 11.11\ \%$

Q. 39. ***A parallel plate capacitor, each with plate area A and separation d, is charged to a potential difference V. The battery used to charge it is then disconnected. A dielectric slab of thickness d and dielectric constant κ is now placed between the plates. What change, if any, will take place in: (i) charge on plates, (ii) electric field intensity between the plates (iii) capacitance of the capacitor. Justify your answer in each case.***

Sol. (i) The charge q_0 on the capacitor plates remains unchanged because the battery has been disconnected, before placing the dielectric slab.

(ii) The surface charge induced on the dielectric slab reduce the electric field intensity to a new value,

$$E = \frac{E_0}{\kappa}$$

(iii) Due to the decrease in potential difference, the capacitance increase.

$$C = \frac{q_0}{V} = \frac{q_0}{V_0/\kappa} = \kappa C_0$$

Q. 40. ***On charging a parallel place capacitor to a potential V, the spacing between the plates is halved, and a dielectric medium***
of $\varepsilon_r = 10$ is introduced between the plates, without disconnecting the d.c. source. Explain, using suitable expressions, how
the (i) capacitance, (ii) electric field and (iii) energy density of the capacitor change.

Sol. As the battery remains connected, the potential V remains unchanged even after the introduction of dielectric medium.

(i) Original capacitance, $C_0 = \frac{\varepsilon_0 A}{d}$

New capacitance,

$$C_0 = \frac{\varepsilon_0 A}{d'} = \frac{\varepsilon_r \varepsilon_0 A}{d/2} = \frac{10\, \varepsilon_0 A}{d/2} = 20 \frac{\varepsilon_0 A}{d} = 20\, C_0$$

(ii) $E = \frac{V}{d/2} = 2 . \frac{V}{d} = 2\, E_0$

(iii) Original energy density, $u_0 = \frac{1}{2} \varepsilon_0 E_0^2$

New energy density, $u = 1/25\ \varepsilon E^2 = \frac{1}{2} \varepsilon_r \varepsilon_0 (2E_0)^2$

$= 4\, \varepsilon_r . \frac{1}{2} \varepsilon_0 E_0^2 = 4 \times 10 \times u_0 = \mathbf{40\ \mu_0}$

.... End.