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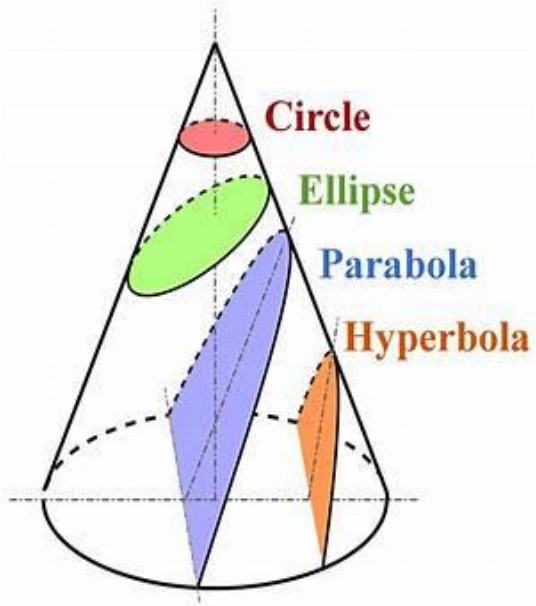
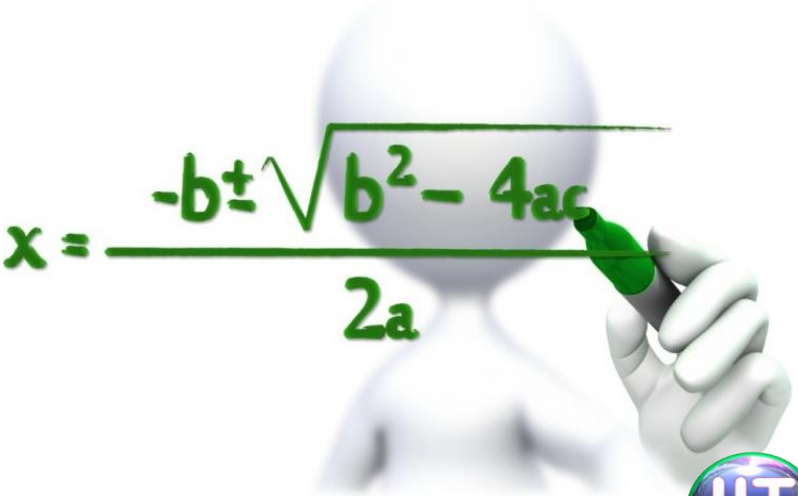
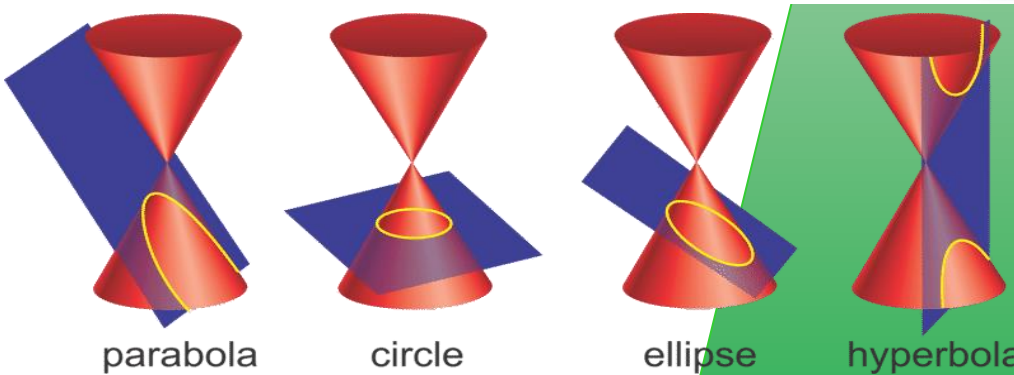
MATHEMATICS
MQUADRATIC EQ.
03

X

CBSE-MATH-QUADRATIC EQUATIONS







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03 MATHEMATICS QUADRATIC EQ.

SET :- 1 **2.**

1. In each of the following determine the given values are the sol^n of the eq^n or not.

(i) $3x^2 + 2x^2 - 1 = 0$: $x = 1$ (ii) $x^2 + 6x + 5 = 0$: $x = -1, x = -5$ (iii) $2x^2 + 6x + 3 = 0$; $x = ½$

(iv) $6x^2 - x - 2 = 0$; $x = -2$, $x = 1/3$ (v) $x^2 + \sqrt{2}x - 4 = 0$; $x = \sqrt{2}$, $x = 2\sqrt{2}$

(vi) $9x^2 - 3x - 2 = 0$; $x = -1/3$, $x = 2/3$ (vii) $6x^2 - 11x - 3 = 0$; $x = -1/3$, $x = 1/3$ (viii) $3x^2 - 2x - 5 = 0$; $x = 3/2$, $x = 1/3$

(ix) $\sqrt{7}y^2 - 16y - 13\sqrt{7} = 0$; $y = 13/\sqrt{7}$, $y = -\sqrt{7}$ (x) $a(x^2 + 1) = x(a^2 + 1)$; $x = -a, x = a, x = 1/a$

SET :-II

1. Factories : (i) $(x - 4)(x + 2) = 0$ (ii) $(2x + 3(3x - 7) = 0$ (iii) $x^2 - 3x - 18 = 0$ (iv) $x^2 - 3x - 10 = 0$ (v) $6x^2 - x - 2 = 0$

(vi) $5x^2 - 3x - 2 = 0$ (vii) $9x^2 - 3x - 2 = 0$ (viii) $2x^2 - ax - a^2 = 0$ (ix) $4x^2 - 13x + 3 = 0$ (x) $8x^2 - 22x + 21 = 0$

(xi) $7x^2 - 25x + 12 = 0$ (xii) $\sqrt{7}y^2 - 6y - 13\sqrt{7} = 0$ (xiii) $4\sqrt{5}x^2 + 7x - 3\sqrt{5} = 0$ (xiv) $48y^2 - 13y - 1 = 0$

(xv) $(x - \frac{1}{2})^2 = \frac{1}{4}$ (Xvi) $a^2b^2x^2 - (a^2 + b^2)x + 1 = 0$ (xvii) $3x^2 - 4x + 1 = 0$

(xviii) $a^2x^2 - 3abx + 2b^2 = 0$ (x ix) $\sqrt{3}y^2 + 11y + 6\sqrt{3} = 0$ (xx) $ax^2 + (4a^2 - 3b)x - 12b = 0$

(xxi) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$ (xxii) $(\sqrt{3}x + 9)(2x + \sqrt{5}) = 0$ (xxiii) $\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

(xxiv) $\sqrt{5}x^2 - 6x - 8\sqrt{5} = 0$ (xxv) $6x^2 + 40 = 31x$ (xxvi) $12x^2 - x - 6 = 0$

(xxvii) $3a^2x^2 - abx - 2b^2 = 0$ (xxviii) $x + \frac{1}{x} = 3\frac{1}{3}$ (xxix) $x(x + a - b) = ab$

(xxx) $x^2 + 48 = 19x$ (xxxi) $(4x^2 - 9) - 2(2x - 3) + x(2x - 3) = 0$ (xxxii) $(2x - 1) - \frac{2}{x - 2} = 0$

(xxxiii) $\frac{1}{p + q + x} = \frac{1}{p} + \frac{1}{q} + \frac{1}{x}$ (xxxiv) $\frac{x}{x + 1} + \frac{x + 1}{x} = \frac{34}{15.}$ (xxxv) $\frac{x+3}{x - 2} - \frac{1 - x}{x} = \frac{17}{4}$

(xxxvi) $(x - 1)(x - 2) = 4x^2 - 5x - 3$ (xxxvii) $1/x-2 + 2/x-1 = 6/x$

DISCRIMINANT

SET :- 3

☑ Nature of roots of QUADRATIC Equation

$ax^2 + bx + c$ depends upon the expression $b^2 - 4ac$ (discriminant) under radical sign As :----

✪ CASE 1
If $b^2 - 4ac$ is positive and a perfect square, then the roots are real, different and rational.

✪ CASE 2:-
If $b^2 - 4ac$ is positive but not a perfect square, then the roots are real, different and irrational.

✪ CASE 3:-
If $b^2 - 4ac = 0$, the roots are real and equal.

✪ CASE 4:-
If $b^2 - 4ac$ is negative then the given quadratic eq^n has no real roots.

1.Write the discriminant of the following eq^n and comment upon the nature of roots. **3.**

(i) $x^2 - 4x + 1 = 0$ **(ii) $x^2 + x + 1 = 0$** **(iii) $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$** **(iv) $\frac{4}{3}x^2 - \frac{2}{x} + \frac{3}{4} = 0$**

(v) $5y^2 - 12y - 9 = 0$ **(vi) $x^2 - 5 - 7 = 0$** **(vii) $9a^2b^2x^2 + 48abcd\,x + 64c^2d^2 = 0$**

SET :- IV----------

1. The eq^n $ax^2 - 2\ 5x + 4 = 0$ has equal roots. Find the value of a
2. $4x^2 - 3kx - 1 = 0$; ----------equal root
3. $9x^2 + 3kx + 4 = 0$; ----------equal root
4. $kx^2 - 2\sqrt{5}x + 4 = 0$; ----------equal root
5. $9x^2 - 24x + k = 0$; ----------equal root
6. $kx^2 - 8x + k = 0$ ----------equal root
7. $kx^2 - 5x + k = 0$; ----------equal root
8. $3x^2 - 2kx + 12 = 0$;---------- equal root
9. $12x^2 + 4kx + 3 = 0$;---------- equal root
10. $3x^2 + 8x - k = 0$; ----------equal root
11) $x^2 - (3k - 1)x + 2k^2 + 2k - 11 = 0$; equal roots
12) $x^2 + 7(3 + 2k) - 2x(1 + 2k) = 0$; equal roots
13) $(k - 2)x^2 + 2(2k - 3)x + 5k - 6 = 0$; equal roots
14) $kx^2 + (k - 1)x + k - 1 = 0$; equal roots
15) $x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$; equal roots
16) $(3k + 1)x^2 + 2(k + 1)x + k = 0$; equal roots
17) $(k + 1)x^2 + 2(k + 3)x + k + 8 = 0$; equal roots
18) $x^2 - 2kx + 17k - 12 = 0$; equal roots
19) $(k + 1)x^2 - 2(3k + 1)x + 8k + 1 = 0$; equal roots
20) $(4 - k)x^2 + (2x + 4)x + 8k + 1 = 0$; equal roots
21) $(2k + 1)x^2 + 2(k + 3)x + k + 5 = 0$; equal roots
22) $kx^2 + kx + 1 = -4x^2 - x$; equal roots
23) $5x^2 - 4x + 2 + k(4x^2 - 2x - 1) = 0$; equal roots
24) $(k + 4)x^2 + (k + 1)x + 4 = 0$; equal roots
25) $(k - 3)x^2 + 4(k - 3)x + 4 = 0$; equal roots
26) $x^2 - (2k + l)x + k^2 = 0$; equal roots
27) $3kx^2 = 4(3kx - 1)$; equal roots
28) $4x^2 - 2(k + 1)x + k + 4 = 0$; equal roots
29) $(k - 12)x^2 + 2(k - 12)x + 2 = 0$; equal roots
30) $x^2 + 4kx + k(k - 1) + 2 = 0$; equal roots

30) $(l + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, then show that $c^2 = a^2(1 + m^2)$

31) The roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then prove that $2b = a + c$

32) If the roots of the quad^r eq^2 $(a^2 + b^2) - x^2 - 2(ac + bd)x + c^2 + d^2 = 0$. Then prove that $\frac{a}{b} = \frac{c}{d}$

33) If -4 is the root of the Quad^r Eq ^{n} $x^2 + px + 4 = 0$ and the equation $x^2 + px + q = 0$ has equal roots find the value of p & q.

SET :- V 4.

To form a Quad^r eq^n from the given roots :

Required Q.E => $x^2 - (\text{sum[S] of the roots})\,x + \text{products[P] of the roots} = 0$

$$x^2 - Sx + p = 0$$

Where

Sum of the roots , $(\alpha+\beta) = -\frac{b}{a}$

Product of the root = $(\alpha\beta) = \frac{c}{a}$

Where $\alpha = \frac{-b + \sqrt{D}}{2a}$

$\beta = \frac{-b - \sqrt{D}}{2a}$

[I] **Find the sun and product of the roots of the follow QE:**

1) $x^2 + 5x + 5 = 0$ 2) $6x^2 + x - 2 = 0$ 3) $2x^2 + 5\sqrt{3}x + 6 = 0$

4) $3x^2 + 2\sqrt{5}x - 5 = 0$ 5) $2x^2 + x - 1 = 0$ 6) $x^2 - 3 = 0$

[2] Construct the quadratic equation whose roots are given below:

[i] $\sqrt{2}$, $-2\sqrt{2}$ [ii] $\sqrt{3}$, $-2\sqrt{3}$ [iii] $3 + 3\sqrt{3}$, $3 - 3\sqrt{3}$ [iv] $\frac{2+\sqrt{5}}{2}$, $\frac{2-\sqrt{5}}{2}$

[v] $-2 + 3\sqrt{5}$, $-2 - 3\sqrt{5}$ [vi] $5 - 2\sqrt{3}$, $5 + 2\sqrt{3}$

[3] Form a Q.E whose one root is $3 - \sqrt{5}$ and the sum of the root is 6 .

[4] **Form a Q.E whose one root is $2 + \sqrt{5}$** and the sum of the root is 4 .

[5] **Form a Q.E whose one root is $1 + \sqrt{2}$** and the sum of the root is 2 .

[6] **Form a Q.E whose one root is** $\sqrt{5}$ and the product of the root is $-2-\sqrt{5}$.

[7] If one of the root of the Q.E $7x^2 - 50x + 7 = 0$, is 7 , Find the other root .

[8] If one of the root of the Q.E $3x^2 - 10x + 3 = 0$, is 3 , Find the other root.

[9] If one of the root of the Q.E $6x^2 - 7x + 2 = 0$, is ½ , Find the other root.

[10] If one of the root the Q.E $2x^2 + px + 4 = 0$ is 2. Find the other root . And also find the value of 'P'.

[11] If one of the root of the Q.E $2x^2 + ax + 6 = 0$ is 2. find the value of 'a' & also find the value of the other root.

[12] If one of the root of the Q.E $2x^2 - 3x + p = 0$ is 3 find the value of 'p' & other root.

[13] If one of the root of Q.E $3x^2 - kx - 2 = 0$ is 2 find the value of 'k' and other root.

[14] If one of the root of Q.E $x^2 - 4x + k = 0$ is 5. find the value of 'k' & other root.

[15] If one of the root of Q.E $2x^2 - 8x - m = 0$ is 5/2 find the value of 'm' & other root.

[16] If one of the root of Q.E. $x^2 - 5x - k = 0$ is 4. find the value of 'k' & other root.

[17] If one of the root of Q.E. $2x^2 - 5x + k = 0$ is 5. Find the value of 'k' & other root.

[18] Find the value of k such that the Q.E. $x^2 - (k+6)x + 2(2k-1) = 0$ has sum of the root as $\frac{1}{2}$ of the product.

[19] Find the value of K so that sum of the root Q.E. $x^2 + 6x - 3k = 0$ is equal to the product of the root.

[20] Find the value of K so that sum of the root Q.E $3x^2 + x(2k + 1) - k + 5 = 0$ is equal to the product of the root.

[21] Find the value of K so that sum of the root Q.E $(k + l)x^2 + 2kx + 4 = 0$; is equal to the product of the root.

[22] Find the value of K so that sum of the root Q.E $(k+l)x^2 + (2k+l)x - 9 = 0$ is equal to the product of the root. 5.

[23] Find the value of K so that sum of the root Q.E $x^2 - (7k+3)x + 3k + 1 = 0$; is equal to twice the product of the root.

[24] Find the value of m so that one of the root of the Q.E.. $4x^2 - 8mx - 9 = 0$ is negative of the other.

[25] Find the value of p so that Q.E. $4x^2 - 8px + 9 = 0$ has roots whose difference is 4.

[26] If the root of the Q.E $ax^2 + bx + c = 0$ are equal .Show that $b^2 - 4ac = 0$.

[27] Find a & b such that Q.E $ax^2 + bx + \frac{3}{4} = 0$ has $\frac{3}{2}$ as sum of one root & $\frac{9}{10}$ as product of the root

[28] If the sum and product of the roots of Q.E. , $ax^2 + 5x + c = 0$ are both equal to 10 . Find the value of a & c.

[29] Find the value of k of QE $x^2 - (2k+l)x + 3k + 7 = 0$ whose sum of the root is $\frac{1}{3}$ of the product

[30] Find the value of k so that the sum of the squares of the root of QE $x^2 - 8x + k = 0$ is 40 also find the roots of the Q.E.

[31] Obtain the conditions so that one root of QE $ax^2 + bx + c = 0$ is double the other. Ans : $2b^2 = 9ac$

[32] Find the value of k so that QE $4x^2 - 2x0 - k - 4 = 0$ has one root as the reciprocal of the other. Find k.

[33] QE $4px^2 + 4qx + 3 = 0$ has $\alpha + \beta = \frac{1}{2}$ and $\alpha\beta = \frac{3}{16}$. find p and q.

[34] QE $x^2 + 3x + k = 0$ has one root double the other . find k

[35]. Given that one root of the QE $ax^2 + bx + 3 = 0$ is 3 times the other. show that $3b^2 = 16ac$.

[36] Find the value of 'k' for which the roots of QE $3x^2 - kx + 14 = 0$ are in ratio 7:6.

[37] $ax^2 - 4x + e = 0$; Sum = product of the root = 8 . Find a &c.

[38] QE $4x^2 + 5x + k = 0$ has one of the root reciprocal of other. Find 'k'.

[39] The coefficient of 'x' of QE $x^2 + bx + c = 0$ was taken as 17 in the place of 13, its root were found to be -2 & -15 . Find the root and quadratic equation.

[40] Two numbers m and n are such that QE $mx^2 + 3x + 2n = 0$ has (-6) as the sum of the roots and also has product of the root. Find m and n

[41] $x^2 + kx + 12 = 0$ has $\alpha - \beta = 1$ find k

SET :- VI

Some Important Relations : -

✪ 1) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

✪2) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha+\beta)$

✪3) $(\alpha - \beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

✪4) $\alpha^3 - \beta^2 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$

✪5) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

✪6) $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)$

✪ 7) $\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta) = (\alpha + \beta)(\sqrt{(\alpha + \beta)^2 - 4\alpha\beta})$

✪8) $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$
$= (\alpha + \beta)^2 - 2\alpha\beta)^2 - 2\alpha^2\beta^2$

✪ 9) $\alpha^4 - \beta^4 = (\alpha^2 - \beta^2)(\alpha^2 + \beta^2) = (\alpha - \beta)(\alpha + \beta)(\alpha^2 + \beta^2)$

Question:- 6.

[1] If α & β are the roots of quadratic equation $3x^2 + 5x - 7 = 0$. Find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

[2]. If α & β are the roots of quadratic equation $x^2 - 2x - 1 = 0$, find the value of $\alpha^2\beta^2 + \alpha\beta^2$

[3]. If α & β are the roots of quadratic equation $ax^2 + bx + c = 0$, find the value of (i) $\alpha^2 + \beta^2$ (ii) $\alpha^3 + \beta^3$ (iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

[4]. If α & β are the roots of quadratic equation $2x^2 + 5x - 6 = 0$ find the value of $\alpha^4\beta + \beta^4\alpha$.

[5]. If α & β are the roots of quadratic equation $x^2 - px + q = 0$, find the value of $\alpha + \beta$, $\alpha\beta$, $\alpha^2 + \beta^2$

[6]. If α & β are the roots of quadratic equation $3x^2 + 8x + 2 = 0$, find the value of $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$

[7]. If α & β are the roots of quadratic equation $x^2 - 3x + 1 = 0$, find the value of $\alpha^2 + \beta^2$

[8]. If α & β are the roots of quadratic equation $x^2 - 5x + 6 = 0$, find the value of (i) $\alpha^2 + \beta^2$ (ii) $\alpha^2 - \beta^2$

[9]. If α & β are the roots of quadratic equation $ax^2 + bx + c = 0$, find the value of (i) $\frac{\alpha + \beta}{\alpha^{-1} + \beta^{-1}}$ (ii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

[10]. If α & β are the roots of quadratic equation $x^2 - 5x + 4 = 0$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

[11]. If α & β are the roots of quadratic equation $2x^2 - 4x + 1 = 0$, find the value (i) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (ii) $\frac{1}{\alpha + 2\beta} + \frac{1}{\beta + 2\alpha}$ (iii) $(\alpha - \beta)^2 + 4\alpha\beta$

[12]. If α & β are the roots of quadratic equation $ax^2 + bx + b = 0$ prove that $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}} = 0$

[13]. If α & β are the roots of Q.E $ax^2 + bx + c = 0$, find(i) $\alpha^2 + \beta^2$ (ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (iii) $\alpha^3 + \beta^3$ (iv) $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ (v) $\alpha^2\beta + \beta^2\alpha$ (vi) $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$ (vii) $\frac{\alpha^3}{\beta} - \frac{\beta^3}{\alpha}$ (viii) $\alpha^3 - \beta^3$ (ix) $\alpha^4 + \beta^4$ (x) $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$

[14]. If α & β are the roots of quadratic equation $x^2 + px + q = 0$, find $\left(\frac{\alpha}{\beta} + 2\right)\left(\frac{\beta}{\alpha} + 2\right)$

[15]. If α & β are the roots of quadratic equation $x^2 - mx + n = 0$, find $(\alpha^2 + 1)(\beta^2 + 1)$

[16]. If α & β are the roots of QE $2x^2 + 5x + k = 0$, find the value of k if $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$

[17]. If α & β are the roots of quadratic equation $5x^2 - px + 1 = 0$ such that $\alpha - \beta = 1$ .find p.

[18]. If α & β are the roots of quadratic equation $x^2 - 8x + p = 0$, then find the value of p and show that $\alpha^2 + \beta^2 = 40$.

[19]. If α & β are the roots of quadratic equation $x^2 - 5x + 3(k - 1) = 0$ such that $\alpha - \beta = 11$, find k.

[20]. If α & β are the roots of quadratic equation $x^2 - 6x + k = 0$ such that $3x + 2\beta = 20$, find 'k'

SET :- VII

[1]. If α and β are the roots of QE $x^2 - 1 = 0$. form a QE whose root are $\frac{2\alpha}{\beta}$ & $\frac{2\beta}{\alpha}$

[2]. If α & β are the roots of quadratic equation $x^2 + x - 2 = 0$ form a QE whose roots are (i) $\alpha + \beta$ and $\alpha\beta$ (ii) $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

[3]. If α &β are the roots of QE $x^2 - x - 2 = 0$, form a QE whose roots are $2\alpha + 1$ & $2\beta + 1$

[4]. If α &β are the roots of QE $x^2 - 3x - 2 = 0$, form a QE whose roots are $\frac{1}{2\alpha+\beta}$ & $\frac{1}{2\beta + \alpha}$

[5]. If α &β are the roots of QE $2x^2 - 5x - 4 = 0$, form a QE whose roots are $\alpha + \frac{1}{\alpha}$, $\beta + \frac{1}{\beta}$

[6]. If α &β are the roots of QE $2x^2 - 3x - 5 = 0$, form a QE whose roots are $\frac{5}{\alpha}$, $\frac{5}{\beta}$.

[7]. If α &β are the roots of QE $x^2 - 3x + 2 = 0$, form a QE whose roots are $\alpha + 1$, $\beta + 1$ 7.

[8]. If α &β are the roots of QE $3x^2 - 4x + 1 = 0$, form a QE whose roots are $\frac{\alpha^2}{\beta}$ & $\frac{\beta^2}{\alpha}$

[9]. If α &β are the roots of QE $2x^2 - 5x + 7 = 0$, form a QE whose roots are $\alpha + 2\beta$, $2\alpha + \beta$

[10]. If α &β are the roots of QE $2x^2 + 5x + 2 = 0$, form a QE whose roots are $\alpha + \frac{\alpha}{\beta}$, $\beta + \frac{\beta}{\alpha}$

[11]. Find a QE whose roots are reciprocal of roots of the QE $4x^2 - 3x - 1 = 0$.

[12]. If α &β are the roots of QE $x^2 + px + q = 0$, form a QE whose roots are $\frac{\alpha}{\beta} + 2$, $\frac{\beta}{\alpha} + 2$

[13]. If α &β are the roots of QE $2x^2 + 2(m+n)x + m^2 + n^2 = 0$ then form of a QE whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$

[14]. If $ax^2 + bx + c = 0$ and α & β are its roots then prove that $a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = b$

[15]. If α &β are the roots of QE $x^2 + px + q$. prove that $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$

[16]. Form an eq where roots are cubes of the roots of the eqⁿ $x^2 + px + q = 0$

[17]. If α &β are the roots of QE $x^2 - 3ax + a^2 = 0$, find a if $\alpha^2 + \beta^2 = \frac{7}{4}$

[18].If α & β are the roots of the eqⁿ such that $\alpha - \beta = 8$ and $\alpha + \beta = 24$. find the quadratic eqⁿ having α & β as roots.

[19].If α & β are the roots of the QE such that $\alpha + \beta = 0$ and $\alpha - \beta = 1$,form a QE where roots are 3α & β.

[20]. Solve the eqⁿ $x^2 + px + 45 = 0$, It has been given that square difference of the root is equal to 144.

SET :- VIII

✪ **$ax^2 + bx + c = 0$, if $b^2 - 4ac \geq 0$ then the given QE ie $ax^2 + bx + c = 0$ can the factorize into a product of real linear factor and is given by $ax^2 + bx + c = a(x - \alpha)(x - \beta)$**

[1]. Determine which of the following can be factorized into product of real linear factors

(i) $x^2 - 6x + 4 = 0$ (ii) $x^2 + x + 1 = 0$ (iii) $2x^2 + 4x + 5 = 0$ (iv) $3x^2 + 2\sqrt{5}x - 5 = 0$

[2]. Factories into a product of real linear factor

(i) $21x^2 - x - 2$ (ii) $x^2 + 4\sqrt{2}x + 6$

(iii) $40 + 3x - x^2$ (iv) $\sqrt{3}x^2 + 11x + 6\sqrt{3}$

(v) $x^2 + 2x - 8$ (vi) $8x^2 + 43x + 15$

(vii) $2x^2 + \sqrt{7}x - 7$ (viii) $\sqrt{7}x^2 - 6x - 13\sqrt{7}$

(ix) $2\sqrt{2}x^2 + 4x + \sqrt{2}$ (x) $6z^2 - 11z + 3$

(xi) $x^2 + \sqrt{2}x + 3$.

TYPE - I

(a) $\left(\frac{x}{x+1}\right)^2 - 5\left(\frac{x}{x+1}\right) + 6 = 0$ 8

(b) $\left(\frac{x}{x-1}\right)^2 - 3\left(\frac{x}{x-1}\right) - 18 = 0$

(c) $2\left(\frac{x}{x+1}\right)^2 - 5\left(\frac{x}{x+1}\right) + 2 = 0$

(d) $\left(\frac{x}{x+1}\right)^2 + 5\left(\frac{x}{x+1}\right) + 6 = 0$

(e) $\left(\frac{2x}{x-5}\right)^2 + \left(\frac{10x}{x-5}\right) - 24 = 0$

(f) $(x^2 - 5x)^2 - 7(x^2 - 5x) + 6 = 0$

(g) $(x^2 + 3x + 2)^2 - 7(x^2 + 3x) - 6 = 0$

(h) $(x^2 + 3x)^2 - (x^2 + 3x) - 6 = 0$

(i) $(x^2 - 3x)^2 - 5(x^2 - x) + 4 = 0$

(j) $\left(\frac{x-2}{x+2}\right)^2 + 3 = 4\left(\frac{x-2}{x+3}\right)$

(k) $\left(\frac{7x-1}{x}\right)^2 + 3\left(\frac{7x-1}{x}\right) - 18 = 0$

(l) $(x^2 + 12x)^2 + 35(x^2 + 12x) + 150 = 0$

(m) $(x^2 - 3x)^2 - 16(x^2 - 3x) - 36 = 0$

(n) $4\left(\frac{7x-1}{x}\right)^2 - 8\left(\frac{7x-1}{x}\right) + 3 = 0$

(Type II, Pattern A)

$$px + \frac{q}{x} = e$$

(i) $y - \frac{3}{y} = \frac{1}{2}$ (ii) $2x - \frac{3}{x} = 5$ (iii) $3y + \frac{5}{16y} = 2$ (iv) $2x + \frac{4}{x} = 9$ (v) $2y + \frac{1}{2y} = \frac{13}{6}$ (vi) $5x + \frac{35}{x} = 8$

(vii) $2x - \frac{3}{x} = 1$ (viii) $x + \frac{1}{x} = \frac{26}{5}$ (ix) $x + \frac{1}{5} + \frac{28}{x+2} = 5$

(Type II, Pattern B)

1) $\left(\frac{2x-3}{x-1}\right) - 4\left(\frac{x-1}{2x-3}\right) = 3$ (2) $\frac{3x+1}{x+1} + \frac{x+1}{3x+1} = \frac{5}{2}$ (3) $\frac{2x+3}{x+1} + 6\frac{x+1}{2x+3} = 7$ (4) $3\sqrt{\frac{x}{5}} + 3\sqrt{\frac{5}{x}} = 10$

(5) $\sqrt{\frac{x}{x-1}} + \sqrt{\frac{x-1}{x}} = \frac{5}{2}$ (6) $\sqrt{\frac{x}{x-1}} + \sqrt{\frac{x-1}{x}} = \frac{13}{6}$ (7) $\frac{12-x}{5} = \frac{3}{2\sqrt{12-x}}$. (8) $\sqrt{\frac{4x-1}{4x+1}} - \sqrt{\frac{4x+1}{4x-1}} = 2\frac{2}{3}$

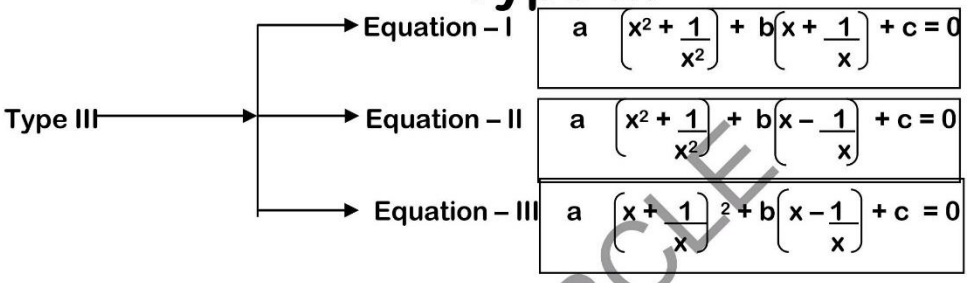
(9) $5\sqrt{\frac{3}{x}} + 7\sqrt{\frac{x}{3}} = 22\frac{2}{3}$ (10) $\frac{x-3}{x+3} - \frac{x+3}{x-3} = 6\frac{6}{7}$ (11) $\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 2\frac{1}{2}$

(12) $2\left(\frac{2x^2+1}{x+5}\right) + 3\left(\frac{x+5}{2x^2+1}\right) = 7$ (13) $\frac{2x-1}{x+1} - 15\left(\frac{x+1}{2x-1}\right) = -2$ (14) $4\sqrt{\frac{x}{x+1}} + 3\sqrt{\frac{x+1}{x}} = 8$

(15) $\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}$ (16) $\sqrt{\frac{x}{x+10}} + \sqrt{\frac{x+10}{x}} = 2\frac{1}{12}$ (17) $8\sqrt{\frac{x}{x+3}} - \sqrt{\frac{x+3}{x}} = 2$ (18) $\frac{x+2}{x-2} + \frac{x-2}{x+2} = \frac{5}{2}$

(19) $6\sqrt{\frac{x}{x+4}} - 2\sqrt{\frac{x+4}{x}} = 11$ (20) $4\sqrt{\frac{x}{x+1}} + 13\sqrt{\frac{x+1}{x}} = 28$

Type III



Type – III - Equation – I

$$A\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)+c=0$$

(1) $2\left(x^2+\frac{1}{x^2}\right)-9\left(x+\frac{1}{x}\right)+14=0$ (2) $3\left(x^2+\frac{1}{x^2}\right)-16\left(x+\frac{1}{x}\right)+26=0$ (3) $2\left(x^2+\frac{1}{x^2}\right)-3\left(x+\frac{1}{x}\right)-1=0$

(4) $4\left(x^2+\frac{1}{x^2}\right)-8\left(x+\frac{1}{x}\right)+3=0$ (5) $6\left(x^2+\frac{1}{x^2}\right)-35\left(x+\frac{1}{x}\right)+62=0$ (6) $9\left(x^2+\frac{1}{x^2}\right)-9\left(x+\frac{1}{x}\right)-52=0$

(7) $\left(x^2+\frac{1}{x^2}\right)+2\left(x+\frac{1}{x}\right)-13=0$ (8) $4\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)-7=0$ (9) $9\left(x^2+\frac{1}{x^2}\right)-24\left(x+\frac{1}{x}\right)-2=0$

(10) $\left(x^2+\frac{1}{x^2}\right)+4\left(x+\frac{1}{x}\right)+6=0$ (11) $2\left(x^2+\frac{1}{x^2}\right)-\left(x+\frac{1}{x}\right)-11=0$ (12) $8\left(x^2+\frac{1}{x^2}\right)+14\left(x+\frac{1}{x}\right)-69=0$

(13) $6\left(x^2+\frac{1}{x^2}\right)+5\left(x+\frac{1}{x}\right)-38=0$ (14) $\left(x^2+\frac{1}{x^2}\right)-7\left(x+\frac{1}{x}\right)-14=0$

➢ Type – III --- Equation – II

$$a\left(x^2+\frac{1}{x^2}\right)\ b\left(x-\frac{1}{x}\right)+c=0$$

(1) $4\left(x^2+\frac{1}{x^2}\right)+8\left(x-\frac{1}{x}\right)-29=0$ [2] $6\left(x^2+\frac{1}{x^2}\right)-25\left(x-\frac{1}{x}\right)+12=0$ [3] $\left(x^2+\frac{1}{x^2}\right)-3\left(x-\frac{1}{x}\right)-2=0$

(4) $\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{1}{x}\right)-17=0$ (5) $2\left(x^2+\frac{1}{x^2}\right)-3\left(x-\frac{1}{x}\right)-81=0$ (6) $8\left(x^2+\frac{1}{x^2}\right)-42\left(x-\frac{1}{x}\right)+29=0$

(7) $\left(x^2+\frac{1}{x^2}\right)-5\left(x-\frac{1}{x}\right)+2=0$ (8) $10\left(x^2+\frac{1}{x^2}\right)-63\left(x-\frac{1}{x}\right)+52=0$ (9) $\left(x^2+\frac{1}{x^2}\right)-\left(x-\frac{1}{x}\right)-4=0$

(10) $2\left(x^2+\frac{1}{x^2}\right)+\left(x-\frac{1}{x}\right)-10=0$

➢ Type – III.... Equation – III

$$a\left(x+\frac{1}{x}\right)^2+b\left(x-\frac{1}{x}\right)+c=0$$

[1] $2\left(x+\frac{1}{x}\right)^2-3\left(x-\frac{1}{x}\right)=8$ [2 $2\left(x+\frac{1}{x}\right)^2-3\left(x+\frac{1}{x}\right)+3=0$ [3] $6\left(x+\frac{1}{x}\right)^2-25\left(x-\frac{1}{x}\right)=0$

[4] $\left(x+\frac{1}{x}\right)^2 - 2\left(x-\frac{1}{x}\right) - 19 = 0$ [5] $4\left(x-\frac{1}{x}\right)^2 + 8\left(x+\frac{1}{x}\right) - 29 = 0$ (6) $2\left(x-\frac{1}{x}\right)^2 - 9\left(x+\frac{1}{x}\right) + 18 = 0$ 10

[7] $4\left(x-\frac{1}{x}\right)^2 - 4\left(x+\frac{1}{x}\right) + 1 = 0$ [8] $3\left(x-\frac{1}{x}\right)^2 - 16\left(x+\frac{1}{x}\right) + 32 = 0$ [9] $\left(x+\frac{1}{x}\right)^2 = 4 + \frac{3}{2}\left(x-\frac{1}{x}\right)$

[10] $\left(x-\frac{1}{x}\right)^2 + 9 = \frac{5}{2}\left(x+\frac{1}{x}+2\right)$ [11] $\left(x+\frac{1}{x}\right)^2 - 2\left(x-\frac{1}{x}\right) + 4 - 11 = 0$ {12} $\left(x-\frac{1}{x}\right)^2 + 8\left(x+\frac{1}{x}\right) - 29 = 0$

Type III – 'Fusion' ..... Mixture of equation:- 1, 2 & 3

[1] $4x^4 - 4x^3 - 7x^2 - 4x + 4 = 0$ [2] $x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$ [3] $2x^4 - x^3 - 11x^2 - x + 2 = 0$

[4] $6x^4 + 25x^3 + 12x^2 - 25x + 6 = 0$ [5] $x^4 + 2x^3 - 13x^2 + 2x + 1 = 0$ [6] $2x^4 + 9x^3 + 8x^2 + 9x + 2 = 0$

[7] $2x^4 - 3x^3 - x^2 - 3x + 2 = 0$

Type IV : Power Based Question

$p^{2x} + r\,p^x + q = 0$

- $a^x \cdot a^y = a^{x+y}$
- $(a^x)^y = a^{xy} = (a^y)^x$
- $\frac{a^x}{a^y} = a^{x-y}$ if $y > x$ then $\frac{a^x}{a^y} = \frac{1}{a^{x-y}}$
- $a^x \cdot b^x = (ab)^x$

[1] $3^{2x} + 9 - 10.3^x = 0$ [2] $2^{2x} + 2^{x+1} = 4 - 2^x$ [3] $4^x - 3.2^{x+3} + 128 = 0$

[4] $16.4^{x+2} - 16.2^{x+1} + 1 = 0$ [5] $3^{x+2} + 3^{-x} = 10$ [6] $5^{2x} - 5^{x+3} + 125 = 5^x$

[7] $9^{x+2} - 6.3^{x+1} + 1 = 0$ [8] $5^{1+x} + 5^{1-x} = 26$ [9] $5^{x+1} + 5^{2-x} = 5^3 + 1$

[10] $5^{4x} - 3.5^{2x+2} = 250$ [11] $2^{2x} - 3.2^{x+2} + 32 = 0$ [12] $5.9^{x+1} - 15.3^{x+2} = 2430$

[13] $4^{x+1} + 4^{1-x} = 10$ [15] $2^{x+1} + 4^x = 8$ [16] $2^{2x+3} = 65(2^x - 1) + 57$

[17] $2^{2x+3} + 2^{x+3} = 1 + 2^x$ [18] $x^{2/3} - 2.x^{1/3} = 15$ [19] $3^{2x} + 9 - 10.3^x = 0$

[20] $4^x - 3.2^{x+2} + 32 = 0$ [21] $4^x - 5.2^{x+4} + 4 = 0$

Type V

$(x+a)(x+b)(x+c)(x+d) + k = 0$

[1] $(x+1)(x+2)(x+3)(x+4) = 120$ [2] $(x+2)^1(x-5)^3(x+1)^4(x-6)^2 = 144$

[3] $(x+1)^{1}(x+2)^{3}(x+3)^{4}(x+4)^{2} = -1$

[4] $x^{1}(x+5)^{3}(x+7)^{4}(x+12)^{2} + 150 = 0$ 11

[5] $x^{1}(x-1)^{2}(x+2)^{3}(x-3)^{4} + 8 = 0$

[6] $(x+4)^{3}(x-5)^{4}(x+6)^{1}(x-7)^{2} = 504$

[7] $(x-1)^{1}(x-3)^{3}(x-5)^{4}(x-7)^{2} = 0$

[8] $x^{1}(x+1)^{3}(x+2)^{4}(x+3)^{2} - 120 = 0$

[9] $(x^2+x-6)(x^2-3x-4) = 0$

[10] $(2x-7)(x^2-9)(2x+5) = 91$

[11] $(x-1)^{1}(x-2)^{3}(3x-2)^{2}(3x+1)^{4} = 21$

Type VI : Square Roots

[1] $\sqrt{ax+b} = c$ → **Single square root** → **Single squaring**

[2] $\sqrt{ax+b} + \sqrt{cx+d} = p$ → **Double root** → **Double squaring**

➢ Single Square :

[1] $\sqrt{x} + 2x = 1$

[2] $\sqrt{217-x} = x-7$

[3] $\sqrt{2x+9} + x = 13$

[4] $\sqrt{13-x^2} = x+5$

[5] $\sqrt{x-3x-6} = 2$

[6] $\sqrt{2x^2-2x+1} - 2x + 3 = 0$

[7] $\sqrt{x+4} = x-2$

[8] $\sqrt{2x+7} = x+2$

[10] $\sqrt{3x^2+x+5} = x-3$

[11] $\sqrt{3x^2-2}+1 = 2x$

[12] $\sqrt{25-x^2} = x-1$

[13] $\sqrt{x+9+3} = x$

[14] $\sqrt{3x+4} + x + 12 = 0$

[15] $\sqrt{4x^2+x-2} + 1 = 2x$

[16] $2\sqrt{2x+1} - 2x = 1$

[17] $\sqrt{2x^2+1} + 7 = 5x$

[18] $x + \sqrt{x+6} = 14$

[19] $\sqrt{y^2-y+2} = y-1$

[20] $\sqrt{3x} = 2x-3$

[21] $\sqrt{x} = x-2$

➢ Double Square Roots :

[1] $\sqrt{3x+10} + \sqrt{6-x} = 6$

[2] $\sqrt{4x-3} + \sqrt{2x+3} = 6$

(3) $\sqrt{4-x} + \sqrt{x+9} = 5$

[4] $\sqrt{2x+9} - \sqrt{x-4} = 3$

[5] $\sqrt{y+1} + \sqrt{2y-5} = 3$

[6] $\sqrt{x+1} + \sqrt{x+3} = 4$

[7] $2\sqrt{x-1} - \sqrt{5+2x} = 1$

[8] $\sqrt{2x+5} + \sqrt{x-2} = -2$

[9] $\sqrt{2x+\sqrt{2x+4}} = 4$

[10] $\sqrt{3x+1} - \sqrt{x-1} = 2$

[11] $\sqrt{2x+3} - \sqrt{x-1} = 1$

[12] $\sqrt{x+1} + \sqrt{2x} = 7$

[13] $\sqrt{x+1} + \sqrt{2x-5} = 3$

[14] $\sqrt{2x^2+x+3} + \sqrt{2x^2+x-6} = 3$

[15] $\sqrt{x^2-3x+16} - \sqrt{x^2-3x+9} = 1$

[16] $\sqrt{3x^2-4x+34} - \sqrt{3x^2-4x-11} = 5$

[17] $\sqrt{2x+6} - \sqrt{x-1} = 2$

[18] $\sqrt{3x+1} - \sqrt{x-1} = 2$

[20] $\sqrt{2x+1} + \sqrt{x+4} = 3$

[21] $\sqrt{3x-5} + \sqrt{x+2} = 3$

[22] $\sqrt{1-5x} + \sqrt{1-3x} = 2$

[23] $\sqrt{x+4} + \sqrt{x+5} = 3$

[24] $\sqrt{x^2+3x+32} + \sqrt{x^2+3x+5} = 9$

➢ Triple Root

[1] $\sqrt{x+5}+\sqrt{x+21}=\sqrt{6x+40}$

[2] $\sqrt{(x-2)(x-3)}-\sqrt{\frac{x-2}{x-3}}-\sqrt{x^2-6x+8}=0$

[3] $\sqrt{x^2+2x-3}+\sqrt{x^2-x}=\sqrt{5(x-1)}$

[4] $\sqrt{4x^2-7x-15}-\sqrt{x^2-3x}=\sqrt{x^2-9}$

[5] $\sqrt{x^2-16}-\sqrt{x^2-8x+16}=\sqrt{x^2-5x+4}$

[6] $\sqrt{x^2+9x-20}-\sqrt{x^2-12x+32}=\sqrt{2x^2-25x+68}$

[7] $\sqrt{x^2-4}-(x-2)=\sqrt{x^2-5x+6}$

[8] $\sqrt{x^2+4x-21}+\sqrt{x^2-x-6}=\sqrt{6x^2-5x-39}$

[9] $\sqrt{x^2+6x+5}+\sqrt{x^2+22x+21}=\sqrt{6x^2+46x+40}$

[10] $\sqrt{2x+9}\sqrt{x-3}-\sqrt{x^2-7x+12}=3\sqrt{x-3}$

[11] $\sqrt{2x+9}\sqrt{x+3}-\sqrt{x^2-x-12}=3\sqrt{x+3}$

[12] $\sqrt{x^2+5x+4}+\sqrt{x^2+8x+7}=\sqrt{x^2-1}$

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[1] $x^2-4x-12\sqrt{x^2-4x+19}+51=0$

[2] $\sqrt{x^2-4x+5}+2x+3=\frac{x^2}{2}$

[3] $8+9\sqrt{(3x-1)(x-2)}=3x^2-7x$

[4] $\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}+\frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}=98$

[5] $\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}=3$

(5) $\sqrt{x^2+ax+b}-\sqrt{x^2+ax+c}=\sqrt{b}-\sqrt{c}$

[6] $\frac{1+x}{1+\sqrt{1+x}}+\frac{1-x}{1-\sqrt{1-x}}=1$

[7] $\frac{1}{\sqrt{x}-\sqrt{x-2}}+\frac{1}{\sqrt{x}+\sqrt{x+2}}=1$

[8] $\frac{x-\sqrt{2x+1}}{x+\sqrt{2x+1}}=\frac{1}{7}$.

(9) $\frac{x-2}{x+2}+\frac{x+2}{x-2}=2\left(\frac{x+3}{x-3}\right)$

[10] $\frac{x+3}{x+2}+\frac{x-3}{x-2}=\frac{2x-3}{x-1}$

WORDS - PROBLEM

➢ **Consecutive Natural Number**
x, x + 1, x + 2, x + 3, x + 4 .....

➢ **Positive Even Integers**
2x, 2x + 2, 2x + 4, 2x + 6 ... .

➢ **Positive Odd Integers**
2x + 1, 2x + 3, 2x + 5, 2x + 7 .....

➢ **Multiple Of Two**
2x, 2x + 2, 2x + 4, 2x + 6 .....

➢ **Multiple Of Three**
3x, 3x + 3, 3x + 6, 3x + 9 .....

➢ **Multiple Of Five**
5x, 5x + 5, 5x + 10, 5x + 15 .....

Set I: Number Type Question

1. The sum of the square of two consecutive natural numbers is 313. Find the numbers.
2. Find the two consecutive natural numbers. Whose product is 20.
3. The sum of the square of three consecutive natural numbers is 149. Find the numbers.
4. Find two consecutive even integers whose square have the sum 340.
5. The sum of the squares of three consecutive numbers is 110. Determine the numbers.
6. The sum of two numbers is 18 and their product is 56. Find the numbers.
7. Find two consecutive positive odd integers whose squares have the sum 130.
8. Find two consecutive even positive integers the sum of whose square is 164.
9. The sum of the square of two consecutive positive odd integers is 290. Find the numbers.
10. The sum of the numbers is 48 and their product is 432. Find the numbers.
11. Find two consecutive natural numbers whose product is 20.
12. Find two consecutive even integers the sum of whose square is 164.
13. The sum of a number and its reciprocal is 17/4. Find the number.
14. An integer when added to its square is equals to 90. Find all the positive values of integer.
15. Divide 42 into two parts whose product is 420.
16. Divide 51 into two parts whose product is 378.

    Hint : $x(51 - x) = 378$
17. Divide 29 into two parts so that the sum of the square of the two part is 425

    Hint : $x^2 + (29 - x)^2 = 425$
18. Find the whole number which when decreased by 20 is equals to 69 times the reciprocal of the number.

    Hint : $x - 20 = 69\ (1/x)$
19. Two numbers differ by 3 and their product is 504. Find the numbers.

    Hint : $x(x + 3) = 504$
20. Divide 57 into two parts whose product is 782. Find the numbers.

    Hint : $x(x + 57) = 782$
21. The sum of two numbers is 15 and the sum of their reciprocal is 3/10. Find the numbers.

    Hint : $1/x + 1/(15 - x) = 3/10$
22. The sum of the square of two consecutive odd integer is 374. Find the numbers.

    Hint : $(2x + 1)^2 = (2x + 3)^2 = 394$
23. Determine two consecutive positive multiple of there whose product is 270.

    Hint : $3x(3x + 3) = 270$
24. The product of two successive positive multiple of there is 180. Find the multiple.

    Hint : $3x(3x + 3) = 180$
25. The product of positive successive integral multiple of 5 is 300. Find the multiple.

    Hint : $5x\ (5x + 5) = 300$
26. Find two consecutive natural numbers whose square have the sum 85.

    Hint : $x^2 + (x + 1)^2 = 85$
27. The sum of the no. and its reciprocal is $3\frac{41}{80}$. Find the number.

    Hint : $x + \frac{1}{x} = 3\frac{41}{80}$
28. Divide 54 into two parts whose product is 629. Find the number. Hint : $x\ (54 - x) = 629$
29. Find the numbers whose sum is 78 and product is 1512.

    Hint : $x(78 - x) = 1512$
30. The sum of the square of three consecutive natural numbers is 194. Find the number.

    Hint : $x^2 + (x + 1)^2 + (x + 2)^2 = 194$
31. The sum of the square of three consecutive natural numbers is 509. Find the numbers.
32. The sum of a number and its positive square root is $\frac{6}{25}$. Find the numbers.
33. The sum of square of two numbers is 233. If one of the number is 3 less than twice the number. Find the number.

    Hint : $x^2 + (2x - 3)^2 = 233$
34. The sum of two numbers is 8 and 15 times their reciprocal is also 8.

    Hint : $x + 8 - x = 15\left(\frac{1}{x} + \frac{1}{8 - x}\right)$
35. Find two natural numbers the sum of their square is 25 times their sum and also equal to 50 times then difference.

36. Find two natural numbers Which differ by 3 and whose square have the sum 117. 14

37. There are three consecutive natural numbers such that the square of the first increased by Product of the other two gives 154. Find the three consecutive nos. Hint : $x^2 + (x + 1)(x + 2) = 154$

➢ 38. There are three consecutive natural numbers such that square of the middle exceed the difference of the square of other two by 96. Find the three consecutive nos. Hint :$(x + 1)^2 = (x + 2)^2 - (x)^2 + 96$

39. The sum of two numbers is 37 and their product is 322. Find the no.

Hint : $x(37 – x) = 322.$

40. Find two consecutive natural numbers whose sure have the sum 221.

41. The sum of two numbers is 40 .Find the number if the sum of their reciprocal is 1/5.

42. Find the successive two positive odd integers, the sum of whose square is 514.

Hint :$(2 x + 1)^2 + (2x + 3)^2 = 514$

43. The square of a number is 15 times the number find the number.

44. The difference of two numbers is 7 and the product of the number is 170. Find the smaller number.

45. Separate 19 into two positive parts, the sum of their square is 193.

46. Find a number such that its square is 20 more than original number.

47. The reciprocal of two consecutive even number is 7/24 one of the number is 6. Find the other number.

Hint : $1 / 2x + 1/ 2x + 2 = 7 /24$

48. Divide 16 into two parts such that twice the square of the greater part exceed the square of the smaller part.

49. The sum of the square of two numbers is 130. The sum of smaller number and twice the larger number is 25. Find the number.

Set : II ---- [Digits problem]

1]. A two digit number is such that the product of he digits is 12. When 36 is added to the number ,the digits interchange their place. Determine the number.

2]. A two digit number is such that the product of its digit is 8. When 63 is subtracted from the number, the digit is interchanging their places. Determine the number.

3]. A two digit number is such that the product of its digit is 18. When 63is subtracted from the number the digit interchange their places. Find the number.

4]. A two digit number is such that the product of its digits is 12. When 9 is added to the number the digit interchange their places find the number.

5]. A two digit no is such that the product of its digit is 18. When 27 is subtracted from the number the digit interchange their places find the number.

6]. A two digit number is 4 times the sum and 3 times the product of its digit. Find the number.

7]. A two digit number is 4 times the sum of its digit and twice the product of its digit. Find the number.

8]. A number consist of two digits the product of its digit is 15. If 18 is added to the number the digit interchange their places . Find the number.

9]. A number consist of two digit the product of its digit is 14. If 45 is subtracted from the number the digit interchanges their places. Find the number.

10]. A two digit number is such that the product of its digit is 16. If 54 is subtracted from the number ,the digit interchanges their places. Find the number.

Set : III [Age Type Question]:

1]. A year ago, a father was 8 times as old as his son. Now his age is square of his son age. Find their present ages.

2]. The sum of the age of son and father is 45 years. 5years ago, the product of their ages is 4 times the age of father at that time. Find their present age.

3]. The product of Ramu's age 5 years ago with his age 9 years later is 15. Find Ramu's present age

4]. Ashu is 'm' years old while his mother Mrs. Veena is 'm^{2}' years old, five years hence, Mrs. Veena will be three times as old as Ashu. Find their present age.

5]. The product of meena's age 5 years ago with her age 8 years later is 30. find her present age.

6]. The sum of the ages of a boy and his brother is 25 years and product of their ages is 126 years. Find their present ag

7]. Asha is twice as old as Usha. After 3 years their age will be one and half (1/2) the age of usha. Find their ages.

8]. A boy and his father's ages among together is 24 years. Fourth part of the product of their ages, exceeds the boy's age by 9 years. Find how old are they. Hint : ¼ x (24 – x) = x + 9.

9]. Two years ago, a man age was three times the square of his sons age, after three years his age will be 4 times his sons age. Find their age.

Hint : 2 years ago ,Son's age was $= x - 2$ 2 years ago ,Father's age was $= 3(x-2)^2$ 15

$\therefore$ Father's present age $= 3(x-2)^2 + 2$; Son's present age $= x - 2 + 2 = x$

A/q, $3(x-2)^2 + 2 + 3 = 4(x+3)$

10]. Five years hence , father's age will be three times the age of the his son . Five years ago, father was 7 times as old as his son . Find their present ages .

Hint : Let son's present age be 'x' years

Now, 5 years hence, father's age will $= 3(x+5)$ ---------- [i]

5 years ago, father's age was $= 7(x-5)$ ---------- [ii]

$\therefore$ Father's present age $= 3(x+5) - 5$ (from [i])

Again, Father's present age $= 7(x-5) + 5$ (from [ii])

$\therefore$ $7(x-5) + 5 = 3(x+5) - 5$

11]. The sum of ages of a father and his son is 45 years . Five years ago , the product of their ages (in years) was 124 . Determine their present ages .

Hint : Let present age of father = x years . $\therefore$ Son's present age $= 45 - x$ years

5 years ago, Father's age was $= x - 5$ & son's age was $= 45 - x - 5 = 40 - x$

A/q , $(40 - x)(x - 5) = 124$

12]. 7 years ago Amit's age was five times the square of Swati's age . Three years hence , Swati's age will be two-fifth of Amit's age . Find their present ages .

Hint : 7 years ago , let Swati's age be 'x' years & Amit's age was $5x^2$ years .

$\therefore$ Swati's present age $= (x + 7)$ years and Amit's present age $= (5x^2 + 7)$ years

3 years hence , Swati's age $= (x + 7 + 3)$ years and Amit's present age $= (5x^2 + 7 + 3)$ years

A/q, $x + 10 = \frac{2}{5}(5x^2 + 10)$

13]. Rohan's Mother is 26 years older than him . The product of their ages 3 years from now will be 360 . We would liked to find Rohan's present age. Hint : Let Rohan's present age be 'x' years

$\therefore$ His mother's age $= (x + 26)$ years

Now, 3 years from now, Rohan's age will be $(x + 3)$ years

& His mother's age will $= (x + 29)$ years

A/q, $(x + 3)(x + 29) = 360$

Set – IV: [Area Type Question]

1]. Perimeter of right angled triangle is 30 cm and its hypotenuse is 13 cm. Find the other two sides.

2]. The hypotenuse of a right angled triangle is 1 less than twice the shortest side. If the side is 1 m more than shortest side. Find the side of triangle.

3]. The perimeter of a right angled triangle is 5 times the shortest side. The area of the triangle is 15 times the shortest side. Find the length of three sides.

4]. The area of a right triangle is 63 cm^2. If the base exceeds that of its altitude by 5 cm. Find the altitude of the triangle.

5]. The side of a right triangle are x, x – 1, x + 1 cm. Find the sides.

6]. The sides of a right triangle containing the right angle are 5x and 3x – 1. If the area of the triangle be 60 cm^2. Find the sides of the triangle.

7]. The hypotenuse of a right triangle is $3\sqrt{10}$ cm. if the smaller leg is tripled and the larger leg is doubled. New hypotenuse will be $9\sqrt{5}$ cm, how long is each side of the triangle.

8]. The hypotenuse of right triangle is $3\sqrt{5}$ if the smaller side is tripled and the larger side is doubled, The new hypotenuse will be 15 cm. Find the length of each side.

9]. The hypotenuse of a right angled triangle is 6 more than twice the shortest side. If the third side be 2 m less than the hypotenuse. Find the side of the triangle.

10] . The area of a right triangle is 600 cm^2, if the base of the triangle exceeds the altitude by 10 cm. Find the dimension of the triangle.

11]. The length of hypotenuse of a right triangle exceed the length of base by 2 cm and exceeds the length of altitude by 1 cm. Find the length of each side.

12]. The area of a right angled triangle is 96 m^2, if the base is 3 times the altitude. Find the base.

13]. The hypotenuse of a right triangle is 5 cm, if the smaller leg is doubled and longer leg is tripled. The new hypotenuse will be $6\sqrt{5}$ m. Find the sides.

Area of Rectangle

14]. The perimeter of rectangle field is 82 m and its area is 400 m^2. Find the breath of the rectangle.

15]. A rectangle field is 16 m long and 10 m wide there is path of uniform width all around it having an area 120 m^2. Find the width of the path.

16]. A rectangular field is 20 m long and 14 m wide. There is a path of equal width all around it having an area 111. Find the width of the path. 16

17]. The length of rectangle is twice its breadth and its area is 288 m^2. Find the dimension of the rectangle.

18]. The length of the rectangular field is 3 times than its breadth, if the area of the field is 147 cm^2. Find the length of the field.

19]. The perimeter of plot is 62 m and its area is 228 m^2. Find the dimension of the rectangular plot.

20]. The length of rectangle is thrice as long as the side of the square. The side of the square is 4 cm more than the width of the rectangle there area being equal. Find the dimension.

21]. A rectangular field is 25 m long and 16 m broad. There is a path of equal width inside it, if the area of the path is 148 m^2. Find the width of the path.

Hint: $25 \times 16 - (25 - 2x)(16 - 2x) = 148\ m^2$

22]. A farmer prepares a rectangular vegetable garden of are 180 m^2, with 39 m of barbed wire he can fence the three sides of the garden having one of the longer side unfenced. Find the dimension of the vegetable garden.

23]. A farmer wises to start a 100 m^2 of a vegetable garden since he has only 30 m barbed wire, he fences the three sides of the rectangular garden. Labeling his house compound wall act as a fourth side fence. Find the dimension of the garden.

24]. Two square have sides x and x + 4. The sum of area is 656 cm^2. Find the sides of the squares.

25]. A footpath of uniform width runs round inside of the rectangular field 32 m long and 24 m wide,if the path occupies 208 m^2. Find the width of the path.

26]. The length of the rectangle is 13/3 unit more than the breadth and area of the rectangle is 350 sq. unit. Find the length.

➢ Set : V [Phase I]

1] . A journey of 192 km from Bombay to Pune takes 2 hr. less by fast train than a slow train is 16 km/hr.less than that of the fast train. Find their original speed.

2]. The time taken by a person to cover 150 km takes 2.5 hr. less than time taken in return journey.He returns at a speed of 10 km/hr. more than the speed of going. What was the speed in km/hr.in each direction.

$$\frac{150}{x} = \frac{150}{x+10} + 2.5$$

4] . An Aeroplane takes 1 hr. less than for a journey of 1200 km, if its speed is increased by 100 km/hr. from its usual speed. Find its usual speed.

$$\frac{1200}{x} = \frac{1200}{x+100} + 1$$

5] . A plane left 20 minutes later due to bad weather and in order to reach its destination 1200 km away in time it has to increase its speed by 120 km/hr. Find the usual speed of the plane.

Hint : $\frac{1200}{x} = \frac{1200}{x+120} + \frac{20}{60}$

6]. A cyclist takes 2 hr. less to cover a distance of 200 km, if he increase his speed by 5 km. Find the speed of the cyclist.

Hint : $\frac{200}{x} = \frac{200}{x+5} + 2$

7] . The time taken by a man to cover 260 km was of $1\frac{1}{2}$ hr. more than the time taken by him during the return of his journey, if his speed in returning was 12 km/hr. more than his speed in going. Find his speed in each direction.

Hint : $\frac{260}{x} = \frac{260}{x+12} + \frac{3}{2}$

8]. A car covers a distance of 2592 km, the number of hr. taken for the journey is one half a number representing the speed in km/hr. Find the time taken to cover the distance.

Hint : $\frac{1}{2} \times \frac{2592}{x} = x$

9]. A car covers a distance of 648 km. The number of hour taken for the journey is one half the number representing the speed in km/hr. Find the time taken to cover the distance.

Hint : $\frac{1}{2} \times \frac{648}{x} = x$

✪✪ Phase II [Upstream and Downstream]:

10]. The speed of the boat in still water is 8 km/hr. it can go 30 km upstream and 44 km downstream in 10 hr. Find the speed of the stream.

Hint : $\frac{30}{8-x} + \frac{44}{8+x} = 10$ 17

11]. The speed of a boat in still water is 11 km /hr. it can go 12 km upstream and return downstream to the original point in 2 hr. 45 m. Find the speed of the stream.

Hint : $\frac{12}{11-x} + \frac{12}{11+x} = 2 + \frac{45}{60}$

12]. Swati can row his boat 5 km/hr. if she takes 1 hr. long to row the boat 5.25 km upstream and then to return downstream. Find the speed of the swati.

Hint : $\frac{5.25}{5-x} = \frac{5.25}{5+x} + 1$

13]. Ram can row 8 km downstream and return in 1 hr 40 m. if the speed of the stream is 2 km/hr.Find the speed of the boat in still water.

Hint : $\frac{8}{x-2} + \frac{8}{x-2} = 1 + \frac{40}{60}$

14]. The speed of the boat in still water is 8 km/hr. it can go 15 km upstream and 22 km downstream is 5 hr. Find the speed of the stream.

$\frac{15}{8-x} + \frac{22}{8+x} = 5$

15]. The speed of the boat in still water is 15 km/hr. it can go 30 km upstream and return back downstream to the original point is 4 hr. 30 m. Find the speed of the stream.

$\frac{30}{15-x} + \frac{30}{15+x} = 4 + \frac{30}{60}$

16]. A motor boat goes 10 km upstream and return back to the starting point is 55 min. if the speed of the boat in still water is 22 km/hr. Find the speed of the current.

$\frac{10}{22-x} + \frac{10}{22+x} = \frac{55}{60}$

17]. The current of a stream runs at a rate of 2 km/hr. a motor boat goes 10 km upstream and back again To the starting point is 55 min. find the speed of the motor boat in still water.

$\frac{10}{x-2} + \frac{10}{x+2} = \frac{55}{60}$

18] . A motor boat whose speed in still water is 15 km/hr. goes 40 km downstream and comes back in total time 6 hr. Find the speed of the stream.

$\frac{40}{x+15} + \frac{40}{x-15} = 60$

☑ Phase: III[Time]

19]. In a flight of 1600 km, an aircraft was slowed down by bad weather. Its average speed for the trip was reduced by 400 km/hr. and time for the flight are increased by 40 min. Find the actual time of the flight.

20] . In a flight of 600 km. an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr. and time increased by 30 min. Find the duration of the flight.

21] . In a flight of 6000 km. an aircraft was slowed down due to bad weather speed of the trip was reduced by 400 km/hr. and the time of the flight increased by 30 min. Find the duration of the flight.

Set : VI [Reasoning]:

1] . The angry arjun carried some arrows for fight bhishm. With half the number of arrows, he cut down the arrows thrown by bhishm and with 6 other arrows he killed the rath driver with one arrow each he knock down respectively the rath, the flag and the bow. Finally with one more than 4 times the square root of the arrows he laid bhishm unconscious on an arrow bed.Find the total number of arrows arjun had.

2]. One fourth of a herd of a camel was seen in the forest, twice the square root of the herd had gone to mountain and the remaining 15 camel were on the bank of river. Find the total number of camel.

3]. Out of certain number of saras bird .one fourth of the number are moving about the lotus plant $1/9^{th}$ coupled with one fourth as well as 7 times the square root of the number are moving on a hill & 56 bird remains on a vacula tree. What is the total number of birds.

Hint : $\frac{x}{4} + \frac{x}{9} + \frac{x}{4} + 7\sqrt{x} + 56 = 0$

4]. In a cricket match kapil took 1 wicket less than twice the number of wickets takes by ravi, if the product of the number of wickets by these two be 15. Find the number of wicket taken by each.

Hint : $x(2x - 1) = 15$ 18

5]. Out of certain number of saras bird, 7 times the square root of the number are swimming in water while 2 remaining are playing on the shore.

Hint : $\frac{7\sqrt{x}}{2} + 2 = x$

6]. A person on tour has 360 for his daily expenses, if he exceeds his tour program by 4 day he must cut down his daily expense by Rs. 3 per day. Find the number of days of his tour program.

7]. A shopkeeper buys a number of books for Rs 80, if he bought 4 more books in the same amount each book would have cost Rs1 less. How many books should he buy.

Hint : $\frac{80}{x} = \frac{80}{x+4} + 1$

8]. Rs. 250 divided equally among certain number of children, if there were 25 more children each have received 50 p. less. Find the number of children.

Hint : $\frac{250}{x} = \frac{250}{x+25} + 0.50$

9]. Some student planed a picnic. The budget for food was Rs 500 but 5 of them failed to go and thus the cost of food for each member increased by Rs. 5 .How many student attend the picnic.

10] . A person on tour has 3600 for his daily expenses, if he exceeds his tour program for 4 days he had to cut down his daily expenses by Rs. 30 per day. Find the original duration of tour.

11. Rs. 9000 was divided equally among certain number of person. Had there were 20 more person each would have got Rs 160. Find the original number of person.

12 .Rs. 6500 was divided equally among certain number of person. Had there were 15 more person each would have got Rs 30 less. Find the original number of person.

13]. A piece of cloth cost Rs 200, if the piece of cloth is 5 m longer each meter of cloth cost Rs 2 less. The cost of cloth remains unchanged. How long is the piece and what is the original rate of cloth per meter.

14]. A shopkeeper buys a certain quantity of tomatoes for Rs 60 and the number of kg that he buy is 5 more than the number of paisa that each kg cost. Find the cost of each kg of tomatoes.

15] . A piece of cloth cost Rs 35, if the piece were 4 m longer and each meter cost Re. 1 less , the cost would remain unchanged. How long is the piece.

16]. The hostel bill for a number of person for overnight stay is Rs 4800, if there were 4 more the bill of each person would have reduced by Rs 200. Find the number of person staying overnight.

17]. A tourist with Rs 300 calculate that he could spend Rs x every day in holyday, he spent Rs x + 10 In holyday and had nothing 5 days before the end of his holiday.

➢ Set :VII [One Day Work]s

1] . A takes 6 days less than the time taken by B to finish a piece of work, if both A and B together can finish it in 4 days. Find the time taken by B to finish the work.

2]. A takes 10 days less than the time taken by B to finish a piece of work, if both A and B can finish it in 12 days. Find the time taken by B to finish a work.

➢ Set :VIII [Miscellaneous]

1]. In the game of card Neeraj scored 3 more than twice than that of the Pankaj score. The product of their score was 65. How many points did each score.

2]. Two circles touches externally, the sum of their area is 130π cm^2. The distance between their center is 14 cm .Find the radii of the given circles.

3]. Two trains leaves New Delhi railway station . The first train travels due west and the 2nd train due north . The 1st train travels 5 km faster than the 2nd train . If after 2h they are 50 km apart . Find the speed of two trains.

4]. Vikram wishes to fit 3 rods together in a shape of a rt. angled triangle . The hypo. is 2cm longer than the base & 4cm longer than the altitude , how long should the rod the be cut ?

5]. A wire of length 96 m is bent to form a rt.angled triangle with hypo. 40 cm . Find the area of a rt.angled triangle. 19

6]. A chess board contains 64 equal squares and the area of each square is 6.25 cm^2 . A border round the board is 2cm wide . Find the length of the side o the chess board .

7]. In a class test , the sum of the shefali's marks in Maths and English is 30 . Had she got 2 marks more in Math and 3 marks less in English , the product of their marks would have been 210 . Find her marks in the two subjects .

8]. A cottage industry produces a certain number of pottery articles in a day . It was observed on a particular day that the cost of production of each particle (in Rs) was 3 more than twice the number of articles produced on that day . If the total cost of production on that day was Rs. 90 , Find the number of articles produced and the cost of each article.

9]. A party of tourists booked a room in a hotel for Rs. 1200 . Three of the members failed to pay ,as a result, other had to pay Rs 20 , more (each) .how many tourists were there in the party ?

10] A peacock is perched on a pillar 9 feet high . From a distance of 27 feet from the pillar , a snake is coming to its hole at the bottom of the pillar. Seeing the snake , the peacock pounces upon it . If their speeds are equal, at what distance from the hole is the snake caught ?