



U-02 CH-05

CBSE - PHYSICS - CIRCULAR MOTION

UNIFORM CIRCULAR MOTION

CBSE - PHYSICS



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UNIFORM CIRCULAR MOTION

UNIT-2 CHAPTER-V

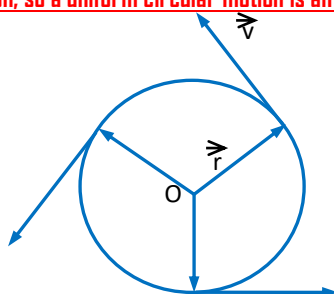
If a particle moves along a circular path with a constant speed (i.e., it covers equal distances along the circumference of the circle in equal intervals of time), then its motion is said to be a uniform circular motion.

- Examples:** (i) Motion of the tip of the second hand of a clock.
 (ii) Motion of a point on the rim of a wheel rotating uniformly.

☐☐ **Uniform circular motion is an accelerated motion:** In uniform circular motion,

⊙ **the speed of the body remains the same but the direction of motion changes at every point.** Fig., shows the different velocity vectors at different positions of the particle. At each position, the velocity vector v is perpendicular to the radius vector r . Thus, the velocity of the body changes continuously due to the continuous change in the direction of motion of the body.

⊙ **As the rate of change of velocity is acceleration, so a uniform circular motion is an accelerated motion.**

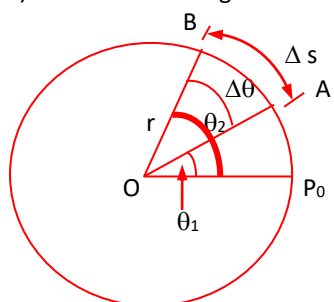


[Direction of velocity changes at every point]

☐☐ **ANGULAR DISPLACEMENT:**

The angular displacement of a particle moving along a circular path is defined as the angle swept out by its radius vector in the given time interval.

Suppose a particle starts from the position P_0 . Its angular position is θ_1 at time t_1 and θ_2 at time t_2 . Let the particle cover a distance Δs in the time interval $t_2 - t_1 (= \Delta t)$. It revolves through and $\theta_2 - \theta_1 (= \Delta\theta)$ in this interval.



[Angular displacement]

The angle of revolution $\Delta\theta$ is the angular displacement of the particle. If r is the radius of the circle, then

$$\Delta\theta = \frac{\Delta s}{r} \quad \left[\because \text{Angle} = \frac{\text{Arc}}{\text{Radius}} \right]$$

▶ The unit of angular displacement is **radian**

▶ It is a **dimensionless quantity**.

☐☐ **ANGULAR VELOCITY:**

The time rate of change of angular displacement of a particle is called its angular velocity.

▶ It is denoted by ω .

▶ Unit: It is measured in **radian per second (rad s^{-1})**

▶ dimensional formula is **$[M^0L^0T^{-1}]$** .

Expression: if $\Delta\theta$ is the angular displacement of a particle in time Δt , then its average angular velocity is

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

When the time interval $\Delta t \rightarrow 0$, the limiting value of the average velocity is called the instantaneous angular velocity, which is given by

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

TIME PERIOD: The time taken by a particle to complete one revolution along its circular path is called its period of revolution.

► It is denoted by **T**

► Unit: measured in **second**.

FREQUENCY: The frequency of an object in circular motion is defined as the number of revolutions completed per unit time.

► It is denoted by **ν (nu)**.

Expression: If ν is the frequency of revolution of a particle, then time taken to complete ν revolutions = 1 second, time taken to complete 1 revolution = $1/\nu$ second

But time taken to complete 1 revolution is the time period T, so

$$T = 1/\nu \quad \text{or} \quad \nu = 1/T$$

Relations between ω , ν and T:

By definition of time period, a particle completes one revolution in time T i.e., it traverses an angle of 2π radian in time T.

\therefore When time $t = T$, Angular displacement $\theta = 2\pi$ radian

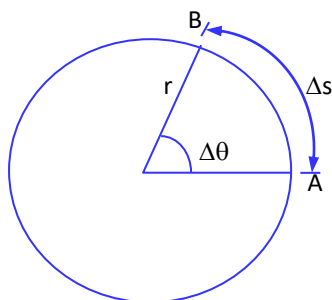
Angular velocity = $\frac{\text{Angular displacement}}{\text{Time}}$

or
$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi\nu \quad [\because 1/T = \nu]$$

Relation between linear velocity and angular velocity:

Consider a particle moving along a circular path of radius r . As shown in Fig., suppose the particle moves from A to B in time Δt covering distance Δs along the arc AB. Hence the angular displacement of the particle is

$$\Delta\theta = \frac{\Delta s}{r}$$



[Velocity in uniform circular motion]

Dividing both sides by Δt , we get

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{r} \frac{\Delta s}{\Delta t}$$

Taking the limit $\Delta t \rightarrow 0$ on both sides,

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{1}{r} \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t}$$

But
$$\lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega$$

the instantaneous angular velocity

and
$$\lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} = v,$$

The instantaneous linear velocity.

$$\therefore \omega = 1/r \cdot v$$

or
$$v = \omega r$$

Linear velocity = Angular velocity \times radius

In vector notation, we have the relation

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$

For a given angular velocity (ω), the linear velocity v of a particle is directly proportional to its distance from the centre.

ANGULAR ACCELERATION:

The time rate of change of angular velocity of a particle is called its angular acceleration.

Expression: If $\Delta\omega$ is the change in angular velocity in time Δt , then the average angular acceleration is

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$$

The instantaneous acceleration is equal to the limiting value of the average acceleration $\Delta\omega/\Delta t$ when Δt approaches zero. It is given by

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

The angular acceleration is measured in **radian per second² (rad s^{-2})**

Dimensions **$[M^0 L^0 T^{-2}]$** .

The relation between linear velocity v and angular velocity ω is **$v = r\omega$**

Differentiating both sides w.r.t. time t , we get

$$\frac{dv}{dt} = r \frac{d\omega}{dt}$$

or $\frac{dv}{dt} = r \left(\frac{d\omega}{dt} \right)$ [$\because r$ is constant]

or $a = \alpha r$

Linear velocity = Angular acceleration \times radius

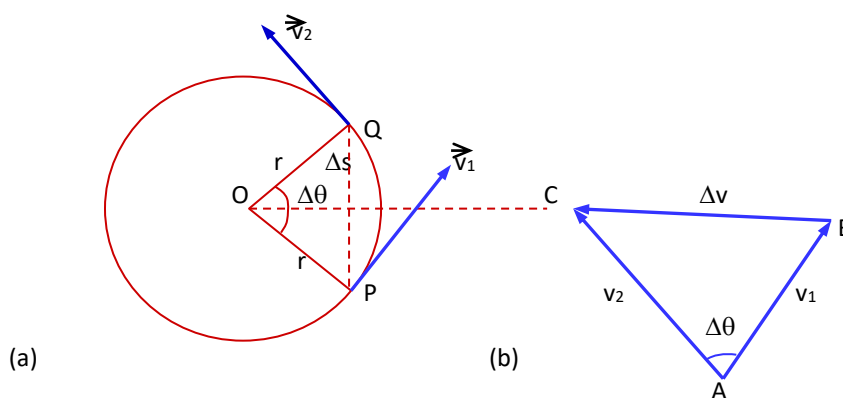
In vector rotation, we have the relation

$$\vec{a} = \vec{\alpha} \times \vec{r}$$

CENTRIPETAL ACCELERATION:

When a body is in uniform circular motion, its speed remains constant but its velocity changes continuously due to the change in its direction. Hence the motion is accelerated. A body undergoing which is directed along the radius towards the centre of the circular path. This acceleration is called centripetal (centre seeking) acceleration.

Expression for centripetal acceleration: Consider a particle moving on a circular path of radius r and centre O , with a uniform speed v . As shown in Fig. (a), suppose at time t the particle is at P and at time $t + \Delta t$, the particle is at Q . Let v_1 and v_2 be the velocity vectors at P and Q , directed along the tangents at P and Q respectively.



[Determination of centripetal acceleration]

To determine the change in velocity, take an external point A . Draw AB equal to and parallel to v_1 and AC equal to and parallel to v_2 . Draw the vector BC to close the triangle, as shown in Fig. (b).

Applying triangle law of vector addition in ΔBAC ,

$$AB + BC = AC \quad \therefore \quad BC = AC - AB = v_2 - v_1$$

Thus, the change in velocity in time Δt is given by

$$BC = \Delta v$$

If Δt is small, the chord PQ becomes equal to arc PQ . Then OPQ , can be considered as a triangle. $\angle POQ = \angle BAC = \Delta\theta$. This is because the angle between the radii PO and QO is same as the angle between the tangents at P and Q .

Also, $OP = OQ = r$, radius of the circle.

$$|v_1| = |v_2| = v \quad \text{i.e.,} \quad AB = AC = v$$

Also $\angle POQ = \Delta\theta$, $\angle BAC = \Delta\theta$
 Thus, the two triangles POQ and BAC are similar. Hence

$$\frac{PQ}{OP} = \frac{BC}{AB}$$

or $\frac{\Delta s}{r} = \frac{\Delta v}{v}$

or $\Delta v = \frac{v}{r} \Delta s$

Dividing both sides by Δt , we get

$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \frac{\Delta s}{\Delta t}$$

Taking the limit $\Delta t \rightarrow 0$ on both sides, we get

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{v}{r} \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t}$$

But $\lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = a$

the instantaneous acceleration

and $\lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} = v$

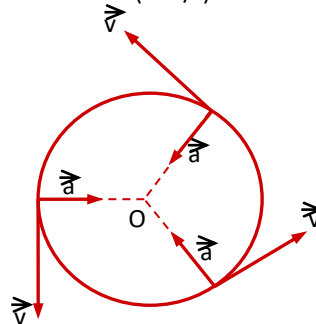
the instantaneous velocity

$\therefore a = \frac{v}{r} \cdot v$

or $a = \frac{v^2}{r} = \omega^2 r$ [$\because v = \omega r$]

This gives the magnitude of the acceleration of a particle in uniform circular motion.

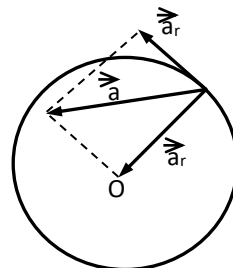
Direction of acceleration: As Δt tends to zero, the angle $\Delta\theta$ also approaches zero. In this limit, as $AB = AC$, so $\angle ABC = \angle ACB = 90^\circ$. Thus, the change in velocity Δv and hence the acceleration a is perpendicular to the velocity vector v_1 . But v_1 is directed along tangent at point P, so acceleration 'a' act along the radius towards the centre of the circle. Such an acceleration is called centripetal acceleration. Its magnitude remains constant ($= v^2/r$) but its direction continuously changes and remains perpendicular to the velocity vector at all positions.



[Centripetal acceleration]

Circular motion with variable speed: Consider a particle moving along a circular path of radius with a variable speed v . As the speed of the particle changes, so acceleration has a tangential component,

$$a_T = \frac{dv}{dt} = r\alpha$$



[Resultant acceleration circular motion]

As the direction of motion changes continuously, so the acceleration has a radial component,

$$a_r \text{ or } a_c = \frac{v^2}{r}$$

The resultant acceleration of the particle will be

$$a = \sqrt{a_T^2 + a_r^2}$$

KNOWLEDGE +

- ☞ It was the Dutch scientist Christian Huygens who first gave a thorough analysis of centripetal acceleration in 1673.
- ☞ In uniform circular motion, the direction of velocity vector which acts along the tangent to the path, changes continuously but its magnitude always remains constant ($v = r\omega$). So circular motion is an accelerated motion.
- ☞ As 'v' and 'r' are constant, so the magnitude of the centripetal acceleration is a constant ($= v^2/r$). But the direction of a_c changes continuously, pointing always towards the centre. So, centripetal acceleration is not a constant vector.
- ☞ The resultant acceleration of an object in circular motion is towards the centre only if the speed is constant.
- ☞ For a body moving with a constant angular velocity, the angular acceleration is zero.
- ☞ In projectile motion, both the magnitude and direction of acceleration (g) remain constant, while in uniform circular motion the magnitude remains constant but the direction continuously changes. Hence the equations of motion $v = u + at$, etc., are not applicable to circular motion. These equations hold only when both the magnitude and direction of acceleration are constant.

Examples based on Uniform Circular Motion

◆ **Formulae Used**

1. Angular displacement, $\theta = \frac{s}{r}$
2. Angular velocity, $\omega = \frac{\theta}{t}$
3. Also, $\omega = \frac{2\pi}{T} = 2\pi v$
4. Linear velocity, $v = r\omega$
5. Centripetal acceleration, $a = \frac{v^2}{r} = r\omega^2$
6. Linear acceleration, $a = r\alpha$

◆ **Units Used:** Here θ is in radian, ω in rad s^{-1} , v in ms^{-1} , a in ms^{-2} and angular acceleration α in rad s^{-2} .

Q. 1. Calculate the angular speed of (i) the hour hand of a watch and (ii) the earth about its own axis.

Sol. (i) The hour hand completes one rotation in 12 hours.

$$\begin{aligned} \therefore \omega &= \frac{\theta}{T} = \frac{2\pi \text{ rad}}{12 \text{ h}} = \frac{2\pi \text{ rad}}{12 \times 60 \times 60 \text{ s}} \\ &= \frac{\pi}{21600} \text{ rad s}^{-1} \end{aligned}$$

(ii) The earth completes one rotation about its axis in 24 hours.

$$\begin{aligned} \therefore \omega &= \frac{\theta}{t} = \frac{2\pi \text{ rad}}{24 \text{ h}} = \frac{2\pi \text{ rad}}{24 \times 60 \times 60} \\ &= \frac{\pi}{43200} \text{ rad s}^{-1} \end{aligned}$$

Q. 2. Calculate the angular speed of flywheel making 420 revolutions per minute.

Sol. Here $v = 420$ revolutions/min

$$= \frac{420}{60} \text{ revolution/s}$$

$$\omega = 2\pi v = 2 \times \frac{22}{7} \times \frac{420}{60} = 44 \text{ rad s}^{-1}$$

Q. 3. A body of mass 10 kg revolves in a circle of diameter 0.40 m, making 1000 revolutions per minute. Calculate its linear velocity and centripetal acceleration.

Sol. Here $m = 10$ kg, $r = 0.40$ m,

$$v = \frac{1000}{60} \text{ s}^{-1}$$

$$\text{Angular speed, } \omega = 2\pi v = 2\pi \times \frac{1000}{60} = \frac{100\pi}{3} \text{ rad s}^{-1}$$

Linear velocity,

$$v = r\omega = 0.20 \times \frac{100\pi}{3} = \frac{20\pi}{3} \text{ ms}^{-1}$$

Centripetal acceleration,

$$a = r\omega^2 = 0.20 \times \left(\frac{100\pi}{3}\right)^2 = \frac{2000\pi^2}{9} \text{ ms}^{-2}$$

Q. 4. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 seconds, what is the magnitude and direction of acceleration of the stone?

Sol. Here $r = 80 \text{ cm}$, $v = \frac{14}{25} \text{ rps}$

$$\therefore \omega = 2\pi v = 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25} \text{ rad s}^{-1}$$

The acceleration of the stone is

$$a = r\omega^2 = 80 \times \left(\frac{88}{25}\right)^2 = 991.2 \text{ cm s}^{-2}$$

This acceleration is directed along the radius of the circular path towards the centre of the circle.

Q. 5. An aircraft executes a horizontal loop of radius 1 km with a steady speed of 900 km h⁻¹. Compare its centripetal acceleration with the acceleration due to gravity.

Sol. Here $r = 1 \text{ km} = 1000 \text{ m}$

$$v = 900 \text{ kmh}^{-1} = \frac{900 \times 5}{18} = 250 \text{ ms}^{-1}$$

Centripetal acceleration,

$$a = \frac{v^2}{r} = \frac{(250)^2}{1000} = 62.5 \text{ ms}^{-2}$$

$$\therefore \frac{\text{Centripetal acceleration}}{\text{Acceleration due to gravity}} = \frac{62.5}{9.8} = 6.38$$

Q. 6. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. (i) What is the angular speed and the linear speed of the motion? (ii) Is the acceleration vector a constant vector? What is the magnitude? (iii) What is its linear displacement?

Sol. Here $r = 12 \text{ cm}$, $v = \frac{7}{100} \text{ s}^{-1}$

(i) Angular speed,

$$\omega = 2\pi v = 2 \times \frac{22}{7} \times \frac{7}{100} = 0.44 \text{ rads}^{-1}$$

Linear speed, $v = r\omega = 12 \times 0.44 = 5.28 \text{ cm s}^{-1}$

(ii) Acceleration is always directed towards the centre of the circular groove. As the insect moves, the direction of the acceleration vector changes. So, acceleration vector is not a constant vector.

Magnitude of acceleration,

$$a = r\omega^2 = 12 \times (0.44)^2 = 2.32 \text{ cm s}^{-1}$$

(iii) After completing 7 revolutions, the insect comes back to its initial position. So its linear displacement is zero.

Q. 7. The radius of the earth's orbit around the sun is $1.5 \times 10^{11} \text{ m}$. Calculate the angular and linear velocity of the earth. Through how much angle does the earth revolve in 2 days?

Sol. Here $r = 1.5 \times 10^{11} \text{ m}$,

Period of revolution of the earth,

$$T = 365 \text{ days} = 365 \times 24 \times 60 \times 60 \text{ s}$$

\therefore Angular velocity,

$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{365 \times 24 \times 60 \times 60} = 1.99 \times 10^{-7} \text{ rad s}^{-1}$$

Linear velocity,

$$v = r\omega = 1.5 \times 10^{11} \times 1.99 \times 10^{-7} = 2.99 \text{ ms}^{-1}$$

In 365 days, the earth revolves through an angle of 2π radians.

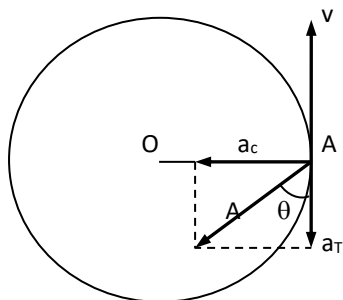
\therefore Angle through which the earth revolves in 2 days

$$= \frac{2\pi}{365} \times 2 = \frac{2 \times 3.14 \times 2}{365} = 0.0344 \text{ rad}$$

Q. 8. A cyclist is riding with a speed of 27 km h⁻¹. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate 0.5 ms⁻¹. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

Sol. Here $r = 80$ m
 $v = 27 \text{ kmh}^{-1} = 27 \times 5 \text{ ms}^{-1} = 7.5 \text{ ms}^{-1}$,
 Centripetal acceleration,
 $a_c = \frac{v^2}{r} = \frac{(7.5)^2}{80} = 0.7 \text{ ms}^{-2}$

Suppose the cyclist applies brakes at the point A of the circular turn, then, tangential acceleration a_T (negative) will act opposite to velocity. Given $a_T = 0.5 \text{ ms}^{-2}$.



As the accelerations a_c and a_T are perpendicular to each other, so the net acceleration of the cyclist is

$$a = \sqrt{a_c^2 + a_T^2} = \sqrt{(0.7)^2 + (0.5)^2}$$

$$= \sqrt{0.49 + 0.25} = \sqrt{.74} = 0.86 \text{ ms}^{-2}$$

If θ is the angle between the total acceleration and the velocity of the cyclist, then,

$$\tan \theta = \frac{a_c}{a_T} = \frac{0.7}{0.5} = 1.4 \quad \text{or} \quad \theta = 54^\circ 28'$$

Q. 9. A particle moves in a circle of radius 4.0 cm clockwise at constant speed of 2 cms^{-1} . If \hat{x} and \hat{y} are unit acceleration vectors along X-axis and Y-axis respectively (in cms^{-2}), find the acceleration of the particle at the instant half way between P and Q. Refer to Fig. (a).

Sol. As shown in Fig. (b), let R be the midpoint of arc PQ. Then $\angle POR = 45^\circ$.

Magnitude of acceleration at R,

$$a = \frac{v^2}{r} = \frac{(2)^2}{4} = 1 \text{ cm s}^{-2}$$

The acceleration a acts along RO.

Magnitude of component of a along X-axis.

$$a_x = a \cos 45^\circ = 1 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \text{ cms}^{-2}$$

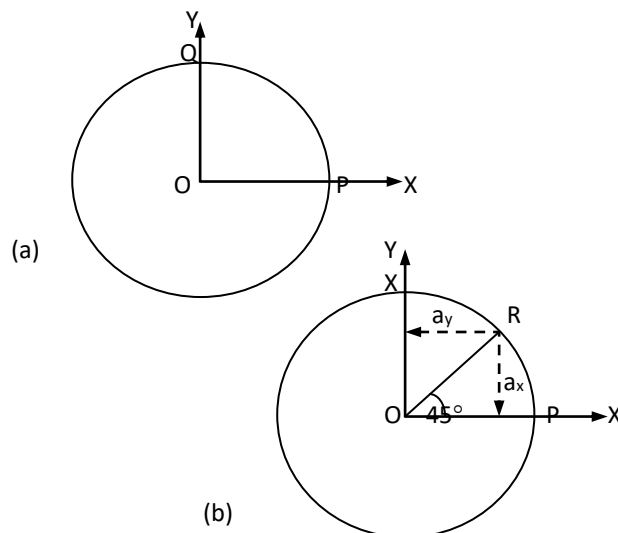
$$\therefore \hat{a}_x = -\frac{1}{\sqrt{2}} \hat{x}$$

Magnitude of component of a along Y-axis,

$$a_y = 1 \times \sin 45^\circ = \frac{1}{\sqrt{2}} \text{ cms}^{-2}$$

$$\therefore \hat{a}_y = -\frac{1}{\sqrt{2}} \hat{y}$$

$$\text{Hence } \hat{a} = \hat{a}_x + \hat{a}_y = -\frac{1}{\sqrt{2}} (\hat{x} + \hat{y})$$



Problems For Practice

Q. 1. What is the angular velocity of a second hand and minute hand of a clock?

Sol. The second hand of a clock completes one rotation in 60 s.

$$t = 60 \text{ s} \quad \text{and} \quad \theta = 2\pi \text{ rad}$$

Hence $\omega = \frac{\theta}{t} = \frac{2\pi}{60} = 2 \times 3.14 = 0.1047 \text{ rad s}^{-1}$

(ii) The minute hand of a clock completes one rotation in 60 minutes.

$$\therefore t = 60 \text{ min} = 3600 \text{ s} \quad \text{and} \quad \theta = 2\pi \text{ rad}$$

$$\text{Hence } \omega = \frac{\theta}{t} = \frac{2\pi}{3600} = \frac{2 \times 3.14}{3600} = 0.0017 \text{ rad s}^{-1}$$

Q. 2. A body of mass 0.4 kg is whirled in a horizontal circle of radius 2m with a constant speed of 10 ms^{-1} . Calculate its (i) angular speed (ii) frequency of revolution (iii) time period and (iv) centripetal acceleration.

Sol. Here $m = 0.4 \text{ kg}$, $r = 2 \text{ m}$, $v = 10 \text{ ms}^{-1}$

(i) Angular speed, $\omega = \frac{v}{r} = \frac{10}{2} = 5 \text{ rad s}^{-1}$

(ii) Frequency, $\nu = \frac{\omega}{2\pi} = \frac{5 \times 7}{2 \times 22} = 0.795 \text{ Hz}$
 $\approx 0.8 \text{ Hz}$

(iii) Time period, $T = \frac{1}{\nu} = \frac{1}{0.8} = 1.25 \text{ s}$

(iv) Centripetal acceleration,
 $a = r\omega^2 = 2 \times (5)^2 = 50 \text{ ms}^{-2}$

Q. 3. A circular wheel of 0.50 m radius is moving with a speed of 10 ms⁻¹. Find the angular speed.

Sol. Here $r = 0.50 \text{ m}$, $v = 10 \text{ ms}^{-1}$

As $v = r\omega$

$\therefore \omega = \frac{v}{R} = \frac{10 \text{ ms}^{-1}}{0.50 \text{ m}} = 20.0 \text{ rad s}^{-1}$

Q. 4. Assuming that the moon completes one revolution in a circular orbit around the earth in 27.3 days, calculate the acceleration of the moon towards the earth. The radius of the circular orbit can be taken as $3.85 \times 10^5 \text{ km}$.

Sol. $a = r\omega^2 = r \left(\frac{2\pi}{T} \right)^2 = 3.85 \times 10^8 \times \left(\frac{2 \times 3.14}{27.3 \times 24 \times 60 \times 60} \right)^2$
 $= 2.73 \times 10^{-3} \text{ ms}^{-2}$

Q. 5. The angular velocity of a particle moving along a circle of radius 50 cm is increased in 5 minutes from 100 revolutions per minute to 400 revolutions per minute. Find (i) angular acceleration and (ii) linear acceleration.

Sol. Here $\nu_1 = \frac{100}{60} \text{ rps}$, $\nu_2 = \frac{400}{60} \text{ rps}$, $r = 50 \text{ cm}$,

$t = 5 \text{ min} = 300 \text{ s}$

(i) Angular acceleration,

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi\nu_2 - 2\pi\nu_1}{t} = 2\pi \left(\frac{\nu_2 - \nu_1}{t} \right)$$

$$= 2\pi \left(\frac{400 - 100}{60 \times 300} \right) = \frac{\pi}{30} \text{ rad s}^{-2}$$

(ii) Linear acceleration, $a = r\alpha = 50 \times \frac{\pi}{30}$
 $= \frac{5\pi}{3} \text{ cm s}^{-2}$

Q. 6. Calculate the linear acceleration of a particle moving in a circle of radius 0.4 m at the instant when its angular velocity is 2 rad s⁻¹ and its angular acceleration is 5 rad s⁻².

Sol. Here $r = 0.4 \text{ m}$, $\omega = 2 \text{ rad s}^{-1}$, $\alpha = 5 \text{ rad s}^{-2}$

Centripetal acceleration,

$a_T = r\alpha = 0.4 \times 5 = 2 \text{ ms}^{-2}$

Centripetal acceleration,

$a_C = r\omega^2 = 0.4 \times (2)^2 = 1.6 \text{ ms}^{-2}$

\therefore Total linear acceleration,

$a = \sqrt{a_T^2 + a_C^2} = \sqrt{2^2 + (1.6)^2} = 2.6 \text{ ms}^{-2}$

If a makes angle θ with a_T , then

$\tan \theta = \frac{a_C}{a_T} = \frac{1.6}{2} = 0.8$

$\therefore \theta = 38^\circ 40'$

Q. 7. A threaded rod with 12 turns per cm and diameter 1.18 cm is mounted horizontally. A bar with a threaded hole to match the rod is screwed onto the rod. The bar spins at the rate of 216 rpm. How long will it take for the bar to move 1.50 cm along the rod?

Sol. Pitch of threaded screw = 1/12 cm

Number of rotations required to move a distance of 1.5 cm,

$n = \frac{\text{Distance}}{\text{Pitch}} = \frac{1.5}{1/12} = 18$

$\therefore \theta = 2\pi n = 2\pi \times 18 = 36\pi \text{ rad}$

Angular speed of the bar,

$\omega = 2\pi\nu = 2\pi \times \frac{216}{60} = 7.2\pi \text{ rad s}^{-1}$

\therefore Required time, $t = \frac{\theta}{\omega} = \frac{36\pi}{7.2\pi} = 5 \text{ s}$