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ELECTROSTATICS

REVIEW OF BASIC CONCEPTS

1. Coulomb's Law

On the basis of his measurements, Coulomb arrived at a law, known after his name as Coulomb's law, which states that the *magnitude of the electric force between two charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them,* i.e.

$$F \propto \frac{q_1 q_2}{r^2}$$

or $$F = k\,\frac{q_1 q_2}{r^2}$$

In the SI system, k is written as $1/4\pi\varepsilon_0$ where ε_0 is called the permittivity of vacuum and its value is

$$\varepsilon_0 = 8.854 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$$

Then $$k = \frac{1}{4\pi\varepsilon_0} \simeq 9 \times 10^9\ \text{Nm}^2\ \text{C}^{-2}$$

The force F is attractive for unlike charges ($q_1 q_2 < 0$) and repulsive for like charges ($q_1 q_2 > 0$).

Coulomb's Law in vector form

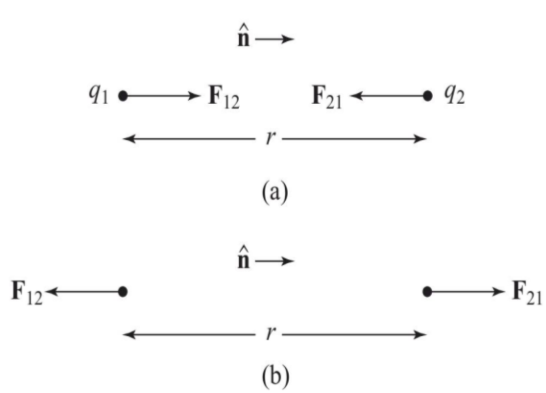


Fig. 11.1

Case (a): Unlike charges ($q_1 q_2 < 0$) [Fig. 11.1(a)]
Force exerted on q_2 by q_1 is

$$\mathbf{F}_{12} = \frac{q_1 q_2\,\hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2}$$

where $\hat{\mathbf{n}}$ is a unit vector directed from q_1 to q_2.
Force exerted by q_1 on q_2 is

$$\mathbf{F}_{21} = -\frac{q_1 q_2\,\hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2}$$

Case (b): Like charges ($q_1 q_2 > 0$) [Fig. 11.1(b)]

$$\mathbf{F}_{12} = -\frac{q_1 q_2\,\hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2}$$

$$\mathbf{F}_{21} = \frac{q_1 q_2\,\hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2}$$

2. Relative Permittivity (or Dielectric Constant)

Relative permittivity of a medium is defined as the ratio of the permittivity of the medium to permittivity of vacuum, i.e.

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

ε_r is also called the dielectric constant (K) of the medium. Thus $K = \dfrac{\varepsilon}{\varepsilon_0}$ or $\varepsilon = K\varepsilon_0$. By definition K for air = 1. If charges q_1 and q_2 are situated in a medium other than air or vacuum, the magnitude of force between them is given by

$$F = \frac{q_1 q_2}{4\pi\varepsilon r^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 K r^2}$$

3. Principle of Superposition

If many charges are present, the total force on a given charge is equal to the vector sum of the individual forces exerted on it by all other charges taken one at a time.

EXAMPLE 1 Two point charges $q_1 = +9\ \mu C$ and $q_2 = (-1)\ \mu C$ are held 10 cm apart. Where should at third charge $+Q$ be placed from q_2 on the line joining them so that charge Q does not experience any net force?

SOLUTION Charge Q will not experience any net force if the forces exerted on it by charges q_1 and q_2 are equal and in opposite directions.

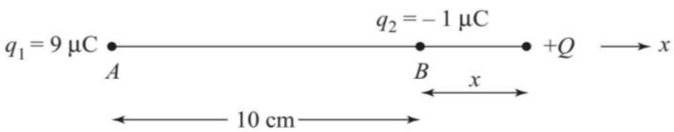


Fig. 11.2

It follows from Fig. 11.2 that charge Q will not experience forces in opposite direction if it lies at any point between AB. Let x be the distance of Q from q_2. Then forces exerted on Q by q_1 and q_2 respectively are

$$\mathbf{F}_1 = \frac{q_1 Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 (0.1+x)^2} = \frac{9\times10^{-6} Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 (0.1+x)^2}$$

and $$\mathbf{F}_2 = -\frac{q_2 Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 x^2} = \frac{1\times10^{-6} Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 x^2}$$

Net force on $Q = \mathbf{F}_1 + \mathbf{F}_2$

Net force on $Q = 0$ if $\mathbf{F}_1 + \mathbf{F}_2 = 0$

$$\Rightarrow \quad \frac{9\times10^{-6} Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 (0.1+x)^2} - \frac{1\times10^{-6} Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 x^2} = 0$$

$$\Rightarrow \quad 9 = \frac{(0.1+x)^2}{x^2}$$

$$\Rightarrow \quad 3 = \frac{0.1+x}{x}$$

$$\Rightarrow \quad x = 0.05 \text{ m} = 5 \text{ cm}$$

EXAMPLE 2 Two charges, each equal to $-4\ \mu C$, are held a certain distance apart. A charge Q is placed exactly mid-way between them. Find the magnitude and sign of Q so that the system of three charges is in equilibrium.

SOLUTION A system of charges is in equilibrium if no charge of the system experiences any net force.

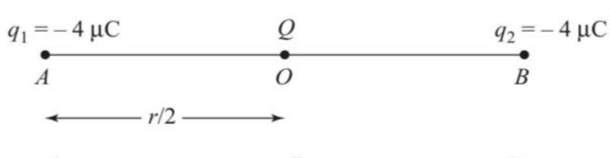


Fig. 11.3

Equilibrium of charge Q at O

Since Q is at the same distance from equal charges q_1 and q_2, it will be equilibrium for any positive or negative value, because it will experience equal and opposite forces.

Equilibrium of charge q_1 at A

If Q is negative, it will repel q_1. Also q_2 will repel q_1. Hence q_1 cannot be in equilibrium if Q is negative. So Q must be positive.

Force exerted on q_1 by Q is

$$\mathbf{F} = \frac{4\times10^{-6} Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 \left(\frac{r}{2}\right)^2}$$

Force exerted on q_1 by q_2 is

$$\mathbf{F}' = -\frac{\left(4\times10^{-6}\right)\times\left(4\times10^{-6}\right)\hat{\mathbf{i}}}{4\pi\varepsilon_0 r^2}$$

Net force on q_1 will be zero if $\mathbf{F} + \mathbf{F}' = 0$, i.e. if

$$\frac{4\times10^{-6} Q \hat{\mathbf{i}}}{4\pi\varepsilon_0 \left(\frac{r}{2}\right)^2} - \frac{\left(4\times10^{-6}\right)\times\left(4\times10^{-6}\right)\hat{\mathbf{i}}}{4\pi\varepsilon_0 r^2} = 0$$

$$\Rightarrow \quad Q = 1\times10^{-6}\ \text{C} = 1\ \mu C$$

It is easy to check that charge q_2 will also be in equilibrium. Hence the system of three charges will be in equilibrium if $Q = +1\ \mu C$.

EXAMPLE 3 Four point charges, each equal to $q = 4\ \mu C$, are held at the corners of a square $ABCD$ of side $a = 10$ cm. Find the magnitude and sign of a charge Q placed at the centre of the square so that the system of charges is in equilibrium.

SOLUTION

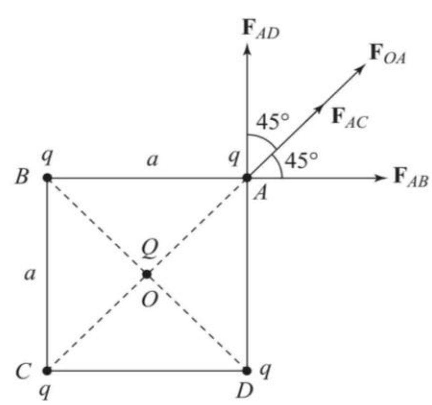


Fig. 11.4

$AC\,(=r) = \sqrt{2}a$. Let us consider the equilibrium of charge q at A (Fig. 11.4)

Force exerted on charge at A by charge at B is

$$\mathbf{F}_{AB} = \frac{kq^2}{a^2}\hat{\mathbf{i}}, \quad \text{where } k = \frac{1}{4\pi\varepsilon_0}$$

Similarly

$$\mathbf{F}_{AD} = \frac{kq^2}{a^2}\hat{\mathbf{j}}$$

$$\mathbf{F}_{AC} = \frac{kq^2}{(a\sqrt{2})^2}(\cos 45°\hat{\mathbf{i}} + \sin 45°\hat{\mathbf{j}})$$

$$= \frac{kq^2}{a^2 2\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

and $$\mathbf{F}_{OA} = \frac{kqQ}{\left(\frac{a}{\sqrt{2}}\right)^2}(\cos 45°\hat{\mathbf{i}} + \sin 45°\hat{\mathbf{j}})$$

$$= \sqrt{2}\frac{kqQ}{a^2}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

Net force on charge q at A in the x-direction is

$$\mathbf{F}_x = \left(\frac{kq^2}{a^2} + \frac{kq^2}{a^2 2\sqrt{2}} + \frac{\sqrt{2}\,kqQ}{a^2}\right)\hat{\mathbf{i}}$$

$\Rightarrow$ $$\mathbf{F}_x = \frac{kq}{a^2}\left(q + \frac{q}{2\sqrt{2}} + \sqrt{2}\,Q\right)\hat{\mathbf{i}} = \alpha\hat{\mathbf{i}}$$

where $$\alpha = \frac{kq}{a^2}\left(q + \frac{q}{2\sqrt{2}} + \sqrt{2}\,Q\right)$$

Similarly net force on charge q at A in the y-direction is

$$\mathbf{F}_y = \alpha\hat{\mathbf{j}}$$

$\therefore$ Resultant force on charge q at A is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{\alpha^2 + \alpha^2} = \sqrt{2}\alpha$$

Charge q will be in equilibrium if $F = 0$, i.e. if

$$\alpha = 0$$

i.e. if $$\frac{kq}{a^2}\left(q + \frac{q}{2\sqrt{2}} + \sqrt{2}\,Q\right) = 0$$

$\Rightarrow$ $$Q = -\frac{q}{4}(1 + 2\sqrt{2})$$

$$= -\frac{4\,\mu\text{C}}{4}(1 + 2\sqrt{2}) = -(1 + 2\sqrt{2})\,\mu\text{C}$$

4. Electric Field

An electric field exists at any point in the space surrounding a charge. To define the electric field, we place a small positive point charge q_0 at the point in space where the electric field is to be found and we measure the coulomb force $\mathbf{F}$ at that point. The electric field $\mathbf{E}$ is then given by

$$\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}}{q_0}$$

If a charge q is placed at a point where the electric field due to other charge or charges is $\mathbf{E}$, then the charge q will experience a force $\mathbf{F}$ given by

$$\mathbf{F} = q\mathbf{E}$$

(1) Electric field due to an isolated point charge Electric field at a distance r from a source charge q is given by

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

For a positive charge $(+q)$, vector $\mathbf{E}$ is directed radially outwards from it and for a negative charge $(-q)$, $\mathbf{E}$ is directed radially inwards it. Because electric field $\mathbf{E}$ is vector quantity, the net electric field due to several charges is given by the vector sum of the electric fields due to the individual charges.

(2) Electric field due to an electric dipole A pair of equal and opposite point charges separated by a certain distance is called an electric dipole.

Case (a): Electric field at a point on the axis of a dipole
Let $2a$ be the separation between point charges $-q$ and $+q$ (Fig. 11.5).

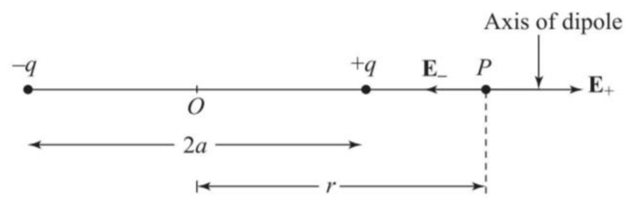


Fig. 11.5

Electric fields at P due to $+q$ and $-q$ respectively are

$$\mathbf{E}_+ = \frac{q\hat{\mathbf{i}}}{4\pi\varepsilon_0(r-a)^2}$$

$$\mathbf{E}_- = -\frac{q\hat{\mathbf{i}}}{4\pi\varepsilon_0(r+a)^2}$$

Electric field at point P is

$$\mathbf{E}_a = \mathbf{E}_+ + \mathbf{E}_-$$

$$= \frac{2q(2\mathbf{a})r}{4\pi\varepsilon_0(r^2 - a^2)^2} = \frac{2\mathbf{p}r}{4\pi\varepsilon_0(r^2 - a^2)^2}$$

where $\mathbf{p} = q(2\mathbf{a})$ is the dipole moment and $2\mathbf{a}$ is the vector distance between charges $-q$ and $+q$. Dipole moment $\mathbf{p}$ is a vector quantity directed from $-q$ to $+q$.
For a very short dipole $(a \ll r)$

$$\mathbf{E}_a = \frac{2\mathbf{p}}{4\pi\varepsilon_0 r^3}$$

Case (b): Electric field at a point on the perpendicular bisector (equatorial plane) of a dipole

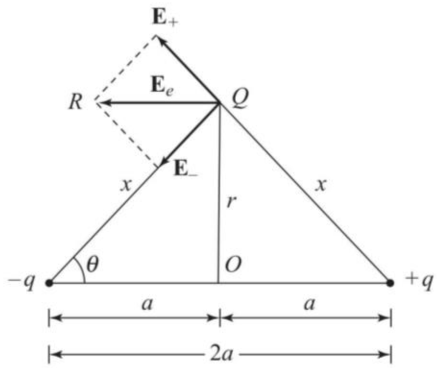


Fig. 11.6

Electric fields at point Q due to $+q$ and $-q$ are (see Fig. 11.6)

$$\mathbf{E}_+ = \frac{q}{4\pi\varepsilon_0 x^2} \quad \text{and} \quad \mathbf{E}_- = \frac{q}{4\pi\varepsilon_0 x^2}$$

The magnitude of the resultant electric field at Q is

$$E_e = E_+\cos\theta + E_-\cos\theta$$

Using $\cos\theta = \dfrac{a}{x}$ and $x = \sqrt{r^2 + a^2}$, we get

$$E_e = \frac{q(2a)}{4\pi\varepsilon_0 (r^2 + a^2)^{3/2}} \quad \text{directed from } Q \text{ to } R$$

In vector form

$$\mathbf{E}_e = -\frac{\mathbf{p}}{4\pi\varepsilon_0 (r^2 + a^2)^{3/2}}$$

For a very short dipole ($a \ll r$)

$$\mathbf{E}_e = -\frac{\mathbf{p}}{4\pi\varepsilon_0 r^3}$$

> **Note**
> (i) The direction the electric field at a point on the axial line of a dipole is along the dipole moment.
> (ii) The direction of the electric field at a point on the equatorial line of a dipole is antiparallel to the dipole moment.

(3) Electric field due to a uniformly charged conducting rod A conducting rod AB of negligible thickness and length L carries a charge a charge Q uniformly distributed on it. To find the electric field at point P at a distance a from end B (Fig. 11.7), we consider a small element of length dx of the rod located at a distance x from end B.

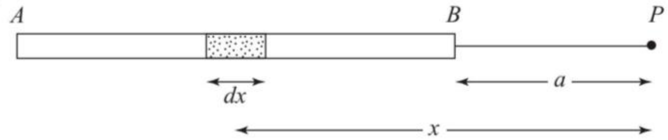


Fig. 11.7

Charge of element is $dq = \dfrac{Q}{L}\,dx = \lambda\,dx$ where $\lambda = \dfrac{Q}{L}$ is the linear charge density. The electric field due to the element at point P is

$$dE = \frac{dq}{4\pi\varepsilon_0 x^2} = \frac{\lambda\,dx}{4\pi\varepsilon_0 x^2}$$

Hence
$$E = \int dE = \frac{\lambda}{4\pi\varepsilon_0}\int_a^{(L+a)} \frac{dx}{x^2} = \frac{\lambda}{4\pi\varepsilon_0}\left|-\frac{1}{x}\right|_a^{(L+a)}$$

$$\Rightarrow \quad E = -\frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{(L+a)} - \frac{1}{a}\right]$$

$$\Rightarrow \quad E = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{L}{a(L+a)}\right]$$

If Q is positive, **E** is directed from left to right.

(4) Electric field due to a uniformly charged ring (or loop) of wire at a point on its axis Consider a ring of radius R carrying a chare Q distributed uniformly on it. To find electric field at a point P on its axis at a distance x from the centre O, consider an element of length dl (Fig. 11.8). The charge of the element is

$$dq = \frac{Q\,dl}{2\pi R}$$

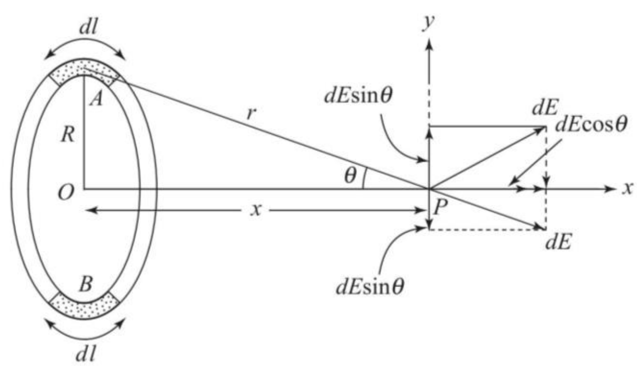


Fig. 11.8

The electric field due to element A is dE given by

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}$$

There is a similar electric field at P due to element a diametrically opposite point B. The x components of electric fields due to these elements add up while the y components cancel. Hence

$$E = \int dE \cos\theta$$

$$= \int \frac{dq \cos\theta}{4\pi\varepsilon_0 r^2}$$

$$= \frac{Q}{2\pi R} \times \frac{1}{4\pi\varepsilon_0} \times \frac{x}{(R^2 + x^2)^{3/2}} \int dl$$

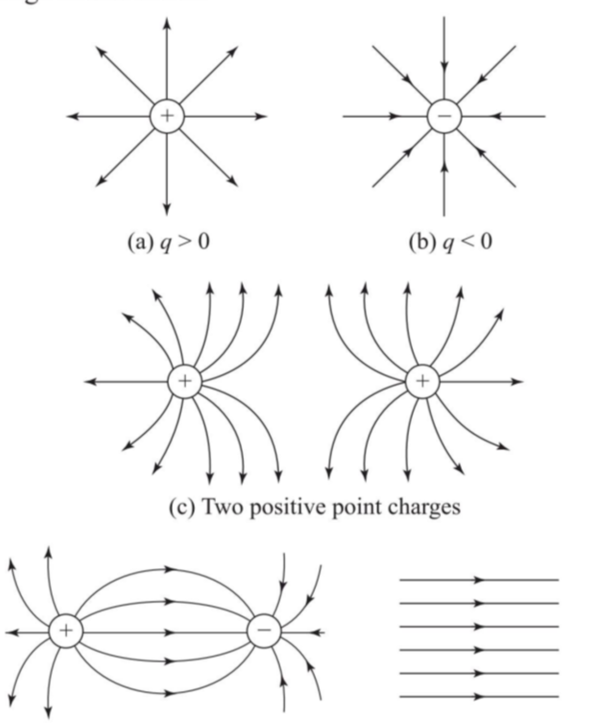
$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qx}{(R^2+x^2)^{3/2}}\right] \quad \left(\because \int dl = 2\pi R\right)$$

The direction of **E** is from O to P if charge Q is positive.

5. Electric Field Lines

Electric field lines of an electrostatic field give a pictorial representation of the field. An electric field line is a curve, the tangent to which at a point gives the direction of the electric field at that point.

Figure 11.9 shows field line patterns around some charge distributions.



(a) $q > 0$ (b) $q < 0$

(c) Two positive point charges

(d) Electric Dipole (e) Uniform electric field

Fig. 11.9

Properties of Electric Field Lines

(i) The tangent to a field line at any point gives the direction of electric field at that point.

(ii) Field lines originate from a positive charge and terminate on a negative charge.

(iii) No two field lines intersect.

(iv) Field lines are closer together in the region where the field is stronger and farther apart where the field is weaker.

(v) The number of field lines originating or ending on a charge is proportional to the magnitude of the charge.

> **Note** Electric field line due to a charge distribution never forms a closed loop. But if the electric field is induced by a time-varying magnetic field, its field line forms a closed loop.

6. Electric Flux

The electric flux through a surface in an electric field is a measure of the number of electric field lines passing through the surface.

For a plane surface of surface area **S** in an electric field **E**, the electric flux ϕ is defined as

$$\phi = \mathbf{E}\cdot\mathbf{S} = ES\cos\theta$$

where **S** is called the area vector, its magnitude is S and its direction is normal to the surface and away from it. Angle θ is the angle between **E** and **S**.

For a curved surface,

$$\phi = \int \mathbf{E}\cdot d\mathbf{S} = \int (\mathbf{E}\cdot\hat{\mathbf{n}})\, dS$$

where $\hat{\mathbf{n}}$ is a unit outward normal to the surface. dS is the surface area of an element of the surface (Fig. 11.10). The SI unit of electric flux is $\text{NC}^{-1}\ \text{m}^2$ or Vm (volt metre).

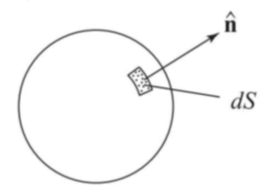


Fig. 11.10

7. Gauss's Law in Electrostatics

Gauss's law states that the electric flux through a closed surface **S** in an electric field **E** is equal to $\frac{q}{\varepsilon_0}$, where q is the net charge enclosed in the surface and ε_0 is electrical permittivity of vacuum.

$$\oint_S \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \qquad (1)$$

Gauss's law is used to obtain the expression for the electric field due to linear, surface and volume charge distributions which are uniform and symmetric so that a proper and convenient closed surface (called the Gaussian surface) can be chosen to evaluate the surface integral in Eq. (1).

Some Important Points about Gauss's Law

(1) Gauss's law holds for any closed surface of any shape or size.

(2) The surface that we choose to evaluate electric flux [i.e. to evaluate the surface integral in Eq. (1)] is called Gaussian surface.

(3) If the Gaussian surface is so chosen that there are some charges outside and some inside the surface, then charge q on the R.H.S. of Eq. (1) is the net charge (taking into account the sign of charges) enclosed inside the surface but electric field $\mathbf{E}$ on the L.H.S. of Eq. (1) is the electric field due to all the charges both inside and outside the surface.

(4) The exact location of charges inside Gaussian surface does not affect the value of the electric flux.

(5) If Coulomb's law did not hold, Gauss's law also would not hold.

8. Applications of Gauss's Law

(1) Electric field due to a thin infinitely long straight charged rod or wire Electric field at a point at a perpendicular distance r from a thin, infinitely long straight rod or wire carrying a uniform linear charge density λ is given by

$$\mathbf{E} = \frac{\lambda \hat{\mathbf{n}}}{2\pi \varepsilon_0 r}$$

where $\lambda = q/L$ is the charge per unit length of the rod and $\hat{\mathbf{n}}$ is a unit vector pointing away from the rod if q is positive and towords it if q is negative.

(2) Electric field due to a thin sheet of charge Electric field at a perpendicular distance r from a thin, flat and infinite sheet carrying a uniform surface charge density σ is given by

$$\mathbf{E} = \frac{\sigma \hat{\mathbf{n}}}{2\varepsilon_0}$$

where $\sigma = q/A$ is charge per unit area and $\hat{\mathbf{n}}$ is a unit vector pointing away from the sheet if q is positive and towards it if q is negative. Notice that $\mathbf{E}$ is independent of r, the distance from the sheet.

(3) Electric field due to a thin charged spherical shell Electric field at a distance r from a spherical shell of radius R carrying a surface charge density σ $(= q/4\pi R^2)$ is given by

$$\mathbf{E} = \frac{q\hat{\mathbf{n}}}{4\pi\varepsilon_0 r^2} = \frac{\sigma R^2 \hat{\mathbf{n}}}{\varepsilon_0 r^2} \quad (\text{for } r > R)$$

$$= \frac{\sigma \hat{\mathbf{n}}}{\varepsilon_0} \quad (\text{for } r = R)$$

$$= \text{zero} \quad (\text{for } r < R)$$

where $\hat{\mathbf{n}}$ is a unit vector pointing radially outwards if q is positive and inwards if q is negative.

9. Electric Potential

The electric potential at a point in an electrostatic field is the work per unit charge that is done to bring a small charge in from infinity to that point along any path.

$$V = \lim_{q_0 \to 0} \frac{W}{q_0}$$

(1) Electric potential due to an isolated point charge Electric potential at a point P in the electric field of a point charge is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

where r is the distance of the point P from the charge. This potential is spherically symmetric around the point, i.e. it depends only on r for a given charge q. Since potential is a scalar function, the spherical symmetry means that the potential at a point does not depend upon the direction of that point with respect to the point charge; it only depends on the distance of the point from the charge.

Notice that the potential due to a positive charge $(q > 0)$ is positive, it is negative in the neighbourhood of an isolated negative charge $(q < 0)$.

(2) Electric potential due to two point charges To find the electric potential at a point in the electric field due to two or more charges, we first calculate the potential due to each charge, assuming that all other charges are absent, and then simply add these individual contributions. Since, unlike electric field, electric potential is a scalar, the addition here is the *ordinary* sum, not a *vector* sum.

The potential at any point due to two point charges q_1 and q_2 is, therefore, simply given by

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

where r_1 and r_2 are the distances of the point in the question from charges q_1 and q_2 respectively.

(3) Electric potential due to many point charges The potential at any point due to a system of N point charges is given by

$$V = V_1 + V_2 + \ldots + V_N = \frac{1}{4\pi\varepsilon_0} \sum_{n=1}^{N} \frac{q_n}{r_n}$$

10. Relation between E and V

Electric field is the negative gradient of potential. This means that the potential decreases along the direction of the electric field.

$$E = -\frac{dV}{dr}$$

11. Electric Potential Energy

The electric potential energy of a system of point charges is defined as the amount of work done to assemble this system of charges by bringing them in from an infinite distance. We assume that the charges were at rest when they were infinitely separated, i.e. they had no initial kinetic energy.

The electric potential energy of two point charges q_1 and q_2 separated by a distance r_{12} as shown in Fig. 11.11 (a) is given by

$$U_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

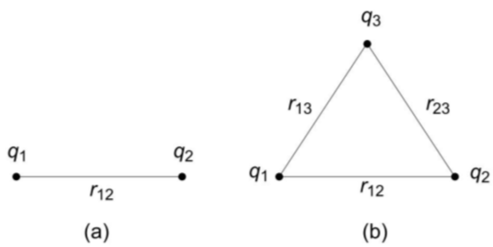


Fig. 11.11

The electric potential energy of a system of three point charges as shown in Fig. 11.11 (b) is given by

$$U = U_{12} + U_{23} + U_{31} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}\right)$$

This expression can be generalized for any number of charges.

12. The Electron-Volt

The SI unit of potential energy is the *joule*. In atomic physics a more convenient unit called the *electron-volt* (written as eV) is used. *An electron-volt is the potential energy gained or lost by an electron in moving through a potential difference of 1 volt.* Since the magnitude of charge on an electron is 1.6×10^{-19} C,

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

13. Potential Energy of an Electric Dipole in an External Electric Field

When a dipole is placed in a uniform electric field E, as shown in Fig. 11.12, it experiences a torque given by

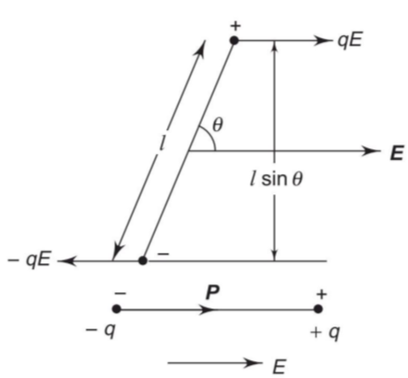


Fig. 11.12

$$\tau = p\,E \sin\theta$$

where θ is the angle between the line joining the two charges and the electric field. In vector form

$$\tau = \mathbf{p} \times \mathbf{E}$$

The torque tends to rotate the dipole to a position where $\theta = 0$, i.e, $\mathbf{p}$ is parallel to $\mathbf{E}$.

The electric potential energy of a dipole is

$$U = -\mathbf{p} \cdot \mathbf{E}$$

14. Additional Useful Formulae

(1) Electric field and potential due to a group of charges

(i) Charge q at each vertex of an equilateral triangle of side a (Fig. 11.13).

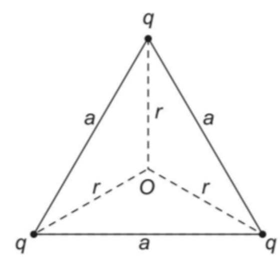


Fig. 11.13

At centroid O, $E_0 = 0$ and $V_0 = \dfrac{3q}{4\pi\varepsilon_0 r}$

where $r = \dfrac{a}{\sqrt{3}}$.

(ii) Charge q at each vertex of a square of side a (Fig. 11.14).

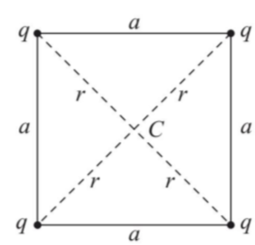


Fig. 11.14

At center C, $E_c = 0$ and $V_c = \dfrac{4q}{4\pi\varepsilon_0 r}$; $r = \dfrac{a}{\sqrt{2}}$

Note In the above two cases, if one of the charges is removed from a vertex, the net electric field at O and C is $E = q/4\pi\varepsilon_0 r^2$, directed towards the empty vertex.

(iii) For Fig. 11.15,

$$V_c = 0 \quad \text{and} \quad E_c = \frac{2\sqrt{2}q}{4\pi\varepsilon_0 r^2}$$

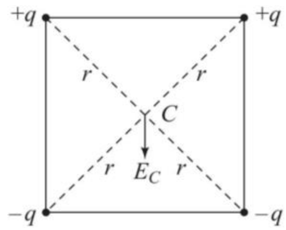


Fig. 11.15

(iv) For Fig. 11.16,

$$V_c = 0$$

$$E_c = 0$$

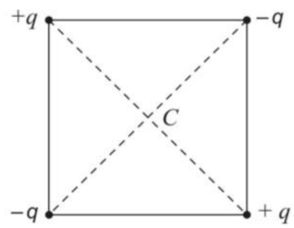


Fig. 11.16

(v) Infinite number of charges, each equal to q, placed on the x-axis at $x = r, x = 2r, x = 4r\ldots$ and so on. Electric field at origin O is

$$E_0 = \frac{q}{4\pi\varepsilon_0 r^2}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+\cdots\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\left(1+\frac{1}{4}+\frac{1}{16}+\cdots\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\times\frac{1}{\left(1-\frac{1}{4}\right)} = \frac{q}{3\pi\varepsilon_0 r^2}$$

Potential at O is

$$V_0 = \frac{q}{4\pi\varepsilon_0 r}\left(1+\frac{1}{2}+\frac{1}{4}+\cdots\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r}\times\frac{1}{\left(1-\frac{1}{2}\right)} = \frac{q}{2\pi\varepsilon_0 r}$$

(vi) A short electric dipole of dipole moment p (Fig. 11.17)

At point A, $E_r = \dfrac{2p\cos\theta}{4\pi\varepsilon_0 r^3}$ and $E_\theta = \dfrac{p\sin\theta}{4\pi\varepsilon_0 r^3}$

Net electric field at A is $E_A = \sqrt{E_r^2 + E_\theta^2}$

$$= \frac{p}{4\pi\varepsilon_0 r^3}(3\cos 2\theta + 1)^{1/2}$$

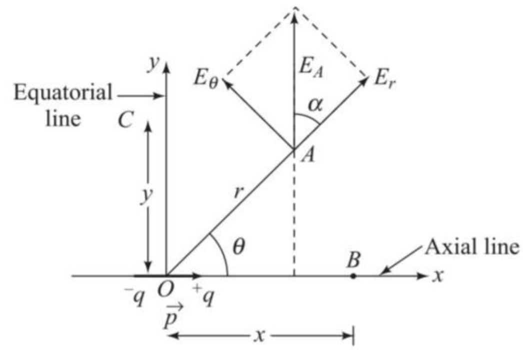


Fig. 11.17

Also $\tan\alpha = \dfrac{1}{2}\tan\theta$. Angle between $\boldsymbol{p}$ and $\boldsymbol{E}_A$ is $(\alpha + \theta)$.

At point B on axial line, $E_B = \dfrac{2p}{4\pi\varepsilon_0 x^3}$ $(\because\ \theta = 0°)$

At point C on equatorial line, $E_C = \dfrac{p}{4\pi\varepsilon_0 y^3}$ $(\because\ \theta = 90°)$

Electric potential at A is $V_A = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

At point B; $V_B = \dfrac{p}{4\pi\varepsilon_0 r^2}$

At point C; $V_C = 0$

(2) Electric field and potential due to some charge distributions $\left(\text{Linear charge density } \lambda = \dfrac{Q}{L}\right)$

(i) Charge Q distributed uniformly on a rod of length L (Fig. 11.18)

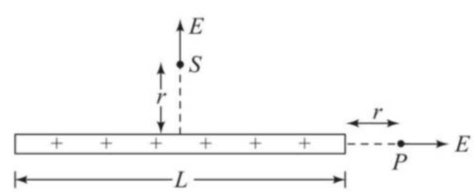


Fig. 11.18

At point P, $E_P = \dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{r(r+L)}\right]$

$$V_P = \frac{Q}{4\pi\varepsilon_0 L}\log_e\left(\frac{r+L}{r}\right)$$

(ii) Charge Q distributed uniformly on a conducting sphere or shell of radius R (Fig. 11.19) $\left(\text{Surface charge densitky } \sigma = \dfrac{Q}{4\pi R^2}\right)$

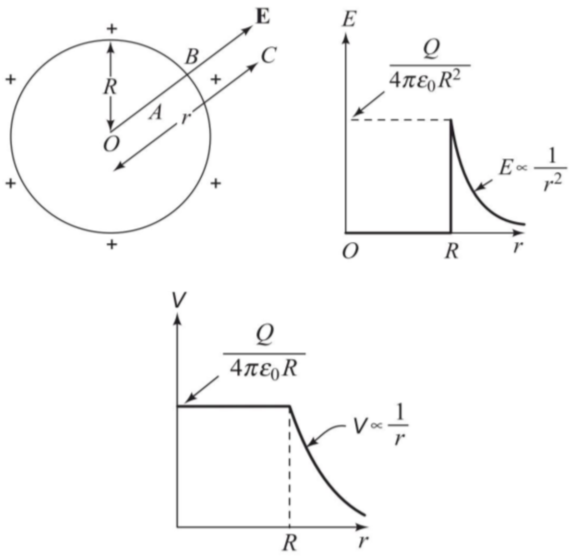


Fig. 11.19

At point C outside the sphere or shell ($r > R$), $E_C = \frac{Q}{4\pi\varepsilon_0 r^2}$

At point B just outside the surface ($r = R$), $E_B = \frac{Q}{4\pi\varepsilon_0 R^2}$

At point A inside the sphere or shell ($r < R$), $E_A = 0$

Potential at center O of sphere or shell, $V_0 = \frac{Q}{4\pi\varepsilon_0 R}$

At points inside ($r < R$), $V_A = \frac{Q}{4\pi\varepsilon_0 R} = V_B$ (at surface)

At point C outside ($r > R$), $V_C = \frac{Q}{4\pi\varepsilon_0 R}$

(iii) Charge Q distributed uniformly on a ring of radius R. (Fig. 11.20) Electric field at a point P on the axis is

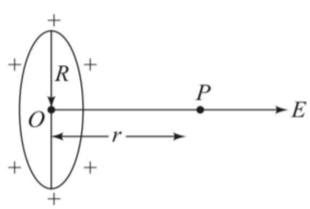


Fig. 11.20

$$E_P = \frac{Q}{4\pi\varepsilon_0} \times \frac{r}{\left(R^2 + r^2\right)^{3/2}}$$

E is maximum at $r = \pm\frac{R}{\sqrt{2}}$ and

$$E_{max} = \frac{1}{4\pi\varepsilon_0} \times \frac{2Q}{3\sqrt{3}R^2}$$

Electric potential at point P is

$$V_P = \frac{Q}{4\pi\varepsilon_0} \times \frac{1}{\left(R^2 + r^2\right)^{1/2}}$$

V is maximum at $r = 0$ (i.e. at centre O) and $V_{max} = \frac{Q}{4\pi\varepsilon_0 R}$

(iv) Consider a semi-circular rod of radius R having a charge $+Q$ distributed uniformly on it. Linear charge density is (Fig. 11.21).

$$\lambda = \frac{Q}{\pi R}$$

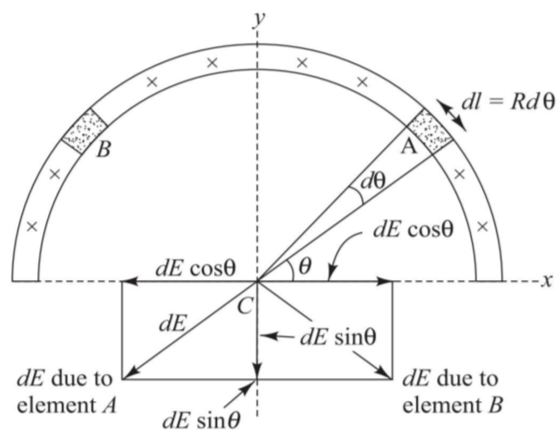


Fig. 11.21

Consider a small element at A of length $dl = Rd\theta$ at A. The charge of the element is $dQ = \lambda dl = \lambda R d\theta$. The electric field due to this element at centre C is

$$dE = \frac{dQ}{4\pi\epsilon_0 R^2} = \frac{\lambda d\theta}{4\pi\epsilon_0 R}$$

The x and y components of dE are $dE\cos\theta$ and $dE\sin\theta$. Now consider an element at a symmetric point B. The x component of electric field of this element will cancel with that of element at A but y component will add up. Hence the electric field at C due to the complete semi-circular rod is

$$E = \int_0^{\pi} dE\sin\theta$$

$$= \frac{\lambda}{4\pi\epsilon_0 R}\int_0^{\pi}\sin\theta\, d\theta$$

$$= \frac{\lambda}{4\pi\epsilon_0 R}\Big|-\cos\theta\Big|_0^{\pi}$$

or, $$E = \frac{\lambda}{2\pi\epsilon_0 R}$$

directed vertically downwards away from the rod. If the rod carries a negative charge ($-Q$), then the electric field is directed vertically upwards towards the rod.

The electric potential at point C is

$$V = \int \frac{dQ}{4\pi \epsilon_0 R}$$

$$= \frac{\lambda}{4\pi \epsilon_0} \int_0^{\pi} d\theta \qquad (\because \quad dQ = \lambda R d\theta)$$

or $$V = \frac{\lambda}{4 \epsilon_0}$$

The above results also hold for a non-conducting rod.

(3) Potential energy of a system of charges

(i) Charge Q kept at each vertex of an equilateral of side a. Potential energy is (Fig. 11.22).

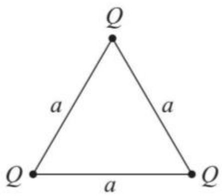


Fig. 11. 22

$$U = \frac{3Q^2}{4\pi \varepsilon_0 a}$$

(ii) Charge Q kept at each vertex of a square of side a (Fig. 11.23)

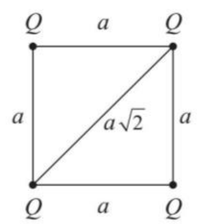


Fig. 11.23

$$U = \frac{4 \times Q^2}{4\pi \varepsilon_0 a} + \frac{2Q^2}{4\pi \varepsilon_0 (a\sqrt{2})}$$

$$= \frac{Q^2}{4\pi \varepsilon_0 a}(4 + \sqrt{2})$$

(iii) An electric dipole of dipole moment **p** placed in uniform electric field **E** with angle θ between **p** and **E** Torque $\tau = \mathbf{p} \times \mathrm{E}$ and potential energy $U = -\mathbf{p}.\mathbf{E}$. The zero of potential energy is taken at $\theta = 90°$.

(a) When $\theta = 0°$, $\tau = 0$, U = minimum = $-pE$ (stable equilibrium)

(b) When $\theta = 90°$, τ = maximum = pE, $U = 0$

(c) When $\theta = 180°$, $\tau = 0$, U = maximum = pE (unstable equilibrium)

(iv) Work done in turning a dipole from angle θ_1 to angle θ_2 is

$$W = pE(\cos\theta_1 - \cos\theta_2)$$

If $\theta_1 = 0°$ and $\theta_2 = 180°$, $W = 2pE$

EXAMPLE 4 An electric dipole AB is placed along the x-axis with its charges placed at distances x_1 and x_2 from a very long thin wire having a uniform linear positive charge density λ as shown in Fig. 11.24. Choose the correct option.

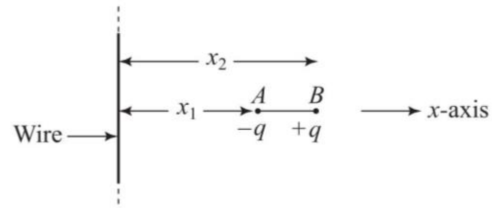


Fig. 11.24

The force experienced by the dipole is

(a) $\frac{\lambda q}{2\pi \varepsilon_0}\left(\frac{1}{x_1} - \frac{1}{x_2}\right)$ along –ve x-direction

(b) $\frac{\lambda q}{4\pi \varepsilon_0}\left(\frac{1}{x_1} - \frac{1}{x_2}\right)$ along +ve x-direction

(c) $\frac{\lambda q}{2\pi \varepsilon_0}(x_2 - x_1)$ along +ve x-direction

(d) $\frac{\lambda q}{2\pi \varepsilon_0}(x_2 + x_1)$ along –ve x-direction

SOLUTION The electric field due to the long wire at a point at a disance x from it is given by

$$E = \frac{\lambda}{2\pi \varepsilon_0 x}$$

As λ is +ve, the direction of E is along +ve x-direction. Hence the force experienced by charge ($-q$) at A is

$$\mathbf{F}_1 = -\frac{\lambda q \hat{\mathbf{i}}}{2\pi \varepsilon_0 x_1}$$

The force experiened by charge ($+q$) at B is

$$\mathbf{F}_2 = \frac{\lambda q \hat{\mathbf{i}}}{2\pi \varepsilon_0 x_2}$$

$\therefore$ Net force on the dipole is

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$$

$$= -\frac{\lambda q}{2\pi \varepsilon_0}\left(\frac{1}{x_1} - \frac{1}{x_2}\right)\hat{\mathbf{i}}$$

So the correct choice is (a).

EXAMPLE 5 A metallic solid sphere A of radius r has a charge q distributed uniformly on its surface. It is surrounded by a concentric metallic hollow sphere B of radius R. The electric field at point P at a distance x from O where $r < x < R$ will be

(a) $\dfrac{2q}{4\pi\varepsilon_0 R^2}$ (b) $\dfrac{q}{4\pi\varepsilon_0 (R-x)r}$

(c) $\dfrac{q}{4\pi\varepsilon_0 x^2}$ (d) zero

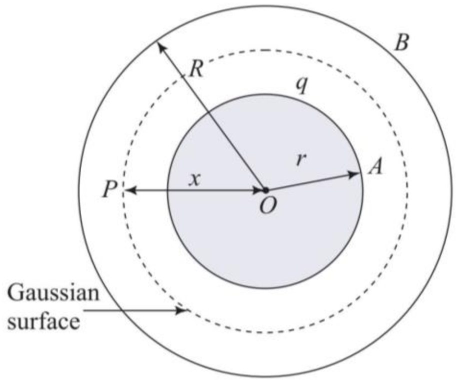


Fig. 11.25

SOLUTION The charge q on A induces a charge $-q$ on the inner side of B and a charge $+q$ on the outer side of B. To find electric field at P we draw a Gaussian spherical surface of radius x, shown dotted in Fig. 11.25. Since point P is inside B, according to Gauss's law, the electric field due to B at P is zero. Now flux through the Gaussian surface due to charge q on A is

$$\phi = \oint \mathbf{E}\cdot d\mathbf{S} = E \times 4\pi x^2$$

From Gauss's law, $\phi = \dfrac{q}{\varepsilon_0}$. Hence

$$\frac{q}{\varepsilon_0} = E \times 4\pi x^2$$

$\Rightarrow$ $$E = \frac{q}{4\pi\varepsilon_0 x^2},$$ which is choice (c).

EXAMPLE 6 In Example 5 above, if a charge Q is given to the spherical shell B, how will the electric field at P be affected?

SOLUTION In this case, the total charge on the outer surface of B will now be $(Q + q)$. Since point P lies inside this sphere, the electric field at P will remain unchanged equal to $\dfrac{q}{4\pi\varepsilon_0 x^2}$.

EXAMPLE 7 Figure 11.26 shows two point charges $+q$ and $+q$ and a metal rod of length L carrying a charge $-q$ with a part $L/3$ of its length inside a box. The electric flux through the box is

(a) $\dfrac{q}{3\varepsilon_0}$ (b) $\dfrac{4q}{3\varepsilon_0}$

(c) $\dfrac{2q}{3\varepsilon_0}$ (d) $\dfrac{5q}{3\varepsilon_0}$

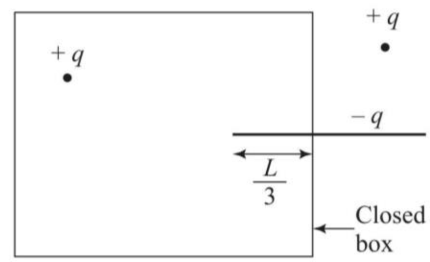


Fig. 11.26

SOLUTION Charge in rod of length $\dfrac{L}{3} = -\dfrac{q}{3}$. According to Gauss's law, the electric flux through a closed surface is equal to $\dfrac{q_{net}}{\varepsilon_0}$ where q_{net} is the net charge enclosed inside the surface; the charge outside the closed surface does not contribute to the flux. Now

$$q_{net} = +q - \frac{q}{3} = \frac{2q}{3}$$

Hence $$\phi = \frac{q_{net}}{\varepsilon_0} = \frac{2q}{3\varepsilon_0},$$ which is choice (c).

EXAMPLE 8 Three thin concentric spherical shells 1, 2 and 3 have radii r_1, r_2 and r_3 respectively. Charge $+q$ is given to shell 1 and charge $-2q$ is given to shell 3 as shown in Fig. 11.27(a). If shell 2 is earthed and $r_3 = 2\,r_2$, the charge on the outer surface of shell 2 will be

(a) zero (b) q

(c) $-\dfrac{2q}{3}$ (d) $-\dfrac{q}{3}$

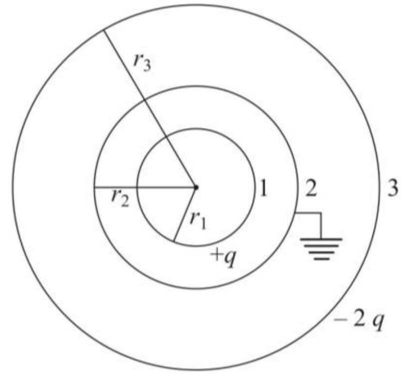


Fig. 11.27(a)

SOLUTION Let q_2 be the charge on the outer surface of shell 2. Charge $+q$ on shell 1 will induce a charge $-q$ on the inner surface of 2. Charge q_2 will induced charge $-q_2$ on the inner surface of 3 and $+q_2$ on its outer surface. Thus the outer surface of 3 will have a charge $(q_2 - 2q)$ as shown in Fig. 11.27(b).

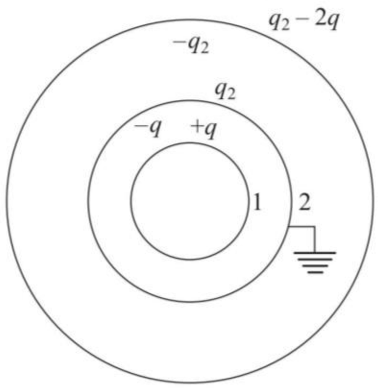


Fig. 11.27(b)

We know that the potential at any point inside a spherical shell is equal to that on its surface. Hence the potential on the outer surface of shell 2 will be

V_2 = potential due to charge $+q$ on 1 + potential due to charge $-q$ on inner surface of 2 + potential due to charge q_2 on outer surface of 2 + potential due to charge $-q_2$ on inner surface of 3 + potential due to charge $(q_2 - 2q)$

$$= \frac{q}{4\pi\varepsilon_0 r_2} - \frac{q}{4\pi\varepsilon_0 r_2} + \frac{q_2}{4\pi\varepsilon_0 r_2} - \frac{q_2}{4\pi\varepsilon_0 r_3} + \frac{(q_2 - 2q)}{4\pi\varepsilon_0 r_3}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2}{r_2} - \frac{q_2}{r_3} + \frac{(q_2 - 2q)}{r_3}\right]$$

Since shell 2 is earthed, its potential V_2 will be zero. Thus

$$\frac{q_2}{r_2} - \frac{q_2}{r_3} + \frac{(q_2 - 2q)}{r_3} = 0$$

$$\Rightarrow \qquad q_2 = \frac{2q\, r_2}{r_3}$$

Given $r_3 = 2\, r_2$. Hence $q_2 = q$, which is choice (b).

EXAMPLE 9 A charge q is placed at the corner of a cube of side l. The electric flux passing through the cube is

(a) $\frac{q}{\varepsilon_0}$ (b) $\frac{q}{6\varepsilon_0}$

(c) $\frac{q}{8\varepsilon_0}$ (d) $\frac{q}{32\varepsilon_0}$

SOLUTION It is clear from Fig. 11.28 that eight cubes, each of side l, are required to form a Gaussian surface so that the charge q at the corner of a small cube appears at the centre of the bigger cube. According to Gauss's law, the electric flux through the bigger cube = $\frac{q}{\varepsilon_0}$. Hence the electric flux through the given small cube = $\frac{q}{8\varepsilon_0}$. So the correct choice is (c).

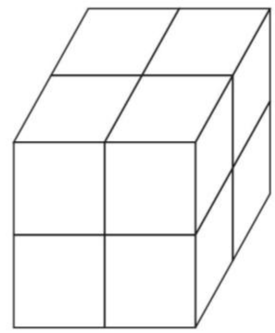


Fig. 11.28

15. Capacitance

When a conductor is given a charge Q, it acquires a potential V which is proportional to the charge given to it, i.e.

$$Q \propto V \quad \text{or} \quad Q = CV \quad \text{or} \quad C = \frac{Q}{V}$$

where C is a constant of proportionality and is called the capacitance which is defined as the amount of charge in coulomb necessary to increase the potential of a conductor by 1 volt. The SI unit of capacitance is farad (symbol F)

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}$$

The farad is a large unit. More practical units are microfarad (μF) and picofarad (pF).

$$1\ \mu\text{F} = 10^{-6}\ \text{F and } 1\ \text{pF} = 10^{-12}\ \text{F}$$

16. Energy of a Charged Conductor

A charged conductor has electric field in the region around it. If additional similar charge is given to the conductor, work has to be done against the electrical repulsive force. This work is stored in the form of potential energy which resides in the electric field. If a charge Q is given to a conductor of capacitance C, the potential energy in its electric field is given by

$$U = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \qquad (\because Q = CV)$$

17. Capacitance of a Single Spherical Conductor

Consider a spherical conductor of radius r having a charge Q. Since the electric field is normal to the surface of the sphere, the lines of force appear to originate from its centre, i.e. the charge Q may be supposed to be concentrated at the centre. Therefore, the potential is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$$

Since $C = \frac{Q}{V}$, the capacitance of the sphere is given by

$$C = 4\pi\varepsilon_0 r$$

Thus, the greater the radius of the sphere, the higher is its capacitance.

18. Capacitors

Any isolated system of two conducting bodies, of any shape and size, separated by a distance is called a capacitor. If two conductors, carrying equal and opposite charge Q have a potential difference V between them, then

$$Q = CV$$

where C is the capacitance of the capacitor and its value depends on the size, the shape, the separation between the conductors and the nature of the medium between them. If C_0 is the capacitance of the capacitor when the medium is air (or vacuum) and C_m its capacitance when the medium is a dielectric other than air, then the dielectric constant of the medium is given by

$$K = \frac{C_m}{C_0}$$

19. Expressions for Capacitance

1. *Parallel Plate Capacitor*

   The capacitance of a parallel plate capacitor is given by

   $$C = \frac{K\varepsilon_0 A}{d}$$

   where A is the area of each plate and d is the distance between them. K is dielectric constant of the material between the plates. For air or vacuum, $K = 1$.

2. *Spherical Capacitor*

   A spherical capacitor consists of a solid charged sphere of radius a surrounded by a concentric hollow sphere of radius b. Its capacitance is given by

   $$C = 4\pi\varepsilon_0 K\left(\frac{ab}{b-a}\right)$$

3. *Cylindrical Capacitor*

   A cylindrical capacitor consists of two co-axial cylinders and its capacitance is given by

   $$C = \frac{2\pi\varepsilon_0 K l}{\log_e\left(\frac{b}{a}\right)}$$

   where l is the length of each cylinder and a and b are the radii of the inner and outer cylinders.

4. If the space between the plates of a parallel plate capacitor is filled with two media of thicknesses d_1 and d_2 having dielectric constants K_1 and K_2, then the capacitance of the capacitor is given by

   $$C = \frac{\varepsilon_0 A}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

20. Capacitors in Parallel and Series

In parallel arrangement of capacitors, the potential difference across individual capacitors is the same and the total charge is shared by them in the ratio of their capacitances.

$$Q = Q_1 + Q_2 + Q_3 + \cdots$$

and $$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3} = \ldots$$

$\therefore$ $$C = C_1 + C_2 + C_3 + \ldots$$

In series arrangement of capacitors, the charge on each capacitor is the same and the total potential difference is shared by them in the inverse ratio of their capacitances.

$$Q = C_1 V_1 = C_2 V_2 = C_3 V_3 = \ldots$$

$$V = V_1 + V_2 + V_3 + \cdots$$

Therefore, the effective capacitance of the combination is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

21. Energy Stored in a Capacitor

As in the case of a charged conductor, the energy stored in a capacitor is given by

$$U = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

where Q = charge on each plate of the capacitor, V = potential difference between plates and C = capacitance of the capacitor. This potential energy resides in the electric field in the medium between the plates.

22. Loss of Energy on Sharing Charges

If two charged bodies carrying charges Q_1 and Q_2 and having capacitances C_1 and C_2 are connected with each other, then their common potential after the sharing of charges is given by

$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

where V_1 and V_2 are the initial potentials of the charged bodies. The loss of energy is given by

$$\Delta E = \frac{1}{2}\,\frac{C_1 C_2}{(C_1 + C_2)}\,(V_1 - V_2)^2$$

23. Force between Plates of a Parallel Plate Capacitor

The plates of a capacitor carry equal and opposite charges. Therefore, they exert an attractive force on each other which is given by

$$F = \frac{Q^2}{2K\varepsilon_0 A}$$

The force per unit area of the plates is

$$f = \frac{F}{A} = \frac{Q^2}{2K\varepsilon_0 A^2} = \frac{\sigma^2}{2K\varepsilon_0}$$

where σ is the charge per unit area.

EXAMPLE 10 A parallel plate capacitor of capacitance 10 μF and plate separation 0.5 mm is connected to a 20 V battery.

(a) What is the charge on each plate?

(b) What is the energy stored in the capacitor?

(c) What is the electric field between the plates?

(d) If the battery is disconnected and then the plate separation is doubled, what are the answers to parts (a), (b) and (c) above?

(e) If the battery is kept connected and the plate separation is doubled, then what are the answers to parts (a), (b) and (c) above?

SOLUTION

(a) $Q = CV = (10 \times 10^{-6}) \times 20 = 200 \times 10^{-6}\ \text{C} = 200\ \mu\text{C}$

(b) $U = \frac{1}{2} CV^2 = \frac{1}{2}(10 \times 10^{-6}) \times (20)^2 = 2 \times 10^{-3}\ \text{J}$

(c) $E = \frac{V}{d} = \frac{20}{(0.5 \times 10^{-3})} = 4 \times 10^4\ \text{Vm}^{-1}$

> **Note** The engery is stored in the electric field between the plates

(d) If the battery is disconnected, the charge on the plates remains the same but the potential difference between the plates will charge. If the separation between the plates is doubled capacitance becomes

$$C' = \frac{\varepsilon_0 A}{2d} = \frac{C}{2} = 5\ \mu\text{F} = 5 \times 10^{-6}\ \text{F}$$

and potential difference between the plates becomes

$$V' = \frac{Q}{C'} = \frac{200\ \mu\text{C}}{5\ \mu\text{F}} = 40\ \text{V}$$

$$\therefore \quad U' = \frac{1}{2} C' V'^2$$

$$= \frac{1}{2} \times 5 \times 10^{-6} \times (40)^2 = 4 \times 10^{-3}\text{J}$$

Alternatively, $U' = \frac{Q^2}{2C'} = \frac{(200 \times 10^{-6})^2}{2 \times (5 \times 10^{-6})} = 4 \times 10^{-3}\text{J}$

$$E' = \frac{V'}{d'} = \frac{40}{1 \times 10^{-3}} = 4 \times 10^4\ \text{Vm}^{-1}$$

(e) If the battery is kept connected, the potential difference between the plates always remains equal to the emf of the battery and hence is constant = 20 V. If d is doubled,

Capacitance becomes $C' = \frac{\varepsilon_0 A}{2d} = \frac{C}{2}$

$$= 5\ \mu\text{F} = 5 \times 10^{-6}\ \text{F}$$

Charge becomes $Q'' = C'V = 5 \times 10^{-6} \times 20 = 10^{-4}\ \text{C}$

Energy stored becomes $U'' = \frac{1}{2} C' V^2$

$$= \frac{1}{2} \times 5 \times 10^{-6} \times (20)^2 = 10^{-3}\text{J}$$

Electric field becomes $= \frac{V}{2d} = \frac{20}{20 \times (0.5 \times 10^{-3})}$

$$= 2 \times 10^4\ \text{Vm}^{-1}$$

EXAMPLE 11 Find the charge on each capacitor shown in Fig. 11.29(a). Given C_1 = 2 μF, and C_2 = 2 μF and C_3 = 1 μF.

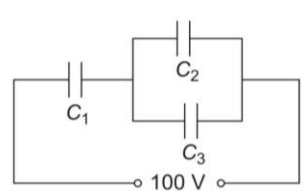


Fig. 11.29(a)

SOLUTION The capacitance of the parallel combination of C_2 and C_3 is $C' = C_2 + C_3 = 3\ \mu\text{F}$

The circuit can be redrawn as shown in Fig. 11.29(b).

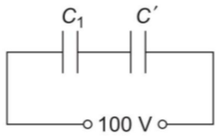


Fig. 11.29(b)

The equivalent capacitance C of series combination of C_1 and C' is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C'}$$

$$\Rightarrow \quad C = \frac{C_1 C'}{C_1 + C'} = \frac{2 \times 3}{2+3} = \frac{6}{5} = 1.2\,\mu\text{F}$$

Let Q_1, Q_2 and Q_3 be the charges on C_1, C_2, and C_3 respectively. Since C_1 and C' are in series, charge on C_1 = charge on $C' = Q_1$. Let V_1 and V' be the potential differences across C_1 and C' respectively. Then

$$Q_1 = C_1 V_1 = C'V' \Rightarrow \frac{V'}{V_1} = \frac{C_1}{C'} = \frac{2}{3}$$

But $\quad V_1 + V' = 100$. Therefore

$$V_1 + \frac{2}{3} V_1 = 100 \Rightarrow V_1 = 60 \text{ V}$$

Therefore $\quad V' = 100 - 60 = 40$ V. Hence

$$Q_1 = C_1 V_1 = 2\,\mu\text{F} \times 60 \text{ V} = 120\,\mu\text{C}$$

Since C_2 and C_3 are in parallel, potential difference V_2 across C_2 = potential difference V_3 across $C_3 = V' = 40$ V. Therefore,

Charge on capacitor $C_2 = C_2 V_2 = 2\,\mu\text{F} \times 40 \text{ V} = 80\,\mu\text{C}$
and Charge on capacitor $C_3 = C_3 V_3 = 1\,\mu\text{F} \times 40 \text{ V} = 40\,\mu\text{C}$
Thus $Q_1 = 120\,\mu\text{C}$, $Q_2 = 80\,\mu\text{C}$ and $Q_3 = 40\,\mu\text{C}$

EXAMPLE 12 In the network shown in Fig 11.29(a) in Example 11, find the total energy stored in the network.

SOLUTION Total (equivalent) capacitance of network is

$$C = 1.2\,\mu\text{F} = 1.2 \times 10^{-6}\text{ F}.$$

Total potential difference across the network is $V = 100$ V. Therefore, total energy stored is

$$U = \frac{1}{2}\,CV^2 = \frac{1}{2} \times (1.2 \times 10^{-6}) \times (100)^2 = 6 \times 10^{-3}\text{ J}$$

Notice that

$$U = U_1 + U_2 + U_3 = \frac{1}{2}\,C_1 V_1^2 + \frac{1}{2}\,C_2 V_2^2 + \frac{1}{2}\,C_3 V_3^2.$$

EXAMPLE 13 Two capacitors of capacitances C_1 and C_2 have a total capacitance of 20 μF when connected in parallel and a total capacitance 4.8 μF when connected in series. Find C_1 and C_2.

SOLUTION

$$C_1 + C_2 = 20 \qquad (1)$$

$$\frac{C_1 C_2}{C_2 + C_2} = 4.8 \qquad (2)$$

Using (1) in (2) we get

$$\frac{C_1\,C_2}{20} = 4.8 \Rightarrow C_1 C_2 = 96\,\mu\text{F}$$

$$\therefore \qquad C_2 = \frac{96}{C_1}$$

Using this in (2) we get

$$C_1^2 - 20C_1 + 96 = 0$$

The two roots of this equation are $C_1 = 8$ μF and 12 μF. Hence the two capacitors have capacitances of 8 μF and 12 μF.

EXAMPLE 14 Four capacitors of capacitances $C_1 = 1$ μF, $C_2 = 2$ μF, $C_3 = 3$ μF and $C_4 = 4$ μF are connected as shown in Fig. 11.30(a).

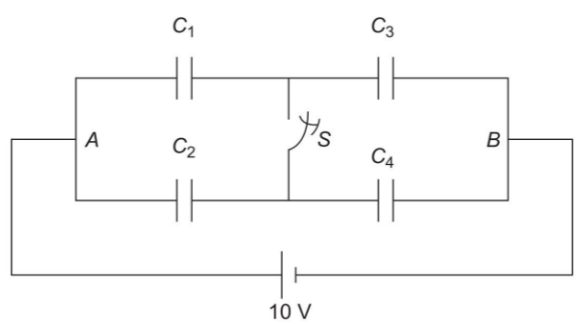


Fig. 11.30(a)

Find the potential difference across C_3 when (a) switch S is open and (b) switch S is closed.

SOLUTION

(a) When switch S is open, the circuit is [see Fig. 11.30(b)]

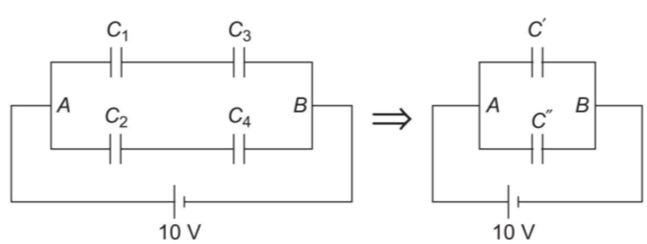


Fig. 11.30(b)

C_1 and C_3 are in series, charge on C_1 = charge on $C_3 = Q$ (say) and potential difference across the combination of C_1 and C_3 is 10 V.

$\therefore \qquad Q = C_1V_1 = C_3V_3 \Rightarrow C_1V_1 = C_3V_3$

$\Rightarrow \qquad V_1 = 3V_3$. Also $V_1 + V_3 = 10$

Hence $\quad V_3 = 2.5$ V.

(b) When switch S is closed, the circuit is as shown in Fig. 11.30(c).

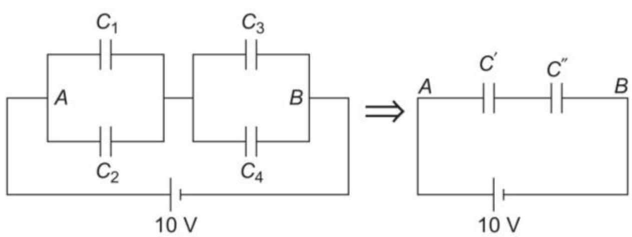


Fig. 11.30(c)

$$C' = 1 + 2 = 3\ \mu\text{F}$$

$$C'' = 3 + 4 = 7\ \mu\text{F}$$

Since C' and C'' are in series, the charge on C' = charge on C''

or $\qquad C'V' = C''V'' \Rightarrow 3V' = 7V''$

$\Rightarrow \qquad V' = \dfrac{7V''}{3}$

Also $\quad V' + V'' = 10 \Rightarrow \dfrac{7V''}{3} + V'' = 10$

$\Rightarrow \qquad V'' = 3$ V

Since C_3 and C_4 are in parallel, potential difference across C_3 = potential difference across C_4 = 3 V.

EXAMPLE 15 In Ex. 13 above, find the energy stored in the network of capacitors shown in Fig. 11.30(a) when (a) Switch S is open and (b) switch S is closed.

SOLUTION

(a) Refer to Fig. 11.30(b). The equivalent capacitance between point A and B is

$$C_a = C' + C''$$

$$= \frac{C_1 C_3}{C_1 + C_3} + \frac{C_2 C_4}{C_2 + C_4}$$

$$= \frac{1 \times 3}{(1+3)} + \frac{2 \times 4}{(2+4)} = \frac{25}{12}\mu\text{F} = \frac{25}{12} \times 10^{-6}\,\text{F}$$

Energy stored is

$$U_a = \frac{1}{2}C_a V^2 = \frac{1}{2} \times \frac{25}{12} \times 10^{-6} \times (10)^2 = 1.04 \times 10^{-4}\,\text{J}$$

(b) Refer to Fig. 11.30(c). The equivalent capacitance between point A and B is

$$C_b = \frac{C'C''}{C' + C''} = \frac{3 \times 7}{3+7} = \frac{21}{10}\mu\text{F} = \frac{21}{10} \times 10^{-6}\,\text{F}$$

Energy stored is

$$U_b = \frac{1}{2}C_b V^2 = \frac{1}{2} \times \frac{21}{10} \times 10^{-6} \times (10)^2 = 1.05 \times 10^{-4}\,\text{J}$$

EXAMPLE 16 The circuit shown in Fig. 11.31 consists of four capacitors C_1, C_2, C_3, and C_4 each of capacitance 4 μF and a battery of emf E = 5V. Find the potential difference between A and B.

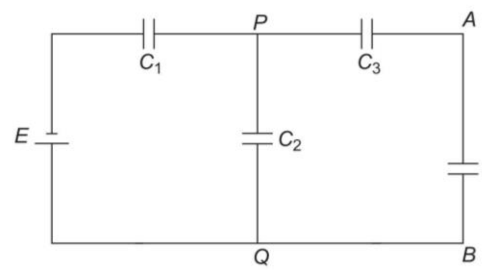


Fig. 11.31

SOLUTION Capacitors C_3 and C_4 are in series, their equivalent capacitance is

$$C' = \frac{C_3 C_4}{(C_3 + C_4)} = \frac{4 \times 4}{(4+4)} = 2\,\mu\text{F}$$

C' is in parallel with C_2.
Their equivalent capacitance is

$$C'' = C_2 + C' = 4 + 2 = 6\,\mu\text{F}$$

Hence the circuit can be redrawn as shown in Fig. 11.32.

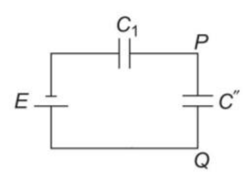


Fig. 11.32

If V_1 is the p.d. across C_1 and V'' across C'', then (since they are in series)

$$C_1 V_1 = C'' V''$$

$\Rightarrow \qquad 4V_1 = 6V'' \Rightarrow V_1 = \dfrac{3V''}{2}$

But $\qquad V_1 + V'' = E = 5$. Hence

$$\frac{3V''}{2} + V'' = 5 \Rightarrow V'' = 2\text{ V}$$

Thus the p.d. between P and Q = 2 V. By symmetry of C_3 and C_4, the p.d. between A and B = 1 V.

EXAMPLE 17 Three capacitors $C_1 = 1\ \mu F$, $C_2 = 2\ \mu F$ and $C_3 = 3\ \mu F$ are connected as shown in Fig. 11.33. Find the equivalent capacitance between points A and B.

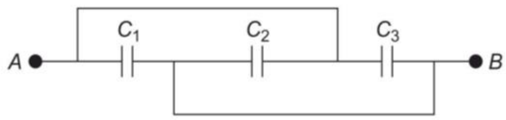


Fig. 11.33

SOLUTION Refer to Fig. 11.34.

Plate 1 of C_1, plate 4 of C_2 and plate 5 of C_3 are connected to point A. Plate 2 of C_1, plate 3 of C_2 and plate 6 of C_3 are connected to point B. Thus C_1, C_2 and C_3 are connected in parallel. Therefore, the equivalent capacitance C_{eq} between point A and B is given by

$$C_{eq} = C_1 + C_2 + C_3$$
$$= 1 + 2 + 3 = 6\ \mu F$$

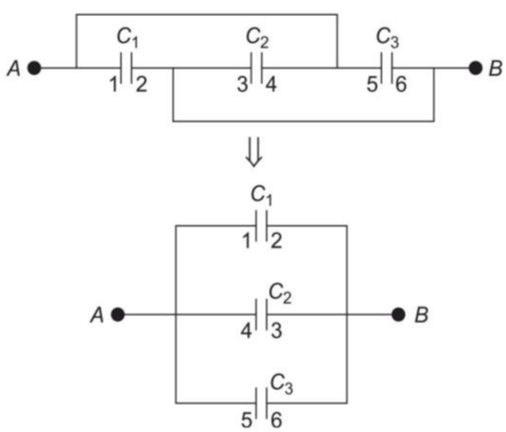


Fig. 11.34

EXAMPLE 18 In the network shown in Fig. 11.35(a), the equivalent capacitance between points P and Q is

(a) $1\ \mu F$ (b) $2\ \mu F$

(c) $3\ \mu F$ (d) $4\ \mu F$

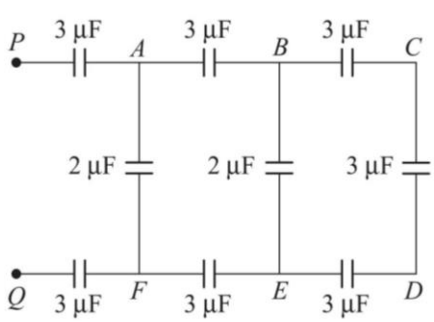


Fig. 11.35(a)

SOLUTION The given network can be simplified as shown in Fig. 11.35(b).

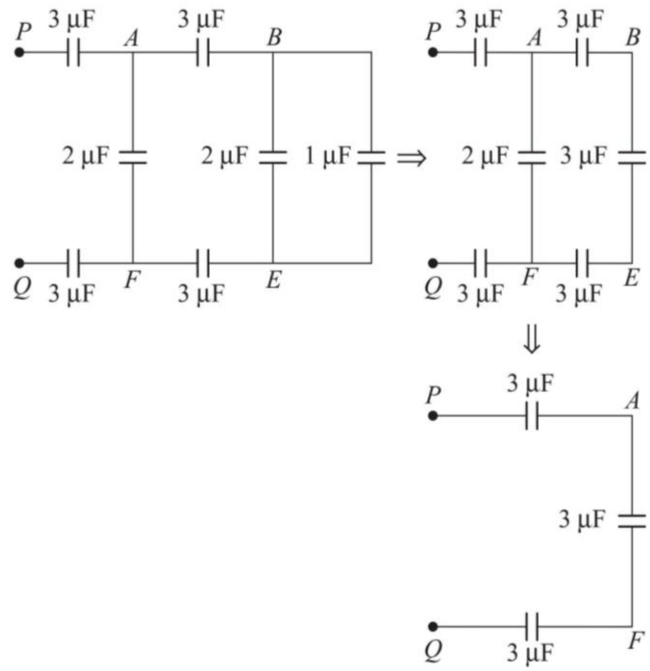


Fig. 11.35(b)

The equivalent capacitance between P and Q is given by

$$\frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1\ \mu F$$

$\Rightarrow$ $C_{eq} = 1\ \mu F$

EXAMPLE 19 In the network shown in Fig. 11.36(a), each capacitor has a capacitance C. The equivalent capacitance between points P and Q is

(a) $\frac{4C}{15}$ (b) $\frac{6C}{19}$

(c) $\frac{8C}{21}$ (d) C

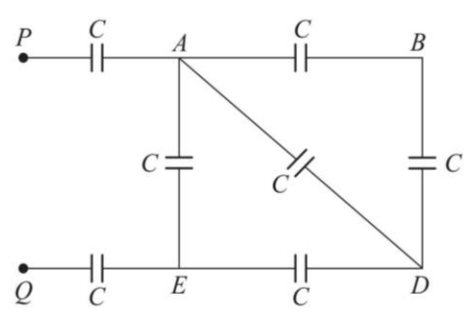


Fig. 11.36(a)

SOLUTION The network can be simplified as shown in Fig. 11.36(b).

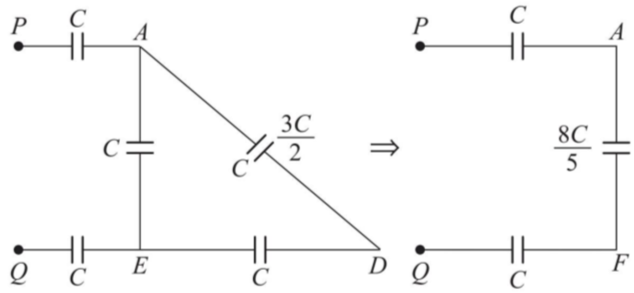


Fig. 11.36(b)

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{5}{8C} + \frac{1}{C}$$

$$\Rightarrow \quad C_{eq} = \frac{8C}{21}$$

EXAMPLE 20 In the circuit shown in Fig. 11.37(a), the charge on the 4 μF capacitor is

(a) zero (b) 15 μC
(c) 20 μC (d) 60 μC

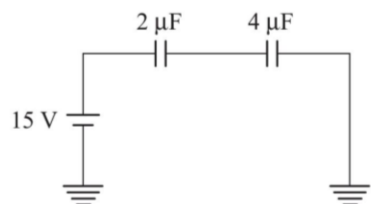


Fig. 11.37(a)

SOLUTION The circuit can be redrawn as shown in Fig. 11.37(b).

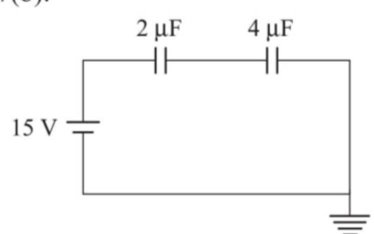


Fig. 11.37(b)

The equivalent capacitance of the series combination is

$$C_{eq} = \frac{2 \times 4}{2+4} = \frac{4}{3}\,\mu F$$

In a series combination, the charge on each capacitor is the same.

$$q = C_{eq} \times V$$

$$= \frac{4}{3}\,\mu F \times 15\ V = 20\,\mu C$$

EXAMPLE 21 In the circuit shown in Fig. 11.38(a), the equivalent capacitance between points A and B is

(a) 3 μF (b) 6 μF
(c) 12 μF (d) 15 μF

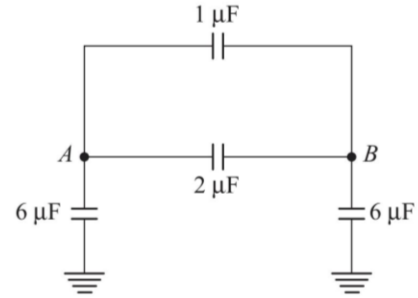


Fig. 11.38(a)

SOLUTION The circuit can be redrawn as shown in Fig. 11.38(b).

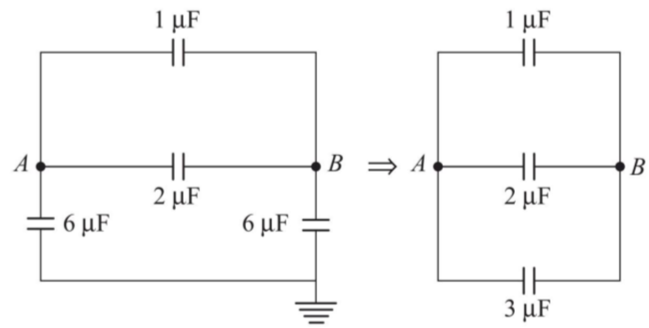


Fig. 11.38(b)

The 6 μF capacitors are connected in series and this combination is in parallel with 1 μF and 2 μF capacitors.

$$C_{eq} = 1 + 2 + 3 = 6\,\mu F$$

24. Wheatstone's Bridge of Capacitors

In the circuit shown in Fig. 11.39(a) in Example 22 below, the network of capacitors form a wheatstone's Bridge. The bridge is balanced if the values of C_1, C_2, C_3 and C_4 satisfy the condition

$$\frac{C_1}{C_2} = \frac{C_3}{C_4}$$

For networks which satisfy this condition, the equivalent capacitance can be found as illustrated in Examples 22, 23 and 24 below.

EXAMPLE 22 Find the equivalent capacitance between A and B in the circuit shown in Fig. 11.39(a). Given $C_1 = 5\,\mu F$, $C_2 = 20\,\mu F$, $C_3 = 10\,\mu F$, $C_4 = 40\,\mu F$ and $C_5 = 30\,\mu F$.

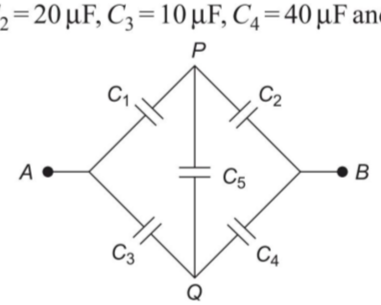


Fig. 11.39(a)

SOLUTION The network shown in the figure is a balanced wheatstone's bridge. If the values of C_1, C_2, C_3 and C_4 are such that the condition

$$\frac{C_1}{C_2} = \frac{C_3}{C_4}$$

is satisfied, the bridge is said to be balanced. The potential at P = potential at Q. Since the potential difference between P and Q is zero (when a battery is connected across A and B), the capacitor C_5 is ineffective as no charge collects on its plates.

Hence the circuit reduces to [see Fig. 11.39(b)]

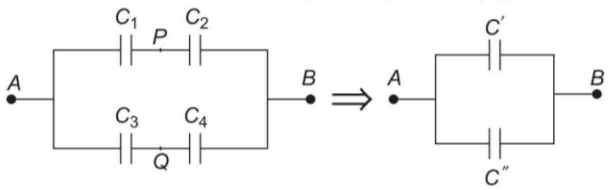


Fig. 11.39(b)

$$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{5 \times 20}{5 + 20} = 4\ \mu\text{F}$$

$$C'' = \frac{C_3 C_4}{C_3 + C_4} = \frac{10 \times 40}{10 + 40} = 8\ \mu\text{F}$$

$$\therefore \quad C_{AB} = C' + C'' = 12\ \mu\text{F}$$

EXAMPLE 23 Each capacitor in the network shown in Fig. 11.40(a) has a capacitance C. The equivalent capacitance between points A and B is

(a) C (b) $2\ C$
(c) $3\ C$ (d) $4\ C$

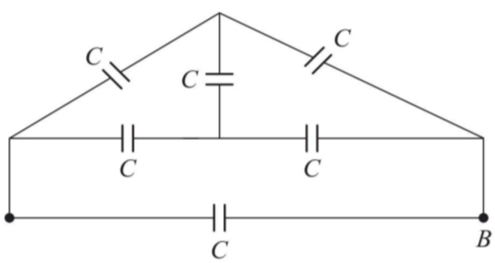


Fig. 11.40(a)

SOLUTION The upper part of the network is a balanced Wheatstone's Bridge. As shown in the above Example 15, the network can be simplified as shown in Fig. 11.40(b).

$$\therefore \quad C_{eq} = C + C = 2\ C$$

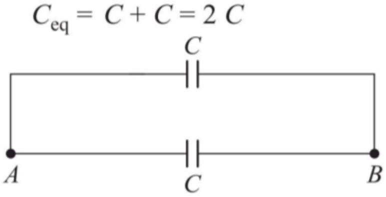


Fig. 11.40(b)

So the correct choice is (b).

EXAMPLE 24 Each capacitor in the network shown in Fig. 11.41(a) has a capacitance C. The equivalent capacitance between points P and Q is

(a) C (b) $\frac{C}{3}$

(c) $\frac{3C}{7}$ (d) $\frac{2C}{5}$

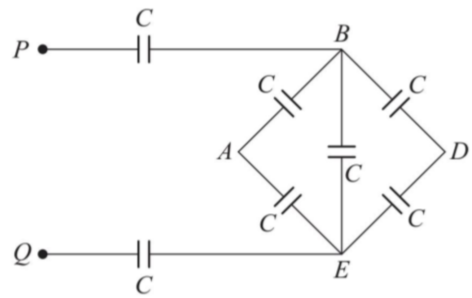


Fig. 11.41(a)

SOLUTION Network $ABDE$ is a balanced Wheatstone's Bridge. As shown above, to find capacitance between A and D, the capacitor between B and E can be ignored. But to find capacitance between B and E, we cannot ignore the capacitor between B and E. Capacitors between B and D and D and E are in series and have a combined capacitance = $C/2$. Capacitors between A and B and A and E have a combined capacitance = $C/2$. Thus the network simplifies to that shown in Fig. 11.41(b).

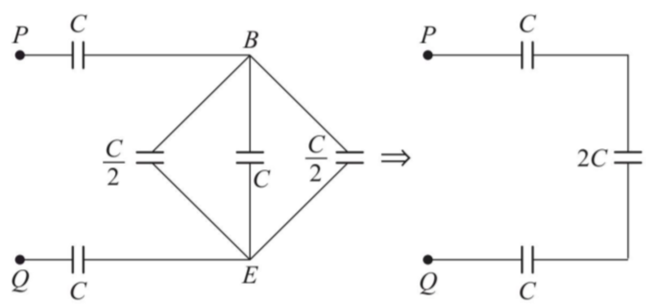


Fig. 11.41(b)

Equivalent capacitance between P and Q is given by

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{C}$$

$$\Rightarrow \quad C_{eq} = \frac{2C}{5}$$

Note For an unbalanced Wheelstone's bridge or for any other more complicated combinations of capacitors, it is not easy to find the equivalent capacitance using the formulae for series and parallel combinations.

For such cases, we should use the following procedure:

(1) Connect an imaginary battery between the points across which the equivalent capacitance is to be found.

(2) Send a positive charge $+Q$ from the positive terminal of the battery and equal negative charge $-Q$ from the negative terminal.

(3) Write the charges on each capacitor plate using the principle of charge conservation. *i.e.*, charges on the two plates must be equal and opposite. Let Q_1, Q_2, ... etc. be the charges on the capacitors in the network and V_1, V_2,... etc. be the respective potential differences.

(4) Use $Q = CV$ for each capacitor. Eliminate Q_1, Q_2, ... etc. and V_1, V_2,... etc. to obtain the equivalent capacitance $C_{eq} = \dfrac{Q}{E}$, where E is the voltage of the battery

EXAMPLE 25 Find the equivalent capacitance between A and B in the circuit shown in Fig. 11.42(a). Given $C = 5\ \mu F$.

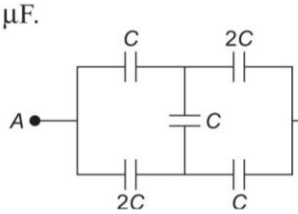


Fig. 11.42(a)

SOLUTION Refer to Fig. 11.42(b).

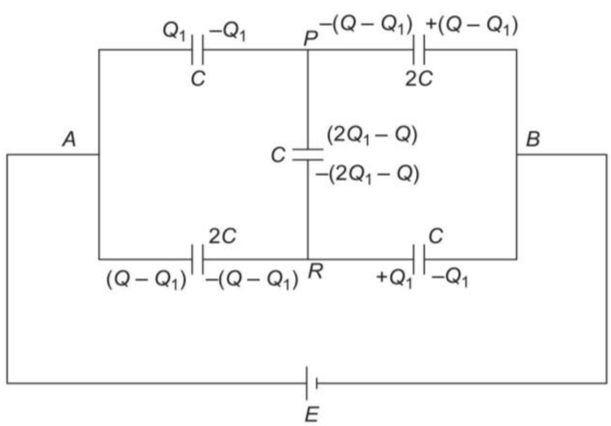


Fig. 11.42(b)

Let us imagine a battery of emf E connected between A and B. Let the positive terminal send a charge $+Q$ and let negative terminal send a charge $-Q$.

Out of charge $+Q$, charge Q_1 appears on the left plate of C and $-Q_1$ on the right plate. The remaining charge $(Q - Q_1)$ appears on the left plate of $2C$ and $-(Q - Q_1)$ appears on its right plate. Similarly, out of charge $-Q$ sent by the negative terminal of the battery, a charge $-Q_1$ appears on the right plate of C (by symmetry) a charge $+Q_1$ appears on its left plate. The remaining charge $+(Q - Q_1)$ appears on the right plate of $2C$ and $-(Q - Q_1)$ appears on its left plate. Now, since the right plate of C, the left plate $2C$ and the upper plate of C are all connected (to P), they form an isolated system. Hence the sum of charges on these plates must be zero. Therefore, the charge on the upper plate of C must be $(2Q_1 - Q)$ and the charge on its lower plate must be $-(2Q_1 - Q)$. The distribution of charges on the capacitor plates is shown in the figure using the principle of charge conservation. Then

$$V_A - V_B = (V_A - V_P) + (V_P - V_B)$$

$$\Rightarrow \quad E = \frac{Q_1}{C} + \frac{(Q - Q_1)}{2C}$$

$$\Rightarrow \quad 2CE = 2Q_1 + (Q - Q_1) = Q_1 + Q \qquad (1)$$

Also $\quad V_A - V_B = (V_A - V_P) + (V_P - V_R) + (V_R - V_B)$

$$\Rightarrow \quad E = \frac{Q_1}{C} + \frac{(2Q_1 - Q)}{C} + \frac{Q_1}{C}$$

$$\Rightarrow \quad CE = Q_1 + (2Q_1 - Q) + Q_1 = 4Q_1 - Q \qquad (2)$$

Eliminating Q_1 from (1) and (2), we get

$$Q = \frac{7CE}{5}$$

$$\therefore \quad C_{eq} = \frac{Q}{E} = \frac{7C}{5} = \frac{7 \times 5\ \mu F}{5} = 7\ \mu F$$

$[\because\ C = 5\ \mu F \text{ (given)}]$

EXAMPLE 26 The circuit shown in Fig. 11.43 consists of two capacitors $C_1 = 2\ \mu F$ and $C_2 = 4\ \mu F$ and two batteries, each of emf $E = 3V$. Find the charge flowing through points A and B when switch S is closed

SOLUTION When switch S is open, the equivalent capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{2 \times 4}{(2 + 4)} = \frac{4}{3}\ \mu F$$

Charge on the R.H.S. plate of C_1 and upper plate of C_2 is

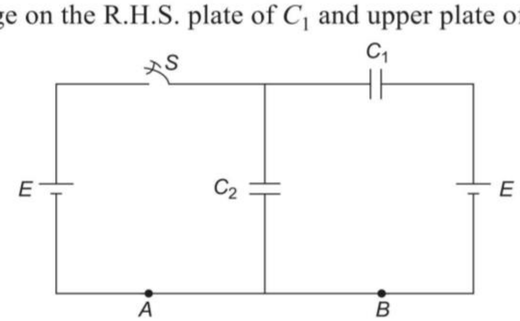


Fig. 11.43

$$Q = CE = \frac{4}{3}\ \mu F \times 3\ V = 4\ \mu C$$

When switch S is closed, the potential difference across C_1 is zero, because the batteries are in opposition. Therefore, charge on the R. H. S. plate of $C_1 = 0$. If the charge flowing through B is q, then

$$Q + q = 0$$

$$\Rightarrow \quad q = -Q = -4\ \mu C$$

Now the charge on the upper plate of $C_2 = C_2E = 4\ \mu F \times 3V = 12\ \mu C$, which is the charge flowing through A.

25. Capacitance of Dielectric Filled Capacitors

(i) Capacitors as shown in Fig. 11.44.

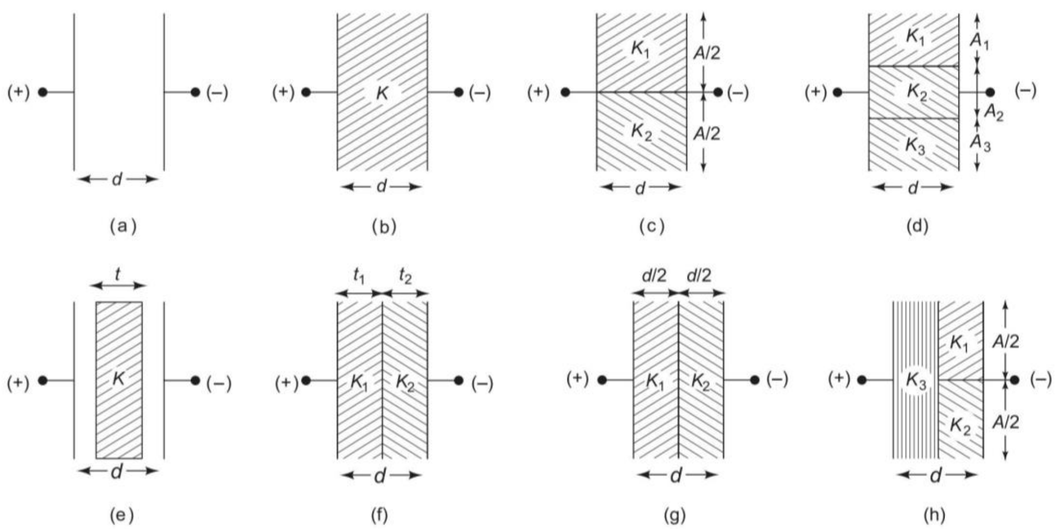


Fig. 11.44

(a) Capacitor with air as dielectric [Fig. 11.45(a)]

$$C_0 = \frac{\varepsilon_0 A}{d}$$

(b) Capacitor completely filled with a dielectric of dielectric constant K [Fig. 11.45(b)]

$$C = KC_0 = \frac{K\varepsilon_0 A}{d}$$

(c) Capacitor filled with two dielectrics as shown in Fig. 11.45(c)

Capacitance of upper dielectric is

$$C_1 = \frac{K_1\varepsilon_0 A/2}{d} = \frac{K_1\varepsilon_0 A}{2d}$$

Capacitance of lower dielectric is $C_2 = \dfrac{K_2\varepsilon_0 A}{2d}$

Since positive plates of C_1 and C_2 are connected together, C_1 and C_2 are in parallel. Hence the capacitance of combination is

$$C = C_1 + C_2 = \left(\frac{K_1 + K_2}{2}\right)\frac{\varepsilon_0 A}{d} = \frac{K_{eq}\varepsilon_0 A}{d}$$

where $K_{eq} = \frac{1}{2}(K_1 + K_2)$ is the equivalent dielectric constant

(d) Capacitor filled with three dielectrics as shown in Fig. 11.45(d)

$$C = C_1 + C_2 + C_3 = \frac{K_{eq}\varepsilon_0 A}{d}$$

$$\text{Where } K_{eq} = \frac{K_1A_1 + K_2A_2 + K_3A_3}{(A_1 + A_2 + A_3)}$$

(e) Capacitor partly filled with a dielectric as shown in Fig. 11.45(e)

$$C = \frac{\varepsilon_0 A}{\left[d - t + \dfrac{t}{K}\right]} = \frac{C_0}{\left[1 + \dfrac{t}{d}\left(\dfrac{1}{K} - 1\right)\right]}$$

where $C_0 = \dfrac{\varepsilon_0 A}{d}$. If $t = d$, $C = KC_0$ as in Fig. 11.45(b)

(f) Capacitor filled with two dielectrics as shown in Fig. 11.45(f)

Capacitance of the left capacitor is $C_1 = \dfrac{K_1\varepsilon_0 A}{t_1}$

Capacitance of the right capacitor is $C_2 = \dfrac{K_2\varepsilon_0 A}{t_2}$

Since C_1 and C_2 are in series, the capacitance of the combination is

$$C = \frac{C_1C_2}{C_1 + C_2} = \frac{K_1K_2\varepsilon_0^2 A^2}{t_1t_2\,\varepsilon_0 A\left(\dfrac{K_1}{t_1} + \dfrac{K_2}{t_2}\right)}$$

$$\Rightarrow \quad C = \frac{K_1 K_2 \varepsilon_0 A}{(K_1 t_2 + K_2 t_1)} = \frac{K_{eq}\varepsilon_0 A}{d}$$

Where $\dfrac{K_{eq}}{d} = \dfrac{K_1 K_2}{(K_1 t_2 + K_2 t_1)}$

$$\Rightarrow \quad \frac{K_{eq}}{t_1 + t_2} = \frac{K_1 K_2}{(K_1 t_2 + K_2 t_1)} \qquad (\because d = t_1 + t_2)$$

$$\Rightarrow \quad K_{eq} = \frac{K_1 K_2 (t_1 + t_2)}{(K_1 t_2 + K_2 t_1)}$$

(g) Capacitor filled with two dielectrics as shown in Fig. 11.45(g)

$$C_1 = \frac{2K_1\varepsilon_0 A}{d}, \; C_2 = \frac{2K_2\varepsilon_0 A}{d}$$

$$\therefore \quad C = \frac{C_1 C_2}{(C_1 + C_2)} = \frac{K_{eq}\varepsilon_0 A}{d}$$

Where $K_{eq} = \dfrac{2K_1 K_2}{(K_1 + K_2)}$

(h) Capacitor filled with three dielectrics as shown in Fig. 11.45(h)

Capacitors with K_1 and K_2 are in parallel and this combination is in series with the capacitor with K_3

$$C = \frac{K_{eq}\varepsilon_0 A}{d}, \text{ Where } K_{eq} \text{ is given by}$$

$$\frac{1}{K_{eq}} = \frac{1}{(K_1 + K_2)} + \frac{1}{2K_3}$$

26. Charging and Discharging of a Capacitor through a Resistance

Consider a capacitor of capacitance C connected in series to a resistor R and a battery of emf E and negligible internal resistance through a two-way key as shown in Fig. 11.45.

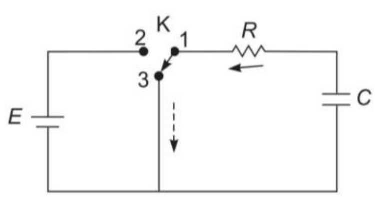


Fig. 11.45

Growth of Charge The battery is introduced in the circuit by connecting terminals 1 and 2 of the key. Initially, i.e. when $t = 0$, the charge on the capacitor plates is zero. As time passes, the charge flows into the capacitor plates and the potential difference q/C (q is the charge at any time t) between the plates rises. The charge on the plates of the capacitor rises till the potential difference between the plates becomes E. The maximum charge collected is $q_0 = CE$. At this stage the current in the circuit becomes zero.

The growth of charge on the capacitor plates as a function of time is given by

$$q = q_0 (1 - e^{-t/RC}) \qquad (1)$$

It is clear that the charge rises exponentially to a steady state maximum value q_0 as shown in Fig. 11.46(a).

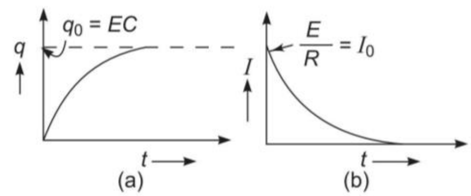


Fig. 11.46

Time Constant The product RC has the dimensions of time. If C is in farad and R in ohm, the product CR will be in seconds. Writing $RC = \tau$ in Eq. (1) we have

$$q = q_0 (1 - e^{-t/\tau})$$

At time $t = \tau$,

$$q = q_0\left(1 - \frac{1}{e}\right) = q_0\left(1 - \frac{1}{2.718}\right) = 0.63 q_0$$

Thus the time constant τ of a CR circuit may be defined as the time during which the charge on the capacitor grows from zero to 0.63 of its maximum value q_0. Whether the charge grows quickly or slowly depends on the value of the time constant, i.e. on the values of R and C. If the product CR (i.e. the time constant) is very small, the charge grows quickly.

The behaviour of current as a function of time t is given by

$$I = \frac{dq}{dt} = I_0 e^{-t/RC}$$

where $I_0 = E/R$ is the maximum current. Therefore, the current decreases exponentially from its maximum value I_0 to zero as shown in Fig. 11.46(b).

Decay of Charge When the charge has attained a steady value EC, the battery is short-circuited by connecting the terminals 1 and 3 of the key K. In such a situation the capacitor starts discharging through the resistor, i.e. the charge on the capacitor starts flowing back through the resistor. The direction of the current is, therefore, reversed.

The decay of charge with time is given by

$$q = q_0\, e^{-t/RC} = q_0\, e^{-t/\tau}$$

Figure 11.47 shows the decay of charge with time.

At time $t = \tau$, $q = q_0 e^{-1} = 0.368\, q_0$. Thus in a time $t = \tau$, the time constant, the charge on the capacitor decays to 0.368 of its initial value q_0. So the charge decays exponentially with time t. Whether the charge decays slowly or quickly depends on the value of the time constant RC.

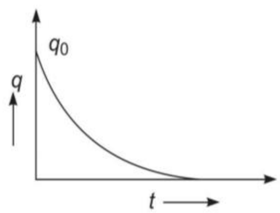


Fig. 11.47

EXAMPLE 27 When the plates of a parallel plate capacitor of capacitance 3.0 μF are connected by a wire of resistance R, the electric field between the plates drops to half of its initial value in 6.9 μs. The value of R is

(a) $\frac{3}{5}\,\Omega$ (b) $\frac{9}{5}\,\Omega$

(c) $\frac{10}{3}\,\Omega$ (d) $\frac{12}{5}\,\Omega$

SOLUTION The voltage between plates is

$$V = \frac{Q}{C} = \frac{Qd}{A\varepsilon_0}$$

Electric field is $E = \frac{V}{d} = \frac{Q}{A\varepsilon_0} = \frac{1}{A\varepsilon_0}(Q_0\, e^{-t/RC})$

$$E = E_0\, e^{-t/RC}$$

where Q_0 is the initial charge and E_0 is the initial electric field. Given that $E = E_0/2$.

$$\frac{E_0}{2} = E_0\, e^{-t/RC}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-t/RC}$$

$$\Rightarrow \quad 2 = e^{t/RC}$$

$$\therefore \quad \ln(2) = \frac{t}{RC}$$

$$\Rightarrow \quad R = \frac{t}{C\ln(2)} = \frac{6.9\,\mu\text{s}}{3.0\,\mu\text{F}\times 0.69}$$

$$= \frac{10}{3}\,\Omega$$

EXAMPLE 28 A capacitor of capacitance C is connected to a 6.0 V battery through a resistance of 5 Ω. The potential difference between the plates rises from zero to 3.0 V in 6.9 μs. The value of C is

(a) 1 μF (b) 2 μF

(c) 3 μF (d) 4 μF

SOLUTION During charging, the charge at time t on the capacitor is given by

$$Q = Q_0\,(1 - e^{-t/RC})$$

Therefore, the potential difference at time t between the capacitor plates is

$$V = \frac{Q}{C} = \frac{Q_0}{C}(1 - e^{-t/RC})$$

$$\Rightarrow \quad V = V_0\,(1 - e^{-t/RC})$$

where $V_0 = \frac{Q_0}{C}$ is the final potential difference = 6.0 V, the voltage of the battery. Given $V = 3.0\text{ V} = \frac{V_0}{2}$. Hence

$$\frac{V_0}{2} = V_0\,(1 - e^{-t/RC})$$

$$\Rightarrow \quad 1 - e^{-t/RC} = \frac{1}{2}$$

$$\Rightarrow \quad e^{-t/RC} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{t}{RC} = \ln(2)$$

$$\Rightarrow \quad C = \frac{t}{R\ln(2)} = \frac{6.9\,\mu\text{s}}{5\times 0.69} = 2\,\mu\text{F}$$

SECTION 1 Multiple Choice Questions with One Correct Choice

Level A

1. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface to 10 V. The potential at the centre of the sphere is
   (a) zero
   (b) 10 V
   (c) the same as that at a point 5 cm away from the surface
   (d) the same as that at a point 25 cm away from the surface

2. Two equal negative charges – q are fixed at points $(0, a)$ and $(0, -a)$ on the y-axis. A positive charge Q is released from rest at a point $(2a, 0)$ on the x-axis. The charge Q will
   (a) execute simple harmonic motion about the origin
   (b) move to the origin and remain at rest there
   (c) move to infinity
   (d) execute oscillatory but not simple harmonic motion.

3. Four capacitors, each of capacitance 50 μF are connected as shown in Fig. 11.48. If the voltmeter reads 100 V, the charge on each capacitor is
   (a) 2×10^{-3} C (b) 5×10^{-3} C
   (c) 0.2 C (d) 0.5 C

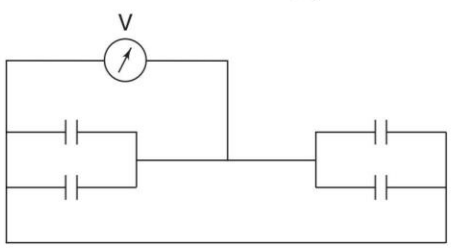


Fig. 11.48

4. Three point charges $4q$, Q and q are placed in a straight line of length l at points distant 0, $l/2$ and l respectively. The net force on charge q is zero. The value of Q is
   (a) $-q$ (b) $-2q$
   (c) $-\frac{1}{2}q$ (d) $4q$

5. Two positive point charges of 12 and 8 microcoulombs respectively are placed 10 cm apart in air. The work done to bring them 4 cm closer is
   (a) zero (b) 3.8 J
   (c) 4.8 J (d) 5.8 J

6. The work done is carrying a charge q once round a circle of radius r with a charge Q at the centre is
   (a) $\frac{qQ}{4\pi\varepsilon_0 r}$ (b) $\frac{qQ}{4\pi\varepsilon_0}\frac{1}{\pi r}$
   (c) $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{2\pi r}\right)$ (d) zero

7. A capacitor of capacitance $C = 2$ μF is connected as shown in Fig. 11.49. If the internal resistance of the cell is 0.5 Ω, the charge on the capacitor plates is
   (a) zero (b) 2 μC
   (c) 4 μC (d) 6 μC

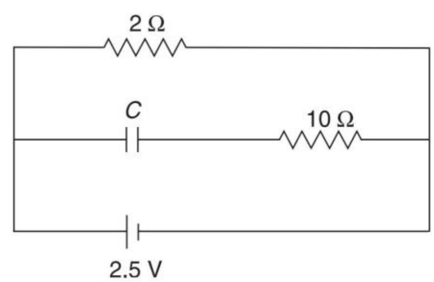


Fig. 11.49

8. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to
   (a) $-\frac{Q}{2}$ (b) $-\frac{Q}{4}$
   (c) $+\frac{Q}{2}$ (d) $+\frac{Q}{4}$

9. The electric potential V (in volt) varies with x (in metre) according to the relation
$$V = 5 + 4x^2$$

The force experienced by a negative charge of 2×10^{-6} C located at $x = 0.5$ m is

(a) 2×10^{-6} N (b) 4×10^{-6} N
(c) 6×10^{-6} N (d) 8×10^{-6} N

10. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to a potential difference V by a battery. The battery is then disconnected and the space between the plates of capacitor C is completely filled with a material of dielectric constant K. The potential difference across the capacitors now becomes

(a) $\frac{V}{K+1}$ (b) $\frac{2V}{K+2}$
(c) $\frac{3V}{K+2}$ (d) $\frac{3V}{K+3}$

11. The force of attraction between the plates of air filled parallel plate capacitor having charge Q and area of each plate A is given by

(a) $\frac{2Q^2}{\varepsilon_0 A}$ (b) $\frac{Q^2}{\varepsilon_0 A}$
(c) $\frac{Q^2}{2\varepsilon_0 A}$ (d) $\frac{Q^2}{4\varepsilon_0 A}$

12. In the network shown in Fig. 11.50, $C_1 = 6\ \mu$F and $C = 9\ \mu$F. The equivalent capacitance between points P and Q is

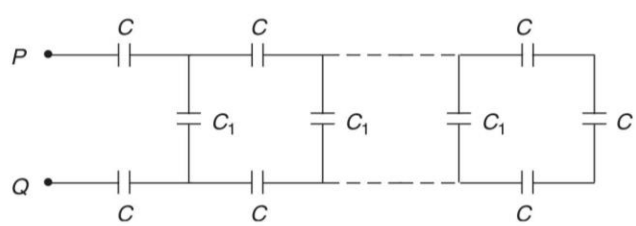


Fig. 11.50

(a) 3 μF (b) 6 μF
(c) 9 μF (d) 12 μF

13. Three capacitors, each of capacitance $C = 3\ \mu$F, are connected as shown in Fig. 11.51. The equivalent capacitance between points P and S is

(a) 1 μF (b) 3 μF
(c) 6 μF (d) 9 μF

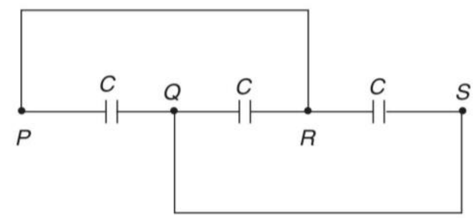


Fig. 11.51

14. A parallel plate capacitor of capacitance 100 pF is to be constructed by using paper sheets of 1.0 mm thickness as dielectric. If the dielectric constant of paper is 4.0, the number of circular metal foils of diameter 2.0 cm each required for this purpose is

(a) 10 (b) 20
(c) 30 (d) 40

15. One thousand spherical water droplets, each of radius r and each carrying a charge q, coalesce to form a single spherical drop. If v is the electrical potential of each droplet and V that of the bigger drop, then

(a) $\frac{V}{v} = \frac{1}{1000}$ (b) $\frac{V}{v} = \frac{1}{100}$
(c) $\frac{V}{v} = 100$ (d) $\frac{V}{v} = 1000$

16. A parallel plate air filled capacitor shown in Fig. 11.52(a) has a capacitance of 2 μF. When it is half filled with a dielectric of dielectric constant $k = 3$ as shown in Fig. 11.52(b), its capacitance becomes

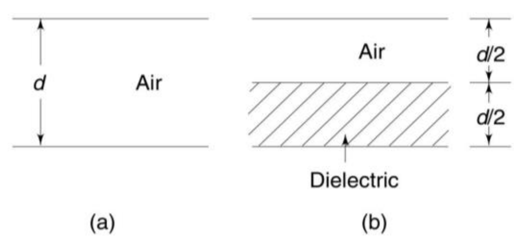


Fig. 11.52

(a) $\frac{1}{3}$ μF (b) 1 μF
(c) 3 μF (d) 9 μF

17. A parallel plate air filled capacitor shown in Fig. 11.53(a) has a capacitance of 2 μF. When it is half filled with a dielectric of dielectric constant $k = 3$ as shown in Fig. 11.53(b), its capacitance becomes

(a) 4 μF (b) 3 μF
(c) 1.5 μF (d) 0.5 μF

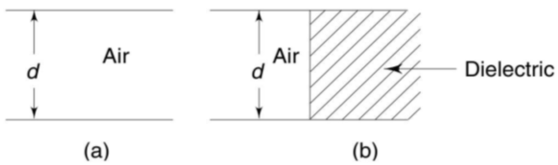


Fig. 11.53

18. Two small identical balls P and Q, each of mass $\sqrt{3}/10$ gram, carry identical charges and are suspended by threads of equal lengths. At equilibrium, they position themselves as shown in Fig. 11.54. What is the charge on each ball. Given $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2} and take $g = 10$ ms^{-2}.
   (a) 10^{-3} C (b) 10^{-5} C
   (c) 10^{-7} C (d) 10^{-9} C

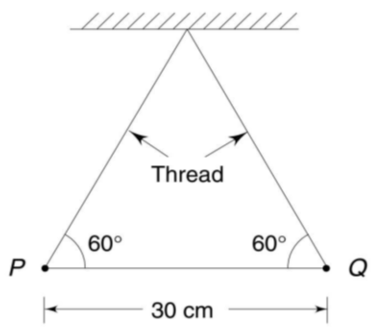


Fig. 11.54

19. Two point charges $q_1 = 2$ µC and $q_2 = 1$ µC are placed at distances $b = 1$ cm and $a = 2$ cm from the origin on the y and x axes as shown in Fig. 11.55. The electric field vector at point P (a, b) will subtend an angle θ with the x-axis given by
   (a) $\tan\theta = 1$ (b) $\tan\theta = 2$
   (c) $\tan\theta = 3$ (d) $\tan\theta = 4$

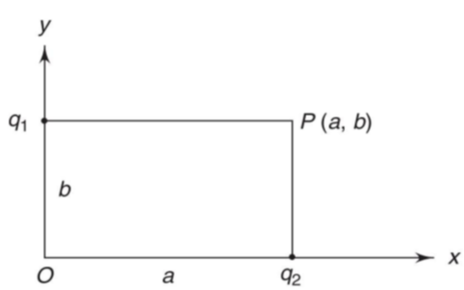


Fig. 11.55

20. An electric dipole placed with its axis in the direction of a uniform electric field experiences
   (a) a force but no torque
   (b) a torque but no force
   (c) a force as well as a torque
   (d) neither a force nor a torque

21. An electric dipole placed with its axis inclined at an angle to the direction of a uniform electric field experiences
   (a) a force but no torque
   (b) a torque but no force
   (c) a force as well as a torque
   (d) neither a force nor a torque

22. An electric dipole placed in a non-uniform electric field experiences
   (a) a force but no torque
   (b) a torque but no force
   (c) a force as well as a torque
   (d) neither a force nor a torque.

23. A cube of side b has a charge q at each of its vertices. What is the electric potential at the centre of the cube?
   (a) $\frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$ (b) $\frac{\sqrt{3}q}{\pi\varepsilon_0 b}$
   (c) $\frac{2q}{\pi\varepsilon_0 b}$ (d) zero

24. In Q. 23, the Electric field at the centre of the cube is
   (a) $\frac{4q}{3\pi\varepsilon_0 b^2}$ (b) $\frac{3q}{\pi\varepsilon_0 b^2}$
   (c) $\frac{2q}{\pi\varepsilon_0 b^2}$ (d) zero

25. Two point charges $-q$ and $+q$ are located at points (0, 0, $-a$) and (0, 0, a) respectively. What is the electric potential at point (0, 0, z)?
   (a) $\frac{qa}{4\pi\varepsilon_0 z^2}$ (b) $\frac{q}{4\pi\varepsilon_0 a}$
   (c) $\frac{2qa}{4\pi\varepsilon_0 (z^2 - a^2)}$ (d) $\frac{2qa}{4\pi\varepsilon_0 (z^2 + a^2)}$

26. In Q. 25, how much work is done in moving a small test charge q_0 from point (5, 0, 0) to a point (– 7, 0, 0) along the x-axis?
   (a) $\frac{5}{7} \times \frac{q_0 q}{4\pi\varepsilon_0 a}$ (b) $\frac{7}{5} \times \frac{q_0 q}{4\pi\varepsilon_0 a}$
   (c) $\frac{1}{12} \times \frac{q_0 q}{4\pi\varepsilon_0 a}$ (d) zero

27. A neutral hydrogen molecule has two protons and two electrons. If one of the electrons is removed we get a hydrogen molecular ion (H_2^+). In the ground state of H_2^+ the two protons are separated by roughly 1.5 Å and the electron is roughly 1 Å from each proton. What is the potential energy of the system?

(a) – 38.4 eV (b) – 19.2 eV
(c) – 9.6 eV (d) zero

28. In a hydrogen atom, the electron and the proton are bound together at a separation of about 0.53 Å. If the zero of potential energy is taken at infinite separation of the electron from the proton, the potential energy of the electron-proton system is

(a) – 54.4 eV (b) – 27.2 eV
(c) – 13.6 eV (d) zero

29. In Q. 28, what is the minimum work required to free the electron from the proton if the kinetic energy of the electron in its orbit is half the potential energy of the electron-proton system?

(a) 2.2×10^{-12} J (b) 2.2×10^{-14} J
(c) 2.2×10^{-16} J (d) 2.2×10^{-18} J

30. In Q. 28, what will be the potential energy of the electron-proton system if the zero of potential energy is taken at a separation of 1.06 Å?

(a) zero (b) – 13.6 eV
(c) – 27.2 eV (d) – 54.4 eV

31. What is the answer to Q. 29 if the zero of potential energy is taken at a separation of 1.06 Å?

(a) zero (b) 1.1×10^{-14} J
(c) 1.1×10^{-16} J (d) 1.1×10^{-18} J

32. What is the equivalent capacitance between A and D of the network shown in Fig. 11.56?

(a) 200 pF (b) 100 pF
(c) $\frac{200}{3}$ pF (d) 50 pF

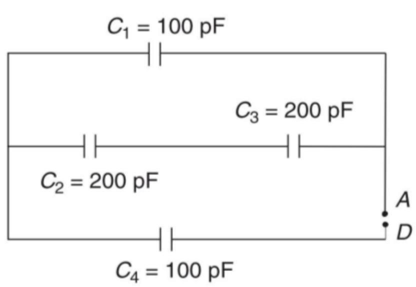


Fig. 11.56

33. Figure 11.57 shows a network of capacitors where the numbers indicate capacitances in microfarad. What must be the value of capacitance C if the equivalent capacitance between points A and B is to be 1 μF?

(a) $\frac{31}{23}$ μF (b) $\frac{32}{23}$ μF
(c) $\frac{33}{23}$ μF (d) $\frac{34}{23}$ μF

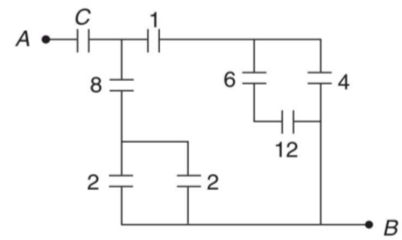


Fig. 11.57

34. A 2 μF capacitor C_1 is charged to a voltage 100 V and a 4 μF capacitor C_2 is charged to a voltage 50 V. The capacitors are then connected in parallel. What is the loss of energy due to parallel connection?

(a) 1.7 J (b) 1.7×10^{-1} J
(c) 1.7×10^{-2} J (d) 1.7×10^{-3} J

35. A positive charge (+ q) is located at the centre of a circle as shown in Fig. 11.58. W_1 is the work done in taking a unit positive charge from A to B and W_2 is the work done in taking the same charge from A to C. Then

(a) $W_1 > W_2$ (b) $W_1 < W_2$
(c) $W_1 = W_2$ (d) $W_1 = W_2 = 0$

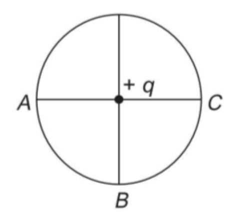


Fig. 11.58

36. Two concentric spheres of radii r_1 and r_2 carry charges q_1 and q_2 respectively. If the surface charge density (σ) is the same for both spheres, the electric potential at the common centre will be

(a) $\frac{\sigma}{\varepsilon_0} \cdot \frac{r_1}{r_2}$ (b) $\frac{\sigma}{\varepsilon_0} \cdot \frac{r_2}{r_1}$
(c) $\frac{\sigma}{\varepsilon_0}(r_1 - r_2)$ (d) $\frac{\sigma}{\varepsilon_0}(r_1 + r_2)$

37. The magnitude of the electric field on the surface of a sphere of radius r having a uniform surface charge density σ is

(a) $\frac{\sigma}{\varepsilon_0}$ (b) $\frac{\sigma}{2\varepsilon_0}$
(c) $\frac{\sigma}{\varepsilon_0 r}$ (d) $\frac{\sigma}{2\varepsilon_0 r}$

38. The electric field due to an extremely short dipole at a distance r from it is proportional to

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$

(c) $\frac{1}{r^3}$ (d) $\frac{1}{r^4}$

39. The electric potential due to an extremely short dipole at a distance r from it is proportional to

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$

(c) $\frac{1}{r^3}$ (d) $\frac{1}{r^4}$

40. A soap bubble of radius r is charged to a potential V. If the radius is increased to nr, the potential on the bubble will become

(a) nV (b) n^2V

(c) $\frac{V}{n}$ (d) $\frac{V}{n^2}$

41. If n drops, each of capacitance C, coalesce to form a single big drop, the capacitance of the big drop will be

(a) n^3C (b) nC

(c) $n^{1/2}C$ (d) $n^{1/3}C$

Level B

42. If n drops, each charged to a potential V, coalesce to form a single drop, the potential of the big drop will be

(a) $\frac{V}{n^{2/3}}$ (b) $\frac{V}{n^{1/3}}$

(c) $Vn^{1/3}$ (d) $Vn^{2/3}$

43. If n drops, each of capacitance C and charged to a potential V, coalesce to form a big drop, the ratio of the energy stored in the big drop to that in each small drop will be

(a) $n : 1$ (b) $n^{4/3} : 1$

(c) $n^{5/3} : 1$ (d) $n^2 : 1$

44. A parallel plate capacitor is made by stacking 10 identical metallic plates equally spaced from one another and having the same dielectric between plates. The alternate plates are then connected. If the capacitor formed by two neighbouring plates has a capacitance C, the total capacitance of the combination will be

(a) $\frac{C}{10}$ (b) $\frac{C}{9}$

(c) $9\,C$ (d) $10\,C$

45. Figure 11.59 shows four capacitors connected to an 8 V power supply. What is the potential difference across each 1 μF capacitor?

(a) 1 V (b) 2 V

(c) 3 V (d) 4 V

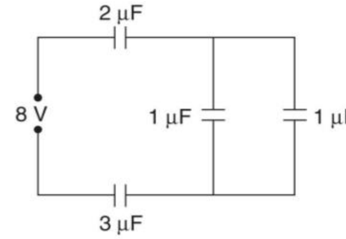


Fig. 11.59

46. Figure 11.60 shows three capacitors connected to a 6 V power supply. What is the charge on the 2 μF capacitor?

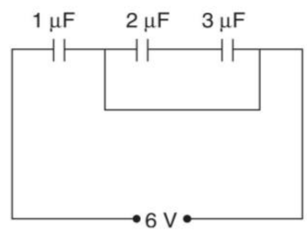


Fig. 11.60

(a) 1 μC (b) 2 μC

(c) 3 μC (d) 4 μC

47. Figure 11.61 shows five capacitors connected across a 12 V power supply. What is the charge on the 2 μF capacitor?

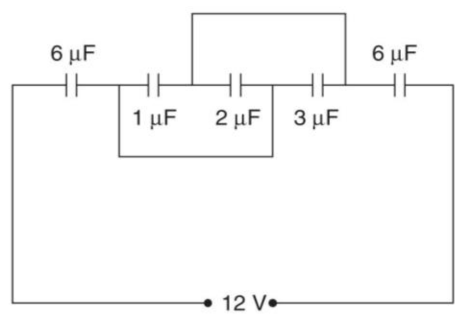


Fig. 11.61

(a) 6 μC (b) 8 μC

(c) 10 μC (d) 12 μC

48. Six charges, each equal to $+q$, are placed at the corners of a regular hexagon of side a. The electric potential at the point where the diagonals of the hexagon intersect will be given by

(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a}$

(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{6q}{a}$ (d) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{\sqrt{3}q}{2a}$

49. In Q. 48, the electric field at the point of intersection of diagonals is

(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$

(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{6q}{a^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{\sqrt{3}q}{2a^2}$

50. A parallel plate capacitor with air as dielectric is charged to a potential V. It is then connected to an uncharged parallel plate capacitor filled with wax of dielectric constant k. The common potential of both capacitors is

(a) V (b) kV

(c) $(1+k)V$ (d) $\frac{V}{(1+k)}$

51. A capacitor of capacitance C is fully charged by a 200 V supply. It is then discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat 2.5×10^2 J kg^{-1} K^{-1} and of mass 0.1 kg. If the temperature of the block rises by 0.4 K, what is the value of C?

(a) 500 μF (b) 400 μF

(c) 300 μF (d) 200 μF

52. A charge having magnitude Q is divided into two parts q and $(Q-q)$ which are held a certain distance r apart. The force of repulsion between the two parts will be maximum if the ratio q/Q is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$

53. A charge Q is given to a hollow metallic sphere of radius R. The electric potential at the surface of the sphere is

(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$

(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^2}$ (d) $4\pi\varepsilon_0 Q/R$

54. In Q. 53, the potential at a distance r from the centre of the sphere where $r < R$ is

(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{(R-r)}$

(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R+r}$ (d) $\frac{4\pi\varepsilon_0 Q}{(R-r)}$

55. The electric potential V at any point (x, y, z) in space is given by $V = 4x^2$ volt where x, y and z are all in metre. The electric field at the point (1 m, 0, 2 m) in Vm^{-1} is

(a) 8 along negative x-axis
(b) 8 along positive x-axis
(c) 16 along negative x-axis
(d) 16 along positive x-axis

56. A charge Q is situated at the centre of a cube. The electric flux through one of the faces of the cube is

(a) $\frac{Q}{\varepsilon_0}$ (b) $\frac{Q}{2\varepsilon_0}$

(c) $\frac{Q}{4\varepsilon_0}$ (d) $\frac{Q}{6\varepsilon_0}$

57. Eight dipoles of charges of magnitude q are placed inside a cube. The total electric flux through the cube will be

(a) $\frac{8q}{\varepsilon_0}$ (b) $\frac{16q}{\varepsilon_0}$

(c) $\frac{q}{\varepsilon_0}$ (d) zero

58. The magnitude of the electric field in the annular region of a charged cylindrical capacitor

(a) is the same throughout
(b) is higher near the outer cylinder than near the inner cylinder
(c) varies as $1/r$ where r is the distance from the axis
(d) varies as $1/r^2$ where r is the distance from the axis.

59. n identical capacitors are joined in parallel and the combination is charged to voltage V. The total energy stored is U. The capacitors are now disconnected and joined in series. The total energy stored in the series combination will be

(a) $\frac{U}{n}$ (b) U

(c) nU (d) n^2U

60. Two spheres of radii r and R carry charges q and Q respectively. When they are connected by a wire, there will be no loss of energy of the system if

(a) $qr = QR$ (b) $qR = Qr$
(c) $qr^2 = QR^2$ (d) $qR^2 = Qr^2$

61. Two equal point charges of 1 μC each are located at points $(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$ m and $(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}})$ m. What is the magnitude of electrostatic force between them?

(a) 10^{-3} N (b) 10^{-6} N
(c) 10^{-9} N (d) 10^{-12} N

62. The introduction of a metal plate between the plates of a parallel plate capacitor increases its capacitance by 4.5 times. If d is the separation of the two plates of the capacitor, the thickness of the metal plate introduced is

(a) $\frac{d}{3}$ (b) $\frac{5d}{9}$
(c) $\frac{7d}{9}$ (d) d

63. If the potential difference between the plates of a capacitor is increased by 20%, the energy stored in the capacitor increases by exactly

(a) 20% (b) 22%
(c) 40% (d) 44%

64. If the potential difference between the plates of a capacitor is increased by 0.1%, the energy stored in the capacitor increases by very nearly

(a) 0.1% (b) 0.11%
(c) 0.144% (d) 0.2%

65. Three capacitors connected in series have an effective capacitance of 2 μF. If one of the capacitors is removed, the effective capacitance becomes 3 μF. The capacitance of the capacitor that is removed is

(a) 1 μF (b) $\frac{3}{2}$ μF
(c) $\frac{2}{3}$ μF (d) 6 μF

66. The effective capacitance of two capacitors of capacitances C_1 and C_2 (with $C_2 > C_1$) connected in parallel is $\frac{25}{6}$ times the effective capacitance when they are connected in series. The ratio C_2/C_1 is

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$
(c) $\frac{5}{3}$ (d) $\frac{25}{6}$

67. Three equal point charges q are placed at the corners of an equilateral triangle. Another charge Q is placed at the centroid of the triangle. The system of charges will be in equilibrium if Q equals

(a) $\frac{q}{\sqrt{3}}$ (b) $-\frac{q}{\sqrt{3}}$
(c) $\frac{q}{3}$ (d) $-\frac{q}{3}$

68. A metallic sphere A of radius a carries a charge Q. It is brought in contact with an uncharged sphere B of radius b. The charge on sphere A now will be

(a) $\frac{aQ}{b}$ (b) $\frac{bQ}{a}$
(c) $\frac{bQ}{a+b}$ (d) $\frac{aQ}{a+b}$

69. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. The potential difference between the surface of the solid sphere and the outer surface of the hollow shell is V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is

(a) V (b) $2V$
(c) $4V$ (d) $-2V$

70. Two identical thin rings, each of radius R are coaxially placed at a distance R apart. If Q_1 and Q_2 are the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to the centre of the other is

(a) zero

(b) $\frac{q}{4\pi\varepsilon_0\sqrt{2}R}(Q_1 - Q_2)(\sqrt{2}-1)$

(c) $\frac{q\sqrt{2}}{4\pi\varepsilon_0 R}(Q_1 + Q_2)$

(d) $\frac{(\sqrt{2}+1)q(Q_1+Q_2)}{\sqrt{2}\,4\pi\varepsilon_0 R}$

71. An electron of mass m_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p, also initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2/t_1 is nearly equal to

(a) 1 (b) $\left(\frac{m_p}{m_e}\right)^{1/2}$
(c) $\left(\frac{m_e}{m_p}\right)^{1/2}$ (d) 1836

72. A metallic solid sphere is placed in a uniform electric field. In Fig. 11.62, which path will the field lines follow?

(a) 1 (b) 2
(c) 3 (d) 4

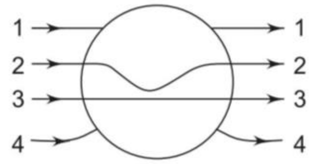


Fig. 11.62

73. A charge $+q$ is fixed at each of the points $x = x_0, x = 3x_0$, $x = 5x_0$ ... upto infinity and a charge $-q$ is fixed at each of the points $x = 2x_0, x = 4x_0, x = 6x_0$ ... up to infinity. Here x_0 is a positive constant. The potential at the origin of this system of charges is

(a) zero (b) $\dfrac{q}{4\pi\varepsilon_0 x_0 \ln(2)}$

(c) infinity (d) $\dfrac{q \ln(2)}{4\pi\varepsilon_0 x_0}$

74. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right-angled isosceles triangle as shown in Fig. 11.63. The net electrostatic energy of the configuration is zero if Q is equal to

(a) $\dfrac{-q}{1+\sqrt{2}}$ (b) $\dfrac{-2q}{2+\sqrt{2}}$

(c) $-2q$ (d) $+q$

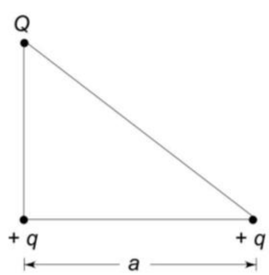


Fig. 11.63

75. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference $2V$. The charging battery is then disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

(a) zero (b) $\dfrac{3}{2}\,CV^2$

(c) $\dfrac{25}{6}\,CV^2$ (d) $\dfrac{9}{2}\,CV^2$

76. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As x goes from 0 to $3d$,

(a) the magnitude of the electric field remains the same
(b) the direction of the electric field changes continuously
(c) the electric potential increases continuously
(d) the electric potential increases at first, then decreases and again increases.

77. Two identical metal plates are given positive charges Q_1 and Q_2 ($< Q_1$) respectively. If they are brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is

(a) $\dfrac{Q_1+Q_2}{2C}$ (b) $\dfrac{Q_1+Q_2}{C}$

(c) $\dfrac{Q_1-Q_2}{C}$ (d) $\dfrac{Q_1-Q_2}{2C}$

78. For the circuit shown in Fig. 11.64, which of the following statements is true?

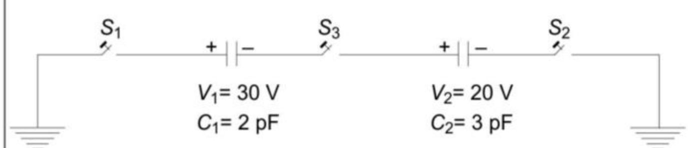


Fig. 11.64

(a) With S_1 closed, $V_1 = 15$ V, $V_2 = 20$ V
(b) With S_3 closed, $V_1 = V_2 = 25$ V
(c) With S_1 and S_2 closed $V_1 = V_2 = 0$
(d) With S_1 and S_3 closed $V_1 = 30$ V and $V_2 = 20$ V

79. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constant K_1, K_2 and K_3 as shown in Fig. 11.65. If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant K is given by

(a) $\dfrac{1}{K} = \dfrac{1}{K_1} + \dfrac{1}{K_2} + \dfrac{1}{2K_3}$

(b) $\dfrac{1}{K} = \dfrac{1}{K_1+K_2} + \dfrac{1}{2K_3}$

(c) $K = \dfrac{K_1K_2}{K_1+K_2} + 2K_3$

(d) $K = K_1 + K_2 + K_3$

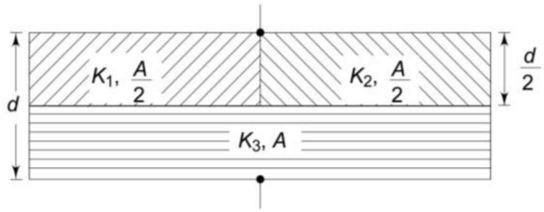


Fig. 11.65

80. A quantity X is given by $\varepsilon_0 L \dfrac{\Delta V}{\Delta t}$ where ε_0 is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of
   (a) resistance (b) charge
   (c) voltage (d) current

81. Consider the situation shown in Fig. 11.66. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is

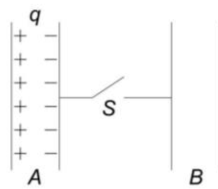


Fig. 11.66

   (a) zero (b) $q/2$
   (c) q (d) $2q$

82. A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm and C be the point on the y-axis at $y = +1$ cm. Then the potentials at the points A, B and C satisfy:
   (a) $V_A < V_B$ (b) $V_A > V_B$
   (c) $V_A < V_C$ (d) $V_A > V_C$

83. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to
   (a) x (b) x^2
   (c) x^3 (d) $1/x$

84. There is a uniform electric field of strength 10^3 Vm^{-1} along the y-axis. A body of mass 1 g and charge 10^{-6} C is projected into the field from the origin along the positive x-axis with a velocity of 10 ms^{-1}. Its speed (in ms^{-1}) after 10 second will be (neglect gravitation)
   (a) 10 (b) $5\sqrt{2}$
   (c) $10\sqrt{2}$ (d) 20

85. Two identical charges are placed at the two corners of an equilateral triangle. The potential energy of the system is U. The work done in bringing an identical charge from infinity to the third vertex is
   (a) U (b) $2U$
   (c) $3U$ (d) $4U$

86. A parallel plate capacitor of capacitance 5 μF and plate separation 6 cm is connected to a 1 V battery and charged. A dielectric of dielectric constant 4 and thickness 4 cm is introduced between the plates of the capacitor. The additional charge that flows into the capacitor from the battery is
   (a) 2 μC (b) 3 μC
   (c) 5 μC (d) 10 μC

87. A capacitor of capacitance 4 μF is charged to 80 V and another capacitor of capacitance 6 μF is charged to 30 V. When they are connected together, the energy lost by the 4 μF capacitor is
   (a) 7.8 mJ (b) 4.6 mJ
   (c) 3.2 mJ (d) 2.5 mJ

88. The magnitude of electric field at a distance x from a charge q is E. An identical charge is placed at a distance $2x$ from it. Then the magnitude of the force it experiences is
   (a) qE (b) $2qE$
   (c) $\dfrac{qE}{2}$ (d) $\dfrac{qE}{4}$

89. The flux of electric field $\mathbf{E} = 200\,\hat{\mathbf{i}}$ NC^{-1} through a cube of side 10 cm, oriented so that its faces are parallel to the co-ordinate axes is
   (a) zero (b) 2 NC^{-1}m^2
   (c) 6 NC^{-1}m^2 (d) 12 NC^{-1}m^2

90. Figure 11.67 shows a spherical Gaussian surface and a charge distribution. When calculating the flux of electric field through the Gaussian surface, the electric field will be due to

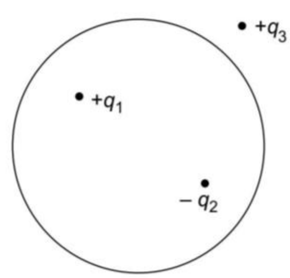


Fig. 11.67

   (a) $+q_3$ alone (b) $+q_1$ and $+q_3$
   (c) $+q_1, +q_3$ and $-q_2$ (d) $+q_1$ and $-q_2$

91. Three infinite long plane sheets carrying uniform charge densities

$$\sigma_1 = -\sigma,\ \sigma_2 = +2\sigma \text{ and } \sigma_3 = +3\sigma$$

are placed parallel to the x–z plane at $y = a$, $y = 3a$ and $y = 4a$ as shown in Fig. 11.68. The electric field at point P is

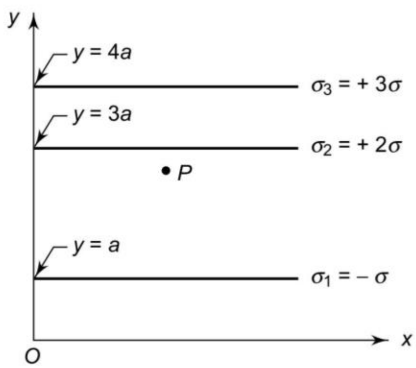


Fig. 11.68

(a) zero (b) $-\dfrac{2\sigma}{\varepsilon_0}\hat{\mathbf{j}}$

(c) $-\dfrac{3\sigma}{\varepsilon_0}\hat{\mathbf{j}}$ (d) $\dfrac{3\sigma}{\varepsilon_0}\hat{\mathbf{j}}$

92. A metallic spherical shell of radius R has a charge $-Q$ distributed uniformly on it. A point charge $+Q$ is placed at the center of the shell. Which graph shown in Fig. 11.69 represents the variation of electric field E with distance r from the centre of the shell?

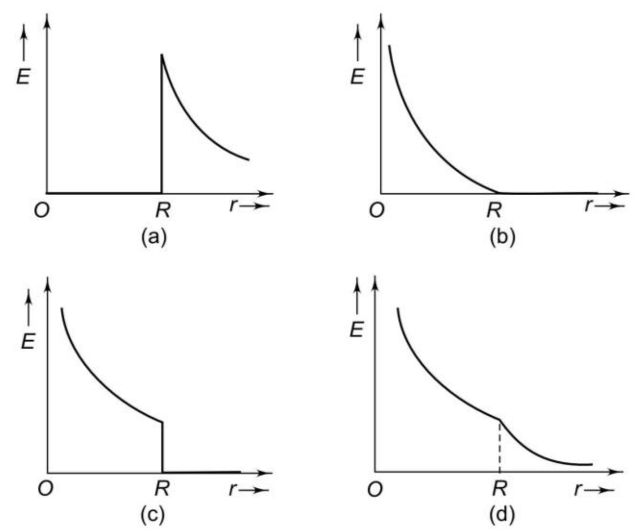


Fig. 11.69

93. A metallic sphere of radius R is charged to a potential V. The magnitude of the electric field at a distance r (> R) from the center of the sphere is

(a) $\dfrac{V}{r}$ (b) $\dfrac{Vr}{R^2}$

(c) $\dfrac{VR}{r^2}$ (d) zero

94. Two point charges $q_1 = 1\mu C$ and $q_2 = 2\mu C$ are placed at points A and B 6 cm apart as shown in Fig. 11.70. A third charge $Q = 5\ \mu C$ is moved from C to D along the arc of a circle of radius 8 cm as shown. The change in the potential energy of the system is

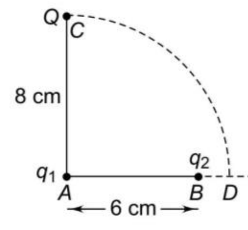


Fig. 11.70

(a) 3.0 J (b) 3.6 J
(c) 5.0 J (d) 7.2 J

95. A partical of mass m and charge $+q$ is midway between two fixed charged particles, each having a charge $+q$ and at a distance $2L$ apart. The middle charge is displaced slightly along the line joining the fixed charges and released. The time period of oscillation is proportional to

(a) $L^{1/2}$ (b) L

(c) $L^{3/2}$ (d) L^2

96. The potential difference between points A and B in the circuit shown in Fig. 11.71 is

(a) 6 V (b) 2 V
(c) 10 V (d) 14 V

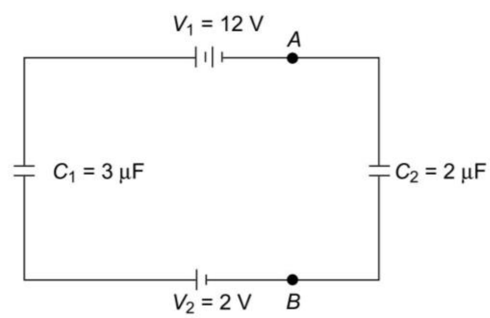


Fig. 11.71

97. An electric field of 200 Vm^{-1} exists in the region between the plates of a parallel plate capacitor of plate separation 5 cm. The potential difference between the plates when a slab of dielectric constant 4 and thickness 1 cm is inserted between the plates is

(a) 7.5 V (b) 8.5 V
(c) 9.0 V (d) 10 V

98. A parallel plate capacitor is maintained at a certain potential difference. When a dielectric slab of thickness 3 mm is introduced between the

plates, the plate separation had to be increased by 2 mm in order to maintain the same potential difference between the plates. The dielectric constant of the slab is

(a) 2 (b) 3
(c) 4 (d) 5

99. A capacitor of capacitance C_1 is charged by connecting it to a battery. The battery is now removed and this capacitor is connected to a second uncharged capacitor of capacitance C_2. If the charge distributes equally on the two capacitors, the ratio of the total energy stored in the capacitors after connection to the total energy stored in them before connection is

(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{3}$

100. Four metal plates numbered 1, 2, 3 and 4 are arranged as shown is Fig. 11.72. The area of each plate is A and the separation between adjacent plates is d. The capacitance of the arrangement is

(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{d}$
(c) $\frac{3\varepsilon_0 A}{d}$ (d) $\frac{4\varepsilon_0 A}{d}$

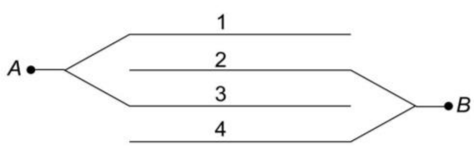


Fig. 11.72

101. Four metal plates numbered 1, 2, 3 and 4 are arranged as shown in Fig. 11.73. The area of each plate is A and the separation between the plates is d. The capacitance of the arrangement is

(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{d}$
(c) $\frac{3\varepsilon_0 A}{d}$ (d) $\frac{4\varepsilon_0 A}{d}$

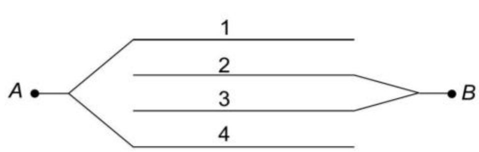


Fig. 11.73

102. The equivalent capacitance between points A and B in the network shown in Fig. 11.74 is ($C_1 = 2\ \mu$F and $C_2 = 3\ \mu$F)

(a) 1 μF (b) 2 μF
(c) 3 μF (d) 4 μF

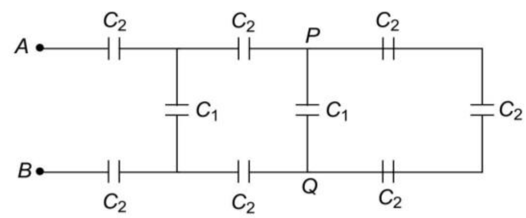


Fig. 11.74

103. A capacitior of capacitance $C_1 = C$ is charged to a voltage V. It is then connected in parallel with a series combination of two uncharged capacitors of capacitances $C_2 = C$ and $C_3 = C$. The charge that will flow through the connecting wires is

(a) $\frac{CV}{3}$ (b) $\frac{2CV}{3}$
(c) CV (d) zero

104. The capacitance of a sphere of radius R_1 is increased 3 times when it enclosed by an earthed sphere of radius R_2. The ratio R_2/R_1 is

(a) 2 (b) $\frac{3}{2}$
(c) $\frac{4}{3}$ (d) 3

105. A parallel plate capacitor of plate area A and plate separation d is charged by a battery of voltage V. The battery is then disconnected. The work needed to pull the plates to a separation $2d$ is

(a) $\frac{AV^2\varepsilon_0}{d}$ (b) $\frac{2AV^2\varepsilon_0}{d}$
(c) $\frac{AV^2\varepsilon_0}{2d}$ (d) $\frac{3AV^2\varepsilon_0}{2d}$

106. One plate of a parallel plate capacitor of plate area A and plate separation d is connected to the positive terminal of a battery of the voltage V. The negative terminal of the battery and the other plate of the capacitor are earthed as shown in Fig. 11.75. The charge that flows from the battery to the capacitor plates is

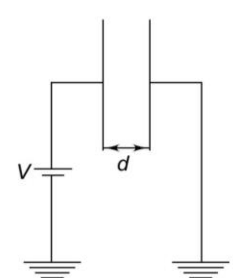


Fig. 11.75

(a) zero (b) $\dfrac{\varepsilon_0 AV}{d}$

(c) $\dfrac{Vd}{\varepsilon_0 A}$ (d) $\dfrac{\varepsilon_0 AV}{2d}$

107. A non-conducting sphere of radius R has a charge Q distributed uniformly throughout its volume. The magnitude of the electric field at a point P inside the sphere at a distance r ($<R$) from the centre is

(a) $\dfrac{Qr}{4\pi \in_0 R^3}$ (b) $\dfrac{Qr}{4\pi \in_0 R^3}$

(c) $\dfrac{Q}{4\pi \in_0 r^2}$ (d) zero

108. In Q. 107 above, the magnitude of the electric field at a point P outside the sphere at a distance r ($>R$) from the centre is

(a) zero (b) $\dfrac{Qr}{4\pi \in_0 R^3}$

(c) $\dfrac{Q}{4\pi \in_0 r^2}$ (d) $\dfrac{Qr^2}{4\pi \in_0 R^4}$

109. A non-conducting sphere of radius R has a charge distributed throughout its volume. The charge density ρ varies with distance r from the centre of the sphere as

$$\rho = \frac{Kr}{R}$$

where K is a constant. The electric field at a point P inside the sphere at a distance r ($< R$) is

(a) $\dfrac{KR^2}{4r \in_0}$ (b) $\dfrac{KR^2}{2\pi \in_0 r}$

(c) $\dfrac{Kr^2}{2\pi \in_0 R}$ (d) $\dfrac{Kr^2}{4R \in_0}$

110. In Q. 109 above, the electric field at a point P outside the sphere at a distance r ($>R$) is

(a) $\dfrac{KR^2}{2 \in_0 r}$ (b) $\dfrac{KR^3}{3\pi \in_0 r^2}$

(c) $\dfrac{Kr^2}{3\pi \in_0 R}$ (d) $\dfrac{KR^3}{3 \in_0 r^2}$

111. A solid metal sphere of radius a is carrying at charge $+q$. It is surrounded by a concentric uncharged hollow metal sphere of inner radius b and outer radius c. Choose the only wrong statement from the following.

(a) Electric field is zero at points for which $r < a$.

(b) Electric field is $\dfrac{q}{4\pi \in_0 r^2}$ for points lying between a and b.

(c) Electric field is zero for points lying between b and c.

(d) Electric field is $\dfrac{q}{2\pi \in_0 r^2}$ at points for which $r > c$.

112. A conducting spherical shell of inner radius a and outer radius b carries a net charge $+2Q$. It is surrounded by a larger spherical shell (concentric with the inner shell) of inner radius c and outer radius d and carries a net charge $+3Q$ as shown in Fig. 11.76. The electric field at point A at a distance r from centre O is

(a) $\dfrac{Q}{4\pi \in_0 r^2}$ (b) $\dfrac{2Q}{4\pi \in_0 r^2}$

(c) $\dfrac{3Q}{4\pi \in_0 r^2}$ (d) zero

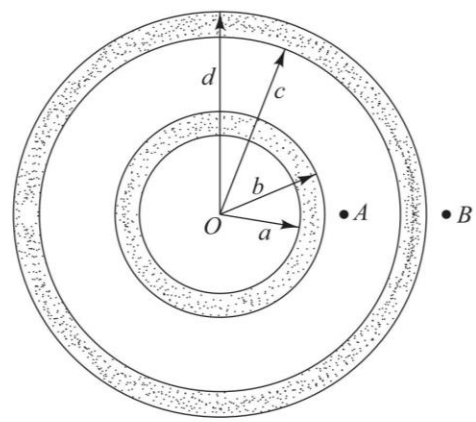


Fig. 11.76

113. In Q. 112 above, the electric field at point B at a distance r from centre O is

(a) zero (b) $\dfrac{2Q}{4\pi \in_0 r^2}$

(c) $\dfrac{3Q}{4\pi \in_0 r^2}$ (d) $\dfrac{5Q}{4\pi \in_0 r^2}$

114. A sphere carrying a charge $+Q$ is fixed. A small sphere carrying a charge $+q$ is placed near the bigger sphere and released from rest. Due to repulsion, the smaller sphere will move away from the bigger sphere with

(a) decreasing velocity and increasing acceleration
(b) increasing velocity and constant acceleration
(c) increasing velocity and decreasing acceleration
(d) increasing velocity and increasing acceleration

115. A thin non-conducting rod of length l carries a positive charge Q uniformly distributed along its length. The rod lies along the y-axis with its mid-point O at the origin. The electric field of the rod at point P at a distance $x = l$ from O is given by

$$E = \frac{kQ}{4\pi \epsilon_0 l^2}$$

where the value of k is

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{2}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{5}}$

116. Two concentric, conducting, thin spherical shells A and B have radii r and R as shown in Fig 11.77. Shell A carries a charge q and B carries a charge Q. Shell A is supported on an insulating stand. The potential of shell A is

(a) $\frac{1}{4\pi \epsilon_0}\left(\frac{q}{r}+\frac{Q}{R}\right)$ (b) $\frac{1}{4\pi \epsilon_0}\left(\frac{q+Q}{R-r}\right)$

(c) $\frac{1}{4\pi \epsilon_0}\left(\frac{Q}{r}+\frac{q}{R}\right)$ (d) $\frac{1}{4\pi \epsilon_0}\frac{q}{r}$

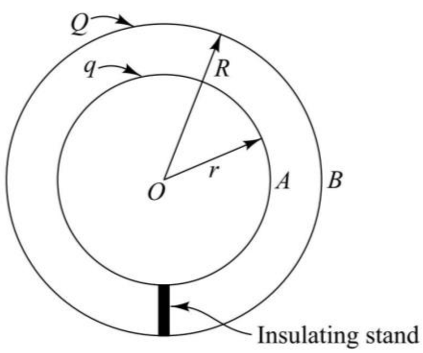


Fig. 11.77

117. In Q. 116 above the potential difference $(V_A - V_B)$ between the inner shell and the outer shell is

(a) $\frac{q}{4\pi \epsilon_0}\left(\frac{1}{r}-\frac{1}{R}\right)$ (b) $\frac{Q}{4\pi \epsilon_0}\left(\frac{1}{r}+\frac{1}{R}\right)$

(c) $\frac{q+Q}{4\pi \epsilon_0}\left(\frac{1}{r}-\frac{1}{R}\right)$ (d) $\frac{q+Q}{4\pi \epsilon_0}\left(\frac{1}{r}+\frac{1}{R}\right)$

118. A long conducting cylinder of radius R has a uniform linear charge density λ. The electric field at a point at a perpendicular distance r from the cylinder is

$$E = \frac{\lambda}{2\pi \epsilon_0 r}$$

The electric potential at that point is

(a) $\frac{\lambda}{2\pi \epsilon_0}$ (b) $-\frac{\lambda R}{2\pi \epsilon_0 r^2}$

(c) $\frac{\lambda}{2\pi \epsilon_0}\ln\left(\frac{r}{R}\right)$ (d) $\frac{\lambda}{2\pi \epsilon_0}\ln\left(\frac{R}{r}\right)$

119. A cylindrical capacitor of length l consists of a solid conducting cylinder of radius a which carries a linear charge density $+\lambda$, surrounded by a concentric outer cylindrical shell of radius b which carries a linear charge density $-\lambda$. The capacitance of the capacitor is

(a) $\frac{2\pi \epsilon_0 l}{\ln\left(\frac{b}{a}\right)}$ (b) $\frac{4\pi \epsilon_0 l}{\ln\left(\frac{b}{a}\right)}$

(c) $\frac{2\pi \epsilon_0 b}{\ln\left(\frac{b}{a}\right)}$ (d) $\frac{4\pi \epsilon_0 a}{\ln\left(\frac{b}{a}\right)}$

120. A spherical capacitor consists of a spherical conducting shell of radius a which is surrounded by a concentric outer spherical shell of radius b. The capacitance of the capacitor is

(a) $4\pi \epsilon_0\left(\frac{ab}{b+a}\right)$ (b) $4\pi \epsilon_0\left(\frac{ab}{b-a}\right)$

(c) $2\pi \epsilon_0\left(\frac{a^2}{b}\right)$ (d) $2\pi \epsilon_0\left(\frac{b^2}{a}\right)$

121. Fig. 11.78 shows two capacitors C_1 = 2μF and C_2 = 4μF connected to a 12V battery and two switches S_1 and S_2. Switch S_2 is initially kept open and S_1 is kept closed until the capacitor C_1 is fully charged. Switch S_1 is now opened and S_2 is closed until electrostatic conditions are restored. In this process, the percentage loss of energy stored in C_1 is

(a) 88.9% (b) 76.5%

(c) 63.4% (d) 55.0%

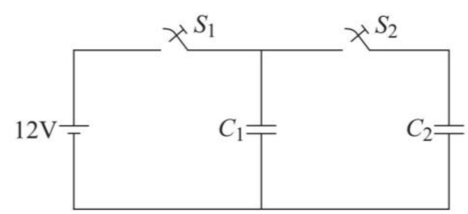


Fig. 11.78

122. Two isolated conducting spheres 1 and 2 each carry a charge $+Q$. Sphere 1 has radius a and sphere B has radius $4a$. If the two spheres are connected by a conducting wire, the charge on sphere 1 is Q_1 and on sphere 2 is Q_2. Then

(a) $Q_1 = Q, \quad Q_2 = Q$

(b) $Q_1 = \frac{2Q}{3}, \quad Q_2 = \frac{4Q}{3}$

(c) $Q_1 = \frac{2Q}{5}, \quad Q_2 = \frac{8Q}{5}$

(d) $Q_1 = \frac{Q}{2}, \quad Q_2 = \frac{3Q}{2}$

123. A solid non-conducting sphere of radius R has a uniform volume charge density ρ. Spherical portion of radius r is removed from the solid sphere as shown in Fig. 11.79. The magnitude of electric field at a point P at a distance $a < r$ from the centre of the cavity is

(a) proportional to a (b) proportional to b
(c) proportional to c (d) zero

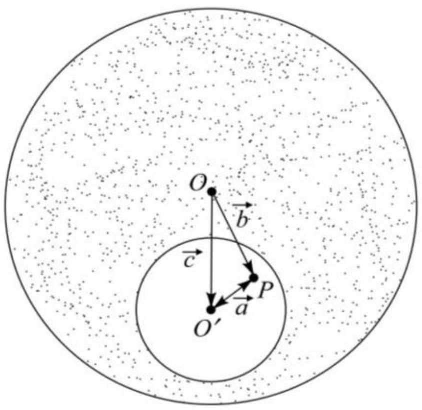


Fig. 11.79

124. A long hollow conducting cylinder A of radius R_1 is kept inside another long hollow conducting cylinder B of radius $R_2 > R_1$. Initially cylinders A and B carry no charge. Which of the following statements is true?

1. When a charge density is given to the outer cylinder A, a potential difference appears between cylinders A and B.
2. When a charge density is given to the inner cylinder B, a potential difference appears between cylinders A and B.
3. When the same charge density is given to both A and B, no potential difference appears between them.

(a) 1 only (b) 2 only
(c) 1 and 3 only (d) 2 and 3 only

125. A negative point charge $-Q$ is placed at distance of r from the surface of an uncharged conducting sphere of radius R. The net charge on the sphere will be

(a) positive
(b) negative
(c) positive if $r < R$ and negative if $r > R$
(d) zero

126. A capacitor of capacitance C_1 is charged to the voltage V of a battery as shown in Fig. 11.80. The fraction of its stored energy dissipated after the switch S is disconnected from 1 and connected to 2 is

(a) $\frac{C_1}{C_1 + C_2}$ (b) $\frac{C_2}{C_1 + C_2}$

(c) $\frac{C_1C_2}{(C_1 + C_2)^2}$ (d) $\frac{C_1C_2}{(C_1 - C_2)^2}$

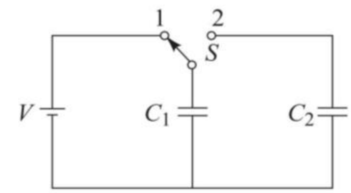


Fig. 11.80

127. Three non-conducting spheres 1, 2 and 3 have radii R, $2R$ and $3R$ respectively. Equal charge Q is uniformly distributed in each sphere. If E_1, E_2 and E_3 are the magnitudes of electric fields at point P at a distance $2R$ from the centre of spheres 1, 2 and 3 respectively as shown in Fig. 11.81, then

(a) $E_1 > E_2 > E_3$ (b) $E_1 < E_2 < E_3$
(c) $E_1 = E_2 = E_3$ (d) $E_2 = E_3 > E_1$

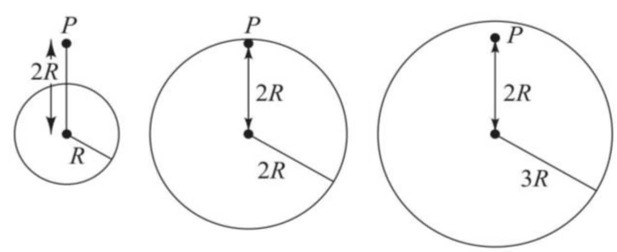


Fig. 11.81

128. In the circuit shown in Fig. 11.82, 7 μC of charge is given to the right plate of the 5 μF capacitor. Then, in the steady state, the charge on the left place of the 4 μF capacitor will be

(a) 1.0 μC (b) 3.5 μC
(c) 2.8 μC (d) 4.0 μC

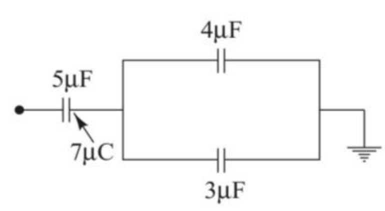


Fig. 11.82

129. The electric potential at a point in three dimensions is given by $V = x^2y + yz$. The magnitude of the electric field at point P (1, 2, 3) is

(a) 6 units (b) 7 units
(c) 8 units (d) $\sqrt{14}$ units

130. The electric potential at a space point $P(x, y, z)$ is

$$V = x^2 + y^2 + z^2$$

The modulus of the electric field at that point is proportional to

(a) $V^{1/2}$ (b) V
(c) $V^{3/2}$ (d) V^2

131. A solid conducting sphere of radius r is surrounded by a conducting hollow spherical shell of radius R with their centres coinciding. When a charge $+Q$ is given to the sphere, a potential difference V appears between the surface of the sphere and the outer surface of the shell. If the shell is now given a charge $-nQ$, the new potential difference between the surface of the sphere and the outer surface of the shell will be

(a) V (b) nV
(c) $(n + 1)V$ (d) $(n - 1)V$

132. Three concentric metallic spherical shells 1, 2 and 3 have radii R, $2R$ and $3R$ respectively. Each shell is given the same charge Q. Their surface charge densities will be in the ratio of

(a) 3 : 2 : 1 (b) 9 : 4 : 1
(c) 6 : 3 : 2 (d) 36 : 9 : 4

133. The energy stored in the network of capacitors shown in Fig 11.83 is

(a) 40 μJ (b) 80 μJ
(c) 120 μJ (d) 160 μJ

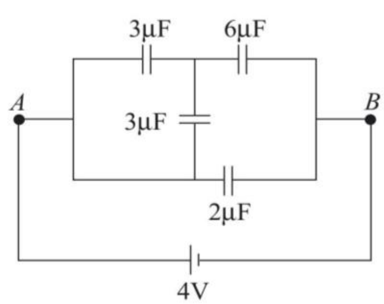


Fig. 11.83

134. In the circuit shown in Fig. 11.84, the charge on the 5 μF capacitor is

(a) 10 μC (b) 20 μC
(c) 30 μC (d) 40 μC

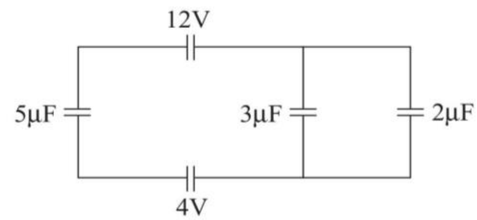


Fig. 11.84

Answers

Level A

1. (b)	2. (d)	3. (b)	4. (a)
5. (d)	6. (d)	7. (c)	8. (b)
9. (d)	10. (c)	11. (c)	12. (a)
13. (d)	14. (a)	15. (c)	16. (c)
17. (a)	18. (c)	19. (b)	20. (d)
21. (b)	22. (c)	23. (a)	24. (d)
25. (c)	26. (d)	27. (b)	28. (b)
29. (d)	30. (b)	31. (a)	32. (c)
33. (b)	34. (d)	35. (d)	36. (d)
37. (a)	38. (c)	39. (b)	40. (c)
41. (d)			

Level B

42. (d)	43. (c)	44. (c)	45. (c)
46. (b)	47. (b)	48. (c)	49. (a)
50. (d)	51. (a)	52. (a)	53. (b)
54. (a)	55. (a)	56. (d)	57. (d)
58. (c)	59. (b)	60. (b)	61. (a)
62. (c)	63. (d)	64. (d)	65. (d)
66. (a)	67. (b)	68. (d)	69. (a)
70. (b)	71. (b)	72. (d)	73. (d)
74. (b)	75. (b)	76. (c)	77. (d)
78. (b)	79. (b)	80. (d)	81. (a)
82. (b)	83. (b)	84. (c)	85. (b)
86. (c)	87. (a)	88. (d)	89. (a)
90. (c)	91. (c)	92. (a)	93. (c)
94. (b)	95. (c)	96. (a)	97. (b)
98. (b)	99. (b)	100. (c)	101. (c)

102. (a)	103. (a)	104. (b)	105. (c)
106. (b)	107. (a)	108. (c)	109. (d)
110. (d)	111. (d)	112. (b)	113. (d)
114. (c)	115. (d)	116. (a)	117. (a)
118. (d)	119. (a)	120. (b)	121. (a)
122. (c)	123. (c)	124. (b)	125. (d)
126. (b)	127. (c)	128. (d)	129. (a)
130. (a)	131. (a)	132. (c)	133. (a)
134. (b)			

Solutions

Level A

1. The potential inside a spherical conductor is constant and is the same as that on the surface. Hence the correct choice is (b).
2. Let the charge Q be at P, with $OP = x$. The resultant force F is along the x-axis directed towards the origin. The charge Q moves to O, and acquires kinetic energy. It will cross O and move to -ve x-axis until it comes to rest. It is again attracted towards O and crosses it and this process continues. Therefore charge Q executes periodic motion (see Fig. 11.85).

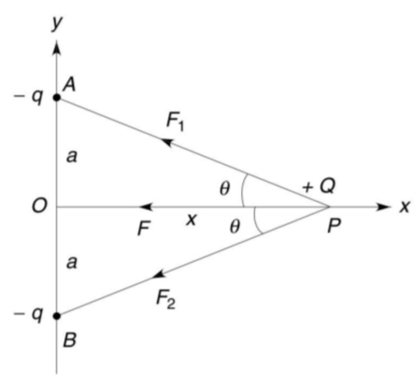


Fig. 11.85

Let $AP = BP = r$. Then

$$F_1 = F_2 = \frac{qQ}{4\pi\varepsilon_0 r^2}$$

The resultant force on Q is

$$F = F_1 \cos\theta + F_2 \cos\theta = \frac{2qQ}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$F = \frac{2qQx}{4\pi\varepsilon_0 r^3} = \frac{2qQ}{4\pi\varepsilon_0}\frac{x}{(a^2+x^2)^{3/2}}$$

Thus F is not of the form $F = kx$ (where k = constant) and hence the motion is not simple harmonic. Hence the correct choice is (d).

3. Each parallel combination of capacitors is equivalent to a capacitance of 100 μF connected in series. Potential drop across each of them will be 50 V. Charge $Q = CV = 100 \times 10^{-6} \times 50 = 5 \times 10^{-3}$ C
Hence the correct choice is (b).
4. Refer to Fig. 11.86.

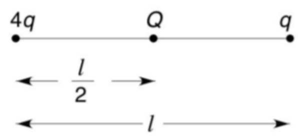


Fig. 11.86

The net force on q will be zero if

$$\frac{q \cdot 4q}{4\pi\varepsilon_0 l^2} + \frac{qQ}{4\pi\varepsilon_0 (l/2)^2} = 0$$

$$4q^2 + 4qQ = 0$$

or $$4q(q+Q) = 0$$

$\therefore$ $$Q = -q$$

Hence the correct choice is (a).

5. Electrostatic potential energy when the charges are 10 cm = 0.1 m apart is

$$W_1 = \frac{q_1 q_2}{4\pi\varepsilon_0 r} = \frac{12\times10^{-6}\times8\times10^{-6}}{4\pi\varepsilon_0\times0.1}$$

$$= \frac{96\times10^{-11}}{4\pi\varepsilon_0}$$

Potential energy when the charges are brought 4 cm closer, i.e., when they are 6 cm = 0.06 m apart is

$$W_2 = \frac{12\times10^{-6}\times8\times10^{-6}}{4\pi\varepsilon_0\times0.06} = \frac{16\times10^{-10}}{4\pi\varepsilon_0}$$

$\therefore$ Work done $= W_2 - W_1 = \frac{10^{-10}}{4\pi\varepsilon_0}(16-9.6)$

$$= 9\times10^{9}\times10^{-10}\times6.4$$

$$= 5.76 \text{ J} \simeq 5.8 \text{ J}$$

Hence the correct choice is (d).

6. The work done in carrying a charge round a closed path is zero. Hence the correct choice is (d).
7. When the capacitor is fully charged, no current flows in the 10 Ω resistor. The current in the circuit is

$$I = \frac{2.5}{2+0.5} = 1 \text{ A}$$

$\therefore$ Potential drop across $2\,\Omega$ resistor $= 2\Omega \times 1\text{A} = 2\text{V}$
This is also the potential drop across the capacitor plates. Therefore, the charge on capacitor plates is

$$Q = CV = 2 \times 10^{-6} \times 2 = 4 \times 10^{-6}\ \text{C} = 4\ \mu\text{C}$$

Hence the correct choice is (c).

8. Refer to Fig. 11.87.

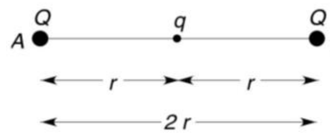


Fig. 11.87

The three charges will be in equilibrium, if no net force acts on each charge. The charge q is in equilibrium because the forces exerted on q by charge Q at A and charge Q at B are equal and opposite. The charge Q at A will be in equilibrium if the forces exerted on it by charge q and charge Q at B are equal and opposite, i.e. if

$$\frac{qQ}{4\pi\varepsilon_0 r^2} = -\frac{Q \times Q}{4\pi\varepsilon_0 (2r)^2}$$

or $\quad q = -\dfrac{Q}{4}$

Similarly, charge Q at B will be in equilibrium if $q = -\dfrac{Q}{4}$. Hence the correct choice is (b).

9. Electric field $E = -\dfrac{dV}{dx} = -\dfrac{d}{dx}(5 + 4x^2) = -8x$

Force on charge $(-q) = -qE = +8qx$
At $x = 0.5$ m, force $= 8 \times 2 \times 10^{-6} \times 0.5 = 8 \times 10^{-6}$ N
Hence the correct choice is (d).

10. Original capacitance of the parallel combination of C and $2C = C + 2C = 3C$. Total charge $Q = 3CV$. When the capacitor C is filled with dielectric, its capacitance becomes KC. Therefore, the capacitance of the combination after the capacitor C is filled with dielectric, $C' = KC + 2C = (K + 2)C$. Since the charge remains the same, $Q = 3CV$, the potential difference across the capacitors will be

$$\frac{Q}{C'} = \frac{3CV}{(K+2)C} = \frac{3V}{K+2}$$

Hence the correct choice is (c).

11. The capacitance of the capacitor is $C = \varepsilon_0 A/x$ where x is the distance between the plates. The energy stored in the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{\varepsilon_0 AV^2}{2x}$$

Differentiating w.r.t x we get

$$\frac{dU}{dx} = \frac{\varepsilon_0 AV^2}{2}\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{\varepsilon_0 AV^2}{2x^2}$$

The force of attraction between the plates is

$$F = -\frac{dU}{dx} = \frac{\varepsilon_0 AV^2}{2x^2} \qquad \text{(i)}$$

Now $\quad Q = CV = \dfrac{\varepsilon_0 AV}{x} \quad$ or $\quad V = \dfrac{Qx}{\varepsilon_0 A} \qquad$ (ii)

Using (ii) in (i) we get

$$F = \frac{Q^2}{2\varepsilon_0 A}$$

Hence the correct choice is (c).

12. The last three capacitors on the right, each of capacitance $C = 9\ \mu$F are in series and are equivalent to a capacitance C' given by

$$\frac{1}{C'} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1}{3} \text{ or } C' = 3\ \mu\text{F}.$$

Since C' is in parallel with C_1, the equivalent capacitance of the last part of the network is $C'' = C' + C_1 = 3 + 6 = 9\ \mu$F. Continuing this process of calculation towards the left, we notice that we are finally left with the combination whose equivalent capacitance is $3\ \mu$F. Hence the correct choice is (a).

13. The three capacitors can be rearranged as shown in Fig. 11.88. The capacitance between points P and S or between points Q and R = sum of the three capacitances $= 3C = 9\ \mu$F. Hence the correct choice is (d).

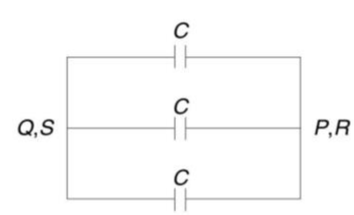


Fig. 11.88

14. $C = 100$ pF $= 100 \times 10^{-12}$ F $= 10^{-10}$ F
Let the number of sheets of foils required be n. They will form $(n - 1)$ capacitors. If K is the dielectric constant of the dielectric, the capacitance is given by

$$C = \frac{K\varepsilon_0 (n-1)A}{d}$$

or $\quad n-1 = \dfrac{Cd}{K\varepsilon_0 A} = \dfrac{Cd}{K\cdot 4\pi\varepsilon_0}\cdot\dfrac{4\pi}{\pi r^2}$

$$= \frac{4Cd}{K\cdot 4\pi\varepsilon_0 r^2}$$

$$= \frac{4\times10^{-10}\times1\times10^{-3}\times9\times10^9}{4\times\left(1.0\times10^{-2}\right)^2} = 9$$

or $n = 10$

Hence the correct choice is (a).

15. If R is the radius of the big drop, we have

$$\frac{4\pi R^3}{3} = 1000\times\frac{4\pi r^3}{3}$$

which gives $R = 10\,r$. The electrical potential of each droplet is

$$v = \frac{q}{4\pi\varepsilon_0 r}$$

and that of the big drop is

$$V = \frac{1000q}{4\pi\varepsilon_0 R}$$

$$\therefore \quad \frac{V}{v} = \frac{1000r}{R} = 100 \qquad (\because R = 10\,r)$$

Hence the correct choice is (c).

16. If A is the area of each plate, the capacitance of the air capacitor shown in Fig. 11.52(a) on page 11.25 is

$$C_0 = \frac{\varepsilon_0 A}{d}, \text{ where } C_0 = 2\ \mu\text{F (given)}.$$

The capacitance of air capacitor in Fig. 11.53 (b) is

$$C_1 = \frac{\varepsilon_0 A}{d/2} = \frac{2\varepsilon_0 A}{d} = 2C_0$$

The capacitance of the dielectric filled capacitor in Fig. 11.53 (b) is

$$C_2 = \frac{k\varepsilon_0 A}{d/2} = \frac{2k\varepsilon_0 A}{d} = 2kC_0$$

where k is the dielectric constant. Now capacitors C_1 and C_2 are in series. Therefore, the capacitance C of the capacitor shown in Fig. 11.52 (b) is given by

$$\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2} = \frac{1}{2C_0}+\frac{1}{2kC_0} = \frac{(k+1)}{2C_0 k}$$

or $\quad C = \dfrac{2C_0 k}{(k+1)} = \dfrac{2\times2\mu\text{F}\times3}{(3+1)} = 3\ \mu\text{F}$

Hence the correct choice is (c).

17. If A is the area of each plate, the capacitance of the air-filled capacitor shown in Fig. 11.53(a) on page 11.26 is

$$C_0 = \frac{\varepsilon_0 A}{d}, \text{ where } C_0 = 2\ \mu\text{F (given)}.$$

The capacitance of air capacitor in Fig. 11.54(b) is

$$C_1 = \frac{\varepsilon_0 A/2}{d} = \frac{\varepsilon_0 A}{2d} = \frac{C_0}{2}$$

The capacitance of dielectric filled capacitor in Fig. 11.53(b) is

$$C_2 = \frac{k\varepsilon_0 A/2}{d} = \frac{k\varepsilon_0 A}{2d} = \frac{kC_0}{2}$$

Since C_1 and C_2 are in parallel, the capacitance C of the capacitor shown in Fig. 11.53(b) is

$$C = C_1 + C_2 = \frac{C_0}{2}+\frac{kC_0}{2}$$

$$= \frac{C_0}{2}(1+k) = \frac{2\mu\text{F}}{2}(1+3) = 4\ \mu\text{F}$$

Hence the correct choice is (a).

18. Refer to Fig. 11.89. Let us consider forces on a ball, say, Q. Three forces act on it: (i) tension T in the thread, (ii) force mg due to gravity and (iii) force F due to Coulomb repulsion along +ve x-direction. For equilibrium, the sum of the x and y components of these forces must be zero,

i.e. $\quad T\cos 60^\circ - F = 0$

and $\quad T\sin 60^\circ - mg = 0$

These equations give $F = mg\cot 60^\circ = \dfrac{\sqrt{3}}{10}\times10^{-3}\times10\times\dfrac{1}{\sqrt{3}} = 10^{-3}$ N. Now

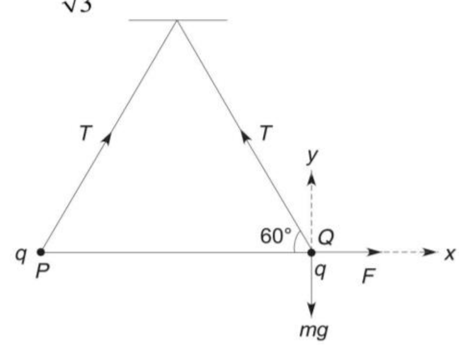


Fig. 11.89

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2}$$

Putting $F = 10^{-3}$ N, $r = 0.3$ m and $\dfrac{1}{4\pi\varepsilon_0} = 9\times10^9$, we get $q = 10^{-7}$ coulomb.

19. Refer to Fig. 11.90. The electric field E_1 at (a, b) due to q_1 has a magnitude

$$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{a^2}$$

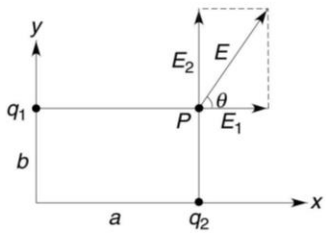


Fig. 11.90

and is directed along $+x$-axis. The electric field E_2 at (a, b) due to q_2 has a magnitude

$$E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{b^2}$$

and is directed along $+y$-axis. The angel θ subtended by the resultant field E with the x-axis is given by

$$\tan\theta = \frac{E_2}{E_1} = \frac{q_2}{q_1}\cdot\frac{a^2}{b^2} = \frac{1}{2}\times\left(\frac{2}{1}\right)^2 = 2$$

Hence the correct choice is (b).

20. The correct choice is (d). The electric field E exerts a force qE on charge $+q$ and a force $-qE$ on charge $-q$ of the dipole. Since these forces are equal and opposite, they add upto zero.

21. The correct choice is (b). A torque acts on the dipole which tends to align it along the field.

22. The correct choice is (c). In a non-uniform electric field, a dipole experiences a force which gives it a translational motion and a torque which gives it a rotational motion.

23. The distance of a vertex from the the centre of the cube of side b is $r = \sqrt{3}\ b/2$. Now the potential due to charge q at the centre is $q/4\pi\varepsilon_0 r$. Hence the potential due to the arrangement of eight charges (each of magnitude q) at the centre is

$$V = \frac{8q}{4\pi\varepsilon_0 r} = \frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$$

24. We know that electric fields are to be added vectorially. From the symmetry of the eight charges with respect to the centre of the cube, it is evident that the electric fields at the centre due to two opposite charges cancel in pairs (being equal and opposite). Hence the net electric field at the centre of the cube will be zero.

25. Refer to Fig. 11.91. The distance of point P_1 from charge $+q$ is $r_1 = z - a$ and from charge $-q$ is $r_2 = z + a$.

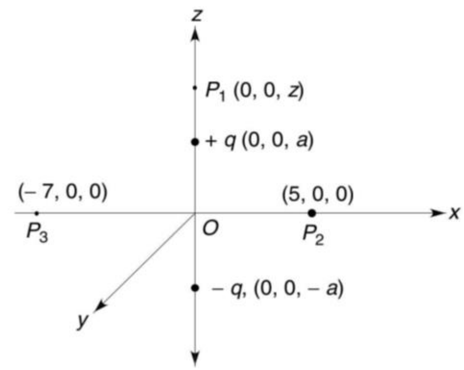


Fig. 11.91

$$\therefore \text{ Potential at } P_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{r_2 - r_1}{r_1 r_2}$$

$$= \frac{2qa}{4\pi\varepsilon_0\left(z^2 - a^2\right)},$$

which is choice (c).

26. Refer to Fig. 11.91 again. Any point on the perpendicular bisector passing through the centre of the dipole is at the same distance from the two charges. Hence the potentials at point $P_2(5, 0, 0)$ and that at point $P_3(-7, 0, 0)$ are zero. Since P_2 and P_3 are at the same potential (zero), the potential difference between them is zero. Hence no work will be done in moving a charge from P_2 to P_3. The answer will not change if the path of the charge is changed because the work done is independent of the path taken.

27. Refer to Fig. 11.92. The total potential energy of the arrangement of charges is the sum of the energies of each pair of charges. The potential energy of the system comprising the three charges q_1, q_2 and q_3 is

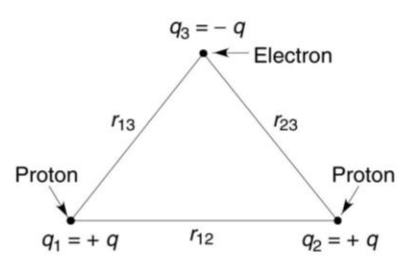


Fig. 11.92

$$U = W_1 + W_2 + W_3$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \quad \text{(i)}$$

Here $q_1 = q_2 = q = +1.6 \times 10^{-19}$ C (proton), $q_3 = -q = -1.6 \times 10^{-19}$ C (electron), $r_{12} = 1.5$ Å $= 1.5 \times 10^{-10}$ m, $r_{13} = r_{23} = 1$ Å $= 1 \times 10^{-10}$ m and $1/4\pi\varepsilon_0 = 9 \times 10^9$ Nm^{2}C^{-2}. Thus

$$U = -\frac{4}{3}\cdot\frac{q^2 \times 10^{10}}{4\pi\varepsilon_0} \text{ joule}$$

$$= -\frac{4}{3}\cdot\frac{q \times 10^{10}}{4\pi\varepsilon_0} \text{ eV} \quad (\because q = 1.6 \times 10^{-19} \text{ C})$$

$$= -\frac{4 \times 1.6 \times 10^{-19} \times 10^{10} \times 9 \times 10^9}{3}$$

$$= -19.2 \text{ eV}$$

28. Charge on electron $(-e) = -1.6 \times 10^{-19}$ C, charge on proton $(e) = 1.6 \times 10^{-19}$ C, separation $r = 0.53$ Å $= 0.53 \times 10^{-10}$ m. If the zero of potential energy is taken to be at infinite separation, the potential energy of the electron-proton system is

$$U = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r} \text{ joule}$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e}{r} \text{ eV}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})}{0.53 \times 10^{-10}} = -27.2 \text{ eV}$$

Hence the correct choice is (b).

29. If the electron was at rest, 27.2 eV of the energy will have to be supplied (or $27.2 \times 1.6 \times 10^{-19}$ J of work will have to be done) to free the electron from the attraction of the proton and remove it to infinity. Since the electron is moving (round the proton) with a kinetic energy $= U/2 = 1/2 \times (-27.2) = -13.6$ eV, the electron itself is supplying an energy of 13.6 eV due to centrifugal action. Hence the minimum amount of work required to free the electron $= 27.2 - 13.6 = 13.6$ eV $= 13.6 \times 1.6 \times 10^{-19} = 2.2 \times 10^{-18}$ J. Hence the correct choice is (d).

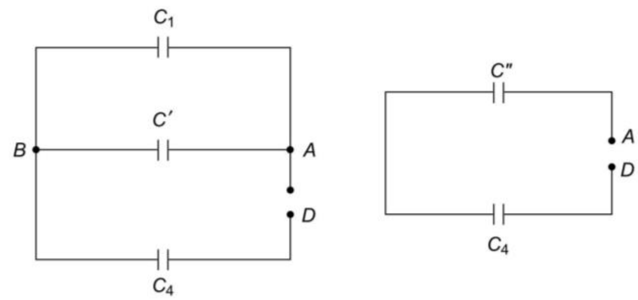
30. The potential energy of electron-proton system at a separation of 1.06 Å = half that at a separation of 0.53 Å = half of – 27.2 eV = – 13.6 eV. If the zero of potential energy at a separation of 1.06 Å is taken to be zero (instead of – 13.6 eV), the potential energy of the electron-proton system would be $= -27.2 - (-13.6) = -13.6$ eV, which is choice (b).

31. Since the potential energy of the system is now – 13.6 eV, the energy supplied by the electron itself is 13.6 eV by virtue of its orbital motion round the proton. Hence the minimum work to pull the electron from the atom will be zero.

32. The series combination of C_2 and C_3 is equivalent to a capacitance C' given by

$$\frac{1}{C'} = \frac{1}{C_2} + \frac{1}{C_3}$$

or $$C' = \frac{C_2 C_3}{C_2 + C_3} = \frac{200 \times 200}{200 + 200} = 100 \text{ pF}$$



(a) (b)

Fig. 11.93

Therefore the circuit reduces to the one shown in Fig. 11.93(a). The equivalent capacitance between points A and B is

$$C'' = C_1 + C' = 100 + 100 = 200 \text{ pF}$$

The circuit may be further simplified to that in Fig. 11.93(b). The equivalent capacitance C of the entire network, i.e., between points A and D, is now that of the series combination of C'' and C_4. Thus

$$\frac{1}{C} = \frac{1}{C''} + \frac{1}{C^4} = \frac{1}{200} + \frac{1}{100}$$

$$= \frac{3}{200} \quad \text{or} \quad C = \frac{200}{3} \text{ pF}$$

Hence the correct choice is (c).

33. The series combination of 6 and 12 is equivalent to 4 and the parallel combination of 2 and 2 is also equivalent to 4. Therefore the network can be simplified as shown in Fig. 11.94.

The parallel combination of 4 and 4 is equivalent to 8 and the series combination of 8 and 4 is equivalent to 8/3. Thus the combination in Fig. 11.94 reduces to that in Fig. 11.95.

The series combination of 1 and 8 in Fig. 11.95 yields 8/9 as shown in Fig. 11.96.

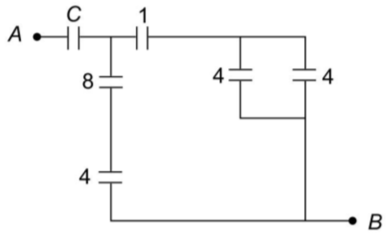


Fig. 11.94

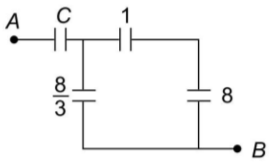


Fig. 11.95

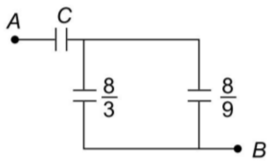


Fig. 11.96

Now 8/3 and 8/9 are in parallel and their equivalent is 32/9. Therefore, the network finally reduces to that in Fig. 11.97. Since the total capacitance between A and B is to be (i.e. 1 μF), we have

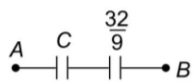


Fig. 11.97

$$1 = \frac{1}{C} + \frac{9}{32}$$

$$\Rightarrow \quad C = \frac{32}{23}\ \mu\text{F. Hence the correct choice is (b).}$$

34. Charge Q_1 and $C_1 = C_1 V_1 = 2 \times 10^{-6} \times 100 = 2 \times 10^{-4}$ C. Charge Q_2 on $C_2 = C_2 V_2 = 4 \times 10^{-6} \times 50 = 2 \times 10^{-4}$ C. Total charge $Q = Q_1 + Q_2 = 4 \times 10^{-4}$ C. Total energy before connection is

$$E_1 = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

$$= \frac{1}{2} \times 2 \times 10^{-6} \times (100)^2 + \frac{1}{2} \times 4 \times 10^{-6} \times (50)^2$$

$$= 1.5 \times 10^{-2}\ \text{J}$$

The common potential difference V after connection is given by

$$C_1 V + C_2 V = Q$$

or $$V = \frac{Q}{C_1 + C_2}$$

Therefore, total energy after connection is

$$E_2 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2} \times \frac{Q^2}{(C_1 + C_2)}$$

$$= \frac{1}{2} \times \frac{(4 \times 10^{-4})^2}{(2+4) \times 10^{-6}} = 1.33 \times 10^{-2}\ \text{J}$$

∴ Loss of energy $= E_1 - E_2 = 0.17 \times 10^{-2}$ J. Hence the correct choice is (d).

35. Points A, B and C are at the same distance from charge $+q$; hence electrical potential is the same at these points, i.e. there is no potential difference between A, B and C. Hence $W_1 = W_2 = 0$.

36. The electric potential at the common centre is

$$V = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2}$$

Now $$\sigma = \frac{q_1}{4\pi r_1^2} = \frac{q_2}{4\pi r_2^2}$$

∴ $$V = \frac{1}{\varepsilon_0}\left[\frac{q_1 r_1}{4\pi r_1^2} + \frac{q_2 r_2}{4\pi r_2^2}\right] = \frac{\sigma}{\varepsilon_0}(r_1 + r_2)$$

Hence the correct choice is (d).

37. If q is charge on the sphere, the electric field on its surface is

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

But $\sigma = \dfrac{q}{4\pi r^2}$. Therefore, $q = 4\pi r^2 \sigma$. Hence

$$E = \frac{4\pi r^2 \sigma}{4\pi\varepsilon_0 r^2} = \frac{\sigma}{\varepsilon_0}$$

Thus the correct choice is (a).

38. The correct choice is (c)

39. The correct choice is (b).

40. If the radius of a bubble is increased by a factor n, its capacitance is also increased by a factor n, i.e. $C' = nC$. Since the charge Q on the bubble remains unchanged, we have

$$Q = CV = C'V'$$

or $$V' = \frac{CV}{C'} = \frac{CV}{nC} = \frac{V}{n}$$

Hence the correct choice is (c).

41. If ρ is the density of a small drop and r its radius, then the mass of each small drop is $m = \dfrac{4\pi}{3} r^3 \rho$. If n such drops coalesce to form a big drop of radius R, then the mass of the big drop is $nm = \dfrac{4\pi}{3} R^3 \rho$. Hence $R = n^{1/3} r$. Now, the capacitance of a sphere is proportional to its radius. Hence the capacitance of the big drop will be $C' = n^{1/3} C$. Hence the correct choice is (d).

Level B

42. If Q is the charge on each small drop, charge on the big drop is $Q' = nQ$. Now $Q' = C'V' = Q = CV$. Therefore

$$\frac{V'}{V}=\frac{Q'}{Q}\times\frac{C}{C'}=\frac{n}{n^{1/3}}=n^{2/3}$$

Hence the correct choice is (d).

43. $E=\frac{1}{2}\ CV^2, E'=\frac{1}{2}\ C'\ V'^2$. Therefore,

$$\frac{E'}{E}=\frac{C'}{C}\cdot\frac{V'^2}{V}=n^{1/3}\times(n^{2/3})^2=n^{5/3}$$

Hence the correct choice is (c).

44. The combination is equivalent to (10 – 1) = 9 capacitors, each of capacitance C connected in parallel. Hence the correct choice is (c).

45. The total capacitance across power supply = $\frac{6}{8}$ μF. The charge on 2 μF capacitor or 3 μF capacitor = $8\times\frac{6}{8}=6$ μC. So the charge on each 1 μF capacitor = 3 μC. Therefore, potential difference across each 1 μF capacitor = charge/capacitance = 3 μC/1 μF = 3 volts.

46. Capacitors of capacitances 2 μF and 3 μF are in parallel and this combination is in series with 1 μF capacitor. Thus, we have 1 μF capacitor in series with 5 μF capacitor and the potential difference across this series combination is 6V. Therefore, the potential differences across 5 μF capacitor (which consists of a parallel combination of 2 μF and 3 μF capacitors) is 1 V. Hence the charge on 2 μF capacitor = 2 μF × 1 V = 2 μC, which is choice (b).

47. Capacitors 1 μF, 2 μF and 3 μF are in parallel, their total capacitance is 6 μF. Thus, we have three capacitors in series each of capacitance 6 μF across the 12 V power supply. So, the potential drop across each is 12/3 = 4 V. This is also the potential across 1 μF capacitor and 2 μF capacitor and 3 μF capacitor, because they are in parallel. Therefore, charge on 2 μF capacitor = 2 μF × 4 V = 8 μC. Hence the correct choice is (b).

48. The distance of the point of intersection of diagonals = side of the hexagon = a. The potential at this point due to each charge = $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a}$. Therefore, total potential $=\frac{1}{4\pi\varepsilon_0}\cdot\frac{6q}{a}$ which is choice (c).

49. The net electric field at the point of intersection of diagonals is zero because the electric field at this point due to equal charges at opposite corners with cancel each other in pairs.

50. If C is the capacitance of the air-filled capacitor, the total charge on its plates, before connection, is $Q = CV$. After it is connected with an uncharged capacitor, let V' be the common potential and Q_1 be the charge on capacitor C and Q_2 on the other capacitor

$Q_1 = V'\ C$ and $Q_2 = V'\ kC$. Also $Q = Q_1 + Q_2$. Therefore,

$$CV = V'\ C + V'\ kC$$

or $\quad V = V'\ (1 + k)$

or $\quad V'=\frac{V}{(1+k)}$. Hence the correct choice is (d).

51. Energy stored in the capacitor is

$$\frac{1}{2}\ CV^2=\frac{1}{2}\times C\times(200)^2=2\times10^4\times C\text{ joule}$$

Energy appearing as heat in the block is

$$m\ c\ \theta = 0.1\times2.5\times102\times0.4=10\text{ J}$$

Therefore,

$$2\times10^4\times C=10$$

or $\quad C=5\times10^{-4}\text{ F}=500\ \mu\text{F}$

52. The force of repulsion between the two parts is given by

$$F=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q(Q-q)}{r^2}$$

For F to be maximum, $\frac{dF}{dq}=0$, i.e.

$$\frac{d}{dq}\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{q(Q-q)}{r^2}\right]=0$$

Since r is fixed, we have

$\frac{d}{dq}\ [q\ (Q-q)]=0$ or $1(Q-q)+q\ (0-1)=0$ or $\frac{q}{Q}=\frac{1}{2}$

Hence the correct choice is (a).

53. For points on the surface of the sphere or outside the sphere, a charged sphere behaves as if the charge is concentrated at its center. Therefore, the potential at the surface of the sphere is given by

$$V=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$$, which is choice (b).

54. At points inside a charged metallic sphere, i.e. for $r < R$, the potential is zero. Hence the correct choice is (a).

55. $E = -\frac{dV}{dx}\hat{\mathbf{i}}$ where $\hat{\mathbf{i}}$ is a unit vector along the positive x-axis. Hence $\mathbf{E}$ at a point whose x-coordinate is $x = 1$ m is

$$\mathbf{E} = -\frac{d}{dx}(4x^2)\hat{\mathbf{i}} = -8x\hat{\mathbf{i}} = -8\hat{\mathbf{i}}\ \mathrm{Vm^{-1}}.$$

The negative sign shows that $\mathbf{E}$ is along the negative x-axis. Hence the correct choice is (a).

56. If a symmetrical closed surface has n identical surfaces and a charge Q is placed at its centre, then the flux through each face $= \frac{Q}{n\varepsilon_0}$. For a cube $n = 6$. Hence the correct choice is (d).

57. Since a dipole consists of two equal and opposite charges, the net charge of a dipole is zero. Hence the correct choice is (d).

58. The correct choice is (c).

59. Let C be the capacitance of each capacitor. For parallel combination, the net capacitance is $C_1 = nC$. Also $V_1 = V$. Therefore, the energy stored in the parallel combination is

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times nC\times V^2 = \frac{1}{2}nCV^2$$

For series combination, we have $C_2 = C/n$ and $V_2 = nV$. Therefore, the energy stored in the series combination is

$$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}\times\frac{C}{n}\times(nV)^2 = \frac{1}{2}nCV^2$$

Hence the correct choice is (b).

60. There will be no loss of energy if the potential of the spheres is the same i.e. if

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{Q}{4\pi\varepsilon_0 R}$$

or $\frac{q}{r} = \frac{Q}{R}$. Hence the correct choice is (b).

61. $\mathbf{r} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) - (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ m. The magnitude of $\mathbf{r}$ is

$$r = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1+4+4} = 3\text{ m}$$

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}$$

$$= \frac{9\times10^9\times10^{-6}\times10^{-6}}{(3)^2} = 10^{-3}\text{ N}$$

Hence the correct choice is (a).

62. Initial capacitance $C = \frac{\varepsilon_0 A}{d}$. When a metal plate of thickness t is introduced, the capacitance becomes $C' = \frac{\varepsilon_0 A}{(d-t)}$. Given $C' = 4.5\,C$

Thus $$\frac{\varepsilon_0 A}{d-t} = \frac{\varepsilon_0 A}{d}\times\frac{9}{2}$$

which gives $9(d-t) = 2d$ or $t = \frac{7d}{9}$ which is choice (c).

63. $U_1 = \frac{1}{2}CV^2,\ U_2 = \frac{1}{2}C(1.2\,V)^2 = \frac{1}{2}CV^2\times1.44$

$$\therefore \quad \frac{U_2 - U_1}{U_1}\times100 = (1.44-1)\times100 = 44\%$$

Thus the correct choice is (d).

64. $U = \frac{1}{2}CV^2$. Therefore, $\delta U = CV\,\delta V$. Therefore,

$$\frac{dU}{U}\times100 = \frac{CV\delta V}{\frac{1}{2}CV^2}\times100 = \frac{2\delta V}{V}\times100$$

$$= \frac{2\times0.1\times100}{100} = 0.2\%$$

Hence the correct choice is (d).

65. Given $\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{2}$ (1)

and $\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3}$ (2)

Using (2) in (1), we have

$\frac{1}{3} + \frac{1}{C_3} = \frac{1}{2}$ which gives $C_3 = 6\ \mu$F

Hence the correct choice is (d).

66. Given $C_1 + C_2 = \frac{C_1C_2}{C_1+C_2}\times\frac{25}{6}$

or $6(C_1+C_2)^2 = 25\,C_1C_2$

or $6C_1^2 + 6C_2^2 + 12\,C_1C_2 = 25\,C_1C_2$

or $6C_1^2 + 6C_2^2 - 13\,C_1C_2 = 0$

Let $C_2 = x\,C_1$. Then, we have

$6C_1^2 + 6x^2C_1^2 - 13\,x\,C_1^2 = 0$

or $6x^2 - 13x + 6 = 0$

which gives $x = \frac{3}{2}$ or $\frac{2}{3}$. Since $C_2 > C_1$, $x = \frac{2}{3}$ is not possible. Hence the correct choice is (a).

67. The system be in equilibrium if the net force on charge q at one vertex due to charges q at the other two vertices is equal and opposite to the force due to charge Q at the centroid, i.e. (here a is the side of the triangle)

$$-\frac{\sqrt{3}\,q^2}{4\pi\varepsilon_0 a^2} = \frac{Qq}{4\pi\varepsilon_0\left(\frac{a}{\sqrt{3}}\right)^2}$$

which gives $Q = -\frac{q}{\sqrt{3}}$. Hence the correct choice is (b).

68. Charge will flow from A to B until their potentials become equal. If charge q flows from A to B, then

$$\frac{Q-q}{4\pi\varepsilon_0 a} = \frac{q}{4\pi\varepsilon_0 b}$$

or $Q - q = \frac{a}{b}\, q$ which gives $q = \frac{bQ}{a+b}$. Hence charge left on $A = Q - q = Q - \frac{bQ}{a+b} = \frac{aQ}{a+b}$.

Hence the correct choice is (d).

69. When any additional negative charge is given to a hollow spherical shell, the potential on its surface falls, but the potential at each point within the shell also falls by the same amount. Hence the potential difference between the given surfaces remains unchanged. Thus the correct choice is (a).

70. Refer to Fig. 11.98.

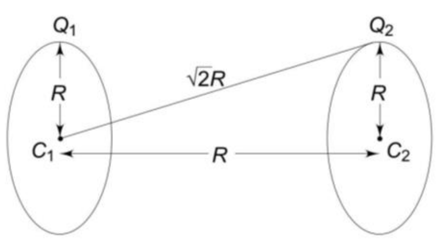


Fig. 11.98

Potential at C_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}\,R}\right)$$

Potential at C_2 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}\,R}\right)$$

$\therefore$ Work done $= q\,(V_1 - V_2)$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}\,R}\right) - \left(\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}\,R}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0\sqrt{2}\,R}\,(Q_1 - Q)\,(\sqrt{2} - 1)$$

Hence the correct choice is (b).

71. Force $F = qE$. Therefore, acceleration $a = qE/m$. Now distance moved in time t is

$$s = \frac{1}{2}\,at^2 = \frac{1}{2}\left(\frac{qE}{m}\right)t^2.$$

For electron: $s_e = \frac{1}{2}\left(\frac{qE}{m_e}\right)t_1^2$

For proton: $s_p = \frac{1}{2}\left(\frac{qE}{m_p}\right)t_2^2$

Given $s_e = s_p$. Therefore,

$$\frac{t_1^2}{m_e} = \frac{t_2^2}{m_p} \text{ or } \frac{t_2}{t_1} = \left(\frac{m_p}{m_e}\right)^{1/2}$$

Hence the correct choice is (b).

72. The electric field is always perpendicular to the surface of a conductor. On the surface of a metallic solid sphere, the electric field is perpendicular to the surface and directed towards the centre of the sphere. Hence the correct choice is (d).

73. $$V = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q}{x_0} + \frac{q}{3x_0} + \frac{q}{5x_0} + \cdots \text{ upto infinity}\right\}$$

$$+ \frac{1}{4\pi\varepsilon_0}\left\{\frac{-q}{2x_0} + \frac{-q}{4x_0} + \frac{-q}{6x_0} + \cdots \text{ upto infinity}\right\}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x_0}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\cdots \text{ upto infinity}\right)$$

$$= \frac{q}{4\pi\varepsilon_0 x_0}\ln(1+1) = \frac{q\ln(2)}{4\pi\varepsilon_0 x_0}$$

Hence the correct choice is (d).

74. Since the hypotenuse side of triangle $= \sqrt{2}\,a$, the net electrostatic energy is

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{Qq}{a} + \frac{Qq}{\sqrt{2}\,a} + \frac{qq}{a}\right)$$

For $U = 0$, we require

$$\frac{Qq}{a} + \frac{Qq}{\sqrt{2}\,a} + \frac{q^2}{a} = 0$$

which gives $$Q = -q\left(\frac{\sqrt{2}}{\sqrt{2}+1}\right) = \frac{-2q}{2+\sqrt{2}}$$

Hence the correct choice is (b).

75. $Q_1 = CV$ and $Q_2 = (2C) \times (2V) = 4CV$. Since the capacitors are connected in parallel such that the plates of opposite polarity are connected together, the common potential is

$$V' = \frac{Q_2 - Q_1}{C_1 + C_2} = \frac{4CV - CV}{C + 2C} = V$$

Equivalent capacitance $C' = C + 2C = 3C$. Therefore, the final energy of the configuration is

$$U' = \frac{1}{2}\, C' V'^2 = \frac{1}{2} \times 3C \times V^2 = \frac{3}{2}\, CV^2,$$

which is choice (b).

76. The insertion of the dielectric slab decreases the electric field without changing its direction. The electric potential increases as we go from the negative to the positive plate. Hence the correct choice is (c).

77. Within the plates electric fields due to charges Q_1 and Q_2 are

$$E_1 = \frac{Q_1}{2\varepsilon_0 A} \text{ and } E_2 = \frac{Q_2}{2\varepsilon_0 A}$$

As these fields are in opposite directions and $Q_1 > Q_2$, the net electric field within the plates is

$$E = E_1 - E_2 = \frac{1}{2\varepsilon_0 A}\,(Q_1 - Q_2)$$

Hence $V = Ed = \frac{d}{2\varepsilon_0 A}(Q_1 - Q_2) = \frac{Q_1 - Q_2}{2C}$ which is choice (d). $\left(\because C = \frac{\varepsilon_0 A}{d}\right)$

78. When switch S_3 is closed, the potential difference across C_1 and C_2 will become equal to the average of V_1 and V_2, i.e. (30 + 20)/2 = 25 V. Hence the correct choice is (b).

79. We have $C_1 = \frac{(A/2)\varepsilon_0 K_1}{(d/2)} = \frac{A\varepsilon_0 K_1}{d}$

$$C_2 = \frac{(A/2)\varepsilon_0 K_2}{(d/2)} = \frac{A\varepsilon_0 K_2}{d} \text{ and}$$

$$C_3 = \frac{A\varepsilon_0 K_3}{(d/2)} = \frac{2A\varepsilon_0 K_3}{d}$$

The capacitors C_1 and C_2 are in parallel and their equivalent capacitance is

$$C' = C_1 + C_2 = \frac{A\varepsilon_0}{d}\,(K_1 + K_2)$$

This combination is in series with C_3. Hence the net capacitance is

$$\frac{1}{C''} = \frac{1}{C'} + \frac{1}{C_3} = \frac{d}{A\varepsilon_0 (K_1 + K_2)} + \frac{d}{2A\varepsilon_0 K_3}$$

$$= \frac{d}{\varepsilon_0 A}\left[\frac{1}{(K_1 + K_2)} + \frac{1}{2K_3}\right]$$

or $C'' = \frac{A\varepsilon_0 K}{d}$ where $\frac{1}{K} = \frac{1}{(K_1 + K_2)} + \frac{1}{2K_3}$

Hence the correct choice is (b).

80. The capacitance of a parallel plane capacitor is given by $C = \varepsilon_0 A/d$. Hence the dimensions of $\varepsilon_0 L$ are the same as those of capacitance.

$\therefore$ Dimensions of $\varepsilon_0 L \frac{\Delta V}{\Delta t}$

$$= \frac{\text{dimension of } C \times \text{dimensions of } V}{\text{time}}$$

$$= \frac{\text{dimension of } Q}{\text{time}} \qquad (\because Q = CV)$$

$$= \frac{\text{charge}}{\text{time}} = \text{current}$$

Hence the correct choice is (d).

81. Since the outer plate of B is free, charge cannot flow from A to B. Hence the correct choice is (a).

82. Electric field is the negative gradient of potential, i.e.

$$E = -\frac{dV}{dx}$$

Thus, V decreases as dx increases in the direction of the field. This implies that $V_A > V_B$, which is choice (b).

83. Potential energy of the system when charge Q is at O is

$$U_0 = \frac{qQ}{a} + \frac{qQ}{a} = \frac{2qQ}{a}$$

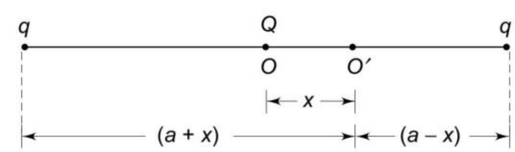


Fig. 11.99

When charge Q is shifted to position O', the potential energy will be (see Fig. 11.99)

$$U = \frac{qQ}{(a + x)} + \frac{qQ}{(a - x)}$$

$$= \frac{qQ(2a)}{(a^2 - x^2)} = \frac{2qQ}{a} \times \left(1 - \frac{x^2}{a^2}\right)^{-1}$$

$$\simeq \frac{2qQ}{a} \times \left(1 + \frac{x^2}{a^2}\right) \qquad (\because x << a)$$

$$\therefore \quad \Delta U = U - U_0$$

$$= \frac{2qQ}{a} \times \left(1 + \frac{x^2}{a^2}\right) - \frac{2qQ}{a} = \frac{2qQ}{a^3}\,(x^2)$$

Hence $\Delta U \propto x^2$ which is choice (b).

84. Given $v_x = 10\ \text{ms}^{-1}$. Since the electric field is directed along the y-axis, the acceleration of the body along the y-direction is

$$a_y = \frac{qE}{m} = \frac{10^{-6} \times 10^3}{10^{-3}} = 1 \text{ ms}^{-2}$$

Therefore, the velocity of the body along the y-axis at time $t = 10$ s is

$$v_y = at = 1 \times 10 = 10 \text{ ms}^{-1}$$

∴ Resultant velocity $v = \sqrt{v_x^2 + v_y^2}$

$$= \sqrt{(10)^2 + (10)^2} = 10\sqrt{2} \text{ ms}^{-1}$$

Hence the correct choice is (c).

85. Let Q be the magnitude of each charge and a the length of each side of the triangle. The potential energy of the system of two equal charges placed at vertex A and B is U (given). This means that U is the work done in bringing a charge Q from infinity to vertex B. Hence the work done in bringing an identical charge Q from infinity to the third vertex C = work done to overcome the force of repulsion of Q placed at A at a distance a + work done to overcome the force of repulsion of Q placed at B at the same distance $a = U + U = 2U$, which is choice (b).

86. Charge on capacitor plates without the dielectric is

$$Q = CV = (5 \times 10^{-6} \text{ F}) \times 1 \text{ V} = 5 \times 10^{-6} \text{ C} = 5 \,\mu\text{C}$$

The capacitance after the dielectric is introduced is

$$C' = \frac{\varepsilon_0 A}{d - \left(t - \frac{t}{K}\right)} = \frac{\varepsilon_0 A/d}{1 - \left(\frac{t - \frac{t}{K}}{d}\right)}$$

$$= \frac{C}{1 - \left(\frac{t - \frac{t}{K}}{d}\right)} = \frac{5 \,\mu\text{F}}{1 - \left(\frac{4 \text{ cm} - \frac{4 \text{ cm}}{4}}{6 \text{ cm}}\right)}$$

$$= \frac{5 \,\mu\text{F}}{1 - \left(\frac{4-1}{6}\right)} = 10 \,\mu\text{F}$$

∴ Charge on capacitor plates now will be

$$Q' = C'V = 10 \,\mu\text{F} \times 1 \text{ V} = 10 \,\mu\text{C}$$

Additional charge transferred = $Q' - Q = 10 \,\mu\text{C} - 5 \,\mu\text{C} = 5 \,\mu\text{C}$, which is choice (c).

87. Common potential is $V = \dfrac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

$$= \frac{(4 \times 10^{-6}) \times 80 + (6 \times 10^{-6}) \times 30}{4 \times 10^{-6} + 6 \times 10^{-6}} = 50 \text{ V}$$

∴ Energy lost by 4 μF capacitor

$$= \frac{1}{2} C_1 V_1^2 - \frac{1}{2} C_1 V^2$$

$$= \frac{1}{2} C_1 (V_1^2 - V^2)$$

$$= \frac{1}{2} \times (4 \times 10^{-6}) \times \{(80)^2 - (50)^2\}$$

$$= 7.8 \times 10^{-3} \text{ J} = 7.8 \text{ mJ}$$

Hence the correct choice is (a).

88. Given $E = \dfrac{q}{4\pi\varepsilon_0 x^2}$. Hence the magnitude of the electric field at a distance $2x$ from charge q is

$$E' = \frac{q}{4\pi\varepsilon_0 (2x)^2} = \frac{q}{4\pi\varepsilon_0 x^2} \times \frac{1}{4} = \frac{E}{4}$$

Therefore, the force experienced by a similar charge q at a distance $2x$ is

$$F = qE' = \frac{qE}{4}$$

Hence the correct choice is (d).

89. Refer to Fig. 11.100. Let S be the surface area of each face of the cube. The flux through surfaces $ABCD$ and $EFGH$ is zero because these surfaces are parallel to the electric field **E** ($\theta = 90°$).

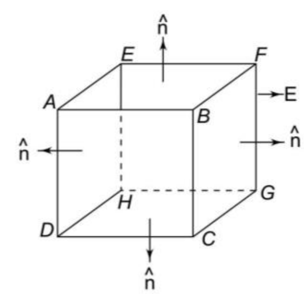


Fig. 11.100

Flux through face $BFGC$ is $\phi_1 = ES \cos 0° = ES$. Flux through face $AEHD$ is $\phi_2 = ES \cos 180° = -ES$. Total flux through the cube = $\phi_1 + \phi_2 = ES - ES = 0$.

Hence the correct choice is (a).

90. The electric flux is given by the surface integral $\int \mathbf{E} . d\mathbf{s}$. Here the electric field **E** is due to all the charges, both inside and outside the Gaussian surface. Hence the correct choice is (c).

91. The electric field at a point P due to an infinite long plane sheet carrying a uniform charge density σ is given by

$$E = \frac{\sigma}{2\varepsilon_0}$$

It is independent of the distance of point P from the sheet and is, therefore, uniform. The direction of the electric field is away from the sheet and perpendicular to it if σ is positive and is towards the sheet and perpendicular to it if σ is negative. Hence

$$E_1 = \frac{\sigma}{2\varepsilon_0}(-\hat{\mathbf{j}}) \text{ along –ve } y\text{–direction}$$

$$E_2 = \frac{2\sigma}{2\varepsilon_0}(-\hat{\mathbf{j}}) \text{ along –ve } y\text{–direction}$$

and $$E_3 = \frac{3\sigma}{2\varepsilon_0}(-\hat{\mathbf{j}}) \text{ along –ve } y\text{–direction}$$

From the superposition principle, the net electric field at point P is

$$E = E_1 + E_2 + E_3$$

$$= \frac{\sigma}{2\varepsilon_0}(-\hat{\mathbf{j}}) + \frac{2\sigma}{2\varepsilon_0}(-\hat{\mathbf{j}}) + \frac{3\sigma}{2\varepsilon_0}(-\hat{\mathbf{j}})$$

$$= -\frac{3\sigma}{\varepsilon_0}\hat{\mathbf{j}}$$, which is choice (c).

92. Electric field due to charge $-Q$ on the shell at a distance r from its center is (for $r > R$)

$$E_1 = \frac{Q}{4\pi\varepsilon_0 r^2}$$

directed towards the centre.

Electric field due to charge $+Q$ at the centre at a distance r is

$$E_2 = \frac{Q}{4\pi\varepsilon_0 r^2}$$

directed away from the centre.

$\therefore$ Net electric field E (for $r > R$) $= E_1 - E_2 = 0$. For $r < R$, the electric field due to the shell is zero. In this region, the electric field due to charge $+Q$ at the centre decreases as $1/r^2$. Hence the correct graph is (a).

93. Let the charge on the sphere be Q. Then

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

which gives $Q = 4\pi\varepsilon_0 RV$

The electric field at a distance r is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{4\pi\varepsilon_0 RV}{4\pi\varepsilon_0 r^2} = \frac{RV}{r^2}$$

thus the correct choice is (c).

94. If charge Q is moved from C to D along the arc, the potential energy between pairs (q_1, Q) and (q_1, q_2) will not change as the distance between them remains unchanged ($\because AC = AD$). The potential energy of the pair of charges q_2 and Q will change.

Now, distance $BC = \sqrt{(8)^2 + (6)^2} = 10$ cm and $BD = 8 - 6 = 2$ cm. Therefore, change in P.E. is

$$\Delta U = \frac{q_2 Q}{4\pi\varepsilon_0}\left[\frac{1}{BD} - \frac{1}{BC}\right]$$

$$= (2 \times 10^{-6}) \times (5 \times 19^{-6}) \times (9 \times 10^{-9})\left(\frac{1}{0.02} - \frac{1}{0.1}\right)$$

$= 3.6$ J, which is choice (b).

95. If the middle charge is displaced by a distance x, the net force acting it, when it is released, is

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{(L+x)^2} - \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{(L-x)^2}$$

$$= \frac{4q^2 Lx}{4\pi\varepsilon_0 (L^2 - x^2)^2}$$

For $x << L$, $F = \frac{q^2 x}{\pi\varepsilon_0 L^3} = -kx$

where $k = \frac{q^2}{\pi\varepsilon_0 L^3}$

Now $T = 2\pi\sqrt{\frac{m}{k}}$

So, the correct choice is (c).

96. The batteries are in opp,sition as their positive terminals are connected together. Hence the effective voltage is

$$V = V_1 - V_2 = 12 - 2 = 10 \text{ V}$$

As the capacitors C_1 and C_2 are in series, the effective capacitance of the circuit is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

or $$C = \frac{6}{5} = 1.2\ \mu\text{F}.$$

Therefore, charge on capacitors is

$$Q = CV = 1.2\ \mu\text{F} \times 10\text{V} = 12\ \mu\text{C}$$

$\therefore$ Potential difference across A and B = potential difference across capacitor C_2

$$= \frac{Q}{C_2} = \frac{12\mu\text{C}}{2\mu\text{F}} = 6 \text{ V}$$

97. Potential difference between the plates before the slab is introduced is

$$V = E \times d = 200 \times 0.05 = 10 \text{ V}$$

The capacitance of the capacitor is given by

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{0.05} \text{ or } \varepsilon_0 A = 0.05\, C$$

When a slab of dielectric constant K and thickness t is introduced, the capacitance becomes

$$C' = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)} = \frac{0.05C}{0.05 - 0.01\left(1 - \frac{1}{4}\right)} = \frac{20C}{17}$$

Now $Q = CV = C'V'$. Therefore,

$$V' = \frac{CV}{C'} = \frac{CV}{20C/17} = \frac{17V}{20} = \frac{17 \times 10}{20} = 8.5 \text{ V}$$

98. The capacitance before the introduction of the slab is $C = \frac{\varepsilon_0 A}{d}$

If Q is the charge on the plates, the potential difference is

$$V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A} \qquad (1)$$

Let d' be the new separation between the plates. When a slab of thickness t and dielectric constant K is introduced, the new capacitance is

$$C' = \frac{\varepsilon_0 A}{d' - t\left(1 - \frac{1}{K}\right)}$$

Since charge Q remains the same, the new potential difference is

$$V' = \frac{Q}{C'} = \frac{Q\left[d' - t\left(1 - \frac{1}{K}\right)\right]}{\varepsilon_0 A} \qquad (2)$$

Given $V' = V$. Equating Eqs. (1) and (2), we get

$$d = d' - t\left(1 - \frac{1}{K}\right) \text{ or } d' - d = t\left(1 - \frac{1}{K}\right)$$

Given $d' = d = 2$ mm and $t = 3$ mm. Thus

$$2 = 3\left(1 - \frac{1}{K}\right)$$

which gives $K = 3$. Hence the correct choice is (b).

99. If Q is the initial charge on capacitor C_1, the initial energy is given by

$$U_i = Q^2/2\, C_1$$

When the two capacitors are connected together, and as the charge is distributed equally, the charge on each capacitor is $Q/2$. Since the potential difference (in a parallel connection) across the two capacitors is also the same, it follows that their capacitances are equal (since $C = Q/V$). Thus $C_1 = C_2 = C$(say). Also, $Q_1 = Q_2 = Q/2$.

Therefore, final energy stored in the two capacitors is

$$U_f = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} = \frac{(Q/2)^2}{2C} + \frac{(Q/2)^2}{2C} = \frac{Q^2}{4C}$$

But $\quad U_i = \frac{Q^2}{2C}$

$$\therefore \quad \frac{U_f}{U_i} = \frac{1}{2}, \text{ which is choice (b).}$$

100. Plate 1 is connected to plate 3 and plate 2 is connected to plate 4. Thus, there are three capacitors in parallel, each of capacitance

$$C = \frac{\varepsilon_0 A}{d}$$

as shown in Fig. 11.101. Hence the equivalent capacitance is

$$C' = 3C = \frac{3\varepsilon_0 A}{d}, \text{ which is choice (c).}$$

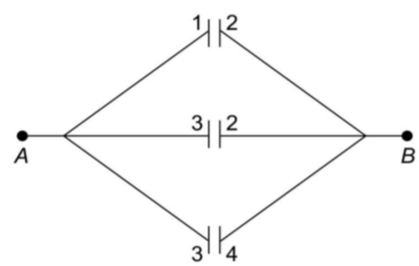


Fig. 11.101

101. The inner plates 2 and 3 are connected together. Hence they act as a single conductor. Since the outer plates 1 and 4 are connected together, there are effectively two capacitors (between plates 1 and 2 and plates 3 and 4) in parallel, each of capacitance $C = \varepsilon_0\, A/d$ as shown in Fig. 11.102. Thus, the equivalent capacitance is

$$C' = 2C = \frac{2\varepsilon_0 A}{d}, \text{ which is choice (b).}$$

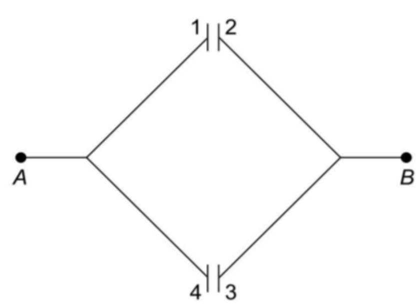


Fig. 11.102

102. The network reduces to that shown in Fig. 11.103. The correct choice is (a).

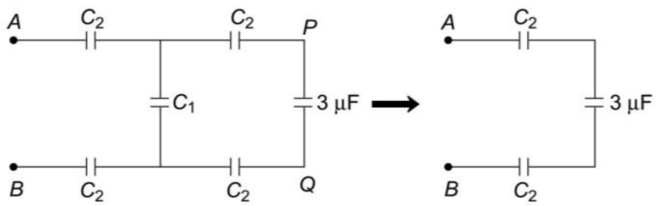


Fig. 11.103

103. Charge on C_1 is $Q_1 = C_1V = CV$. This charge is shared by the three capacitors. The equivalent capacitance of the series combination of C_2 and C_3 is

$$C' = \frac{C_2C_3}{C_2 + C_3} = \frac{C}{2} \qquad (\because\ C_2 = C_3 = C)$$

The common potential of C_1 and C' is

$$V' = \frac{Q_1}{C_1 + C'} = \frac{C_1V}{C_1 + C'}$$

$$= \frac{CV}{C + C/2} = \frac{2V}{3}$$

$\therefore$ Final charge on C_1 is

$$Q_1' = C_1V' = \frac{2CV}{3}$$

$\therefore$ Charge that will flow through the connecting wires is

$$Q'' = Q_1 - Q_1' = CV - \frac{2CV}{3} = \frac{CV}{3}$$, which is choice (a).

104. $C_1 = 4\pi\varepsilon_0 R_1$

$$C_2 = \frac{4\pi\varepsilon_0(R_1R_2)}{(R_2 - R_1)}$$

Given $C_2 = 3C_1$ Hence

$$\frac{4\pi\varepsilon_0(R_1R_2)}{(R_2 - R_1)} = 3 \times 4\pi\varepsilon_0 R_1$$

which gives $\dfrac{R_2}{R_1} = \dfrac{3}{2}$, which is choice (b).

105. Energy stored initially is $U_i = \dfrac{Q^2}{2C}$. if d is doubled, C becomes $C/2$. Hence, energy stored when d is doubled is $U_f = \dfrac{Q^2}{C}$

$\therefore$ Work needed is

$$W = U_f - U_i = \frac{Q^2}{C} - \frac{Q^2}{2C} = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

$(\because\ Q = CV)$

Now $C = \dfrac{A\varepsilon_0}{d}$. Hence

$$W = \frac{A\varepsilon_0V^2}{2d}$$, which is choice (c)

106. The circuit can be redrawn as shown in Fig. 11.104.

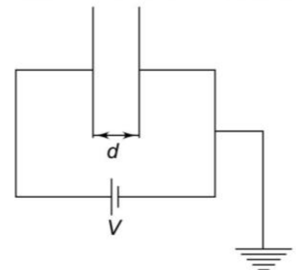


Fig. 11.104

The charge on the capacitor plates is

$$Q = CV = \frac{A\varepsilon_0V}{d}$$

So the correct choice is (b).

107. Volume charge density ρ is given by

$$\rho = \frac{Q}{\frac{4\pi}{3}R^3}$$

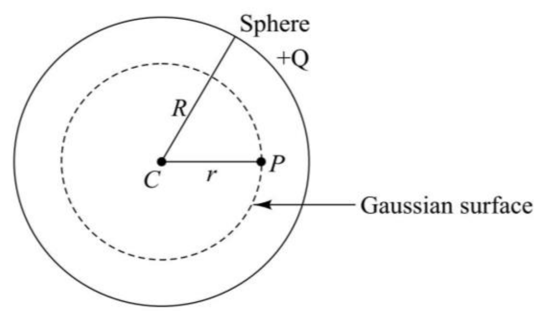


Fig. 11.105

Charge enclosed within the Gaussian surface is

$$q = \rho \times \frac{4\pi}{3}r^3$$

$$= \frac{Q}{\frac{4\pi}{3}R^3} \times \frac{4\pi}{3}R^3 = \frac{Qr^3}{R^3}$$

Electric flux through the Gaussian surface is

$$\phi = \int_s \mathbf{E.ds} = E \times \int_s ds = E \times 4\pi r^2$$

From Gauss's law

$$\phi = \frac{q}{\epsilon_0}$$

$$\Rightarrow \quad E \times 4\pi r^2 = \frac{Qr^3}{\epsilon_0 R^3}$$

$$\Rightarrow \quad E = \frac{Qr}{4\pi\epsilon_0 R^3}$$

So the correct choice is (a). Since the charge is positive, the direction of **E** at point P is radially outward. If the charge is negative, the direction of **E** will be radially inward. Note that in terms of ρ, $E = \dfrac{\rho r}{3 \in_0}$.

108. Refer to Fig. 11.106.

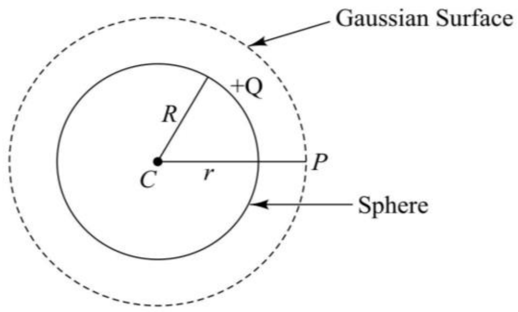


Fig. 11.106

Charge enclosed inside the Gaussian surface is

$$q = \text{charge on the sphere} = Q$$

From Gauss's law

$$\phi = \frac{q}{\in_0}$$

or $\quad E \times 4\pi r^2 = \dfrac{Q}{\in_0} \qquad (\because \quad q = Q)$

$$\Rightarrow \qquad E = \frac{Q}{4\pi \in_0 r^2}$$

So the correct choice is (c). **E** is radially outward if Q is positive and radially inward if Q is negative.

109. We assume the Gaussian surface of radius r. To find the charge q enclosed inside this surface, we consider a thin spherical shell of radius a and thickness da (see Fig. 11.107).

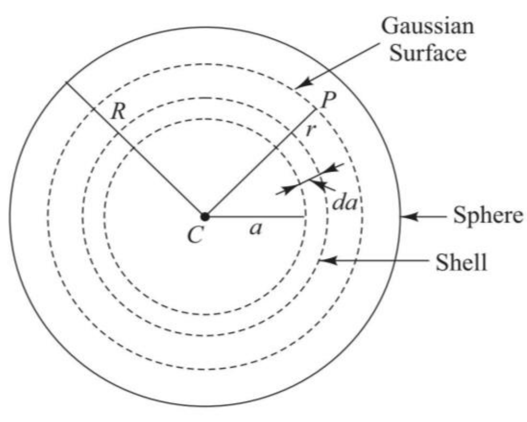


Fig. 11.107

Volume of the shell is $dV = 4\pi a^2 da$. So the charge contained in the shell is

$$dq = \rho \times 4\pi a^2 da$$

$$= \frac{Ka}{R} \times 4\pi a^2 da \qquad (\because \quad r = a)$$

Therefore, the total charge enclosed inside the Gaussian surface is

$$q = \int_{a=0}^{a=r} dq = \frac{4\pi K}{R} \int_0^r a^3 da$$

$$= \frac{4\pi K r^4}{4R} = \frac{\pi K r^4}{R}$$

Electric flux through the Gaussian surface is

$$\phi = EA = E \times 4\pi r^2$$

From Gauss's law,

$$\phi = \frac{q}{\in_0}$$

or $\quad E \times 4\pi r^2 = \dfrac{\pi K r^4}{R \in_0}$

$$\Rightarrow \qquad E = \frac{K r^2}{4R \in_0}$$

Hence $\quad E \propto r^2$, which is choice (d).

110. In this case, the charge inside the Gaussian surface = charge on the entire sphere. Since now $r = R$

$$q = \int_{a=0}^{a=R} \frac{KR}{R} \times 4\pi a^2 dR$$

$$= \frac{4\pi K R^3}{3}$$

$$\therefore \quad E \times 4\pi r^2 = \frac{q}{\in_0} = \frac{4\pi K R^3}{3 \in_0}$$

$$\Rightarrow \qquad E = \frac{K R^3}{3 \in_0 r^2}$$

So the correct choice is (d).

111. At points for which $r < a$, the electric field is zero because there can be no electric field within the body of a conductor.

At points for which r lies between b and c, the inner sphere acts as if all its charge were concentrated at its centre. Hence $E = \dfrac{q}{4\pi \in_0 r^2}$.

At points for which r lies between b and c, the electric field is again zero because these points are in the body of a conductor.

The charge $+q$ on the inner sphere induces a charge $-q$ on the inner surface of the outer sphere and a charge $+q$ on the outer surface of the outer sphere. At points for which $r > c$, the net charge enclosed inside the Gaussian spherical surface of radius r ($> c$) is $+q + (-q) + (+q) = +q$. So the electric field for $r > c$ is $E = \frac{q}{4\pi \epsilon_0 r^2}$.

So the only incorrect statement is (d).

112. Figure 11.108 shows the distribution of charges on the surfaces of the shells.

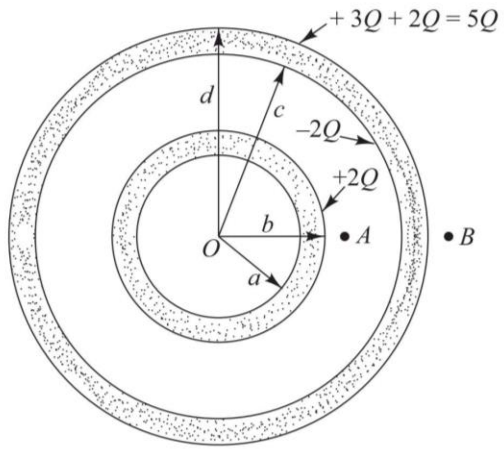


Fig. 11.108

We consider the Gaussian surface to be a sphere of radius $OA = r$. The net charge enclosed inside the surface is

$$q = 0 + 2Q = +2Q$$

because the inner surface of the inner shell has no charge.

From Gauss's law, the electric field at A will be given by

$$\phi = \frac{q}{\epsilon_0}$$

or $$E \times 4\pi r^2 = \frac{+2Q}{\epsilon_0}$$

$$E = \frac{2Q}{4\pi \epsilon_0 r^2}$$

So the correct choice is (b).

113. In this case the radius of the Gaussian surface is $OB = r$. The net charge enclosed inside this surface is

$$q = 5Q + (-2Q) + 2Q + 0 = 5Q$$

From Gauss's law,

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0} = \frac{5Q}{\epsilon_0}$$

$$\Rightarrow \quad E = \frac{5Q}{4\pi \epsilon_0 r^2}, \quad \text{which is choice (d).}$$

114. The acceleration of the smaller sphere is

$$a = \frac{F}{m} = \frac{1}{4\pi \epsilon_0} \frac{Qq}{r^2 m}$$

As the smaller sphere moves away, r increases. Hence a decreases. Since a is always positive, the speed of the smaller sphere is always increasing. So the correct choice is (c).

115. Refer to Fig. 11.109. The linear charge density is

$$\lambda = \frac{Q}{l}$$

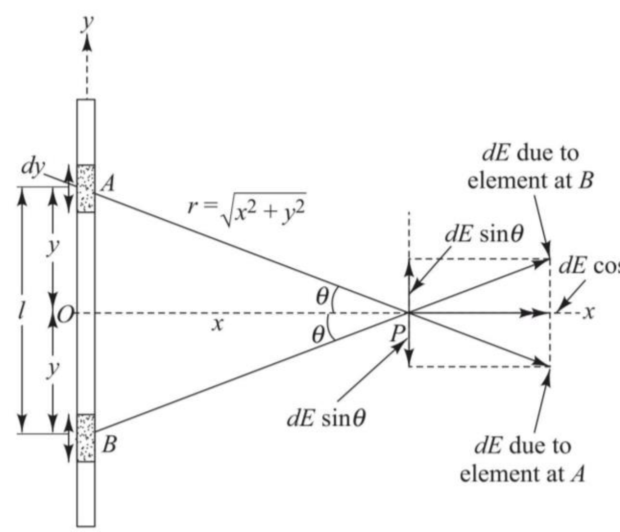


Fig. 11.109

Consider two symmetrically located line elements at A and B, each of small length dy at distance y from origin O. The charge of the element is $dq = \lambda dy$. The electric field at P due to the element at A is

$$dE = \frac{dq}{4\pi \epsilon_0 r^2} = \frac{\lambda dy}{4\pi \epsilon_0 (x^2 + y^2)}$$

It follows from the figure that the x components $dE \cos\theta$ add up but the y components $dE \sin\theta$ cancel each other. Therefore, the electric field at P due to the complete rod is

$$E = \int_{-l/2}^{+l/2} dE \cos\theta = 2\int_0^{l/2} dE \cos\theta$$

$$= \frac{2\lambda}{4\pi \epsilon_0} \int_0^{l/2} \frac{dy}{x^2 + y^2} \times \frac{x}{\sqrt{x^2 + y^2}}$$

$$= \frac{\lambda x}{2\pi \in_0} \int_0^{l/2} \frac{dy}{(x^2 + y^2)^{3/2}}$$

$$= \frac{\lambda x}{2\pi \in_0} \left| \frac{y}{x^2 (x^2 + y^2)^{1/2}} \right|_{y=0}^{y=l/2}$$

or
$$E = \frac{\lambda}{2\pi \in_0 x} \left[\frac{l/2}{\left[x^2 + \left(\frac{l}{2}\right)^2 \right]^{1/2}} \right] \quad (1)$$

Putting $x = l$ and $\lambda = \frac{Q}{l}$ in Eq. (1) we get

$$E = \frac{Q}{2\sqrt{5}\pi \in_0 l^2}$$

So the value of $k = \frac{2}{\sqrt{5}}$ which is choice (d).

116. The potential at every point inside the outer shell B is $V_1 = \frac{Q}{4\pi \in_0 R}$, which is also the potential on the surface of inner shell A due to outer shell B. The potential on the surface of shell A due to its own charge is $V_2 = \frac{q}{4\pi \in_0 r}$. Therefore, the total potential of shell A is

$$V_A = V_1 + V_2 = \frac{Q}{4\pi \in_0 R} + \frac{q}{4\pi \in_0 r}$$

$$= \frac{1}{4\pi \in_0}\left(\frac{Q}{R} + \frac{q}{r}\right)$$

So the correct choice is (a).

117. For points outside the outer shell, the two shells behave as if all their charge were concentrated at their common centre O. At a point $x > R$,

$$V = \frac{q+Q}{4\pi \in_0 x}$$

If we put $x = R$, we get the potential on the surface of B. Thus

$$V_B = \frac{q+Q}{4\pi \in_0 R}$$

$$\therefore \quad V_A - V_B = \frac{1}{4\pi \in_0}\left(\frac{q}{r} + \frac{Q}{R}\right) - \frac{1}{4\pi \in_0}\left(\frac{q+Q}{R}\right)$$

$$= \frac{q}{4\pi \in_0}\left(\frac{1}{r} - \frac{1}{R}\right)$$

So the correct choice is (a). Note that the potential difference $(V_A - V_B)$ does not depend on the charge on the outer shell.

118.
$$E = -\frac{dV}{dr}$$

$$\Rightarrow \quad dV = -E\,dr$$

$$\Rightarrow \quad V = -\int_R^r E\,dr$$

$$= -\frac{\lambda}{2\pi \in_0}\int_R^r \frac{dr}{r}$$

$$= -\frac{\lambda}{2\pi \in_0} \ln\left(\frac{r}{R}\right)$$

$$= \frac{\lambda}{2\pi \in_0} \ln\left(\frac{R}{r}\right)$$

So the correct choice is (d).

119. The electric field at a distance r from a conducting cylinder is given by

$$E = \frac{\lambda}{2\pi \in_0 r}$$

Therefore, the potential difference between the inner cylinder and the outer cylinder is

$$V = -\int E\,dr$$

$$= -\frac{\lambda}{2\pi \in_0}\int_b^a \frac{dr}{r}$$

$$= \frac{\lambda}{2\pi \in_0}\int_a^b \frac{dr}{r}$$

$$\Rightarrow \quad V = \frac{\lambda}{2\pi \in_0} \ln\left(\frac{b}{a}\right)$$

Now
$$C = \frac{Q}{V} = \frac{\lambda l}{\frac{\lambda}{2\pi \in_0} \ln\left(\frac{b}{a}\right)}$$

$$= \frac{2\pi \in_0 l}{\ln\left(\frac{b}{a}\right)},$$ which is choice (a).

120. Let $+Q$ be the charge on the inner shell and $-Q$ on the outer shell. In the space between the two shells, the electric field is due to the inner shell alone. At a distance r from the centre of the inner shell, the electric field is

$$E = \frac{1}{4\pi \in_0}\frac{Q}{r^2}$$

Let V be the potential difference between the two shells. Then

$$V = -\int_b^a E\,dr = \int_a^b E\,dr$$

or $$V = \frac{Q}{4\pi \in_0} \int_a^b \frac{dr}{r^2}$$

$$= \frac{Q}{4\pi \in_0} \left| -\frac{1}{r} \right|_a^b$$

$$= \frac{Q}{4\pi \in_0} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{Q\,(b-a)}{4\pi \in_0 ab}$$

or $$Q = \left[\frac{4\pi \in_0 ab}{(b-a)} \right] V$$

Comparing this equation with $Q = CV$, we get

$$C = 4\pi \in_0 \left(\frac{ab}{b-a} \right),$$ which is choice (b).

121. When C_1 is fully charge, the voltage V_1 across it = voltage of the battery = 12 V. The charge on C_1 is

$$Q = C_1 V_1 = (2 \times 10^{-6}) \times 12 = 24 \times 10^{-6}\ \text{C} = 24\ \mu\text{C}$$

When S_1 is opened and S_2 is closed, charge will flow from C_1 to C_2 until they have the same potential V which is given by

$$V = \frac{Q}{C_1 + C_2} = \frac{24 \mu C}{(2+4)\mu F} = 4\ \text{V}$$

Since C_1 and C_2 are in parallel, they have the same common potential. Now

$$U_i = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times (2 \times 10^{-6}) \times (12)^2 = 1.44 \times 10^{-4}\ \text{J}$$

$$U_f = \frac{1}{2} C_1 V^2 = \frac{1}{2} \times (2 \times 10^{-6}) \times (4)^2 = 0.16 \times 10^{-4}\ \text{J}$$

$\therefore$ Loss of energy stored in $C_1 = 1.44 \times 10^{-4} - 0.16 \times 10^{-4}$ $= 1.28 \times 10^{-4}$ J. The percentage loss of energy is $\frac{1.28 \times 10^{-4}}{1.44 \times 10^{-4}} \times 100 = 88.9\%$, which is choice (a).

122. The moment the two spheres are connected by a conducting wire, they quickly form an equipotential surface. Charge will flow from sphere 1 (which was at higher potential $= \frac{Q}{4\pi \in_0 a}$) to sphere 2 (which was at lower potential $= \frac{Q}{4\pi \in_0 (4a)}$) until their potentials are equalized.

It Q_1 is charge on sphere 1 and Q_2 on sphere 2, then

$$\frac{Q_1}{4\pi \in_0 (a)} = \frac{Q_2}{4\pi \in_0 (4a)}$$

$$\Rightarrow \quad Q_2 = 4Q_1 \qquad (1)$$

But $Q_1 + Q_2 = Q + Q = 2Q$ (2)

From (1) and (2) we get $5Q_1 = 2Q$ or $Q_1 = \frac{2Q}{5}$ and $Q_2 = 2Q - Q_1 = 2Q - \frac{2Q}{5} = \frac{8Q}{5}$. So the correct choice is (c).

123. Refer to the solution of Q. 106 and to Fig. 11.105 where $\overrightarrow{O'P} = \vec{a}$, $\overrightarrow{OP} = \vec{b}$ and $\overrightarrow{OO'} = \vec{c}$.

Electric field at P due to the complete sphere is

$$\vec{E}_1 = \frac{\rho \vec{b}}{3 \in_0}$$

Electric field at P due to cavity is

$$\vec{E}_2 = \frac{\rho \vec{a}}{3 \in_0}$$

Therefore, the net electric field at P is

$$\vec{E} = \vec{E}_1 - \vec{E}_2 .$$

$$= \frac{\rho \vec{b}}{3 \in_0} - \frac{\rho \vec{a}}{3 \in_0} = \frac{\rho}{3 \in_0} (\vec{b} - \vec{a})$$

$$= \frac{\rho \vec{c}}{3 \in_0} \qquad (\because\ \vec{c} + \vec{a} = \vec{b})$$

So the correct choice is (c). Note that the electric field at any point in the emptied space is finite and constant.

124. When a surface charge density is given to the outer cylinder, the same potential appears on both A and B. Hence statement 1 is false. When a charge density is given to the inner cylinder, an electric field is produced between A and B. Hence a potential difference appears between A and B. So statement 2 is true. When the same charge density $\sigma = Q/4\pi R^2$ is given to A and B, the charge on A is greater than that on B. Hence, a potential difference appears between them. So statement 3 is false. Thus the correct choice is (b).

125. When a negative charge $-Q$ is placed out side a neutral conducting sphere, it will induce a positive charge $+Q$ on the side closer to it and an equal negative charge $-Q$ on the opposite side as shown in Fig. 11.110. Hence the net charge on the sphere will be zero. So the correct choice is (d).

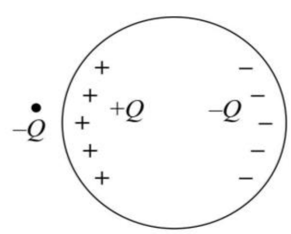


Fig. 11.110

126. Initial energy stored in C_1 is

$$U_i = \frac{1}{2}C_1V^2$$

when the switch S is turned to position 2, charge will flow from one capacitor to the other until their potentials are equalised. The common potential is

$$V_1 = \frac{q}{C_1+C_2} = \frac{C_1 V}{C_1+C_2}$$

Since C_1 and C_2 are in parallel, the equivalent capacitance = $C_1 + C_2$. Therefore, the energy stored now is

$$U_f = \frac{1}{2}(C_1+C_2)V_1^2 = \frac{1}{2}(C_1+C_2)\times\left(\frac{C_1V}{C_1+C_2}\right)^2$$

$$= \frac{C_1^2V^2}{2(C_1+C_2)}$$

∴ Loss of energy is

$$\Delta U = U_i - U_f$$

$$= \frac{1}{2}C_1V^2 - \frac{C_1^2V^2}{2(C_1+C_2)} = \frac{C_1C_2V^2}{2(C_1+C_2)}$$

Fraction of energy stored in C_1 dissipated is

$$\frac{\Delta U}{U_i} = \frac{C_2}{C_1+C_2},$$ which is choice (b)

127. Volume charge densities of spheres 1, 2 and 3 are

$$\rho_1 = \frac{Q}{\frac{4\pi}{3}R^3} = \frac{3Q}{4\pi R^3}$$

$$\rho_2 = \frac{Q}{\frac{4\pi}{3}(2R)^3} = \frac{3Q}{4\pi\times 8R^3} = \frac{\rho_1}{8}$$

$$\rho_3 = \frac{Q}{\frac{4\pi}{3}(3R)^3} = \frac{\rho_1}{27}$$

Now refer to the solution of Q. 127. For points outside the sphere, the entire charge Q can be assumed to be concentrated at its centre. Hence

$$E_1 = \frac{Q}{4\pi\epsilon_0(2R)^2} = \frac{QR}{4\pi R^3\epsilon_0}\times\frac{1}{4} = \frac{\rho_1 R}{12\epsilon_0}$$

For points inside the sphere, $E = \frac{\rho r}{3\epsilon_0}$, where r is the distance of the point P from the centre of the sphere. Hence

$$E_2 = \frac{\rho_2\times(2R)}{3\epsilon_0} = \frac{\rho_1}{8}\times\frac{2R}{3\epsilon_0} = \frac{\rho_1 R}{12\epsilon_0}$$

$$E_3 = \frac{\rho_3\times(2R)}{3\epsilon_0} = \frac{\rho_1}{27}\times\frac{2R}{3\epsilon_0} = \frac{2}{81}\frac{\rho_1 R}{\epsilon_0}$$

It is clear that $E_1 = E_2 < E_3$. So the correct choice is (c).

128. Let q μC be the charge on the left plate of the 4 μF capacitor. Then the charge on the left plate of the 3 μF capacitor will be $(7-q)$ μC. Since the potential difference across the 3 μF capacitor = potential difference across the 4 μF capacitor,

$$\frac{7-q}{3} = \frac{q}{4}$$

$$\Rightarrow \quad q = 4.0\ \mu\text{C}$$

So the correct choice is (d).

129. The x, y and z components of the electric field are

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}(x^2y+yz) = -2xy$$

$$E_y = -\frac{dV}{dy} = -\frac{d}{dy}(x^2y+yz) = -x^2-z = -(x^2+z)$$

$$E_z = -\frac{dV}{dz} = -\frac{d}{dz}(x^2y+yz) = -y$$

The magnitude of the electric field is

$$E = \sqrt{E_x^2+E_y^2+E_z^2} = \sqrt{4x^2y^2+(x^2+z)^2+y^2}$$

At space point P (1, 2, 3),

$$E = \sqrt{4\times1^2\times2^2+(1^2+3)^2+2^2}$$

$$= \sqrt{16+16+4} = 6 \text{ units}$$

So the correct choice is (a).

130. $E_x = -2x$, $E_y = -2y$ and $E_z = -2z$

$$\therefore \quad |E| = \sqrt{E_x^2+E_y^2+E_y^2}$$

$$= \sqrt{4x^2+4y^2+4y^2}$$

$$= 2\sqrt{(x^2+y^2+z^2)} = 2\sqrt{V}$$

So the correct choice is (a).

131. V = potential on the surface of the sphere of radius r due to + Q – potential due to + Q at a distance R from the centre of the sphere (which is potential on the outer surface of the shell)

$$= \frac{Q}{4\pi\epsilon_0 r} - \frac{Q}{4\pi\epsilon_0 R}$$

$$\Rightarrow \quad V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r}-\frac{1}{R}\right) \qquad (1)$$

If a charge $-nQ$ is given to the shell, the potential V_1 on the surface of sphere = potential due to + Q on the surface + potential due to $-nQ$ on the surface of the sphere. Therefore,

$$V_1 = \frac{Q}{4\pi \in_0 r} + \frac{-nQ}{4\pi \in_0 R} = \frac{Q}{4\pi \in_0}\left(\frac{1}{r} - \frac{n}{R}\right)$$

Potential on the outer surface of the shell = potential due to + Q at a distance R from + potential due to $-nQ$ on its surface. Therefore,

$$V_2 = \frac{Q}{4\pi \in_0 R} - \frac{nQ}{4\pi \in_0 R} = \frac{Q}{4\pi \in_0 R}(1-n)$$

Therefore, the new potential difference is

$$V' = V_1 - V_2$$

$$= \frac{Q}{4\pi \in_0}\left(\frac{1}{r} - \frac{n}{R}\right) - \frac{Q}{4\pi \in_0 R}(1-n)$$

$$= \frac{Q}{4\pi \in_0}\left(\frac{1}{r} - \frac{1}{R}\right) \quad (2)$$

From (1) and (2) we find that $V' = V$. So the correct choice is (a).

132. Charge Q on shell 1 induces a charge $-Q$ on the inner surface of shell 2 and a charge $+Q$ on its outer surface, so that the total charge on the outer surface of shell 2 is $Q + Q = 2Q$ [see Fig. 11.111]

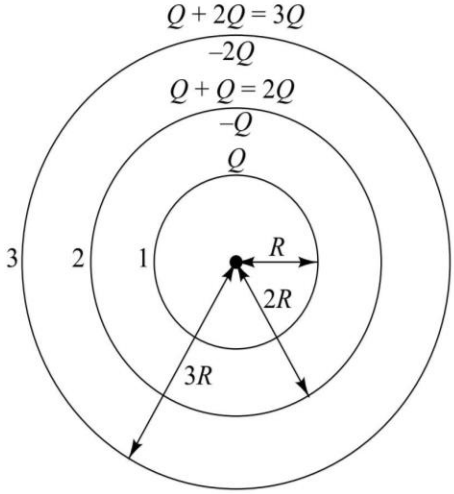


Fig. 11.111

Charge $2Q$ on the outer surface of 2 induced a charge $-2Q$ on the inner surface of 3 and a charge $+2Q$ of its outer surface so that the total charge on the outer surface of 3 is $Q + 2Q = 3Q$. Hence surface charge densities on shells 1, 2 and 3 respectively are

$$\sigma_1 = \frac{Q}{4\pi R^2}$$

$$\sigma_2 = \frac{2Q}{4\pi(2R)^2} = \frac{\sigma_1}{2}$$

and $$\sigma_3 = \frac{3Q}{4\pi(3R)^2} = \frac{\sigma_1}{3}$$

$$\therefore \sigma_1 : \sigma_2 : \sigma_3 = 1 : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2$$

So the correct choice is (c).

133. The circuit can be redrawn as shown in Fig. 11.112.

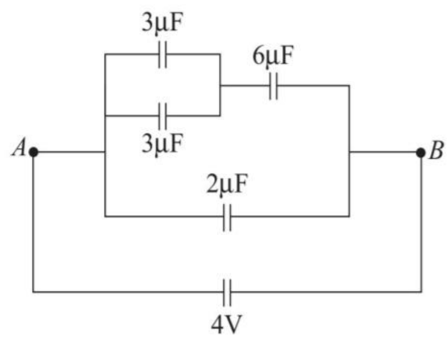


Fig. 11.112

The equivalent capacitance between A and B is

$$C = \frac{6 \times 6}{12} + 2 = 5\ \mu\text{F}$$

$$\therefore \text{ Energy stored} = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 5\mu F \times (4\text{V})^2 = 40\ \mu\text{J}$$

134. Since the cells are in opposition, the circuit can be redrawn as shown in Fig. 11.113.

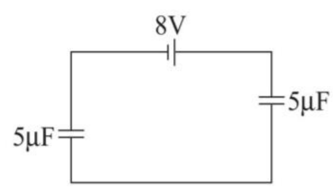


Fig. 11.113

The potential difference across each capacitor = $\frac{8}{2}$ = 4V.

$\therefore$ Charge on 5 μF capacitor = (5 μF) × (4V) = 20 μC

So the correct choice is (b).

2 SECTION Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage.

Passage I

An infinite number of charges, equal to q, are placed along the x-axis at $x = 1$, $x = 2$, $x = 4$, $x = 8$ .... and so on.

1. The electric potential at the point $x = 0$ due to this set of charges is

   (a) $\frac{q}{\pi\varepsilon_0}$ (b) $\frac{q}{2\pi\varepsilon_0}$

   (c) $\frac{q}{3\pi\varepsilon_0}$ (d) $\frac{q}{4\pi\varepsilon_0}$

2. The electric field at point $x = 0$ is

   (a) $\frac{q}{3\pi\varepsilon_0}$ (b) $\frac{q}{4\pi\varepsilon_0}$

   (c) $\frac{q}{5\pi\varepsilon_0}$ (d) $\frac{q}{6\pi\varepsilon_0}$

3. If the consecutive charges have opposite sign, the electric potential at $x = 0$ would be

   (a) $\frac{q}{3\pi\varepsilon_0}$ (b) $\frac{q}{4\pi\varepsilon_0}$

   (c) $\frac{q}{5\pi\varepsilon_0}$ (d) $\frac{q}{6\pi\varepsilon_0}$

4. If the consecutive charges have opposite sign, the electric field at $x = 0$ would be

   (a) $\frac{q}{3\pi\varepsilon_0}$ (b) $\frac{q}{4\pi\varepsilon_0}$

   (c) $\frac{q}{5\pi\varepsilon_0}$ (d) $\frac{q}{6\pi\varepsilon_0}$

1. The potential at $x = 0$ due to a set of infinite number of charges placed on the x-axis as shown in Fig. 11.101, is

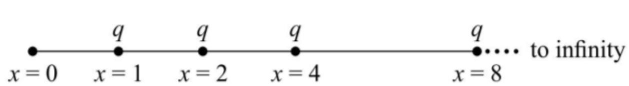


Fig. 11.101

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1}+\frac{q}{2}+\frac{q}{4}+\frac{q}{8}+\cdots\text{to}\,\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\text{to}\,\infty\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1-\left(\frac{1}{2}\right)^{\infty}}{1-\frac{1}{2}}\right]t$$

$$= \frac{q}{4\pi\varepsilon_0}\times\frac{(1-0)}{\left(\frac{1}{2}\right)} = \frac{q}{2\pi\varepsilon_0}$$, which is choice (b).

2. Since the charges are placed along the same straight line, the electric field at $x = 0$ will be directed along the x-axis and its magnitude is given by

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1^2}+\frac{q}{2^2}+\frac{q}{4^2}+\frac{q}{8^2}+\cdots\text{to}\,\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots\text{to}\,\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1-\left(\frac{1}{4}\right)^{\infty}}{1-\frac{1}{4}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\times\frac{(1-0)}{\left(\frac{3}{4}\right)} = \frac{q}{3\pi\varepsilon_0}$$, which is choice (a).

3. If the consecutive charges have opposite sign, the potential at $x = 0$ is given by

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1}-\frac{q}{2}+\frac{q}{4}-\frac{q}{8}+\frac{q}{16}-\frac{q}{32}\cdots\text{to}\,\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(1+\frac{1}{4}+\frac{1}{16}+\cdots\text{to}\,\infty\right)-\left(\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\cdots\text{to}\,\infty\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(\frac{1}{1-\frac{1}{4}}\right)-\frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{4}{3}-\frac{1}{2}\times\frac{4}{3}\right] = \frac{q}{6\pi\varepsilon_0}$$

Hence the correct choice is (d).

4. $$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1^2} - \frac{q}{2^2} + \frac{q}{4^2} - \frac{q}{(8)^2} + \frac{q}{(16)^2} - \frac{q}{(32)^2} + \cdots \text{to}\,\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(1 + \frac{1}{16} + \frac{1}{256} + \cdots \text{to}\,\infty\right) - \left(\frac{1}{4} + \frac{1}{64} + \frac{1}{1024} + \cdots \text{to}\,\infty\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\left(\frac{1}{1-\frac{1}{16}}\right) - \frac{1}{4}\left(\frac{1}{1-\frac{1}{16}}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{16}{15} - \frac{1}{4}\times\frac{16}{15}\right] = \frac{q}{5\pi\varepsilon_0}$$

Hence the correct choice is (c).

Questions 5 to 7 are based on the following passage.

Passage II

A point particle of mass M is attached to one end of a massless rigid non-conducting rod of length L. Another point particle of the same mass is attached to the other end of the rod. The two particles carry charges $+q$ and $-q$. This arrangement is held in a region of a uniform electric field **E** such that the rod makes a small angle θ (say of about 5°) with the field direction as shown in Fig. 11.114.

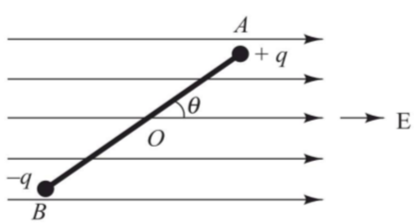


Fig. 11.114

5. The magnitude of the torque acting on the rod is
   (a) $qEL \sin\theta$ (b) $qEL \cos\theta$
   (c) qEL (d) zero
6. When the rod is released, it will rotate with an anular frequency ω equal to
   (a) $\left(\frac{qE}{ML}\right)^{1/2}$ (b) $\left(\frac{2qE}{ML}\right)^{1/2}$
   (c) $\left(\frac{qE}{2ML}\right)^{1/2}$ (d) $\frac{1}{2}\left(\frac{qE}{ML}\right)^{1/2}$
7. The minimum time taken by the rod to align itself parallel to the electric field after it is set free is given by
   (a) $\frac{\pi}{2}\left(\frac{ML}{2qE}\right)^{1/2}$ (b) $2\pi\left(\frac{ML}{qE}\right)^{1/2}$
   (c) $2\pi\left(\frac{2ML}{qE}\right)^{1/2}$ (d) $2\pi\left(\frac{ML}{2qE}\right)^{1/2}$

Solutions

5. A non-conducting rigid rod having equal and opposite charges at the ends is an electric dipole. When it is placed in a uniform electric field, it experiences a torque which tends to align it with the field lines. Referring to Fig. 11.115, the electric forces $F = qE$ each acting at A and B constitute a couple whose torque is given by

$$\tau = \text{force} \times \text{perpendicular distance}$$
$$= F \times AC = F \times AB \sin\theta = qEL \sin\theta$$

So the correct choice is (a).

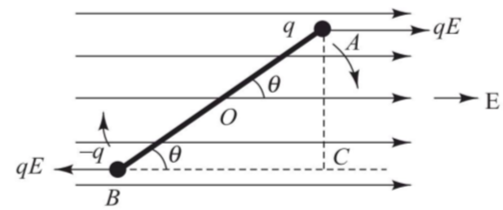


Fig. 11.115

6. Since θ is small, $\sin\theta \simeq \theta$, where θ is expressed in radian. Thus $\tau = qEL\theta$

$\therefore$ Restoring torque $\tau = -qEL\theta$ (1)

If α is the angular acceleration of the rotatory motion, $\tau = I\alpha$

where I is the moment of inertia of the two masses at A and B about an axis passing through the centre O and perpendicular to the rod. Since the rod is massless,

$$I = M \times (AO)^2 + M \times (BO)^2$$
$$= M \times \left(\frac{L}{2}\right)^2 + M \times \left(\frac{L}{2}\right)^2 = \frac{ML^2}{2}$$

Thus $\tau = \frac{ML^2\alpha}{2}$ (2)

Using Eq. (2) in Eq. (1), we get

$$\alpha = -\left(\frac{2qE}{ML}\right)\theta = -\omega^2\theta$$

where $\omega = \left(\frac{2qE}{ML}\right)^{1/2}$, which is choice (b).

7. The time period of oscillation is

$$T = \frac{2\pi}{\omega} = 2\pi\left(\frac{ML}{2qE}\right)^{1/2}$$

Rotating in the clockwise sense, the minimum time taken by the rod to align itself parallel to the electric field is the time it takes to complete one-fourth of angular oscillation, i.e.

$$t_{\min} = \frac{T}{4} = \frac{\pi}{2}\left(\frac{ML}{2qE}\right)^{1/2}$$

So the correct choice is (a).

Questions 8 to 11 are based on the following passage.

Passage III

Two identical particles A and B of mass m carry a charge Q each. Initially particle A is at rest on a smooth horizontal plane and the partical B is projected with a speed v along the horizontal plane from a large distance directly towards the first particle. The distance of closest approach is x.

8. At the closest approach, speed v_1 of particle A is
   (a) $v_1 = v$ (b) $v_1 = \sqrt{2}v$
   (c) $v_1 = v/2$ (d) $v_2 = v/\sqrt{2}$
9. At the closest approach, speed v_2 of particle B is
   (a) $v_2 = v/2$ (b) $v_2 = v/\sqrt{2}$
   (c) $v_2 = v$ (d) $v_2 = 2v$
10. At the distance of closest approach, the total energy of the system is
   (a) $\frac{mv^2}{2}$ (b) $\frac{mv^2}{2} + \frac{Q^2}{4\pi\varepsilon_0 x}$
   (c) $mv^2 + \frac{Q^2}{4\pi\varepsilon_0 x}$ (d) $\frac{mv^2}{4} + \frac{Q^2}{4\pi\varepsilon_0 x}$
11. The distance of closest approach is
   (a) $x = \frac{Q^2}{\pi\varepsilon_0 mv^2}$ (b) $x = \frac{Q^2}{2\pi\varepsilon_0 mv^2}$
   (c) $x = \frac{Q^2}{4\pi\varepsilon_0 mv^2}$ (d) $x = \frac{2Q^2}{\pi\varepsilon_0 mv^2}$

Solutions

8. Initially particle A is at rest and particle B moves towards it with a velocity v from a large distance. Therefore, the initial momentum of the system is

$$p_i = mv + 0 = mv$$

Since the initial distance is large, the particle B exerts negligible repulsive force of the first particle. But, as the distance decreases, the first particle begins to move in the same direction as the second particle under the action of the force of repulsion. The distance between them, therefore, keeps decreasing until it attains a certain minimum value. Let v_1 and v_2 be the velocities of particles 1 and 2 and let t be the time taken by them to acquire these velocities. Then the distances travelled by them will be $x_1 = v_1 t$ and $x_2 = v_2 t$. The separation between them at this time t is

$$x = x_1 - x_2 = (v_1 - v_2)t$$

This separation will be minimum if $dx/dt = 0$. Now

$$\frac{dx}{dt} = v_1 - v_2$$

$\therefore$ For closest approach, $v_1 - v_2 = 0$ or $v_1 = v_2$.

Therefore, the final momentum of the system at the closest approach is

$$p_f = mv_1 + mv_2 = m(v_1 + v_2)$$

From the law of conservation of momentum,

$$p_i = p_f \text{ or } mv = m(v_1 + v_2) \text{ or } v = v_1 + v_2$$

Since the particles are identical, it follows that

$$v_1 = v_2 = \frac{v}{2}.$$

So the correct choice is (c).

9. The correct choice is (a).
10. If x is the distance of closest approach, the final energy of the system is

$$E_f = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{x}$$

or $$= \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{x}$$

or $$E_f = \frac{mv^2}{4} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{x}$$

which is choice (c).

11. The initial energy of the system is

E_i = KE of first particle + KE of second particle + PE due to charge on each particle.

$$= 0 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{\infty}$$

or $$E_i = \frac{1}{2}mv^2$$

From the law of conservation of energy, $E_i = E_f$. Therefore

$$\frac{1}{2}mv^2 = \frac{1}{4}mv^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{x}$$

which gives

$$x = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4Q^2}{mv^2} = \frac{Q^2}{\pi\varepsilon_0 mv^2}$$

Questions 12 to 15 are based on the following passage.

Passage IV

A parallel plate capacitor consists of two metal plates, each of area A, separated by a distance d. A dielectric slab of the same surface area A and thickness t and dielectric constant K is introduced with its faces parallel to the plates as shown in Fig. 11.116.

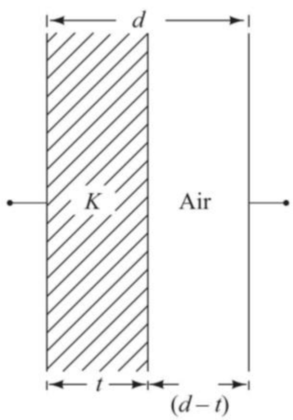


Fig. 11.116

12. The capacitance of the system is

(a) $C = \dfrac{\varepsilon_0 AK}{(d-t)}$

(b) $C = \dfrac{\varepsilon_0 A}{\left(d - \dfrac{t}{K}\right)}$

(c) $C = \dfrac{\varepsilon_0 A}{\left[d + t\left(\dfrac{1}{K} - 1\right)\right]}$

(d) $C = \dfrac{\varepsilon_0 A}{\left[d + t\left(1 - \dfrac{1}{K}\right)\right]}$

13. If $K = 3$, for what value of t/d will the capacitance of the system be twice that of the air capacitor alone ?

(a) $\dfrac{1}{2}$ (b) $\dfrac{2}{3}$

(c) $\dfrac{3}{4}$ (d) $\dfrac{4}{5}$

14. If $K = 3$ and $t/d = 1/2$, the ratio of the energy stored in the system shown in the figure and the air capacitor alone if the charge is the same in both capacitors is

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{3}$

(c) $\dfrac{2}{3}$ (d) $\dfrac{1}{4}$

15. In Q.14 above, the loss of energy is due to

(a) heating of the connecting wires which connect the capacitor with the battery.

(b) the flow of charge from the capacitor to the battery.

(c) leakage of the capacitor.

(d) polarization of the dielectric.

Solutions

12. The system is equivalent to a series combination of two capacitor one of thickness t filled with a dielectric of dielectric constant K and the other of thickness $(d - t)$ with air as dielectric. Their capacitances respectively are

$$C_1 = \frac{\varepsilon_0 KA}{t} \quad (1)$$

and $$C_2 = \frac{\varepsilon_0 A}{(d-t)} \quad (2)$$

The capacitance C of the system is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

or $$C = \frac{C_1 C_2}{C_1 + C_2} \quad (3)$$

Using Eqs. (1) and (2) in Eq. (3) and simplifying we get

$$C = \frac{\varepsilon_0 A}{\left[d + t\left(\frac{1}{K} - 1\right)\right]} \quad (4)$$

So the correct chioce is (c).

13. The capacitance of the air capacitor alone is

$$C_a = \frac{\varepsilon_0 A}{d} \quad (5)$$

Dividing Eq. (4) by Eq. (5), we get

$$\frac{C}{C_a} = \frac{1}{\left[1 + \frac{t}{d}\left(\frac{1}{K} - 1\right)\right]}$$

Given $K = 3$ and $C/C_a = 2$. Thus

$$2 = \frac{1}{\left[1 + \frac{t}{d}\left(\frac{1}{3} - 1\right)\right]}$$

which gives $\dfrac{t}{d} = \dfrac{3}{4}$, which is choice (c).

14. Putting $K = 3$ and $t/d = \dfrac{1}{2}$ in Eq. (4), we get $C = \dfrac{3\varepsilon_0 A}{d}$.

Now $C_a = \dfrac{\varepsilon_0 A}{d}$. Hence $\dfrac{C_a}{C} = 3$. If Q is the same,

$U_a = \dfrac{Q^2}{2C_a}$ and $U = \dfrac{Q^2}{2C}$.

Thus $\dfrac{U}{U_a} = \dfrac{1}{3}$, which is choice (b).

15 The correct choice is (d).

Questions 16 to 18 are based on the following passage

Passage V

A parallel plate capacitor, of plate area A, contains two dielectric slabs of equal dimensions and having dielectric constants K_1 and K_2 as shown in Figs. 11. 117 (a) and (b).

16. The capacitance of the capacitor shown in Fig. 11.117 (a) is

(a) $\dfrac{\varepsilon_0 A}{2d}(K_1+K_2)$

(b) $\dfrac{2\varepsilon_0 A}{d}(K_1+K_2)$

(c) $\dfrac{2\varepsilon_0 A}{d}\left(\dfrac{K_1K_2}{K_1+K_2}\right)$

(d) $\dfrac{\varepsilon_0 A}{2d}\left(\dfrac{K_1K_2}{K_1+K_2}\right)$

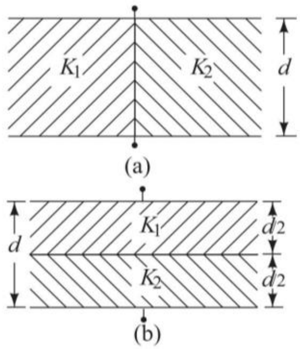


Fig. 11.117

17. The capacitance of the capacitor shown in Fig. 11.117 (b) is

(a) $\dfrac{2\varepsilon_0 A}{d}(K_1+K_2)$ (b) $\dfrac{\varepsilon_0 A}{2d}(K_1+K_2)$

(c) $\dfrac{2\varepsilon_0 A}{d}\left(\dfrac{K_1K_2}{K_1+K_2}\right)$ (d) $\dfrac{\varepsilon_0 A}{2d}\left(\dfrac{K_1K_2}{K_1+K_2}\right)$

18. If $K_1 = 2$ and $K_2 = 3$, the ratio of the capacitances of capacitors in Fig. 11.117 (a) and (b) is

(a) $\dfrac{9}{4}$ (b) $\dfrac{4}{9}$

(c) $\dfrac{24}{25}$ (d) $\dfrac{25}{24}$

Solutions

16. The system shown in Fig. 11.117 (a) is equivalent to a parallel combination of two capacitors—one of plate area $A/2$ filled with a dielectric constant K_1 and the other of plate area $A/2$ filled with a dielectric of dielectric constant K_2, each having the same thickness d. Their respective capacitances are

$$C_1 = \frac{K_1\varepsilon_0 (A/2)}{d} = \frac{K_1\varepsilon_0 A}{2d}$$

and $$C_2 = \frac{K_2\varepsilon_0 (A/2)}{d} = \frac{K_2\varepsilon_0 A}{2d}$$

The capacitance of the parallel combination of C_1 and C_2 is

$$C_a = C_1 + C_2 = \frac{\varepsilon_0 A}{2d}(K_1+K_2) \quad (1)$$

So the correct choice is (a).

17. The system shown in Fig. 11.117 (b) is equivalent to a series combination of the capacitors—one filled with a dielectric of dielectric constant K_1 and of thickness $d/2$ and the other filled with a dielectric of dielectric constant K_2 and of thickness $d/2$, each having the same plate area A. Their respective capacitance are

$$C_1' = \frac{K_1\varepsilon_0 A}{d/2} = \frac{2K_1\varepsilon_0 A}{d}$$

and $$C_2' = \frac{K_2\varepsilon_0 A}{d/2} = \frac{2K_2\varepsilon_0 A}{d}$$

The capacitance of the series combination of C_1' and C_2' is

$$\frac{1}{C_b} = \frac{1}{C_1'} + \frac{1}{C_2'} = \frac{d}{2\varepsilon_0 A}\left(\frac{1}{K_1+K_2}\right)$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{K_1+K_2}{K_1K_2}\right)$$

or $$C_b = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1K_2}{K_1+K_2}\right) \quad (2)$$

Thus the correct choice is (c).

18. Using Eqs. (1) and Eqs. (2) we get

$$\frac{C_a}{C_b} = \frac{(K_1+K_2)^2}{4K_1K_2}$$

If $K_1 = 2$ and $K_2 = 3$, we have

$$\frac{C_a}{C_b} = \frac{(2+3)^2}{4\times 2\times 3} = \frac{25}{24},$$ which is choice (d).

Questions 19 to 21 are based on the following passage.

Passage VI

Fig. 11.118 shows a network of seven capacitors. The charge on the 5 μF capacitor is 10 μC.

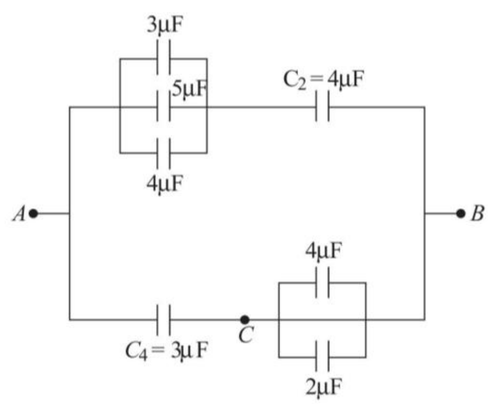


Fig. 11.118

19. The equivalent capacitance between points A and B is
(a) 2.5 μF (b) 5 μF
(c) 7.5 μF (d) 10 μF

20. The potential difference between points A and B is
(a) 2 V (b) 4 V
(c) 6 V (d) 8 V

21. The potential difference between points A and C is
(a) $\frac{16}{3}$ V (b) $\frac{12}{5}$ V
(c) $\frac{8}{3}$ V (d) zero

Solutions

19. The equivalent capacitance C_1 of the parallel combination of 3 μF, 5 μF and 4 μF capacitors is

$$C_1 = 3 + 5 + 4 = 12\ \mu\text{F}$$

The equivalent capacitance C_3 of the parallel combination of 4 μF and 2 μF capacitors is

$$C_3 = 4 + 2 = 6\ \mu\text{F}$$

Now C_1 and C_2 are in series. Their equivalent capacitance C' is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{12} + \frac{1}{4} = \frac{1}{3}$$

or $C' = 3$ μF. The equivalent capacitance C'' of the series combination of C_3 and C_4 is given by

$$\frac{1}{C''} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

or $C'' = 2$ μF. The equivalent capacitance between A and B = capacitance of a parallel combination of C' and $C'' = C' + C'' = 3 + 2 = 5$ μF.

Thus the correct choice is (b).

20. Given, charge on 5 μF capacitor = 10 μC = 10×10^{-6} C.

∴ Potential difference across 5 μF capacitor

$$= \frac{10\times10^{-6}}{5\times10^{-6}} = 2\ \text{V}$$

As the 3 μF, 4 μF and 5 μF capacitors are joined in parallel, the potential difference across each is the same = 2 V. Therefore,

Charge on 3 μF capacitor = $3 \times 10^{-6} \times 2 = 6 \times 10^{-6}$ = 6 μC

Charge on 4 μF capacitor = $4 \times 10^{-6} \times 2 = 8 \times 10^{-6}$ C = 8 μC

∴ Total charge flowing through C_1 and C_2 = 10 μC + 6 μC + 8 μC = 24 μC.

∴ Potential difference across $C_2 = \frac{24\,\mu\text{C}}{4\,\mu\text{F}} = 6$ V

∴ Total potential differenc across AB = 2 V + 6 V = 8 V

So the correct choice is (d).

21. The equivalent capacitance C_3 and C_4 is $C'' = 2$ μF. Therefore, charge flowing throught C_3 and C_4 = 8 V × 2 × 10^{-6} F = 16 × 10^{-6} C = 16 μC. Hence the potential between A and C is = $\frac{16\,\mu\text{C}}{3\,\mu\text{F}}$ = 16/3 V, which is choice (a).

Questions 22 to 25 are based on the following passage.

Passage VII

Two capacitors A and B with capacitances 3 μF and 2 μF are charged to a potential difference of 100 V and 180 V respectively. They are connected to an uncharged 2 μF capacitor C through a switch S as shown in Fig. 11.119.

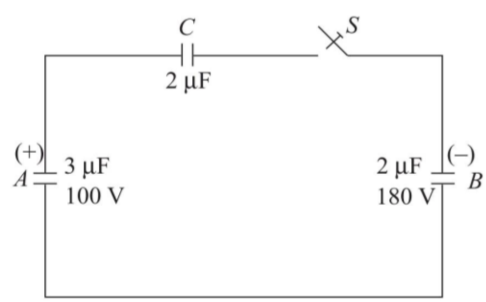


Fig. 11.119

22. When the switch is pressed, the charge flowing through the circuit is
(a) 180 μC (b) 190 μC
(c) 200 μC (d) 210 μC

23. When the switch is pressed, the final charge on capacitor A will be
(a) 60 μC (b) 90 μC
(c) 120 μC (d) 150 μC

24. When the switch is pressed, the final charge on capacitor B will be
(a) 150 μC (b) 160 μC
(c) 180 μC (d) 200 μC

25. When the switch is pressed, the final charge on capacitor C will be
(a) 90 μC (b) 150 μC
(c) 210 μC (d) 300 μC

Solutions

26. Refer to Fig. 11.120. Let Q be the charge flowing through the circuit. When the switch is pressed, the voltages developed on capacitors A, B and C are $V_A = \frac{Q}{3\times10^{-6}}$ volt, $V_B = \frac{Q}{2\times10^{-6}}$ volt and $V_C = \frac{Q}{2\times10^{-6}}$ volt.

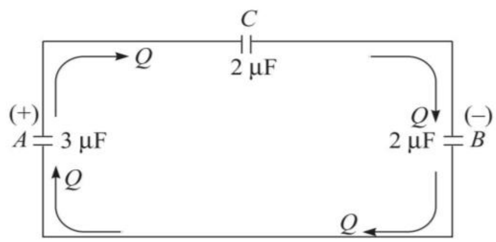


Fig. 11.120

Applying Kirchhoff's law to the loop, we have

$\Delta V_A + \Delta V_B - \Delta V_C = 0$ or

$$\left(100-\frac{Q}{3\times10^{-6}}\right)+\left(180-\frac{Q}{2\times10^{-6}}\right)-\frac{Q}{2\times10^{-6}}=0$$

which gives $Q = 210 \times 10^{-6}$ C = 210 μC. So the correct choice is (d).

23. The initial charge on capacitor A is $(Q_i)_A = (V_i\,C_i)_A$ = 100 V × 3 μF = 300 μC

∴ Final charge on A is

$(Q_i)_A = (Q_i)_A - Q = 300\,\mu C - 210\,\mu C = 90\,\mu C$, which is chioce (b).

24. Similarly, the final charge on B is

$(Q_f)_B = (Q_i)_B - Q$ = (180 V) × (2 μCF) – 210 μC

= 360 μC – 210 μC = 150 μC

So the correct choice is (a).

25. From conservation of charge, we have

$(Q_f)_A + (Q_f)_C$ = 300 μC

∴ $(Q_f)_C$ = 300 μC – $(Q_f)_A$

= 300 μC – 90 μC = 210 μC,

which is chioce (c).

Questions 26 to 29 are based on the following passage.

Passage VIII

In the circuit shown in Fig. 11.121, emf E_1 = 14 V (internal resistance r_1 = 1 Ω), R_1 = 6 Ω, R_2 = 3.5 Ω, emf E_2 = 12 V (internal resistance r_2 = 0.5 Ω), C_1 = 4 μF and C_2 = 2 μF.

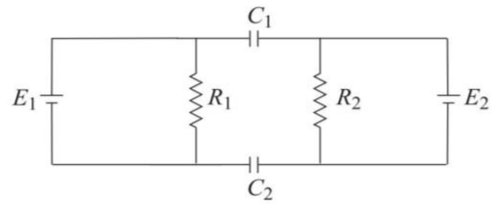


Fig. 121

26. The potential difference acress R_1 is
    (a) 2 V (b) 12 V
    (c) 14 V (d) 26 V
27. The potential difference across R_2 is
    (a) 10.5 V (b) 12.5 V
    (c) 15.0 V (d) 26 V
28. The effective capacitance of the circuit is
    (a) 2 μF (b) $\frac{2}{3}$ μF
    (c) 3 μF (d) $\frac{4}{3}$ μF
29. The charge on capacitor C_2 is
    (a) 20 μC (b) 30 μC
    (c) 40 μC (d) 60 μC

Solutions

26. Current through R_1 is

$$I_1 = \frac{E_1}{R_1 + r_1} = \frac{14}{6+1} = 2\text{ A}$$

∴ Potential difference across R_1 is $V_1 = I_1\,R_1 = 2 \times 6$ = 12 V, which is choice (b).

27. Current through R_2 is

$$I_2 = \frac{E_2}{R_2 + r_2} = \frac{12}{3.5+0.5} = 3\text{ A}$$

∴ Potential difference across R_2 is $V_2 = I_2\,R_2 = 3 \times 3.5$ = 10.5 V

So the correct choice is (a).

28. Effective capacitance $C = \frac{C_1C_2}{C_1+C_2} = \frac{4\times2}{4+2} = \frac{4}{3}$ μF.

The correct choice is (d).

29. Total voltage $V = V_1 + V_2$ = 12 + 10.5 = 22.5 V

Charge $Q = CV = \frac{4}{3}\times10^{-6}\times22.5 = 30\times10^{-6}\,\text{C} = 30\,\mu\text{C}$

Thus the correct choice is (b).

SECTION 3 Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1

(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

1. **Statement-1**

Figure 11.122 shows the tracks of two charged partices A and B in a uniform electric field between two charged plates. The charge to mass ratio of B is greater than that of A. Neglect the effect of gravity.

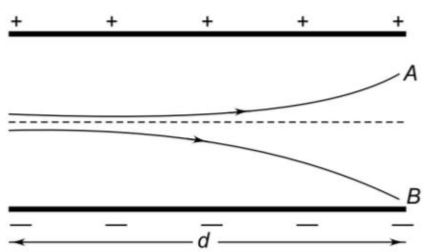


Fig. 11.122

Statement-2

The vertical acceleration of particle B is greater than that of particle A.

2. **Statement-1**

A positive charge $+q$ is located at the centre of a circle as shown in Fig. 11.123. W_1 is the work done in taking a small positive charge $+q_0$ from A to B and W_2 is the work done in taking the same charge from A to C. Then $W_2 > W_1$

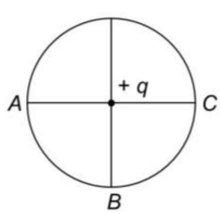


Fig. 11.123

Statement-2

Work done = charge × potential difference.

3. **Statement-1**

A small test charge is initially at rest at a point in an electrostatic field of an electric dipole. When released, it will move along the line of force passing though that point.

Statement-2

The tangent at a point on a line of force gives the direction of the electric field at that point.

4. **Statement-1**

In a non-uniform electric field, a dipole will have translatory as well as rotatory motion.

Statement-2

In a non-uniform electric field, a dipole experiences a force as well as a torque.

5. **Statement-1**

If electric field is zero at a point, the electric potential must also be zero at that point.

Statement-2

Electric field is equal to the negative gradient of potential.

6. **Statement-1**

If electric potential is constant in a certain region of space, the electric field in that region must be zero.

Statement-2

Electric field is equal to the negative gradient of potential.

7. **Statement-1**

If an electron is moved from P to Q, its potential energy increases (see Fig. 11.124)

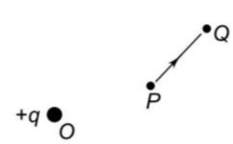


Fig. 11.124

Statement-2

Potential at Q is less than that at P.

8. **Statement-1**

In Q. 7 above, the work done to move an electron from P to Q and then back to P is zero.

Statement-2

Electric field is conservative.

9. **Statement-1**

The work done by the electric field of a nucleus in moving an electron around it in a complete orbit is greater if the orbit is elliptical than if the orbit is circular.

Statement-2

Electric field is conservative.

10. **Statement-1**

Electrons move from a region of higher potential to a region of lower potential.

Statement-2

An electron has less potential energy at a point where the potential is higher and vice versa.

11. **Statement-1**

The electric field is always tangential to the surface of a conductor.

Statement-2

The potential at every point on the surface of a conductor is the same.

12. **Statement-1**

The equipotential surfaces corresponding to a constant electric field along the x-direction are equidistant planes parallel to the y-z plane.

Statement-2

Electric is normal to every point on an equipotential surface

13. **Statement-1**

The electric field in the region around a point charge is uniform.

Statement-2

The equipotential surface of the electric field of a point charge is a sphere with the charge at its centre.

14. **Statement-1**

If a metallic sphere A of radius r carrying a charge Q is brought in contact with an uncharged metallic sphere B of radius $2r$, the charge on sphere A reduces to $Q/3$.

Statement-2

Charge flows from A to B until their potentials are equalised.

15. **Statement-1**

A parallel plate capacitor is charged by a d.c. source supplying a constant voltage V. If the plates are kept connected to the source and the space between the plates is filled with a dielectric, the charge on the plates will increase.

Statement-2

Additional charge will flow from the source to the plates.

16. **Statement-1**

A parallel plate capacitor is charged by a battery of voltage V. The battery is then disonnected. If the space between the plates is filled with a dielectric, the energy stored in the capacitor will decrease.

Statement-2

The capacitance of a capacitor increases due to the introduction of a dielectric between the plates.

17. **Statement-1**

A parallel plate capacitor is charged by a battery. The battery is then disconnected. If the distance between the plates is increased, the energy stored in the capacitor will decrease.

Statement-2

Work has to be done to increase the separation between the plates of a charged capacitor.

18. **Statement-1**

Two adjacent conductors when given the same charge will have a potential difference between them if they are of different shape and size.

Statement-2

The potential to which a conductor is raised depends not only on the amount of charge but also on the shape and size of the conductor.

Solutions

1. The correct choice is (a). Let E be the electric field between the plates; It is directed vertically downwords. If q is the charge of the particle, it will experience a force $F = qE$. Hence its acceleration (in the vertical direction) is

$$a = \frac{F}{m} = \frac{qE}{m}$$

where m is the mass of the particle. If t is time spent by the particle between the plates (i.e in the region of the electric field), the vertical distance travelled (i.e. deflection) of the particle is

$$y = \frac{1}{2}at^2 = \frac{qEt^2}{2m}$$

Thus $\frac{q}{m} \propto y$. Since particle B has a higher deflection, it has a higher charge to mass ratio.

2. The correct choice is (d). Points A, B and C are at the same distance from charge $+q$. Hence electric potential difference between points A, B and C is zero. Hence $W_1 = W_2 = 0$.

3. The correct choice is (d). The test charge will move along the line of force if the line of force is straight (as in the case of a single charge). If the lines of force is curved, the charge will not move along the line of force. The reason is that the line of force does not give the direction of velocity, it gives the direction of the force which is along the tangent to the curve at that point.

4. The correct choice is (a).

5. The correct choice is (d). Since $E = -dV/dr$, if $E = 0$, V = constant not necessarily equal to zero.

6. The correct choice is (a).

7. The correct choice is (c). Since charge of an electron is negative, P.E at P and Q is

$$U_P = -\frac{eq}{4\pi\varepsilon_0(OP)}$$

$$U_Q = -\frac{eq}{4\pi\varepsilon_0(OQ)}$$

Since $OQ > OP$, U_Q is less negative than U_P, i.e. $U_Q > U_P$. For the same reason, $V_Q > V_P$.

8. Since charge $+q$ will attract the electron, work is done to move the electron from P to Q is negative because the work is done against the field. To move it from Q back to P an equal positive work is done by the field because electric field is conservative. So the correct choice is (a).

9. The correct choice is (d). The work done by the electric field in moving a charge around a closed path of any shape (circular or elliptical) is zero.

10. The correct choice is (d). Since the electron has a negative charge, it has less energy at a point where the potenial is higher and vice versa. Hence in an electric field an electron moves from a region of lower potential to a region of higher potential.

11. The correct choice is (d). Since E along the surface $= 0$, $dV/dr = 0$

12. The electric field is always normal to the equipotential surface. Therefore, for a constant electric field in the x-direction, the equipotential surfaces are planes parallel to the y-z plane. Since the field is constant, the equipotential surfaces are equidistant from each other. The correct choice is (a).

13. The correct choice is (d). The electric field in a region around a point charge varies with distance r:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

14. The correct choice is (a). Charge will flow from A to B until their potentials become equal. If charge q flows from A to B, then

$$\frac{Q-q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0(2r)}$$

which gives $Q - q = \frac{q}{2} \Rightarrow q = \frac{2Q}{3}$. Hence charge left on $A = Q - \frac{2Q}{3} = \frac{Q}{3}$.

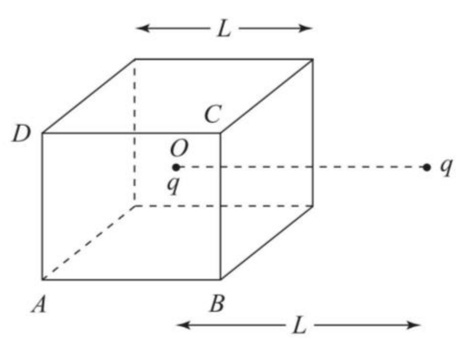
15. The correct choice is (a). Since the source supplies a constant voltage V, the potential difference between the plates remains equal to V because the source is not disconnected. The capacitance C increases due to the introduction of the dielectric. Since $Q = CV$, the charge Q on the capacitor plates will increase.
16. The correct choice is (b). The charge Q on the capacitor plates remains unchanged because there is no source to supply extra charge as the battery is disconnected. The capacitance C increases due to the introduction of the dielectric. Now, energy stored $U = Q^2/2C$. Since Q remains unchanged and C increases, U will decrease.
17. The correct choice is (d). The charge Q remains unchanged as the battery is disconnected. The capacitance C decreases if the separation between the plates is increased. Now, energy stored $U = Q^2/2C$. Since Q remains the same and C is decreased, U will increase.
18. The correct choice is (a).

SECTION 4 Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. 2J of work is done in moving a charge of 20 C from one point to another 2 cm apart. The potential difference between the points is

(a) 0.1 V (b) 8 V
(d) 2 V (d) 0.5 V **[2002]**

2. A charged particle q is placed at the centre O of a cube of side L. Another equal charge q is placed at a distance L from O. The electric flux through face $ABCD$ is



(a) $\frac{q}{4\pi\varepsilon_0 L}$ (b) zero
(c) $\frac{q}{2\pi\varepsilon_0 L}$ (d) $\frac{q}{3\pi\varepsilon_0 L}$ **[2002]**

3. If a parallel combination of n capacitors, each of capacitance C, is connected to a source of voltage V, the energy stored in the system is

(a) CV (b) $\frac{1}{2}nCV^2$
(c) CV^2 (d) $\frac{1}{2n}CV^2$ **[2002]**

4. If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium, then the value of q is

(a) $\frac{Q}{2}$ (b) $-\frac{Q}{2}$
(b) $\frac{Q}{4}$ (d) $-\frac{Q}{4}$ **[2002]**

5. The capacitance (in F) of a spherical conductor having radius 1 m is

(a) 1.1×10^{-10} (b) 10^{-6}
(c) 9×10^{-9} (d) 10^{-3} **[2002]**

6. To identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is

(a) $\frac{1}{4}C(V_1^2 - V_2^2)$ (b) $\frac{1}{4}C(V_1^2 + V_2^2)$
(c) $\frac{1}{4}C(V_1 - V_2)^2$ (d) $\frac{1}{4}C(V_1 + V_2)^2$

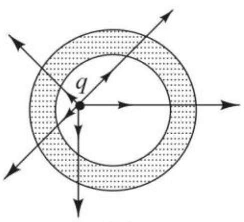
[2002]

7. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, When it is displaced by a small distance x along the x-axis, is approximately proportional to

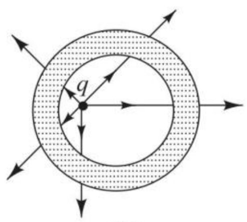
(a) x　　(b) x^2

(c) x^3　　(d) $1/x$　　**[2002]**

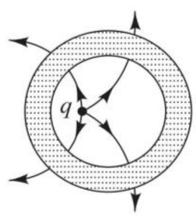
8. A metallic shell has a point charge q kept inside its circular cavity. The charge is not exactly at the centre of the cavity. Which of the diagrams correctly represents the electric lines of force?



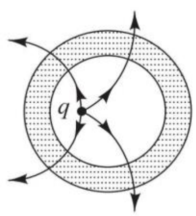
(a)



(b)



(c)



(d)

[2002]

9. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be

(a) $\dfrac{(\phi_2 - \phi_1)}{\varepsilon_0}$　　(b) $\dfrac{(\phi_1 + \phi_2)}{\varepsilon_0}$

(c) $(\phi_2 - \phi_1)\varepsilon_0$　　(d) $(\phi_1 + \phi_2)\varepsilon_0$　　**[2003]**

10. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor

(1) decreases　　(2) remains unchanged

(3) becomes infinite　　(4) increases　　**[2003]**

11. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P at a distance $R/2$ from the centre of the shell is

(a) $\dfrac{2Q}{4\pi\varepsilon_0 R}$　　(b) $\dfrac{2Q}{4\pi\varepsilon_0 R} - \dfrac{2q}{4\pi\varepsilon_0 R}$

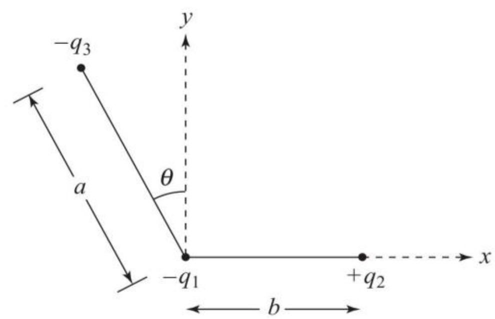
(c) $\dfrac{2Q}{4\pi\varepsilon_0 R} + \dfrac{q}{4\pi\varepsilon_0 R}$　　(d) $\dfrac{2(q+Q)}{4\pi\varepsilon_0 R}$　　**[2003]**

12. The work done in placing a charge of 8×10^{-18} C on a capacitor of capacitance 100 μF is

(1) 16×10^{-32} J　　(2) 3.1×10^{-26} J

(3) 4×10^{-10} J　　(4) 32×10^{-32} J　　**[2003]**

13. Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to



(a) $\dfrac{q_2}{b^2} - \dfrac{q_3 \cos\theta}{a^2}$　　(b) $\dfrac{q_2}{b^2} + \dfrac{q_3 \cos\theta}{a^2}$

(c) $\dfrac{q_2}{b^2} + \dfrac{q_3 \sin\theta}{a^2}$　　(d) $\dfrac{q_2}{b^2} - \dfrac{q_3 \sin\theta}{a^2}$

[2003]

14. Four charges, each equal to $-Q$, are placed at the corners of a square and a charge q is at its centre. If the system is in equilibrium, the value of q is

(a) $-\dfrac{Q}{4}(1+2\sqrt{2})$　　(b) $\dfrac{Q}{4}(1+2\sqrt{2})$

(c) $-\dfrac{Q}{2}(1+2\sqrt{2})$　　(d) $\dfrac{Q}{2}(1+2\sqrt{2})$　　**[2004]**

15. Two spherical conductors A and B having equal radii and carrying equal charges repel each other with a force F when kept a certain distance apart. A third identical spherical conductor but uncharged is brought in contact with B and removed. It is then brought in contact with A and removed. The new force of repulsion between A and B is

(a) $\dfrac{F}{4}$　　(b) $\dfrac{3F}{4}$

(c) $\dfrac{F}{8}$　　(d) $\dfrac{3F}{8}$　　**[2004]**

16. A charged particle q is shot towards another charged particle Q which is fixed with an initial speed v. The particle q reaches upto the closest distance r before it is repelled back. What will be the closest distance of approach if the particle q was shot with a velocity $2v$?

(a) r (b) $2r$

(c) $\frac{r}{2}$ (d) $\frac{r}{4}$ **[2004]**

17. A charged oil drop of mass 9.9×10^{-15} kg is suspended in a uniform electric field of 3×10^4 Vm^{-1}. If $g = 10$ ms^{-2}, the charge on the oil drop will be

(a) 3.3×10^{-18} C (b) 3.2×10^{-18} C

(c) 1.6×10^{-18} C (d) 4.8×10^{-18} C

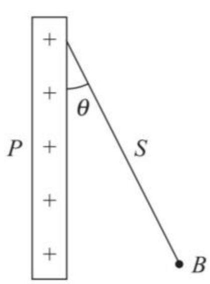
[2004]

18. A fully charged capacitor of capacitance C is discharged through a coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperature of the block is raised by ΔT, the potential difference across the capacitor is

(a) $\sqrt{\frac{2mC\Delta T}{s}}$ (b) $\frac{mC\Delta T}{s}$

(c) $\frac{ms\Delta T}{C}$ (d) $\sqrt{\frac{2ms\Delta T}{C}}$ **[2005]**

19. A charged ball B hangs from a silk thread S, which makes an angle θ with a large conducting sheet P as shown in the figure. The surface charge density σ of the sheet is proportional to



(a) $\cos\theta$ (b) $\cot\theta$

(c) $\sin\theta$ (d) $\tan\theta$ **[2005]**

20. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x-axis at which the net electric field due to the two point charges is

(a) $2L$ (b) $\frac{L}{4}$

(c) $8L$ (d) $4L$ **[2005]**

21. Two thin wire rings, each of radius R, are placed at a distance d apart with their axes coinciding. The charges on the rings are $+q$ and $-q$. The potential difference between the centres of the two rings is

(a) $\frac{qR}{4\pi\varepsilon_0 d^2}$

(b) $\frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2 + d^2}}\right]$

(c) zero

(d) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2 + d^2}}\right]$ **[2005]**

22. An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience

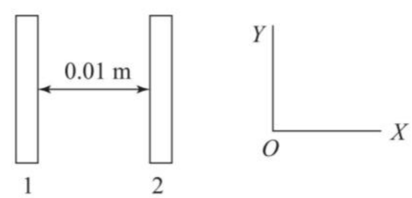
(a) a torque as well as a translational force

(b) a torque only

(c) a translational force only in the direction of the field

(d) a translational force only in a direction normal to the direction of the field **[2006]**

23. Two insulating plates are both uniformly charged in such a way that the potential difference them is $V_2 - V_1 = 20$ V. (i.e. plate 2 is at a higher potential). The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate is 1. What is its speed when it hits plate 2?
($e = 1.6 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg.)



(a) 1.87×10^6 ms^{-1} (b) 3.2×10^{-18} ms^{-1}

(c) 2.65×10^6 ms^{-1} (d) 7.02×10^{12} ms^{-1}

[2006]

24. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of sphere A and B is

(a) 2 : 1 (b) 1 : 4

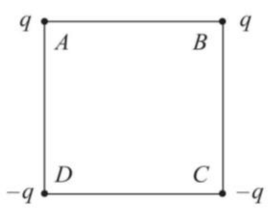
(c) 4 : 1 (d) 1 : 2 **[2006]**

25. An electric charge 10^{-3} μC is placed at the origin (0, 0) of $X - Y$ co-ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be

(a) 9 volt (b) zero
(c) 2 volt (d) 4.5 volt **[2007]**

26. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be
(a) 1 (b) 2
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$ **[2007]**

27. Charges are placed on the vertices of a square as shown in the figure. Let E be the electric field and V the potential at the centre of the square. If the charges on A and B are interchanged with those on D and C respectively, then



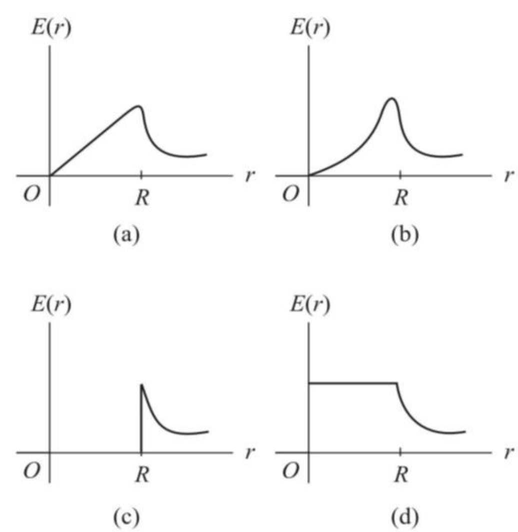
(a) E remains unchanged but V will change
(b) both E and V will change
(c) E and V will remain unchanged
(d) V remains unchanged but E will change

28. The potential at a point x due to some charges situated on the x-axis is given by
$$V = \frac{20}{(x^2 - 4)} \text{ volt}$$
where x is measured in μm. The electric field E at x = 4 μm is
(a) $\frac{5}{3}$ volt/μm in the –ve x-direction
(b) $\frac{5}{3}$ volt/μm in the +ve x-direction
(c) $\frac{10}{9}$ volt/μm in the –ve x-direction
(d) $\frac{10}{9}$ volt/μm in the +ve x-direction **[2007]**

29. A parallel plate capacitor with a dielectric of dielectric constant K between the plates has a capacitance C and is changed to a voltage V. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is
(a) $\frac{1}{2}(K-1)CV^2$ (b) $\frac{CV^2(K-1)}{K}$
(c) $(K-1)\,CV^2$ (d) zero **[2007]**

30. A parallel plate capacitor with air between the plates has a capacitance of 9pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One dielectric has dielectric constant $k_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $k_2 = 6$ and $\frac{2d}{3}$ thickness. Capacitance of the capacitor is now
(a) 40.5 pF (b) 20.25 pF
(c) 1.8 pF (d) 45pF **[2008]**

31. A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell?



32. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then Q/q equals:
(a) 1 (b) $-\frac{1}{\sqrt{2}}$
(c) $-2\sqrt{2}$ (d) –1 **[2009]**

33. *This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.*
Statement-1: For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement-2: The net work done by a conservative force on an object moving along a closed loop is zero.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.

(b) Statement-1 is false, Statement-2 is true.

(c) Statement-1 is true, Statement-2 is false.

(d) Statament-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1. **[2009]**

34. Two points P and Q are maintained at the potentials of 10 V and – 4V, respectively. The work done in the moving 100 electrons from P to Q is:

(a) -2.24×10^{-16} J (b) 2.24×10^{-16} J

(c) -9.60×10^{-17} J (d) 9.60×10^{-17} J **[2009]**

35. Let $\rho(r) = \dfrac{Qr}{\pi R^4}$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point P inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is

(a) $\dfrac{Qr_1^2}{4\pi \in_0 R^4}$ (b) $\dfrac{Qr_1^2}{3\pi \in_0 R^4}$

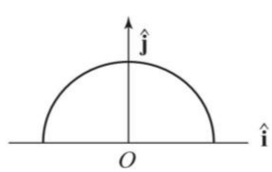
(c) zero (d) $\dfrac{Q}{4\pi\eta_0 r_1^2}$ **[2009]**

36. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be

(a) 1 (b) $\dfrac{1}{2}$

(c) $\dfrac{1}{4}$ (d) 2 **[2010]**

37. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is



(a) $\dfrac{q}{4\pi^2 \varepsilon_0 r^2}\hat{\mathbf{j}}$ (b) $-\dfrac{q}{4\pi^2 \varepsilon_0 r^2}\hat{\mathbf{j}}$

(c) $-\dfrac{q}{2\pi^2 \varepsilon_0 r^2}\hat{\mathbf{j}}$ (4) $\dfrac{q}{2\pi^2 \varepsilon_0 r^2}\hat{\mathbf{j}}$ **[2010]**

38. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\dfrac{5}{4} - \dfrac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance $r (r < R)$ from the origin is given by

(a) $\dfrac{\rho_0 r}{3\varepsilon_0}\left(\dfrac{5}{4} - \dfrac{r}{R}\right)$ (b) $\dfrac{4\pi\rho_0 r}{3\varepsilon_0}\left(\dfrac{5}{4} - \dfrac{r}{R}\right)$

(c) $\dfrac{\rho_0 r}{4\varepsilon_0}\left(\dfrac{5}{3} - \dfrac{r}{R}\right)$ (d) $\dfrac{4\rho_0 r}{3\varepsilon_0}\left(\dfrac{5}{4} - \dfrac{r}{R}\right)$

[2010]

39. Two identically charged spheres are suspended by strings of equal lengths. The strings make an angle of 30º with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the spheres is 1.6 g cm^{-3}, the dielectric constant of the liquid is

(a) 4 (b) 3

(c) 2 (d) 1 **[2010]**

40. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is

(a) $-24\pi a\varepsilon_0 r$ (b) $-6\pi a\varepsilon_0 r$

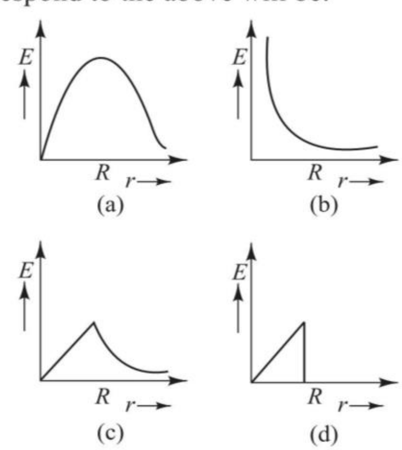
(c) $-24\pi a\varepsilon_0$ (d) $-6a\varepsilon_0$ **[2011]**

41. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d<<l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them

(a) $v \propto x^{-1/2}$ (b) $v \propto x^{-1}$

(a) $v \propto x^{1/2}$ (b) $v \propto x$ **[2011]**

42. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be:



[2012]

43. This questions has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describe the two statements.

An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinite is zero.

Statement-1: When a change 'q' is taken from the centre of the surface of the sphere its potential energy changes by $q\rho/3\varepsilon_0$

Statement-2: The electric field at a distante $r(r < R)$ from the centre of the sphere is $\rho r/3\varepsilon_0$

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true, Statement 2 is the correct explanation of Statement 1.

[2012]

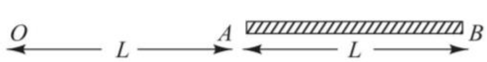
44. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = q/2$ is placed at the origin. If charge q_0 is given a small displacement ($y << a$) along the y-axis, the net force acting on the particle is proportional to

(a) –y (b) $\frac{1}{y}$
(c) $-\frac{1}{y}$ (d) y **[2013]**

45. Two capacitors C_1 and C_2 are charged to 120 V and 200 V, respectively. It is found that by connecting them together the potential on each one can be made zero. Then

(a) $3C_1 = 5C_2$ (b) $3C_1 + 5C_2 = 0$
(c) $9C_1 = 4C_2$ (d) $5C_1 = 3C_2$ **[2013]**

46. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electricpotential at the point O lying at a distance L from the end A is:



(a) $\frac{3Q}{4\pi\varepsilon_0 L}$ (b) $\frac{Q}{4\pi\varepsilon_0 L \ln 2}$
(c) $\frac{Q \ln 2}{4\pi\varepsilon_0 L}$ (d) $\frac{Q}{8\pi\varepsilon_0 L}$ **[2013]**

47. An electric dipole is placed at an angle of 30º to a non-uniform electric field. The dipole will experience
(a) a torque as well as a translational force
(b) a torque only
(c) a translational force only in the direction of the field
(d) a translational force only in a direction normal to the direction of the filed **[2014]**

48. A parallel plate capacitor with air between the plates has a capacitance of 9pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One dielectric has dielectric constant $K_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $K_2 = 6$ and thickness $\frac{2d}{3}$. Capcitance of the capacitor is now
(a) 40.5 pF (b) 20.25 pF
(c) 1.8 pF (d) 45 pF **[2014]**

Answers

1. (a)	2. none	3. (b)	4. (d)
5. (a)	6. (c)	7. (b)	8. (c)
9. (c)	10. (b)	11. (c)	12. (d)
13. (c)	14. (a)	15. (d)	16. (d)
17. (a)	18. (d)	19. (d)	20. (a)
21. (b)	22. (a)	23. (c)	24. (a)
25. (b)	26. (d)	27. (d)	28. (d)
29. (d)	30. (a)	31. (c)	32. (c)
33. (d)	34. (b)	35. (a)	36. (c)
37. (c)	38. (c)	39. (c)	40. (d)
41. (a)	42. (c)	43. (c)	44. (d)
45. (a)	46. (c)	47. (a)	48. (a)

Solutions

1. $\Delta V = \frac{W}{q} = \frac{2}{20} = 0.1$ V

2. Charge outside the cube does not contribute to electric flux. From Gauss's law, electric flux through the cube is $\frac{q}{\varepsilon_0}$. Since a cube has 6 faces, the flux through one face $ABCD$ is $\frac{q}{6\varepsilon_0}$. Hence no choice

given in the question is correct. Moreover, choices (a), (c) and (d) do not have the dimensions of electric flux.

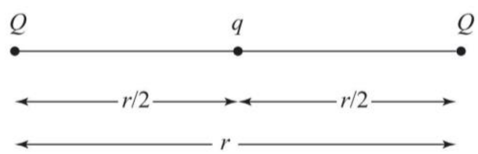
3. Equivalent capacitance is $C_{eq} = nC$. Therefore, energy stored is

$$U = \frac{1}{2}\, C_{eq}\, V^2 = \frac{1}{2} nCV^2$$

4. Let r be the distance between equal charges Q.
Since q is exactly mid-way between equal charges (Q), charge q will always be in equilibrium. The charge Q will be in equilibrium if
$\mathbf{F}_{Qq} = \mathbf{F}_{QQ} = 0$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{\left(\frac{r}{2}\right)^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{r^2} = 0$$

$$\Rightarrow \quad q = -\frac{Q}{4}$$



5. $C = 4\pi\varepsilon_0 r = \dfrac{1}{9\times 10^9} \times 1 = 1.1 \times 10^{-10}$ F.

6. Initial Energy $U_i = U_1 + U_2 = \dfrac{1}{2}\ CV_1^2 + \dfrac{1}{2}\, CV_2^2$

When they are connected, the potential across each is $V = \dfrac{1}{2}\ (V_1 + V_2)$. Final energy is

$$U_f = \frac{1}{2}\, CV^2 + \frac{1}{2}\, CV^2 = CV^2$$

$$= C\left(\frac{V_1 + V_2}{2}\right)^2$$

$\therefore$ Decrease in energy $= Ui - U_f$

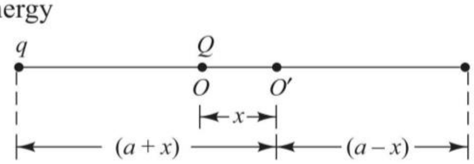
$$= \frac{1}{2}\ C(V_1^2 + V_2^2) - \frac{1}{4}\, C(V_1 + V_2)^2$$

$$= \frac{1}{4}\, C(V_1 - V_2)^2$$

7. Potential energy of the system when charge Q is at O is

$$U_0 = \frac{qQ}{a} + \frac{qQ}{a} = \frac{2qQ}{a}$$

When charge Q is shifted to position O', the potential energy



$$U = \frac{qQ}{(a+x)} + \frac{qQ}{(a-x)} = \frac{qQ(2a)}{(a^2 - x^2)}$$

$$= \frac{2qQ}{a} \times \left(1 - \frac{x^2}{a^2}\right)^{-1}$$

$$\simeq \frac{2qQ}{a} \times \left(1 + \frac{x^2}{a^2}\right) \qquad (x << a)$$

$$\therefore\ \Delta U = U - U_0 = \frac{2qQ}{a}\left(1 + \frac{x^2}{a^2}\right) - \frac{2qQ}{a}$$

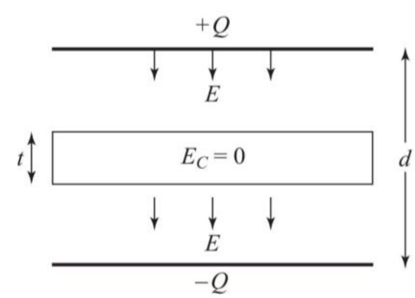
$$= \frac{2qQ}{a^3}(x^2)$$

Hence $\Delta U \propto x^2$ Which is choice (b).

8. Because the charge is not located at the centre of the cavity, inside the cavity the lines of force are skewed. Hence choice (a) and (b) are incorrect. Outside the shell, the lines of force are the same as if the charge were located at the centre of the cavity. Also there can be no line of force in the metallice body of the shell. Hence choice (d) also incorrect. Thus, the correct pattern is shown in (c).

9. According to Gauss's law, the net electric flux leaving a closed surface $= \dfrac{q}{\varepsilon_0}$. Hence
$(\phi_2 - \phi_1) = \dfrac{q}{\varepsilon_0} \Rightarrow q = \varepsilon_0(\phi_2 - \phi_1)$

10. In electrostatics, the electric field inside a conductor $(E_c) = 0$. The potential between the plates of the capacitor is



$$V = E(d - t) = E_C t$$

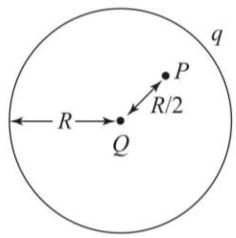
$$= E(d - t) \qquad (\because E_C = 0)$$

$$= \frac{\sigma}{\varepsilon_0}(d - t) = \frac{Q}{A\varepsilon_0}(d - t)$$

$$\therefore\ C = \frac{Q}{V} = \frac{A\varepsilon_0}{(d - t)}$$

If t is negligible compared to d, $C = \dfrac{A\varepsilon_0}{d}$. So the correct choice is (b).

11. Potential at P due to charge q on the shell is



$$V_1 = \frac{q}{4\pi\varepsilon_0 R}$$

Potential at P due to charge Q at the centre of the shell is

$$V_2 = \frac{Q}{4\pi\varepsilon_0(R/2)} = \frac{2Q}{4\pi\varepsilon_0 R}$$

Therefore, total potential at P is

$$V = V_1 + V_2 = \frac{q}{4\pi\varepsilon_0 R} + \frac{2Q}{4\pi\varepsilon_0 R}$$

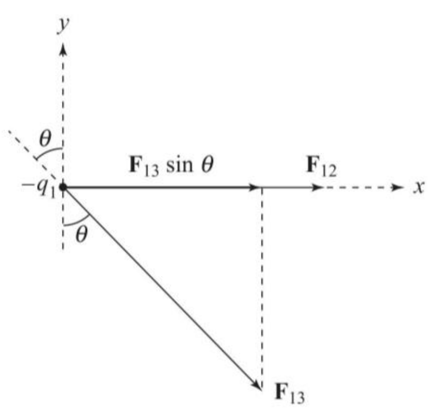
12. $W = \frac{Q^2}{2C} = \frac{(8\times10^{-18})^2}{2\times(100\times10^{-6})} = 32\times10^{-32}$ J

13. Force exerted on $-q_1$ by $+q_2$ is

$$F_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 b^2} \text{ along } +x \text{ axis}$$

Force exerted on $-q_1$ by $-q_3$ is

$$F_{13} = \frac{q_1 q_3}{4\pi\varepsilon_0 a^2} \text{ along the direction shown in figure.}$$



The x-component of F_{13} is

$$F_{13}\cos(90^\circ - \theta) = F_{13}\sin\theta$$

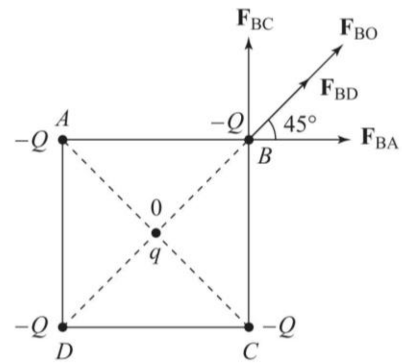
Net force on $-q_1$ is

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 b^2} + \frac{q_1 q_3 \sin\theta}{4\pi\varepsilon_0 a^2}$$

$$\Rightarrow F = \frac{q_1}{4\pi\varepsilon_0}\left(\frac{q_2}{b^2} + \frac{q_3\sin\theta}{a^2}\right)$$

For a given q_1, $F \propto \left(\frac{q_2}{b^2} + \frac{q_3\sin\theta}{a^2}\right)$

14. The system is in equilibrium if each experiences no force. It is clear that charge q at the centre would be in equilibrium for any value of q.



Let $AB = BC = BD = DA = a$. Then

$$DB = \sqrt{2}a \text{ and } OB = \frac{a}{\sqrt{2}}$$

$$F_{BA} = F_{BC} = \frac{Q^2}{4\pi\varepsilon_0 a^2}$$

$$F_{BD} = \frac{Q^2}{4\pi\varepsilon_0(\sqrt{2}a)^2} = \frac{Q^2}{4\pi\varepsilon_0(2a)^2}$$

$$F_{BO} = \frac{qQ}{4\pi\varepsilon_0\left(\frac{a}{\sqrt{2}}\right)^2} = -\frac{2qQ}{4\pi\varepsilon_0 a^2}$$

Net force on $-Q$ at B will be zero, if

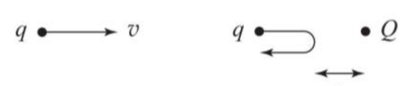
$$F_{BA}\cos45^\circ + F_{BC}\cos45^\circ + F_{BD} + F_{BO} = 0$$

$$\Rightarrow \frac{Q^2}{4\pi\varepsilon_0 a^2\sqrt{2}} + \frac{Q^2}{4\pi\varepsilon_0 a^2\sqrt{2}} + \frac{Q^2}{4\pi\varepsilon_0(2a^2)} - \frac{2qQ}{4\pi\varepsilon_0 a^2} = 0$$

$$\Rightarrow q = \frac{Q}{4}(1+2\sqrt{2})$$

15. The correct choice is (d).

16. Particle q will momentarily come to rest at a distance r from Q and will then be repelled back.



At the closest distance of approach, the kinetic energy of q is converted into potential energy of the system which is q at a distance r from Q. Hence

$$\frac{1}{2}mv^2 = \frac{qQ}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \qquad r = \frac{2qQ}{4\pi\varepsilon_0 mv^2}$$

Hence $r \propto \frac{1}{v^2}$. If v is doubled, r reduces to $\frac{r}{4}$.

17. $mg = qE$

$$\Rightarrow q = \frac{mg}{E} = \frac{9.9\times10^{-15}\times10}{3\times10^4} = 3.3\times10^{-18}\text{ C.}$$

18. $\frac{1}{2}CV^2 = ms\Delta T$

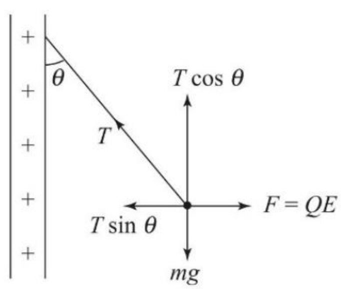
$$\Rightarrow \qquad V = \sqrt{\frac{2ms\Delta T}{C}}$$

19. Electric field due to sheet is

$$E = \frac{\sigma}{2\varepsilon_0}$$

where $\sigma = \frac{Q}{A}$; A = area of sheet

It follows from the figure that



$T\cos\theta = mg$ (1)

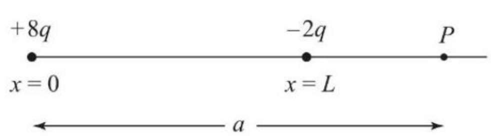
$T\sin\theta = QE = \sigma AE$ (2)

Dividing (2) by (1), we get

$$\tan\theta = \frac{\sigma AE}{mg}$$

Since E is uniform and A, m and g are fixed, $\sigma \propto \tan\theta$.

20. It is clear that the net electric field cannot be zero at any point between $x = 0$ and $x = L$. Let the electric field be zero at a point P at a distance a from $x = 0$.



Electric field at P due to charge $+8q$ is

$$\mathbf{E}_1 = \frac{8q\hat{\mathbf{i}}}{4\pi\varepsilon_0 a^2}$$

Electric field at P due to charge $-2q$ is

$$\mathbf{E}_2 = \frac{2q\hat{\mathbf{i}}}{4\pi\varepsilon_0(a-L)^2}$$

Net electric field at P is $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$. $\mathbf{E} = 0$ if

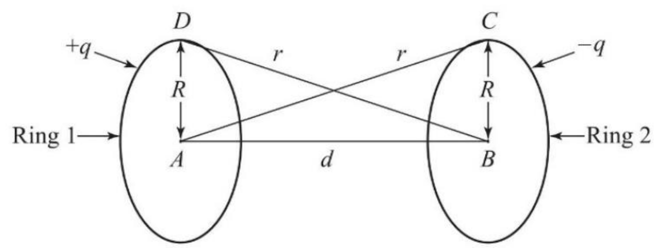
$$\frac{8q\hat{\mathbf{i}}}{4\pi\varepsilon_0 a^2} - \frac{2q\hat{\mathbf{i}}}{4\pi\varepsilon_0(a-L)^2} = 0$$

$$\Rightarrow \frac{4}{a^2} = \frac{1}{(a-L)^2}$$

$$\Rightarrow \frac{2}{a} = \frac{1}{(a-L)}$$

$$\Rightarrow a = 2L$$

21. Let $AC = BD = r = \sqrt{R^2 + d^2}$



Potential at A is

V_A = potential at A due to charge $+q$ on ring 1 + potential at A due to charge $-q$ on ring 2

$$= \frac{q}{4\pi\varepsilon_0 R} - \frac{q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right)$$

Potential at B is

V_B = potential at B due to charge $-q$ on ring 2 + potential at B due to charge $+q$ on ring 1

$$= \frac{-q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$$

$$\therefore\ V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right) - \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{2}{R} - \frac{2}{r}\right)$$

$$= \frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2 + d^2}}\right]$$

22. The correct choice is (a).

23. Potential difference between plates 1 and 2 is

$V = V_2 - V_1 = 20$ V

Energy gained by the electron is eV. If v is its velocity with which it hits plate 2, then

$$\frac{1}{2}mv^2 = eV$$

or $v = \sqrt{\dfrac{2eV}{m}}$

$$= \left[\frac{2\times(1.6\times10^{-19})\times 20}{9.11\times10^{-31}}\right]^{1/2}$$

$= 2.65 \times 10^6$ ms^{-1}.

24. Let Q_1 and Q_2 be the charges on spheres A and B and R_1 and R_2 their respective radii. Their potentials are

$$V_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_1}{R_1} \text{ and } V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_2}{R_2}$$

If $V_1 > V_2$, charge will flow from A to B when the spheres are connected by a wire, until their potentials are equalized. If charge q flows from A to B, then

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{(Q_1-q)}{R_1} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(Q_2+q)}{R_2}$$

$$\Rightarrow \frac{(Q_1-q)}{(Q_2+q)} = \frac{R_1}{R_2} \quad (1)$$

The electric fietlds on the surfaces of the spheres are

$$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(Q_1-q)}{R_1^2} \text{ and } E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(Q_2+q)}{R_2^2}$$

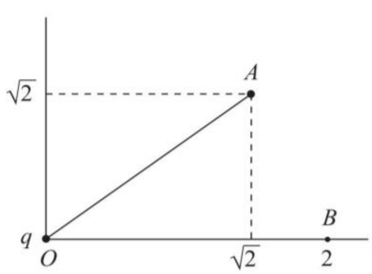
$$\therefore \frac{E_1}{E_2} = \frac{(Q_1-q)}{(Q_2+q)}\cdot\left(\frac{R_2}{R_1}\right)^2 = \frac{R_2}{R_1} = \frac{2}{1} \quad \text{[use Eq. (1)]}$$

Hence the correct choice is (a). The same result is obtained if $V_2 > V_1$.

25. Refer to the following figure.

$OA = 2$, $OB = 2$

Now $\quad V_A = \dfrac{q}{4\pi\varepsilon_0(OA)}, \quad V_B = \dfrac{q}{4\pi\varepsilon_0(OB)}$



$V_A - V_B = 0 \quad (\because OA = OB)$

26. Let Q be the charge on the capacitor plates and V be the voltage of the battery.

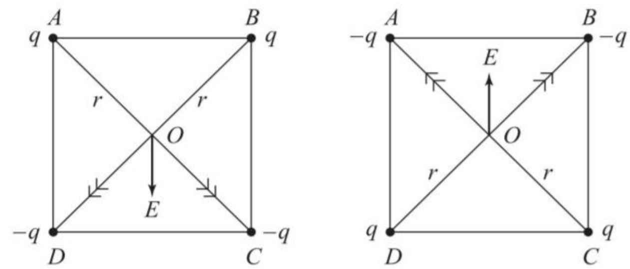
Work done by the battery is W = charge × potential difference

$= Q \times V = CV \times V = CV^2$

Energy stored in the capacitor is $U = \dfrac{1}{2}CV^2$

$$\therefore \frac{U}{W} = \frac{\frac{1}{2}CV^2}{CV^2} = \frac{1}{2}$$

27. Refer to the following figures.



(a) (b)

Case 1: The potential at centre O is [see Fig. (a)]

$$V = V_A + V_B + V_C + V_D$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} + \frac{q}{r} - \frac{q}{r} - \frac{q}{r}\right] = 0$$

The electric fields at O due to charges at A, B, C and D respectively are

$E_A = \dfrac{q}{4\pi\varepsilon_0 r^2}$ directed from O to C

$E_B = \dfrac{q}{4\pi\varepsilon_0 r^2}$ directed from O to D

$E_C = \dfrac{q}{4\pi\varepsilon_0 r^2}$ directed from O to C

$E_D = \dfrac{q}{4\pi\varepsilon_0 r^2}$ directed from O to D

It is clear that the resultant electric field E at point O is directed vertically downwards.

Case 2: It follows from Fig. (b) that the potential V at point O is still zero but the resultant electric field E is directed vertically upwards.

28. $$E = -\frac{dV}{dx}$$

$$= -\frac{d}{dx}\left[\frac{20}{(x^2-4)}\right]$$

$$= \frac{20\times(2x)}{(x^2-4)^2} \text{ volt}$$

At $x = 4$ μm, $\quad V = \dfrac{20\times(2\times4)}{\left[(4)^2-4\right]^2}$ volt

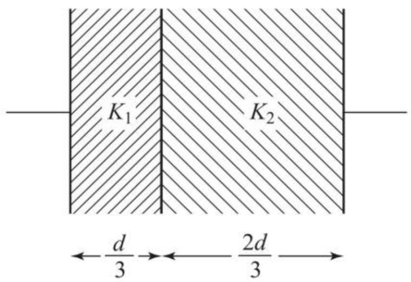
$$= \frac{10}{9} \text{ volt/μm}$$

Since V decreases as x increases, it follows from $E = -dV/dx$ that $\vec{E}$ is along the +ve x-direction.

29. In this process, the energy stored in the capacitor remains unchanged equal to $\dfrac{1}{2}CV^2$ where

$C = \dfrac{K\varepsilon_0 A}{d}$. Hence the net work done by the system is zero.

30. Given $C_0 = \dfrac{\varepsilon_0 A}{d} = 9$ pF.



$$C_1 = \frac{k_1\varepsilon_0 A}{d/3} = \frac{3\varepsilon_0 A}{d/3} = \frac{9\varepsilon_0 A}{d} = 9C_0$$

$$C_2 = \frac{k_2\varepsilon_0 A}{2d/3} = \frac{6\varepsilon_0 A}{2d/3} = \frac{9\varepsilon_0 A}{d} = 9C_0$$

Capacitors C_1 and C_2 are in series. The equivalent capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{9C_0 \times 9C_0}{18C_0} = \frac{9}{2}C_0$$

$$= \frac{9}{2} \times 9\text{pF} = 40.5\text{pF}$$

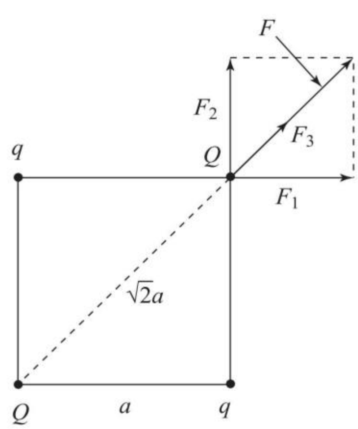
31. Inside a spherical shell (i.e., for $r < R$), the electric field is zero. Outside the shell, i.e., for $r > R$,

$$E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Hence the correct graph is (c).

32. Let a be the side of the square and $k = \dfrac{1}{4\pi\varepsilon_0}$.

Assuming that Q and q have the same sign, the forces acting on Q are as shown in the figure.



$$F_1 = \frac{kqQ}{a^2}$$

$$F_2 = \frac{kqQ}{a^2}$$

$$F_3 = \frac{kQ^2}{2a^2}$$

The resultant of F_1 and F_2 is

$$F = \sqrt{F_1^2 + F_2^2} = \frac{\sqrt{2}\,kqQ}{a^2}$$

The net force on Q will be zero if $F + F_3 = 0$, i.e.

$$\frac{\sqrt{2}\,kqQ}{a^2} + \frac{kQ^2}{2a^2} = 0$$

$\Rightarrow \dfrac{Q}{q} = -2\sqrt{2}$, whtich is choice (c). Note that changes Q and q must have opposite sign.

33. Electrostatic field is conservative. For a conservative field, the work done to take a charged particle from a point P to another point Q is independent of the path followed by the particle to go from P to Q. In addition, work done to take the particle from Q to P is equal and opposite in sign. Hence the net work done in moving the particle from P to Q and then back to P is zero. So the correct choice is (d).

34. $q = 100\,e = 100 \times (-1.6 \times 10^{-19}) = -1.6 \times 10^{-17}$ C

$W_{P\to Q} = q\,(V_Q - V_P) = -1.6 \times 10^{-17} \times (-4 - 10) = 2.24 \times 10^{-16}$ J

35. Using Gauss's theorem $\int \vec{E}\cdot\vec{dA} = \dfrac{q}{\varepsilon_0}$, where q is the charge enclosed inside the Gaussian spherical surface of radius r_1,

$$E \times 4\pi r_1^2 = \frac{q}{\varepsilon_0} \qquad (1)$$

$$q = \int_0^{r_1} \rho\, dV$$

$$= \int_0^{r_1} \frac{Qr}{\pi R^4} \times 4\pi r^2 dr \qquad (\because dV = 4\pi r^2 dr)$$

$$= \frac{4Q}{R^4}\int_0^{r_1} r^3 dr = \frac{Qr_1^4}{R^4} \qquad (2)$$

Using (2) in (1) we get

$$E = \frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$$, which is choice (a).

36. The charge decays according to the equation

$$Q = Q_0 e^{-t/\tau}$$

where $\tau = RC$ and Q_0 = charge at $t = 0$.

Energy stored is $U = \dfrac{Q^2}{2C} = \dfrac{Q_0^2}{2C}e^{-2t/\tau} = U_0 e^{-2t/\tau}$

where U_0 = initial energy at $t = 0$.

Given $U = \dfrac{U_0}{2}$ at $t = t_1$.

i.e. $\dfrac{U_0}{2} = U_0 e^{-2t_1/\tau} \Rightarrow e^{2t_1/\tau} = 2$

or $\dfrac{2t_1}{\tau} = \ln(2) \Rightarrow t_1 = \dfrac{\tau}{2}\ln(2)$

Given $Q = \dfrac{Q_0}{4}$ at $t = t_2$. Hence

$$\frac{Q_0}{4} = Q_0 e^{-t_2/\tau}$$

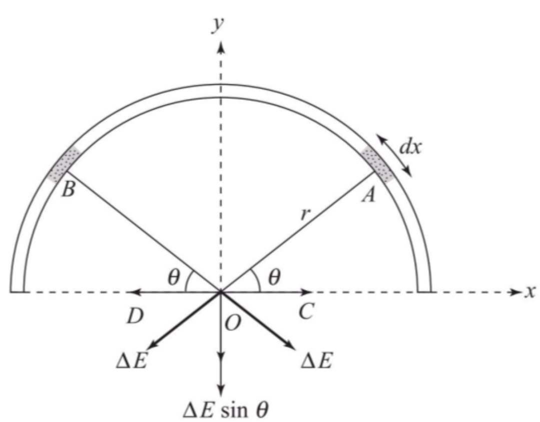
which gives $t_2 = \tau\ln(4) = 2\tau\ln(2)$

$\therefore$ $\dfrac{t_1}{t_2} = \dfrac{1}{4}$, which is choice (c).

37. Linear charge density is

$$\lambda = \frac{q}{\pi r}$$

To find the net electric field at O, we divide the ring into a large number of very small elements each of length dx. The charge of an element is $dq = \lambda dx = \lambda r d\theta$, where $d\theta$ is the angle subtended by the element at centre O. The electric field at O due to the element at A is



$\Delta E = \dfrac{dq}{4\pi\varepsilon_0 r^2}$ directed radially outwards

Similarly, the electric field at O due to an element B symmetrically opposite to A is also ΔE. It is clear that the horizontal components $OC = OD = \Delta E\cos\theta$ cancel but he vertical components $= \Delta E\sin\theta$ add up and are directed along the negative y-direction. Hence the net electric field at O is

$$\mathbf{E} = \int \Delta\mathbf{E}\sin\theta = \frac{\lambda r(-\hat{j})}{4\pi\varepsilon_0 r^2}\int_0^\pi \sin\theta\, d\theta \quad (\because dq = \lambda r d\theta)$$

$$= \frac{\lambda\hat{j}}{4\pi\varepsilon_0 r}\left|-\cos\theta\right|_0^\pi$$

$$= \frac{\lambda\hat{j}}{4\pi\varepsilon_0 r}(\cos\pi - \cos 0)$$

$$= \frac{\lambda\hat{j}}{2\pi\varepsilon_0 r}$$

$$= -\frac{q\hat{j}}{2\pi^2 r^2\varepsilon_0} \quad \left(\because \lambda = \frac{q}{\pi r}\right)$$

38. For $r < R$,

$$q = \int_0^r \rho\, dV = \int_0^r \rho_0\left(\frac{5}{4} - \frac{r}{R}\right) \times 4\pi r^2 dr$$

$$= 4\pi\rho_0 \int_0^r \left(\frac{5}{4}r^2 dr - \frac{r^3}{R}dr\right)$$

$$= 4\pi\rho_0\left(\frac{5r^3}{12} - \frac{r^4}{4R}\right)$$

For Gauss's law

$$\int \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E \times 4\pi r^2 = \frac{4\pi\rho_0}{\varepsilon_0}\left(\frac{5r^3}{12} - \frac{r^4}{4R}\right)$$

$$\Rightarrow E = \frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$$

39. At equilibrium,

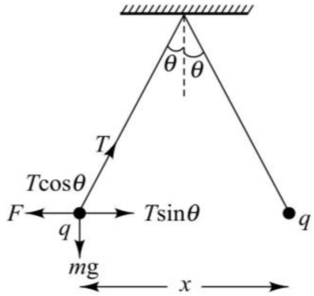
$$F = T\sin\theta$$

and $mg = T\cos\theta$

$$\frac{F}{mg} = \tan\theta \quad (1)$$

Where F is the force of repulsion between the sphere. If r is the distance between them,

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2}$$



When suspended in a liquid of density ρ (since q and hence r remains the same)

$$F_l = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\ ,\quad K = \text{dielectric constant of liquid}$$

$$\therefore\ \frac{F}{F_l} = K$$

Due to upthrust (since θ remains the same), the apparent weight of the sphere $= mg\left(1-\frac{\rho}{\sigma}\right)$ where $\sigma =$ density of the material of the sphere. Hence

$$\frac{F_l}{mg\left(1-\frac{\rho}{\sigma}\right)} = \tan\theta \qquad (2)$$

Equating (1) and (2) we get

$$\frac{F}{F_l} = \left(1-\frac{\rho}{\sigma}\right)$$

$$\Rightarrow\quad K = \frac{1}{\left(1-\frac{\rho}{\sigma}\right)} = \frac{1}{\left(1-\frac{0.8}{1.6}\right)} = 2$$

40. $$\phi = ar^2 + b$$

$$E = -\frac{d\phi}{dr} = -2ar$$

From Gusss's law

$$\Rightarrow\quad \oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow\quad E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow\quad (-2ar) \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow\quad q = -8\pi\varepsilon_0 a\, r^3$$

Charge density $$\rho = \frac{q}{\frac{4\pi}{3}r^3} = \frac{-8\pi\varepsilon_0 ar^3}{\frac{4\pi}{3}r^3}$$

$$= -6a\ \varepsilon_0$$

41. As shown in the solution of Q. 39 above,

$$F = mg\tan\theta = \frac{q^2}{4\pi\varepsilon_0\, x^2} = mg\tan\theta$$

$$\Rightarrow\quad \tan\theta = \frac{q^2}{4\pi\varepsilon_0 mg\, x^2}$$

If l is the length of the string, $\tan\theta = \frac{x/2}{l} = \frac{x}{2l}$

$$\therefore\quad \frac{x}{2l} = \frac{q^2}{4\pi\varepsilon_0 mgx^2}$$

$$\Rightarrow\quad q^2 = \left(\frac{2\pi\varepsilon_0 mg}{l}\right)x^3$$

$$\Rightarrow\quad q \propto x^{3/2}$$

$$\therefore\quad \frac{dq}{dt} \propto \frac{3}{2}x^{1/2}\frac{dx}{dt} = \frac{3}{2}x^{1/2}v \qquad \left(\because v = \frac{dx}{dt}\right)$$

Since $\frac{dq}{dt}$ = constant (given), $v \propto x^{-1/2}$

42. For a uniformaly charged sphere, the electric field inside the sphere is directly proportional to R and outside the sphere, it falls as $1/R^2$. Hence the correct graph is (c).

43. Potential energy at the centre of the sphere is

$$U_c = \frac{3}{2}\times\frac{Qq}{4\pi\varepsilon_0 R}$$

and on its surface, the P. E. is

$$U_s = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R}$$

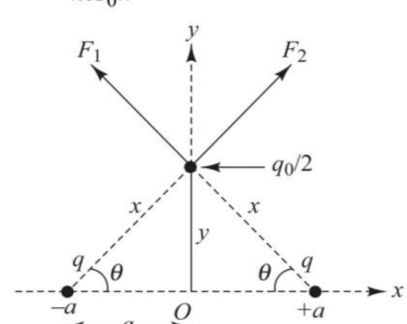
$$\therefore\quad \Delta U = U_c - U_s = \frac{Qq}{8\pi\varepsilon_0 R}$$

Now $$Q = \frac{4\pi}{3}R^3\rho$$

$$\therefore\quad \Delta U = \frac{q}{8\pi\varepsilon_0 R}\times\frac{4\pi}{3}R^3\rho = \frac{\rho R^2 q}{6\varepsilon_0}$$

so the correct choice is (c).

44. $$F_1 = F_2 = \frac{q\, q_0}{4\pi\varepsilon_0 x^2} = F$$



The x-components of F_1 and F_2 cancel and the y-components add up. The net force on $q_0/2$ is

$$F_{net} = 2F\cos\theta = 2F \times \frac{y}{x} = \frac{2q\,q_0}{4\pi\varepsilon_0} \times \frac{y}{x^3}$$

Now $\quad x = (a^2 + y^2)^{1/2}$

$$F_{net} = \frac{2q\,q_0\,y}{4\pi\varepsilon_0 \times (a^2 + y^2)^{3/2}}$$

Since $y << a$

$$F_{net} = \frac{2q\,q_0\,y}{4\pi\varepsilon_0 \times 2a^3}$$

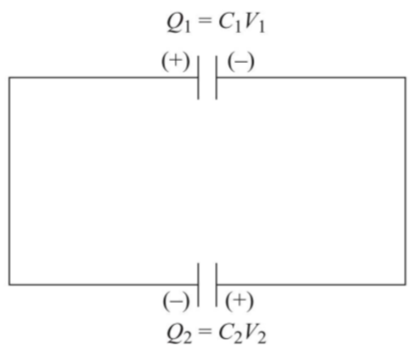
Hence $F_{net} \propto y$. So the correct choice is (d).

45. The potential on each capacitor will be zero if the capacitors are connected as shown in the figure, i.e.

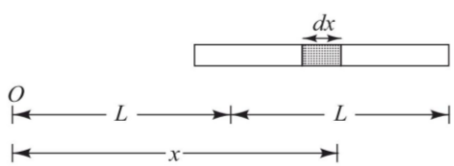
$$Q_1 = Q_2$$

$\Rightarrow \quad 120\,C_1 = 200\,C_2$

$\Rightarrow \quad 3C_1 = 5\,C_2$



46.



Consisder a small element of length dx at distance x from O. Charge of the element is

$$q = \frac{Qdx}{L}$$

The electric potential at O due to the charged rod is

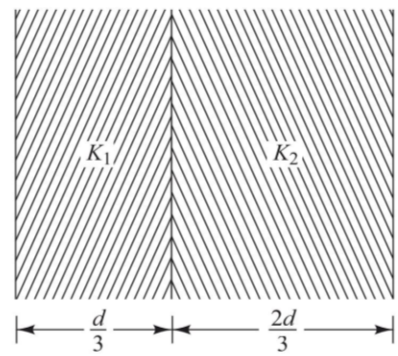
$$V = \int_{x=L}^{x=2L} \frac{1}{4\pi\varepsilon_0}\frac{q}{x} = \int_{x=L}^{x=2L} \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\frac{d}{x}$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\int_{x=L}^{x=2L}\frac{dx}{x}$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\left|\ln x\right|_L^{2L} = \frac{Q\ln(2)}{4\pi\varepsilon_0 L}$$

47. The correct choice is (a)

48. Given $\quad C_0 = \dfrac{\varepsilon_0 A}{d} = 9\text{ pF}$



$$C_1 = \frac{k_1\varepsilon_0 A}{d/3} = \frac{3\varepsilon_0 A}{d/3} = \frac{9\varepsilon_0 A}{d} = 9\,C_0$$

$$C_2 = \frac{k_2\varepsilon_0 A}{2d/3} = \frac{6\varepsilon_0 A}{2d/3} = \frac{9\varepsilon_0 A}{d} = 9\,C_0$$

Capacitors C_1 and C_2 are in series. The equivalent capacitance is

$$C = \frac{C_1C_2}{C_1 + C_2} = \frac{9C_0 \times 9C_0}{18C_0} = \frac{9}{2}C_0$$

$$= \frac{9}{2} \times 9\text{pF} = 40.5\text{ pF}$$