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UNIFORM ACCELERATION

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UNIFORM ACCELERATION

UNIFORM MOTION

As we have discussed earlier also, in uniform motion velocity of the particle is constant and acceleration is zero. Velocity is constant means its magnitude (called speed) is constant and direction is fixed. Therefore, motion is 1-D in same direction. If velocity is along positive direction, then displacement is also along positive direction. Therefore, distance travelled (d) is equal to the displacement (s). If velocity is along negative direction then displacement is also negative and distance travelled in this case is the magnitude of displacement. Equations involved in this motion are

(i) Velocity (may be positive or negative) = constant
(ii) Speed, v = constant
(iii) Acceleration = 0
(iv) Displacement (may be positive or negative) = velocity × time
(v) Distance = speed × time or $\boxed{d = vt}$
(vi) Distance and speed are always positive, whereas displacement and velocity may be positive or negative.

One Dimensional Motion with Uniform Acceleration

As we have discussed in article 6.3 that equations like $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ etc. can be applied directly with constant (or uniform) acceleration. Further, in one dimensional motion, all vector quantities (displacement, velocity and acceleration) can be treated like scalars by using sign convention method. In this method, one direction is taken as positive and the other as the negative and then all vector quantities are written with paper signs. In most of the cases, we will take following sign convention.

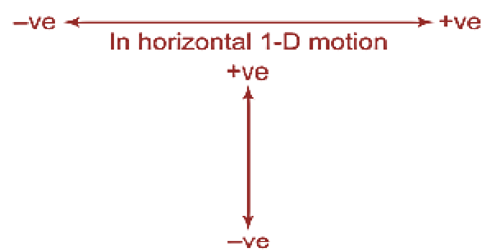


Fig. 6.10

The equations used in 1-D motion with uniform acceleration are

$$v = u + at \quad ...(i)$$

$$v^2 = u^2 + 2as \quad ...(ii)$$

$$s = ut + \frac{1}{2}at^2 \quad ...(iii)$$

$$s_1 = s_0 + ut + \frac{1}{2}at^2 \quad ...(iv)$$

$$s_t = u + at - \frac{1}{2}a \quad ...(v)$$

In the above equations, u = initial velocity, v = velocity at time t, a = constant acceleration
s = displacement measured from the starting point

Here, starting point means the point where the particle was at $t = 0$. It is not the point where $u = 0$.
s_1 = displacement measured from any other point, say P, where P is not the starting point.
s_0 = displacement of the starting point from P.
s_t = displacement (not the distance in t^{th} second).

We know that moving objects like buses, car, train etc all do not move with uniform velocity. Sometimes, their velocity increases & some time their velocity decreases. Therefore, we can say that the motion of the vehicle or other moving objects in non-uniform.

□ **In non- uniform motion the**, The velocity of the object changes with time. Such motion of the object in said to be an accelerated uniform.

□ **Acceleration ($\vec{a}$):** **"The time rate of change of velocity of moving object is called an acceleration".**

or simply. **"The change in velocity per unit time is called acceleration.**

i.e., $$\text{Accel}^r = \frac{\text{Change in velocity}}{\text{Time taken}}$$

► Unit (S.I) = m/s^2
(cgs) = cm/s^2.

► **Dimensional formula:** -[$M^0L^1T^2$]

► Acceleration is a vector quantity. It is characterised by both magnitude & direction.

□ **Accelerated motion of an object is of two types: -**

- **(i) Uniformly accelerated motion.**
- **(ii) Non uniformly accelerated motion.**

□ **(I) Uniformly Accelerated Motion**: - If the change in velocity of an object in each time is constant. Then, the Accel ^{r} of object is constant.

Thus,

"The motion of a body whose Accel ^{r} is constant is known as uniformly Accelerated motion".

□ **(ii) Non- uniformly Accelerated motion:** - If the change in velocity of an object in each unit time is not constant then the Accel ^{r} of the object is variable.

Thus,

"The motion of an object having variable Accel ^{r} is known as non – uniform accelerated motion.

$$\therefore \quad \text{Accel}^r \quad a = \frac{\text{Change in velocity}}{\text{Time taken for this change.}}$$

Let, v_1 & v_2 be the initial & final velocities at time t_1 & t_2 respectively.

$$\therefore \quad a = \frac{v_2 - v_1}{t_2 - t_1}$$

► **Physical meaning of Accel^r:** - "The physical meaning of the quantity Accel ^{r} that it tells us how much amount the velocity of an object is changed per unit time."

► **Positive Acceleration:** - Accel^r is **positive** if the velocity is $\boxed{\text{Increasing}}$

if the final velocity (v_2) of an object is greater than its initial velocity(v_1), then

$$\text{Accelr, } a = \frac{v_2 - v_1}{t_2 - t_1} = +\text{ive}$$

◙ This means that the object is speeding up.

► **Negative Acceleration**: - Accelr is **negative** of the velocity is Decreasing

Then the final velocity (v_2) of an object in less them its initial velocity (v_1).

$$\text{Accel}^{r}: \quad a = \frac{v_2 - v_1}{t_2 - t_1} = -\text{ive}$$

This means that object is slowing down - Negative Retardation – Retardation or deceleration.

► **Average Accel^r & Instantaneous Accel^r:**

If an object has non – uniform Accel^r Motion, its velocity changes in each unit time.
In this case average Accel^r of the subject is calculated.

◙ **Average Accel^r:** - **"Average Accelr is defined as the change in velocity of the object divided by the total time taken for this change in velocity."**

i.e., Average Accelr, $a_{av} = \dfrac{\text{Change in velocity}}{\text{Time taken}}$

Acceleration is the time rate of change of velocity. The average acceleration over a time interval Δt is equal to change of velocity during the time interval divided by Δt.

$$\text{Average acceleration, } \bar{a} = \frac{\Delta v}{\Delta t}$$

The instantaneous acceleration a of a body is defined as the limit of the average acceleration at a particular time as $\Delta t \to 0$.

$$\therefore \quad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- In most of the cases, displacement is measured from the starting point, therefore Eq. (iii) or $s = ut + \frac{1}{2}at^2$ is used.
- For small heights, if the motion is taking place under gravity then acceleration is always constant (= acceleration due to gravity). This is $9.8\ m/s^2$ ($\approx 10\ m/s^2$) in downward direction. According to our sign convention downward direction is negative. Therefore,

$$a = g = 9.8\ \text{m/s}^2 \approx -10\ \text{m/s}^2$$

- One-dimensional motion (with constant acceleration) can be observed in following three cases:
  Case 1 Initial velocity is zero.
  Case 2 Initial velocity is parallel to constant acceleration.
  Case 3 Initial velocity is antiparallel to constant acceleration.

  In first two cases, motion is only accelerated and direction of motion does not change. In the third case motion is first retarded (till the velocity becomes zero) and then accelerated in opposite direction.

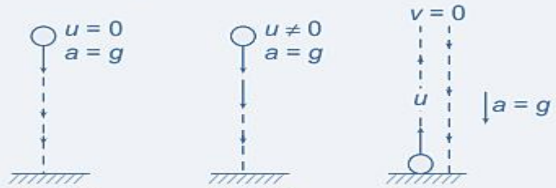


Fig. 6.11

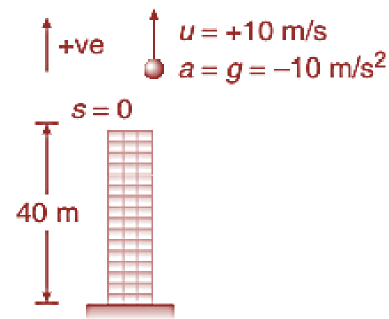
- In most of the problems of time calculations, $s = ut + \frac{1}{2}at^2$ equation is useful. But s has to be measured from the starting point.
- In case 3 (of point 3), we need not to apply two separate equations, one for retarded motion (when motion is upwards) and other for accelerated motion (when motion is downwards). Problem can be solved by applying the equations only one time, provided s (in $s = ut + \frac{1}{2}at^2$) is measured from the starting point and all vector quantities are substituted with proper signs.

Example *A ball is thrown upwards from the top of a tower 40 m high with a velocity of 10 m/s. Find the time when it strikes the ground. Take $g = 10\ m/s^2$.*

Solution In the problem, $u = +10$ m/s, $a = -10$ m/s^2

and $s = -40$ m (at the point where stone strikes the ground)

Substituting in $s = ut + \frac{1}{2}at^2$, we have



$$-40 = 10\,t - 5t^2$$

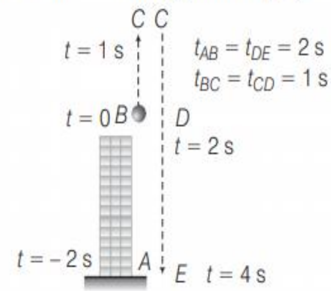
or $5t^2 - 10t - 40 = 0$

or $t^2 - 2t - 8 = 0$

Solving this, we have $t = 4$ s and -2 s.

Taking the positive value $t = 4$ s.

The significance of $t = -2$ s can be understood by following figure



Example 6.14 *A ball is thrown upwards from the ground with an initial speed of u. The ball is at a height of 80 m at two times, the time interval being 6 s. Find u. Take $g = 10\ m/s^2$.*

Solution Here, $u = u$ m/s, $a = g = -10$ m/s^2 and $s = 80$ m.

Substituting the values in $s = ut + \frac{1}{2}at^2$,

we have $80 = ut - 5t^2$ or $5t^2 - ut + 80 = 0$

or $t = \dfrac{u + \sqrt{u^2 - 1600}}{10}$ and $\dfrac{u - \sqrt{u^2 - 1600}}{10}$

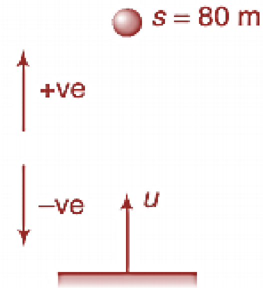


Fig. 6.14

Now, it is given that $\dfrac{u + \sqrt{u^2 - 1600}}{10} - \dfrac{u - \sqrt{u^2 - 1600}}{10} = 6$

or $\dfrac{\sqrt{u^2 - 1600}}{5} = 6$ or $\sqrt{u^2 - 1600} = 30$ or $u^2 - 1600 = 900$

$\therefore$ $u^2 = 2500$ or $u = \pm 50$ m/s

Ignoring the negative sign, we have $u = 50$ m/s

- In motion under gravity, we can use the following results directly in objective problems:

(a) If a particle is projected upwards with velocity u, then

(i) maximum height attained by the particle, $h = \dfrac{u^2}{2g}$

(ii) time of ascent = time of descent = $\dfrac{u}{g}$ $\Rightarrow$ $\therefore$ Total time of flight = $\dfrac{2u}{g}$

Fig. 6.15

(b) If a particle is released from rest from a height h (also called free fall), then

(i) velocity of particle at the time of striking with ground, $v = \sqrt{2gh}$

(ii) time of descent (also called free fall time) $t = \sqrt{\dfrac{2h}{g}}$

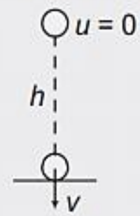


Fig. 6.16

Note *In the above results, air resistance has been neglected and we have already substituted the signs of u, g etc. So, you have to substitute only their magnitudes.*

EQUATION OF UNIFORMLY ACCELERATED LINEAR MOTION

Consider an object moving with a uniform acceleration 'a' along a straight line OX with origin at O as shown in Fig. 7.1. Let at $t = 0$, the position of the object be x_0 (initial displacement from O) and its velocity v_0 (initial velocity). Let at $t = t$, the position of the object be x and its velocity v (final velocity). Clearly, in time t, the displacement of the object is $x - x_0$.

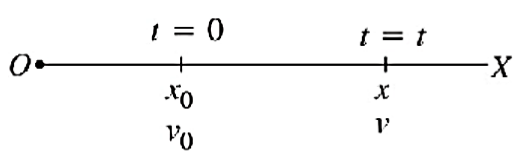


Fig. 7.1

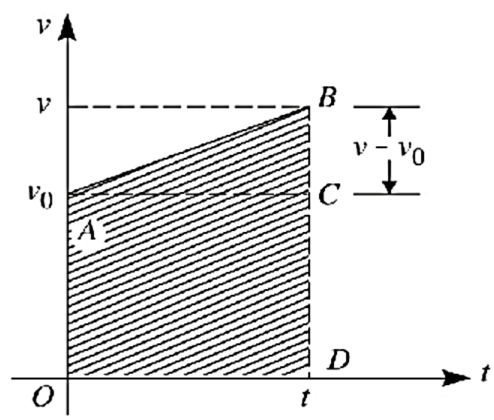


Fig. 7.2

(*i*) **Displacement – Average Velocity relation.** The velocity-time ($v - t$) graph for the motion in Fig. 7.1 is AB as shown in Fig. 7.2. The velocity increases uniformly from v_0 at $t = 0$ to v in time t (= OD). We know that displacement during any time interval is equal to the area under velocity-time graph during that time interval. In this case, the time interval is from 0 to t so that displacement is equal to the shaded area in Fig. 7.2.

$\therefore$ Displacement = Shaded area = Area of trapezium $OABD$

$$= \frac{AO + BD}{2} \times OD = \frac{v_0 + v}{2} \times t$$

$\therefore$ Average velocity, $$\bar{v} = \frac{\text{Displacement}}{\text{Time interval}} = \frac{v_0 + v}{2} \times t \div t$$

or $$\bar{v} = \frac{v_0 + v}{2}$$

Note that for the case of constant acceleration *and only for this case,* the average velocity is equal to one-half the sum of initial and final velocities.

Now Displacement = Average velocity × Time interval

or $$x - x_0 = \left(\frac{v_0 + v}{2}\right)t \qquad \ldots(i)$$

(ii) **Velocity–Time relation.** Since object's velocity has changed by an amount $v - v_0$ during time t, the constant acceleration 'a' during this time is

$$a = \frac{v - v_0}{t}$$

or $$v = v_0 + at \qquad \ldots(ii)$$

(iii) **Position–Time relation.** This can be found as under :

Average velocity, $$\bar{v} = \frac{\text{Displacement}}{\text{Time}} = \frac{x - x_0}{t}$$

or $$x - x_0 = \bar{v}\, t$$

or $$x - x_0 = \left(\frac{v_0 + v}{2}\right)t \qquad \left(\because \bar{v} = \frac{v_0 + v}{2}\right)$$

or $$x - x_0 = \left(\frac{v_0 + v_0 + at}{2}\right)t \qquad (\because v = v_0 + at)$$

or $$x - x_0 = v_0 t + \frac{1}{2}at^2$$

or $$x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad \ldots(iii)$$

If $x - x_0 = S$, the displacement of the object in time t, then eq. (*iii*) can be written as :

$$S = v_0 t + \frac{1}{2}at^2$$

(iv) **Velocity–Displacement relation.** This can be found as under :

$$x - x_0 = \left(\frac{v_0 + v}{2}\right)t \qquad \text{[From eq. (}i\text{)]}$$

or $$x - x_0 = \left(\frac{v_0 + v}{2}\right)\left(\frac{v - v_0}{a}\right) \qquad \left[\text{From eq. (}ii\text{)}, t = \frac{v - v_0}{a}\right]$$

or $$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

$$\therefore \qquad v^2 - v_0^2 = 2a(x - x_0) \qquad \ldots (iv)$$

If $x - x_0 = S$, the displacement of the object in time t, then eq. (iv) can be written as :

$$v^2 - v_0^2 = 2\ aS$$

The *four equations* of motion for uniform acceleration are given in the table below. Each equation is also identified by the variable (x, v, a or t) that *does not* appear in it.

Equation	Variable not appearing
$v = v_0 + at$	x
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
$x - x_0 = \left(\frac{v_0 + v}{2}\right)t$	a
$v^2 - v_0^2 = 2a(x - x_0)$	t

(i) The *first two equations* respectively give the final velocity v and final displacement x in terms of their initial values v_0 and x_0, the acceleration 'a' and time interval t.

(ii) The *last two equations* are very important because they are usually sufficient to solve problems for constant acceleration.

DISPLACEMENT IN Nth second FOR UNIFORM ACCELERATION

Consider an object moving with a uniform acceleration 'a' along a straight line OX with origin at O as shown in Fig. 7.3. Let at $t = 0$, the position of the object be x_0 and its velocity v_0 (initial velocity). Let at $t = t$, the position of the object be x and its velocity v. Clearly, in time t, the displacement of the object is $x - x_0 = S$. The displacement of the object in time t is given by ;

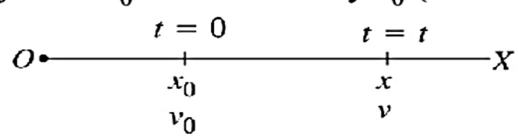


Fig. 7.3

$$S = v_0 t + \frac{1}{2}at^2$$

If S_n and S_{n-1} are the displacements of the object in n and $n - 1$ seconds, then displacement of the object in the nth second is

$$\begin{aligned} S_{nth} &= S_n - S_{n-1} \\ &= \left[v_0 n + \frac{1}{2}an^2\right] - \left[v_0(n-1) + \frac{1}{2}a(n-1)^2\right] \\ &= \left[v_0 n + \frac{1}{2}an^2\right] - \left[v_0 n - v_0 + \frac{1}{2}an^2 - an + \frac{a}{2}\right] \\ &= v_0(n - n + 1) + \frac{a}{2}(n^2 - n^2 + 2n - 1) \\ &= v_0 + \frac{a}{2}(2n - 1) \end{aligned}$$

$$\therefore \qquad S_{nth} = v_0 + \frac{a}{2}(2n-1) \qquad \ldots (i)$$

Eq. (i) gives the expression for the displacement in the nth second for uniform acceleration.

EQUATIONS OF UNIFORM ACCELERATION FROM v – t GRAPH

Fig. 7.4 shows the velocity-time graph for an object moving along a straight line with constant acceleration a. The velocity-time graph is a straight line because a (= slope of graph) is constant. Note that graph starts from v_0 because at t = 0, the initial velocity is assumed to be v_0.

(*i*) Velocity at any time *t*. From velocity-time graph, we can derive the relation for final velocity v of the object at any time t. Thus in Fig. 7.4, the final velocity v is the sum of two parts — initial velocity v_0 and the velocity change at during the time t. (Note that time interval = $t - 0 = t$).

$$\therefore \qquad v = v_0 + at$$

It is the same result that we derived earlier from algebraic considerations. Note that how clearly this relation is indicated in the $v - t$ graph.

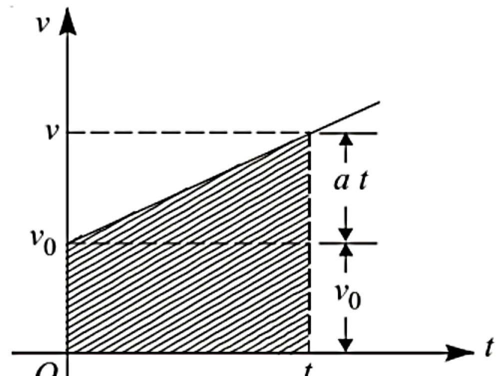


Fig. 7.4

(*ii*) Final position *x* at any time *t*. From the velocity-time graph in Fig. 7.4, we can derive the relation for the position x of the object as a function of time. We know that displacement $*x - x_0$ of the object during the time t is equal to the area of the graph for that time interval (*i.e.*, 0 to t in this case). The area under the graph for time interval 0 to t is the shaded area. It consists of a rectangle of width t and height v_0 plus right angled triangle of base t and height $a\ t$.

$$\therefore \qquad x - x_0 = \text{Shaded area}$$

$$\text{or} \qquad x - x_0 = v_0\, t + \frac{1}{2}\, a\, t^2$$

$$\text{or} \qquad x = x_0 + v_0\, t + \frac{1}{2}\, at^2$$

(*iii*) $\qquad v^2 - v_0^{\,2} = 2aS$

We shall derive the above relation graphically. Referring to Fig 7.4, we have,

$$x - x_0 = \text{Area of trapezium (shaded portion)}$$

$$= \frac{1}{2}\,(v + v_0)t \qquad \text{...}(i)$$

$$\text{Acceleration, } a = \text{Slope of velocity-time graph}$$

$$= \frac{v - v_0}{t}$$

$$\text{or} \qquad t = \frac{v - v_0}{a}$$

Putting the value of t in eq. (i), we have,

$$x - x_0 = \frac{1}{2}\left(v + v_0\right) \times \frac{v - v_0}{a}$$

$$\therefore \qquad v^2 - v_0^2 = 2a\,(x - x_0)$$

If $x - x_0 = S$, the displacement of the particle in time t, then, $v^2 - v_0^2 = 2aS$.

(iv) **Average velocity.** Compare the velocity-time graphs in Fig. 7.5 (*i*) and in Fig. 7.5 (*ii*). The shaded area under $v - t$ graph is the same for both cases for the same time interval.

In Fig. 7.5 (*i*), the area is under the trapezoid and in Fig. 7.5 (*ii*), the same area is under the rectangle. Both areas are **equal to the change in displacement $x - x_0$. The horizontal line

* At $t = 0$, velocity is v_0 and the position of the object (*i.e.*, its displacement from the origin) is x_0.

** The velocity-time graphs are with identical areas under the graph. In Fig. 7.5 (*i*), the area is with constant acceleration and in Fig. 7.5 (*ii*), the same area is shown with constant velocity.

(solid line) in Fig. 7.5 (*ii*) has the value $(v_0 + v)/2$; it is in fact the average velocity $\overline{v}$.

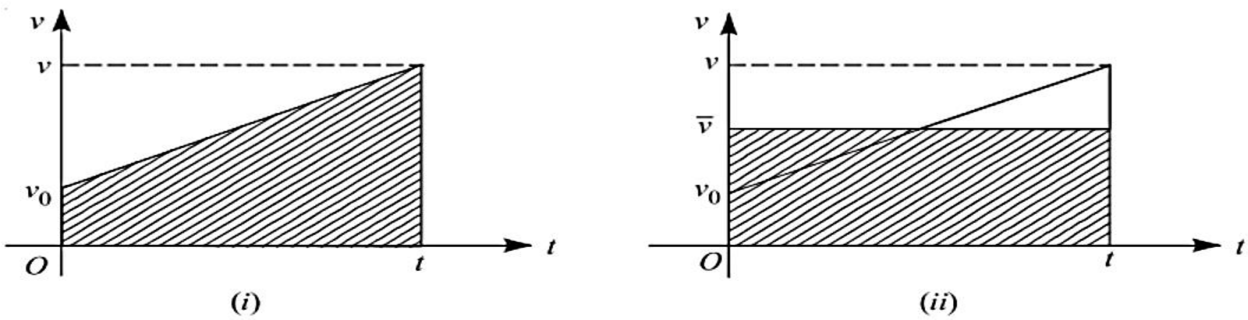


Fig. 7.5

$$\therefore \qquad \text{Average velocity, } \overline{v} = \frac{v_0 + v}{2}$$

Referring to Fig. 7.5 (*ii*), the area under the $v - t$ graph for $0 - t$ time interval is equal to the width t of the rectangle multiplied by height $\overline{v}$ $[= (v_0 + v)/2]$.

$$\therefore \qquad x - x_0 = \left(\frac{v_0 + v}{2}\right)t$$

(iv) **Distance travelled by a particle in n^th Second:** – Consider velocity time graph of a uniform accelerated particle.

Now select two points A and B on v – t graph corresponding to time t_{n-1} and t_n.

Let, v_{n-1} = velo. of the particle at A

v_n = velo. of the particle at B

Now,

Distance travelled by particle in n^{th} second is

S_{nth} = Area of trapezium ABCD.

= ½ (sum of | | sides) × $\perp^{ar}$ distance

= ½ (AD + BC) × AE

= ½ [(v_{n-1} + v_n) × (n – (n – 1)]

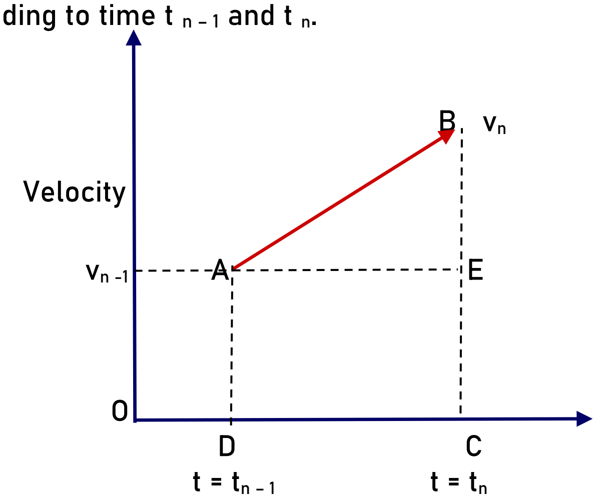
= ½ (v_{n-1} + v_n) × 1 .... (1)

Now, we know that v = u + at

When t = n–1, v_{n-1} = u + a (n –1).

When t = n, v_n = u + an

Putting the values of v_n and v_{n-1} in Eq (1)



S_{nth} = ½ [u + a (n–1) + u + an]

= ½ [2u + a (n – 1) + n)]

S_{nth} = u + ½ a (2n – 1)

POSITION – TIME GRAPH FOR POSITIVE ACCELERATION

The position–time relation for uniformly accelerated motion in a straight line is given by ;

$$x = x_0 + v_0 t + \frac{1}{2} at^2 \quad \text{...}(i)$$

Eq. (*i*) shows that x is a quadratic function of time t. Therefore, the x –t graph is a ***parabola** (See Fig. 7.6). Note that slope of the tangent at $t = 0$ is equal to the initial velocity v_0 and slope of the tangent at time t is equal to the velocity v at time t. It is clear that the slope of the graph is continuously increasing (*i.e.*, velocity is continuously increasing). The measurements would show that the rate of increase of slope is constant.

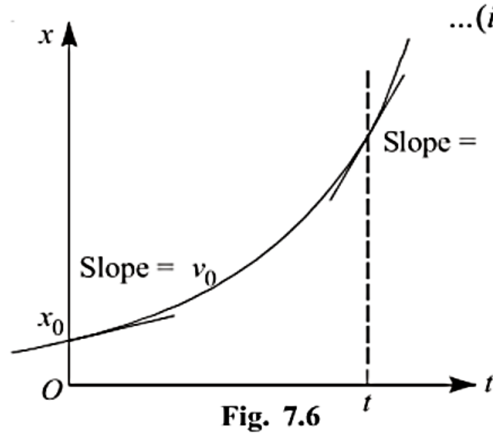


Fig. 7.6

Note. $x = x_0 + v_0 t + \frac{1}{2} at^2$

At first sight, it may appear that in this equation, acceleration is not constant. However, by differential calculus, it can be easily shown that acceleration is constant.

$$\frac{dx}{dt} = \frac{d}{dt}\left(x_0 + v_0 t + \frac{1}{2}at^2\right) = v_0 + at$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}(v_0 + at) = a$$

VERTICAL MOTION UNDER GRAVITY

One important example of linear motion with constant acceleration is the vertical motion (downward or upward) of an object under gravity. Objects that move under gravity alone are called freely falling bodies. *A freely falling body always has an acceleration of 9.8 ms^{-2} vertically downward.* Note that magnitude (9.8 ms^{-2}) as well as direction (vertically downward) of the acceleration is constant.

A freely falling body has a constant acceleration of 9.8 ms^{-2} acting vertically downward. It does not matter whether the body is rising or falling; its acceleration is still g (= 9.8 ms^{-2}) downward. When a body is rising, its velocity decreases at the rate of 9.8 ms^{-1} every second until it reaches at the top of its motion. At this instant, the velocity of the body is zero and it starts downward journey. As the body is falling, its velocity increases at the rate of 9.8 ms^{-1} every second until it hits the ground.

Equations of Motion. Since vertical motion under gravity alone is the linear motion with constant acceleration, all the equations of motion derived in Art. 7.2 are applicable to freely falling bodies. The only thing to be done is to change the nomenclature *i.e.*, displacement S is replaced by h (vertical displacement) and a by g. Therefore, equations of motion for a freely falling body are :

(*i*) $$h = \left(\frac{v_0 + v}{2}\right)t$$

(*ii*) $$v = v_0 + g\,t$$

(*iii*) $$h = v_0\,t + \frac{1}{2}\,g\,t^2$$

(*iv*) $$v^2 - v_0^2 = 2\,g\,h$$

(*v*) $$h_{nth} = v_0 + \frac{g}{2}(2n - 1)$$

Sign convention. While solving problems regarding freely falling bodies, it is very important to adopt one sign convention for displacements and velocities. *It is a usual practice to choose h to be positive in the upward direction from the selected origin and negative in the downward direction from the origin.* Therefore, you must specify the origin of measurement in problems involving motion under gravity.

DIFFERENCE BETWEEN DISTANCE (d) AND DISPLACEMENT (D)

The s in equations of motion ($s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$) is really the displacement not the distance. They have different values only when u and a are of opposite sign or $u \uparrow\downarrow a$.

Let us take the following two cases :

Case 1 When u is either zero or parallel to a, then motion is simply accelerated and in this case distance is equal to displacement. So, we can write

$$\boxed{d = s = ut + \frac{1}{2}at^2}$$

Case 2 When u is antiparallel to a, the motion is first retarded then accelerated in opposite direction. So, distance is either greater than or equal to displacement ($d \geq |s|$). In this case, first find the time when velocity becomes zero. Say it is t_0.

$$0 = u - at_0 \quad \Rightarrow \quad \therefore \quad t_0 = \left|\frac{u}{a}\right|$$

Now, if the given time $t \leq t_0$, distance and displacement are equal. So, $d = s = ut + \frac{1}{2}at^2$

For $t \leq t_0$, (with u positive and a negative)

For $t > t_0$, distance is greater than displacement. $d = d_1 + d_2$

Here, d_1 = distance travelled before coming to rest $= \left|\frac{u^2}{2a}\right|$

d_2 = distance travelled in remaining time $t - t_0 = \frac{1}{2}|a(t - t_0)^2|$

$$\therefore \quad \boxed{d = \left|\frac{u^2}{2a}\right| + \frac{1}{2}|a(t - t_0)^2|}$$

Note *The displacement is still* $s = ut + \frac{1}{2}at^2$ *with u positive and a negative.*

Example 6.15 *A particle is projected vertically upwards with velocity 40 m/s. Find the displacement and distance travelled by the particle in*
(a) 2 s *(b) 4 s* *(c) 6 s*
Take $g = 10\ m/s^2$.

Solution Here, u is positive (upwards) and a is negative (downwards). So, first we will find t_0, the time when velocity becomes zero.

$$t_0 = \left|\frac{u}{a}\right| = \frac{40}{10} = 4\text{ s}$$

(a) $t < t_0$. Therefore, distance and displacement are equal.

$$d = s = ut + \frac{1}{2}at^2 = 40 \times 2 - \frac{1}{2} \times 10 \times 4 = 60\text{ m}$$

(b) $t = t_0$. So, again distance and displacement are equal.

$$d = s = 40 \times 4 - \frac{1}{2} \times 10 \times 16 = 80\text{ m}$$

(c) $t > t_0$. Hence, $d > s$, $\quad s = 40 \times 6 - \frac{1}{2} \times 10 \times 36 = 60\text{ m}$

While

$$d = \left|\frac{u^2}{2a}\right| + \frac{1}{2}|a(t - t_0)^2|$$

$$= \frac{(40)^2}{2 \times 10} + \frac{1}{2} \times 10 \times (6 - 4)^2$$

$$= 100\text{ m}$$

ONE DIMENSIONAL MOTION AND NON- UNIFORM ACCELERATION

When acceleration of a particle is not constant we take help of differentiation or integration.

Equations of Differentiation

(a) $$\boxed{v = \frac{ds}{dt}} \qquad \text{...(i)}$$

If the motion is taking place along x-axis, then this equation can be written as,

$$v = \frac{dx}{dt}$$

Here, v is the instantaneous velocity and x, the x co-ordinate at a general time t.

(b) $$\boxed{a = \frac{dv}{dt}} \qquad \text{...(ii)}$$

Here, a is the instantaneous acceleration of the particle. Further, a can also be written as

$$a = \frac{dv}{dt} = \left(\frac{ds}{dt}\right)\left(\frac{dv}{ds}\right) = v\left(\frac{dv}{ds}\right) \qquad \left[\text{as } \frac{ds}{dt} = v\right]$$

$\therefore$ $$\boxed{a = v\left(\frac{dv}{ds}\right)} \qquad \text{...(iii)}$$

Equations of Integration

(c) $$\int ds = \int v dt \quad ...(iv)$$

or $$s = \int v dt \quad ...(v)$$

In the above equations, v should be either constant or function of t

(d) $$\int dv = \int a\, dt \quad ...(vi)$$

or $$\Delta v = v_f - v_i = \int a\, dt \quad ...(vii)$$

In the above equations a should be either constant or function of time t.

(e) $$\int v\, dv = \int a\, ds \quad ...(viii)$$

In the above equation a should be either constant or function of s.

(i) To convert s-t equation into v-t equation or v-t equation into a-t equation differentiation will be done.

$s\text{-}t \to v\text{-}t \to a\text{-}t$ *(differentiation)*

(ii) To convert a-t equation into v-t equation or v-t equation into s-t equation, integration equations (with some limits) are required. By limit we mean the value of physical quantity which we will get after integration should be known at some given time.

For example, after integrating v (w.r.t time) we will get displacement s. Therefore, to get complete s function value of s should be known at some given time. Otherwise constant of integration remains as an unknown.

Thus, $a\text{-}t \to v\text{-}t \to s\text{-}t$ *(integration with limits)*

EQUATIONS FOR UNIFORMLY ACCELERATED LINEAR MOTION

(CALCULUS METHOD)

(*i*) **Velocity-Time relation.** The instantaneous acceleration of a moving object is given by ;

$$a = \frac{dv}{dt}$$

or $$dv = a\ dt$$

When $t = 0$; $v = v_0$ and when $t = t$, $v = v$.

We integrate velocity from the initial value v_0 to the final value v, the corresponding limits on the time being 0 and t.

$$\therefore \quad \int_{v_0}^{v} dv = \int_0^t a\, dt$$

Since acceleration a is constant, it can be taken out of the integral sign.

$$\therefore \quad \int_{v_0}^{v} dv = a\int_0^t dt$$

or $$|\, v\,|_{v_0}^{v} = a\,|\, t\,|_0^t$$

or $$v - v_0 = at$$

$$\therefore \quad v = v_0 + a\ t \quad ...(i)$$

(ii) Position-Time relation. $v = v_0 + a\,t$

or $$\frac{dx}{dt} = v_0 + at$$

or $$dx = v_0\,dt + at\,dt$$

When $t = 0$; $x = x_0$ and when $t = t$; $x = x$.

Integrating the above equation for dx through proper limits, we get,

$$\int_{x_0}^{x} dx = \int_0^t v_0 dt + \int_0^t a\,t\,dt$$

Since v_0 (= initial velocity) and acceleration a are constant, they can be taken out of the integral sign.

$$\therefore \quad \int_{x_0}^{x} dx = v_0 \int_0^t dt + a \int_0^t t\,dt$$

or $$|\,x\,|_{x_0}^{x} = v_0\,|\,t\,|_0^t + a\left|\frac{t^2}{2}\right|_0^t$$

$$\therefore \quad x - x_0 = v_0\,t + \frac{1}{2}at^2$$

or $$x = x_0 + v_0\,t + \frac{1}{2}a\,t^2 \qquad ...(ii)$$

If $x - x_0 = S$, the displacement of the object in time t, then eq. (*ii*) can be written as :

$$S = v_0\,t + \frac{1}{2}\,a\,t^2$$

(iii) Velocity-Displacement relation.

$$a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt}$$

or $$a = \frac{dv}{dx}\cdot v \qquad \left(\because \frac{dx}{dt} = v\right)$$

or $$a\,dx = v\,dv$$

When $x = x_0$, $v = v_0$ and when $x = x$, $v = v$.

$\therefore$ Integrating above equation with proper limits, we have,

$$\int_{x_0}^{x} a\,dx = \int_{v_0}^{v} v\,dv$$

Since acceleration a is constant, it can be taken out of the integral sign.

$$\therefore \quad a\int_{x_0}^{x} dx = \int_{v_0}^{v} v\,dv \quad \text{or} \quad a\,|\,x\,|_{x_0}^{x} = \left|\frac{v^2}{2}\right|_{v_0}^{v}$$

or $$a(x - x_0) = \frac{v^2}{2} - \frac{v_0^2}{2} \quad \therefore \quad v^2 - v_0^2 = 2a(x - x_0) \qquad ...(iii)$$

If $x - x_0 = S$, the displacement of the object in time t, then eq. (*iii*) can be written as :

$$v^2 - {v_0}^2 = 2\,a\,S$$

can verify the correctness of these equations of uniformly accelerated linear motion by differential calculus.

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

(i) $$v = \frac{dx}{dt} = \frac{d}{dt}\left(x_0 + v_0 t + \frac{1}{2}at^2\right) = v_0 + at$$

$\therefore$ $$v = v_0 + a\ t$$

(ii) $$a = \frac{dv}{dt} = \frac{d}{dt}(v_0 + at) = a = \text{constant}$$

EXAMPLE *Displacement-time equation of a particle moving along x-axis is*

$$x = 20 + t^3 - 12t \text{ (SI units)}$$

(a) Find, position and velocity of particle at time $t = 0$.
(b) State whether the motion is uniformly accelerated or not.
(c) Find position of particle when velocity of particle is zero.

Solution (a) $$x = 20 + t^3 - 12t \quad \ldots\text{(i)}$$

At $t = 0$, $x = 20 + 0 - 0 = 20$ m

Velocity of particle at time t can be obtained by differentiating Eq. (i) w.r.t. time i.e.

$$v = \frac{dx}{dt} = 3t^2 - 12 \quad \ldots\text{(ii)}$$

At $t = 0$, $v = 0 - 12 = -12$ m/s

(b) Differentiating Eq. (ii) w.r.t. time t, we get the acceleration $a = \frac{dv}{dt} = 6t$

As acceleration is a function of time, the motion is non-uniformly accelerated.

(c) Substituting $v = 0$ in Eq. (ii), we have $0 = 3t^2 - 12$

Positive value of t comes out to be 2 s from this equation. Substituting $t = 2$ s in Eq. (i), we have $x = 20 + (2)^3 - 12(2)$ or $x = 4$ m

EXAMPLE . **A body is dropped from rest at a height of 150 m and simultaneously another body is dropped from rest from a point 100 m above the ground. What is the difference between their heights after they have fallen for 3 seconds? Take g = 10 ms^{-2}.**

Solution. Since initial velocity in the two cases is zero, $h = \frac{1}{2} gt^2$.

Downward distance covered by *first body* in 3 seconds

$$= \frac{1}{2} \times 10 \times (3)^2 = 45 \text{ m}$$

$\therefore$ Height of first body above the ground = 150 – 45 = 105 m

Downward distance covered by *second body* in 3 seconds

$$= \frac{1}{2} \times 10 \times (3)^2 = 45 \text{m}$$

$\therefore$ Height of second body above the ground = 100 – 45 = 55 m

$\therefore$ Difference in height between the two after 3 seconds = 105 – 55 = 50 m

EXAMPLE . **A stone is dropped from the top of a tower 400 m high and at the same time another stone is projected upward vertically from the ground with a velocity of 100 ms^{-1}. Find when and where the two will meet.**

Solution. Suppose the two stones meet after t seconds when the stone from the top of the tower has covered a distance of h. Then the distance of the meeting point from the ground is $(400 - h)$ as shown in Fig. 7.15. Take the downward motion positive.

For downward motion, $g = +9.8\ \text{ms}^{-2}; v_0 = 0$.

$$\therefore \quad h = v_0 t + \frac{1}{2}gt^2 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$$

or $\quad h = 4.9\ t^2 \quad \ldots (i)$

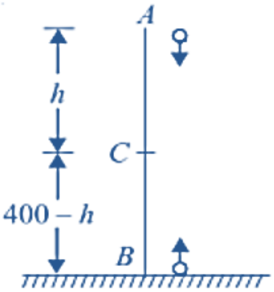


Fig. 7.15

For upward motion, $g = -9.8\ \text{ms}^{-2}$; $v_0 = 100\ \text{ms}^{-1}$.

$$\therefore \quad 400 - h = v_0 t + \frac{1}{2}gt^2 = 100 \times t + \frac{1}{2}(-9.8)\ t^2$$

or $\quad 400 - h = 100\ t - 4.9\ t^2 \quad \ldots (ii)$

Adding eqs. (i) and (ii), we have,

$$400 = 100\ t \quad \therefore \quad t = 400/100 = 4\ \text{s}$$

Putting $t = 4$ s in eq. (i), $h = 4.9 \times (4)^2 = 78.4$ m.

Therefore, the two stones meet 78.4 m below the top of tower after 4 s.

EXAMPLE **A balloon is ascending at the rate of 12 ms^{-1}. When it is at a height of 65 m from the ground, a packet is dropped from it. After how much time and with what velocity does the packet reach the ground? Take $g = 10$ ms^{-2}.**

Solution. $h = v_0\ t + \frac{1}{2}gt^2$

Taking downward motion as positive ; $v_0 = -12\ \text{ms}^{-1}$; $g = +10\ \text{ms}^{-2}$

$$\therefore \quad 65 = -12 \times t + \frac{1}{2} \times 10 \times t^2$$

or $\quad 5\ t^2 - 12\ t - 65 = 0 \quad$ or $\quad (5t + 13)(t - 5) = 0$

$\therefore \quad t = -13/5$ sec ; $t = 5$ sec

As the time cannot be negative ; $t = 5$ sec.

$$\therefore \quad v = v_0 + gt = -12 + 10 \times 5 = 38\ \text{ms}^{-1}$$

EXAMPLE . **A bottle dropped from a balloon reaches the ground in 20 s. Determine the height of the balloon if (i) it was at rest in the air (ii) it was ascending with a speed of 50 ms^{-1}, when the bottle was dropped.**

Solution. (i) $h = v_0\ t + \frac{1}{2}gt^2$

Taking upward motion as positive, $g = -9.8\ \text{ms}^{-2}$

$$\therefore \quad h = 0 \times t - \frac{1}{2} \times 9.8 \times (20)^2 = -1960\ \text{m} \qquad \therefore \ |h| = 1960\ \text{m}$$

(ii) Taking upward motion as positive, $g = -9.8\ \text{ms}^{-2}$

$$\therefore \quad h = v_0 t + \frac{1}{2}gt^2 = 50 \times 20 + \frac{1}{2} \times (-9.8) \times (20)^2 = -960\ \text{m}$$

$$\therefore \quad |h| = 960\ \text{m}$$

EXAMPLE **A parachutist bails out from an aeroplane and after dropping through a distance of 40 m opens the parachute and decelerates at 2 ms^{-2}. If he reaches the ground with a speed of 2 ms^{-1}, how long he is in the air? At what height did he bail out from the plane?**

Solution. Time for first 40 m journey.

Now $\quad v_0 = 0$; $h = -40$ m ; $g = -9.8$ m/s^2 ; $v = ?$

$$(-v)^2 - v_0^2 = 2gh \quad \text{or} \quad v^2 - (0)^2 = 2 \times (-9.8) \times (-40)$$

$$\therefore \quad v = \sqrt{2 \times 9.8 \times 40} = 28\,\text{m/s}$$

Now $\quad v = v_0 + gt_1 \quad \therefore \quad t_1 = \dfrac{v - v_0}{g} = \dfrac{-28-0}{-9.8} = 2.9$ s

Time for second journey.

$$v_0 = -28 \text{ m/s} \; ; \; v = -2 \text{ m/s} \; ; \; a = +2 \text{ m/s}^2$$

$$v = v_0 + at_2 \quad \text{or} \quad -2 = -28 + 2t_2 \quad \therefore \quad t_2 = 13\text{s}$$

Total time t for which the parachute is in air is

$$t = t_1 + t_2 = 2.9 + 13 = 15.9 \text{ s}$$

Height from which parachute has fallen during second journey.

$$v^2 - v_0^2 = 2ah \quad \text{or} \quad (-2)^2 - (-28)^2 = 2 \times (+2) \times -h \quad \therefore \quad h = 195 \text{ m}$$

$\therefore$ Height at which the parachutist bailed out = 40 + 195 = 235 m

EXAMPLE . **Water drops fall at regular intervals from the nozzle of a shower on to the floor 3.24 m below. The first drop strikes the floor at the instant the fourth one begins to fall from the nozzle. Find the positions of the drops when a drop just strikes the floor.**

Solution. Fig. 7.14 shows the positions of the four drops *A, B, C* and *D*. When drop *A* strikes the floor, the drop *D* just beigns to fall from the nozzle. The positions of the second and third drops are mid-way between them. Let t be the time interval between two successive drops. Referring to Fig. 7.14,

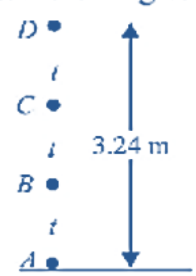


Fig. 7.14

$$CD = \frac{1}{2}gt^2 \; ; \; BD = \frac{1}{2}g\,(2t)^2 = 2gt^2 \; ; \; AD = \frac{1}{2}\,g\,(3t)^2 = 4.5\,gt^2$$

It is given that $\quad AD = 3.24$ m.

$$\therefore \quad 4.5\,gt^2 = 3.24$$

$$\text{or} \quad gt^2 = 3.24/4.5 = 0.72$$

$$\therefore \quad CD = \frac{1}{2}gt^2 = \frac{1}{2} \times 0.72 = 0.36 \text{ m}$$

$$BD = 2gt^2 = 2 \times 0.72 = 1.44 \text{ m} \; ; \quad AD = 3.24 \text{ m}$$

EXAMPLE **A person throws a ball vertically upward with an initial velocity of 15 ms^{-1}. Calculate (*i*) how high it goes and (*ii*) how long the ball is in air before it comes to his hand.**

Solution. We select earth as the origin so that $g = -9.8$ ms^{-2}.

(*i*) At the highest point, velocity is zero.

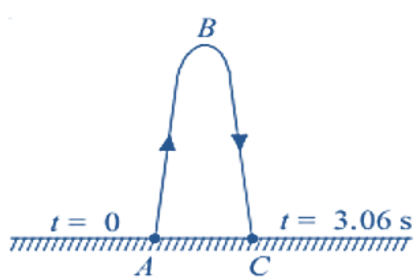


Fig. 7.12

Now $\quad v^2 - v_0^2 = 2\,g\,h$

Here $\quad v = 0$; $v_0 = +15$ ms^{-1} ; $g = -9.8$ ms^{-2}

$$\therefore \quad (0)^2 - (+15)^2 = 2 \times (-9.8) \times h$$

$$\therefore \quad \text{Maximum height, } h = \frac{-(+15)^2}{2\times(-9.8)} = 11.5 \text{ m}$$

Example 7.8. **A ball is thrown vertically upward with a velocity of 29.4 ms^{-1}. Describe the motion of the ball.**

Solution. Let us take origin at the ground. Then upward displacement and velocity will be positive and downward displacement and velocity will be negative. As g is always directed downward, $g = -9.8$ ms^{-2}.

Upward Journey. Since the ball is under constant acceleration ($= -9.8$ m s^{-2}), its velocity will decrease by 9.8 ms^{-1} every second.

Velocity of ball after 1s = 29.4 – 9.8 = 19.6 ms^{-1}

Velocity of ball after 2s = 19.6 – 9.8 = 9.8 ms^{-1}

Velocity of ball after 3s = 9.8 – 9.8 = 0

Thus the ball will reach the highest point in 3 s. At the highest point, the velocity of the ball is zero. Now the ball starts *downward journey.

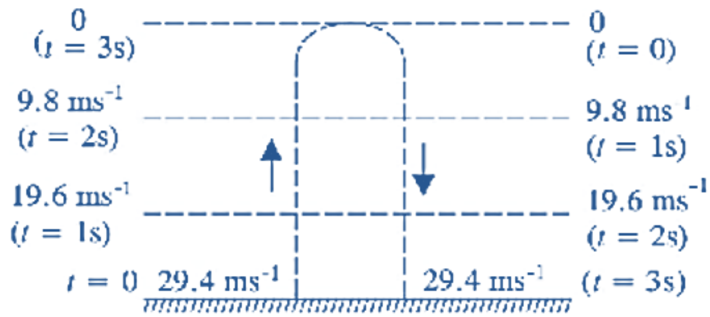


Fig. 7.11

Downward Journey. Note that during the downward journey of the ball, g is again -9.8 ms^{-2}. But displacement and velocity are negative.

Velocity after 1 s = 0 – 9.8= –9.8 ms^{-1}

Velocity after 2 s = $-(v_0 + g\,t)$ = – (9.8 + 9.8)= –19.6 ms^{-1}

Velocity after 3 s = – (19.6 + 9.8) = –29.4 ms^{-1}

Thus the ball reaches the ground in 3 s and strikes it with a velocity of 29.4 m s^{-1}.

Conclusions. Three important conclusions can be drawn about a freely falling body.

(*i*) The time taken for upward journey is equal to the time taken for the downward journey.

(*ii*) The magnitude of velocity (*i.e.* speed) with which the ball was thrown upward, it hits the ground with the same speed.

(*iii*) The magnitude of velocity of the ball at any point in its upward and downward journey is the same.

The above behaviour of a freely falling body is due to the fact that value of g is constant.

Example 7.6. **A tennis ball is dropped on to the floor from a height of 10 m. It rebounds to a height of 2.5 m. If the ball is in contact with the floor for 0.01 second, what is the average acceleration during the contact ? Take g = 10 ms^{-2}.**

Solution. Let v_1 be the velocity of the ball just before striking the floor and v_2 be the velocity of the ball just after striking the floor. Since the two velocities are acting in the opposite directions, the change in velocity $\Delta v = v_1 - (-v_2) = v_1 + v_2$ and time for the change Δt = 0.01 second.

$$\therefore \quad \text{Average acceleration, } \bar{a} = \frac{v_1 + v_2}{\Delta t}$$

$$\text{Now } v_1 = \sqrt{2gh_1} = \sqrt{2\times 10\times 10} = \sqrt{200}\text{ ms}^{-1};\ v_2 = \sqrt{2gh_2} = \sqrt{2\times 10\times 2.5} = \sqrt{50}\text{ ms}^{-1}$$

$$\therefore \quad \bar{a} = \frac{\sqrt{200}+\sqrt{50}}{0.01} = 2121\text{ ms}^{-2}$$

EXAMPLE **A car starts from rest and accelerates unifomiily at a rate of 2 ms^{-2} for 20 seconds. It then maintains a constant velocity for 10 seconds. The brakes are then applied and the car is uniformly retarded and comes to rest in 5 seconds. Draw the velocity-time graph for the motion and find (*i*) the maximum velocity (*ii*) the retardation in the last 5 seconds (*iii*) total distance travelled and (*iv*) average velocity.**

Solution. Fig. 7.10 shows the velocity-time graph for the motion of the car. The graph is *OABC*.

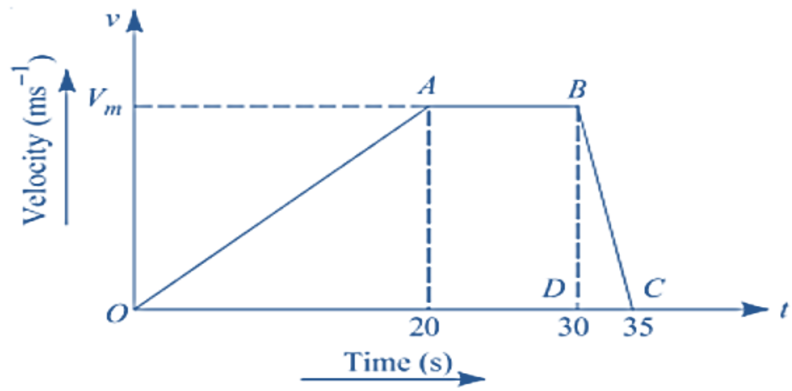


Fig. 7.10

(*i*) $a = 2\ \text{ms}^{-2}$; $v_0 = 0$; $t = 20$ s ; $v_m = ?$

$$v_m = v_0 + at = 0 + 2 \times 20 = 40\ \text{ms}^{-1}$$

(*ii*) Retardation = Slope of line $BC = -\dfrac{BD}{DC} = -\dfrac{40}{5} = -8\ \text{ms}^{-2}$

Alternatively; Retardation $= \dfrac{v - v_m}{t} = \dfrac{0-40}{5} = -8\ \text{ms}^{-2}$

(*iii*) Total distance travelled, S = Area of trapezium *OABC*

$$= \frac{1}{2}(AB + OC) \times BD$$

$$= \frac{1}{2}(10 + 35) \times 40 = 900\ \text{m}$$

(*iv*) Average velocity $= \dfrac{S}{t} = \dfrac{900}{35} = 25.71\ \text{ms}^{-1}$

EXAMPLE **A body is moving with uniform acceleration. Its velocity after 5 seconds is 25 ms^{-1} and after 8 seconds, it is 34 ms^{-1}. Calculate the distance it will travel in 12th second.**

Solution. $v = v_0 + a\,t$

For the first case : $v = 25\ \text{ms}^{-1}$; $t = 5$ s

$\therefore$ $25 = v_0 + 5a$ ... (*i*)

For the second case : $v = 34\ \text{ms}^{-1}$; $t = 8$ s

$\therefore$ $34 = v_0 + 8a$ ... (*ii*)

Solving eqs. (*i*) and (*ii*), we get, $v_0 = 10\ \text{ms}^{-1}$; $a = 3\ \text{ms}^{-2}$.

The distance travelled in the 12th second is given by ;

$$S_{12th} = v_0 + \frac{a}{2}(2n-1) = 10 + \frac{3}{2}(2 \times 12 - 1) = 44.5\ \text{m}$$

EXAMPLE **A car starts from rest and accelerates uniformly for 10 s to a velocity of 36 km/h. It then runs at constant velocity and is finally brought to rest in 50 m with a constant retardation. The total distance covered by the car is 500 m. Find the value of acceleration, retardation and total time taken.**

Solution. Fig. 7.7 shows the velocity-time graph for the motion. The car uniformly accelerates for time t_1 (= 10 s) and attains a constant velocity v (= 10 ms^{-1}). In the time inverval t_1 to t_2, it moves with this constant velocity (*i.e.*,10 ms^{-1}). In time interval t_2 to t_3, it retards uniformly and comes to stop.

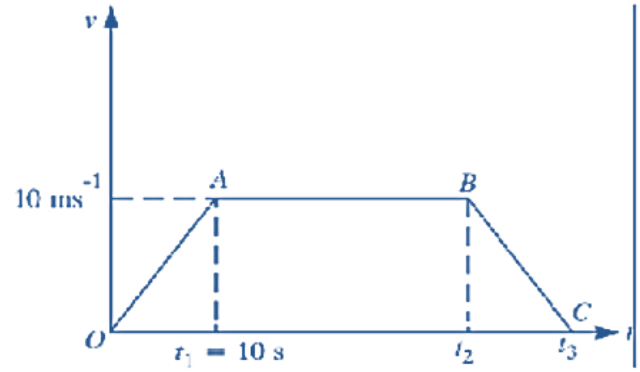


Fig. 7.7

(*i*) Constant Acceleration part (*OA*)

$v_0 = 0$; v = 36 km/h = 10 ms^{-1} ; t_1 = 10 s

$$\therefore \quad \text{Constant acceleration,} \quad a = \frac{v - v_0}{t_1} = \frac{10-0}{10} = 1\,\text{ms}^{-2}$$

$$\text{Average velocity,} \quad \bar{v} = \frac{v_0 + v}{2} = \frac{0+10}{2} = 5\ \text{ms}^{-1}$$

$$\text{Distance covered,} \quad S_1 = \bar{v}\, t_1 = 5 \times 10 = 50\ \text{m}$$

(*ii*) Constant Retardation part (*BC*)

v_0 = 10 ms^{-1} ; v = 0 ; $t_3 - t_2$ = ? ; S_3 = 50 m

$$\text{Average velocity,} \quad \bar{v} = \frac{v_0 + v}{2} = \frac{10+0}{2} = 5\,\text{ms}^{-1}$$

$$\text{Distance covered,} \quad S_3 = \bar{v}\,(t_3 - t_2)$$

$$\therefore \quad t_3 - t_2 = \frac{S_3}{\bar{v}} = \frac{50}{5} = 10\,\text{s}$$

$$\text{Retardation,} \quad a = \frac{v - v_0}{t_3 - t_2} = \frac{0-10}{10} = -1\,\text{ms}^{-2}$$

(*iii*) Constant-velocity part (*AB*)

$$\text{Distance covered,} \quad S_2 = S - S_1 - S_3 = 500 - 50 - 50 = 400\ \text{m}$$

$$\therefore \quad \text{Time taken,} \quad t_2 - t_1 = \frac{S_2}{\text{constant velocity}} = \frac{400}{10} = 40\ \text{s}$$

$$\therefore \quad \text{Total time for journey} = t_1 + (t_2 - t_1) + (t_3 - t_2)$$

$$= 10 + 40 + 10 = 60\ \text{s}$$

EXAMPLE **A body moving with uniform acceleration covers 24 m in the 4th second and 36 m in the 6th second. Calculate acceleration and initial velocity.**

Solution. The distance covered in the nth second is given by :

$$S_{nth} = v_0 + \frac{a}{2}(2n-1) \therefore\ 24 = v_0 + \frac{a}{2}(2 \times 4 - 1) \quad ...(i)$$

Also
$$36 = v_0 + \frac{a}{2}(6 \times 2 - 1) \quad ...(ii)$$

Subtracting eq. (i) from eq. (ii), we get,

$$12 = 2a \quad \therefore\ a = 12/2 = 6 \text{ ms}^{-2}$$

Putting the value of a in eq. (i), we get,

$$24 = v_0 + \frac{6}{2}(2 \times 4 - 1) \quad \therefore\ v_0 = 3 \text{ ms}^{-1}$$

CONCEPTUALS Q AND A

Q.1. When is an object said to be accelerating?

Ans. Whenever the velocity (magnitude or direction or both) of an object changes, we say that the object is accelerating. However, an object moving at a constant speed in a straight line is not accelerating because its velocity is not changing *i.e.*, magnitude as well as direction of velocity is the same.

Q.2. What do you mean by uniformly accelerated linear motion?

Ans. Uniformly accelerated linear motion means that the magnitude of the acceleration is constant and the motion is in a straight line.

Q.3. Give one most common example of uniformly accelerated linear motion.

Ans. One of the most common and practically important examples of uniformly accelerated linear motion is the vertical movement of an object in the gravitational field of the earth. The effect of gravitational attraction is to impart all unsupported bodies an equal downward acceleration of 9.8 ms^{-2} (neglecting the retarding effect of air).

Q.4. We say that acceleration due to gravity is constant only if we ignore air resistance. For what situations can air resistance be ignored?

Ans. The answer to this question depends upon several factors :

(i) If you do not require greater accuracy, you can ignore air resistance.

(ii) If the body is moving through air at a moderate speed, air resistance can be ignored. It is because the faster an object moves, the more air resistance affects its acceleration.

(iii) If the body is heavy and/or streamlined, you can ignore air resistance. It is because shape and surface area of an object greatly affect air resistance. Air resistance is much more important for a light object, such as a ping-pong ball, than for a heavier object of the same size, such as a golf ball.

Q.5. Is the value of g positive or negative?

Ans. The acceleration due to gravity always acts vertically downward whether the body is rising or falling. The value of g will be negative or positive depending upon whether we choose upward direction positive or negative. If we choose upward direction positive, g, being directed downward, is $-9.8\ ms^{-2}$. If we choose downward direction positive, then g is $+9.8\ ms^{-2}$.

Q.6. If a body moves with constant acceleration, what is the average velocity?

Ans. If a body moves with constant acceleration, the velocity-time graph is a straight line: the slope of graph = constant acceleration. Therefore, the average velocity lies midway between the initial velocity (v_0) and the final velocity (v) *i.e.*,

$$\bar{v} = \frac{v_0 + v}{2}$$

Note that it is true *only* for constant acceleration.

Q.7. What is a freely falling body?

Ans. When an object is under the action of gravity alone, it is called a freely falling body. The presence of other forces, in particular air resistance, may alter dramatically the statement that all objects fall with the same constant acceleration g ($= 9.8\ m\ s^{-2}$).

Q.8. The position of a stone dropped from rest from a cliff is given by $x = 4.9\ t^2$ where x is in metres and t is in seconds. What is the acceleration of the stone?

Ans. $$x = 4.9\ t^2 \quad \text{or} \quad \frac{dx}{dt} = 2(4.9)t = 9.8t$$

or $$\frac{d^2x}{dt^2} = 9.8\ ms^{-2} \quad \text{or} \quad a = 9.8\ ms^{-2}$$

We arrive at an important conclusion that if the displacement of a body varies as the square of elapsed time, the acceleration of the body is constant.

Q.9. In one dimensional motion, can a particle have zero speed at an instant but non-zero acceleration at that instant?

Ans. The answer is yes. For example, when a body starts falling freely, its speed is zero but acceleration is $9.8\ ms^{-2}$.

Q.10. In the position-time ($x - t$) graph shown in Fig. 7.16, what is the acceleration?

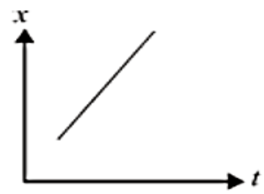


Fig. 7.16

Ans. The $x - t$ graph is a straight line. Therefore, slope is constant. This means that velocity of the body is constant. Therefore, acceleration is zero.

Q.11. The distance travelled by a body is found to be directly proportional to time. Is the body moving with uniform velocity?

Ans. $$s \propto t \quad \text{or} \quad s = k\ t \quad \therefore \quad v = \frac{ds}{dt} = k \text{ (constant)}$$

Therefore, the body is moving with uniform velocity.

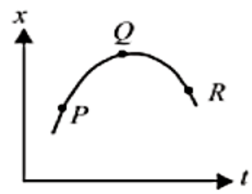


Fig. 7.17

Q.12. Fig. 7.17 shows $x - t$ graph for a moving body. At what point velocity is zero?

Ans. The velocity at any point on the graph is the slope of the tangent line at that point. The slope of the tangent line at Q is zero. Therefore, velocity is zero at point Q.

VERY SHORT ANSWER QUESTIONS

Q.1. A ball is thrown straight up. What is its velocity and acceleration at the top?

Ans. Velocity at top = 0 ; Acceleration at top = 9.8 ms^{-2} downwards.

Q.2. Can a body have zero velocity and still be accelerating?

Ans. Yes. In case of a body projected vertically upwards, the velocity at the highest point is zero but acceleration is g (= 9.8 ms^{-2}) downwards.

Q.3. A body is thrown vertically upwards with a velocity of 30 m/s. What is its velocity when it hits the ground?

Ans. 30 m/s because the velocity with which the body hits the ground is equal to the velocity with which it is thrown up. The reason for this is that acceleration due to gravity g(= 9.8 ms^{-2}) remains constant during upward or downward journey.

Q.4. Two balls of different masses (one lighter and other heavier) are thrown vertically upward with the same initial speed. Which one will rise to greater height?

Ans. Both the balls will rise to the same height. It is because in the formula $v^2 - v_0^2 = 2gh$, no mass term is involved.

Q.5. What does the area under acceleration-time graph for any time interval represent?

Ans. Area under $a - t$ graph = $\frac{1}{2}at = \frac{1}{2} \times \frac{v - v_0}{t} \times t = \frac{v - v_0}{2}$.

Therefore, area under $a - t$ graph during time interival t represents change in velocity during that time interval.

Q.6. If the acceleration of a particle is constant in magnitude but not in direction, what type of path does the particle follow?

Ans. It will follow a non-linear path.

Q.7. A ball is thrown vertically upwards and takes 5 s to reach the highest point. What is the time for downward journey?

Ans. 5 s because the value of g (= 9.8 ms^{-2} downwards) is constant for upward and downward journey.

Q.8. A man standing on the edge of a cliff throws a stone straight up with initial velocity u and then throws another stone straight down with same initial speed u and from the same position. Find the ratio of the speeds, the two stones would have attained when they hit ground at the base of the cliff.

Ans. When the stone is thrown up with speed u, it will return to the thrower with speed u. Since both stones are falling under gravity with the same initial speed u, they will hit the ground with the same speed. Hence the ratio of the speeds of the two stones will be 1 : 1.

Q.9. A stone is thrown vertically upward from the ground at 30 ms^{-1}. What is the time of flight?

Ans. t = 6.1s (Refer to Example 7.10).

Q.10. The initial and final velocities of a uniformly accelerated body are 20 ms^{-1} and 60 ms^{-1}. What is the average velocity?

Ans. Average velocity = $\frac{\text{Initial vel.} + \text{Final vel.}}{2} = \frac{20 + 60}{2} = 40 \text{ ms}^{-1}$

Q.11. How will you find the distance covered by a uniformly accelerated body from its $v-t$ graph?

Ans. The distance covered by a body in a given interval of time is equal to the *total area* (positive and negative) under v–t graph during that time interval. Note that negative area (due to negative velocity) is also taken as positive.

Note. The displacement of a body during a given time interval is the *net area* (with due regard to sign to area) under v–t graph during that interval.

Q.12. A car starting from rest has an aceleration of 5 ms^{-2}. How fast will it be after 10 s?

Ans. $v = v_0 + at = 0 + 5 \times 10 = 50 \text{ ms}^{-1}$

Q.13. A body is thrown up vertically with velocity *u*. How high will it rise?

Ans. $v^2 - u^2 = 2gh$ or $0^2 - u^2 = 2(-g)h$ $\therefore$ $h = u^2/2g$.

Q.14. Velocity and acceleration are interlinked but is it possible that one is zero while other is not?

Ans. Yes it is possible. A body thrown vertically upward has zero velocity at the highest point but it has acceleration g (= 9.8 ms^{-2} downwards). On the other hand, acceleration is zero for motion with uniform velocity.

Q.15. A body is thrown vertically upward from the surface of earth. What is the direction of acceleration of the body?

Ans. The acceleration is g (= 9.8 ms^{-2}) acting vertically downward whether the body is going upward or coming downward.

Q.16. What is the unique characteristic of uniformly accelerated motion?

Ans. In case of uniformly accelerated motion (*and only for this case*), the average velocity is equal to one-half of sum of initial and final velocities.

Q.17. What is the ratio of the distances travelled by a body falling freely from rest during first, second and third second of its fall?

Ans. $S_{nth} = v_0 + \frac{a}{2}(2n-1) = 0 + \frac{g}{2}(2n-1) = \frac{g}{2}(2n-1)$

$$\therefore\ S_1 : S_2 : S_3 = \frac{g}{2}(2\times 1-1) : \frac{g}{2}(2\times 2-1) : \frac{g}{2}(2\times 3-1) = 1 : 3 : 5$$

Q.18. Can you cite an example where a body subjected to uniform acceleration does not move in a straight line?

Ans. A body subjected to uniform acceleration in one dimensional motion moves in a straight line. However, the projectile is subjected to uniform acceleration and has a parabolic path (motion in a plane).

Q.19. What is the acceleration of a body when its $v-t$ graph is (*i*) perpendicular to time-axis (*ii*) parallel to time-axis?

Ans. (*i*) Inifinity (*ii*) zero.

Q.20. A stone is thrown vertically upward with an initial velocity of 14 ms^{-1}. What is the maximum height reached? (g = 9.8 ms^{-2})

Ans. $h = v_0^2/2g = 14^2/2 \times 9.8 = 10$ m.

SHORT ANSWER QUESTIONS

Q.1. What are the three main characteristics of a freely falling body?

Ans. A body that moves under gravity alone is called a freely falling body.

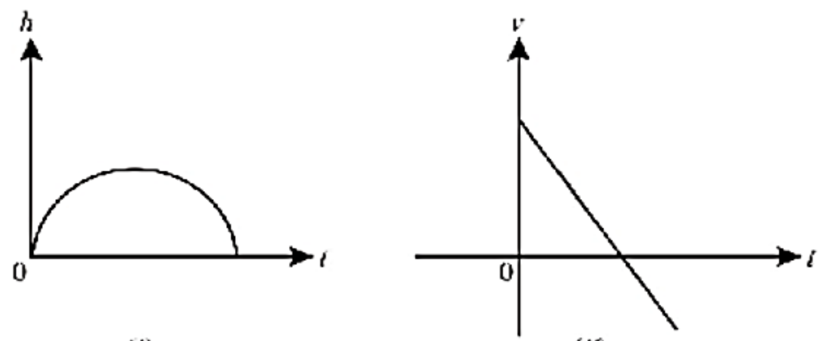
(*i*) Time taken for upward journey is equal to that for downward journey.

(*ii*) The speed with which body is thrown upward, it hits the ground with the same speed.

(*iii*) The speed of the body at any point in its upward and downward journey is the same.

Q.2. A ball is thrown vertically upwards. Draw its height-time and velocity-time graph.

Ans. The $h - t$ graph for the motion is a parabola and is shown in Fig. 7.18 (*i*). The $v - t$ graph for the motion is a straight line [See Fig. 7.18 (*ii*)] because acceleration is constant during the entire motion of the body.



(*i*) (*ii*)

Fig. 7.18

Q.3. A ball is thrown vertically upward from the ground at 30 ms^{-1}. What is the time of flight?

Ans. The displacement $h = 0$ when the ball strikes the ground.

Now $h = v_0 t + \frac{1}{2} g t^2$ or $0 = 30 \times t - \frac{1}{2} \times 9.8\ t^2$ $\therefore$ $t = 6.1$ s

Q.4. Two balls of different masses (one lighter and other heavier) are thrown vertically upwards with the same speed. Which one will pass through the point of projection in their downward journey with greater speed?

Ans. Suppose v_0 is the velocity of projection of the ball and v is the velocity of the ball while passing downward through the point of projection. Since displacement $h = 0$,

$\therefore$ $v^2 - v_0^2 = 2gh$ or $v^2 - v_0^2 = 2g \times 0$ $\therefore$ $v = v_0$

Hence both the balls will have the same speed while passing through the point of projection in their downward journey. Alternatively, there is no mass term in the equation of motion.

Q.5. For a particle in one dimensional motion having uniform acceleration, the instantaneous speed is always equal to the magnitude of instantaneous velocity. Why?

Ans. In one dimensional motion having uniform acceleration, the direction of velocity does not change; only the magnitude of velocity changes (*i.e.* speed). Therefore, the instantaneous speed is always equal to the magnitude of instantaneous velocity of the particle in one dimensional (*i.e.*, in a straight line) motion.

Q.6. What will be the distance moved by a freely falling body from rest in the 5th second of its journey?

Ans. $S_{nth} = v_0 + \frac{a}{2}(2n-1)$. Here $v_0 = 0$; $a = 9.8\text{ms}^{-2}$ and $n = 5$.

$\therefore$ $S_5 = 0 + \frac{9.8}{2}(2 \times 5 - 1) = 4.9\ (9) = 44.1$ m

Q.7. What is the magnitude and direction of acceleration due to gravity?

Ans. A freely falling body always has an acceleration of 9.8 ms^{-2} downward. It does not matter whether the body is rising or falling ; its acceleration is still g (= 9.8 ms^{-2}) downward.

Q.8. The position of a particle moving along a straight line is given by : $x = 2 - 5t + 6t^2$. What is the acceleration of the particle at t = 2s ?

Ans. a = 12 (Refer to Example 6.22).

Q.9. The acceleration due to gravity is 9.8 ms^{-2}. What does it mean?

Ans. It means that when the motion of a body is under gravity *alone*, its speed decreases at the rate of 9.8 ms^{-1} every second during upward motion. During downward journey, the speed of the body increases at the rate of 9.8 ms^{-1} every second until it hits the ground.

Q.10. A body starts from rest and has a uniform acceleration a. What will be the shape of its (*i*) $x - t$ graph (*ii*) $v - t$ graph (*iii*) $a - t$ graph?

Ans. (*i*) Parabola (*ii*) It will be a straight line passing through the origin (*iii*) It will be a straight line parallel to the time-axis.

Q.11. A body is dropped from a height h. In how much time will it reach the ground?

Ans. $h = v_0 t + \frac{1}{2}at^2$. Here $v_0 = 0$; $a = g$.

$\therefore\ h = 0 \times t + \frac{1}{2}gt^2$ or $t = \sqrt{\frac{2h}{g}}$

Q.12. The velocity of a particle in ms^{-1} moving along a straight line is given by : $v = 10 + 3t^2$. What is instantaneous acceleration at t = 4s ?

Ans. $a = \frac{dv}{dt} = \frac{d}{dt}\left(10+3t^2\right) = 6t$. Therefore, at t = 4s ; $a = 6 \times 4 = 24\ ms^{-2}$.

Q.13. Why does time occur twice in the unit of acceleration?

Ans. Acceleration $= \frac{\text{Change in velocity}}{\text{Time}} = \frac{\text{Displacement/Time}}{\text{Time}} = \frac{\text{Displacement}}{(\text{Time})^2}$

Q.14. A food packet is released from a helicopter which is rising steadily at 2 ms^{-1}. What is the velocity of the packet after 2 seconds? (Take g = 10 ms^{-2}).

Ans. $v = v_0 + gt$. Taking upward direction positive, we have, $v_0 = 2\ ms^{-1}$; $g = -10\ ms^{-2}$. Therefore, $v = 2 - 10 \times 2 = -18\ ms^{-1}$. Hence the packet is falling downward with a velocity of 18 ms^{-1} after 2 s.

Q.15. Can you round a curve with zero acceleration?

Ans. No. It is because when you round a curve, the direction of motion is changing from instant to instant. This is possible only by a force and hence by an acceleration.

Q.16. Is the rate of change of acceleration with time important to mechanics? Comment.

Ans. The rate of change of acceleration with time is not important to mechanics. It is because the laws of motion involve acceleration only and to solve problems relating to motion, we do not require rate of change of acceleration with time.

Q.17. A car travelling at 36 km/h takes a U turn in 5 seconds. What is the acceleration of the car?

Ans. Change in velocity in 5 seconds, $\Delta v = 36 - (-36) = 72$ km/h = 20 ms^{-1}

$\therefore$ Acceleration of car, $a = \Delta v/(\Delta t) = 20/5 = 4\ ms^{-2}$.