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+91-9939586130
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www.aepstudycircle.com

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2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122-JH

PHYSICS

MOTION IN ONE DIMENSION

INTRODUCTION AND PARAMETERS

What you already know

- Functions
- Differentiation
- Integration
- Vectors

What you will learn

- Rest and Motion
- Frame of reference
- Motion Parameters
- Position Time graphs
- Instantaneous speed and velocity

Mechanics

Mechanics is the study of cause and effect of motion of bodies.

Kinematics

Branch of mechanics which analyses the effects i.e., parameters and properties of motion.

Dynamics

Branch of mechanics which analyses the cause of motion of bodies i.e., the force(s).

Rest and Motion - Motion is a combined property of the object and the observer. There is no meaning of rest or motion without the observer. Nothing is in absolute rest or in absolute motion. An object is said to be in motion with respect to an observer, if its position changes with respect to that observer. For motion, it may happen that the observer moves with respect to the object or the object moves with respect to the observer.

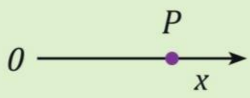
Motion is always relative.

So, we will now define motion by defining the observer, the frame (frame of reference) in which the observer sits and other related parameters.

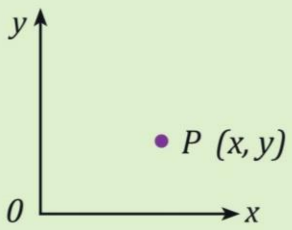
Frame of reference : The frame attached with the observer.

Reference point : The point from where measurements are taken.

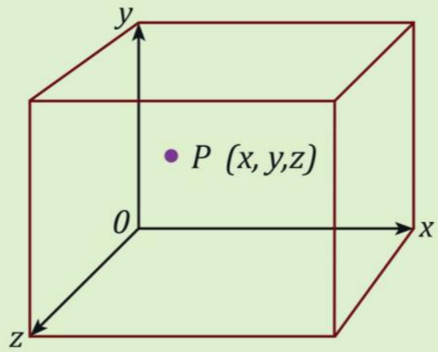
Dimension : The dimension of a mathematical space is the minimum number of independent information needed to specify any point within it.



1-Dimension



2-Dimension



3-Dimension

Motion in One Dimension

The particle is constrained to move in a straight line and can only change its direction of motion opposite to its original direction of motion.

Parameters related to Motion

Position

Acceleration

Velocity

Distance

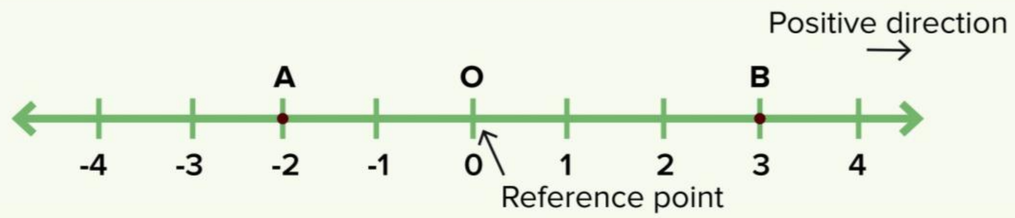
Displacement

Time

Speed

Parameters related to Motion

Position of a point : The position of a point is the expression of its accurate location from a pre-chosen reference point called the origin.



Position of *A* with respect to *O* (denoted by X_{AO}) is *-2*.

$X_{AO} = X_A - X_O = -2$

Position of *B* with respect to *O* (denoted by X_{BO}) is *3*.

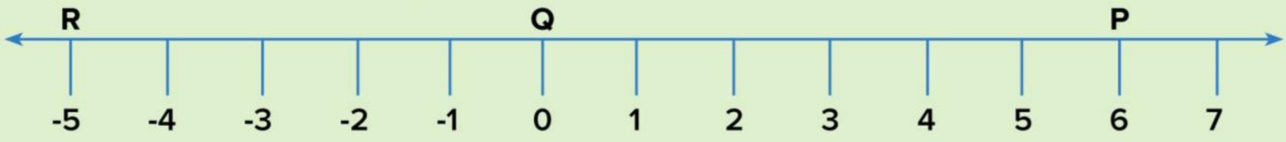
$X_{BO} = X_B - X_O = 3$

Distance : Distance covered by a body is the total length of the actual path covered in travelling from its initial to final position.

Distance is a scalar quantity.

Displacement : Displacement is the shortest distance from the initial to the final position of a body undergoing motion.

Displacement is a vector quantity.



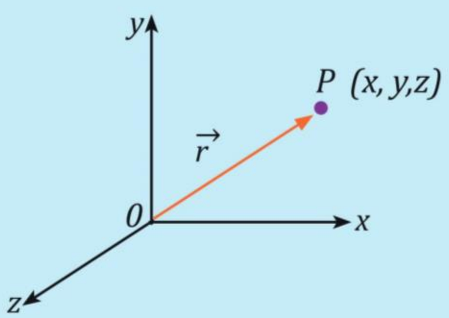
If a particle travels from R to P and then returns back to Q

Distance	Displacement
length of RQ + length of QP + length of PQ = 5 + 6 + 6 = 17 Units	Shortest distance between R and Q= 5 Units

Position and Displacement vector

Position vector

A vector which represents the **position of a point** in magnitude and direction w.r.t the origin of a coordinate system is called **position vector.**

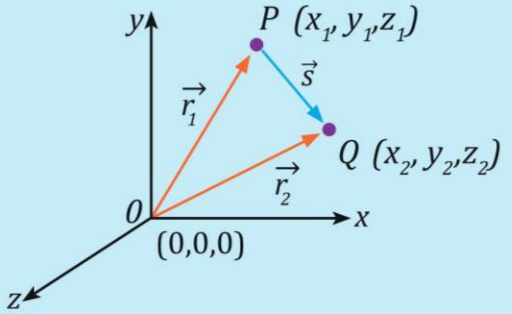


Position vector of Point 'P' w.r.t origin is

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Displacement vector

- A displacement vector represents a straight line directed from initial to final position.
- It is the difference between the final and initial position vectors.



$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$
$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$
$$\vec{r}_1 + \vec{s} = \vec{r}_2$$
$$\vec{s} = \vec{r}_2 - \vec{r}_1$$

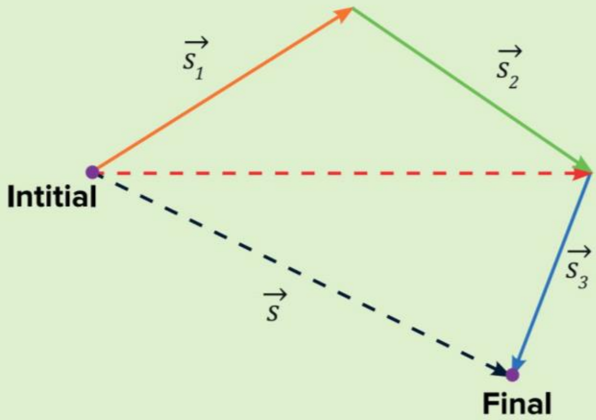
$$\vec{s} = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\,\hat{k}$$

BOARDS

Distance and Displacement

If a particle passes through two different points before reaching the final position, then

Displacement: $\vec{s} = \vec{s}_1 + \vec{s}_2 + \vec{s}_3$



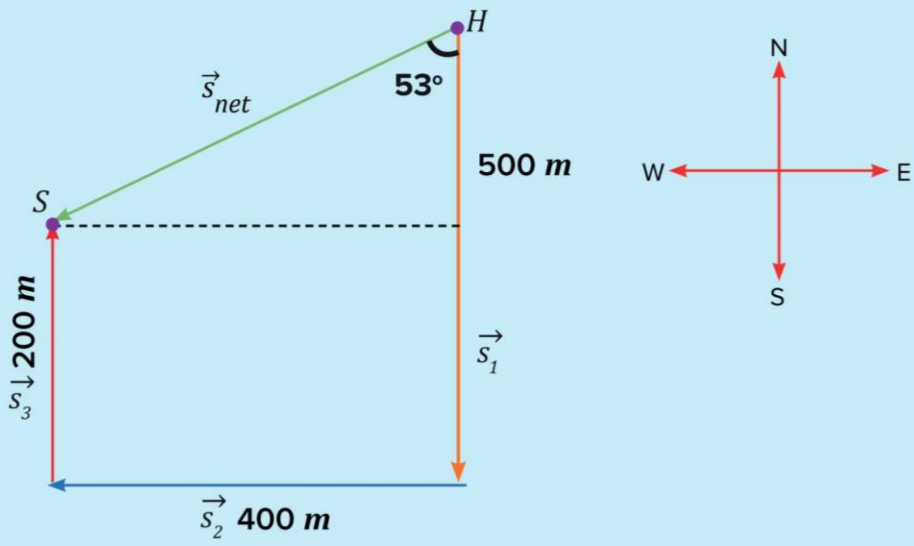
Distance: $d = |\vec{s}_1| + |\vec{s}_2| + |\vec{s}_3|$

A boy starts from his home and walks for 500m due south, 400m due west and 200m due north to reach his school. What is the distance covered by the boy, and the displacement from his home to school?

Solution

Step 1

Visualize the situation with a diagram.



Step 2

Recall the definitions and apply them.

(i) Distance, $d = |\vec{s}_1| + |\vec{s}_2| + |\vec{s}_3|$

$d = 500m + 400m + 200m$

$d = 1100m$

(ii) Displacement, $\vec{s}_{net} = \vec{s}_1 + \vec{s}_2 + \vec{s}_3$

$\vec{s}_{net} = -500\hat{j} - 400\hat{i} + 200\hat{j}$

$= -400\hat{i} - 300\hat{j}$

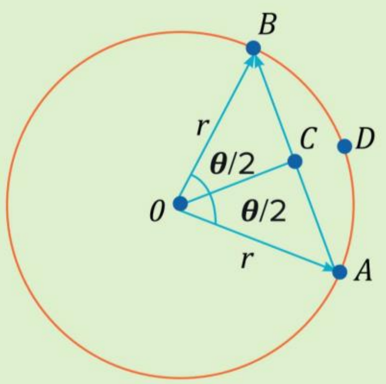
$|\vec{s}_{net}| = \sqrt{300^2 + 400^2} = 500\ m$

$\tan\theta = \dfrac{400}{300} = \dfrac{4}{3}$

$\theta = \tan^{-1}\dfrac{4}{3} = 53^\circ$

Therefore, the displacement $\vec{s}_{net}$ is $500m$, 53° west of south.

Displacement along circular path



Distance covered = ADB (arc length) = $r\theta$

Linear Displacement = AB

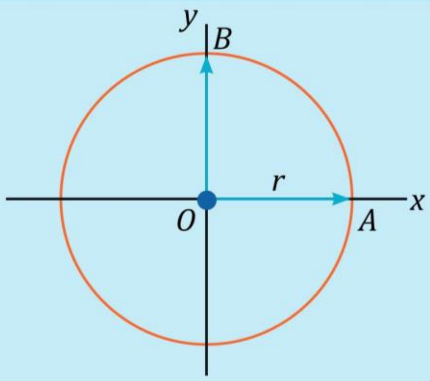
$AC = OA \sin\dfrac{\theta}{2}$

$\therefore AB = 2AC = 2r\sin\dfrac{\theta}{2}$

A body moves over one-fourth of a circular arc from A to B in a circle of radius r. The magnitudes of distance travelled and displacement will be, respectively

(a) $\dfrac{\pi r}{2}, r\sqrt{2}$ (b) $\dfrac{\pi r}{2}, r$ (c) $\pi r, \dfrac{r}{\sqrt{2}}$ (d) $\pi r, r$

Solution



Distance travelled = l

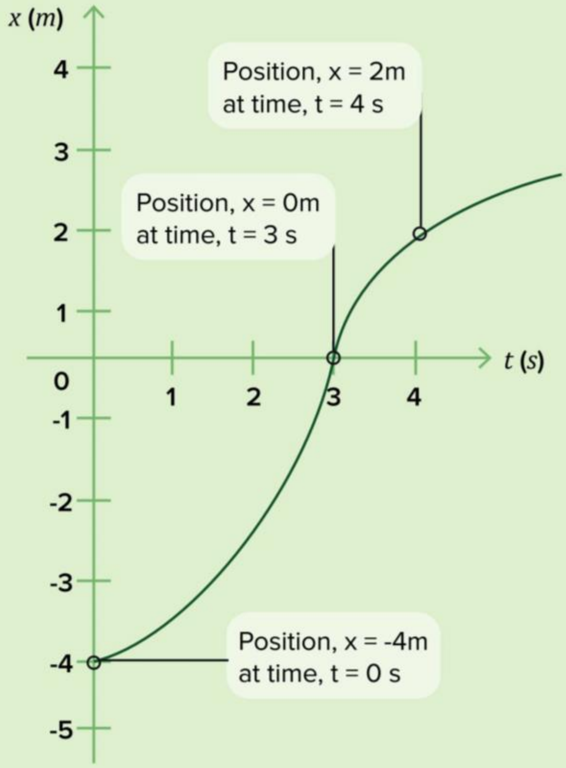
$l = r\theta$

$l = \dfrac{\pi r}{2}$

Displacement $= \sqrt{r^2 + r^2} = r\sqrt{2}$

Position-Time Graph

It is a pictorial representation of the variation of the position of a body with time. For example, a particle P is at $x = -4m$ with respect to the point of reference (origin) at $t = 0$. And as it starts moving towards the positive direction, it reaches $x = 0m$ at $t = 3s$ and $x = 2m$ at $t = 4s$. The motion can be represented graphically like this:

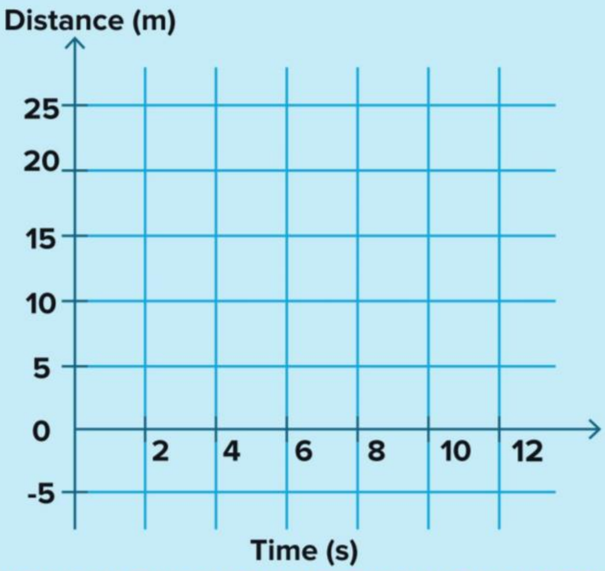


A man walks 10 m due positive direction in 5 seconds, then stops for 2 seconds and finally walks 15 m due negative direction in 5 seconds. Draw his displacement - time graph and distance-time graph. (Assume distance covered changes linearly between given positions.)

BOARDS

Solution

Step 1: Choose the right scale and draw the axes



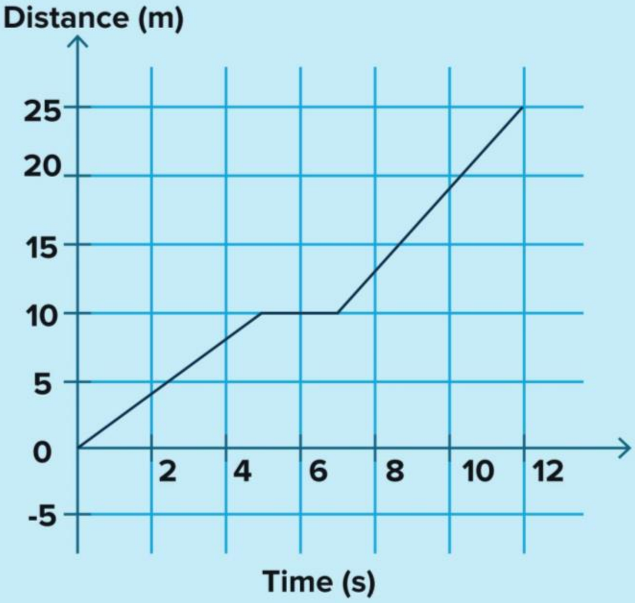
Step 2: Break down the movement of the man.

0 s to 5 s: 10 m along positive direction

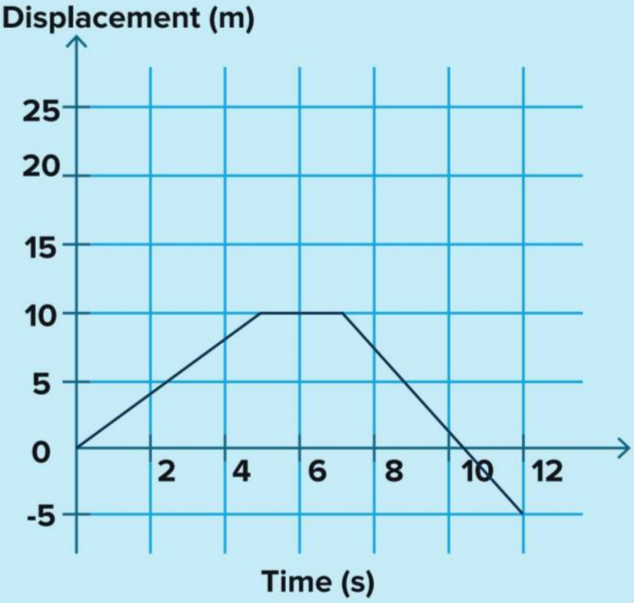
5 s to 7 s: 0 m at rest

7 s to 12 s: 15 m along negative direction

Step 3: Draw the distance vs. time graph.



Step 4: Draw the displacement vs. time graph.



Speed

1. Average Speed

Average speed of a particle over a certain time interval is defined as the ratio of total distance of path travelled to the time interval.

$$\langle v \rangle = \frac{\text{total distance covered}}{\text{time taken}}$$

It is a **scalar** quantity.

2. Instantaneous Speed

It is defined as the limit of the average speed when the considered time interval approaches zero.

$$\text{Speed} = \frac{\Delta s}{\Delta t}$$

$$\text{Instantaneous Speed} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

$$\text{Instantaneous Speed} = \frac{ds}{dt}$$

MAIN

Velocity

1. Average Velocity

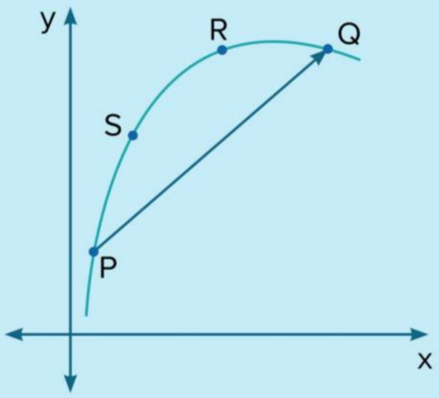
The average velocity of a particle over a certain time interval is defined as the ratio of net displacement to the time interval.

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time taken}}$$

It is a **vector** quantity.

$$<\vec{v}> = \frac{\vec{\Delta s}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

Direction of Average Velocity



The direction of average velocity is the **direction of the net displacement** for the time interval it is calculated.

$$<\vec{v}> = \frac{\vec{\Delta s}}{\Delta t}$$

$$\Rightarrow <\vec{v}> \parallel \vec{\Delta s}$$

2. Instantaneous Velocity

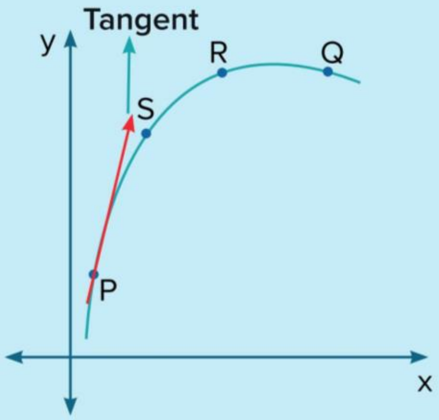
- Instantaneous rate of change of position with respect to time i.e. **velocity at an instant.**
- It is the ratio of displacement to an infinitesimally small interval of time.

$$<\vec{v}> = \frac{\vec{\Delta s}}{\Delta t}$$

$$<\vec{v}> = \lim_{\Delta t \to 0} \frac{\vec{\Delta s}}{\Delta t}$$

$$<\vec{v}> = \frac{\vec{ds}}{dt}$$

Direction of instantaneous velocity



The direction of instantaneous velocity at any instant is tangential to the path at that instant.

$$<\vec{v}> = \frac{d\vec{s}}{dt}$$

$$\therefore \ \vec{v} \parallel d\vec{s}$$

Instantaneous speed of a particle is the magnitude of its instantaneous velocity.
$\Rightarrow |\vec{v}| = $ Speed

BOARDS

Graphical interpretation of velocity

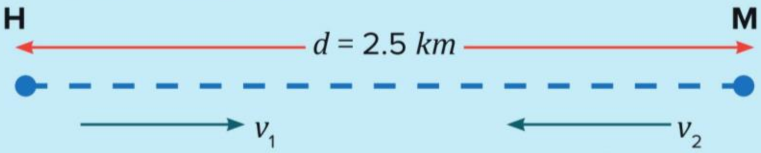
Slope of tangent at any time, $t_i = \tan\theta_i$ = Instantaneous velocity at time t_i

A man walks on a straight road from his home to the market 2.5km away with a speed of 5km/h. Finding the market closed, he instantly turns and walks back home with a speed of 7.5km/h. What is the average speed of the man over the interval of time 0 to 40 min?

Solution

Step 1

Visualize the situation.



Step 2

Calculate the time taken for the first half of the journey.

Time taken to go to market,

$t_1 = \frac{d}{v_1} = \frac{2.5}{5} = \frac{1}{2}\, hr = 30\ mins$

Step 3

Calculate the remaining time.

Remaining time,

$t_2 = 40 - 30 = 10\ mins$

Step 4

Calculate the distance travelled in the remaining time.

Distance travelled while returning home,

$= v_2 \times t_2 = 7.5 \times \frac{10}{60} = 1.25 km$

Step 5

Recollect the formula for average speed and apply it.

Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{2.5 + 1.25}{\frac{40}{60}} = 5.625\ km/h$

A particle travels from point A to B in a straight line with uniform speed of 60 km/h. It immediately returns from B to A with uniform speed of 40 km/h. Find the magnitudes of average velocity and average speed of particle over the whole journey.

Solution

Magnitude of displacement, $|\vec{s}| = 0$

Hence magnitude of average velocity

$$\frac{|\vec{s}|}{t} = |<\vec{v}>| = 0 \ km/h$$

$$\text{Average speed} = <v> = \frac{\text{Total distance}}{\text{Total time}}$$

$$v = \frac{s+s}{t_1+t_2} = \frac{2s}{s\left[\frac{1}{v_1}+\frac{1}{v_2}\right]} = \frac{2v_1 v_2}{v_1+v_2}$$

(Given $v_1 = 60 \ km/h$ and $v_2 = 40 \ km/h$)

$$= \frac{2 \times 40 \times 60}{40+60} = 48 \ km/h$$

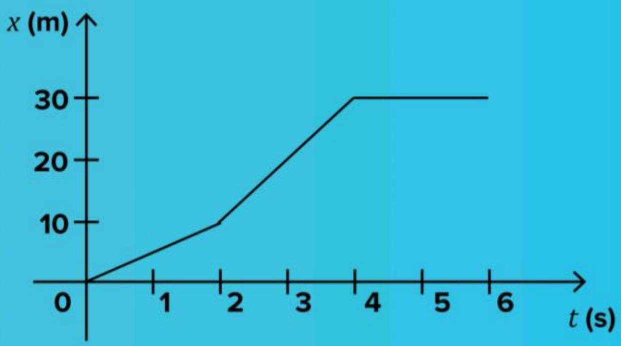
The position-time graph of an object moving in a straight line is given.

What is the average speed of the object during the time interval

(i) $t = 2s$ to $t = 4s$

(ii) $t = 4s$ to $t = 6s$

Also find the instantaneous speed at $t = 1s$ and $t = 3s$



MAIN

Solution

Step 1

Observe the graph and note down your observations.

- The curve from $t = 0s$ to $t = 2s$ is a single straight line.
- The curve from $t = 2s$ to $t = 4s$ is another single straight line.
- The curve from $t = 4s$ to $t = 6s$ is a horizontal line.

Step 2

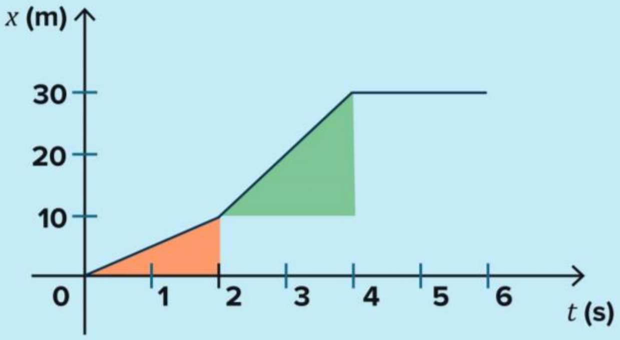
Calculate the distance travelled and average speed based on these observations.

(i) From $t = 2s$ to $t = 4s$,
Total distance covered = 30 - 10 = 20m

$$\text{So, Average speed} = \frac{\text{distance covered}}{\text{time taken}}$$

$$= \frac{20m}{2s} = 10 \ m/s$$

(ii) From $t = 4s$ to $t = 6s$, the object is at rest. So average speed is **zero.**



Instantaneous speed at $t = 1s$

$$v|_{t=1s} \Rightarrow \text{slope}|_{t=1s} = \frac{\Delta x}{\Delta t} = \frac{10 - 0}{2 - 0} = 5\ m/s$$

Instantaneous speed at $t = 3s$

$$v|_{t=3s} \Rightarrow \text{slope}|_{t=3s} = \frac{\Delta x}{\Delta t} = \frac{30 - 10}{4 - 2} = 10\ m/s$$

MOTION IN ONE DIMENSION

MOTION UNDER ACCELERATION

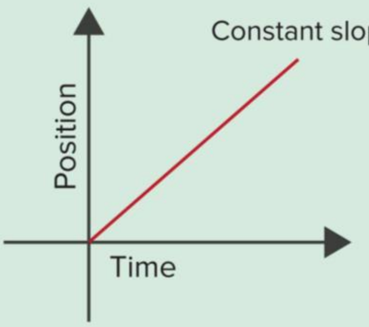
What you already know

- Rest and motion
- Frame of reference
- Motion parameters
- Position time graphs
- Instantaneous speed and velocity

What you will learn

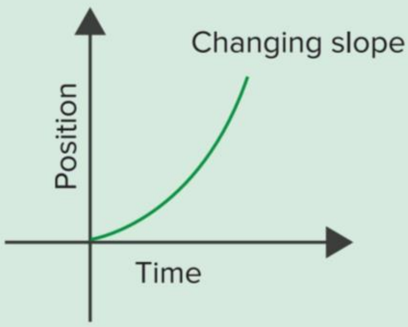
- Uniform and non-uniform motion
- Acceleration
- Graphs
- Equations of motion

Uniform motion



- If a body travels equal distances in equal intervals of time, the motion is **uniform**.
- Graphically, if the curve on a position vs time graph is a straight line, then the motion is uniform.

Non-uniform motion

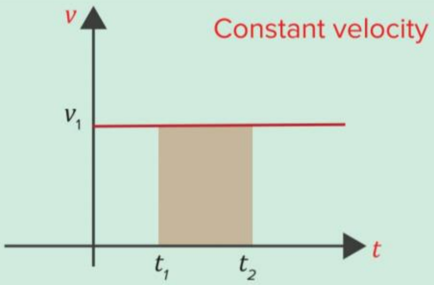


- If a body travels unequal distances in equal intervals of time, the motion is **non-uniform**.
- Graphically, if the curve on a position vs time graph is a not straight line, then the motion is non-uniform.

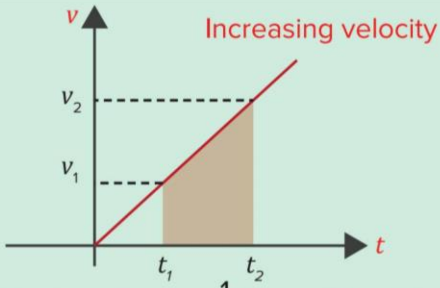
MAIN

Velocity vs Time Graph

- The area under the curve on a velocity vs time graph gives us the change in position or displacement in that time interval.
- For the interval t_1 to t_2 , the displacement is given by the area between t_1 , t_2 and the velocity-time curve.

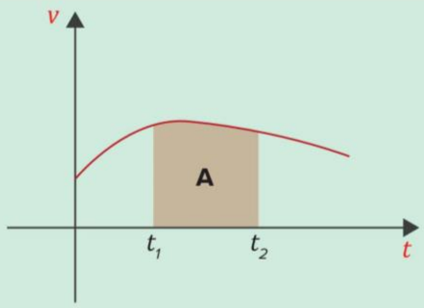


$$Displacement = v_1(t_2 - t_1)$$



$$Displacement = \frac{1}{2}(v_1 + v_2)(t_2 - t_1)$$

- If the curve is irregular, then



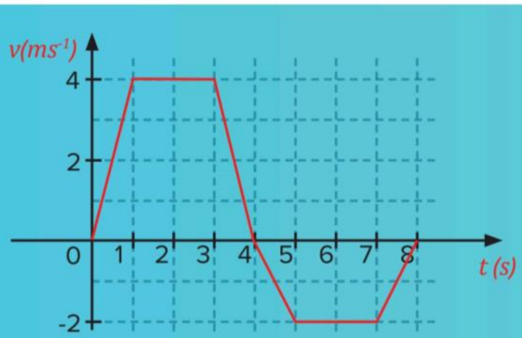
$$A = \int_{t_1}^{t_2} v\,dt = \text{displacement covered}$$

Area is a vector quantity. While calculating area from a graph, you may get a positive or a negative value depending on the location of said area in the graph. See the example given below for better understanding.

The velocity vs time graph of a linear motion is shown in the figure. For the particle, find the net

(i) distance covered.

(ii) displacement at the end of 8 sec.



Solution

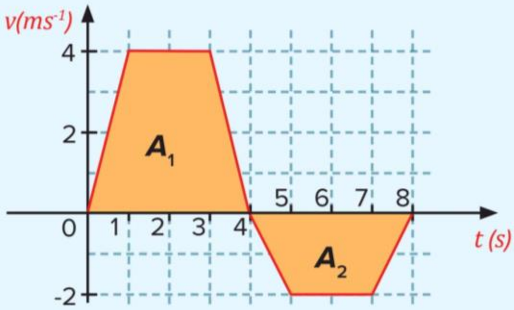
Step 1:
Observe the graph and note down your observations.

- The curve projects two trapeziums on the x axis.
- The length of the parallel lines of the first trapezium is 2 and 4 and the height is 4 units.
- The length of the parallel lines of the second trapezium is 2 and 4 and the height is -2 units.

Step 2: Calculate the area under the v - t curves.

$$A_1 = \frac{1}{2} \times (2 + 4) \times 4 = 12$$

$$A_2 = \frac{1}{2} \times (2 + 4) \times (-2) = -6$$



Step 3: Calculate the distance and displacement

(i) Distance travelled by the particle

$$S = |A_1| + |A_2|$$
$$= 12 + 6$$
$$S = 18m$$

(ii) The displacement of the particle is given by

$$x = A_1 + A_2$$
$$= 12 - 6$$
$$x = 6m$$

In the above example dealing in linear motion, the v-t curve crosses the time axis, i.e, velocity went from positive to negative. From this you can infer that the particle in motion reversed its direction at that point. Which also means the final or net displacement of the particle will be less than the total distance travelled.

Acceleration

Acceleration is the rate of change of velocity of an object per unit time.

Average Acceleration

$$\text{Average acceleration} = \frac{\text{Change in velocity}}{\text{time taken}}$$

$$\langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

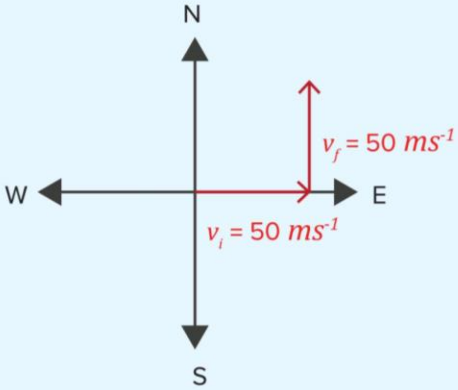
It is a vector quantity. Its direction is always along the "change in velocity vector".

$$\langle \vec{a} \rangle \parallel \Delta \vec{v}$$

A particle initially moving towards east with 50 ms^{-1}, takes left and moves along north with 50 ms^{-1} in an interval of 1 sec. What is the magnitude and direction of its average acceleration?

Solution

Step 1: Visualize the situation with a diagram.



Step 2: Recollect the formula and apply it.

$$\langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

$$\Delta \vec{v} = \vec{v}_f - \vec{v}_i = 50\hat{j} - 50\hat{i}$$

$$|v_f - v_i| = |50\hat{j} - 50\hat{i}| = \sqrt{50^2 + 50^2} = 50\sqrt{2}\ ms^{-1}$$

$$|\vec{a}| = \left|\frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}\right|$$

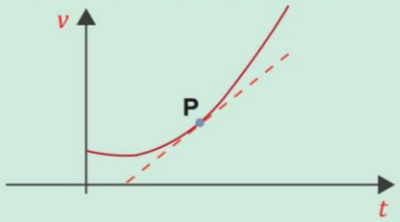
$$= \frac{50\sqrt{2}}{1}$$

$$= 50\sqrt{2}\ ms^{-2}$$

And as we know, the direction of average acceleration and change in velocity is the same. So, the direction of $\langle \vec{a} \rangle$ is along the North-West direction.

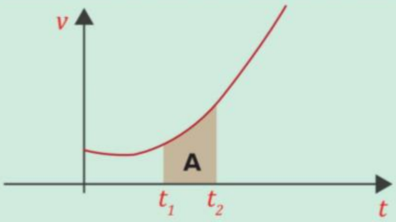
Instantaneous Acceleration

It is the rate of change in velocity at a particular instant of time. It is given by the slope of the tangent to the *v-t* graph at that instant of time.



$$Slope = \frac{dv}{dt} = instantaneous\ acceleration$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$



We already know that the area under the $v - t$ graph gives displacement.

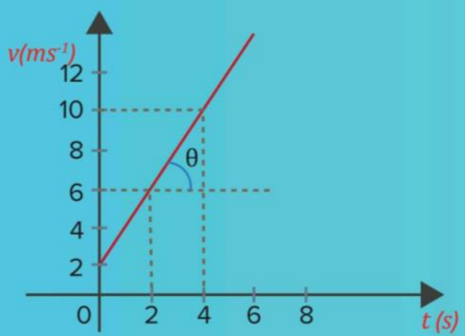
$$Area = \int_{t_1}^{t_2} v\,dt = displacement\ covered$$

- When a body is speeding up, or its velocity is increasing, it has positive acceleration.
- When a body is slowing down, or its velocity is decreasing, it has negative acceleration.
- When the direction of motion of the body changes, its velocity changes. This causes the acceleration to change as well.

The average acceleration of the object over the time interval 2s to 4s will be

a. 4 ms^{-2} b. 2 ms^{-2}

c. 8 ms^{-2} d. 6 ms^{-2}



Solution

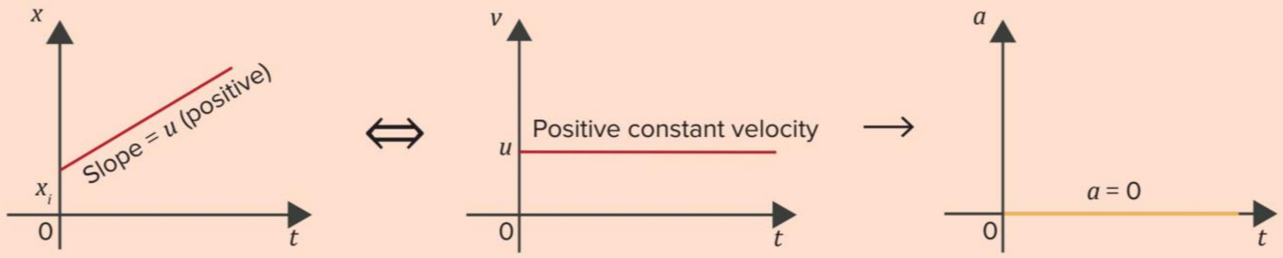
$$a = \tan\theta = \frac{\Delta v}{\Delta t} = \frac{10 - 6}{4 - 2}$$

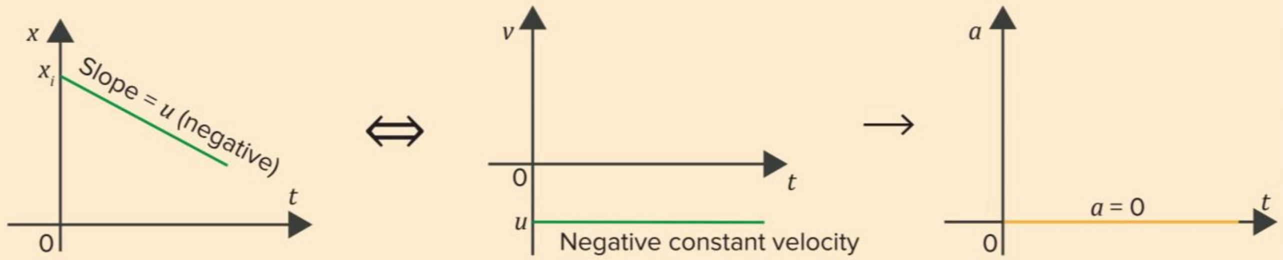
$$= 2\ ms^{-2}$$

Here, average acceleration is equal to the instantaneous acceleration as the acceleration is constant. You can check this by finding the slope at any other point in the graph.

Acceleration vs Time Graphs

Everything we have learnt about uniform velocity (both positive and negative) can be summarised using these graphs.



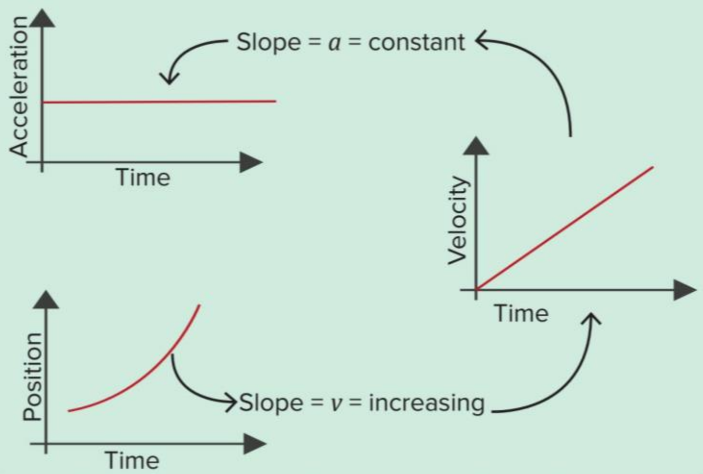


Uniform velocity ⇒ zero acceleration.

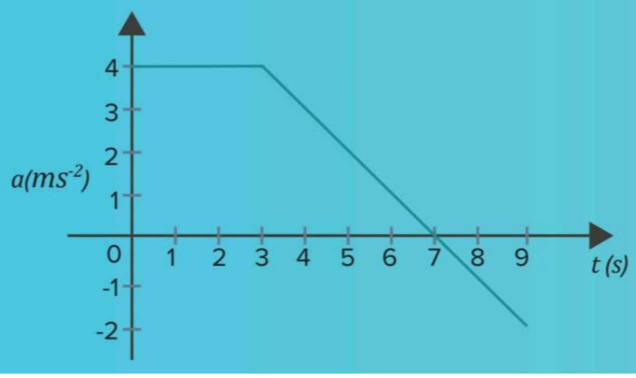
Here acceleration is uniform but zero. Now, when the acceleration is non zero and uniform, we'll have something like this.

Slope of acceleration and time graph gives jerk. The area under a-t graph gives change in velocity.

$$\int_{t_1}^{t_2} a\,dt = |\vec{v}_2 - \vec{v}_1| = |\Delta\vec{v}|$$



Vishnu is cycling in a straight line with a velocity of 5 m/s. Then, at time t = 0 s, a stiff wind blows causing him to accelerate as seen in the diagram. What is his cycling velocity after the wind has blown for 9s?



Solution

As the area under the a-t graph gives change in velocity,

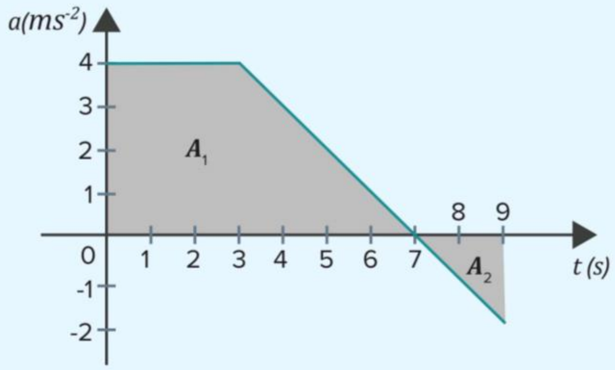
$A_1 = (4)(3) + \frac{1}{2}(4)(4) = 20$

$A_2 = \left[\frac{1}{2}(2)\times(-2)\right] = -2$

$v_f - v_i = A_1 + A_2 = 18\ ms^{-1}$

$v_f = 18\ ms^{-1} + v_i$

$v_f = 18\ ms^{-1} + 5\ ms^{-1} = 23\ ms^{-1}$



MAIN

Position-time $(x - t)$ graph

- Slope $= \frac{dx}{dt}$ = velocity

Velocity-time $(v - t)$ graph

- Slope $= \frac{dv}{dt}$ = acceleration
- Area $= \int_{t_1}^{t_2} v\, dt$ = displacement

Acceleration-time $(a - t)$ graph

- Slope $= \frac{da}{dt}$ = jerk
- Area $= \int_{t_1}^{t_2} a\, dt$ = change in velocity

BOARDS

Equation Of Motion For Uniform Acceleration

For uniformly accelerated motion along a straight line, we have simple equation that relates

u = *initial velocity (at t = 0)*

a = *acceleration (constant)*

v = *final velocity (at time t)*

s = *displacement (change in position / coordinate* $\Rightarrow \Delta x = x_f - x_i$*)*

x_f = *final coordinate (position)*

x_i = *initial coordinate (position)*

1st: $v = u + at$

2nd: $\Delta x = s = ut + \frac{1}{2}at^2$

3rd: $v^2 = u^2 + 2as = u^2 + 2a(\Delta x)$

An object moving at 8 ms^{-1}, accelerated at a constant rate of 5 ms^{-2}. The velocity of this object after 10 sec is

Solution

Step 1: Note down the given data.

Initial speed, u = 8 ms^{-1}

Acceleration, a = 5 ms^{-2}

Time period, t = 10 sec

Final velocity, v = ?

Step 2: Recollect the equations of motion.

$v = u + at$

Step 3: Implement the equation of motion.

$v = u + at$

$v = 8 + 5 \times 10$

$v = 58\ ms^{-1}$

A particle moving with 10 m/s along a straight line is subjected to constant acceleration. After some time, its velocity is observed to be 30 m/s. Find out velocity of the particle at the midpoint of its subsequent path.

Solution

Step 1: Note down the given data.

Let, total distance = 2x

Midpoint distance = x

Velocity at midpoint = v = ?

Initial speed = u = 10 ms^{-1}

Acceleration = a = 5 ms^{-2}

Time period = t = 10 s

Final velocity = v_f = 30 ms^{-1}

Step 2: Recollect the equations of motion.

$v^2 = u^2 + 2as$

Step 3: Apply the equation of motion in stages.

For the particle's motion from starting point to midpoint of the total distance,

$v^2 = 10^2 + 2ax$ .....(i)

For the particle's motion from midpoint to the end,

$30^2 = v^2 + 2ax$ .....(ii)

Subtracting (ii) from (i)

$v^2 = 10^2 + 2ax$ .....(i)

$30^2 = v^2 + 2ax$ .....(ii)

$v^2 - 30^2 = 10^2 - v^2$

$2v^2 = 900 + 100$

$v^2 = 500$

$v = 10\sqrt{5}\ ms^{-1}$

A man is standing 40 m behind a bus when it starts moving away with 1 ms^{-2} constant acceleration. At the same instant, the man starts running behind with a constant speed of 9 ms^{-2}. Find the time taken by the man to catch the bus.

Solution

Step 1: Visualise the given situation: ; at t = 0

Step 2: Note down the given data:

Speed of man = u = 9 ms^{-1}

Acceleration of bus = a = 1 ms^{-2}

Time period = t = ?

Distance between the bus and man = x_0 = 40 m

Step 3: Recollect the equation of motion.

$$s = ut + \frac{1}{2}at^2$$

Step 4: Implement the equation of motion.

Let the man catch the bus after 't' seconds.

Distance travelled by the bus during this time:

$x_0 = 40m,\ a = 1\ ms^{-2}$

$$x - 40 = 0 \times t + \frac{1}{2}(1)\,t^2$$

$$x = 40 + \frac{t^2}{2}\ \text{....(i)}$$

$$\Delta x = ut + \frac{1}{2}at^2$$

For man:

$x_0 = 0m,\ u = 9\ ms^{-1}$

$x = 9t$....(ii)

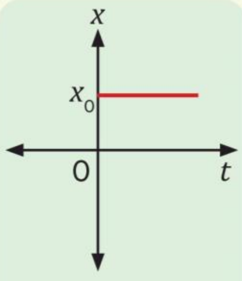
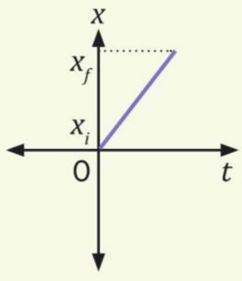
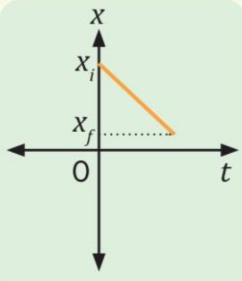
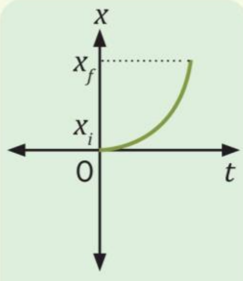
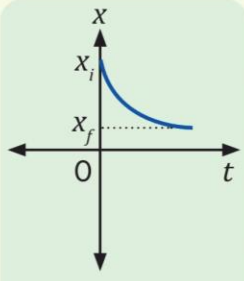
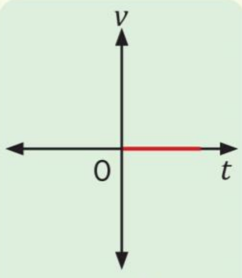
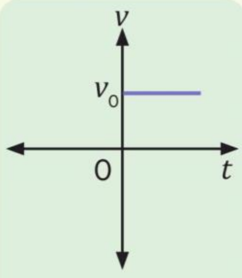
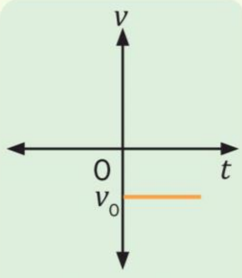
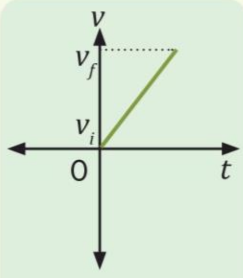
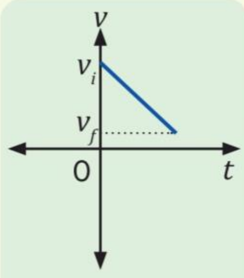
As in time t both the bus and the man should be at same position for the man to catch the bus, equate (i) and (ii).

$$40 + \frac{t^2}{2} = 9t$$

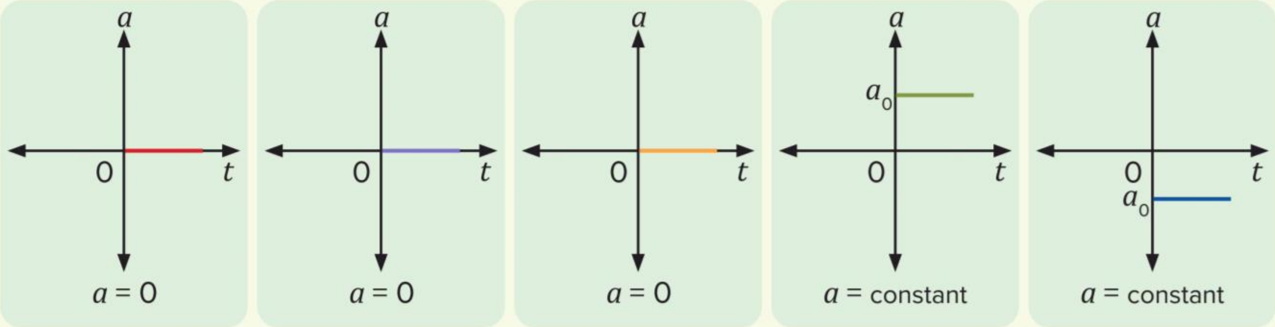
Step 5: Solve the quadratic equation. Solving this quadratic equation, we get, $t = 8s$ or $10s$.

Hence, the time taken by the man to catch the bus is 8 seconds. At 10s the bus will overtake the man once again.

MAIN ADVANCED

Graph	Rest	Unifrom velocity (+ve)	Unifrom velocity (-ve)	Unifrom acceleration (+ve)	Unifrom acceleration (-ve)
x - t	 x = constant				
v - t	 v = 0	 v = constant	 v = constant		

Graph	Rest	Unifrom velocity (+ve)	Unifrom velocity (-ve)	Unifrom acceleration (+ve)	Unifrom acceleration (-ve)
a - t	$a = 0$	$a = 0$	$a = 0$	a = constant	a = constant



MOTION IN ONE DIMENSION

MOTION UNDER GRAVITY

What you already know

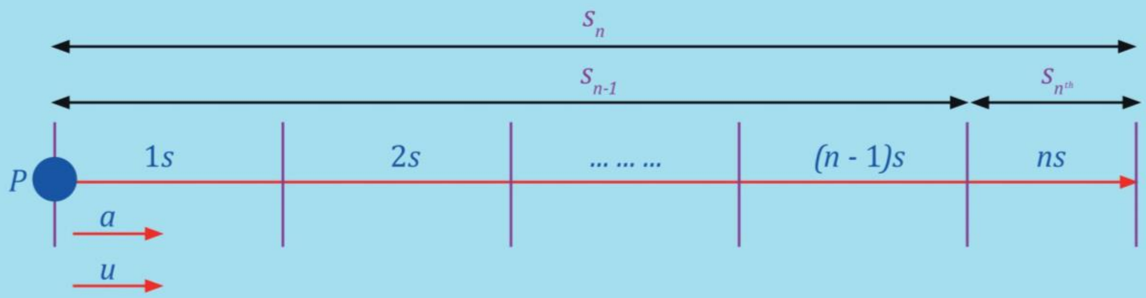
- Uniform and non-uniform motion
- Acceleration
- Graphs
- Equations of motion

What you will learn

- Displacement in the n^{th} second
- Motion under gravity
- Average speed and velocity

Displacement in n^{th} Second

An object *P* with initial velocity *u* and constant acceleration *a* undergoes a displacement s_n in *n* sec and s_{n-1} in $(n-1)$ sec. The displacement in n^{th} sec can be calculated like this :



Displacement in *n* sec is

$$s_n = un + \frac{1}{2}an^2 \dots \dots (1)$$

Displacement in $(n - 1)$ sec is

$$s_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2 \dots \dots (2)$$

Displacement in n^{th} sec

$$s_{nth} = s_n - s_{n-1}$$

$$s_{nth} = un - u(n-1) + \frac{1}{2}an^2 - \frac{1}{2}a(n-1)^2$$

$$= u(n-(n-1)) + \frac{1}{2}a(n^2 - (n-1)^2)$$

$$= u + \frac{1}{2}a(n^2 - n^2 + 2n - 1)$$

$$s_{n^{th}} = u + \frac{a}{2}(2n-1)$$

Example

The displacement of a body in the 4^{th} second of motion is twice the displacement in the 2^{nd} second. If the acceleration of the body is $3\ ms^{-2}$, what was its initial velocity?

Solution

$$s_{nth} = u + \frac{a}{2}(2n - 1)$$

$$s_{2nd} = u + \frac{1}{2}(3)(2 \times 2 - 1)$$

$$s_{2nd} = u + \frac{9}{2}$$

$$s_{4th} = u + \frac{1}{2}(3)(2 \times 4 - 1)$$

$$s_{4th} = u + \frac{21}{2}$$

$$s_{4th} = 2\ s_{2nd}$$

$$u + \frac{21}{2} = 2\left(u + \left(\frac{9}{2}\right)\right)$$

$$u + \frac{21}{2} = 2u + \frac{18}{2}$$

$$u = \frac{3}{2}\ ms^{-1}$$

Motion Under Gravity

For objects moving vertically near the surface of the earth, the only force acting on it is its weight (mg), i.e., the gravitational pull of the earth. The acceleration due to gravity ($a = g$) has a fixed value, irrespective of the mass of the object.

$$a = 9.8\ ms^{-2} \approx 10\ ms^{-2} \text{ (in SI or MKS)}$$

OR

$$a = 32.2\ ft\ s^{-2} \approx 32\ ft\ s^{-2} \text{ (in FPS)}$$

The underlying assumption to analyse motion under gravity is that "air resistance is negligible".

Example

A particle is thrown up with velocity 30 ms^{-1}. Find

(i) Time of fight

(ii) Maximum height reached by the particle

(iii) Landing speed of the particle

(assume acceleration due to gravity $g = 10\ ms^{-2}$)

Solution

Taking upwards as positive and starting point as origin.

Given:

a = g	$-10\ ms^{-2}$
u	$30\ ms^{-1}$
v	0

(i) Time of flight

Ascent

$v = u + at_a$

$0 = 30 - 10 \times t_a$

$t_a = \frac{30}{10} = 3s$

Descent

Since time of descent (t_d) is same as time of ascent (t_a)

$t_a = t_d$

Time of flight = $t_a + t_d$

$t = 6s$

OR

$s = ut + \frac{1}{2}at^2$

$0 = 30t + \frac{1}{2}(-10)t^2$

$t^2 - 6t = 0$

$t(t - 6) = 0$

$t = 0s, 6s$

This means the displacement was zero at the start (t = 0s) and end (t = 6s) of the motion.

(ii) Maximum height

$v^2 = u^2 + 2as$

$0 = 30^2 - 2 \times 10 \times s$

$900 = 20 \times s$

$s = \frac{900}{20}$

$s = 45\ m$

(iii) Landing speed

$v_f = u + at$

$= 30 - 10 \times 6$

$v_f = -30\ ms^{-1}$

Landing speed = $30\ ms^{-1}$

Basic steps to follow while solving these questions:

Step 1: Choose origin and one direction as positive.

Step 2: Tabulate the given data with correct sign.

Step 3: Select appropriate equation, insert the given values and get the answer.

Example

A ball is thrown upwards from the top of a tower of height 60 *m* with a velocity of 20 ms^{-1}. Find the landing speed on the ground.

Solution

Taking upwards as positive and starting point as origin.

Given:

a = g	$-10\ ms^{-2}$
u	$20\ ms^{-1}$
s	$-60\ m$

$v^2 = u^2 + 2gs$

$v^2 = 20^2 + 2 \times (-10) \times (-60)$

$v^2 = 1600$

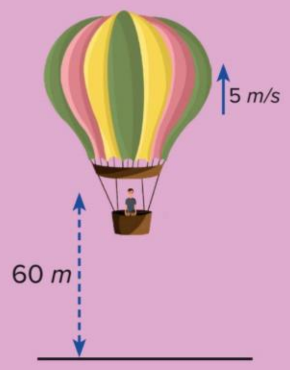
$v = \pm 40\ ms^{-1}$

Landing speed = $40\ ms^{-1}$

Example

BOARDS

A stone is dropped from a balloon going up with a uniform velocity of 5 ms^{-1}. If the balloon was at 60 m height when the stone was dropped, find the time after which the stone hits the ground. (Take $g = 10\ ms^{-2}$)

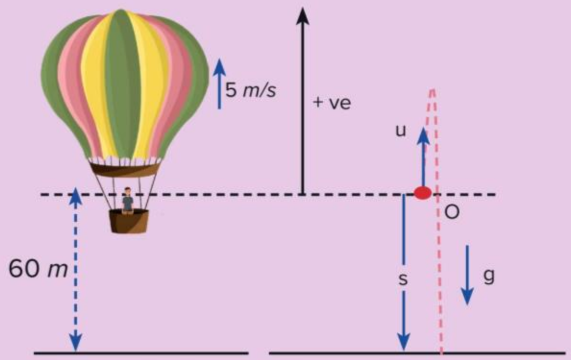


Solution

Choosing origin at point of detachment and upwards as positive.

Given:

u	$+5\ ms^{-1}$
a = g	$-10\ ms^{-2}$
s	$-60\ m$



$s = ut + \frac{1}{2}at^2$

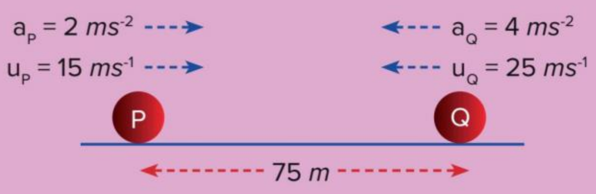
$-60 = 5(t) + \frac{1}{2}(-10)t^2$

$t = 4s$

When a body detaches itself from a system, it retains its velocity by virtue of inertia of motion.

Example

Two particles P and Q are separated by 75 m. Both are moving **towards** each other in a straight line as shown in the figure with $a_P = 2\ ms^{-2}$, $u_P = 15\ ms^{-1}$, $a_Q = 4\ ms^{-2}$, $u_Q = 25\ ms^{-1}$. Calculate the time when they meet.



Solution

Choosing the origin at P and rightward as positive.

At $t = t_0$ (i.e., the time of convergence)

Final position of point P = Final position of point Q

$\left(x_0 + ut + \frac{1}{2}at^2\right)_P = \left(x_0 + ut + \frac{1}{2}at^2\right)_Q$

$u_P = +15\ ms^{-1}$	$u_Q = -25\ ms^{-1}$
$a_P = +2\ ms^{-2}$	$a_Q = -4\ ms^{-2}$
x_0 for P = 0 m	x_0 for Q = + 75 m

$$15t + \frac{1}{2}(2)\,t^2 = 75 + (-25)\,t + \frac{1}{2}(-4)\,t^2$$

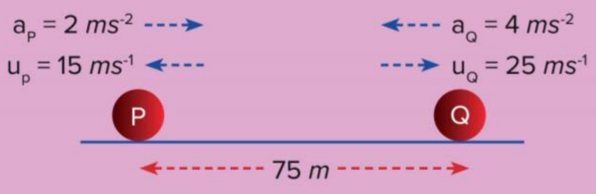
$$3t^2 + 40t - 75 = 0$$

$$t = \frac{5}{3}\ s$$

Example

MAIN

Two particle *P* and *Q* are separated by 75 *m*. Both are moving **away** from each other in a straight line as shown in the figure with $a_P = 2\ ms^{-2}$, $u_P = 15\ ms^{-1}$, $a_Q = 4\ ms^{-2}$, $u_Q = 25\ ms^{-1}$. Calculate the time when they meet.



Solution

Choosing the origin at *P* and rightward as positive.

At $t = t_0$ (i.e., the time of convergence)

Final position of point *P* = Final position of point *Q*

$$\left(x_0 + ut + \frac{1}{2}at^2\right)_P = \left(x_0 + ut + \frac{1}{2}at^2\right)_Q$$

$u_P = -15\ ms^{-1}$	$u_Q = 25\ ms^{-1}$
$a_P = +2\ ms^{-2}$	$a_Q = -4\ ms^{-2}$
x_0 for P = 0 m	x_0 for Q = + 75 m

$$-15t + \frac{1}{2}(2)\,t^2 = 75 + (25)\,t + \frac{1}{2}(-4)\,t^2$$

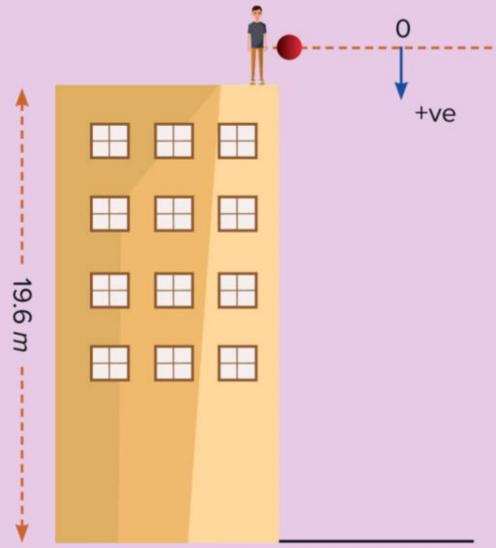
$$3t^2 - 40t - 75 = 0$$

$$t = 15\ s$$

Example

A ball is dropped from a height of 19.6 *m* above the ground. It rebounds from the ground and raises itself up to the same height. Take the starting point as the origin and vertically downward as the positive. Draw approximate plots of position vs time, velocity vs time, and acceleration vs time. Neglect the small interval during which the ball was in contact with the ground. (Take $g = 9.8\ ms^{-2}$).

Solution



Choosing the origin at the initial position of the ball and vertical downward direction as positive.

$h = x = +19.6\ m$, $u = 0\ ms^{-1}$, $a = g = +9.8\ ms^{-2}$

$$x = ut + \frac{1}{2}at^2$$

$$19.6 = (4.9)\,t^2$$

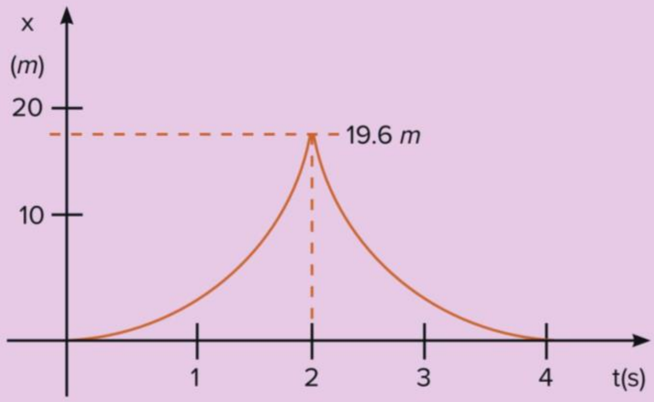
$$t = 2\ s$$

In 2 sec, ball reaches the ground and it takes another 2 sec to bounce back to the initial point.

$x = ut + \frac{1}{2}at^2$

$x = 4.9\ t^2$

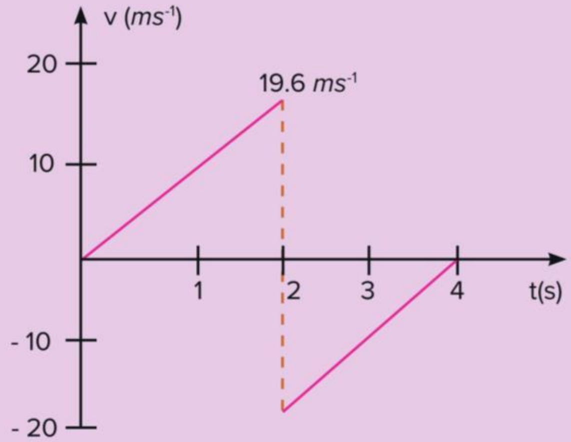
t = 0 s	x = 0 m
t = 1 s	x = 4.9 m
t = 2 s	x = 19.6 m
t = 3 s	x = 4.9 m
t = 4 s	x = 0 m



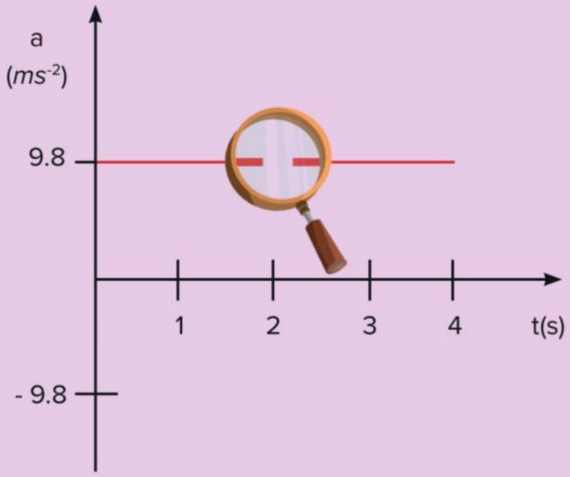
$v = u + gt$

$v = 9.8t$

t = 0 *s*	v = 0
t = 1 *s*	v = 9.8 ms^{-1}
t = 3 *s*	v = - 9.8 ms^{-1}
t = 4 *s*	v = 0



The ball is in contact with the ground at t = 2 *s*. The velocity and acceleration at that instant is **not defined**, as the direction of velocity or acceleration can't be determined.



Example

ADVANCED

An elevator starts descending with uniform acceleration. To measure the acceleration, a person in the elevator drops a coin at the moment the elevator starts. The coin is *6 ft* above the floor of the elevator at the time it is dropped. The person observes that the coin strikes the floor in one second. Calculate from this data, the acceleration of the elevator.

Solution

Assuming the initial position of the coin as the origin and downward direction as positive.

For elevator and coin u = 0

Acceleration of coin g = 32 fts^{-2}

Acceleration of elevator = a

Time taken to strike the floor (coin), t = 1 s.

Distance covered by the coin:

$s_c = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}g(1)^2 = \frac{g}{2}$

Distance covered by the elevator:

$s_e = ut + \frac{1}{2}at = 0 + \frac{1}{2}a(1)^2 = \frac{a}{2}$

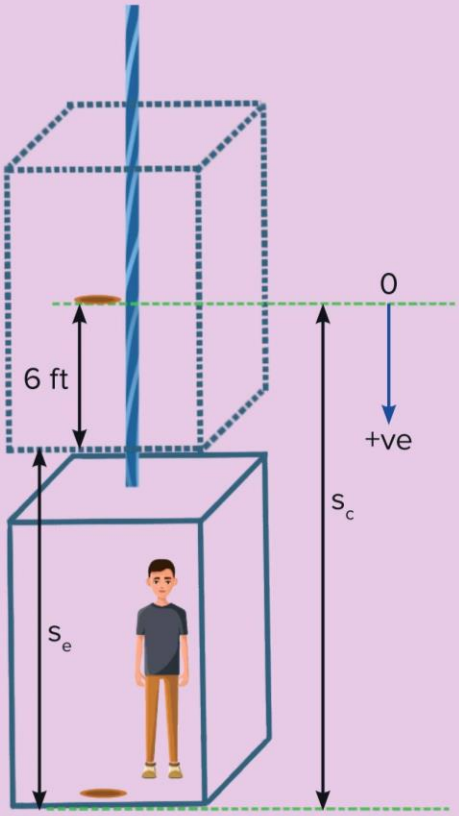
Total distance covered by coin is given by

$s_c = 6 + s_e$

$6 + \frac{1}{2}a = \frac{1}{2}g$

$\Rightarrow 6 + \frac{1}{2}a = 16$

$\Rightarrow a = 20\ fts^{-2}$



Some Questions on Average Speed and Velocity

Example

A particle travels half of the total distance with speed v_1 and next half with speed v_2 along a straight line. Find out the average speed of the particle.

Solution

Let total distance travelled by the particle be 2s.

Time taken to travel first half, $t_1 = \frac{s}{v_1}$

Time taken to travel next half, $t_2 = \frac{s}{v_2}$

Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

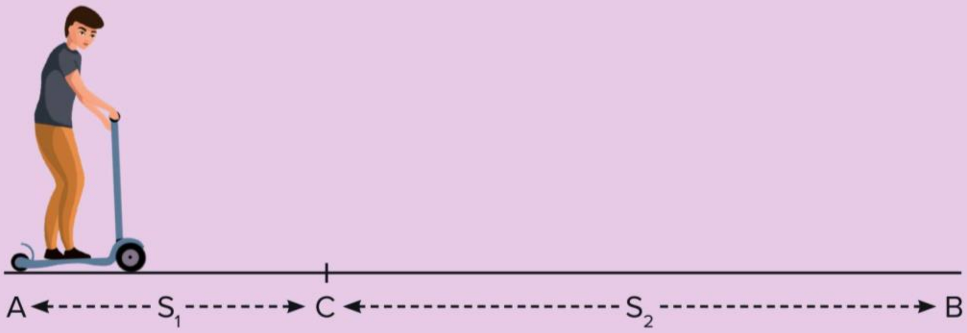
$\langle v \rangle = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$

Example

A person travelling on a straight line moves with uniform velocity v_1 for some time and with uniform velocity v_2 for the next equal time. The average velocity is given by

Solution

Let the person travel from A to B as per the given data, where first part S_1 is travelled in time $\frac{t}{2}$ and next S_2 also in time $\frac{t}{2}$



So according to the condition:

$$v_1 = \frac{S_1}{\frac{t}{2}} \text{ and } v_2 = \frac{S_2}{\frac{t}{2}}$$

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{S_1 + S_2}{t}$$

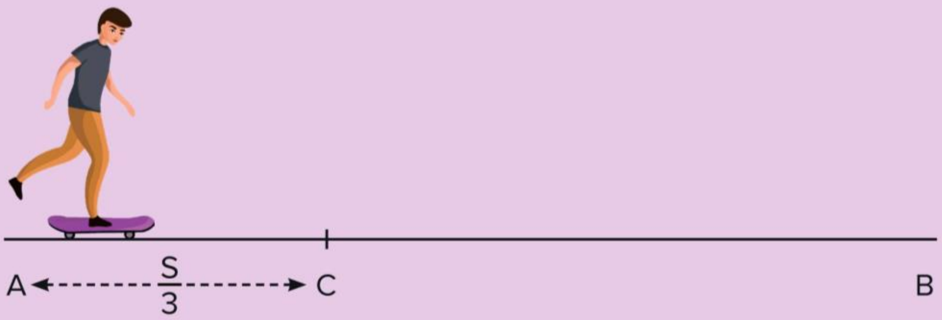
$$= \frac{\frac{v_1 t}{2} + \frac{v_2 t}{2}}{t} = \frac{v_1 + v_2}{2}$$

Example

A skate-boarder moving along a straight line travels one-third of the total distance with a speed of 3 ms^{-1} and the remaining is covered with a speed of 4 ms^{-1} for half the time and 5 ms^{-1} for the other half of the time. The average speed during his motion is.

Solution

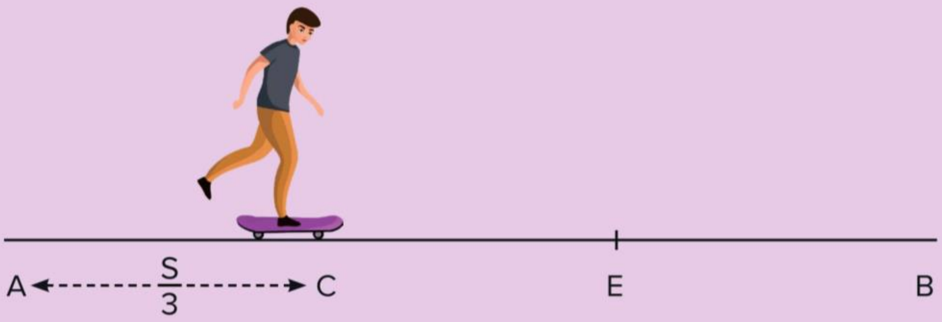
Given, $v_{AC} = 3\ ms^{-1}$ and let S be the total distance travelled by him



Let t_1 be the time taken for first one-third distance

$\Rightarrow t_1 = t_{AC} = \dfrac{\text{Distance}}{\text{Speed}} = \dfrac{S}{3 \times 3} = \dfrac{S}{9}$ sec

Now the remaining distance $= \dfrac{2s}{3}$. Let t_2, be the time taken for the remaining half

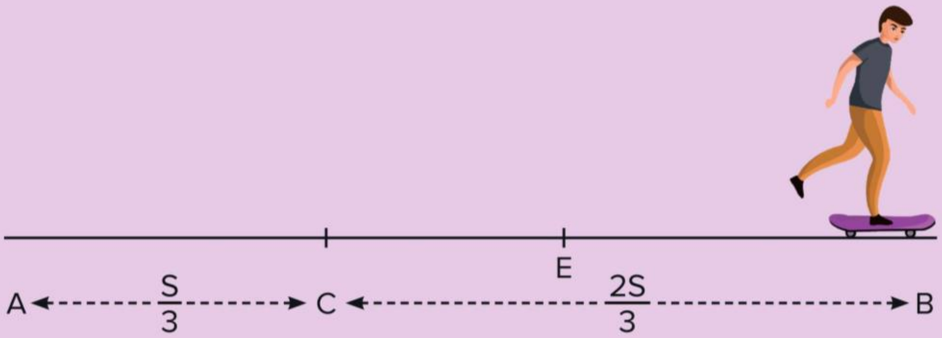


From given data, he travelled CE with a speed of 4 ms^{-1} in first $\dfrac{t_2}{2}$ and then EB with a speed of 5 ms^{-1} in next $\dfrac{t_2}{2}$

Hence, we can write that the remaining distance

$\dfrac{2s}{3} = 4 \times \dfrac{t_2}{2} + 5 \times \dfrac{t_2}{2} = \dfrac{9t_2}{2}$

$t_2 = \dfrac{4S}{27}$



So thus the total time $T = t_1 + t_2$

$= \dfrac{S}{9} + \dfrac{4S}{27} = \dfrac{7S}{27}$

$\therefore$ Average speed $= \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{S}{\dfrac{7S}{27}} = 3.85\ ms^{-1}$

MOTION IN ONE DIMENSION

EQUATIONS OF MOTION

What you already know

- Displacement in the n^{th} second
- Motion under gravity
- Average speed and velocity

What you will learn

- Illustrations
- Equations of motion: Derivation
- Relation between motion parameters

A police inspector in a car is chasing a pickpocket on a straight road. The car is going at its maximum speed v (assumed uniform). The pickpocket starts to ride on the motorcycle of a waiting friend when the car is at a distance d away. The motorcycle starts with a constant acceleration a. Show that the pickpocket will be caught if $v \geq \sqrt{2ad}$.

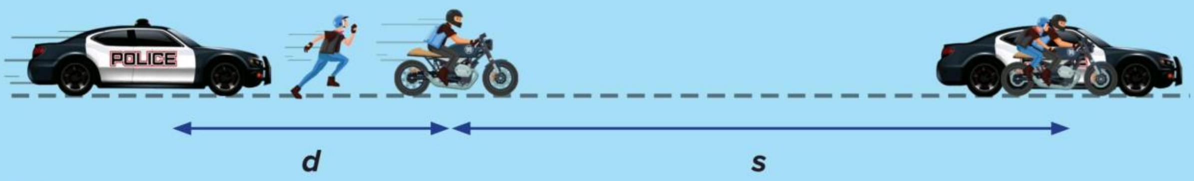
Solution

Suppose the pickpocket is caught at a time *t* after the motorcycle starts. As the initial velocity *u* of the pickpocket is zero, the distance travelled by the motorcycle during this interval is

$$s = \frac{1}{2}at^2 \; .... \text{(i)}$$

During this interval, the car travels a distance

$$s + d = vt \; .... \text{(ii)}$$



By (i) and (ii)

$$\frac{1}{2}at^2 + d = vt$$

$$t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}$$

The pickpocket will be caught if time is real and positive.
This will be possible when the term in square root is positive.

$$\therefore v^2 - 2ad \geq 0$$

$$v^2 \geq 2ad \text{ or } v \geq \sqrt{2ad}$$

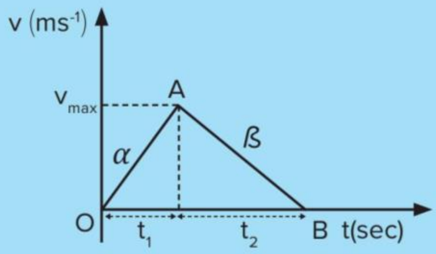
A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t, find the following:

(a) Maximum velocity acquired by the car (b) Displacement of the car in time t

MAIN

Solution

(a) The v-t curve for given problem will be as shown:



Total time of motion will be

$t = t_1 + t_2$

As we know, the slope of a v-t curve gives acceleration for that period, slope of OA curve

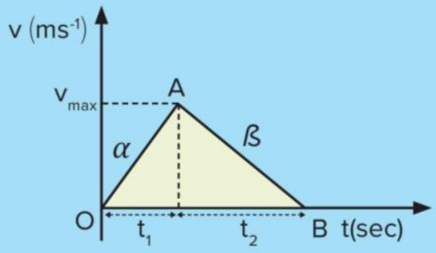
$\tan\theta = \alpha = \frac{v_{max}}{t_1}$

Similarly, slope of AB curve $\beta = \frac{v_{max}}{t_2}$

$$t = t_1 + t_2 = \frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} \Rightarrow v_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)t$$

(b) From the v-t graph

Area of triangle enclosed = Area under the graph = Displacement



$$= \frac{1}{2}(v_{max})(t) = \frac{1}{2}\frac{\alpha\beta t^2}{(\alpha+\beta)}$$

$$s = \frac{\alpha\beta t^2}{2(\alpha+\beta)}$$

A particle accelerates from rest with acceleration 5 ms^{-2}. After some time, the particle moves with constant velocity. Then it decelerates with 5 ms^{-2} and finally comes to rest. The whole journey takes 25 s and the average speed of the journey is 72 km h^{-1}. Which of the following statements is/are correct?

(A) The time interval during which it has moved with constant velocity is 15 seconds.
(B) The maximum velocity acquired by it is 25 ms^{-1}.
(C) The time interval during which it has moved with constant velocity is 25 seconds.
(D) Displacement of the particle in constant velocity phase is 325 m.

MAIN **Solution**

Let the time intervals be t_1, t_2 and t_3

Given, $t_1 + t_2 + t_3 = 25$ s

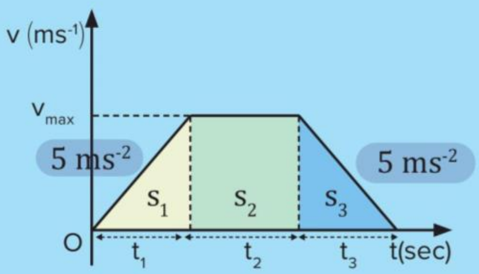
Average speed of journey is

$$\langle v \rangle = \frac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3} = 72 \times \frac{5}{18} = 20\,ms^{-1}$$

Then, the total distance covered will be

$\therefore s_1 + s_2 + s_3$

$= 20 \times 25 = 500\ m$



So, by area method,

$$s_1 = \frac{1}{2}(v_{max})t_1 = \frac{1}{2}(at_1)t_1 = \frac{1}{2}5t_1^2$$

$$s_2 = v_{max}t_2 = (at_1)t_2 = 5t_1t_2$$

$$s_3 = \frac{1}{2}(v_{max})t_3 = \frac{1}{2}(at_1)t_3 = \frac{1}{2}at_3^2 = 2.5t_3^2$$ (∵ by symmetry $t_1 = t_3$)

On solving, we get two values for quadratic equation roots.

$t_1 = t_3 = 20$ s and $t_2 = -15$ s , or
$t_1 = t_3 = 5$ s and $t_2 = 15$ s

Since time cannot be negative, the value of $t_2 = -15$ s is not possible.

So, correct solution is

$t_1 = t_3 = 5$ s and $t_2 = 15$ s

This shows that statement (A) is correct and statement (C) is incorrect.

The displacement will be:

$s_1 = s_3 = 62.5\ m$ and $s_2 = 375\ m$

This shows the statement (D) is incorrect.

$v_{max} = 25\ ms^{-1}$

This shows the statement (B) is correct.

An insect starts moving with a velocity 10 ms^{-1} towards the East and has an acceleration of 5 ms^{-2} towards the West. Find its distance and displacement in 4 s.

Solution

Taking positive along the East.

Using the formula for displacement,

$$s = ut + \frac{1}{2}at^2$$

$$s = 10 \times 4 - \frac{1}{2} \times 5 \times 16$$

$$s = 0\ m$$

This means that in 4 seconds, the insect moves towards the East and turns back towards the West on the same path, and returns back to its original position. Hence the displacement will be zero.

Now, let us find the total distance travelled by the insect.

We know that,

$u = +10\ ms^{-1}$

Time for velocity to become 0,

$$t = \frac{v - u}{a} = \frac{0 - 10}{-5} = 2\ s$$

So, displacement = 0 m,

But distance = 2x

Where,

$$x = 10 \times 2 - \frac{1}{2} \times 5 \times 2^2 = 10\ m$$

So, total distance travelled = 20 m

Equations of Motion

First equation of motion

Slope of the velocity vs time graph represents the acceleration of the object. As the graph is a straight line from A to B, the acceleration is uniform for this time period.

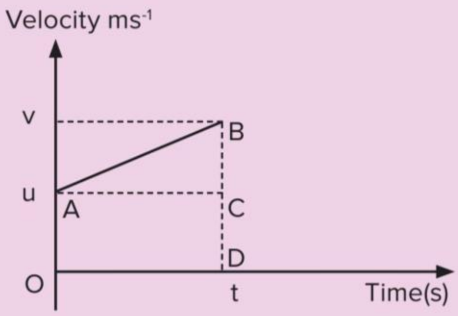
Acceleration = slope of graph AB

$$a = \frac{BC}{AC}$$

$$a = \frac{BD - CD}{AC}$$

$$a = \frac{v - u}{t}$$

$$v = u + at$$



Second equation of motion

Area included between the velocity vs time graph and time axis is the displacement covered by the object in the given time interval *t*.

Area of the trapezium ABCDOA is

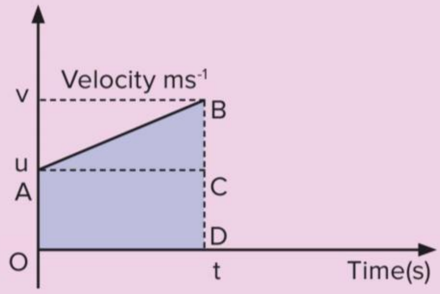
$$s = \frac{1}{2}(OA + BD) \times OD$$

$$s = \frac{1}{2}(u + v) \times t$$

But, from the first equation, v = u + at

$$\Rightarrow s = \frac{1}{2}(u + u + a\,t) \times t$$

$$s = ut + \frac{1}{2}at^2$$



Third equation of motion

As we know, the displacement is

$$s = \frac{1}{2}(OA + BD) \times OD$$

And acceleration is

$$a = \frac{BC}{AC} = \frac{BD - CD}{AC}$$

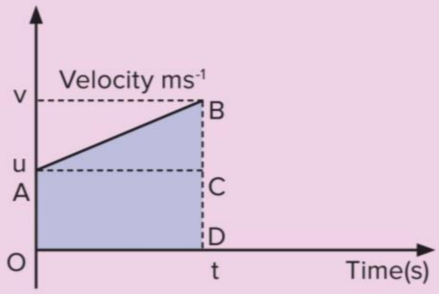
$$a = \frac{v - u}{t}$$

$$\Rightarrow t = \frac{v - u}{a}$$

$$\Rightarrow s = \frac{1}{2}(u + v) \times \frac{v - u}{a}$$

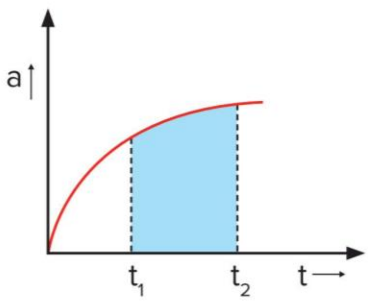
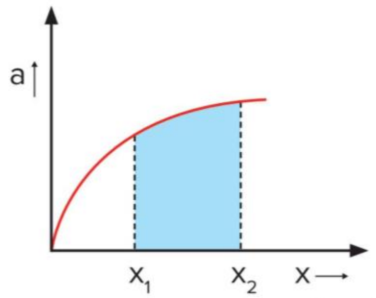
$$s = \frac{(v^2 - u^2)}{2a}$$

$$v^2 - u^2 = 2as$$



ADVANCED

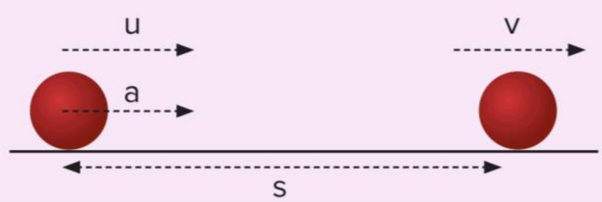
Relation Between Motion Parameters

Acceleration when the velocity is a function of time	Acceleration when the velocity is a function of position
$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$ $a = \frac{d^2x}{dt^2}$	$a = \frac{dv}{dt}\frac{dx}{dx}$ $a = \frac{dv}{dx}\left(\frac{dx}{dt}\right) = \frac{v\,dv}{dx}$
For a-t graph	For a-x graph
 Area = $\int_{t_1}^{t_2} a\,dt = v_2 - v_1 = \Delta v$	 Area = $\int_{x_1}^{x_2} a\,dx = \frac{v_2^2 - v_1^2}{2}$

BOARDS

Equations of Motion Using Calculus

First equation of motion



We know that,

$$a = \frac{dv}{dt}$$

Integrating it over the time interval *0 s* to *t s*, wherein the velocity changes from initial velocity *u* to some final velocity *v*, we get

$$\int_0^t a\,dt = \int_u^v dv$$

Since acceleration is constant

$$a[t]_0^t = [v]_u^v$$

a(t - 0) = (v - u)

v = u + at

Second equation of motion

Velocity is

$$\frac{dx}{dt} = v = u + at$$

Integrating it over the time interval *0 s* to *t s*, where the position changes from initial position x_i to some final position x_f, we get,

$$\int_{x_i}^{x_f} dx = \int_0^t (u + at)\,dt$$

$$[x]_{x_i}^{x_f} = u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

$$(x_f - x_i) = u(t - 0) + a\left(\frac{t^2}{2} - 0\right)$$

$s = \Delta x = ut + \frac{1}{2}at^2$, where $\Delta x = x_f - x_i$

Third equation of motion

When velocity is a function of position, i.e., v(x), the acceleration is written as follows:

$$a = v\frac{dv}{dx}$$

Cross multiplying and integrating it, we get,

$\Rightarrow v\,dv = a\,dx$

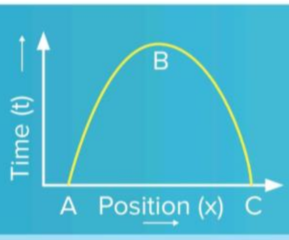
$$\int_u^v v\,dv = \int_{x_i}^{x_f} a\,dx$$

$$\left[\frac{v^2}{2}\right]_u^v = a[x]_{x_i}^{x_f}$$

$$\left(\frac{v^2}{2} - \frac{u^2}{2}\right) = a(x_f - x_i)$$

$v^2 = u^2 + 2a(\Delta x)$

Is the time variation of position shown in the figure observed in nature?

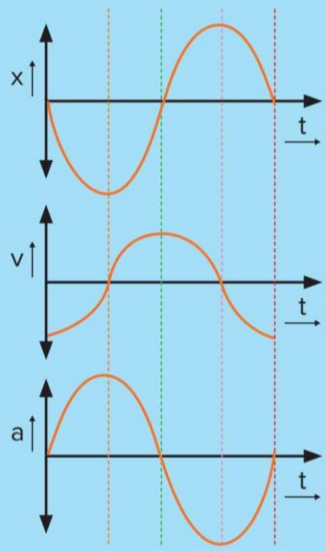


Solution

From A to B
As position increases, the time increases.
From B to C
As position increases, the time decreases.
Since time is decreasing, the given time position variation is not possible.
Also at a specific time, there could not be two positions; this also shows the graph is impossible.

If the position-time graph of a particle is a reverse (negative) sine curve as shown, draw its velocity vs time graph and acceleration vs time graph.

Solution



Given, position vs time graph is a reverse (negative) sine curve. Its velocity vs time graph can be obtained by taking the derivative of the position time graph.

$\Rightarrow \frac{dx}{dt} = -\cos t$ $\qquad v = \frac{dx}{dt}$

$v = -\cos t$

Thus, the velocity vs time graph is a reverse (negative) cosine curve. The derivative of velocity is acceleration.

$\Rightarrow \frac{dv}{dt} = -(-\sin t)$ $\qquad a = \frac{dv}{dt}$

$a = \sin t$

Thus, acceleration vs time graph is a sine curve.

A particle located at $x = 0$ *m* at time $t = 0$ *s* starts moving along the positive *x* direction with a velocity *v* that varies as $v=\alpha\sqrt{x}$. How does acceleration of the particle vary with time? What is the average velocity of the particle after the first *s m* of its path?

Solution

We know that, $a = v\frac{dv}{dx}$

$$\frac{dv}{dx} = \frac{d}{dx}\left[\alpha\sqrt{x}\right] = \alpha\frac{1}{2\sqrt{x}}$$

$$a = v\frac{dv}{dx} = \left(\alpha\sqrt{x}\right)\left(\alpha\frac{1}{2\sqrt{x}}\right)$$

$$\Rightarrow a = \frac{\alpha^2}{2}ms^{-2}$$

Thus, the acceleration is constant.
As the acceleration is constant, we can use all equations of motion.
We have,

$s = ut + \frac{1}{2}at^2$, as u = 0 and $a = \frac{\alpha^2}{2}$

$$s = \frac{1}{2}\left(\frac{\alpha^2}{2}\right)t^2$$

$$\Rightarrow t = \frac{2\sqrt{s}}{\alpha}$$

$$\Rightarrow \langle v\rangle = \frac{s}{t} = \frac{s}{\left(\frac{2\sqrt{s}}{\alpha}\right)} = \frac{1}{2}\alpha\sqrt{s}\,ms^{-1}$$

For a particle moving along x-axis, acceleration is given as a = v. Find the position as a function of time. (Given, at t = 0 s, x = 0 m, and u = 1 ms^{-1})

Solution

Given, the magnitude of acceleration is equal to the magnitude of velocity.

a = v ....(i)

We know that,

$a = \frac{dv}{dt}$ ....(ii)

So, from (i) and (ii) we have

$$\frac{dv}{dt} = v$$

Rearranging like terms, and integrating from time *0 s* to *t s*, when velocity changes from 1 ms^{-1} to v.

$$\int_1^v \frac{dv}{v} = \int_0^t dt$$

We know, from the standard integration formula, $\int \frac{1}{x}dx = ln\,x + c$, where In is the natural log.

Thus,

$$[ln\,v]_1^v = [t]_0^t$$

Putting the limits,

$$(ln\,v - ln1) = (t - 0)$$

We know that, *ln 1 = 0*

Thus,
(In v - 0) = (t - 0)
In v = t
$\Rightarrow v = e^t$

We have, from the definition of logarithm,
If *In a = b* then, $a = e^b$

Further;

$$v = \frac{dx}{dt} = e^t$$

$$\frac{dx}{dt} = e^t$$

Integrating from time 0 s to t s,

$$\int_0^x dx = \int_0^t e^t\, dt$$

$$[x]_0^x = [e^t]_0^t$$

On putting limits we get,

$(x - 0) = (e^t - e^0)$

Thus the position of the particle as a function of time is

$\Rightarrow x = e^t - 1$

For a particle moving along a positive x-axis, acceleration is given as a = x. Find the position as a function of time. (Given, at t = 0 s, x = 1 m and u = 1 ms^{-1})

MAIN

Solution

Given, the magnitude of acceleration is equal to the magnitude of displacement.

a = x (i)

$$a = v\frac{dv}{dx} \quad \text{(ii)}$$

Rearranging like terms from (i) and (ii),

$\Rightarrow v\, dv = x\, dx$

Integrating from initial velocity u = 1 ms^{-1} to final velocity v,

$$\int_1^v v\, dv = \int_1^x x dx$$

$$\left[\frac{v^2}{2}\right]_1^v = \left[\frac{x^2}{2}\right]_1^x$$

On putting the limits in above equation,

$$\left(\frac{v^2}{2} - 1\right) = \left(\frac{x^2}{2} - 1\right)$$

On solving, we get,

v = x

Also, velocity can be written as

$$v = \frac{dx}{dt} = x$$

Rearranging like terms and integrating, we get

$$\int_1^x \frac{dx}{x} = \int_0^t dt$$

We have, from the standard integration formula, $\int \frac{1}{x} dx = \ln x + c$, where In is the natural log.

$$[\ln x]_1^x = [t]_0^t$$

On putting limits, we get,

In x = t

$x = e^t$

We have, from the definition of logarithm, If $\ln a = b$ then, $a = e^b$

This is the position as a function of time.

MOTION IN ONE DIMENSION

NON-UNIFORM ACCELERATION AND CALCULUS APPROACH

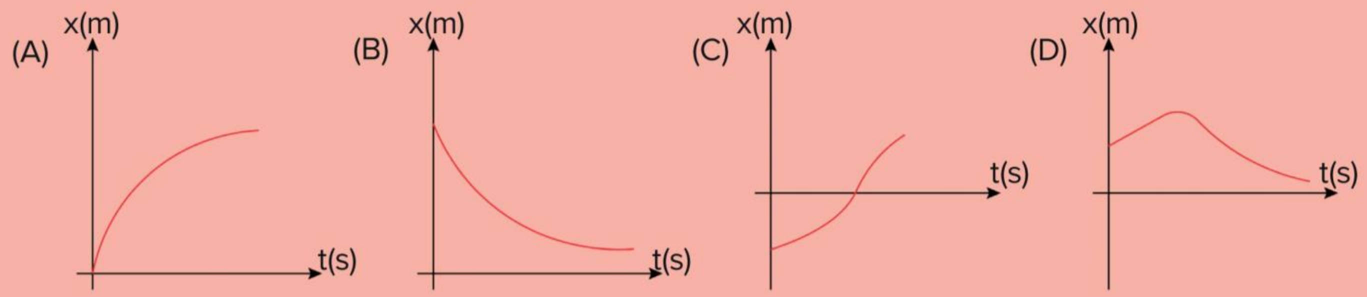
What you already know

- Motion under gravity
- Equations of motion
- Basic application of calculus in physics

What you will learn

- Advanced interpretations of graphs
- Conversion of graphs
- Application of graphs in real life physics problems

Among the four graphs given, identify the graph for which average velocity over the time interval (0, T) can vanish for a suitably chosen (0, T).

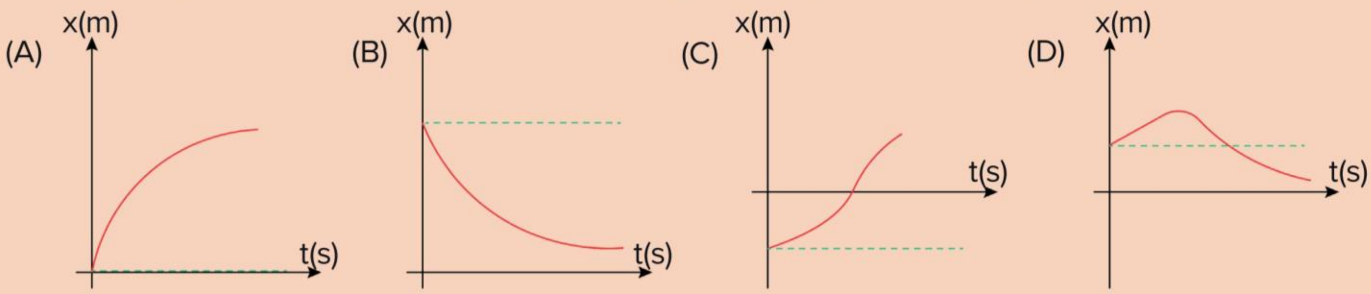


Solution

Average velocity of a particle becomes zero when the total displacement over a time interval becomes zero.

$$\text{Average Velocity} = \frac{\text{Total Displacement}}{\text{Total Time Taken}}$$

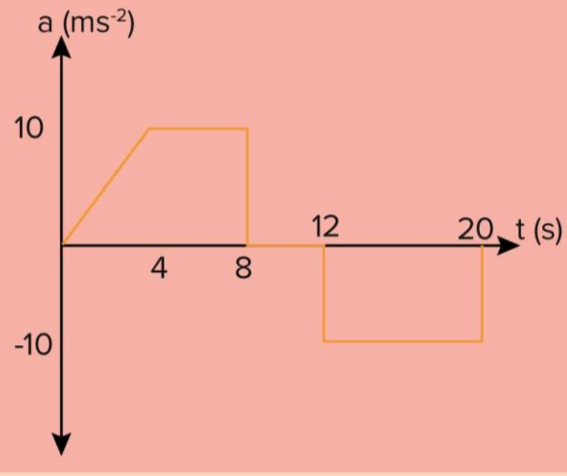
Total displacement in an interval is zero if a line drawn parallel to the time axis intersects the x-t curve at two points, which implies that the average velocity is zero in the given interval.



In option D, the line drawn parallel to the time axis intersects the x-t curve at two different points. So, option (D) is the correct answer.

At time, t = 0s, a particle is at rest at origin. The acceleration of the particle travelling along a straight line is shown in the figure. Draw the $v-t$ graph and then find the following:

(a) Maximum speed of the particle
(b) Distance travelled between 12 s to 20 s
(c) Displacement from 12 s to 20 s



MAIN

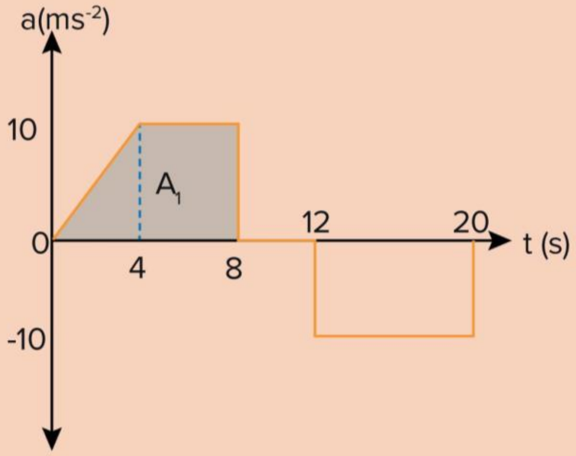
Solution

Acceleration vs time graph of the particle is given. We know that the area under a-t graph gives the change in velocity.

From the graph, we can see that,

Time interval	Acceleration	Velocity
(1) 0 s – 4 s	Linearly varying	Parabolic
(2) 4 s – 8 s	Constant	Linearly increasing
(3) 8 s – 12 s	Zero	Constant
(4) 12 s – 20 s	Constant but negative	Linearly decreasing

(1) & (2) Between 0 s to 8 s



Area, $A_1 = v_8 - v_0$

$\Rightarrow A_1 = \frac{1}{2} \times (4+8) \times 10 = 60 ms^{-1}$

$v_0 = 0$, because particle starts from rest

$\Rightarrow v_8 - 0 = 60 ms^{-1}$

$\Rightarrow v_8 = 60 ms^{-1}$.....(i)

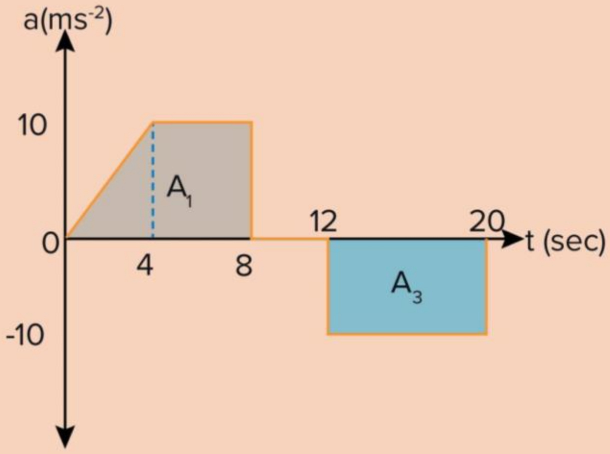
Area of Trapezium = (½) * (Sum of parallel sides) * Perpendicular distance

(3) Between 8 s to 12 s

Acceleration is zero, hence the velocity is constant. In other words, as the area A_2 is zero, the change in velocity is zero. So, the velocity at 8[th] second remains the same upto 12[th] second.

$$v_8 = v_{12} \ldots\ldots (ii)$$

(4) Between 12 s to 20 s



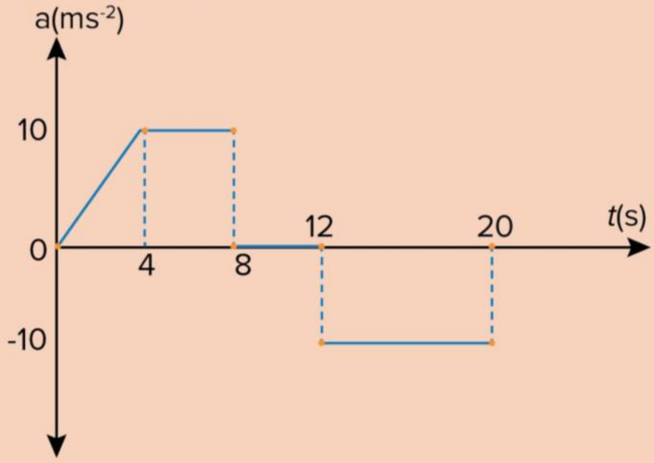
Area, $A_3 = -10 \times 8 = -80 ms^{-1}$

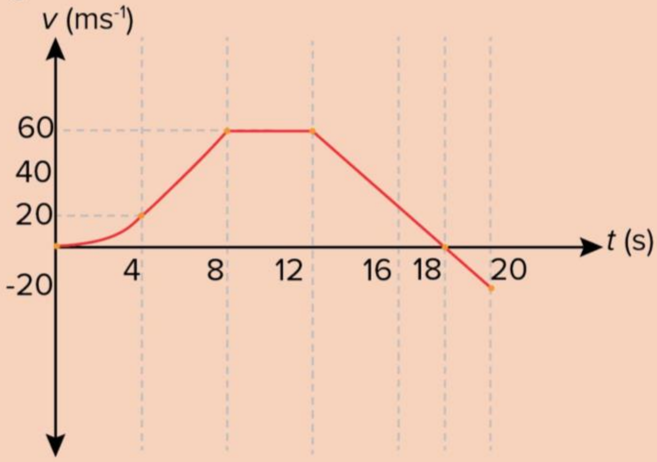
$\Rightarrow \Delta v = v_{20} - v_{12} = v_{20} - v_8 = -80 ms^{-1}$

$\Rightarrow v_{20} - 60 = -80 ms^{-1}$

$\Rightarrow v_{20} = -20 \ ms^{-1}$

Now, we have the velocity of the particle for different instants. We will make a velocity vs time graph from the given data by joining the points using suitable curves.

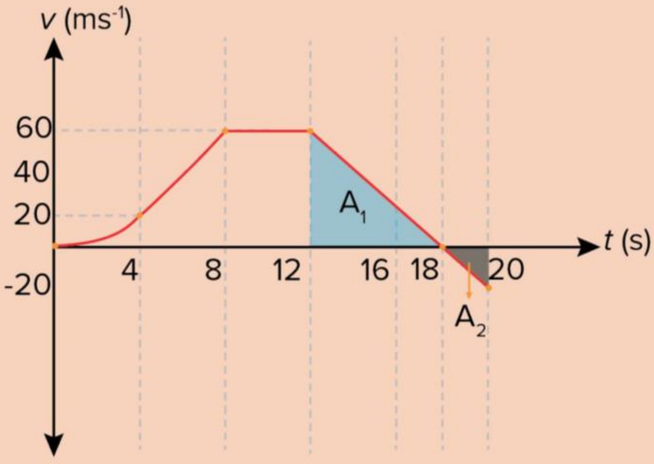




From the v-t graph,

Area under the v-t graph gives the displacement.

So, the displacement from 12 s to 20 s is $A_1 + A_2$.



$$A_1 = \frac{1}{2} \times 60 \times (18 - 12) = 180m$$

$$A_2 = \frac{1}{2} \times (-20) \times (20 - 18) = -20m$$

(a) Maximum speed is 60 ms^{-1}

(b) Distance between 12 s to 20 s is

$|A_1| + |A_2| = |180| + |-20| = 200m$

(c) Displacement between 12 s to 20 s is

$A_1 + A_2 = 180 + (-20) = 160 \ m$

If the velocity of a particle moving along a straight line changes with time as $v = 2\sin\left(\frac{\pi t}{2}\right)$ its average velocity over time interval t = 0 s to t = 2(2n - 1) s, n being any positive integer, is:

(A) $\frac{8}{\pi\times(2n-1)}ms^{-1}$ (B) $\frac{4}{\pi\times(2n-1)}ms^{-1}$ (C) zero (D) $\frac{16}{\pi\times(2n-1)}ms^{-1}$

BOARDS

Solution

Given,

Velocity, $v = 2\sin\left(\frac{\pi\times t}{2}\right)$

Time interval, t = 0 s to t = 2(2n - 1) s

We know that,

$$\text{Average Velocity} = v_{avg} = \frac{\int_{t_1}^{t_2} v\,dt}{\int_{t_1}^{t_2} dt}$$

In this case,

$$v_{avg} = \frac{\int_0^{2\times(2n-1)} v\,dt}{\int_0^{2\times(2n-1)} dt}$$

$$= \frac{\int_0^{2\times(2n-1)} 2\times\sin\left(\frac{\pi t}{2}\right)dt}{\int_0^{2\times(2n-1)} dt}$$

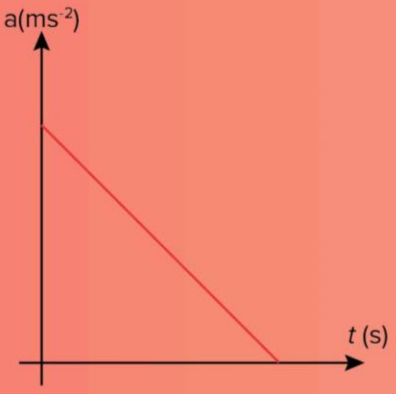
$$= \frac{-\frac{4}{\pi}\left[\left\{\cos\left(\frac{\pi\times t}{2}\right)\right\}\right]_0^{2\times(2n-1)}}{[t]_0^{2\times(2n-1)}} \text{; using chain rule}$$

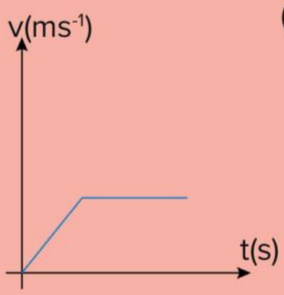
$$= \frac{-\frac{4}{\pi}\times[-1-1]}{2\times(2n-1)} = \frac{\frac{8}{\pi}}{2\times(2n-1)}\text{; cos of even multiples of }\left(\frac{\pi}{2}\right) = -1$$

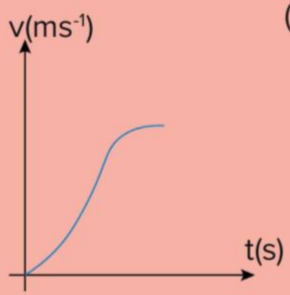
$$\Rightarrow v_{avg} = \frac{4}{\pi\times(2n-1)}$$

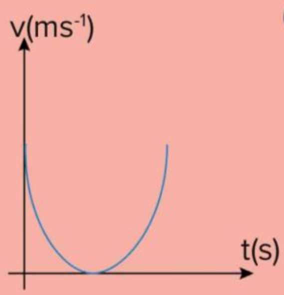
Ans : B

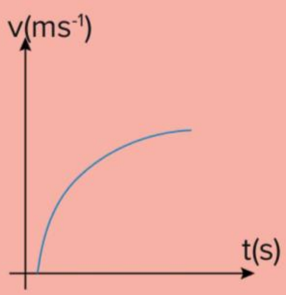
Acceleration vs velocity graph of a particle moving in a straight line from rest is shown in the figure. The corresponding velocity vs time graph would be:



(A)


(B)


(C)


(D)


Solution

Equation of a straight line can be represented as, $a = -mv + c$

Now,

$$a = -mv + c$$

$$\Rightarrow \frac{dv}{dt} = -mv + c$$

$$\Rightarrow dv = (-mv + c)dt$$

Integrating both sides,

$$\Rightarrow \int \frac{dv}{(-mv+c)} = \int dt; \quad \left\{\int \frac{dx}{x} = \ln(x) + C\right\}$$

$$\Rightarrow \frac{-\ln(c-mv)}{m} = t + C$$

$$\Rightarrow -\ln(c-mv) = m \times (t+C)$$

$$\Rightarrow c - mv = e^{-m\times(t+C)}$$

$$\Rightarrow v = \frac{c - e^{-m\times(t+C)}}{m}$$

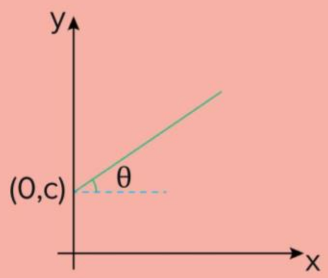
$$\Rightarrow v = \frac{c - e^{-mt} \times e^{-mC}}{m}$$

$$\Rightarrow v = \frac{e^{-mC}\left\{ce^{mC} - e^{-mt}\right\}}{m} = \frac{\left\{ce^{mC} - e^{-mt}\right\}}{\frac{m}{e^{-mC}}}$$

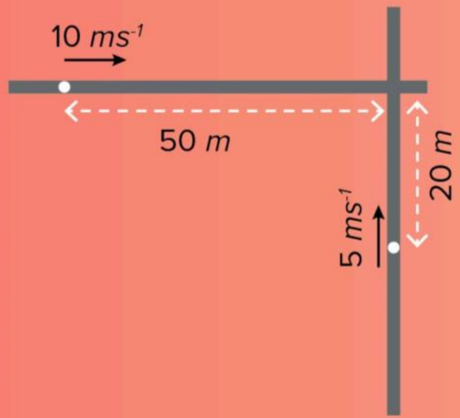
$$\Rightarrow v = \frac{k - e^{-mt}}{m'}; \text{ where } ce^{mC} = k(\text{constant}) \text{ and } \frac{m}{e^{-mC}} = m'$$

Option (D) represents this equation, since it is an inverted exponential function graph.

General equation of a straight line is $y = mx + c$,
where m = slope of line
c = intercept on y-axis



Two particles A and B move with constant velocities 10 ms^{-1} and 5 ms^{-1}, respectively, along the path shown in the figure. Find the time when the particles are closest to each other.

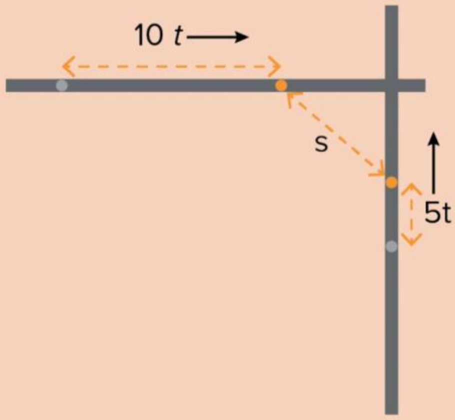


BOARDS

Solution

After time t, the particles reach the point as shown in the figure. Now, the distance between the particles is s. We need to find t for which s is minimum.

At time t, the horizontal particle moves a distance of 10t, while the vertical particle moves 5t from their initial position. From the figure,



$$s^2 = (50-10t)^2 + (20-5t)^2$$

$$\Rightarrow s^2 = 5^2\left\{(10-2t)^2 + (4-t)^2\right\}$$

$$\Rightarrow s^2 = 25 \times \left(116-48t+5t^2\right)$$

$$\Rightarrow s = 5\sqrt{116-48t+5t^2}$$

For s to be minimum, we have,

$$\frac{ds}{dt} = 0 \Rightarrow \frac{d}{dt}\left(5\sqrt{116-48t+5t^2}\right) = 0$$

Using chain rule of differentiation,

$$\Rightarrow \frac{5}{2} \times \frac{10t-48}{\sqrt{116-48t+5t^2}} = 0$$

$$\Rightarrow 10t - 48 = 0$$

$$\Rightarrow t = 4.8 \text{ s}$$

So, at time, t = 4.8 s, both the particles are at minimum distance from each other.

A particle moves in a straight line with an initial velocity v_0 and retardation βv, (v is velocity at any time t). Check if,

(A) the particle will cover a total distance of $\frac{v_0}{\beta}$

(B) the velocity of particle will become $\frac{v_0}{2}$ after time $\frac{1}{\beta}$

(C) the particle will continue to move for a long time

(D) the particle will stop shortly

MAIN

Solution

Given,

Retardation is βv,

Acceleration, $a = -\beta v$

$$a = -\beta v = v\frac{dv}{dx} \text{ ;} x = \text{displacement}$$

$$\Rightarrow dv = -\beta dx$$

$$\text{At } v = v_0 \Rightarrow x = 0$$

$$\text{and } v = 0 \Rightarrow x = x$$

Integrate both side with limits,

$$\int_{v_0}^{0} dv = \int_0^x -\beta dx$$

$$\Rightarrow [v]_{v_0}^{0} = -\beta[x]_0^x$$

$$\Rightarrow 0 - v_0 = -\beta(x-0)$$

$$\Rightarrow x = \frac{v_0}{\beta}$$

which means that the particle will cover a distance of x before the velocity becomes 0. Since the acceleration is proportional to the velocity, as soon as the velocity becomes zero, the acceleration also becomes zero and thus the particle will not move any further.

Now, for finding the time after which the particle comes to rest,

$$a = \frac{dv}{dt} = -\beta v$$

$$\Rightarrow \frac{dv}{v} = -\beta dt$$

Integrate both side with limits,

$$\int_{v_0}^{v} \frac{dv}{v} = \int_0^t -\beta dt$$

$$\Rightarrow [\ln(v)]_{v_0}^{v} = -\beta[t]_0^t$$

$$\Rightarrow \ln\left(\frac{v}{v_0}\right) = -\beta t$$

$$\Rightarrow \frac{v}{v_0} = e^{-\beta t}$$

$$\Rightarrow v = v_0 \times e^{-\beta t}$$

When $v = 0$

$$\Rightarrow 0 = \frac{v_0}{e^{\beta t}}$$

$$\Rightarrow t \to \infty$$

So, the particle moves for a long time and covers a distance $x = \frac{v_0}{\beta}$ as in option (A).
So, (A) and (C) are correct.

The equation of motion of a particle moving along a straight line is given as $x = \frac{1}{2}vt$, where x, v, and t have the usual meaning. Comment about acceleration.

Solution

We have,

$$x = \frac{vt}{2}$$

$$\Rightarrow x = \frac{t}{2} \times \frac{dx}{dt}$$

$$\Rightarrow 2\frac{dt}{t} = \frac{dx}{x}$$

Integrate both side with limits,

$$\Rightarrow 2\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\Rightarrow 2(\ln(t) + c) = \ln(x)$$

$$\Rightarrow \ln(t)^2 - \ln(x) = -2c = 2c'; \{\ln(x^n) = n\ln(x)\}$$

$$\Rightarrow \ln\left(\frac{t^2}{x}\right) = \ln(2c');\ \text{Using}\ \ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$$

$$\Rightarrow \frac{t^2}{x} = 2c'$$

$$\Rightarrow x = \frac{t^2}{2c'}$$

$$\Rightarrow x = t^2 \times C;\ \text{taking}\ \frac{1}{2c'}\ \text{as another constant}\ C$$

Now,

$$v = \frac{dx}{dt} = 2 \times t \times C, \text{and}$$

$$a = \frac{dv}{dt} = 2C = \text{constant}$$

Therefore, the acceleration is constant.

If the position of a particle moving in a straight line is given by $x = t^3 - 4t^2 - 3t$. Find the distance and displacement of the particle from 0 s to 4 s.

Solution

We have,

$$\text{Displacement} = x_4 - x_0 = \left[t^3 - 4t^2 - 3t\right]_0^4$$
$$\Rightarrow x_4 - x_0 = \left\{\left(4^3 - 4\times 4^2 - 3\times 4\right) - (0)\right\} = -12\text{m}$$
$$\text{Displacement} = -12\text{m}$$

For distance, we need to check if the velocity in that time interval (between 0 s to 4 s) becomes zero anywhere.

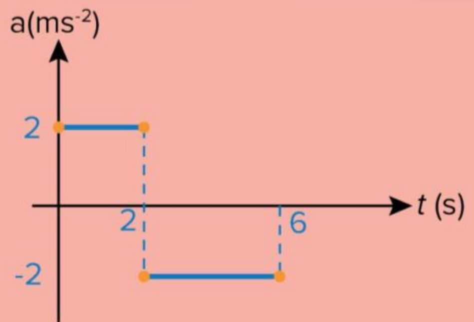
$$\text{Velocity, } v = \frac{dx}{dt} = \frac{d\left(t^3 - 4t^2 - 3t\right)}{dt}$$
$$\Rightarrow v = 3t^2 - 8t - 3$$
$$\text{Now, } v = 0 \Rightarrow 3t^2 - 8t - 3 = 0$$
$$\text{On solving, we get, } t = 3\text{s} \ \& \ t = -\frac{1}{3}\text{s}$$
$$\text{Neglecting negative value of time, we get, } t = 3\text{s, at which } v = 0 \text{ ms}^{-1}$$

Now, the distance is as follows:

$$\text{Distance} = |x_3 - x_0| + |x_4 - x_3|$$
$$= \left|\left[t^3 - 4t^2 - 3t\right]_0^3\right| + \left|\left[t^3 - 4t^2 - 3t\right]_3^4\right|$$
On solving, we get,
$$\text{Distance} = |-18|\text{m} + |6|\text{m} = 24\text{m}$$

At time, t = 0s, a particle is at rest at origin. It is now moving in a straight line with an acceleration of 2 ms^{-2} for the first 2 s and −2 ms^{-2} for the next 4 s as shown in the a-t graph.
Plot graphs for the following:

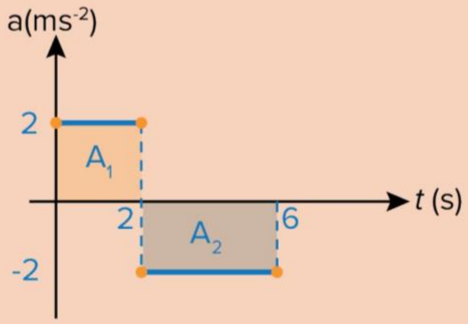
(a) Speed *vs* time
(b) Distance *vs* time



MAIN

Solution

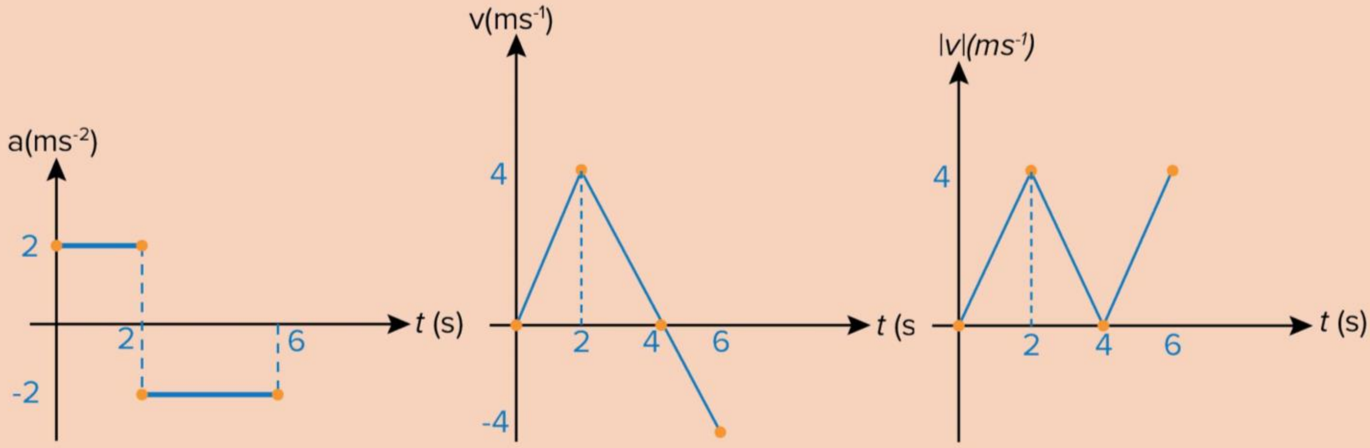
We know that the **area under a-t graph gives us the change in velocity.** So, we will find velocity at different instant then join them with suitable curves.



From the figure,

$$\text{Area } A_1 = 2 \times 2 = 4\text{ms}^{-1}$$
$$\Rightarrow v_2 - v_0 = 4\text{ms}^{-1};$$
$$\Rightarrow v_2 - 0 = 4\text{ms}^{-1}$$
$$\Rightarrow v_2 = 4\text{ms}^{-1}$$
$$\text{Also, Area } A_2 = (6-2) \times (-2) = -8\text{ms}^{-1}$$
$$\Rightarrow v_6 - v_2 = -8\text{ms}^{-1}$$
$$\Rightarrow v_6 - 4 = -8\text{ms}^{-1}$$
$$\Rightarrow v_6 = -4\text{ms}^{-1}$$

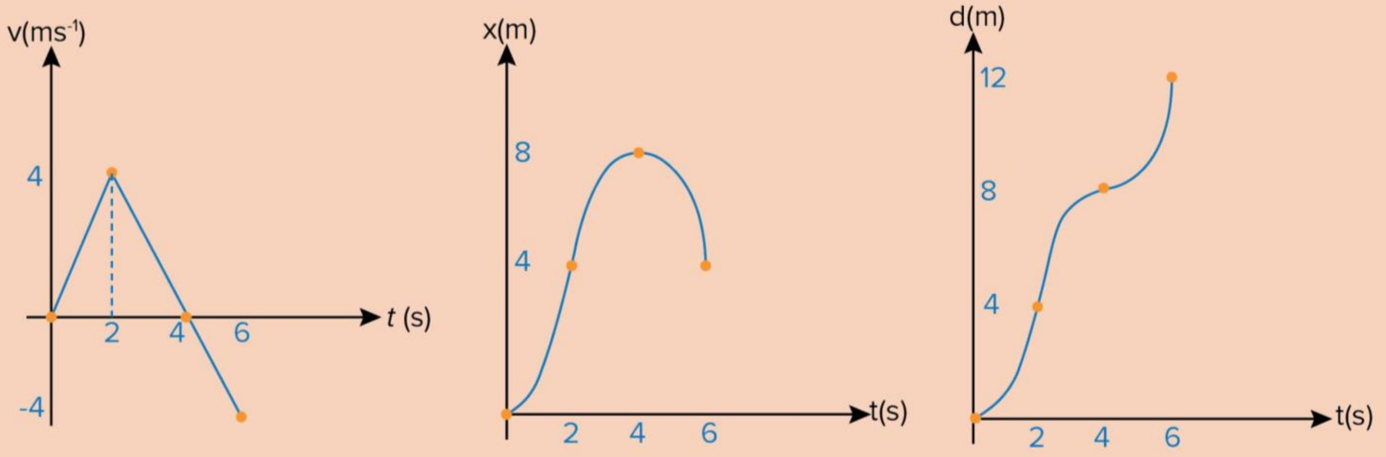
Now, we got velocity at different instants and we know that acceleration is constant in every interval. So the change in velocity will be linear.
For the speed vs time graph, we just reflect the negative part of the velocity vs time graph (which is below x-axis) and get the speed vs time graph.



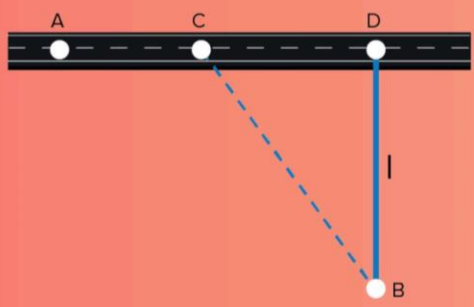
For the distance vs time graph, we know that the area under the velocity vs time graph gives us the change in displacement. So, we first plot the displacement vs time graph and then flip the negative slope sections to get the distance vs time graph.

From v-t graph,

$$A_1 = x_2 - 0 = \frac{1}{2} \times 2 \times 4$$
$$\Rightarrow x_2 = 4\text{m}$$
$$A_2 = x_4 - x_0 = \frac{1}{2} \times 4 \times 4$$
$$\Rightarrow x_4 = 8\text{m}$$
$$A_3 = x_6 - x_4 = \frac{1}{2} \times 2 \times (-4)$$
$$\Rightarrow x_6 = 4\text{m}$$



From point A, located on the highway, as shown in the figure, one must get by car as soon as possible to point B, located in the field at a distance l from the highway. It is known that the car moves in the field n times slower than the highway. At what distance from point D one must turn off the highway?



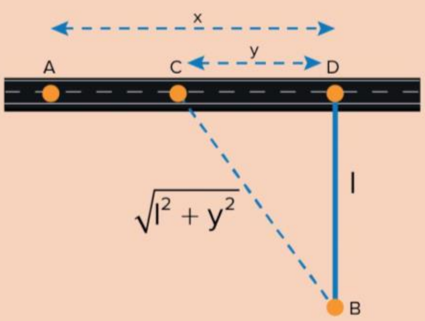
Solution

Let,

Speed of car on highway be v ms^{-1}

Speed of car on the field = $\frac{v}{n}\ ms^{-1}$

Let the distance between point A and D be x and the car get off the highway at point C, which is located at a distance y from D.
Given, the distance between D and B is l m.



From the figure,

$$AC = x - y$$

$$\&CB = \sqrt{CD^2 + DB^2} = \sqrt{y^2 + l^2}$$

Total time to reach B = $t_{AC} + t_{CB}$

$$\Rightarrow t = \frac{AC}{v} + \frac{CB}{v/n};$$

$$\Rightarrow t = \frac{(x-y)}{v} + \frac{\sqrt{y^2+l^2}}{v/n}$$

Here, x, l, n, and v are constants and we need to find y.

For time to be minimum,

$$\Rightarrow \frac{dt}{dy} = 0 \Rightarrow \frac{d\left\{\frac{(x-y)}{v} + \frac{\sqrt{y^2+l^2}}{v/n}\right\}}{dy} = 0$$

$$\Rightarrow \frac{dt}{dy} = \frac{-1}{v} + \frac{n}{v} \times \frac{y}{\sqrt{y^2+l^2}} = 0; \text{ Using } \frac{d}{dx}\left\{\sqrt{a^2+x^2}\right\} = \frac{x}{\sqrt{a^2+x^2}}$$

$$\Rightarrow \frac{n}{v} \times \frac{y}{\sqrt{y^2+l^2}} = \frac{1}{v}$$

$$\Rightarrow \frac{y}{\sqrt{y^2+l^2}} = \frac{1}{n}$$ Squaring both sides, we get,

$$\frac{y^2}{y^2+l^2} = \frac{1}{n^2}$$

$$\Rightarrow n^2y^2 = y^2 + l^2$$

$$\Rightarrow y^2 \times (n^2-1) = l^2$$

$$\Rightarrow y^2 = \frac{l^2}{(n^2-1)}$$

$$\Rightarrow y = \frac{l}{\sqrt{n^2-1}}$$

Therefore, at a distance of $\frac{l}{\sqrt{n^2-1}}$, one must turn off the highway to reach point B in minimum time.