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POLYNOMIAL

★ INTRODUCTION

In class IX, we have studied the polynomials in one variable and their degrees. We have also learnt about the values and the zeros of a polynomial. In this chapter, we wil discuss more about the zeros of a polynomial and the relationship between the zeros and the coefficients of a polynomial with particular reference to quadratic polynomials. In addition, statement and simple problems on division algorithm for polynomials with real coefficients will be discussed.

★ HISTORICAL FACTS


Rene Descartes

Determining the roots of polynomials, or "solving algebraic equations", is among the oldest problems in mathematics. However, the elegant and practical notation we use today only developed beginning in the 15th century. Before that, equations were written out in words. For example, an algebra problem from the Chinese Arithmetic in Nine Sections, begins "Three sheafs of good crop, two sheafs of mediocre crop, and one sheaf of bad crop are sold for 29 dou". We would write $3x + 2y + z = 29$.

The earliest known use of the equal sign is in Robert Recorde's The Whetstone of Witte, 1557. The signs + for addition, – for subtraction, and the use of a letter for an unknown appear in Michael Stifel's Arithmetica Integra, 1544. **Rene Descartes**, in La geometrie, 1637, introduced the concept of the graph of a polynomial equation. He popularized the use of letters from the beginning of the alphabet to denote constants and letters from the end of the alphabet to denote variables, as can be seen in the general formula for a polynomial, where the a's denote constants and x denotes a variable. Descartes introduced the use of superscripts to denote exponents as well.

★ RECALL

(i) **Polynomials** : An algebraic expression of the form $p(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots\ldots\ldots + a_1x^1 + a_0x^0$ where $a_n \neq 0$ and $a_0, a_1, a_2, \ldots\ldots a_n$ are real numbers and each power of x is a positive integer, is called a polynomial.

Hence, a_n, a_{n-1}, a_{n-2}, are coefficients of $x^n, x^{n-1} \ldots\ldots\ldots\ldots x^0$ and $a_nx^n, a_{n-1}x^{n-1}, a_{n-2}x^{n-2}, \ldots\ldots\ldots\ldots$ are terms of the polynomial. Here the term a_nx^n is called the *leading term* and its coefficient a_n, the *leading coefficient.*

For example : $p(u) = \frac{1}{2}u^3 - 3u^2 + 2u - 4$ is a polynomial in variable u.

$\frac{1}{2}u^3, -3u^2, 2u, -4$ are known as terms of polynomial and $\frac{1}{2}, -3, 2, -4$ are their respective coefficients.

$6x^{-2}$	This is NOT a polynomial term	Because the variable has a negative exponent
$\frac{1}{x^2}$	This is NOT a polynomial term	Because the variable is in the denominator
sqrt(x)	This is NOT a polynomial term	Because the variable is inside a radical
$4x^2$	This IS a polynomial term	Because it obeys all the rules

(ii) **Types of Polynomials** : Generally we divide the polynomials in three categories.

Types of Polynomials
- Based on number of distinct variables
- Based on degree
- Based on number of terms

Polynomials classified by number of distinct variables

Number of distinct variables	Name	Example
1	Univariate	$x + 9$
2	Bivariate	$x + y + 9$
3	Trivariate	$x + y + z + 9$

Generally, a polynomial in more than one variable is called a **multivariate polynomial.**

A second major way of classifying polynomials is by their degree. Recall that the degree of a term is the sum of the exponents on variables, and that the degree of a polynomial is the largest degree of any one term.

Polynomials classified by degree

Degree	Name	Example
$-\infty$	Zero	0
0	(non-zero) constant	1
1	Linear	$x + 1$
2	quadratic	$x^2 + 1$
3	cubic	$x^3 + 2$
4	quartic (or biquadratic)	$x^4 + 3$
5	quintic	$x^5 + 4$
6	sextic (or hexic)	$x^6 + 5$
7	septic (or heptic)	$x^7 + 6$
8	octic	$x^8 + 7$
9	nonic	$x^9 + 8$
10	decic	$x^{10} + 9$

Usually, a polynomial of degree n, for n greater than 3, is called a polynomial of degree n, although the phrases *quartic polynomial* and *quintic polynomial* are sometimes used.

The polynomial 0, which may be considered to have no terms at all, is called the **zero polynomial.** Unlike other constant polynomials, its degree is not zero. Rather the degree of the zero polynomial is either left explicitly undefined, or defined to be negative (either -1 or $-\infty$)

Polynomials classified by number of non-zero terms		
Number of non-zero terms	Name	Example
0	zero polynomial	0
1	monomial	x^2
2	binomial	$x^2 + 1$
3	trinomial	$x^2 + x + 1$

If a polynomial has only one variable, then the terms are usually written either from highest degree to lowest degree ("descending powers") or from lowest degree to highest degree ("ascending powers").

(iii) **Value of a Polynomial :** If p(x) is a polynomial in variable x and α is any real number, then the value obtained by replacing x by α in p(x) is called value of p(x) at $x = \alpha$ and is denoted by $p(\alpha)$.

For example : Find the value of $p(x) = x^3 - 6x^2 + 11x - 6$ at $x = -2$

$\Rightarrow \quad p(-2) = (-2)^3 - 6(-2)^2 + 11(-2) - 6 = -8 - 24 - 22 - 6 \quad \Rightarrow p(-2) = -60$

(iv) **Zero of a Polynomial :** A real number α is a zero of the polynomial p(x) if $p(\alpha) = 0$.

For example : Consider $p(x) = x^3 - 6x^2 + 11x - 6$

$p(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$

$p(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$

$p(3) = (3)^3 - 6(3)^2 + 11(3) - 6 = 27 - 54 + 33 - 6 = 0$

Thus, 1,2 and 3 are called the zeros of polynomial p(x).

★ GEOMETRICAL MEANING OF THE ZEROS OF A POLYNOMIAL

Geometrically the zeros of a polynomials f(x) are the x-co-ordinates of the points where the graph y = f(x) intersects x-axis. To understand it, we will see the geometrical representations of linear and quadratic polynomials.

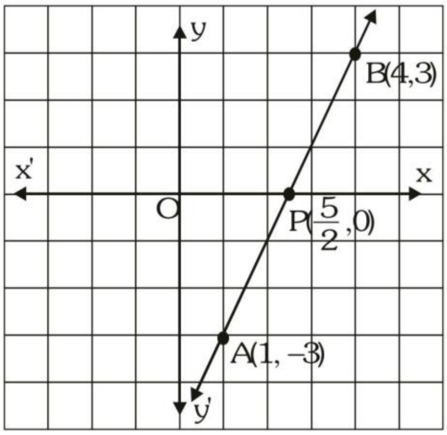
Geometrical Representation of the zero of a Linear Polynomial

Consider a linear polynomial, $y = 2x - 5$.

The following table lists the values of y corresponding to different values of x.

x	1	4
y	–3	3

On plotting the points A(1, –3) and B(4, 3) and joining them, a straight line is obtained.



From, graph we observe that the graph of $y = 2x - 5$ intersects the x-axis at $\left(\frac{5}{2}, 0\right)$ whose x-coordinate is $\frac{5}{2}$.

Also, zero of $2x - 5$ is $\frac{5}{2}$.

Therefore, we conclude that the linear polynomial ax + b has one and only one zero, which is the x-coordinate of the point where the graph of y = ax + b intersects the x-axis

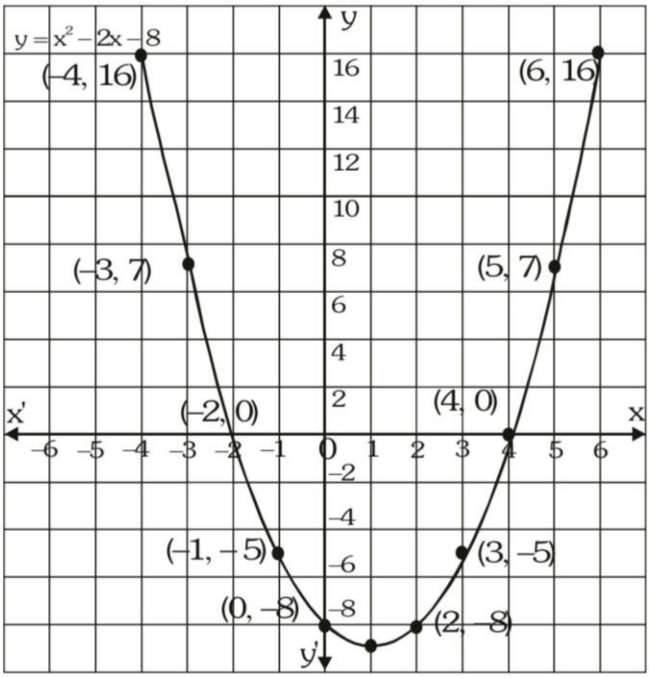
Geometrical Representation of the zero of a Quadratic Polynomial :

Consider a quadratic polynomial, $y = x^2 - 2x - 8$,

The following table gives the values of y or f(x) for various values of x.

x	−4	−3	−2	−1	0	1	2	3	4	5	6
$y = x^2 - 2x - 8$	16	7	0	-5	−8	−9	−8	−5	0	7	16

On plotting the points (–4, 16), (–3, 7) (–2, 0), (–1, –5), (0, –8), (1, –9) (2, –8), (3, –5), (4, 0), (5, 7) and (6, 16) on a graph paper and drawing a smooth free hand curve passing through these points, the curve thus obtained represents the graph of the polynomial $y = x^2 - 2x - 8$. This is called a parabola.

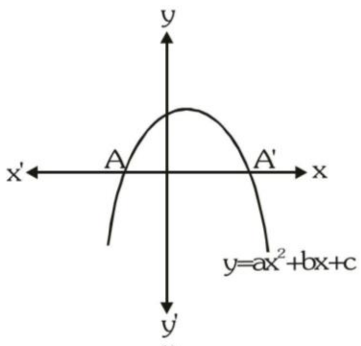


It is clear from the table that –2 and 4 are the zeros of the quadratic polynomial $x^2 - 2x - 8$. Also, we observe that –2 and 4 are the x-coordinates of the points where the graph of $y = x^2 - 2x - 8$ intersects the x-axis.

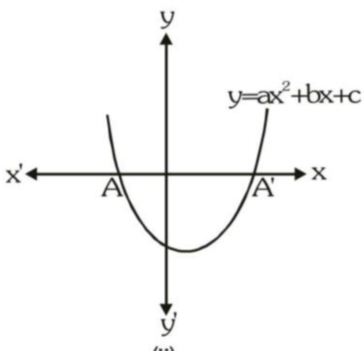
Consider the following cases –

Case-I : Here, the graph cuts x-axis at two distinct points A and A'.

The x-coordinates of A and A' are two zeroes of the quadratic polynomial $ax^2 + bx + c$.

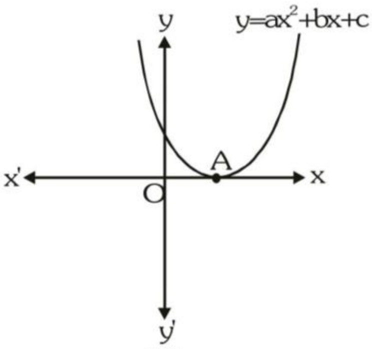


(i)

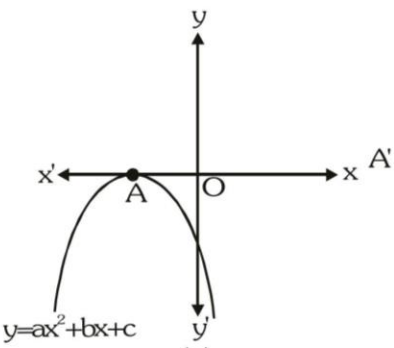


(ii)

Case-II : Here, the graph cuts the x-axis at exactly one point, i.e., at two coincident points. So, the two points A and A' of Case (i) coincide here to become one point A.



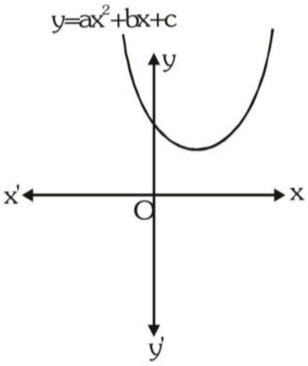
(iii)



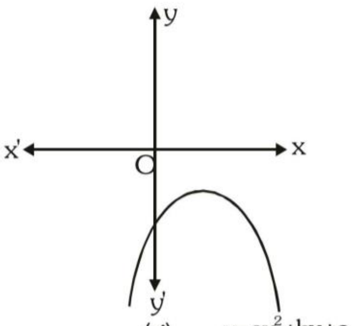
(iv)

The x-coordinate of A is the only zero for the quadratic polynomial $ax^2 + bx + c$ in this case.

Case-III : Here, the graph is either completely above the x-axis or completely below the x-axis. So, it does not cut the x-axis at any point.



(v)



(vi)

So, the quadratic polynomial $ax^2 + bx + c$ has no zero in this case.

So, you can see geometrically that a quadratic polynomial can have either two distinct zeroes or one zero, or no zero. This also means that a polynomial of degree 2 has atmost two zeroes.

Remark : In general given a polynomial p(x) of degree n, the graph of y = p(x) intersects the x-axis at atmost n points. Therefore, a polynomial p(x) of degree n has atmost n zeros.

★ RELATIONSHIP BETWEEN THE ZEROS AND COEFFICIENTS OF A POLYNOMIAL

For a linear polynomial ax + b, $(a \neq 0)$, we have,

$$\text{zero of a linear polynomial} = -\frac{b}{a} = -\frac{(\text{constant term})}{(\text{coefficient of } x)}$$

For a quadratic polynomial $ax^2 + b + c$ $(a \neq 0)$, with α and β as it's zeros, we have

$$\text{Sum of zeros} = \alpha + \beta = -\frac{b}{a} = -\frac{(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeros} = \alpha\,\beta = \frac{c}{a} = \frac{(\text{constant term})}{(\text{coefficient of } x^2)}$$

If α and β are the zeros of a quadratic polynomial f(x). Then polynomial f(x) is given by

$$f(x) = K\{x^2 - (\alpha + \beta)x + \alpha\beta\}$$

or $\quad f(x) = K\{x^2 - (\text{sum of the zeros})\, x + \text{product of the zeros}\}$

where K is a constant.

COMPETITION WINDOW

RELATIONSHIP BETWEEN THE ZEROS AND COEFFICIENTS OF A CUBIC POLYNOMIAL

For a cubic polynomial $ax^3 + bx^2 + cx + d$ $(a \neq 0)$, with α, β and γ as its zeros, we have :

$$\text{Sum of three zeros} = \alpha + \beta + \gamma = \frac{-b}{a}$$

$$\text{Sum of the product of its zeros taken two at a time} = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\text{Product of its zeros} = \alpha\beta\gamma = \frac{-d}{a}$$

The cubic polynomial whose zeros are α, β and γ is given by $f(x) = \{x^3 - (\alpha + \beta + \gamma)\, x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)\, x - \alpha\beta\gamma\}$

RELATIONSHIP BETWEEN THE ZEROS AND COEFFICIENTS OF A BI-QUADRATIC POLYNOMIAL

For a bi-quadratic polynomial $ax^4 + bx^3 + cx^2 + dx + e$ $(a \neq 0)$, with α, β, γ and δ as its zeros, we have :

Sum of four zeros $= \alpha + \beta + \gamma + \delta = \frac{-b}{a}$

Sum of the product of its zeros taken two at a time $= \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$

Sum of the product of its zeros taken three at a time $= \alpha\beta\gamma + \alpha\beta\delta + \beta\gamma\delta + \gamma\delta\alpha = \frac{-d}{a}$

Product of all the four zeros $= \alpha\beta\gamma\delta = \frac{e}{a}$

The bi-quadratic polynomial whose zeros are α, β, γ and δ is given by

$$f(x) = \{x^4 - (\alpha + \beta + \gamma + \delta)\,x^3 + (\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)\,x^2 - (\alpha\beta\gamma + \alpha\beta\delta + \beta\gamma\delta + \gamma\delta\alpha)\,x + \alpha\beta\gamma\delta\}$$

Ex.1 Find the zeros of the quadratic polynomial $x^2 + 7x + 12$, and verify the relation between the zeros and its coefficients.

Sol. We have,

$f(x) = x^2 + 7x + 12 = x^2 + 4x + 3x + 12$

$\Rightarrow$ $f(x) = x(x + 4) + 3(x + 4)$

$\Rightarrow$ $f(x) = (x + 4)(x + 3)$

The zeros of f(x) are given by

$f(x) = 0$

$\Rightarrow$ $x^2 + 7x + 12 = 0$

$\Rightarrow$ $(x + 4)(x + 3) = 0$

$\Rightarrow$ $x + 4 = 0$ or, $x + 3 = 0$

$\Rightarrow$ $x = -4$ or $x = -3$

Thus, the zeros of $f(x) = x^2 + 7x + 12$ are $\alpha = -4$ and $\beta = -3$

Now, sum of the zeros $= \alpha + \beta = (-4) + (-3) = -7$

and $-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{7}{1} = -7$

$\therefore$ Sum of the zeros $= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= \alpha\beta = (-4)$ Ч $(-3) = 12$

and, $\frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{12}{1} = 12$

$\therefore$ Product of the zeros $= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Ex.2 Find the zeros of the quadratic polynomial $f(x) = abx^2 + (b^2 + ac)\,x + bc$ and verify the relationship between the zeros and its coefficients.

Sol. $f(x) = abx^2 + (b^2 + ac)\,x + bc = abx^2 + b^2x + acx + bc$

$= bx\,(ax + b) + c\,(ax + b) = (ax + b)\,(bx + c)$

So, the value of f(x) is zero when $ax + b = 0$ or $bx + c = 0$, i.e. $x = \frac{-b}{a}$ or $x = \frac{-c}{b}$

Therefore, $\frac{-b}{a}$ and $\frac{-c}{b}$ are the zeros (or roots) of f(x).

Now, sum of zeros $= \left(\frac{-b}{a}\right) + \left(\frac{-c}{b}\right) = \frac{-b^2 - ac}{ab} = \frac{-(b^2 + ac)}{ab} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of zeros $= \left(\frac{-b}{a}\right)\left(\frac{-c}{b}\right) = \frac{bc}{ab} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

★ SYMMETRIC FUNCTIONS OF THE ZEROS

Let α,β be the zeros of a quadratic polynomial, then the expression of the form $\alpha + \beta$; $(\alpha^2 + \beta^2)$; $\alpha\beta$ are called the functions of the zeros. By symmetric function we mean that the function remain invariant (unaltered) in values when the roots are changed cyclically. In other words, an expression involving α and β which remains unchanged by interchanging α and β is called a symmetric function of α and β.

Some useful relations involving α and β are :-

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

(ii) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

(iii) $\alpha^2 - \beta^2 = (\alpha + \beta)\,(\alpha - \beta) = (\alpha + \beta)\,\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

(iv) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta)$

(v) $\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta\,(\alpha - \beta)$

(vi) $\alpha^4 - \beta^4 = (\alpha^2 + \beta^2)\,(\alpha + \beta)\,(\alpha - \beta) = [(\alpha + \beta)^2 - 2\alpha\beta]\,(\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

(vii) $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

(viii) $\alpha^5 + \beta^5 = (\alpha^3 + \beta^3)\,(\alpha^2 + \beta^2) - \alpha^2\beta^2\,(\alpha + \beta) = [(\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta)]\,[(\alpha + \beta)^2 - 2\alpha\beta] - (\alpha\beta)^2\,(\alpha + \beta)$

Ex.3 If r and s are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$ then calculate :

(i) $r^2 + s^2$ (ii) $\frac{r^c}{s} < \frac{s^c}{r}$

Sol. Since α and β are the zeros of the quadratic polynomial

$f(x) = ax^2 + bx + c$

∴ $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

(i) We have,

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = \left(\frac{-b}{a}\right)^2 - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$

(ii) We have, $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta} = \frac{\left(-\frac{b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)}{\frac{c}{a}} \Rightarrow \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{3abc - b^3}{a^2c}$

Ex.4 If r and s are the zeros of the quadratic polynomial p(s) = $3s^2 - 6s + 4$, find the value of $\frac{r}{s}+\frac{s}{r}+2\left(\frac{1}{r}+\frac{1}{s}\right)+3rs$

Sol. Since α and β are the zeros of the polynomial p(s) = $3s^2 - 6s + 4$.

$\therefore \quad \alpha+\beta = \frac{-(-6)}{3} = 2$ and $\alpha\beta = \frac{4}{3}$

We have $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta = \frac{\alpha^2+\beta^2}{\alpha\beta}+2\left(\frac{\beta+\alpha}{\alpha\beta}\right)+3\alpha\beta$

$= \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}+\frac{2(\alpha+\beta)}{\alpha\beta}+3\alpha\beta = \frac{(2)^2-2\times\frac{4}{3}}{\frac{4}{3}}+\frac{2\times 2}{\frac{4}{3}}+3\times\frac{4}{3} = 8$

Ex.5 If r1and1s are the roots (zeros) of the polynomial f(x) = $x^2 - 3x + k$ such that r1–1s = 1, find the value of k.

Sol. Since α and β are the roots (zeros) of the polynomial f(x) = $x^2 - 3x + k$.

$\therefore \quad \alpha+\beta = \frac{-(-3)}{1} = 3$ and $\alpha\beta = k$.

We have $\alpha - \beta = 1 \Rightarrow (\alpha-\beta)^2 = (1)^2 \Rightarrow \alpha^2 - 2\alpha\beta + \beta^2 = 1$

$\Rightarrow \quad (\alpha^2+\beta^2) - 2\alpha\beta = 1 \Rightarrow \{(\alpha+\beta)^2 - 2\alpha\beta\} - 2\alpha\beta = 1$

$\Rightarrow \quad (\alpha+\beta)^2 - 4\alpha\beta = 1 \Rightarrow (3)^2 - 4$ Ч $k = 1$

$\Rightarrow \quad 9 - 4k = 1 \Rightarrow 4k = 8 \Rightarrow k = 2$

Hence, the value of k is 2.

Ex.6 If r=s are the zeros of the polynomial f(x) = $2x^2 + 5x + k$ satisfying the relation $r^2 + s^2 + rs = \frac{21}{4}$, then find the value of k for this to be possible.

Sol. Since α and β are the zeros of the polynomial f(x) = $2x^2 + 5x + k$.

$\therefore \quad \alpha + \beta = \frac{-5}{2}$ and $\alpha\beta = \frac{k}{2}$

Now, $\quad \alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$

$\Rightarrow \quad (\alpha^2 + \beta^2 + 2\alpha\beta) - \alpha\beta = \frac{21}{4}$

$\Rightarrow \quad (\alpha+\beta)^2 - \alpha\beta = \frac{21}{4}$

$\Rightarrow \quad \frac{25}{4} - \frac{k}{2} = \frac{21}{4} \qquad \left[\because \alpha+\beta = -\frac{5}{2} \text{ and } \alpha\beta = \frac{k}{2}\right]$

$\Rightarrow \quad -\frac{k}{2} = -1$

$\Rightarrow \quad k = 2$

Ex.7 Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

(i) $\frac{1}{4}, -1$ (ii) $\sqrt{2}, \frac{1}{3}$

Sol. We know that a quadratic polynomial when the sum and product of its zeros are given is given by –

f(x) = k {x^2 – (Sum of the zeros) x + Product of the zeros}, where k is a constant.

(i) Required quadratic polynomial f(x) is given by

$$f(x) = k\left(x^2 - \frac{1}{4}x - 1\right)$$

(ii) Required quadratic polynomial f(x) is given by

$$f(x) = k\left(x^2 - \sqrt{2}x + \frac{1}{3}\right)$$

Ex.8 If r=s are the zeros of the polynomial $ax^2 + bx + c$, find a polynomial whose zeros are $\frac{1}{a\alpha+b}$ and $\frac{1}{a\beta+b}$

Sol. Since α and β are the zeros of the polynomial $ax^2 + bx + c$.

m $\alpha+\beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

Since $\frac{1}{a\alpha+b}$ and $\frac{1}{a\beta+b}$ are the zeros of the required polynomial.

m sum of the zeros $= \frac{1}{a\alpha+b} + \frac{1}{a\beta+b} = \frac{a\beta+b+a\alpha+b}{(a\alpha+b)(a\beta+b)}$

$$= \frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2} = \frac{a\times\left(\frac{-b}{a}\right)+2b}{a^2\times\left(\frac{c}{a}\right)+ab\times\left(\frac{-b}{a}\right)+b^2} = \frac{b}{ac}$$

Product of the zeros $= \left(\frac{1}{a\alpha+b}\right)\left(\frac{1}{a\beta+b}\right) = \frac{1}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$

$$= \frac{1}{a^2\times\frac{c}{a}+ab\times\left(\frac{-b}{a}\right)+b^2} = \frac{1}{ac}$$

Hence, the required polynomial = x^2 – (sum of zeros) x + product of zeros = $x^2 - \left(\frac{b}{ac}\right)x + \frac{1}{ac}$

★ DIVISION ALGORITHM FOR POLYNOMIALS

If f(x) is a polynomial and g(x) is a non-zero polynomial, then there exist two polynomials q(x) and r(x) such that f(x) = g(x) Ч q(x) + r(x) , where r(x) = 0 or degree r(x) < degree g(x). In other words,

Dividend = Divisor Ч Quotient + Remainder

Remark : If r(x) = 0 , then polynomial g(x) is a factor of polynomial f(x).

Ex.9 Divide the polynomial $2x^2 + 3x + 1$ by the polynomial $x + 2$ and verify the division algorithm.

Sol. We have

$$\begin{array}{r|l} & 2x-1 \\ \hline x+2 & 2x^2+3x+1 \\ & 2x^2+4x \\ & -\quad - \\ \hline & -x+1 \\ & -x-2 \\ & +\quad + \\ \hline & 3 \end{array}$$

Clearly, quotient = $2x - 1$ and remainder = 3

Also, $(x + 2)(2x - 1) + 3 = 2x^2 + 4x - x - 2 + 3 = 2x^2 + 3x + 1$

i.e., $2x^2 + 3x + 1 = (x + 2)(2x-1) + 3$. Thus, Dividend = Divisor Ч Quotient + Remainder.

Ex.10 Check whether the polynomial $t^2 - 3$ is a factor of the polynomial $2t^4 + 3t^3 - 2t^2 - 9t - 12$, by dividing the second polynomial by the first polynomial.

Sol. We have

$$\begin{array}{r|l} & 2t^2+3t+4 \\ \hline t^2-3 & 2t^4+3t^3-2t^2-9t-12 \\ & 2t^4 \qquad -6t^2 \\ & - \qquad\quad + \\ \hline & 3t^3+4t^2-9t-12 \\ & 3t^3 \qquad -9t \\ & - \qquad\quad + \\ \hline & 4t^2 \qquad -12 \\ & 4t^2 \qquad -12 \\ & - \qquad\quad + \\ \hline & 0 \end{array}$$

Since the remainder is zero, therefore, the polynomial $t^2 - 3$ is a factor of the polynomial $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

Ex.11 Find all the zeros of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

Sol. Let $p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$ be the given polynomial. Since two zeros are $\sqrt{2}$ and $-\sqrt{2}$ so, $(x - \sqrt{2})$ and $(x + \sqrt{2})$ are both factors of the given polynomial p(x).

Also, $(x - \sqrt{2})(x + \sqrt{2}) = (x^2 - 2)$ is a factor of the polynomial. Now, we divide the given polynomial by $x^2 - 2$.

$$\begin{array}{r|l|l} x^2-2 & 2x^4-3x^3-3x^2+6x-2 & 2x^2-3x+1 \\ & 2x^4 \qquad -4x^2 & \\ & - \qquad\quad + & \\ \hline & -3x^3+x^2+6x-2 & \\ & -3x^3 \qquad +6x & \\ & + \qquad\quad - & \\ \hline & x^2 \qquad -2 & \\ & x^2 \qquad -2 & \\ & - \qquad\quad + & \\ \hline & 0 & \end{array}$$

By division algorithm, we have

$2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x^2 - 2)(2x^2 - 3x + 1)$

$\Rightarrow \quad 2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x - \sqrt{2})(x + \sqrt{2})(2x^2 - 2x - x + 1)$

$\Rightarrow \quad 2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x - \sqrt{2})(x + \sqrt{2})\{2x(x - 1) - (x - 1)\}$

$\Rightarrow \quad 2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x - \sqrt{2})(x + \sqrt{2})(x - 1)(2x - 1)$

When p(x) = 0, $x = \sqrt{2}, -\sqrt{2}, 1$ and $\frac{1}{2}$

Hence, all the zeros of the polynomial $2x^4 - 3x^3 - 3x^2 + 6x - 2$ are $\sqrt{2}, -\sqrt{2}, 1$ and $\frac{1}{2}$

Ex.12 On dividing $f(x) = x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $- 2x + 4$, respectively. Find g(x).

Sol. Here, Dividend = $x^3 - 3x^2 + x + 2$,

Quotient = $x - 2$,

Remainder = $- 2x + 4$ and Divisor = g(x).

Since Dividend = Divisor Ч Quotient + Remainder

So, $x^3 - 3x^2 + x + 2 = g(x)$ Ч $(x - 2) + (-2x + 4)$

$\Rightarrow$ g(x) Ч $(x - 2) = x^3 - 3x^2 + x + 2 + 2x - 4$

$$\Rightarrow \quad g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2} = \frac{(x-2)(x^2 - x + 1)}{x - 2} = x^2 - x + 1$$

Hence, $g(x) = x^2 - x + 1$.

★ SYNOPSIS

1. The highest power of the variable (x) in a polynomial p(x) is called a degree of polynomial p(x).
2. A polynomial of degree one is called linear polynomial :

   $p(x) = ax + b$, where $a \neq 0$ | a = coefficient of x ;
   | b = constant term
3. A polynomial of a degree two is called quadratic polynomial :

   $p(x) = ax^2 + bx + c$, where $a \neq 0$ | a = coefficient of x^2
   | b = coefficient of x
   | c = constant term
4. A polynomial of degree three is called a cubic polynomial : $p(x) = ax^3 + bx^2 + cx + d,\ a \neq 0$.
5. The zeros of a polynomial p(x) are precisely the x-coordinates of the point where the graph of $y = p(x)$ intersects the x-axis.
6. The graph of the quadratic function $y = ax^2 + bx + c,\ a \neq 0$ is a parabola.
7. The parabola opens upwards if $a > 0$ and opens downwards if $a < 0$.
8. A polynomial of degree n can have at most n zeros. So the quadratic polynomial can have at most two zeros and a cubic polynomial can have at most three zeros.
9. If α, β are the zeros of a quadratic polynomial $ax^2 + bx + c$, $a \neq 0$ then

   Sum of its zeros = $\alpha + \beta = -\frac{b}{a}$ and Product of its zeros = $\alpha\beta = \frac{c}{a}$.
10. If α,β,γ are the zeros of a cubic polynomial $ax^3 + bx^2 + cx + d,\ a \neq 0$ then

    Sum of its zeros = $\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

    Sum of the products of zeros taken two at a time = $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

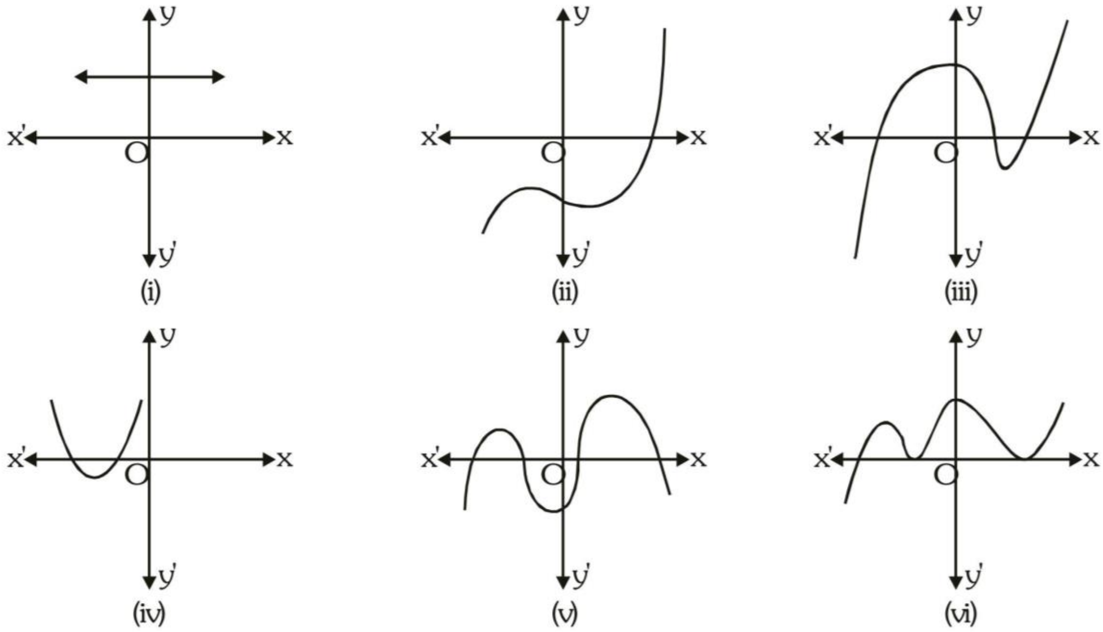
    Product of its zeros = $\alpha\beta\gamma = -\frac{d}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$
11. The division algorithm states that given any polynomial p(x) and any non-zero polynomial g(x) then we can find quotient polynomial q(x) and remainder polynomial r(x) such that :

    $p(x) = g(x).\ q(x) + r(x)$ where deg. of $r(x) <$ degree of g(x), deg of $r(x) = 0$.

SOLVED NCERT EXERCISE

EXERCISE : 2.1

1. The graph of y = p(x) are given in fig below, for some polynomials p(x). Find the number of zeros of p(x), in each case.



Sol. (i) Graph of y = p(x) does not intersect the x-axis. Hence, polynomial p(x) has no zero.

(ii) Graph of y = p(x) intersects the x-axis at one and only one point.

Hence, polynomial p(x) has one and only one real zero.

[Rest Try Yourself]

EXERCISE : 2.2

1. Find the zeros of the following quadratic polynomials and verify the relationship between the zeros and the coefficients.

(i) $x^2 - 2x - 8$ (ii) $4s^2 - 4s + 1$ (iii) $6x^2 - 3 - 7x$ (iv) $4u^2 + 8u$

(v) $t^2 - 15$ (vi) $3x^2 - x - 4$

Sol. (i) $^2 - 2x - 8 = x^2 - 4x + 2x - 8 = x(x - 4) + 2(x - 4) = (x + 2)(x - 4)$

Zeroes are – 2 and 4.

Sum of the zeros $= (-2) + (4) = 2 = \frac{-(-2)}{1} = \frac{-(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$

Product of the zeros $= (-2)(4) = -8 = \frac{(-8)}{1} = \frac{(\text{Constant term})}{(\text{Coefficient of } x^2)}$

(ii) $4s^2 - 4s + 1 = (2s - 1)^2$

The two zeros are $\frac{1}{2}, \frac{1}{2}$

Sum of the two zeros $= \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$

Product of two zeros $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4} = \frac{(\text{Constant term})}{(\text{Coefficient of } x^2)}$

[Rest Try Yourself]

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

(i) $\frac{1}{4}, -1$ (ii) $\sqrt{2}, \frac{1}{3}$ (iii) $0, \sqrt{5}$ (iv) 1, 1 (v) $-\frac{1}{4}, \frac{1}{4}$ (vi) 4, 1

Sol. (i) Let the quadratic polynomial be $ax^2 + bx + c$

Then $-\frac{b}{a} = \frac{1}{4}$ and $\frac{c}{a} = -1$

i,e, $\frac{b}{a} = \frac{-1}{4}$ and $\frac{c}{a} = \frac{-1}{1}$

We select a = LCM (4, 1) = 4

Then $\frac{b}{4} = \frac{-1}{4}$ and $\frac{c}{4} = -1 \Rightarrow b = -1$ and $c = -4$.

Substituting $a = 4, b = -1, c = -4$ in $ax^2 + bx + c$, we get the required polynomial $4x^2 - x - 4$

(ii) $-\frac{b}{a} = \sqrt{2}$, $\frac{c}{a} = \frac{1}{3}$

$\Rightarrow \frac{b}{a} = \frac{-\sqrt{2}}{1}$, $\frac{c}{a} = \frac{1}{3}$

Select a = LCM (1, 3) = 3.

Then $\frac{b}{3} = -\sqrt{2}$ and $\frac{c}{3} = \frac{1}{3} \Rightarrow b = -3\sqrt{2}$ and $c = 1$.

Substituting $a = 3, b = -3\sqrt{2}$ and $c = 1$ in $ax^2 + bx + c$, we get the required polynomial $3x^2 - 3\sqrt{2}x + 1$

[Rest Try Yourself]

EXERCISE : 2.3

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following :

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$.

Sol. (i) $x^2 - 2 \overline{)\, x^3 - 3x^2 + 5x - 3\,(}\; q(x) = (x - 3)$

$$\begin{array}{l} \quad x^3 \qquad\; -2x \\ \underline{\;-\qquad\quad\;\; +\;} \\ \quad -3x^2 + 7x - 3 \\ \quad -3x^2 \qquad +6 \\ \underline{\quad +\qquad\quad\;\; -\;} \\ \qquad r(x) = (7x - 9) \end{array}$$

Hence, Quotient $q(x) = x - 3$ and Remainder $r(x) = 7x - 9$

[Rest Try Yourself]

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$ (ii) $x^2 + 3x + 1,\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1,\ x^5 - 4x^3 + x^2 + 3x + 1$

Sol. (i) $t^2 - 3 \overline{\big)\, 2t^4 + 3t^3 - 2t^2 - 9t - 12 \,\big(}\ q(t)\ 2t^2 + 3t + 4$

$$\begin{array}{l}
2t^4 \quad - 6t^2 \\
- \qquad + \\
\hline
3t^3 + 4t^2 - 9t - 12 \\
3t^3 \quad - 9t \\
- \qquad + \\
\hline
4t^2 - 12 \\
4t^2 - 12 \\
- \qquad + \\
\hline
\text{Remainder} = 0
\end{array}$$

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$

[Rest Try Yourself]

3. Obtain all other zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol. Two of the zeros of $3x^4 + 6x^3 - 2x^2 - 10\,x - 5$ are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

$\Rightarrow \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)$ is a factor of the polynomial.

i.e., $x^2 - \frac{5}{3}$ is a factor.

i.e., $(3x^2 - 5)$ is a factor of the polynomial. Then we apply the division algorithm as below :

$3x^2 - 5 \overline{\big)\, 3x^4 + 6x^3 - 2x^2 - 10x - 5 \,\big(}\ q(x) = x^2 + 2x + 1$

$$\begin{array}{l}
3x^4 \quad - 5x^2 \\
- \qquad + \\
\hline
6x^3 + 3x^2 - 10x - 5 \\
6x^3 \quad - 10x \\
- \qquad + \\
\hline
3x^2 - 5 \\
3x^2 - 5 \\
- \qquad + \\
\hline
4
\end{array}$$

The other two zeros will be obtained from the quadratic polynomial $q(x) = x^2 + 2x + 1$

Now $x^2 + 2x + 1 = (x + 1)^2$.

Its zeros are $-1, -1$.

Hence, all other zeros are $-1, -1$.

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

[Try Yourself]

5. Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg p(x) = deg q(x) (ii) deg q(x) = deg r(x) (iii) deg r(x) = 0.

Sol. (i) $p(x) = 2x^2 + 2x + 8$, $g(x) = 2x^0 = 2$; $q(x) = x^2 + x + 4$; $r(x) = 0$

(ii) $p(x) = 2x^2 + 2x + 8$; $g(x) = x^2 + x + 9$; $q(x) = 2$; $r(x) = -10$

(iii) $p(x) = x^3 + x + 5$; $g(x) = x^2 + 1$; $q(x) = x$; $r(x) = 5$.

EXERCISE-1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. Quadratic polynomial having zeros 1 and –2 is –

(A) $x^2 - x + 2$ (B) $x^2 - x - 2$

(C) $x^2 + x - 2$ (D) None of these

2. If (x–1) is a factor of $k^2x^3 - 4kx + 4k - 1$, then the value of k is –

(A) 1 (B) – 1

(C) 2 (D) –2

3. For what value of a is the polynomial $2x^4 - ax^3 + 4x^2 + 2x + 1$ divisible by $1 - 2x$?

(A) a = 25 (B) a = 24 (C) a = 23 (D) a = 22

4. If one of the factors of $x^2 + x - 20$ is (x + 5), then other factor is –

(A) (x – 4) (B) (x – 5) (C) (x – 6) (D) (x – 7)

5. If α,β be the zeros of the quadratic polynomial $2x^2 + 5x + 1$, then value of $\alpha + \beta + \alpha\beta$ =

(A) –2 (B) –1 (C) 1 (D) None of these

6. If α,β be the zeros of the quadratic polynomial $2 - 3x - x^2$, then $\alpha + \beta$ =

(A) 2 (B) 3 (C) 1 (D) None of these

7. Quadratic poolynomial having sum of it's zeros 5 and product of it's zeros – 14 is –

(A) $x^2 - 5x - 14$ (B) $x^2 - 10x - 14$

(C) $x^2 - 5x + 14$ (D) None of these

8. If x = 2 and x = 3 are zeros of the quadratic polynomial $x^2 + ax + b$, the values of a and b respectively are :

(A) 5, 6 (B) – 5, – 6 (C) –5, 6 (D) 5, 6

9. If 3 is a zero of the polynomial $f(x) = x^4 - x^3 - 8x^2 + kx + 12$, then the value of k is –

(A) – 2 (B) 2 (C) – 3 (D) $\frac{3}{2}$

10. The sum and product of zeros of the quadratic polynomial are – 5 and 3 respectively the quadratic polynomial is equal to –

(A) $x^2 + 2x + 3$ (B) $x^2 - 5x + 3$ (C) $x^2 + 5x + 3$ (D) $x^2 + 3x - 5$

11. On dividing $x^3 - 3x^2 + x + 2$ by polynomial g(x), the quotient and remainder were x – 2 and 4 – 2x respectively then g(x) :

(A) $x^2 + x + 1$ (B) $x^2 + x - 1$

(C) $x^2 - x - 1$ (D) $x^2 - x + 1$

12. If the polynomial $3x^2 - x^3 - 3x + 5$ is divided by another polynomial $x - 1 - x^2$, the remainder comes out to be 3, then quotient polynomial is –

(A) $2 - x$ (B) $2x - 1$ (C) $3x + 4$ (D) $x - 2$

13. If sum of zeros = $\sqrt{2}$, product of its zeros = $\frac{1}{3}$. The quadratic polynomial is –

(A) $3x^2 - 3\sqrt{2}x + 1$
(B) $\sqrt{2}x^2 + 3x + 1$
(C) $3x^2 - 2\sqrt{3}x + 1$
(D) $\sqrt{2}x^2 + x + 3$

14. If $-\frac{1}{3}$ is the zero of the cubic polynomial $f(x) = 3x^3 - 5x^2 - 11x - 3$ the other zeros are :

(A) – 3, – 1 (B) 1, 3 (C) 3, – 1 (D) – 3, 1

15. If α and β are the zeros of the polynomial $f(x) = 6x^2 - 3 - 7x$ then $(\alpha + 1)(\beta + 1)$ is equal to –

(A) $\frac{5}{2}$ (B) $\frac{5}{3}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

16. Let $p(x) = ax^2 + bx + c$ be a quadratic polynomial. It can have at most –

(A) One zero
(B) Two zeros
(C) Three zeros
(D) None of these

17. The graph of the quadratic polynomial $ax^2 + bx + c$, $a \neq 0$ is always –

(A) Straight line
(B) Curve
(C) Parabola
(D) None of these

18. If 2 and $-\frac{1}{2}$ as the sum and product of its zeros respectively then the quadratic polynomial f(x) is –

(A) $x^2 - 2x - 4$
(B) $4x^2 - 2x + 1$
(C) $2x^2 + 4x - 1$
(D) $2x^2 - 4x - 1$

19. If α and β are the zeros of the polynomial $f(x) = 16x^2 + 4x - 5$ then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to –

(A) $\frac{2}{5}$
(B) $\frac{5}{2}$
(C) $\frac{3}{5}$
(D) $\frac{4}{5}$

20. If α and β are the zeros of the polynomial $f(x) = 15x^2 - 5x + 6$ then $\left(1 + \frac{1}{\alpha}\right)\left(1 + \frac{1}{\beta}\right)$ is equal to –

(A) $\frac{13}{3}$ (B) $\frac{13}{2}$ (C) $\frac{16}{3}$ (D) $\frac{15}{2}$

OBJECTIVE — ANSWER KEY — EXERCISE -1

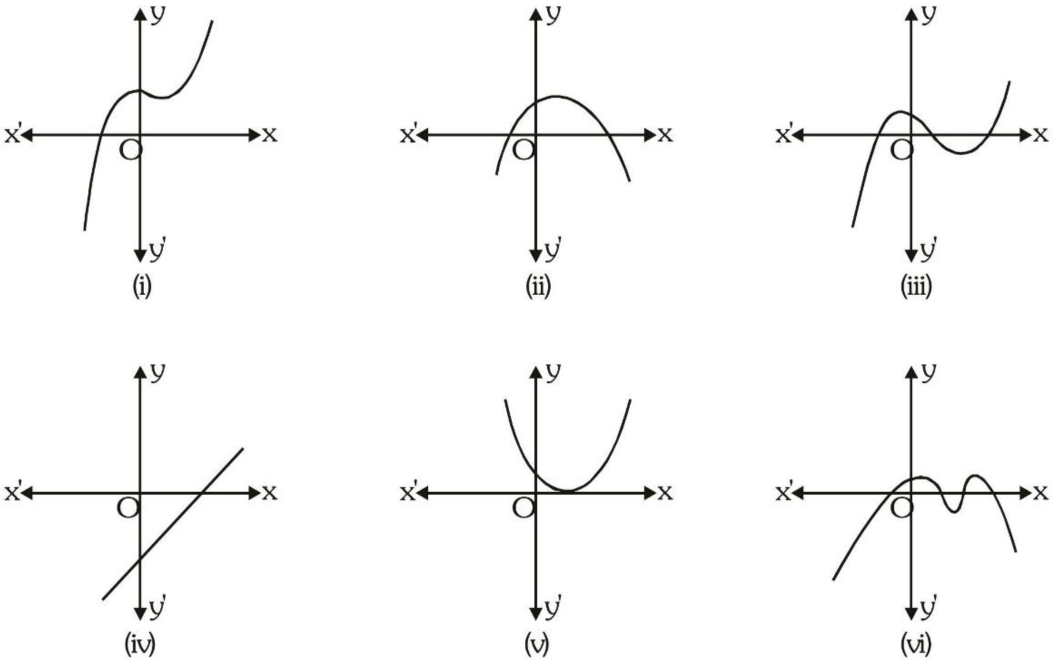
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	A	A	A	A	D	A	C	B	C	D	D	A	C	B
Que.	16	17	18	19	20										
Ans.	B	C	D	D	A										

EXERCISE-2 (FOR SCHOOL/BOARD EXAMS)

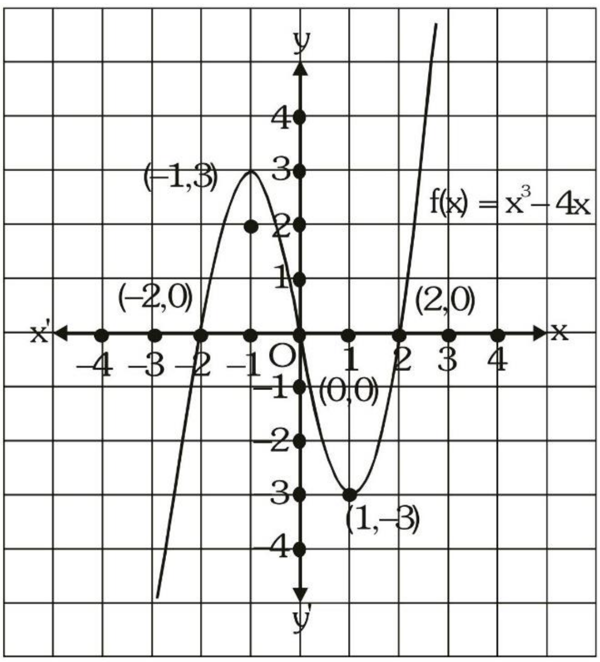
SUBJECTIVE TYPE QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. Look at the graph in fig given below. Each is the graph of $y = p(x)$, where $p(x)$ is a polynomial. For each of the graph, find the numbre of zeros of $p(x)$.



2. Consider the cubic polynomial $f(x) = x^3 - 4x$. Find from the fig, the number of zeros of the above stated polynomials.

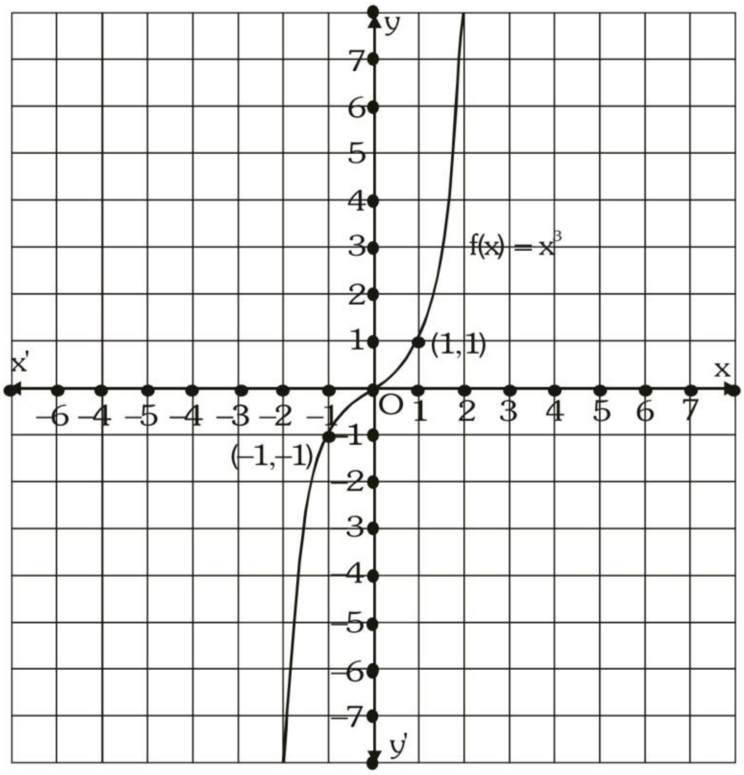


3. Let $f(x) = x^3$

The graph of the polynomial is shown in fig.

(i) Find the number of zeros of polynomial f(x).

(ii) Determine the co-ordinates of the points, at which the graph intersects the x-axis.



SHORT ANSWER TYPE QUESTIONS

1. Find the zeros of the following quadratic polynomials and verify the relationship between the zeros and their coefficients.

(i) $6x^2 - x - 1$ (ii) $25x\,(x + 1) + 4$ (iii) $4x^2 + 4x + 1$ (iv) $48y^2 - 13y - 1$ (v) $63 - 2x - x^2$
(vi) $2x^2 - 5x$ (vii) $49x^2 - 81$ (viii) $4x^2 - 4x - 3$

2. Find a quadratic polynomial each with the given numbers as the zeros of the polynomials.

(i) $3+\sqrt{7}, 3-\sqrt{7}$ (ii) $2\sqrt{3}, -2\sqrt{3}$ (iii) $-\frac{3}{7}, -\frac{2}{3}$ (iv) $\sqrt{3}, 3\sqrt{3}$ (v) $2+3\sqrt{2}, 2-3\sqrt{2}$ (vi) $\frac{8}{3}, \frac{5}{2}$

3. Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

(i) $4\sqrt{3}, 9$ (ii) $2\sqrt{3}-1, 3-\sqrt{3}$ (iii) $0, -\frac{1}{4}$ (iv) $\frac{-10}{\sqrt{3}}, 7$ (v) $\frac{5}{6}, \frac{25}{9}$ (vi) $\frac{-2\sqrt{5}}{3}, -\frac{5}{3}$ (vii) $-\sqrt{3}, \frac{1}{4}$ (viii) $-\frac{6}{5}, \frac{9}{25}$

(ix) $\sqrt{2}, -12$

4. If α and β are the zeros of the polynomial $f(x) = 5x^2 + 4x - 9$ then evaluate the following :

(i) $\alpha - \beta$ (ii) $\alpha^2 + \beta^2$ (iii) $\alpha^2 - \beta^2$ (iv) $\alpha^3 + \beta^3$ (v) $\alpha^3 - \beta^3$ (vi) $\alpha^4 - \beta^4$

5. If one of the zeros of the quadratic polynomial $2x^2 + px + 4$ is 2, find the other zero. Also find the value of p.

6. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is the reciprocal of the other, find the value of a.

7. If the product of zeros of the polynomial $ax^2 - 6x - 6$ is 4, find the value of a.

8. Find the zeros of the quadratic polynomial $5x^2 - 4 - 8x$ and verify the relationship between the zeros and the coefficients of the polynomial.

9. Determine if 3 is a zero of $p(x) = \sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} - \sqrt{4x^2 - 14x + 6}$

10. If α and β be two zeros of the quadratic polynomial $ax^2 + bx + c$, then evaluate :

(i) $a^2 + b^2$ (ii) $\alpha^3 + \beta^3$ (iii) $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ (iv) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

11. Find the value of k :
(i) If α and β are the zeros of the polynomial $x^2 - 5x + k$ where $\alpha - \beta = 1$.
(ii) If α and β are the zeros of the polynomial $x^2 - 8x + k$ such that $\alpha^2 + \beta^2 = 40$.
(iii) If α and β are the zeros of the polynomial $x^2 - 6x + k$ such that $3\alpha + 2\beta = 20$.

12. If 2 and 3 are zeros of polynomial $3x^2 - 2kx + 2m$, find the values of k and m.

13. If one zero of polynomial $3x^2 = 8x + 2k + 1$ is seven times the other, then find the zeros and the value of k.

14. If α and β are the zeros of the polynomial $2x^2 - 4x + 5$. Form the polynomial where zeros are :
(i) $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ (ii) $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ (iii) $1+\frac{\beta}{\alpha}$ and $1+\frac{\alpha}{\beta}$

15. If α and β are the zeros of the quadratic polynomial $x^2 - 3x + 2$, find a quadratic polynomial whose zeros are :
(i) $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$ (ii) $\frac{\alpha-1}{\alpha+1}$ and $\frac{\beta-1}{\beta+1}$

16. If the sum of the squares of zeros of the polynomial $5x^2 + 3x + k$ is $-\frac{11}{25}$, find the value of k.

17. If one zero of the quadratic polynomial $2x^2 - (3k + 1)x - 9$ is negative of the other, find the value of k.

18. If α and β are the two zeros of the quadratic polynomial $x^2 - 2x + 5$, find a quadratic polynomial whose zeros are $\alpha + \beta$ and $\frac{1}{\alpha}+\frac{1}{\beta}$

19. If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - px + q$, prove that $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{p^4}{q^2}-\frac{4p^2}{q}+2$

20. Apply the division algorithm to find the quotient q(x) and remainder r(x) on dividing p(x) by g(x) as given below :
(i) $p(x) = 3x^3 + 2x^2 + x + 1$; $g(x) = x^3 + 3x + 2$
(ii) $p(x) = x^6 + x^4 - x^2 - 1$; $g(x) = x^3 - x^2 + x - 1$
(iii) $p(x) = 2x^5 + 3x^4 + 4x^3 + 4x^2 + 3x + 2$; $g(x) = x^3 + x^2 + x + 1$
(iv) $p(x) = x^3 - 3x^2 - x + 3$; $g(x) = x^2 - 4x + 3$

21. Find the quotient q(x) and remainder r(x) of the following when f(x) is divided by g(x). Verify the division algorithm.
(i) $f(x) = x^6 + 5x^3 + 7x + 3$; $g(x) = x^2 + 2$
(ii) $f(x) = x^4 + 2x^2 + 1$; $g(x) = x^3 + 1$
(iii) $f(x) = 4x^4 - 7x^2 + 18x - 1$; $g(x) = 2x + 1$
(iv) $f(x) = 5x^3 - 70x^2 + 153x - 342$; $g(x) = x^2 - 10x + 6$

22. Check whether g(y) is a factor of f(y) by applying the division algorithm :
(i) $f(y) = 2y^4 + 3y^3 - 2y^2 - 9y - 12$, $g(y) = y^2 - 3$
(ii) $f(y) = 3y^4 + 5y^3 - 7y^2 + 2y + 2$, $g(y) = y^2 + 3y + 1$
(iii) $f(y) = y^5 - 4y^3 + y^2 + 3y + 1$, $g(y) = y^3 - 3y + 1$

23. (a) If 1 is the zero of $f(x) = k^2x^2 - 3kx + 3k - 1$ then find the value(s) of k.
(b) If 1 and – 2 are the zeros of $f(x) = x^3 + 10x^2 + ax + b$, then find the values of a and b.
(c) Find p and q such that 3 and – 1 are the zeros of $f(x) = x^4 + px^3 + qx^2 + 12x - 9$.
(d) If 3 is the zero of $f(x) = x^4 - x^3 - 8x^2 + kx + 12$, then find the value of k.
Also show that – 2 is the zero of $x^3 - 2x + 4$

24. (a) Find all the zeros of $3x^3 + 16x^2 + 23x + 6$ if two of its zeros are – 3 and – 2.
(b) Determine all the zeros of $4x^3 + 12x^2 - x - 3$ if two of its zeros are $-\frac{1}{2}$ and $\frac{1}{2}$.
(c) Determine all the zeros of $x^3 + 5x^2 - 2x - 10$ if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$
(d) Determine all the zeros of $4x^3 + 12x^2 - x - 3$ if one of its zeros is $\frac{5}{2}$
(e) Determine all the zeros of $4x^3 + 5x^2 - 180x - 225$ if one of its zeros is $-\frac{5}{4}$.

25. (a) Find all the zeros of $3x^4 - 10x^3 + 5x^2 + 10x - 8$ if three of its zeros are 1, 2 and – 1.
(b) Obtain all the zeros of $2x^4 + 5x^3 - 8x^2 - 17x - 6$ if three of its zeros are –1, –3, 2.
(c) Determine all the zeros of $x^4 - x^3 - 8x^2 + 2x + 12$ if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

26. (a) Obtain all other zeros of the polynomial $2x^3 - 4x - x^2 + 2$ if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.
(b) Find all the zeros of $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeros are $2+\sqrt{3}$ and $2-\sqrt{3}$.
(c) Find all the zeros of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeros are 2 and – 2.
(d) Find all the zeros of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

27. (a) On dividing $f(x) = 3x^3 + x^2 + 2x + 5$ by a polynomial $g(x) = x^2 + 2x + 1$, the remainder $r(x) = 9x + 10$. Find the quotient polynomial q(x).
(b) On dividing f(x) by a polynomial $x - 1 - x^2$, the quotient q(x) and remainder r(x) are (x – 2) and 3 respectively. Find f(x).
(c) On dividing $x^5 - 4x^3 + x^2 + 3x + 1$ by polynomial g(x), the quotient and remainder are $(x^2 - 1)$ and 2 respectively. Find g(x).
(d) On dividing $f(x) = 2x^5 + 3x^4 + 4x^3 + 4x^2 + 3x + 2$ by a polynomial g(x), where $g(x) = x^3 + x^2 + x + 1$, the quotient obtained as $2x^2 + x + 1$. Find the remainder r(x).

POLYNOMIALS **ANSWER KEY** EXERCISE-2 (X)-CBSE

- **Very Short Answer Type Questions**

1. (i) One zero, (ii) Two zero, (iii) Three zeros, (iv) One zero, (v) One zero, (vi) Four zeros

2. Three zeros 3. (i) One zero, (ii) (0, 0)

- **Short Answer Type Questions**

1. (i) $-\frac{1}{3},\frac{1}{2}$, (ii) $-\frac{1}{5},\frac{-4}{5}$, (iii) $\frac{-1}{2},\frac{-1}{2}$, (iv) $\frac{1}{3},\frac{-1}{16}$, (v) 7, –9, (vi) $0,\frac{5}{2}$, (vii) $\frac{9}{7},\frac{-9}{7}$, (viii) $\frac{3}{2},\frac{-1}{2}$

2. (i) $x^2 - 6x + 2$, (ii) $x^2 - 12$, (iii) $21x^2 + 23x + 6$, (iv) $x^2 - 4\sqrt{3}x + 9$, (v) $x^2 - 4x - 14$, (vi) $6x^2 - 31x + 40$

3. (i) $x^2 - 4\sqrt{3}x + 9$, (ii) $x^2 - (2\sqrt{3}-1)x + (3-\sqrt{3})$, (iii) $4x^2 - 1$, (iv) $3x^2 + 10\sqrt{3}x + 21$, (v) $18x^2 - 15x + 50$, (vi) $3x^2 + 2\sqrt{5}x - 5$, (vii) $4x^2 + 4\sqrt{3}x + 1$, (viii) $25x^2 + 30x + 9$, (ix) $x^2 - \sqrt{2}x - 12$

4. (i) $\frac{14}{5}$, (ii) $\frac{106}{25}$, (iii) $\frac{-56}{25}$, (iv) $\frac{-604}{125}$, (v) $\frac{854}{125}$, (vi) $\frac{-5936}{125}$ 5. p = – 6, other zero = 1 6. a = 3 7. a = $\frac{-3}{2}$

8. 2 and $\frac{-2}{5}$ 9. Yes 10. (i) $\frac{b^2-2ac}{a^2}$ (ii) $\frac{3abc-b^3}{a^3}$ (iii) $\frac{3abc-b^3}{c^3}$ (iv) $\frac{3abc-b^3}{a^2c}$ 11. (i) 6 (ii) 12 (iii) – 16

12. k = $\frac{15}{2}$, m = 9 13. $\frac{1}{3},\frac{7}{3}, k=\frac{-5}{3}$ 14. (i) $\frac{1}{5}(5x^2 - 4x + 2)$ (ii) $\frac{1}{25}(25x^2 + 4x + 4)$ (iii) $\frac{1}{5}(5x^2 - 8x + 8)$

15. (i) $20x^2 - 9x + 1$ (ii) $3x^2 - x$ 16. 2 17. $-\frac{1}{3}$ 18. $5x^2 - 12x + 4$

20. (i) q(x) = 3, r = $2x^2 - 8x - 5$, (ii) q(x) = $x^3 + x^2 + x + 1$, r(x) = 0, (iii) q(x) = $2x^2 + x + 1$, (x) = x + 1, (iv) q(x) = x + 1, r(x) = 0

21. (i) q(x) = $x^4 - 2x^2 + 5x + 4$, r(x) = – (3x + 5), (ii) q(x) = x, r(x) = $2x^2 - x + 1$, (iii) q(x) = $2x^3 - x^2 - 3x + \frac{11}{2}$, r(x) = $-\frac{13}{2}$, (iv) q(x) = 5x – 20, r(x) = – 127x – 22

22. (i) g(y) is a factor of f(y), (ii) g(x) is a factor of f(x), (iii) g(t) is not a factor of f(t)

23. (a) k = ± 1, (b) a = 7, b = – 18, (c) p = – 8, q = 12, (d) k = 2

24. (a) –2, –3, $-\frac{1}{3}$, (b) $\frac{1}{2},-\frac{1}{2},3$, (c) $\sqrt{2},-\sqrt{2},-5$, (d) $\frac{5}{2},\frac{3}{2},\frac{1}{2}$, (e) $-\frac{5}{4},3\sqrt{5},-3\sqrt{5}$

25. (a) 1, 2, –1, $\frac{4}{3}$ (b) $-1,-3,2,-\frac{1}{2}$, (c) $\sqrt{2},-\sqrt{2},3,-2$

26. (a) $\frac{1}{2}$, (b) $2 \pm \sqrt{3},1,-\frac{1}{2}$, (c) 2, –2, 5 and – 6, (d) $\pm\sqrt{2},\frac{3}{2}$ and – 5

27. (a) q(x) = 3x – 5, (b) f(x) = $-x^3 + 3x^2 - 3x + 5$, (c) g(x) = $x^3 - 3x + 1$, (d) r(x) = x + 1,

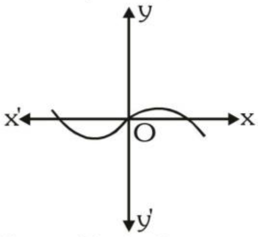
MATHEMATICS

EXERCISE-3 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

QUESTIONS CARRYING 1 MARK

1. Write the zeros of the polynomial $x^2 + 2x + 1$. [Delhi-2008]
2. Write the zeros of the polynomial, $x^2 - x - 6$. [Delhi-2008]
3. Write a quadratic polynomial, the sum and product of whose zeros are 3 and – 2 respectively. [Delhi-2008]
4. Write the number of zeros of the polynomial $y = f(x)$ whose graph is given in figure. [AI-2008]



5. If $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$, find a [Foreign-2008]
6. For what value of k, (–4) is a zero of the polynomial $x^2 - x - (2k + 2)$? [Delhi-2009]
7. For what value of p, (–4) is a zero of the polynomial $x^2 - 2x - (7p + 3)$? [Delhi-2009]
8. If 1 is a zero of the polynomial $p(x) = ax^2 - 3(a - 1)x - 1$, then find the value of a. [AI-2009]
9. Write the polynomial, the product and sum of whose zeros $-\frac{9}{2}$ and $-\frac{3}{2}$ respectively [Foreign-2009]
10. Write the polynomial, the product and sum of whose zeros are $-\frac{13}{5}$ and $-\frac{3}{5}$ respectively. [Foreign-2009]

QUESTIONS CARRYING 2 MARKS

11. Find the zeros of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeros and the co-efficients of the polynomial. [Delhi-2008]
12. Find the zeros of the quadratic polynomial $5x^2 - 4 - 8x$ and verify the relationship between the zeros and the coefficients of the polynomial. [Delhi-2008]
13. Find the quadratic polynomial sum of whose zeros is 8 and their product is 12. Hence, find the zeros of the polynomial. [AI-2008]
14. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other. Find the vlaue of 'a' [AI-2008]
15. If the product of zeros of the polynomial $ax^2 - 6x - 6$ is 4, find the value of 'a' [AI-2008]
16. Find all the zeros of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of it's zeros are 2 and – 2. [Foreign-2008]
17. Find all the zeros of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of it's zeros are $\sqrt{2}$ and $-\sqrt{2}$ [Foreign-2008]
18. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $(ax + b)$, find a and b. [Delhi-2009]
19. If the polynomial $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by another polynomial $x^2 + 5$, the remainder comes out to be $px + q$. Find the values of p and q. [Delhi-2009]
20. Find all the zeros of the polynomial $x^3 + 3x^2 - 2x - 6$, if two of it's zeros are $-\sqrt{2}$ and $\sqrt{2}$.[AI-2009]
21. Find all the zeros of the polynomial $2x^3 + x^2 - 6x - 3$, if two of it's zeros are $-\sqrt{3}$ and $\sqrt{3}$. [AI-2009]
22. If the polynomial $6x^4 + 8x^3 - 5x^2 + ax + b$ is exactly divisible by polynomial $2x^2 - 5$, then find the value of a and b. [Foreign-2009]

POLYNOMIALS **ANSWER KEY** EXERCISE-3 (X)-CBSE

1. $x = -1$ **2.** 3, – 2 **3.** $x^2 - 3x - 2$ **4.** 3 **5.** 2 **6.** 9 **7.** 3 **8.** a = 1 **9.** $2x^2 + 3x - 9$ **10.** $5x^2 + 3x - 13$

11. $\left[\frac{-1}{3}, \frac{3}{2}\right]$ **12.** $\left[\frac{-2}{5}, 2\right]$ **13.** $x^2 - 8x + 12$; (6, 2) **14.** 3 **15.** $\frac{-3}{2}$ **16.** 2, – 2, – 6 and 5 **17.** $\sqrt{2}, -\sqrt{2}, -5$ and $\frac{3}{2}$

18. a = 1, b = 2 **19.** p = 2, q = 3 **20.** $-\sqrt{2}, \sqrt{2}$ and – 3 **21.** $-\sqrt{3}, \sqrt{3}$ and $-\frac{1}{2}$ **22.** a = – 20, b = – 25

EXERCISE-4 (FOR OLYMPIADS)

Choose The Correct One

1. If α, β and γ are the zeros of the polynomial $2x^3 - 6x^2 - 4x + 30$, then the value of $(\alpha\beta + \beta\gamma + \gamma\alpha)$ is
(A) -2 (B) 2 (C) 5 (D) -30

2. If α, β and γ are the zeros of the polynomial $f(x) = ax^3 + bx^2 + cx + d$, then $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=$
(A) $-\frac{b}{a}$ (B) $\frac{c}{d}$ (C) $-\frac{c}{d}$ (D) $-\frac{c}{a}$

3. If α, β and γ are the zeros of the polynomial $f(x) = ax^3 - bx^2 + cx - d$, then $\alpha^2 + \beta^2 + \gamma^2 =$
(A) $\frac{b^2 - ac}{a^2}$ (B) $\frac{b^2 + 2ac}{b^2}$ (C) $\frac{b^2 - 2ac}{a}$ (D) $\frac{b^2 - 2ac}{a^2}$

4. If α, β and γ are the zeros of the polynomial $f(x) = x^3 + px^2 - pqrx + r$, then $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha} =$
(A) $\frac{r}{p}$ (B) $\frac{p}{r}$ (C) $-\frac{p}{r}$ (D) $-\frac{r}{p}$

5. If the parabola $f(x) = ax^2 + bx + c$ passes through the points $(-1, 12)$, $(0, 5)$ and $(2, -3)$, the value of $a + b + c$ is –
(A) -4 (B) -2 (C) Zero (D) 1

6. If a, b are the zeros of $f(x) = x^2 + px + 1$ and c, d are the zeros of $f(x) = x^2 + qx + 1$ the value of $E = (a - c)(b - c)(a + d)(b + d)$ is –
(A) $p^2 - q^2$ (B) $q^2 - p^2$ (C) $q^2 + p^2$ (D) None of these

7. If α, β are zeros of $ax^2 + bx + c$ then zeros of $a^3x^2 + abcx + c^3$ are –
(A) $\alpha\beta, \alpha + \beta$ (B) $\alpha^2\beta, \alpha\beta^2$ (C) $\alpha\beta, \alpha^2\beta^2$ (D) α^3, β^3

8. Let α, β be the zeros of the polynomial $x^2 - px + r$ and $\frac{\alpha}{2}, 2\beta$ be the zeros of $x^2 - qx + r$. Then the value of r is –
(A) $\frac{2}{9}(p-q)(2q-p)$ (B) $\frac{2}{9}(q-p)(2p-q)$ (C) $\frac{2}{9}(q-2p)(2q-p)$ (D) $\frac{2}{9}(2p-q)(2q-p)$

9. When $x^{200} + 1$ is divided by $x^2 + 1$, the remainder is equal to –
(A) $x + 2$ (B) $2x - 1$ (C) 2 (D) -1

10. If $a(p + q)^2 + 2bpq + c = 0$ and also $a(q + r)^2 + 2bqr + c = 0$ then pr is equal to –
(A) $p^2 + \frac{a}{c}$ (B) $q^2 + \frac{c}{a}$ (C) $p^2 + \frac{a}{b}$ (D) $q^2 + \frac{a}{c}$

11. If a,b and c are not all equal and α and β be the zeros of the polynomial $ax^2 + bx + c$, then value of $(1 + \alpha + \alpha^2)(1 + \beta + \beta^2)$ is :
(A) 0 (B) positive (C) negative (D) non-negative

12. Two complex numbers α and β are such that $\alpha + \beta = 2$ and $\alpha^4 + \beta^4 = 272$, then the polynomial whose zeros are α and β is –
(A) $x^2 - 2x - 16 = 0$ (B) $x^2 - 2x + 12 = 0$ (C) $x^2 - 2x - 8 = 0$ (D) None of these

13. If 2 and 3 are the zeros of $f(x) = 2x^3 + mx^2 - 13x + n$, then the values of m and n are respectively –
(A) $-5, -30$ (B) $-5, 30$ (C) $5, 30$ (D) $5, -30$

14. If α,β are the zeros of the polynomial $6x^2 + 6px + p^2$, then the polynomial whose zeros are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$ is –
(A) $3x^2 + 4p^2x + p^4$ (B) $3x^2 + 4p^2x - p^4$
(C) $3x^2 - 4p^2x + p^4$ (D) None of these

15. If c, d are zeros of $x^2 - 10ax - 11b$ and a, b are zeros of $x^2 - 10cx - 11d$, then value of a + b + c + d is –
(A) 1210 (B) – 1 (C) 2530 (D) – 11

16. If the ratio of the roots of polynomial $x^2 + bx + c$ is the same as that of the ratio of the roots of $x^2 + qx + r$, then –
(A) $br^2 = qc^2$ (B) $cq^2 = rb^2$ (C) $q^2c^2 = b^2r^2$ (D) $bq = rc$

17. The value of p for which the sum of the squares of the roots of the polynomial $x^2 - (p-2)x - p - 1$ assume the least value is –
(A) – 1 (B) 1 (C) 0 (D) 2

18. If the roots of the polynomial $ax^2 + bx + c$ are of the form $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$ then the value of $(a + b + c)^2$ is –
(A) $b^2 - 2ac$ (B) $b^2 - 4ac$ (C) $2b^2 - ac$ (D) $4b^2 - 2ac$

19. If α, β and γ are the zeros of the polynomial $x^3 + a_0x^2 + a_1x + a_2$, then $(1 - \alpha^2)(1 - \beta^2)(1 - \gamma^2)$ is
(A) $(1 - a_1)^2 + (a_0 - a_2)^2$ (B) $(1 + a_1)^2 - (a_0 + a_2)^2$
(C) $(1 + a_1)^2 + (a_0 + a_2)^2$ (D) None of these

20. If α,β,γ are the zeros of the polynomial $x^3 - 3x + 11$, then the polynomial whose zeros are (α+β), (β+γ) and (γ+α) is –
(A) $x^3 + 3x + 11$ (B) $x^3 - 3x + 11$
(C) $x^3 + 3x - 11$ (D) $x^3 - 3x - 11$

21. If α,β,γ are such that $\alpha + \beta + \gamma = 2$, $\alpha^2 + \beta^2 + \gamma^2 = 6$, $\alpha^3 + \beta^3 + \gamma^3 = 8$, then $\alpha^4 + \beta^4 + \gamma^4$ is equal to–
(A) 10 (B) 12 (C) 18 (D) None of these

22. If α,β are the roots of $ax^2 + bx + c$ and α + k, β + k are the roots of $px^2 + qx + r$, then k =
(A) $-\frac{1}{2}\left[\frac{a}{b}-\frac{p}{q}\right]$ (B) $\left[\frac{a}{b}-\frac{p}{q}\right]$ (C) $\frac{1}{2}\left[\frac{b}{a}-\frac{q}{p}\right]$ (D) (ab – pq)

23. If α,β are the roots of the polynomial $x^2 - px + q$, then the quadratic polynomial, the roots of which are $(\alpha^2 - \beta^2)(\alpha^3 - \beta^3)$ and $(\alpha^3\beta^2 + \alpha^2\beta^3)$:
(A) $px^2 - (5p + 7q)x - (p^6q^6 + 4p^2q^6) = 0$
(B) $x^2 - (p^5 - 5p^3q + 5pq^2)x + (p^6q^2 - 5p^4q^3 + 4p^2q^4) = 0$
(C) $x^2 - (p^3q - 5p^5 + p^4q) - (p^6q^2 - 5p^2q^6) = 0$
(D) All of the above

24. The condition that $x^3 - ax^2 + bx - c = 0$ may have two of the roots equal to each other but of opposite signs is :
(A) ab = c (B) $\frac{2}{3}a = bc$ (C) $a^2b = c$ (D) None of these

25. If the roots of polynomial $x^2 + bx + ac$ are α,β and roots of the polynomial $x^2 + ax + bc$ are α,γ then the values of α,β,γ respectively are –
(A) a,b,c (B) b,c,a (C) c,a,b (D) None of these

26. If one zero of the polynomial $ax^2 + bx + c$ is positive and the other negative then (a,b,c ∈R, a ≠ 0)
(A) a and b are of opposite signs. (B) a and c are of opposite signs.
(C) b and c are of opposite signs. (D) a,b,c are all of the same sign.

27. If α,β are the zeros of the polynomial $x^2 - px + q$, then $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}$ is equal to –

(A) $\frac{p^4}{q^2}+2-\frac{4p^2}{q}$ (B) $\frac{p^4}{q^2}-2+\frac{4p^2}{q}$ (C) $\frac{p^4}{q^2}+2q^2-\frac{4p^2}{q}$ (D) None of these

28. If α,β are the zeros of the polynomial $x^2 - px + 36$ and $\alpha^2 + \beta^2 = 9$, then p =

(A) ± 6 (B) ± 3 (C) ± 8 (D) ± 9

29. If α,β are zeros of $ax^2 + bx + c$, $ac \neq 0$, then zeros of $cx^2 + bx + a$ are –

(A) $-\alpha, -\beta$ (B) $\alpha,\frac{1}{\beta}$ (C) $\beta,\frac{1}{\alpha}$ (D) $\frac{1}{\alpha},\frac{1}{\beta}$

30. A real number is said to be algebraic if it satisfies a polynomial equation with integral coefficients. Which of the following numbers is not algebraic :

(A) $\frac{2}{3}$ (B) $\sqrt{2}$ (C) 0 (D) π

31. The bi-quadraic polynomial whose zeros are 1, 2, $\frac{4}{3}$, – 1 is :

(A) $3x^4 - 10x^3 + 5x^2 + 10x - 8$ (B) $3x^4 + 10x^3 - 5x^2 + 10x - 8$
(C) $3x^4 + 10x^3 + 5x^2 - 10x - 8$ (D) $3x^4 - 10x^3 - 5x^2 + 10x - 8$

32. The cubic polynomials whose zeros are 4, $\frac{3}{2}$ and – 2 is :

(A) $2x^3 + 7x^2 + 10x - 24$ (B) $2x^3 + 7x^2 - 10x - 24$
(C) $2x^3 - 7x^2 - 10x + 24$ (D) None of these

33. If the sum of zeros of the polynomial $p(x) = kx^3 - 5x^2 - 11x - 3$ is 2, then k is equal to :

(A) $k = -\frac{5}{2}$ (B) $k = \frac{2}{5}$ (C) $k = 10$ (D) $k = \frac{5}{2}$

34. If $f(x) = 4x^3 - 6x^2 + 5x - 1$ and α,β and γ are its zeros, then $\alpha\beta\gamma$ =

(A) $\frac{3}{2}$ (B) $\frac{5}{4}$ (C) $-\frac{3}{2}$ (D) $\frac{1}{4}$

35. Consider $f(x) = 8x^4 - 2x^2 + 6x - 5$ and $\alpha,\beta,\gamma,\delta$ are it's zeros then $\alpha + \beta + \gamma + \delta$ =

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) $-\frac{3}{2}$ (D) None of these

36. If $x^2 - ax + b = 0$ and $x^2 - px + q = 0$ have a root in common and the second equation has equal roots, then –

(A) $b + q = 2ap$ (B) $b + q = \frac{ap}{2}$ (C) $b + q = ap$ (D) None of these

OBJECTIVE — ANSWER KEY — EXERCISE -4

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	D	B	C	B	B	D	C	B	D	C	B	C	A
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	B	B	B	D	C	C	B	A	C	B	A	D	D	D
Que.	31	32	33	34	35	36									
Ans.	A	C	D	D	D	B									

EXERCISE-5 (FOR IIT-JEE/AIEEE)

CHOOSE THE CORRECT ONE

1. If the sum of the two zeros of $x^3 + px^2 + qx + r$ is zero, then pq = [EAMCET-2003]
   (A) – r (B) r (C) 2r (D) – 2r
2. Let $a \neq 0$ and p(x) be a polynomial of degree greater than 2. If p(x) leaves remainders a and – a when divided respectively by x + a and x – a, the remainder when p(x) is divided by $x^2 - a^2$ is [EAMCET-2003]
   (A) 2x (B) – 2x (C) x (D) – x
3. If one root of the polynomial $x^2 + px + q$ is square of the other root, then – [IIT-Screening-2004]
   (A) $p^3 - q(3p - 1) + q^2 = 0$ (B) $p^3 - q(3p + 1) + q^2 = 0$
   (C) $p^3 + q(3p - 1) - q^2 = 0$ (D) $p^3 + q(3p + 1) - q^2 = 0$
4. If α,β are the zeros of $x^2 + px + 1$ and γ,δ be those of $x^2 + qx + 1$, then the value of $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) =$ [DCE-2000]
   (A) $p^2 - q^2$ (B) $q^2 - p^2$ (C) p^2 (D) q^2
5. The quadratic polynomial whose zeros are twice the zeros of $2x^2 - 5x + 2 = 0$ is – [Kerala Engineering-2003]
   (A) $8x^2 - 10x + 2$ (B) $x^2 - 5x + 4$ (C) $2x^2 - 5x + 2$ (D) $x^2 - 10x + 6$
6. The coefficient of x in $x^2 + px + q$ was taken as 17 in place of 13 and it's zeros were found to be – 2 and – 15. The zeros of the original polynomial are – [Kerala Engineering-2003]
   (A) 3, 7 (B) – 3, 7 (C) – 3, – 7 (D) – 3, – 10
7. If $\alpha + \beta = 4$ and $\alpha^3 + \beta^3 = 44$, then α,β are the zeros of the polynomial. [Kerala Engineering-2003]
   (A) $2x^2 - 7x + 6$ (B) $3x^2 + 9x + 11$ (C) $9x^2 - 27x + 20$ (D) $3x^2 - 12x + 5$
8. If α,β,γ are the zeros of the polynomial $x^3 + 4x + 1$, then $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1} =$
   (A) 2 (B) 3 (C) 4 (D) 5 [EAMCET-2003]
9. If α,β are the zeros of the quadratic polynomial $4x^2 - 4x + 1$, then $\alpha^3 + \beta^3$ is –
   (A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) 16 (D) 32
10. The value of 'a', for which one root of the quadratic polynomial $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2$ is twice as large as the other, is – [AIEEE-2003]
    (A) $-\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $-\frac{2}{3}$ (D) $\frac{1}{3}$
11. Let α,β be the zeros of $x^2 + (2 - \lambda)x - \lambda$. The values of λ for which $\alpha^2 + \beta^2$ is minimum is –
    (A) 0 (B) 1 (C) 2 (D) 3 [AMU-2002]
12. If 1 + 2i is a zero of the polynomial $x^2 + bx + c$, b, c $\in$ R, then (b,c) is given by –
    (A) (2, – 5) (B) (– 3, 1) (C) (–2, 5) (D) (3, 1)
13. If 2 + i is a zero of the polynomial $x^3 - 5x^2 + 9x - 5$, the other zeros are –
    (A) 1 and 2 – i (B) – 1 and 3 + i (C) 0 and 1 (D) None of these
14. The value of λ for which one zero of $3x^2 - (1 + 4\lambda)x + \lambda^2 + 2$ may be one-third of the other is –
    (A) 4 (B) $\frac{33}{8}$ (C) $\frac{17}{4}$ (D) $\frac{31}{8}$
15. If 1 – i is a zero of the polynomial $x^2 + ax + b$, then the values of a and b are respectively.
    (A) 2, 1 (B) – 2, 2 [Tamil Nadu Engineering 2002]
    (C) 2, 2 (D) 2, – 2
16. If the sum of the zeros of the polynomial $x^2 + px + q$ is equal to the sum of their squares, then –
    (A) $p^2 - q^2 = 0$ (B) $p^2 + q^2 = 2q$
    (C) $p^2 + p = 2q$ (D) None of these

17. Let α,β be the zeros of the polynomial $(x - a)(x - b) - c$ with $c \neq 0$. Then the zeros of the polynomial $(x - \alpha)(x - \beta) + c$ are : [IIT-1992, AIEEE-2002]
(A) a, c (B) b, c (C) a, b (D) a + c, b + c

18. If p,q are zeros of $x^2 + px + q$, then – [AIEEE-2002]
(A) p = 1 (B) p = 1 or 0 (C) p = – 2 (D) p = – 2 or 0

19. If $\alpha \neq \beta$ and $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the polynomial whose zeros are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is : [AIEEE-2002]
(A) $3x^2 - 25x + 3$ (B) $x^2 - 5x + 3$
(C) $x^2 + 5x - 3$ (D) $3x^2 - 19x + 3$

20. If $\alpha \neq \beta$ and the difference between the roots of the polynomials $x^2 + ax + b$ and $x^2 + bx + a$ is the same, then [AIEEE-2002]
(A) a + b + 4 = 0 (B) a + b – 4 = 0 (C) a – b + 4 = 0 (D) a – b – 4 = 0

21. If the zeros of the polynomial $ax^2 + bx + c$ be in the ratio m : n, then
(A) $b^2 mn = (m^2 + n^2) ac$ (B) $(m + n)^2 ac = b^2 mn$
(C) $b^2 (m^2 + n^2) = mnac$ (D) None of these

Comprehension Based Questions

Maximum and Minimum value of a quadratic expression :

At $x = \frac{-b}{2a}$, we get the maximum or minimum value of the quadratic expression, $y = ax^2 + bx + c$

(i) When a > 0, the expression $ax^2 + bx + c$ gives minimum value $= \frac{4ac - b^2}{4a}$

(ii) When a < 0, the expression $ax^2 + bx + c$ gives maximum value $= \frac{4ac - b^2}{4a}$

Based on above information, do the following questions :

22. The minimum value of the expression $4x^2 + 2x + 1$ $(x \in R)$ is –
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

23. If x be real, the maximum value of $7 + 10x - 5x^2$ is –
(A) 12 (B) 15 (C) 16 (D) 18

24. If p and q ($\neq$ 0) are the zeros of the polynomial $x^2 + px + q$, then the least value of $x^2 + px + q$ $(x \in R)$ is –
(A) $-\frac{1}{4}$ (B) $\frac{1}{4}$ (C) $-\frac{9}{4}$ (D) $\frac{9}{4}$

25. If x is real, the minimum value of $x^2 - 8x + 17$ is –
(A) – 1 (B) 0 (C) 1 (D) 2

OBJECTIVE — ANSWER KEY — EXERCISE -5

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	D	A	B	B	D	D	C	A	B	B	C	A	D	B
Que.	16	17	18	19	20	21	22	23	24	25					
Ans.	C	C	B	D	A	B	C	A	C	C					