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FUNCTIONS

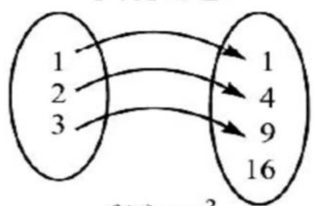
IIT-MATHS (MAINS & ADV)

01

Review of Key Notes and Formulae

Definition: If A and B are two non-empty sets, then the rule that, for each and every element of set A is uniquely associate with set B.

$$f : A \to B$$



$$f(x) = x^2$$

Domain: All elements of set A

$$D_f = \{1, 2, 3\}$$

Co-domain: All elements of Set B

$$Co\text{-}D_f = \{1, 4, 9, 16\}$$

Range: Elements of set B which are involved in mapping.

$$R_f = \{1, 4, 9\}$$

Different Types of Functions

1. **Polynomial function:** Function in the form of:

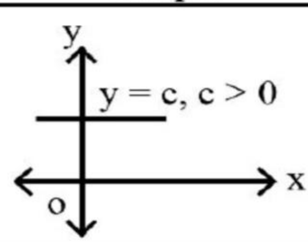
$$f(x) = a_0x^n + a_1x^{n-1} + ... + a_n\ ;\ a_0 \neq 0\ ;\ \text{Degree} = n$$

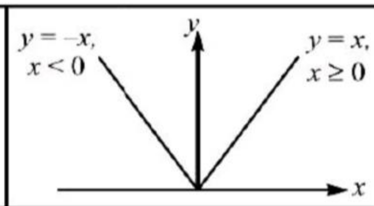
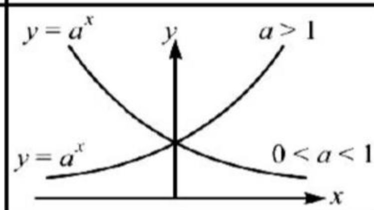
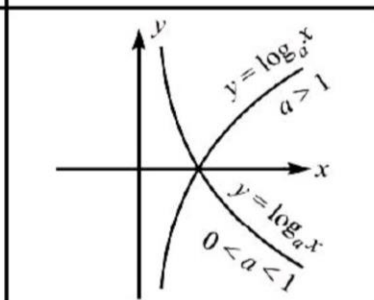
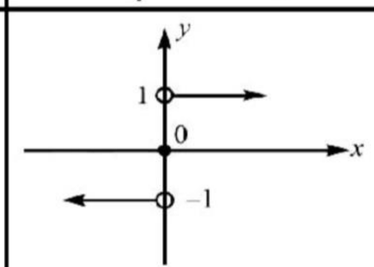
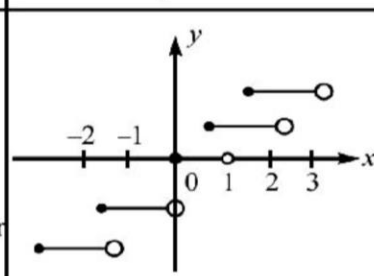
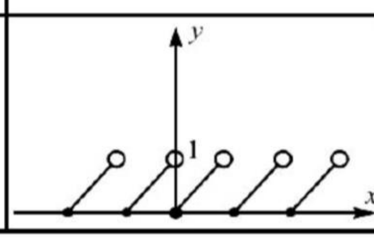
where, $n, n-1, n-2$ .... are non-negative integers. Domain of $f(x) = R$

2. **Rational function:** Functions in form of

$$f(x) = \frac{p(x)}{q(x)};\ q(x) \neq 0$$

where, $P(x)$ and $q(x)$ are polynomial in x. Domain of $f(x) = R - \{x : q(x) = 0\}$

	Function	Graph	Domain & Range
3.	**Constant function:** $y = f(x) = c \quad \forall x \in R$, where c is a constant		Dom : $x \in R$ Range : $y = \{c\}$

4.	**Modulus function:** $y = f(x) = \lvert x \rvert$		Dom: $x \in R$ Range: $y \in [0, \infty]$
5.	**Exponential function:** $y = f(x) = a^x$, where $a > 0, a \neq 1$		Dom: $x \in R$ Range: $y \in (0, \infty)$
6.	**Logarithmic function:** $y = f(x) = \log_a x$ where $a > 0, a \neq 1$		Dom: $x \in (0, \infty)$ Range: $y \in R$
7.	**Signum function:** $y = f(x) = \text{Sgn}(x)$ $\Rightarrow \quad f(x) = \begin{cases} \frac{\lvert x \rvert}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$		Dom: $x \in R$ Range: $y \in \{-1, 0, 1\}$
8.	**Greatest integer function:** $y = f(x) = [x]$ → x if x is integer → greatest integer less than x		Dom: $x \in R$ Range = $\{z\}$
9.	**Fractional part function:** $y = f(x) = \{x\}$ $\{x\} = x - [x]$		Dom: $x \in R$ Range: $y \in [0, 1)$
10.	**Trigonometric function:**		
	Functions	**Domain**	**Range**
	$y = \sin x$	$x \in R$	$y \in [-1, 1]$
	$y = \cos x$	$x \in R$	$y \in [-1, 1]$

$y = \tan x$	$x \in R - \left\{(2n+1)\frac{\pi}{2}\right\}$	$y \in R$
$y = \cot x$	$x \in R - \{n\pi\}$	$y \in R$
$y = \operatorname{cosec} x$	$x \in R - \{n\pi\}$	$y \in (-\infty, -1] \cup [1, \infty)$
$y = \sec x$	$x \in R - \{(2n+1)\frac{\pi}{2}\}$	$y \in (-\infty, -1] \cup [1, \infty)$

Equal or Identical Functions:

Two functions $f(x)$ and $g(x)$ are said to be identical if.

(i) Domain of $f(x)$ = Domain of $g(x)$

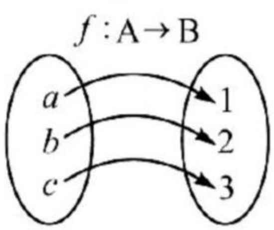
(ii) Co-domain of $f(x)$ = Co-domain of $g(x)$

(iii) $f(x) = g(x)$ for every x belonging to their domain.

Classification of Functions:

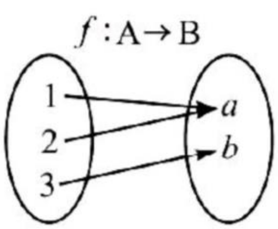
1. **One-one function:** The mapping $f : A \to B$

(Injective Function) is one-one function if different elements in A have different images in B.



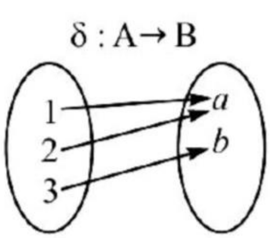
$x_1, x_2 \in A$
$x_1 \neq x_2 \;\Rightarrow\; f(x_1) \neq f(x_2)$

2. **Many-one function:** The mapping $f : A \to B$ is many-one two or more than two different elements in A have the same image in B.



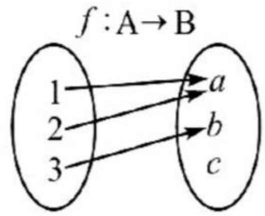
$x_1, x_2 \in A$
$x_1 \neq x_2 \;\Rightarrow\; f(x_1) - f(x_2)$

3. **Onto (surjective) function:** A function is said to be onto if



Range = Co-domain

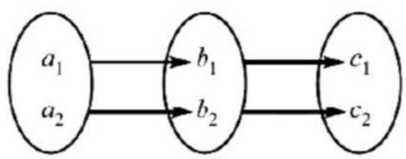
4. **Into function:** A function is said to be into if $\boxed{\text{Range} \neq \text{Co-domain}}$



★**Note** $\Rightarrow$ $\boxed{\text{Bijective function} \Leftrightarrow \text{One-one} + \text{Onto}}$

Composition of Function:

If $f : A \to B$ and $g : B \to C$ are two functions, then composite function of f and g are $go'f : A \to C$ will be defined as $g(f(x)) = gof(x)\ \forall\ x \in A$



Even and Odd Functions

(i) If $f(x) + f(-x) = 0\ \forall\ x \in$ Domain of $f(x)$

$\Rightarrow$ Odd function $\Rightarrow$ Symmetric about origin

(ii) If $f(x) = f(-x)\ \forall\ x \in$ Domain of $f(x)$

$\Rightarrow$ Even function $\Rightarrow$ Symmetric about y-axis

★**Note** $f(x) = 0$ is even as well as odd function.

Homogeneous Function:

Functions consists of variables both x & y such that $f(x, y)$ is homogeneous if:

$$f(tx, ty) = t^n f(x, y)$$

$$\downarrow$$

Homogeneous of degree 'n'

Periodic Function:

A function is periodic if its each value is repeated after a definite interval. So a function is periodic if there exists a positive real number 'T' such that

$$f(x + T) = f(x)\ \forall\ x \in D_f$$

★ Period $= nT$; $n \in I$

$\therefore$ $\boxed{f(x + nt) = f(x)}$; $n \in I$

★**Note:** Constant function has no fundamental period.

Inverse of a Function:

If $f : A \to B$ is a one-one and onto fn. both then we can define the inverse of the function as $g : B \to A$, such that $f(x)\ y \Rightarrow g(y) = x$

$\downarrow$

Inverse of $f(x)$.

Properties of Invertible Function:

(i) Inverse of bijective function is unique and bijective.

(ii) $(f^{-1})^{-1} = f$

(iii) $(gof)^{-1} = f^{-1}og^{-1}$

(iv) Inverse of a function is a mirror image about $y = x$ line.

TIPS AND TRICKS: (T-1)

There are only two polynomial functions exists, which satisfies the condition $f(x) + f\left(\frac{1}{x}\right) = f(x) \cdot f\left(\frac{1}{x}\right)$

are $\boxed{f(x) = 1 \pm x^n}$; $n \in I^+ : x \in R$

Illustration 1

Find the polynomial function which satisfies the condition $f(x) + \left(\frac{1}{x}\right) = f(x).\left(\frac{1}{x}\right)$ of degree 3 and is always increasing function.

Short-cut solution :

Using T-1 $\quad f(x) = 1 \pm x^3 \quad$ ($\because$ degree is 3)

Now, for function is always ↑sing

So, $\quad f(x) = 1 + x^3$ or $f(x) = 1 - x^3$

Differentiate, $\quad f'(x) = 3x^2 > 0$ & $f'(x) = -3x^2 < 0$

$\Rightarrow$ ↑sing $\quad \Rightarrow$ ↑sing *fn*

Hence, we conclude that the required function is

$f(x) = 1 + x^3$

TIPS AND TRICKS: (T-2)

If $y = f(x) = [x]$ is a greatest integer function then,

$$[x] + [-x] = \begin{cases} 0 \ ; & x \in I \\ -1 ; & x \notin I \end{cases}$$

Illustration 2

Find the value of $\int_a^{3\pi}[2\cos x]\,dx$, where [ ] is the greatest integer function.

Short-cut solution :

Using T-2 $\int_a^{3\pi}[2\cos x]\,dx$, where [ ] is greatest integer function.

Let $I = \int_a^{3\pi}[2\cos x]\,dx$ ... (1)

Apply, King Property : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$\Rightarrow \quad I = \int_0^{3\pi}[-2\cos x]\,dx$ ... (2)

Now, add (1) and (2)

$\Rightarrow \quad 2I = \int_0^{3\pi}[2\cos x]+[-2\cos x]\,dx$

Since 2 cos x is not always '0' or Integer

$\Rightarrow \quad 2I = \int_0^{3\pi}(-1)\,dx$

$\therefore I = \dfrac{-3\pi}{2}$

TIPS AND TRICKS: (T-3)

If $y = f(x) = \{x\}$ is a fractional part function.

then, $\{x\} + \{-x\} = \begin{cases} 0 \; ; & x \in I \\ 1 \; ; & x \notin I \end{cases}$

TIPS AND TRICKS: (T-4)

${}^nC_r$ and ${}^rC_n$, simultaneously possible only when $n = r$

Illustration 3

Let $f(x) = {}^{x+1}C_{2x-8}$, $g(x) = {}^{2x-8}C_{x+1}$ if $h(x) = f(x) \cdot g(x)$.

Then find domain and range of $h(x)$.

Short-cut solution :

Using T-4 $x + 1 = 2x - 8$

$\Rightarrow \quad x = 9$

Hence domain is $x \in \{9\}$ and $R_f = 1$.

TIPS AND TRICKS: (T-5)

Range of the function $f(x) = \dfrac{ax+b}{cx+d}; x \neq \dfrac{-d}{c}$ is $R - \left(\dfrac{d}{c}\right)$

Illustration 4

Find the range of the function $f(x) = \dfrac{2x+1}{5x-2}; x \neq \dfrac{2}{5}$

Short-cut solution :

Using T-5 $\quad y \in R - \left\{\dfrac{2}{5}\right\}$

TIPS AND TRICKS: (T-6)

To find Range of $f(x) = \cos(K \sin x)$

where $K \in R^+$ then,

If $K \in [0, \pi) \Rightarrow$ Range is $y \in [\cos K, 1]$

If $K \in [\pi, \infty) \Rightarrow$ Range $\rightarrow y \in [-1, 1]$

Illustration 5

Find range of the function $y = \cos(2 \sin x)$

Short-cut solution :

Using T-6 $\quad \because K = 2$ (case IInd) and $K < \pi$

$\Rightarrow$ Range $\rightarrow y \in [\cos 2, 1]$

Illustration 6

Find range of the function $y = \cos(3 \sin x)$

Short-cut solution :

Using T-6 $\quad \because K = 3 \Rightarrow K < \pi$ (case IInd)

$\Rightarrow$ Range $\rightarrow y \in [\cos 3, 1]$

Illustration 7

Find the range of the function $f(x) = \cos(4 \sin x)$

Short-cut solution :

Using T-6 $\quad \because K = 4 \Rightarrow K > \pi$ (case Ist)

Hence, $\quad$ Range $\rightarrow y \in [\cos -1, 1]$

TIPS AND TRICKS: (T-7)

Identification of function using graph:

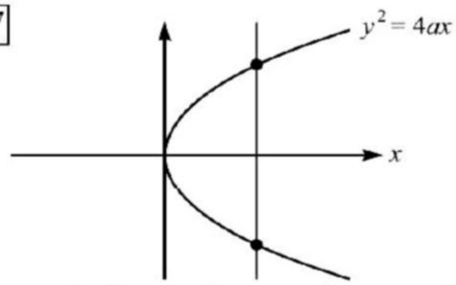
If it is possible to draw lines parallel to y-axis which cuts the curve more than one point then the given relation is not a function and when the line cuts the curve at only one point than it is a function.

Illustration 8

Check whether it is a function or not $y^2 = 4\,ax$

Short-cut solution :

Using T-7



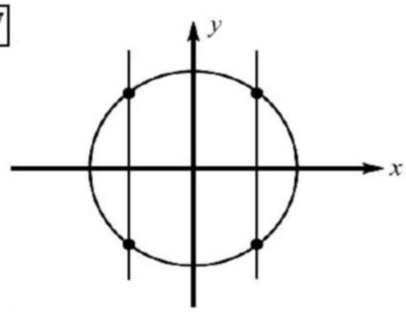
Vertical line cuts the graph more than one time then it is not a function.

Illustration 9

Check whether $x^2 + y^2 = a^2$ is a function or not

Short-cut solution :

Using T-7



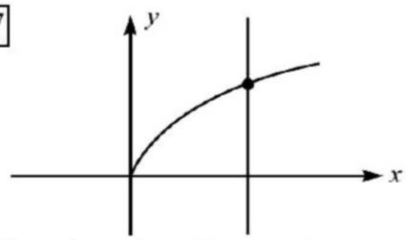
$\Rightarrow$ It is not a function.

Illustration 10

Check whether $y = \sqrt{x}$ is a function or not.

Short-cut solution :

Using T-7



$\Rightarrow$ It is a function since it cuts the curve once.

TIPS AND TRICKS: (T-8(i))

Graphical approach to check one-one or many-one function

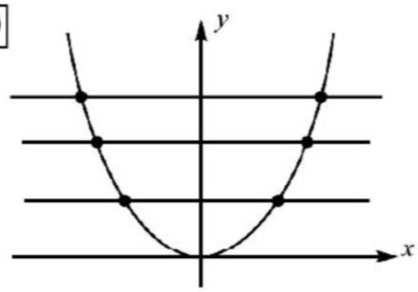
Construct the graph and draw lines parallel to x-axis, if it cuts the graph one time then it is a function and if it cuts more than one time then it is a many-one function.

Illustration 11

Check whether $y = x^2$ is a one-one or many-one function.

Short-cut solution :

Using T-8(i)



It is a many-one function.

TIPS AND TRICKS: (T-8(ii))

Calculus approach to check one-one or many-one function

Differentiate the function. $y = f(x)$ {$f(x)$ must be differentiable}

★ If $\frac{dy}{dx} > 0 \Rightarrow$ Monotonically Increases $\Rightarrow$ one-one function

★ If $\frac{dy}{dx} < 0 \Rightarrow$ Monotonically Decreases $\Rightarrow$ one-one function

★ If $\frac{dy}{dx} > 0 \Rightarrow$ for some 'x' and $\frac{dy}{dx} > 0$ for some x then function is many-one function.

Illustration 12

Check whether one-one or many-one function.

(i) $f(x) = x^3$ (ii) $f(x) = x^2$

Short-cut solution :

Using T-8(ii)

(i) $f(x) = x^3 \Rightarrow f'(x) = 3x^2 > 0 \Rightarrow$ one-one function

(ii) $f(x) = x^2 \Rightarrow f'(x) = 2x$ $\begin{cases} x > 0 \Rightarrow \uparrow \text{ses} \\ x < 0 \Rightarrow \downarrow \text{ses} \end{cases}$ Many- one function

★**Note:** (1) All trigonometric functions are many-one function.

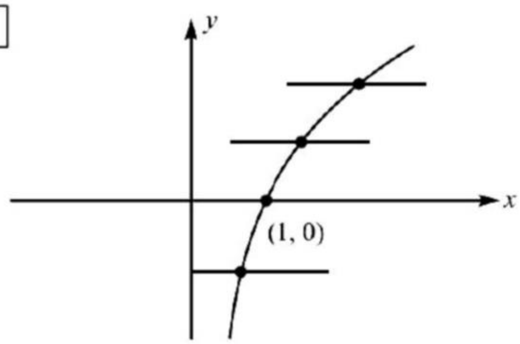
(2) All inverse trigonometric function are one-one function.

Illustration 13

Check for one-one or many-one function for $f(x) = \log_e x$

Short-cut solution :

Using T-8(ii)



It is one-one function.

Illustration 14

Check whether the following functions $f(x)$ are either one-one or many-one function.

(i) $f(x) = e^x$

(ii) $f(x) = \sin x$

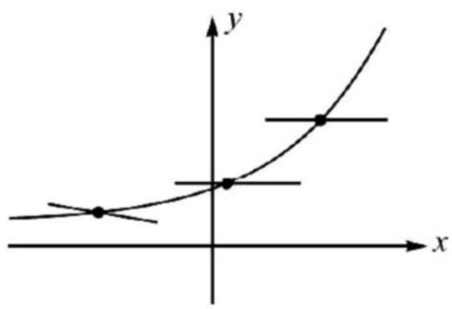
(iii) $f(x) = |x|$

(iv) $f(x) = \tan x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Short-cut solution :

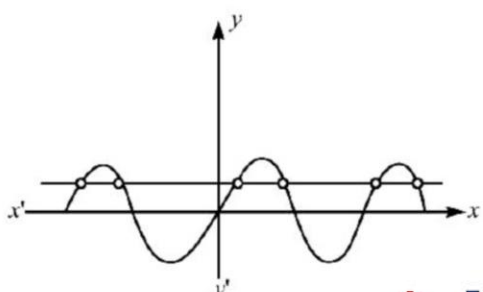
Using T-8(ii)

(i) $f(x) = e^x$



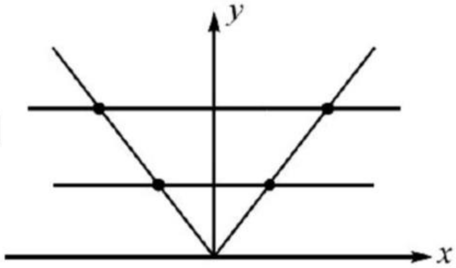
One-one function.

(ii) $f(x) = \sin x$



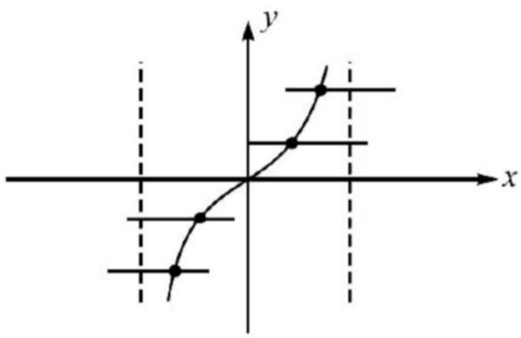
$\Rightarrow$ Many-one function.

(iii) $f(x) = |x|$



$\Rightarrow$ Many-one function.

(iv) $f(x) = \tan x$ in $x \in \left(-\frac{\pi}{2}, -\frac{\pi}{2}\right)$



$\Rightarrow$ One-one function.

Illustration 15

Check whether one-one or many-one function for

$f(x) = f : R \to R$

$f(x) = x^3 + x^2 + 7x + \sin x$

Short-cut solution :

Using T-8(ii)

Differentiate $\quad f'(x) = 3x^2 + 2x + 7 + \cos x$

$= (x^2 + 2x + 1) + (2x^2 + 6) + \cos x$

$$f'(x) = \underbrace{(x+1)^2}_{\geq 0} + \underbrace{(2x^2 + 6)}_{\geq 6} + \underbrace{\cos x}_{[-1,1]}$$

$\Rightarrow \quad f'(x) > 0 \Rightarrow$ one-one function

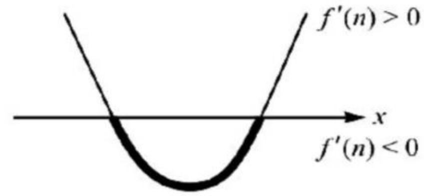
Illustration 16

Check whether one-one or many-one function for

$f(x) = x^3 + 6x^2 + 11x$

Short-cut solution :

Using T-8(ii)



Differentiate $f'(x) = 3x^2 + 12x + 11$

This is a parabola open upwards

Now, we find discriminant.

$$D = 144 - 132$$

$\Rightarrow$ $D > 0$ (two distinct roots)

So this implies $f'(x) > 0$ and $f'(x) < 0$ both.

$\Rightarrow$ Many-one function.

TIPS AND TRICKS: (T-9)

Short trick to check whether onto or into function for polynomial functions.

For $x \in R$

Put, $x \to \infty$ If $\Rightarrow$ $f(x) \to \infty$

Put, $x \to -\infty$ If $\Rightarrow$ $f(x) \to -\infty$

Hence, onto function.

Illustration 17

Check whether onto or into function.

(i) $f(x) = x^3$

(ii) $f(x) = x^2$

Short-cut solution :

Using T-9

(i) Since $f(x)$ is odd function.

$$\Rightarrow \left.\begin{array}{ll} x \to +\infty, & f(x) \to +\infty \\ x \to -\infty, & f(x) \to -\infty \end{array}\right\} \Rightarrow \text{One- one function and onto function.}$$

(ii) $f(x) = a_0 x^{2n} + a_1 x^{2n-2} + \ldots\ldots\ldots$

$$\text{Put, } \left.\begin{array}{ll} x \to +\infty, & f(x) \to \infty \\ x \to -\infty, & f(x) \to \infty \end{array}\right\} \Rightarrow \text{Many- one function and into function.}$$

Illustration 18

Check whether the given function is bijective or not

$f(x) = x^3 + 5x + 1$ **[AIEEE 2009]**

Short-cut solution :

Using T-8(ii)

On differentiating $\to$ $f(x) = 3x^2 + 5 > 0s$

$\Rightarrow$ one-one function.

Using T-9

$$\left.\begin{matrix} x \to +\infty, & f(x) \to \infty \\ x \to -\infty, & f(x) \to -\infty \end{matrix}\right\} \Rightarrow \text{Onto function}$$

Hence, above function is bijective.

TIPS AND TRICKS: (T-10)

If set A contains 'm' elements and another set B contains 'n' elements, then the

(i) Total number of functions $= (n)^m$
For $f : A \to B$

(ii) Number of one-one function is ^nP_m for $n \geq m$ and 0 (zero) for $n < m$

(iii) Number of many-one function = Total number of functions – One-one function

(iv) Number of onto functions are
- (a) If $n < m \Rightarrow n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - {}^nC_3(n-3)^m + \ldots$
- (b) If $n = m \Rightarrow n!$
- (c) If $n > m \Rightarrow 0$

(v) Number of into function are
- (a) If $n \leq m \Rightarrow$ Total number of functions – Onto functions
- (b) If $n > m \Rightarrow (n)^m$ [Total – Onto]

(vi) Number of constant functions $= n$.

(vii) If A and B are two sets having n-elements and '2' elements respectively.
Then number of onto functions from A to B
is $2^n - 2$ if $n \geq 2$ and '0' if $n > 2$

Illustration 19

If $A = \{1, 5, 9, 7, 14, 22\}$ and $B = \{2, 3, 5, 6\}$, then number of:
- (a) Total functions
- (b) One-one functions
- (c) Many-one functions
- (d) Onto function
- (e) Into functions
- (f) Constant functions

Short-cut solution :

Since, $m = 6$ and $n = 4$

(a) Using T-10(i) $\Rightarrow (n)^m = (4)^6 = 4096$

(b) Using T-10(ii) $\Rightarrow$ Since $n < m \Rightarrow$ One-one function = 0

(c) Using T-10(iii) $\Rightarrow$ Many-one function = Total – (One-one functions)
$= 4096 - 0 = 4096$

(d) Using T-10(iv) Since $n < m$

$= (n^m) - {}^nC_1\,(n-1)^m + {}^nC_2\,(n-2)^m + \ldots\ldots$

$= 4^6 - {}^4C_1 \cdot 3^6 + {}^4C_2 \cdot (2)^6 - {}^4C_3 \cdot (1)^6 + {}^4C_4 \cdot (0)^6$

$= 4096 - 4 \times 729 + 6 \times 64 - 4 + 0$

$= 1560$

(e) Using T-10(v) Total – Onto

$4096 - 1560 = 2536$

(f) Using T-10(vi) Constant function $= n = 4$

TIPS AND TRICKS: (T-11)

If A and B are finite sets and $f : A \to B$ is a bijection, then A and B have same number of the elements. If A has n elements, then number of the bijections from A to B is $\boxed{n!}$

Illustration 20

If $A = \{2, 3, 4, 5, 6\}$, then the total number of bijection function in $f : A \to A$ is

(a) 110 (b) 115 (c) 120 (d) 125

Short-cut solution :

Using T-11 Since $n = 5$

$\Rightarrow 5! = 120$ **Ans. (c)**

TIPS AND TRICKS: (T-12)

In order to find fundamental period of

(i) $\sin(2\,n\pi\,\{x\}) \longrightarrow \boxed{\frac{1}{n}}$; where $\{x\}$ is fractional part function and $n \in I^+$

(ii) $\sin(2n+1)\,\pi\,\{x\} \longrightarrow \boxed{1}$

(iii) If $f(x)$ is periodic function with fundamental period 'T' then $\frac{}{f(x)}$ and $\sqrt{f(x)}$ will also be periodic with fundamental period 'T'.

Illustration 21

Find the fundamental period of

(i) $\sin(2\pi(\{x\}))$ (ii) $\sin(4\pi\{x\})$

Short-cut solution :

Using T-12(i) $\frac{1}{n} = \frac{1}{1} \Rightarrow \boxed{T = 1}$

(ii) $\sin(4\pi\ \{x\})$

Using T-12(ii) $\dfrac{1}{n} = \dfrac{1}{2} \Rightarrow \boxed{T = \dfrac{1}{2}}$

Illustration 22

Find the fundamental period of $f(x) = \sec x$

Short-cut solution :

Using T-12(i) $f(x) = \dfrac{1}{\cos x}$

Since period of $\cos x$ is 2π

Hence period of $f(x) = \sec x$ is also 2π.

Illustration 23

Find the fundamental period of $f(x) = \dfrac{1}{\sqrt{\tan x}}$

Short-cut solution :

Using T-12(iii) $f(x) = \sqrt{\cot x}$

Since fundamental period of $\cot x$ = is 'π'

Then, fundamental period of $\sqrt{\cot x} = \dfrac{1}{\sqrt{\tan x}}$ is also 'π'

TIPS & TRICKS!

TIPS AND TRICKS: (T-13)

If $f(x)$ is periodic with fundamental period 'T' than $f(ax + b)$ is also periodic with fundamental period $\dfrac{T}{|a|}$

Illustration 24

Find the fundamental period of $f(x) = \sin(2x + 3)$

Short-cut solution :

Using T-13 Since fundamental period of $\sin x$ is 2π

$\Rightarrow \quad T = \dfrac{2\pi}{|2|} \quad \Rightarrow \quad T = \pi$

Illustration 25

Find the period of $f(x) = \{-3x + 5\}$ where $\{*\}$ is fractional part function.

Short-cut solution :

Using T-13 Since period of $y = \{x\}$ is 1

Hence, $T = \dfrac{1}{|-3|} \quad \Rightarrow \quad T = \dfrac{1}{3}$

TIPS AND TRICKS: (T-14)

Let $f(x)$ and $g(x)$ be the two functions which are periodic then period of $h(x) = f(x) + g(x)$ is $\boxed{\text{LCM of } T_1 \text{ and } T_2}$ as one its periods.

where, T_1, T_2 are period of $f(x)$ and $g(x)$ respectively.

★**Note:** LCM of irrational number $= \dfrac{\text{LCM of numerator}}{\text{HCF of denominator}}$

Illustration 26

Find period of $f(x) = \cos(\sin x) + \cos(\cos x)$

Short-cut solution :

$\boxed{\text{Using T-14}}$ Since, period of $\cos(\sin x)$ is π and priod of $\cos(\cos x)$ is π

Hence the priod of $f(x) = \text{LCM}(\pi, \pi) = \pi$.

Illustration 27

Find period of $f(x) = -\sin\dfrac{2x}{7} + \cos\dfrac{3x}{5}$.

Short-cut solution :

$\boxed{\text{Using T-13}}$ Since period of $-\dfrac{\sin 2x}{7}$ is 7π

and period of $\dfrac{\cos 2x}{5}$ is $\dfrac{10\pi}{3}$

$\boxed{\text{Using T-14}}$

Hence, the period of $f(x) = \text{LCM}\left(7\pi, \dfrac{10\pi}{3}\right)$

$$\Rightarrow \frac{\text{LCM of } N^r}{\text{HCF of } D^r} = \frac{\text{LCM}(7\pi, 10\pi)}{\text{HCF}(1, 3)} = \frac{70\pi}{1}$$

TIPS AND TRICKS: (T-15)

Let $y = f(x)$ is a function and it satisfies the relation $f(n+a) + f(n+b) =$ constant then period of this junction is $2|b-a|$.

Illustration 28

If $f(x) + f(x+5) = 12$, the period of $f(x)$ is:

Short-cut solution :

Using T-15 $\quad 2\,|5-0| = 10.$

Illustration 29

If $f(x+2)+f(x+9)=30$, then period of $f(x)$ is:

Short-cut solution :

Using T-15 $\quad 2\,|9-2| = 14.$

TIPS AND TRICKS: (T-16)

$$[x]+\left[x+\frac{1}{n}\right]+\left[x+\frac{2}{n}\right]+\ldots+\left[x+\frac{n-1}{n}\right]=[nx]$$

where, $n \in N$ and [*] is greatest integer function.

Illustration 30

Find the value of

$$\left[\frac{1}{5}\right]+\left[\frac{1}{5}+\frac{1}{100}\right]+\left[\frac{1}{5}+\frac{2}{100}\right]+\ldots\ldots+\left[\frac{1}{5}+\frac{99}{100}\right]$$

Short-cut solution :

Using T-16 $\quad [nx]=\left[100\times\frac{1}{5}\right]=20$

SHORTCUTS: (SC-1)

To find number of solution of $f(x) = g(x)$ where $f(x)$ and $g(x)$ are functions. Then draw graph for both LHS and RHS on same Cartesian plane and find number of point of intersection.

Number of point of intersection = Number of solutions/ Roots

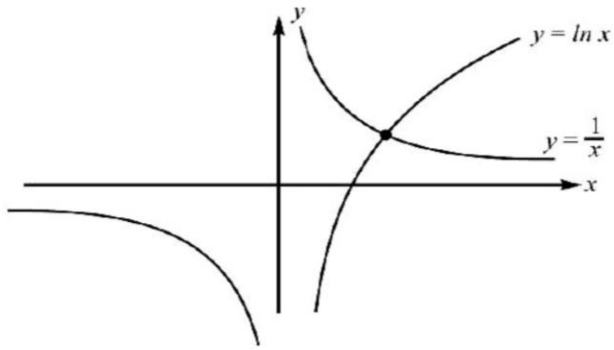
Illustration 31

Find number of solution of $x \cdot \ln x = 1$

Short-cut solution :

Using SC-1 $\quad \because \quad \ln x = \dfrac{1}{x}\,;\, x>0$ and $x \neq 0$

Now, we draw graph for $y = \ln x$ and $y = \dfrac{1}{x}\,;\, x > 0$



Number of intersection = 1 = Number of solutions

Illustration 32

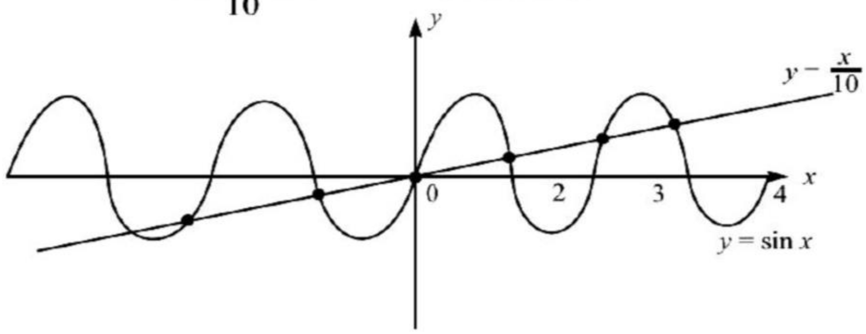
Find number of solutions of $\sin x = \frac{x}{10}$.

Short-cut solution :

Using SC-1 Now, we will draw graphs for $y = \sin x$ and $y = \frac{x}{10}$

$\because \quad \sin x \in [-1, 1]$

$\Rightarrow \quad -1 \le \frac{x}{10} \le 1 \quad \Rightarrow \quad -10 \le x \le 10$



We have to sketch the curve when $x \in [-10, 10]$

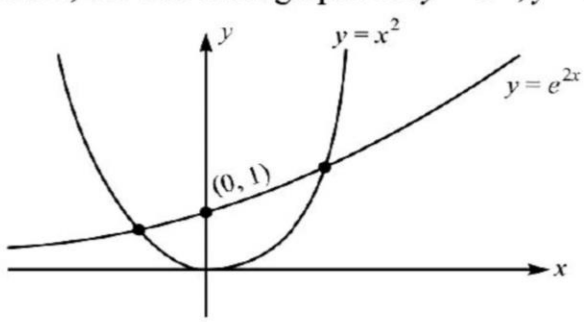
Hence, number of intersection = 7 = Number of solutions.

Illustration 33

Find the number of solutions of $e^{2x} = x^2$.

Short-cut solution :

Using SC-1 Now, we will draw graphs for $y = e^{2x}$, $y = x^2$



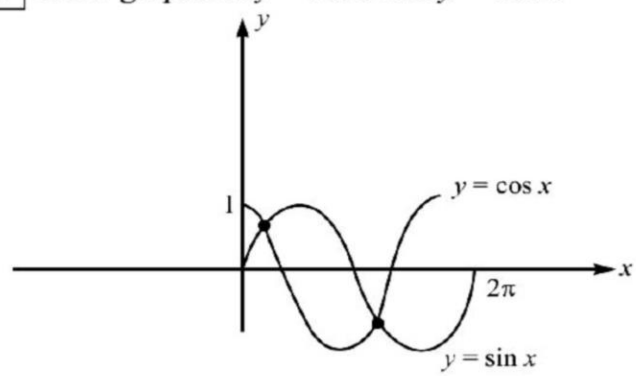
Number of intersection = Number of solutions = 2.

Illustration 34

Find the number of solutions of $\sin x = \cos x$ in $x \in [0, 2\pi]$.

Short-cut solution :

Using SC-1 Draw graphs of $y = \sin x$ and $y = \cos x$



Number of intersection in $x \in [0, 2\pi] = 2 =$ Number of solutions

SHORTCUTS: (SC-2)

In order to find domain and range of function, the shortest way is to draw graph. Graphs are also useful for one-one, many-one, onto and into.

Domain = Existence of graph along x - axis

Range = Existence of graph along y - axis

Illustration 35

Find the domain and range also check function is one-one or not in its domain. $f(x) = \dfrac{x^2 - 5x + 4}{x^2 + 2x - 3}$

Short-cut solution :

Using SC-2 $f(x) = \dfrac{x^2 - 5x + 4}{x^2 + 2x - 3} = \dfrac{(x-4)(x-1)}{(x+3)(x-1)}; \quad x \neq 1, -3$

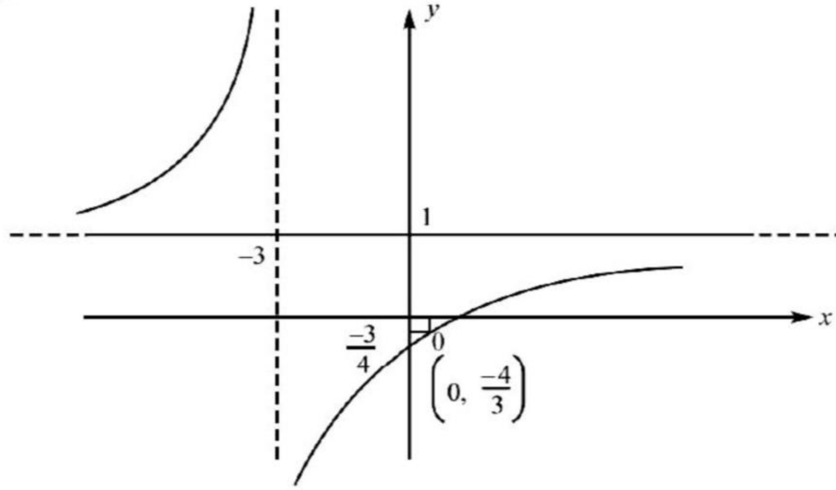
First of all we will check monotonicity.

Differentiating $f(x)$ w.r.t. x, we get

$$\Rightarrow \quad f'(x) = \frac{d}{dx}\left(\frac{x-4}{x+3}\right) = \frac{(x+3)-(x-4)}{(x+3)^2} = \frac{7}{(x+3)^2}$$

$$\Rightarrow \quad f'(x) > 0 \Rightarrow \text{Increasing function}$$

Graph



$\Rightarrow$ Domain : $x \in R - \{-3, 1\}$

Range : $x \in R - \left\{1, \frac{-3}{4}\right\}$

As shown function is **one-one**.

Illustration 36

$f : [2, \infty] \to Y$

$f(x) = x^2 - 4x + 5$ is both one-one and onto if

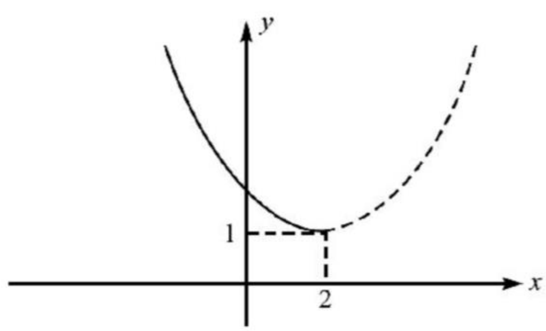
(a) $Y = R$ (b) $Y = [1, \infty)$

(c) $Y = [4, \infty)$ (d) $Y = [5, \infty)$

Short-cut solution :

Using SC-2 Rewrite, $f(x) = (x - 2)^2 + 1 \Rightarrow$ Parabola with vertex (2, 1)

Graph



$y_{min} = 1$; $y_{max} = \infty$

Hence, $Y = [1, \infty)$.

Illustration 37

$f : R \to R$

$f(x) = \begin{cases} x^2 + 2mx - 1 ; x \le 0 \\ mx - 1 ; x > 0 \end{cases}$ then find the value of m

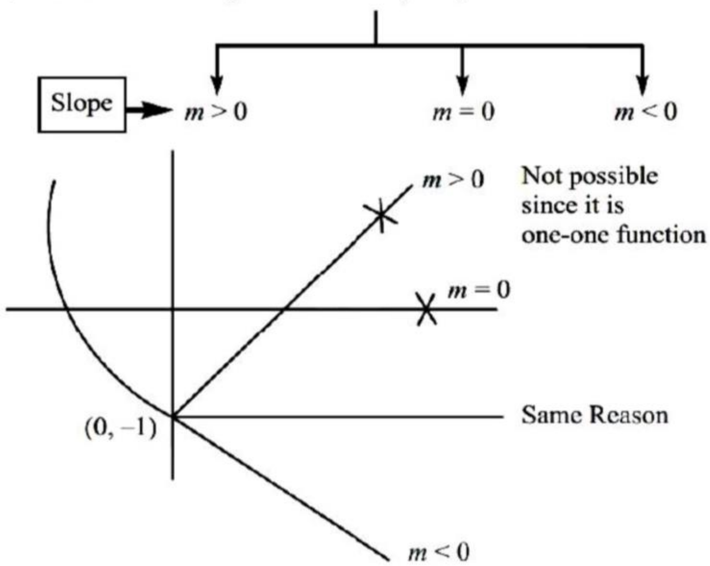
Short-cut solution :

Using SC-2

Graph

For $x < 0$, it is parabola open upwards

For $x > 0$, there are three possibilities (line)



Hence, $m < 0 \Rightarrow m \in (-\infty, 0)$.

SHORTCUTS: (SC-3)

Inverse of many-one function does not exist, only one-one onto function are invertible.

$$f'(x) \ge 0 \quad \text{or} \quad f(x) \le 0$$

Illustration 38

If $f : R \to R$, $f(x) = x^3 + (a + 2)x^2 + 3ax + 5$ is invertible mapping. Find 'a'.

Short-cut solution :

Using SC-3 Invertible $\Rightarrow$ One-one + Onto

Hence $f'(x) \ge 0$ or $f'(x) \le 0$

$\Rightarrow$ $f'(x) = 3x^2 + 2x(a + 2) + 3a \ge 0$ or $\le 0 \; \forall \; n \in R$

$\Rightarrow \quad \boxed{D \le 0}$

Hence, $4(a+2) - 4.9a \le 0$

$$a^2 - 5a + 4 \le 0$$

$$(a-1)(a-4) \le 0$$

$\Rightarrow \quad a \in [1, 4]$

SHORTCUTS: (SC-4)

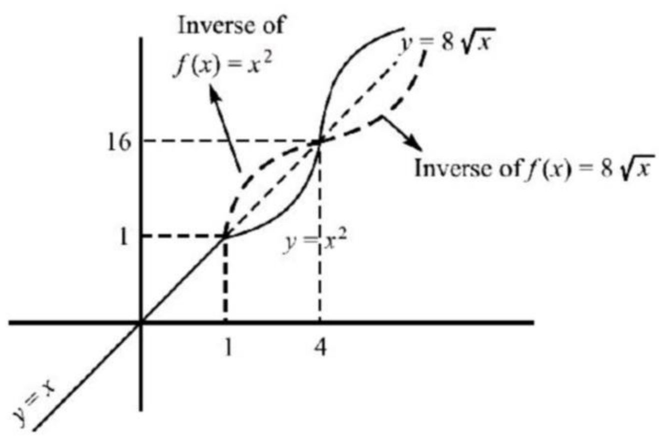
Inverse of a function is a mirror image about $y = x$ line. So copy the graph along other side of $y = x$ line.

Illustration 39

If $f(x) = \begin{cases} x & ; \quad x < 1 \\ x^2 & ; \quad 1 \le x \le 4 \\ 8\sqrt{x} & ; \quad x > 4 \end{cases}$. Then find $f^{-1}(x)$

Short-cut solution :

$\boxed{\text{Using SC-4}}$ **Draw Graph**



Hence, $f^{-1}(x) = \begin{cases} x & ; \quad x < 1 \\ \sqrt{x} & ; \quad 1 \le x \le 16 \\ \dfrac{x^2}{64} & ; \quad x > 16 \end{cases}$

SHORTCUTS: (SC-5)

Even functions are symmetric about y-axis odd functions are symmetric about origin.

Illustration 40

If $f(x) = \begin{cases} x^2 & ; \quad x \in (0,1] \\ 2-x; & x>1 \end{cases}$. Define $f(x)$ for $x<0$ if $f(x)$ is

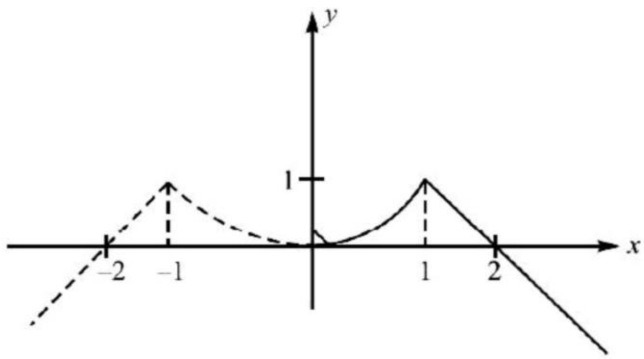
(i) Even function (ii) Odd function

Short-cut solution :

Using SC-5

(i) **Even Function**

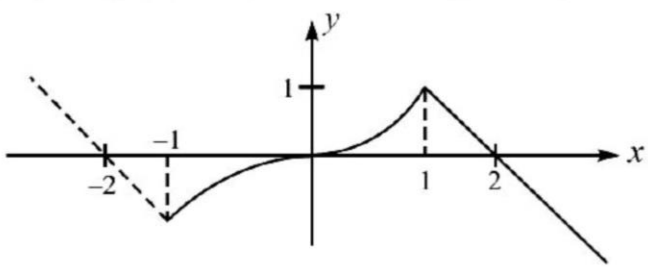
Draw graph of $f(x)$ using short cut $\Rightarrow$ Symmetry about y-axis



Hence, $f(x) = \begin{cases} x+2 & ; \quad x \in (-\infty, -1) \\ x^2 & ; \quad x \in (-1, 0) \end{cases}$

(ii) **Odd Function**

Draw graph of $f(x)$ using shortcut $\Rightarrow$ Symmetry about origin



Hence, $f(x) = \begin{cases} -x-2 & ; \quad x \in (-\infty, -1) \\ -x^2 & ; \quad x \in (-1, 0) \end{cases}$

TECHNIQUE

If f^{-1} be the inverse of bijective function $f(x)$ then $f(f^{-1}(x)) = x$.
Apply the formula of f on $f^{-1}(x)$ and use of the identity $f(f^{-1}(x)) = x$ to solve for $f^{-1}(x)$

Illustration 41

Find the inverse of the function $f(x) = \log_a\left(x + \sqrt{x^2+1}\right); a > 1$

Short-cut solution :

Using Tech.

$f\left(f^{-1}(x)\right)=x$

$\Rightarrow \log_a\left(f^{-1}(x)+\sqrt{\left(f^{-1}(x)\right)^2+1}\right)=x$

$\Rightarrow f^{-1}(x)+\sqrt{\left(f^{-1}(x)\right)^2+1}=a^x$ ...(i)

and $-f^{-1}(x)+\sqrt{\left(f^{-1}(x)\right)^2+1}=a^{-x}$ ..(ii)

From (i) and (ii), $f^{-1}(x)=\left(\frac{a^x-a^{-x}}{2}\right)$

Concept Booster Exercise

1. Find a polynomial of degree '5' which satisfies the relation $f(x)+f\left(\frac{1}{x}\right)=f(x)\cdot f\left(\frac{1}{x}\right)$ which is always decreasing function.

2. Find a polynomial which satisfies $f(x)+f\left(\frac{1}{x}\right)=f(x)\cdot f\left(\frac{1}{x}\right)\ \forall x\in R-\{0\}$ and the condition $f(3)=-26$, then determine $f'(1)$.

3. Find: $I=\int_0^{3\pi}\{2\cos x\}\,dx$; where {} is fractional part function.

4. The range of the function $f(x)={}^{7-x}P_{x-3}$ is **[AIEEE 2004]**

(a) {1, 2, 3} (b) {1, 2, 3, 4, 5, 6} (c) {1, 2, 3, 4} (d) {1,2,3,4,5}

5. If $f: R\to S$, defined by $f(x)=\sin x-\sqrt{3}\cos x+1$, is onto, then the interval of 'S' is: **[AIEEE 2004]**

(a) [0, 3] (b) [–1, 1] (c) [0, 1] (d) [–1, 3]

6. The range of the function $f(x)=\frac{2+x}{2-x}$, $x\neq 2$ is **[AIEEE 2002]**

(a) R (b) $R-\{-1\}$ (c) $R-\{1\}$ (d) $R-\{2\}$

7. The range of $f(x)=\frac{3x-1}{2x+1}$; $x\neq\frac{-1}{2}$

(a) $R-\left\{\frac{2}{3}\right\}$ (b) $R-\left\{\frac{3}{2}\right\}$ (c) R (d) $R-\left\{\frac{2}{5}\right\}$

8. The range of the function $f(x)=\cos\left(\frac{1}{2}\sin x\right)$

(a) [cos 2, 1] (b) $\left[\cos\frac{1}{2},1\right]$

(c) [–1, 1] (d) None of these

9. The range of the function $f(x)=\cos(5\sin x)$

(a) [cos 5, 1] (b) [–1, 1] (c) $\left[\cos\frac{1}{5},1\right]$ (d) [–1, 1)

10. Which of the following is/are the functions **[AIEEE 2002]**

(a) $y^2=4x$ (b) $x^2=8y$ (c) $x^2+y^2=4$ (d) $\left[\cos\frac{1}{5},1\right]$

11. The function $f: R\to R$ defined by $f(x)=\sin x$ is:

(a) Into (b) Onto (c) One-one (d) Many-one

12. The period of the function $f(x)$ = if $f(x+10) + f(x+20) = 50$

(a) 30 (b) 40 (c) 60 (d) 20

13. The period of $f(x) = \cos\frac{\pi}{4}x + \sin\frac{\pi x}{3}$.

(a) 24 (b) 12 (c) 36 (d) 6

14. The function $f: R \to R$ defined by $f(x) = x^2 - 3x + 2$

(a) Onto (b) Into (c) Many-one (d) One-one

15. Given $X = \{1, 2, 3, 4\}$, find all one-one, onto mappings, $f: X \to X$ such that, ...

16. Let $E = \{1, 2, 3, 4\}$ and $F = \{1, 2\}$. The number of onto functions from E to F is **[AIEEE 2002]**

(a) 14 (b) 16 (c) 12 (d) 8

17. Suppose $f(x) = (x+1)^2$ for $x \geq -1$, If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line $y = x$, then $g(x)$ equals **[AIEEE 2002]**

(a) $-\sqrt{x} - 1, x \geq 0$ (b) $\frac{1}{(x+1)^2}, x \geq -1$

(c) $\sqrt{x+1}, n \geq -1$ (d) $\sqrt{x} - 1, x \geq 0$

18. Let $f: R \to R$ be defined by $f(x) = 2x + \sin x; x \in R$. Then f is **[AIEEE 2002]**

(a) one to one and onto (b) one to one but not onto

(c) onto but not one-one (d) neither one-to one nor onto

19. The function $f: [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$ is **[AIEEE 2012]**

(a) one-one and onto (b) onto but not one-one

(c) one-one but not onto (d) neither one-one nor onto

20. The inverse function of $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$, $x \in (-1, 1)$ is **[JEE M 2020]**

(a) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$ (b) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

(c) $\frac{1}{4}\left(\log_8^e\right)\log_e\left(\frac{1+x}{1-x}\right)$ (d) $\frac{1}{4}\left(\log_8^e\right)\log_e\left(\frac{1-x}{1+x}\right)$

21. Let $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to R$ given by $f(x) = [\log(\sec x + \tan x)]^3$, then **[JEE M 2014]**

(a) $f(x)$ is odd function
(b) $f(x)$ is an one-one function
(c) $f(x)$ is onto function
(d) $f(x)$ is even function

22. Let $f(x) = ax^7 + bx^3 + cx - 5$; $a, b, c \in$ constant then find $f(+7)$, if $f(-7) = 7$.

(a) -16
(b) -15
(c) -17
(d) -20

23. Find the period of the function $f(x) = 2 + 3\cos(3x - 2)$

(a) 2π
(b) π
(c) —
(d) $\frac{\pi}{3}$

24. Find the domain of the function $f(x) = \dfrac{\sqrt{\cos - \frac{1}{\sqrt{2}}}}{\sqrt{\frac{3}{2}x - x^2 - \frac{1}{2}}}$

25. Let $f(x) = \dfrac{x}{1+x}$ defined from $(0, \infty) \to [0, \infty)$ then $f(x)$ is **[AIEEE 2003]**

(a) one-one but not onto
(b) one-one and onto
(c) many-one but not onto
(d) many-one and onto

Solutions

1. **($1-x^5$)** Using T-1 $f(x) = 1 \pm x^5$

Since function is always decreasing is

So, $f'(x) < 0$

$\Rightarrow f(x) = 1 + x^5$ or $f(x) = 1 - x^5$

$f'(x) = 5x^4 > 0$ or $f'(x) = -5x^4 < 0$

$\Rightarrow$ increasing function $\Rightarrow$ decreasing function

Hence, $f(x) = 1 - x^5$ is our required answer.

2. **(–3)** Using T-1 $f(x) = 1 \pm x^n$

$\because$ $f(3) = -26$

$f(3) = 1 + 3^n$	$f(3) = 1 - 3^n$
$-26 = 1 + 3^n$	$-26 = 1 - 3^n$
$-27 = 3^n$ $\Rightarrow$	$3^n = 27$
Not possible $\Rightarrow$	$n = 3$

Hence, the required function is

$f(x) = 1 - x^3 \Rightarrow f'(x) = -3x^2$

$\Rightarrow f'(1) = -3$

3. $\left(\dfrac{3\pi}{2}\right)$ Using T-3

$$I = \int_0^{3\pi} \{-2\cos x\}\, dx \quad \text{... (1)}$$

Apply king property

$$\Rightarrow I = \int_0^{3\pi} \{2\cos x\}\, dx \quad \text{... (2)}$$

Add eqns. (1) and (2)

$$\Rightarrow 2I = \int_0^{3\pi} (\{2\cos x\} + \{-2\cos x\})\, dx$$

$$\Rightarrow 2I = \int_0^{3\pi} (1)\, dx \Rightarrow I = \frac{3\pi}{2}$$

4. **(a)** We know that $x - 3 \geq 0 \Rightarrow x \geq 3$

And $7 - x \geq x - 3 \Rightarrow 2x \leq 10 \Rightarrow x \leq 5$

Hence, $x = 3, 4, 5$

Now At $x = 3 \Rightarrow y = {}^4P_0 = 1$

At $x = 4 \Rightarrow y = {}^3P_1 = 3$

At $x = 5 \Rightarrow y = {}^2P_2 = 2$

Hence, range = $\{1, 2, 3\}$.

5. **(d)** As we know that $-\sqrt{a^2+b^2} \le a \sin x + b \cos x \le \sqrt{a^2+b^2}$

Hence, $\quad -2 \le \sin x - \sqrt{3}\cos x \le 2$

$\quad -1 \le \sin x - \sqrt{3}\cos x + 1 \le 3$

Onto $\Rightarrow$ Range = Codomain $\Rightarrow$ $S \in [-1, 3]$.

6. **(b)** Using T-5 Range $\rightarrow y \in R - \left\{\frac{1}{-1}\right\}$

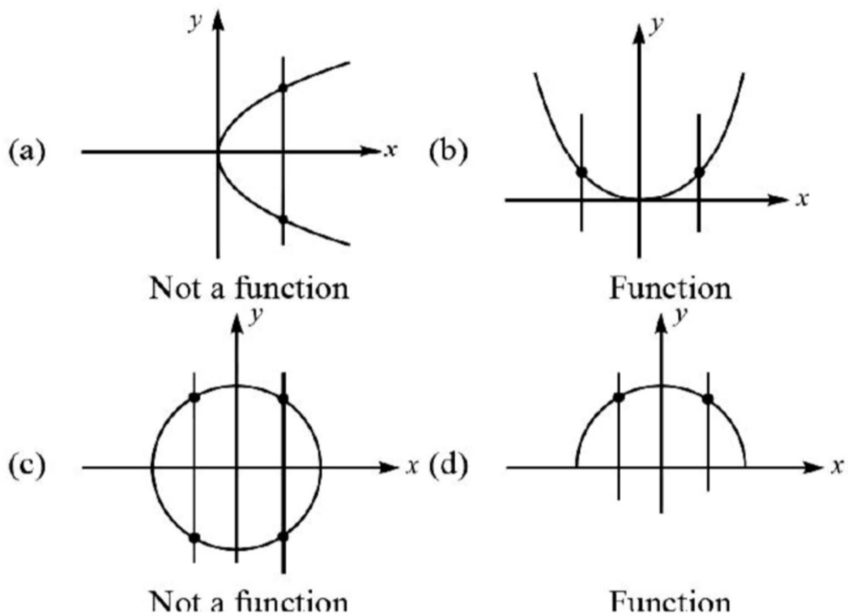
$\Rightarrow \quad y \in R - \{-1\}$

7. **(b)** Using T-5 Range $\rightarrow y \in R - \left\{\frac{3}{2}\right\}$

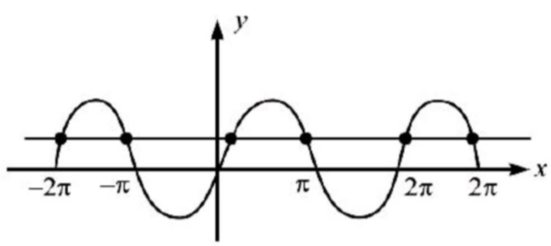
8. **(b)** Using T-6 $\because \frac{1}{2} < \pi \Rightarrow$ Range is $y \in \left[\cos\frac{1}{2}, 1\right]$

9. **(b)** Using T-6 $\because 5 > \pi \Rightarrow$ Range is $y \in [-1, 1]$

10. **(b, d)** Using T-7



11. **(a, d)** Using T-8



$\Rightarrow$ Many-one

Range of $\sin x = y$ is $y \in [-1, 1]$
But given co-domain is $y \in R$ $\Bigg\}$ $\Rightarrow$ Range $\neq$ Codomain

Hence, into function.

12. **(d)** Using T-15 $\quad a = 10, \quad b = 20$

$\Rightarrow \quad$ Period $= 2\,|20 - 10| = 20.$

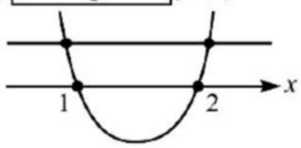
13. **(a)** $f(x) = \underbrace{\sin\frac{\pi x}{4}}_{T_1} + \underbrace{\sin\frac{\pi x}{3}}_{T_2}$, Since period of $\sin x = 2\pi$

Using T-13 $\quad T_1 = \dfrac{2\pi}{\frac{\pi}{4}}$ and $T_2 = \dfrac{2\pi}{\frac{\pi}{3}}$

Using T-14 $\quad$ Period = LCM (T_1, T_2)

$= \text{LCM}(8, 6) = 24.$

14. **(b, c)** Using T-8 $\; f(x) = x^2 - 3x + 2 \;\Rightarrow\;$ Parabola open upwards



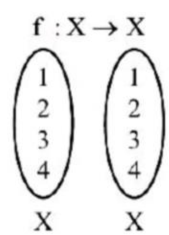
$\Rightarrow$ Many-one function

Using T-9 $\quad \left.\begin{array}{l} x \to \infty \;\Rightarrow\; f(x) \to \infty \\ x \to -\infty \;\Rightarrow\; f(x) \to \infty \end{array}\right\} \Rightarrow$ Into function

15. **(24)** Using T-10(ii) $\; m = 4$ and $n = 4$

$^{n}P_{m} = {}^{4}P_{4} = \dfrac{4!}{0!}$

$n = m$



$\Rightarrow$ No. of one-one functions = 24

Using T-10(iv) Since, $n = m = 4$

$\Rightarrow$ No. of onto functions $= n! = 4! = 24.$

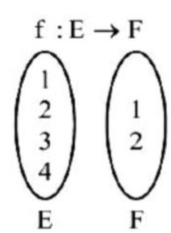
16. **(a)** Using T-10(iv) $\; m = 4$ and $n = 2$

Since, $n < m$

$\Rightarrow n^m - {}^{n}C_1 (n-1)^m + {}^{n}C_2 (n-2)^m - {}^{n}C_3 (n-3)^m + \ldots$

$= \; 2^4 - {}^{2}C_1 (1)^4 + {}^{2}C_2 \times 0$

$= 16 - 2 = 14.$



17. **(d)** Using SC-3

Since, $g(x)$ is reflection about $y = x$ line of $f(x)$

$\Rightarrow g(x)$ is inverse of $f(x)$

Hence, $\quad y = (x+1)^2 \;\Rightarrow\; x + 1 = \sqrt{y} \;\Rightarrow\; x = -1 + \sqrt{y}$

$\Rightarrow f^{-1}(x) = -1 + \sqrt{x}\;;\; x \geq 0.$

18. **(a)** Using T-8(ii) Differentiate $f(x)$

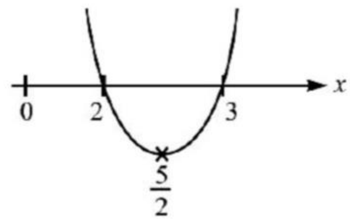
$$\Rightarrow \quad f'(x) = 2 + \underbrace{\cos x}_{[-1,1]} \Rightarrow f'(x) > 0 \Rightarrow \text{one-one function}$$

Using T-9 $f(x) = 2x + \underbrace{\sin x}_{[-1,1]}$

$$\left.\begin{array}{l} x \to \infty \quad \Rightarrow \quad f(x) \to \infty \\ x \to -\infty \quad \Rightarrow \quad f(x) \to -\infty \end{array}\right\} \Rightarrow \text{Onto function.}$$

19. **(d)** Using T-8(ii) Differentiate $f(x)$

$$f'(x) = 6x^2 - 30x + 36$$
$$= 6(x^2 - 5x + 6)$$



$$\left.\begin{array}{l} \text{For, } \quad x \in [0,2] \Rightarrow f'(x) > 0 \\ \text{For, } \quad x \in [2,3] \Rightarrow f'(x) < 0 \end{array}\right\}$$

Hence, many-one function.

20. **(c)** $\frac{y}{1} = f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}; x \in (-1, 1)$

Apply componendo and dividendo

$$\Rightarrow \frac{y+1}{y-1} = \frac{2.8^{2x}}{-2.8^{-2x}} = -8^{4x}$$

Take $\log_8$ to both sides

$$\Rightarrow 4x = \log_8\left(\frac{y+1}{1-y}\right) \Rightarrow f^{-1}(x) = \frac{1}{4}\log_8\left(\frac{y+1}{1-y}\right) \quad \{\text{Change base}\}$$

$$\Rightarrow f^{-1}(x) = \frac{1}{4}\frac{\log_e\left(\frac{x+1}{1-x}\right)}{\log_e 8}$$

21. **(a, b, c)** For even/odd function

$$\Rightarrow f(-x) = [\log(\sec x - \tan x)]^3$$

$$= \left(\log\left[\frac{\sec^2 x - \tan^2 x}{\sec x + \tan x}\right]\right)^3 = \left[\log\left(\frac{1}{\sec x + \tan x}\right)\right]^3$$

$$= -[\log(\sec x + \tan x)]^3$$

$f(-x) = -f(x) \Rightarrow$ Odd function

For one-one function

Using T-8(ii) $f'(x) = 3[\log(\sec x + \tan x)]^2 \times \frac{1 \cdot \sec x(\sec x + \tan x)}{(\sec x + \tan x)}$

$\Rightarrow f'(x) = 3[\log(\sec x + \tan x)]^3$ $\underbrace{\sec x}_{>0 \text{ for } \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)}$

Since, $f'(x) > 0$

$\Rightarrow$ One-one function

Since, range = codomain $\in R$. Hence, onto function.

22. **(c)** Put $x = -7$

$\Rightarrow f(-7) = -(a{\cdot}7^7 + b{\cdot}7^3 + c{\cdot}7) - 5 \Rightarrow (a{\cdot}7^7 + b{\cdot}7^3 + c{\cdot}7) = -12$

Now, put $x = 7$

$f(7) = \underbrace{a\cdot 7^7 + b\cdot 7^3 + c\cdot 7} - 5 = -12 - 5 = -17.$

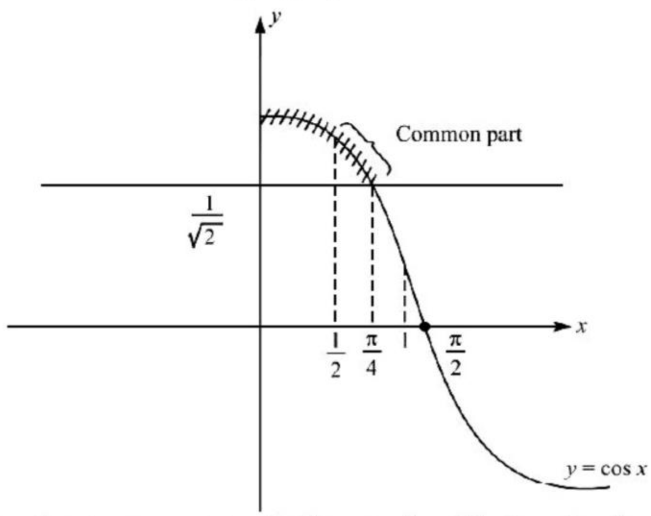
23. **(c)** Using T-13 Since period of $\cos x$ is 2π

$\Rightarrow$ Period of $f(x)$ is $\frac{2\pi}{3}$.

24. $x \in \left(\frac{1}{2}, \frac{\pi}{4}\right]$

(1) $\cos x - \frac{1}{\sqrt{2}} \geq 0$ (2) $\frac{3}{2}x - x^2 - \frac{1}{2} > 0$

$\left(x - \frac{1}{2}\right)(x-1) < 0$ $x \in \left(+\frac{1}{2}, 1\right)$



Hence, the common part is the graph will give the domain of $f(x)$

So the domain is $x \in \left(\frac{1}{2}, \frac{\pi}{4}\right]$.

25. **(a)** Using T-8(ii) $f'(x) = \frac{(1+x) - x}{(1+x)^2} > 0$

Hence $f(x)$ is one-one function

Since, in the co-domain $\to [0, \infty)$; '0' is included

But in domain $x \neq 0 \Rightarrow f(x) \neq 0$

Hence, Range $\neq$ Co-domain $\Rightarrow$ Not onto