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CURRENT ELECTRICITY

1. ELECTRIC CURRENT, OHM'S LAW, RESISTANCE
2. KIRCHOFF'S LAW, BRIDGE AND POTENTIOMETRE (electrical measurement)
3. HEATING EFFECT OF CURRENT
4. CHEMICAL EFFECT OF CURRENT
5. THERMO EFFECT OF CURRENT

"The branch of physics which deals with the dynamics (motion) of charge is called Current electricity".

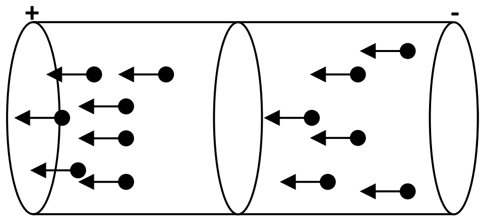
☑ **ELECTRIC CURRENT (I):** *"Electric current is defined as amount of charge flowing through any cross-section of a substance in a unit time".*

In other words , *It is defined as the rate of flow of electric charge through any cross –section of a wire".*

i.e., Electric current $= \frac{\text{Total charge flowing}}{\text{Time taken}}$ => $\boxed{I = \frac{q}{t}}$

➢ **Instantaneous Current:** If the net charge 'dq' flows through a wire in small time 'dt' then,

$I = dq / dt$



If 'n' carriers of electricity each having charge 'e', through any cross-section of the conductor in time 't, then,

$\boxed{I = \frac{ne}{t}}$

☞ **UNIT of Electric current: 'Ampere'. (A) (SI)**

As, $I = q / t$

i.e., Ampere (A) = 1 coulomb / 1 second = C / s

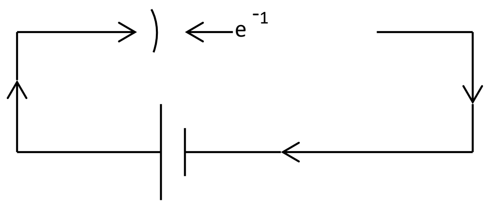
☞ **1 Ampere:** *"The current through a wire is said to be 1 ampere, if 1C of charge is flowing per second through a section of the wire".* ***or***

"The current through a wire is called 1 ampere if 1C of charge flows through the wire in 1 second".

------ milli ampere (1 mA) = 10^{-3} A
------ Micro ampere (1 μA) = 10^{-6} A
------ Electromagnetic Unit of current (emu) or (ab ampere) 1A = 1 / 10 emu.
------ Electrostatic Unit of current (esu) or (stat ampere) 1A = 3×10^{9} esu.

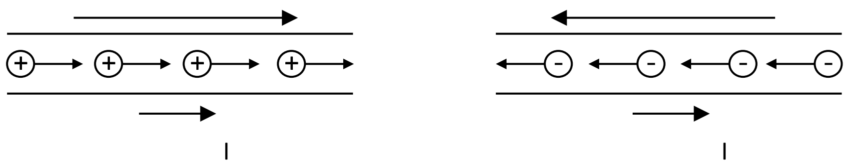
➢ **Direction of electric current:** Conventionally, direction of flow of positive charge gives the direction of conventional current (i.e., **from high potential to low potential**).

---- - The direction of flow of electric current (i.e., negative charge) gives the direction of electronic current (i.e., **from low potential to high potential**).



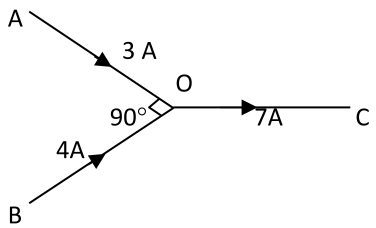
Conventional and electronic currents: By convention, the direction of motion of positive charges is taken as the direction of electric current. However, a negative charge moving in one direction is equivalent to an equal positive charge moving in the opposite direction, as shown in Fig. As the electrons are negatively charged particles, so the direction of electronic current (i.e. the current constituted by the flow of electrons) is opposite to that of the conventional current.

Conventional current   electronic current



✪ **The direction of electronic current is opposite to that of conventional current.**

✪ **Electric current is a scalar quantity:** Although electric current has both magnitude and direction, yet it is a scalar quantity. This is because the laws of ordinary algebra are used to add electric currents and the laws of vector addition are not applicable to the addition of electric currents. For example, i–n Fig. two different currents of 2 A and 3 A flowing in two mutually perpendicular wires AO and BO meet at the junction O and then flow along wire OC. The current junction O and then flow along wire OC. The current in wire OC is 7 A which is the scalar addition of 3 A and 4 A and not 5 A as required by vector addition.



✪ The arrows represent the sense or direction of the flow of charges in a conductor .

Examples based on Definition of Electric current

❖ **Formulae Used**

1. Electric current = $\frac{\text{Charge}}{\text{Time}}$ or $I = \frac{q}{t}$

2. As q = ne, so $I = \frac{ne}{t}$

3. In case of an electron revolving in a circle of radius r with speed v, period of revolution of the electron is

$$T = \frac{2\pi r}{v}$$

Frequency of revolution, $\nu = \frac{1}{T} = \frac{v}{2\pi r}$

Current at any point of the orbit is

I = Charge flowing in 1 revolution × No. of revolutions per second

or $I = e\nu = \frac{ev}{2\pi r}$

❖ **Units Used**

Electric charge is in coulomb (C), time in second (s), and current in ampere (A)

❖ **Constant Used** Charge on an electron, $e = 1.6 \times 10^{-19}$ C.

Q. 1. ***10^{20} electrons, each having a charge of 1.6×10^{-19} C, pass from a point A towards another point B in 0.1 s. What is the current in ampere? What is its direction?***

Sol. Here $n = 10^{20}$, $e = 1.6 \times 10^{-19}$ C, t = 0.1 s

Current, $I = \frac{q}{t} = \frac{ne}{t}$

$= \frac{10^{20} \times 1.6 \times 10^{-19}\ \text{C}}{0.1\ \text{s}} = 160$ A

the direction of current is from B to A.

Q. 2. ***Show that one ampere is equivalent to a flow of 6.25 × 10^18 elementary charges per second.***

Sol. Here I = 1 A, t = 1 s, e = 1.6 × 10^{-19} C

As $I = \frac{q}{t} = \frac{ne}{t}$

∴ Number of electrons,

$n = \frac{It}{e} = \frac{1 \times 1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$

Q. 3. ***How many electrons pass through a lamp in one minute, if the current is 300 mA?***

Sol. I = 300 mA = 300 × 10^{-3} A,

t = 1 minute = 60 s, e = 1.6 × 10^{-19} C

As $I = \frac{q}{t} = \frac{ne}{t}$

∴ Number of electrons,

$n = \frac{It}{e} = \frac{300 \times 10^{-3} \times 60}{1.6 \times 10^{-19}} = 1.125 \times 10^{20}$

Q. 4. ***How many electrons per second flow through a filament of a 120 V and 60 W electric bulb? Given electric power is the product of voltage and current.***

Sol. Here V = 120 V, P = 60 W, t = 1 s

As P = VI, there $I = \frac{P}{V} = \frac{60}{120} = 0.5$ A

Number of electrons,

$n = \frac{It}{e} = \frac{0.5 \times 1}{1.6 \times 10^{-19}} = 3.125 \times 10^{18}$

Q. 5. ***In the Bohr model of hydrogen atom, the electron revolves around the nucleus in a circular path of radius 5.1 × 10^{-11} m at a frequency of 6.8 × 10^{15} revolutions per second. Calculate the equivalent current.***

Sol. Here r = 5.1 × 10^{-11} m, v = 6.8 × 10^{15} rps, e = 1.6 × 10^{-19} C

Current, I = eV = 1.6 × 10^{-19} × 6.8 × 10^{15}= 1.088 × 10^{-3} A

Q. 6. ***In a hydrogen atom, an electron moves in an orbit of radius 5.0 × 10^{-11} m with a speed of 2.2 × 10^{6} ms^{-1}. Find the equivalent current. (Electronic charge = 1.6 × 10^{-19} coulomb).***

Sol. Here r = 5.0 × 10^{-11} m, v = 2.2 × 10^{6} ms^{-1}, e = 1.6 × 10^{-19} C

Period of revolution of electron,

$T = \frac{2\pi r}{v} = \frac{2\pi \times 5.0 \times 10^{-11}}{2.2 \times 10^{6}}$ s

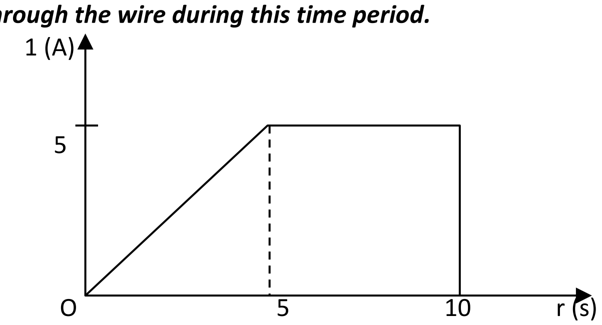
Frequency, $\nu = \frac{1}{T} = \frac{2.2 \times 10^{6}}{2\pi \times 5.0 \times 10^{-11}}$

$= \frac{2.2 \times 7 \times 10^{17}}{2 \times 22 \times 5} = 7 \times 10^{15}\ s^{-1}$

Current, $I = e\nu = 1.6 \times 10^{-19} \times 7 \times 10^{15}$

$= 1.12 \times 10^{-3}$ A

Q. 7. ***Fig. shows a plot of current I through the cross-section of a wire over a time interval of 10 s. Find the amount of charge that flows through the wire during this time period.***



Sol. Amount of charge that flows in 10 s

= Area under the I – t graph

= ½ × 5 × 5 + (10 – 5) 5 = 37.5 C

Q. 8. ***The amount of charge passing through cross-section of a wire is***

$q(t) = at^2 + bt + c$

(i) Write the dimensional formulae for a, b and c.

(ii) If the values of a, b and c in SI units are 5, 3 and 1 respectively, find the value of current at t = 5 second.

Sol. (i) Given $q(t) = at^2 + bt + c$

Dimension of a = $\left[\frac{q}{t^2}\right] = \frac{AT}{T^2} = AT^{-1}$

Dimension of b = $\left[\frac{q}{t}\right] = \frac{AT}{T} = A$

Dimension of c = [q] = AT

(ii) Current, $I = \frac{dq}{dt} = \frac{d}{dt}(at^2 + bt + c) = 2at + b$

At t = 5 s, $I = 2 \times 5 \times 5 + 3 = 53$ A

Q. 9. ***The current through a wire depends on time as $I = I_0 + \alpha t$, where $I_0 = 10$ A and $\alpha = 4\ As^{-1}$. Find the charge that flows across a section of the wire in 10 seconds.***

Sol. As $I = \frac{dq}{dt} = I_0 + \alpha t$

∴ $q = \int (I_0 + \alpha t)\, dt = I_0 t + \frac{\alpha t^2}{2} + C$

At t = 0, q = 0, so constant of integration C = 0

∴ $q = I_0 t + \frac{\alpha t^2}{2}$

At t = 10 s, $q = 10 \times 10 + \frac{4 \times (10)^2}{2} = 300$ C

➤ CURRENT CARRIERS in different materials:

" ***The charges particles whose flow in a definite direction constitute the electric current are called current carrier."***

(* except semi conductor)

☑ 1.] **Current carries in solid conductor:** In solid conductor like metals the atoms are quite close to each other and are strongly bound to one another. But the valence electrons of the atom do not remain attached to individual atoms but are free to move through out the volume of the conductor. Such valence electrons are called free electron (or conduction electron).

--- Under the effect of an external electric field the conduction electron moves in a definite direction causing electric current in the conductor.

✪ Conduction **electrons** are the current carriers in **solid conductors**.

✪ In an insulator all the electrons are tightly bound to their parent atoms hence they do not have free electrons as such there are practically no current carriers in an insulator.

☑ 2.] **Current carries in liquid**: -- Liquids which dissociate into ions (called electrolyte) allows electric current to pass through them Ex: $CuSO_4$, AgCl.

This electrolyte dissociates into positive and negative ions (Cu^{2+}, SO_4^{2-} , Ag^+ Cl^-, Ag NO_3^- etc)

-- Under the influence of external electric field, the positive and negative ions of the electrolyte move in definite direction to constitute electric current.

✪ **POSITIVE ION (Cation) & NEGATIVE ion (Anion) are the current carriers in the liquids.**

☑ 3.] **Current carries in Gases**: Generally, Gases behave **as** insulators of electricity but they get ionized and become conductor at low pressure when high potential difference is applied across them. Ionized gas contains positive ions and electrons.

✪ **positive ions and electrons are the current carriers in gases.**

☑ TYPES OF CURRENT:

(a) ***Steady current :(Steady direct current)*** A B

"An electric current is said to be steady direct current if its magnitude and direction do not change with time".

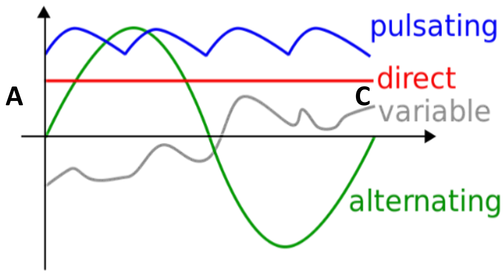
-- Since the magnitude and direction of current do not change with the time so, a straight-line AB || to time axis represent steady current.

(b) ***Variable current :(Varying current)***: ***"An electric current is said to be Varying direct current if its magnitude changes with and polarity remains the same.***

(c) ***Alternate current*****: *"The current whose magnitude changes continuously with time and direction changes periodically is called alternating current***

--- AC current is represented by a semi- curve and called sinusoidal AC.

--- Current for short duration = Transient Current



☑ **ELECTROMOTIVE FORCE (emf)**: ***"Electromotive force is not a force but is maximum work done in taking a unit charge once round the closed circuit".***

--- The emf is associated with an arrangement or mechanism which can supply energy or does work to move the electric charge from lower potential to higher potential. Such an arrangement is called source of emf (EX: cell, battery, etc).

Therefore,

"Emf of a cell is defined as the maximum P.D between two electrodes of the cell when no current is drawn from the cell or cell in the open circuit". ***or***

"Energy supplied by the cell to drive a unit charge once round the complete circuit is called emf of the cell"

$$e = \oint \mathbf{E} \cdot d\mathbf{l}$$

dl → small path represents - vector

E → Electric field = F /q

✪ **SI unit** = volt or joule /coulomb.

☞* **1 volt** -- "If ***1 joule of energy is supplied by the cell to drive 1 C of charge once round the whole circuit, the emf of cell is said to be 1 volt."***

EMF VS. POTENTIAL DIFFERENCE

Electromotive force	*Potential difference*
1.It is the work done by a source in taking a unit charge once round and complete circuit.	**1.** It is the amount of work done in taking a unit charge from one point of a circuit to another.
2. It is equal to the maximum potential difference between the two terminals of a source when it is in an open circuit.	**2.** Potential difference may exist between any two points of a closed circuit.
3. It exists even when the circuit is not closed.	**3.** It exists only when the circuit is closed.
4. It has non-electrostatic origin.	**4.** It originates from the electrostatic field set up by the charges accumulated on the two terminals of the source.
5. It is a cause. When emf is applied in a circuit, potential difference is caused.	**5.** It is an effect.
6. it is equal to the sum of potential differences across all the components of a circuit including the p.d. required to send current through the cell itself.	**6.** Every circuit component has its own potential difference across its ends.
7. It is larger than the p.d. across any circuit element.	**7.** It is always less than the emf.
8. It is independent of the external resistance in the circuit.	**8.** It is always less than the emf.

☑ DRIFT VELOCITY:

"Drift velocity is defined as the average velocity with which free electrons in a conductor gets drifted in a direction opposite to the direction of the applied electric field."

Explanation: -- In a conductor (say metal) , these are large no. of free electrons (or conduction electrons) which are in state of continuous rapid zig – zag motion within the body of the conductor.

Now, assume **1 free electron / atom,** the no. of free electron / m^3 will be of the order 10^{29}. The average thermal speed $V = \sqrt{\frac{3KT}{m}}$ of the free electron is of the order 10^5 m /s at room temperature. But their direction of motion is so randomly such that the average thermal velocity of the electrons is zero.

If $\vec{U}_1, \vec{U}_2, \vec{U}_3$ - - - - - - - - $\vec{U}_n$ are random thermal velocities of n electrons in a metal .

∴ average thermal velocity of electron = 0.

i.e., $\frac{\vec{U}_1 + \vec{U}_2 + \vec{U}_3 - - - - - - - - + \vec{U}_n}{n} = 0$

i.e., (There will be no net flow of electrons). But when P.d is applied across the ends of a conductor, an electric field is set up. Each free electrons in the conductor ***experiences a force in a direction opposite to the electric field (F = - q E).***

Now, the free electron accelerated from negative end to the positive end of the conductor.

In moving, they suffer frequent collision against the ion and lose energy. ***During the short time between every two successive collision, the electron accelerated towards the positive end***. But extra velocity so gained is destroyed at each subsequent collision. *The net result is that in addition to their random motion, the electron acquired a small velocity towards the positive end of the conductor*.

Thus, **"Drift velocity is defined as the average velocity with which conduction electron get drifted towards the positive end of the conductor under the influence of ext. electric field."**

☞ **Drift velocity of electron** is of the order of **10^{-5} m/ s**.

Expression: -- Suppose e = electronic charge

m = mass of each electron

V = P.d across the end of the conductor

l = Length of the conductor

$$\mathbf{E = \frac{V}{l}}$$

Each electron experiences a force, F = --qE = --eE

∴ **acceleration, a = F / m = -eE / m.** *Due to this acceleration, the free electron acquires, in addition to its thermal velocity a velocity component in a direction opposite to the direction of E*. The gain in velocity due to $\vec{E}$ is very small & is lost in the next collision (As a result, the acceleration of an electron is not proportional to $\vec{E}$). (The positive ions also experience a force due to $\vec{E}$ but they cannot move as they are heavy and tightly bound in the metal).

✪☞**"The short time for which a free electron accelerates before it under goes a collision with positive ion in the conductor is called Relaxation time." (τ).**

∴ If an electron having random thermal velocity the accelerates for time τ_1(before it suffers a collision) then it will attain a velocity.

$$\vec{v}_1 = \vec{U}_1 + \vec{a}\tau_1$$

lly, Velocity acquired by other electrons,

$$v_2 = U_2 + a\tau_2$$
$$v_1 = U_1 + a\tau_2$$

— — — — — — — — — — — — — —

— — — — — — — — — — — — — —

$$v_n = U_n + a\tau_n$$

As per definition of Drift Velocity, Average velocity of all the 'n' electron i.e.,

Drift velocity $V_d = \frac{U_1 + a\tau_1 + U_2 + a\tau_2 + \ldots\ldots\ldots\ldots + U_n + a\tau_n}{n}$

or, $V_d = \frac{U_1 + U_2 + - - - - - - - - + U_n}{n} + \frac{a(\tau_1 + \tau_2 + \ldots\ldots\ldots\ldots \tau_n)}{n}$

$V_d = 0 + a\tau$ where, $\tau = \frac{(\tau_1 + \tau_2 + \ldots\ldots\ldots\ldots \tau_n)}{n}$

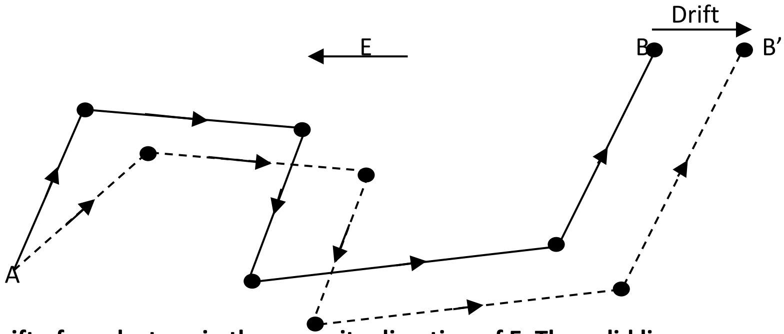
$\mathbf{V_d = a\tau}$ **= average relaxation time**

$$V_d = -\frac{eE\tau}{m}$$

Magnitude of Drift velocity , $V_d = \frac{-eE\tau}{m}$

Relaxation time , $\tau = \frac{\lambda}{v_{rms}} = \frac{\text{mean free path}}{\text{root means square velocity of an electron}}$

► The electric field accelerates an electron between two collisions, yet it does not produce any net acceleration. This is because the electrons keep colliding with the positive metal ions. The velocity gained by it due to the electric field is lost in next collision. As a result, it acquires field is lost in next collision. As a result, it acquires a constant average velocity v_d in the opposite direction of E. The motion of the electron is similar to that of a small spherical metal ball rolling down a long flight of stairs. As the ball falls from one stair to the next, it acquires acceleration due to the force of gravity. The moment it collides with the stair, it gets decelerated. The net effect is that after falling through a number of steps, the ball begins to roll down the stairs with zero average acceleration i.e., at constant average speed. Moreover, as the average time τ between two successive collisions is small, an electron slowly and steadily drifts in the opposite direction of E, as shown in Fig.



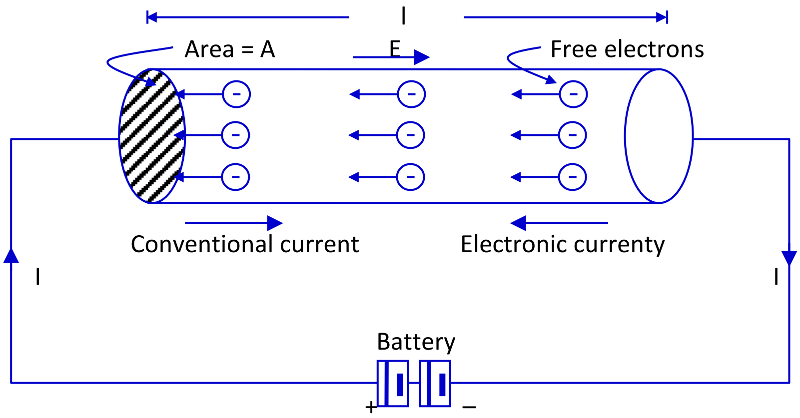
[Slow an steady drift of an electron in the opposite direction of E. The solid lines represent the path in the absence of E and dashed lines in the presence of E.]

☑ Relation between Drift velocity and Electric current:

: Suppose a potential difference V is applied across a conductor of length l and of uniform cross-section A. The electric field E set up inside the conductor is given by

$$E = \frac{V}{l}$$

Under the influence of field E, the free electrons begin to drift in the opposite direction E with an average drift velocity v_d.



[Drift of electrons and electric Field inside a conductor]

Let the number of electrons per unit volume or Electron density = n

Charge on an electron = e

Number of electrons in length l of the conductor

= n × Volume of the conductor = n A *l*

Total charge contained in length l of the conductor, q = n e A l

All the electrons which enter the conductor at the right end will pass through the conductor at the left end in time t

$$t = \text{distance} = \frac{l}{v_d}$$

∴ Current, $I = \frac{q}{t} = \frac{enAl}{l/v_d}$ or $I = enAv_d$

This equation relates the current I with the drift velocity v_d.

Hence, ***Current flowing through a Conductor is directly proportional to the drift velocity.***

The electric bulb glows immediately when switch is on

Explanation: When we close the circuit then E is established in the circuit instantly with the speed of electro – - magnetic wave which carries electrons to drift from any portion of the circuit towards high potential Due to which the current is set up in the entire circuit instantly. The current so set up does not wait for the electrons to flow from one end of the conductor to another end. Due to this bulb glows immediately when switch is on.

** Since $V_d = -\frac{eE\tau}{m}$

$$I = \frac{ne^2 A\tau}{m} . E$$

☑ Electron Mobility (μ):

"The mobility of free electron in a conductor is defined as the drift velocity acquired per unit strength of electric field across the conductor."

In other word,

"Mobility is the ratio of the drift velocity (V_d) of current carrier and the applied electric field (E)."

$$\mu = \frac{V_d}{E} \qquad \therefore \qquad V_d = \mu E$$

or, $\mu = \frac{eE\tau}{Em} = \frac{e}{m}$; $\mu = \frac{e\tau}{m}$

Since, $I = A n e V_d$ (proved)

$$\therefore \qquad I = A n e \mu E$$

➢ **S I Unit of μ = $m^2 V^{-1} S^{-1}$**

☑ Electric current and Current density

"Current density of a conductor is a defined as the current flowing per unit area of the conductor held perpendicular to the flow of current."

i.e., Current density, $J = \frac{\text{Current (I)}}{\text{Area (A)}} = \frac{I}{A}$

➢ Current density is the current through an element of the surface area $d\vec{S}$ of a conductor than $dI = \vec{J} . d\vec{S}$.

∴ total current flowing through the surface S is -------

$$\int dI = \int_S \vec{J} . d\vec{S}$$

$$I = \int_S \vec{J} . d\vec{S}$$

Current is scalar (being dot product of $\vec{J}$ and $d\vec{S}$).

Again, $J = \frac{I}{A}$ $\quad J = \frac{A n e V_d}{A}$ $\quad$ [∵ $I = A n e V_d$]

∴ $J = n e \frac{e\tau}{m} E$ $\quad$ [∵ $V_d = \frac{e\tau}{m} E$]

or, $J = \frac{ne^2 E\tau}{m}$

➢ **S I Unit of current density is A / m².**

☑ OHM' S LAW (given by George Simon Ohm in 1828)

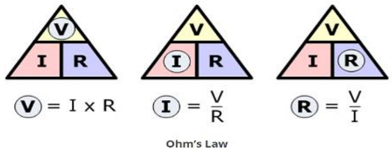
"It states that the current 'I' flowing through a conductor is directly proportional to the p.d 'V' across the end of the conductor provided physical condition of the conductor such as temperature mechanical strain etc are kept constant."

i.e., $I \propto V$

or, $V \propto I$ => $V = IR$,

where, $R = \frac{V}{I}$ = Constant called electrical resistance of simple resistance.

➢ The value of resistance depends upon
--- **the nature of the conductor,**
--- **its dimension,**
--- **the physical condition.**



✪ Derivation of Ohm's law:

Let V_d = drift velocity of electrons through a section of the conductor of its length '*l*' and cross section area (A).

V = P.d across the section of the conductor.

E = Electric field.

∴ Current in the conductor, $I = A n e V_d$

But, $V_d = \frac{e E \tau}{m} = \frac{e V \tau}{m l}$ $\left[\because E = \frac{V}{l} \right]$

∴ $I = A n e \left[\frac{e V \tau}{m l} \right]$

$I = \left[\frac{A n e^2 \tau}{m l} \right] V$ Or, $\frac{V}{I} = \frac{m l}{A n e^2 \tau}$ = **R = a constant for a given conductor.**

$V = IR$ Ohm's Law.

☑ RESISTANCE OF CONDUCTOR:

The resistance of a conductor is the property by virtue of which it opposes the flow of charges through it. The more the resistance, the less is the current I for a given potential difference. It is equal to the ratio of the potential difference applied across the conductor to the current flowing through it. Thus

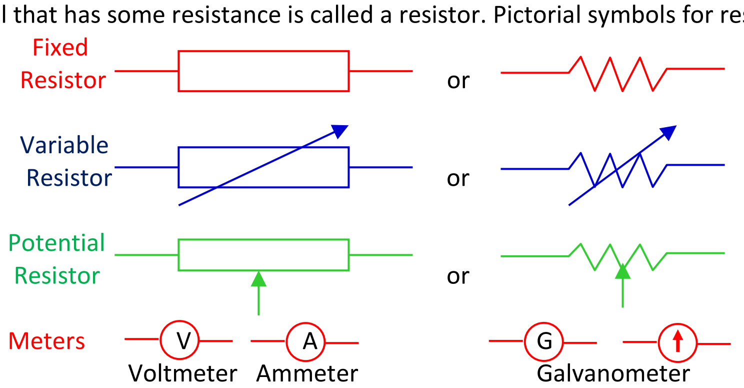
"Resistance of a conductor is defined as the ratio of the P.d (V) applied across the ends of the conductor to the current (I) flowing through it."

i.e., $R = \frac{V}{I}$

➢ **S I Unit -- Ohm** $1\,\Omega = \frac{1\text{ Volt}}{1\text{ Amp}} = V/A.$

☞ ***"Resistance of a conductor is said to be 1 Ω if current of I A flow through it when P.d 1 V is applied across it."***

► Any material that has some resistance is called a resistor. Pictorial symbols for resistors and meters are given below:



[Symbols for resistors and meters]

➢ Dimensional Formula

$$R = \frac{V}{I} = \frac{\text{work}}{\text{charge} \times \text{current}}$$

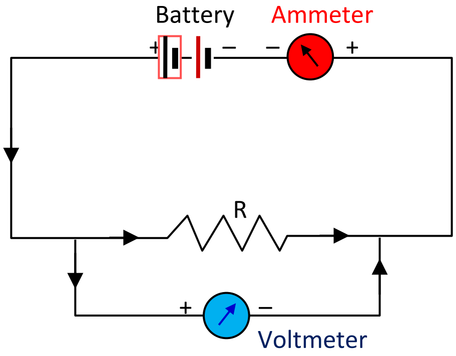
$$= \frac{\text{work}}{(I \times T) \times I} = \frac{M L^2 T^{-2}}{(A \times T) \times A} = \frac{M L^2 T^{-2}}{A^2\ T}$$

$$= [M L^2 T^{-3}\ A^{-2}].$$

- **Cause of Resistance:** --

Resistance of a given conducting wire is due to the collision of free electrons with the ions or atoms of the conductor while drifting towards the positive ends of the conductor which in terns depends upon the arrangement of the atom in the conducting material as well as length & thickness of the wire. The ***opposition offered*** by the ions and atoms is termed as the resistance of the conductor.

Measurement of resistance: Fig. shows a simple circuit for measuring the resistance of a wire. Here the battery and ammeter is connected in series with the wire and the voltmeter in parallel with it. The ratio of the voltmeter reading (V) and the ammeter reading (I) gives the resistance (R) of the wire.



[To measure resistance of wire]

- **Resistivity (or Specific Resistance):**

We know that, $R = \frac{V}{I} = \frac{m\,l}{A\,n\,e^2\tau} = \left[\frac{m}{n\,e^2\,\tau}\right]\frac{l}{A}$

$R = \rho\frac{l}{A}$ Where, $\rho = \frac{m}{n\,e^2\,\tau}$ = constant called specific resistance of the material of the conductor.

Again, ➤ $R \propto l$ and ➤ $R \propto \frac{1}{A}$

Combining these two, $R \propto \frac{l}{A}$ => $R = \rho\frac{l}{A}$ → Electrical Resistivity (or Resistivity)

- If area $A = 1$, $l = 1$, then $R = \rho$

∴ ***"Resistivity of a conductor is the resistance offered by a wire of unit length and unit area of cross-section".***

- **Unit of ρ:** $\rho = R\frac{A}{l} = \frac{\Omega\, m^2}{m} = \Omega - m$

- **INTERNATIONAL OHM:** **"It is defined as the resistance of 106.3 cm long Mercury column of 1mm² cross-section area and mass 14.4521 gm at 0^0C".**

☑ **CONDUCTANCE (G):** **"The inverse of resistance (R) is called Conductance of conductor".**

- **Unit:** Ω^{-1} **or 'mho' or 'seimen'**

- **ELECTRICAL Conductivity (σ) :** Conductivity of a substance is the inverse of its Resistivity . i.e.,

$$\sigma = \frac{1}{\rho}$$

- **Unit:** $\Omega^{-1}m^{-1}$ **or 'mho m^{-1}' or 'seimen m^{-1}'**

- **Dimensional Formula of** $G = \frac{1}{R} = [M^{-1} L^{-2} T^{3} A^{2}]$

- **Dimensional Formula of** $\sigma = \frac{1}{\rho} = [M^{-1} L^{-3} T^{3} A^{2}]$

✪ **Relation between J (current density), Conductivity (σ) and E**

We know that $I = n\,A\,e\,V_d$

$$= A\,n\,e\left[\frac{e\,E}{m}\right]\tau$$

$$= \frac{A\,n\,e^2\,\tau\,E}{m}$$

$$\frac{I}{A} = \frac{n e^2 \tau E}{m}$$

$$J = \frac{1}{\rho} E \qquad \left(\text{since, } \rho = \frac{m}{n e^2 \tau} \right)$$

∴ $J = \sigma E$ [since, $\sigma = 1/\rho$]

☑ Factors Affecting electrical Resistivity

We know that, $\rho = \dfrac{m}{n e^2 \tau}$

1.) i.e. ρ is ***inversely proportional*** to the ***no. of free electrons per unit volume (n)*** of the conductor.

☞ Since value of n depends upon the nature of the material therefore, ρ of a conductor depends upon the nature of the material.

2.) Again, ρ is inversely proportional to the average relaxation time (τ) of the free electrons in the conductor.

☞ Now, the value of τ decreases with increase in temperature of the conductor. Also, $\rho \propto \dfrac{1}{\tau}$

∴ *The Resistivity of conductor depends upon its temperature and it increases with increase in temperature of conductor.* (➢ **Mass of electron (m) and charge of conductor are constant.)**

➢ Variation of Resistivity with temperature and temperature coefficient of resistance:

As 'n' dose not change with temperature, so variation in ρ with temperature depends only on the relaxation time (τ). when the temperature of the conductor increases, the amplitude of vibration of ions increases and hence the collision between electron and ions become more frequent. Therefore, the opposition to the flow of electron increases. Thus, Resistance of the metallic conductor increases and decreases with the increase or decrease of the temperature respectively.

Let R_1 = Resistance of the conductor at $t_1{}^0C$.

R_2 = Resistance of the conductor at $t_2{}^0C$. $(t_2 > t_1)$

∴ Increase in resistance = $R_2 - R_1$

Now, $R_2 - R_1 \propto R$ and $R_2 - R_1 \propto (t_2 - t_1)$

Combining the above,

$R_2 - R_1 \propto R_1 (t_2 - t_1)$

$R_2 - R_1 = \alpha R_1 (t_2 - t_1)$ -------- (i)

Also,

$$\alpha = \frac{R_2 - R_1}{R_1 (t_2 - t_1)} \quad \text{from (i)}$$

$$\alpha = \frac{\text{Increase in resistance}}{\text{Original resistance} \times \text{Increase in temperature}}.$$

∴ ***Temperature coefficient of resistance (α) is defined as the change in resistance per unit original resistance per degree rise in temperature.***

➢ **UNIT of α** = $\dfrac{\text{Ohm}}{\text{Ohm Kelvin}}$ = Kelvin^{-1}

➢ **α** is positive for metallic conductor as $R_2 > R_1$ i.e., their resistance increases with the rise in temperature.

➢ **α** is negative for insulator. **α** is very small for high Resistivity alloys like manganin (≃ $10^{-5}\ {}^0C^{-1}$) their resistance dose not change appreciably with change in temperature -- Manganin [{84% Cu, 12% Mn, 4% Ni} constant 60% Cu, 40% Ni] are used standard resistance.

➢ Variation of Resistivity with temperature

Resistivity of a Material, $\rho = \dfrac{m}{n e^2 \tau}$

∴ $\rho \propto \dfrac{1}{n \tau}$ [as m and e are constant]

i.e., ρ is related to -- 1] **no. density 'n' of free electrons in the material**; 2] **Relaxation time 'τ'**.

☞ In most of the metal 'n' does not change with temperature but an increase in temperature increases the amplitude of vibration of ions of the metal. As a result of it, the collision of free electrons with ions / atoms while drifting towards the positive end resulting decrease the relaxation time and hence Resistivity increases as $\rho \propto \dfrac{1}{\tau}$.

Thus, temp^r dependence of Resistivity is given by

$$\rho = \rho_0 [1 + \alpha_r (T - T_0)]$$

Where, ρ = Resistivity at temp^r T.
ρ_0 = Resistivity at temp^r to (273 K or 0^0C). α_r = Temperature coefficient of resistance.

∴ Temperature coefficient $\alpha_r = \dfrac{\rho - \rho_0}{\rho_0 (T - T_0)}$

Thus, ***"Temperature Coefficient is defined as change in Resistivity per unit original Resistivity per unit original change in temperature".***

- **For Conductors:** α_r is **positive** i.e., ρ increases with increase in temperature.
- **For Semi- Conductors: α_r is negative i.e., ρ decreases with increase in temperature**
- **For Insulators: ρ increases exponentially with decrease in temperature. It becomes infinitely large at temperature near absolute zero, i.e., the conductivity is almost zero at 0 Kelvin.**

- Temperature dependence of Resistivity of semi-conductors and insulators is $\rho = \rho_0 e^{E_g / 2KT}$
  where, E_g = Energy gap between C B and VB (or activation energy for Conduction).
  K = Boltzman's Constant = 1.381×10^{-23} J/mol / K
  T = absolute temperature.
- **For $E_g \approx$ 1eV, the material is semi – conductor (ρ is not very high)**
- **For $E_g \approx$ 1eV, the material is Insulators (ρ is very high)**

☑ **Non-Ohmic Conductor**: **Conductors** which do not obeys Ohm's law are called non-ohmic conductors.

- Relation V / I = R holds good for both ohmic and non-ohmic conductors.
  ----- For Conductors, R = Constant
  ----- For non- ohmic Conductors, R = Constant (dynamic resistance)

Ex: (i) Rectifier (converts AC into DC) Semi conductor diode, Vacuum tube, Liquid electrolyte, transistors etc

- Resistance of a material depends upon the nature of the material. If current flowing through conductor is increased, the conductor becomes hot and temperature does not remain same as per required condition of ohm's law.

☑ **THERMISTOR**: **"A thermistor is a heat sensitive resistor usually made of a semi –conducting material and Resistivity changes very rapidly with change of temperature".**

- The temperature coefficient of a thermistor is very high.
- The resistance of a thermistor changes very rapidly with change in temperature,
- The temperature coefficient of a thermistor can be both positive or negative.

✪ **TYPES OF THEMISTOR**: [1] ***With negative temperature coefficient of Resistivity*** i.e., Resistivity of this type of thermistor decreases with rise in temperature. Such a thermistor has wide range of application.
[2] ***With positive temperature coefficient of Resistivity*** i.e., Resistivity of this type of thermistor increases with rise in temperature.

- Thermistor are prepared from oxides of metal like Nickel, Iron, copper, cobalt. Thermistor are very small size and are generally, in the form of beads, disc, rods etc. A pair of platinum leads are attached at the two ends for electric connection This arrangement sealed in a bulb.
- The thermistor can be used over a wide range or temperature i.e., from very low temperature to 1100^0C.
- A thermistor can have a resistance in the range of 0.1 **Ω** to 10^7 **Ω** depending upon its composition
- The change in the value of a thermistor can be either due to change in temperature of its surrounding or by passing current.

✪ **APPLICATION: 1]**. Thermistor are used to detect small temperature changes and to measure very small temperature (≈ 10K)

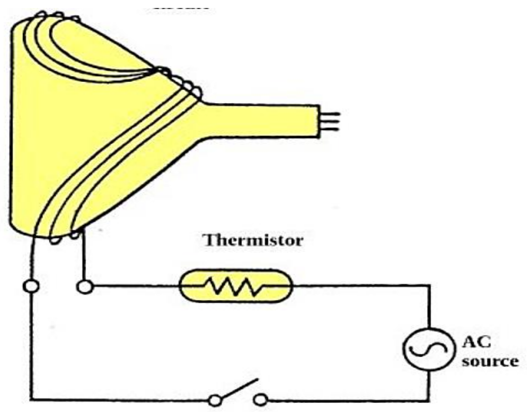
[2]. Thermistor are used in temperature control units of industry.

[3]. Thermistor are used in the protective circuit of electrical equipment like transformer, motor & generator.

[4]. ***Thermistor are used to safe guard the filament of the picture – tube of TV against the variation of current.***

Explanation: *Thermistor with negative temperature coefficient is used to avoid the burning of the filament of a TV. A thermistor (T) is put in series with the filament of the TV tube. When a supply voltage is switched on, the thermistor has a high resistance in the initial stage because it is cold. Therefore, it controls the current to moderate value. When the thermistor gets heated up , its resistance decreases considerably and hence normal current flows through the filament of the picture tube.*

[5] Thermistor is used for voltage stabilization.



- A **thermistor** is a resistor whose value of the resistance alters with the change of the temperature.
- The word thermistor is a combination of the two words first is thermal that means heat and second is resistance.
- In electrical circuits, the thermistors are used to stop the inrush current from the circuitries, as they are also sensors with the variation of the resistance they indicate about current.
- There are the 2 main categories of the thermistors first one is the NTC (negative temperature coefficient) and second is PTC (positive temperature coefficient).

☑ **SUPERCONDCTIVITY: (Discovered by *K. ONNES* in 1911)**

"The property by virtue of which a metal, alloy, oxides etc shows almost zero resistance when cooled its critical temperature (or transition temperature) is called Superconductivity".

➢➢ The substance which shows the property of superconductivity is called superconductors.

➢➢ The temperature below which a material becomes superconductor is called ***transition temperature or Criticaltemperature*** (T_C).

➢➢ Resistance of metallic conductor decreases with decrease in temperature, but when the temperature reaches T_C, the resistance of the material completely disappears and than there will be flow of electron without resistance.

✪✪ The critical temperature for different material is different: [a] Mercury = 4.2 K; [b] Lead = 7.25 K [c] Aluminium = 0.18 K [d] Nirobium = 9.2 K

✪ Alloy of platinum, cobalt and Gallium = 18. 5 K.

➢➢ **Cause of Superconductivity:** The cause of superconductivity is that electronS in superconductors are no longer independent but become mutually dependent. When critical temperature reaches. The ionic vibration which could deflects electrons in a metal are unable to defect the electron around and therefore this coherent cloud of electron makes no collision with ions of the superconductor and as such there is no resistance.

✪ **Application: [1]** used for making very strong electromagnet. [2] Used to produce very high-speed computers. [3] Super conducting electric power transmission for long distance with no power loss. [4] For high level research purpose.

CLASSIFICATION OF MATERIALS IN TERMS OF RESISTIVITY

The electrical resistivity of substances varies over a vary wide range, as shown in Table. Various substances can be classified into three categories:

1. Conductors: The materials which conduct electric current fairly well are called conductors. Metals are good conductors. They have low resistivities in the range of 10^{-8} Ω m to 10^{-6} Ω m. Copper and aluminium have the lowest resistivities of all the metals, so their wires are used for transporting electric current over large distances without the appreciable loss of energy. On the other hand nichrome has a resistivity of about 60 times that of copper. It is used in the elements of electric heater and electric iron.

2. Insulators: The materials which do not conductor electric current are called insulators. They have high resistivity, more than 10^4 Ω m. Insulators like glass, mica, bekelite and hard rubber have very high resistivities in the range 10^{14} Ω m to 10^6 Ω m. So they are used for blocking electric current between two points.

3. Semiconductors: These are the materials whose resistivities lie in between those of conductors and insulators i.e., between 10^{-6} Ω m to 10^4 Ω m. Germanium and silicon are typical semiconductors. For moderately high resistance in the range of k Ω, resistors made of carbon (graphite) or some semiconducting materials are used.

Table: Electrical resistivities of some substances

Material	Resitivity at 0°C ρ (Ω m)	Temperature coefficient of resistivity at 0°C, $\alpha = 1/\rho\ [d\rho/dT]$ (°C^{-1})	Number of valence electrons per unit cell
A. Conductors			
Silver	1.6×10^{-8}	0.0041	1
Copper	1.7×10^{-8}	0.0068	1
Aluminium	2.7×10^{-8}	0.0043	3
Tungsten	5.6×10^{-8}	0.0045	6
Iron	10×10^{-8}	0.0065	8
Platinum	11×10^{-8}	0.0039	10
Mercury	98×10^{-8}	0.0009	2
Nichrome (Alloy of NI, Fe, Cr)	100×10^{-8}	0.0004	
Manganin (alloy of Cu, Ni, Fe, Mn)	48×10^{-8}	0.002×10^{-3}	
B. Semiconductors			
Carbon (graphite)	3.5×10^{-5}	– 0.0005	4
Germanium	0.46	– 0.05	4
Silicon	2300	– 0.07	4
C. Insulators			
Pure water	2.5×10^{5}		—
Glass	$10^{10} - 10^{14}$		—
Hard Rubber	$10^{13} - 10^{16}$		—
NaCl	$\sim 10^{14}$		8
Fused quartz	$\sim 10^{16}$		—

►**Common commercial resistors**: The commercial resistors are of two major types:

1. Wire-bound resistors: These are made by winding the wires of an alloy like manganin, constantan or nichrome on an insulating base. The advantage of using these alloys is that they are relatively insensitive to temperature. But inconveniently large length is required for taking a high resistance.

2. Carbon resistors: They are made from mixture of carbon black, clay and resin binder which are pressed and then moulded into cylinder rods by heating. The rods are enclosed in a ceramic or plastic jacket.

The carbon resistors are widely used in electronic circuits of radio receivers, amplifiers, etc. They have the following advantages:

(i) They can be made with resistance values ranging from few ohms to several million ohms.
(ii) They are quite cheap and compact.
(iii) They are good enough for many purposes.

☑ Colour code for CARBON RESISTORS:

Resistor is a component of an electrical circuit offering certain opposition to the flow of current in that circuit.

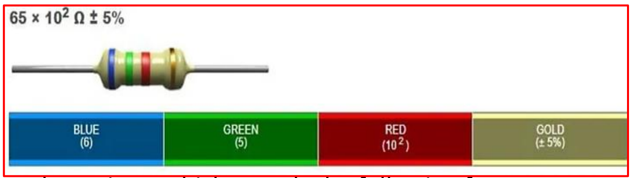
➢ **Value of CARBON RESISTOR:** The value of the resistance and their percentage of accuracy are indicated on carbon resistor or code printed on them. These colour band or code can be translated into a number by using the standard colour code.

Colour code for resistors: A colour code is used to indicate the resistance value of a carbon resistor and its percentage accuracy. The colour code used throughout the world is shown in Table.

Colour	Letter as an aid to memory	Number	Multiplier	Colour	Tolerance
Black	B	0	10^0	Gold	5%
Brown	B	1	10^1	Silver	10%
Red	R	2	10^2	No fourth band	20%
Orange	O	3	10^3		
Yellow	Y	4	10^4		
Green	G	5	10^5		
Blue	B	6	10^6		
Violet	V	7	10^7		
Grey	G	8	10^8		
White	W	9	10^9		

►**How to remember colour code:** ....

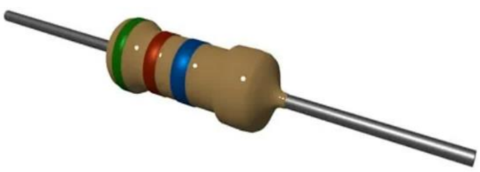
B B R O Y of Great Britain had Very Good Wife
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
0 1 2 3 4 5 6 7 8 9

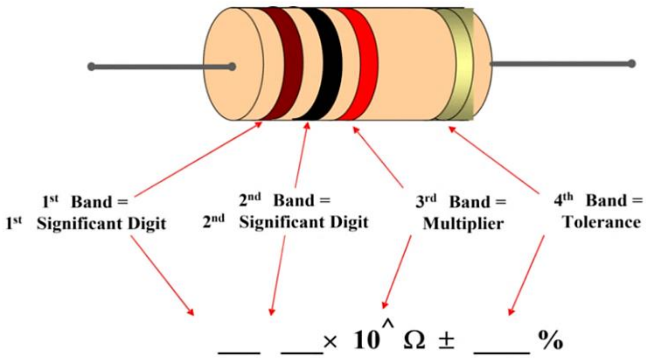


There are two systems of marking the colour codes:

☑ **First system:** A set of coloured co-axial rings or bands is printed on the resistor which reveals the following facts:

1. The first band indicates the first significant figure.
2. The second band indicates the second significant figure.
3. The third band indicates the power of ten with which the above two significant figures must be multiplied to get the resistance value in ohms.
4. The fourth band indicates the tolerance or possible variation in percent of the indicated value. If the fourth band is absent, it implies a tolerance of ± 20 %.

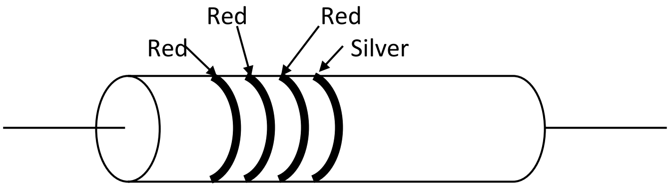




Illustrations: 1. In Fig. the colours of the four bands are red, red, red and silver; the resistance value is

Red Red Red Silver
↓ ↓ ↓ ↓
2 2 2 ± 10%

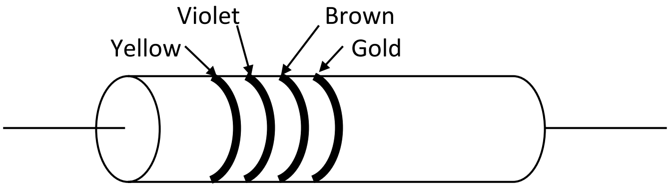
R = 22 × 10^2 Ω ± 10 %.



2. In Fig. the colours of the four bonds are yellow, violet, brown and gold; the resistance value is

Yellow Violet Brown Gold
↓ ↓ ↓ ↓
4 7 1 ± 5%

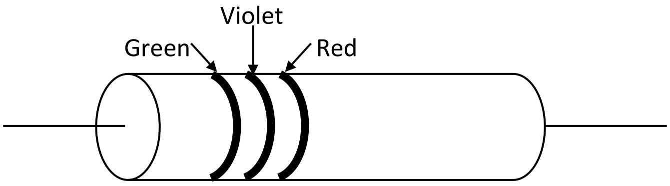
R = 47 × 10^1 Ω ± 5 %.



2. When there are only three coloured bands printed on a resistor and there is no gold or silver band, the tolerance is 20%. In fig. there are only three bands of green, violet and red colours; the resistance value is

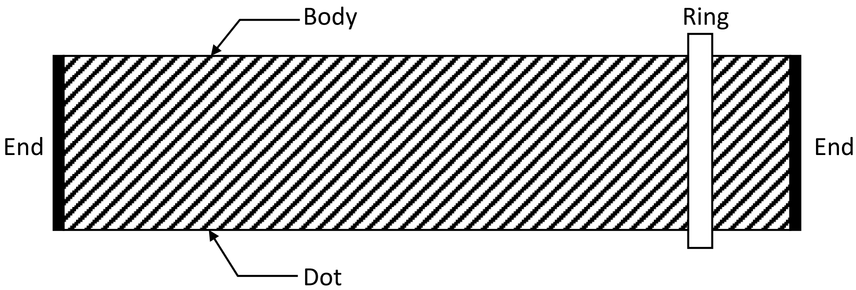
Green	Violet	Red	No fourth band
↓	↓	↓	↓
5	7	2	± 20%

$R = 57 \times 10^2\ \Omega \pm 20\%$.



☑ **Second system:**

1. The colour of the body gives the first significant figure.



2. The colour of the end gives the second significant figure.
3. The colour of the dot gives the number of zeroes to be placed after the second figure.
4. The colour of the ring gives the tolerance or percent accuracy of the indicated value.

Illustration: Suppose for a given resistor, the body colour is yellow, end colour is violet, dot colour is orange and the ring colour is silver.

Body	End	Dot	Ring
Yellow	Violet	Orange	Silver
↓	↓	↓	↓
4	7	3	± 10 %

∴ $R = 47 \times 10^3\ \Omega \pm 10\ \% = 47\ k\Omega \pm 10\ \%$

➢➢➢➢ Memory Aids

✪ B **B** **R** **O** **Y** has **V**ery **G**ood **W**ife **W**earing **G**old **S**ilver **N**ecklace.

✪ B **B** **R** **O** **Y** **G**reat **B**ritain **V**ery Good Wife.

✪ **Black** **Br**own **Red** of **Your** **Gate** **Became** **Very** **Good** **When** **Given** **Silver Colour.**

✪ Black Brown Roy Got Blue Van & Good Wife wearing Gold Silver Necklace.

Ex – Suppose the colour band sequence of a resistor is given ., brown , yellow & gold. Then what is the effective resistance of the resistors as per colour code solution here code is green – yellow – gold .

Resistance $= 51 \times 10^4 + 5\ \%$

$= 51 \times 10^4 \pm (51 \times 10^4 \times \frac{5}{100})$

$= 51 \times 10^4 \pm 2.55 \times 10^4$

$= (51 \pm 2.55) \times 10^4\ \Omega.$

Examples based on Ohm's law, Resistance, Resistivity, Conductance, Conductivity, Current Density and Colour Code of Carbon Resistance

- **Formulae Used**

1. Ohm's law, $R = \frac{V}{I}$ or $V = IR$
2. Resistance of a uniform conductor, $R = \rho \frac{l}{A}$
3. Resistivity of specific resistance, $\rho = \frac{RA}{l}$
4. Conductance = $\frac{1}{R}$
5. Conductivity = $\frac{1}{\text{Resistivity}}$ or $\sigma = \frac{1}{\rho} = \frac{l}{RA}$
6. Current density = $\frac{\text{Current}}{\text{Area}}$ or $j = \frac{I}{A}$

- **Units Used**

Potentials difference V is in volt (V), current I in ampere (A), resistance R in ohm (Ω), resistivity ρ in Ωm, conductance in ohm^{-1} or mho or siemens (S), conductivity in $\Omega^{-1}\,m^{-1}$ or Sm^{-1} and current density j in Am^{-2}.

Q. 1. ***In a discharge tube, the number of hydrogen ions (i.e., protons) drifting across a cross-section per second is 1.0×10^{18}, while the number of electrons drifting in the opposite direction across another cross-section is 2.7×10^{18} per second. If the supply voltage is 230 V, what is the effective resistance of the tube?***

Sol. The current carried by a negatively charged electrons is equivalent to the current carried by a proton in the opposite direction, therefore, total current in the direction of protons is

I = Total charge flowing per second = $(n_e + n_p)\, e$

$= [2.7 \times 10^{18} + 1.0 \times 10^{18}] \times 1.6 \times 10^{-19}$

$= 3.7 \times 1.6 \times 10^{-1} = 0.592$ A

Effective resistance,

$$R = \frac{V}{I} = \frac{230}{0.592}\,\Omega = 388.5\,\Omega = 3.9 \times 10^2\,\Omega$$

Q. 2. ***An electrons beam has an aperture of $1.0\ mm^2$. A total of 6×10^{16} electrons flow through any perpendicular cross-section per second. Calculate (i) the current and (ii) the current density in the electron beam.***

Sol. Here $A = 1.0\ mm^2 = 10^{-6}\ m^2$,

$n = 6 \times 10^{16}$, $t = 1$ s

(i) $I = \frac{q}{t} = \frac{ne}{t} = \frac{6 \times 10^{16} \times 1.6 \times 10^{-19}}{1}$

$= 9.6 \times 10^{-3}$ A

(ii) Current density, $j = \frac{I}{A} = \frac{9.6 \times 10^{-3}}{10^{-6}}$

$= 9.6 \times 10^{3}\ Am^{-2}$

Q. 3. ***A copper wire of radius 0.1 mm and resistance 1 k Ω is connected across a power supply of 20 V. (i) How many electrons are transferred per second between the supply and the wire at one end? (ii) Write down the current density in the wire.***

Sol. Here $r = 0.1\ mm = 0.1 \times 10^{-3}$ m, $R = 1\ k\Omega = 10^3\ \Omega$, $V = 20$ V

(i) Current, $I = \frac{V}{R} = \frac{20}{10^3} = 0.02$ A

No. of electrons, $n = \frac{q}{e} = \frac{It}{e}$

$= \frac{0.02 \times 1}{1.6 \times 10^{-19}} = 1.25 \times 10^{17}$

(ii) Current density,

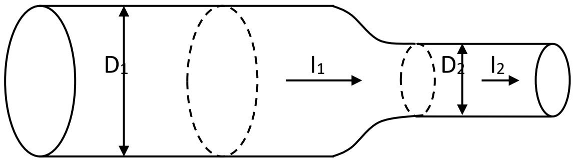
$$j = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{0.02}{3.14 \times (0.1 \times 10^{-3})^2}$$

$= 6.37 \times 10^{5}\ Am^{-2}$

Q. 4. ***Current flows through a constricted conductor, as shown in Fig. The diameter $D_1 = 2.0$ mm and the current density to the left of the constriction is $j = 1.27 \times 10^6\ Am^{-2}$. (i) What current flows into the constriction? (ii) If the current density is doubled as it emerges from the right side of the construction, what is diameter D_2?***

Sol. Here $D_1 = 2.0$ mm, $j_1 = 1.27 \times 10^6\ Am^{-2}$, $j_2 = 2\, j_1$

(i) Current flowing into the construction,



$I_1 = j_1 A = j_1 \times \pi\left(\frac{D_1}{2}\right)^2$

$= 1.27 \times 10^6 \times 3.14 \times (1 \times 10^{-3})^2$

$= 3.987$ A

(ii) For a steady flow of current,

$I_1 = I_2$ or $j_1 A_1 = j_2 \times A_2$

or $j_1 \times \pi\left(\frac{D_1}{2}\right)^2 = j_2 \times \pi\left(\frac{D_2}{2}\right)^2$

or $j_1 \times \pi\left(\frac{D_1}{2}\right)^2 = 2j_1 \times \pi\left(\frac{D_2}{2}\right)^2$ $[\because j_2 = 2 j_1]$

or $D_2 = \frac{1}{\sqrt{2}} D_1 = 0.707 D_1$

$= 0.707 \times 2.0$ mm $= 1.414$ mm.

Q. 5. ***A uniform wire is cut into 10 segments increasing in length in equal steps, the resistance of the shortest segment is R and the resistances of the other segments increase in steps of 8 Ω. If the resistance of the longest segment is 2 R, find the value of R and hence find the resistance of the original wire.***

Sol. Resistance of first or shortest segment = R Ω

Resistance of second segment = R + 8 = R + 8 × 1 Ω

Resistance of third segment = R + 8 × 9 = R + 72 Ω

But resistance of longest segment = 2 R

$\therefore$ 2R = R + 72 or R = 72 Ω

Resistance of the original wire

= R + (r + 8) + (R + 16) + … + (R + 72)

= 10 R + 8 (1 + 2 + 3 + … + 9) = 10 R + 8 × 45 = 10 × 72 + 8 × 45 = 1080 Ω

Q. 6. ***A current of 2 mA is passed through a colour coded carbon resistor with first, second and third rings of yellow, green and orange colours. What is the voltage drop across the resistor?***

Sol.

Yellow	Green	Orange
↓	↓	↓
4	5	3

$\therefore$ $R = 45 \times 10^3\ \Omega$

Given $I = 2$ mA $= 2 \times 10^{-3}$ A

$\therefore$ $V = RI = 45 \times 10^3 \times 2 \times 10^{-3}$ V $= 90$ V

Q. 7. ***An arc lamp operates at 80 V, 10 A. Suggest a method to use it with a 240 V d.c. source. Calculate the value of the electric component required for this purpose.***

Sol. Resistance of the arc lamp is

$R = \frac{V}{I} = \frac{80}{10} = 8\ \Omega$

In order to use arc lamp with a source of 240 V, a resistance R' should be connected in series with it so that current through the circuit does not exceed 10 A. Then

$I(R + R') = V$ or $10(8 + R') = 240$

or $R' = 24 - 8 = 16\ \Omega$

Q. 8. ***Calculate the resistivity of a material of a wire 10 m long, 0.4 m in diameter and having a resistance of 2.0 Ω.***

Sol. Here $l = 10$ m, $r = 0.2$ mm $= 0.2 \times 10^{-3}$ m, $R = 2\ \Omega$

Resistivity, $\rho = \frac{RA}{l} = \frac{R \times \pi r^2}{l}$

$= \frac{2 \times 3.14 \times (0.2 \times 10^{-3})^2}{10}$

$= 2.513 \times 10^{-8}\ \Omega$m

Q. 9. ***The external diameter of a 5-metre-long hollow tube is 10 cm and the thickness of its wall is 5 mm. If the specific resistance of copper be 1.7 × 10⁻⁵ ohm-metre, then determine its resistance.***

Sol. The cross-sectional area of the tube is

$A = \pi(r_2^2 - r_1^2)$

$= 3.14 \times [(5 \times 10^{-2})^2 - (4.5 \times 10^{-2})^2] = 14.9 \times 10^{-4}\ m^2$

Also, $\rho = 1.7 \times 10^{-8}\ \Omega$m, $l = 5$ m

$\therefore$ Resistance, $R = \rho \frac{l}{A} = \frac{1.7 \times 10^{-8} \times 5}{14.9 \times 10^{-4}} = 5.7 \times 10^{-5}\ \Omega$

Q. 10. ***Find the resistivity of a conductor in which a current density of 2.5 Am⁻² is found to exist, when an electric field of 15 Vm⁻¹ is applied on it.***

Sol. Here $j = 2.5\ Am^{-2}$, $E = 15\ Vm^{-1}$

Resistivity, $\rho = \frac{RA}{\rho} = \frac{V}{I} \cdot \frac{A}{l}$

$= \frac{V/I}{I/A} = \frac{E}{j} = \frac{15}{2.5} = 6\ \Omega\,m$

Q. 11. ***Calculate the electrical conductivity of the material of a conductor of length 3 m, area of cross-section 0.02 mm² having a resistance of 2 Ω.***

Sol. Here $l = 3$ m, $R = 2\ \Omega$, $A = 0.02$ mm² 0.02×10^{-6} m²

Electrical conductivity $= \frac{1}{\text{Resistivity}}$

or $\sigma = \frac{1}{\rho} = \frac{l}{RA} = \frac{3}{2 \times 0.02 \times 10^{-6}}$ $\left(R = \rho \cdot \frac{l}{A}\right)$

$= 75 \times 10^{6}\ \Omega^{-1}\ m^{-1}$

Q. 12. ***A wire of resistance of 4 Ω is used to wind a coil of radius 7 cm. The wire has a diameter of 1.4 mm and the specific resistance of its material is 2 × 10⁻⁷ Ωm. Find the number of turns in the coil.***

Sol. Let n be the number of turns in the coil. Then total length of wire used

$= 2\pi R \times n = 2\pi \times 7 \times 10^{-2} \times n$ metre

Total resistance, $R = \rho \frac{l}{A}$

or $4 = \frac{2 \times 10^{-7} \times 2\pi \times 7 \times 10^{-2} \times n}{\pi (0.7 \times 10^{-3})^2}$

∴ $n = 70$

Q. 13. ***A wire of 10-ohm resistance is stretched to trice its original length. What will be its (i) new resistivity, and (ii) new resistance?***

Sol. (i) Resistivity ρ remains unchanged because it is the property of the material of the wire.

(ii) In both case, volume of wire is same. So

$V = A' l' = Al$

or $\frac{A'}{A} = \frac{l}{l'} = \frac{l}{½ l} = \frac{2}{1}$

∴ $\frac{R'}{R} = \frac{\rho l'/A'}{\rho l/A} = \frac{l'}{l} \times \frac{A}{A'} = ½ \times ½ = ¼$

or $R' = ¼ R = ¼ \times 16 = 4\ \Omega$

Change in resistance

$= \frac{R - R'}{R} \times 100 = \frac{12}{16} \times 100 = 75\%$

Q. 14. ***The resistance of a wire is R ohm. What will be its new resistance if it is stretched to n times its original length?***

Sol. In both case, volume of the wire is same.

∴ $V = Al = A' l'$

or $\frac{A}{A'} = \frac{l'}{l} = n$ [∵ $l' = nl$]

∴ $\frac{R'}{R} = \frac{\rho l'/A'}{\rho l/A} = \frac{l'}{l} \cdot \frac{A}{A'} = n \cdot n = n^2$

or $R' = n^2 R$.

Q. 15. ***A cylindrical wire is stretched to increase its length by 10 %. Calculate the percentage increase in resistance.***

Sol. New length, $l' = l + 10\%$ of l

$= l + 0.1\, l = 1.1\, l$

or $\frac{l'}{l} = 1.1$

$Al = A' l'$

or $\frac{A}{A'} = \frac{l'}{l}$

∴ $\frac{R'}{R} = \frac{l'}{l} \times \frac{A}{A'} = \left(\frac{l'}{A'}\right)^2 = (1.1)^2 = 1.21$

The percentage increase in resistance,

$\frac{R' - R}{R} \times 100 = \left(\frac{R'}{R} - 1\right) \times 100$

$= (1.21 - 1) \times 100 = 21\%$

Q. 16. ***A piece of silver has a resistance of 1 Ω. What will be the resistance of a constantan wire of one-third length and one-half diameter, if the specific resistance of constantan is 30 times that of silver?***

Sol. For silver,

$R = \frac{4\rho l}{\pi d^2} = 10\ \Omega$

For constantan,

$R' = \frac{4\rho' l'}{\pi d'^2} = \frac{4 \times 30\rho \times l/3}{\pi\left(\frac{d}{2}\right)^2}$

$= \frac{40 \times 4\rho l}{\pi d^2} = 40R = 40 \times 1 = 40\ \Omega$

Q. 17. ***On applying the same potential difference between the ends of wires of iron and copper of the same length, the same current flows in them. Compare their radii. Specific resistance of iron and copper and respectively 1.0×10^{-7} and 1.6×10^{-8} Ωm. Can their current-densities be made equal by taking appropriate radii?***

Sol. On applying same potential difference, same current flows in the two wires. Hence the resistance of the two wires should be equal.

But $R = \rho \frac{l}{A} = \rho \frac{l}{\pi r^2}$

For the two wires of same length l, we have

$R_1 = \rho_1 \frac{l}{\pi r_1^2}$

$R_2 = \rho_2 \frac{l}{\pi r_2^2}$

As $R_1 = R_2$

∴ $\frac{\rho_1}{r_1^2} = \frac{\rho_2}{r_2^2}$

$\frac{r_1}{r_2} = \sqrt{\frac{\rho_1}{\rho_2}}$

∴ $\frac{r_{iron}}{r_{copper}} = \sqrt{\frac{\rho_{iron}}{\rho_{copper}}}$

$= \sqrt{\frac{1.0 \times 10^{-7}}{1.6 \times 10^{-8}}} = 2.5$

No, current densities cannot be equal because they depend on nature of the metals.

☑ Grouping of RESISTORS:

The resistors may be connected in three ways --------

----1] In Series, **----2] In parallel,** **----3] In mixed grouping.**

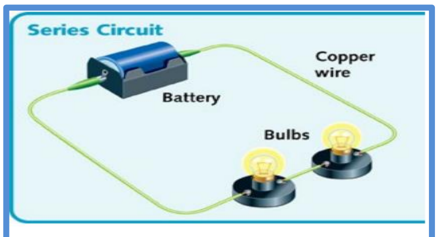
➢➢ EQUIVALENT RESISTORS

A single resistor which draws the same current as the given combination of resistor (in series or parallel) when the same p.d is applied across its end points is called equivalent resistor (or effective resistor or net resistors or total resistor).

✪ 1] SERIES COMBINAITON OF RESISTORS:

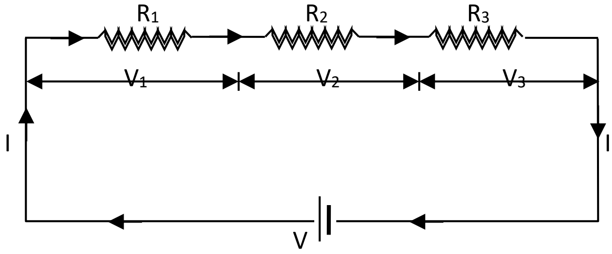
Resistors are said to be connected in series between two points if they provide only a single path between two points.

"Resistors are said to be connected in series if the same current is flowing through each resistor when same p.d is applied across the combination."



Series circuit

Resistance in series: If a number of resistances are connected end to end so that the same current flows through each one of them in succession, then they are said to be connected in series. Fig. shows three resistances R_1, R_2 and R_3 connected in series. When a potential difference V is applied across the combination, the same current I flows through each resistance.



[Resistance in series]

By ohm's law, the potential drops across the three resistances are

$V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3$

If R_S is the equivalent resistance of the series combination, then we must have

$V = IR_S$

But V = Sum of the potential drops across the individual resistance

or $V = V_1 + V_2 + V_3$

or $IR_S = IR_1 + IR_2 + IR_3$

or $R_S = R_1 + R_2 + R_3$

The equivalent resistance of n resistances connected in series will be

$R_S = R_1 + R_2 + R_3 + ... + R_n$

Thus, when a number of resistance of resistances are connected in series, their equivalent resistance is equal to the sum of the individual resistances.

♦ Laws of resistances in series:

(i) Current through each resistance is same.
(ii) Total potential drop = Sum of the potential drops across the individual resistances.
(iii) Individual potential drops are directly proportional to individual resistances.
(iv) Equivalent resistance = Sum of the individual resistances.
(v) Equivalent resistance is larger than the largest individual resistances.

➢ If there are 'n' resistors in series, then

$$R_s = R_1 + R_2 + R_3 + \text{--------} + R_n.$$

or,

$$R_s = \sum_{i=1}^{n} R_i$$

➢ *When the resistors are connected in series the equivalent resistance of the series (R_S) is equal to sum of the individual resistance.*

➢ *Value of equivalent Resistance in series is always greater than the individual resistance.*

➢ *The current in every resistor is same.*

➢ *The current in circuit is independent an relative position of the various resistors in the series.*

✪ **2] RESISTANCE IN PARALLEL: --** ***"Two or more resistors are said to be connected in parallel, if p.d, across each of them is equal to the applied p.d."***

Expression: If a number of resistances are connected in between two common points so that each of them provides a separate path for current, then they are said to be connected in parallel. Fig. shows three resistance R_1, R_2 and R_3 connected in parallel between points A and B. Let V be the potential difference applied across the combination.

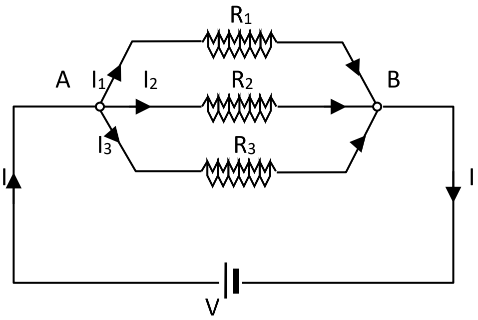
Let I_1, I_2 and I_3 be the currents through the resistances R_1, R_2 and R_3 respectively. Then the current in the main circuit must be

$I = I_1 + I_2 + I_3$

Since all the resistances have been connected between the same two points A and B, therefore, potential drop V is same across each of them. By Ohm's law, the currents through the individual resistance will be

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}$$

If R_p is the equivalent resistance of the parallel combination, then we must have



[Resistance in parallel]

$$I = \frac{V}{R_p}$$

But $I = I_1 + I_2 + I_3$

or $\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$ or $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

The equivalent resistance R_p of n resistances connected in parallel is given by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$

Thus, when a number of resistances are connected in parallel, the reciprocal of the equivalent resistance of the parallel combination is equal to the sum of the reciprocal of the individual resistances.

Laws of resistances in parallel:

(i) Potential drop across each resistance is same.
(ii) Total current = sum of the currents through individual resistances.
(iii) Individual currents are inversely proportional to the individual resistances.
(iv) Reciprocal of equivalent resistance = sum of the reciprocals of the individual resistances.
(v) Equivalent resistance is less than the smallest individual resistance.

- If there are 'n' resistors in parallel, then

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \text{---------} + \frac{1}{R_n}$$

i.e.,

$$\frac{1}{R_p} = \sum_{i=1}^{n} \frac{1}{R_i}$$

- *When resistors are connected in parallel, reciprocal of equivalent resistance of the parallel combination is equal to the sum of the reciprocal of individual resistance.*
- *In parallel combination, R_p is less than the least individual resistance.*
- *P.d. across resistor is the same and is equal to applied p.d.*
- *Total current through parallel combination is the sum of the individual current.*
- *The current through each resistor is inversely proportional to the resistance of resistor.*

If the effective resistance in the circuit is to be increased ➲ resistors are connected in series.

If the effective resistance in the circuit is to be decreased ➲resistors are connected in parallel.

Examples based on Combination of Resistance in Series and Parallel

- **Formulae Used**

1. The equivalent resistance R_S of a number of resistances connected in series is given by

$$R_S = R_1 + R_2 + R_3 + ...$$

2. The equivalent resistance R_p of a number of resistances connected in parallel is given by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$

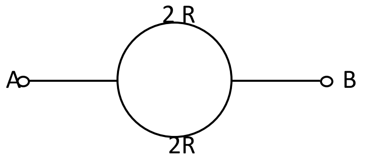
3. For two resistances in parallel,

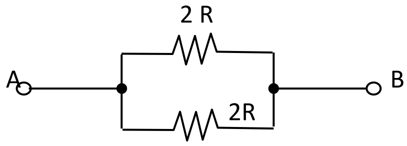
Currents through the two resistors will be

$$I_1 = \frac{R_2 I}{R_1 + R_2} \quad \text{and} \quad I_2 = \frac{R_1 I}{R_1 + R_2}$$

- **Units Used** All resistances are in ohm (Ω).

Q. 1. ***A wire of resistance 4 R is bent in the form of a circle [Fig.]. What is the effective resistance between the ends of the diameter?***

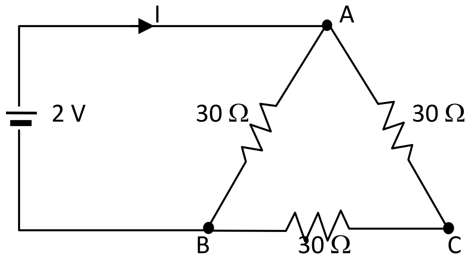




Sol. As shown in Fig. the two resistances of value 2 R each are in parallel with each other. So, the resistance between the ends A and B of a diameter is

$$R' = \frac{2R \times 2R}{2R + 2R} = R$$

Q. 2. ***Find the value of current I in the circuit shown in Fig.***



Sol. In the given circuit, the resistance of arm ACB (30 + 30 = 60 Ω) is in parallel with the resistance of arm AB (= 30 Ω) Hence the effective resistance of the circuit is

$$R = \frac{30 \times 60}{30 + 60} = 20\ \Omega$$

Current, $I = \frac{V}{R} = \frac{2}{20} = 0.1$ A

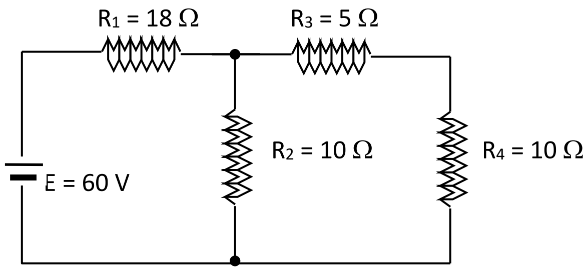
Q. 3. ***Determine the voltage drop across the resistor R_1 in the circuit given below with E = 60 V, R_1 = 18 Ω, R_2 = 10 Ω.***

Sol. As the resistances R_3 and R_4 are in series, their equivalent resistance

$= 5 + 10 = 15\ \Omega$

The series combination of R_3 and R_4 is in parallel with R_2. Their equivalent resistance is

$$R' = \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6\ \Omega$$



The combination R' is in series with R_1.

∴ Total resistance of the circuit,

$R = 6 + 18 = 24\ \Omega$

Current, $I = \frac{E}{R} = \frac{60}{24} = 2.5$ A

∴ Voltage drops across R_1

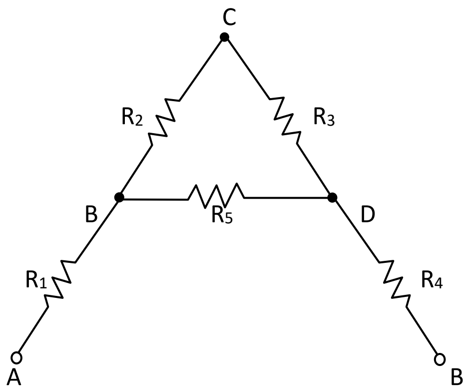
$= IR_1 = 2.5 \times 18$ V $= 45$ V

Q. 4. ***A letter A consists of a uniform wire of resistance 1 ohm per cm. The sides of the letter are each 20 cm long and the cross-piece in the middle is 10 cm long while the apex angle is 60°. Find the resistance of the letter between the two ends of the legs.***

Sol. Clearly,

AB = BC = CD = DE = BD = 10 cm

∴ $R_1 = R_2 = R_3 = R_4 = R_5 = 10\ \Omega$



As R_2 and R_3 are in series, their combined resistance = 10 + 10 = 20 Ω. This combination is in parallel with R_5 (= 10 Ω) Hence resistance between points B and D is given by

$$\frac{1}{R} = \frac{1}{20} + \frac{1}{10} = \frac{3}{20} \quad \text{or} \quad R = \frac{20}{3}\ \Omega$$

Now resistance R_1, R and R_4 form a series combination. So, resistance between the ends A and E is

$$R' = 10 + \frac{20}{3} + 10 = 26.67\ \Omega$$

Q. 5. ***A set of n identical resistors, each of resistance R Ω, when connected in series have an effective resistance XΩ and when the resistors are connected in parallel, their effective resistance is Y Ω. Find the relation between R, X and Y.***

Sol. The effective resistance of the n resistors connected in series is

$$X = R + R + R + \dots \text{ n terms} = nR$$

The effective resistance Y of the n resistors connected in parallel is given by

$$\frac{1}{Y} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \dots \text{ n terms} = \frac{n}{R} \quad \text{or} \quad Y = \frac{R}{n}$$

$$\therefore \quad XY = nR . \frac{R}{n} = R^2$$

Q. 6. ***A parallel combination of three resistors takes a current of 7.5 A from a 30 V supply. If the two resistors are 10 Ω and 12 Ω, find the third one.***

Sol. Here $R_p = \frac{V}{I} = \frac{30}{7.5} = 4\,\Omega$

But $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ or $\frac{1}{4} = \frac{1}{10} + \frac{1}{12} + \frac{1}{R_3}$

or $\frac{1}{R_3} = \frac{1}{4} - \frac{11}{60} = \frac{1}{15}$ $\therefore$ $R_3 = 15\,\Omega$

Q. 7. ***When a current of 0.5 A is passed through two resistances in series, the potential difference between the ends of the series arrangement is 12.5 V. On connecting them in parallel and passing a current of 1.5 A, the potential difference between their ends is 6 V. Calculate the two resistances.***

Sol. For series combination, V = 12.5 V, I = 0.5 A

$\therefore$ $R_1 + R_2 = 12.5 = 25.0\,\Omega$ ... (1)

For parallel combination, V = 6.0 V, I = 1.5 A

$\therefore$ $R_p = \frac{V}{I}$ or $\frac{R_1 R_2}{R_1 + R_2} = \frac{6.0}{1.5} = 4.0$

or $R_1 R_2 = 4 (R_1 + R_2) = 4 \times 25 = 100$

$(R_1 - R_2)^2 = (R_1 + R_2)^2 - 4 R_1 r_2$

$= (25)^2 - 4 \times 100 = 225$

$R_1 - R_2 = 15$ ... (2)

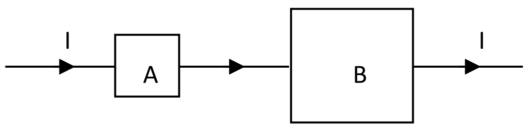
Solving (1) and (2), $R_1 = 20\,\Omega$, $R_2 = 5\,\Omega$

Q. 8. ***Two square metal plates A and B are of same thickness and material. The side of B is twice that of A. These are connected in series, as shown in Fig. Find the ratio R_A/R_B of the resistance of the two plates.***

Sol. let l be the side of the square plate A and 2l that of square plate B. Let d be the thickness of each plate.

$R_A = \frac{\rho l}{A} = \frac{\rho l}{l \times d} = \frac{\rho}{d}$, $R_B = \frac{\rho \times 2l}{2l \times d} = \frac{\rho}{d}$

$\therefore$ $\frac{R_A}{R_B} = \frac{\rho/d}{\rho/d} = 1 : 1$



Q. 9. ***Three conductors of conductance G_1, G_2 and G_3 are connected in series. Find their equivalent conductance.***

Sol. As conductance is reciprocal of resistance, therefore

$R_1 = \frac{1}{G_1}$, $R_2 = \frac{1}{G_2}$, $R_3 = \frac{1}{G_3}$

For the series combination, $R = R_1 + R_2 + R_3$

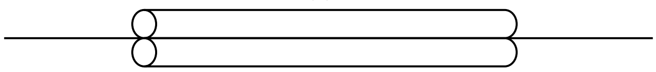
$$\therefore \quad \frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} = \frac{G_2 G_3 + G_1 G_3 + G_1 G_2}{G_1 G_2 G_3}$$

or equivalent conductance,

$$G = \frac{G_1 G_2 G_3}{G_2 G_3 + G_1 G_3 + G_1 G_2}$$

Q. 10. ***A copper rod of length 20 cm and cross-sectional area 2 mm^2 is joined with a similar aluminium rod as shown in Fig. Find the resistance of the combination between the ends. Resistivity of copper = 1.7×10^{-8} Ωm and resistivity of aluminium = 2.6×10^{-8} Ωm.***

Copper



Aluminium

Sol. For copper rod, $\rho = 1.7 \times 10^{-8}\,\Omega$m, l = 20 cm = 20×10^{-2} m, A = 2 mm^2 = 2×10^{-6} m^2

$\therefore$ Resistance, $R_1 = \frac{\rho l}{A} = \frac{1.7 \times 10^{-8} \times 20 \times 10^{-2}}{2 \times 10^{-6}}$

$= 1.7 \times 10^{-3}\,\Omega$

For aluminium rod, $\rho = 2.6 \times 10^{-8}\ \Omega$m,

$l = 20 \times 10^{-2}$ m, $A = 2 \times 10^{-6}$ m^2

∴ Resistance, $R_2 = \dfrac{2.6 \times 10^{-8} \times 20 \times 10^{-2}}{2 \times 10^{-6}}$

$= 2.6 \times 10^{-3}\ \Omega$

As the two rods are joined in parallel, their equivalent resistance is

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{1.7 \times 10^{-3} \times 2.6 \times 10^{-3}}{1.7 \times 10^{-3} + 2.6 \times 10^{-3}}$$

$$= \frac{1.7 \times 2.6 \times 10^{-3}}{4.3} = 1.028 \times 10^{-3}\ \Omega = 1.028\ \text{m}\ \Omega$$

Q. 11. ***The length and radii of three wires of same metal are in the ratios 2 : 3 : 4 and 3 : 4 : 5 respectively. They are joined in parallel and included in a circuit having 5 A current. Find current in each wire.***

Sol. Let R_1, R_2, R_3 be the resistances of the wires, Then

$$R_1 : R_2 : R_3 = \frac{l_1}{r_1^2} : \frac{l_2}{r_2^2} : \frac{l_3}{r_3^2} = \frac{2}{9} : \frac{3}{16} : \frac{4}{25}$$

The ratio of the currents in the three wires must be inverse of the above ratio.

$$\therefore \quad I_1 : I_2 : I_3 = \frac{9}{2} : \frac{16}{3} : \frac{25}{4} = 54 : 64 : 75$$

As total current = 5 A, therefore

$$I_1 = \frac{5 \times 54}{193} = 1.40\ \text{A}, \quad I_2 = \frac{5 \times 64}{193} = 1.66\ \text{A},$$

$$I_3 = \frac{5 \times 75}{193} = 1.94\ \text{A}$$

Q. 12. ***A wire of uniform cross-section and length l has a resistance of 16 Ω. It is cut into four equal parts. Each part is stretched uniformly to length l and all the four stretched parts are connected in parallel. Calculate the total resistance of the combination so formed. Assume that stretching of wire does not cause any change in the density of its material.***

Sol. Resistance of each of the four parts of length $l/4 = 4\ \Omega$. When each part is stretched to length l, its volume remains same.

$$V = A'l' = Al \quad \text{or} \quad \frac{A'}{A} = \frac{l}{l'} = \frac{l/4}{l} = \frac{1}{4}$$

$$\therefore \quad \frac{R}{R'} = \frac{l}{l'} \times \frac{A'}{A} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

or $R' = 16 \times R = 16 \times 4 = 64\ \Omega$

i.e., resistance of each stretched part is 64 Ω. When these four parts are connected in parallel, the total resistance R of the combination is given by

$$\frac{1}{R} = \frac{1}{64} + \frac{1}{64} + \frac{1}{64} + \frac{1}{64} = \frac{4}{64} = \frac{1}{16}$$

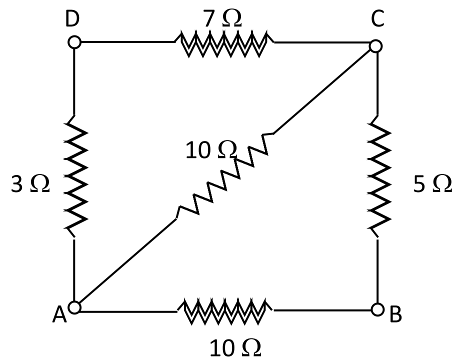
or $R = 16\ \Omega$

Q. 13. ***Find, in the given network of resistors, the equivalent resistance between the points A and B, between A and D, and between A and C.***

Sol. The resistors AD (= 3 Ω) and DC (= 7Ω) are in series to give a total resistance R' = 10 Ω. The resistance R' (= 10 Ω) and the resistor AC (= 10 Ω) are in parallel. Their equivalent resistance is

$$R'' = \frac{10 \times 10}{10 + 10} = 5\ \Omega$$

Now R'' (= 5 Ω) and CB (= 5 Ω) are in series, their total resistance R''' = 10 Ω. Finally, R''' (= 10 Ω) and AB (= 10 Ω) are in parallel between A and B. Hence the equivalent resistance between points A and B is



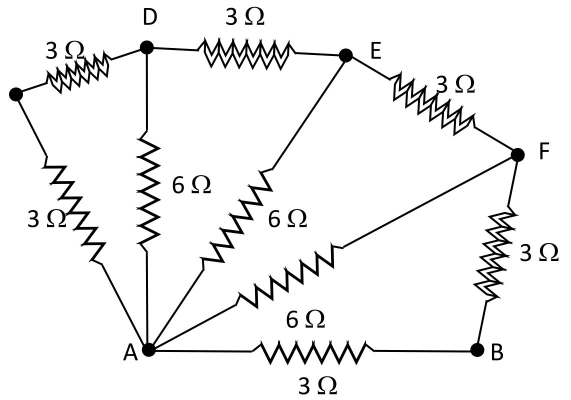
$$R_{AB} = \frac{10 \times 10}{10 + 10} = 5\ \Omega$$

Similarly, $R_{AD} = \frac{39}{16}\ \Omega$ and $R_{AC} = \frac{15}{4}\ \Omega$

Q. 14. Find the effective resistance between points A and B for the network shown in Fig.

Sol. At points A and D, a series combination of 3 Ω, 3 Ω resistances (along AC and CD) is in parallel with 6 Ω resistance (Along AD), therefore, resistance between A and D

$$= \frac{1}{\frac{1}{3+3} + \frac{1}{6}}\ \Omega = 3\ \Omega$$



Similarly, resistance between A and E

$$= \frac{1}{\frac{1}{3+3} + \frac{1}{6}} = 3\ \Omega$$

Resistance between A and F

$$= \frac{1}{\frac{1}{3+3} + \frac{1}{6}} = 3\ \Omega$$

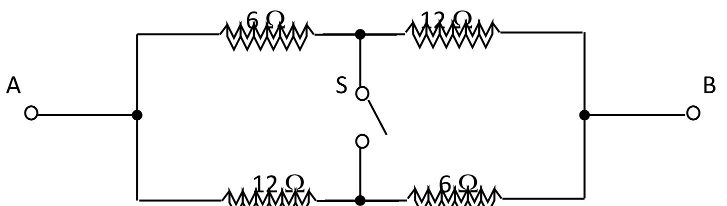
Finally, resistance between A and B

$$= \frac{1}{\frac{1}{3+3} + \frac{1}{3}} = 2\ \Omega$$

Thus, the effective resistance between A and B is 2 Ω.

Q. 15. Find the effective resistance of the network shown in Fig. between the point A and B when (i) the switch S is open (ii) Switch S is closed.

Sol.



(i) When the switch S is open, the resistances of 6 Ω and 12 Ω in upper portion are in series, the equivalent resistance is 18 Ω. Similarly, resistances in the lower portion have equivalent resistance of 18 Ω. Now the two resistance of 18 Ω are in parallel between points A and B.

∴ Effective resistance between points A and B = $\frac{18 \times 18}{18 + 18} = 9\ \Omega$

(ii) When the switch S is closed, the resistances of 6 Ω and 12 Ω on the left are in parallel. Their equivalent resistance is

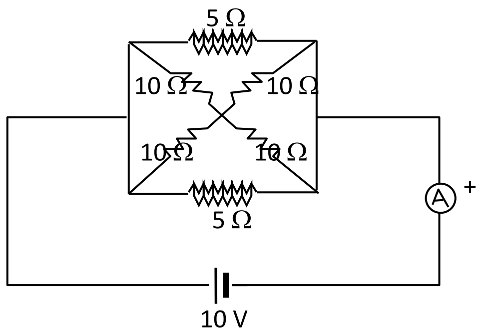
$$\frac{6 \times 12}{6 + 12} = 4\ \Omega$$

Similarly, the resistances on the right have equivalent resistance of 4 Ω. Now the two resistances of 4 Ω are in series.

∴ Effective resistance between points A and B = 4 + 4 = 8 Ω

Q. 16. Calculate the current shown by the ammeter A in the circuit shown in Fig.

Sol. The equivalent circuit is shown in Fig.



For the two 10 Ω resistances connected in parallel,

Equivalent resistance = $\frac{10 \times 10}{10 + 10} = 5\ \Omega$

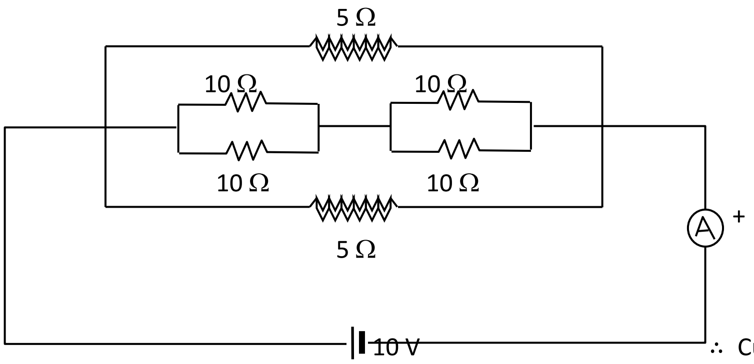
For two such combinations connected in series,

Equivalent resistance = 5 + 5 = 10 Ω

Now we have resistances of 5 Ω, 10 Ω and 5 Ω connected in parallel, so

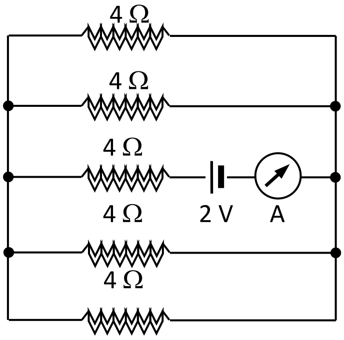
$$\frac{1}{R} = \frac{1}{5} + \frac{1}{10} + \frac{1}{5} = \frac{1}{2} \quad \text{or} \quad R = 2\ \Omega$$

Also V = 10 V



∴ Current, I = V = 10 = 5 A

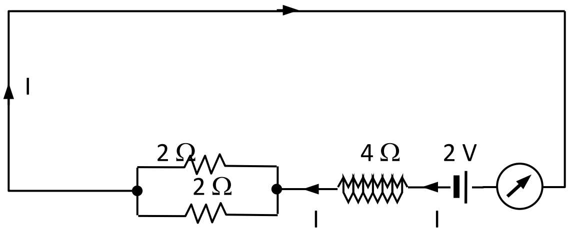
Q. 17. ***Find 4 Ω resistances, a 2 V battery and an ammeter are connected as shown in Fig. Find the ammeter reading.***



Sol. The equivalent circuit is shown in Fig.

Equivalent resistance, $R = \frac{2 \times 2}{2+2} + 4 = 5\ \Omega$

∴ Ammeter reading, $I = \frac{\mathcal{E}}{R} = \frac{2}{5} = 0.4\ A$



Q. 18. ***Calculate the steady-state current 2 Ω resistor in the circuit shown in Fig. The internal resistance of the battery is negligible and C = 2 μF.***

Sol. In the steady state, capacitor offers infinite resistance to d.c. so on current flows through 4 Ω resistor, which is thus ineffective.

Effective resistance between points A and B is

$R' = \frac{2 \times 3}{2+3} = \frac{6}{5} = 1.2\ \Omega$

Total resistance of the circuit,

$R = 1.2 + 2.8 = 4.0\ \Omega$

Current drawn from the battery,

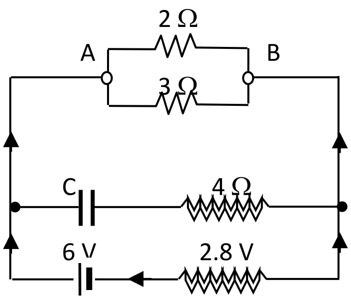
$I = \frac{\mathcal{E}}{R} = \frac{6}{4} = 1.5\ A$

P.D. between points A and B

$V = I R' = 1.5 \times 1.2 = 1.8\ V$

∴ Current through 2 Ω resistor

$= \frac{V}{2} = \frac{1.8}{2} = 0.9\ A$



Q. 19. ***Find the potential difference between the points A and B in the circuit shown in Fig. Internal resistances of the cells are negligible.***

Sol. Net emf = 5 – 2 = 3 V. This sends a current in clockwise direction as shown in Fig. The point B is at a higher potential than the point A.

Total resistance = 10 + 20 = 30 Ω

∴ Circuit in the circuit is

$I = \frac{\mathcal{E}}{R} = \frac{3}{30} = 0.1\ A$

P.D. across 20 Ω resistor = R I = 20 × 0.1 = 2 V

∴ $V_B - V_A = 2 + 2 = 4\ V$

P.D. across 10 Ω resistor = 10 × 0.1 = 1 V

∴ $V_A - V_B = 1 - 5 = -4\ V$

10 Ω 5 V
A B
2 V 20 V
1 V

10 Ω 5 V
A B
2 V 20 Ω
2 V

Q. 20. ***In the circuit shown in Fig. find the potential difference across the capacitor.***

Sol. In the steady state (when the capacitor is fully charged), no current flows through the branch CEF. The given circuit then reduces to the equivalent circuit shown in Fig.

B 3 Ω C 3 μF E
3 Ω 3 Ω
3 Ω 3 Ω
A F
D

The equivalent resistance of the circuit is

$$R = \frac{6 \times 3}{6+3} + 3 = 5\ \Omega$$

Current drawn from the battery,

$$I = \frac{15\ V}{5\ \Omega} = 3\ A$$

Current through branch BCD,

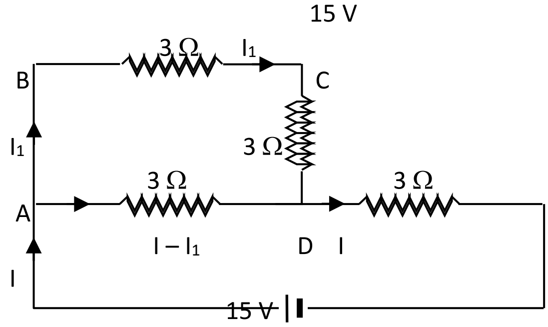
$$I_1 = \frac{3}{6+3} \times I = \frac{3}{9} \times 3 = 1\ A$$

Current through the arm DF = I = 3 A

P.D. across the capacitor = P.D. between points C and F

= P.D. across CD + P.D. across DF

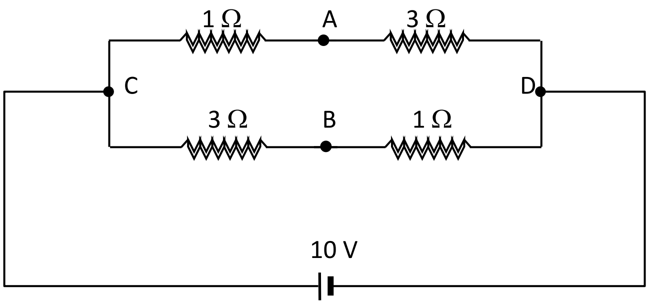
= 3 × 1 + 3 × 3 = 12 V



Q. 21. A battery of emf 10 V is connected to resistances as shown in Fig. Find the potential difference between the points A and B.

Sol. Total resistance, $R = \frac{4 \times 4}{4+4} = 2\ \Omega$

Current, $I = \frac{V}{R} = \frac{10\ V}{2\ \Omega} = 5\ A$



As each of the two parallel branches has same resistance (4 Ω), so the current of 5 A is divided equally through them.

Current through each branch = 5/2 = 2.5 A

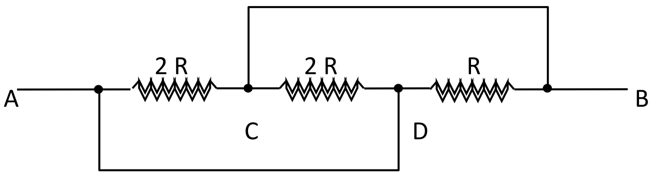
Now $V_C - V_A = 2.5 \times 1 = 2.5\ V$

and $V_C - V_B = 2.5 \times 3 = 7.5\ V$

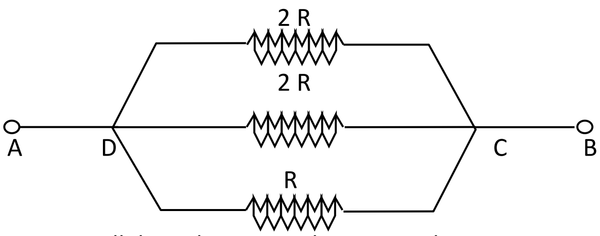
∴ $V_A - V_B = (V_C - V_B) - (V_C - V_A)$

$= 7.5 - 2.5 = 5.0\ V$

Q. 22. ***What is the equivalent resistance between points A and B of the circuit shown in Fig.***



Sol. Obviously, the points A and D are equipotential points. Also, the points B and C are equal potential points. So the given network of resistances reduces to the equivalent circuit shown in Fig.



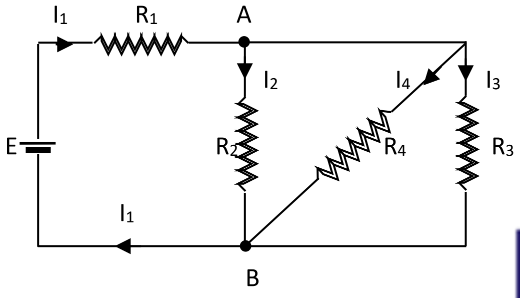
The three resistances form a parallel combination. Their equivalent resistance R_{eq} is given by

$$\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{R} = \frac{1+1+2}{2R} = \frac{2}{R}$$

or $R_{eq} = R/2$

Q. 23. ***In the circuit shown in Fig. $R_1 = 100\ \Omega$, $R_2 = R_3 = 50\ \Omega$, $R_4 = 75\ \Omega$ and E = 4.75 V. Work out the equivalent resistance of the circuit and the current in each resistor.***

Sol. The resistances R_2, R_3 and R_4 are in parallel. Their equivalent resistance R' is given by



$$\frac{1}{R'} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{50} + \frac{1}{50} + \frac{1}{75}$$

$$= \frac{8}{150} = \frac{4}{75}$$

or $R' = \frac{75}{4}\,\Omega$

The resistance R_1 is in series with R'. Hence total resistance of the circuit is

$$R = R_1 + R' = 100 + \frac{75}{4} = \frac{475}{4}\,\Omega$$

The current I_1 is the current sent by the cell E in the whole circuit.

$\therefore$ $I_1 = \frac{E}{R} = \frac{4.75}{475/4} = 0.04$ A

Potential drop between A and B,

$$V = I_1 R' = 0.04 \times \frac{75}{4} = 0.75 \text{ V}$$

This is the potential drop across each of the resistances R_1, R_2 and R_3 in parallel. Therefore, current through these resistances are

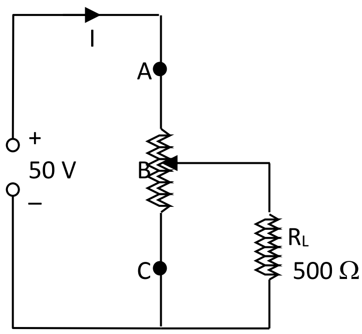
$$I_2 = \frac{V}{R_2} = \frac{0.75}{50} = 0.015 \text{ A}; \quad I_3 = \frac{V}{R_3} = \frac{0.75}{50} = 0.015 \text{ A}$$

And $I_4 = \frac{V}{R_4} = \frac{0.75}{75} = 0.10$ A

Q. 24. ***As shown in Fig. a variable rheostat of 2 k Ω is used to control the potential difference across a 500 Ω load.***
(i) If the resistance AB is 500 Ω, what is the potential difference across the load?
(ii) If the load is removed, what should be the resistance at BC to get 40 V between B and C?

Sol. Here $R_{AC} = 2\text{ k}\Omega = 2000\,\Omega$, $R_L = 500\,\Omega$, R_{AB} 500 Ω, $R_{BC} = R_{AC} - R_{AB} = 2000 - 500 = 1500\,\Omega$

Total resistance of the parallel combination of R_{BC} and R_L,



$$R' = \frac{1500 \times 500}{1500 + 500} = 375\,\Omega$$

Total resistance of the circuit,

$$R = R_{AB} + R' = 500 + 375 = 875\,\Omega$$

Current in the circuit,

$$I = \frac{V}{R} = \frac{50 \text{ V}}{875\,\Omega} = \frac{2}{35} \text{ A}$$

(i) The potential drop across R_L will be the same as the potential drop across the parallel combination of R_{BC} and R_L.

$\therefore$ Potential drop across $R_L = V - V_{AB}$

$$= 50 - \frac{2}{35} \times 500 = 50 - 28.57 = 21.43 \text{ V}$$

(ii) If the load is removed, then the current will flow through entire resistance R_{AC} of the rheostat.

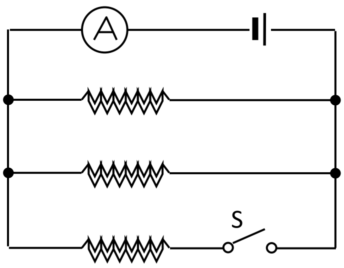
$\therefore$ Current, $I' = \frac{50 \text{ V}}{2000\,\Omega} = \frac{1}{40}$ A

To obtain a potential drop of 40 V between B and C, the required resistance BC must be

$$R'BC = \frac{40 \text{ V}}{1/40 \text{ A}} = 1600\,\Omega$$

Q. 25. ***In the circuit shown in Fig. both the ammeter and the cell have negligible resistance. Three external resistors are identical. When the switch S is opened, the ammeter reads 0.6 A. What will the ammeter read when the switch S is closed?***

Sol. Let r be the resistance of each resistor. When the switch S is opened, the parallel combination of upper two resistors in in the circuit. Their equivalent resistance is



$$R = \frac{r \times r}{r + r} = \frac{r}{2}$$

Potential drop across the cell,

$$V = IR = 0.6 \times \frac{r}{2} = 0.3\,r$$

When switch S is closed, the parallel combination of all the three resistors is in the circuit. The equivalent resistance R' of the combination is given by

$$\frac{1}{R'} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{3}{r} \text{ or } R' = \frac{r}{3}$$

$\therefore$ Ammeter reading, $I' = \frac{V}{R'} = \frac{0.3\,r}{r/3} = 0.9$ A

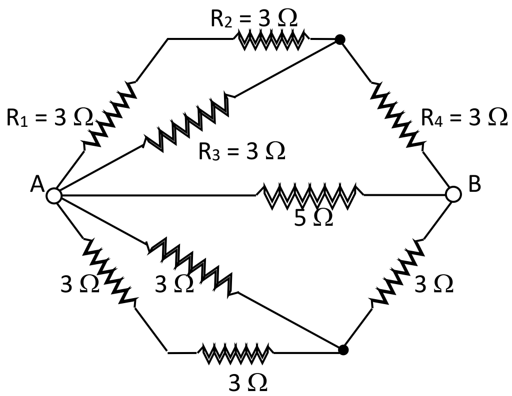
Q. 26. Find the equivalent resistance between the points A and B of the network of resistors shown in Fig.

Sol. The resistors R_1 and R_2 are in series. The equivalent resistance

$$= 3 + 3 = 6\ \Omega$$

The 6 Ω resistances is in parallel with R_3, so that their equivalent resistance

$$= \frac{6 \times 3}{6 + 3} = 2\ \Omega$$



Now the 2 Ω resistance is in series with R_4. So the total resistance of the upper portion = 2 + 3 = 5 Ω

Similarly, total resistance of the lower portion

$$= 5\ \Omega$$

Now we have three 5 Ω resistors connected in parallel between the point A and B. Hence the equivalent resistance R of the entire network is given by

$$\frac{R}{R} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5} \qquad \text{or} \qquad R = \frac{5}{3}\ \Omega$$

Internal Resistance and Terminal P.d of a Cell:

➢ ***"A cell is a device which produces the necessary P.d to an electric circuit to maintain a continuous flow of current in it."***

Internal resistance: **--(r) is applied as the resistance offered by the electrolyte of the cell to the flow of current through it.**

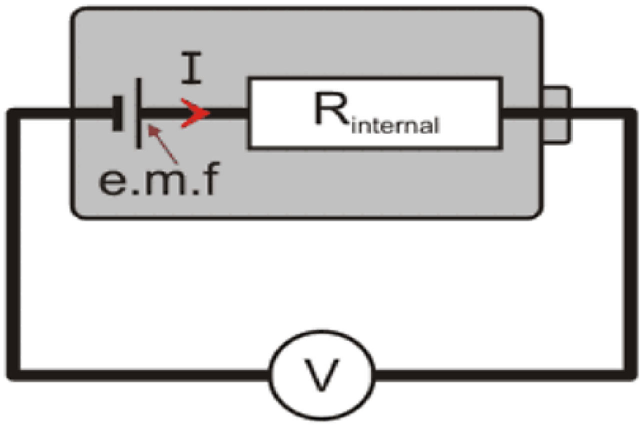
➢ **Unit -- Ω .**

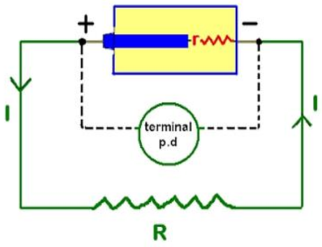
Internal resistance: When the terminals of a cell are connected by a wire, an electric current flow in the wire from positive terminal of the cell towards the negative terminal. But inside the electrolyte of the cell, the positive ions flow from the lower to the higher potential (or negative ions from the higher to the lower potential) against the background of other ions and neutral atoms of the electrolyte. So the electrolyte offers some resistance to the flow of current inside the cell.

The resistance offered by the electrolyte of a cell to the flow of current between its electrodes is called internal Resistance of the cell. **The internal resistance of a cell depends on following factors:**

1. Nature of the electrolyte. **2. It is directly proportional to the concentration of the electrolyte.**
3. It is directly proportional to the distance between the two electrodes.
4. It varies inversely as the common area of the electrodes immersed in the electrolyte.
5. It increases with the decreases in temperature of the electrolyte.

The internal resistance of a freshly prepared cell is usually low but its value increases as we draw more and more current from it.





$IR - \xi + Ir = 0$ (By Kirchhoff voltage law)

$IR = \xi - Ir$

$V = \xi - Ir$

terminal p.d. — emf — lost voltage

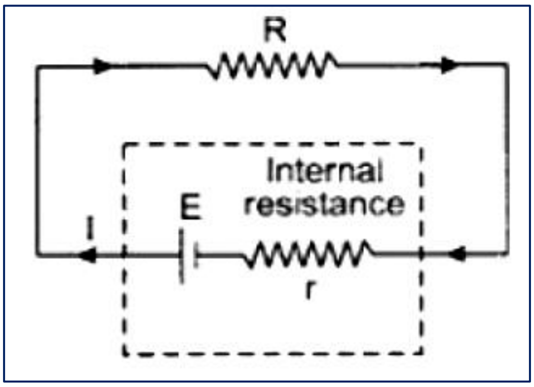
➢ **Terminal P.d: -- (V) Terminal P.d** or **closed-circuit voltage of a cell is defined as the P.d between its terminal in a closed circuit.**

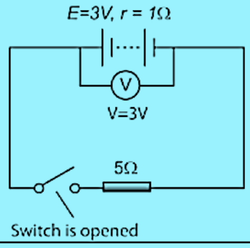
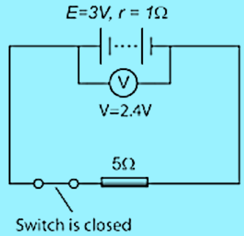
➢ **Unit – Volt (V).**

➢ When the electric cell (source of emf) is in a closed circuit, the current flows through the circuit. There is a fall of potential across the internal resistance of the cell. The terminal P.d between two electrodes of the cell is less than the emf (E) of the cell by an amount equal to potential drop across the internal resistance of the cell.
Therefore, the ***terminal P.d is always less than the emf of the cell.***

✪ **Relation between emf (E) and terminal P.d (V): --**

Consider a cell of emf 'E' and internal resistance 'r' , connected to an external resistance 'R' through a key (K) .



Open Circuit	Close Circuit
	
In open circuit (when the switch is off), the voltmeter shows the reading of the e.m.f.	In close circuit (when the switch is on), the voltmeter shows the reading of the potential difference across the cell.
With the presence of internal resistance, the potential difference across the cell is always less than the e.m.f..	

☞ **When key (K) is open [- ()-] no current is drawn from the cell. So, the voltmeter connected across the cell gives the value of emf (E).**

☞ **When key (K) is closed [-(∗)-] , current is drawn from the cell by the circuit and is ,**

$$I = \frac{E}{R + r}$$ [since, R and r is in series therefore, total resistance = R + r].

$$E = IR + Ir \quad \text{-----------------(i)}$$

Since 'R' is connected in parallel to the electrode of the cell, so the terminal p.d of the cell is equal to the p.d across the resistance. $V = IR$.

From (i) $E = V + Ir$ ∴ $$V = E - Ir$$

i.e., Terminal **p.d is less than emf of the cell**.

▶ ***Terminal p.d is equal to Emf of the cell if and only if circuit is open (or no current is drawn from the cell).***

i.e., if $I = 0$

then $V = E - 0\,r$ ∴ $V = E$

➢➢ **Determination of Internal resistance 'r'** :

We know that, $I = \frac{E}{R + r}$

Since, $V = IR$ ∴ $V = \left[\frac{E}{R + r}\right] R$

or, $R + r = \frac{E}{V} R$ or, $r = \frac{E}{V} R - R$

∴ $r = \left[\frac{E}{V} - 1\right] R$

➢➢ **FACTS**

1] The direction of current sent by a cell an electric circuit is from positive terminal to negative terminal outside the cell & negative terminal to positive terminal inside the cell.

--- It means inside the cell; the positive charge flows from negative electrode to positive electrode.

2] When current is drawn from the cell than terminal p.d is less than emf of the cell **V < E.**

3] When no current is drawn from the cell **V = E**.

4] During charging of a cell. **V > E**.

EXPALNATION: - During charging the +ive electrodes of the cell is connected to positive terminal of battery charger and negative of the cell is connected to negative terminal of the battery charger, In this process current flows from +ive electrode of the cell to the negative electrode.

∴ $V = E + I R$.

- ** $E = V + I r$ or, $I r = E - V$ → Lost voltage (or potential drop).

1. Although we call e.m.f. as force but actually it is energy which is given by the cell to unit charge for flowing through the circuit.
2. If external resistance is connected between the terminals of the cell the the potential difference between the terminals is 'Terminal voltage' of the cell.
3. When there is no external resistance connected between the terminals the potential difference is equal to the e.m.f. of the cell.

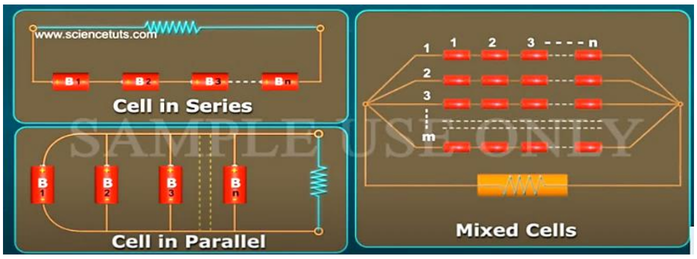
☑ GROUPING OF CELLS

The e.m.f and current which can be obtained from a single cell are generally small. The number of cells may be suitably grouped or connected together for increasing the e.m.f or current. Such a combination of cells is called a battery. Following are the three different ways of grouping the cells.

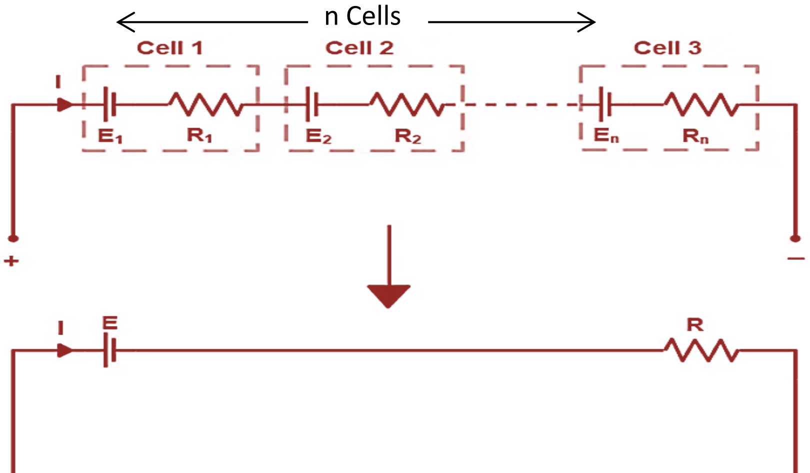
(i) Series grouping

(ii) Parallel grouping

(iii) Series parallel grouping



[I] **CELL IN SERIES**: -- The cells are said to be connected in series if **positive terminal** of cell is connected to **-ive terminal** of second cell and so on. The external resistance (R) is connected to the free end of 1st cell.



Series Grouping Of Cell

Let 'n' = no. of identical cell connected in series each of emf E and internal resistance 'r'.

Total internal resistance = $n r$ (since internal resistance of all the cells are connected in series.)

Total Resistance in the circuit = $R + n r$ (since R and n r are in series.)

Effective emf of the cell = $E + E + E + \text{- - - - - -} + R_n$ up to n terms = $n E$.

∴ Current in the resistance R is

$$I = \frac{n E}{R + n r}$$

✪Special case

[1] If $R >> n r$ (i.e., external resistance is very very large than total internal resistance) than 'n r' can be neglected.

∴ $R + n r \simeq R$ or, current $I = n \frac{E}{R} = n \frac{E}{R}$

Since, $\frac{E}{R}$ = current due to a single cell. Thus, the current in the external Resistance is **n times**, due to a single cell.

[2] If **$R << nr$**, then R can be neglected, i.e., $R + nr \simeq nr$

∴ current in external resistance, $I = \frac{nE}{nr} = \frac{E}{r}$

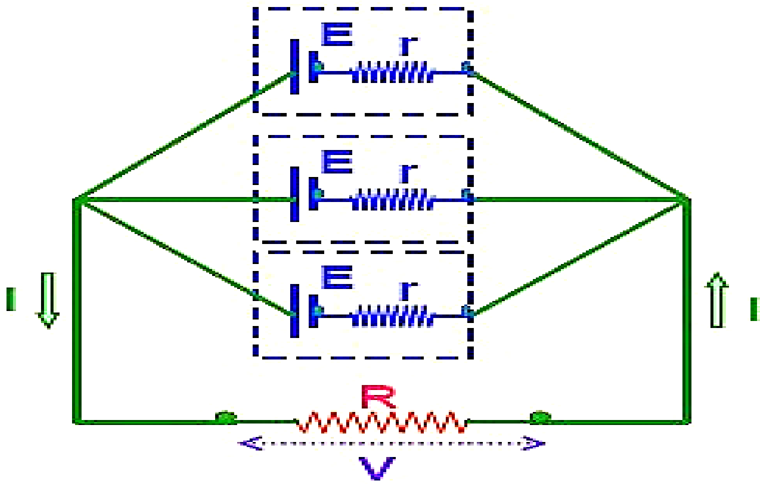
Thus, current in the external resistance is same as due to a single cell.

✪➢ **In order to get a large amount of current from the cells connected in series, the external resistance should be very large as compared to the net internal resistance of the cell.**

✪➢ If large no. of cells in series are connected to a very small external resistance, then the current from these cells will be equal to a single cell. (It means, there is no use of such a combination).

[II]**CELL IN PARALLEL** – The cells are said to connected in parallel if positive terminal of all the cells is connected together at one point A and their negative terminal at another point B.

Consider 'n' identical cells of emf E and internal resistance 'r' connected in parallel to an external resistance 'R'.



Total emf of all cells = emf of a single cell = E. [Since, cells are in parallel].

Total internal resistance $\frac{1}{r_P} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \text{------}$ up to n terms. [Since, cells are parallel]

∴ $\frac{1}{r_P} = \frac{n}{r}$ or, $r_P = \frac{r}{n}$.

Total resistance in the circuit = $R + \frac{r}{n}$ [since, R and r / n are in series]

∴ current in the Resistance R is

$$I = \frac{E}{R + \frac{r}{n}} = \frac{nE}{nR + r}$$

✪✪ Special case:

➢ **[1] IF** **$R >> r$**, then r can be neglected ∴ $nR + r \simeq nR$.

∴ $I = \frac{nE}{nR} = \frac{E}{R}$.

The current in the external resistance is same as due to a single cell.

➢ **[2] If** **$r >> R$**, then nR can be neglected as compared to r.

∴ $nR + r \simeq r$.

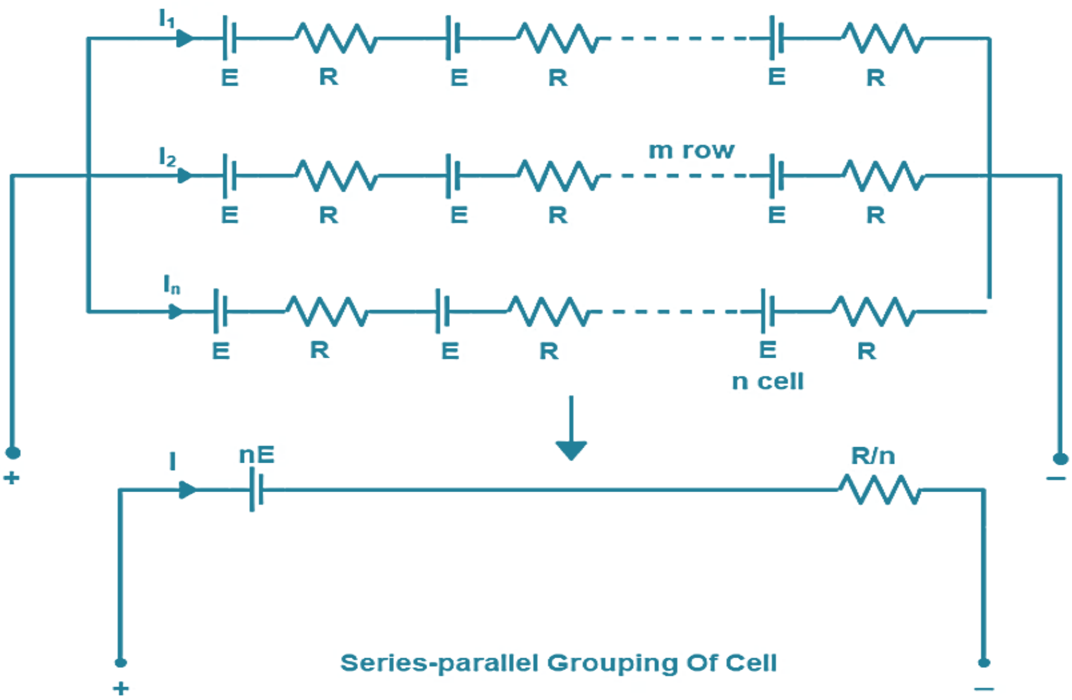
$$I = \frac{nE}{r} = n\left[\frac{E}{r}\right]$$

Thus, the current in the external resistance is 'n' times the current due to a single cell.

➢➢ **In order to get large amount of current from the cell connected in parallel, the net internal resistance should be very large as compared to external resistance**

➢➢ If a large no. of cell connected in parallel to a very large external resistance, then the current from these cells will be equal to the current due to single cell. Such an arrangement of cell is of no use.

☑ **MIXED GROUPING OF CELL**: Consider a no. of cells each of emf E and internal resistance ‘r’.
Let these cells be arranged in ‘m’ rows and each row contains ‘n’ cells.



Series-parallel Grouping Of Cell

In each row, total emf = n E.

In each row, n cells are in series. ∴ Total internal resistance = n r.

Since, there are ‘m’ rows each having internal resistance ‘n r’.

∴ Total internal resistance of al the cell (r_P)

$$\frac{1}{r_P} = \frac{1}{nr} + \frac{1}{nr} + \text{------ up to m terms.}$$

$$\frac{1}{r_P} = \frac{m}{nr}$$

$$\therefore \quad r_P = \frac{nr}{m.}$$

Now, the internal resistance $\frac{nr}{m}$ and R are in series.

∴ The current flowing through the circuits $I = \dfrac{nE}{R + \frac{nr}{m.}}$

$$I = \frac{mnE}{mR + nr} = \frac{NE}{mR + nr}$$ [Where N = m n total no. of cells]

The current I will be maximum if (m R + n r) is minimum or m R - n r = 0.

or, R = n r /m = internal resistance of mixed grouping of cell.

➢Thus, in order to get the maximum current in the circuit the mixed grouping of cells must be does in such a way that **the external resistance is equal to the net internal resistance of all cells.**

Examples based on EMF, Internal Resistance, Terminal Potential Difference and Grouping of Cells

- **Formulae Used**

1. EMF of a cell, $\mathcal{E} = \frac{W}{q}$
2. For a cell of internal resistance r, the emf is
$\mathcal{E} = V + Ir = I(R + r)$
3. Terminal p.d. of cell, $V = IR = \frac{\mathcal{E} R}{R + r}$
4. Terminal p.d. when a current is being drawn from the cell,
$V = \mathcal{E} - Ir$
5. Terminal p.d. when the cell si being charged,
$V = \mathcal{E} + Ir$
6. Internal resistance of a cell, $r = R\left(\frac{\mathcal{E} - V}{V}\right)$

- **Units Used**

EMF $\mathcal{E}$ and terminal p.d. V are in volt (V), internal resistance r and external resistance R in Ω and current I in ampere (A).

Q. 1. ***For driving a current of 3 A for 5 minutes in an electric circuit, 900 J of work is ot be done.***

Sol. The amount of charge that flows through the circuit in 5 minutes is

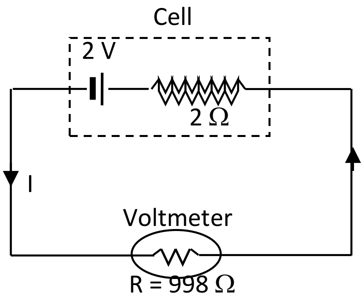
$q = I \times t = 3 \times 5 \times 60 = 900$ C

As emf is the work done in flowing in unit charge in the closed circuit, therefore

$\mathcal{E} = \frac{W}{q} = \frac{900 \text{ J}}{900 \text{ C}} = 1.0$ V

Q. 2. ***A voltmeter of resistance 998 Ω is connected across a cell of emf 2 V and internal resistance 2 Ω. Find the p.d. across the voltmeter, that across the terminals of the cell and percentage error in the reading of the voltmeter.***

Sol. Here $\mathcal{E} = 2$ V, $r = 2\ \Omega$



Resistance of voltmeter, R = 998 Ω

Current in the circuit is

$I = \frac{\mathcal{E}}{R + r}$

$= \frac{2 \text{ V}}{(998 + 2)\ \Omega} = 2 \times 10^{-3}$ A

The p.d. across the voltmeter is

$V = IR$

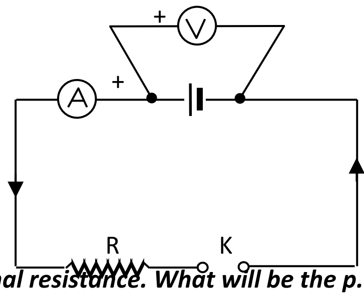
$= 2 \times 10^{-3} \times 998 = 1.996$ V

The same will be the p.d. across the terminals of the cell. The voltmeter used to measure the emf of the cell will read 1.996 volt. Hence the percentage error is

$\frac{\mathcal{E} - V}{\mathcal{E}} \times 100 = \frac{2 - 1.996}{2} \times 100 = 0.2$ %

Q. 3. ***In the circuit shown in fig., the voltmeter reads 1.5 V, when the key is open. When the key is closed, the voltmeter reads 1.35 V and ammeter reads 1.5 A. Find the emf and the internal resistance of the cell.***

Sol. When the key is open, the voltmeter reads almost the emf of the cell.



$\therefore$ $\mathcal{E} = 1.5$ V

When the key is closed, voltmeter reads the terminal potential difference V.

$V = 1.35$ V, $I = 1.5$ A, $r = ?$

$r = \frac{\mathcal{E} - V}{I} = \frac{1.5 - 1.35}{1.5} = 0.1\ \Omega$

Q. 4. ***A cell of emf 2 V and internal resistance 0.1 Ω is connected to a 3.9 Ω external resistance. What will be the p.d. across the terminals of the cell?***

Sol. Here $\mathcal{E} = 2$ V, $r = 0.1\ \Omega$, $R = 3.9\ \Omega$

Current, $I = \frac{\mathcal{E}}{R + r} = \frac{2}{3.9 + 0.1} = 0.5$ A

P.D. across the terminals of the cell, $V = IR = 0.5 \times 3.9 = 1.95$ V

Q. 5. ***The reading on a high resistance voltmeter when a cell is connected across it is 2.2 V. When the terminals of a cell are also connected to a resistance of 5 Ω, the voltmeter reading drops to 1.8 V. Find the internal resistance of the cell.***

Sol. Here $\mathcal{E} = 2.2$ V, $R = 5\ \Omega$, $V = 1.8$ V

Internal resistance,

$$r = R\left(\frac{\mathcal{E} - V}{V}\right)$$
$$= 5\left(\frac{2.2 - 1.8}{1.8}\right)\Omega = 1.1\ \Omega$$

Q. 6. ***A dry cell of emf 1.6 V and internal resistance 0.10 Ω is connected to a resistance of R ohm. The current drawn from the cell is 2.0 A. Find the voltage drop across R.***

Sol. Here $\mathcal{E} = 1.6$ V, $r = 0.10\ \Omega$, $I = 2.0$ A

Voltage drop across R will be , $V = \mathcal{E} - Ir = 1.6 - 2.0 \times 0.10 = 1.4\ \Omega$

Q. 7. ***A battery of emf 'ℰ', and internal resistance 'r', gives a current of 0.5 A with an external resistor of 12 ohm and a current of 0.25 A with an external resistor of 25 ohm. Calculate (i) internal resistance of the cell and (ii) e.m.f. of the cell.***

Sol. EMF of the cell, $\mathcal{E} = I(R + r)$

In first case, $\mathcal{E} = 0.5\,(12 + r)$

In second case, $\mathcal{E} = 0.25\,(25 + r)$

∴ $0.5\,(12 + r) = 0.25\,(25 + r)$

On solving, we get $r = 1\ \Omega$

Hence $\mathcal{E} = 0.5\,(12 + 1) = 6.5$ V

Q. 8. ***A battery of emf 3 volt and internal resistance r is connected in series with a resistor of 55 Ω through an ammeter of resistance 1 Ω. The ammeter reads 50 mA. Draw the circuit diagram and calculate the value of r.***

Sol. Total resistance

$= 55 + 1 + r\ \Omega = 56 + r\ \Omega$

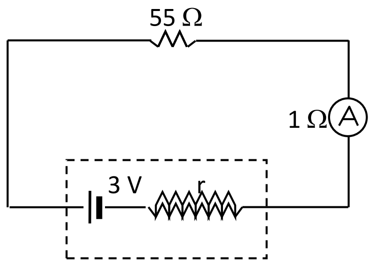
Current = 50 mA = 50×10^{-3} A

emf = 3 V

Resistance = $\dfrac{\text{emf}}{\text{Current}}$

$56 + r = \dfrac{3}{50 \times 10^{-3}} = 60$

$r = 60 - 56 = 4\ \Omega$



Q. 9. ***(a) A car has a fresh storage battery of emf 12 V and internal resistance 5.0 × 10⁻² Ω. If the starter motor draws a current of 90 A, what is the terminal voltage of the battery when the starter is on?***

(b) After long use, the internal resistance of the storage battery increase to 500 Ω. What maximum current can be drawn from the battery? Assume the emf of the battery to remain unchanged.

(c) If the discharged battery is charged by an external emf source, is the terminal voltage of the battery during charging greater or less than its emf 12 V?

Sol. (a) Here $\mathcal{E} = 12$ V, $I = 90$ A, $r = 5.0 \times 10^{-2}\ \Omega$

∴ Terminal voltage,

$V = \mathcal{E} - Ir = 12 - 4.5 = 7.5$ V

(b) The maximum current can be drawn from a battery by shorting it.

Then $V = 0$

and $I_{max} = \dfrac{\mathcal{E}}{r} = \dfrac{12}{500}$ A = 24 mA

Clearly, the battery is useless for starting the car and must be charged again.

(c) During discharge of the accumulator, the current inside the cells (of the accumulator) is opposite to what it is when the accumulator discharges. That is, during charging, current flows from the + ve to – ve terminal inside the cells. Consequently, during charging

$V = \mathcal{E} + Ir$ Hence V must be greater than 12 V during charging.

Q. 10. ***A battery of emf 12.0 V and internal resistance 0.5 Ω is to be charged by a battery charger which supplies 110 V d.c. How much resistance must be connected in series with the battery to limit the charging current to 5.0 A? What will be the p.d. across the terminals of the battery during charging?***

Sol. For charging, the positive terminal of the charger is connected to the positive terminals of the battery.

∴ Net emf $\mathcal{E}' = 110 - 12.0 = 98$ V

If R is the series resistor, then the charging current will be

$I = \dfrac{\mathcal{E}'}{R + r} = \dfrac{98}{R + 0.5}$ A

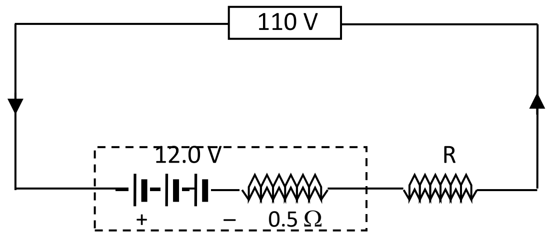
Given I = 5.0 A, therefore

$\dfrac{98}{R + 0.5} = 5.0$ or $R = 19.1\ \Omega$

The terminal p.d. of the battery during charging is

$V = \mathcal{E} + Ir = 12.0 + 5.0 \times 0.5$

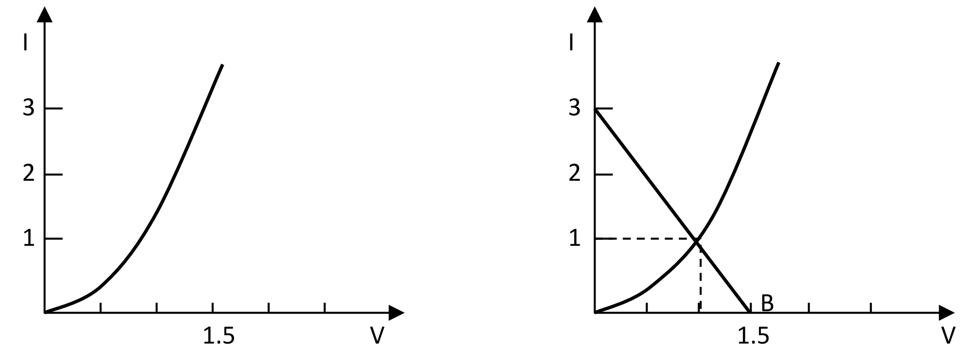
$= 14.5$ V



If the series resistor R were not included in the charging circuit, the charging current would be 98/0.5 = 196 A, which is dangerously high.

Q. 11. ***A cell of emf 1.5 V and internal resistance 0.5 Ω is connected to a (non-linear) conductor whose V – I graph is shown in Fig. (a). Obtain graphically the current drawn from the cell and its terminal voltage.***

Sol.



(a) (b)

Here $\mathcal{E} = 1.5$ V, $r = 0.5\ \Omega$

Terminal voltage of the cell, $V = \mathcal{E} - Ir$

For different currents, terminal voltages can be determined as follows:

$I = 0$, $V = 1.5 - 0 = 1.5$ V $\quad$ $I = 1$ A, $V = 1.5 - 1 \times 0.5 = 1.0$ V

$I = 2$ A, $V = 1.5 - 2 \times 0.5 = 0.5$ V $\quad$ $I = 3$ A, $V = 1.5 - 3 \times 0.5 = 0$

The V – I graph for the cell is a straight-line AB, as shown in Fig. (b). This straight-line graph intersects the given non-linear V – I graph at current = 1 A and at voltage = 1 V

∴ Current drawn from the cell = 1 A

Terminal voltage of the cell = 1 V

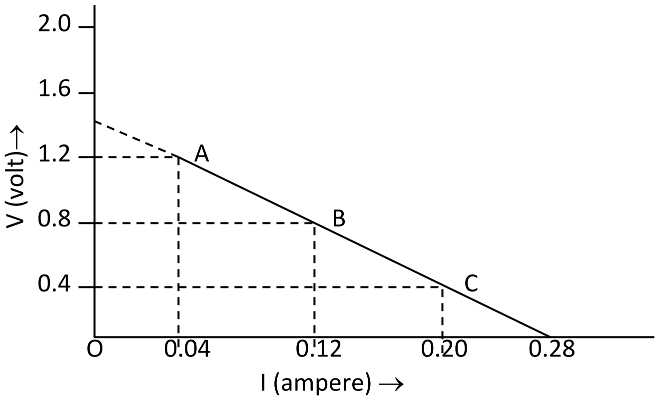
Q. 12. ***Potential differences across terminals of a cell were measured (in volt) against different currents (in ampere) flowing through a cell. A graph was drawn which was straight line ABC, as shown in Fig. Determine from the graph***

(i) emf of the cell

(ii) maximum current obtained from the cell and

(iii) internal resistance of the cell.

Sol.



(i) The potential difference corresponding to zero current equals the emf of the cell. ∴ EMF of the cell, $\mathcal{E} = 1.4$ V

(ii) Maximum current is drawn from the cell when the terminal potential difference is zero. ∴ $I_{max} = 0.28$ A

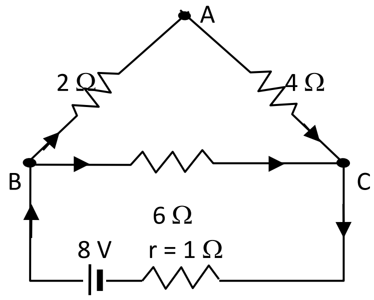
(iii) Internal resistance,

$$r = \frac{\mathcal{E}}{I_{max}} = \frac{1.4\ V}{0.28\ A} = 5\ \Omega$$

Q. 13. ***A uniform wire of resistance 12 Ω is cut into three pieces in the ratio 1 : 2 : 3 and the three pieces are connected to form a triangle. A cell of emf 8 V and internal resistance 1 Ω is connected across the highest of the three resistors. Calculate the current through each part of the circuit.***

Sol. In Fig. $R_{AB} = 2\ \Omega$, $R_{BC} = 4\ \Omega$ and $R_{AC} = 6\ \Omega$

The series combination of 2 Ω and 4 Ω (of equivalent resistance 6 Ω) is in parallel with the 6 Ω resistance . The equivalent resistance is



$$R = \frac{6 \times 6}{6 + 6} = 3\ \Omega$$

$$\text{Current, } I = \frac{\mathcal{E}}{R + r} = \frac{8\ V}{(3 + 1)\ \Omega} = 2\ A$$

The resistance R_{BAC} and R_{BC} of the parallel braches are equal.

∴ $I_{BAC} = I_{BC} = 1$ A

Examples based on Grouping of Cells

♦ **Formulae Used**

1. For n cells in series, $I = \dfrac{n\mathcal{E}}{R + nr}$
2. For n cells in parallel, $I = \dfrac{n\mathcal{E}}{nR + r}$
3. For mixed grouping, $I = \dfrac{mn\mathcal{E}}{mR + nr}$

 where n = no. of cells in series in one row, m = no. of rows of cells in parallel.
4. For maximum current, the external resistance must be equal to the total internal resistance.

i.e., $\dfrac{nr}{m} = R$

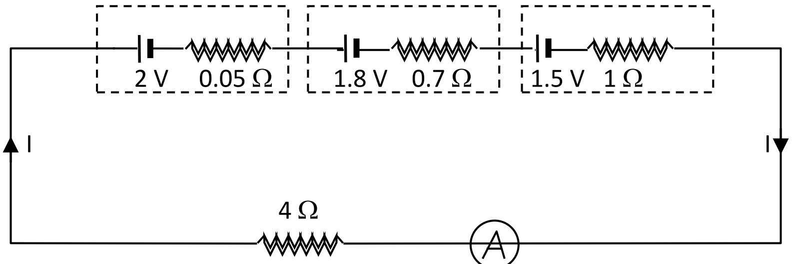
or $nr = mR$

♦ **Units Used**

EMF and terminal p.d. are in volt (V), internal resistance ® and external resistance R in Ω, current in ampere (A).

Q. 1. ***(a) Three cells of emf 2.0 V, 1.8 V and 1.5 V are connected in series. Their internal resistances are 0.05 Ω, 0.7 Ω and 1 Ω respectively. If the battery is connected to an external resistor of 4 Ω via a very low resistance ammeter, what would be the reading in the ammeter?***
(b) If the three cells above were joined in parallel, would they be characterized by a definite emf and internal resistance (independent of external circuit)? If not, how will you obtain currents in different braches?

Sol. (a) The circuit diagram is shown in Fig.



As the three cells have been connected in series to an external resistor of 4 Ω, therefore

Total emf = (2.0 + 1.8 + 1.5) V = 5.3 V

Total resistance = (0.05 + 0.7 + 1 + 4) Ω = 5.75 Ω

Current, $I = \dfrac{\text{emf}}{\text{resistance}} = \dfrac{5.3}{5.75}\text{ A} = 0.92\text{ A}$

(b) No, there is no formula for emf and internal resistance of non-similar cells, joined in parallel. For this situation, we must use Kirchhoff's law.

Q. 2. ***A cell of emf 1.1 V and internal resistance 0.5 Ω is connected to a wire of resistance 0.5 Ω. Other cells of the same emf is connected in series but the current in the wire remains the same. Find the internal resistance of the second cell.***

Sol. In first case:

Total emf, $\mathcal{E} = 1.1$ V

Total resistance, R = 0.5 + 0.5 = 1 Ω

∴ Current, $I = \dfrac{\mathcal{E}}{R} = \dfrac{1.1}{1} = 1.1$ A

In second case:

Total emf, $\mathcal{E} = 1.1 + 1.1 = 2.2$ V

Total resistance, R = 0.5 + 0.5 + r = (1 + r) Ω   Where r is the internal resistance of the second cell.

∴ Current, $I = \dfrac{2.2}{1 + r} = 1.1$ or r = 1 Ω

Q. 3. ***Two identical cells of emf 1.5 V each joined in parallel provide supply to an external circuit consisting of two resistances of 17 Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of cells to be 1.4 V. Calculate the internal resistance of each cell.***

Sol. Here $\mathcal{E} = 1.5$ V, V = 1.4 V

Resistance of external circuit = Total resistance of two resistances of 17 Ω connected in parallel

or $R = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{17 \times 17}{17 + 17}\,\Omega = 8.5\,\Omega$

Let r' be the total internal resistance of the two cells. Then

$r' = R\left(\dfrac{\mathcal{E} - V}{V}\right) = 8.5\left(\dfrac{1.5 - 1.4}{1.4}\right)\Omega = 0.6\,\Omega$

As the two cells of internal resistance r Ω each have been connected in parallel, therefore.

$\dfrac{1}{r'} = \dfrac{1}{r} + \dfrac{1}{r}$ or $\dfrac{1}{0.6} = \dfrac{2}{r}$

or r = 0.6 × 2 = 1.2 Ω

17 Ω
17 Ω
I   I
1.5 V   r
1.5 V   r

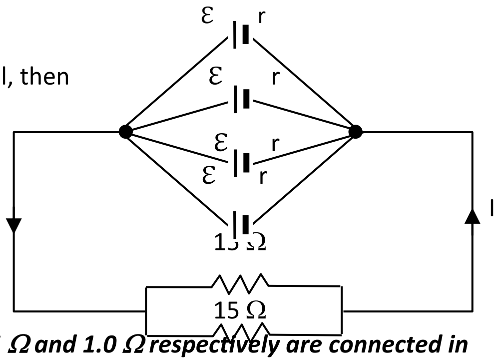
Q. 4. ***Four identical cells, each of emf 2 V, are joined in parallel providing supply of current to external circuit consisting of two 15 Ω resistors joined in parallel. The terminal voltage of the cells, as read by an ideal voltmeter is 1.6 volt. Calculate the internal resistance of each cell.***

Sol. As shown in fig. four cell are connected in parallel to the parallel combination of two 15 Ω resistors.

Here $\mathcal{E} = 2$ V, V = 1.6 V

The external resistance provided by two 15 Ω resistors connected in parallel is

$$R = \frac{15 \times 15}{15 + 15} = 7.5\ \Omega$$



If r' is the total internal resistance of the four cells connected in parallel, then

$$r' = R\left(\frac{\mathcal{E} - V}{V}\right) = 7.5\left(\frac{2 - 1.6}{1.6}\right) = \frac{15}{8}\ \Omega$$

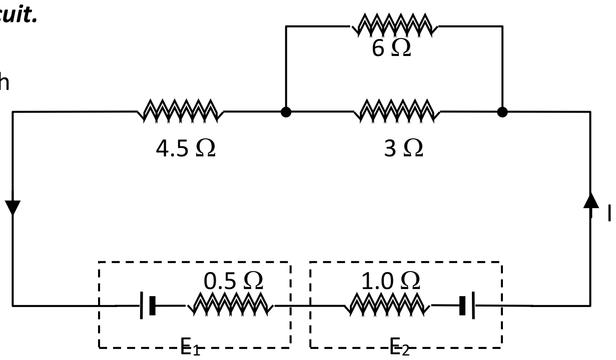
If r is the internal resistance of each cell, then

$$\frac{1}{R'} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{4}{r}$$

or $r = 4r' = 4 \times \frac{15}{8} = 7.5\ \Omega$

Q. 5. ***Two cells E_1 and E_2 of emfs 4 V and 8 V having internal resistances 0.5 Ω and 1.0 Ω respectively are connected in opposition to each other. This combination is connected in series with resistances of 4.5 Ω and 3.0 Ω. Another resistance is connected in parallel across the 3 Ω resistor.***
(a) Draw the circuit diagram.
(b) Calculate the total current flowing through the circuit.

Sol. (a) The circuit diagram is shown below.



(b) As the resistances of 6 Ω and 3 Ω are in parallel with each other, their equivalent resistance is

$$R' = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

Total resistance in the circuit

= 4.5 + 2 + 0.5 + 1.0 = 8 Ω

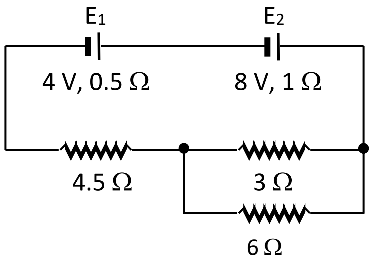
Net emf = 8 – 4 = 4 V

Current, $I = \frac{4}{8} = 0.5$ A

Q. 6. ***In the circuit diagram given in Fig. the cells E_1 and E_2 have emfs 4V and 8 V and internal resistances 0.5 Ω and 1.0 Ω respectively. Calculate the current in each resistance.***

Sol. Effective emf of the circuit

$= \mathcal{E}_2 - \mathcal{E}_1 = 8 - 4 = 4$ V



Total resistance of the circuit

$$= 1 + 0.5 + 4.5\ \Omega + \frac{3 \times 6}{3 + 6} = 8\ \Omega$$

∴ Current in the circuit, $I = \frac{4}{8} = 0.5$ A

Current through 4.5 Ω resistance = I = 0.5 A

P.D. across the parallel combination of 3 Ω and 6 Ω resistance is

$$V = R'I = \frac{3 \times 6}{3 + 6} \times 0.5 = 1\ V$$

Current through 3 Ω resistance = $\frac{1\ V}{3\ \Omega} = \frac{1}{3}$ A ; Current through 6 Ω resistance = $\frac{1\ V}{6\ \Omega} = \frac{1}{6}$ A

Q. 7. ***In Fig. $\mathcal{E}_1$ and $\mathcal{E}_2$ are respectively 2.0 V and 4.0 V and the resistances r_1, r_2 and R are respectively 1.0 Ω, 2.0 Ω and 5.0 Ω. Calculate the current in the circuit. Also calculate (i) potential difference between the points b and a, (ii) potential difference between a and c.***

Sol. As emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ are opposing each other and $\mathcal{E}_2 > \mathcal{E}_1$, so

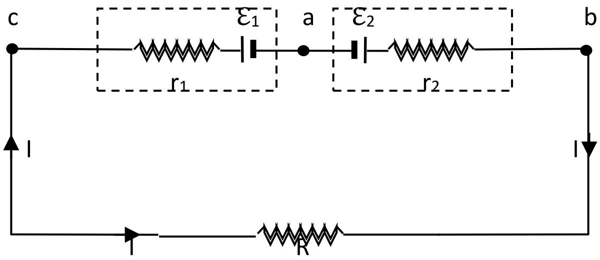
Net emf = $\mathcal{E}_2 - \mathcal{E}_1 = 4 - 2 = 2$ V

This emf sends circuit I in the anticlockwise direction.

Total resistance = $R + r_1 + r_2 = 5 + 1 + 2 = 8\ \Omega$

Current in the circuit

$$= \frac{\text{Net emf}}{\text{Total resistance}} = \frac{2}{8} = 0.25 \text{ A}$$



(i) Current inside the cell $\mathcal{E}_2$ flows from –ve to +ve terminal, so the terminal p.d. of this cell is $V_a - V_b = \mathcal{E}_2 - Ir_2 = 4.0 - 0.25 \times 2.0 = 3.5$ V

(ii) Current inside the cell $\mathcal{E}_1$ flows from + ve to –ve terminal. Hence the terminal p.d. of this cell is

$V_a - V_c = \mathcal{E}_1 + Ir_1 = 2.0 + 0.25 \times 1.0 = 2.25$ V

Q. 9. ***36 cells each of internal resistance 0.5 Ω and emf 1.5 V each are used to send current through an external circuit of 2 Ω resistance. Find the best mode of grouping them and the current through the external circuit.***

Sol. Here $\mathcal{E} = 1.5$ V, r = 0.5 V, r = 0.5 Ω, R = 2 Ω

Total number of cells, mn = 36 ... (1)

For maximum current in the mixed grouping,

$$\frac{nr}{m} = R \quad \text{or} \quad \frac{n \times 0.5}{m} = 2 \quad \text{... (2)}$$

Multiplying equations (1) and (2), we get

$$0.5\,n^2 = 72 \quad \text{or} \quad n^2 = 144$$

$$\therefore \quad n = 12 \quad \text{and} \quad m = \frac{36}{12} = 3$$

Thus, for maximum current there should be three rows in parallel, each containing 12 cells in series.

∴ Maximum current

$$= \frac{mn\,\mathcal{E}}{mR + nr} = \frac{36 \times 1.5}{3 \times 2 + 12 \times 1.5} = 4.5 \text{ A}$$

Q. 11. ***Two cells of EMFs 1 V, 2 V and internal resistance 2 Ω and 1 Ω respectively are connected in (i) series, (ii) parallel. What should be the external resistance in the circuit so that the current through the resistance be the same in the two cases? In which case more heat is generated in the cells?***

Sol. Current in series circuit is given by

$$I_S = \frac{\mathcal{E}_1 + \mathcal{E}_2}{r_1 + r_2 + R} = \frac{1+2}{2+1+2} = \frac{3}{3+R}$$

When the two cells are connected in parallel,

$$\mathcal{E}_{eq} = \frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 + r_2} = \frac{1 \times 1 + 2 \times 2}{2+1} = \frac{5}{3}$$

$$\mathcal{E}_{eq} = \frac{r_1 r_2}{r_1 + r_2} = \frac{1 \times 2}{1+2} = \frac{2}{3}$$

Current in the parallel circuit is given by

$$I_p = \frac{\mathcal{E}_{eq}}{r_{eq} + R} = \frac{5/3}{2/3 + R} = \frac{5}{2 + 3R}$$

As $I_S = I_P \quad \therefore \quad \frac{3}{3+R} = \frac{5}{2+3R}$

or $6 + 9R = 15 + 5R$

or $R = 9/4 = 2.25\ \Omega$

More heat will be generated in series case due to larger resistance.

Q. 12. ***A cell of unknown emf E and internal resistance r, two unknown resistance R_1 and R_2 ($R_2 > R_1$) and a perfect ammeter are given. The current in the circuit is measured in five different situations:***

(i) Without any external resistance in the circuit, (ii) With resistance R_1 only,

(iii) With resistance R_2 only, (iv) With both R_1 and R_2 used in series combination, and

(v) With R_1 and R_2 used in parallel combination.

The current obtained in the five cases are: 0.42 A, 0.6 A, 1.05 A, 1.4 A, and 4.2 A

but not necessarily in that order. Identify the currents in the five cases listed above the calculate E, r, R_1 and R_2.

Sol. Total resistance in the five cases are:

(i) r, (ii) $r + R_1$, (iii) $r + R_2$, (iv) $r + R_1 + R_2$, (v) $r + \frac{R_1 R_2}{R_1 + R_2}$

As $R_2 > R_1$, these resistance in increasing order are

$$r,\ r + \frac{R_1 R_2}{R_1 + R_2},\ r + R_1,\ r + R_2,\ r + R_1 + R_2$$

The currents in decreasing order are:

4.2 A, 1.4 A, 1.05 A, 0.6 A, 0.42 A

$$\therefore \quad \frac{\mathcal{E}}{r} = 4.2 \quad \text{... (1)}$$

$$\frac{\mathcal{E}}{r + R_1} = 1.05 \text{ A} \quad \text{... (2)}$$

$$\frac{\mathcal{E}}{r + R_2} = 0.6 \text{ A} \quad \text{... (3)}$$

$$\frac{\mathcal{E}}{r + R_1 + R_2} = 0.42 \text{ A} \quad \text{... (4)}$$

$$\frac{\mathcal{E}}{r + \frac{R_1 R_2}{R_1 + R_2}} = 1.4 \text{ A} \quad \text{... (5)}$$

On dividing, (1) and (2)

$$\frac{r + R_1}{r} = \frac{4.2}{1.05} \quad \text{or} \quad 1 + \frac{R_1}{r} = 4 \quad \text{or} \quad R_1 = 3r$$

On dividing (1) by (3),

$r + R_2 = 4.2$ or $1 + R_2 = 7$ or $R_2 = 6r$

From (1), $\mathcal{E} = 4.2\ r$

Putting the above values in (4), we get

$$\frac{4.2\ r}{r + 3r + 6r} = 0.42 \quad \text{or} \quad r = 1\ \Omega$$

Hence $\mathcal{E} = 4.2$ V, $r = 1\ \Omega$, $R_1 = 3\ \Omega$ and $R_2 = 6\ \Omega$

Q. 13. ***A battery consists of 12 cells in series, each having an emf $\mathcal{E}$ and internal resistor r. Some of the cells in the battery are connected with wrong polarity. This battery is connected to another with wrong polarity. This battery is connected to another source of emf $2\mathcal{E}$ and internal resistance 2r. An ammeter in the circuit reads 3 A when battery and the source aid each other and 2 A in the same direction when they oppose each other. Find how many cells in the battery are connected with wrong polarity.***

Sol. Suppose n cells are connected wrongly in the battery. Then (12 – n) cells give forward emf and n cells give reverse emf.

∴ Effective emf of the battery

$$= (12 - n)\ \mathcal{E} - n\mathcal{E} = (12 - 2n)\ \mathcal{E}$$

Total resistance of the circuit with battery and source in both cases

$$= 12\ r + 2\ r = 14\ r$$

Currents in the two cases must be proportional to the emfs in the two cases

∴ $$\frac{(12 - 2n)\ \mathcal{E} + 2\ \mathcal{E}}{(12 - 2n)\ \mathcal{E} - 2\ \mathcal{E}} = \frac{3}{2} \quad \text{or} \quad \frac{(14 - 2n)\ \mathcal{E}}{(10 - 2n)\ \mathcal{E}} = \frac{3}{2}$$

or $$\frac{7 - n}{5 - n} = \frac{3}{2} \quad \text{or} \quad 14 - 2n = 15 - 3\ n$$

∴ $n = 1$

i.e., one cell has been connected with wrong polarity in the battery.

Examples based on Temperature Variation of Resistance

♦ **Formulae Used**

Temperature coefficient of resistance

$$\alpha = \frac{R_2 - R_1}{R_1\ (t_2 - t_1)}$$

If $t_1 = 0°C$ and $t_2 = t°C$, then

$$\alpha = \frac{R_t - R_o}{R_o \times t} \quad \text{or} \quad R_t = R_o\ (1 + \alpha t)$$

♦ **Units Used** Resistance are in Ω, temperature in °C or K.

Q. 1. ***(i) At what temperature would the resistance of a copper conductor be double its resistance at 0 ℃? (ii) Dies this temperature hold for all copper conductors regardless of shape and size? Given α for Cu = 3.9×10^{-3} ℃$^{-1}$.***

Sol. (i) $$\alpha = \frac{R_2 - R_1}{R_1\ (t_2 - t_1)} = \frac{2\ R_0 - R_0}{R_0\ (t - 0)} = \frac{1}{t}$$

∴ $$t = \frac{1}{\alpha} = \frac{1}{3.9 \times 10^{-3}} = 256°\ C$$

Thus the resistance of copper conductor becomes double at 256°C.

(ii) Since α does not depend on size and shape of the conductor, so the above result holds for all copper conductors.

Q. 2. ***The resistance of the platinum wire of a platinum resistance thermometer at the ice point is 5 Ω and at steam point is 5.39 Ω. When the thermometer is inserted in a hot bath, the resistance of the platinum wire is 5.975 Ω. Calculate the temperature of the bath.***

Sol. Here $R_0 = 5\ \Omega$, $R_{100} = 5.23\ \Omega$, $R_t = 5.795\ \Omega$

As $R_t - R_0\ (1 + \alpha t)$

∴ $R_t - R_0 = R_0\ \alpha t$ ... (i)

and $R_{100} - R_0 = R_0\ \alpha \times 100$ ... (ii)

On dividing (i) by (ii), we get

$$\frac{R_t - R_0}{R_{100} - R_0} = \frac{t}{100}$$

or $$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 = \frac{5.795 - 5}{5.23 - 5} = \frac{0.795}{0.23} \times 100 = 345.56°\ C$$

Q. 3. ***A nichrome heating element connected to a 220 V supply draws an initial current of 2.2 A which settles down after a few seconds to a steady value of 2.0 A. Find the steady temperature of the heating element. The room temperature is 30 ℃ and the average temperature coefficient of resistance of nichrome is 1.7×10^{-4} per ℃.***

Sol. Here $V = 220$ V, $I_1 = 2.2$ A, $I_2 = 2.0$ A, $\alpha = 1.7 \times 10^{-4}\ °C^{-1}$

Resistance at room temperature of 30°C,

$$R_1 = \frac{V}{I_1} = \frac{220}{2.2} = 100\ \Omega$$

Resistance at steady temperature,

$$R_2 = \frac{V}{I_2} = \frac{220}{2.0} = 110\ \Omega$$

As $$\alpha = \frac{R_2 - R_1}{R_1\ (t_2 - t_1)}$$

∴ $$t_2 - t_1 = \frac{R_2 - R_1}{R_1\ \alpha} = \frac{110 - 100}{100 \times 1.7 \times 10^{-4}} = 588°C$$

Steady temperature, $t_2 = 588 + t_1 = 588 + 30 = 618°\ C$

Q. 4. *An electric toaster used nichrome (an alloy of nickel and chromium) for its heating element. When a negligibly small current passes through it, its resistance at room temperature (27.0 ℃) is found to be 75.3 Ω. When the toaster is connected to a 230 V supply, the current settles after a few seconds to a steady value of 2.68 A. What is the steady temperature of the nichrome element? The temperature coefficient of resistance of nichrome averaged over the temperature range involved is 1.70 × 10^{-4} ℃$^{-1}$.*

Sol. Here $R_1 = 75.3\ \Omega$, $t_1 = 27°C$ $\qquad R_2 = \frac{230}{2.68} = 85.8\ \Omega$, $t_2 = ?$

$$t_2 - t_1 = \frac{R_2 - R_1}{R_1\alpha} = \frac{85.8 - 75.3}{75.3 \times 1.70 \times 10^{-4}} = 820°C$$

Steady temperature, $t_2 = 820 + t_1 = 820 + 27 = 847°$ C

At the steady temperature, the heating effect due to the current equals heat loss to the surroundings.

Q. 5. *The resistance of a tungsten filament at 150 °C is 133 ohm. What will be its resistance at 500 °C? The temperature coefficient of resistance of tungsten is 0.0045 per ℃.*

Sol. Here $R_{150} = 133\ \Omega$, $\alpha = 0.0045°$ C, $R_{500} = ?$

Now $R_t = R_0(1 + \alpha t)$

∴ $R_{150} = R_0(1 + \alpha \times 150)$

or $133 = R_0(1 + 0.0045 \times 150)$ ... (1)

and $R_{500} = R_0(1 + \alpha \times 500)$

or $R_{500} = R_0(1 + 0.0045 \times 500)$ ... (2)

Dividing (2) by (1), we get

$$\frac{R_{500}}{133} = \frac{1 + 0.0045 \times 500}{1 + 0.0045 \times 150} = \frac{3.25}{12.675}$$

or $R_{500} = \frac{3.25}{1.675} \times 133 = 258\ \Omega$

Q. 6. *The resistance of a conductor at 20 °C is 3.15 Ω and at 100 °C is 3.75 Ω. Determine the temperature coefficient of resistance of the conductor. What will be the resistance of the conductor at 0 ℃?*

Sol. $R_1 = R_0(1 + \alpha t_1)$ and $R_2 = R_0(1 + \alpha t_2)$

On dividing, $\frac{R_1}{R_2} = \frac{1 + \alpha t_1}{1 + \alpha t_2}$

or $R_1(1 + \alpha t_2) = R_2(1 + \alpha t_1)$

or $\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$

Here $t_1 = 20°C$, $R_1 = 3.15\ \Omega$, $t_2 = 100°$ C, R_2 3.75 Ω

∴ $\alpha = \frac{3.75 - 3.15}{(3.15 \times 100) - (3.75 \times 20)}$

$= \frac{0.60}{315 - 75} = \frac{0.60}{240} = 0.0025°\ C^{-1}$

$R_0 = \frac{R_t}{1 + \alpha t_1} = \frac{3.15}{1 + 0.0025 \times 20} = 3.0\ \Omega$

Q. 7. *A standard coil marked 2 Ω is found to have a resistance of 2.118 Ω at 30 °C. Calculate the temperature at which the marking is correct. The temperature coefficient of the resistance of the material of the coil is 0.0042 ℃$^{-1}$.*

Sol. $R_1 = R_0(1 + \alpha t_1)$ and $R_2 = R_0(1 + \alpha t_2)$

∴ $\frac{R_1}{R_2} = \frac{1 + \alpha t_1}{1 + \alpha t_2}$

Here, $R_1 = 2\Omega$, $R_2 = 2.118\ \Omega$, $t_2 = 30°$ C, $t_1 = ?$

∴ $\frac{2}{2.118} = \frac{1 + 0.0042 \times t_1}{1 + 0.0042 \times 30} = \frac{1 + 0.0042 \times t_1}{1.126}$

or $1 + 0.0042\ t_1 = \frac{2 \times 1.126}{2.118} = \frac{2.252}{2.118}$

∴ $t_1 = \frac{1}{0.0042}\left(\frac{2.252}{2.118} - 1\right) = \frac{0.104}{0.0042 \times 2.118} = 15°$ C $\qquad$ i.e., the marking will be correct at 15° C.

Q. 8. *A potential difference of 200 V is applied to a coil at a temperature of 15 ℃ and the current is 10 A. What will be the mean temperature of the coil when the current has fallen to 5 A, the applied voltage being same as before? Given $\alpha = \frac{1}{234}$ ℃$^{-1}$ at 0 °C.*

Sol. In the second case, the current decreases due to the increase in resistance on heating.

Now, $R_{15} = \frac{V}{I} = \frac{200}{10} = 20\ \Omega$

Let t be the temperature at which current falls to 5 A. Then

$R_t = \frac{200}{5} = 40\ \Omega$

As $R_t = R_0(1 + \alpha t)$

$R_{15} = R_0\left(1 + \frac{15}{234}\right)$

or $20 = \frac{R_0 \times 249}{234}$ ... (1)

$R_t = R_0\left(1 + \frac{t}{234}\right)$

or $40 = R_0\left(\frac{234 + t}{234}\right)$ ... (2)

Dividing (2) by (1), $2 = \frac{234 + t}{249}$

or $t = 498 - 234 = 264°\ C$

Q. 9. ***The resistance of iron and copper wires at 20°C are 3.9 Ω and 4.1 Ω respectively. At what temperature will the resistance be equal? Temperature coefficient of resistivity for iron is $5.0 \times 10^{-3}\ K^{-1}$ and for copper it is $4.0 \times 10^{-3}\ K^{-1}$. Neglect any thermal expansion.***

Sol. Let resistance of iron wire at t°C = Resistance of copper wire at t°C

∴ $R_{20}\,[1 + \alpha\,(t - 20)] = R_{20}'\,[1 + \alpha'\,(t - 20)]$

$3.9\,[1 + 5.0 \times 10^{-3}\,(t - 20)] = 4.1\,[1 + 4.0 \times 10^{-3}\,(t - 20)]$

$[3.9 \times 5 - 4.1 \times 4] \times 10^{-3} \times (t - 20) = 4.1 - 3.9$

$t - 20 = \frac{0.2}{3.1 \times 10^{-3}} = 64.5$

$t = 64.5 + 20 = 84.5°C$

Q. 10. ***A carbon filament has a resistance of 100Ω at 0℃. What must be the resistance of a copper filament placed in series with carbon so that the combination has the same resistance at all temperature? Temperature coefficient of resistance of carbon = – 0.0007 C^{-1} and that of copper is 0.004 C^{-1}.***

Sol. As $\alpha = \frac{R - R_0}{R_0\,t}$

∴ Change in resistance = $\alpha\,R_0\,t$

Let the resistance of copper placed in series with carbon at 0°C be R, so that the combination has the same resistance of copper per °C = Decrease in resistance of carbon per °C

$(\alpha\,R_0\,t)$ for copper = $(\alpha\,R_0\,t)$ for carbon

$0.004 \times R \times t = 0.0007 \times 100 \times t$

$R = \frac{0.0007 \times 100}{0.0004} = 17.5\ \Omega$

Q. 11. ***A metal wire of diameter 2 mm and length 100 m has a resistance of 0.5475 Ω at 20°C and 0.805 Ω at 150°C. Find (i) the temperature coefficient of resistance (ii) resistance at 0℃ (iii) resistivities at 0° and 20°C.***

Sol. Here $r = 1\text{ mm} = 10^{-3}\text{ m},\ l = 100\text{ m},\ t_1 = 20°\text{ C},\ R_1 = 0.5475\ \Omega,\ t_2 = 150°\text{ C},\ R_2 = 0.805\ \Omega$

(i) Temperature coefficient of resistance is

$\alpha = \frac{R_2 - R_1}{R_1\,(t_2 - t_1)} = \frac{0.805 - 0.5475}{0.547\,(150 - 20)}$

$= 3.6 \times 10^{-3}\ °C^{-1}$

(ii) Resistance at °C is

$R_0 = \frac{R_1}{1 + \alpha t_1} = \frac{0.5475}{1 + 3.6 \times 10^{-3} \times 20} = \frac{0.5475}{1.072}$

$= 0.5107\ \Omega$

(iii) Resistivity at 0°C, $\rho = \frac{R_0\,A}{l} = \frac{R_0 \times \pi r^2}{l}$

$= 0.5107 \times 3.14 \times (10^{-3})^2 = 1.60 \times 10^{-8}\ \Omega m$

Resistivity at 20°C is

$\rho_{20} = \rho_0\,(1 + \alpha t) = 1.60 \times 10^{-8}\,(1 + 3.6 \times 10^{-3} \times 20)$

$= 1.60 \times 10^{-8} \times 1.072 = \mathbf{1.72 \times 10^{-8}\ \Omega m.}$