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HEATING EFFECT OF CURRENT

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HEATING EFFECT OF CURRENT

✪ Some important effects of current are:

- 1) Heating effect
- II) Chemical effect
- II) Thermo electric effect
- IV) Magnetic effect

✪ HEATING EFFECT OF CURRENT & JOULES LAW

When an electric current through a conducting wire, the electric energy converted in to heat energy. This effect is known as heading effect of current.

Electric bulb, Heater, Toaster, Geyser are some of electric appliances based on the heating effect of current.

"The heating effect of a conductor by the flow of an electric current through it is called **Joule Heating**".

Joule stated that when a current "I" is made to flow through a conductor of resistance "R" for time "t" and Heat "Q" is produced such that $Q = I^2Rt$ → **Joule's law of Heating**

It is clear that the amount of heat developed in a conductor by the passage of steady electric current through it is proportional to.............

- **I) The square of the electric current (I)**
- **II) The resistance of conductors (R)**
- **III) The time for which events follow (I)**

☑ CAUSE OF HEATING EFFECT OF CURRENT

When a battery is connected to the ends of a conductor, an electric field is up in the conductor. The large no. of free electrons present in the conductor gets accelerated towards the positive end i.e., in a direction opposite to the electric field and acquire extra kinetic energy in addition to their own K. E (due to their thermal motion). Due to which electric current flows through the conductor. These accelerated electron on their way suffer frequent collisions with the ions or atoms of the conductor and transfer their gained (extra) KE to them. As a result, average KE of vibration of ions or atoms of the conductor, rises i.e ***amplitude of vibration of ions & atoms increases***.

So, the number collision between free electrons and ions increases. thus, more work is done to carry free electrons from one end of the conductor to the other end. This ***work done is converted into heat energy.***

☑ HEAT PRODUCED BY ELETRIC CURRENT: (Derivation of joule's law).

The heat produced in a conductor depends upon the square of the current flowing through its residence of the conductor i.e., time for which the current follows through it.

i.e. $Q = H = I^2 Rt$

Consider a conductor AB of resistance R.

Let, P.d applied across AB = V

Current following through the conductor = I

& Time for which the current following = 't'

∴ total change flowing through AB in time t_1, q = It

We know, $\text{P.d} = \frac{\text{work done}}{\text{charge}}$

=> $V = \frac{W}{q}$

∴ work done in carrying a charge a from A to B. $W = V \times q = V I t$ [∵ q = I t]

$= (IR) \times I t$ [∵ V = I R]

$= I^2 Rt$

This work done is equal to the heat produced.

∴ **$Q \text{ (or H)} = W = I^2 Rt$** **$H = I^2RT$ Joule** $H = \frac{I^2Rt}{4.18}$ Calorie [1 Cal = 4.18 J]

- **Law of electric current** => H α I^2 (keeping R & t constant)
- **Law of resistance** => H α R (keeping I & t constant)
- **Law of time** => H α t (keeping R & I constant)

Different form of Heat energy

$$H = VIt = I^2Rt = (V/R)^2\,Rt = V^2/R^2 \times Rt = V^2/R\ t.$$

Heat produced in Series combination

$H_S = (V^2/(2R))\,t$ ∵ $R_1 = R + R = 2R$

Heat produced in parallel combination

$H_P = 2V^2/R\ t$

$H_P = 4H_S$ [∵ $R_P = \frac{R\times R}{R+R} = R/2$]

☞ The rate of production of heat in a conductor is not the same thing as the rate of increasing temperature of conductor

Explanation: The rate of increase of temperature depends on the heat capacity of the conductor. It also depends upon the rate at which heat can escapes from the conductor by conduction, convection or radiation. The rate of loss of heat increase as the temperature of the conductor increase. The temperature of a current carrying conductor continuous to rise till the rate of production of heat is equal to the rate of loss of heat after which the temperature remains constants. Thus, when the circuit closed through a conductor (say lamp). There is a rapid rise in the temp^r of the filament until the rate of loss of heat (mainly by radiation) equals to the rate of production of Heat i.e., I^2 Rt.

▶ However a fuse is constructed so that when the current in it exceeds a certain predetermined value, the fuse melts before is final equilibrium temperature can be attained.

☑ ELECTRIC POWER

"The rate at which work is done by the source of emf in maintaining the current in electronic circuit is called electric power of the circuit."

☞(The electric power of an electric circuit is the **rate of consumption of energy**).

Energy consumed in time "t", W = VIt

Electric power= $\frac{\text{work done}}{T}$ = VI t / t = V I

i.e., **P = VI** Joule's / sec.

or **P = VI** ———— (1)

again, P = VI =(IR) I

∴ **$P = I^2R$** ———— (2) (in terms of I & R)

also $P = V^2/R^2 \times R$ [∵ I = V/R]

$P = V^2/R$ ———— (3) (in terms of V and R)

from (1), (2) & (3)

$P = VI = I^2R = V^2/R^2$

from (1) P = V I

(Watt) = volt × Ampere.

Now, V = 1 volt. & I = 1 A.

∴ P (watt) = 1 volt × 1 Amp.

* **1 Watt = "Power of an electric circuit is said to be watt if IA current flows in it against a p.d of 1 volt."**

▶ **Bigger unit of power** (1) 1 kilo watt (1 kW) = 1000W
(2) Mega Walt (M W) = 10^6w

▶ **Commercial unit** Horse power (HP) = 746 Walt

* **Power in kilo Walt = V (in volt) × I (in amp) / 1000**

➢ Electric energy:

"Electric energy is the total work done by an electric current in given time."

In other's words,

"The total work done or energy supplied by the source of emf in maintaining the current in an electric circuit for a given time is called electric energy consumed in the circuit."

Electric energy, W = V I t
= P t

Thus,

The total amount of electrical energy consumed by an electric circuit **depends upon its electric power** and **the time for which the power is used.**

➢ **S . I Unit**: Joule.

1 Joule = 1 volt × 1 amp × sec
= 1 walt × sec

➢ **Commercial unit** : **Kilo watt hour (kwh) or BOTO (Board of trade unit).**

"1 kwh is defined as the amount of work done when a power 1 kilowatt is consumed for 1 hour "

i.e.,1 kwh = 1 kw × 1 h
= 1000 w × 3600 sec
= 3.6×10^6 J/~~S~~ × ~~S~~
= 3.6×10^6 Joule
= 3.6×10^{15} erg (since. 1 J = 10^7 erg)

➢ **Other Relation of Electric Energy**:

$$E = W = I^2 Rt$$

$$E = W = V^2t/R$$

➢ **Relation Between Electric Energy & Electric Power**:

E = V I t
P = V I
∴ E/P = V I t / V I

P = E / t

➢ Some Important Aspects & Applications of Heating Effects of currents:

1) Incandescent Electric Bulbs

It is based on the fact that when a metal having high melting points is heated to a high temp. It becomes White hot i.e., incandescent.

A common electric bulb has tungsten filaments placed in an inert gas (like N_2 or Ar) and enclosed by a glass bulb. Tungsten has a large value of resistivity & high melting point 13300 K. When current is passed through it. It gets heated up to a high temp. which makes it glow (at ≃ 2700k) and limits light immediately.

☞If the resistances are connecting in series the current, I' is same through each resistor, then.

P α R & V α R (as V = IR)

=> It means, in series combination, the power consumed will be more in higher resistance.

☞ If the resistances are connected in parallel the P.d (V) is same across each resister then.

P α *1*/ R

and I α *1*/R (as V = *I*R)

=> It means in parallel connections the current and power consumed will be more as smaller resistance.

✪ **In parallel grouping of bulbs**, higher power (or wattage) will give more bright light and will pass greater current through it. It will have lesser resistance and same Pd across it. (it ***one bulb gets fused other bulbs will work***.)

✪ **In series grouping of bulbs**, the bulb of higher power (or voltage) will give less bright light and will have smaller resistance & P.d across it (if ***one bulb gets fused another bulb will not work***.)

(i) In case of incandescent lamp.

since, P = V I

i.e., current drawn by an electric bulb is directly proportional to its power.

Again,

since, $P = V^2 / R$

i.e., Resistance of the filament of an electric bulb is inversely proportional to its power.

(ii) Electric Iron, Electric heater, Electric oven, Immersion rod, geyser etc.

In these appliances, the heating element is made from Nichrome (high Resistivity = 100×10^{-8} m at 0^0 C). High melting point 3300 k, high malleability Nichrome does not get oxidise, Nichrome wire acquires steady state when red hot (at 800^0C) Its temp. coefficient of Resistivity = $0.4 \times 10^{-3} k^{-1}$

Now, as electric power, P = V I

∴ for given voltage V, P α I

➢ Higher is the power of electric appliances the larger is the current drawn by it.

Since, heat produced H α I^2

∴ heat produced due to current is high in both of them.

=> since, $P = V^2/R$ for given voltage, P α 1/R.

Resistance of high electric power instrument is smaller than that of low electric power.

=> The heater wire must be of high Resistivity & of high melting point as.

$H = V^2t / R = V^2 A t / \rho l$

i.e., **l = Small ; ρ= high**.

(iii) The current supply wires are not heated up out the filament of the lamp becomes white hot

We know that heat produced H α R. The filament of lamp and the current supply wire are in series. so, same current flows through the wires and the filament. But the resistance of the wires is very small as compared to the resistance of the filament. So, the current supply wires are not heated up but the filament by the lamp becomes very hot.

✪(iv) Fuse Wire: Electric fuse is a protective device used in series with an electric circuit or an electric appliance to wire. It from damage due to overheating wherever strong current passed through the circuit.

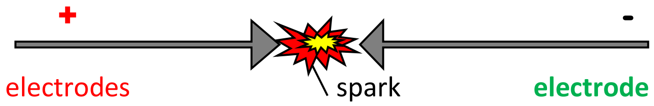
➢ A fuse is prepared from tin lead alloy (63% in +37 % lead), High resistance, low melting point.

As soon as the safe limit of current exceeds due to some fault, the excessive heat produced in the fuse melts it & The current supply to the electric circuit is cut off.

✪ (v) Carbon arc light of film projector

If high P.d is applied across two electrodes having smaller gap between them, a spark occurs in the air gap.
In film projector an arc is produced in gap between carbon electrodes. This arc is used to produce light which

is used to project film in cinema hall.



☑ Current rating of fuse (safe limit of current)

Consider a wire,

length = l, radius = r, Resistivity = ρ, current following through the wire = I

Rate of heat produced in wire, power, P^2R.

But, $R = \rho L/A = \rho L/\pi r^2$

$$\therefore \quad P = I^2 (\rho L / \pi r^2)$$

This heat increases the temperature of wire at the same time due to radiation, heat also lost to the surrounding of the wire Which decreases the temperature of fuse wire. The temp, of fuse wire becomes constant when the heat lost per second from the surface of the wire is equal to the rate at which the heat produced in wire.

let "H" be the amount of heat lost per sec. from unit area of the surface of the wire.

.'. Heat lost / second from the surface of wire,

$P' = H \times$ surface area of wire

$= H \times 2\pi rL$

At steady rate (equilibrium)

$$P' = P$$

$$H \times 2\pi r \cancel{L} = \frac{I^2 (\rho \cancel{L})}{\pi^2 r}$$

$$I^2 = \frac{2\pi^2 r^3 H}{\rho}$$

$$I = \left(\frac{2\pi^2 r^3}{\rho}\right)^{1/2}$$

$\because \quad \frac{2\pi^2}{\rho}$ = (constant)

$$I \propto r^{3/2}$$

Where I = safe limit of current.

➢ EFFCIENCY OF AN ELECTRIC DEVICE (η):

"Efficiency of an electric device is defined as the Ratio of its output power to the input power".

$$\eta = \frac{\text{output power}}{\text{input power}}$$

In case of an electric power $\eta = \frac{\text{Output mechanical power}}{\text{input mechanical power}}$

Here, input power = output mechanical power × power lost in Heat

➢ MAXIMUM POWER TRANSFER THEOREM:-

"It states that the power output across load due to a cell or battery is maximum if the load resistance is equal to the effective internal resistance of cell or battery."

=>It simply means, when the effective internal resistance of cell or battery is equal to external load resistance in a circuit, the efficiency of a battery or cell is maximum.

➢ POWER RATING OF A RESISTOR: -

Let us consider a pure resistance R through which current I is flowing. The potential difference the terminals a & b is given by

$V_{ab} = IR$ or $P = I^2R = \frac{V^2}{R}$

The potential at a is necessarily higher than at *b* and there is a power input to the resistor. The circulating charges give up energy to the atoms of the resistor when they collide with them. The temperature of the resistor increases unless there is a flow of heat of the resistor. We say that energy is dissipated in the resistor of a rate I^2R. Because of this heat, every resistor has a **maximum power rating. It is the maximum power that can be dissipated without overheating the device.** When When this rating is exceeded, the resistance may change unpredictably. In more extreme cases, the resistor may melt or even explode. In practical applications, the power rating of a resistor is just as important a characteristic as its resistance value.

KNOWLEDGE Plus :Conceptuals :

- The equation: W = VIt is applicable to the conversion of electrical energy into any other form, by the equation: $H = I^2Rt$ is applicable only to the conversion of electrical energy into heat energy in an ohmic resistor.
- Joule's law of heating holds good even for a.c. circuits. Only current and voltage have to be replaced by their rms values.
- If the circuit is purely resistive, the energy expanded by the source entirely appears as heat. But if the circuit has an active element like a motor, then a part of the energy supplied by the source goes to do useful work and the rest appears as heat.
- The emission of light by a substance when heated to a high temperature is called incandescence.
- A heater wire is made from a material of large resistivity and high melting point while a fuse wire is made from a material of large resistivity and low melting point.
- The load in an electric circuit refers to the current drawn by the circuit from the supply line. If the current in a circuit exceeds the safe value, we say that the circuit is overloaded.
- The temperature upto which a wire gets heated (i.e., steady state temperature θ) is directly proportional to the square of the current and is inversely proportional to the cube of its radius but is independent of its length.

$$\theta \propto \frac{I^2}{r^3}$$

- When the resistances are connected in series, the current I through each resistance is same. Consequently,

$$P \propto R \qquad (\because P = I^2R)$$

and $$V \propto R \qquad (\because V = IR)$$

Hence in a series combination of resistance, the potential difference, power consumed and hence heat produced will be larger in the higher resistance.

- When the resistances are connected in parallel, the potential difference, V is same across each resistance. Consequently,

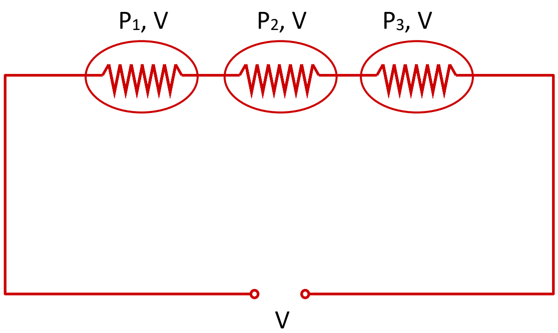
$$P \propto \frac{1}{R} \qquad \left(\because P = \frac{V^2}{R}\right)$$

And, $$I \propto 1 \qquad \left(\because I = \frac{V}{R}\right)$$

Hence in a parallel combination of resistances, the current, power consumed and hence heat produced will be larger in the smaller resistance.

ILLUSTRATIVE EXAMPLES 01 Prove that the reciprocal of the total power consumed by a series combination of appliances is equal to the sum of the reciprocals of the individual powers of the appliances.

As shown in Fig. consider a series combination of three bulbs of powers P_1, P_2 and P_3; which have been manufactured for working on the same voltage V.



[Series combination of bulbs]

The resistance of the three bulbs will be

$$R_1 = \frac{V^2}{P_1},\ R_2 = \frac{V^2}{P_2},\ R^2 = \frac{V^2}{P_3}$$

As the bulbs are connected in series, so their equivalent resistance is

$R = R_1 + R_2 + R_3$

If P is the effective power of the combination, then

$\frac{V^2}{P} = \frac{V^2}{P_1} + \frac{V^2}{P_2} + \frac{V^2}{P_3}$ or $\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}$

Thus, for a series combination of appliances, the reciprocal of the effective power is equal to the sum of the reciprocals of the individual powers of the appliances.

Clearly, when N bulbs of same power P are connected in series,

$P_{eff} = \frac{P}{N}$

As the bulbs are connected in series, the current I through each bulb will be same.

$I = \frac{V}{R_1 + R_2 + R_3}$

The brightness of the three bulbs will be

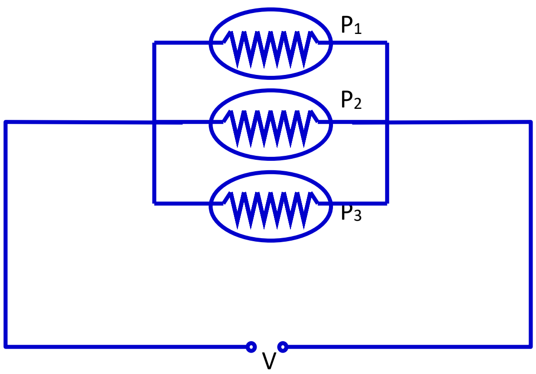
$P'_1 = I^2R_1$, $P'_2 = I^2R_2$, $P'_3 = I^2R_3$

As $R \propto \frac{1}{P}$, the bulb of lowest wattage (power) will have maximum resistance and it will glow with maximum brightness. When

the current in the circuit exceeds the safety limit, the bulb of lowest wattage will be fused first.

ILLUSTRATIVE EXAMPLES 02 **Prove that when electrical appliances are connected in parallel, the total power consumed is equal to the sum of the powers of the individual appliances.**

Power consumed by a parallel combination of appliances. As shown in Fig., consider a parallel combination of three bulbs of powers P_1, P_2 and P_3, which have been manufactured for working on the same voltage V.



[Parallel combination of bulbs]

The resistances of the three bulbs will be

$R_1 = \frac{V^2}{P_1}$, $R_2 = \frac{V^2}{P_2}$, $R_3 = \frac{V^2}{P_3}$

As the bulbs are connected in parallel, their effective resistance R is given by

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Multiplying both sides by V^2, we get

$\frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^2}{R_3}$

or $P = P_1 + P_2 + P_3$

Thus, for a parallel combination of appliances, the effective power is equal to the sum of the powers of the individual appliances.

If N bulbs, each of power P, are connected in parallel, then

$P_{eff} = NP$

The brightness of the three bulbs will be

$P_1 = \frac{V^2}{R_1}$, $P^2 = \frac{V^2}{R_2}$, $P_3 = \frac{V^2}{R_3}$

As the resistance of the highest wattage (power) bulb is minimum, it will glow with maximum brightness. If the current in the circuit exceeds the safety limit, the bulb with maximum wattage will be fused first. For this reason, the appliances in houses are connected in parallel.

Efficiency of a source of emf: The efficiency of a source of emf is defined as the ratio of the output power to the input power. Suppose a source of emf $\mathcal{E}$ and internal resistance r is connected to an external resistance R. Then its efficiency will be

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{VI}{\mathcal{E}I} = \frac{V}{\mathcal{E}} = \frac{IR}{I(R+r)}$$

or $$\eta = \frac{R}{R+r}$$

ILLUSTRATIVE EXAMPLES 03 **(a) A battery of emf $\mathcal{E}$ and internal resistance r is connected across a pure resistive device (e.g., an electric heater or an electric bulb) of resistance R. Show that the power output of the device is maximum when there is a perfect 'matching' between the external resistance and the source resistance (i.e., where R = r). Determine the maximum power output.**
(b) What is power output of the source above if the battery is shorted? What is the power dissipation inside the battery in that case?

Maximum power theorem: It states that the output power of a source of emf is maximum when the external resistance in the circuit is equal to the internal resistance of the source.

Let emf of the battery = $\mathcal{E}$
Internal resistance = r
Resistance of the device = R

∴ Current through device,

$$I = \frac{\text{Total emf}}{\text{Total resistance}} = \frac{\mathcal{E}}{R+r}$$

∴ Power output of the resistive device will be

$$P = I^2 R = \left(\frac{\mathcal{E}}{R+r}\right)^2 R$$

$$= \frac{\mathcal{E}^2 R}{(R+r)^2} = \frac{\mathcal{E}^2 R}{(R-r)^2 + 4Rr} \quad \text{... (i)}$$

Obviously, the power output will be maximum when

R – r = 0 or R = r

Thus, the power output of the device is maximum when there is perfect matching between the external resistance and the resistance of the source, i.e, when R = r. This proves maximum power theorem.

Maximum power output of the source is

$$P_{max} = \frac{\mathcal{E}^2 r}{(r+r)^2} = \frac{\mathcal{E}^2}{r} \quad \text{[Putting R = r in Eq. (i)]}$$

(b) When the battery is shorted, R becomes zero, therefore, power output = 0. In the case, entire power of battery is dissipated as heat inside the battery due to its internal resistance.

Power dissipation inside the battery

$$= I^2 r = \left(\frac{\mathcal{E}}{r}\right)^2 r = \frac{\mathcal{E}^2}{r}$$

ILLUSTRATIVE EXAMPLES 04 **Show that the efficiency of a battery when delivering maximum power is only 50%.**

Maximum efficiency of a source of emf. For a source of emf,

Input power = $\mathcal{E}I$
Output power = VI

∴ Efficiency $$\eta = \frac{VI}{\mathcal{E}I} = \frac{V}{\mathcal{E}} = \frac{IR}{I(R+r)} = \frac{R}{R+r}$$

When the source delivers maximum power, R = r

∴ $$\eta = \frac{r}{r+r} = \frac{1}{2} = 50\,\%$$

Thus, the efficiency of a source of emf is just 50% when it is delivering maximum power.

ILLUSTRATIVE EXAMPLES 05 **Define efficiency of an electric device. Write an expression for the efficiency of an electric motor.**

Efficiency of an electric device: The efficiency of an electric device is defined as the ratio of the output power to the input power

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

For an electric motor, we can write

$$\eta = \frac{\text{Output mechanical power}}{\text{Input electric power}}$$

Here, input electric power = Output mechanical power + Power lost as heat

ILLUSTRATIVE EXAMPLES 06 **(a) An electric motor runs on a d.c. source of emf $\mathcal{E}$ and internal resistance r. Show that the power output of the source is maximum when the current drawn by the motor is $\mathcal{E}/2r$.**

(b) Show that power output of electric motor is maximum when the back emf is one-half the source emf, provided the resistance of the windings of the motor is negligible.

(a) Output power from a source connected to an electric motor: Let the current drawn by the motor by I. Then

Power output of the source, $P = \mathcal{E}I - I^2 r$

P is maximum when $\frac{dP}{dI} = 0$

or $\mathcal{E} - 2Ir = 0$ or $I = \frac{\mathcal{E}}{2r}$

Hence the power output of the source is maximum when the current drawn by the motor is $\mathcal{E}/2r$.

(b) Here, emf of source = $\mathcal{E}$

Internal resistance of source = r

Back emf of motor = $\mathcal{E}'$

Resistor of motor = $R \simeq 0$

As the external resistance R is negligible,

therefore, current in the circuit $= \frac{\mathcal{E} - \mathcal{E}'}{r}$

And power output of the motor = Power output of the source = $\mathcal{E}I - I^2 r$

From part (a), this is maximum when

$$I = \frac{\mathcal{E}}{2r} \quad \text{or} \quad \frac{\mathcal{E} - \mathcal{E}'}{r} = \frac{\mathcal{E}}{2r} \quad \text{or} \quad \mathcal{E}' = \frac{\mathcal{E}}{2}$$

Hence the power output of electric motor is maximum when the back emf is one-half the source emf.

ILLUSTRATIVE EXAMPLES 04 **Explain why electric power is transmitted at voltages and low currents to distant places.**

High voltage power transmission: Electric power is transmitted from power stations to homes and factories through transmission cables. These cables have resistance. Power is wasted in them as heat. Let us see how can we minimise this power loss.

Suppose power P is delivered to a load R via transmission cables of resistance R, If V is the voltage across load R and I the current through it, then

$$P = VI$$

The power wasted in transmission cables is

$$P_t = I^2 R_t = \frac{P^2 R_t}{V^2}$$

Thus, the power wasted in the transmission cables is inversely proportional to the square of voltage. Hence to minimise the power loss, electric power is transmitted to distant placed at high voltages and low currents. These voltages are stepped down by transformers before supplying to homes and factories.

Problems BASED ON HEATING EFFECT OF CURRENT

♣♠ **FORMULAE**

1. Heat produced by electric current,

$$H = I^2Rt \text{ joule} = \frac{I^2Rt}{4.18} \text{ cal}$$

or $$H = VIt \text{ joule} = \frac{VIt}{4.18} \text{ cal}$$

2. Electric power, $P = \frac{W}{t} = VI = I^2R = \frac{V^2}{R}$

3. Electric energy, $W = Pt = VIt = I^2Rt$

Current I is in ampere, resistance R in ohm, time t in second, power P in watt, electric energy in joule or ink Wh.

Q. 1. An electric current of 4.0 A flows through a 12 Ω resistor. What is the rate at which heat energy is produced in the resistor?

Sol. Here I = 4 A, R = 12 Ω

Rate of production of heat energy, $P = I^2R = 4^2 \times 12 = 192$ W.

Q. 2. How many electrons flow through the filament of a 120 V and 60 W electric lamp per second? Given e = 1.6 × 10^{-19} C.

Sol. Here P = 60 W, V = 120 V, t = 1 s

$$I = \frac{P}{V} = \frac{60}{120} = 0.5 \text{ A}$$

But $I = \frac{q}{t} = \frac{ne}{t}$

∴ No. of electrons flowing per second is

$$n = \frac{It}{e} = \frac{0.5 \times 1}{1.6 \times 10^{-19}} = 3.125 \times 10^{18}$$

Q. 3. A heating element is marked 210 V, 630 W. What is the current drawn by the element when connected to a 210 V d.c. mains? What is the resistance of the element?

Sol. Here P = 630 W, V = 210 V

Current drawn, $I = \frac{P}{V} = \frac{630}{210} = 3$ A.

Resistance of the element, $R = \frac{V}{I} = \frac{210}{3} = 70\ \Omega$

Q. 4. Calculate the current through a lamp of 60w operating at 220 v

Sol. current, $I = \frac{P}{V} = \frac{60w}{220v} = \frac{3}{11}\text{A} = 0.273$ A

Q. 5. Two registers of 2 Ω & 4 Ω are connected in parallel of a constant DC voltage. In which resistor is more heat produced?

Sol. We know that, $H = \frac{V^2}{R}t$

In the given problem, 'V' & 't' are constants,

.'. $H \propto \frac{1}{R}$

i.e., lesser the resistance, more will be the Heat produced. So, more Heat will be produced in 2 Ω resistance.

Q. 6. A 10 V storage battery of negligible internal resistance is connected across a 50 Ω resistor made of alloy manganin. How much heat energy is produced in the resistor in 1 h? What is the source of this energy?

Sol. Here V = 10 V, R = 50 Ω, t = 1 h = 3600 s

Heat energy produced in 1 h is

$$H = \frac{V^2t}{R} = \frac{10 \times 10 \times 3600}{50} = 7200 \text{ J}$$

The source of this energy is the chemical energy stored in the battery

Q. 7. An electric motor operates on a 50 V supply and draws a current of 12 A. If the motor yields a mechanical power of 150 W, what is the percentage efficiency of the motor?

Sol. Input power = VI = 50 × 12 = 600 W

Output power = 150 W

Efficiency of motor

$= \frac{\text{Output power}}{\text{Input power}} \times 100 = \frac{150 \times 100}{600} = 25\ \%$

Q. 8. The maximum power rating of a 20 Ω resister is 2 KW, [that is, this the maximum power, the resister dissipate (as heat) without melting or changing in some other under desirable way] Would you connects this resister directly across a 300 V d.c. source of negligible internal resistance? Explain your answer.

Sol When the given resister is connected directly across a 300 V d.c. source, the rate at which Heat is produced,

$= \frac{V^2}{R} = \frac{300 \times 300}{20} = 4500\text{ KW} = 4.5\text{ KW}$

This value is more than the maximum power rating of the given resister. So, It is not advisable to connect the given resister directly across the 300 V d.c. source.

Q. 9. Bulb with raring 250 V, 100 W is connected to a power supply of 220 V situated 10 m away. Using a copper wire of area of cross section 5 mm². How much power will be consumed by the connecting wires? Resistivity of copper = 1.7×10^{-8} Ω m

Sol. Resistance of copper wire = $\rho \frac{l}{A} = 1.7 \times 10^{-8} \times \frac{20}{5 \times 10^{-6}}\ \Omega = \frac{1.7}{250}\ \Omega = 0.068\ \Omega$

Resistance of bulb = $\frac{V^2}{\rho} = \frac{250 \times 250}{100} = 625\ \Omega$

Total resistance = (625 + 0.068) Ω = 625. 068 Ω

Current, I = $\frac{220}{625.068}$ A = 0.35 A

Power consumed by connecting wire = 0.35 × 0.35 × 0.068 Ω = 8.33×10^{-3} W.

Q. 10. An electric motor operating on a 50 V d.c. supply draws a current of 12 A. If the efficiency of the motor is 30 %, estimate the resistance of the windings of the motor.

Sol. Here V = 50 V, I = 12 A, η = 30 %

As the efficiency of electric motor is 30%, therefore, power dissipated as heat is

$P = 70\%\text{ of }VI = \frac{70}{100} \times 50 \times 12\text{ W} = 420\text{ W}$

But power dissipated as heat, $P = I^2R$

∴ $I^2R = 420$ or $R = \frac{420}{I^2} = \frac{420}{144} = 2.9\ \Omega$

Q. 11. (a) A nichrome heating element across 230 V supply consumes 1.5 kW of power and heats up to a temperature of 750°C. A tungsten bulb across the same supply operates at a much higher temperature of 1600°C in order to be able to emit light. Does it mean that the tungsten bulb necessarily consumes greater power? (b) Which of the two has greater resistance: a 1 kW heater or a 100 W tungsten bulb, both marked for 230 V?

Sol. (a) No, the steady temperature acquired by a resistor depends not only on the power consumed but also its characteristics such as surface area, emotivity, etc., which determine its power loss due to radiation.

(b) Here V = 230 V, P_1 = 1 kW = 1000 W,

P_2 = 100 W

$R_1 = \frac{V^2}{P_1} = \frac{230 \times 230}{1000}\ \Omega = 52.9\ \Omega$

$R_2 = \frac{V^2}{P_2} = \frac{230 \times 230}{}\ \Omega = 529\ \Omega$

Thus the 100 W bulb has a greater resistance.

Q. 12. Lamp of 100 W works at 220 volts. What as its resistance & current capacity?

Sol. Power of lamp, P = 100 W

Operating voltage, V = 220 Volt

Now, current capacity of the lamp, $I = \frac{P}{V} = \frac{100}{220} = 0.0455$A.

Q. 13. Calculate the numbers of electrons moving per second through the filament of a lamp of 100 Watt, operating at 200 Volt. given: - on an electron, 'e' = 1.6×10^{-19} C.

Sol. Power of the lamp, P = 100 W

Operating voltage, V = 200 Volt.

Now, P = V I

$\therefore \quad I = \frac{P}{V} = \frac{100}{200} = 0.5$ A

Charge passing through the lamp in 1 sec.,

q = I t = 0.5 × 1 = 0.5 C.

∴ no. of electron moving through the filament per second,

$n = \frac{q}{e} = \frac{0.5}{1.6 \times 10^{-19}} = 3.125 \times 10^{18}$

Q. 14. Two identical heaters, each rated 220 V, 1000 W are placed in series with each other across 220 V line. What is their combined power?

Sol. Resistance of each heather $= \frac{V^2}{P} = \frac{220 \times 220}{1000} = \frac{484}{10}\ \Omega$

total resistance, $R_s = 2 \times \frac{484}{10} = \frac{484}{5}\ \Omega$

Combined power, $P = \frac{V^2}{R_s} = \frac{220 \times 220 \times 5}{484} = 500$ W .

Q. 15. An electric power station (100 MW) transmits power to a distant load through long and thin cables. Which of the two modes of transmission would result in lesser power wastage: power transmission of : (i) 20, 000 V or (ii) 200 V?

Sol. Let R be the resistance of transmission cables.

Here P = 100 MW = 100×10^6 W

(i) V_1 = 20, 000 V

∴ Current, $I_1 = \frac{P}{V_1} = \frac{100 \times 10^6}{20,000} = 5000$ A

Rate of heat dissipation at 20, 000 V is

$P_1 = I_1^2R = (5000)^2R = 25 \times 10^6$ R watt.

(ii) V_2 = 200 V

∴ Current, $I_2 = \frac{100 \times 10^6}{200} = 5 \times 10^5$ A

Rate of heat dissipation at 200 V is

$P_2 = I_2^2R = (5 \times 10^5)^2R = 25 \times 10^{10}$ R watt

Clearly, $P_1 < P_2$ Hence, there will be lesser power wastage when the power is transmitted at 20, 000 V.

Q. 9. Two ribbons are given with the following particulars:

Ribbon	A	B
Alloy	Constantan	Nichrome
Length (m)	8.456	4.235
Width (mm)	1.0	2.0
Thickness (mm)	0.03	0.06
Temp. coefficient of Resistivity (°C^{-1})	Negligible	Negligible
Resistivity	4.9	11

For a fixed voltage supply, which of the two ribbons corresponds to a greater rate of heat production?

Sol. Since $R = \rho \frac{l}{A}$

∴ Resistance of constantan ribbon,

$R_1 = \frac{4.9 \times 10^{-7} \times 8.456}{1.0 \times 10^{-3} \times 0.03 \times 10^{-3}}\ \Omega = 138.1\ \Omega$

Let V be the fixed supply voltage. Then the rate of production of heat of constantan ribbon,

$P_1 = \frac{V^2}{R_1} = \frac{V^2}{138.1}$ Watt

Resistance of nichrome ribbon,

$R_2 = \frac{1.1 \times 10 - 6 \times 4.235}{2.0 \times 10^{-3} \times 0.06 \times 10^{-3}}\ \Omega = 28.8\ \Omega$

Rate of production of heat in nichrome ribbon,

$P_2 = \frac{V^2}{R_2} = \frac{V^2}{38.8}$ watt

Clearly nichrome ribbon has greater rate of production of heat because of its lesser resistance.

Q. 16. A heater coil is rated 100 W, 200 W. It cut into two identical parts. Both parts are connected together in parallel, to the same source of 200 V. Calculate the energy liberated per second in the new combination.

Sol. Resistance of heater coil,

$$R = \frac{V^2}{P} = \frac{200 \times 200}{100} = 400\ \Omega$$

Resistance of either half part = 200 Ω

Equivalent resistance when both parts are connected in parallel,

$$R' = \frac{200 \times 200}{200 + 200} = 100\ \Omega$$

Energy liberated per second when combination is connected to a source of 200 V

$$= \frac{V^2}{R'} = \frac{200 \times 200}{100} = 400\ J$$

Q. 17. An electric bulb is marked 100 W, 230 V. If the supply voltage drops to 115 V, what is the heat and light energy produced by the bulb in 20 min? Calculate the current flowing through it.

Sol. If the resistance of the bulb be R, then Rate of production of heat and light energy,

$$P = \frac{V^2}{R}$$

$$\therefore \quad R = \frac{V^2}{P} = \frac{230 \times 230}{100} = 529\ \Omega$$

When the voltage drops to V′ = 115 V, the total heat and light energy produced by the bulb in 20 min will be

$$H = P \times t = \frac{V'^2}{R} \times t$$

$$= \frac{115 \times 115}{529} \times 20 \times 60 = 30,000\ J = 30\ kJ$$

$$\text{Current, } I = \frac{V'}{R} = \frac{115}{529} = \frac{5}{23}\ A$$

Q. 18. Determine the percentage by which the illumination of a lamp will decrease if the current drops by 20 %.

Sol. If R is the resistance of the lamp and I is the current flowing for time t, then heat produced is

$$H = I^2Rt$$

When current drops by 20%, current in the circuit

$$= 80\ \%\ \text{of } I = \frac{80}{100} I = \frac{4}{5} I$$

$$\text{Heat produced, } H' = \left(\frac{4}{5} I\right)^2 Rt = \frac{16}{25} I^2Rt = \frac{16}{25} H$$

Decrease in heat production,

$$\frac{H - H'}{H} \times 100 = \left(1 - \frac{H'}{H}\right) \times 100 = \left(1 - \frac{16}{25}\right) \times 100 = 36\ \%$$

Q. 19. An electric bulb rated for 500 W at 100 V is used in circuit having a 200 v supply. Calculate the resistance R that must be put in series with the bulb, so that the bulb delivers 500 W.

Sol. Resistance of the bulb,

$$R = \frac{V^2}{P} = \frac{100 \times 100}{500} = 20\ \Omega$$

$$\text{Current through the bulb, } I = \frac{V}{R} = \frac{100}{20} = 5\ A$$

For the same power dissipation, the current through bulb must be 5 A.

When the bulb is connected to 200 V supply, the safe resistance of the circuit should be

$$R' = \frac{V'}{I} = \frac{200}{5} = 40\ \Omega$$

∴ Resistance required to be put in series with the bulb is

$$R' - R = 40 - 20 = 20\ \Omega$$

Q. 20. The maximum power rating of a 20 Ω resistor is 2.0 kW. (That is this is the maximum power the resistor can dissipate (as heat) without melting or changing in some other undesirable way). Would you connect this resistor directly across a 300 V d.c. source of negligible internal resistance? Explain your answer.

Sol. Maximum power rating of the given 20 Ω resistor,

$$P' = 2.0\ kW$$

When connected to 300 V d.c. supply, the power consumption or rate of production of heat would be

$$P = \frac{V^2}{R} = \frac{300 \times 300}{20}\ W = 4500\ W = 4.5\ kW$$

This power consumption exceeds the maximum power rating of the resistor. Hence the 20 Ω resistor must not be connected directly across the 300 V d.c. source. For doing so, a small resistance of 10 Ω should be connected in series with it.

Q. 21. An electric heater and an electric bulb are rated 500 W, 220 V and 100 W, 220 V respectively. Both are connected in series to a 220 V d.c. mains. Calculate the power consumed by (i) the heater and (ii) electric bulb.

Sol. Resistances of heater and bulb are

$$R_1 = \frac{V^2}{P_1} = \frac{220 \times 220}{500} = \frac{484}{5} = 96.8\ \Omega$$

$$R_2 = \frac{V^2}{P^2} = \frac{220 \times 220}{100} = 484\ \Omega$$

Total resistance of series combination is

$$R_1 + R_2 = 96.8 + 484 = 580.8\ \Omega$$

Current, $I = \frac{V}{R} = \frac{220}{580.8} = 0.38$ A

(i) Power consumed by heater is,

$$P_1 = I^2R_1 = 0.38^2 \times 96.8 = 13.8 \text{ W}$$

(ii) Power consumed by bulb,

$$P_2 = I^2R_2 = 0.38^2 \times 484 = 69.89 \text{ W}$$

Q. 21. Two heaters are marked 200 V, 300 W and 200 V, 600 W. If the heaters are combined in series and the combination connected to a 220 V d.c. supply, which heater will produce more heat?

Sol. Resistance of the two heaters are

$$R_1 = \frac{V^2}{P_2} = \frac{200 \times 200}{300} = \frac{400}{3}\ \Omega$$

$$R_2 = \frac{V^2}{P_2} = \frac{200 \times 200}{600} = \frac{200}{3}\ \Omega$$

For series combination, $R_1 + R_2 = \frac{600}{3} = 200\ \Omega$

∴ Current, $I = \frac{V}{R} = \frac{200}{200} = 1$ A

Power dissipations in the two heaters are

$$P_1' = I^2 R_1 = 1^2 \times \frac{400}{3} = \frac{400}{3} \text{ W}$$

$$P_2' = I^2 R_2 = 1^2 \times \frac{200}{3} = \frac{200}{3} \text{ W}$$

∴ $P_1' = 2 P_2'$ The first heater (of 300 W) produces more heat than the second heater.

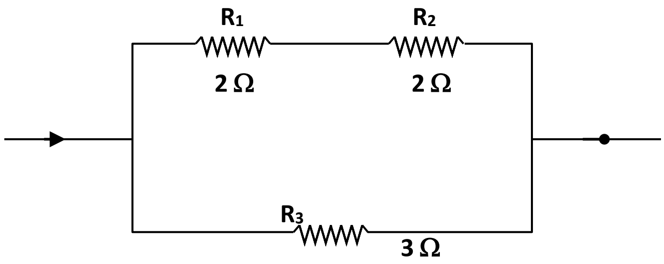
Q. 22. To produce 10^3 Joule of Heat on 10 seconds, how much voltage should be applied to Ω resistance?

Sol. $H = 10^3$ J, $t = 10$ sec., $R = 100\ \Omega$

Now, $H = \frac{V^2}{R} t$ ∴ $V^2 = \frac{HR}{t} = \frac{103 \times 100}{10} = 10^4$

∴ $V = \sqrt{10^4} = 10^2 = 100$ V.

Q. 23. In a part of the circuit shown in the Fig. 3.136, the rate of heat dissipation in 4 Ω resistor is 100 J/s. Calculate the heat dissipated in the 3 Ω resistor in 10 seconds.



Sol. Let I_1 be the current through the series combination of R_1 and R_2 and I_2 be the current through R_3.

P.D. across $(R_1 + R_2)$ = P.D. across R_3

∴ $(4 + 2) I_1 = 3 I_2$ or $I_2 = 2I_1$

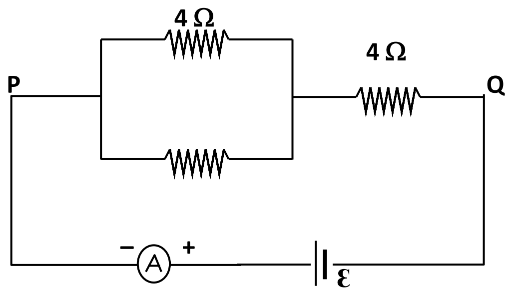
Rate of heat dissipation in 4 Ω resistor

$= I_1^2R_1 = I_1^2 \times 4 = 100 \text{ Js}^{-1}$

∴ $I_1 = \sqrt{\frac{100}{4}} = \sqrt{25} = 5$ A and $I_2 = 2I_1 = 10$ A

Heat dissipated in 3 Ω resistor in 10 s $= I_2^2 R_2 t = (10)^2 \times 3 \times 10 = 3000$ J

Q. 24. The resistance of each of the three wires, shown in Fig. is 4 Ω. This combination of resistors is connected to a source of emf Ɛ. The ammeter shows a reading of 1 A. Calculate the power dissipated in the circuit.



Sol. Total resistance between the points P and Q,

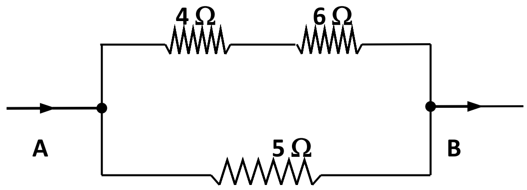
$$R = \frac{4 \times 4}{4 + 4} + 4 = 2 + 4 = 6\ \Omega$$

Current in the circuit, $I = 1$ A

Power dissipated in the circuit,

$$P = I^2 R = 1^2 \times 6 = 6 \text{ W}$$

Q. 25. In the circuit shown in Fig. the heat produced in 5 Ω resistor, due to the current flowing through it is 10 calorie per second.
Find the heat produced in 4 Ω resistor.



Sol. Heat produced per second in 5 Ω resistor,

$$P = 10 \text{ cal s}^{-1} = 10 \times 4.2 \text{ Js}^{-1}$$

As $\frac{V^2}{R} = P$

∴ Potential difference between points A and B is

$$V = \sqrt{PR} = \sqrt{10 \times 4.2 \times 5} = \sqrt{210} \text{ V}$$

Current through 4 Ωresistors,

$$I = \frac{\sqrt{210}}{4 + 6} = \frac{\sqrt{210}}{10} \text{ A}$$

Heat produced per second in 4 Ω resistor

$$= I^2 R = \left(\frac{\sqrt{210}}{10}\right)^2 \times 4 \text{ Js}^{-1}$$

$$= \frac{210}{100} \times \frac{4}{4.2} \text{ cal s}^{-1} \quad = 2 \text{ cal s}^{-1}$$

Q. 26. A heater is designed to operate with a power of 1000 W in a 100 V line. It is connected in combination with a resistance of 10 Ω and a resistance R, to a 100 V mains, as shown in Fig. What should be the value of R so that the heater operates with a power of 62.5 W?

A 10 Ω B C
Heater

R

100 V

Sol. Resistance of heater,

$$R' = \frac{V^2}{P} = \frac{100^2}{1000} = 10\ \Omega$$

When the heater is connected as shown and the power drops to 62.5 W, the p.d. across the heater would be

$$V' = \sqrt{P'R'} = \sqrt{62.5 \times 10} = 25 \text{ V}$$

∴ P. D. across 10 Ω resistor

$$= 100 - 25 = 75 \text{ V}$$

Current in 10 Ω resistor, $I = \frac{75}{10} = 7.5$ A

Current through heater, $I' = \frac{V'}{R'} = \frac{25}{10} = 2.5$ A

∴ Current through resistance

$R = I - I' = 7.5 - 2.5 = 5$ A

P.D. across R = P.D. across heater = V' = 25 V

∴ Resistance, $R = \frac{V'}{I - I'} = \frac{25}{} = 5\ \Omega$

Q. 27. A house is fitted with 20 lamps of 60 W each, 10 fans consuming 0.5 A each and an electric kettle of resistance 110 Ω. If the energy is supplied at 220 V and costs 75 paise per unit, calculate the monthly bill for running appliances for 6 hours a day. Take 1 month = 30 days.

Sol. Power of 20 lamps of 60 W each

$= 20 \times 60 = 1200$ W

Power consumed by 10 fans at 0.5 A current

$= 10 \times VI = 10 \times 220 \times 0.5 = 1100$ W

Power consumed by electric kettle of 110 Ω resistance

$= \frac{V^2}{R} = \frac{220 \times 220}{110} = 440$ W

Total power of the appliances

$= 1200 + 1100 + 440 = 2740$ W = 2.74 kW

Total time for which appliances are used

$= 6 \times 30 = 180$ h

Total energy consumed

$= P . t = 2.74$ kW × 180 h = 493.2 kWh or units

∴ Monthly bill = 493.2 × 0.75 = Rs. 369.90

Q. 28. There are two electric bulbs rated 60 W, 110 V and 100 W, 110 V. They are connected in series with a 220 V d.c. supply. Will any bulb fuse? What will happen if they are connected in parallel with the same supply?

Sol. Currents required by the two bulbs for the normal glowness are

$I_1 = \frac{P_1}{V} = \frac{60}{} = 0.55$ A

and $I_2 = \frac{P_2}{V} = \frac{100}{110} = 0.91$ A

The resistance of the two bulbs are

$R_1 = \frac{V}{I_1} = \frac{110}{0.55} = 202\ \Omega$

and $R_2 = \frac{V}{I_2} = \frac{110}{0.91} = 121\ \Omega$

When the bulbs are connected in series across the 220 V supply, the current through each bulb will be

$I = \frac{V}{R_1 + R_2} = \frac{220}{202 + 121} = 0.68$ A

As $I_1 < I$ and $I_2 > I$, so that 60 W bulb will fuse while the 100 W bulb will light up dim.

When the bulbs are joined in parallel, their equivalent resistance is

$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{202 \times 121}{202 + 121} = 76\ \Omega$

Current drawn from the 220 V supply will be

$I' = V = 220 \simeq 3$ A

In the two bulbs of resistance $R_1 = (\simeq 202\ \Omega)$ and R_2 (= 120 Ω), the current of 3 A will split up into roughly 1 A and 2 A respectively. Hence both the bulbs will fuse.

Q. 29. Twenty-one electric bulbs are connected in series with the mains of a 220 V supply. After on bulb is fused, the remaining 20 bulbs are again connected in series, across the same mains. By what percentage will the illumination of (i) a bulb change and (ii) all the bulbs change?

Sol. Let R be the resistance of each bulb. When all the 21 bulbs are connected in series, total resistance = 21 R

∴ Current in the circuit, $I = \frac{220}{21\ R}$

Illuminating power of each bulb,

$p = I^2R = \left(\frac{220}{21\ R}\right)^2 \times R = \left(\frac{220}{21}\right)^2 \frac{1}{R}$

Illuminating power of all the 21 bulbs,

$P = \frac{220}{21\ R}^2 \times 21\ R = \frac{(220)^2}{21\ R}$

When one bulb gets fused and remaining 20 bulbs are connected in series, the current becomes

$$I' = \frac{220}{20\,R}$$

Illuminating power of each bulb becomes

$$p' = I'^2 R = \left(\frac{220}{20\,R}\right)^2 \times R = \left(\frac{220}{20}\right)^2 \frac{1}{R}$$

Illuminating power of the 20 bulbs will be

$$P' = \left(\frac{220}{20\,R}\right)^2 \times 20\,R = \frac{(220)^2}{20\,R}$$

(i) Percentage increase in the illumination of one bulb

$$= \left(\frac{p' - p}{p}\right) \times 100 = \left(\frac{p'}{p} - 1\right) \times 100$$

$$= \left[\frac{\frac{(220)^2}{20\,R}}{\frac{(220)^2}{21\,R}} - 1\right] \times 100 = \left(\frac{21 - 1}{20}\right) \times 100 = 5\ \%$$

Q. 30. The resistance of a 240 V and 200 W electric bulb when hot is 10 times the resistance when cold. Find its resistance at room temperature. If the working temperature of the filament is 2000°C, find the temperature coefficient of the filament.

Sol. Resistance of the hot bulb is given by

$$R' = \frac{V^2}{P} = \frac{240 \times 240}{200} = 288\ \Omega$$

Resistance of bulb at room temperature,

$$R = \frac{R'}{10} = \frac{288}{10} = 28.8\ \Omega$$

Since $R' = R(1 + \alpha t)$ $\quad\therefore\quad$ $288 = 28.8(1 + \alpha \times 2000)$

or $\quad \alpha = \dfrac{9}{2000}\ °C^{-1} = 4.5 \times 10^{-3}\ °C^{-1}$

Q. 31. A thin metallic wire of resistance 100 Ω is immersed in a calorimeter containing 250 g of water at 10 °C and current of 0.5 ampere is passed through it form half an hour. If the water equivalent of the calorimeter is 10 g, find the rise of temperature.

Sol. Here $\quad m = 250$ g, $\quad I = 0.5$ A, $t = 30$ min $= 1800$ s, $w = 10$ g

$\therefore$ Heat produced $= I^2Rt = (0.5)^2 \times 100 \times 1800$ J $= 45000$ J

Heat gained by water and calorimeter

$= (m + w)\, c\, \theta = (250 + 10) \times 1 \times \theta$ cal $= 260 \times 4.2\ \theta$ joule

$\therefore \quad 2600 \times 4.2 \times \theta = 45000$

Rise in temperature, $\theta = \dfrac{45000}{2600 \times 4.2} = 41.2°C$

Q. 32. A copper electric kettle weighing 1000 g contains 900 g of water at 20°C. It takes 12 minutes to raise the temperature to 100 °C. If electric energy is supplied at 210 V, calculate the strength of the current, assuming that 10% heat is wasted. Specific heat of copper is 0.1.

Sol. Water equivalent of copper kettle is

w = Mass × Specific heat $= 1000 \times 0.1 = 100$ g

Also, $m = 900$ g, $\theta = \theta_2 - \theta_1 = 100 - 20 = 80°C$

Heat required, $\quad H = (m + w)\, c\, \theta = (900 + 100) \times 1 \times 80 = 80,000$ cal

Heat produced

$$= \frac{V\,It}{4.2} = \frac{210 \times I \times 12 \times 60}{4.2}\ \text{cal} = 36000\ I\ \text{cal}$$

Useful heat = 90 % of $36000\ I = \dfrac{90 \times 36000\ I}{100} = 32400\ I$ cal

$\therefore \quad 32400\ I = 80,000$

Current $\quad I = \dfrac{80000}{32400} = 2.469$ A

Q. 33. A coil is enamelled copper wire of resistance 50 Ω is embedded in a black of ice and a potential difference of 210 V applied across it. Calculate the rate at which ice melts. Latent heat of ice is 80 cal per gram.

Sol. Here R = 50 Ω, V = 210 V, t = 1 s,

$L = 80 \text{ cal g}^{-1}$

Heat produced,

$$H = \frac{V^2 t}{4.2\,R} = \frac{210 \times 210 \times 1}{4.2 \times 50} = 210 \text{ cal.}$$

Suppose m gram of ice melts per second. Then

mL = H

or $m = \frac{H}{L} = \frac{210}{80} = 2.62 \text{ g s}^{-1}$.

Q. 34. An electric kettle has two heating coils, when one of the coils is switched on, the kettle begins to boil in 6 minutes and when the other is switched on, the boiling begins in 8 minutes, In what time will the boiling begin if both the coils are switched on simultaneously (i) in series and (ii) in parallel?

Sol. Let R_1 and R_2 be the resistances of the two coils, V the supply voltage and H, the heat required to boil the water.

For the first coil, $H = \frac{V^2 t_1}{JR_1} = \frac{V^2 \times 6 \times 60}{4.2\,R_1}$ cal

For the second coil, $H = \frac{V^2 t^2}{JR_2} = \frac{V^2 \times 5 \times 60}{4.2\,R_2}$ cal

$\therefore \frac{V^2 \times 6 \times 60}{4.2\,R_1} = \frac{V^2 \times 8 \times 60}{4.2\,R_2}$ or $\frac{R_2}{R_1} = \frac{8}{6} = \frac{4}{3}$

(i) When the coils are in series, effective resistance = $R_1 + R_2$. Let the boiling occur in time t_1 min.

Then $\frac{V^2 t_1 \times 60}{4.2\,(R_1 + R_2)} = H = \frac{V^2 \times 6 \times 60}{4.2\,R_1}$

or $t_1 = 6\left(\frac{R_1 + R_2}{R_1}\right) = 6\left(1 + \frac{R_2}{R_1}\right)$

$= 6\left(1 + \frac{4}{3}\right)$ min = 14 min.

(ii) When the two coils are in parallel,

effective resistance = $\frac{R_1 R_2}{R_1 + R_2}$

Let the boiling occur in time t_2 min. Then

$$\frac{V^2 t_2 \times 60}{4.2\left(\frac{R_1 R_2}{R_1 + R_2}\right)} = H = \frac{V^2 \times 6 \times 60}{4.2\,R_1}$$

or $t_2 = 6 \times \frac{R_1 R_2}{(R_1 + R_2) R_1} = 6 \times \frac{1}{\left(1 + \frac{R_1}{R_2}\right)}$

$= 6 \times \frac{1}{\left(1 + \frac{3}{4}\right)}$ min = 3.43 min.

Q. 35. The heater coil of an electric kettle is rated at 2000 W, 200 V. How much time will it take in raising the temperature 10of 1 litre of water from 20°C to 100°C, assuming that only 80% of the total heat energy produced by the heater coil is used in raising the temperature of water. Density of water = 1 g cm^{-3} and specific heat of water = 1 cal g^{-1} °C^{-1}.

Sol. Here P = 2000 W,

Volume of water = 1 litre = 1000 cm^3

Mass of water, m = Volume × density = 1000 cm^3 × 1 g cm^{-3} = 1000 g

Rise in temperature,

$\theta = \theta_2 - \theta_1 = 100 - 20 = 80°C$

Heat gained by water

$= mc\,\theta = 1000 \times 1 \times 80 = 80,000$ cal

Let t be the time taken to increase the temperature from 20° to 100 °C.

Then total heat produced by heating coil

= Pt = 2000 t joule

Useful heat produced

$= 80\%\ 2000\,t = \frac{80 \times 2000\,t}{100}$ j

$= \frac{80 \times 2000\,t}{100 \times 4.2}$ cal

Useful heat produced = Heat gained by water

$\frac{80 \times 2000\,t}{100 \times 4.2} = 80000$

or $t = \frac{80000 \times 100 \times 4.2}{80 \times 2000} = 210$ s

Q. 36. One kilowatt electric heater is to be used with 220 V d.c. supply. (i) What is the current in the heater? (ii) What is its resistance? (iii) What is the power dissipated in the heater? (iv) How much heat in calories is produced per second? (v) How many grams of water at 100°C will be converted per minute into steam at 100°C, with the heater? Assume that the heat losses due to radiation are negligible. Latent heat of steam = 540 cal per gram.

Sol. Here P = 1 kW = 1000 W, V = 220 V

(i) Current, $I = \frac{P}{V} = \frac{1000}{220} = 4.55\ A$

(ii) Resistance, $R = \frac{V^2}{P} = \frac{220 \times 220}{1000} = 48.4\ \Omega$

(iii) Power dissipated in heater = 1000 W.

(iv) Heat produced per second,

$H = \frac{VIt}{J} = \frac{P.t}{J} = \frac{1000 \times 1}{4.2} = 240\ cal\ s^{-1}$

(v) Heat produced per minute,

H = 240 × 60 = 14400 cal

We know that 540 cal of heat convert 1 g water at 100° C into steam at 100° C.

∴ Mass of water converted into steam = $\frac{14400}{540} = 26.67$ g.

Q. 37. Two walls of a closed cubical box of edge 50 cm are made of a material of thickness 1 mm and thermal conductivity 4×10^{-4} cal s^{-1} cm^{-1} °C^{-1}. The interior of the box maintained at 100°C above the outside temperature by a heater placed inside the box and connected across a 400 V d.c. source. Calculate the resistance of the heater.

Sol. Here, $K = 4 \times 10^{-4}$ cal s^{-1} cm^{-1} °C^{-1}, $\theta_2 - \theta_1 = 100$ °C, d = 1 mm = 0.1 cm

Surface area of the six faces of the cubical box,

$A = 6 \times (50 \times 50) = 15000\ cm^2$

The amount of heat conducted out per second through the walls of the cubical box is

$H_1 = \frac{K A (\theta_2 - \theta_1)}{d} = \frac{4 \times 10^{-4} \times 15000 \times 100}{0.1}$

= 6000 cal = 6000 × 4.2 J

If R is the resistance of the heater, then heat produced per second

$H_2 = I^2 Rt = \frac{V^2}{R} = \frac{(400)^2}{R}$

Temperature inside the box will be maintained by the heater if

$H_1 = H_2$ or $\frac{(400)^2}{R} = 6000 \times 4.2$

or $R = \frac{400 \times 400}{6000 \times 4.2} = 6.35\ \Omega$

Q. 38. A 10 V battery of negligible internal resistance is charged by a 200 V d.c. supply. If the resistance in the charging 11circuit is 38 Ω, what is the value of charging current?

Sol. As the battery emf opposes the charging emf, therefore,

net emf = 200 – 10 = 190 V

Charging current,

$I = \frac{\text{Net emf}}{\text{Resistance}} = \frac{200 - 10}{} = 5\ A$

Q. 39. A dry cell of emf 1.6 V and internal resistance 0.10 ohm is connected to a resistor of resistance R ohm. If the current drawn from the cell is 2 A, then
(i) What is the voltage drop across R?
(ii) What is the energy dissipation in the resistor?

Sol. Here $\mathcal{E} = 1.6$ V, r = 0.10 Ω, I = 2.0 A

$R + r = \frac{\mathcal{E}}{I} = \frac{1.6}{2.0} = 0.8\ \Omega$

R = 0.8 – 0.10 = 0.70 Ω

(i) Voltage drop across R, V = IR = 2 × 0.70 = 1.4 V.

(ii) Rate of energy dissipation inside the resistor = VI = 1.4 × 2.0 = 2.8 W.

Q. 40. A dry cell of emf 1.5 V and internal resistance 0.10 Ω is connected across a resistor in series with a very low resistance ammeter. When the circuit is switched on, the ammeter reading settles to a steady value of 2.0 A. What is the steady
(a) Rate of chemical energy consumption of the cell, (b) Rate of energy dissipation inside the cell,
(c) Rate of energy dissipation inside the resistor, (d) Power output of the source?

Sol. Here $\mathcal{E}$ = 1.5 V, r = 0.10 Ω, I = 2.0 A

(a) Rate of chemical energy consumption of the cell = $\mathcal{E}I$ = 1.5 V × 2.0 A = 3.0 W

(b) Rate of energy dissipation inside the cell = $I^2r = (2)^2 \times 0.10$ W = 0.40 W

(c) Rate of energy dissipation inside the resistor = $\mathcal{E}I = I^2r$ = 3.0 – 0.40 = 2.6 W

(d) Power output of the source= Power input to the external circuit

$= \mathcal{E}I = I^2r$ = 2.6 W.

Q. 41. A series battery of 10 lead accumulators, each of emf 2 V and internal resistance 0.25 ohm, is charged by a 220 V d.c. mains. To limit the charging current, a resistance of 47.5 ohm is used in series in the charging circuit. What is (a) the power supplied by the mains and (b) power dissipated as heat? Account for the difference of power in (a) and (b).

Sol. emf of the battery = 10 × 2 = 20 V

Internal resistance of the battery = 10 × 0.25 = 2.5 Ω

Total resistance = r + R = 2.5 + 47.5 = 50.0 Ω

As the battery emf opposes the charging emf,

∴ Effective emf = $\mathcal{E}$ – V = 220 – 20 = 200 V

$$\text{Charging current} = \frac{\text{Effective emf}}{\text{Total resistance}} = \frac{200}{} = 4 \text{ A}$$

(a) Power supplied by the mains = VI = 220 × 4 = 880 W

(b) Power dissipated as heat $= I^2 (R + r) = 4^2 \times 50$ = 800 W

The difference of power = 880 – 800 = 80 W, is stored in the battery in the form of chemical energy.

Q. 42. A series battery of 6 lead accumulators each of emf 2.0 V and internal resistance 0.50 Ω is charged by a 100 V d.c. supply. What series resistance should be used in charging circuit in order to limit the current to 8.0 A? Using the required resistor, obtain (a) the power supplied by the d.c. source

(b) the power supplied by the d.c. energy stored in the battery in 15 min.

-Sol. Here $\mathcal{E}$ = 2.0 V, r = 0.50 Ω, V = 100 V, I = 8.0 A

As the battery emf opposes the charging emf,

∴ Effective emf = 100 – 2.0 × 6 = 88 V

Let the required series resistance be of R Ω. Then

total resistance = (0.50 × 6 + R) Ω = (3 + R) Ω

$$\text{Now } I = \frac{\text{Total emf}}{\text{Total resistance}} \quad \therefore 8 = \frac{88}{3 + R}$$

or 24 + 8R= 88 or $R = \frac{64}{8}\ \Omega = 8\ \Omega$

(a) Power supplied by d.c. source= VI = 100 V × 8 A = 800 W.

(b) Power dissipated as heat

$= I^2 (R + r) = 8^2 (8 + 0.50 \times 6)$ W= 64 × 11 W = 704 W

(c) Power supplied by the d.c. energy stored in the battery in 15 min

= (800 – 704) W × 15 min = 96 W × 900 s = 86400 J.

Q. 43. Power from a 64 V d.c. supply goes to charge a battery of 8 lead accumulators each of emf 2.0 V and internal resistance 1/8 Ω. The charging current also runs an electric motor placed in series with the battery. If the resistance of the windings of the motor is 7.0 Ω and the steady supply current is 3.5 A, obtain

(a) The mechanical energy yielded by the motor, (b) The chemical energy, stored in the battery during charging in 1 h.

Sol. emf of the battery,

$\mathcal{E}_b$ = 2.0 × 8 V = 16 V , d.c. supply voltage, $\mathcal{E}_S$ = 64 V

Internal resistance of the battery

$$r = \frac{1}{8} \times 8\ \Omega = 1\ \Omega$$

Resistance of motor, R = 7.0 Ω

Let back emf of motor, = $\mathcal{E}_m$

Both the back emf $\mathcal{E}_m$ of the motor and the emf $\mathcal{E}_b$ of the battery act in the opposite direction of the supply emf $\mathcal{E}_S$.

Therefore, net current in the circuit must be

$$I = \frac{\text{Net emf}}{\text{Net resistance}} = \frac{\mathcal{E}_S - \mathcal{E}_b - \mathcal{E}_m}{}$$

$$\text{or} \quad 3.5 = \frac{64 - 16 - \mathcal{E}_m}{8}$$

or $\mathcal{E}_m$ = 48 – 28 = 20 V.

(a) Mechanical energy yielded by motor in 1 h = $\mathcal{E}_m$. It = 20 × 3.5 × 3600 J= 252000 J.

(b) Chemical energy stored in the battery in 1 h = $\mathcal{E}_b$. It = 20 × 3.5 × 3600 J= 252000 J.