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PHYSICS

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PROJECTILE MOTION

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PHYSICS

MOTION IN TWO DIMENSIONS

ENTERING TWO DIMENSION

What you already know

- Motion in 1D
- Equations of motion

What you will learn

- Basics of 2D motion
- Introduction to projectile motion
- Parameters of projectile motion

2D Motion

When the motion of an object is restricted within a plane, it is said to undergo a motion in **2D.**

2D motion can be studied as two independent 1D motions. (One along x-axis and the other along y-axis)

Example: Motion of a carrom coin

3D Motion

When the motion of an object is permitted all over the space, it is said to undergo a motion in **3D**.

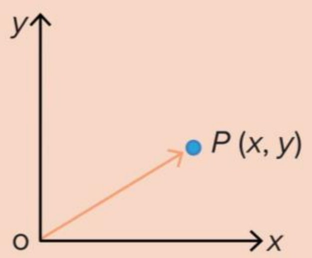
3D motion can be studied as three independent 1D motions. (Along x-axis, y-axis, and z-axis)

Example: Motion of a fish in an aquarium

2D Motion

- 2D motion can be studied as two **simultaneous** and **independent** 1D motions.

  (**Note: Projection** of x-axis on the y-axis is **zero** and vice versa. Hence, two vectors which are perpendicular to each other have no effect on each other. In other words, they are independent of each other. The motion along *x*-axis and *y*-axis are connected by time which is a scalar quantity. In other words, time is the only parameter common for both *x* and *y* components of motion.)
- 2D motion is a vector superimposition of two 1D motions along *x* and *y* directions.
- Position of a body under 2D motion can be specified by independent *x* and *y* coordinates. Position of point *P* is (x, y) and the position vector of *P* is $x\hat{i} + y\hat{j}$.



$\vec{r} = x\hat{i} + y\hat{j}$

- So, in 2D motion, position, velocity, and acceleration are divided into x and y components which are studied separately.

Position ($\vec{r}$) → x, y

Velocity ($\vec{v}$) → $v_x = \dfrac{dx}{dt}$, $v_y = \dfrac{dy}{dt}$

Acceleration ($\vec{a}$) → $a_x = \dfrac{d(v_x)}{dt}$, $a_y = \dfrac{d(v_y)}{dt}$

Steps to analyse 2D motion

(i) Choose an origin and establish the positive sense of x and y axes.

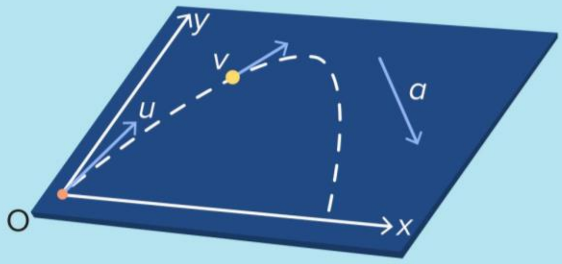
(ii) Resolve every given vector along the x and y axes.

(iii) Tabulate every single data separately for x and y.

(iv) Apply suitable equations of motion along x and y separately.

(v) Relate the two, as the time variable is common for both.

- Consider a 2D motion as shown in the figure. A particle is following a parabolic path with an initial velocity u and constant acceleration a. Let v be is the instantaneous velocity at any time t.



- Acceleration can be resolved along x and y directions as a_x and a_y respectively. As a is a constant, a_x and a_y are also constants.
- Equations of motion should be applied separately for x and y directions.

x-axis	*y*-axis
$v_x = u_x + a_x t$	$v_y = u_y + a_y t$
$\Delta x = u_x t + \dfrac{1}{2} a_x t^2$	$\Delta y = u_y t + \dfrac{1}{2} a_y t^2$
$v_x^2 = u_x^2 + 2a_x \Delta x$	$v_y^2 = u_y^2 + 2a_y \Delta y$

- Velocity and position can be dealt separately in x and y directions as shown in the table given below:

x-axis	y-axis	Resultant
$v_x = u_x + a_x t$	$v_y = u_y + a_y t$	$\vec{v} = v_x\hat{i} + v_y\hat{j}$; $\lvert\vec{v}\rvert = \sqrt{v_x^2 + v_y^2}$
$\Delta x = u_x t + \frac{1}{2}a_x t^2$	$\Delta y = u_y t + \frac{1}{2}a_y t^2$	$\vec{r} = x\hat{i} + y\hat{j}$; $\lvert\vec{r}\rvert = \sqrt{x^2 + y^2}$

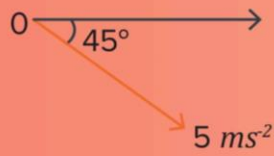
- Similar to 1D motion, in 2D motion, we have,

$v_x = \frac{dx}{dt}$	$v_y = \frac{dy}{dt}$
$\frac{d(v_x)}{dt} = a_x = v_x \frac{dv_x}{dx}$	$\frac{d(v_y)}{dt} = a_y = v_y \frac{dv_y}{dy}$

A roller coaster goes down a 45° incline with an acceleration of 5.0 ms^{-2}. (Starts from rest)

(a) How far will the roller coaster travel in 10 seconds horizontally?

(b) How far will the roller coaster travel in 10 seconds vertically?



Solution

The motion of the roller coaster can be considered as a 1D motion along the direction of acceleration. Taking the direction of acceleration as r - direction.

$$x = x_0 + ut + \frac{1}{2}at^2$$

In r-direction,

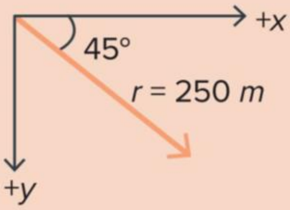
$$r = r_0 + ut + \frac{1}{2}at^2$$

$$= 0 + 0 \times t + \frac{1}{2}(5)(10)^2$$

$$= 250\,m$$

Taking right direction as positive x axis and downwards as positive y axis,

Displacement along y-direction



$$\Delta y = r\sin\theta$$

$$= 250 \sin 45^0$$

$$= \frac{250}{\sqrt{2}}\,m$$

Displacement along x-direction

$$\Delta x = r\cos\theta$$

$$= 250 \cos 45^0$$

$$= \frac{250}{\sqrt{2}}\,m$$

The position of a particle at time, $t = 0$ is P (-1, 2, -1). It starts moving with an initial velocity $\vec{u} = 3\hat{i} + 4\hat{j}$ ms^{-1} and with a uniform acceleration $\vec{a} = \left(-4\hat{i} + 3\hat{j}\right)$ ms^{-2}. Find the final position and the magnitude of displacement after 4 seconds.

BOARDS

Solution

$u_x = 3\,ms^{-1}$	$u_y = 4\,ms^{-1}$
$a_x = -4\,ms^{-2}$	$a_y = 3\,ms^{-2}$

Initial position vector of the particle = $-\hat{i} + 2\hat{j} - \hat{k}$

Final position of particle after 4 seconds

$$\Delta\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{s_f} = \vec{s_i} + \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$= \left(-\hat{i} + 2\hat{j} - \hat{k}\right) + \left[\left(3\hat{i} + 4\hat{j}\right)\times 4\right] + \left[\frac{1}{2}\left(-4\hat{i} + 3\hat{j}\right)\times 4^2\right]$$

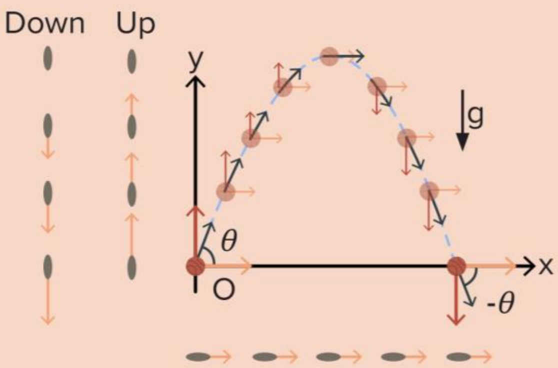
$$= -21\hat{i} + 42\hat{j} - \hat{k}$$

Displacement = $\vec{s_f} - \vec{s_i} = -20\hat{i} + 40\hat{j}$

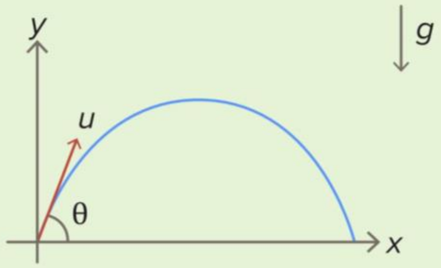
Displacement magnitude = $\sqrt{(-20)^2 + 40^2} = 20\sqrt{5}\,m$

Projectile Motion

- A particle projected at an angle to the Earth's surface with some initial velocity moves along a curved path because of the acceleration due to gravity (g). This motion is known as **projectile motion**.
- Projectile motion is a special type of 2D motion in which trajectory of motion is parabolic.
- The projectile motion can be resolved into two 1D motions, a uniform velocity motion along x-axis and a uniformly accelerated motion along y-axis.



A ground to ground projectile motion is a special type of projectile motion such that the point of projection and the point of landing will be at the same level (ground).



Terms associated with projectile motion

1. **Launch angle (θ):** It is the angle with the horizontal at which a projectile is launched.
2. **Initial velocity (u):** It is the velocity with which the projectile is launched.
3. **Instantaneous velocity of the projectile (v):** It is always tangential to the path of the projectile for the entire trajectory and can be resolved into components at any point.
4. **Acceleration due to gravity (g):** It acts constantly on the projectile in a vertically downward direction and gives it a parabolic trajectory. It constantly changes the projectile's instantaneous velocity.
5. **Time of flight (T):** It is the total time for which the projectile remains in air, i.e., the time interval between launching and landing.
6. **Maximum height (H):** It is the distance of the highest point of the trajectory from the ground. Here, the instantaneous velocity is horizontal as its vertical component vanishes.
7. **Range (R):** It is the maximum displacement of the projectile in the horizontal direction.

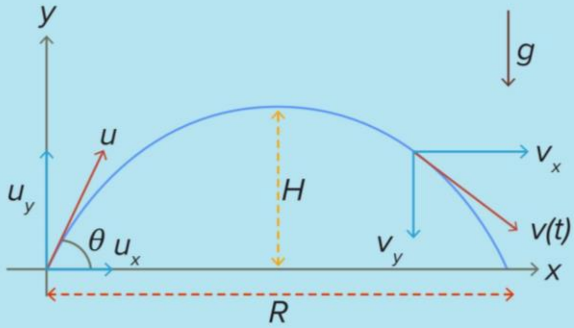
Assumptions in projectile motion

- We consider only those trajectories that are of sufficiently **short range and height**, so that the gravitational force can be considered constant.
- Earth's surface is assumed to be **flat** over the range of a projectile.
- Air resistance is ignored.

MAIN

Equations of Projectile Motion

- Let's resolve the projectile motion into two 1D motions, a uniform velocity motion along x-axis and a uniformly accelerated motion along *y*-axis.



Along x-axis (Uniform velocity)	Along y-axis (Uniform acceleration)
$a_x = 0$	$a_y = -g$ ($\because$ constant acceleration)
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$x = (u\cos\theta)t$	$y = (u\sin\theta)t - \frac{1}{2}gt^2$
$v_x = u\cos\theta$ ($\because$ *horizontal velocity is constant*)	$v_y = u\sin\theta - gt$

- **Magnitude of velocity at any point**

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$
$$= \sqrt{u^2 + g^2t^2 - 2(u\sin\theta)gt}$$

Time of flight (T)

The displacement along y-direction after landing for a ground to ground projectile motion is zero

$$u_yT - \frac{1}{2}gT^2 = 0$$
$$T = \frac{2u_y}{g}$$
$$T = \frac{2u\sin\theta}{g} \quad (\because u_y = u\sin\theta)$$

Maximum height (H)

At maximum height, the vertical velocity of a projectile becomes zero.

$$v_y^2 = u_y^2 - 2gH = 0$$
$$H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g} \qquad \because u_y = u\sin\theta$$

Range (R)

Motion along x-direction is uniform.

Hence, for a ground to ground projectile,

Range, $R = u_xT$

$$= (u\cos\theta) \times \frac{2u\sin\theta}{g}$$
$$= \frac{u^2\sin 2\theta}{g} \qquad (\because 2\sin\theta\cos\theta = \sin 2\theta)$$

A ball is thrown at a speed of **50** ms^{-1} at an angle of **60°** with the horizontal. Find the following: (Take $g = 10\ ms^{-2}$)

(a) Time of flight (b) The maximum height reached (c) The range of the ball

Solution

$$\text{Time of flight, } T = \frac{2u\sin\theta}{g}$$

$$= \frac{2\times 50\times \sin 60^0}{10}$$

$$= 5\sqrt{3}\ s$$

$$\text{Maximum height, } H = \frac{u^2\sin^2\theta}{2g}$$

$$= \frac{50^2\times\left(\frac{\sqrt{3}}{2}\right)^2}{2\times 10}$$

$$= 93.75\ m$$

$$Range, R = \frac{u^2\sin 2\theta}{g}$$

$$= \frac{50^2\sin 120^0}{10}$$

$$= 216.5\ m$$

PHYSICS

MOTION IN TWO DIMENSION

SPECIFICS OF PROJECTILE MOTION

What you already know

- Motion in 1D
- Introduction to 2D motion
- Introduction to projectile motion

What you will learn

- Ground to ground projectile motion
- Landing velocity
- Equation of trajectory

In a soccer practice session, the football is kept at the centre of the field, 40 yards from the 10 ft high goalpost. A goal is attempted by kicking the football at a speed of 64 fts^{-1} at an angle of 45° to the horizontal. Will the ball reach the goalpost?

Solution

We have, $g = 9.8ms^{-2} = 32\ fts^{-2},\ x = 40\ yd = 120\ ft,\ u = 64\ fts^{-1}$

It will be a goal when height of the projectile is less than the height of the goalpost, i.e., 10ft at the location of the goalpost.

i.e., at $x = 120\ ft$

Time when ball will reach the location of goalpost,

$$x = (u\cos\theta)t \Rightarrow t = \frac{x}{u\cos\theta} = \frac{120}{64\cos(45^\circ)} = \frac{15\sqrt{2}}{8}\ s$$

$$y = u\sin\theta(t) - \frac{1}{2}gt^2$$

Height of projectile at the location of goalpost,

$$y = 64\frac{1}{\sqrt{2}}\left(\frac{15\sqrt{2}}{8}\right) - \frac{1}{2}(32)\left(\frac{15\sqrt{2}}{8}\right)^2$$

$= 7.5\ ft$

As 7.5 ft is less than 10 ft, the ball will be passing in between goal post and the ground, hence it is a goal.

1. Range is maximum when $\theta = 45°$ and $R_{max} = \frac{u^2}{g}$

2. For objects projected at complementary launch angles, range will be the same.

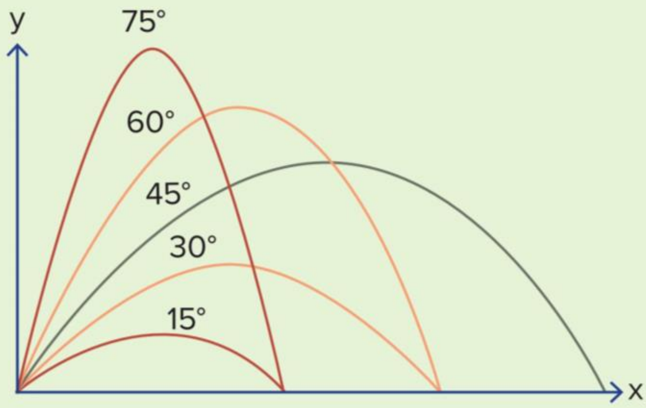
   Let $R_1 = R_2$

   Equation of range is

   $$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 2\alpha}{g}$$

   $$2\theta = 180^o - 2\alpha \Rightarrow \alpha = 90^o - \theta$$

   The particle shot with the **same speed** but at complimentary **projection angles will have the same range.**



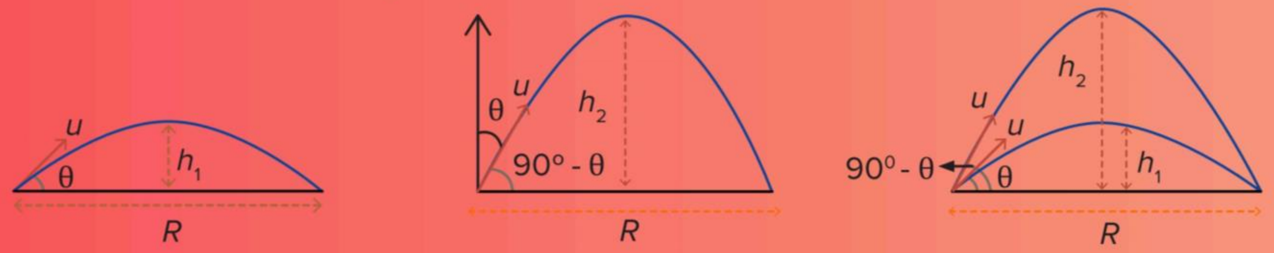
3. When range is maximum, height H reached by projectile is,

   $$H = \frac{u^2 \sin^2 45^o}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

4. If R = H,

   $$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow \tan\theta = 4$$

A stone is projected from the ground with a certain speed at an angle **θ with horizontal** and it attains a maximum height **h_1**. When it is projected with the same speed at an angle **θ with vertical**, it attains a height **h_2**. Find the horizontal range of the projectile.



Solution

When angles are complementary, range is the same.

$$h_1 = \frac{u^2 \sin^2 \theta}{2g}$$

$$h_2 = \frac{u^2 \sin^2 (90^o - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$$

Then,

$$h_1 h_2 = R^2 \left(\frac{1}{16}\right)$$

$$\Rightarrow R^2 = 16 h_1 h_2$$

$$\Rightarrow R = 4\sqrt{h_1 h_2}$$

An object projected with the same speed at two different angles covers the same horizontal range R. If the times of flight are t_1 and t_2, prove that $R = \frac{g}{2} t_1 t_2$.

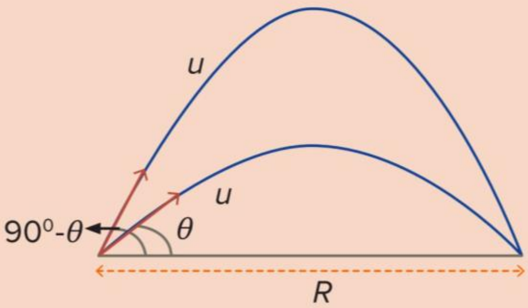
Solution

Range of projectile is,

$$R = \frac{u^2 \sin 2\theta}{g}$$

Time of flight for two cases,

$$t_1 = \frac{2u\sin\theta}{g}, t_2 = \frac{2u\sin(90^o - \theta)}{g} = \frac{2u\cos\theta}{g}$$



From t_1 and t_2,

$$t_1 t_2 = 4u^2 \frac{\sin\theta\cos\theta}{g^2} = \frac{2R}{g},$$

$$R = \frac{2\ u^2 \sin\theta\ \cos\theta}{g}$$

Rearranging the equations, we get,

$$\Rightarrow R = \frac{1}{2} g t_1 t_2$$

Landing Velocity

During a projectile motion,

Journey of ascent: It is the journey from the ground to the maximum height.

Journey of descent: It is the journey from the maximum height to the ground.

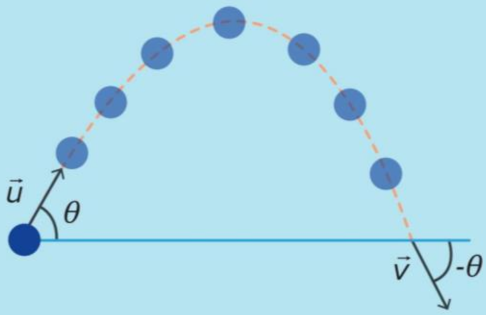
Final velocity,

$$v_y^2 = u_y^2 - 2gy$$

$$\Rightarrow v_y^2 = u^2 \sin^2\theta$$

$$\Rightarrow v_y = \pm u\sin\theta \Rightarrow v_y = u_y$$

Also, $v_x = u_x \Rightarrow v = u$

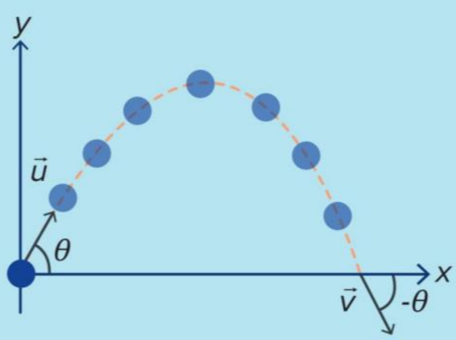


Symmetry in Ground to Ground Projectile Motion

A. Maximum height occurs halfway through the flight of the projectile.

B. Launch angle is symmetric with the landing angle.

C. Projectile spends half its time travelling upwards and the other half travelling downwards, also known as time of ascent and time of descent, respectively.



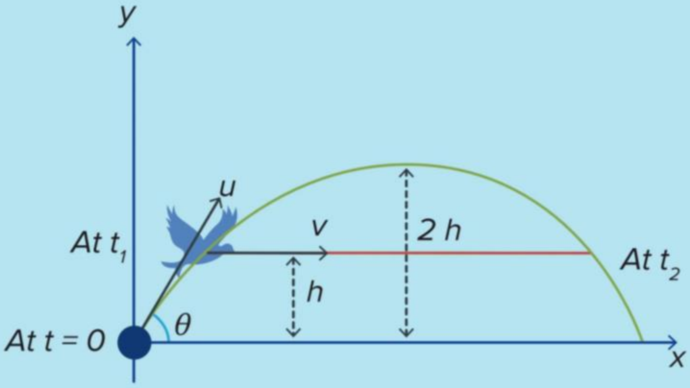
A stone is projected from a point on the ground in a direction to hit a bird on the top of a telegraph post of height **h**, it attains a maximum height **2h** above the ground otherwise. If at the instant of projection, the bird were to fly away horizontally with a uniform speed, find the ratio of the horizontal velocities of the bird and the stone if the stone still hits the bird.

MAIN

Solution

Let t_1 and t_2 be the time at which the stone crosses the height h. And according to the question, at t_2 the stone hits the bird, i.e., both the bird and the stone are at the same place. So the ratio of horizontal distances covered will be,

$$vt_2 = (u\cos\theta)(t_2 - t_1) \Rightarrow \frac{v}{u\cos\theta} = \frac{t_2 - t_1}{t_2}$$



Maximum height of projectile is given by,

$$H = \frac{u^2\sin^2\theta}{2g}$$

$$\Rightarrow 2h = \frac{u^2\sin^2\theta}{2g} \Rightarrow u\sin\theta = 2\sqrt{gh}$$

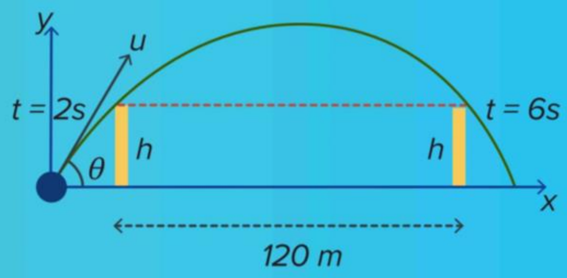
$$\Rightarrow y = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$\Rightarrow t = \frac{4\sqrt{gh} \pm 2\sqrt{2gh}}{2g}$$

$$\Rightarrow t_1 = \sqrt{\frac{h}{g}}\left(2-\sqrt{2}\right) \quad \text{and} \quad t_2 = \sqrt{\frac{h}{g}}\left(2+\sqrt{2}\right) \qquad \Rightarrow \frac{v}{u\cos\theta} = \frac{t_2 - t_1}{t_2} = \frac{2}{\sqrt{2}+1}$$

If a projectile crosses two walls of equal height h symmetrically, as shown in figure, choose the correct statement among the following. ($g = 10\ ms^{-2}$)

(A) Time of flight is 8 s.
(B) Height of each wall is 60 m.
(C) The maximum height of the projectile is 80 m.
(D) All of the above



Solution

It is given that the projectile will reach height h at an instance of 2 *s*. It will take the same time to land on ground from the same height. To travel the distance between two poles, it takes 4 s. Hence, the total time of flight will be 8 *s*,
T = 8 *s*

$$\Rightarrow \frac{2u\sin\theta}{g} = 8$$

$$\Rightarrow u\sin\theta = 40$$

$$y = u_y t - \frac{1}{2}gt^2$$

Height of each wall,

$$H_{wall} = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$= 40(2) - \frac{1}{2}(10)(2)^2 = 60m$$

Maximum height of projectile $= \dfrac{u^2\sin^2\theta}{2g}$

$$= \frac{40\times 40}{2\times 10} = 80m$$

Hence option D is correct.

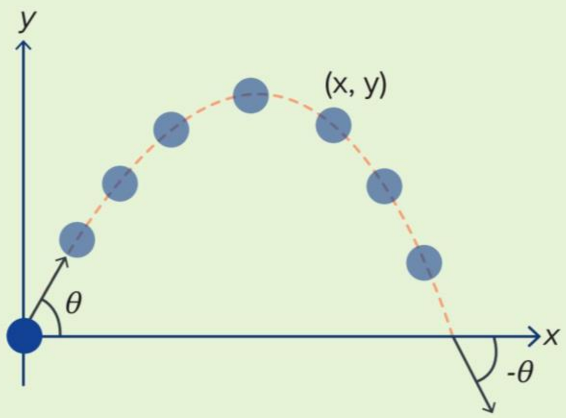
MAIN

Equation of Trajectory

The trajectory is the path travelled by any projectile. It is plotted on a X-Y graph. Mathematically, it is derived by eliminating time from the equations of motion, which gives a parabola.

$$x = (u\cos\theta)t + 0$$

$$\Rightarrow t = \frac{x}{u\cos\theta}$$



$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x^2}{u^2\cos^2\theta}\right)$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

We know that,

$$\sin 2\theta = 2\sin\theta\cos\theta \quad and \quad R = \frac{u^2\sin 2\theta}{g}$$

$$y = x\tan\theta - \frac{gx^2\sin\theta}{2u^2\cos^2\theta\sin\theta}$$

$$y = x\tan\theta - \frac{x^2\tan\theta}{\left(\dfrac{u^2\sin 2\theta}{g}\right)}$$

$$y = x\tan\theta\left(1 - \frac{x}{R}\right)$$

A particle moves in the x-y plane with a constant acceleration 'a' directed along the negative y-axis. The equation of motion of the particle has the form $y = \alpha x - \beta x^2$, where α and β are positive constants. Find the velocity of the particle at the origin.

Solution

Comparing this equation with the equation of trajectory,

$$y = x\tan\theta - \frac{ax^2}{2u^2\cos^2\theta}$$

$$\tan\theta = \alpha \;\; and \;\; \beta = \frac{a}{2u^2}sec^2\theta$$

This gives

$$\Rightarrow u = \sqrt{\frac{a}{2\beta}\left(1+\alpha^2\right)}$$

The path followed by a body projected in the x-y plane is given by $y = \sqrt{3}x - \frac{1}{2}x^2$

If $g = 10\ ms^{-2}$, what will be the initial velocity of the projectile (x and y are in m)?

Solution

Given

$$y = \sqrt{3}x - \frac{1}{2}x^2$$

Comparing with the equation of trajectory,

$$y = x(\tan\theta) - \frac{1}{2}\frac{g}{u^2\cos^2\theta}x^2$$

$$\frac{1}{2} = \frac{1}{2}\frac{g}{u^2\cos^2\theta}$$

This gives,

$$\Rightarrow u^2\cos^2\theta = g$$

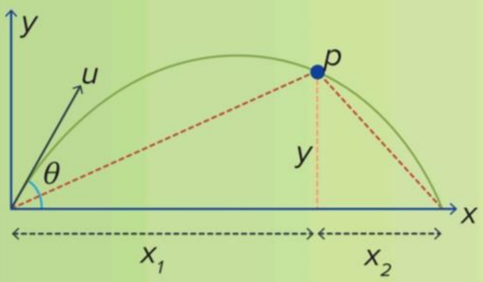
$$\Rightarrow u^2\cos^2 60^o = 10$$

$$\Rightarrow u = 2\sqrt{10}\ ms^{-1}$$

Find the equation of trajectory for point P in the given figure for a projectile.

(A) $y=\left[\dfrac{x_1x_2}{x_1-x_2}\right]\tan\theta$ (C) $y=\left[\dfrac{2x_1x_2}{x_1+x_2}\right]\cos\theta$

(B) $y=\left[\dfrac{x_1x_2}{x_1+x_2}\right]\tan\theta$ (D) $y=\left[\dfrac{2x_1x_2}{x_1+x_2}\right]\tan\theta$



BOARDS

Solution

The equation of trajectory for point P can be written as,

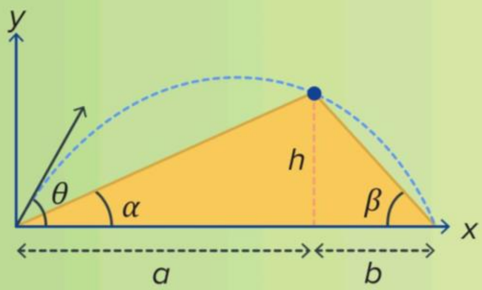
$$y = x\tan\theta\left(1-\frac{x}{R}\right) = x_1\tan\theta\left(1-\frac{x_1}{x_1+x_2}\right)$$

$$= x_1\tan\theta\left(\frac{x_1+x_2-x_1}{x_1+x_2}\right)$$

$$y=\frac{x_1x_2}{x_1+x_2}\tan\theta$$

Option (B) is correct.

A particle is projected over a triangle from one extremity of its horizontal base. Grazing over the vertex, it falls on the other extremity of the base. If α and β are the base angles of the triangle and θ is the angle of projection, prove that tanθ = tanα + tanβ.



Solution

Equation of trajectory can be written as,

$$y = x\tan\theta\left(1-\frac{x}{R}\right)$$

$$\Rightarrow h = a\tan\theta\left(1-\frac{a}{a+b}\right) \qquad [R=a+b]$$

$$\frac{h}{a} = \tan\theta\left[\frac{b}{a+b}\right] \Rightarrow \tan\theta = \frac{h}{a}+\frac{h}{b}$$

$$\tan\alpha = \frac{h}{a};\ \tan\beta = \frac{h}{b}$$

$$\Rightarrow \tan\theta = \tan\alpha + \tan\beta$$

Note: Since (a, h) lies on the trajectory of the projectile, it should satisfy equation 1.

PHYSICS

MOTION IN TWO DIMENSION

PROJECTILE : VARIOUS ANGLES AND COLLISION

What you already know

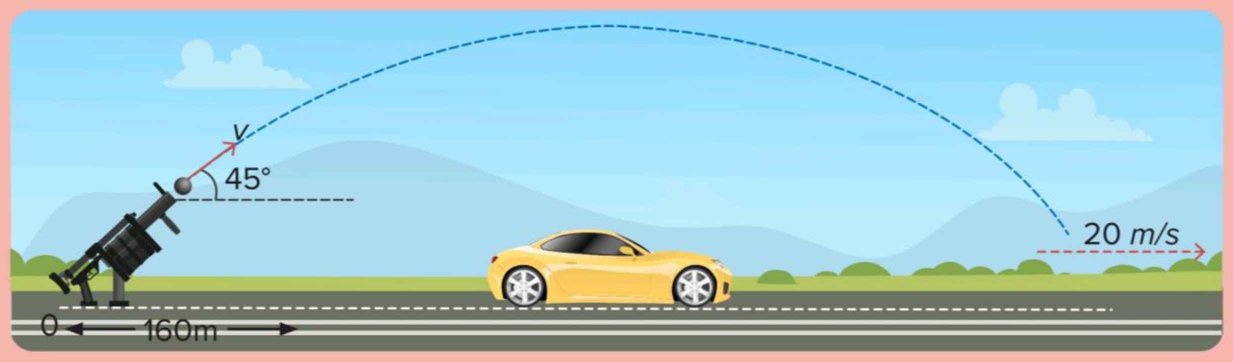
- Equations of motion
- Flowchart to handle projectile motion
- Ground to ground projectile motion
- Motion parameters for projectile motion

What you will learn

- General projectile motion
- Horizontal projection
- Elastic collision of projectile with a wall
- Flowchart to handle general projectile motion
- Projection from an elevation at different angles

Example

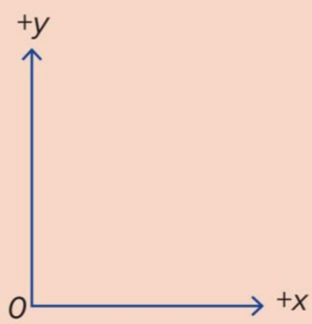
A gun kept on a straight horizontal road is used to hit a car, travelling along the same roadway with a uniform speed of **20 ms^{-1}**. The car is at **160 m** from the gun when the gun is fired at an angle of **45^0** with the horizontal. Find the distance of the car from the gun when the shell hits it, and the speed of projection of the shell from the gun. (**g** = **10 ms**$^{-2}$)



Solution

Motion of the bullet is ground to ground projectile motion.

Taking origin as the position of the gun, and positive X and Y axes as shown,



Velocity of car, $v_x = 20\ ms^{-1}$
Initial position of car, $x_0 = 160$ m

Angle of projection of bullet, $\theta = 45^0$
Acceleration of bullet, $a_x = 0$, $a_y = -g = -10\ ms^{-2}$

Let the speed of projection of the bullet be *u*.

$$Time\ of\ flight, T = \frac{2u\sin\theta}{g}$$
$$= \frac{2u\sin 45^0}{g}$$
$$= \frac{\sqrt{2}u}{10}$$
$$Range, R = \frac{u^2\sin 2\theta}{g}$$
$$= \frac{u^2\sin 90^0}{g}$$
$$= \frac{u^2}{10}$$

Distance covered by the car during the time of flight,

$$D = Velocity\ of\ car \times Time\ of\ flight$$
$$= 20 \times \frac{u\sqrt{2}}{10}$$
$$= 2\sqrt{2}u$$

When the bullet hit the car,

$$Range, R = x_0 + D$$
$$\frac{u^2}{10} = 160 + 2\sqrt{2}u$$
$$u^2 - 20\sqrt{2} - 1600 = 0$$
$$u^2 - 40\sqrt{2} + 20\sqrt{2} - 1600 = 0$$
$$\left(u - 40\sqrt{2}\right)\left(u + 20\sqrt{2}\right) = 0$$
$$u = 40\sqrt{2}\ or\ u = -20\sqrt{2}$$

As *u* is the speed of the projectile, it only takes positive value.

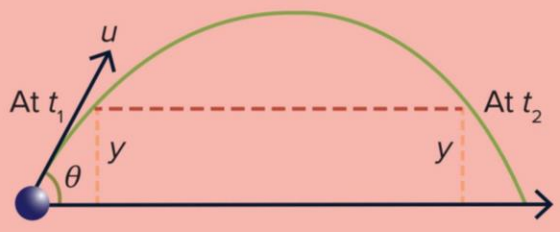
$$\therefore u = 40\sqrt{2}\ ms^{-1}$$

Distance of the car from the gun when the shell hits the car,

$$= x_0 + 20T$$
$$= 160 + \frac{20 \times \sqrt{2}u}{g}$$
$$= 160 + \frac{20 \times \sqrt{2} \times 40\sqrt{2}}{10}$$
$$= 320m$$

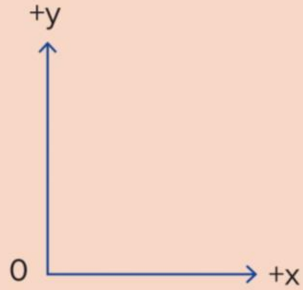
Example

If a projectile is fired from the ground with velocity u, prove that sum of the two time instants at a given height is equal to the time of flight. (g = 10 ms^{-2})



Solution

It is a ground to ground projectile motion.
We have to prove, $t_1 + t_2$ = time of flight
Taking origin as starting point, and positive axes as shown,



t_1 and t_2 are the time corresponding to elevation = y

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$= u\sin\theta(t) - \frac{1}{2} g t^2$$

$$2y = 2u\sin\theta(t) - gt^2$$

$$t^2 - \left(\frac{2u\sin\theta}{g}\right)t + \frac{2y}{g} = 0$$

Roots of above quadratic equation are t_1 and t_2.

$$\text{Sum of roots}, t_1 + t_2 = \frac{-\left(\frac{2u\sin\theta}{g}\right)}{-1} = \frac{2u\sin\theta}{g} = T$$

If roots of quadratic equation ax2 + bx + c = 0 are **α** and ***β***, then sum of roots **α** + ***β*** = -(b/a)]

Equation of Trajectory

The following factors influence the time of flight, range, and maximum height reached by a projectile.

- Velocity of release
- Angle of release
- Point of projection

General Projectile Motion

- When projectile motion is not ground to ground projectile it is called as general projectile motion.
- In general projectile motion, point of projection and point of landing are at different heights.

Flowchart for general projectile

- Choose an origin and establish the positive sense of two axes.
- Resolve every given vector along *X* and *Y* axes.
- Tabulate every single data separately for *X* and *Y*.
- Apply suitable equations of motion along *X* and *Y* separately.
- Relate the two, as the time variable is common for both.

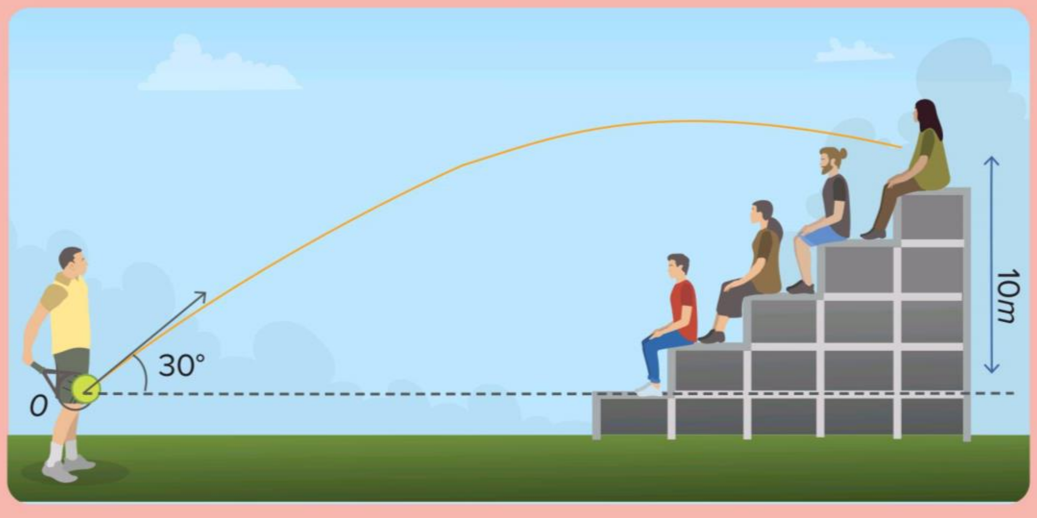
Flowchart of a general projectile motion is the same as the flowchart of a ground to ground projectile motion.

Example

A tennis player wins a match and hits a ball into the stands at 30 ms^{-1} and at an angle 30^0 above the horizontal. On its way down, the ball is caught by a spectator 10 m above the point where the ball was hit.

i. Calculate the time it takes the tennis ball to reach the spectator.

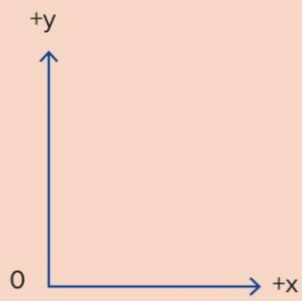
ii. What is the magnitude and direction of the velocity of the ball at the impact?



Solution

It is not a ground to ground projectile motion. Hence, it should be treated as a general projectile motion.

Choosing origins as the player, and positive *X* and *Y* directions as shown.



Initial speed of the ball = 30 ms^{-1}
Angle of projection, $\theta = 30_0$

Along X-direction	Along Y-direction
$u_x = 30\cos 30^0 = 15\sqrt{3}\,ms^{-1}$	$u_y = 30\sin 30 = 15\,ms^{-1}$
$a_x = 0$	$u_y = -g$

(i) Let t be the time at which the ball is at an elevation of 10 m.

$$y = u_y t + \frac{1}{2}a_y t^2$$
$$10 = (30\sin 30)t - \frac{1}{2}gt^2$$
$$= 15t - \frac{1}{2}\times 10\times t^2$$
or
$$t^2 - 3t + 2 = 0$$
$$t = 1s\, or\, 2s$$

So, at $t = 1$ s, the ball was at a height 10 m on the way up and at $t = 2$ s, the ball was at a height 10 m on the way down.

As the spectator caught the ball on the way down, time taken by the tennis ball to reach the spectator is 2 s.

(ii) Let v be the magnitude of velocity at the impact.

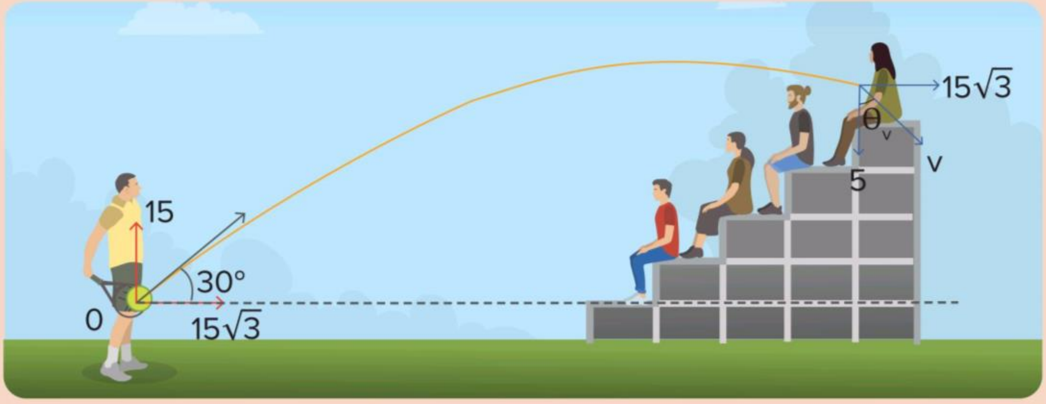
$$v_x = v\cos\theta = u\cos\theta = 15\sqrt{3}\,ms^{-1} \quad (\because \textit{horizonatal velocity is constant})$$
$$v_y = u_y - gt$$
$$= 15 - 10\times 2$$
$$= -5\,ms^{-1}$$

The negative sign implies that the velocity is in negative Y-direction (downward).

Magnitude of velocity at impact,

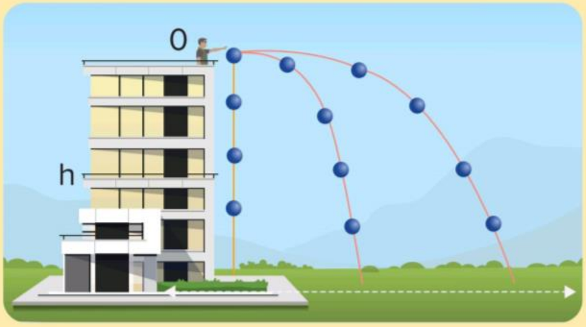
$$v = \sqrt{{v_x}^2 + {v_y}^2}$$
$$= \sqrt{(15\sqrt{3})^2 + (-5)^2}$$
$$= \sqrt{700} = 10\sqrt{7}$$

Let θ_v be the angle between the impact velocity vector and the vertical as shown.



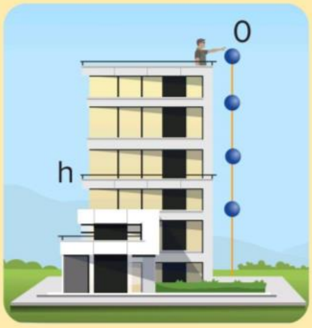
$$\theta_v = \tan^{-1}\left(\frac{15\sqrt{3}}{5}\right)$$
$$= \tan^{-1}\left(3\sqrt{3}\right)$$

Horizontal Projection

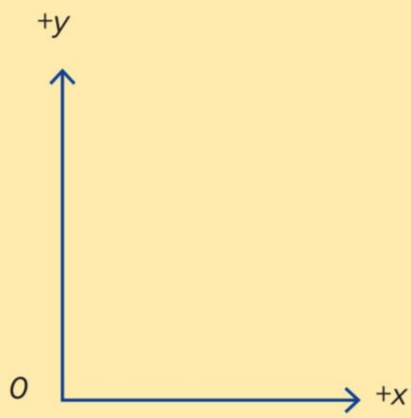


- It is a special type of general projectile motion with angle of launch = 0
- Time of flight of horizontal projection from a height h is equal to the time taken by the body to reach the ground when dropped from a height *h*.

Proof



Case 1: If an object is dropped from a height *h* as shown



It is a 1D motion.
Taking launch point as origin and positive *X* and *Y* directions as shown

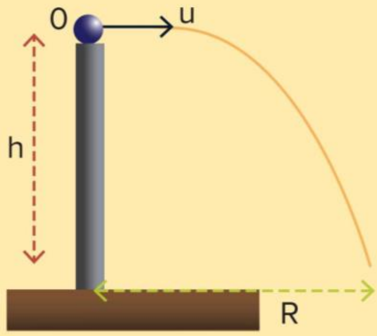
Equation of motion, $s = ut + \frac{1}{2}at^2$

At the bottom point, $-h = ut - \frac{1}{2}gt^2$

$= -\frac{1}{2}gt^2$

Time taken to reach the ground, $t = \sqrt{\frac{h}{2g}}$

Case 2: If an object is thrown with a horizontal velocity



Along X-direction	Along Y-direction
$u_x = u$	$u_y = 0$
$a_x = 0$	$a_y = -g$

Equation of motion, $s = ut + \frac{1}{2}at^2$

Along vertical direction, $-h = u_y t - \frac{1}{2}gt^2$

$= -\frac{1}{2}gt^2$

Time of flight, $t = \sqrt{\frac{h}{2g}}$

In horizontal projection, the time of flight is independent of initial velocity and only depends on the height of projection.

- Velocity at a general point, $v_p = \sqrt{u^2 + g^2t^2}$

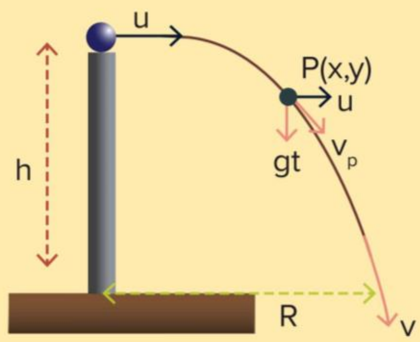
Proof

At any point, P(x, y)

$v_x = u$ (Constant)

$$v_y = u_y + a_y t$$
$$= 0 - gt = -gt$$
$$v_p = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{u^2 + g^2t^2}$$

- Landing velocity, $v = \sqrt{u^2 + 2gh}$



Proof

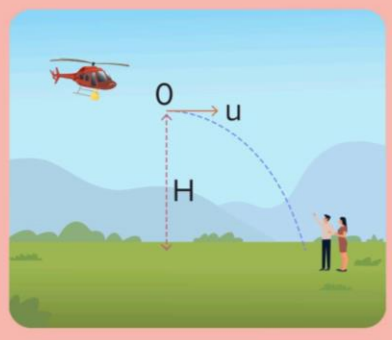
At the time of landing,

$v_x = u$ (Constant)

$$v_y^2 = u_y^2 + 2a_y s_y$$
$$= 0 - 2g(-h)$$
$$v_y = \sqrt{2gh}$$
$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{u^2 + 2gh}$$

Example

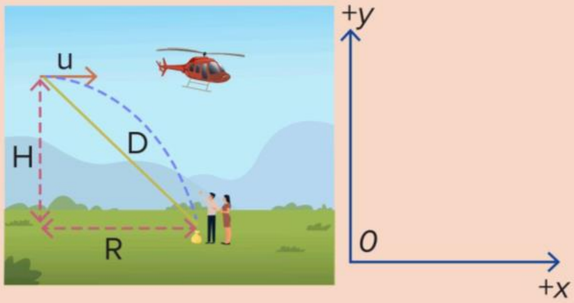
A helicopter on a flood relief mission, flying horizontally at a speed u at an altitude H, has to drop a food packet for a victim on the ground. At what distance from the victim should the packet be dropped? The victim stands in the vertical plane of the helicopter's motion.



Solution

It is a case of horizontal projection.

Launch speed = Speed of helicopter = u



Height of projection = H

Time of flight, $T = \sqrt{\dfrac{2H}{g}}$

Horizontal distace, $R = u \times T$

$$= u\sqrt{\frac{2H}{g}}$$

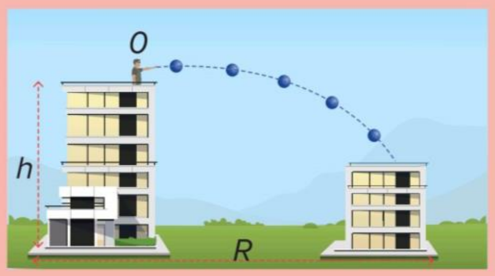
Distance between the helicopter and the person,

$$D = \sqrt{R^2 + H^2}$$

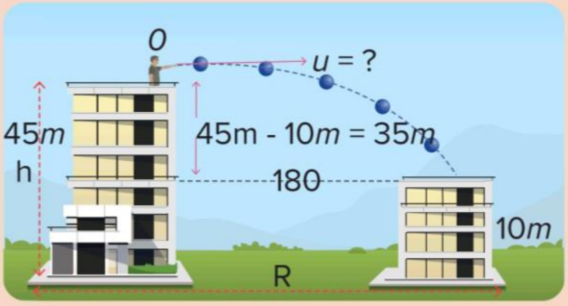
$$= \sqrt{\frac{2u^2H}{g} + H^2}$$

Example

An object is thrown between two tall buildings 180 m from each other. The object thrown horizontally from the top of a building, which is 45 m high, lands on the top of another building which is 10 m high. Find the speed of projection. (Use $g = 10\ ms^{-2}$)



Solution



Vertical distance covered,
h = Height of taller building - Height of shorter building
= 45 - 10 = 35 m

Time of flight,

$$T = \sqrt{\frac{2h}{g}}$$

$$= \sqrt{\frac{2 \times 35}{10}}$$

$$= \sqrt{7}\,s$$

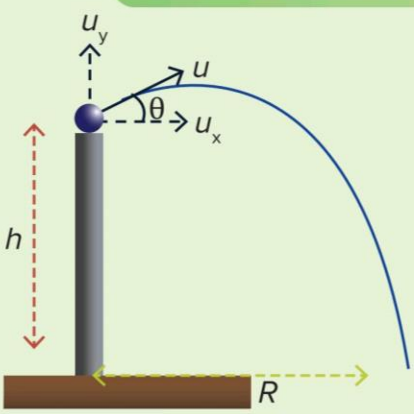
Horizontal distance covered
= Distance between the buildings
$R = 180$ m

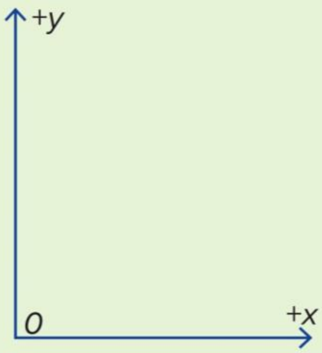
$$R = uT$$

$$u = \frac{R}{T}$$

$$= \frac{180}{\sqrt{7}}\,ms^{-1}$$

Projection at an angle θ above horizontal





Along X-direction	Along Y-direction
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = -g$

Equation of motion,

$s = ut + \frac{1}{2}at^2$

In vertical direction,

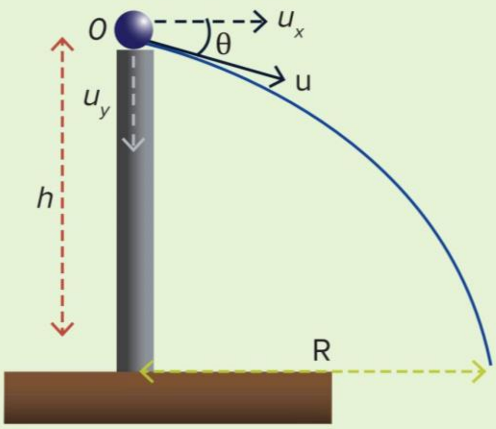
$y = u_y t + \frac{1}{2}a_y t^2$

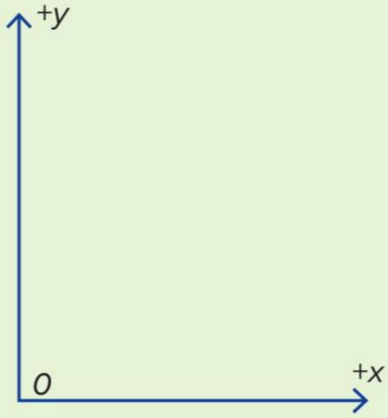
At the landing point

$-h = (u\sin\theta)T - \frac{1}{2}gT^2 \rightarrow$ *This equation gives time of flight T.*

Horizontal distance coverd, $R = u_x T = u\cos\theta T$

Projection at an angle θ above horizontal





Along X-direction	Along Y-direction
$u_x = u\cos\theta$	$u_y = -u\sin\theta$
$a_x = 0$	$a_y = -g$

Equation of motion,

$$s = ut + \frac{1}{2}at^2$$

In vertical direction,

$$y = u_y t + \frac{1}{2}a_y t^2$$

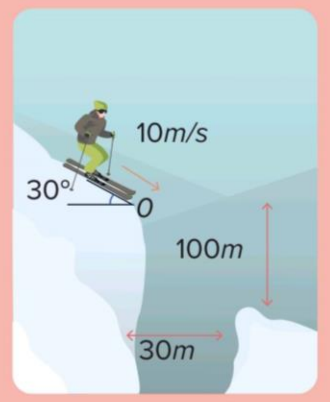
At the landing point

$$-h = (-u\sin\theta)T - \frac{1}{2}gT^2 \rightarrow \text{This equation gives time of flight } T.$$

Horizontal distance coverd, $R = u_x T = u\cos\theta\, T$

Example

In a high-speed ski chase, a secret agent skis off a slope inclined at 300 below the horizontal at **10**ms^{-1}. In order to land safely on the snow **100**m below, the agent must clear a valley **30** m wide. Does he make it? Ignore the air resistance.



Solution

$u = 10,\ \theta = 30^0$

X	Y
$u_x = u\cos 30$	$u_y = -u\sin 30$
$a_x = 0$	$a_y = -g$

$$\Delta y = u_y t + \frac{1}{2} a_y t^2$$
$$-100 = -5t + \frac{1}{2}(-10)t^2$$
$$t^2 + t - 20 = 0$$
$$(t+5)(t-4) = 0$$
$$t = -5 \text{ or } 4$$

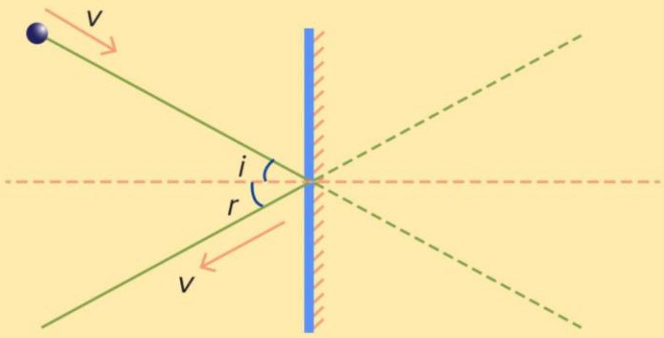
As time cannot be negative,
Time of flight, $T = 4\ s$
Horizontal distance covered during jump,

$$R = u\cos\theta \times T$$
$$= 10 \times \frac{\sqrt{3}}{2} \times 4$$
$$= 20\sqrt{3} \approx 34.6\ s$$

Horizontal distance covered is greater than 30 m. Hence landing is safe.

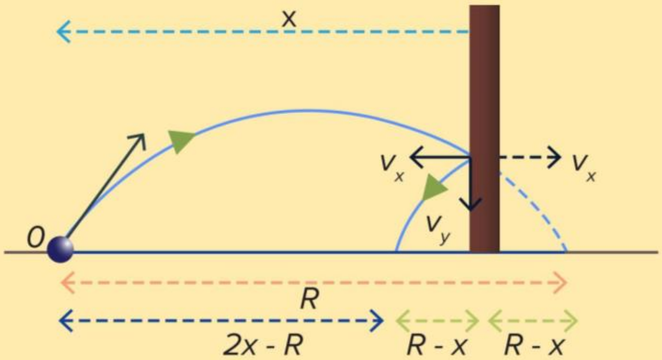
Elastic collision with a wall

In case of elastic collision,
1. Angle of incidence (i) = Angle of reflection (r)
2. Speed remains the same after collision.



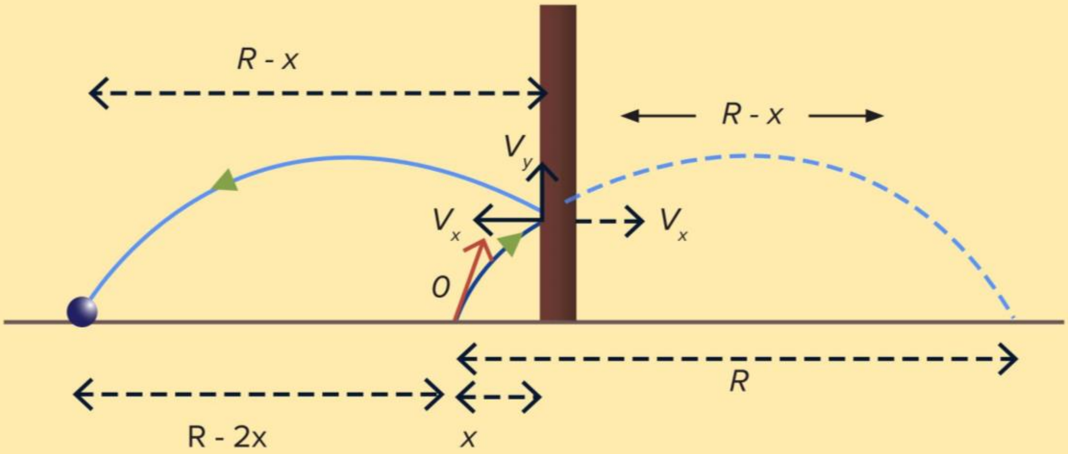
Elastic collision of projectile with a wall

- **Case I: $x \geq R/2$ (On its way down)**



Time of flight and maximum height remains the same but range changes.
Range = x - (R - x) = 2x - R (From the figure)

- **Case II: $x < R/2$ (On its way up)**



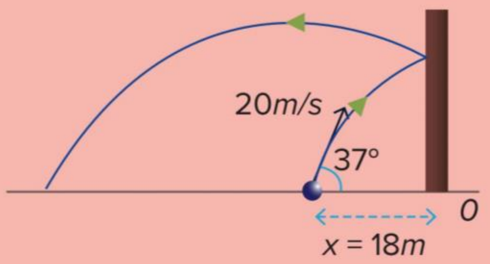
Time of flight and maximum height remains the same but range changes.

Range = (R - x) - X = R - 2x (From the figure)

Example

A ball is thrown at an angle of 37^0 from a point which is at a distance 18 m from a wall. The ball undergoes elastic collision with the wall and rebounds. Take $g = 10\ ms^{-2}$ and find the following:

I. The distance between the launching and landing point
II. The maximum height reached by the ball
III. Time of flight



Solution

I. Range, if there was no collision,

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{20^2 \times 2 \sin\theta \cos\theta}{g}$$

$$= \frac{400 \times 2 \sin 37 \cos 37}{10}$$

$$= 80 \times \frac{3}{5} \times \frac{4}{5} = 38.4\,m$$

$R/2 > x$ → Collision happened on way up

$$Actual\ range = R - 2x = 38.4 - (2 \times 18) = 2.4m$$

II. Maximum height does not change.

$$H=\frac{(u\sin\theta)^2}{g}$$
$$=\frac{\left(20\times\frac{3}{5}\right)^2}{10}$$
$$=7.2\,m$$

III. Time of flight does not change.

$$T=\frac{2u\sin\theta}{g}$$
$$=\frac{2\times20\sin37}{10}$$
$$=\frac{2\times20\times\frac{3}{5}}{10}=2.4\,s$$

PHYSICS

MOTION IN TWO DIMENSIONS

PROJECTILE ON INCLINED PLANE

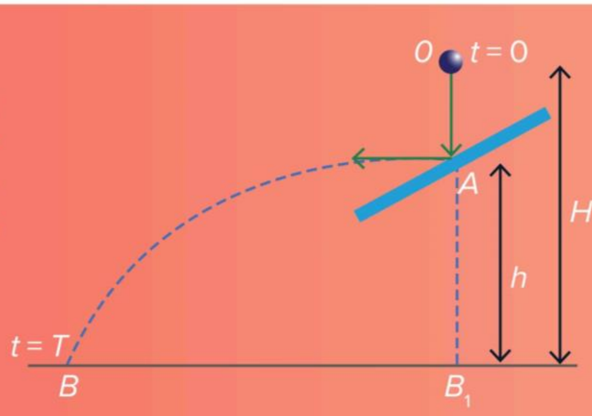
What you already know

- Projectile motion
- Horizontal projectile motion
- Parameters of projectile motion

What you will learn

- Projectile on an inclined plane
- Projectile motion from bottom to top and top to bottom of an inclined plane
- Parameters of projectile motion on an incline plane

A body falling freely from a given height H hits an inclined plane in its path at a height h. As a result of this impact, the velocity of the body becomes horizontal. For what value of $\frac{h}{H}$, will the body take maximum time to reach the ground?



Solution

The body is under free fall from O to A. Then, it undergoes horizontal projectile motion from A to B.

Height covered during fall from O to A = H − h

Time taken to fall from O to A,

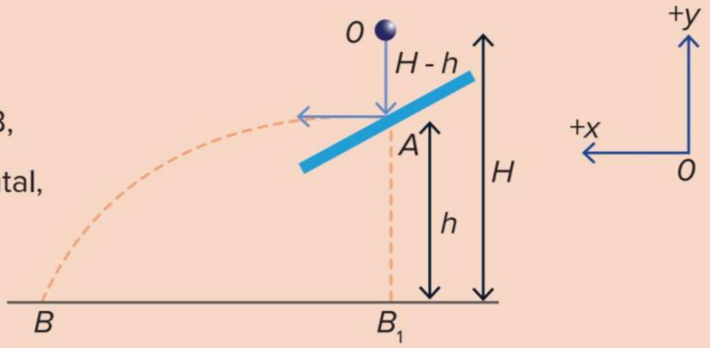
$$t_1 = \sqrt{\frac{2(H-h)}{g}}$$

Time taken for horizontal projection from A to B,

$$t_2 = \sqrt{\frac{2h}{g}}$$ {as the velocity at A becomes horizontal, $u_y=0$}

Total time,

$$T = t_1 + t_2$$



$$= \sqrt{\frac{2(H-h)}{g}} + \sqrt{\frac{2h}{g}}$$

$$= \sqrt{\frac{2}{g}}\left(\sqrt{H-h} + \sqrt{h}\right)$$

At maximum value of T,

$$\frac{dT}{dh} = 0$$

$$\text{or } \sqrt{\frac{2}{g}}\left(\frac{-1}{2\sqrt{H-h}} + \frac{1}{2\sqrt{h}}\right) = 0 \qquad \because \frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}}$$

$$\frac{1}{2\sqrt{H-h}} = \frac{1}{2\sqrt{h}}$$

$$H - h = h$$

$$\frac{h}{H} = \frac{1}{2}$$

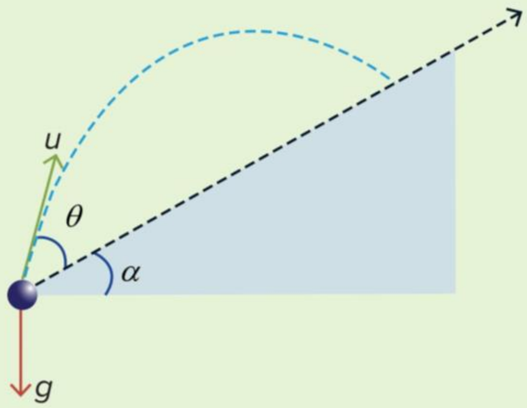
Time of flight of horizontal projection from a height h is the same as that of free fall from height h.

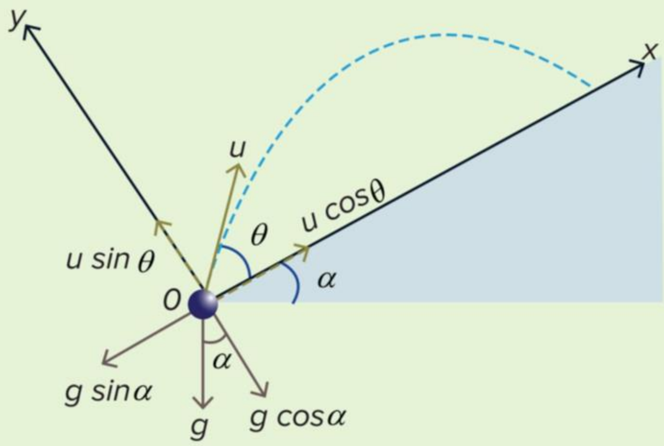
MAIN

Projectile on an Inclined Plane

Case I: Bottom to Top

- $\boldsymbol{\alpha}$ is the angle of inclination and $\boldsymbol{\theta}$ is the projection angle measured from the inclined plane.
- It is not a ground to ground projectile motion.
- Choosing projection point as origin and x-axis along the inclined plane as shown.





Along x-axis	Along y-axis
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = -g\sin\alpha$	$a_y = -g\cos\alpha$

Time of flight (T)

When time, t = Time of flight (T), $\Delta y = 0$,

$$\Delta y = u_y t + \frac{1}{2} a_y t^2$$

$$0 = (u \sin\theta)t + \frac{1}{2}(-g\cos\alpha)t^2$$

t = 0 and

$$t = \frac{2u\sin\theta}{g\cos\alpha}$$ Since t cannot be 0

$$\therefore \text{Time of flight, } T = \frac{2u\sin\theta}{g\cos\alpha} = \frac{2u_y}{a_y}$$

In case of ground to ground projectile motion, time of flight, $T = \dfrac{2u\sin\theta}{g} = \dfrac{2u_y}{a_y}$

Maximum distance from incline (y_{max})

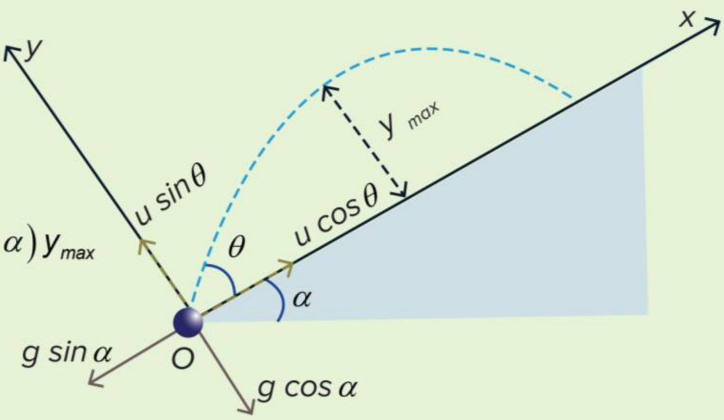
At y = y_{max} $v_y = 0$,

$$\text{Equation of motion, } v_y^2 = u_y^2 + 2a_y s_y$$

At top point, velocity along y − axis = 0

$$0 = (u\sin\theta)^2 + 2(-g\cos\alpha)y_{max}$$

$$y_{max} = \frac{u^2\sin^2\theta}{2g\cos\alpha} = \frac{u_y^2}{2a_y}$$



- In case of ground to ground projectile motion, maximum height

$$H_{max} = \frac{(u\sin\theta)^2}{2g} = \frac{(u_y)^2}{2a_y}$$

- The term height is not used in projection on an inclined plane, as height is conventionally measured from horizontal ground.

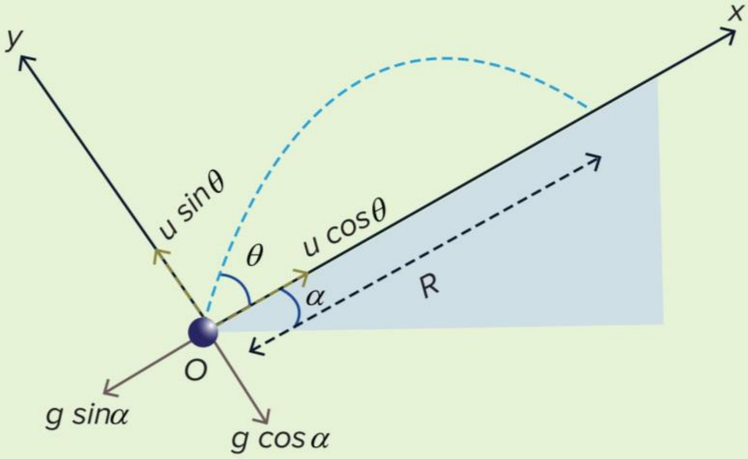
Range (R)

At $y = y_{max}$ $v_y = 0$,

$$\text{Equation of motion, } s_x = u_x t + \frac{1}{2} a_x t^2$$

At $t = T, s_x = R$ *where* T = *time of flight*

$$R = (u\cos\theta)T - \frac{1}{2}(g\sin\alpha)T^2$$



$$R = (u\cos\theta)\left(\frac{2u\sin\theta}{g\cos\alpha}\right) - \frac{1}{2}(g\sin\alpha)\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2 \qquad \because T = \left(\frac{2u\sin\theta}{g\cos\alpha}\right)$$

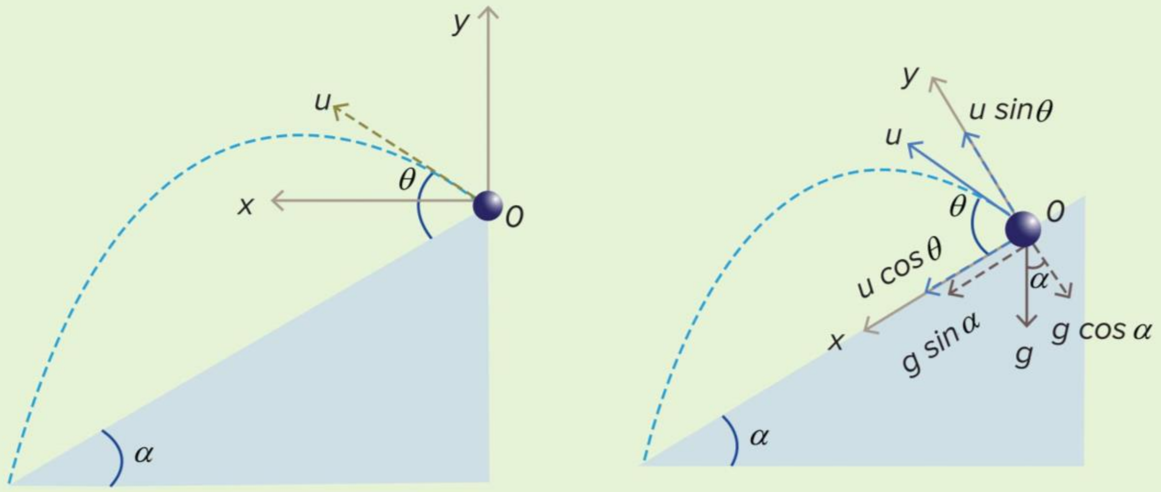
$$R = \left(\frac{2u^2\sin\theta}{g\cos\alpha}\right)\left(\cos\theta - \frac{\sin\theta\sin\alpha}{\cos\alpha}\right)$$

$$R = \left(\frac{2u^2\sin\theta}{g\cos\alpha}\right)\left(\frac{\cos\theta\cos\alpha - \sin\theta\sin\alpha}{\cos\alpha}\right)$$

$$R = \frac{2u^2\sin\theta\cos(\theta+\alpha)}{g\cos^2\alpha} \qquad \because \cos\theta\cos\alpha - \sin\theta\sin\alpha = \cos(\theta+\alpha)$$

Case II: Top to Bottom

- $\boldsymbol{\alpha}$ is the angle of inclination and $\boldsymbol{\theta}$ is the projection angle measured from the inclined plane.
- Choosing projection point as origin and x-axis along the inclined plane as shown.



Along *x*-axis	Along *y*-axis
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = g\sin\alpha$	$a_y = -g\cos\alpha$

Time of flight (T)

When time, t = Time of flight (T), $\Delta y = 0$,

$$\Delta y = u_y t + \frac{1}{2}a_y t^2$$

$$0 = (u\sin\theta)t + \frac{1}{2}(-g\cos\alpha)t^2$$

$$t = 0, \text{ or, } \frac{2u\sin\theta}{g\cos\alpha}$$

$$\therefore \text{Time of flight, } T = \frac{2u\sin\theta}{g\cos\alpha} = \frac{2u_y}{a_y}$$

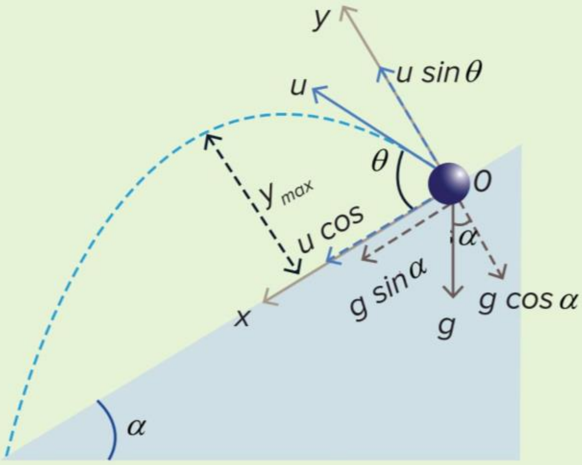
Maximum distance from incline (y_{max})

At $y = y_{max}$ $v_y = 0$,

Equation of motion, $v_y^2 = u_y^2 + 2a_y s_y$

$$0 = (u\sin\theta)^2 + 2(-g\cos\alpha)y_{max}$$

$$y_{max} = \frac{u^2\sin^2\theta}{2g\cos\alpha} = \frac{u_y^2}{2a_y}$$



Range (R)

Equation of motion, $s_x = u_x t + \frac{1}{2}a_x t^2$

At $t = T, s_x = R$ where T = time of flight

$$R = (u\cos\theta)T + \frac{1}{2}(g\sin\alpha)T^2$$

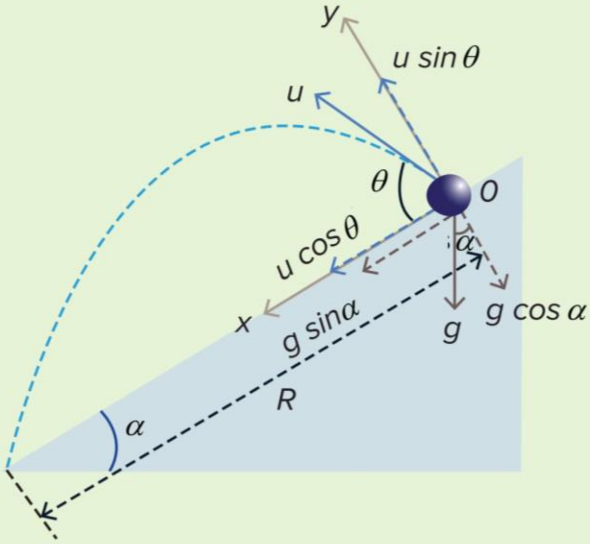
$$\because T = \left(\frac{2u\sin\theta}{g\cos\alpha}\right)$$

$$= (u\cos\theta)\left(\frac{2u\sin\theta}{g\cos\alpha}\right) + \frac{1}{2}(g\sin\alpha)\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2$$

$$= \left(\frac{2u^2\sin\theta}{g\cos\alpha}\right)\left(\cos\theta + \frac{\sin\theta\sin\alpha}{\cos\alpha}\right)$$

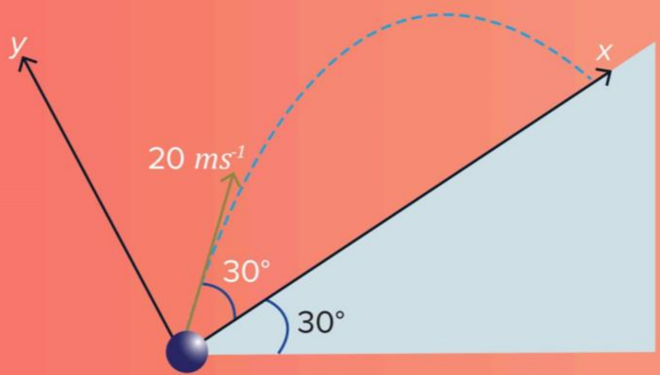
$$= \left(\frac{2u^2\sin\theta}{g\cos\alpha}\right)\left(\frac{\cos\theta\cos\alpha + \sin\theta\sin\alpha}{\cos\alpha}\right)$$

$$= \frac{2u^2\sin\theta\cos(\theta-\alpha)}{g\cos^2\alpha} \quad \because \cos\theta\cos\alpha + \sin\theta\sin\alpha = \cos(\theta-\alpha)$$



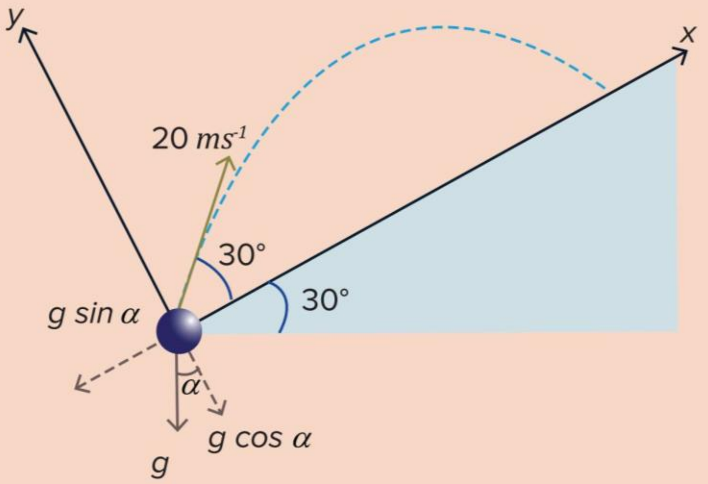
A projectile is thrown from the base of an inclined plane of angle 30° as shown in the figure. It is thrown at an angle of 30° from the incline at a speed of 20 ms^{-1}. Take $g = 10\ ms^{-2}$. Find the following:

(a) The total time of flight of the projectile.
(b) The maximum distance from the incline.



BOARDS

Solution



The projection is from bottom to top.

Angle of inclination, $\alpha = 30°$

Angle of projection measured from the inclined plane, $\theta = 30°$

Speed of projection, $u = 20\ ms^{-1}$

Along *x*-axis	**Along *y*-axis**
$u_x = u\cos\theta = 20\cos 30^0 = 10\sqrt{3}\,ms^{-1}$	$u_y = u\sin\theta = 20\sin 30^0 = 10\,ms^{-1}$
$a_x = -g\sin\alpha = -10\times\sin 30^0 = -5\,ms^{-2}$	$a_y = -g\cos\alpha = -10\times\cos 30^0 = -5\sqrt{3}\,ms^{-2}$

(a) Time of flight,

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

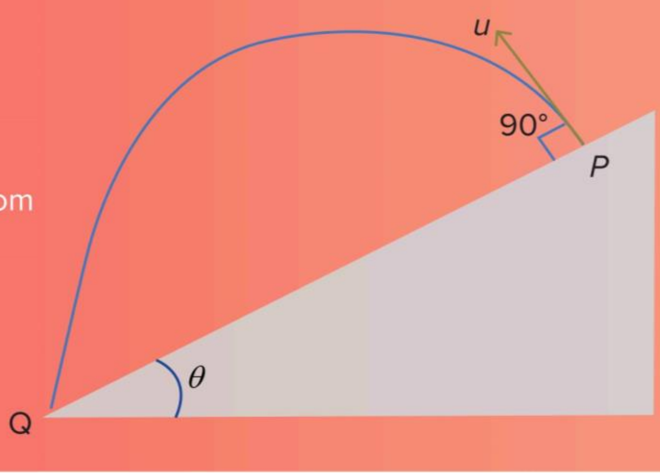
$$= \frac{2\times 10}{5\sqrt{3}} = \frac{4}{\sqrt{3}}\,s$$

(b) Maximum distance from the incline,

$$y_{max} = \frac{(u\sin\theta)^2}{2g\cos\alpha}$$

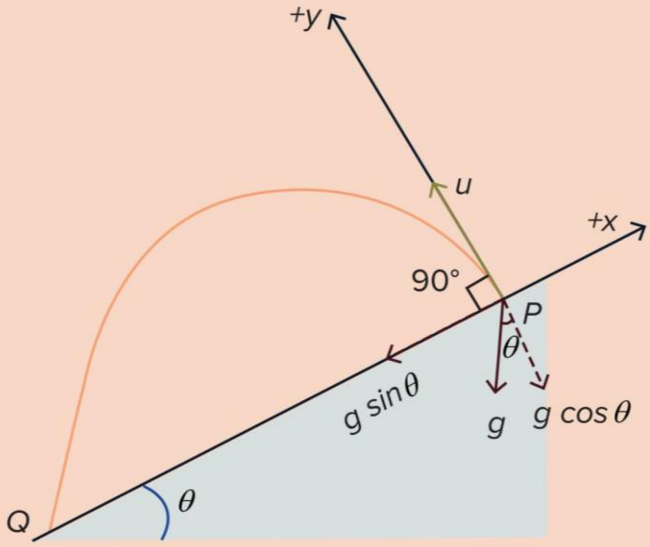
$$= \frac{10^2}{2\times 5\sqrt{3}} = \frac{10}{\sqrt{3}}\,m$$

If the time taken by the projectile to reach from point P to point Q is T, then compute PQ.



MAIN

Solution



The projection is from top to bottom.

Angle of inclination = $\boldsymbol{\theta}$

Angle of projection measured from the inclined plane = 90°

Along x-axis	**Along y-axis**
$u_x = 0$	$u_y = u$
$a_x = -g \sin\theta$	$a_y = -g\cos\theta$

When t = Time of flight (T), $\Delta y = 0$

$$\Delta y = u_y t + \frac{1}{2}a_y t^2$$

$$0 = uT + \frac{1}{2}(-g\cos\theta)T^2$$

$$\text{Time of flight, } T = \frac{2u}{g\cos\theta}$$

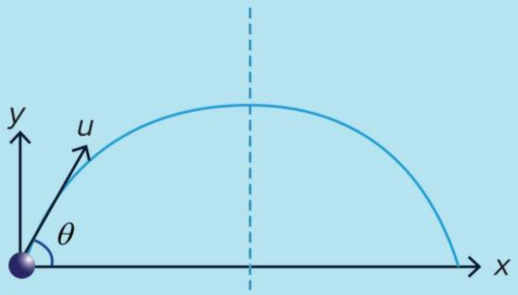
$$PQ = u_x T + \frac{1}{2}a_x T^2$$

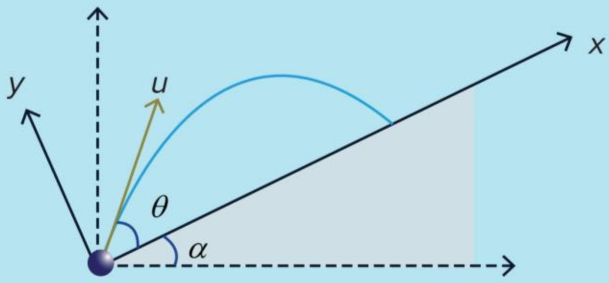
$$= 0 - \frac{1}{2}g\sin\theta\left(\frac{2u}{g\cos\theta}\right)^2$$

$$= \frac{-2u^2\sin\theta}{g\cos^2\theta} \qquad (-\text{ve sign as } PQ \text{ is measured in} -\text{ve } x-\text{direction})$$

$$|PQ| = \frac{2u^2\sin\theta}{g\cos^2\theta}$$

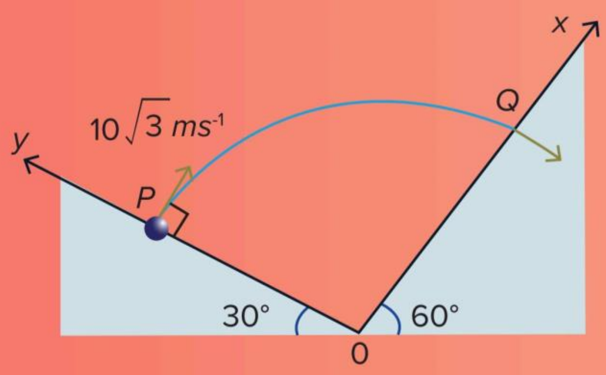
Symmetry of Projectile Trajectory





- In case of ground to ground projectile motion, trajectory is a parabola with an axis of symmetry. But in case of incline projectile motion, trajectory is a parabola without an axis of symmetry.
- In case of ground to ground projectile motion, acceleration along the horizontal direction is zero. Hence, it undergoes a uniform motion along x-direction. So, it has an axis of symmetry.
- However, in case of a projectile on an inclined plane, it has a non-zero acceleration along x and y direction. Hence, it does not have an axis of symmetry.

Two inclined planes of angles 30° and 60° are placed touching each other at the base as shown in the figure. A projectile is projected at right angle with a speed of $10\sqrt{3}\ ms^{-1}$ from point P and hits the other incline at point Q normally. Find the following:

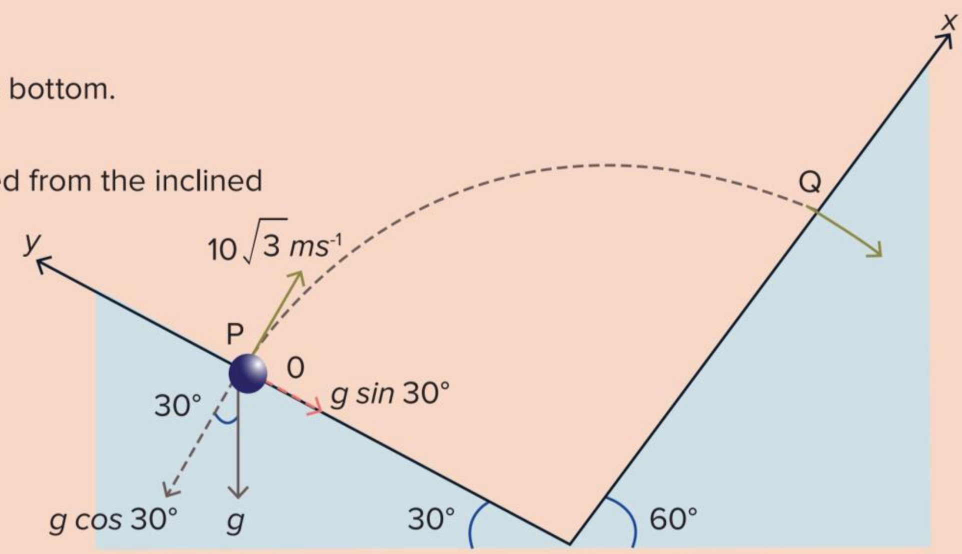


Solution

The projection is from top to bottom.

Angle of inclination = θ

Angle of projection measured from the inclined plane = 90°



Along *x*-axis	Along *y*-axis
$u_x = 10\sqrt{3}\, ms^{-1}$	$u_y = 0$
$a_x = -5\sqrt{3}\, ms^{-2}$	$a_y = -5\, ms^{-2}$

Let v be the landing velocity at Q.

Landing velocity is perpendicular to X-axis, so $v_x = 0$

At $t = T$ (Time of flight); $v_x = 0$

$$v_x = u_x + a_x t$$

$$0 = 10\sqrt{3} - 5\sqrt{3}\,T$$

$$T = 2\,s$$

$$v_y = u_y + a_y T$$

$$= 0 - 5 \times 2 = -10\ ms^{-1}$$

$\therefore$ Landing speed $= 10\ ms^{-1}$