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UNIT:II KINEMATICS

- CH:01 UNIFORM MOTION
- CH:02 UNIFORMLY ACCELARATED MOTION
- CH:03 MOTION IN A PLANE PROJECTILE MOTION
- CH:04 VECTORS
- CH:05 UNIFORM CIRCULAR MOTION

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UNIFORM MOTION

MOTION IN A STRIAGHT LINE

UNIT-I : CH -01

1. **Mechanics**: It is the branch of physics that deals with the conditions of rest or motion of the material objects around us.
2. **Statics:** It is the branch of mechanics that deals with the study of objects at rest or in equilibrium.
3. **Kinematics**: It is the branch of mechanics that deals with the study of motion of objects without considering the cause of motion.
4. **Dynamics**: It is the branch of mechanics that deals with the study with the study of motion of objects taking into consideration the cause of their motion.
5. **Rest**: An object is at rest if it does not change its position w.r.t. its surroundings with the passage of time.
6. **Motion**: An object is in motion if it change its position w.r.t. its surroundings with the passage of time.
7. **Rest and motion are relative terms**: Nobody can exist in a state of absolute rest or of absolute motion.
8. **Point object**: If the position of an object changes by distances much greater than its own size in a reasonable duration of time, then the object may be regarded as a point object.
9. **One dimensional motion**: The motion of an object is said to be one dimensional motion if only one out of the three coordinates specifying the position of the object changes with time. In such a motion, an object motion along a straight line path.
10. **Two-dimensional motion**: The motion of an object is said to be two-dimensional motion if two out of the three coordinates specifying the position of the object change with time. In such a motion, the object moves in a plane.
11. **Three-dimensional motion**: The motion of an object is said to be three-dimensional motion if all the three coordinates specifying the position of the object change with time. In such a motion, the object moves in space.
12. **Distance or path length**: It is the length of the actual path traversed by a body between its initial and final positions. It is a scalar quantity. Its SI unit is metre. It is always positive or zero.
13. **Displacement**: It is defined as the change in the position of an object in a fixed direction. It is given by the vector drawn from the initial position to the final position of the object. It is a vector quantity. It can be positive, negative or zero. Its SI unit is metre.
14. **Speed:** It is rule of change of position of a body in any direction.

    $$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

15. **Uniform speed:** An object is said to be moving with uniform speed if it covers equal distances in equal intervals of time, however small these time intervals of time.
16. **Variable speed**: An object is said to be moving with variable speed if it covers unequal distances in equal intervals of time.
17. **Average speed**: It is equal to the total distance travelled by the object divided by the total time taken to cover that distance.

    $$\text{Average speed: } \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

18. **Instantaneous speed:** The speed of an object an any particular instant of time or at a particular point of its path is called the instantaneous speed of the object.

$$\text{Instantaneous speed, } v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

19. **Velocity:** It is the rate of change of position of an object in a particular direction. It is equal to the displacement covered by a body per unit time.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

Velocity of a vector quantity. Its SI unit is ms^{-1} and dimensional formula is $[M^{o}L^{1}T^{-1}]$.

20. **Uniform velocity:** A body is said to be moving with uniform velocity if it covers equal displacements in equal intervals of
time, however small these time intervals may be.

21. **Variable velocity:** A body is said to be moving with variable velocity if either its speed changes or directional of motion changes or both change with time.

22. **Average velocity:** It is equal to the net displacement covered divided by the total time taken.

$$v_{av} = \frac{\text{Net displacement}}{\text{Total time taken}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

23. **Instantaneous velocity:** The velocity of an object at a particular instant of time or at a particular point of its path is called its instantaneous velocity.

$$v = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{\Delta \vec{x}}{dt}$$

24. **Uniform motion:** An object is said to be in uniform motion if it covers equal distances in equal intervals of time, however small these intervals may be, in the same fixed direction.

25. **Displacement in uniform motion:** If at time t = 0, the displacement is x_0, then

$$x = x_0 + vt \quad \text{or} \quad x - x_0 = vt \quad \text{or} \quad s = vt$$

26. **Non-uniform motion:** A body is said to be in non-uniform motion if its velocity changes with time.

27. **Acceleration:** The rate of change of velocity of an object is called its acceleration.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

Acceleration is a vector quantity. Its SI unit is ms^{-2} and its dimensional formula is $[M^{o}L^{1}T^{-2}]$.

28. **Uniform acceleration:** If the velocity of an object changes by equal amounts in equal intervals of time, however small these intervals may be, then the object is said to move with uniform or constant acceleration.

29. **Variable acceleration:** If the velocity of an object changes by unequal amounts in equal intervals of time, then the object is said to be in variable acceleration.

30 **Average acceleration:** The average acceleration of an object between two points on the path of its motion is defined as ratio of the change in velocity to the total time interval in which that change has taken place.

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

31. **Instantaneous acceleration:** The acceleration of an object at a given instant of time or at a given point of its motion is **called** its instantaneous acceleration.

$$a = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{x}}{dt^2}$$

32. **Positive acceleration:** If the velocity of an object increases with time, its acceleration is positive.

33. **Negative acceleration:** If the velocity of an object decreases with time, its acceleration is negative. Negative acceleration is also called retardation or deceleration.

34. **Equations of motion for constant acceleration**

(a) In conventional form: Let u be the initial velocity of a particle, the uniform acceleration, v its velocity after time t and s is the distance travelled in time t, then the following equations hold good:

(i) $v = a + at$ (ii) $s = \frac{u + u}{2} \times t$ (iii) $s = ut + ½ at^2$ (iv) $v^2 - u^2 = 2as$

(v) $s = vt - ½ at^2$ (vi) $s_{n\ th} = u + a/2\ (2n - 1)$

where $s_{n\ th}$ = the distance travelled in n^{th} second.

(b) In Cartesian form: Suppose a particle moves with uniform acceleration a along X-axis. Let x_0 and x be its position co-ordinates and v_0 and v be its velocities at time t = 0 and t respectively. Then the following equations hold good:

(i) $v = v_0 + at$ (ii) $x = x_0 + v_0t + ½ at^2$ or $s = v_0t = ½ at^2$

(iii) $v^2 = v_0^2 + 2a\ (x - x_0) = v_0^2 + 2as$ (iv) $s_{n\ th} = v_0 + a/2\ (2n - 1)$

35. **Free fall:** In the absence of air resistance, all bodies fall with the same acceleration near the surface of the earth. This

motion of a body falling towards the earth from a small height is called free fall. The acceleration with a body falls is called acceleration due to gravity and is denoted by g. Near the surface of the earth, $g = 9.8\ ms^{-2}$.

36. **Motion under gravity:** For a freely falling body, the following equations of motion hold good:

(i) $v = u + gt$ (ii) $s = ut + ½ gt^2$ (iii) $v^2 - u^2 = 2\ gs$

When a body falls freely under the action of gravity, its velocity increases and the value of g is taken positive.

When a body is thrown vertically upward, its velocity decreases and the value of g is taken negative.

For a body thrown vertically upward with initial velocity u, we have

(i) Maximum height reached, $h = \frac{u^2}{2g}$

(ii) Time of ascent = Time of descent = $\frac{u}{g}$

(iii) Total time of flight to come back to the point of projection = $\frac{2u}{g}$

(iv) Velocity of fall at the point of projection = u

(v) Velocity attained by a body dropped through height h, $v = \sqrt{2gh}$

37. **Position-time graph:** It is the graph between the time t and position x of a particle relative to a fixed origin. Its slope at any point gives the instantaneous velocity at that point.

(i) For a stationary object, the position-time graph is a straight line parallel to the time axis.

(ii) For a body in uniform motion, the position-time graph is a straight line inclined to the time axis.

(iii) For uniformly accelerated motion, the position-time graph is a parabola.

38. **Velocity-time graph:** It is a graph of time versus velocity. Its slope at any point gives the acceleration at the corresponding instant. Distance covered in time t equals area under the velocity-time graph bounded by the time-axis.

(i) For uniform motion, the velocity-time graph is a straight line parallel to the time axis.

(ii) For uniform acceleration, the velocity-time graph is a straight line inclined to the time axis.

(iii) For variable acceleration, the velocity-time graph is a curve.

39. Relative velocity: The relative velocity of an object B with respect to object A when both are in motion is the rate of change of position of object B with respect to object A,

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

Relative velocity of object B w.r.t. object B,

$\vec{V}_{AB} = \vec{v}_A - \vec{v}_B$

When both the objects A and B move in the same direction,

$v_{AB} = v_A - v_B$

When the objects B moves in the opposite direction of A,

$v_{AB} = v_A + v_B$

When v_A and v_B are inclined to each other at angle θ,

$$v_{AB} = \sqrt{v^2{}_A + v^2B + 2v_Av_B \cos(180^\circ - \theta)}$$

$$= \sqrt{v^2{}_A + v^2{}_B - 2v_A v_B \cos\theta}$$

If v_{AB} makes angle β with v_A, then

$$\tan\beta = \frac{v_B \sin\theta}{v_A - v_B \cos\theta}.$$

THEORY

We commonly observe that bodies are either in motion or at rest. A body is said to be in motion if it changes its position w.r.t. its surroundings with the passage of time. A car speeding along a highway, a train running on a railway track, a man walking on the road — all these are the examples of objects in motion. A body is said to be at rest if it does not change its position *w.r.t.* its surroundings with the passage of time. A book placed on the table remains on the table and we say that the book is at rest. Although the study of objects at rest is important, a large part of our everyday experience concerns the things that move. For this reason, the study of motion is one of the basic studies in physics. In this chapter, we shall discuss the motion in a straight line *i.e.,* rectilinear motion.

MECHANICS

The branch of physics which deals with the study of forces and motion and their relationship is called **mechanics.**

MECHANICS

STATICS

KINEMATICS

DYNAMICS

(*i*) **Statics.** *The branch of mechanics which deals with objects at rest is called statics.* Statics is of central importance in the design of bridges, buildings and other structures.

(*ii*) **Kinematics.** *The branch of mechanics which deals with the motion of objects only without considering the cause of motion is called kinematics.* In other words, kinematics does not consider the size, shape, mass *etc.* of the body; it is restricted to properties of the motion. Note that the word kinematics is derived from the Greek word kinema meaning "*motion*".

(*iii*) **Dynamics.** *The branch of mechanics which deals with the cause of motion is called dynamics.* In fact, in dynamics, we study the effect of forces in causing motion. Therefore, mass of the object must be considered. Note that the word dynamics is derived from Greek word dynamis meaning "*power*".

Knowledge Plus: ....

✪ **CLASSICAL MECHANICS**: (Newtonian Mechanics): Mechanics which deals with the study of motion of the moving bodies with speed much less than the speed of light in air / vacuum (3×10^8m/s).

Ex: Motion of bus, Motion of Bike, Motion of train etc.

✪ **RELATIVISTIC MECHANICS**: Mechanics which deals with the study of motion of the bodies moving with the speeds comparable to the speed of light is known as Relativistic mechanics.

REST AND MOTION

Rest. *An object is said to be at rest if it does not change its position w.r.t. its surroundings with the passage of time.* For example, a book placed on the table remains on the table *i.e.*, it does not change its position with time *w.r.t.* its surroundings. We say that the book is at rest.

Motion. *An object is said to be in motion if it changes its position w.r.t. its surroundings with the passage of time.* For example, a speeding car is in motion because as the time passes, the car changes its position *w.r.t.* its surroundings *e.g.*, electric poles, trees, buildings *etc.*

REST AND MOTION ARE RELATIVE TERMS

For example : A passenger sitting in a moving train is at rest with respect to the fellow passengers but at the same time he is in motion with respect to the objects outside train. So, the object may be at rest w.r.t. one object and at the same time it may be in motion w.r.t. another object. Hence, rest and motion are relative terms.

Absolute Rest and Motion are Unknown. In order to know whether the position of any object changes with time or not, a point which is absolutely fixed in space has to be chosen as reference point. But there is no such point in space. The earth revolves around sun, the solar system travels through our galaxy, the milkyway and cluster of galaxies with respect to other clusters. Hence, no object in the universe is in the state of absolute rest. So, the absolute motion cannot be realised in practice. Thus, we consider only the relative rest and relative motion in practice.

Absolute rest is complete absence of motion . Also, *Absolute rest is beyond our imagination because all heavenly bodies are moving w.r.t each other.*

Absolute rest *is practically not possible because we do not have any reference point which is absolutely fixed (rest) in space .*

CONCEPT OF POINT OBJECT OR PARTICLE

When any moving object covers a very large distance as compared to the size or dimensions of the object, then to study the motion of that object its dimensions or size may be neglected and the object is regarded as a point object.

"An object is said to be point object if its dimensions (i.e., Length, breadth and thickness) are negligible as compared to the distance travelled by it".

"An object is said to be a point object if it changes its position by distance which are much greater than its size".

For example : *(i)* Earth can be considered as a point object for studying its motion around the sun.

(ii) A car travelling several hundred kilometres can be considered as a point object.

(iii) Train going from Delhi to Chennai, then train is considered as a point object.

Explanation: Actual motion of definite size (or large size) objects is quite complicated. This is because, the actual motion of these bodies is in fact combination of Translational and Rotatory motion. But if we consider a point mass body than it has only one type of motion at a time.

. TYPES OF MOTION

In general, the motion of a body can be of three types *viz.*

(*i*) Translatory motion
(*ii*) Rotational motion
(*iii*) Oscillatory motion

(*i*) **Translatory motion.** *A body is said to have transalational motion if each particle of the body has the same displacement in the same time interval.* In other words, in this type of motion, every particle of the body covers a definite distance in definite times in linear paths. The motion of the body of a car moving on a straight road is an example of translational motion.

If a single particle or a point mass moves in a straight line, it is called *rectilinear motion.*

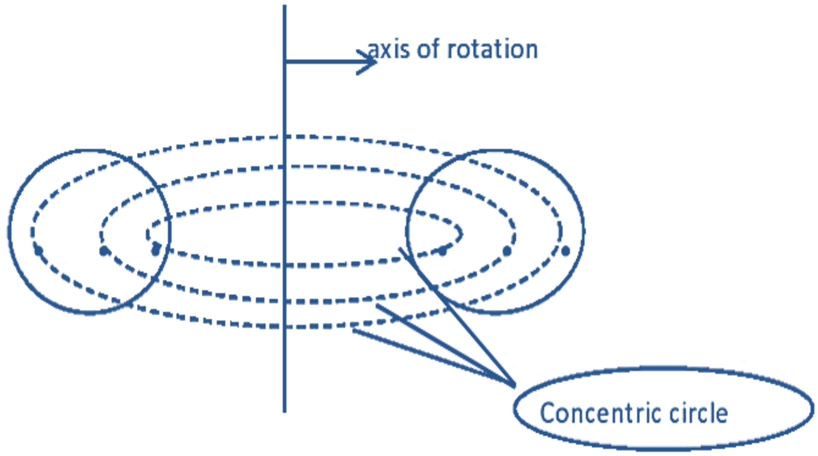
(*ii*) **Rotational motion.** *A body is said to have rotational motion or rotatory motion if each particle of the body (except those on the axis of rotation) travels in a circle.* The *axis of rotation* is a straight line that consists of the centres of the circular motion of the particles. The motion of a door being opened or closed is an example of rotational motion. The line along the hinges is the axis of rotation.

If a single particle or a point mass moves in a circle, it is called *circular motion.*

(*iii*) **Oscillatory motion.** *A body is said to have an oscillatory motion if it moves to and fro repeatedly about a fixed point called mean position e.g.*, swinging of a simple pendulum. If the amplitude of oscillatory motion is extremely small, the motion is called *vibratory motion.*

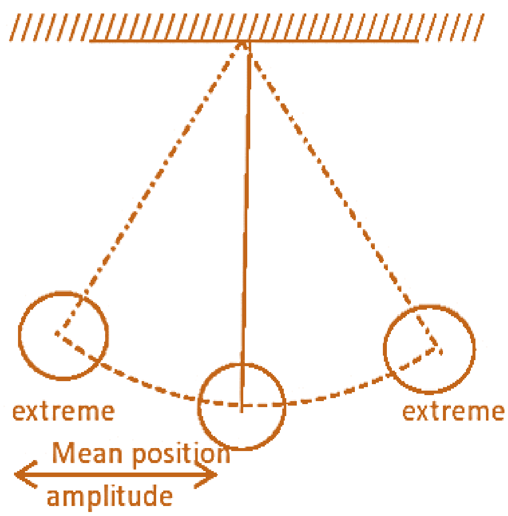
- Rectilinear motion can be uniform or non uniform .
- In translation motion , a line joining any two points on the body remains parallel to itself through out the motion of the body.
- During T. motion of the body , there is change in the location of the body .

- C.M and R.M can be uniform or non uniform . If C.M and R.M are ***uniform*** then these are called as ***periodic motion*** .



- During rotational motion of the body , there is ***change is orientation*** of the body , but there is ***no change in the location*** of the from of the axis of rotation .

If in the O.M , the amplitude is ***very small*** (microscopic), the motion of the body is said to be ***vibratory motion*** .



LOCATION OF POINT IN SPACE

The space we live in is three dimensional *i.e.*, we can specify the position of any point in space with three numbers. To do so, we choose a convenient point O as the origin. Through O, we draw three mutually perpendicular axes OX, OY and OZ as shown in Fig. 6.1. This is known as rectangular (cartesian) coordinate system. The three numbers – the distances along OX axis, OY axis and OZ axis – will specify the position of the point P in space. We say that coordinates of point P are x, y and z. If all the three coordinates x, y and z of an object remain unchanged with time, we say the object is at rest *w.r.t.* the coordinate system. If any one or more of the object's coordinates change with time, we say that the object is moving *w.r.t.* the coordinate system.

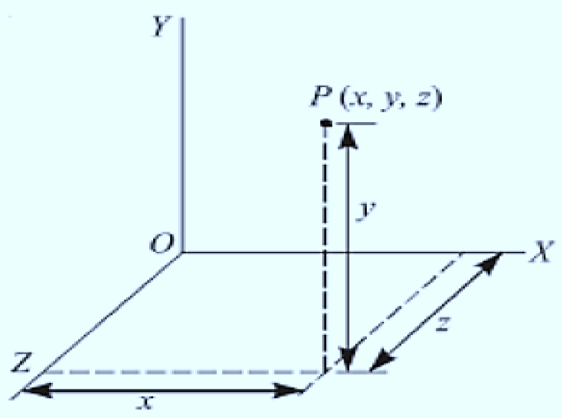


Fig. 6.1

FRAME OF REFERENCE

When we want to specify the position of a point object which is in motion, we need to use a reference mark and a system having a set of axes. The most convenient system for this is rectangular co-ordinate system which consists of three mutually perpendicular axes, labelled X–, Y– and Z–axis. The point of intersection of these three axes is called as a reference mark or point. This point acts as a reference point or the position of the observer. The observer has a clock with him to record the time. The position of object at a given instant of time can be described in terms of position co-ordinates $(x, y, z,)$. This co-ordinate system along with a clock constitutes a frame of reference.

Hence, *the frame of reference is a system of co-ordinate axes attached to observer (having clock with him) with respect to which the observer can describe the position, velocity, acceleration etc. of moving object.*

OR

The fixed point or place or reference mark w.r.t. which the position, displacement, velocity, acceleration of a body or an object is measured.

Types of Frame of Reference

1. Inertial frame of reference : *A frame of reference which is either at rest or moving with uniform velocity is called as inertial frame of reference*. In this frame of reference, Newton's law of motion hold good.

For example : A frame of reference attached to a person in bus at rest or bus is moving with uniform velocity along a straight line. Fixed stars in sky form an inertial frame of reference.

2. Non-inertial frame of reference : *An accelerated or moving frame of reference is known as non-inertial frame of reference*. In this frame of reference, Newton's laws of motion does not hold good.

For example : A frame of reference for person in a bus moving with variable velocity or moving with some acceleration along a straight line or straight path. All rotating frames are non-inertial.

Earth revolves around the sun in almost circular orbit and also spins about its own axis. Due to this, the velocity of earth keeps on changing with time. Hence, Newton's first law of motion does not hold good for earth. Thus, the frame of reference attached to a person on earth for observing the things outside the earth is non-inertial frame of reference. However, for the motion of objects on earth, the earth is considered to be at rest. This frame of reference attached to a person on the earth is taken as inertial frame of reference .

MOTION IN ONE. TWO AND THREE DIMENSIONS

The position of a particle in space is expresses in terms of three rectangular co-ordinates x , y & z .(three mutually perpendicular lines, also called as co-ordinate frame) When these co-ordinates change with time , then the particle is said to be in motion (it is not necessary then all the co-ordinates may change with time .) Even if one or two co-ordinate change with time , the particle is said to be in motion . Accordingly , **we have three types of motion : -**

1. One Dimensional Motion : *The motion of any object is said to be one dimensional if only one out of the three co-ordinates is required to specify the position of the object with the passage of time.*

In one dimensional motion, object moves along a straight line or well defined straight path. Hence, one dimensional motion is sometimes called as linear or rectilinear motion.

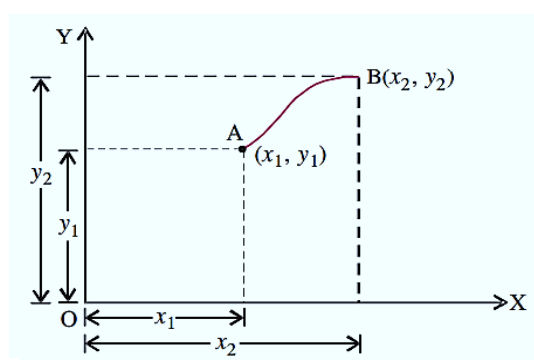
Examples of one dimensional motion:
(i) Motion of a train along a straight track.
(ii) Motion of a freely falling body.

Let any object is moving along X-axis in a straight line. At time t object is at point A such that OA = x. So, in order to know the position of object at any time t only one co-ordinate is required, *i.e.* distance x. Hence, *the motion of an object in which only one co-ordinate is changing with time is called as one dimensional motion.*

The motion of an object is said to be **one dimensional** *if only one of the three coordinates of the object changes with time.*

2. Two Dimensional Motion : *The motion of any object is said to be two dimensional motion if two out of the three co-ordinates are required to specify the position of object with the passage of time in plane.*

Let a particle moves from position A to B in plane, then its x co-ordinate changes from x_1 to x_2 and y co-ordinate from y_1 to y_2.



Examples of two dimensional motion:
(i) Motion of planets around the sun.
(ii) A car moving along a zig-zag path on a level road.

When an object moves in plane , then its position can be described by knowing its distance from two mutually $\perp^r$ co-ordinate axes , say x and y .

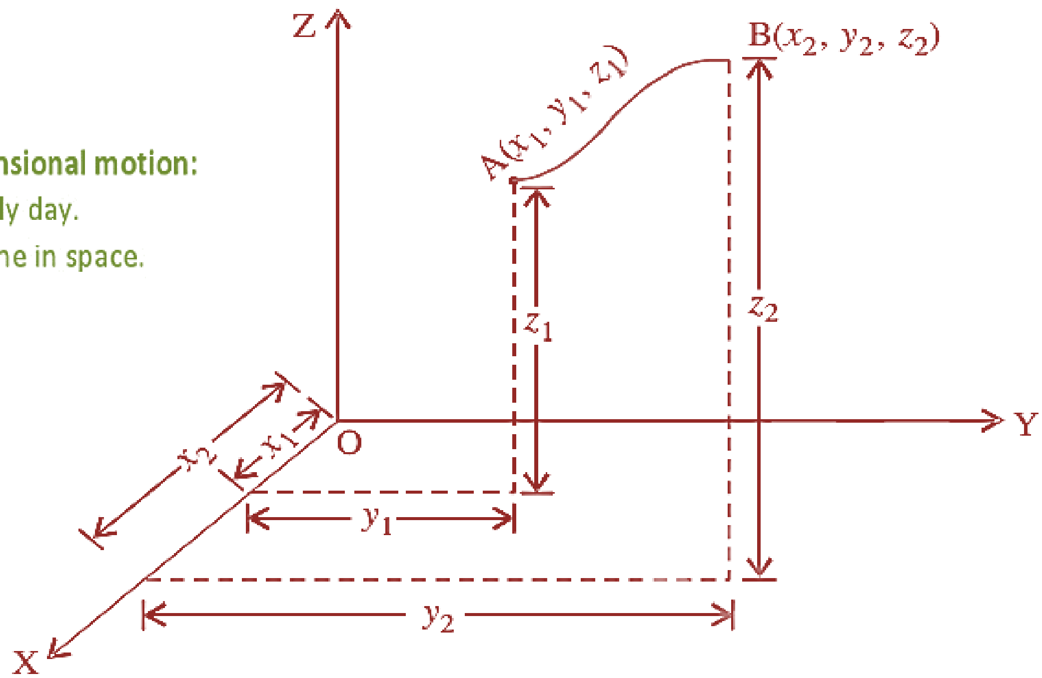
Three Dimensional Motion : *The motion of any object is said to be three dimensional if all the three co-ordinates are required to specify the position of object with the passage of time in space.*

In this motion, three co-ordinates are changing with time so called as three dimensional motion. Let a particle moves in space from position A to B. Here, the co-ordinates (x_1, y_1, z_1) change to (x_2, y_2, z_2) as the particle goes from A to B.

Examples of three dimensional motion:

(i) A kite flying on a widely day.

(ii) Motion of an aeroplane in space.



- In 3 – D motion three co-ordinates (x, y, & z) are required to describe the position of the object an instant.
- When a particle moves from A to B , the rectangular co –ordinate (x, y, z) changes from (x_1, y_1, z_1) to (x_2, y_2, z_2).

SCALAR AND VECTOR QUANTITIES

1. Scalar quantities : The *scalar quantities are those physical quantities which have only magnitude and no direction. e.g.,* mass, length, time, speed, work, power etc. are scalar quantities.

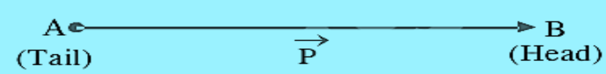
Scalars can be added, subtracted, multiplied and divided by simple algebraic methods. A scalar may be positive or negative. A scalar is specified simply by number and unit, where number represents its magnitude.

2. Vector quantities : *The vector quantities are those physical quantities which have both magnitude as well as direction. e.g.*, displacement, velocity, acceleration, force, torque, momentum etc. are vector quantities.

> *Vectors cannot be added, subtracted, multiplied by simple algebraic methods*. For these operations, we have to use the laws of vectors. *The division of a vector by another vector is not a valid operation in vector algebra, because the division of a vector by direction is not possible.*

A vector can be represented by a single letter in bold face or by a single letter having an arrowhead on it.

$$\vec{P} = \vec{AB}$$



Length AB of vector gives magnitude of vector $\vec{P}$ and arrowhead gives the direction. Point A of the arrowed line is called tail or origin of the vector. Point B which lies at the end of the arrowed line is called as tip or terminus or head of vector.

MOTION IN A STRAIGHT LINE

1-D motion can be represented graphically by plotting a graph between the position of a body and time taken by it . This graph is called ***position time graph***.

i) Origin , unit and sense of (passage of time)

- The starting point of the particle along a straight line is called ***origin (O***).
- The instant of time taken when the motion of the particle is taken as ***zero time***.
- The choice of origin of lime is purely arbitrary (Swachhand).
- The ***time interval*** between events ***does not change*** with the ***change of origin of time*** .

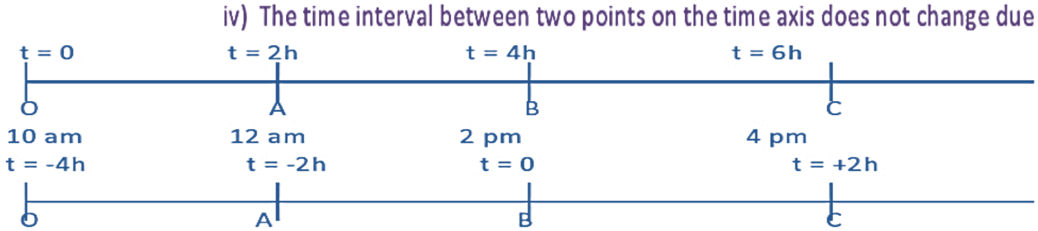
Unit of Time :- Second , minute , hour , day & year .

Conclusion : - i) The origin of time axis can be shifted to any point on the time axis.

ii) The time measured after the origin of time axis (**right of origin**) is taken as **positive .**

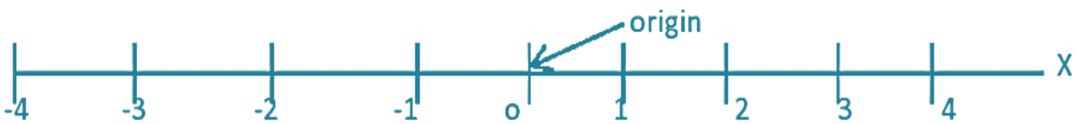
iii)The time measured after the origin before the origin of the time axis (**left of origin**) is taken as **negative** .

iv) The time interval between two points on the time axis does not change due to shift on the origin of time axis .



•• Origin unit & Direction for position measurement

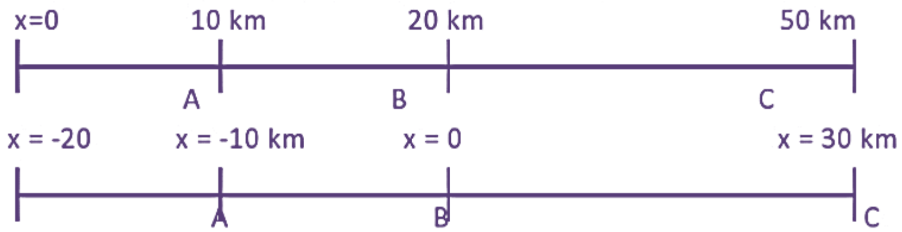
Consider an object moving along a straight lime . The starting point of the object is called origin (O) . The point O is considered as the reference point for the measurement of the position of the object.



▫▫ Conclusion

i) The location & position of the object or any instant of time is measured from the origin .

ii) The origin of position can be shifted to any point in the position axis .



iii) The right side of the origin (O) is known as **positive side** & the origin is taken as **negative side** .

iv) In case of vertical motion (y- axis)

----- Distance measured above again = **+ive**

----- Distance measured below origin = **-ive**

v) The distance between location on position axis does not change due to shift in the origin of position axis .

✪ The position of a moving point object can be written as 'x'. where x = position of the particle at any instant from the origin O . As the position goes on changing with passage of time , so the position 'x' depends on time .
i.e, ***position is a function of time (t).***

∴ $\boxed{x = f(t)}$ → Equation of motion of a point object .

DESCRIPTION OF MOTION

To describe the motion of an object, we need the concepts of *displacement, velocity* and *acceleration*. These are vector quantities *i.e.*, they possess magnitude as well as direction. For the motion of an object in two or three dimensions, the detailed knowledge of vectors is essential. However, for one dimensional motion, we need only one axis (*e.g.*, *x*-axis or *y*-axis) to indicate the position of the object. For such a restricted motion, there are only two *possible directions *viz.* positive and negative. Therefore, vector symbols are not needed for one dimensional motion. The sign (+ or –) indicates the direction of motion along the axis. In this entire unit, we shall study the concepts of displacement, velocity, acceleration and related topics considering one dimensional motion.

DISTANCE AND DISPLACEMENT

DISTANCE (PATH LENGTH) *Distance or path length of an object in motion in a given time is the length of actual path travelled/traversed by the object in that time.*

OR

It is the length of actual path travelled by the object in the given time during the motion.

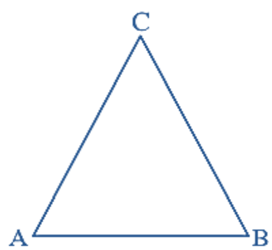


Fig. 6

Units of Distance :

SI unit of distance is **metre (m).**

CGS unit of distance is **centimetre (cm).**

But in our daily life, we come across the bigger units of distance *e.g.* kilometre.

Dimensional formula of distance is $[M^0LT^0]$

1. Illustrations for Distance : If object goes from A to C, C to B and B to A in time *t*, then total distance/path length of object in time $t = AC + CB + BA$.

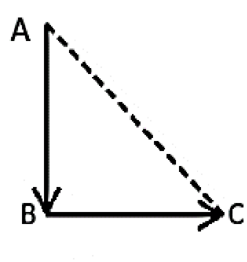
Characteristics :

1. Distance is a scalar quantity.
2. Distance travelled by a body in a given interval of time is always positive.
3. Distance travelled by vehicles is measured by a device ODOMETER, fitted on the dash board of vehicles.
4. It can never be zero or negative.

DISPLACEMENT

It is the shortest distance travelled by the object between initial to final positions of object.

The *displacement of the object is the change in the position of object in a particular direction.* It is represented by a vector drawn from initial to final position of the object.



Displacement is a vector quantity as it posses both magnitude and direction .

Displacement $|\vec{AC}| = \sqrt{3^2 - 4^2} = \sqrt{25} = 5$ cm .

Displacement $= \vec{AC}$

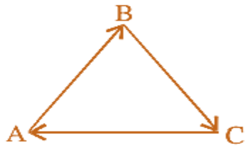
Here, arrow head are indicating that the object is displaced C to A , then displacement $= \vec{CA}$

i.e. $\vec{AC} = -\vec{CA}$ (Mag. same but opposite direction)

Illustrations for Displacement :

1. If the object goes along path ABC, then displacement of the object is $\overrightarrow{AC}$. The arrow head at AC shows that the object is displaced from point A to C but if the object is displaced from point C to A, then displacement is $\overrightarrow{CA}$.

2. If the object goes from A to B, B to C and back to A in time t, then displacement of the object in time t will be $\overrightarrow{AA} = \vec{O}$.



Units of Displacement

SI unit of displacement is **metre (m)**.

CGS unit of displacement is **centimetre** (cm).

Dimensional formula of displacement is $[M^0LT^0]$.

Displacement of an object in motion can be positive, negative or zero.

Let an object is going along positive X-axis in a straight line. At time $t = 0$, object is at point O and at time $t = t$ object is at point A having displacement x_i and at time $t = t'$ displacement is x_f

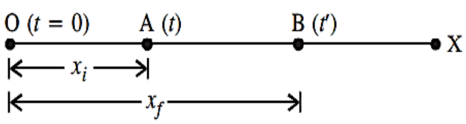


Fig. 8

$\therefore \quad \Delta d = x_f - x_i$

Δd = Final position – Initial position

($\Delta d \rightarrow$ displacement of particle. x_i and x_f be initial and final positions of the particle.)

(i) Displacement of an object is positive if $x_f > x_i$.

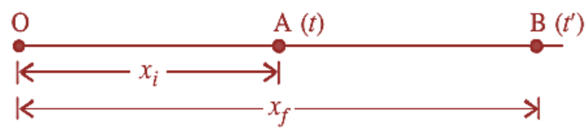


Fig. 9

$\Delta d = x_f - x_i$

As $x_f > x_i \quad \Delta d = +\text{ve}$

(ii) Displacement of an object is negative if $x_f < x_i$.

Let the object goes from point A to B to the left in time t to t', then displacement is negative.

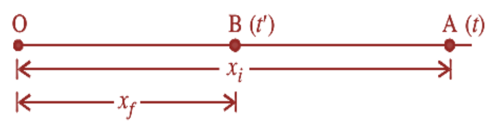


Fig. 10

$\Delta d = x_f - x_i$

As $x_f < x_i \quad \Delta d = -\text{ve}$.

Zero displacement : The displacement of the object is zero if the object remains stationary or it moves from point A to B towards right and then moves back from point B to A along negative X–axis

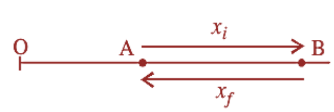


Fig. 11

Here $\quad x_i = x_f$

$\Delta d = x_f - x_i$

$\therefore \quad \Delta d = 0$

Characteristics of displacement

1. Displacement has the units of length.
2. Displacement of any object in a given interval of time may be positive, negative or zero.
3. Displacement of any object is a vector quantity as it has both magnitude and direction.
4. Displacement of any object in the given interval of time is independent of choice of origin.

Let the origin is at point O shifted from O to O′. The particle goes from A to B in time t, then displacement of particle/object in time $t = x_2 - x_1$.

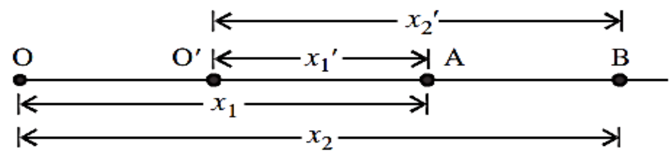


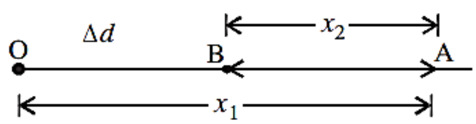
Fig. 12

Now, the origin shifts from O to O′ and the object moves from point A to B in same time t. So, the displacement in this case is again same, *i.e.* $= x'_2 - x'_1 = AB$

Hence, $x_2 - x_1 = AB = x'_2 - x'_1$

5. The actual distance travelled by the object in the given interval of time is always greater than or equal to displacement.

Let an object goes from O to A along the +ve X–axis and then from A to B along –ve X–axis, then displacement is $\overrightarrow{OB}$



Actual distance = OA + AB is greater than displacement $\overrightarrow{OB}$.

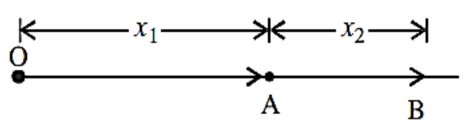


Fig. 14

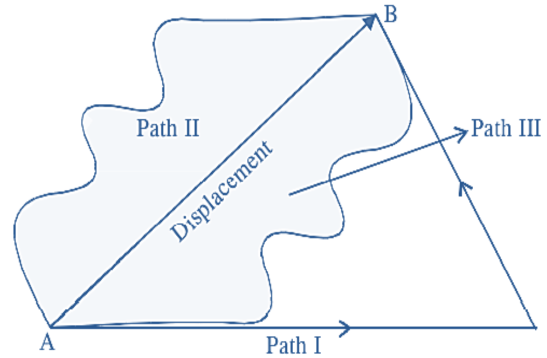
Secondly,

Actual distance moved $= OA + AB$

Displacement $= \overrightarrow{OB}$

Actual distance = Displacement.

6. Displacement of an object between any two points is the unique path between these points.
7. Displacement of an object between two positions does not give any information regarding the path followed by object.
8. Displacement of any object is the shortest path between two positions.



9. Displacement between any two positions is independent of the actual path followed by object in moving from one position to other.

	Distance	Displacement
1.	It is the actual path travelled by object.	It is the shortest path travelled by the object between initial and final positions.
2.	Distance is a scalar physical quantity.	Displacement is a vector physical quantity.
3.	Distance travelled by the object is always positive.	Displacement of the object in given interval of time may be positive, negative or zero.
4.	Distance travelled by the object depends on the shape of the path followed by object.	Displacement is independent of the path followed by the object between initial and final positions.

- *Distance is always positive . It never decreases with time .*
- *Displacement can decrease or increase with time .*

▶ ix) Displacement is a single value function of time i.e. a particle can not be as two position or two different position at the same the same time .

SPEED

Speed *of any object is the rate of change of position of the object with time in any direction. It is equal to the distance travelled by the object in motion per unit time.*

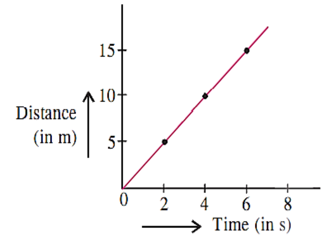
Speed of any object in motion is the ratio of total path length (distance) and the corresponding time taken by the object i.e.,

$$\text{Speed} = \frac{\text{Total path length or total distance}}{\text{Time taken}}$$

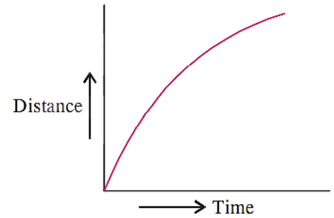
- Speed has only magnitude and no direction, so it is a scalar quantity.
- Also the distance travelled by an object is either positive or zero, so the speed may be positive or zero but never negative.
- The SI unit of speed is ms^{-1}.
- The CGS unit of speed is cms^{-1}.
- The dimensional formula of speed is $[M^0L^1T^{-1}]$.

Different Types of Speed :

1. **Uniform speed.** *An object is said to be moving with a uniform speed (i.e., constant speed) if it covers equal distances in equal intervals of time, however small these time intervals may be.*
   Thus if a car moves with a uniform speed of 60 km/hr for 2 hours, the distance covered by the car in 2 hours = 60 × 2 = 120km.



2. **Variable speed.** *An object is said to be moving with a variable speed if it covers equal distances in unequal intervals of time, however small these time intervals may be.*



3. **Average speed.** When an object moves with a variable speed, we generally describe its motion in terms of average speed.
   The average speed of an object is the total distance covered divided by the time taken to cover the distance i.e.,

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Time taken}}$$

For example, if a car covers a distance of 200 km in 4 hours, then average speed of the car = 200/4 = 50 km/hr. Note that average speed tells us nothing about the different speeds and variations that may have taken place during the journey.

If an object covers distance Δx in time Δt, then

$$\text{Average speed } v_{av} = \frac{\Delta x}{\Delta t}. \qquad (\Delta x = x_f - x_i;\ \Delta t = t_f - t_i)$$

4. **Instantaneous speed.** When an object is moving with a variable speed, it has different speeds at different instants of time.

The speed of an object at a given instant of time is called its instantaneous speed.

Instantaneous speed of any object is the limiting value of the average speed when the time interval approaches zero.

Let Δx be the distance travelled by any object in time Δt

$$\therefore \text{Average speed, } \upsilon_{av} = \frac{\Delta x}{\Delta t}$$

So $$\upsilon_{ins} = \underset{\Delta t \to 0}{\text{Lt}} (\upsilon_{av}) = \underset{\Delta t \to 0}{\text{Lt}} \left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

$$\upsilon_{ins} = \frac{dx}{dt}$$

where $\frac{dx}{dt}$ is differential co-efficient of x w.r.t. t *i.e.,* first derivative of distance with respect to time.

1. Speedometer of an automobile indicates the instantaneous speed of object at that instant.
2. If the object is moving with uniform motion, then average speed of object = instantaneous speed of object.

Average Speed in Different Situations :

I. If a body travels equal distances with speed υ_1, υ_2 and υ_3 respectively, then average speed

$$\upsilon_{av} = \frac{3\,v_1\,v_2\,v_3}{v_1 v_2 + v_2 v_3 + v_1 v_3}.$$

II. A body covers different distances with different velocity.

Let a body covers distances x_1, x_2, x_3 .... etc.

with speeds $\upsilon_1, \upsilon_2, \upsilon_3$ .... etc.

total distance travelled $= x_1 + x_2 + x_3 + ...$

$$\text{Total time taken} = \frac{x_1}{\upsilon_1} + \frac{x_2}{\upsilon_2} + \frac{x_3}{\upsilon_3} + ...$$

$$\text{average speed, } \upsilon_{av} = \frac{x_1 + x_2 + x_3 + ...}{\left(\frac{x_1}{\upsilon_1} + \frac{x_2}{\upsilon_2} + \frac{x_3}{\upsilon_3} + ...\right)}$$

If $x_1 = x_2 = x$ say *i.e.,* the body covers equal distances with different velocities, then

$$\upsilon_{av} = \frac{x + x}{x\left(\frac{1}{v_1} + \frac{1}{v_2}\right)} = \frac{2x}{x\left(\frac{v_1 + v_2}{v_1\,v_2}\right)}$$

$$\upsilon_{av} = \left(\frac{2\,v_1\,v_2}{v_1 + v_2}\right)$$

So, average speed is harmonic mean of individual speeds.

III. Suppose a body travels with speed υ_1, υ_2, υ_3 .... in times t_1, t_2, t_3, ... then

Total distance travelled $= \upsilon_1 t_1 + \upsilon_2 t_2 + \upsilon_3 t_3 + ...$

total time taken $= t_1 + t_2 + t_3 ....$

$$\therefore \quad \upsilon_{av} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + ...}{t_1 + t_2 + t_3 + ...}$$

If $t_1 = t_2 = t_3 = t$ say

$$\upsilon_{av} = \frac{(v_1 + v_2 + ... + v_n)t}{n\,t}$$

$$\upsilon_{av} = \frac{v_1 + v_2 + ...v_n}{n}$$

$\therefore$average speed is equal to arithmatic mean of the individual speeds.

VELOCITY

It is the *time rate of change of position or displacement of the object in a given direction.*

Velocity of an object is defined as the displacement divided by the time taken for the displacement i.e.,

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

Units of Velocity

SI unit of velocity is m s^{-1} and CGS unit of velocity is cm s^{-1}

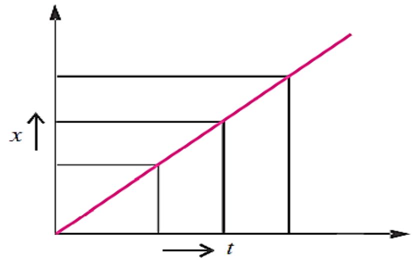
It is also expressed in km h^{-1}.

Dimensional formula of velocity = $[M^{o}LT^{-1}]$

- As *velocity has both magnitude as well as direction, so velocity is a vector quantity.*
- Velocity of an object can be positive, negative or zero according to the displacement which can be positive, negative or zero.
- As distance $\geq$ displacement *i.e., distance is greater than or equal to the magnitude of displacement so speed of the body is also greater than or equal to magnitude of velocity.*

Types of Velocity

1. Uniform velocity : *An object is said to be moving with uniform velocity if it covers equal displacements in equal to intervals of time howsoever small the interval may be.*



2. Variable velocity/Non-uniform velocity : *An object is said to be moving with variable velocity if it covers equal displacements in unequal intervals of time or unequal displacement in equal intervals of time.*

OR

An object *is said to be moving with variable velocity if either its speed or the direction of motion or both changes with time.*

3. Average velocity : *Average velocity of any object moving with variable velocity is the ratio of its total displacement to the total time interval in which that displacement occurs.*

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

Let $\vec{x}_1$ and $\vec{x}_2$ be the displacements or positions of any object at time t_1 and t_2, then

$$\upsilon_{av} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1} = \frac{\Delta \vec{x}}{\Delta t}.$$

4. Instantaneons velocity : It is *the velocity of the object at a particular instant of time.*

OR

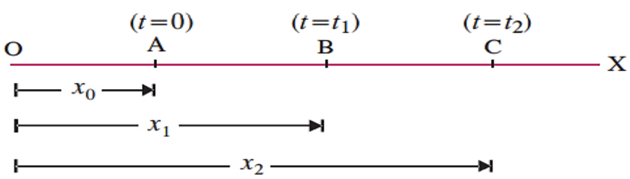
Instantaneous velocity of any object is the limiting value of the average velocity when the time interval approaches zero.

$$\boxed{\upsilon_{\text{ins}} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

where $\frac{dx}{dt}$ is the differential co-efficient of x w.r.t. time (t).

FORMULAE FOR UNIFORM MOTI

The formulae which connects position, velocity and time of an object are called formulae for uniform motion.



Let an object is moving with uniform velocity $\vec{\upsilon}$ along the +ve X-axis, origin for the measurement of its position is O, and the origin for the measurement of time is the instant object is at point A.

At time $t = 0$ object is at point A having displacement from origin x_0 :

At time $t = t_1$ object reaches point B having displacement $\vec{x}_1$ from O.

At time $t = t_2$ object reaches point C having displacement $\vec{x}_2$ from O, Displacement of object in time $(t_1 - 0) = \vec{x}_1 - \vec{x}_0$

$$\therefore \quad \vec{\upsilon} = \frac{\vec{x}_1 - \vec{x}_0}{t_1 - 0} \text{ so } \vec{x}_1 - \vec{x}_0 = \vec{\upsilon}\,(t_1)$$

$$\Rightarrow \quad \vec{x}_1 = \vec{x}_0 + \vec{\upsilon}\, t_1 \qquad \text{...(1)}$$

And displacement in time $(t_2 - 0) = \vec{x}_2 - \vec{x}_0$

$$\therefore \quad \vec{\upsilon} = \frac{\vec{x}_2 - \vec{x}_0}{t_2 - 0} \text{ so } \vec{x}_2 - \vec{x}_0 = \vec{\upsilon}\,(t_2)$$

$$\Rightarrow \quad \vec{x}_2 = \vec{x}_0 + \vec{\upsilon}\, t_2 \qquad \text{...(2)}$$

Subtracting eqn. (1) from eqn. (2)

$$\vec{x}_2 - \vec{x}_1 = \vec{\upsilon}\, t_2 - \vec{\upsilon}\, t_1$$

$$\vec{x}_2 - \vec{x}_1 = \vec{\upsilon}\,(t_2 - t_1)$$

$$\vec{x}_2 = \vec{x}_1 + \vec{\upsilon}\,(t_2 - t_1) \qquad \text{...(3)}$$

Relation (1), (2) and (3) represents formulae for uniform motion along straight line.

$\vec{x} - \vec{x}_0$ = Displacement in time internal $(t_1 - 0)$

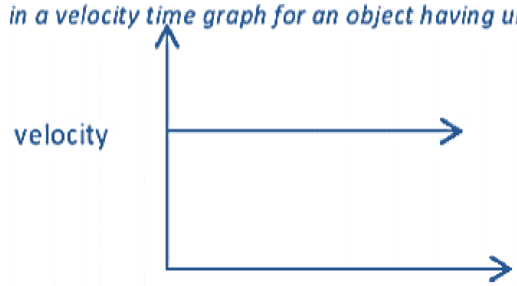
$\vec{x} - \vec{x}_0 = s$ (say) so $s = \upsilon t_1$

This equation can be used to calculate the distance travelled by object in uniform motion.

☑ Some Important Features of Uniform motion :-

- i) *For a uniform motion along a straight line in a given direction , the magnitude of displacement is equal to the actual distance covered by the object .*
- ii) *The velocity in uniform motion is independent of choice of origin .*
- iii) *The velocity in uniform motion does not depends upon the time interval ($t_2 - t_1$)*
- iv) *The velocity of an object is taken to be positive if the object is moving towards the right of the origin and is taken to be negative if the object is moving towards the left of the origin .*
- v) ***No force** is required for an object to be in uniform motion .*
- vi) *The average & instantaneous velocity have same value in a uniform motion .*

--- *The velocity shown in a velocity time graph for an object having uniform velocity is a straight line parallel to time axis .*



KNOWLEDGE PLUS

1. Average speed of any object is equal to average velocity if it travels in a straight line without change in its direction of motion.
2. Average speed of any object is greater than average velocity if it travels in a straight line but changes its direction of motion.
3. Average speed and instantaneous speed of object are equal if the object moves with constant speed.
4. Speedometer of a vehicle measures its instantaneous speed.
5. Magnitude of average velocity may or may not be equal to average speed.
6. When any object is moving along a straight line in definite direction, then distance travelled by object is equal to the magnitude of displacement. Hence, both instantaneous and average speeds are equal.

	Speed	Velocity
1.	Speed of a body is the distance travelled by it per unit time.	Velocity of a body is the displacement of the body per unit time.
2.	Speed is a scalar physical quantity.	Velocity is a vector physical quantity.
3.	Speed of a body is always positive.	Velocity of a body may be positive, negative or zero.
4.	Speed tells nothing about the direction of motion of body.	Velocity tells about the direction of motion of body.
5.	Speed of any body is always equal to or greater than the velocity of body.	Velocity of any body is equal to or less than the speed of body.

The distinction between speed and velocity can be made more clear if we refer to Fig. 6.11. Here a car is moving round a circular track at a constant speed of 40 km/hr. At every point on the track, the speed is the same *i.e.*, 40 km/hr. But at every point (such as *A*, *B* and *C*) the velocity is different. Thus the magnitude of velocity at points *A*, *B* and *C* is the same but the direction of velocity is different. Therefore, we note that velocities at all points (*e.g.*, *A*, *B* and *C*) are different, even though the magnitudes of the velocities are same.

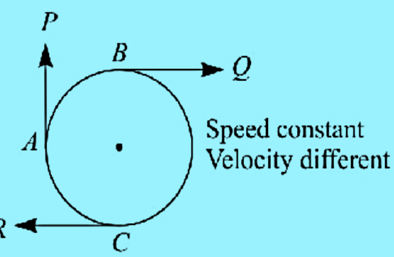


Fig. 6.11

Examples Based on Distance, Displacement, Speed and Velocity

Formulae used

(i) Distance covered = Length of actual path covered by body.

(ii) Displacement = vector drawn from initial to final position of body.

(iii) $$\text{Average speed} = \frac{\text{Total Distance Covered}}{\text{Total time taken}}$$

(iv) $$\text{Average velocity } v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

(v) $$v_{ins} = \underset{\Delta t \to 0}{\text{Lt}} (v_{av}) = \underset{\Delta t \to 0}{\text{Lt}} \left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

Units used

All distances and displacements are in metre or (kilometre), average speed, average velocity are in m s^{-1} or sometime km h^{-1}, time in second (or hour)

All velocities are in m s^{-1}

Example 6.1. An object travels different distances with different speeds in the same direction. Find the expression for the average speed.

Solution. Suppose an object travels distances S_1, S_2, S_3 etc. with speeds v_1, v_2, v_3 etc. in the same direction respectively.

$$\text{Total distance travelled} = S_1 + S_2 + S_3$$

$$\text{Total time taken} = \frac{S_1}{v_1} + \frac{S_2}{v_2} + \frac{S_3}{v_3}$$

$$\therefore \quad \text{Average speed} = \frac{S_1 + S_2 + S_3}{\frac{S_1}{v_1} + \frac{S_2}{v_2} + \frac{S_3}{v_3}}$$

Special case. If the object travels two equal distances ($S_1 = S_2 = S$) with different speeds v_1 and v_2, then,

$$\text{Average speed} = \frac{S + S}{\frac{S}{v_1} + \frac{S}{v_2}} = \frac{2\,v_1 v_2}{v_1 + v_2}$$

Example 6.2. An object travels with different speeds during different time intervals in the same direction. Find the expression for the average speed.

Solution. Suppose an object travels with speeds v_1, v_2, v_3 etc. during time intervals t_1, t_2, t_3 etc. in the same direction.

$$\text{Total distance travelled} = v_1\, t_1 + v_2\, t_2 + v_3\, t_3$$

$$\text{Total time taken} = t_1 + t_2 + t_3$$

$$\therefore \quad \text{Average speed} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3}{t_1 + t_2 + t_3}$$

Special case. If $t_1 = t_2 = t_3 = t$, then,

$$\text{Average speed} = \frac{v_1 + v_2 + v_3}{3}$$

EXAMPLE 1. A body covers a circular path of radius R in 10 seconds. Calculate the distance and displacement of the body at the end of 30 seconds and 35 seconds.

Solution. Case I. Body covers a circular path in 10 s. So, in 30 s it will cover three circular paths.

$\therefore$ distance travelled in 30 s $= 3 \times 2\,\pi R = 6\,\pi R$

whereas after covering three circular paths the body will be at the starting point so displacement = 0.

Case II. In 35 seconds, the body will cover three complete circular paths and one half circular path $(3 \times 10 + 5)$ s.

$\therefore$ distance travelled in 35 s $= 3 \times 2\,\pi R + \frac{1}{2} \times 2\pi R = 7\pi R.$

In 35 s, the displacement will be $R + R = 2\,R$.

EXAMPLE 2. Calculate the average speed of a man if he walks for 1 minute at a speed of 1 m s^{-1} and then runs for 1 minute at a speed of 3 m s^{-1} along a straight track.

Solution : Given

$$t_1 = 1 \text{ min} = 60 \text{ s}, \ \upsilon_1 = 1 \text{ m s}^{-1}$$

$$t_2 = 1 \text{ min} = 60 \text{ s}, \ \upsilon_2 = 3 \text{ m s}^{-1}$$

$$\text{Total distance} = \upsilon_1 t_1 + \upsilon_2 t_2 = 1 \times 60 + 3 \times 60 = 240 \text{ m}$$

$$\text{Total time taken} = t_1 + t_2 = 60 + 60 = 120 \text{ s}$$

$$\therefore \text{average speed} = \frac{240}{120} = \mathbf{2 \ m \ s^{-1}}.$$

EXAMPLE 4. A car covers one third of the total distance with a speed of 30 km h^{-1} and remaining distance with a speed of 54 km h^{-1}. Find average speed of car.

Solution : Let total distance $= x$

$\frac{x}{3}$ distance travelled with speed of 30 km h^{-1}.

Remaining $\left(x - \frac{x}{3}\right) = \frac{2x}{3}$ distance with a speed of 54 km h^{-1}

$$\text{Time taken to cover } \frac{x}{3} \text{ distance } (t_1) = \frac{x/3}{30} = \frac{x}{90} \text{ h}$$

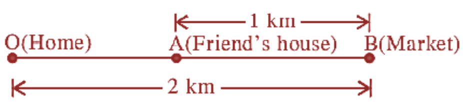
$$\text{Time taken to cover } \frac{2x}{3} \text{ distance } t_2 = \frac{2x/3}{54} = \frac{x}{81} \text{ h}$$

$$\therefore \text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{x}{\frac{x}{90} + \frac{x}{81}}$$

$$= \frac{x(90 \times 81)}{81x + 90x} = \frac{90 \times 81x}{171x}$$

$$= \mathbf{42{\cdot}63 \ km \ h^{-1}}$$

EXAMPLE 6. A boy runs from his home to the market 2 km away. He reaches the market in 60 minutes. Seeing the market closed, he at once runs back to his home and stops after 30 minutes at his friend's house which is 1 km away from the market. Calculate the average speed and magnitude of average velocity during the journey.



Solution. Let O be the home of boy. Point A is friend's house and B is market.

Total distance travelled by boy $=$ OB $+$ BA $= 2 + 1 = 3$ km

$$\text{Total time taken} = 60 + 30 = 90 \text{ minutes}$$

$$= \frac{90}{60} = \frac{3}{2} \text{ hours}$$

$$\therefore \text{ Average speed} = \frac{3}{3/2} = \frac{3 \times 2}{3} = \mathbf{2 \ km \ h^{-1}}$$

EXAMPLE 3. A car covers first half of total distance with a speed of 54 km h^{-1} and second half with speed 72 km h^{-1}. Find average speed of car.

Solution : $\upsilon_1 = 54 \text{ km h}^{-1}; \ \upsilon_2 = 72 \text{ km h}^{-1}$

$$\upsilon_{av} = \frac{2\upsilon_1\upsilon_2}{\upsilon_1 + \upsilon_2} = \frac{2 \times 54 \times 72}{54 + 72}$$

$$= \frac{7776}{126} \text{ km h}^{-1} = \mathbf{61{\cdot}7 \ km \ h^{-1}}$$

EXAMPLE 5. A body is moving in a straight line along X-axis. Its distance from origin is given by equation $x = 8t - t^2$ where x is in m and t in s. Find average speed of the object in the time interval $t = 0$ and $t = 2$ s. Calculate its instantaneous speed at $t = 2$ s.

Solution. Given $x = 8t - t^2$

Case I : When $t = 0 \ \therefore x_0 = 0$

$$t = 2, x = 8 \times 2 - (2)^2 = 16 - 4 = 12 \text{ m}$$

$$\therefore \quad \Delta x = x - x_0 = 12 \text{ m} - 0 = 12 \text{ m}$$

$$\Delta t = 2 - 0 = 2 \text{ s}$$

So, average speed in the given time interval,

$$\upsilon_{av} = \frac{\Delta x}{\Delta t} = \frac{12}{2} = 6 \text{ m s}^{-1}$$

Case II : Instantaneous Speed

$$\upsilon = \frac{dx}{dt} = \frac{d}{dt}(8t - t^2)$$

$$\Rightarrow \quad \upsilon = 8\frac{dt}{dt} - \frac{d}{dt}(t^2)$$

$$\Rightarrow \quad \upsilon = 8 - 2t$$

But $t = 2$ s $\therefore \ \upsilon = 8 - 2 \times 2 = \mathbf{4 \ m \ s^{-1}}$

$$\therefore \text{ Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$\text{Total displacement} = \text{OB} - \text{BA} = 2 - 1 = 1 \text{ km}$$

$$\text{Total time} = 90 \text{ minutes} = \frac{90}{60} = \frac{3}{2} \text{ hr}$$

$$\therefore \text{ Average velocity} = \frac{1}{3/2} = \frac{2}{3} \text{ km h}^{-1} = \mathbf{0{\cdot}67 \ km \ h^{-1}}$$

EXAMPLE 7. A body travels from point A to point B at 40 km h^{-1} and from B to A at 60 km h^{-1}. Calculate average speed and average velocity.

Solution. Let t_1 is time taken by body in travelling from A to B and t_2 be the time taken in travelling from B to A, then

$$t_1 = \frac{AB}{40}, \quad t_2 = \frac{BA}{60}$$

$$\therefore \quad t_1 + t_2 = \frac{AB}{40} + \frac{BA}{60}$$

$$= AB\left(\frac{1}{40} + \frac{1}{60}\right) = \frac{AB}{24}\text{ s.}$$

Total distance $= AB + BA = AB + AB = 2\,AB$

$$\therefore \text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{2\,AB}{AB/24} = \frac{2 \times 24\,AB}{AB} = \mathbf{48\ km\ h^{-1}}$$

As the initial position and final position of the body is same (*i.e.*, A) so net displacement = 0.

$$\therefore \text{Average velocity} = \frac{\text{Net displacement}}{\text{Net time}} = \frac{0}{AB/24} = \mathbf{0}.$$

EXAMPLE 9. A body covers one-third of its journey with speed 'u', next one-third with speed 'υ' and the last one- third with speed w. Calculate the average speed of the body during the entire journey.

Solution. Let each distance is x

$\therefore$ total distance $= x + x + x = 3x$

Time taken by body to cover $\frac{1}{3}$ rd distance $= \frac{x}{u}$

Next $\frac{1}{3}$ rd distance $= \frac{x}{\upsilon}$

Last $\frac{1}{3}$ rd distance $= \frac{x}{w}$

$\therefore$ total time taken for entire journey $= \frac{x}{u} + \frac{x}{\upsilon} + \frac{x}{w}$

$$\therefore \text{ Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{3x}{\frac{x}{u} + \frac{x}{\upsilon} + \frac{x}{w}} = \frac{3x}{x\left(\frac{1}{u} + \frac{1}{\upsilon} + \frac{1}{w}\right)}$$

$$= \frac{3}{\frac{\upsilon w + uw + u\upsilon}{u\upsilon w}} = \frac{\mathbf{3u\upsilon w}}{\mathbf{u\upsilon + \upsilon w + uw}}$$

EXAMPLE 8. On a track of 60 km, a train travels the first 30 km with the uniform speed of 30 km h^{-1}. How fast must the train travel next 30 km so that the average speed will be 40 km h^{-1} for the complete trip ?

Solution. Total length of track = 60 km

Given $\quad S_1 = 30$ km and $S_2 = 30$ km

So $\quad S = S_1 = S_2 = 30$ km

$\upsilon_1 = 30$ km h^{-1}, $\upsilon_{av} = 40$ km h^{-1}

Let υ_2 be the speed of train for next 30 km.

We know $$\upsilon_{av} = \frac{S_1 + S_2}{t_1 + t_2} = \frac{S + S}{\frac{S}{\upsilon_1} + \frac{S}{\upsilon_2}}$$

$$= \frac{2S}{S\left(\frac{1}{\upsilon_1} + \frac{1}{\upsilon_2}\right)} = \frac{2}{\frac{\upsilon_1 + \upsilon_2}{\upsilon_1\upsilon_2}} = \frac{2\upsilon_1\upsilon_2}{\upsilon_1 + \upsilon_2}$$

$$\Rightarrow \quad 40 = \frac{2 \times 30 \times \upsilon_2}{30 + \upsilon_2}$$

$$\Rightarrow \quad 40 = \frac{60\upsilon_2}{30 + \upsilon_2} \Rightarrow 40\,(30 + \upsilon_2) = 60\,\upsilon_2$$

$$\Rightarrow \quad 30 + \upsilon_2 = \frac{3}{2}\upsilon_2 \quad \Rightarrow \quad \upsilon_2 = \frac{3}{2}\upsilon_2 - 30$$

$$\Rightarrow \upsilon_2 - \frac{3}{2}\upsilon_2 = -30 \quad \Rightarrow \left(\frac{2-3}{2}\right)\upsilon_2 = -30$$

$$\Rightarrow \quad -\frac{1}{2}\upsilon_2 = -30 \quad \Rightarrow \upsilon_2 = 60\text{ km h}^{-1}$$

So, the train must travel with speed 60 km h^{-1} in driving next 30 km.

EXAMPLE 10. A car travels at a rate of 20 km h^{-1} for 10 minutes and then 30 km h^{-1} for 20 minutes. Calculate *(i)* Total distance travelled by car and *(ii)* Average speed of the car during whole journey.

Solution. Given $\upsilon_1 = 20$ km h^{-1},

$$t_1 = 10\text{ min} = \frac{10}{60} = \frac{1}{6}\text{ h} \quad \left(\upsilon = \frac{D}{t}\right)$$

$$\therefore \text{distance travelled } S_1 = \upsilon_1 t_1 = 20 \times \frac{1}{6} = \frac{10}{3}\text{ km}$$

Also $\upsilon_2 = 30$ km h^{-1}, $t_2 = 20\text{ min} = \frac{20}{60} = \frac{1}{3}$ h

$$\therefore \text{distance travelled } S_2 = \upsilon_2 t_2 = 30 \times \frac{1}{3} = 10\text{ km}$$

(i) Total distance travelled by car (S) $= S_1 + S_2 = \frac{10}{3} + 10$

$$= \frac{40}{3}\text{ km} = \mathbf{13{\cdot}33\ km}$$

(ii) Average speed $= \frac{\text{Total distance}}{\text{Total time}} = \frac{S_1 + S_2}{t_1 + t_2}$

$$= \frac{13{\cdot}33}{\frac{1}{6} + \frac{1}{3}} = \frac{13{\cdot}33}{1/2} = \mathbf{26{\cdot}66\ km\ h^{-1}}$$

EXAMPLE 11. A car covers the first half of the distance between stations at a speed of 40 km h^{-1} and the second half at a speed of 50 km h^{-1}. What is average speed of car ?

Solution. Given $\upsilon_1 = 40$ km h^{-1}, $\upsilon_2 = 50$ km h^{-1}

Average speed of car,

$$\upsilon_{av} = \frac{2\upsilon_1\upsilon_2}{\upsilon_1 + \upsilon_2} = \frac{2 \times 40 \times 50}{40 + 50}$$

$$= \frac{400}{90} = \mathbf{44{\cdot}4\ km\ h^{-1}}$$

EXAMPLE 13. A person walks on a straight road from his home to a market 3 km away with a speed of 6 km h^{-1}. Finding the market closed, he instantly turns back and travels with a speed of 9 km h^{-1}. What is *(a)* magnitude of average velocity and *(b)* average speed of the person over time interval *(i)* 0 to 40 min *(ii)* 0 to 50 min.

Solution. Distance of market from home = 3 km

Speed of person in going from home to market = 6 km h^{-1}

$\therefore$ time taken by person in going from home to market

$$t_1 = \frac{\text{Distance}}{\text{Speed}} = \frac{3\text{ km}}{6\text{ km/h}} = \frac{1}{2}\text{ h} = 30\text{ min.}$$

Similarly, time taken by person in coming back from market to home

$$t_2 = \frac{3\text{ km}}{9\text{ km h}^{-1}} = \frac{1}{3}\text{ h} = 20\text{ min}$$

$$\therefore \text{ total time taken} = t_1 + t_2 = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}\text{ h} = 50\text{ min.}$$

Case I. Time from 0 to 40 min

Distance moved by person in 30 min (from home to market) at a speed 6 km/h = 3 km

Distance covered in next 10 min (in coming back from market to home) at a speed 9 km/h $= 9 \times \frac{10}{60} = \frac{3}{2} = 1{\cdot}5$ km.

So, displacement $= 3{\cdot}0 - 1{\cdot}5 = \mathbf{1{\cdot}5\ km}$

Total distance travelled by person $= 3 + 1{\cdot}5 = \mathbf{4{\cdot}5\ km}$

(a) Average velocity $= \dfrac{\text{Total displacement}}{\text{Total time}}$

$$= \frac{1{\cdot}5}{40/60} = \frac{90}{40} = \mathbf{2{\cdot}25\ km/h}$$

(b) Average speed $= \dfrac{4{\cdot}5\text{ km}}{40/60} = 4{\cdot}5 \times \dfrac{3}{2} = \mathbf{6{\cdot}75\ km/h}$

Case II. Distance moved in 30 min = 3 km (from home to market) at speed 6 km/h

Distance moved in 20 minutes (from market $= 9 \times \frac{20}{60} = 3$ km to home) at the speed 9 km/h

Hence, Displacement $= 3 - 3 = 0$

Distance $= 3 + 3 = 6$ km.

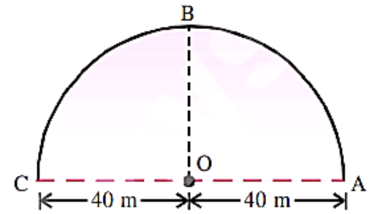
(a) Average velocity $= \dfrac{\text{Total displacement}}{\text{Total time}} = \dfrac{0}{5/6} = 0$

(b) Average speed $= \dfrac{\text{Total distance travelled}}{\text{Total time}}$

$$= \frac{6}{\frac{5}{6}\text{ h}} = \frac{6 \times 6}{5} = \mathbf{7{\cdot}2\ km/h}$$

EXAMPLE 12. A man moves in a semicircular track of radius 40 m. If he starts at one end of track and reaches the other end, find the distance covered and displacement.

Solution. Radius of the semicircular track (R) = 40 m



$\therefore$ distance covered by the man = length of semicircular path ABC

$= \pi R = 3{\cdot}14 \times 40 = 126$ m

Displacement of man = Diameter of semicircle (AC)

$= (AO + OC) = 40 + 40 = \mathbf{80\ m}$

EXAMPLE 14. A man starts from his home at 8 : 00 a.m. to his office. He walks with a speed of 2 m s^{-1} on a road up to his office 3·0 km away from home. He stays in the office up to 4 : 00 p.m. and returns to his home by bus which moves non-stop with a speed of 10 m s^{-1}. Calculate

(i) **time taken by the man to reach the office**

(ii) **time taken by man to reach his home. Also plot $(x - t)$ graph for his motion.** ***(H.P.B.O.S.E. 2009)***

Solution. Distance of the office from home,

S = 3 km = 3000 m

Case I. Speed of man while going from home to office (υ) = 2 m s^{-1}

$\therefore$ time taken by man to reach office

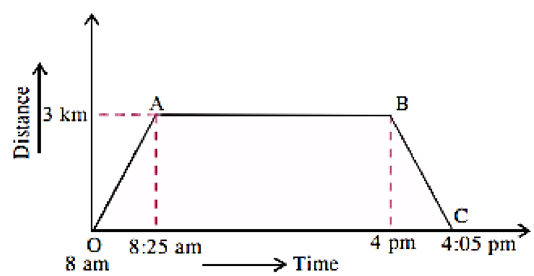
$$t = \frac{S}{\upsilon} = \frac{3000}{2} = 1500\text{ s} \qquad \left(\upsilon = \frac{S}{t}\right)$$

$$t = \frac{1500}{60} = \mathbf{25\ min}$$

Hence, man reaches from home to office in **25 minutes**

i.e., man must reach at **8:25 a.m.** in the office.

Stay of the man in the office is from 8:25 a.m. to 4:00 p.m. so no distance is travelled by man during this time.



Case II. While returning to his home from office by bus

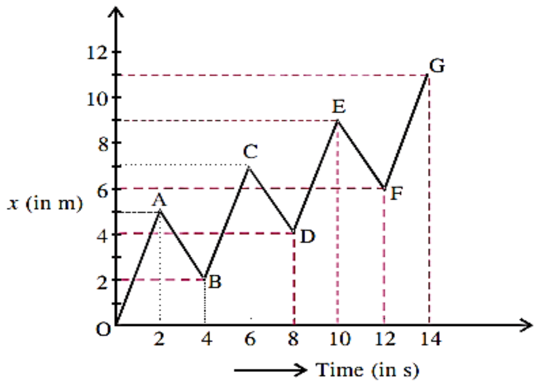
Speed of bus (υ_1) = 10 m s^{-1}

$$\text{Time taken } t_1 = \frac{\text{Distance}}{\text{Velocity}} = \frac{3000\text{ m}}{10\text{ m s}^{-1}}$$

$= 300$ s $= 5$ min

So man must reaches home at **4:05 p.m.**

EXAMPLE 15. A man wants to climb to the top of a vertical pole of height 11 m. He climbs 5 m in 2 s and then slips 3 m in 2 s. This process repeats. Plot $x-t$ graph for the motion of the man. Find the time taken by man to reach the top of the pole and total distance covered by man.



Solution. Let at time $t = 0$ man strats climbing the pole. After 2 s man climbs 5 m and slips 3 m in next 2 s. After 6 s the man will be at 7 m height but after 8 s he will come down to 4 m. After 10 s, he will be at 9 m height and after 12 s he will be at 6 m height (9 – 3 = 6 m). After 14 s he will be at height 11 m.

$\therefore$ time taken by person to reach the highest point = 14 s.

Total distance travelled

$$= OA + AB + BC + CD + DE + EF + FG$$
$$= 5 + 3 + 5 + 3 + 5 + 3 + 5 = \mathbf{29\ m.}$$

EXAMPLE 16. A body moves in a straight line along X-axis. Its displacement from the origin at any time is given by $x = 8t - 3t^2$. (x is in metre, t in seconds)

***(i)* Calculate the average velocity in time interval $t = 0$ to $t = 2$ s.**

***(ii)* Find the instantaneous velocity of the body at the instant $t = 0$, $t = 2$ s and $t = 4$ s.**

Solution. Given $x = 8t - 3t^2$.

(i) Calculation of average velocity in the interval $t = 0$ to $t = 2$ s.

$$v_{av} = \frac{x(t = 2s) - x(t = 0)}{2 - 0}$$

$$x(t = 2s) = 8 \times 2 - 3 \times 4 = 16 - 12 = 4\ m$$

$$x(t = 0) = 0$$

$$\therefore \quad v_{av} = \frac{4 - 0}{2 - 0} = \frac{4\ m}{2\ s} = 2\ m\ s^{-1}$$

(ii) $$v_{ins} = \frac{dx}{dt} = \frac{d}{dt}(8t - 3t^2)$$

$$v_{ins} = 8 - 6t$$

$$v_{ins}(t = 0) = 8 - 6 \times 0 = \mathbf{8\ m\ s^{-1}}$$

$$v_{ins}(t = 2\ s) = 8 - 6 \times 2 = 8 - 12 = \mathbf{-4\ m\ s^{-1}}$$

$$v_{ins}(t = 4\ s) = 8 - 6 \times 4 = 8 - 24 = \mathbf{-16\ m\ s^{-1}}$$

◆ From here it is clear that when velocity of the body is variable (function of time) then average and instantaneous velocities are different.

Problems for Practice

1. A body travels the first half of the total distance with a velocity v_1 and the second half with a velocity v_2. Calculate the average velocity. [**Ans.** $\frac{2v_1v_2}{v_1 + v_2}$]
2. A car covers the first half of the distance between two places at a speed of 40 km h^{-1} and second half with a speed of 60 km h^{-1}. What is average speed of car ? [**Ans.** 48 km h^{-1}]
3. A train moves with a speed of 30 km h^{-1} in the first 15 minutes, with a speed of 40 km h^{-1} in next 15 minutes and with speed 60 km h^{-1} in last 30 minutes. Calculate the average speed of the train for this journey. [**Ans.** 47·5 km h^{-1}]
4. A cyclist moving on a circular track of radius 100 m completes one revolution in 4 minutes. What is
   (i) Average speed
   (ii) Average velocity in one full revolution ?
   [**Ans.** *(i)* 50 π metre/minute, *(ii)* 0]
5. If a car travels a distance S_1 with velocity v_1 and distance S_2 with velocity v_2 in same direction, then what is average velocity of the car. [**Ans.** $\frac{(S_1 + S_2)v_1v_2}{S_1v_1 + S_2v_2}$]
6. A gun is fired from a distance of 1·2 km from a hill. The echo of the sound is heard back at the same place after 8 seconds. Find velocity of sound. [**Ans.** 300 m s^{-1}]
7. Two cars A and B are at positions 100 m and 200 m from the origin at time $t = 0$. They start simultaneously with velocities 10 m s^{-1} and 5 m s^{-1} respectively. Determine the positions and time at which they will overtake one another. Assume that they are moving in same direction. [**Ans.** 300 m, 20 s]
8. A boy reached a railway station 4 km away from his house running with a uniform speed in 1·0 hour. He took rest for 0·5 hour at the station and then came back to his house walking with uniform speed in 1·5 hour. Represent the whole journey of the boy by a time-displacement graph and determine his average speed. [**Ans.** 2·67 km h^{-1}]
9. A car moving on a straight road covers one-third of the distance with 20 km/h and the rest with 60 km/h. What is the average speed of car ? [**Ans.** 36 km/h]
10. A train 600 m long crosses a bridge of 1000 m in 10 s. Find the average speed of the train when it just crosses the bridge. [**Ans.** 160 m s^{-1}]
11. What is the distance in km of a quasar from which light takes 3×10^9 years to reach the earth ? Speed of light is $c = 3 \times 10^8$ m s^{-1}. [**Ans.** $2{\cdot}838 \times 10^{22}$ km]

12. The velocity-time graph for a body moving along a straight line is as shown in figure. Find the displacement of the body in 10 s of its motion. **[Ans.** 8 m]

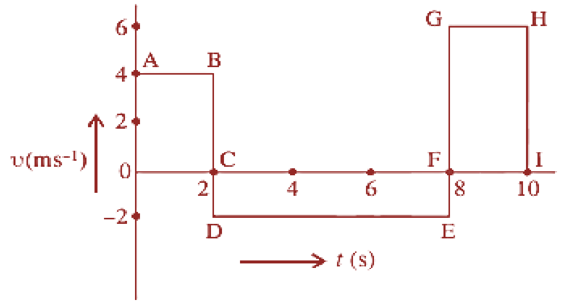


Fig. 21

13. A car travelled the first third of a distance x at a speed of 10 km/h, the second third at a speed of 20 km/h and the last third at a speed of 60 km/h. Determine the average speed of car over entire distance x. **[Ans.** 18 km h^{-1}]

14. A table clock has its minute hand 5 cm long. Find the average velocity of the tip of the minute hand *(a)* between 6:00 am and 6:15 am and *(b)* between 6 : 00 am and 6:30 pm. **[Ans.** *(a)* $7{\cdot}86 \times 10^{-3}$ cm s^{-1} *(b)* $2{\cdot}2 \times 10^{-4}$ cm s^{-1}]

15. The velocity-time graph of an object moving along a straight line is shown below, calculate the distance covered by object between

(a) $t = 0$ to $t = 5$ s *(b)* $t = 0$ to $t = 10$ s.

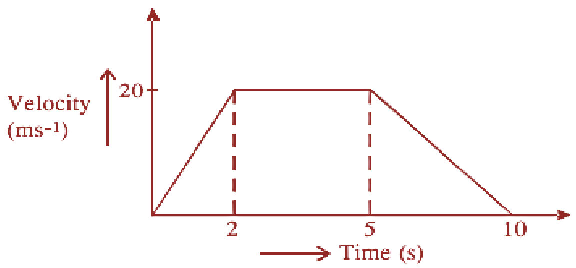


Fig. 22

[Ans. *(a)* 80 m, *(b)* 130 m]

16. A car starts from rest in a certain direction, a scooter moving with the uniform speed overtakes the car. Their velocity-time graphs are shown in figure. Calculate *(i)* the difference between the distance travelled by car and scooter in 15 s *(ii)* the time when the car will catch up the scooter *(iii)* the distance of car and scooter from starting point at that instant. **[Ans.** *(i)* 112·5 m *(ii)* 22·5 s *(iii)* 675 m]

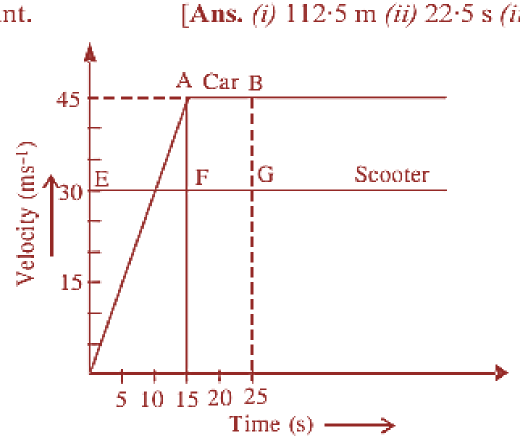


Fig. 23

Hints / Solutions

1. Let distance x is travelled with velocity υ_1 and other part of distance x with velocity υ_2

$\therefore$ total distance $= x + x = 2x$

$$\text{Total time taken} = \frac{x}{\upsilon_1} + \frac{x}{\upsilon_2} = x\left(\frac{1}{\upsilon_1} + \frac{1}{\upsilon_2}\right)$$

$$= x\left(\frac{\upsilon_1 + \upsilon_2}{\upsilon_1\upsilon_2}\right)$$

$$\therefore \text{ Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{2x}{x\left(\frac{\upsilon_1+\upsilon_2}{\upsilon_1\upsilon_2}\right)} = \frac{2\upsilon_1\upsilon_2}{\upsilon_1 + \upsilon_2}$$

4. Radius of circular track $= 100$ m

Time taken for completing one revolution = 4 minutes

Distance travelled in one revolution $= 2\pi r$

$= 2 \times \pi \times 100 \text{ m} = 200\pi$ m

$$\therefore \text{ Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{200\,\pi}{4}$$

$= \mathbf{50\,\pi}$ **m/minutes**

Also displacement in 4 minutes = 0

$$\therefore \text{ Average velocity} = \frac{\text{Displacement}}{\text{Time}} = \mathbf{0}$$

7. Given $x_{OA} = 100$ m, $x_{OB} = 200$ m

$\upsilon_A = 10$ m s^{-1}, $\upsilon_B = 5$ m s^{-1}

Positions of cars after time t

$x = x_{OA} + \upsilon_A\, t$ ...(1)

(x is same in both cases during overtake)

$x = x_{OB} + \upsilon_B\, t$ ...(2)

From eqn. (1) and eqn. (2)

$100 + 10t = 200 + 5t$

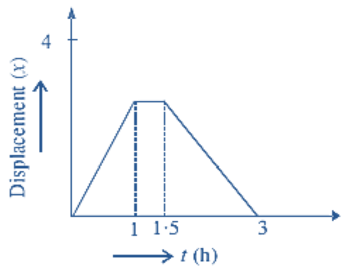
$\Rightarrow$ $5t = 100$ $\Rightarrow$ $\mathbf{t = 20}$ **s**

Using t in eqn. (1)

$x = 100 + 10 \times 20 = 100 + 200 = \mathbf{300}$ **m**

So, after 20 seconds and travelling 300 m distance the cars overtake each other.

8. Average speed $= \dfrac{8}{3} = \mathbf{2{\cdot}67}$ **km h^{-1}**



10. Length of train – 600 m, Length of bridge – 1000 m

Total distance to be travelled by train to cross the bridge

$= 600 + 1000 = 1600$ m

Time $= 10$ s

$$v_{av} = \frac{1600 \text{ m}}{10 \text{ s}} = \mathbf{160 \text{ m s}^{-1}}$$

11. Using distance $= c \times t$

12. Displacement of body $=$ Area under $(v - t)$ graph

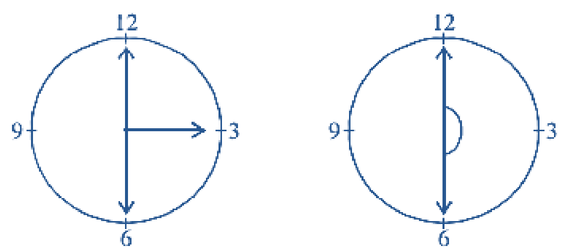
$x =$ Area OABC + Area CDEF + Area FGHI

$x = \text{OA} \times \text{OC} + \text{CD} \times \text{CF} + \text{FG} \times \text{FI}$

$x = 4 \times 2 + (-2 \times 6) + 6 \times 2$

$x = 8 - 12 + 12 = \mathbf{8 \text{ m}}$

14. *(i)* At 6:00 a.m. the tip of minute hand will be at 12 mark and 6:15 a.m. at 90° away.



(a) *(b)*

Let R be the length of minute hand

$\therefore$ displacement of needle

$= \sqrt{\text{R}^2 + \text{R}^2} = \sqrt{2\text{R}^2} = \sqrt{2}\text{R} = \sqrt{2} \times 5$ cm

Time taken to go from 6:00 a.m. to 6:15 a.m.

$= 15$ minutes $= 15 \times 60$ s

$$\therefore \text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$= \frac{\sqrt{2} \times 5}{15 \times 60} = \frac{\sqrt{2}}{180} \text{ cm s}^{-1}$$

$$= \mathbf{7{\cdot}86 \times 10^{-3} \text{ cm s}^{-1}}$$

(ii) At 6:00 a.m. minute hand is at 12:00 mark and at 6:30 p.m. it is 180° away.

$\therefore$ displacement $= \text{R} + \text{R} = 2\text{R} = 2 \times 5 \text{ cm} = 10$ cm.

Time taken from 6:00 a.m. to 6:30 p.m. = 12 hours 30 minutes $= [12 \times 60 + 30] \times 60 = 45000$ s

$$\therefore \text{Average velocity} = \frac{10}{45000} = \mathbf{2{\cdot}2 \times 10^{-4} \text{ cm s}^{-1}}$$

16. *(i)* Distance travelled by car in 15 s = Area of ΔOAC

$$= \frac{1}{2} \times \text{OC} \times \text{AC} = \frac{1}{2} \times 15 \times 45 = 337{\cdot}5 \text{ m}$$

Distance travelled by scooter in 15 s

$=$ Area of rectangle OEFC

$= \text{OE} \times \text{OC} = 30 \times 15 = 450$ m

GRAPHICAL REPRESENTATION OF UNIFORM MOTION

The graphical representation of uniform motion is very helpful because it provides more physical insight than the mathematical relations. We shall study the following graphs :

1. Displacement-time ($x - t$) graph
2. Velocity-time ($v - t$) graph

While interpreting the graphs for uniform motion, the following points may be kept in mind :

(*i*) The displacement (x) and velocity (v) will be taken along the Y-axis and time (t) along the X-axis.

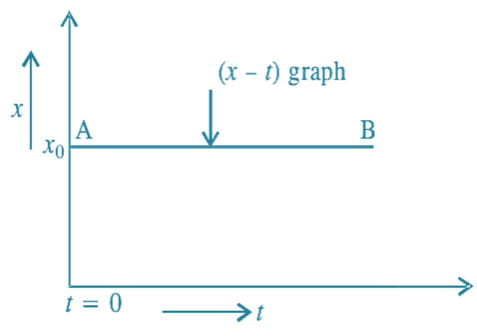
(*ii*) Positive velocity (or displacement) means that the motion of the object is to the right of the origin O. Negative velocity (or displacement) means that the motion is to the left of the origin O. *Therefore, if $x - t$ or $v - t$ graph is above X-axis (i.e., time axis), the velocity (v) is positive and if it is below X-axis, velocity is negative.*

(*iii*) If at $t = 0$, the initial displacement of the object from the origin is zero (*i.e.*, $x_0 = 0$), the $x - t$ graph will start from the origin of the graph. If at $t = 0$, the initial displacement is x_0, the $x - t$ graph will start at x_0.

(*iv*) Zero velocity of the object means the object is at rest.

POSITION-TIME GRAPHS

1. Position-time ($x - t$) graph for Stationary Object : *An object is said to be a stationary if its position does not change with the passage of time.* So, the object remains at constant distance $x = x_0$ from the origin at all times. Thus, **position time ($x - t$) graph for a stationary object is a straight line parallel to time axis.**



Such a graph described by a train standing at railway station or a bus standing on a road side.

2. If the object is moving uniformly along a straight line starting from origin O.

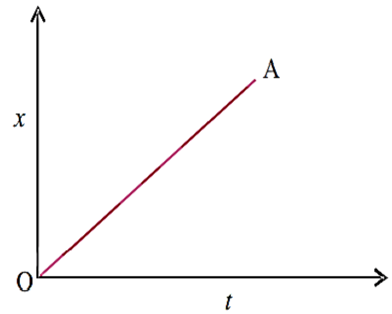


Fig. 27 (I)

When the object is in uniform motion along a straight line starting from origin, then position time graph is a straight line OA, inclined to time axis.

3. Position-time graph of uniform motion in one dimension for an object *(i)* object starts its journey from left of origin *(ii)* from origin *(iii)* from right of origin.

In these three situations, the $(x - t)$ graph is as shown in fig. 27 (II)

(i) Object starting from left of origin then graph is a straight line BC starts from point B

(ii) When object starts journey from origin, then $(x - t)$ graph is a straight line OA starts from origin O and inclined to time axis.

(iii) When the journey is from the right of origin then $(x - t)$ graph is a line DE which starts from point D.

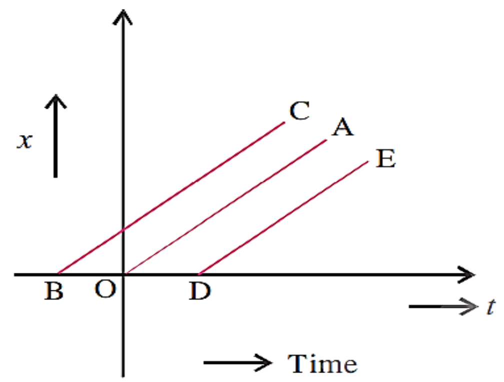


Fig. 27 (II)

4. If the object is moving with a constant negative velocity starting from the positive position :

When the object is moving with –ve velocity, then position-time graph for such motion is a straight line (FG) inclined to time axis as shown in figure.

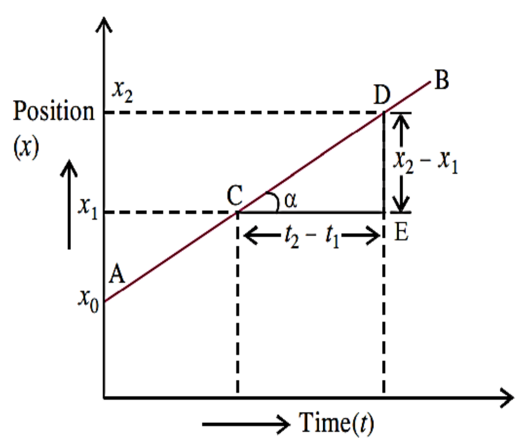
It is to be noted that position time graph cannot be a straight line parallel to position axis, as it will indicate infinite velocity.

POSITION-TIME GRAPH FOR UNIFORM MOTION

An *object in uniform motion covers equal displacements in equal intervals of time. So, position-time graph for an object in uniform motion is a straight line AB inclined to time axis.* Let at time $t = 0$, t_1 and t_2 the position co-ordinates of the object are x_0, x_1 and x_2 respectively.

Let C and D are two points on position-time graph corresponding to time t_1 and t_2.

Change in the displacement of the object in time $t_2 - t_1 = x_2 - x_1$



$$\therefore \text{ velocity of object } (\upsilon) = \frac{x_2 - x_1}{t_2 - t_1} = \frac{DE}{CE}$$

But $$\frac{DE}{CE} = \tan\alpha$$

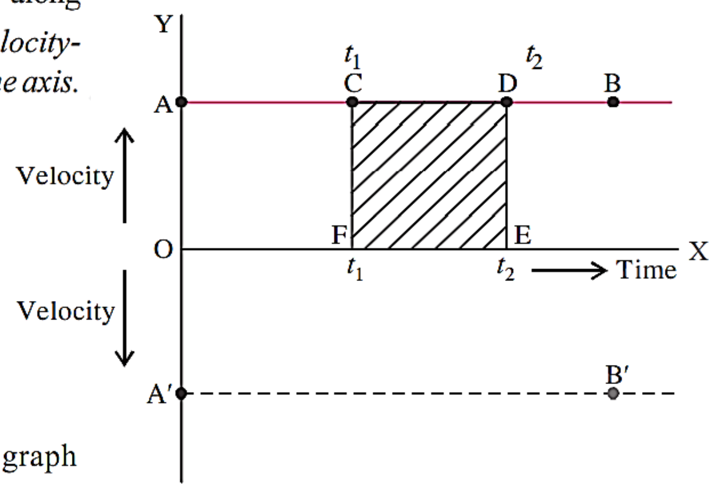
$\therefore$ velocity of object $= \tan\alpha$ (slope of position-time graph)

VELOCITY-TIME GRAPH FOR UNIFORM MOTION

When an object is moving uniformly it moves with uniform velocity (υ) in the same fixed direction. So, *velocity time graph for uniform motion is a straight line parallel to time axis.*

Let an object is moving with uniform velocity $\vec{\upsilon}$ along positive X-axis, then velocity of object is positive. So, *velocity-time graph of the object is a straight line AB parallel to time axis.*

The *velocity-time graph for uniform motion helps us to calculate the displacement of object in a given interval of time geometrically.*



But if the object is moving along negative X-axis, then again the ($\upsilon - t$) graph is a straight line parallel to time axis but lies below the time axis.

Let C and D be two points on velocity-time graph corresponds to time t_1 and t_2

$$\therefore \quad OA = FC = ED = \upsilon$$

$$OF = t_1, OE = t_2$$

Displacement of the object in the time interval $t_2 - t_1$ = Uniform velocity × Time interval $= \upsilon(t_2 - t_1)$ ... (1)

Area under velocity-time graph $= CF \times FE = \upsilon(t_2 - t_1)$ ... (2)

From eqns. (1) and (2)

Area under velocity-time graph = Displacement of the object in time interval $t_2 - t_1$.

Hence, *displacement of the object in time interval ($t_2 - t_1$) is numerically equal to area under velocity-time graph between the instants t_1 and t_2.*

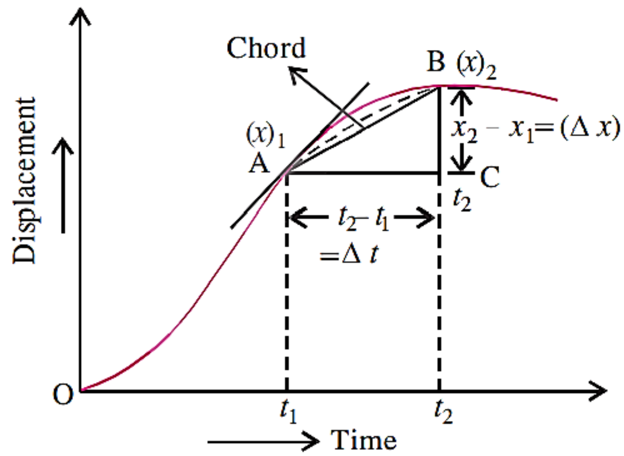
NON-UNIFORM MOTION

An object is said to be in non-uniform motion if it undergoes equal displacements in unequal intervals of time or unequal displacements in equal intervals of time.

Clearly in non-uniform motion, the velocity of the object is different at different points.

Calculation of average velocity and instantaneous velocity

(i) Average velocity



Let A and B are two points on position-time graph representing non-uniform motion of object. x_1 and x_2 be the position of the object corresponding to time t_1 and t_2.

Then BC $= x_2 - x_1 = \Delta x$

AC $= t_2 - t_1 = \Delta t$

Join points A and B so as to get chord AB.

Change in position of object in time $t_2 - t_1 = x_2 - x_1$

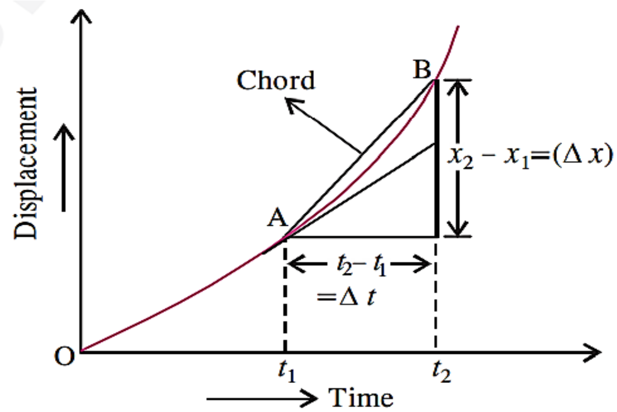
$$\therefore \text{ average velocity, } \upsilon_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{BC}{AC} \quad \text{...(1)}$$

$$\text{Slope of chord AB} = \frac{BC}{AC} = \frac{\Delta x}{\Delta t} \quad \text{...(2)}$$

From eqns. (1) and (2)

Average velocity (υ_{av}) of object = Slope of chord AB.

Hence, *the average velocity of the object between any two points is the slope of the chord (slope of x – t graph) corresponding to these two points on position-time graph.*



Instantaneous velocity : Let the point B approaches A (B $\to$ A) so $\Delta t \to 0$. Here, the chord AB after passing through successive stages (shown by dotted lines) become the tangent to the graph at A.

$\therefore$ the *instantaneous velocity of the object at any instant of time is the limiting value of average velocity when the time interval approaches zero.*

$$\underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta x}{\Delta t} = \underset{\Delta t \to 0}{\text{Lt}} \text{ (Slope of chord AB).}$$

$$= \text{Slope of the tangent at A} \quad \text{...(3)}$$

$$\text{But } \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \upsilon \quad \text{...(4)}$$

(instantaneous velocity of object at A)

From eqns. (3) and (4)

υ = Slope of tangent at A

So, *the instantaneous velocity at any point is equal to the slope of the tangent to the position-time graph at that time.*

1. If the motion of body is uniform, its instantaneous velocity at any instant of time is equal to the velocity of uniform motion. In other words, if the body possesses uniform velocity, both average and instantaneous velocities are equal.

2. If the motion of object is non-uniform, instantaneous velocity is a varying quantity which assumes different values at different instant of time.

3. The position-time graph of uniform motion is a straight line, its slope at all the points is same. Thus, in uniform motion the instantaneous velocity at all the points is same (*i.e.*, it does not depend on time.)

4. In uniform motion, the magnitude of the instantaneous velocity is equal to the instantaneous speed at the given instant.

5. A **speedometer** in an automobile measures the **instantaneous speed** and **odometer** measures the **distance covered** by the automobile.

6. Cheetah is the fastest land animal and can achieve a peak speed of 100 km/h is less than 500 m.

RELATIVE VELOCITY IN ONE DIMENSION

Velocity is not an absolute quantity but is measured relative to other objects. Thus to measure the velocity of an object, we specify the frame of reference. Ordinarily, we consider the surface of earth as our frame of reference and specify the velocity of body with respect to it (*i.e.*, earth). Thus, when we say that a car is travelling at *60 km/h, we mean to say it is going 60 km/h relative to the road or earth's surface.

When the two objects A and B are moving with different velocities, then *the velocity of one object A w.r.t. another object B is called as relative velocity of object A w.r.t. object B.*

The relative velocity of one object w.r.t. another object is the velocity with which one object moves w.r.t. another object. Hence, *relative velocity is the time rate of change of relative position of one object w.r.t. another object.*

Suppose the two cars A and B are travelling on a straight level road in the same direction as shown in Fig. 6.23. Let the uniform velocities of A and B be v_A and v_B respectively. We want to find the velocity of car A with respect to car B. Note that v_A is the velocity of car A *w.r.t.* earth. But we want to find v_{AB} – now our frame of reference is car B. If we superpose a velocity $-v_B$ on the two cars, then car B comes to rest (*i.e.*, car B and earth become identical frames of reference), then the velocity of car A *w.r.t.* car B is $v_{AB} = v_A - v_B$.

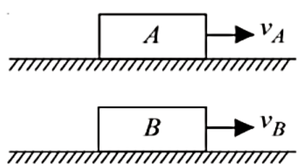


Fig. 6.23

Suppose two cars A and B having velocities v_A and v_B are moving along a straight road in the same direction. In order to find the relative velocity of A *w.r.t.* B, we add velocity $-v_B$ to both the cars. This effectively brings the car B to rest and the resultant velocity of car A becomes $v_A - v_B$.

$\therefore$ Relative velocity of car A *w.r.t.* car B is

$$v_{AB} = v_A - v_B$$

The direction of the relative velocity will depend upon the direction of $(v_A - v_B)$.

Note. $\quad v_{AB} = v_A - v_B$

But v_A and v_B are the velocities of the cars *w.r.t.* earth and can be written as v_{AE} and v_{BE} respectively.

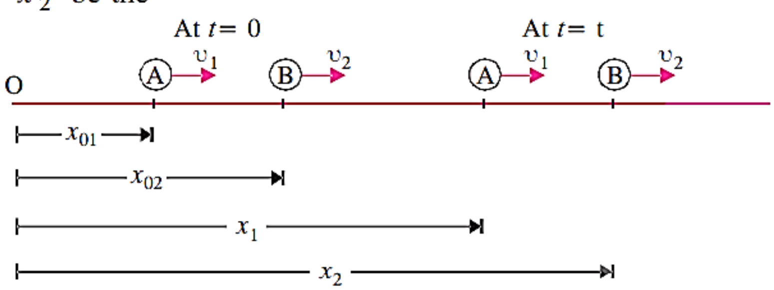
$\therefore \quad v_{AB} = v_{AE} - v_{BE}$

or $\quad v_{AB} = v_{AE} + v_{EB} \quad \ldots(i) \quad (\because v_{BE} = -v_{EB})$

Eq. (i) is a very useful relation. By writing the equation in this form, we see that inner subscripts on the right-hand of eq. (i) are the same (the two E's) and the outer subscripts on the right of eq. (i) (the A and B) are the same as the subscripts of the term v_{AB} on the left-hand side of eq. (i).

MATHEMATICAL TREATMENT OF RELATIVE VELOCITY

Expression of Relative Velocity : Let us consider two objects A and B moving with uniform velocities $\vec{\upsilon}_1$ and $\vec{\upsilon}_2$ along straight line track. At time $t = 0$, $\vec{x}_{01}$ and $\vec{x}_{02}$ be the displacements of objects from origin and $\vec{x}_1$ and $\vec{x}_2$ be the displacements of the objects at time $t = t$.



Positions of objects A and B after time t are given by

$$\vec{x}_1 = \vec{x}_{01} + \vec{\upsilon}_1 t \quad \ldots(1)$$

$$\vec{x}_2 = \vec{x}_{02} + \vec{\upsilon}_2 t \quad \ldots(2)$$

Subtracting eqn.(1) from eqn. (2)

$$\vec{x}_2 - \vec{x}_1 = \vec{x}_{02} - \vec{x}_{01} + \vec{\upsilon}_2 t - \vec{\upsilon}_1 t$$

$$\vec{x}_2 - \vec{x}_1 = \vec{x}_{02} - \vec{x}_{01} + \left(\vec{\upsilon}_2 - \vec{\upsilon}_1\right) t \quad \ldots(3)$$

Let $\vec{x}_{02} - \vec{x}_{01} = \vec{x}_0$, Relative displacement of object B w.r.t. object A at time $t = 0$.

$\vec{x}_2 - \vec{x}_1 = \vec{x}$, Relative displacement of object B w.r.t. object A at time $t = t$

$\therefore$ eqn. (3) becomes

$$\vec{x} = \vec{x}_0 + \left(\vec{\upsilon}_2 - \vec{\upsilon}_1\right) t$$

$$\Rightarrow \quad \vec{x} - \vec{x}_0 = \left(\vec{\upsilon}_2 - \vec{\upsilon}_1\right) t$$

or $$\vec{\upsilon}_2 - \vec{\upsilon}_1 = \frac{\vec{x} - \vec{x}_0}{t} \quad \ldots(4)$$

Equation *(4) gives the time rate of change of relative position of object B w.r.t. object A i.e. relative velocity of object B w.r.t. object A.*

Equation *(4) gives the time rate of change of relative position of object B w.r.t. object A i.e. relative velocity of object B w.r.t. object A.*

Hence, relative velocity of object B w.r.t. object A

$$\vec{\upsilon}_{BA} = \vec{\upsilon}_2 - \vec{\upsilon}_1$$

= Velocity of object B – Velocity of object A

Similarly, $$\vec{\upsilon}_{AB} = \vec{\upsilon}_1 - \vec{\upsilon}_2 \quad \ldots(5)$$

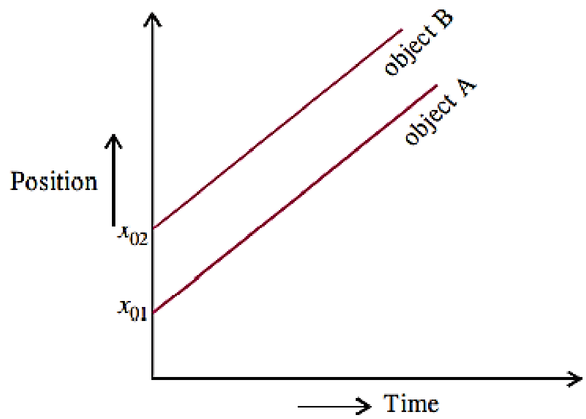
= Velocity of object A – Velocity of object B.

If the objects are in one dimensional motion along same direction, then magnitude of displacement is equal to the total distance travelled by object in given time then arrow heads can be ignored in eqn. (4)

$$\therefore \quad \upsilon_2 - \upsilon_1 = \frac{x - x_0}{t} = \upsilon_{BA} \quad \ldots(6)$$

POSITION-TIME GRAPH OF RELATIVE VELOCITY

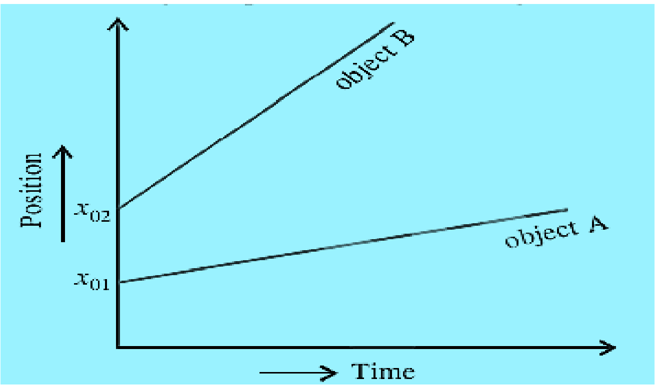
(i) **If two objects are moving with same velocities :** Here $\upsilon_1 = \upsilon_2$ or $\upsilon_2 - \upsilon_1 = 0$, then from eqn. (4) $x - x_0 = 0$ or $x = x_0$.



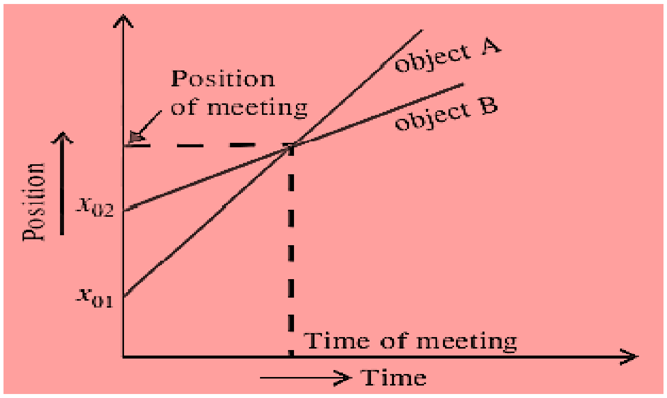
So, the two objects remain at constant distance apart, which is equal to the relative separation between two objects initially *i.e.* at $t = 0$. Thus, the position-time graphs are parallel straight lines.

(ii) **If two objects move with unequal velocities *i.e.*, $\upsilon_1 \neq \upsilon_2$**

In this situation, two subcases arise.



(a) If $\upsilon_2 > \upsilon_1$, then $\upsilon_2 - \upsilon_1$ = positive, so from equation (4), $x - x_0$ = positive *i.e.*, the relative separation between two objects goes on increasing with time. So, the objects will never meet with each other and their position-time graphs will open out gradually as shown in figure 35.



(b) If $\upsilon_1 > \upsilon_2$, then $\upsilon_2 - \upsilon_1$ = negative so from eqn. (4), $x - x_0$ = negative *i.e.*, the relative separation between two objects goes on decreasing with time. After some time the two objects meet each other and then the object moving faster (object A) move more and more away (left) from the object moving slower (object B) as shown in fig. 36. In the position-time graphs for these objects the time co-ordinate corresponding to point of intersection gives their time of meeting and corresponding position co-ordinate gives position of meeting.

(iii) **When two objects are moving in opposite direction**

Let object A is moving with velocity υ_1 along positive X-axis and object B is moving with velocity υ_2 along negative X-axis thus υ_1 is positive and υ_2 is negative.

$$\therefore \quad \upsilon_{AB} = \upsilon_1 - \upsilon_2 \quad (\upsilon_2 \text{ is } -\text{ve})$$

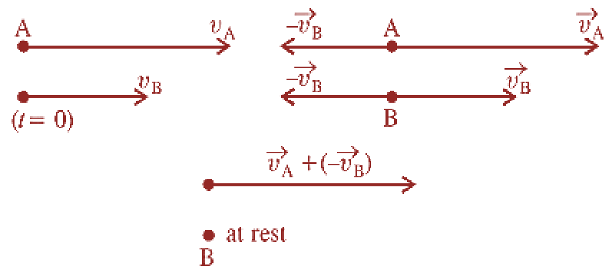
$$\upsilon_{AB} = \upsilon_1 - (-\upsilon_2) = \upsilon_1 + \upsilon_2$$

$$\text{if } \upsilon_1 = \upsilon_2 = \upsilon \text{ then } \upsilon_{AB} = \upsilon + \upsilon = 2\upsilon$$

$$\upsilon_{AB} = 2\upsilon$$

DETERMINATION OF RELATIVE VELOCITIES

***(i)* When the two objects are moving along parallel straight lines in the same direction *i.e.* Angle between them is 0° :**



Let objects A and B are moving towards the right. At time t = 0 both the objects are just parallel and in front to each other.

Object A is moving with velocity $\vec{\upsilon}_A$ and B with $\vec{\upsilon}_B$. To calculate relative velocity of object A w.r.t. object B superimpose $-\vec{\upsilon}_B$ on objects A and B. Due to this velocity of object B becomes zero. *i.e.* object B is brought to rest and relative velocity of object A becomes $\vec{\upsilon}_A + (-\vec{\upsilon}_B) = \vec{\upsilon}_A - \vec{\upsilon}_B$

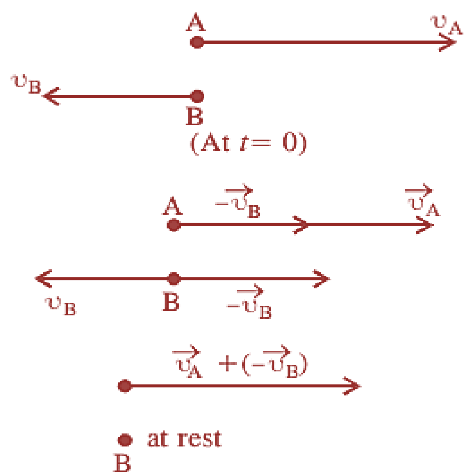
Hence, relative velocity of object A w.r.t. object B is given by

$$\vec{\upsilon}_{AB} = \vec{\upsilon}_A - \vec{\upsilon}_B$$

As $\vec{\upsilon}_A$, $\vec{\upsilon}_B$ and $\vec{\upsilon}_{AB}$ are all along same direction, so

$$\upsilon_{AB} = \upsilon_A - \upsilon_B$$

***(ii)* When the two objects are moving along parallel straight lines in opposite directions :**



Here, object A is moving to the right and B to the left so angle between them is 180°. $\vec{\upsilon}_A$ and $\vec{\upsilon}_B$ be the velocities of two objects. To find relative velocity of object A w.r.t. object B superimpose $-\vec{\upsilon}_B$ on objects A and B. Due to this, the velocity of object B becomes zero *i.e.* object B comes to rest and velocity of object A becomes $\vec{\upsilon}_A + (-\vec{\upsilon}_B) = \vec{\upsilon}_A - \vec{\upsilon}_B$

So, the relative velocity of object A w.r.t. object B is given by

$$\vec{\upsilon}_{AB} = \vec{\upsilon}_A - \vec{\upsilon}_B$$

As $\vec{\upsilon}_B$ (direction of $\vec{\upsilon}_B$) is opposite to $\vec{\upsilon}_A$. then the magnitude of $\vec{\upsilon}_{AB}$ will be

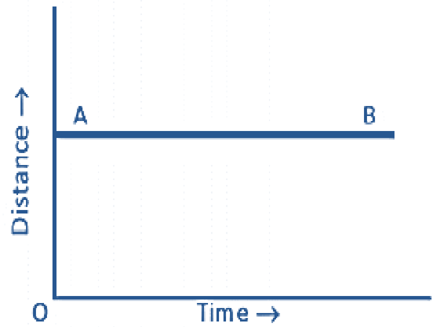
$$\upsilon_{AB} = \upsilon_A + \upsilon_B$$

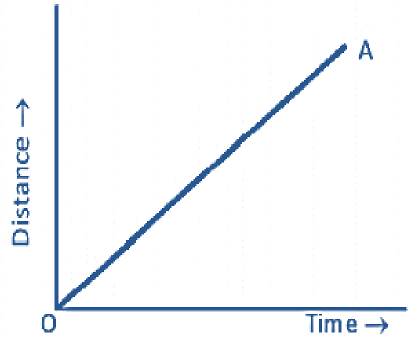
ANALYSING NATURE OF MOTION FROM VARIOUS GRAPHS

Different types of distance-time graphs:

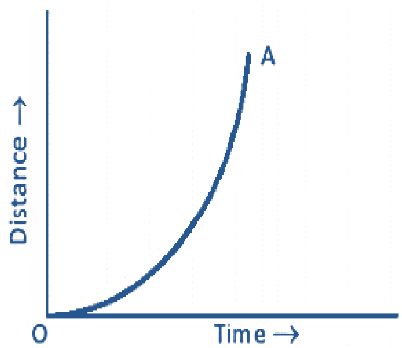
(i) For a body at rest, the distance-time graph is a straight line AB, as shown in Fig. As the slope of AB is zero, so speed of the body is zero.

(ii) For a body moving with uniform speed, the distance-time graph is a straight line inclined to the time axis, as shown in Fig. As the graph passes through O, so distance travelled at t = 0 is also zero.

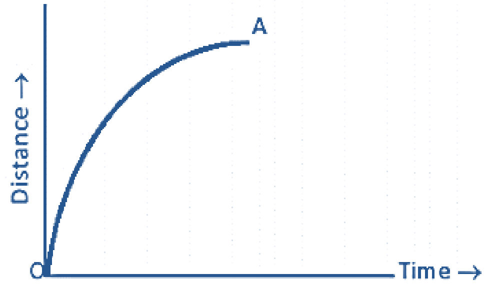




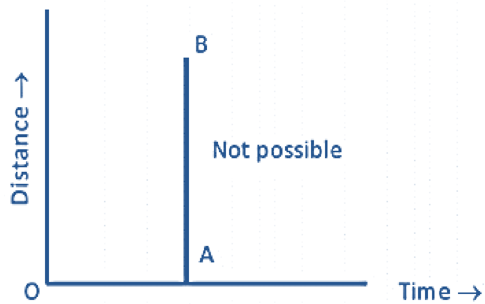
(iii) The distance-time graph in Fig. represents accelerated (speeding up) motion, because the slope of the graph is increasing with time.



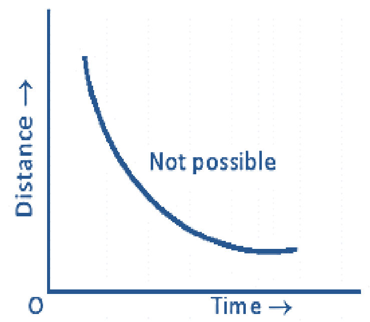
(iv) the distance-time graph in Fig. represents decelerated (speeding down) motion, because slope of the graph is decreasing with time.



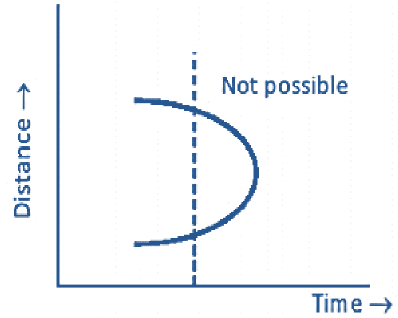
(v) In Fig. distance-time graph is a straight line parallel to distance axis. It represents infinite speed which is not possible.



(vi) The distance covered by a body cannot decrease with the increase of time. So the distance-time graph of the type shown in Fig., is not possible.

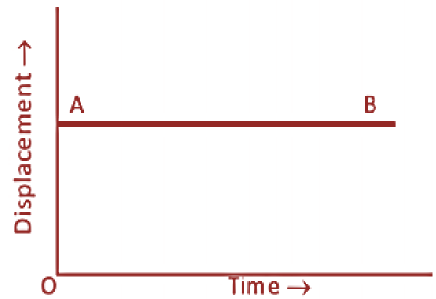


(vii) The distance-time graph shown in Fig., is not possible because it represents two different positions of the body at the same instant which is not possible.

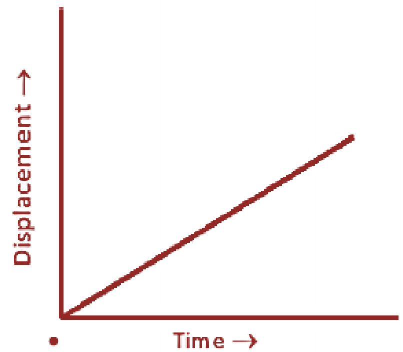


Different types of displacement-time graphs:

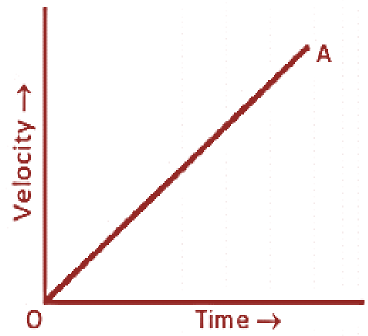
(i) For a stationary body, the displacement-time graph is a straight line AB parallel to time-axis. The zero slope of line AB indicates zero velocity.



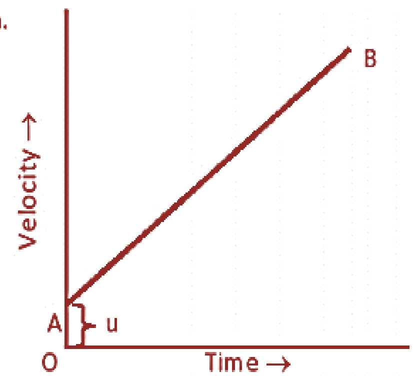
(ii) In Fig. the displacement-time graph is a straight line OA inclined to time axis. It has a single slope. So it represents a constant velocity and hence zero acceleration.



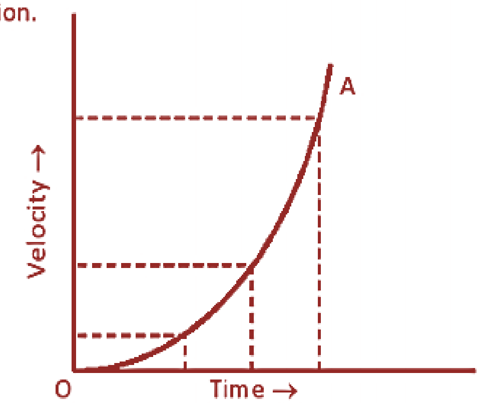
(ii) When a body starts from rest and moves with uniform acceleration, its v – t graph is straight line OA inclined to the time-axis and passing through the origin O. Greater is the slope of the v – t graph, greater will be the acceleration.



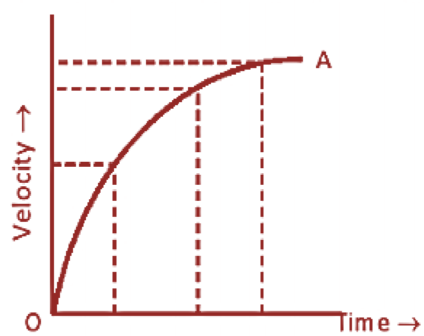
(iii) In Fig. the straight line v – t graph does not pass through origin O. The body has an initial velocity u (= OA) and then it moves with a uniform acceleration.



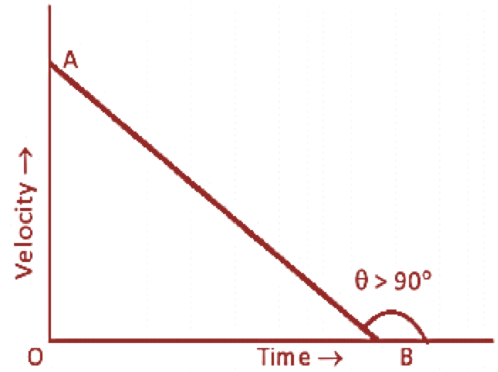
(iv) In Fig., greater changes in velocity are taking place in equal intervals of time. So the v – t graph bending upwards represents an increasing acceleration.



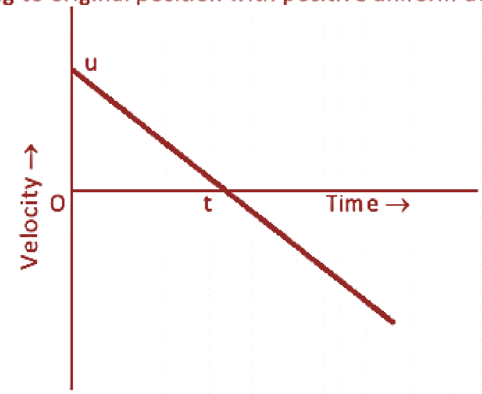
(v) In Fig., smaller changes in velocity are taking place in equal intervals of time. So the v – t graph bending downwards represents a decreasing acceleration.



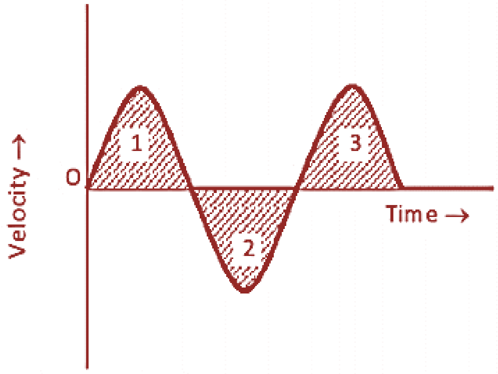
(vi) In Fig. the body starts with an initial velocity u. The velocity decreases uniformly with time, becoming zero after some time. As $\theta > 90°$, the graph has a negative slope. The v – t graph represents uniform negative acceleration.



(vii) In Fig., the v – t graph represents a body projected upwards with an initial velocity u. The velocity decreases 21
with time (negative uniform acceleration), becoming zero after certain time t. Then the velocity becomes negative and increases in magnitude, showing body is returning to original position with positive uniform acceleration.



(viii) The area between the velocity-time graph and the time-axis gives the displacement. In Fig. the v – t graph represents variable acceleration.

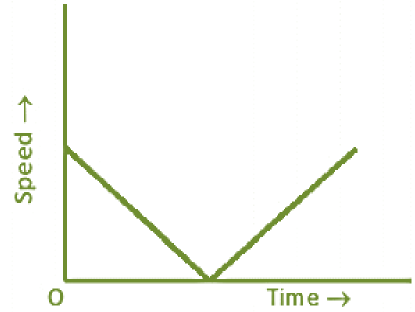


Displacement covered = Area 1 – Area 2 + Area 3

Distance covered = Area 1 + Area 2 + Area 3

Different types of speed-time graphs:

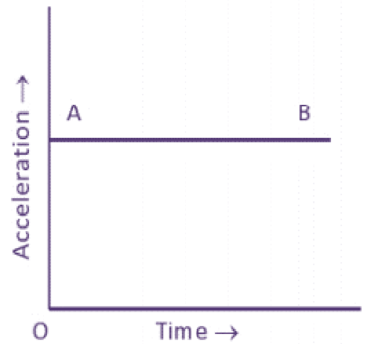
(i) For a body projected upwards, the speed-time graph is of the type shown in Fig. When the body moves, its speed decreases uniformly, becoming zero at the highest point. As the body moves down, its speed increases uniformly. It returns with the same speed with which it was thrown up.



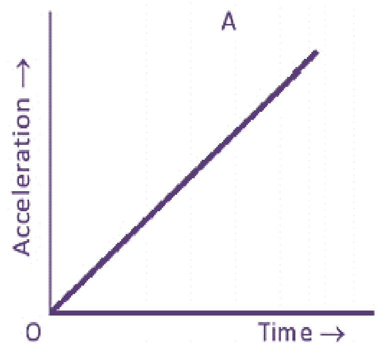
(ii) For a ball dropped on the ground from a certain height, the speed-time graph is of the type shown in Fig. As the ball falls, its speed increased. As the ball bounces back, its speed decreases uniformly and becomes zero at the highest point.

Different types of acceleration-time graphs:

(i) For a body moving with constant acceleration, the acceleration-time graph is a straight line AB parallel to the time-axis, as shown in Fig.

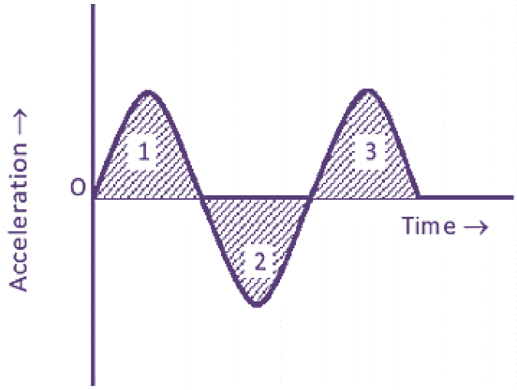


(ii) When the acceleration of a body increases uniformly with time, its a – t graph is a straight line OA inclined to the time-axis.



(iii) For a body moving with variable acceleration, the a – t graph is a curve. The area between the a – t graph and the time-axis gives the change in velocity, as shown in Fig.

Change in velocity = Area 1 – Area 2 + Area 3



Quick Review of Chapter

- Mechanics is the branch of Physics which deals with the study of the motion of the objects.
- Frame of reference is a fixed point or place w.r.t. which the position of the object is measured.
- Frames of references are of two types : inertial and non-inertial frame of reference.
- An inertial frame of reference is the frame of reference which is either at rest or moving with uniform velocity.
- An accelerated frame of reference is non-inertial frame of reference.
- Distance is the length of actual path travelled by a body.
- Displacement is the shortest distance travelled by the body between initial and final positions.
- Speedometer measures the instantaneous speed of the vehicle.
- Odometer measures the distance travelled by vehicle.
- Distance is a scalar while displacement is a vector quantity.
- If the motion of the object or body is straight line or in one direction, then distance travelled by object = Displacement of the object.
- Distance travelled by any object in given time interval > the displacement if the object changes its direction of motion.
- Distance ≥ Displacement.
- Speed of Body $= \dfrac{\text{Distance}}{\text{Time}}$
- Speed of body is always positive.
- Average speed of a body travelling distance S_1 and S_2 with velocity υ_1 and υ_2.

$$\upsilon_{av} = \frac{S_1 + S_2}{\left(\dfrac{S_1}{\upsilon_1} + \dfrac{S_2}{\upsilon_2}\right)}$$

- Newtonian or classical mechanics deals with the study of the motion of objects with speeds less than speed of light.
- Average speed of a body travelling equal distances with different speeds υ_1 and υ_2 is

$$\upsilon_{av} = \frac{2\upsilon_1\upsilon_2}{\upsilon_1 + \upsilon_2}$$

- Average speed of a body travelling equal distances with different speeds υ_1, υ_2 and υ_3 respectively

$$\upsilon_{av} = \frac{3\upsilon_1\upsilon_2\upsilon_3}{\upsilon_1\upsilon_2 + \upsilon_2\upsilon_3 + \upsilon_1\upsilon_3}$$

Velocity :

- Velocity $= \dfrac{\text{Displacement}}{\text{Time}}$.
- Velocity is a vector quantity.
- Slope of the displacement-time graph in uniform motion = uniform velocity of body.
- Speed of a body is maximum at a point where the slope of distance-time graph is maximum.
- Velocity of a body is maximum at a point where the slope of displacement-time graph is maximum.
- Displacement of a body moving with uniform velocity = Area under velocity-time graph.
- Average speed = Average velocity of body if the body moves along one direction.
- Average velocity of body < Average speed of body if the direction of motion of body changes.
- Average velocity of body = Instantaneous velocity of body if the body moves with uniform/constant velocity
- SI unit of speed and velocity are same *i.e.* m s^{-1}.

Examples Based on Relative Velocity

Formulae used

(i) Relative velocity of object A w.r.t. object B

$$\upsilon_{AB} = \upsilon_A - \upsilon_B$$

(ii) Relative velocity of object B w.r.t. object A

$$\upsilon_{BA} = \upsilon_B - \upsilon_A$$

Sign conventions used

(i) If the objects A and B are moving along positive X-axis then both υ_A and υ_B are taken as positive.

(ii) If the objects A and B are moving along negative X-axis then υ_A and υ_B are taken as negative.

UNITS USED

All velocities are in ms^{-1} or kmh^{-1}

Q. 1. ***A car A moving at 10 ms^{-1} on a straight road, is ahead of car B moving in the same direction at 6 ms^{-1}. Find the velocity of A relative to B and vice versa.***

Sol. Here $v_A = 10\ ms^{-1}$, $v_B = 6\ ms^{-1}$

Velocity of A relative to B,

$$v_{AB} = v_A - v_B = 10 - 6 = 4\ ms^{-1}$$

Positive velocity indicates that the driver or car B sees car A moving ahead from him at the rate of 4 ms^{-1}.

Velocity of B relative to A

$$v_{BA} = v_B - v_A = 6 - 10 = -4\ ms^{-1}$$

Negative velocity indicates that the driver of car A (when looks back) sees the car B lagging behind at the rate of 4 ms^{-1}.

Q. 2. ***A jet airplane travelling at the speed of 500 kmh^{-1} ejects its products of combustion at the speed of 1500 kmh^{-1} relative to the jet plane. What is the speed of the latter with respect to an observer on the ground?***

Sol. Here speed of jet airplane, $v_1 = 500\ kmh^{-1}$

Let v_2 be the speed of products w.r.t. the ground. Suppose the direction of motion of the jet plane is positive. Then the relative velocity of products w.r.t. jet plane is

$$v_2 - v_1 = -1500$$

or $$v_2 = v_1 - 1500 = 500 - 1500 = -1000\ kmh^{-1}$$

Negative sign shows that the direction of products of combustion is opposite to that of the jet plane.

$\therefore$ Speed of products of combustion w.r.t. ground = 1000 kmh^{-1}

Q. 3. ***A police van moving on a highway with a speed of 30 kmh^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 kmh^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car?***

Sol. Speed of police van,

$$v_p = 30\ kmh^{-1} = \frac{25}{3}\ ms^{-1}$$

Speed of bullet, $v_b = 150\ ms^{-1}$

Speed of the police van is shared by the bullet.

$\therefore$ Relative speed of bullet w.r.t. ground

$$= v_b + v_p = 150 + \frac{25}{3} = \frac{475}{3}\ ms^{-1}$$

Speed of thief's car,

$$v_t = 192\ kmh^{-1} = \frac{160}{3}\ ms^{-1}$$

Relative speed of bullet w.r.t. thief's car

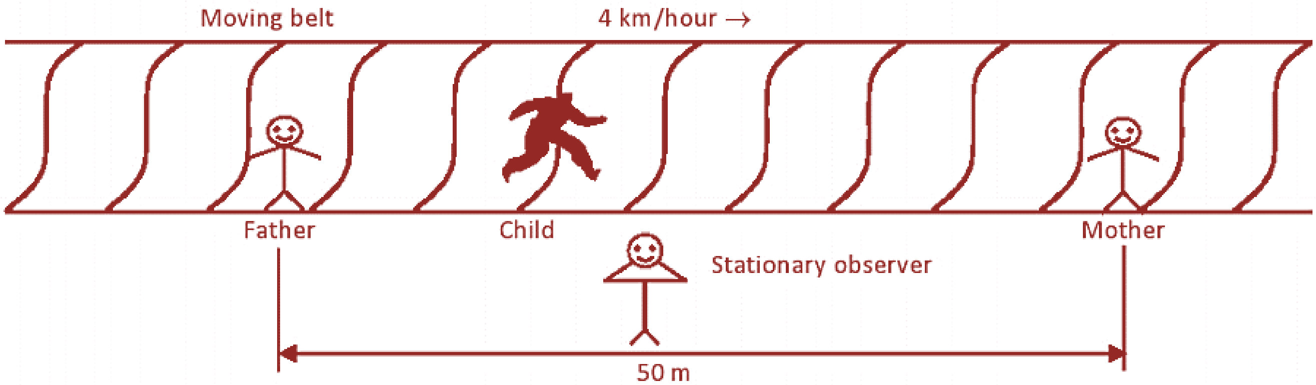
$$= (v_b + v_p) - v_t = \frac{475}{3} - \frac{160}{3} = 105\ ms^{-1}$$

Hence the speed of the bullet with which it hits the thief's car = 105 ms^{-1}

Q. 4. ***On a long horizontally moving belt, a child runs to and fro with a speed 9 kmh^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 kmh^{-1}. For an observer on a stationary platform outside, what is the***
(i) Speed of the child running in the direction of motion of the belt,
(ii) Speed of the child running opposite to the direction of motion of the belt, and
(iii) Time taken by the child in (i) and (ii)?
Which of the answers alter if motion is viewed by one of the parents?

Sol. (i) Speed of the child running in the direction of motion of the belt

$= (9 + 4)\ \text{kmh}^{-1} = 13\ \text{kmh}^{-1}$



(ii) Speed of the child running opposite to the direction of motion of the belt

$= (9 - 4)\ \text{kmh}^{-1} = 5\ \text{kmh}^{-1}$

(iii) Speed of the child w.r.t. either parent

$= 9\ \text{kmh}^{-1} = 2.5\ \text{ms}^{-1}$

Distance to be covered = 50 m

Time taken $= \dfrac{50}{2.5} = 20$ s

If the motion is viewed by one of the parents, answers to (i) and (ii) are altered but answer to (iii) remains unchanged.

Q. 5. ***Two parallel rail tracks run north south. Train A moves north with a speed of 54 kmh^{-1} and train B moves south with a speed of 90 kmh^{-1}. What is the***
(i) relative velocity of B with respect to A?
(ii) relative velocity of ground with respect to B?
(iii) velocity of a monkey running on the roof of the train .
Against its motion (with a velocity of 18 kmh^{-1} with respect to the train A) as observed by a man standing on the ground?

Sol. Taking south to north direction as the positive direction of x-axis, we have

$v_A = +54\ \text{km/h} = 54 \times 1000\ \text{ms}^{-1} = 15\ \text{ms}^{-1}$

$v_B = -90\ \text{km/h} = \dfrac{-90 \times 1000}{3600}\ \text{ms}^{-1} = -25\ \text{ms}^{-1}$

(i) Relative velocity of B with respect to A

$= v_B - v_A = -25 - 15 = -40\ \text{ms}^{-1}$

So to an observer in train A, the train B appears to move with a speed of 40 ms^{-1} from north to south.

(ii) Relative velocity of ground with respect to B.

$= 0 - v_B = 0 + 25 = 25\ \text{ms}^{-1}$

Q. 6. ***Two trains 120 m and 80 m in length are running in opposite directions with velocities 42 kmh^{-1} and 30 kmh^{-1}. In what time they will completely cross each other?***

Sol. Relative velocity of one train w.r.t. the other

$= 42 - (-30) = 72\ \text{kmh}^{-1} = 20\ \text{ms}^{-1}$

Total distance to be travelled by each train to cross other train

$= 120 + 80 = 200$ m

Time taken by each train to cross other train

$= \dfrac{200}{20} = 10$ s

Q. 7. ***Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20 kmh^{-1} in the direction A to B notices that a bus goes past him every 180 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road?***

Sol. Let speed of each bus = v kmh^{-1}

For buses going from town A to B

Relative speed of a bus in the direction of motion of the man = (v – 20) kmh^{-1}

Buses plying in this direction go past the cyclist after every 18 min.

$\therefore$ Distance covered = $(v-20)\frac{18}{60}$ km

Since a bus leaves the town after every T min, so the above distance covered

$= v \times \frac{T}{60}$ km

$\therefore$ $(v-20)\frac{18}{60} = v \times \frac{T}{60}$ ... (i)

For buses going from town B to A

Relative speed of bus in the direction opposite to the motion of the man

= (v + 20) kmh^{-1}

Buses going in this direction go past the cyclist after every 6 min, therefore

$(v+20)\frac{6}{60} = v \times \frac{T}{60}$

Dividing (i) by (ii), we get

$\frac{(v-20)\,18}{(v+20\,v)\,6} = 1$ or $3v - 60 = v + 20$

$v = 40$ kmh^{-1}

From equation (ii), $(40+20)\frac{6}{60} = \frac{40 \times T}{60}$

or $T = \frac{60 \times 6}{40} = 9$ min

Q. 8. ***The speed of a motor launch with respect to still water is = 7 ms^{-1} and the speed of stream is u = 3 ms^{-1}. When the launch began travelling upstream, a float was dropped from it. The launch travelled 4.2 km upstream, turned about the caught up with the float. How long is it before the launch reaches the float?***

Sol. For upstream motion of launch

Relative velocity = 7 – 3 = 4 ms^{-1}

Distance moved = 4.2 km = 4200 m

Time taken, $t_1 = \frac{4200}{4} = 1050$ s

For downstream motion of launch

Distance moves downstream by float in 1050 s

$= 3 \times 1050 = 3150$ m

Distance between float and launch turned about

$= 4200 + 3150 = 7350$ m

This distance is to be covered by launch with its own velocity (7 ms^{-1}) because stream velocity is being shared by both.

$\therefore$ Time taken, $t_2 = \frac{7350}{7} = 1050$ s

Total time taken, $t = t_1 + t_2 = 1050 + 1050 = 2100$ s = 35 min.

Q. 9. ***Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 kmh^{-1} in the same direction, with A ahead of B. The driver of B decides to overtaken A and accelerates by 1 ms^{-2}. If after 5 s, the guard of B just brushes past the driver of A, what was the original distance between them?***

Sol. Let x be the distance between the driver of train A and the guard of train B. Initially, both trains are moving 26
in the same direction with the same speed of 72 kmh^{-1}. So relative velocity of B w.r.t. A = $v_B - v_A = 0$. Hence the train B needs to cover a distance with

$$a = 1\ ms^{-2},\ t = 50\ s,\ u = 0$$

As $s = ut + ½\ at^2$

∴ $x = 0 \times 50 + ½ \times 1 \times (50)^2 = 1250$ m

Q. 10. ***On a two-lane road, car A is travelling with a speed of 36 kmh^{-1}. Two cars B and C approach car A in opposite directions with a speed of 54 kmh^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?***

Sol. At the instant when B decides to overtake A, the speeds of three cars are

$$v_A = 36\ kmh^{-1} = 36 \times \frac{5}{18} = +10\ ms^{-1}$$

$$v_B = +54\ kmh^{-1} = +54 \times \frac{5}{18} = +15\ ms^{-1}$$

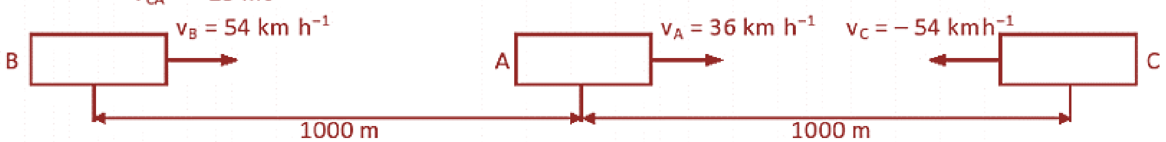
$$v_C = -54\ kmh^{-1} = -15\ ms^{-1}$$

Relative velocity of C w.r.t. A,

$$v_{CA} = v_C - v_A = -15 - 10 = -25\ ms^{-1}$$

∴ Time that C requires to just cross A

$$= \frac{1\ km}{v_{CA}} = \frac{1000\ m}{25\ ms^{-1}} = 40\ s$$



In order to avoid the accident, B must overtake A in a time less than 40 s. So, far train B we have

Relative velocity of train B w.r.t. A,

$$v_{BA} = v_B - v_A = 15 - 10 = 5\ ms^{-1}$$

∴ $s = 1$ km $= 1000$ m, $u = 5\ ms^{-1}$, $t = 40$ s

As $s = ut + ½\ at^2$ ∴ $1000 = 5 \times 40 + ½\ a \times (40)^2$

or $1000 = 200 + 800\ a$ or $a = 1\ ms^{-2}$

Thus 1 ms^{-2} is the minimum acceleration that car B requires to avoid an accident.

PROBLEMS FOR PRACTICE

Q. 1. ***A car A is moving with a speed of 60 kmh^{-1} and car B is moving with a speed of 75 kmh^{-1}, along parallel straight paths, starting from the same point. What is the position of car A w.r.t. B after 20 minutes?***

Sol. Relative speed of A w.r.t. B = 60 – 75 = – 15 kmh^{-1}

Distance of A and B after 20 min = $-15 \times \frac{20}{60} = -5$ km

Q. 2. ***Two buses start simultaneously towards each other from towns A and B which are 480 km apart. The first bus takes 8 hours to travel from A and B while the second bus takes 12 hours to travel from B to A. Determine when and where the buses will meet.***

Sol. Speed of first bus = $\frac{480}{8} = 60\ kmh^{-1}$

Speed of second bus = $\frac{480}{12} = 40\ kmh^{-1}$

Suppose the two buses meet after time t. Then

$$60\,t + 40\,t = 480 \quad \text{or} \quad t = 4.8\ h$$

Distance from A = 60 × 4.8 = 288 km

Q. 3. ***Two trains A and B, each of length 100 m, are running on parallel tracks. One overtakes the other in 20 s and one crosses the other in 10 s. Calculate the velocities of each train.***

Sol. Let u and v be the velocities of trains A and B respectively.

During overtaking, relative velocity of A w.r.t. B = u – v

During crossing, relative velocity of A w.r.t. B = u + v

Total distance to be covered by A during overtaking or crossing

= 100 + 100 = 200 m

$\therefore \quad \frac{200}{u-v} = 20 \quad \text{and} \quad \frac{200}{u-v} = 0$

or $\quad u - v = 10 \quad \text{and} \quad u + v = 20$

On solving, $u = 15\ ms^{-1}$, $v = 5\ ms^{-1}$

Q. 4. ***A man swims in a river with and against water at the rate of 15 kmh⁻¹ and 5 kmh⁻¹. Find the man's speed in still water and the speed of the river.***

Sol. Let u be the speed of man in still water and v be the speed of river. Then

$u + v = 15\ kmh^{-1}$ and $u - v = 5\ kmh^{-1}$

On solving, $u = 10\ kmh^{-1}$, $v = 5\ kmh^{-1}$

Q. 5. ***A motorboat covers the distance between the two spots on the river in 8 h and 12 h downstream and upstream respectively. Find the time required by the boat to cover this distance in still water.***

Sol. Let u and v be the velocity of boat in still water and velocity of river respectively. If x is the distance between the two spots, then

$u + v = \frac{x}{8}$ (for upstream)

$u - v = \frac{x}{12}$ (for downward)

On adding, $2u = \frac{20}{96}x$ or $u = \frac{10}{96}x$

Time required by boat in still water

$= \frac{x}{v} = \frac{x}{10x/96} = 9.6\ h$

Q. 6. ***A car A is travelling on a straight level road with a speed of 60 kmh⁻¹. It is followed by another car B which is moving with a speed of 70 kmh⁻¹. When the distance between them is 2.5 km, the car B is given a deceleration of 20 kmh⁻². After what distance and time will the car B catch up with car A?***

Sol. Relative velocity of car B w.r.t. A

$= 70 - 60 = 10\ kmh^{-1}$

$\therefore$ For car B, $u = 10\ kmh^{-1}$, $s = 2.5\ km$,

$a = -20\ kmh^{-1}$

As $\quad s = ut + \frac{1}{2}at^2$

$\therefore 2.5 = 10t - \frac{1}{2} \times 20 \times t^2 \quad$ or $\quad t = 0.5\ h$

Actual distance travelled by car B during this time,

$s = ut + \frac{1}{2}at^2 = 70 \times 0.5 - \frac{1}{2} \times 20 \times (0.5)^2$

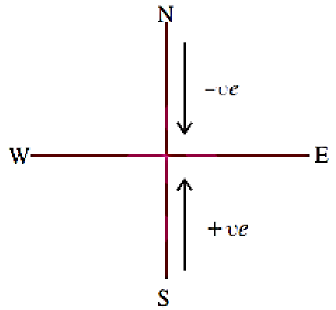
$= 35 - 2.5 = 32.5\ km$

EXAMPLE 17. **Two parallel rail tracks run north-south. Train A moves north with a speed of 54 km h^{-1} and train B moves south with a speed 90 km h^{-1}. Calculate**

(a) Relative velocity of B w.r.t. A

(b) Relative velocity of ground w.r.t. train B.

(c) Velocity of a monkey running on the roof of train A against the motion with a velocity of 18 km h^{-1} w.r.t. the train A as observed by man standing on ground ?



Sol. Let the motion from south to north is taken as positive, then

$$\upsilon_A = +54 \text{ km h}^{-1} = \frac{54 \times 1000}{3600} = 15 \text{ m s}^{-1}$$

$$\upsilon_B = -90 \text{ km h}^{-1} = -\frac{90 \times 1000}{3600} = -25 \text{ m s}^{-1}$$

(a) Relative velocity of train B w.r.t. train A

$$\upsilon_{BA} = \upsilon_B - \upsilon_A = -25 - 15 = \mathbf{-40 \text{ m s}^{-1}}$$

Hence, train B appears to move with speed of 40 m s^{-1} from north to south.

(b) Ground is at rest so $\upsilon_g = 0$

$\therefore$ relative velocity of ground *w.r.t.* train B

$$\upsilon_{gB} = \upsilon_g - \upsilon_B = 0 - (-25) = \mathbf{25 \text{ m s}^{-1}} \text{ due north}$$

(c) Let velocity of monkey w.r.t. ground be υ_m.

$\therefore$ relative velocity of monkey w.r.t. train A

$$= \upsilon_m - \upsilon_A = -18 \text{ km h}^{-1}$$

$$= \frac{-18 \times 1000}{3600} = -5 \text{ m s}^{-1}$$

$$\therefore \quad \upsilon_m = \upsilon_A - 5$$

$$\upsilon_m = 15 - 5 = \mathbf{10 \text{ m s}^{-1}} \text{ due north.}$$

EXAMPLE 18. **A car travelling with a speed of 90 km h^{-1} on a straight road is ahead of a scooter travelling with a speed of 50 km h^{-1}. How would the relative velocity be altered, if scooter is ahead of car ?**

Solution. Let the velocities of car and scooter are υ_c and υ_s respectively.

$$\upsilon_c = 90 \text{ km h}^{-1}, \upsilon_s = 50 \text{ km h}^{-1}$$

Relative velocity of car w.r.t. scooter

$$\upsilon_{cs} = \upsilon_c - \upsilon_s$$

Case I. When car is ahead of scooter

$\upsilon_{cs} = 90 - 50 = \mathbf{40 \text{ km h}^{-1}}$ (away from scooter)

Case II. When the scooter is ahead of car

$\upsilon_{cs} = 90 - 50 = \mathbf{40 \text{ km h}^{-1}}$ (towards the scooter)

EXAMPLE 19. **How long will a boy sitting near the window of a train travelling 54 km h^{-1} saw a train passing by in the opposite direction with a speed of 36 km h^{-1}. The length of slow moving train being 100 m.**

Solution. Velocities of the trains are respectively 54 km h^{-1} and 36 km h^{-1}

$\therefore$ relative velocity of the slow moving train w.r.t. boy

$$= (54 + 36) \text{ km h}^{-1} = 90 \text{ km h}^{-1} = 25 \text{ m s}^{-1}$$

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}} \Rightarrow \text{Time} = \frac{100}{\text{velocity}} = \frac{100}{25}$$

$$\Rightarrow \quad t = \mathbf{4 \text{ s}}$$

EXAMPLE 20. **A train 100 m long is travelling at 60 km h^{-1}. In what time it will cross a cyclist moving at 6 km h^{-1} (a) in the same direction (b) in the opposite direction.**

Solution. Given velocity of train, $\upsilon_t = 60$ km h^{-1}

Velocity of cyclist, $\upsilon_c = 6$ km h^{-1}

Case I. Both train and cyclist move in same direction

Relative velocity of train w.r.t. cyclist

$$\upsilon_{tc} = \upsilon_t - \upsilon_c$$

$$\upsilon_{tc} = 60 - 6 = 54 \text{ km h}^{-1} = 15 \text{ m s}^{-1}$$

$$\upsilon = \frac{x}{t}$$

$$15 = \frac{100}{t} \quad \Rightarrow \quad t = \frac{100}{15} = 6{\cdot}67 \text{ s.}$$

Relative velocity of train w.r.t. cyclist (when both moving away from each other), $\upsilon_{tc} = \upsilon_t + \upsilon_c$

$$\upsilon_{tc} = (60 + 6) \text{ km h}^{-1} = 66 \text{ km h}^{-1} = 18{\cdot}33 \text{ m s}^{-1}$$

$$\Rightarrow \quad 18{\cdot}33 = \frac{100}{t} \Rightarrow t = \frac{100}{18{\cdot}33} = \mathbf{5{\cdot}45 \text{ s}}$$

EXAMPLE 21. **Two trains are moving in the same direction with velocities 30 m s^{-1} and 25 m s^{-1}. Find the relative velocity of the second train w.r.t. first. If at $t = 0$ s, the second train is 400 m ahead of first train then after what time the first train will overtake the second train ?**

Solution. Given velocity of first train, $\upsilon_1 = 30$ m s^{-1}

Velocity of second train, $\upsilon_2 = 25$ m s^{-1}

Relative velocity of second train w.r.t. first $\upsilon_{21} = \upsilon_2 - \upsilon_1$, $\upsilon_{21} = 25 - 30 = -5$ m s^{-1}

We know relative separation between two objects at any time t is given by

$$x_2(t) - x_1(t) = x_2(0) - x_1(0) + (\upsilon_2 - \upsilon_1)\,t \qquad ...(1)$$

When the first train overtake the second train,

Then $x_2(t) - x_1(t) = 0$

$$0 = x_2(0) - x_1(0) + (\upsilon_2 - \upsilon_1)t$$

$$\Rightarrow \quad 0 = 400 + (-5)t$$

$$\Rightarrow \quad -400 = -5t$$

$$\Rightarrow \quad t = \mathbf{80\ s}$$

Hence, the first train will overtake the second in 80 s.

Problems for Practice

1. Two trains, each of length 200 m, are running on parallel tracks. One overtakes the other in 20 seconds and one crosses the other in 10 seconds. Calculate the velocities of two trains. [**Ans.** 30 m s^{-1}, 10 m s^{-1}]
2. A car A is moving with a speed of 60 km h^{-1} and car B is moving with a speed of 75 km h^{-1}, along parallel straight paths, starting from the same point. What is the position of car A w.r.t. car B after 20 minutes ? [**Ans.** 5 km behind]
3. Two cars started simultaneously towards each other from two towns A and B which are 480 km apart. It took the first car travelling from A to B eight hours to cover the distance and second car travelling from B to A, ten hours. Determine when the cars will meet after starting and at what distance from A ? [**Ans.** 4·4 h, 264 km]
4. If a man's speed along and against the water current in a river is 15 km h^{-1} and 5 km h^{-1}, then find the man's speed in still water and the speed of river. [**Ans.** 10 km h^{-1}, 5km h^{-1}]
5. Two buses are moving in the same direction with the same speed 30 km h^{-1}. They are separated by a distance of 4 km. What is the speed of a bus moving in opposite direction if it meets these two buses at an interval of 5 minutes ? [**Ans.** 18 km h^{-1}]
6. A man from a van moving with a speed of 36 km h^{-1} fires a bullet at a car moving in the same direction with a speed of 108 km h^{-1}. If the speed of bullet is 504 km h^{-1}, then with what speed the bullet strikes the car ? [**Ans.** 120 m s^{-1}]
7. A car is moving with a velocity 10 m s^{-1}. A motorcyclist wishes to overtake the car in 60 s. With what velocity the motor cyclist should chase the car which is 1 km ahead of him ? [**Ans.** 26·67 m s^{-1}]

Hints / Solutions

1. Let the velocities of trains A and B are u and υ

While overtaking

Relative velocity of train A w.r.t. train B = $u - \upsilon$

While crossing

Relative velocity of train A w.r.t. train B = $u + \upsilon$

Total distance to be travelled by train A while crossing = 200 + 200 = 400 m

Hence $$20 = \frac{400}{u-\upsilon} \qquad \left\{\because \upsilon = \frac{D}{t}, t = \frac{D}{\upsilon}\right\}$$

$$u - \upsilon = 20 \qquad ...(1)$$

Also $$10 = \frac{400}{u+\upsilon}$$

$$u + \upsilon = 40 \qquad ...(2)$$

Adding eqn. (1) and (2), $2u = 60 \Rightarrow u = \mathbf{30\ m\ s^{-1}}$

From (2), $30 + \upsilon = 40$, $\upsilon = \mathbf{10\ m\ s^{-1}}$.

2. $\upsilon_A = 60$ km h^{-1}, $\upsilon_B = 75$ km h^{-1}

$\upsilon_{AB} = 60 - 75 \Rightarrow \upsilon_{AB} = -15$ km h^{-1}

$\therefore$ distance of car A from car B after 20 minutes

$$= -15 \times \frac{20}{60} = -5 \text{ km}$$

$\therefore$ car **A is 5 km** behind car B.

3. Distance between two towns = 480 km *i.e.*, S = 480 km

Speed of first car $= \dfrac{S}{8} = \dfrac{480}{8} = 60$ km h^{-1}

Speed of second car $= \dfrac{S}{10} = \dfrac{480}{10} = 48$ km h^{-1}

Let the two cars meet after time t so

$$\frac{S}{8} \times t + \frac{S}{10} \times t = S \quad \Rightarrow 60t + 48t = 480$$

$$\Rightarrow \quad 108t = 480,\ t = \frac{480}{108} = 4{\cdot}44 \text{ h.}$$

Distance from A = 60 × 4·4 = **264 km.**

6. Speed of car (υ_c) = 108 km h^{-1} = 108 × 5/18 = 30 m s^{-1}

Speed of bullet (υ_b) = Speed of van + Speed of the bullet with which it is fired

= 36 + 504 = 540 km h^{-1} = 150 m s^{-1}

$\therefore$ speed with which bullet strikes the car

$$\upsilon_{bc} = \upsilon_b - \upsilon_c = 150 - 30 = \mathbf{120\ m\ s^{-1}}$$

7. Velocity of car $\upsilon_c = 10$ m s^{-1}

Let the velocity of motor cyclist = υ_m

$\therefore$ relative velocity of motor cycle w.r.t. car = $(\upsilon_m - 10)$ m s^{-1}

Time to overtake car = 60 s

Separation between car and motorcycle initially 1 km = 1000 m

Using formula $\upsilon_m - \upsilon_c = \dfrac{x - x_0}{t}$

$$\upsilon_m - 10 = \frac{1000}{60}$$

$$\upsilon_m - 10 = 16{\cdot}67 \Rightarrow \upsilon_m = \mathbf{26{\cdot}67\ m\ s^{-1}}$$

HOTS

◆ **Problem 1. The displacement of a particle moving in one dimension, under the action of constant force is related to time t by equation $t = \sqrt{x} + 3$ where x is in metres and t is in seconds. Find the displacement of the particle when its velocity is zero.**

Solution. Given $t = \sqrt{x} + 3$

$$\therefore \quad \sqrt{x} = t - 3 \qquad ...(1)$$

Squaring eqn.(1) both sides

$$x = t^2 + 9 - 6t \text{ or } x = t^2 - 6t + 9 \qquad ...(2)$$

So instantaneous velocity $v_{ins} = \dfrac{dx}{dt} = \dfrac{d}{dt}(t^2 - 6t + 9)$

$$v_{ins} = \frac{dx}{dt} = 2t - 6 \qquad ...(3)$$

Now when velocity of particle is zero.

$$v_{ins} = 0$$

$$\therefore \quad 2t - 6 = 0$$

$$\Rightarrow \quad 2t = 6 \Rightarrow t = 3\text{s}$$

$\therefore$ Displacement of particle at $t = 3$ s

$$x = 9 - 18 + 9 = 0$$

Hence displacement = 0

◆ **Problem 2. The velocity of a particle is given by $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then what is displacement after $t = 1$ s ?** ***(A.I.E.E.E. 2007)***

Solution. Given $v = v_0 + gt + ft^2$

But $\quad v = \dfrac{dx}{dt}$

$$\therefore \quad \frac{dx}{dt} = v_0 + gt + ft^2 \qquad ...(1)$$

$$\therefore \quad dx = (v_0 + gt + ft^2)\, dt \qquad ...(2)$$

Integrating eqn. (2) both sides within limits at $t = 0, x = 0$ and at $t = 1$ s, $x = x$

$$\int_{x=0}^{x} dx = \int_{t=0}^{t=1} \left(v_0 + gt + ft^2\right) dt$$

$$\Rightarrow (x)_0^x = \int_{t=0}^{t=1\text{s}} v_0 dt + \int_{t=0}^{t=1\text{s}} gt\, dt + \int_{t=0}^{t=1\text{s}} ft^2 dt$$

$$\Rightarrow \quad x - 0 = v_0\,(t)_0^1 + g\left(\frac{t^2}{2}\right)_0^1 + f\left(\frac{t^3}{3}\right)_0^1$$

$$\Rightarrow \quad x = v_0(1-0) + \frac{g}{2}(1-0) + \frac{f}{3}(1-0)$$

$$\Rightarrow \quad x = v_0 + \frac{1}{2}g + \frac{1}{3}f$$

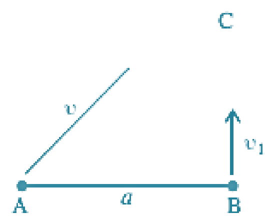
◆ **Problem 3. Two boys are standing at the ends A and B of a ground where AB = a. The boy at B starts running in a direction perpendicular to AB with velocity v_1. The boy at A starts running simultaneously with velocity v and catches the other boy in time t. Find the value of t.** ***(C.B.S.E. (Med.) 2005)***

Solution. Given

Boys are standing at ends A and B such that AB = a

Boy at B runs perpendicular to AB with velocity v_1

Let the boy at A catches the boy B at point C after time t



Distance AC $= vt$

Distance BC $= v_1 t$

Using Pythagorus theorem in ΔCAB

$$AC^2 = AB^2 + BC^2$$

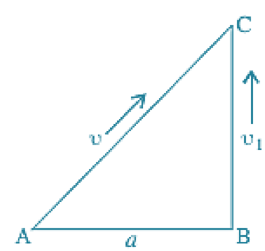
$$\Rightarrow \quad v^2t^2 = a^2 + v_1 t^2$$

$$\Rightarrow v^2 t^2 - v_1^2 t^2 = a^2$$

$$\Rightarrow (v^2 - v_1^2)\, t^2 = a^2$$

$$\Rightarrow \quad t^2 = \frac{a^2}{\left(v^2 - v_1^2\right)}$$

Hence $\quad t = \dfrac{a}{\sqrt{v^2 - v_1^2}}$



◆ **Problem 4. A bus is moving with a speed of 10 m s^{-1} on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what speed could it chase the bus ?**

(C.B.S.E. A.I.P.M.T. 2010)

Solution. Given $\upsilon_b = 10 \text{ m s}^{-1}$

Let υ_{sb} be the relative velocity of scooterist w.r.t. bus

$$\upsilon_{sb} = \upsilon_s - \upsilon_b \qquad \therefore \quad \upsilon_s = \upsilon_{sb} + \upsilon_b$$

$$\upsilon_{sb} = \frac{\text{Displacement}}{\text{time}} = \frac{1 \text{ km}}{100 \text{ s}}$$

$$= \frac{1000 \text{ m}}{100 \text{ s}} = 10 \text{ m s}^{-1}$$

$$\therefore \quad \upsilon_s = 10 \text{ m s}^{-1} + 10 \text{ m s}^{-1} = 20 \text{ m s}^{-1}$$

◆ **Problem 5. A particle moving along X-axis has acceleration f at time t is given by $f = f_0\left(1 - \frac{t}{T}\right)$ where f_0 and T are constants. The particle at $t = 0$ has zero velocity. Find the velocity of the particle in the time interval between $t = 0$ and the instant when $f = 0$.** *(C.B.S.E. (Med.) 2007)*

Solution. Given $f = f_0\left(1 - \frac{t}{T}\right)$ ...(1)

at $f = 0$ and $t = T$

Eqn.(1) can be written as

$$\frac{d\upsilon}{dt} = f_0\left(1 - \frac{t}{T}\right)$$

$$\therefore \quad d\upsilon = f_0\left(1 - \frac{t}{T}\right)dt \qquad \text{...(2)}$$

Integrating equation (2) within proper limits *i.e.*, at $t = 0, \upsilon = 0$ and at $t = T$, velocity $= \upsilon$

$$\int_0^{\upsilon} d\upsilon = \int_{t=0}^{t=T} f_0\left(1 - \frac{t}{T}\right)dt$$

$$\Rightarrow \quad (\upsilon)_0^{\upsilon} = \int_0^{T} f_0\,dt - \int_0^{T} \frac{f_0 t}{T}\,dt$$

$$\upsilon - 0 = (f_0 t)_0^T - \frac{f_0}{T}\int_0^T t\,dt$$

$$= f_0(T - 0) - \frac{f_0}{T}\left(\frac{t^2}{2}\right)_0^T$$

$$\upsilon = f_0 T - \frac{f_0}{2T}(T^2 - 0)$$

$$= f_0 T - \frac{f_0}{2T}T^2 = f_0 T - \frac{f_0}{2}T$$

$$\Rightarrow \quad \upsilon = \frac{1}{2} f_0 T$$

◆ **Problem 6. Two cyclists A and B return from picnic spot O to their homes P and Q respectively. The position-time graphs are shown in figure given below. Tell**

***(a)* Which of the two lines closer to the picnic spot ?**

***(b)* Which of the two started earlier ?**

***(c)* Which of the two was faster ?**

***(d)* Do both of them reach their home at the same time ?**

***(e)* Who overtakes whom and how many times ?**

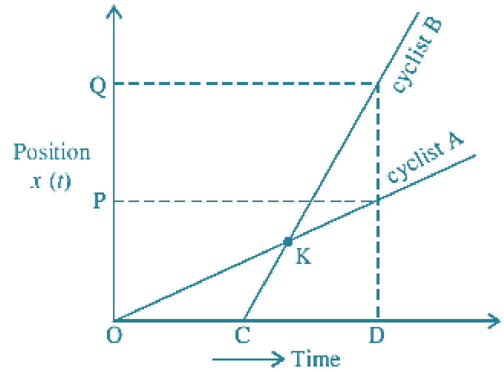


Fig. 41

Solution. *(a)* From position-time graph, it is clear that OQ > OP.

So, line A is closer to the picnic spot.

(b) The position-time graph for cyclist A starts from origin (t = 0) while position-time graph of B starts from C, which indicates that B started later than A after a time interval so A started earlier than B.

(c) Speed = slope of position-time graph. As slope of position-time graph for B is more than the slope of position-time graph for A, hence B is faster than A.

(d) Corresponding to both P and Q, the time interval is same *i.e.* equal to OD. It indicates that both A and B reach their homes at the same time.

(e) The position-time graph intersects at point K, which indicates that cyclist B crosses A and after this there is no point of intersection. Thus, two cyclists cross each other only once.

◆ **Problem 7. Figure shown here shows the distance-time graph of two trains, which start moving simultaneously in the same direction. From the graph, find**

***(i)* How much ahead the train B is from train A when motion starts ?**

***(ii)* What is speed of train B ?**

***(iii)* When and where will A catch B ?**

***(iv)* What is difference between speeds of A and B ?**

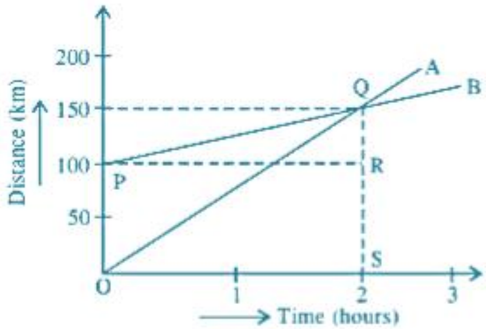


Fig. 42

Solution. *(i)* From distance-time graph, it is clear that train B is ahead of train A by the distance OP = 100 km when the motion starts.

(ii) Speed of train B $= \frac{QR}{PR} = \frac{50}{2} = \mathbf{25\ km\ h^{-1}}$

(iii) Two trains intersect each other at point Q so train A will catch the train B after 2 hours and at a distance 150 km from the origin.

(iv) Speed of train A $= \frac{QS}{OS} = \frac{150-0}{2-0} = \frac{150}{2}$

$= \mathbf{75\ km\ h^{-1}}$.

◆ **Problem 8. The speed-time graph of a particle moving along a fixed direction is as shown. Find**

***(i)* Distance travelled by particle between 0 second and 10 second.**

***(ii)* Average speed between this interval.**

***(iii)* Time when speed was minimum.**

(iv)* Time when speed was maximum.** ***(Delhi 2006)

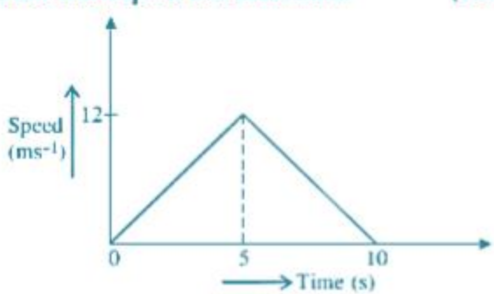


Fig. 43

Solution. Distance travelled by particle between 0 s and 10 s

Distance = Area under speed-time graph

$= \frac{1}{2}(10-0)(12-0) = \frac{1}{2} \times 10 \times 12 = \mathbf{60\ m}$

(ii) Average speed $= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{60}{10} = \mathbf{6\ m\ s^{-1}}$

(iii) Speed of particle is minimum at times $t = 0$ and $t = 10$ s.

(iv) Speed is maximum at $t = \mathbf{5\ s}$.

◆ **Problem 9. A particle is moving along X-axis. The position of the particle at any instant of time is given by**

$x = a + bt^2$ **where** $a = 6$ **m and** $b = 3{\cdot}5$ **m s^{-2}**

t **is measured in second. Find *(i)* velocity of particle at t = 0 s and t = 3 s.**

***(ii)* Average velocity between t = 3 s and t = 6 s.**

Solution. Given $x = a + bt^2$, $a = 6$, $b = 3{\cdot}5$ m s^{-2}

$\therefore \quad x = 6 + 3{\cdot}5t^2 \qquad ...(1)$

Differentiating eqn.(1) both sides w.r.t. t

$$\frac{dx}{dt} = \frac{d}{dt}(6 + 3{\cdot}5t^2) = 0 + 2 \times 3{\cdot}5t = 7t.$$

$$\frac{dx}{dt} = 7t \Rightarrow v = 7t \qquad ...(2)$$

(i) At $t = 0$, $v = 0$ m s^{-1}

At $t = 3$ s, $v = 7 \times 3 = 21$ m s^{-1}

(ii) Average velocity $= \frac{\text{Total displacement of particle}}{\text{Total time taken}}$

$$= \frac{x(t = 6\,s) - x(t = 3\,s)}{(6-3)\,s}$$

$$= \frac{\{6 + 3{\cdot}5(36)\} - \{6 + 3{\cdot}5(9)\}}{3s}$$

$$= \frac{(6+126) - (6+31{\cdot}5)}{3}$$

$$= \frac{(132 - 37{\cdot}5)\,m}{3\,s} = \frac{94{\cdot}5\,m}{3\,s} = \mathbf{31{\cdot}5\ m\ s^{-1}}$$

◆ **Problem 10. A person starts from the centre O of a circular ground of radius 1 km, reaches the end A of the circle and travels along the circumference up to B and then reaches the centre along BO as shown in Fig. If he takes 15 minutes to complete this journey, then find *(i)* net displacement *(ii)* average velocity *(iii)* average speed of person.**

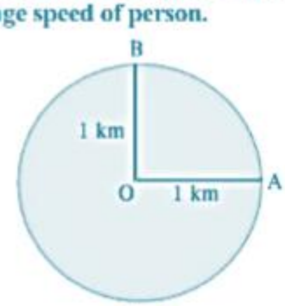


Fig. 44

Solution. *(i)* As person starts the journey from point O and again reaches the point O to complete the journey, then

Net displacement = 0

(ii) Average velocity $= \frac{\text{Net displacement}}{\text{Net time}} = \frac{0}{15} = 0$

(iii) Average speed $= \dfrac{\text{Net distance travelled}}{\text{Time taken}}$

Now, Net distance travelled

$$= OA + \frac{1}{4} \times \text{Circumference of circle} + BO$$

$$= 1\text{ km} + \frac{1}{4} \times 2\pi r + 1$$

$$= 1\text{ km} + \frac{\pi \times 1}{2} + 1\text{ km}$$

$$= 2\text{ km} + \frac{3{\cdot}14}{2}\text{ km} = \mathbf{3{\cdot}57\ km}$$

Time taken for travelling this distance = 15 minutes

$$= \frac{15}{60}\text{ h} = \frac{1}{4}\text{ h}$$

$\therefore$ average speed of person $= \dfrac{3{\cdot}57}{1/4\text{ h}}$ km $= \mathbf{14{\cdot}28\ km\ h^{-1}}$

◆ **Problem 11. A point traversed half the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half of the time, and with velocity v_2 for the other half of the time. Find the mean velocity of the point average over the whole time of motion.**

Solution. Let total distance traversed by point = S

then by question $\dfrac{S}{2} = v_0 t_1 \Rightarrow t_1 = \dfrac{S}{2v_0}$ ...(1)

If t be the time taken by the point to travel remaining distance $\dfrac{S}{2}$, then

$$\frac{S}{2} = v_1 \frac{t}{2} + v_2 \frac{t}{2} = (v_1 + v_2)\frac{t}{2}$$

$$t = \frac{2S}{2(v_1 + v_2)},\ t = \frac{S}{v_1 + v_2} \quad ...(2)$$

$\therefore$ Average speed $= \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{S}{t_1 + t}$

$$= \frac{S}{\dfrac{S}{2v_0} + \dfrac{S}{v_1 + v_2}}$$

Average speed $= \dfrac{S}{\dfrac{S(v_1 + v_2) + 2Sv_0}{2v_0(v_1 + v_2)}}$

$$= \frac{2v_0(v_1 + v_2)S}{S[v_1 + v_2 + 2v_0]}$$

Average speed $= \mathbf{\dfrac{2v_0(v_1 + v_2)}{v_1 + v_2 + 2v_0}}$

◆ **Problem 12. Two persons P and Q are standing 54 m apart on a long moving belt. Person P rolls a round stone towards person Q with a speed of 9 m s^{-1} with respect to belt. If the belt is moving with speed 4 m s^{-1} in the direction from P to Q *(a)*. What will be the speed of stone w.r.t. an observer on stationary platform ? *(b)* What is the time taken by stone to travel from P to Q ? *(c)* What will be the speed of stone w.r.t. an observer on a stationary platform if person Q rolls the stone with a velocity of 9 m s^{-1} w.r.t. the belt towards person P and the time taken by stone to travel from Q to P ?**

Solution. *(a)* Let the direction from P to Q is taken as positive.

Given, speed of belt, $v_b = 4$ m s^{-1}

Speed of stone w.r.t. belt $v_s = 9$ m s^{-1}

Hence, speed of stone w.r.t. stationary observer $= v_b + v_s$

$= (9 + 4)$ m s^{-1} = 13 m s^{-1}

(b) Distance between persons P and Q = 54 m

Speed of stone, $v_s = +9$ m s^{-1}

$\therefore$ time taken by stone in travelling from P to Q, time

$$t = \frac{\text{Distance}}{\text{Velocity}} = \frac{54\text{ m}}{9\text{ m s}^{-1}} = 6\text{ s}$$

$\therefore$ time = 6 s

(c) Given $v_b = +4$ m s^{-1}

$v_s = -9$ m s^{-1} (stone rolls from person Q to P)

$\therefore$ speed of stone w.r.t. stationary observer

$= v_s + v_b$

$= -9$ m s^{-1} + 4 m s^{-1} = -5 m s^{-1}

Negative sign shows that the stone is moving in a direction opposite to the direction in which belt is moving.

Speed of stone w.r.t. P or Q = 9 m s^{-1}

$\therefore$ Time taken, $t = \dfrac{\text{Distance}}{\text{Speed}} = \dfrac{54\text{ m}}{9\text{ m s}^{-1}} = \mathbf{6\ s}$

CONCEPTUAL QUESTIONS

Q.1. What is translation motion?

Ans. Rigid objects that move without rotation are said to undergo translation motion. When we are interested only in translation motion and the object's size is not significant, we can treat the object as a particle.

Q.2. Which is generally our frame of reference for studying motion of objects?

Ans. Ordinarily our frame of reference for studying motion of objects is a coordinate system attached to the surface of earth. Unless stated otherwise, this is our frame of reference.

Q.3. An aeroplane is flying in the sky. Is its motion three dimensional?

Ans. Yes. It is because it may be changing its direction as well as the height from the ground during its flight.

Q.4. Which is the simplest type of motion?

Ans. The uniform motion in a straight line (one dimensional motion) is the simplest type of motion.

Q.5. Why is time period repeated twice in the unit of acceleration?

Ans. The velocity of a particle is the rate at which its displacement changes with time; its acceleration is the rate at which its velocity changes with time. Thus acceleration is the rate of a rate. So time is repeated twice.

Q.6. Fig. 6.35 shows $v - t$ graph for various situations. What does each graph indicate?

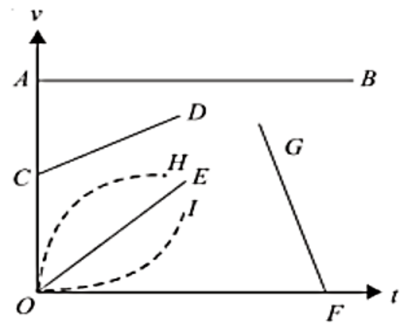


Fig. 6.35

Ans. Acceleration = Slope of $v - t$ graph

(*i*) Straight line AB indicates that the acceleration of the moving body is *zero*. Clearly, the body is moving with constant velocity.

(*ii*) Straight line CD indicates that the body has *constant positive acceleration* with initial velocity OC. In this case, the velocity of the body is increasing.

(*iii*) Straight line OE indicates that the body has positive constant acceleration with zero initial velocity.

(*iv*) Dotted curve OI shows the increasing acceleration. Here the slope of the graph increases with time.

(*v*) Dotted curve OH indicates decreasing acceleration. Here the slope of the graph decreases with time.

(*vi*) The straight line FG indicates that the body is moving with constant negative acceleration. Here the slope of the graph is negative. It means the velocity of the body is decreasing at a constant rate.

VERY SHORT ANSWER QUESTIONS

Q.1. Are rest and motion absolute or relative terms?

Ans. Rest and motion are relative terms.

Q.2. Which speed is measured by the speedometer of a car?

Ans. Instantaneous speed *i.e.,* speed at a given instant of time.

Q.3. A body is moving with a uniform velocity. What does it mean?

Ans. It means that the body is moving with a constant speed in a straight line.

Q.4. Can a body have a constant velocity but a variable speed?

Ans. No. It is because speed is equal to the magnitude of velocity.

Q.5. Can a body have a constant speed but variable velocity?

Ans. Yes. When a body is rotated along a circular path with constant linear speed, the velocity is changing (because direction of motion is changing) but speed is constant.

Q.6. What does the slope of position-time graph of a body represent?

Ans. The slope of $x - t$ graph of a body represents its velocity.

Q.7. Under what condition will the distance and displacement of a moving object will have the same magnitude?

Ans. When the object moves in a straight line in the same direction.

Q.8. What is the nature of position-time graph for a stationary object?

Ans. The $x - t$ graph for a stationary object is a straight line parallel to time-axis.

Q.9. Can the speed of a body be negative?

Ans. No. It is because speed = distance/time and distance can never be negative.

Q.10. Can the slope of $x - t$ graph be negative?

Ans. Yes. It is so when the velocity of the body is negative.

Q.11. What is meant by a point object in physics?

Ans. An object is considered as a point object when its size is negligible compared to the scale of observation.

Q.12. What does slope of velocity-time graph represent?

Ans. The slope of velocity-time graph represents acceleration.

Q.13. How will you calculate distance travelled from velocity-time graph?

Ans. The area under $v - t$ graph for a given time interval gives the distance covered during that time interval.

Q.14. What will be the nature of $x - t$ graph for a uniform motion?

Ans. It will be a straight line inclined to the time-axis.

Q.15. Can an object be accelerated without speeding up or slowing down?

Ans. Yes, it can be so in case of uniform circular motion of an object.

Q.16. If the acceleration of a body is zero, what is the nature of motion?

Ans. If the acceleration of a body is zero, the body is either moving with constant velocity in a straight line or remains at rest.

Q.17. What is the basic difference between speed and velocity?

Ans. Speed indicates how fast the body is moving whereas velocity tells us how fast and in which direction it is moving.

Q.18. What does area under velocity-time graph represent?

Ans. The net area under velocity-time graph represents displacement for a given time interval.

Q.19. Can the displacement be greater than distance travelled by an object?

Ans. Displacement is the shortest distance between two positions. Therefore, displacement cannot be greater than the distance travelled.

Q.20. What will be the nature of velocity-time graph for a uniform motion?

Ans. It will be a straight line parallel to time-axis.

Q.21. When will the relative velocity of two moving objects be zero?

Ans. When the two objects are moving with same speed in the same direction (*i.e.*, equal velocities).

Q.22. How can a body be simultaneously at rest and in motion?

Ans. In relative motion, it is possible. A passenger in a train is at rest *w.r.t.* train but is in motion *w.r.t.* ground.

SHORT ANSWER QUESTIONS

Q.1. If the displacement of a body is zero, is the distance covered by it necessarily zero? Comment.

Ans. No. For example, when a body is thrown vertically upward, it comes back to the same point after some time. In this case, displacement of the body is zero but the distance travelled by the body is $2h$ where h is the maximum height to which the body rises. Similarly, when a body moving in a circle of radius r completes one revolution, the displacement of the body is zero but distance travelled by it is $2\pi r$.

Q.2. Explain that rest and motion are always relative.

Ans. Rest and motion are relative terms. For example, a book on the table is at rest *w.r.t.* table and other objects in the room. If an observer is located on the moon, he will observe that the book and other objects in the room are moving. Thus the book is at rest if viewed from the room but is moving if viewed from the moon.

Q.3. A body can have zero average velocity but not zero average speed. Comment.

Ans. The statement is true.

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}\ ;\ \text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

If a body moving in a circular path of radius r completes one revolution in t seconds, then displacement is zero but distance travelled is $2\pi r$. Therefore, the average velocity of the body is zero while average speed is $2\pi r/t$.

Q.4. A car covers a distance S_1 with velocity v_1 and distance S_2 with velocity v_2. What is the average velocity of car?

Ans.
$$\text{Average velocity} = \frac{S_1 + S_2}{(S_1/v_1) + (S_2/v_2)}$$
... Refer to Example 6.1.

Q.5. During a given interval of time, can the average velocity of a body be greater than its average speed?

Ans. No. It is because during a given time interval, the total displacement can be equal to or smaller than the total distance travelled.

Q.6. A cyclist completes one round of circular track of radius R = 20 m in 20 seconds. What will be the displacement at the end of 20 s and 30 s?

Ans. At the end of 20 s, the cyclist completes one revolution and the displacement is zero. However, at the end of 30 s, the cyclist completes 1.5 revolutions and the displacement = diameter of circle = $2R = 2 \times 20 = 40$ m.

Q.7. In the above question, what is the average speed and average velocity?

Ans. After 20 s, average speed = $2\pi R/t = 2\pi \times 20/20 = 2\pi$ m/s and average velocity = 0 m/s.

After 30 s, average speed = $1.5 \times 2\pi \times 20/30 = 2\pi$ m/s and average velocity = $2\ R/t = 40/30 = 4/3\text{ms}^{-1}$.

Q.8. The velocity-time graph of two bodies A and B make angles of 30° and 60° with the time-axis. What is the ratio of their accelerations?

Ans. $a_A/a_B = \tan 30°/\tan 60° = 1/3$.

Q.9. The displacement-time graph of two bodies A and B make angles of 45° and 30° with time-axis. What is the ratio of their velocities?

Ans. $v_A/v_B = \tan 45°/\tan 30° = \sqrt{3}$.

Q.10. A car travels half the distance with velocity v_1 and the second half with a velocity v_2. If the total distance is S, what is the average velocity of the car?

Ans. $$\text{Average velocity} = \frac{2v_1v_2}{v_1+v_2}$$ ... Refer to Example 6.1.

Q.11. Does velocity or acceleration decide the direction of motion of a body?

Ans. It is the velocity and not acceleration that decides the direction of motion of a body. Thus when a body is thrown upward, the direction of motion of the body and its velocity are in the upward direction. However, acceleration due to gravity (g) acts vertically downward. Therefore, direction of motion of the body is in the direction of velocity and not that of acceleration.

Q.12. If the distance (x) covered by a moving body is directly proportional to time (t), what conclusions you draw?

Ans. $x \propto t$ or $x = kt$ when k is constant of proportionality.

Now, $$v = \frac{dx}{dt} = \frac{d}{dt}(kt) = k = \text{constant}$$

Therefore, the body is moving with uniform velocity *i.e.,* with a constant speed in a straight line.

Q.13. If the displacement of a body is directly proportional to the square of time, what conclusions you draw?

Ans. The body is moving with uniform acceleration (Refer to Example 6.20).

Q.14. If the displacement of a body is proportional to t^3, what conclusions you draw?

Ans. Acceleration $\propto t$ (Refer to Example 6.21).

Q.15. A ship S_1 is sailing due east with a velocity of 30 km/h and another ship S_2 is sailing due south with the same velocity. What is the velocity of S_2 relative to S_1?

Ans. Impress a velocity of 30 km/h due west on both ships. This brings ship S_1 to rest. The velocity v of S_2 *w.r.t.* S_1 is the resultant of two velocities, each 30 km/h, one due west and the other due south.

$$\therefore \qquad v = \sqrt{30^2 + 30^2} = 30\sqrt{2} \text{ km/h south-west}$$

Q.16. When two cars A and B move towards each other with constant velocities v_A and v_B, the distance between them decreases by 10 m/s. When they move in same direction with the same speed, the distance between them increases by 5 m/s. What are their velocities ?

Ans. $v_A + v_B = 10$ m/s and $v_A - v_B = 5$ m/s. Therefore, $v_A = 7.5$ m/s ; $v_B = 2.5$ m/s.

Fill in the Blanks

1. The branch of Physics that deals with the motion of material objects is called as ....
2. An object can be at rest as well as in motion ....
3. .... of a particle varies even if its .... is constant.
4. Displacement of an object can be .... distance.
5. When a ball is thrown upward then velocity of body at .... point is ....
6. A motion in which the distance of the moving particle from fixed point is always constant during its motion is called ....
7. If a particle covers distances x_1 and x_2 with speeds v_1 and v_2 in the same direction then the average speed of the particle is ....
8. When the position-time graph for the motion of particle is a straight line parallel to position axis then its velocity is ....
9. Motion of an aeroplane during its flight is called as ....
10. The revolution of earth around the sun is called ....
11. If the distance covered by a particle is zero then its displacement must be ....
12. Slope of the position-time graph represents ....

ANSWERS

1. mechanics **2.** at the same time **3.** Velcotiy, speed **4.** greater than or equal to 5. highest, zero **6.** circular motion.

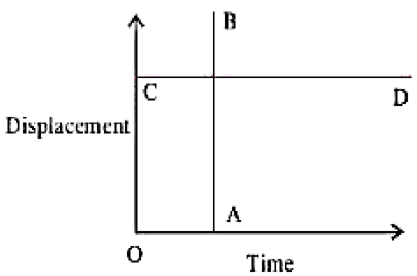
7. $\dfrac{x_1 + x_2}{\left(\dfrac{x_1}{v_1} + \dfrac{x_2}{v_2}\right)}$ **8.** infinite **9.** three dimensional motion

10. two dimensional motion **11.** zero **12.** uniform velocity of object.

Very Short Answer Concept Based Problems

(Carry 1 Mark)

1. **Can the displacement be greater than the distance travelled by an object ? Give reason.**
   Ans. No, the displacement of the object can never be greater than distance. The displacement can be either equal to or less than the distance travelled by object because the displacement is the shortest distance between initial and final locations where as distance is the actual path travelled between initial and final locations.
2. **Can earth be regarded as a point object when it is describing its yearly journey around the sun ?**
   Ans. Yes, because the size of the earth is very small as compared to the radius of the orbital path of earth around sun.
3. **Which speed is measured by the speedometer of your scooter ?**
   Ans. Speedometer of the scooter measures the instantaneous speed of the scooter at a given instant of time.
4. **Can the speed of a body be negative ?**
   Ans. No, speed can never be negative as speed is the distance travelled by a body per unit time and distance travelled is never negative.
5. **What is the nature of velocity-time graph for a uniform motion ?**
   Ans. It will be a straight line parallel to time axis.
6. **What is the nature of position-time graph for a uniform motion ?**
   Ans. It will be a straight lime inclined to time axis.
7. **If the displacement-time graph for a particle is parallel to *(i)* Displacement axis *(ii)* the time axis**
   What will be velocity of the particle ?



   Ans. *(i)* Velocity will be infinite. It is not a practical situation.
   (ii) Zero.
8. **Can position-time graph have negative slope ?**
   Ans. Yes, when velocity of the object is negative.
9. **Can a body have a constant speed and still have varying velocity ?**
   Ans. Yes, when a particle is in uniform circular motion then speed of particle is constant whereas velocity is varying as there is the change in direction of motion at every point.
10. **Can a body have constant velocity but still have varying speed ?**
    Ans. No, a body never possess constant velocity if the speed is varying. Whenever speed changes velocity also changes.
11. **Can a particle in one-dimensional motion have zero speed and a non zero velocity ?**
    Ans. No, if the speed is zero, then velocity will be necessarily zero.
12. **Under what conditions is the average velocity equal to instantaneous velocity ?**
    Ans. It happens when the body is moving with uniform velocity *i.e.*, motion of body is uniform in one direction.
13. **What do you understand by positive and negative time ?**
    Ans. The instant of time which is taken after the origin of time is called positive time and the instant of time which is taken before the origin of time is negative time.
14. **What is the common between the two graphs shown in figure ?**

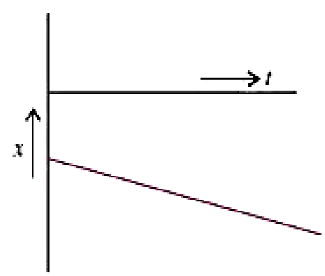


Fig. 46

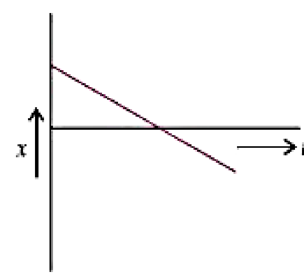


Fig. 47

    Ans. Both the graphs represent negative velocity.

15. Is the speed-time graph shown in the given figure possible ?

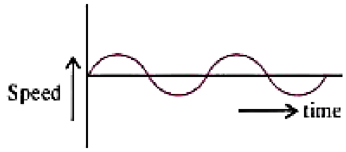


Fig. 48

Ans. No, the speed-time graph shown in figure is not possible because speed can never be negative.

16. What is common between the two graphs shown in figure ?

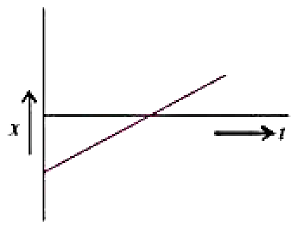


Fig. 49

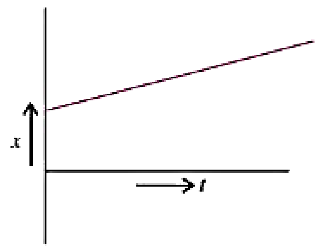


Fig. 50

Ans. Both the graphs represent positive velocity.

17. What does the area under velocity-time graph represent ?

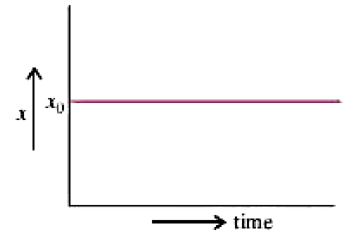
Ans. Displacement.

18. Under what conditions the magnitude of the average velocity is equal to average speed ?

Ans. The magnitude of the average velocity will always be equal to average speed when the object moves with constant velocity.

19. Draw the position-time graph for a stationary car or an object.

Ans. Position-time graph for a stationary car is as shown.



20. Identify the types of motion in the following cases :

(a) **A carrom coin striking against the side of a board and hopping up as it rebounds without rebounding smoothly.**

(b) **A car going on a zig-zag path through traffic on a busy highway.**

(c) **A car on a straight highway**

(d) **An ant crawling on a large sphere.**

(e) **The earth revolving around the sun.**

Ans. *(a)* Three dimensional motion

(b) Two dimensional motion

(c) One dimensional motion

(d) Two dimensional motion

(e) Two dimensional motion.

Short Answer Concept Based Problems

(Carry 2 or 3 Marks)

1. Can a body is said to be at rest as well as in motion at the same time ?

Ans. Yes, an object may be at rest relative to one object and at the same time it may be in motion relative to another object. For example, a passenger sitting in a moving bus is at rest w.r.t. another passengers in the bus and at the same time he is in motion w.r.t. the surrounding objects (Buildings, trees on the road side). Hence rest and motion are relative terms.

2. When an object is considered as a point object ? Explain briefly.

Ans. An object is considered as a point object if during motion in a given time its dimensions are negligible as compared to the distance travelled by it. For example, if a bus of size 10 metres is moving few hundred kilometers distance then in order to study its motion, the bus can be considered as a point object as 10 m is negligible as compared to the few hundred km distance.

3. Is the magnitude of displacement of object and total distance covered by it in certain time interval same ? Explain.

Ans. It is not necessary because if an object covers a complete circular loop of radius. r, then the displacement of object is zero but the distance covered is $2\pi r$.

4. Is it true that in one dimensional motion a particle with zero speed may have non-zero velocity ? Explain.

Ans. It will never be possible because velocity = Speed + Direction, so if speed is zero velocity will also be zero.

5. Explain that a body can have zero average velocity but not zero average speed

Ans. $$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$\text{Average velocity} = \frac{\text{Total or Net displacement}}{\text{Total time taken}}$$

Now, if the body is moving along a circular path of radius r, then displacement of the body is zero in one complete rotation but distance is $2\pi r$.

$\therefore$ average speed of body $= \dfrac{2\pi r}{T}$ and average velocity = 0.

6. Show that the average velocity of a body over a complete time interval is either less than or equal to the average speed of particle over the same interval.

Ans. $$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

Since displacement is less than or equal to the distance therefore average velocity is less than or equal to average speed.

7. Under what circumstances, the relationship $\Delta x = v\, \Delta t$ hold exactly ?

Ans. The given relation hold good only if the object is moving with uniform velocity.

8. An athlete completes one round of a circular track of radius R in 30 seconds. What will be the displacement at the end of 2 min. 15 second ?

Ans. No. of rotation completed in 30 s = 1

$$\text{rotations completed in 2 min. 15 s} = \frac{2 \times 60 + 15}{30} = 4{\cdot}5$$

$\therefore$ the no. of rounds covered in 2 min. 15 s = 4·5 rounds which means the athlete covers four complete rounds and one half round so finally he will be at the opposite end of the diameter from starting point.

Hence, displacement $= 2\vec{R}$.

9. A body travels with velocity v_1 for time t_1 and with velocity v_2 for time t_2 seconds in the same direction, find the average velocity of body.

Ans. Total time taken by body $= t_1 + t_2$

Net displacement in time $t_1 + t_2 = v_1 t_1 + v_2 t_2$

$$\therefore \quad \text{average velocity} = \frac{v_1 t_1 + v_2 t_2}{(t_1 + t_2)}$$

10. The displacement x of the body in motion is given by $x = A \sin(\omega t + \theta)$. Determine the time at which displacement will be maximum.

Ans. Given $x = A \sin(\omega t + \theta)$

The displacement will be maximum if $\sin(\omega t + \theta)$ is maximum *i.e.*,

$$\sin(\omega t + \theta) = 1$$
$$\sin(\omega t + \theta) = \sin \pi/2$$
$$(\omega t + \theta) = \pi/2$$
$$\omega t = \frac{\pi}{2} - \theta \text{ or } t = \frac{1}{\omega}\left(\frac{\pi}{2} - \theta\right)$$
$$t = \left(\frac{\pi}{2\omega} - \frac{\theta}{\omega}\right)$$

11. If a person travels a distance S_1 with velocity v_1 and distance S_2 with velocity v_2 in the same direction, then what should be the average velocity of person ?

Ans. Distance S_1 is travelled by person with velocity v_1

So time taken by person in travelling distance S_1 *i.e.*,

$$t_1 = \frac{S_1}{v_1}$$

Similarly time taken by person in travelling distance S_2 *i.e.*,

$$t_2 = \frac{S_2}{v_2}$$

Total distance travelled by person $= S_1 + S_2$

Total time taken by person to travel this distance $= t_1 + t_2$

$$\therefore \quad \text{Average velocity} = \frac{\text{Total distance}}{\text{Total time taken}} = \frac{S_1 + S_2}{t_1 + t_2}$$

$$\therefore \quad v_{av} = \frac{S_1 + S_2}{t_1 + t_2} = \frac{S_1 + S_2}{\frac{S_1}{v_1} + \frac{S_2}{v_2}}$$

$$= \frac{S_1 + S_2}{\frac{S_1 v_2 + S_2 v_1}{v_1 v_2}} = \frac{(S_1 + S_2)\, v_1 v_2}{S_1 v_2 + S_2 v_1}$$

12. The speedometer of a car A moving eastward reads 50 km/h. It passes another car B which travels westward at 50 km/h. *(i)* Do both the cars have speed ? *(ii)* Do they have same velocity ? *(iii)* What is relative velocity of car *A* w.r.t., car B ?

Ans. *(i)* Speedometer measures the instantaneous speed of the car hence both cars possess same speed.

(ii) Because velocity is a vector quantity so both cars are having opposite velocities and move in opposite direction.

(iii) $v_{AB} = v_A - v_B = 50 - (-50)$

$= 100$ km/h due east.

13. If the distance covered by a moving object varies directly as the time, what conclusion could you draw about the motion and the forces ?

Ans. Let the distance covered is x

$\therefore \quad x \propto t,\ x = kt$

where k is constant of proportionality.

$$\therefore \quad v = \frac{dx}{dt} \Rightarrow v = \frac{d}{dt}(kt) = k$$

So, the body must be moving with constant speed, also net force on body in direction of motion.

$$F = ma \Rightarrow F = 0$$

14. Four persons K, L, M and N are initially at the four corners of a square of side a. Each person now moves with a uniform speed v in such a way that K always travel directly towards L, L directly towards M, M directly towards N and N directly towards K. What is the time after which they will meet ?

Ans. Motion of four persons K, L, M and N is as shown in figure

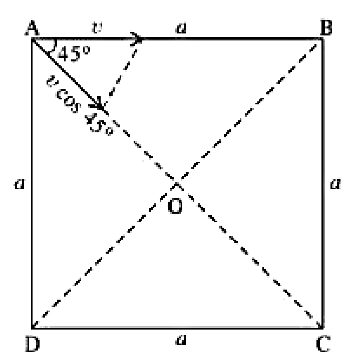


Fig. 86

18. A body covers a distance of x metre along a semicircular path. Calculate the magnitude of displacement of the body and the ratio of distance to displacement.

Ans. Let r be the radius of the semicircular path.

$$\therefore\ x = \frac{2\pi r}{2} = \pi r$$

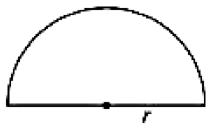


Fig. 57

$$r = \frac{x}{\pi}\ \therefore \text{ Diameter} = 2r = \frac{2x}{\pi}$$

Magnitude of displacement = Diameter of semicircle

$$= \frac{2x}{\pi}$$

Ratio of distance and displacement $= \dfrac{x}{2x/\pi} = \dfrac{x.\pi}{2x} = \dfrac{\pi}{2}$

19. Usually average speed means the ratio of total distance travelled to the total time elapsed. However, some times the phrase 'average speed' can mean the magnitude of average velocity. Are the two same ? Explain.

Ans. It is not correct to say that the average speed of a body is equal to the magnitude of average velocity because both have different meanings.

Average speed, $v_{av} = \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

Magnitude of average velocity $= \left|\dfrac{\text{Total displacement}}{\text{Total time}}\right|$

As distance travelled by a body is always positive, whereas the displacement may be positive, negative or zero, so distance $\geq$ displacement

$$\therefore \quad v_{av} \geq \left|\vec{v}_{av}\right|$$

20. Is the time variation of position, as shown in figure observed in nature ?

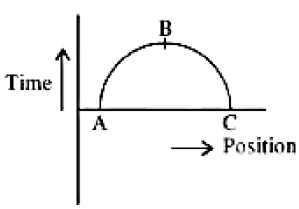


Fig. 58

Ans. It is not possible because in the given variation with the increase of position-time firstly increases from A to B and then with the increase of position-time decreases along BC which is never possible.

21. The graph between total path length and time for a particle moving along a straight line as shown in figure is not possible. Explain why ?

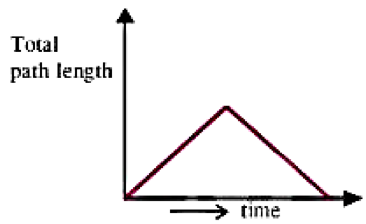


Fig. 59

Ans. It is clear from the figure that with the passage of time, the total path length firstly increases and then it decreases. But with the passage of time path length must always increases or remains constant. Hence, this graph is not possible.

22. If velocity is constant, does the average velocity over an interval of time differ from the instantaneous velocity.

Ans. If the velocity of body is constant/uniform, then average velocity of body will never be different from instantaneous velocity.

23. Displacement-time graph for the motion of a particle is shown in figure. What can you say about the instantaneous velocity of the particle at points A, B and C ?

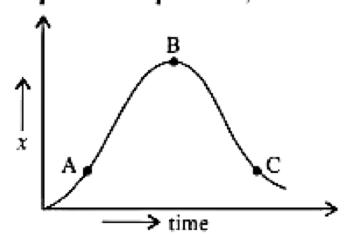


Fig. 60

Ans. We know that the slope of the displacement-time graph is equal to velocity of particle.

At point A slope of displacement-time graph is positive, so instantaneous velocity at point A will be positive.

At point B slope of displacement-time graph is zero, so instantaneous velocity at point B is zero. At point C slope of displacement-time graph is negative so velocity will be negative.