AEP STUDY CIRCLE

SINCE 2001...

IIT-NEET-CBSE

ACCENTS EDUCATIONAL PROMOTERS

24 YEARS OF LEGACY

XI CBSE

PHYSICS NLM

UNIT-II

ONLINE-OFFLINE LEARNING ACADEMY

YOUR GATEWAY TO EXCELLENCE IN

IIT-JEE, NEET AND CBSE EXAMS

UNIT:III
NEWTON'S LAWS OF MOTION

IIT-JEE

NEET

CBSE

CONTACT US:

+91-9939586130

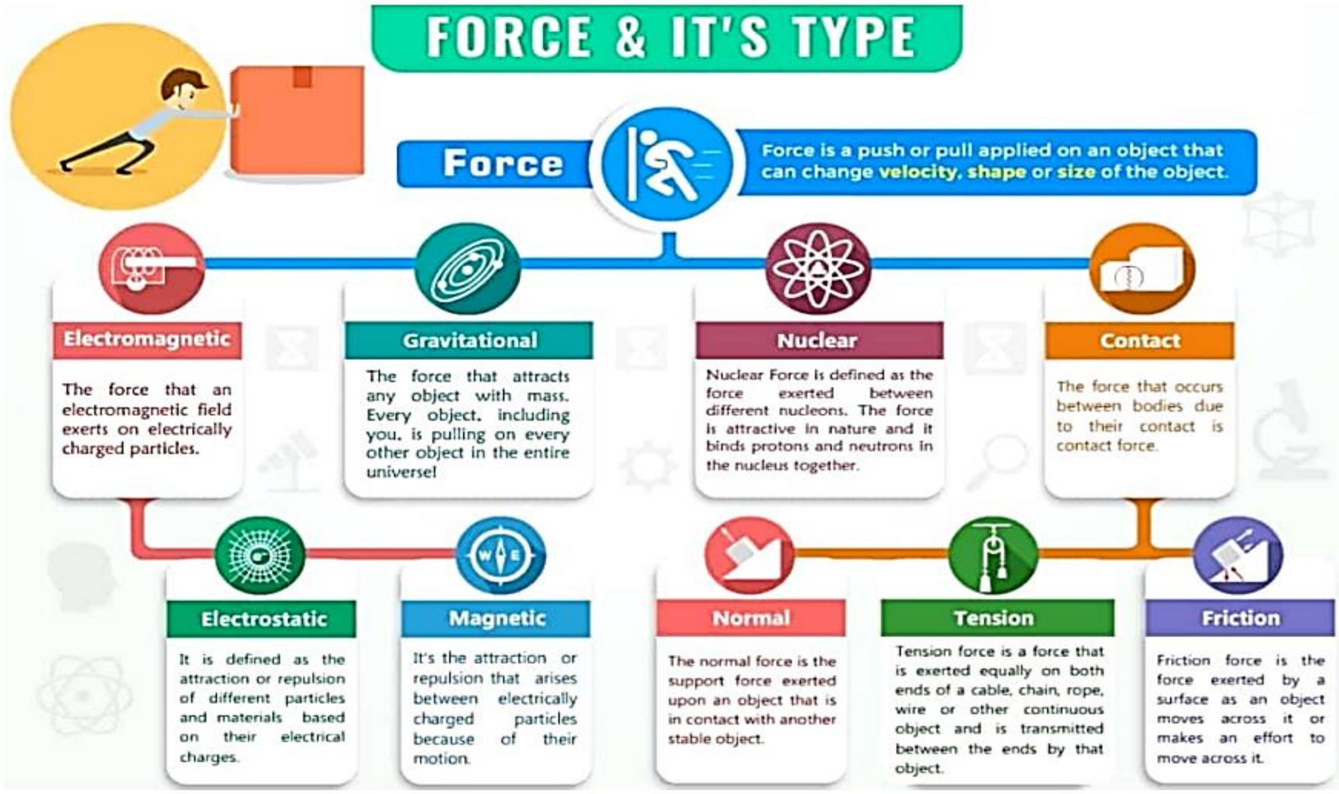
+91-9955930311

www.aepstudycircle.com

aepstudycircle@gmail.com

2ND FLOOR, SATKOUDI COMPLEX, THANA CHOWK, RAMGARH - 829122 JH

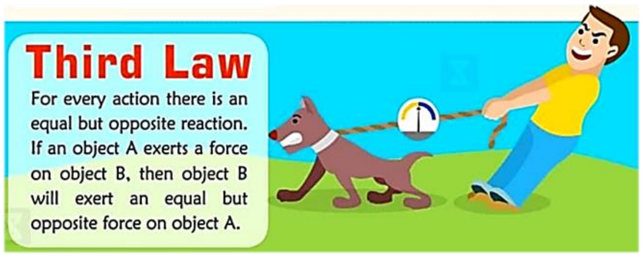
NEWTON'S LAWS OF MOTION



Law 1: Everybody will remain at rest or continue to move with uniform velocity unless an external force is applied to it.

Law 2: When an external force is applied to a body of constant mass, the force produces an acceleration, which is directly proportional to the force and inversely proportional to the mass of the body.

Law 3: When a body A exerts a force on another body B, B exerts an equal and opposite force on A.



NEWTON'S LAWS OF MOTION

Because it is often necessary to compare masses of such dissimilar bodies as a sample of sugar, sample of air, an electron, and Moon, it is necessary to define mass in terms of a property that not only is inherent and permanent but is also universal in that it is possessed by all known forms of matter. All matter possesses two properties, gravitation and inertia The property of gravitation is that every material body attracts every other material body. The property of inertia is that every material body resists any attempt to change its motion.

Rocket propulsion is based upon the law of conservation of linear momentum. The fuel is burnt in the ignition chamber of the rocket engine. The gases so produced are ejected from an orifice in the rear of the rocket. A thrust acts on the rocket due to reaction of the movement of gases and thus the rocket starts moving in the forward direction.

FORCE: **"Force is a push or pull which changes or tends to change the state of rest or uniform motion or direction of Motion of an object"**

- ► **i.e, Force ---produce or tries to produce motion.,**
- ► **i.e ,Force--- stops or tries to stops a moving body.**
- ► **I.e, Force ---changes or tries to change the direction of a body.**

EFFECTS PRODUCED BY A FORCE: **A force applied on an object can produce three types of changes:**

1. Force can change speed of an object, making it to move slower or faster. For example, a horse by exerting a force on a cart, pulls it from rest and subsequently exerting a larger force, the horse makes the cart move with a larger speed. Similarly a force exerted by the brakes slows or stops a moving train.

2. Force can change the direction of motion of an object. For example, the force exerted on the steering wheel of a car changes the direction of motion.

3. Force can change the shape of an object. For example, if we hold a rubber ball between our palms and push the two palms against each other, the ball no longer remains round but gets oblong.

☺ **It is not necessary that a force always set a body in motion or bring the moving body to rest or changes the direction of motion of the body . In such cases , the force may not be sufficient to cause the desired change.**
Ex: Pushing a wall , Pushing a train , etc.

- Force is a polar vector as it has a point of application.
- Forces can be classified as positive and negative. A positive force represents repulsion (e.g., between two like charges) and a negative force represents attraction (e.g., between two unlike charges).

Aristotelian law of motion: The Greek philosopher (684-329 B.C.) asserted that the natural state of a body is\ the state of rest. Every body in motion tends to slow down and comes to rest. An external force is necessary to maintain its motion. For example a cart on a road has to be constantly pushed to keep it in motion. A single push will not take it far. From such observations Aristotle concluded the following law:
According to Aristotelian law of motion, an external force is necessary to keep a body moving with uniform velocity.

However,

Aristotle's views were proved wrong by Galileo Galileo(1564 – 1642) about two thousand years later on.

It was observed that external forces were necessary to counter the opposing forces of friction to keep bodies in uniform motion. If there were no friction, no external force would be needed to maintain the state of uniform motion of a body.

THE LAW OF INERTIA

In the absence of external force , the <u>**inability**</u> of a body to change its state by itself is termed as **inertia** i.e,

" Inertia is inherent property of the object that they do not change their state unless acted upon by an external force, is called INERTIA". **OR**

" The property of a body by virtue of which it opposes any change in its state of rest or uniform motion is called INERTIA".

◙ GALILEO'S LAW OF INERTIA : In the absence of an external force , no body can change on its own , its state of rest or the state of uniform motion along a straight line .

► Galileo was the first to establish that **no force is required to keep a body moving uniformly along a straight .**

► **Force is required only for changing the state of the body.**

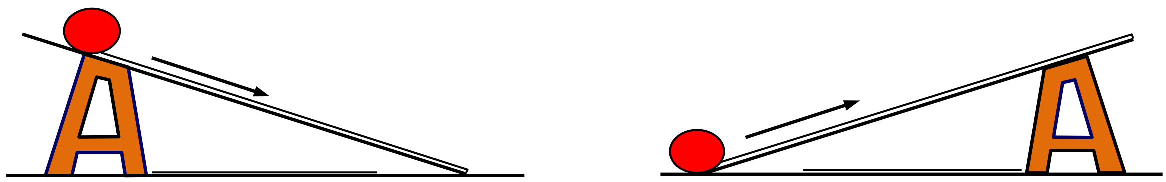
Galileo's experiments on the motion of objects: It was Galileo who first asserted that objects move with constant speed when no external forces act on them. He arrived at this revolutionary conclusion on the basis of following simple experiments.

(a) <u>Galileo's experiments with single inclined plane</u>: Galileo first studied the motion of objects on an inclined plane.

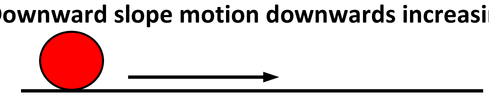
(i) Galileo observed that when an object moves down an inclined plane, its speed increases.

(ii) When the object is moved up the inclined plane, its speed decreases.

(iii) From the above two observations, Galileo argued that when the plane slopes neither upward nor downwards, there should be neither acceleration nor retardation.



[Downward slope motion downwards increasing speed] **[Upward slope motion upwards decreasing speed]**



[No slope horizontal motion constant speed]

[Galileo's observations of motion on a single inclined plane]

Galileo, therefore, concluded that **on a horizontal plane an object should move with a constant velocity in a straight line path.**

(b) Galileo's experiments on two inclined planes combined together: Galileo noted that in the case of an oscillating pendulum, the bob always reaches the same height on either side of the mean position. From this observation, Galileo thought of an imaginary experiment. In this experiment, two inclined planes are arranged facing each other, as shown in Fig. (a).

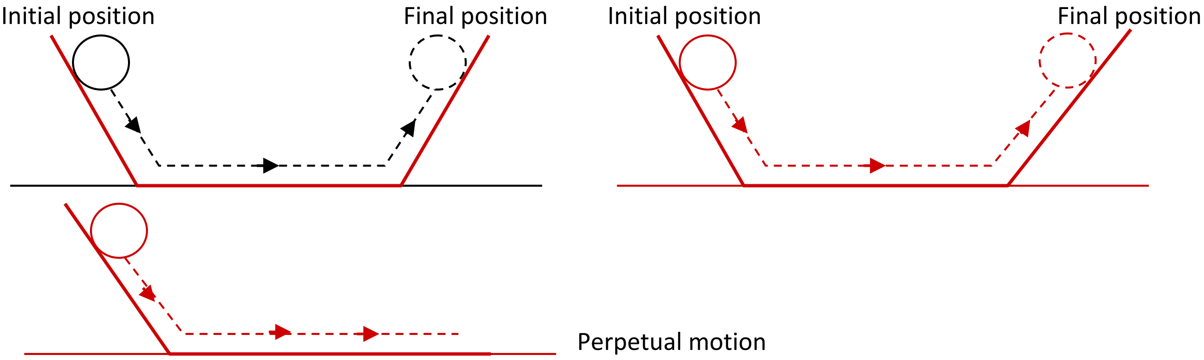
(i) When an object rolls down one of the inclined planes, it climbs up the other. It almost reaches the same height but not completely because of the presence of friction. If the friction were absent, the object must have reached the same height as the initial height, as shown in Fig. (a).

(ii) When the slope of the upward inclined plane is decreased, the object has to travel a longer distance to reach the maximum height, as shown in Fig. (b). The more we decrease the slope of the upward inclined plane, the longer would be the distance that the object is needed to travel to reach the same height.

(iii) From the above two observations, Galileo argued that if the second plane is made horizontal [Fig. (c)], the object will have to travel an infinite distance to reach the same height. This is possible only if the object moves for ever with uniform velocity on the horizontal surface.

From the above series of experiments, Galileo formulated the following law of inertia:

A body moving with a certain speed along a straight path will continue to move with same speed along the same straight path in the absence of external forces.



[Galileo's observations of motion on a double inclined plane]

MASS OF THE BODY IS A MEASURE OF INERTIA:

Explanation :- Consider two stationary object , one is heavier than the other .Now if we apply the same force on these object , then it is observed that it is difficult to move the heavier object than that of lighter one . It means , the heavier object resists more to the change in its state of rest than the lighter object .Hence , mass of a body is a measure of inertia .i.e,

▶ .... MORE IS THE MASS, MORE THE INERTIA .

TYPES OF INERTIA:

(A) INERTIA OF REST:– "The inherent property of a body that it remains at rest unless an external force is applied on it Is called inertia of rest".

EXAMPLE:- If a carpet is beaten with a stick , the fibres of the carpet come in motion and hence move forward .But the dust Particle due to inertia rest remain at rest .

(B) INERTIA OF MOTION:–"The inherent property of a body that it remains moving in a straight line with velocity unless a Force is applied on it , is called inertia of motion".

EXAMPLE:-- A person falls forward while getting down from a bus . This because as his feet touch on the ground , the lower Part of the body comes to rest . On the other hand , the upper part remains in the motion due to inertia of motion and hence he falls forward .

(C) INERTIA OF DIRECTION:- "The property of body due to which a body moving in particular direction can never itself CHANGES its direction unless force is applied on it ,is called inertia of direction"

EXAMPLE :-- The spark produced during sharpening of knife against grinding wheel leave the rim of the wheel tangentially .Because of inertia of direction .

LINEAR MOMENTUM:

Concept of Momentum: When a small piece of stone is dropped from a small height on a glass pane placed on a table, it does not break the glass pane. But when a heavy stone is dropped from the same height, the glass pane breaks. Here the small and the heavy stones have the same velocity when they fall on the glass pane.
On the other hand, a greater effort is required to stop a bullet fired from the gun than to stop a bullet of the same mass when just thrown by the hand. In the former case, the bullet has large velocity.
The above examples show that the effect of motion of a body depends both on its masses and velocity. The product of mass and velocity of a body is called its momentum. Thus,

"The quantity of motion contained in a body is called linear momentum of the body and is measure as the product of mass and velocity".

▶ **:MOMENTUM** of a body is measured by the FORCE required to stop the body in unit time . Now, the force required to stop to a moving body depends upon--- **(i) Mass of the body (m) (ii) Velocity if the body (v)**

▶ As the momentum is scalar (mass) times the vector(velocity), it is therefore vector quantity is therefore denoted by P .
If the body of mass 'm' is moving with a velocity(v),its linear momentum, $P = m v$

DIRECTION :Direction of P is the same as the direction of velocity of the body.

UNIT:- As, $P = m\,v$ = **kg m /s** (S.I) = g cm /s (cgs)

DIMENSIONAL FORMULA :- As $P = m\,v = M\frac{L}{T} = [M^1 L^1 T^{-1}]$

- DISCUSSION:

CASE –I : Consider two object of **equal masses (say m)** .Let one object be moving with a velocity v_1 & other be moving with velocity v_2 such that $v_1 > v_2$, then,

$P_1 = m\,v_1$ and $P_2 = m\,v_2$

$\therefore \quad \frac{P_1}{P_2} = \frac{m\,v_1}{m\,v_2} = \frac{v_1}{v_2}$ i.e, $P_1 \propto v_1$, $P_2 \propto v_2$

Since, $v_1 > v_2$, therefore, $P_1 > P_2$

Conclusion: **Linear momentum of the object moving faster will be more than the object of the same mass moving slow.**

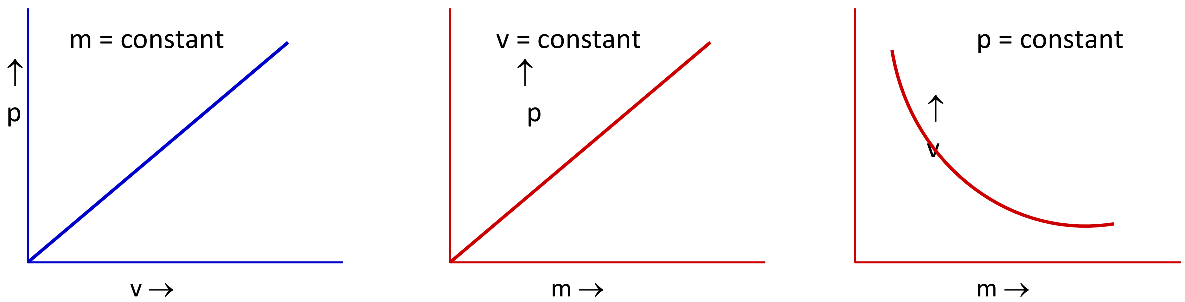
CASE-II: Consider two object ,one is heavier than the other moving with the **same velocity (say v)**.Let the mass of heavy object be m_1 & m_2 be the mass of the lighter one, then

$P_1 = m_1 v$ and $P_2 = m_2 v$

$\therefore \quad \frac{P_1}{P_2} = \frac{m_1 v}{m_2 v} = \frac{m_1}{m_2}$ i.e, $P_1 \propto m_1$, $P_2 \propto m_2$

Since ,$m_1 > m_2$, $\therefore$ $P_1 > P_2$

Conclusion: **Linear momentum of heavy object will be more than that of the light object if both are moving with same velocity.**



NEWTON'S LAWS OF MOTION

Newton's laws of motion: Sir Issac Newton (1642-1727) made a systematic study of motion and extended the ideas of Galileo. He arrived at three laws of motion which are called Newton laws of motion. These laws are as follows:

First law: Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by some external force to change that state.

Second law: The rate of change of linear momentum of a body is directly proportional to the applied force and the change takes place in the direction of the applied force.

Third law: To every action, there is always an equal and opposite reaction.

NEWTON'S 1ST LAWOF MOTION OR LAWS OF INERTIA :-

A/ This law- **"Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by some external force to change that state".**

In other words –

"If no net force acts on the body then the velocity of the body cannot change i.e, the body can't accelerates".

►►►► N'S 1ST law of the motion is divided into three parts:-

B. Based on inertia of motion.

(i) **When a horse running fast suddenly stops, the rider is thrown forward if he is not firmly seated.** This is because the lower part of the rider's body, which is in contact with the horse, comes to rest while the upper part of his body tends to keep moving due to inertia.

(ii) **A person getting out of a moving bus or train falls in the forward direction.** As the man jumps out of a moving bus, his feet suddenly come to rest on touching the ground while the upper part of his body continues to move forward. That is why he falls with his head forward. In order to save himself from falling down, he should run in the forward direction through some distance.

(iii) **An athlete runs for a certain distance before taking a long jump**. The inertia of motion gained by him at the time of jumping adds to his muscular effort and helps him in taking a longer jump.

(iv) **A fireman in a railway engine quickly moves his coal shovel near the furnace and then suddenly stops it.** The shovel near the furnace and then suddenly stops it. The shovel comes to rest but the coal continues moving due to inertia and falls into the furnace.

(v) **A ball thrown upward in a moving train comes back into the thrower's hands.** The ball acquires the horizontal velocity of the train and maintains it (inertia of motion) during its upward and downward motion. In this period the ball covers the same horizontal distance as the train, so it comes back into the thrower's hands.

C. Based on inertia of direction.

(i) Consider a stone being rotated in a circle at the end of a string. The velocity of the stone at any instant is along the tangent to the circle. When the string is released, the centripetal force whirling the stone vanishes. Due to directional inertia, the stone flies off tangentially.

(ii) During the sharpening of a knife, **the sparks coming from the grind stone fly off tangentially** to the rim of the rotating stone. This is due to the inertia of direction.

(iii) ***When a vehicle moves, the mud sticking to its wheels flies off tangentially:*** This is due to inertia of direction. In order that the flying mud does not spoil the clothes of the passerby, the wheels are provided with mudguards.

(iv) ***When a dog chases a hare, the hare runs along a zig-zag path:*** It becomes difficult for the long to catch the hare. This is because the dog has more mass and hence has more inertia of direction than that of hare.

NEWTON'S 2ND LAW OF MOTION:-

Clue to the second law of motion: Suppose a fixed force is applied on two bodies of different masses for the same duration. The lighter body gains a higher speed than the heavier one. However, the change in momentum in both cases is found to be the same. This shows that the same force for the same time causes the same change in momentum for bodies of different masses. This fact was first recognised by Newton who expressed it as his second law of motion.

According to 2nd law , **"The time rate of change of linear momentum of a body is Directly proportional to the applied force and the change takes place in the direction of applied force".**

This law can be divided into two parts:

► ***(i) The rate of change of momentum is directly proportional to the applied force.***

Explanation : The larger the force acting on a body, greater is the change in its momentum. Since change in momentum is equal to the product of mass and the change in velocity and the product of mass and the change in velocity and the mass of the body remains constant, so the rate of change of momentum is directly proportional to the rate of change of velocity i.e., acceleration. Hence force F is proportional to both mass (m) and acceleration (a).

$$F \propto ma$$

► ***(ii) The chance of momentum occurs in the direction of the force.***

Explanation :If a body is at rest, a force will set it in motion. If a body is moving with a certain velocity, a force will increase or decrease this velocity accordingly as the force acts in its same or opposite direction.

▶ MEASUREMENT OF FORCE: from Newton's second law of motion, if a body of mass m is moving with velocity $\vec{v}$, then its linear momentum is $\vec{p} = m\vec{v}$

Differentiating both sides w.r.t. time t, we get

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$$

where $\vec{a}$ is the acceleration product in the body. According to Newton's second law,

Applied force ∝ Rate of change of momentum

or $\vec{F} \propto \frac{d\vec{p}}{dt}$ or $\vec{F} \propto m\vec{a}$

∴ $\vec{F} = k\, m\vec{a}$

The units of m, a and F are so chosen that the proportionally constant k = 1. Suppose m=1, a=1 and F=1, then

$1 = k \times 1 \times 1$ or $k = 1$

∴ $\vec{F} = m\vec{a}$

The above equation can be used to measure force.

In scalar form, Newton's second law can be expressed as

F = ma

∴ 1 unit force = 1 unit mass × 1 unit acceleration

Hence, **a unit force may be defined as the force which produces unit acceleration in a body of unit mass.**

F = m a [Magnitude] , Hence 2nd of motion gives measure of force.

◘ FORCE AND ACCELERATION CAN BE WRITTEN IN TERM OF THEIR RECTANGLE COMPONENT AS:--

$\vec{F} = f_x i + f_y j + f_z k$ and $\vec{a} = a_x i + a_y j + a_z k$

From , $\vec{F} = m\,\vec{a}$, we get, $f_x i + f_y j + f_z k = m(a_x i + a_y j + a_z k)$

$f_x i + f_y j + f_z k = m a_x i + m a_y j + m a_z k$

Comparing the co-efficient of i , j and k, $f_x = m a_x$; $f_y = m a_y$; $f_z = m a_z$

The **direction** of **'f'** is the same as the direction of **a**

$$F_x = \frac{dp_x}{dt} = ma_x$$
$$F_y = \frac{dp_y}{dt} = ma_y$$
$$F_z = \frac{dp_z}{dt} = ma_z$$

▶ Significance of component form: The component form of Newton's second law indicates if the applied force makes some angle with the velocity of the body, it changes the component of velocity along the direction of the force. The component of velocity normal to the force remains unchanged. For example, in the motion of a projectile under the vertical gravitational force, the horizontal component of velocity remains unchanged.

☻ When the Force is written without direction .then,

- ▶ Positive force means REPULSIVE FORCE.
- ▶ Negative force means ATTRACTIVE FORCE.

☻ **DIMENSINAL FORMULA** :- $F = m\,a = [M][LT^{-2}] = [MLT^{-2}]$

▶ **UNIT OF FORCE** : – – [1] Absolute unit
[2] Gravitational unit

□ **ABSOLUTE UNITS OF FORCE**: An absolute unit of force is that force which produces a unit acceleration in a body of unit mass.

(i) **In SI,** the absolute unit of force is newton (N).
One newton is defined as that much force which produces an acceleration of 1 ms^{-2} in a body of mass 1 kg. **1 N = 1 kg × 1 ms^{-2} = 1 kg ms^{-2}**

(ii) **In CGS system**, the absolute unit of force is dyne (dyn): **One dyne is that much force which produces an acceleration of 1 cm s^{-2} in a body of mass 1 gram.**

1 dyne = 1 g × 1 cm s^{-2} = 1 g cm s^{-2}

□**GRAVITATIONAL UNITS OF FORCE**: **A gravitational unit of force is that force which produces acceleration equal to 'g' (acceleration due to gravity) in a body of unit mass.** It may also be defined as the weight of a body of unit mass.

(i) **In SI, the gravitational unit of force is kilogram weight (kg wt) or kilogram force (kg f). It is defined as that much force which produces an acceleration of 9.80 ms^{-2} in a body of mass 1 kg.**
1 kg wt = 1 kg f = 9.8 N

(ii) **In CGS system,** the gravitational unit of force is gram weight (g wt) or gram force (g f**). It is defined as that force which produces on acceleration of 980 cm s^{-2} in a body of mass 1 gram,**

1 g wt = 1 g f = 980 dyne

□**RELATION BETWEEN NEWTON AND DYNE**:

$1\ N = 1\ kg \times 1\ ms^{-2} = 1000\ g \times 100\ cm\ s^{-2}$
$= 10^5\ g\ cm\ s^{-2}$

or $1\ N = 10^5$ dyne

□□□□□□**DIFFERENCE BETWWEN ABSOLUTE & GRAVITATIONAL UNIT OF FORCE** ::

The Absolute unit of force remains the same through out the universe .But the gravitational unit of force vary from place to place or from planet to planet as these units depends upon the value of 'g' which is different at different place.

- Gravitational unit of force is 'g' times the absolute unit.
- The gravitation unit of force are used to express weight of the body
  Ex. .Weight of a body of mass10 kg is 10kgf or 10 kgwt.

► Gravitation unit are practical units.

► Some important Applications of Newton's second law:

(i) The second of motion is a vector law expressed as $\vec{F} = m\vec{a}$

(ii) In the second law, if F = 0, then a = 0. This indicates that a body moves with a uniform velocity in the absence of any external force. Thus second law of consistent with the first law.

(iii) The second law is strictly applicable to a point particle, but is also applicable to a body or a system of particles, providedF is the total external force on the system and a is the acceleration of the system as a whole.

(iv) Second law of motion is a local relation. This means F at a space point at a certain instant determines a at the same point and the same instant.

► Second law of motion is a local relation: The acceleration a at any point (location of the particle) at any instant is determined by the force F at the same point at the same instant. That is, acceleration is determined here and now by the force here and now, not by the earlier history of motion of the particle. For example, the moment a stone is released out of an accelerated train, there is no horizontal force or acceleration on the stone, if air resistance is neglected. The stone has only the vertical force of gravity. It carries no memory of its acceleration with the train a moment ago.

► CONSEQUENCE OF 2ND LAW OF MOTION ;-

[I] Concept of INERTIAL MASS:

We know that the acceleration produced in a body is greater if a large force is applied on it and vice versa .
Now, Consider two bodies of masses m_1 & m_2 on which the same force is applied . Then acceleration produced in the bodies Are, $a_1 = \frac{F}{m_1}$ and $a_2 = \frac{F}{m_2}$,

$\therefore \quad \frac{a_1}{a_2} = \frac{F/m_1}{F/m_2} = \frac{F\ m_2}{F\ m_1} = \frac{m_2}{m_1}$ **i.e,** If $m_1 > m_2$, then $a_1 < a_2$ as $a_1 \propto \frac{1}{m_1}$

☻ i.e, More is thee mass of the body , lesser will be the acceleration produced .Thus ,mass of the body is the of the resistance offered by the body to the change in velocity which applied force tends to produce , i.e, mass of the body is the measure of its inertia . For this reason ,mass given the equation F= ma is called ***inertial mass*** **.**

[II] An accelerated motion is always due to External Force :-

□**The accelerated motion of a body can occur in three ways**:-

{a}*Due to change in its direction only* -- The force must be acting perpendicularly to the direction of motion of the body .Such a force makes the body to move along circular path &is called '***Centripetal Force'***.

{b} ***Due to change in its speed only*** :- The force must be acting on the body along the direction of which opposite to the direction of motion.

{c} ***Due to the change in both speed & direction of motion:-*** The force must be acting at an angle with the direction of motion. The component of force along the direction of motion produces a change in speed while that along the normal produces a change in the direction .

[III] No force is required to move a body with uniform velocity :-

We know that , F = m a ,Now when a body moves uniform Speed or velocity , its acceleration/ retardation= 0

$\therefore \quad F = m \times 0 = 0$

[IV] Force can be measured from N'S 2ND law of motion:

As, $F = ma = m\frac{dv}{dt}$, Thus , if the inertial mass(m) of the body is known, then by measuring change in velocity produced and the for which theforce acts on the body ,'F"can be found

▶ INERTIAL MASS :- [DEFINATION]-

We know that , F= m a , if a = 1, then F= m × **1** = m

"Inertial mass of a body is the FORCE required to produce unit acceleration in the body".

▶ IMPULSE:- [Measure of action of the force]

"Impulse (I)is defined as the product of the average force and the time interval for which the force act the body".

➔ **Impulse is a vector quantity.**

EXPRESSION:-- Consider a constant force F which acts on a body for time 'dt'. The impulse(total effect of the force) Of this force is

$$dI = F\,dt$$

If the time interval of the force is t_1 to t_2 ,then

IMPULSE, $\int dI = \int F\,dt$

$I = F\int dt = F\,[\,t\,] = F\,[t_2 - t_1]$

$\mathbf{I = F\,\Delta t}$ [Since, $\Delta t = t_2 - t_1$]

▶ UNIT OF IMPULSE : $I = F\,\Delta t$ = N-s =newton –second

▶ DIMENSIONAL FORMULA :

$I = F\,\Delta t = [MLT^{-2}]\,[T] = [MLT^{-1}]$

IMPULSE—MOMENTUM THEOREM ::-- A/N'S 2ND law of motion , $F = \frac{dp}{dt}$ or, $dp = F\,dt$ --------[1]

Let at t = 0 , $P = P_1$ & at t = t, $P = P_2$

Integrating equation [1] within these limits ,we have

$$\int dP = \int F\,dt$$

or, $F\int dt = \int dP$

or, $F[\,t\,] = [\,P\,]$

or, $F[\,t - 0\,] = P_2 - P_1$

or, $F\,\Delta t = P_2 - P_1$

But, $F\,\Delta t = I$ **{impulse}**

$\therefore$ **I {impulse} =** $P_2 - P_1$ **[Impulse – Momentum theorem]**

Thus , **Impulse is equal to the change in momentum.**

Conclusion: Since , $F\,\Delta t = P_2 - P_1$ or, $F = \frac{P_2 - P_1}{\Delta t}$, ***If*** **Δ t is small ,F is large and vice-versa.0**

UNIT OF IMPULSE :- I = N s = newton –second

Or

I = change in momentum = $kg\,m/s^2$ [in S. I]

= $g\,cm/s^2$ [in cgs]

▶ The direction of impulse is the same as that of the force.

APPLICATIONS OF THE CONCEPT OF IMPULSE

Impulse of a force = Force × time = Change in momentum

If two forces F_1 and F_2 act on a body to produce the same impulse (or change in momentum), then their time durations t_1 and t_2 should be such that

$$\vec{F}_1 t_1 = \vec{F}_2 t_2$$

Clearly, if the time duration of an impulse is large, the force exerted will be small.

[1] A cricket player moves his hands backward while catching a ball.

Explanation:-- The player has to apply a retarding force to stop the moving ball If the player does not move his hand Backwards while catching the ball, the time to stop the ball is small, then a large retarding force has to Be applied to change the momentum of the ball . when the player moves his hand backwards while Catching the ball , the time to stop the ball is increased and hence less retarding force has to be applied to cause the same change in momentum of the ball . Therefore , the hands of the player are not injured.

[2] A person falling on a cemented floor gets injured but a person falling on a heap of sand in not injured.

On the cemented floor, the person is stopped abruptly. So the cemented floor exerts a large force of reaction causing him severe injuries. When the person falls on a heap of sand, the sand yields (gets depressed) under his weight. The person takes longer time to stop. This decreases the force exerted by the floor on the person.

[3] Huge damage to the train takes place when it suddenly collides against a stationary train.

[4] An athlete is advised to come to stop slowly.

[5] The vehicle are fitted with shockers .

When a vehicle moves on an uneven road, it receives a jerk. The shocker increases the time of jerk and hence reduces its force. This makes journey comfortable and saves the automobile from damage due to bumps.

[6] Glassware's and chinaware are wrapped in straw pieces before transportation .

The straw paper between the chinawares increases the time of experiencing the jerk during transportation. Hence, they strike against each other with a lesser force and are less likely to be damage

[7] Compartment of a train are provided with the buffer to increase the time of impact during shunting of the train.

Buffers increase the time of jerk during shunting. This decreases the force of impact between the bogies. The bogies are thus prevented from receiving severe jerks.

**IMPULSE

Impulsive force: A large force acting for a short time to produce a finite change in momentum is called an impulsive force

Examples: (i) Force exerted by a bat while hitting a ball

(ii) Blow of a hammer on a nail.

(iii) Force experienced by a person when he falls from a certain height on a marble floor.

** It is difficult to measure force and time duration separately in such situations. But the product of force and time, which is equal to the change in momentum of a body, is a measurable quantity. This product is given the same impulse.

Impulse of a force: Impulse is the total effect of a large force which acts for a short time to produce a finite change in momentum. Impulse is defined as the product of the force and the time for which its acts and is equal to the total charge in momentum.

Impulse = Force × time duration

= Total change in momentum

Impulse is a vector quantity denoted by $\vec{I}$. Its direction is same as that of force or the change in momentum. The impulse of a force is positive, negative or zero depending on the momentum of the body increases, decreases or remains unchanged.

Impulse as the product of force and time: Suppose a force F acts for a small time dt. The impulse of the force is given by

$$d\vec{I} = \vec{F}\,dt$$

If we consider a finite interval of time from t_1 to t_2, then the impulse will be

$$\vec{I} = \int d\vec{I} = \int_{t_1}^{t_2} \vec{F}\,dt$$

If $\vec{F}_{av}$ is the average force, then

t_2 t_2

$$\vec{I} = \int_{t_1} \vec{F}_{av}\, dt = \vec{F}_{av} \int_{t_1} dt$$

$$= \vec{F}_{av}\,[\,t\,]_{t_1}^{t_2} = \vec{F}_{av}\,(t_2 - t_1)$$

or $\vec{I} = \vec{F}_{av} \times \Delta t$, where $\Delta t = t_2 - t_1$

Thus, the impulse of a force is equal to the product of the average force and the time interval for which it acts.

SI unit of impulse = kg ms^{-1}

CGS unit of impulse = g cm s^{-1}

Dimensions of impulse = $[MLT^{-1}]$

MEASUREMENT OF IMPULSE BY GRAPHICAL METHOD:

A. When a constant force acts on a body: Suppose a constant force F acts on a body from time t_1 to t_2. The force-time graph is a straight line AB parallel to the time axis, as shown in Fig.

Area of rectangle ABCD = AD × AB = F $(t_1 - t_2)$ = F × t

= Magnitude of impulse of force F in time interval t

Force →

A B

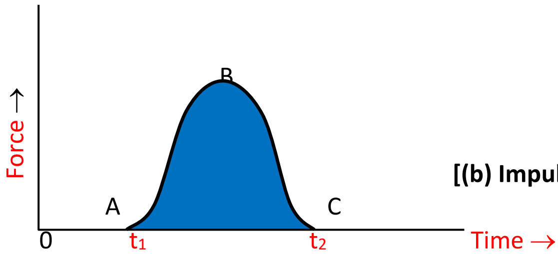
F Ft

D C_2

0 t_1 t_2 Time →

[(a) Impulse of a constant force]

B. When a variable force acts on the body: Suppose a force varying in magnitude acts on a body for time $t_2 - t_1 = t$. The force-time graph is a curve ABC is shown in Fig.



[(b) Impulse of a variable force]

Impulse of force F in time interval t,

$$I = \int_0^t F\,dt = \text{Area under the force-time curve ABC.}$$

Thus, the area under the force-time graph gives the magnitude of the impulse of the given force in the given time interval.

□knowledge+.................

Force is not always in the direction of motion. It may be along v, opposite to $\vec{v}$, normal to $\vec{v}$ or may make some angle with $\vec{v}$.

- In every motion, force $\vec{F}$ is parallel to acceleration $\vec{a}$.
- No force is required to move a body with a uniform velocity. In that case, a = 0, so F = ma = 0.
- Force can be measured from Newton's second law of motion.

  $F = ma = m\dfrac{\Delta v}{\Delta t}$ By knowing mass m and change in velocity Δv in time Δt, the force F can be determined.
- The cause of every accelerated motion is an external force. Internal forces have no role to play.
- If $\vec{v} = 0$ at an instant, i.e., if a body is momentarily at rest, it does not mean that force or acceleration are necessarily zero at that instant. For example, when a ball thrown up reaches its maximum height, $\vec{v} = 0$ but the force continues to be its weight mg and acceleration equal to g.
- The straight line along which a force acts is called the line of action of the force.

Examples based on Linear Momentum and Newton's Second Law of Motion

✖Formulae Used

1. Linear momentum, $p = mv$
2. According to Newton's second law,

Applied force = Rate of change of linear momentum

or $F = \frac{dp}{dt} = ma = m\left(\frac{v-u}{t}\right)$

✖Units Used

Velocities u and v are in ms^{-1}, time t in second, momentum p in $kgms^{-1}$, acceleration a in ms^{-2} and force F in newton (N).

✖Conversions Used

$1\ N = 10^5$ dyne, 1 kg wt = 9.8 N, 1 g wt = 980 dyne

Q. 1. Give the magnitude and direction of the net force acting on
(i) a stone of mass 0.1 kg just after it is dropped from the window of a stationary train.
(ii) the same stone as above just after it is dropped from the window of a train running at a constant velocity of 36 kmh^{-1}.
(iii) the same stone as above just after it is dropped from the window of a train accelerating with 1 ms^{-2}.
(iv) the same stone as above lying on the floor of a train which is accelerating with 1 ms^{-2}, the stone being at rest relative to the train. Neglect air resistance throughout, and take g = 10 ms^{-2}.

Sol. Here m = 0.1 kg, g = 9.8 ms^{-2}

(i) When the stone is just dropped from the window of a stationary train,

$F = mg = 0.1 \times 10 = 1$ N, vertically downwards

(ii) When the stone is dropped from the window of a train running at a constant velocity, no force acts on the stone due to the motion of the train. ∴ $F = mg = 1$ N, vertically downwards.

(iii) In the train accelerating with 1 ms^{-2}, the stone experiences an additional force,

$F' = ma = 0.1 \times 1 = 0.1$ N, along horizontal.

As the stone is dropped, the force F' no longer acts on the stone and so net force on the stone is

$F = mg = 1$ N, vertically downwards

(iv) Here weight of the stone is balanced by the normal reaction of the floor.

Acceleration of the stone = Acceleration of the train = 1 ms^{-2}

∴ $F = ma = 0.1 \times 1 = 0.1$ N, along horizontal.

Q. 2. A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction and magnitude of the net force on the pebble.
(i) during its upward motion.
(ii) during the downward motion.
(iii) at the highest point were it is momentarily at rest.
Do your answers alter if the pebble were thrown at an angle of say 45° with the horizontal direction? Take g = 10 ms^{-2}.

Sol. Here m = 0.05 kg, g = 10 ms^{-2}

(i) Net force on the pebble = mg $= 0.05 \times 10 = 0.5$ N, vertically downwards

(ii) Net force on the pebble = mg $= 0.05 \times 10 = 0.5$ N, vertically downwards

(iii) Net force on the pebble= $mg = 0.05 \times 10 = 0.5$ N, vertically downwards

The answers will not alter if the pebble were thrown at an angle of 45° with the horizontal because the horizontal component of velocity remains constant.

Q. 3. A car of mass 1000 kg is moving with a velocity of 10 ms^{-1} and is acted upon by a forward force of 1000 N due to engine and retarding force of 500 N due to friction. What will be its velocity after 10 seconds?

Sol. Here m = 1000 kg, u = 10 ms^{-1}, t = 10s , u = ?

Net forward force, F = Forward force – Retarding force = 1000 – 500 = 500 N

Acceleration, $a = \frac{F}{m} = \frac{500}{1000} = \frac{1}{2}\ ms^{-2}$

∴ $u = v + at = 1000 = 10 + ½ \times 10 = 15\ ms^{-1}$.

Q. 4. A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 ms^{-1}. How long does the body take to stop?

Sol. Here $F = -50$ N, m = 20 kg, u = 15 ms^{-1}, v = 0

As $F = ma$

∴ $a = \frac{F}{m} = \frac{-50}{20} = -2.5\ ms^{-2}$

Also, $v = u + at$ ∴ $0 = 15 - 2.5 \times t$

or $t = 6$ s.

Q. 5. A constant force acting a body of mass 3 kg changes it speed from 2 ms^{-1} to 3.5 ms^{-1} in 25 s. The direction of motion of the force remains unchanged. What is the magnitude and the direction of the force?

Sol. Here $m = 3$ kg, $u = 2\ ms^{-1}$, $v = 3.5\ ms^{-1}$, $t = 25$ s

As $v = u + at \quad \therefore \quad 3.5 = 2 + a \times 25$

or $a = \frac{3.5 - 2}{25} = 0.06\ ms^{-2}$

Force, $F = ma = 3 \times 0.06 = 0.18$ N .As the applied force increases the speed of the body, it acts in the direction of motion of the body.

***Q. 6.* A bullet of mass 0.04 kg moving with a speed of 90 ms^{-1} enters a heavy wooden block and is stopped after a distance of 60 cm. What is the average resistive force by the block on the bullet?**

Sol. Here $m = 0.04$ kg, $u = 90\ ms^{-1}$, $v = 0$, $s = 60$ cm $= 0.60$ m

As $v^2 - u^2 = 2as \quad \therefore \quad 0 - (90)^2 = 2a \times 0.60$

or $a = -6750\ ms^{-2}$

i.e., Retardation $= 6750\ ms^{-2}$ $\quad\therefore\quad$ Retarding force = Mass × retardation $= 0.04 \times 6750 = 270$ N

Q. 7. A force of 72 dyne is inclined to the horizontal at an angle of 60°. Find the acceleration in a mass of 9 g, which moves in a horizontal direction.

Sol. Here $m = 9$ g, $F = 72$ dyne, $\theta = 60°$

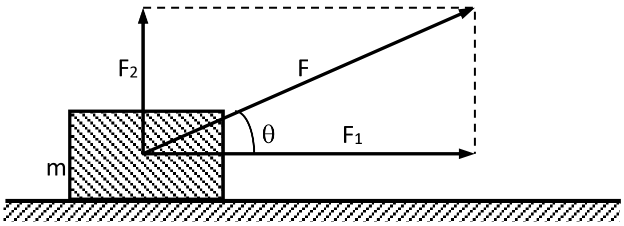
$F_x = F \cos\theta = 72 \times \cos 60°$

$= 72 \times 0.5 = 36$ dyne

Acceleration, $a = \frac{F_x}{m} = \frac{36}{9} = 4\ cms^{-2}$

Q. 8. A body of mass 5 kg is acted upon by two perpendicular forces of 8 N and 6 N. Give the magnitude and direction of the acceleration of the body.

Sol. As shown in Fig.



$F_1 = 8$ N, $F_2 = 6$N, $m = 5$ kg

The magnitude of the resultant force,

$F = \sqrt{F_1^2 + F_2^2} = \sqrt{8^2 + 6^2} = 10$ N

The magnitude of the acceleration produced,

$a = \frac{F}{m} = \frac{10}{5} = 2\ ms^{-2}$

If the force F makes angle θ with F_1, then

$\cos\theta = \frac{F_1}{F} = \frac{8}{10} = 0.8$

$\therefore \quad \theta = \cos^{-1}(0.8) = 36.87°$, with the 8 N force.

Q. 9. The driver of a three wheeler moving with a speed of 36 kmh^{-1} sees a child standing in the middle of the road and brings his vehicle to rest in 4 s just in time to save the child. What is the average retarding force on the vehicle? The mass of the three-wheeler is 400 kg and the mass of the driver is 65 kg.

Sol. Here $u = 36\ kmh^{-1} = 10\ ms^{-1}$, $v = 0$,

$t = 4$ s, $m = 400 + 65 = 465$ kg

As $v = u + at \quad \therefore \quad 0 = 10 + a \times 4$

or $a = -2.5\ ms^{-2}$

Magnitude of the retarding force on the vehicle is $F = ma = 465 \times 2.5 = 1162.5$ N.

Q. 10. A truck starts from rest and accelerates uniformly with 2.0 ms^{-2}. At t = 10 s, a stone is dropped by a person standing on the top of the truck (6 m high from the ground). What are the (i) velocity, and (ii) acceleration of the stone at t = 11 s? Neglect air resistance. Take g = 10 ms^{-2}.

Sol. Here $u = 0$, $a = 2\ ms^{-2}$, $g = 9.8\ ms^{-2}$, $t = 10$ s.

(i) During first 10 s, the horizontal component of the velocity is

$v_x = u + at = 0 + 2 \times 10 = 20\ ms^{-1}$

From 10 s to 11 s (i.e., for 1 s), the vertical component of the velocity is

$v_y = u + gt = 0 + 10 \times 1 = 10\ ms^{-1}$

$\therefore$ Resultant velocity, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20)^2 + (10)^2} = 10\sqrt{5} = 22.4\ ms^{-1}$

The direction of resultant velocity with the horizontal is given by

$\tan\theta = \frac{v_y}{v_x} = \frac{10}{20} = \frac{1}{2}$ or $\theta = \tan^{-1}(1/2) = 26.7°$

(ii) As there is no horizontal acceleration, the only acceleration is vertical.

$\therefore$ Vertically downward acceleration $= g = 9.8\ ms^{-2}$

Q. 11. A body of mass 0.4 kg moving with a constant speed of 10 ms^{-1} to the north is subjected to a constant force of 8 N directed towards the south for 30 s. Take the instant the force is applied to be t = 0, the position of the body at that time to be x = 0 and predict its position at t = – 5 s, 25 s and 100 s.

Sol. We take south to north as the positive direction. Then u = + 10 ms^{-1} (due north), F = –8N (due north), F = – 8 N (due south), t = 30 s, m = 0.4 kg

$$a = \frac{F}{m} = \frac{-8}{0.4} = -20\ ms^{-2}$$

(i) At t = – 5s, no force acts on the particle.

∴ $x = ut = 10 \times (-5) = -50$ m

(ii) AT t = 25 s, the position of the particle will be

$x = ut + ½ at^2 = 10 \times 25 - ½ \times 20 \times (25)^2$

$= 250 - 6250 = -6000$ m $= -6$ km

(iii) At t = 100 s, there is no force because force stops acting after t = 30 s.

∴ Distance covered during first 30 s is

$x_1 = ut + ½ at^2 = 10 \times 30 - ½ \times 20 \times (30)^2$

Velocity acquired at t = 30 s will be

$v = u + at = 10 - 20 \times 30 = -590\ ms^{-1}$

Distance covered in next 70 s with constant velocity of – 590 ms^{-1} is

$x_2 = vt = -590 \times 70 = -41300$ m

∴ Position of the particle at t = 30 s is

$x_1 + x_2 = -8700 - 41300 = -50,000$ m $= -50$ km

Q. 12. A stream of water flowing horizontally with a speed of 15 ms^{-1} gushes out of a tube of cross-sectional area 10^{-2} m^2, and hits at a vertical wall nearby. What is the force exerted on the wall by the impact of water, assuming it does not rebound?

Sol. Here u = 15 ms^{-1}, v = 0, t = 1 s, A = 10^{-2} m^2

Density of water = 1000 kgm^{-3}

m = Mass of water gushed out per second

$= \frac{\text{Volume} \times \text{density}}{\text{Time}} = \frac{\text{Area} \times \text{distance} \times \text{density}}{\text{Time}}$

$=$ Area × velocity × density $= Au\rho = 10^{-2} \times 15 \times 1000 = 150$ kg

Force exerted by the wall on water,

$F = ma = m\left(\frac{v-u}{t}\right) = 150 \times \frac{0-15}{1} = -2250$ N

Force exerted on the wall by the impact of water, $F' = -F = 2250$ N

Q. 13. A block of metal weighing 2 kg is resting on a frictionless plane. It is struck by a jet releasing water at the rate of 1 kg s^{-1} and at the speed of 5 ms^{-1}. Calculate the initial acceleration of the block.

Sol. Force exerted by the jet of water on the block is

$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = v\frac{dm}{dt} = 5\ ms^{-1} \times 1\ kgs^{-1} = 5$ N

Mass of block, m = 2 kg

∴ Acceleration of the block, $a = \frac{F}{m} = \frac{5}{2} = 2.5\ ms^{-2}$

Q. 14. A body of mass m moves along X-axis such that its position co-ordinate at any instant t is $x = at^4 - bt^3 + ct$, where a, b and c are constants. What is the force acting on the particle at any instant t?

Sol. Position co-ordinate, $x = at^4 - bt^3 + ct$

Velocity = dx = $4at^3 - 3bt^2 + c$

Acceleration $= \frac{d^2x}{dt^2} = \frac{d}{dt}(4at^3 - 3bt^3 + c) = 12at^2 - 6bt$

Force = Mass × acceleration = $m(12at^2 - 6bt)$

Problems For Practice

Q. 1. A force acts for 10 s on a body of mass 10 kg after which the force ceases and the body describes 50 m in the text 5 s. Find the magnitude of the force.

Sol. After the force ceases, the body covers 50 m in 5 s, so final velocity of the body is

$v = \frac{\text{Distance}}{\text{Time}} = \frac{50\ m}{5\ s} = 10\ ms^{-1}$

But $v = u + at$ ∴ $10 = 0 + a \times 10$

or $a = 1\ ms^{-2}$

∴ $F = ma = 10\ kg \times 1\ ms^{-2} = 10$ N

Q. 2. **A truck starts from rest and rolls down a hill with constant acceleration. It travels a distance of 400 m in 20 s. Calculate the acceleration and the force acting on it if its mass is 7 metric tonnes.**

Sol. As $s = ut + \frac{1}{2}at^2$ ∴ $400 = 0 + \frac{1}{2}a(20)^2$

or $a = 2\ ms^{-2}$ and $F = ma = 7000 \times 2 = 14{,}000\ N$

Q. 3. **A force of 5 N gives a mass m_1 an acceleration of 8 ms^{-2} and a mass m_2 and acceleration of 24 ms^{-2}. What acceleration would it give if both the masses are tied together?**

Sol. Here $m_1 = \frac{F}{a_1} = \frac{5}{8}$ kg and $m_2 = \frac{F}{a_2} = \frac{5}{24}$ kg

∴ $m_1 + m_2 = \frac{5}{8} + \frac{5}{24} = \frac{5}{6}$ kg

Acceleration of the tied masses $= \frac{F}{m_1 + m_2} = \frac{5}{5/6} = 6\ ms^{-2}$

Q. 4. **In an X-ray machine, an electron is subjected to a force of 10^{-23} N. In how much time the electron will cover a distance of 0.1 m? Take mass of the electron = 10^{-30} kg.**

Sol. Here $a = \frac{F}{M} = \frac{10^{-23}}{10^{-30}} = 10^7\ ms^{-2}$

As $s = ut + \frac{1}{2}at^2$ ∴ $0.1 = 0 + \frac{1}{2} \times 10^7 \times t^2$

or $t = 2 \times 10^{-8}\ s^2$ or $t = 1.4 \times 10^{-4}$ s

Q. 5. **A stone of mass 5 kg falls from top of a cliff 50 m high and buries 1 m in sand. Find the average resistance offered by the sand and the time it takes to penetrate.**

Sol. Velocity attained by the stone as it falls through a height of 50 m is given by

$v^2 - u^2 = 2as$ or $v^2 - 0^2 = 2 \times 9.8 \times 50$

or $v = \sqrt{980}\ ms^{-1}$

Now the stone starts burying into sand with a velocity of $\sqrt{980}\ ms^{-1}$ and finally comes to rest after travelling a distance, s = 1 m

∴ $0^2 - 980 = 2a \times 1$ or $a = -490\ ms^{-2}$

Average resistance offered by sand,

$F = ma - 5 \times 490 = 2450$ N

Time taken by stone to penetrate sand,

$t = \frac{u - v}{a} = \frac{0 - \sqrt{980}}{-490} = 0.064$ s

Q. 6. **A bus starts from rest accelerating uniformly with 4 ms^{-2}. At t = 10 s, a stone is dropped out of a window of the bus 2 m high. What are the (i) magnitude of velocity and (ii) acceleration of the stone at 10.2 s? Take g = 10 ms^{-2}.**

Sol. (i) Horizontal velocity of the bus or the stone at t = 10 s is

$v_x = u + at = 0 + 4 \times 10 = 10\ ms^{-1}$

For vertical motion of the stone,

$u = 0,\ a = g = 10\ ms^{-2},\ \ t = 10.2 - 10 = 0.2$ s

∴ $v_y = 0 + 10 \times 0.2 = 2\ ms^{-1}$

Magnitude of the resultant velocity of the stone is $v = \sqrt{v_x^2 - v_y^2} = \sqrt{40^2 + 2^2} = \sqrt{1604} = 40.04\ ms^{-1}$

(iii) After the stone is dropped, its acceleration along horizontal is zero. It has only a vertical acceleration of 10 ms^{-2}.

Q. 7. **A motor car running at the rate of 7 ms^{-1} can be stopped by applying brakes in 10 m. Show that total resistance to the motion, when brakes are on, is on fourth of the weight of the car.**

Sol. Here $u = 7\ ms^{-1},\ v = 0,\ s = 10\ m,\ a = ?$

As $v^2 - u^2 = 2as$ ∴ $0 - 7^2 = 2a \times 10$

or $a = -2.45\ ms^{-2} = -\frac{9.8\ ms^{-1}}{4} = -\frac{g}{4}$

Total resistance to motion $= -ma = \frac{mg}{4} = \frac{1}{4} \times$ Weight of car

Q. 8. **A force of 50 N is inclined to the vertical at an angle of 30°. Find the acceleration it produces in a body of mass 2 kg which moves in the horizontal direction.**

Sol. Horizontal component of force = $F \cos(90° - \theta) = F \sin\theta$

∴ $a = \frac{F \sin\theta}{m} = \frac{50 \sin 30°}{2} = \frac{50 \times 1}{2 \times 2} = 12.5\ ms^{-2}$

Q. 9. **A scooterist moving with a speed of 36 kmh^{-1} sees a child standing in the middle of the road. He applies the brakes and brings the scooter to rest in 5 s just in time to save child. Calculate the average retarding force on the vehicle, if mass of the vehicle and driver is 300 kg.**

Sol. Here u = 36 kmh^{-1} = 10 ms^{-1}, v = 0, t = 5 s

$$a = \frac{v-u}{t} = \frac{0-10}{5} = -2\ ms^{-2}$$

∴ Retardation = 2 ms^{-2}

Average retarding force = 300 × 2 = 600 N

Q. 10. **A ship of mass 3 × 10^7 kg and initially at rest can be pulled through a distance of 3 m by means of a force of 5 × 10^4 N. The water resistance is negligible. Find the speed attained by the ship.**

Sol. Acceleration, $a = \frac{F}{m} = \frac{5\times10^4}{3\times10^7} = \frac{5}{3}\times10^{-3}\ ms^{-2}$

Also u = 0, s = 3 m

As $v^2 - u^2 = 2as$ ∴ $v^2 - 0 = 2\times\frac{5}{3}\times10^{-3}\times3$

or $v = 10^{-1}\ ms^{-1} = 0.1\ ms^{-1}$

Q. 11. **Force of 5√2 and 6√2N are acting on a body of mass 1000 kg at an angle to 60° to each other. Find the acceleration, distance covered and the velocity of the mass after 10 s.**

Sol. $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$= \sqrt{(5\sqrt2)^2 + (6\sqrt2)^2 + 2\times5\sqrt2\times6\sqrt2\cos60^\circ}$

$= \sqrt{50+72+60} = \sqrt{182} = 13.49$ N

$a = \frac{F}{m} = \frac{13.49}{1000} = 0.01349\ ms^{-2}$

$v = u + at = 0 + 0.01349\times10 = 1.1349\ ms^{-1}$

$s = 0 + \frac{1}{2}\times0.01349\times(10)^2 = 0.6745$ m

Q. 12. **A balloon has a mass of 5 g in air. The air escapes from the balloon at a uniform rate with a velocity of 5 cms^{-1}. If the balloon shrinks completely in 2.5 s, find the average force acting on the balloon.**

Sol. Force, $F = \frac{dp}{dt} = \frac{d}{dt}(mv) = v\frac{dm}{dt}$ [∵ v is constant]

But $\frac{dm}{dt} = \frac{5\ g}{2.5\ g} = 2\ gs^{-1}$ and v = 5 cms^{-1}

∴ F = 5 × 2 = 10 dyne.

Examples based on Impulse of a Force

✖ Formulae Used

1. Impulse = Force × time = Change in momentum or J = F × t = m (v – u)

2. $\vec{J} = \int_{t_1}^{t_2} \vec{F}\cdot dt$ = Area under force-time (F-t) graph

✖ Units Used

Velocities u and v are in ms^{-1}, mass m in kg, time t in second, force F in newton and impulse J in Ns or kg ms^{-1}.

Q. 1. **A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 ms^{-1}. If the mass of the ball is 0.15 kg, determine the impulse imparted to the ball. (Assume linear motion of the ball)**

Sol. Here m = 0.15 kg, u = 12 ms^{-1}, v = – 12 ms^{-1}

Impulse = m (v – u) = 0.15 (– 12 – 12) = – 3.6 Ns

The negative sign indicates that the direction of the impulse is from the batsman to the bowler.

Q. 2. **A cricket ball of mass 150 g moving with a velocity of 15 ms^{-1} is brought to rest by a player in 0.05 s. Calculate the impulse and the average force exerted by the player.**

Sol. Here m = 150 g = 0.15 kg, u = 15 ms^{-1} v = 0, t = 0.05 s

Impulse = m (u – v) = 0.15 (0 – 15) = – 2.25 Ns

Average force = $\frac{Impulse}{Time} = \frac{-2.25}{0.05} = -45$ N

The negative sign indicates the retarding nature of the force.

Q. 3. **Two billiard balls each of mass 0.05 kg moving in opposite directions with speed of 6 ms^{-1} collide and rebound with the same speed. What is the impulse imparted to each ball by the other?**

Sol. Fig. (a) and (b) show the situations of the two balls before and after the collision.

Impulse imparted to one ball by the other = Change in momentum

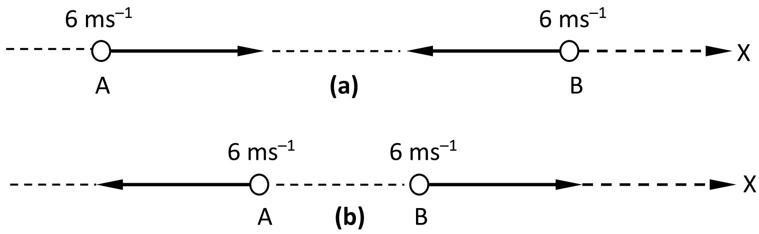
For ball A:

p_i = Momentum before collision

$= 0.05 \times 6 = 0.3$ kg ms^{-1}

p_f = Momentum after collision

$= 0.05 \times (-6) = -0.3$ kg ms^{-1}



∴ Impulse imparted to ball B due to ball A

$= p_f - p_i = -0.3 - 0.3 = -0.6$ kg ms^{-1}

For ball B:

p_i = Momentum before collision $= 0.05 \times (-6) = -0.3$ kg ms^{-1}

p_f = Momentum after collision $= 0.05 \times (6) = 0.3$ kg ms^{-1}

∴ Impulse imparted to ball B due to ball A $= p_f - p_i = 0.3 - (-0.3) = 0.6$ kg ms^{-2}

Q. 4. **A rubber ball of mass 50 g falls from a height of 1 m and rebounds to a height of 0.5 m. Find the impulse and the average force between the ball and the ground if the time for which they are in contact was 0.1 s.**

Sol. Refer to Fig. Initial velocity of ball at A = 0

Final velocity of ball at B = 0 , s = 1

As $v^2 - u^2 = 2as$ => $v^2 - 0^2 = 2 \times 9.8 \times 1$ or $v = \sqrt{19.6}$ ms^{-1}

Let u' be the velocity of rebound of the ball.

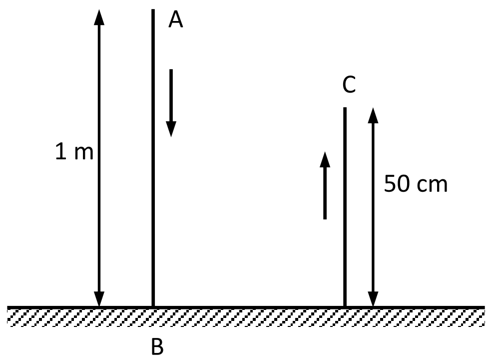
$s' = 50$ cm $= 0.5$ m, $g = -9.8$ ms^{-2}

Final velocity at C = v' = 0

As $v^2 - u'^2 = 2as$

∴ $0^2 (u')^2 = 2 \times (-9.8) \times 0.5$

or $u' = \sqrt{9.8}$ ms^{-1}



Now, Impulse = Change in momentum

$= mv - (-mu') = mv + mu' = m(v + u')$

$= \frac{50}{1000}(\sqrt{19.6} + \sqrt{9.8}) = \frac{1}{20}(4.427 + 3.130)$

$= 0.378$ Ns

Average force $= \frac{\text{Impulse}}{\text{Time}} = \frac{0.378}{0.1} = 3.78$ N

Q. 5. **While launching a rocket of mass 2×10^4 kg, a force of 5×10^5 N is applied for 20 s. Calculate the velocity attained by the rocket at the end of 20 s.**

Sol. Here $m = 2 \times 10^4$ kg, $F = 5 \times 10^5$ N, t = 20 s, u = 0, v = ?

Impulse of force $= F \times t = m(v - u)$

∴ $5 \times 10^4 \times 20 = 2 \times 10^4 (v - 0)$

or $v = \frac{5 \times 10^4 \times 20}{2 \times 10^4} = 500$ ms^{-1}

Q. 6. **A machine gun fires a bullet of mass 40 g with a speed of 1200 ms^{-1}. The person holding the gun can exert a maximum force of 144 N on it. What is the number of bullets that can be fired from the gun per second?**

Sol. Let maximum number of bullets that can be fired per second = n

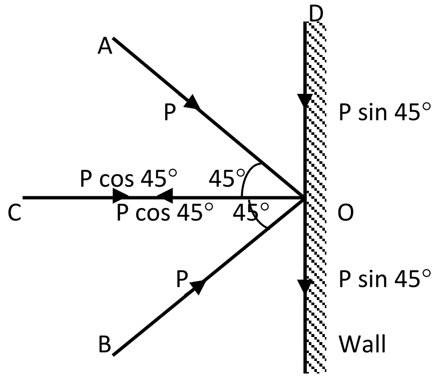
∴ Change in momentum of n bullets = nm (v – u)

$= n \times \frac{40}{1000}(1200 - 0) = 48n$ kg ms^{-1}

As impulse = Change in momentum ∴ $Ft = 48n$ or $n = \frac{Ft}{48} = \frac{144 \times 1}{48} = 3$ bullets/s

Q. 7. **A ball moving with a momentum of 5 kg ms^{-1} strikes against a wall at an angle of 45° and is reflected at the same angle. Calculate the change in momentum.**

Sol. Initial momentum is along AO. It has two rectangular components: **p cos 45° along CO and p sin 45° along DO**
Final momentum p is along OB. It has two rectangular components: **p cos 45° along OC and p sin 45° along OE**



Change in momentum along vertical direction v= Final momentum – Initial momentum = p sin 45° – p sin 45° = 0
Change in momentum along horizontal direction

$= -p\cos 45° - p\cos 45° = -2p\cos 45° = -2\times 5\times 1/\sqrt{2} = -2\times 5\times 0.707 = -7.07$ kg ms^{-1}

Negative sign indicates that the direction of change in momentum is away from the well.

Q. 8. A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 54 kmh^{-1}. What is the impulse imparted to the ball? Mass of the ball is 0.15 kg.

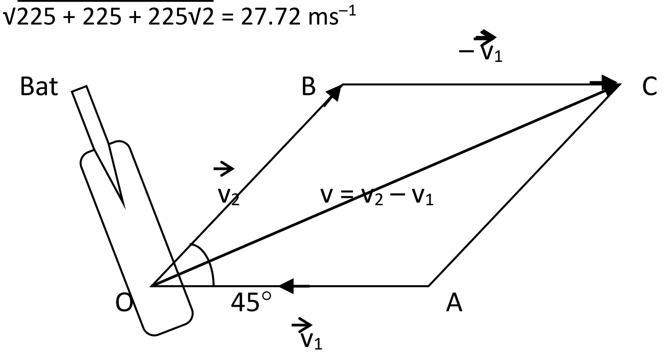
Sol. Speed of the ball = 54 kmh^{-1} = 15 ms^{-1}
Let v_1 and v_2 be the velocities of the ball before and after deflection.
As the speed of the ball remains unchanged even after deflection, so $|\vec{v}_1| = |\vec{v}_2| = 15$ ms^{-1}
In Fig. $\vec{AO} = \vec{v}_1$ and $\vec{OB} = \vec{v}_2$. Clearly, the change in velocities of the ball is

$\vec{v}_2 - \vec{v}_1 = \vec{v}_2 + (-\vec{v}_1) = \vec{OB} + \vec{BC} = \vec{OC} = \vec{v}$ (say)

Then $v = \sqrt{v_1^2 + v_2^2 + 2v_1 v_2\cos 45°}$

$= \sqrt{(15)^2 + (15)^2 + 2\times 15\times 15\times (1/\sqrt{2})}$

$= \sqrt{225 + 225 + 225\sqrt{2}} = 27.72$ ms^{-1}



Impulse imparted to the ball= Mass × Change in velocity of the ball = 0.15 × 27.72 = 4.16 kg ms^{-1}
Impulse is imparted along v. As the velocity v is the resultant of two velocities – v_1, and v_2, which have equal magnitude, so v equally divides the angle between v_1 and v_2 i.e, impulse is directed along the bisector of initial and final directions.

Q. 9. Fig. shows the position time graph of a particle of mass 4 kg. What is the (i) force acting on the particle for $t < 0$, $t > 4$s, $0 < t < 4$ s? (ii) Impulse at t = 0 and t = 4 s? Assume that the motion is one dimensional.

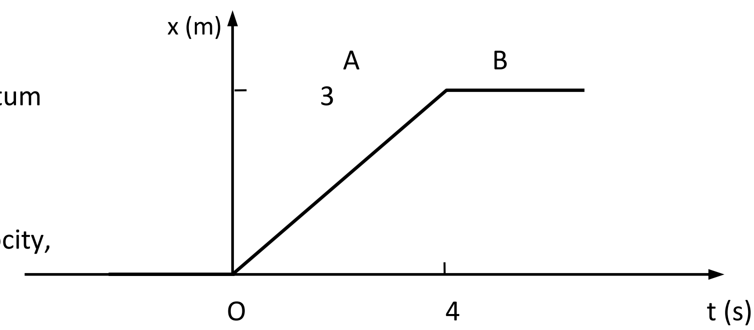
Sol. (i) For t < 0 and t > 4 s, the position of the particle is not changing i.e., the particle is at rest. So no force is acting on the particle during these intervals.
For 0 < t < 4s, the position of the particle is continuously changing. As the position-time graph is a straight line, the motion of the particle is uniform, so acceleration, a = 0. Hence no force acts on the particle during this interval also.
(ii) Before t = 0, the particle is at rest, so u = 0
After t = 0, the particle has a constant velocity,

v = Slope of OA = ¾ ms^{-1}

∴ At t = 0, impulse = Change in momentum

$= m(v - u)$

$= 4\left(\frac{3}{4} - 0\right) = 3$ kg ms^{-1}



Before t = 4 s, the particle has a constant velocity,

u = Slope of OA = ¾ ms^{-1}

After t = 4 s, the particle is at rest, so v = 0
At t = 4s

Impulse = m (v – u) = 4 $\left(0 - \frac{3}{4}\right)$ = – 3 kg ms^{-1}

Q. 10. Fig. below shows the position-time graph of a particle of mass 0.04 kg. Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the particle? What is the magnitude of each impulse?

Sol. Fig. shows that (i) the direction of motion of the particle changes after every 2 s and (ii) in both directions, the particle moves with a uniform speed.

Before t = 2s, velocity of the particle,

u = Slope of x-t graph

$= \frac{(2-0)\text{ cm}}{(2-0)\text{ s}} = 1\ cms^{-1} = 0.01\ ms^{-1}$

After t = 2s, velocity of the particle,

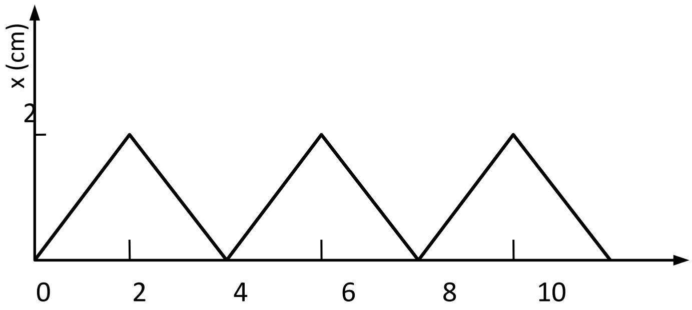
$v = \frac{(0-2)\text{ cm}}{(4-2)\text{ s}} = -1\ cms^{-1} = -0.01\ ms^{-1}$

Mass of particle, m = 0.04 kg

At t = 2 s, magnitude of impulse

= Change in momentum = m (u – v)

= 0.04 [0.01 – (– 0.01)] kg ms^{-1} = 8×10^{-4} kg ms^{-1}



The given x-t graph may represent the repeated rebounding of a particle between two walls situated at x = 0 and x = 2 cm. The particle receives an impulse of 8×10^{-4} kg ms^{-1} after every 2 s.

Q. 11. The initial speed of a body of mass 2.0 kg is 5.0 ms^{-1}. A force acts for 4 s in the direction of motion of the body. The force-time graph is shown in Fig. Calculate the impulse of the force and the final speed of the body.

Sol. Impulse of a force = Area between the force-time graph and the time-axis

= Area of triangle OA'A + Area of rectangle AA'B'B + Area of trapezium BB'C'C + Area of rectangle CC'ED

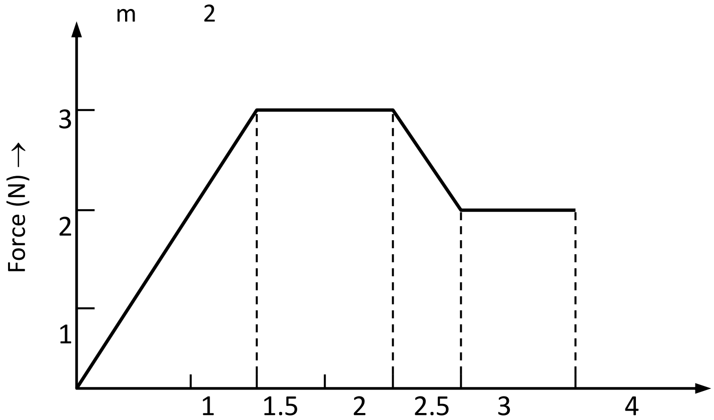
= ½ × 1.5 × 3 + 1 × 3 + ½ (3 + 2) (3 – 2.5) + 2 × 1

= 2.25 + 3 + 1.25 + 2 = 8.50 Ns.

As impulse = Change in momentum = mΔv

∴ Change in velocity,

$\Delta v = \frac{\text{Impulse}}{m} = \frac{8.50}{2} = 4.25\ ms^{-1}$



Final speed of the body = Initial speed + Δv = 5.0 + 4.25 = 9.25 ms^{-1}

Problems For Practice

Q. 1. A cricket ball of mass 150 g is moving with a velocity of 12 ms^{-1}, and is hit by a bat, so that the ball is turned back with a velocity of 20 ms^{-1}. The force of the blow acts for 0.01 second on the ball. Find the average force exerted by the bat on the ball.

Sol. Here m = 150 g = 0.15 kg, u = 12 ms^{-1} v = – 20 ms^{-1}, t = 0.01 s

Impulse = m (v – u) = 0.15 (– 20 – 12) = – 4.8 kg ms^{-1}

Average force = $\frac{\text{Impulse}}{\text{Time}} = \frac{4.8}{0.01}$ = 480 N

Q. 2. A hammer weighing 1 kg moving with the speed of 10 ms^{-1} strikes the head of a nail driving it 10 cm into a wall. Neglecting the mass of the nail, calculate (i) the acceleration during impact (ii) the time interval of the impact and (iii) the impulse.

Sol. Here u = 10 ms^{-1}, v = 0, s = 10 cm = 0.1 m

(i) As $v^2 - u^2 = 2as$

∴ $0^2 - 10^2 = 2a \times 0.1$ or $a = \frac{-100}{2 \times 0.1} = -500\ ms^{-2}$

(ii) As $v = u + at$

$\therefore \quad t = \frac{v-u}{a} = \frac{0-10}{-500} = \frac{10}{500} = 0.02$ s

(iii) Impulse = $F \times t = ma \times t = 1 \times (-500) \times 0.02 = -10$ N

Q. 3. A machine gun has a mass of 20 kg. It fires 30 g bullets at the rate of 400 bullets per second with a speed of 400 ms^{-1}. What force must be applied to the gun to keep it in position?

Sol. For downward motion of the ball:

$u = 0$, $h = 40$ m, $g = +9.8$ ms^{-1}

As $v^2 - u^2 = 2gh \quad \therefore \quad v^2 - 0 = 2 \times 9.8 \times 40$

or $v = \sqrt{784} = 28$ ms^{-1}

For upward motion of the ball:

$u = ?$, $h = 10$ m, $v = 0$ (at the highest point), $g = -9.8$ ms^{-2}

As $v^2 - u^2 = 2gh \quad \therefore \quad 0 - u^2 = -2 \times 9.8 \times 10$

or $u = \sqrt{196} = 14$ ms^{-1}

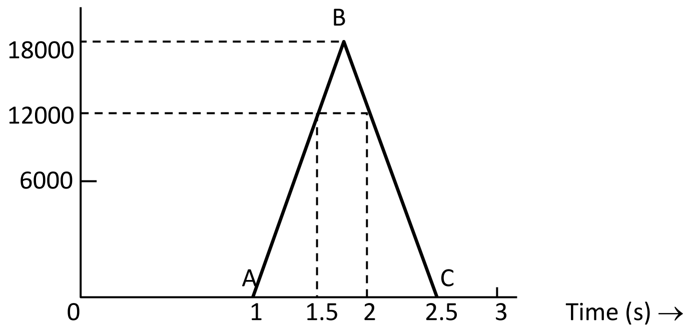
$\therefore$ Change in velocity of the ball $= 28 - (-14) = 42$ ms^{-1}

Impulse = Change in momentum = Mass × Change in velocity

$= \frac{10}{1000}$ kg $\times 42$ ms^{-1} $= 0.42$ Ns

Average force = $\frac{\text{Impulse}}{\text{Time}} = \frac{0.42}{0.1} = 4.2$ N

Q. 4. Fig. shown an estimated force-time graph for a base ball struck by a bat.



From this curve, determine (i) impulse delivered to the ball (ii) force exerted on the ball (iii) the maximum force on the ball.

Sol. (i) Impulse = Area ABC= ½ × 18000 × (2.5 – 1)= 1.35×10^4 kg ms^{-1}

(ii) Force = $\frac{\text{Impulse}}{\text{Time}} = \frac{1.35 \times 10^4}{(2.5-1)} = 9000$ N

(iii) Maximum force = 18000 N

NEWTON'S 3RD LAW OF MOTION :

A/ this law , " To every action , there is an equal and opposite reaction".

- **This simply** means that when two bodies interact , the forces on the bodies from each other always equal in magnitude and opposite in direction .

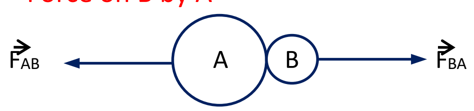
In simple terms, third law can be stated as follows:

Forces in nature always occur between pairs of bodies. Force on body A by body B is equal and opposite of the force on the body B by A.

As shown in fig., if F_{BA} is the force exerted by body A on B and F_{AB} is the force exerted by B on A, then according to Newton's third law.

$F_{AB} = -F_{BA}$

Force on A by B = – Force on B by A



[Newton's third law]

For example, while swimming a man pushes water backward and in turn; he is pushed forward, due to the reaction of water.

The above discussion shows that **a single force can never exist.**

The forces always exist in pairs. The two forces act simultaneously. Any one of them may be called the action and the other reaction.

- No cause-effect relationship exists between action and reaction.

- ***Action and Reaction acts in two different bodies interacting each other (i.e. influencing each other).***

  Consider two bodies A and B press against each other.

  Let, F_1 = Force exerted by 'A' [action]

  F_2 = Force exerted by ' B'[reation]

  Then, $F_1 = --F_2$

- ***A single isolated force cannot exist .***

- ***Force in nature always occurs in pair .***

- ***Action and Reaction force never act on the same body .they act on different bodies .***

- If Action and reaction act on the same body, ,*the resultant force on the body will be zero i.e. the body will be in equilibrium* **.**

- ***Action and Reaction are acting on different never cancel each other .***

- ***Action and Reaction are equal in magnitude but opposite in direction .***

- ***Action & Reaction force act along the line joining the centers of two bodies.***

- ***N's 3rd law of motion is applicable whether the bodies are at rest or in motion.***

- ***N's 3rd law of motion applies to all types of force.***

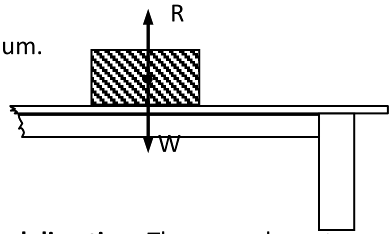
Example:- **[1]** **It** is difficult to derive a nail into a wooden block without holding the block.

[2] Launching a Rocket :-- The gases produced in the combustion chamber of a rocket engine comes out of the rear of the rocket in the downward direction..

[3] The Earth also moves backward when the foot of the man presses it. But the motion of the Earth is so small that we cannot detect it. This is because , the mass of the earth is very large and hence accel^r produced in it due to the force of push is negligible .(i.e., very very small.)

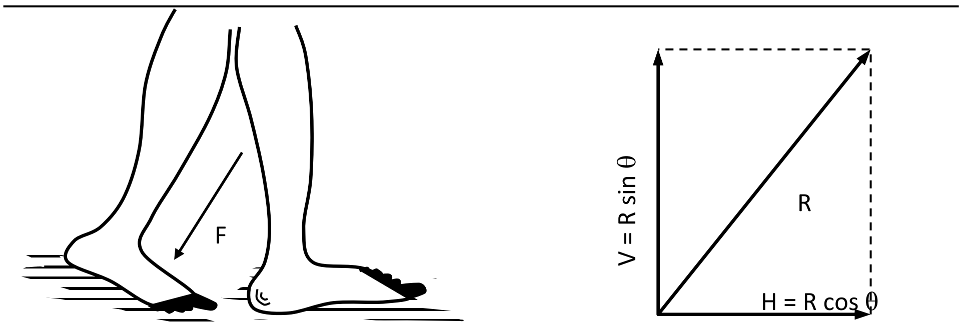
[4]. Book kept on a table: Consider a body of weight N resting on a table top. The body exerts a downward force (action) on the table equal to its own weight W. According to Newton's third law, the table also exerts an equal and upward force R (reaction) on the book such that $R = -W$

As the book is under action of two equal and opposite forces, it remains in equilibrium.



[Forces of action and reaction]

5. While walking, we press the ground (action) with out feet slightly slanted in the backward direction: The ground exerts an equal and opposite force on us. The vertical component of the force of reaction balances our weight and the horizontal component enables us to move forward, as shown in Fig.

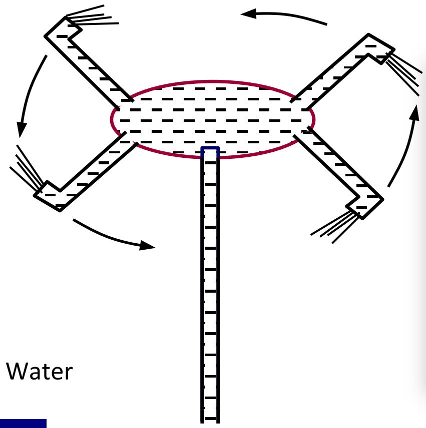


6. It is difficult to walk on a slippery ground or sand because we are unable to push such a ground sufficiently hard: As a result, the force of reaction is not sufficient to help us move forward.

7. It is difficult to drive a nail into a wooden block without supporting it: When we hit the nail with a hammer, the nail and unsupported block together move forward as a single system. There is no reaction. When the block is rested against a support, the reaction of the support holds the block in position and the nail is driven into the wooden block.

8. While swimming, a person pushes water with his hands in the backward direction (action) and water, in turn, pushes him forward due to the force of reaction.

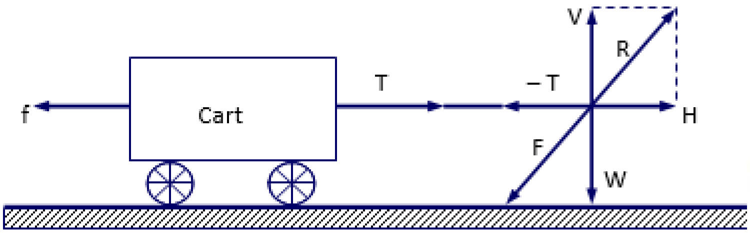
9. Rotatory lawn sprinker: The action of a rotator lawn sprinkler is based on third law of motion. As water issues out of the nozzle, it exerts an equal and opposite force in the backward direction, causing the sprinkler to rotate in the opposite direction. Thus water is scattered in all directions.



[Lawn Sprinkle] Water

HORSE AND CART PROBLEM

Horse and cart problem: As shown in Fig., consider a cart connected to a horse by a string. The horse while pulling the cart produces a tension T in the string in the forward direction (action). The cart, in turn, pulls the horse by an equal force T in the opposite direction.



Initially, the horse presses the ground with a force F in an inclined direction. The reaction R of the ground acts on the horse in the opposite direction. The reaction R has two rectangular components:

□(i) The vertical component V which balances the weight W of the cart.

□(ii) The horizontal component H which helps the horse to move forward.

Let f be the force of friction.

The horse moves forward if H > T. In that case,

net force acting on the horse = H – T.

If the acceleration of the horse is a and m is its mass, then

$H - T = ma$ ... (1)

The cart moves forward it T > f. In that case,

net force acting on the cart = T – f.

The weight of the cart is balanced by the reaction of the ground acting on it.

Obviously, the acceleration acting on the cart is also a. If M is the mass of the cart, then

$T - f = Ma$ ... (2)

Adding (1) and (2), we get

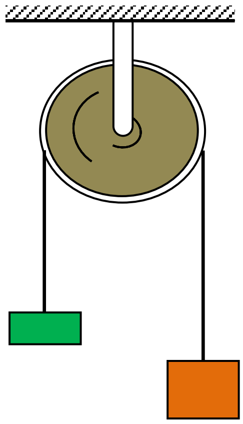
$H - f = (M + m)\,a$ or $\mathbf{a = \frac{H - f}{M + m}}$

Obviously, a is positive if H – f is positive, or if H > f

Thus the system moves if H > f.

► CASE OF PULLEY & MASSES :

Consider two masses M & m connected to the two free ends ends of an inextensible string which passes over a smoothPulley. Let 'T' be the tension in the string . The light mass 'm' moves upward with an accel ʳ 'a' and the heavy mass 'M' moves Downward with an accel ʳ 'a' .



Equation of motion of mass M :-

Resultant downward force acting on mass M is

$F = Mg - T$

But, $F = Ma$ $\therefore$ $Ma = Mg - T$ --------(1)

Equation of motion of mass 'm':-

Resultant upward force acting on mass 'm' is

$F = T - mg$

But, $F = ma$ $\therefore$ $ma = T - mg$ --------- (2)

Adding (1) & (2)

$Ma + ma = Mg - mg$

$a(M + m) = g(M - m)$

$a = \dfrac{(M - m)}{(M + m)} g$ (clearly $a < g$)

putting the value of a in equation (1)

$M \dfrac{(M - m)}{(M + m)} g = Mg - T.$

$T = Mg - Mg \dfrac{(M - m)}{(M + m)}$

$T = Mg \left[1 - \dfrac{(M + m)}{(M + m)}\right]$

$T = Mg \left(\dfrac{M + m - M + m)}{(M+m)}\right) = \dfrac{(2Mm)}{(M + m)} g$

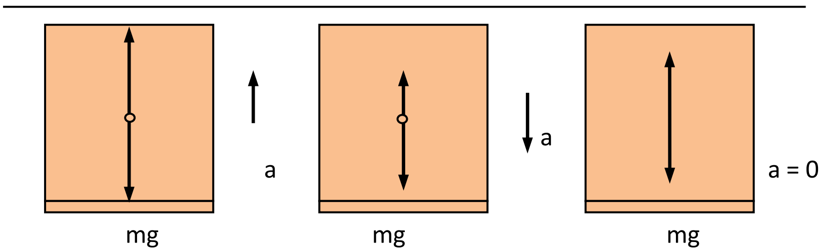
APPARENT WEIGHT OF A MAN IN AN ELEVATOR/LIFT

Consider a man of mass m standing on a weighing machine place in a lift. The actual weight of a man is mg. It acts vertically downwards though the centre of gravity G of the man, it acts on the weighing machine which offers resistance R. The weighing machine reads the reaction R and which is the force experienced by the man. So R is the apparent weight of the man.

(i) When the lift moves upwards with acceleration a: As shown in Fig. (a), the net upward force on the man is

$R - mg = ma$

$\therefore$ Apparent weight, $\boxed{R = m(g + a)}$



[Apparent weight of a man in a lift]

So when a lift accelerates upwards, the apparent weight of the man inside it increases.

(ii) **When the lift moves downwards with acceleration a**: As shown in Fig. (b), the net downward force on the man is

$mg - R = ma$

$\therefore$ Apparent weight, $\boxed{R = m(g - a)}$

So when a lift accelerates downwards, the apparent weight of a man inside it decreases.

(iii) When the lift is at rest or moving with uniform velocity v downward/upward. As shown in Fig. (c), the acceleration a = 0. Net force on the man is $R - mg = m \times 0 = 0$

$R = mg$

or Apparent weight = Actual weight

(iv)When the lift falls freely: If the supporting cable of the lift brakes, the lift falls freely under gravity. Then a = g.

The net downward force on the man is

$R = m(g - g) = 0$

Thus the apparent weight of the man becomes zero. This is because both the man and the lift are moving downwards with the same acceleration 'g' and so there are no forces of action and reaction between so there are no forces of action and reaction between the man and the lift. Hence a person develops a feeling of ***weightlessness*** when he falls freely under gravity.

2nd law is the Real Law of Motion:

(I) First Law is contained in 2 nd Law :

A/2 nd law of motion , $F = ma$.

If $F = 0$ i.e. no external force acts on the body , then $ma = 0$ or, $a = 0$ [as m is not = 0]

This shows that the body at rest will remain at rest or a body moving with uniform motion will continue to move if no External force acts on it .

This is the def n of 1 st law .Hence first law is contained in the 2 nd law.

(II) Third law is contained in the 2 nd law :-

Consider an isolated system (i.e. system in which no external force acts) consisting of two particles 1 & 2.

Let $\vec{F}_{12}$ = Force exerted on the body 1 by the body 2(action)

$\vec{F}_{21}$ = ,, ,, ,, ,, ,, 2 ,, ,, ,, 1. (reaction)

If $\frac{d\vec{p}_1}{dt}$ = Rate of change of momentum of body 1

& $\frac{d\vec{p}_2}{dt}$ = Rate of change of momentum of body 2

then $\vec{F}_{12} = \frac{d\vec{p}_1}{dt}$ & $\vec{F}_{21} = \frac{d\vec{p}_2}{dt}$

On adding , $\vec{F}_{12} + \vec{F}_{21} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = \frac{d}{dt}(\vec{P}_1 + \vec{P}_2)$

Since , no external force acts on the system , then A/N's 2nd law of motion , $\frac{d}{dt}(P_1 + P_2) = 0$

$\therefore$ $\mathbf{F_{12} + F_{21} = 0}$ **or** $\mathbf{F_{12} = -- F_{21}}$ **, which 3rd law of motion .**

➔ **As both the 1st & 3rd law of motion are contained in 2nd law , therefore, 2nd law of motion is the real law .**

LAW OF COSERVATION OF MOMENTUM: According to this law --

"The total linear momentum of an isolated system is conserved if the net external force acting in the system is zero".

Derivation (from N's 2nd law)

A/ N's 2nd law motion , $F = \frac{dp}{dt}$

If $F = 0$, then $\frac{dp}{dt} = 0$ therefore , P = constant [differential coefficient of an isolated constant is 0]

Consider a system having 'n' particles , Let P_1 , P_2 , P_3-----------P_n be the linear momentum of various particles of the system.

$\therefore$ linear momentum , $P = P_1 + P_2 + P_3 + P_4 \text{------------} + P_n$.

If F(ext.) ,be the force acting on the system , then, $F(ext) = \frac{dp}{dt} = \frac{d}{dt}(P_1 + P_2 + P_3 + P_4 \text{------------} + P_n)$

If, F(ext), = 0 then $\frac{d}{dt}(P_1 + P_2 + P_3 + P_4 \text{------------} + P_n) = 0$

Or, $P_1 + P_2 + P_3 + P_4 \text{------------} + P_n$ =constant.

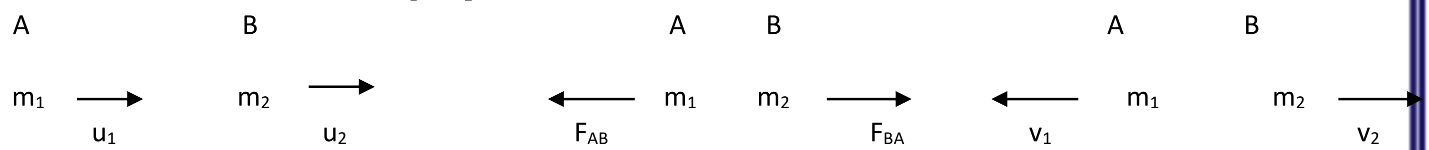
Thus, In the absence of external forces , the total momentum of the system is conserved.

Derivation (from N's 3rd law)

Consider an isolated system of two bodies of masses m_1 & m_2

Let the bodies are moving with velocities u_1 & u_2 resp. in a straight line in the same direction ($u_1 > u_2$), after collision ,

Let these bodies move with velocities v_1 & v_2



Before collision During collision After collision

Total momentum of the system (before collision),

= Linear momentum of A + linear momentum of B. = $\mathbf{m_1 u_1}$ + $\mathbf{m_2 u_2}$ ------------------(i)

Total momentum of the system (after the collision),

= Linear momentum of A + linear momentum of B. = $\mathbf{m_1v_1}$ + $\mathbf{m_2 v_2}$ ------------------(ii)

.'. Change of momentum of A (after collision) = $m_1v_1 - m_1 u_1$

,, ,, ,, ,, B (,, ,,) = $m_2 v_2 - m_2 u_2$

During the collision , let the body B exert a force F_{AB} on the body A . A/N's 3rd law , the body A will exert a force F_{BA} on the body B such that ,

$\mathbf{F_{BA} = - F_{AB}}$ ---------------------(iii)

If 't' be the time for which the collision takes place then impulse acting on A = $F_{AB} \times t$

Similarly, ,, ,, ,, B = $F_{BA} \times t$

but, Impulse = change in momentum

.'. $\mathbf{F_{AB} \times t = m_1v_1 - m_1 u_1}$ & $\mathbf{F_{BA} \times t = m_2 v_2 - m_2 u_2}$

from (iii) $m_2 v_2 - m_2 u_2 = -(m_1v_1 - m_1 u_1)$

$m_1 u_1 + m_2 u_2 = m_1v_1 + m_2 v_2$

i.e. momentum before collision = momentum after collision

Hence, for an isolated system , the total linear momentum remains conserved

Derivation of Newton's third law of motion from the law of conservation of momentum:

Consider two bodies of masses m_1 and m_2 moving along a straight line and colliding against each other. The velocities and hence momenta of the two bodies undergo a change. Let Δp_1 and Δp_2 be the changes in the momenta produced in time Δt.

According to the law of conservation of momentum, the net change in the linear momentum in the absence of external force is zero. ∴ $\Delta p_1 + \Delta p_2 = 0$ or $\Delta p_2 = -\Delta p_1$

or $\dfrac{\Delta p_2}{\Delta t} = -\dfrac{\Delta p_1}{\Delta t}$

or Rate of change of momentum of m_2 = – Rate of change of momentum of m_1

or Force on m_2 = – Force on m_1

or Action = – Reaction ; This proves the Newton's third law of motion.

PRACTICAL APPLICATION OF THE LAW OF CONSERVATION OF MOMENTUM

(i) Recoil of a gun: Let 'M' be the mass of the gun and 'm' be the mass of the bullet. Before firing, both the gun and the bullet are at rest. After firing, the bullet moves with velocity v and the gun moves with velocity V. As no external force acts on the system, so according to the principle of conservation of momentum,

Total momentum before firing = Total momentum after firing

or $0 = m\vec{v} + M\vec{V}$

or $M\vec{V} = -m\vec{v}$ or $\vec{V} = -\dfrac{m}{M}\vec{v}$

The negative sign shows that V and v are in opposite directions i.e., the gun gives a kick in the backward direction or the gun recoils with velocity V. Further, as M >> m, so V << v i.e., the recoil velocity of the gun is much smaller than the forward velocity of the bullet.

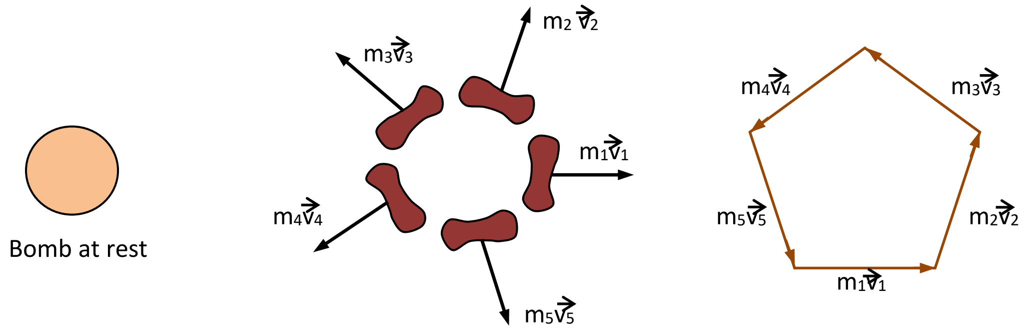
(ii) While firing a bullet, the gun should be held tight to the shoulder: The recoiling gun can hurt the shoulder. If the gun is held tightly against the shoulder, then the body and the gun together will constitute one system. Total mass becomes large and the recoil velocity becomes small.

(iii) When a man jumps out of a boat to the shore, the boat slightly moves away from the shore: Initially, the total momentum of the boat and the man is zero. As the man jumps from the boat to the shore, he gains a momentum in the forward direction. To conserve momentum, the boat also gains an equal momentum in the opposite direction. So the boat slightly moves backwards.

(iv) An astronaut in open space, who wants to return to the spaceship, throws some object in a direction opposite to the direction of motion of the spaceship: By doing so, he gains a momentum equal and opposite to that of the thrown object and so he moves towards the spaceship.

(v) Rocket and jet planes work on the principle of conservation of momentum: Initially, both the rocket and its fuel are at rest. Their total momentum is zero. For rocket propulsion, the fuel is exploded. The burnt gases are allowed to escape through a nozzle with a very high downward velocity. The gases carry a large momentum in the downward direction. To conserve momentum, the rocket also acquires an acquires and equal momentum in the upward direction and hence starts moving upwards

(vi) Explosion of a bomb: Before explosion, suppose the bomb is at rest. Its total momentum is zero. At is explodes, it breaks up into many parts of masses m_1, m_2, m_3, etc., which fly off in different direction with velocities $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, etc.



[An exploding bomb $\sum m\vec{v} = 0$]

The different parts have definite momenta $m_1\vec{v}_1$, $m_2\vec{v}_2$, $m_3\vec{v}_3$, etc.

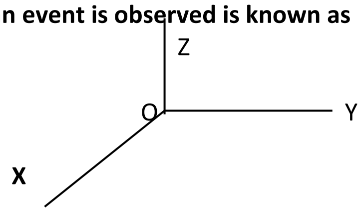
The momenta of the various parts can be represented by the sides of a closed polygon taken in the same order. This indicates that the total momentum after explosion is zero i.e., momentum is conserved. If bomb explodes into two parts, they will fly off in opposite directions with equal momenta.

◘ FRAME OF REFERENCE :- [OBSERVER]

The state of rest and motion of a body are relative term. In order to measure the position of a body in space ,it Is essential to specify reference mark w.r.t we can take observation .This reference mark is called frame of reference.

DEFINATION:--"**The system of co-ordinate axes or any reference mark w.r.t which an event is observed is known as Frame of reference".**

► Three mutually perpendicular axes i.e. OX, OY, & OZ forms frame of reference .



► **Types of frame of reference ——**

(I) Inertial frame of reference

(II) Non- inertial frame of reference.

I. Inertial of frame of reference

: "Any frame of reference which is either at rest or moving with uniform velocity is called inertial frame of reference".

EXAMPLES— (I) Consider a ball lying on the floor of a bus which is at rest . This ball remains till the bus is stationary Or moving with uniform velocity . Therefore , Bus is an example of an Inertial of reference .

(II) The fixed star in the sky are inertial frame of reference .

(III) For all practical purposes is a frame of reference fixed on the Earth can be considered as inertial frame .

☻ Newton's 1st law of motion hold good.

II. Accelerated or Non – Inertial frame of reference:--

"A frame of reference which is accelerated is called accelerated or non- inertial frame of reference".

EX: Consider a ball lying on the floor of the bus which is at rest. This ball starts moving when the bus is accelerated or Retarded .Therefore, bus (in this particular case) is an example of non – inertial frame of reference.

➔ The force which causes the motion in non – inertial frame of reference is known FICTITOUS force .

➔ Rotating reference frame are non- inertial frame.

➔ Since , Earth revolves round the sun and also about its own axis , so it is a rotating frame of reference .Hence as such , Earth is a non – inertial frame of reference.
However ,If we do not take large scale motion such as Wind & ocean current into consideration , we can make the Approximation that the Earth is an inertial frame .

➔ The non-inertial character of the earth is evident from the fact that a falling body does not straight down but slight deflect to the East.

☻ NEWTON"S 1st law is not valid.

Explanation : Suppose a train 'A' goes past the platform with some acceleration 'a'. For an observer on this train , the box Placed in the platform appear to move with an acceleration '-a' .
Here, Net force = 0 , But net force does not imply ZERO acceleration .

Examples based on Newton's Third law and Motion in a Lift

✖FORMULAE USED

1. Reaction = – Action
2. The apparent weight of a man in a lift:
   (i) When the lift moves upwards with acceleration a,
   R = m (g + a)
   (ii) When the lift moves downwards with acceleration a, R = m (g – a)
   (iii) When the lift falls freely, a = g, so R = m (g – a) = m (g – g) = 0
   (iv) When the lift is at rest or moves with uniform velocity, a = 0, so R = m (g – 0) = mg

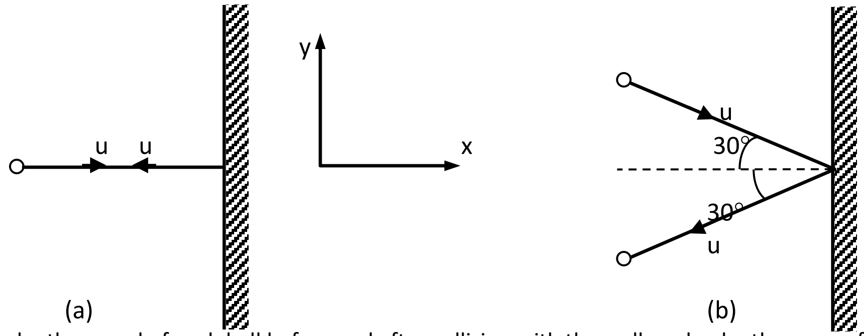
✖UNITS USED

The absolute SI unit of force is newton and CGS unit is dyne. The gravitational SI unit of force is kg f or kg wt and CGS unit is gf or gwt.

Q. 1. Ten one-rupee coins are put on top of each other on a table. Each coin has a mass m kg. Give the magnitude and direction of (i) the force on the 7th coin (counted from the bottom) due to all the coins on its top.
(ii) the force on the 7th coin by the eighth coin.
(iii) the reaction of the 6th coin on 7th coin.

Sol. (i) Force on the 7th coin = Force due to 3 coins on/its top = 3 mg
(ii) Force on the 7th coin by the 8th coin = Masses of 8th, 9th and 10th coins × g = 3 mg
(iv) Reaction of the 6th coin on the 7th coin = Force on the 6th coin due to 7th coin = 4 mg
(v)

Q. 2. Two identical billiard balls strike a rigid wall with the same speed but at different angles, and get reflected without any loss of speed, as shown in Fig. What is (i) the direction of the force of the wall due to each ball? and (ii) the ratio of the magnitude of the impulses imparted on the two balls by the wall?



(a) (b)

Sol. (i) Let u be the speed of each ball before and after collision with the wall, and m be the mass of each ball. Choose x– and y– axes as shown. In Fig. (a),

$p_x^{\text{initial}} = mu$, $p_y^{\text{initial}} = 0$
$p_x^{\text{final}} = -mu$, $p_y^{\text{final}} = 0$

As impulse = change in momentum
∴ x-component of impulse = – mu – mu = – 2mu
y-component of impulse = 0 – 0 = 0

Clearly, the direction of the impulse is along the negative x-direction of motion.
As the direction of the force is same as that of impulse, so the force on the ball due to the wall is normal to the wall, along the negative x-direction of motion.
By Newton's third law of motion, the force on the wall due to the ball is normal to the wall along the positive x-direction. In Fig. (b),

$p_x^{\text{initial}} = mu \cos 30°$, $p_y^{\text{initial}} = -mu \sin 30°$
$p_x^{\text{final}} = -mu \cos 30°$, $p_y^{\text{final}} = -mu \sin 30°$

∴ x – component of impulse = – mu cos 30° – mu cos 30° = – 2 mu cos 30°
y – component of impulse = – mu sin 30° + my sin 30° = 0

Again, the direction of the impulse is normal to the wall along the negative x-direction. By newton's third law, the force on wall due to the ball is normal to the wall along the positive x-direction.

Q. 3. An elevator weighs 4000 kg. When the upward tension in the supporting cable is 48000 N, what is the upward acceleration? Starting from rest, how far does it rise in 3 s?

Sol. Mass of elevator, M = 4000 kg
Weight of elevator = Mg = 4000 kg wt = 4000 × 9.8 = 392000 N
Upward tension, T = 48000 N
Net upward force on the elevator,

F′ = T – Mg = 48000 – 39200 = 8800 N

Upward acceleration, $a = \frac{F'}{M} = \frac{8800}{4000} = 2.2 \text{ ms}^{-2}$

For upward motion: $u = 0$, $a = 2.2\ ms^{-2}$, $t = 3$ s

∴ $s = ut + ½\ at^2 = 0 \times 3 + ½ \times 2.2 \times 3^2 = 9.9$ m

Q. 4. A man weights 70 kg. He stands on a weighing machine in a lift, which is moving.
(i) upwards with a uniform speed of 10 ms^{-1}, (ii) downwards with a uniform acceleration of 5 ms^{-2}.
(iii) upwards with a uniform acceleration of 5 ms^{-2}.
What would be the readings on the scale in each case? What would be the reading, if the lift mechanism failed and it came down freely under gravity?

Sol. The apparent weight measured by the weighting machine is the measure of the reaction R exerted on the man due to the lift.

(i) When the lift moves upward with uniform velocity, reaction of the lift is equal to the weight of the man.

∴ Apparent weight, $R = mg = 70 \times 9.8 = 686$ N = 70 kg wt.

(ii) When the lift moves downwards with uniform acceleration, $a = 5\ ms^{-2}$.

Resultant downward force, $F = mg - R$

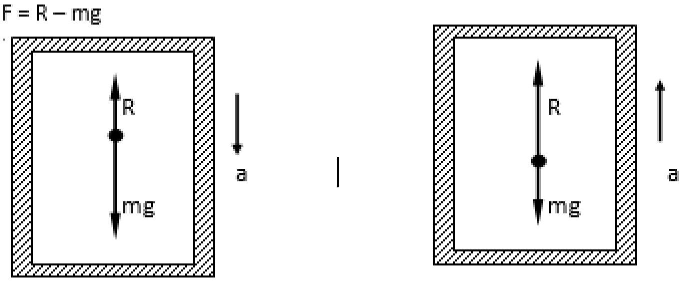
or $ma = mg - R$

∴ Apparent weight, $R = m (g - a) = 70 (9.8 - 5) = 70 \times 4.8 = 336$ N = 34.29 kg wt.

(iii) When the lift moves upwards with uniform acceleration, $a = 5\ ms^{-2}$

Resultant upward force, $F = R - mg$

or $ma = R - mg$



∴Apparent weight, $R = m (g + a) = 70 (9.8 + 5) = 70 \times 14.8 = 1036$ N = 105.7 kg wt

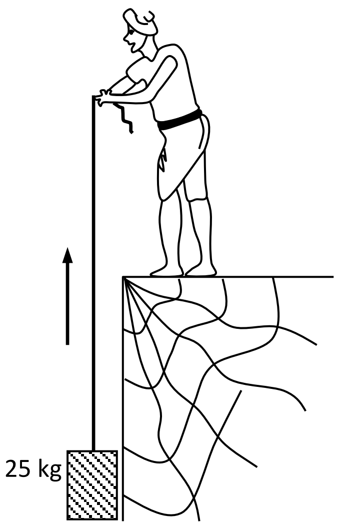
When the lift falls freely under gravity, $a = g$

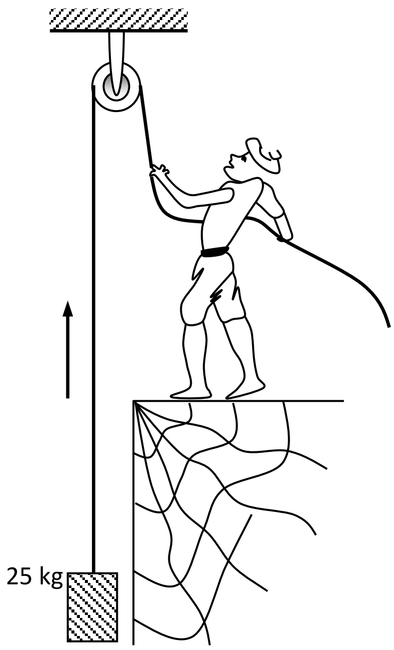
∴ Apparent weight, $R = m (g - a) = m (g - g) = 0$ This is the condition of weightlessness.

Q. 5. A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in Fig. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding?

Sol. In mode (a), the man applies force equal to 25 kg wt in the upward direction. According to Newton's third law of motion, there will be a downward force of reaction on the floor.

∴ Total action on the floor by the man
= 50 kg wt + 25 kg wt = 75 kg wt
= 75×9.8 N = 735 N



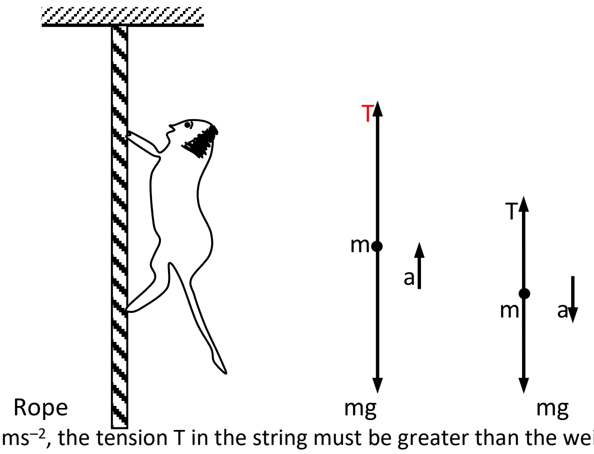


In mode (b), the man applies a downward force equal to 25 kg wt. According to Newton's third law, the reaction will be in the upward direction.

∴ Total action on the floor by the man = 50 kg wt – 25 kg wt = 25 wt = 25×9.8 N = 245 N

As the floor yields to a downward force of 700 N, so the man should adopt mode (b).

Q. 6. A monkey of mass 40 kg climbs on a rope which can stand a maximum tension of 600 N [Fig]. In which of the following cases will the rope break: the monkey
(i) climb up with an acceleration of 6 ms^{-2}
(ii) climbs down with an acceleration of 4 ms^{-2}
(iii) climbs up with a uniform speed of 5 ms^{-1}
(iv) falls down the rope nearly free under gravity.
Take = g = 10 ms^{-2}. Ignore the mass of the rope.



Sol. (i) When the monkey climbs up with an acceleration a = 6 ms^{-2}, the tension T in the string must be greater than the weight of the monkey [Fig. (a)],

$T - mg = ma$

or $T = m(g + a) = 40(10 + 6) = 640$ N

(ii) When the monkey climbs down with an acceleration, a = 4 ms^{-2} [Fig. (b)]

$mg - T = ma$

or $T = m(g - a) = 40(10 - 4) = 240$ N

(iii) When the monkey climbs up with uniform speed, $T = mg = 40 \times 10 = 400$ N

(iv) When the monkey falls down the rope nearly freely, a = g

$\therefore$ $T = m(g - a) = m(g - g) = 0$

As the tension in the rope of case (i) is greater than the maximum permissible tension (600 N), so the rope will break in case (i) only.

Q. 7. A helicopter of mass 1000 kg rises with a vertical acceleration of 15 ms^{-2}. The crew and the passengers weigh 300 kg. Give the magnitude and direction of
(i) force on the floor by the crew and passengers. (ii) action of the rotor of the helicopter on the surrounding air.
(iii) force on the helicopter due to the surrounding air. [Take g = 10 ms^{-2}]

Sol. Mass of helicopter, M = 1000 kg

Mass of the crew and passengers, m = 300 kg   Vertically upward acceleration, a = 15 ms^{-2}

(i) Force on the floor by the crew and passengers, F = Apparent weight = m (g + a)= 300 (10 + 15) = 7500 N, vertically downwards.

(ii) Action of the rotor of the helicopter on the surrounding air = Apparent weight of the helicopter, crew and passengers

$= (M + m)(g + a) = (1000 + 300)(10 + 15)$ = 21500 N, vertically downwards

(iii) Force on the helicopter due to the surrounding air is equal and opposite to the action of the rotor of the helicopter on the surrounding air.

$\therefore$ Force on surrounding air = 32500 N, vertically upwards

Q. 8. A lift of mass 2000 kg is supported by thick steel ropes. If maximum upward acceleration of the lift be 1.2 ms^{-2}, and the breaking stress for the ropes be 2.8×10^8 Nm^{-2}, what should be the minimum diameter of rope?

Sol. Here m = 2000 kg, a = 1.2 ms^{-2}, breaking stress = 2.8×10^8 Nm^{-2}.

As the lift moves upwards, so the tension in the rope is

$T = m(g + a) = 2000(9.8 + 1.2) = 22{,}000$ N.

Now, breaking stress = $\frac{Force}{Area} = \frac{T}{\pi D^2/4} = \frac{4T}{\pi D^2}$

or $2.8 \times 10^4 = \frac{4 \times 22{,}000 \times 7}{22 \times D^2}$

or $D^2 = \frac{4 \times 22{,}000 \times 7}{22 \times 2.8 \times 10^8} = 10^{-4}$

or $D = 10^{-2}$ m = 1 cm

Problems For Practice

Q. 1. An elevator weighing 5000 kg is moving upward and tension in the supporting cable is 50,000 N. Find the upward acceleration. How far does it rise in a time of 10 seconds starting from rest?

Sol. Use T – mg = ma

Q. 2. A woman weighing 50 kgf stands on a weighing machine placed in lift. What will be the reading of the machine, when the lift is (i) moving upwards with a uniform velocity of 5 ms^{-1} and (ii) moving downwards with a uniform acceleration of 1 ms^{-2}? Take g = 10 ms^{-2}.

Sol. R = m (g + a) = 75 (10 + 2) N = 900 N = 90 kg f.

Q.3. **Find the apparent weight of a man weighting 49 kg on earth when he is standing in a lift which is (i) rising with an acceleration of 1.2 ms^{-2} (ii) going down with the same acceleration (iii) falling freely under the action of gravity and (iv) going up or down with uniform velocity. Given g = 9.8 ms^{-2}.**

Sol. (i) $R = m(g + a) = 49(1.2 + 9.8) = 49 \times 11$ N

$= \frac{49 \times 11}{9.8} = 55$ kg f

(ii) $R = m(g - a) = 49(9.8 - 1.2)$ N = 43 kg f.

(iii) $R = m(g - g) = 0$

(iv) $R = m(g - 0) = mg = 49$ kg f

Q. 4. **A body of mass 15 kg is hung by a spring balance in a lift. What would be the reading of the balance when (i) the lift is ascending with an acceleration of 2 ms^{-2} (ii) descending with the same acceleration (iii) descending with a constant velocity of 2 ms^{-1}? Take g = 10 ms^{-2}.**

Sol. (i) $R = m(g + a) = 15(10 + 2) = 180$ N = 18 kg f (ii) $R = m(g - a) = 15(10 - 2) = 120$ N = 12 kg f

(iii) Here a = 0 ∴ R = mg = 15 kg f

Q. 5. **The strings of a parachute can bear a maximum tension of 72 kg wt. By what minimum acceleration can a person of 96 kg descend by means of this parachute?**

Sol. For the person to descend, $T = m(g - a)$

∴ $72 \times 9.8 = 96(9.8 - a)$ or $a = 2.45$ ms^{-2}

Q. 6. **A 70 kg man is sea is being lifted by a helicopter with the help of a rope which can bear a maximum tension of 100 kg wt. With what maximum acceleration the helicopter should rise so that the rope may not break?**

Sol. For the rising helicopter, $T = m(g + a)$

∴ $100 \times 9.8 = 70(9.8 + a)$ or $a = 4.2$ ms^{-2}

Q. 7. **An elevator and its load weigh a total of 800 kg. Find the tension T in the supporting cable when the elevator, originally moving downwards at 20 ms^{-1} is brought to rest with constant retardation in a distance of 50 m.**

Sol. Here m = 800 kg, u = 20 ms^{-1}, v = 0, s = 50 m, T = ?

As $v^2 - u^2 = 2as$ ∴ $0^2 - (20)^2 = 2a \times 50$

or $a = -4$ ms^{-2}

For the elevator moving downwards,

$T = m(g - a) = 800(9.8 + 4) = 1.104 \times 10^4$ N.

Examples based on Conservation of Linear Momentum

✖Formulae Used

1. In the absence of any external force, vector sum of the linear momenta of a system of particles remains constant.

$m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + ... + m_n \vec{v}_n = \text{constant}$

2. For a two body system, $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

3. Recoil velocity of a gun, $V = -\frac{mv}{M}$

where M is the mass of the gun, m the mass of bullet and v is the velocity of the bullet.

✖Units Used All masses are in kg, velocities in ms^{-1} and linear momenta in kg ms^{-1}.

Q. 1. **A shell of mass 0.02 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 ms^{-1}, what is the recoil speed of the gun?**

Sol. Mass of shell, m = 0.02 kg

Mass of gun, M = 100 kg

Speed of shell, v = 80 ms^{-1}

Let V be the recoil speed of gun. According to the law of conservation of momentum,

Initial momentum = Final momentum

or $0 = mv + MV$

∴ $V = -\frac{mv}{M} = \frac{0.02 \times 80}{100} = -0.016$ ms^{-1} Negative sign indicates that the gun moves backward as the bullet moves forward.

Q. 2. **A 30 kg shell is flying at 48 ms^{-1}. When it explodes, its one part of 18 kg stops, while the remaining part files on. Find the velocity of the later.**

Sol. Mass of shell, M = 30 kg Velocity of shell, u = 48 ms^{-1}

After explosion, mass of first part $m_1 = 18$ kg Velocity of first part, $v_1 = 0$

Mass of second part, $m_2 = 30 - 18 = 12$ kg

If v_2 is the velocity of second part, then from the law of conservation of momentum,

$Mu = m_1 v_1 + m_2 v_2$

or $30 \times 48 = 18 \times 0 + 12 \times v_2$

or $v_2 = \frac{30 \times 48}{12} = 120\ ms^{-1}$

Q. 3. A nucleus is at rest in the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei, the products must be emitted in opposite directions.

Sol. Let M be the mass of the nucleus at rest. Suppose it disintegrates into two smaller nuclei of masses m_1 and m_2 which move with velocities v_1 and v_2 respectively.

∴ Momentum before disintegration = $M \times 0 = 0$

Momentum after disintegration = $m_1 v_1 + m_2 v_2$

According to the law of conservation of momentum, $m_1 v_1 + m_2 v_2 = 0$

or $v_2 = -\frac{m_2}{m_1} . v_1$

As masses m_1 and m_2 cannot be negative, the above equation shows that v_1 and v_2 must have opposite signs i.e., the two products must be emitted in opposite directions.

Q. 4. A neutron having a mass 1.67×10^{-27} kg and moving at $10^8\ ms^{-1}$ collides with a deuteron at rest and sticks to it. If the mass of deuteron at rest and sticks to it. If the mass of deuteron is 3.34×10^{-27} kg, find the speed of the combination.

Sol. For neutron: $m_1 = 1.67 \times 10^{-27}$ kg, $u_1 = 10^8\ ms^{-1}$

For deuteron: $m_2 = 3.34 \times 10^{-27}$ kg, $u_2 = 0$

Let v be the speed of the combination. Then by conservation of momentum,

$m_1u_1 + m_2 u_2 = (m_1 + m_2) v$

$1.67 \times 10^{-27} \times 10^8 + 3.34 \times 10^{-27} \times 0 = (1.67 + 3.34) \times 10^{-27} \times v$

$v = \frac{1.67 \times 10^{-27} \times 10^8}{5.01 \times 10^{-27}} = 0.333 \times 10^8\ ms^{-1}$

Q. 5. A car of mass 1000 kg travelling at 32 ms^{-1} dashes into the rear of a truck of mass 8000 kg moving in the same direction with a velocity of 4 ms^{-1}. After the collision, the car bounces with a velocity of 8 ms^{-1}. What is the velocity of truck after the impact?

Sol. For the car: $m_1 = 1000$ kg, $u_1 = 32\ ms^{-1}$, $v_1 = -8\ ms^{-1}$

For the truck: $m_2 = 8000$ kg, $u_2 = 4\ ms^{-1}$, $v_2 = ?$

By conservation of linear momentum, $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$1000 \times 32 + 8000 \times 4 = 1000 \times (-8) + 8000 v_2$

or $64000 + 8000 = 8000 v_2$

or $v_2 = \frac{72000}{8000} = 9\ ms^{-1}$ **[In the same direction]**

Q. 6. A hunter has a machine gun that can fire 50 g bullets with a velocity of 150 ms^{-1}. A 60 kg tiger springs at him with a velocity of 10 ms^{-1}. How many bullets must be hunter fire into the tiger in order to stop him in track?

Sol. Mass of bullet, m = 50 g = 0.05 kg ; Velocity of bullet, v = 150 ms^{-1}

Mass of tiger, M = 60 kg ; Velocity of tiger = 10 ms^{-1}

Let n be the number of bullets required to be pumped into the tiger to stop him in his track.

According to the law of conservation of momentum,

Magnitude of the momentum of n bullets = Magnitude of the momentum of tiger

or $n \times mv = MV$ or $n = \frac{MV}{mv} = \frac{60 \times 10}{0.05 \times 150} = 80$

Q. 7. A disc of mass 10 g is kept floating horizontally by throwing 10 marbles per second against it from below. If the mass of each marble is 5 g, calculate the velocity with which the marbles are striking the disc. Assume that the marbles strike the disc normally and rebound downward with the same speed.

Sol. Mass of each marble piece, $m = 5\ g = 5 \times 10^{-3}$ kg

Number of marbles thrown per second = 10

Let velocity of impact of each marble = v

Change in momentum of each marble = mv – (– mv) = 2 mv

Change in momentum per second = 2mv × 10

∴ Force exerted by marbles on the disc = 20 mv

But the disc can be kept floating if this force balances the weight of the disc. ∴ 20 mv = Mg

$20 \times 5 \times 10^{-3} \times v = 10 \times 10^{-3} \times 9.8$ => $v = \frac{10 \times 9.8}{100} = 0.98\ ms^{-1} = 98\ cms^{-1}$

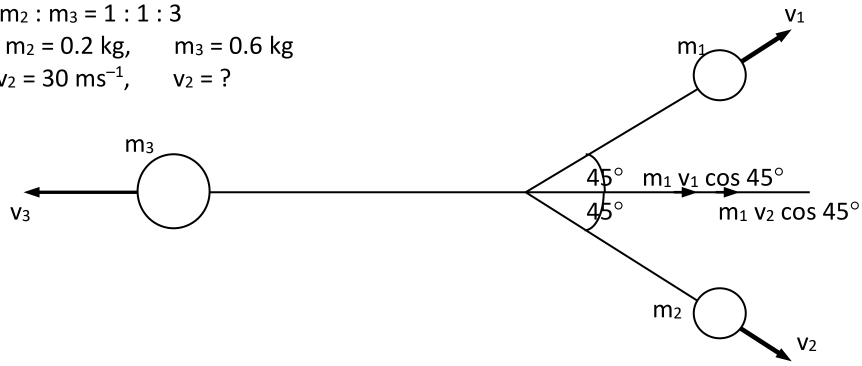
Q. 8. A body of mass 1 kg initially at rest explodes and breaks into three fragments of masses in the ratio 1:1:3. The two pieces of equal mass fly off perpendicular to each other with a speed of 30 ms^{-1} each. What is the velocity of the heavier fragment?

Sol. Here $m_1 + m_2 + m_3 = 1$ kg

As $m_1 : m_2 : m_3 = 1 : 1 : 3$

$\therefore$ $m_1 = m_2 = 0.2$ kg, $m_3 = 0.6$ kg

$v_1 = v_2 = 30$ ms^{-1}, $v_2 = ?$



Applying the law of conservation of momentum to the momenta along horizontal direction,

$$m_3 v_3 = m_1 v_1 \cos 45° + m_2 v_2 \cos 45°$$

$$0.6\, v_3 = 0.2 \times 30 \times 0.707 + 0.2 \times 30 \times 0.707$$

or $$v_3 = \frac{2 \times 0.2 \times 30 \times 0.707}{0.6} = 14.14 \text{ ms}^{-1}$$

~~Problems For Practice~~

Q. 1. A bomb at rest explodes into three fragments of equal masses. Two fragments fly off at right angles to each other with velocities 9 ms^{-1} and 12 ms^{-1} respectively. Calculate the speed of the third fragment.

Sol. Let $\vec{p}_1$, $\vec{p}_2$ and $\vec{p}_3$ be the momenta and $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$ be the velocities of the three fragments respectively

Then $\vec{p}_1 = m\vec{v}_1$, $\vec{p}_2 = m\vec{v}_2$, $\vec{p}_3 = m\vec{v}_3$,

$v_1 = 9$ ms^{-1}, $v_2 = 12$ ms^{-1}, $v_3 = ?$

As $\vec{p}_1 \perp \vec{p}_2$, so the magnitude of their resultant is

$$p = \sqrt{p_1^2 + p_2^2} = m\sqrt{v_1^2 + v_2^2}$$

$$= m\sqrt{9^2 + 12^2} = 15 \text{ m kg ms}^{-1}$$

By conservation of linear momentum, $\vec{p} + \vec{p}_3 = 0$

In magnitude, $p_3 = p$ or $mv_3 = p$

$\therefore$ $$v_3 = \frac{p}{m} = \frac{15\,m}{m} = 15 \text{ ms}^{-1}$$

Q. 2. A man weighing 60 kg runs along the rails with a velocity of 18 kgh^{-1} and jumps into a car of mass 1 quintal standing on the rails. Calculate the velocity with which the car will start travelling along the rails.

Sol. Here $m_1 = 60$ kg, $u_1 = 18$ kmh^{-1} = 5 ms^{-1},

$m_2 = 1$ quintal = 100 kg, $u_2 = 0$, $v = ?$

By conservation of linear momentum,

$$(m_1 + m_2)\, v = m_1 u_1 + m_2 u_2$$

or $$(60 + 100)\, v = 60 \times 5 + 100 \times 0$$

or $$v = \frac{60 \times 5}{160} = \frac{15}{8} = 1.88 \text{ ms}^{-1}$$

Q. 3. A machine gun of mass 10 kg fires 20 g bullets at the rate of 10 bullets per second with a speed of 500 ms^{-1}. What force is required to hold the gun in position?

Sol. Change in the momentum of one bullet = m (v – u) = 2×10^{-3} (500 – 0) = 10 kg ms^{-1}

Change in momentum of 10 bullets = $10 \times 10 = 100$ kg ms^{-1}

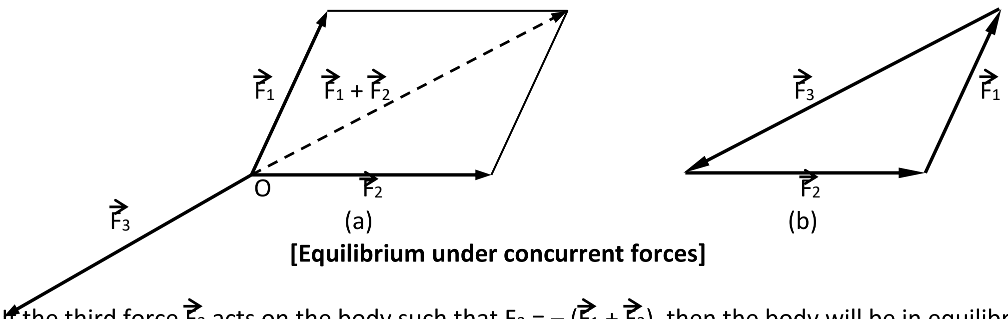
Force required to hold the gun

$$= \frac{\text{Change in momentum}}{\text{Time taken}} = \frac{100 \text{ kg ms}^{-1}}{1\text{s}} = 100 \text{ N}$$

EQUILIBRIUM OF CONCURRENT FORCES

Forces acting at the same point on a body are called concurrent forces. When a number of forces act on a body at the same point and the net unbalanced force is zero, the body will continue in its state of rest or of uniform along a straight line and is said to be in equilibrium.

Consider three concurrent forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ acting at the same point O of body, as shown in Fig. (a). By parallelogram law, Resultant of $\vec{F}_1$ and $\vec{F}_2 = \vec{F}_1 + \vec{F}_2$



[Equilibrium under concurrent forces]

If the third force $\vec{F}_3$ acts on the body such that $\vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$, then the body will be in equilibrium.

i.e., $\vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$

or $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

As shown in Fig. (b), these three forces in equilibrium can be represented by the sides of a triangle taken in the same order.

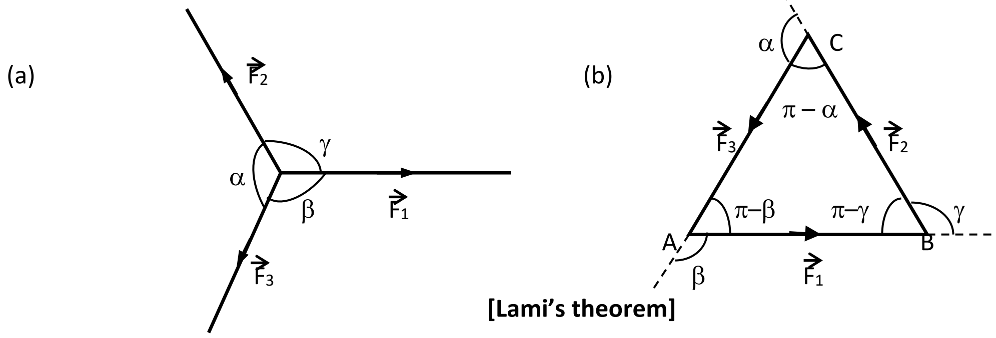
Thus the condition for the equilibrium of a number of forces acting at the same point is that the vector sum of all these forces is equal to zero.

i.e., $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + ... + \vec{F}_n = 0$

In general, particle is in equilibrium under the action of n forces if these forces can be represented by the sides of closed n-sides polygon taken in the same order.

Lami's theorem: Fig. (a) shows a particle O under the equilibrium of three concurrent forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$.

Let α be angle between $\vec{F}_2$ and $\vec{F}_3$, β between $\vec{F}_3$ and $\vec{F}_1$; and γ between $\vec{F}_1$ and $\vec{F}_2$.



[Lami's theorem]

As shown in fig. (b), the forces F_1, F_2 and F_3 can be represented by the sides of ΔABC, taken in the same order. Applying law of sines to ΔABC, we get

$$\frac{F_1}{\sin(\pi-\alpha)} = \frac{F_2}{\sin(\pi-\beta)} = \frac{F_3}{\sin(\pi-\gamma)}$$

or
$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

This is Lami's theorem which states that if three forces acting on a particle keep it in equilibrium, then each force is proportional to the sine of the angle between the other two forces.

Examples based on Equilibrium of Concurrent Forces

✖Formulae Used 1. A number of forces acting at the same point are called concurrent forces.

2. A number of concurrent forces are said to be in equilibrium if their resultant is zero.

$F = F_1 + F_2 + F_3 + ... + F_n = 0$

3. If F_1, F_2 and F_3 are three concurrent forces in equilibrium

(i) $F_1 + F_2 + F_3 = 0$

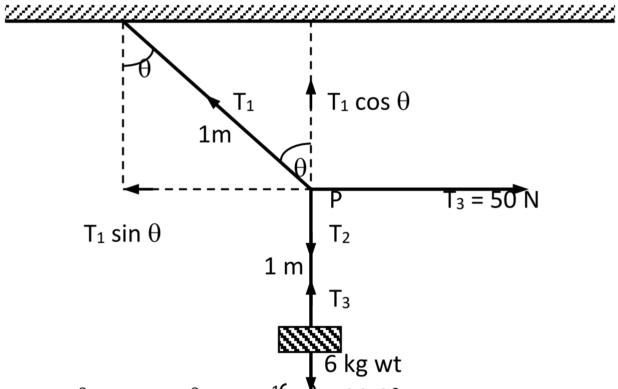
(ii) $\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$ (Lami's theorem)

✖Units Used All forces are in newton.

Q. 1. **A mass of 6 kg is suspended by a rope of length 2 m from a ceiling. A force of 50 N in the horizontal direction is applied at the midpoint of the rope as shown in Fig. What is the angle the rope makes with the vertical in equilibrium? Take g = 10 ms^{-2}. Neglect mass of the rope.**

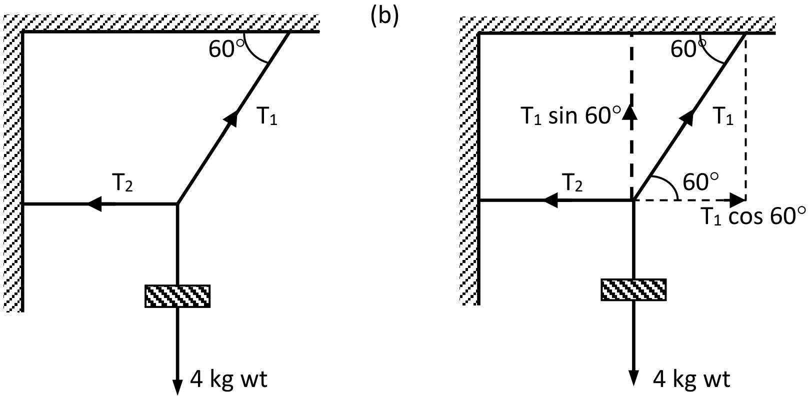
Sol. As shown in Fig., there are three forces acting on the midpoint P of the rope. Suppose the rope makes on angle θ with the vertical in equilibrium. Resolving the force horizontally and vertically, we get

$T_1 \sin\theta = T_3 = 50$ N ... (i) $T_1 \cos\theta = T_2 = 6$ kg wt = 60 N ... (ii)



Dividing (i) by (ii), we get, $\tan\theta = \frac{5}{6}$ or $\theta = \tan^{-1}\left(\frac{5}{6}\right) = 39.8°$

Q. 2. ***Determine the tension T_1 and T_2 in the strings shown in Fig. (a).***



Sol. As shown in Fig. (b) resolve the tension T_1 along horizontal the vertical directions. As the body is in equilibrium,

$T_1 \sin 60° = 4$ kg wt $= 4 \times 9.8$ N ... (i)

$T_1 \cos 60° = T_2$ ... (ii)

From (i), $T_1 = \frac{4 \times 9.8}{\sin 60°} = \frac{4 \times 9.8 \times 2}{\sqrt{3}} = 45.26$ N

From (ii), $T_2 = T_1 \cos 60° = 45.26 \times 0.5 = 22.63$ N

Q. 3. **A train is moving along a horizontal track. A pendulum suspended from the roof makes an angle of 4° with the vertical. Obtain the acceleration of the train. Take g = 10 ms^{-2}.**

Sol. Fig. shows the equilibrium position of a pendulum suspended in a train which is moving towards right with acceleration a along a horizontal track.

Two forces acting on the bob are

(i) Weight mg acting vertically downwards,

(ii) Tension T acting along the string.

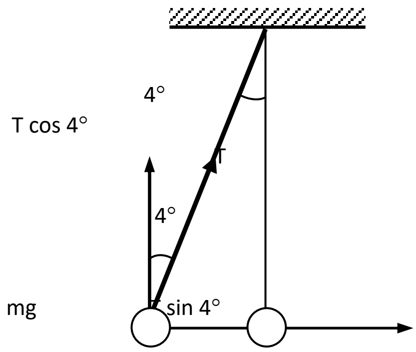
Resolving T along horizontal and vertical directions, we find that

$T \cos 4° = mg$ ... (i)

As the acceleration a of the train and hence that of the pendulum is responsible for tension T sin 4°, so

$T \sin 4° = ma$ ... (ii)

Dividing (ii) by (i), we get, $\tan 4° = \frac{a}{g}$



or $a = g \tan 4° = 10 \times 0.07 = 0.7$ ms^{-2}

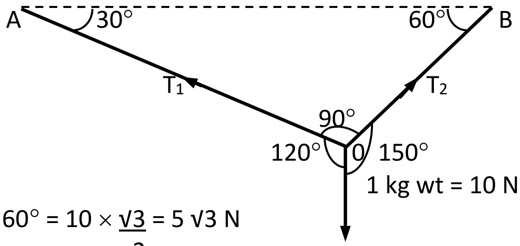
Q. 4. **A ball of mass 1 kg hangs in equilibrium from two strings OA and OB as shown in Fig. What are the tension in strings OA and OB? Take g = 10 ms^{-2}.**

Sol. Various forces acting at the point O are as shown in Fig. The three forces are in equilibrium. Using Lami's theorem,

$$\frac{T_1}{\sin 150°} = \frac{T_2}{\sin 120°} = \frac{10}{\sin 90°}$$

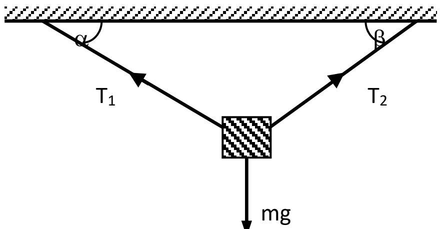
or $\dfrac{T_1}{\sin 30^\circ} = \dfrac{T_2}{\sin 60^\circ} = \dfrac{10}{1}$

∴ $T_1 = 10 \sin 30^\circ = 10 \times 0.5 = 5$ N



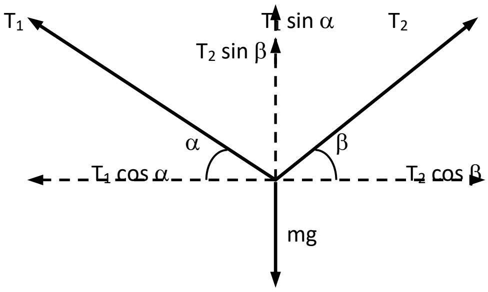
And $T_2 = 10 \sin 60^\circ = 10 \times \dfrac{\sqrt{3}}{2} = 5\sqrt{3}$ N

Q. 5. A body of mass m is suspended by two strings making angles α and β with the horizontal as shown in Fig. Find the tensions in the strings.



Sol. The free-body diagram is shown in Fig. As the body is in equilibrium, the various forces must add to zero. Taking horizontal components,

$T_1 \cos\alpha = T_2 \cos\beta$ or $T_2 = T_1 \dfrac{\cos\alpha}{\cos\beta}$



Taking vertical components,

$T_1 \sin\alpha + T_2 \sin\beta = mg$

or $T_1 \sin\alpha + T_1 \dfrac{\cos\alpha}{\cos\beta} \sin\beta = mg$

or $T_1 = \dfrac{mg}{\sin\alpha + \cos\alpha \sin\beta} = \dfrac{mg \cos}{\sin\alpha\cos\beta + \cos\alpha\sin\beta}$

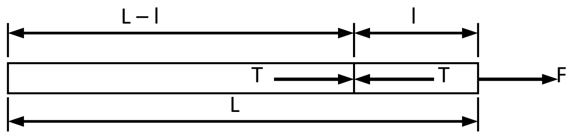
or $T_1 = \dfrac{mg\cos\beta}{\sin(\alpha+\beta)}$

and $T_2 = T_1 \dfrac{\cos\alpha}{\cos\beta} = \dfrac{mg\cos\beta}{\sin(\alpha+\beta)} \cdot \dfrac{\cos\alpha}{\cos\beta} = \dfrac{mg\cos\alpha}{\sin(\alpha+\beta)}$

Q. 6. A uniform rope of length L, resting on a frictionless horizontal surface is pulled at one end by a force F. What is the tension in the rope at a distance l from the end where the force is applied?

Sol. Let M be the mass of uniform rope of length L. Then

Mass per unit length of rope = $\dfrac{M}{L}$



Acceleration in the rope = $\dfrac{F}{M}$

Let T be the tension in the rope at a distance l from the end where the force F is applied.

Mass of length (L – l) of the rope is

$M' = M (L - l)$

As tension T is the only force on the length (L – l) of the rope, so

$T = M' \times \dfrac{F}{M} = \dfrac{M}{L}(L - l) \times \dfrac{F}{M} = \left(1 - \dfrac{l}{L}\right)F$

Problems For Practice

Q. 1. A mass of 10 kg is suspended vertically by a rope of length 2 m from a ceiling. A force of 60 N is applied at the middle point of the rope in the horizontal direction, as shown in fig. Calculate the angle the rope makes with the vertical. Neglect the mass of the rope and take g = 10 ms⁻².

Sol. For equilibrium of the body,

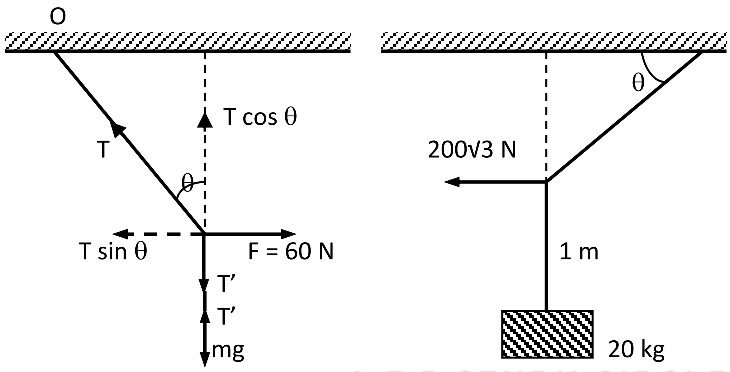
$T \sin\theta = F = 60$ N ... (i)

$T \cos\theta = T' = mg$ ... (ii)

Dividing (i) by (ii), we get

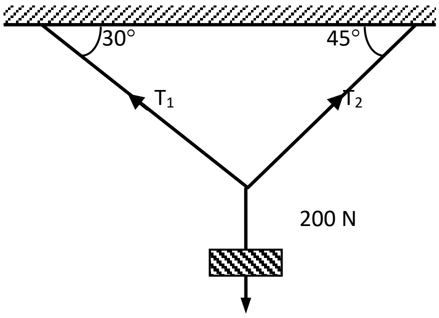
$\dfrac{T\sin\theta}{T\cos\theta} = \dfrac{60}{mg} = \dfrac{60}{10 \times 10} = 0.6$

or $\tan\theta = 0.6$ ∴ $\theta = 31^\circ$



Q. 2. **A body of weight 200 N is suspended with the help of strings as shown in Fig. Find the tensions T_1 and T_2.**

Sol. The free-body diagram is shown in fig.



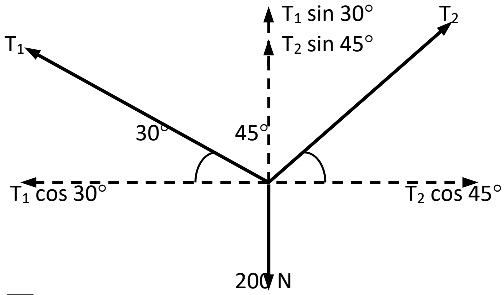
Resolution of forces along horizontal direction gives

$$T_1 \cos 30° = T_2 \cos 45°$$

$$T_1 \times \frac{\sqrt{3}}{2} = T_2 \times \frac{1}{\sqrt{2}} \quad \text{or} \quad T_2 = \sqrt{\frac{3}{2}}\, T_1$$

Resolution of forces along vertical direction gives

$$T_1 \sin 30° + T_2 \sin 45° = 200 \text{ N}$$



$$T_1 \times ½ + \sqrt{\frac{3}{2}}\, T_1 \times \frac{1}{\sqrt{2}} = 200 \quad \text{or} \quad T_1 (1 + \sqrt{3}) = 400$$

$$T_1 = \frac{400}{2.732} = 146.4 \text{ N}; \quad T_2 = \sqrt{\frac{3}{2}} \times 146.4 = 179.3 \text{ N}$$

Examples based on Motion of Connected Bodies

✖Formulae Used

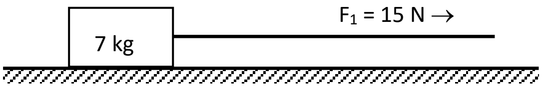
1. When a number of bodies are connected together by strings, rods, etc., It is convenient to draw a free body diagram for each body separately by showing all the forces acting on it.
2. Equation of motion for each body is written by equating the net force acting on the body to its mass times the acceleration produced.

✖Units Used All forces are in newton (N).

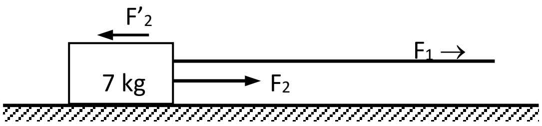
Q. 1. **A pull of 15 N is applied to a rope attached to a block to mass 7 kg lying on a smooth horizontal surface. The mass of the rope is 0.5 kg. What is the force exerted by the block on the rope?**

Sol. The situation is shown in Fig. If a acceleration is produced in the block on applying a force of 15 N, then

$$(7 + 0.5)\, a = 15 \quad \text{or} \quad a = \frac{15}{7.5} = 2 \text{ ms}^{-2}$$



As shown in Fig., let F_2 be the force exerted by the block on the rope and F_2' be reaction of the rope on the block. According to Newton's third law of motion,

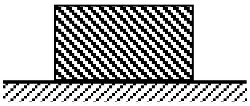


$$F_2 = F_2'$$

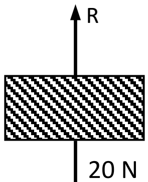
As F_2' is the only force acting on the block which has an acceleration of 2 ms^{-2}, so $F_2 = F_2' = 7 \times 2 = 14$ N

Q. 2. **A wooden block of mass 2 kg rests on a soft horizontal floor. When an iron cylinder of mass 25 kg is placed on top of the block, the floor yields steadily and the block and the cylinder together go down with an acceleration of 0.1 ms^2. What is the action of the block on the floor (a) before and (b) after the floor yields? Tage g = 10 ms^{-2}.**

Sol. (a) In Fig. (a), the block is at rest. Its free-body diagram [Fig. (b)] shows two forces on the block:

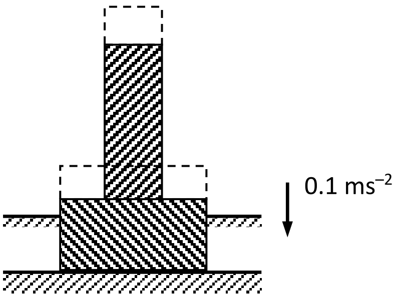


(a)

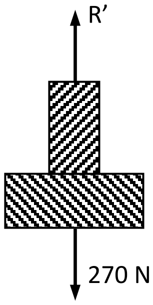


(b)

Free-body diagram of the block



(c)



(d)

(i) Force of gravitational attraction of the earth

= Weight of block = mg = 2 × 10 = 20 N

(ii) Normal reaction R of the floor on the block. By the first law, the net force on the block is zero, so R = 20 N

By the third law, the action of the block 1 i.e., the force exerted by the block on the floor.

= 20 N, vertical downwards.

(b) In Fig. (c), the system (block + cylinder) accelerates downwards with 0.1 ms^{-2} due to the yielding of the floor. The free-body diagram [Fig. (d)] shows two forces on system: 30

(i) Force of gravity due to the earth = Weight of block + system = (25 + 2) × 10 = 270 N

(ii) Normal reaction R' of the floor.

Applying second law to the system, we get

$270 - R' = (25 + 2) \times 0.1$ or $R' = 270 - 2.7 = 267.3$ N

By the third law, action of the system on the floor.= 267.3 N vertically downwards.

Q. 3. Two bodies of masses 10 kg and 20 kg respectively kept on a smooth, horizontal surface are tied to the ends of a light string. A horizontal force F = 600 N is applied to (i) B (ii) A along the direction of string. What is the tension in the string in each case?

Sol. Here F = 600 N, $m_1 = 10$ kg, $m_2 = 20$ kg

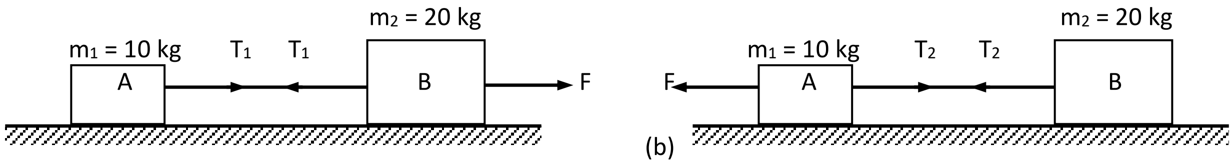
Let T be the tension in the string and a be the acceleration produced in the system, in the direction of applied force F. Then

$$a = \frac{F}{m_1 + m_2} = \frac{600}{10 + 20} = 2\ ms^{-2}$$

(i) Suppose the pull F is applied on the body B of mass 20 kg.

Let T_1 be the tension in the string. As T_1 is the only force acting on mass 10 kg, so

$T_1 = m_1 a = 10 \times 20 = 200$ N

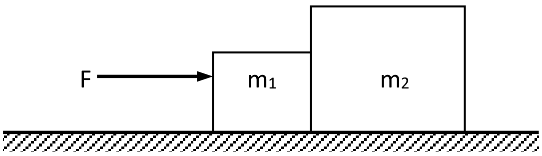


(ii) When the pull F is applied on body A of mass 10 kg [fig. (b)], tension in the string will be

$T_2 = m_2 a = 20 \times 20 = 400$ N

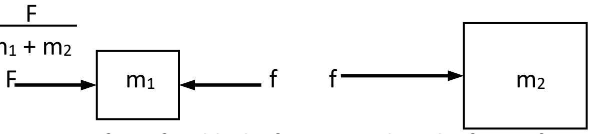
Clearly, the tension depends on which mass end the pull is applied.

Q. 4. Two blocks of masses m_1 and m_2 in contact lie on a horizontal smooth surface, as shown in Fig. The blocks are pushed by a force F. If the two blocks are always in contact, what is the force at their common interface?



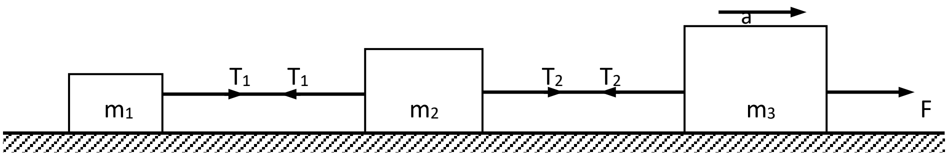
Sol. From Newton's second law, the common acceleration produced in the system will be

$$a = \frac{F}{m_1 + m_2}$$



If block of mass m_1 exerts force f on block of mass m_2, then the force of reaction on block of mass m_1 will be equal and opposite to f. These forces are shown in the free body diagrams of Fig. As the block of mass m_2 has acceleration a, so $f = m_2 a = \frac{m_2 F}{m_1 + m_2}$

Q. 5. As shown in Fig., three blocks connected together lie on a horizontal frictionless table and pulled to the right with a force F = 50 N. If m_1 = 5 kg, m_2 = 10 kg and m_3 = 15 kg, find the tension T_1 and T_2.



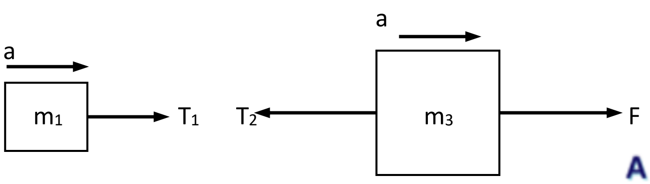
Sol. All the blocks move with common acceleration a under the force F = 50 N

∴ $F = (m_1 + m_2 + m_3)\, a$

or $a = \frac{F}{m_1 + m_2 + m_3} = \frac{50}{5 + 10 + 15} = \frac{5}{3}\ ms^{-2}$

To determine T_1. Refer to the free-body diagram for m_1 shown in Fig. Clearly, the tension T_1 produces acceleration a in mass m_1.

∴ $T_1 = m_1 a = 5 \times \frac{5}{3} = \frac{25}{3} = 8.33$ N



To determine T_2. Refer to the free-body diagram for m_3 shown in Fig. Force F acts towards right and tension T_2 acts towards left.

∴ $F - T_2 = m_3 a$ or $50 - T_2 = 15 \times 5/3 = 25$ N

Q. 6. Two masses 8 kg and 12 kg are connected at the two ends of light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses and the tension in the string when the masses are released.

Sol. Here $m = 8$ kg, $M = 12$ kg, $g = 10$ ms^{-2}

From the derivation of connected motion, We have

$$a = \frac{M - m}{M + m} . g = \frac{12 - 8}{12 + 8} \times 10 = 2 \text{ ms}^{-2}$$

$$T = \frac{2\,Mm}{M + m} . g = \frac{2 \times 12 \times 8}{12 + 8} \times 10 = 96 \text{ N}$$

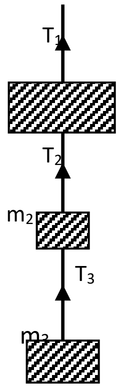
Q. 7. The masses m_1, m_2 and m_3 of the three bodies shown in Fig. are 5, 2 and 3 kg respectively. Calculate the values of the tensions T_1, T_2 and T_3 when (i) the whole system is going upward with an acceleration of 2 ms^{-2} and (ii) the whole system is stationary. Give g = 9.8 ms^{-2}.

Sol. The three bodies together are moving upward with an acceleration of 2 ms^{-2}. The force pulling the system upward is T_1 and the downward force of gravity is $(m_1 + m_2 + m_3)$ g. According to Newton's second law,

$T_1 - (m_1 + m_2 + m_3)\, g = (m_1 + m_2 + m_3)\, a$

or $T_1 = (m_1 + m_2 + m_3)(a + g)$

$= (5 + 2 + 3)(2 + 9.8) = 10 \times 11.8 = 118$ N



Similarly, for the motion of the system $m_2 + m_3$, we can write

$T_2 = (m_2 + m_3)(a + g)$p

$= (2 + 3)(2 + 9.8) = 5 \times 11.8 = 59$ N

For the motion of body of mass m_3, we have

$T_3 = m_3(a + g) = 3\,(2 + 9.8) = 35.4$ N

(ii) When the whole system is stationary, a = 0. From the above equations, we get

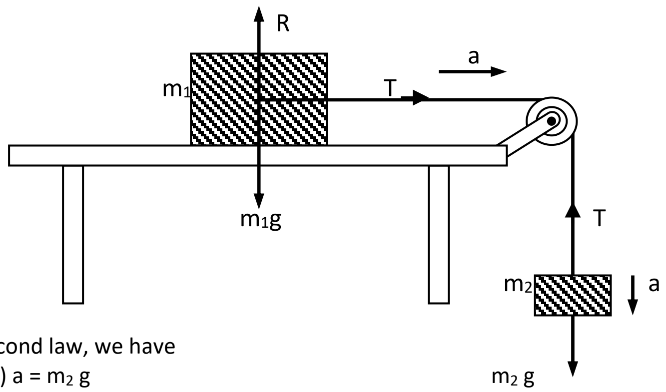
$T_1 = (m_1 + m_2 + m_3) = 10 \times 9.8 = 98$ N

$T_2 = (m_2 + m_3)\, g = 5 \times 9.8 = 49$ N

$T_3 = m_3\, g = 3 \times 9.8 = 29.4$ N

Q. 8. A body m_1 of mass 5 kg is placed on a smooth horizontal table. It is connected to a string which passes over a frictionless pulley and carries at the other end, a body m_2 of mass 5 kg. What acceleration will be produced in the bodies when the nail fixed on the table is removed? What will be the tension in the string during the motion of the bodies? What when the bodies stop? Take g = 9.8 N kg^{-1}.

Sol. The situation is shown in Fig. When the nail fixed on the table is removed, the system of two bodies moves with an acceleration a in the direction as shown.



From Newton's second law, we have

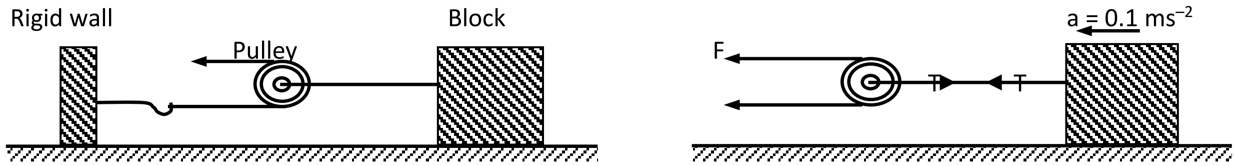
$(m_1 + m_2)\, a = m_2\, g$

$$a = \frac{m_2\, g}{m_1 + m_2} = \frac{5 \times 9.8}{(10 + 5)} = 3.27 \text{ ms}^{-2}$$

Also, $T = m_1 \times a = 10 \times 3.27 = 32.7$ N

When the bodies stop, acceleration, a = 0. Suppose the tension in the string becomes T'. As the net force on each body is zero, so for body m_2, we can write $T' = m_2 g = 5 \times 9.8 = 49$ N

Q. 9. A block of mass 100 kg is set into motion on a frictionless horizontal surface with the help of frictionless pulley and a rope system as shown in Fig. (a). What horizontal force should be applied to produce in the block an acceleration of 10 cms^{-2}.



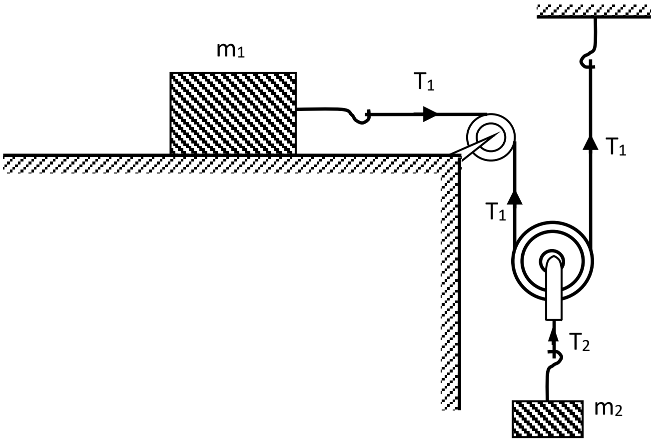
(a) (b)

Sol. As shown in Fig. (b), when force F is applied at the end of the string, the tension in the lower part of the string is also F. If T is the tension in string connecting the pulley and the block, then from Newton's third law,

$T = 2F$

But $T = ma = 100 \times 0.1 = 10$ N ∴ $2F = 10$ N or $F = 5$ N

Q. 10. In terms of masses m_1, m_2 and g, find the acceleration of both the blocks shown in Fig. Neglect all friction and masses of the pulley.



Sol. As the mass m_1 moves towards right through distance x, the mass m_2 moves down through distance x/2.
Clearly, if the acceleration of m_2 is a, then that of m_2, we have

$$T_1 = m_1 a \quad \text{or} \quad m_2 g - T_2 = m_2 \cdot \frac{a}{2}$$

Also $T_2 = 2T_1 = 2m_1 a$

∴ $m_2 g - 2m_1 a = m_2 . \frac{a}{2}$

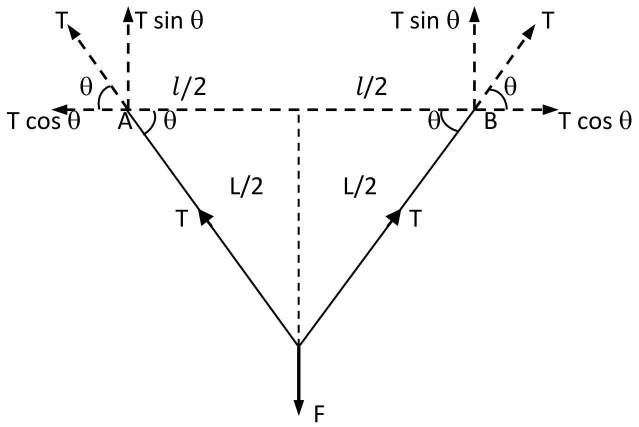
or $2m_2 g - 4m_1 a = m_2 a$ or $2m_2 g = (4m_1 + m_2) a$

∴ Acceleration of $m_1 = a = \frac{2m_2 g}{4m_1 + m_2}$

Acceleration of $m_2 = \frac{a}{2} = \frac{m_2 g}{4m_1 + m_2}$

Q. 11. Two identical point masses, each of mass M are connected to one another by a massless string of length L. A constant force F is applied at the mid-point of the string. If l be the instantaneous distance between the two masses, what will be the acceleration of each mass?

Sol. Fig. shows the position of string at any instant after the application of a force F at the mid point. It also shows the various force acting on the two masses at any instant. If tension T in the string is resolved into horizontal and vertical components, then



$$F = 2T \sin\theta \quad \text{... (i)}$$

and $$Ma = T\cos\theta \quad \text{... (ii)}$$

Where a is the acceleration of each mass.
Dividing (ii) by (i), we get

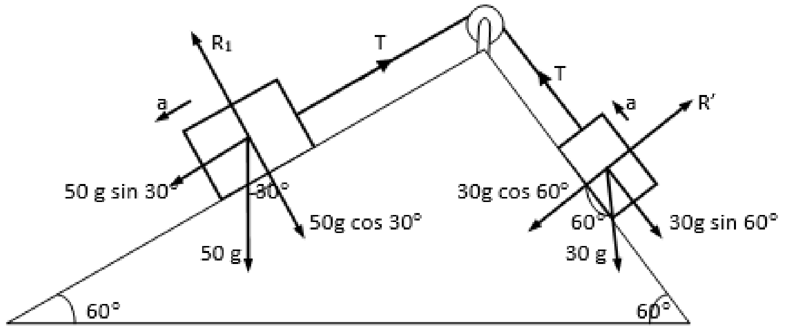
$$\frac{\cos\theta}{2\sin\theta} = \frac{Ma}{F} \quad \text{or} \quad \cot\theta = \frac{2Ma}{F}$$

or $$\frac{l/2}{\sqrt{(L/2)^2 - (l/2)^2}} = \frac{2Ma}{F} \quad \text{or} \quad \frac{2Ma}{F} = \frac{l}{\sqrt{L^2 - l^2}}$$

or $$a = \frac{F}{2M}\left(\frac{l}{\sqrt{L^2 - l^2}}\right)$$

Q. 12. Two blocks of masses 50 kg and 30 kg connected by a massless string pass over a light frictionless pulley and rest on two smooth planes inclined at angles 30° and 60° respectively with the horizontal. Determine the acceleration of the two blocks and the tension in the string. Take g = 10 ms^{-2}.

Sol. Suppose the mass of 50 kg slides down with an acceleration a. The forces acting on the two blocks are shown in Fig. The components of the two weights perpendicular to the inclined planes are balanced by the normal reaction R_1 and R_2.



The tension T of each part of the string is same and also the acceleration a of each block is same.

∴ $50\,g \sin 30° - T = 50\,a$ ... (i)

and $T - 30\,g \sin 60° = 3a$ ... (ii)

Adding (i) and (ii), we get

$(50 \sin 30° - 30 \sin 60°)\,g = (50 + 30)\,a$

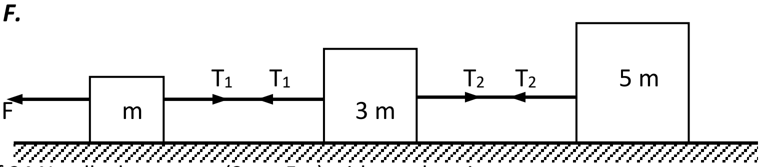
or $a = \dfrac{(50 \times 0.5 - 30 \times 0.866) \times 10}{80} = -0.12\ \text{ms}^{-2}$

The –ve sign indicates that the 50 kg block, instead of sliding down, actually slides up. Hence the 30 kg block slides down and that of 50 kg slides up the inclined plane with $a = 0.12\ \text{ms}^{-2}$.

From (i), $T = 50\,g \sin 30° - 50\,a = 50\,(10 \times 0.5 - 0.12) = 50 \times 4.88 = 244\ \text{N}$

☙ Problems For Practice

Q. 1. ***As shown in Fig. three masses m, 3m and 5 m connected together lie on a frictionless horizontal surface and pulled to the left by a force F. the tension T_1 in the first string is 24 N. Find (i) acceleration of the system, (ii) tension in the second string, and (iii) force F.***



Sol. (i) Tension T_1 of 24 N pulls the masses (3m + 5m) with acceleration a.

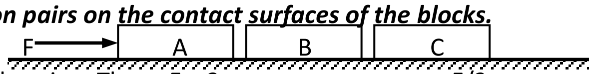
∴ $24 = (3m + 5m)\,a$ or $a = 3/m$

(ii) Tension T_2 pulls mass 5 m with acceleration 3/m.

∴ $T_2 = 5\,m \times \dfrac{3}{m} = 15\ \text{N}$

(iii) $F = (m + 3m + 5\,m)\,a = 9\,m \times \dfrac{3}{m} = 27\ \text{N}$

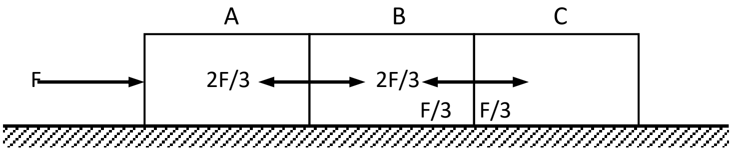
Q. 2. ***Three identical blocks, each having a mass m are pushed by a force F on a frictionless table as shown in Fig. What is the acceleration of the blocks? What is the net force on the block A? What force does A apply on B? What force does B apply on C? Show action-reaction pairs on the contact surfaces of the blocks.***



Sol. Let a be the common acceleration. Then $F = 3\,m \times a$ or $a = F/3\,m$

Net force on block A will be

$F_1 = m \times a = m \times \dfrac{F}{3m} = \dfrac{F}{3}$

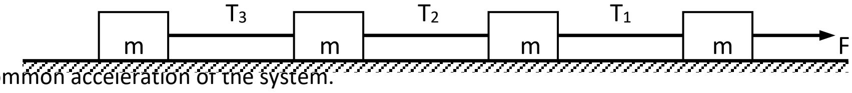


Force applied by A on B,

$F_2 = (m_1 + m_2)\,a = 2\,m \times \dfrac{F}{3m} = \dfrac{2F}{3}$

Force applied by B on C, $F_3 = m \times a = m \times \dfrac{F}{3m} = \dfrac{F}{3}$ The action-reaction forces are shown in Fig.

Q. 3. ***Four blocks of the same mass m connected by cords are pulled by a force F on a smooth horizontal surface, as shown in Fig. Determine the tensions T_1, T_2 and T_3 in the cords.***

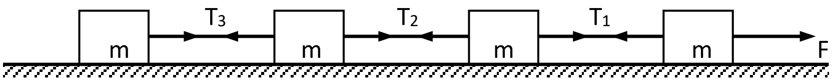


Sol. Let a be the common acceleration of the system.

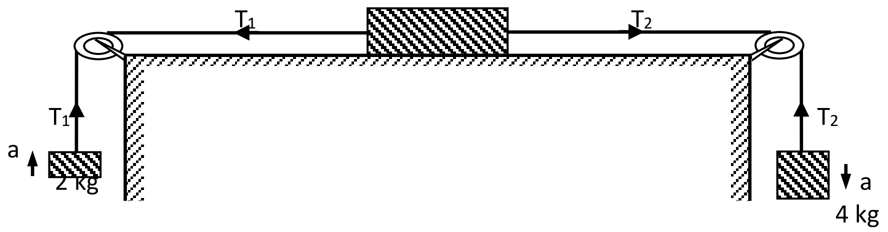
Then $F = (m + m + m + m)\,a = 4\,ma$ or $a = \dfrac{F}{4\,m}$

Applying Newton's 2nd law separately for each block in Fig., we get $F - T_1 = ma$, $T_1 - T_2 = ma$, $T_2 - T_3 = ma$, $T_3 = ma$

On solving the above equations, we get , $T_1 = ¾\,F$, $T_2 = ½\,F$, $T_3 = ¼\,F$

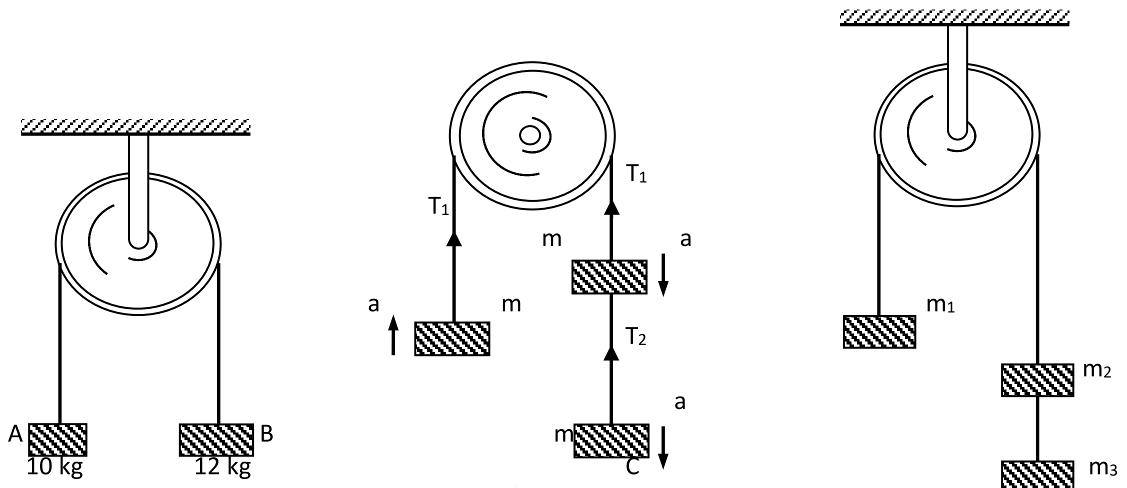


Q. 4. *In Fig., find the acceleration a of the system and the tensions T_1 and T_2 in the strings. Assume that the table and the pulleys are frictionless and the strings are massless. Take g = 9.8 ms^{-2}*



Sol. Here $4g - T_2 = 4a$, $T_1 - 2g = 2a$ and $T_2 - T_1 = 8a$ On solving, a = 1.4 ms^{-2}, T_1 = 22.4 N, T_2 = 33.6 N

Q. 5. *In the Atwood's machine [Fig.], the system starts from rest. What is the speed and distance moves by each mass at t = 3s?*



Sol. Acceleration, $a = \frac{M - m}{M + m} g = \frac{12 - 10}{12 + 10} \times 9.8 = 0.89\ ms^{-2}$

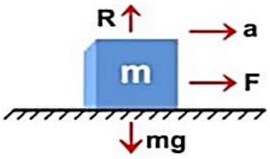
$v = u + at = 0 + 0.89 \times 3 = 2.67\ ms^{-1}$ therefore $s = ut + ½ at^2 = 0 + ½ \times 0.89 \times 9 \simeq 4$ m

APPLICATION OF N.L.M Part II

1 Motion of a Block on a Horizontal Smooth Surface

Case (i) Horizontal pull

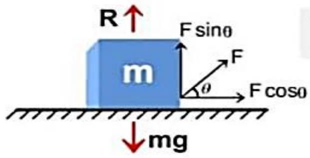
$F = ma$ or $a = \frac{F}{m}$



Case (ii) Pull acting at an angle (θ)

$R + F\sin\theta = mg$

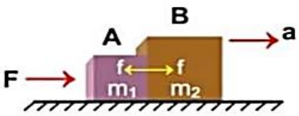
$a = \frac{F\cos\theta}{m}$



2 Motion of Bodies in Contact

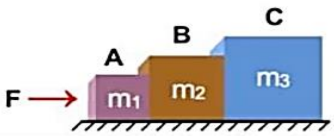
Case (i) Two Body System

$\Rightarrow a = \frac{F}{m_1 + m_2}$ & $f = \frac{m_2 F}{m_1 + m_2}$



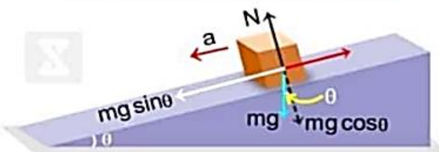
Case (ii) Three Body System

$\Rightarrow a = \frac{F}{m_1 + m_2 + m_3}$



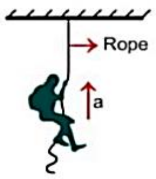
3 Motion of a Body on a Smooth Inclined Plane

$a = g\sin\theta$ $N = mg\cos\theta$



4 Climbing on the Rope

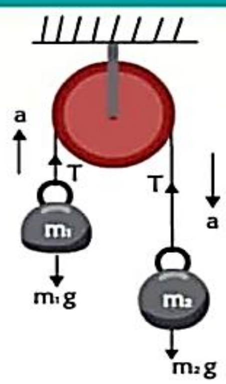
- T > mg, man accelerates in upward direction
- T < mg, man accelerates in downward direction



F.B.D of man

PULLEY BLOCK SYSTEM

1

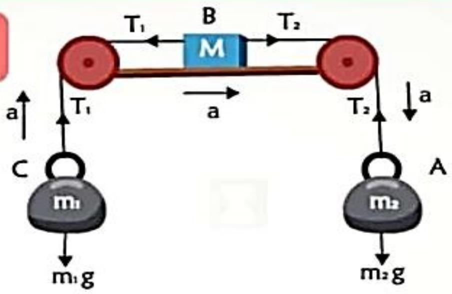


$m_2 > m_1$

$m_2 g - T = m_2 a$

$T - m_1 g = m_1 a$

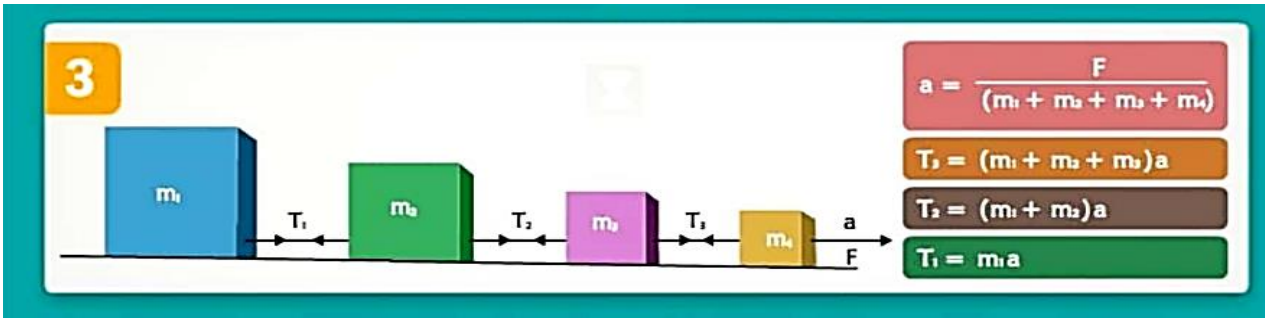
2

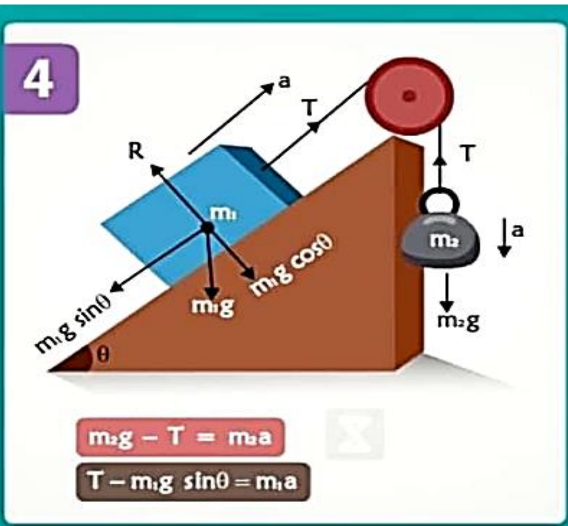


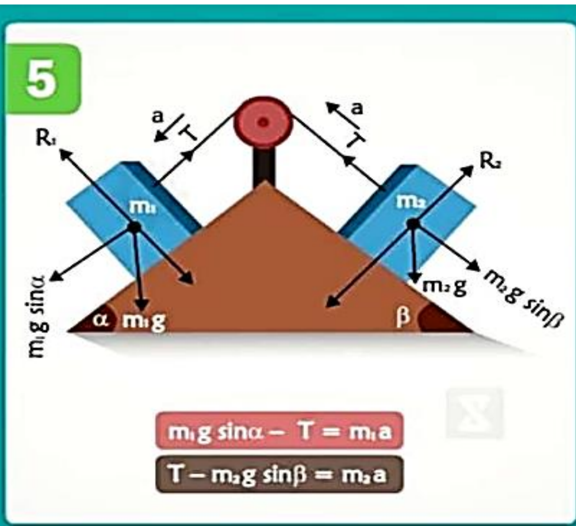
For body A, $m_2 g - T = m_2 a$

For body B, $T_2 - T_1 = Ma$

For body C, $T_1 - m_1 g = m_1 a$



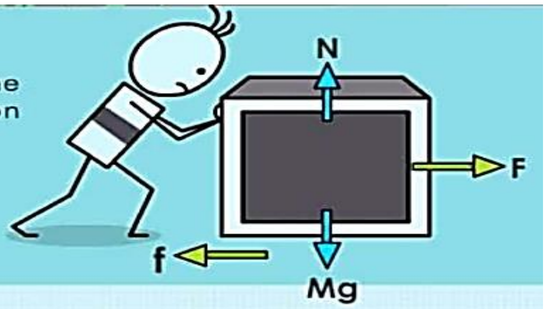




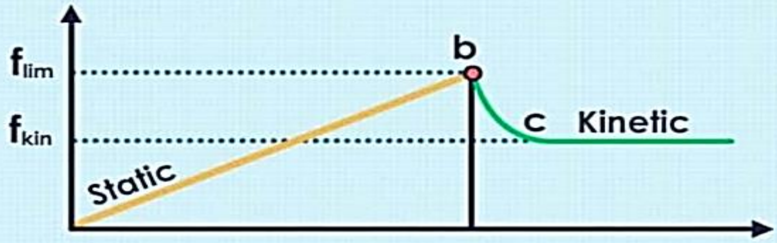
FRICTION

Friction is a contact force that opposes the relative motion or tendency of relative motion between two bodies.

$$f = \mu N = \mu mg$$



TYPES OF FRICTION FORCES



1. STATIC FRICTIONAL FORCE

The opposing force due to which there is no relative motion between the bodies in contact is called static friction force. It's a self-adjusting force.
Coefficient of static friction is μ_s.

2. LIMITING FRICTIONAL FORCE

The maximum frictional force that acts when the body is about to move is called limiting frictional force.

3. KINETIC FRICTIONAL FORCE

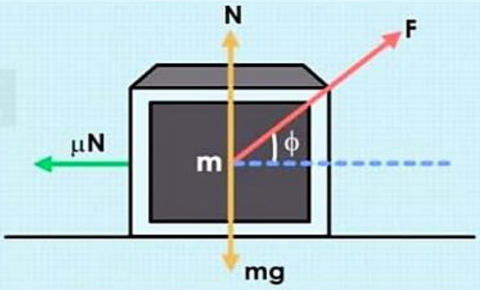
The frictional force between the surfaces in contact when relative motion starts between them is called Kinetic Frictional Force. Coefficient of kinetic friction is μ_k.

$$\mu_k < \mu_s$$

FRICTION

Part II

MINIMUM FORCE REQUIRED TO MOVE THE BODY



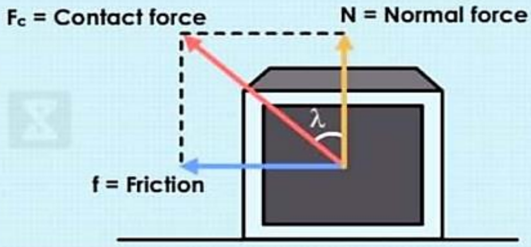
$$F_{min} = \frac{\mu mg}{1+\mu^2}$$

N = Normal force

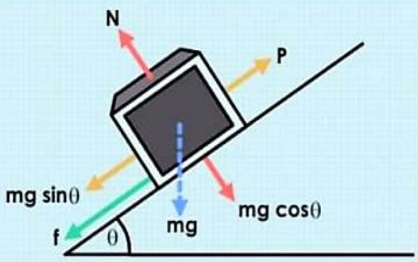
FRICTION AS A COMPONENT OF CONTACT FORCE

$$F_{c\,max} = \sqrt{\mu^2 N^2 + N^2} \quad \{\because f_{max} = \mu N\}$$

$$F_{c\,max} = N\sqrt{\mu^2 + 1}$$



MOTION ON A ROUGH INCLINED PLANE



Balancing Vertical Forces

$$N = mg\cos\theta$$

Balancing Horizontal Forces

$$f = \mu N = \mu\, mg\cos\theta$$

When sliding with acceleration 'a'

$$mg\sin\theta - \mu mg\cos\theta = ma$$

ANGLE OF REPOSE

The angle of repose is the maximum angle that a surface can be tilted from the horizontal, such that an object on it is just able to stay on the surface without moving.

or $\tan\theta_c = \mu$

where θ_c is called angle of repose.