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Introducing the CBSE Class 10 Mathematics Area Related to Circles Mastery Kit - your go-to resource for mastering the concepts of circles and their associated areas. Specifically curated for CBSE Class 10 students, this kit is designed to simplify complex geometrical ideas and enhance your problem-solving skills.

KEY FEATURES:

1. **Complete Syllabus Coverage:** **The Mastery Kit ensures comprehensive coverage of the CBSE Class 10 Mathematics syllabus for the topic "Area Related to Circles." All key concepts, theorems, and formulas are included for thorough preparation.**

2. **Conceptual Clarity:** **Enjoy clear, concise explanations of concepts related to the area of circles. Complex theorems and formulas are broken down into simple, understandable components, making it easy for you to grasp and apply them.**

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4. **Practice Exercises:** **Reinforce your learning with a range of practice exercises that progress in difficulty. From basic problems to advanced applications, these exercises are designed to build your confidence and proficiency in dealing with problems related to the area of circles.**

5. **CBSE Exam Focus:** **Tailored to align with CBSE examination patterns, the Mastery Kit equips you with the knowledge and skills necessary to excel in your Class 10 Mathematics exams.**

6. **Visual Aids and Diagrams:** **Visual representations, diagrams, and illustrations are used to simplify abstract concepts, ensuring better comprehension and retention. Geometrical figures are visually presented to enhance your understanding.**

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AREA RELATED TO CIRCLE

Syllabus Reference

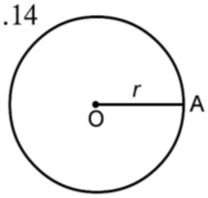
IMPORTANT DEFINITIONS AND FORMULAE

- ***Circumference of a circle Or Perimeter of a circle:***

  The distance around the circle or the length of a circle is called its circumference or perimeter.

  Circumference (perimeter) of a circle = πd or $2\pi r$, where d is a diameter and r is a radius of the circle

  and $\pi = \frac{22}{7}$ or 3.14



- ***Area of a circle:***

  Area of a circle = πr^2

- ***Area of a semicircle:***

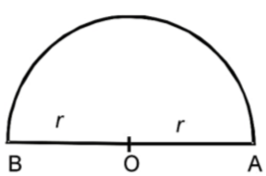
  Area of semicircle = $\frac{1}{2}\pi r^2$

- ***Area of a quadrant:***

  Area of a quadrant (quarter circle) = $\frac{\pi r^2}{4}$

- ***Perimeter of a semicircle:***

  Perimeter of a semicircle or protractor = $\pi r + 2r$



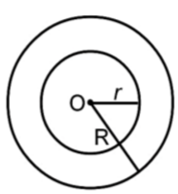
- ***Area of the ring:***

  Area of the ring or an annulus

  $= \pi R^2 - \pi r^2$

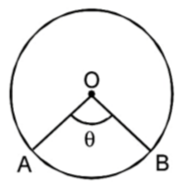
  $= \pi (R^2 - r^2)$

  $= \pi (R + r)(R - r)$



- **Length of the arc AB**

  $$= \frac{2\pi r\theta}{360^\circ} \text{ or } \frac{\pi r\theta}{180^\circ}$$

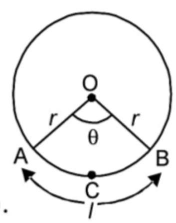


- ***Area of a sector:***

  Area of sector OACBO = $\frac{\pi r^2\theta}{360^\circ}$

  Or

  Area of sector OACBO = $\frac{1}{2}(r \times l)$.

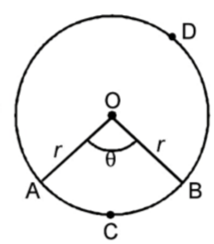


- ***Perimeter of a sector:***

  Perimeter of sector OACBO

  = Length of arc AB + $2r$

  $= \frac{\pi r\theta}{180^\circ} + 2r.$



- ***Other important formulae:***

  (*i*) Distance moved by a wheel in 1 revolution

  = Circumference of the wheel

  (*ii*) Number of revolutions in one minute

  $$= \frac{\text{Distance moved in 1 minute}}{\text{Circumference}}$$

  (*iii*) Angle described by minute hand in 60 minutes = 360°

  (*iv*) Angle described by hour hand in 12 hours = 360°

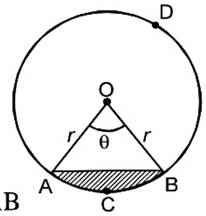
- The mid-point of the hypotenuse of a right triangle is equidistant from the vertices of the triangle.
- Angle subtended at the circumference by a diameter is always a right angle.

Area of a segment:

(*i*) Area of minor segment ACBA

= Area of sector OACBO – Area of ΔOAB

$$= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin\theta.$$



(*ii*) Area of major segment BDAB

= Area of the circle – Area of minor segment ACBA

= πr^2 – Area of minor segment ACBA.

- If a chord subtends a right angle at the centre, then

  Area of the corresponding segment $= \left(\frac{\pi}{4} - \frac{1}{2}\right) r^2$.

- If a chord subtends an angle of 60° at the centre, then

  Area of the corresponding segment $= \left(\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right) r^2$.

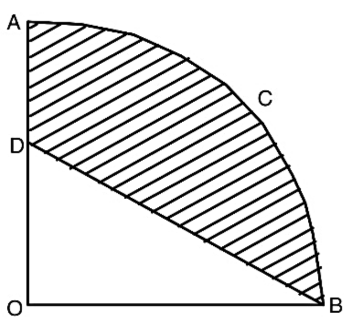
- If a chord subtends an angle of 120° at the centre, then

  Area of the corresponding segment $= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) r^2$.

NCERT & BOARD QUESTIONS CORNER
(Remembering & Understanding Based Questions)

Short Answer Type-I Questions

1. ***In the given figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the shaded region.***



Sol. Here, OACB is a quadrant of circle with centre O and radius OB = 3.5 cm.

OD = 2 cm

$$\text{Area of } \Delta ODB = \frac{1}{4} \times OD \times OB$$

$$= \frac{1}{2} \times 3.5 \times 2$$

$$= 3.5 \text{ cm}^2$$

$$\text{Area of quadrant OACB} = \frac{1}{4} \times \pi \times r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= \frac{11 \times 3.5 \times 0.5}{2 \times 7}$$

$$= 9.625 \text{ cm}^2$$

Area of shaded region = Area of quadrant OACB – Area of ΔODB

$= 9.625 \text{ cm}^2 - 3.5 \text{ cm}^2$

$= 6.125 \text{ cm}^2$

2. ***If the perimeter of a semicircular protractor is 66 cm, find the radius of the protractor.***

Sol. Let *r* cm be the radius of protractor

Perimeter of semicircle $= 2r + \pi r$

$\therefore \quad 2r + \pi r = 66$

$\Rightarrow \quad r\left(2 + \frac{22}{7}\right) = 66$

$\Rightarrow \quad r \times \frac{36}{7} = 660$

$\Rightarrow \quad r = \frac{66 \times 7}{36} = \frac{77}{6}$

$= 12.83 \text{ cm}$

3. ***The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.***

Sol. Circumference of the circle having radius 19 cm $= 2\pi \times 19 = 38\pi$ cm

Circumference of the circle having radius 9 cm $= 2\pi \times 9 = 18\pi$ cm

Let *r* be the radius of the required circle.

$\therefore$ Circumference of new circle = Sum of circumference of two circles

$$2\pi r = 38\pi + 18\pi$$

$$\Rightarrow \quad 2\pi r = 56\pi$$

$$\Rightarrow \quad r = \frac{56\pi}{2\pi} = 28$$

Hence, the radius of the required circle is 28 cm.

4. ***A race track is in the form of a ring whose inner circumference is 352 m and outer circumference is 396 m. Find the width of the track.***

$$\left[\textit{Use } \pi = \frac{22}{7}\right]$$

Sol. Let R and r be radii of outer and inner circles respectively.

$\therefore$ We have, $2\pi r = 352$

$$\Rightarrow \quad r = \frac{352}{2\pi}$$

and $2\pi R = 396 \Rightarrow R = \frac{396}{2\pi}$

$\therefore$ Width of the track $= R - r$

$$= \frac{396}{2\pi} - \frac{352}{2\pi} = \frac{44}{2\pi}$$

$$= \frac{44 \times 7}{2 \times 22} = 7 \text{ m}$$

5. ***The length of the minute hand of the clock is 14 cm. Find the area swept by the minute hand from 9:00 to 9:35.***

Sol. Angle described by the minute hand in 60 minutes $= 360°$

Angle described by the minute hand from 9 : 00 to 9:35 *i.e.*, in 35 minutes $(\theta) = \frac{360°}{60°} \times 35 = 210°$

Length of minute hand (radius r) = 14 cm

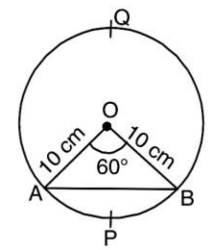
Area swept by the minute hand $= \frac{\theta}{360°} \times \pi r^2$

$$= \frac{210°}{360°} \times \frac{22}{7} \times 14 \times 14 = \frac{1078}{3} \text{ cm}^2$$

$$= 359.33 \text{ cm}^2$$

6. ***Find the area of the sector of a circle with radius 10 cm and of central angle 60°. Also, find the area of the corresponding major sector.***

Sol. Here, r = 10 cm and $\theta = 60°$



Area of minor sector

$$\text{OAPB} = \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{60°}{360°} \times \frac{22}{7} \times 10 \times 10$$

$$= \frac{1100}{21} \text{ cm}^2$$

Area of major sector

$$\text{OAQB} = \frac{300°}{360°} \times \frac{22}{7} \times 10 \times 10$$

$$[\because \theta = 360° - 60° = 300°]$$

$$= \frac{5500}{21} \text{ cm}^2$$

7. ***Find the area of ΔPQR such that ∠Q = 90°, PR = 10 cm and ∠PRQ = 30°.***

[Take $\sqrt{3}$ = 1.73]

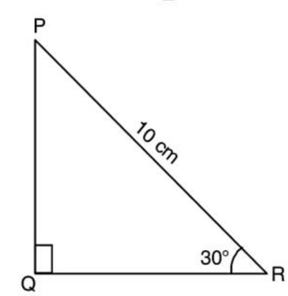
Sol. Here, $\frac{PQ}{PR} = \sin 30°$

$$\Rightarrow \quad \frac{PQ}{10} = \frac{1}{2} \Rightarrow PQ = 5 \text{ cm}$$

And $\frac{QR}{PR} = \cos 30°$

$$\Rightarrow \quad \frac{QR}{10} = \frac{\sqrt{3}}{2} \Rightarrow QR = 5\sqrt{3} \text{ cm}$$

$\therefore$ Area of $\Delta PQR = \frac{1}{2} \times PQ \times QR$

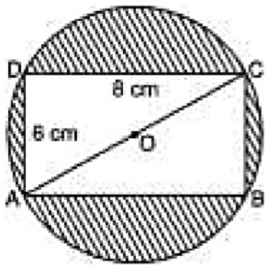


$$= \frac{1}{2} \times 5 \times 5\sqrt{3} = \frac{25}{2} \times 1.73$$

$$= \frac{43.25}{2} = 21.625 \text{ cm}^2$$

Short Answer Type-II Questions

8. ***Find the area of the shaded region in fig., if ABCD is a rectangle with sides 8 cm and 6 cm and O is the centre of circle. (Take $\pi = 3.14$)***



Sol. Given that ABCD is a rectangle with AD = 6 cm, DC = 8 cm and $\angle D = 90°$

$\therefore \quad AC^2 = AD^2 + DC^2 = 6^2 + 8^2$

$= 36 + 64 = 100$

$AC = 10$ cm

$AO = OC = \frac{10}{2} = 5$ cm

Now, area of the shaded region = Area of circle – Area of rectangle

$= 3.14 \times 5 \times 5 - 6 \times 8$

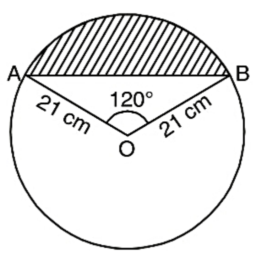
$= 78.5 - 48$

$= 30.5 \text{ cm}^2$

Hence, the area of the shaded region is 30.5 cm^2.

9. ***Find the area of the segment shown in fig., if radius of the circle is 21 cm and $\angle AOB = 120°$.***

$\left(\textit{Use } \pi = \frac{22}{7}\right)$



Sol. Area of the segment AYB

= Area of sector OAYB – Area of ΔOAB ...(*i*)

Now, area of the sector OAYB

$= \frac{120°}{360°} \times \frac{22}{7} \times 21 \times 21$

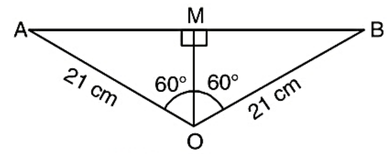
$= 462 \text{ cm}^2$ ...(*ii*)

For finding the area of ΔOAB, darw $OM \perp AB$ as shown in figure.

Note that OA = OB. Therefore, by RHS congruence, $\Delta AMO \cong \Delta BMO$.

So, M is the mid-point of AB

and $\angle AOM = \angle BOM = \frac{1}{2} \times 120° = 60°$.



Let $\quad OM = x$ cm

So, from ΔOMA, $\frac{OM}{OA} = \cos 60°$

$\Rightarrow \quad \frac{x}{21} = \frac{1}{2} \quad \left[\cos 60° = \frac{1}{2}\right]$

$\Rightarrow \quad x = \frac{21}{2}$

So, $\quad OM = \frac{21}{2}$ cm

Also, $\quad \frac{AM}{OA} = \sin 60° = \frac{\sqrt{3}}{2}$

So, $\quad AM = \frac{21\sqrt{3}}{2}$ cm

Therefore, $\quad AB = 2AM$

$= \frac{2 \times 21\sqrt{3}}{2} \text{ cm} = 21\sqrt{3} \text{ cm}$

So, area of $\Delta OAB = \frac{1}{2} AB \times OM$

$= \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2} \text{ cm}^2$

$= \frac{441}{4}\sqrt{3} \text{ cm}^2$ ...(*iii*)

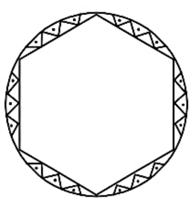
Therefore, area of the segment AYB

$= \left[462 - \frac{441}{4}\sqrt{3}\right] \text{cm}^2$

[From (*i*), (*ii*) and (*iii*)]

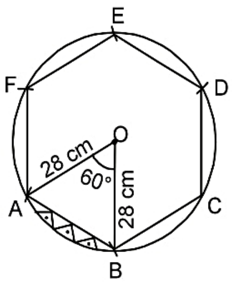
$= \frac{21}{4}(88 - 21\sqrt{3}) \text{ cm}^2$

10. ***A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm². [Use $\sqrt{3} = 1.7$]***



Sol. Here, OA = OB = r = 28 cm, $\angle AOB = \frac{360^\circ}{6} = 60^\circ$

$\left[\begin{array}{l}\because \text{ angle subtend by each side of a regular} \\ \text{polygon at the centre is given by } \frac{360^\circ}{n}\end{array}\right]$



Now, area of one shaded designed portion

= Area of sector AOB – Area of ΔAOB

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 28 \times 28 - \frac{\sqrt{3}}{4} \times 28 \times 28$$

[∵ ΔAOB is an equilateral Δ having side 28 cm]

$= 410.67 - 333.2 = 77.47 \text{ cm}^2$

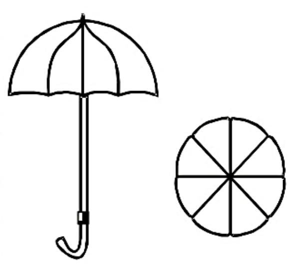
∴ Area of all six shaded designed portion

$= 6 \times 77.47 = 464.82 \text{ cm}^2$

Total cost of making the design = ₹ 464.82 × 0.35

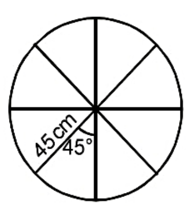
= ₹ 162.69.

11. ***An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.***



Sol. Here, umbrella is a flat circle of radius 45 cm and consisting 8 ribs.

∴ r = 45 cm

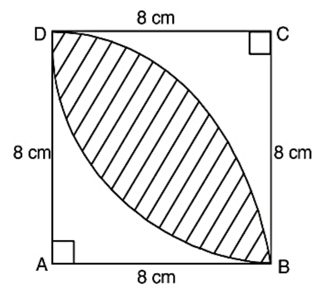


Angle between two consecutive ribs $= \frac{360^\circ}{8} = 45^\circ$

∴ Area between two consecutive ribs $= \frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 45 \times 45 = \frac{22275}{28} \text{ cm}^2$$

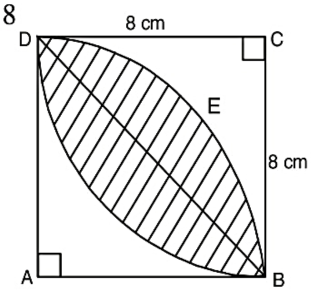
12. ***Calculate the area of the shaded region in the figure common between two quadrants of circle of radius 8 cm each.***



Sol. Area of quadrant ABED $= \frac{1}{4} \times \pi \times 8^2$

$= \frac{1}{4} \times \frac{22}{7} \times 8 \times 8$

$= \frac{352}{7} \text{ cm}^2$



Area of ΔABD $= \frac{1}{2} \times AB \times AD$

$= \frac{1}{2} \times 8 \times 8 = 32 \text{ cm}^2$

Area of shaded region

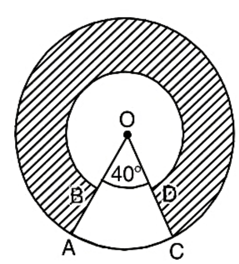
= 2 [Area of quadrant ABED – Area of ΔABD]

$= 2\left[\frac{352}{7} - 32\right] = 2\left[\frac{352 - 224}{7}\right]$

$= \frac{2 \times 128}{7} = \frac{256}{7}$

$= 36.57 \text{ cm}^2$

13. ***In fig., find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm, where ∠AOC = 40°. [Use $\pi = \frac{22}{7}$]***



Sol. Here, R = OA = 14 cm, r = OB = 7 cm and ∠AOC = 40°

∴ Reflex angle ∠AOC = Reflex angle ∠BOD

= 360° – 40 = 320°

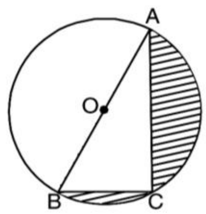
Area of the shaded region = Area of major sector AOC – Area of major sector BOD

$$= \frac{320^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14 - \frac{320^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{8}{9} \times \frac{22}{7}(196 - 49)$$

$$= \frac{8}{9} \times \frac{22}{7} \times 147 = 410.67 \text{ cm}^2$$

14. ***In fig., O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region. [Take π = 3.14]***



Sol. Here, AB = 13 cm and AC = 12 cm and AOB is the diameter.

Since angle in a semicircle is right angle *i.e.*, ∠BCA = 90°

∴ By using Pythagoras Theorem, we have

$$BC = \sqrt{AB^2 - AC^2} = \sqrt{169 - 144}$$

$$= \sqrt{25}$$

$$= 5 \text{ cm}$$

Radius of the circle $= \frac{13}{2}$ cm

Area of the shaded region

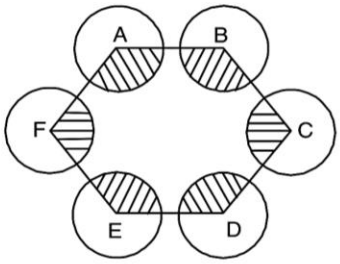
= Area of semicircle – Area of ΔABC

$$= \frac{1}{2} \times 3.14 \times \frac{13}{2} \times \frac{13}{2} - \frac{1}{2} \times 5 \times 12$$

$$= \frac{530.66}{8} - \frac{60}{2}$$

$$= 66.33 - 30 = 36.33 \text{ cm}^2$$

15. ***In figure, ABCDEF is any regular hexagon with different vertices A, B, C, D, E and F as the centres of circles with same radius 'r' are drawn. Find the area of the shaded portion.***



Sol. Each interior angle of hexagon

$$ABCDEF = \frac{(6-2)\times 180^\circ}{6} = 4 \times 30^\circ$$

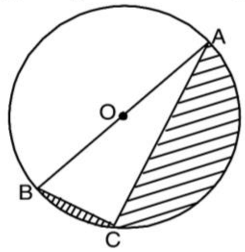
$$= 120^\circ$$

[∵ Each interior angle of regular polygon of n sides $= \frac{(n-2)\times 180^\circ}{n}$]

Area of 6 shaded regions = 6 × area of sector with radius r

$$= 6 \times \frac{120^\circ}{360^\circ} \times \pi r^2 = 2\pi r^2 \text{ sq. units}$$

16. ***In figure, AC = 24 cm, BC = 10 cm and O is the centre of the circle. Find the area of the shaded region. [Use π = 3.14]***



Sol. Here, AC = 24 cm, BC = 10 cm and AOB is a diameter.

Since angle in semicircle is a right angle *i.e.*, ∠ACB = 90°

$$\therefore \quad AB = \sqrt{AC^2 + BC^2}$$

$$= \sqrt{24^2 + 10^2} = \sqrt{576 + 100}$$

$$= \sqrt{676} = 26 \text{ cm}$$

∴ Radius of circle $= \frac{26}{2} = 13$ cm

Now, Area of shaded region

= Area of semicircle – Area of ΔABC

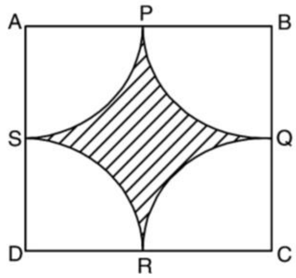
$$= \frac{1}{2}\pi r^2 - \frac{1}{2} \times BC \times AC$$

$$= \frac{1}{2} \times 3.14 \times 13 \times 13 - \frac{1}{2} \times 24 \times 10$$

$$= \frac{530.66}{2} - 120$$

$$= 265.33 - 120 = 145.33 \text{ cm}^2$$

17. ***Find the area of the shaded region in figure, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square ABCD of side 12 cm. [Use $\pi = 3.14$]***



Sol. Here, side of the square ABCD = 12 cm

Since P, Q, R and S are the mid-points of AB, BC, CD and DA

$\therefore$ AP = PB = BQ = QC = CR = RD = DS

$= SA = \frac{12}{2} = 6$ cm

Area of the shaded region

= Area of square ABCD – 4 × Area of quadrant APS

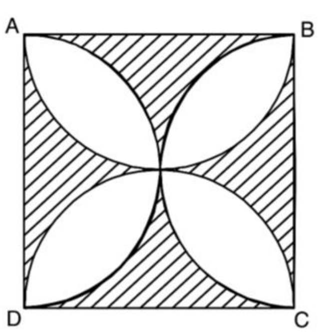
$= 12 \times 12 - 4 \times \frac{1}{4} \times 3.14 \times 6 \times 6$

$= 144 - 113.04$

$= 30.96\ \text{cm}^2$

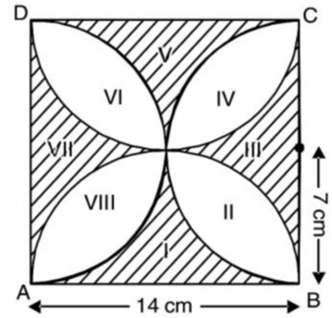
Hence, area of the shaded region is $30.96\ \text{cm}^2$.

18. ***In fig., ABCD is a square of side 14 cm. Semicircles are drawn with each side of square as diameter. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$***



Sol. Here, ABCD is a square of side 14 cm, semicircles are drawn with each side of square as diameter.

$\therefore$ Radius of semicircle = 7 cm



Let the square ABCD be divided into eight parts as shown in the diagram.

Now, area of one semicircle $= \frac{1}{2} \times \pi \times 7^2$

$\Rightarrow \quad \text{VIII} + \text{I} + \text{II} = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 77\ \text{cm}^2 \quad \ldots(i)$

Similarly, $\text{II} + \text{III} + \text{IV} = 77\ \text{cm}^2 \quad \ldots(ii)$

$\text{IV} + \text{V} + \text{VI} = 77\ \text{cm}^2 \quad \ldots(iii)$

$\text{VI} + \text{VII} + \text{VIII} = 77\ \text{cm}^2 \quad \ldots(iv)$

Adding (*i*), (*ii*), (*iii*) and (*iv*), we obtain

$\Rightarrow$ (I + II + III + IV + V + VI + VII + VIII) + (II + IV + VI + VIII) = 4 × 77

$\Rightarrow$ Area of square ABCD + area of unshaded region = 308

$\Rightarrow$ 14 × 14 + area of unshaded region = 308

$\Rightarrow$ Area of unshaded region

$= 308 - 196 = 112\ \text{cm}^2$

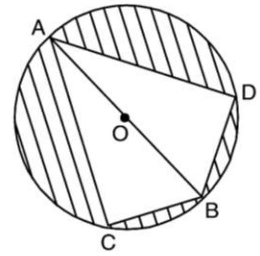
Area of shaded region *i.e.*, I + III + V + VII

= Area of square – Area of unshaded region

$= 196 - 112 = 84\ \text{cm}^2$

Hence, the area of the shaded region is $84\ \text{cm}^2$.

19. ***Find the area of the shaded region in figure, if BC = BD = 8 cm, AC = AD = 15 cm and O is the centre of the circle. [Take $\pi = 3.14$]***



Sol. AB is the diameter of the circle.

In ΔABC, $\angle ACB = 90°$

$\therefore \quad AB = \sqrt{AC^2 + BC^2} = \sqrt{15^2 + 8^2}$

$= \sqrt{225 + 64} = \sqrt{289} = 17$ cm

$\therefore$ Radius of the circle $= \frac{17}{2}$ cm

Now, area of $\Delta ACB = \frac{1}{2} \times AC \times BC$

$= \frac{1}{2} \times 15 \times 8 = 60\ cm^2$

Similarly, area of $\Delta ADB = 60\ cm^2$

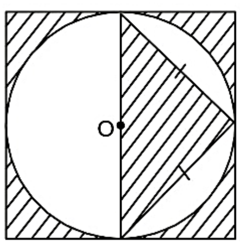
$\therefore$ Area of shaded region

$=$ Area of the circle $- [ar(\Delta ACB) + ar(\Delta ADB)]$

$= 3.14 \times \frac{17}{2} \times \frac{17}{2} - [60 + 60]$

$= 226.87 - 120 = 106.87\ cm^2$

20. ***In figure, a circle of radius 7 cm is inscribed in a square. Find the area of the shaded region.***



Sol. AB is the diameter of the circle

$\therefore \angle ACB = 90°$, Also, $AC = BC = x$ (say)

$\therefore$ ΔACB is an isosceles right triangle

$\therefore \quad AB = 14$ cm

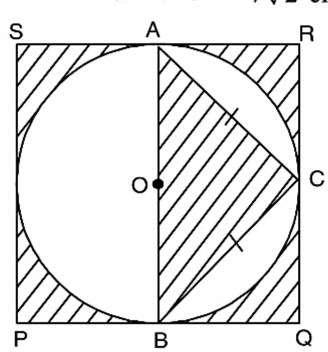
$AB^2 = AC^2 + BC^2$

$\Rightarrow \quad 14^2 = x^2 + x^2$

$\Rightarrow \quad 196 = 2x^2$

$\Rightarrow \quad 14 = \sqrt{2}x \Rightarrow x = \frac{14}{\sqrt{2}} = 7\sqrt{2}$ cm

$\therefore \quad AC = BC = 7\sqrt{2}$ cm



Area of shaded region

$=$ Area of square PQRS $-$ Area of circle $+$ Area of ΔACB

$= (\text{side})^2 - \pi r^2 + \frac{1}{2} \times AC \times BC$

$= 14 \times 14 - \frac{22}{7} \times 7 \times 7 + \frac{1}{2} \times 7\sqrt{2} \times 7\sqrt{2}$

$= 196 - 154 + 49 = 91\ cm^2$

21. ***Nandini made a design on a square chart paper ABCD, made of squares, semicircular arcs and arcs of quadrant of circles (see figure). Calculate the total shaded area in given figure.***

Sol. Here, ABCD is a square of side $= 28$ cm

There are four small squares each of side $= 14$ cm

Now, in each small square, we have a quadrant of radius 14 cm and a semicircle of radius 7 cm.

Now, required shaded area

$= 4 \times \{\text{area of quadrant} - \text{area of semicircle}\}$

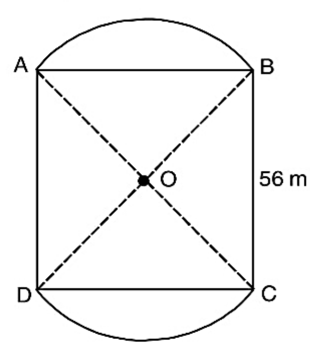
$= 4 \times \left\{\frac{1}{4} \times \pi \times 14 \times 14 - \frac{1}{2} \times \pi \times 7 \times 7\right\} cm^2$

$= 4\pi\left\{49 - \frac{49}{2}\right\} cm^2$

$= 4 \times \frac{22}{7} \times \frac{49}{2}$

$= 308\ cm^2$

22. ***In figure, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the area of the lawn and flower beds.***



Sol. $\because$ The diagonals of the square are perpendicular bisector of each other.

$$\angle AOB = 90° = \angle COD$$

$\therefore$ Radius of sector OAB $= \frac{1}{2}$ AC

$= \frac{1}{2} \times 56\sqrt{2}$

$= 28\sqrt{2}$ m

[$\because$ Diagonal of square $= \sqrt{2} \times$ side]

Also, radius of sector COD $= 28\sqrt{2}$ m

$\therefore$ Area of (sector AOB + sector COD)

$= 2 \times \frac{90°}{360°} \times \frac{22}{7} \times 28\sqrt{2} \times 28\sqrt{2}$

$= 2464$ cm^2

Now, area of (ΔAOD + ΔBOC)

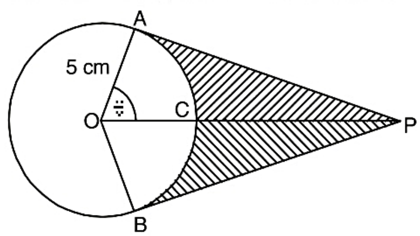
$= \frac{1}{2} \times$ area of square ABCD

$= \frac{1}{2} \times 56 \times 56 = 1568$ cm^2

$\therefore$ Total area $= 2464$ cm^2 $+ 1568$ cm^2

$= 4032$ cm^2

23. ***An elastic belt is placed around the rim of a pulley of radius 5 cm (see figure). From one point C on the belt, the elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also, find the shaded area. [Use $\pi = 3.14$ and $\sqrt{3} = 1.73$]***



Sol. Here, radius of the pulley *i.e.*, OA = OB = 5 cm and OP = 10 cm

Since tangent is perpendicular to the radius at the point of contact.

$\therefore$ $\angle OAP = \angle OBP = 90°$

Consider rt. $\angle$ed ΔOAP, we have

$$\sin(\angle OPA) = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2} = \sin 30°$$

$\Rightarrow$ $\angle OPA = 30°$

$\therefore$ $\theta = 180° - 90° - 30° = 60°$

[using $\angle$ sum property of a Δ]

$\therefore$ $\angle AOB = 2\theta = 2 \times 60° = 120°$

or Reflex $\angle AOB = 360° - 120° = 240°$

Now, the length of the belt in contact with the pulley

$= \frac{240°}{360°} \times 2 \times \frac{22}{7} \times 5$

$= \frac{2}{3} \times \frac{44}{7} \times 5 = \frac{440}{21} = 20.95$ cm.

Area of the shaded region

$= 2$(area of ΔOAP) $-$ area of sector OACBO

$= 2 \times \frac{1}{2} \times 5\sqrt{3} \times 5 - \frac{120°}{360°} \times \frac{22}{7} \times 5 \times 5$

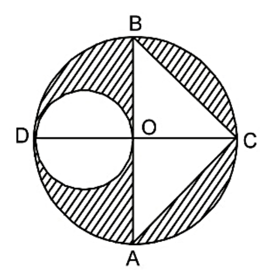
[$\because$ in rt. $\angle$ed ΔOAP, AP $= \sqrt{OP^2 - OA^2}$

$= \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}$ cm]

$= 43.25 - \frac{1}{3} \times \frac{22}{7} \times 25 = 43.25 - 26.19$

$= 17.06$ cm^2

24. ***In the given figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.***



Sol. Here, two diameters AB and CD are perpendicular to each other.

$\therefore$ OA = OB = OC = OD = 7 cm

Radius of smaller circle with OD as diameter $= \frac{7}{2}$ cm

$\therefore$ Area of the shaded region

= Area of big circle $-$ Area of smaller circle $-$ Area of ΔABC

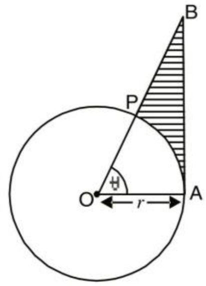
$= \frac{22}{7} \times 7 \times 7 - \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} - \frac{1}{2} \times 14 \times 7$

$= 154 - 38.5 - 49 = 66.5$ cm^2.

Long Answer Type Questions

25. *In figure, is shown a sector OAP of a circle with centre O, containing ∠θ. AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is*

$$r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180^\circ} - 1\right]$$



Sol. Since tangent is perpendicular to the radius at the point of contact *i.e.*, OA ⊥ AB

$\therefore \quad \angle OAB = 90°$

Now, in rt. ∠ed ΔOAB

$$\frac{AB}{r} = \tan\theta$$

$\Rightarrow \quad AB = r\tan\theta \quad \ldots(i)$

Also, $\frac{OB}{OA} = \sec\theta$

$\therefore \quad OB = OA\sec\theta$

$\therefore \quad OP + PB = r\sec\theta$

$\therefore \quad r + PB = r\sec\theta$

$\therefore \quad PB = r\sec\theta - r \quad \ldots(ii)$

Length of the arc AP $= \frac{\theta}{360°} \times 2\pi r$

$= \frac{\theta\pi r}{180°} \quad \ldots(iii)$

Now, the perimeter of the shaded region

$= AB + PB + AP$

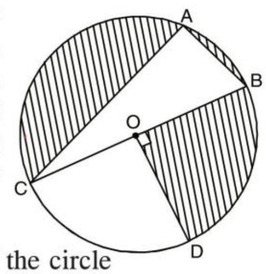
$= r\tan\theta + r\sec\theta - r + \frac{\theta\pi r}{180°}$

[using (*i*), (*ii*) and (*iii*)]

$= r(\tan\theta + \sec\theta - 1 + \frac{\theta\pi}{180°})$

$= r(\tan\theta + \sec\theta + \frac{\pi\theta}{180°} - 1)$

26. *In figure, O is the centre of the circle with AC = 24 cm, AB = 7 cm and ∠BOD = 90°. Find the area of the shaded region. [Use π = 3.14]*



Sol. ∵ BOC is a diameter of the circle

$\therefore \quad \angle BAC = 90°$

[angle in a semicircle is right angle]

So, by using Pythagoras Theorem, we have

$BC^2 = AB^2 + AC^2 = (7)^2 + (24)^2$

$= 49 + 576$

$= 625$

$\Rightarrow \quad BC = 25$

∴ Radius of circle with centre O $= \frac{25}{2}$ cm

$\Rightarrow \quad r = 12.5$ cm

Now, Area of shaded region = Area of circle – Area of quadrant COD – Area of ΔBAC

$= \pi r^2 - \frac{\pi r^2}{4} - \frac{1}{2} \times AB \times AC$

$= \frac{3}{4}\pi r^2 - \frac{1}{2} \times AB \times AC$

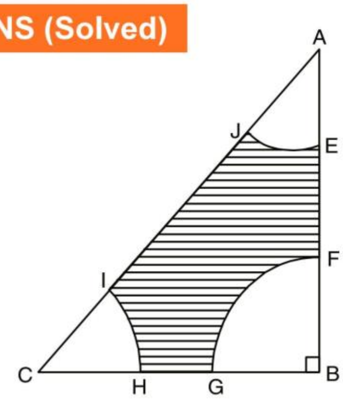
$= \left(\frac{3}{4}\right) 3.14 \times 12.5 \times 12.5 - \frac{1}{2} \times 7 \times 24$

$= 367.97 - 84 = 283.97 \text{ cm}^2$

Hence, area of shaded region = 283.97 cm²

APPLICATION BASED QUESTIONS (Solved)

1. *In the given figure, ABC is a right-angled triangle, right-angled at B, with AB = 14 cm and BC = 24 cm. With vertices A, B and C as centres arcs are drawn each of radius 7 cm. Find the area and perimeter of the shaded region.*



Sol. Here, ΔABC is a right-angled triangle, right-angled at B, with AB = 14 cm and BC = 24 cm

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{196 + 576}$$
$$= \sqrt{772} = 27.78 \text{ cm}$$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} BC \times AB$

$$= \frac{1}{2} \times 24 \times 14 = 168 \text{ cm}^2$$

For three sectors at A, B and C, we have

$$r = 7 \text{ cm}$$

and $\angle A + \angle B + \angle C = 180°$

$\therefore$ Area of three sectors $= \frac{180°}{360°} \times (7)^2 \times \frac{22}{7}$

$$= \frac{1}{2} \times 7 \times 7 \times \frac{22}{7}$$
$$= 77 \text{ cm}^2$$

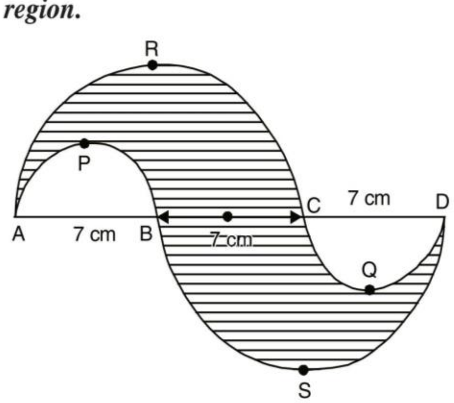
$\therefore$ Area of shaded region = 168 – 77 = 91 cm^2

Perimeter of the shaded region = EF + GH + IJ + circumferences at three vertices

$$= 14 + 24 + 27.78 - 6 \times 7 + \frac{180°}{360°} \times 2 \times \frac{22}{7} \times 7$$

$$= 65.78 - 42 + 22 = 45.78 \text{ cm}$$

2. ***In the given figure, APB and CQD are semi-circles of diameters 7 cm each, while ARC and BSD are semicircles of diameter 14 cm each. Find the perimeter and area of the shaded region.***



Sol. For bigger semicircles ARC and BSD,

$$\text{radii } (r) = 7 \text{ cm}$$

$\therefore$ Perimeter of two bigger semicircles

$$= 2 \times \frac{1}{2} \times 2\pi r$$
$$= 2 \times \frac{22}{7} \times 7 = 44 \text{ cm}$$

Area of two bigger semicircles $= 2 \times \frac{1}{2} \pi r^2$

$$= \pi r^2 = \frac{22}{7} \times 7 \times 7$$
$$= 154 \text{ cm}^2$$

For smaller semicircles APB and CQD,

$$\text{radii } (r) = \frac{7}{2} \text{ cm}$$

$\therefore$ Perimeter of two smaller semicircles

$$= 2 \times \frac{1}{2} \times 2\pi r$$
$$= 2 \times \frac{22}{7} \times \frac{7}{2} = 22 \text{ cm}$$

Area of two smaller semicircles $= 2 \times \frac{1}{2} \pi r^2$

$$= \pi r^2$$
$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$
$$= 38.5 \text{ cm}^2$$

Hence, perimeter of the shaded region is 44 + 22 *i.e.,* 66 cm and area of the shaded region is 154 + 38.5 *i.e.,* 192.5 cm^2.

3. ***In the given figure, three semicircles A, B and C having diameter 3 cm each, another semicircle E having diameter 9 cm and a circle D of diameter 4.5 cm. Find the area of the shaded region. [Use π = 3.14]***

Sol. For semicircles A, B and C, we have

diameter = 3 cm, radius = 1.5 cm

Area of semicircle A $= \frac{1}{2}\pi(1.5)^2$ cm^2

Area of semicircle B $= \frac{1}{2}\pi(1.5)^2$ cm^2

Area of semicircle C $= \frac{1}{2}\pi(1.5)^2$ cm^2

For circle D, we have

diameter = 4.5 cm $\Rightarrow$ radius $= \frac{4.5}{2}$ cm

Area of circle D $= \pi\left(\frac{4.5}{2}\right)^2$ cm^2

For semicircle E, we have

diameter = 9 cm $\Rightarrow$ radius = 4.5 cm

Area of semicircle E = $\frac{1}{2}\pi(4.5)^2$ cm^2

Now, Area of shaded region

= area of E + area of B – area of A – area of C – area of D

$= \frac{1}{2}\pi(4.5)^2 + \frac{1}{2}\pi(1.5)^2 - \frac{1}{2}\pi(1.5)^2 - \frac{1}{2}\pi(1.5)^2 - \pi\left(\frac{4.5}{2}\right)^2$

$= \frac{1}{2}\pi(4.5)^2 - \frac{1}{2}\pi(1.5)^2 - \frac{1}{2}\times\frac{\pi}{2}\times(4.5)^2$

$= \frac{1}{2}\times\pi\left\{(4.5)^2 - (1.5)^2 - \frac{(4.5)^2}{2}\right\}$

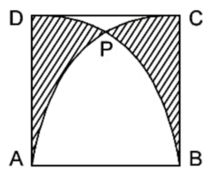
$= \frac{1}{2}\times 3.14\,[20.25 - 2.25 - 10.125]$

$= 1.57\,\{7.875\}$

$= 12.36375$ cm^2

ANALYZING, EVALUATING & CREATING TYPE QUESTIONS (Solved)

1. ***In the adjoining figure, ABCD is a square of side 6 cm. Find the area of the shaded region.***



Sol. From P, draw PQ ⊥ AB, join PA and PB.

In ΔAPQ and ΔBPQ, we have

AP = BP = 6 cm [radii of equal circles]

PQ = PQ [common]

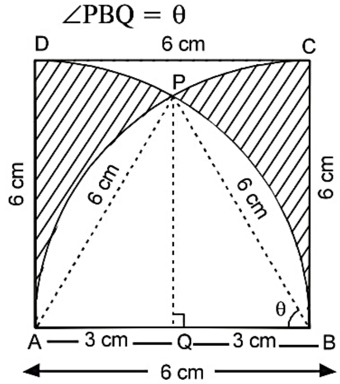
∠AQP = ∠BQP = 90° [by const.]

⇒ ΔAPQ ~ ΔBPQ [by RHS congruency axiom]

Therefore, $AQ = BQ = \frac{1}{2}AB$

⇒ AQ = BQ = 3 cm

Let ∠PBQ = θ



∴In rt. ΔQBP, ∠Q = 90°

⇒ $\cos\theta = \frac{QB}{PB}$

$\cos\theta = \frac{3}{6} = \frac{1}{2}$

⇒ cos θ = cos 60°

⇒ θ = 60°

Also, $\frac{PQ}{PB} = \sin 60°$

⇒ PQ = PB sin 60°

$PQ = 6\times\frac{\sqrt{3}}{2} = 5.196$ cm ...(*i*)

Now, area of sector BPA $= \frac{60°}{360°}\times\frac{22}{7}\times 6\times 6$

$= \frac{1}{6}\times\frac{22}{7}\times 6\times 6$

$= 18.857$ cm^2

Area of ΔBPQ $= \frac{1}{2}QB\times PQ$

$= \frac{1}{2}\times 3\times 5.196$ [using (*i*)]

$= 7.794$ cm^2

Area of the portion APB

= 2 × [Area of sector BPA – Area of ΔBPQ]

= 2 × [18.857 – 7.794] = 22.126 cm^2

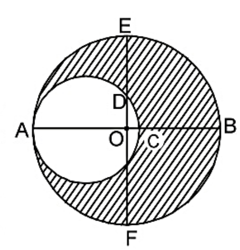
Area of the shaded portion = 2 × [Area of quadrant ABC – Area of the portion APB]

$= 2\times\left[\frac{90°}{360°}\times\frac{22}{7}\times 6\times 6 - 22.126\right]$

$= 2\times[28.286 - 22.126] = 12.32$ cm^2

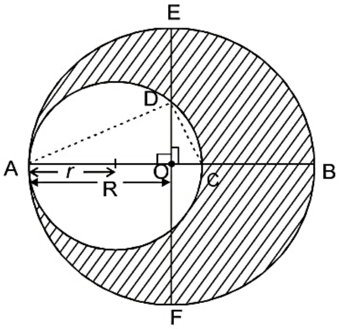
Hence, the required area of the shaded portion is 12.32 cm^2.

2. ***In the figure alongside, crescent is formed by two circles which touch at the point A, O is the centre of the bigger circle. If CB = 9 cm and ED = 5 cm, find the area of the shaded region. [Take π = 3.14]***



Sol. Suppose R and r be the radii of bigger and smaller circles, respectively

$$\Rightarrow \quad AB - AC = CB$$



$$\Rightarrow \quad 2R - 2r = 9$$

$$\Rightarrow \quad R - r = \frac{9}{2} = 4.5 \text{ cm} \qquad \ldots(i)$$

Join AD and CD

$$\Delta AOD \sim \Delta DOC$$

$$\Rightarrow \quad \frac{OD}{OA} = \frac{OC}{OD}$$

$$\Rightarrow \quad OD^2 = OA \times OC$$

$$\Rightarrow \quad (R - 5)^2 = R \times (R - 9)$$

$$[\because \text{ DE} = 5 \text{ cm and CB} = 9 \text{ cm}]$$

$$\Rightarrow \quad R^2 + 25 - 10R = R^2 - 9R$$

$$\Rightarrow \quad R = 25 \text{ cm}$$

From (i), we have $R - r = 4.5$

$$\Rightarrow \quad r = R - 4.5$$

$$= 25 - 4.5 = 20.5 \text{ cm}$$

Now, area of the shaded portion $= \pi R^2 - \pi r^2$

$$= \pi (R^2 - r^2) = \pi (R + r)(R - r)$$

$$= 3.14 \times (25 + 20.5)(25 - 20.5)$$

$$= 3.14 \times 45.5 \times 4.5 = 642.915 \text{ cm}^2$$

Hence, the required area of the shaded portion is 642.915 cm^2.

3. ***A car has two wipers do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle 120°. Find the total area cleaned at each sweep of the blades. (Take*** $\pi = \frac{22}{7}$***)***

Sol. Here, area cleaned by each blade of the wipers of a car is a sector of radius 21 cm and angle subtended at the centre is 120°.

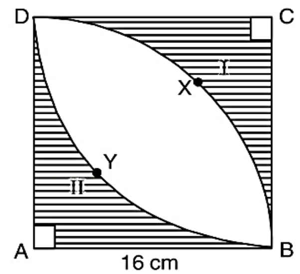
$\therefore$ Total area swept by two wipers in each sweep

$$= 2 \times \frac{\theta}{360^\circ} \times \pi r^2$$

$$= 2 \times \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$$

$$= 924 \text{ cm}^2$$

4. ***Calculate the area other than the area common between two quadrants of the circle of radius 16 cm each, which is shown as the shaded region in the figure.***



Sol. Let ABCD be the given square of side 16 cm.

Here, we have two quadrants of the circle of radius 16 cm each, say ABXD and CDYB.

Area of sector ABXD $= \frac{1}{4}\pi r^2 \quad [\because \angle DAB = 90°]$

$$= \frac{1}{4} \times \frac{22}{7} \times 16 \times 16 = \frac{1408}{7} \text{ cm}^2$$

$\therefore$ Area of the shaded region I

= Area of square ABCD – Area of sector ABXD

$$= 16 \times 16 - \frac{1408}{7}$$

$$= \frac{1792 - 1408}{7}$$

$$= \frac{384}{7} \text{ cm}^2$$

Similarly, area of the shaded region II $= \frac{384}{7} \text{ cm}^2$

$\therefore$ Total area of the shaded region

$$= \frac{384}{7} + \frac{384}{7}$$

$$= \frac{768}{7} = 109.71 \text{ cm}^2$$

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ASSIGNMENT-I

Q.1. Find the area of sector of a circle with radius 6 cm, if angle of sector is 60°.

(*a*) 18 cm^2 (*b*) 18.85 cm^2
(*c*) 19 cm^2 (*d*) 17.85 cm^2

Q.2. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then find a relation between R, R_1 and R_2.

(*a*) $R_1 + R_2 = R$ (*b*) $R_1 - R_2 = R$
(*c*) $R_1^2 + R_2^2 = R^2$ (*d*) $R_1^2 - R_2^2 = R^2$

Q.3. Find the area of the circle that can be inscribed in a square of side 6 cm.

(*a*) 6π cm^2 (*b*) 3π cm^2
(*c*) 9π cm^2 (*d*) π cm^2

Q.4. What is the perimeter of the sector with radius 10.5 cm and sector angle 60°?

(*a*) 24 cm (*b*) 36 cm
(*c*) 32 cm (*d*) 28 cm

Q.5. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

(*a*) 1254 cm^2 (*b*) 1254.96 cm^2
(*c*) 1250 cm^2 (*d*) 1245.60 cm^2

Q.6. The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 km per hour?

(*a*) 150 (*b*) 220
(*c*) 250 (*d*) 349

Q.7. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

(*a*) 15 cm^2 (*b*) 15.6 cm^2
(*c*) 14.6 cm^2 (*d*) 16 cm^2

Q.8. The side of a square is 10 cm. Find the area between inscribed and circumscribed circles of the square.

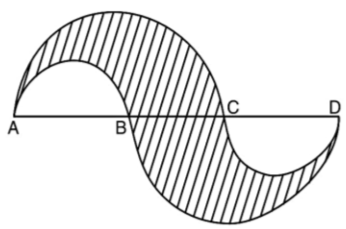
(*a*) 10π cm^2 (*b*) 20π cm^2
(*c*) 25π cm^2 (*d*) 15π cm^2

Q.9. Find area of the largest triangle that can be inscribed in a semicircle of radius '*r*' units.

(*a*) *r* square units (*b*) 2*r* square units
(*c*) r^2 square units (*d*) 4*r* square units

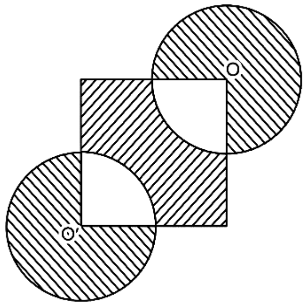
Q.10. Two circular pieces of equal radii and maximum areas, touching each other are cut from a rectangular cardboard of dimensions 14 cm × 7 cm. Find the area of the remaining cardboard. (Use $\pi = 22/7$)

Q.11. In figure, AC = BD = 7 cm and AB = CD = 1.75 cm. Semicircles are drawn as shown in the figure. Find the area of the shaded region. (Use $\pi = 22/7$)

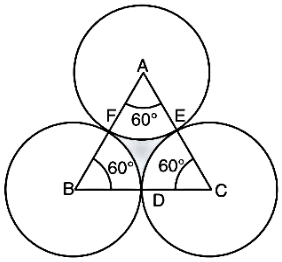


Q.12. A park is of the shape of a circle of diameter 7 m. It is surrounded by a path of width of 0.7 m. Find the expenditure of cementing the path. If its cost is ₹ 110 per square metre.

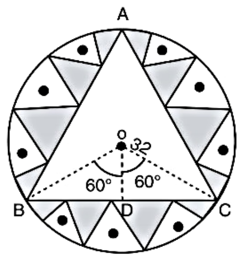
Q.13. In the given figure, the side of square is 28 cm and radius of each circle is half of the length of the side of the square, where O and O′ are centres of the circle. Find the area of shaded area.



Q.14. The area of an equilateral triangle is 1732.05 cm^2. About each angular point as centre, a circle is described with radius equal to half the length of the side of the triangle. Find the area of the triangle not included in the circles. (Use $\pi = 3.14$)



Q.15. On a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in figure. Find the area of the design (shaded region).



ASSIGNMENT-II

Q.1. Find area of a sector of angle p (in degrees) of a circle with radius R.

(*a*) $\frac{p}{360^\circ}\times\pi R^2$ (*b*) $\frac{p}{720^\circ}\times 2\pi R^2$

(*c*) Both (*a*) and (*b*) (*d*) Neither (*a*) nor (*b*)

Q.2. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then find the relation between R, R_1 and R_2.

(*a*) $R_1 + R_2 = R$ (*b*) $R_1 - R_2 = R$

(*c*) $R_1^2 + R_2^2 = R^2$ (*d*) $R_1^2 - R_2^2 = R^2$

Q.3. If area of sector of a circle is 112.04 cm^2 and area of the triangle formed by two radii and the chord is 72 cm^2, then find area of the corresponding segment.

(*a*) 40.04 cm^2 (*b*) 40 cm^2

(*c*) 44 cm^2 (*d*) 40.44 cm^2

Q.4. If area of major segment of a circle is 584.02 cm^2 and area of circle is 605 cm^2, then find the area of minor segment.

(*a*) 20 cm^2 (*b*) 20.20 cm^2

(*c*) 20.08 cm^2 (*d*) 200 cm^2

Q.5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the area of segment formed by corresponding chord of the arc.

(*a*) $441\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)\text{cm}^2$ (*b*) $441\left(\frac{\pi}{4}-\frac{\sqrt{3}}{6}\right)\text{cm}^2$

(*c*) $44\left(\frac{\pi}{6}-\frac{3}{4}\right)\text{cm}^2$ (*d*) $441\left(\frac{\pi}{6}-\frac{3}{4}\right)\text{cm}^2$

Q.6. Find the area of the square that can be inscribed in a circle of radius 8 cm.

(*a*) 120 cm^2 (*b*) 200 cm^2

(*c*) 128 cm^2 (*d*) 182 cm^2

Q.7. If the perimeter and the area of a circle are numerically equal, then find the radius of the circle.

(*a*) 3 units (*b*) 2 units

(*c*) 2.5 units (*d*) 4 units

Q.8. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. If the area of segment formed by corresponding chord of the arc is $441\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)\text{cm}^2$, then find the area of the major segment.

(*a*) $441\left(\frac{\sqrt{3}}{4}-\frac{\pi}{6}\right)\text{cm}^2$ (*b*) $441\left(\frac{\sqrt{3}}{4}-\frac{5\pi}{6}\right)\text{cm}^2$

(*c*) $441\left(\frac{3}{4}-\frac{5\pi}{6}\right)\text{cm}^2$ (*d*) $441\left(\frac{3}{4}-\frac{\pi}{6}\right)\text{cm}^2$

Q.9. A drain cover is made from a square metal plate of side 40 cm having 441 holes of diameter 1 cm each drilled in it. Find the area of the remaining square plate.

(*a*) 1253.5 cm^2 (*b*) 1543.8 cm^2

(*c*) 8790 cm^2 (*d*) None of these

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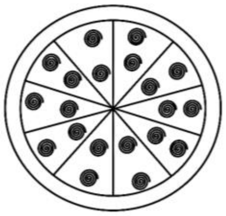
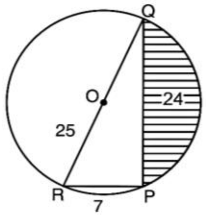
Q.10. A chord AB of a circle of radius 15 cm makes an angle of 60° at the centre. Find the area of major and minor segment. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

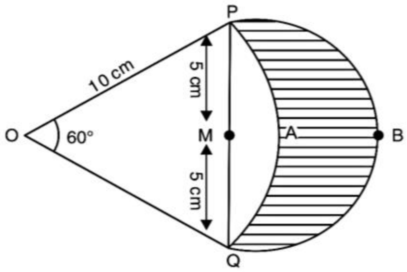
Q.11. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find:
(*i*) the total length of the silver wire required.
(*ii*) the area of each sector of the brooch.



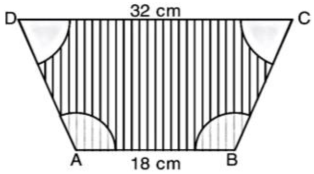
Q.12. Find the area of the shaded region in figure, if PQ = 24 cm and PR = 7 cm and O is the centre of the circle.



Q.13. Figure shows two arcs PAQ and PBQ. Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semicircle drawn on PQ as diameter with centre M. If OP = OQ = PQ = 10 cm, find the area of shaded region.



Q.14. In the given figure, ABCD is a trapezium with AB || DC, AB = 18 cm and DC = 32 cm and the distance between AB and DC is 14 cm. If arcs of equal radii 7 cm taking A, B, C and D have been drawn, then find the area of the shaded region.



Q.15. In the figure, ABC is an equilateral triangle inscribed in a circle of radius 4 cm with center O. Find the area of the coloured region.