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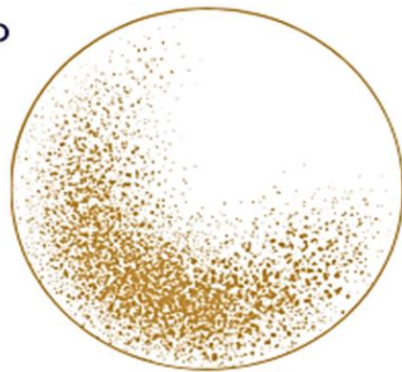


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Introducing the CBSE Class 10 Mathematics Area Related to Circles Mastery Kit - your go-to resource for mastering the concepts of circles and their associated areas. Specifically curated for CBSE Class 10 students, this kit is designed to simplify complex geometrical ideas and enhance your problem-solving skills.



### Key Features:

- 1. Complete Syllabus Coverage:** The Mastery Kit ensures comprehensive coverage of the CBSE Class 10 Mathematics syllabus for the topic "Area Related to Circles." All key concepts, theorems, and formulas are included for thorough preparation.
- 2. Conceptual Clarity:** Enjoy clear, concise explanations of concepts related to the area of circles. Complex theorems and formulas are broken down into simple, understandable components, making it easy for you to grasp and apply them.
- 3. Step-by-Step Solutions:** The kit provides detailed, step-by-step solutions to a variety of problems, guiding you through the application of formulas and theorems. This helps you understand the problem-solving process effectively.
- 4. Practice Exercises:** Reinforce your learning with a range of practice exercises that progress in difficulty. From basic problems to advanced applications, these exercises are designed to build your confidence and proficiency in dealing with problems related to the area of circles.
- 5. CBSE Exam Focus:** Tailored to align with CBSE examination patterns, the Mastery Kit equips you with the knowledge and skills necessary to excel in your Class 10 Mathematics exams.
- 6. Visual Aids and Diagrams:** Visual representations, diagrams, and illustrations are used to simplify abstract concepts, ensuring better comprehension and retention. Geometrical figures are visually presented to enhance your understanding.

Prepare for your CBSE Class 10 Mathematics exams confidently with the Area Related to Circles Mastery Kit. Elevate your mathematical prowess and set the stage for academic success.

# AREA RELATED TO CIRCLE



## Basic Concepts

□ **Perimeter and Area of a Circle:** A circle is collection of all such points of a plane, which are at constant distance from a fixed point of that plane. Fixed point 'O' is called centre and constant distance  $OA$  is called radius of the circle.

□ Perimeter of a circle is generally known as its circumference. Since the ratio of circumference 'C' of a circle to its diameter always remain constant and this constant is denoted the greek letter  $\pi$  (Pie).

$$\text{Therefore, } \frac{\text{circumference}}{\text{Diameter}} = \pi \Rightarrow \frac{C}{2r} = \pi$$

Hence,  $C = 2\pi r$ , where circumference =  $C$ , radius =  $r$

$$\text{Area of circle} = \pi r^2$$

$$\text{Area of semicircle} = \frac{\pi r^2}{2}$$

$$\text{Area of quadrant of circle} = \frac{\pi r^2}{4}$$

□ **Sector of a Circle and its Area:** The portion of the circular region enclosed by two radius and the corresponding arc is called a sector of the circle. In Fig. 11.2  $\angle AOB$  is called **angle of the sector** and  $OAPB$  is called sector.

□ **Minor Sector and Major Sector:** A sector in which angle of sector is less than  $180^\circ$  is called **Minor sector** while a sector having angle of sector more than  $180^\circ$  is called **Major sector**. In other way we can say if  $\theta$  is angle of sector of minor sector then  $(360^\circ - \theta)$  is the angle of sector of major sector.

□ **Area of Sector:** Let  $OAPB$  be a sector of a circle with centre 'O', radius  $r$  and angle of sector  $\theta$ . Now we assume a circular region to be a sector forming an angle of  $360^\circ$ . By applying the unitary methods, we can derive formula for area of sector as follows:

$$\therefore \text{When angle of sector is } 360^\circ, \text{ area of sector} = \pi r^2$$

$$\therefore \text{When angle of sector is } 1^\circ \text{ area of sector} = \frac{\pi r^2}{360^\circ}$$

$$\therefore \text{When angle of sector is } \theta^\circ \text{ area of sector} = \frac{\pi r^2 \theta}{360^\circ}$$

Hence, area of sector of angle  $\theta = \frac{\pi r^2 \theta}{360^\circ}$ , where  $r$  is radius and  $\theta$  is angle of sector in degree measure.

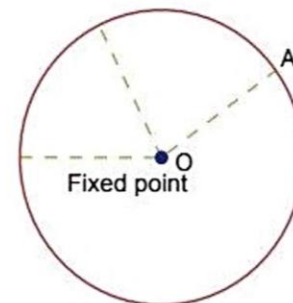


Fig. 11.1

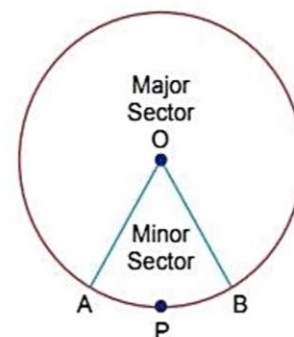


Fig. 11.2

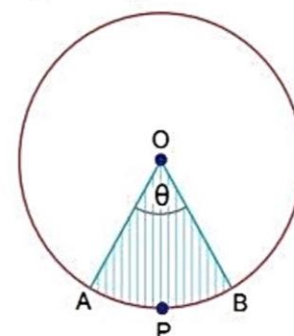


Fig. 11.3

- Length of an Arc of Sector of Angle ' $\theta$ ': By applying the unitary methods and taking the whole length of the circle as  $2\pi r$ , we can obtain the following result as follows:

Length of an arc of a sector of angle  $\theta$  ( $l$ )

$$\begin{aligned} &= \frac{\theta}{360^\circ} \times 2\pi r \\ &= \frac{2\pi r \theta}{360^\circ}, \text{ where } r \text{ is radius and } \theta \text{ is angle of corresponding sector.} \end{aligned}$$

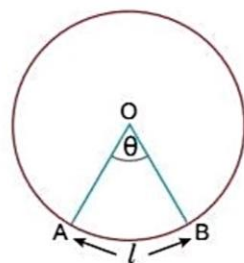


Fig. 11.4

- Area of Combination of Plane Figures:** We can find the area of combination of plane figures by adding the area of all such plane figures, one by one with the combination of which given plane figure formed.

For example, suppose we have to find the area of plane Fig. 11.5  $ABCDEF$  given alongside.

Area of figure  $ABCDEF$  = Area of square  $ABCE$  + ar ( $\Delta DCE$ )  
+ area of semicircle  $AFE$ .

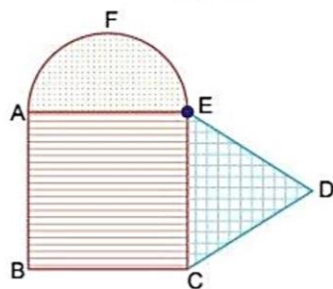


Fig. 11.5

- Segment:** The portion of the circular region enclosed between a chord and the corresponding arc is called a segment of the circle. In Fig. 11.6  $APBA$  (shaded area) is the segment.

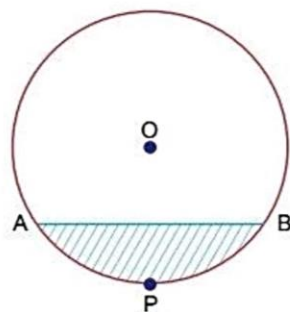


Fig. 11.6

- Minor Segment and Major Segment:** In Fig. 11.7,  $APB$  is minor segment while  $BQA$  is major segment.

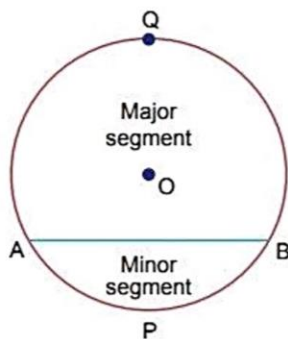


Fig. 11.7

- Area of Segment:** Let  $APB$  be segment of circle with centre  $O$  and radius  $r$ . From Fig. 11.8, area of segment  $APB$  = area of sector  $OAPB$  - ar ( $\Delta OAB$ )

$$\begin{aligned} &= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} \times AB \times OM \\ &= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} \times 2r \sin \frac{\theta}{2} \cdot r \cos \frac{\theta}{2} \\ &= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 2 \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2} \\ &= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin \theta \\ &= \frac{\pi r^2 \theta}{360^\circ} - r^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \quad \left[ \because 2 \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2} = \sin \theta \right] \end{aligned}$$

$[\angle AOB = \theta]$

$\sin \frac{\theta}{2} = \frac{AM}{r}$

$\Rightarrow AM = r \sin \frac{\theta}{2} \Rightarrow AB = 2r \sin \frac{\theta}{2}$

Also,  $\cos \frac{\theta}{2} = \frac{OM}{r} \Rightarrow OM = r \cos \frac{\theta}{2}$

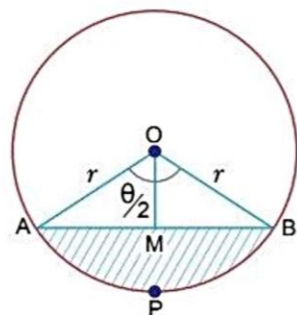


Fig. 11.8

### Important Facts/Tips:

- (i) When we write segment and sector, we will mean the minor segment and the minor sector respectively.
- (ii)  $\pi$  is an irrational number and its decimal expansion is non-terminating and non-repeating but for practical purpose, we will take the value of  $\pi$  as  $\frac{22}{7}$  or 3.14
- (iii)  $\frac{1}{4}$ th part of a circle or sector with central angle  $90^\circ$  is called quadrant of a circle.
- (iv) A minute hand of a clock makes  $6^\circ$  in 1 minute at the centre of the clock.
- (v) To find the perimeter of a semicircular disc (fig. 11.9) we add the length of diameter in the half circumference.

$$\begin{aligned} \text{i.e. Perimeter of disc } ACBA &= \text{Length of } ACB + \text{length of } AB \\ &= \pi r + d \text{ (diameter)} \end{aligned}$$

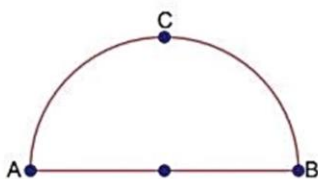


Fig. 11.9

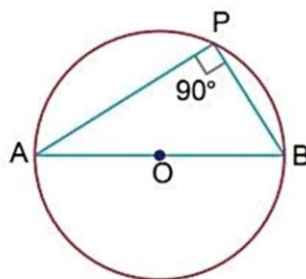


Fig. 11.10

(vi) Angle described by hour hand in one hour =  $\frac{360^\circ}{12} = 30^\circ$

(vii) Angle of semicircle is right angle. In fig. 11.10  $\angle APB = 90^\circ$ .

### Selected NCERT Questions

1. Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol. Let  $r$  be the radius of circle, then circumference = 22 cm

$$\Rightarrow 2\pi r = 22 \quad \Rightarrow \quad r = \frac{22}{2 \times \frac{22}{7}} = \frac{22 \times 7}{2 \times 22} = \frac{7}{2}$$

$$\begin{aligned} \text{Now, area of a quadrant of a circle} &= \frac{\pi r^2}{4} = \frac{\frac{22}{7} \times \left(\frac{7}{2}\right)^2}{4} = \frac{\frac{22}{7} \times \frac{49}{4}}{4} \\ &= \frac{154}{16} = \frac{77}{8} = 9\frac{5}{8} = 9.625 \text{ cm}^2 \end{aligned}$$

2. Figure 11.11 depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

Sol. The area of Gold region =  $\pi(10.5)^2 = \frac{22}{7} \times 110.25$   
 $= \frac{2425.5}{7} \text{ cm}^2 = 346.5 \text{ cm}^2$

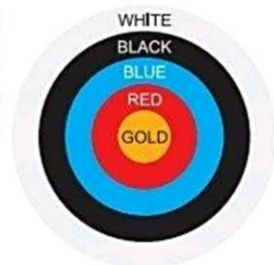


Fig. 11.11

$$= \frac{330}{360} \times 3.14 \times 16 \text{ cm}^2 = 46.05 \text{ cm}^2$$

$$= 46.1 \text{ cm}^2 \text{ (approx.)}$$

5. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Sol. Let  $R$  be the radius of required circle. Then, we have

$$\pi R^2 = \pi(8)^2 + \pi(6)^2$$

$$\Rightarrow \pi R^2 = 64\pi + 36\pi \Rightarrow \pi R^2 = 100\pi$$

$$\therefore R^2 = \frac{100\pi}{\pi} = 100 \Rightarrow R = 10 \text{ cm}$$

Hence, radius of required circle is 10 cm.

6. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Sol. Let  $R$  be the radius of required circle. Then, we have

$$2\pi R = 2\pi(19) + 2\pi(9)$$

$$\Rightarrow 2\pi R = 2\pi(19 + 9) \Rightarrow R = \frac{2\pi \times 28}{2\pi} = 28$$

Hence, the radius of required circle is 28 cm.

7. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Sol. Since the minute hand rotates through  $6^\circ$  in one minute, therefore, area swept by the minute hand in one minute is the area of a sector of angle  $6^\circ$  in a circle of radius 14 cm.

$$\text{Hence, the area swept in 5 minutes} = \frac{\theta}{360^\circ} \times \pi r^2 \times 5$$

$$= \frac{6^\circ}{360^\circ} \times \frac{22}{7} \times (14)^2 \times 5 = \frac{1}{60} \times 22 \times 28 \times 5$$

$$= \frac{154}{3} \text{ cm}^2 = 51 \frac{1}{3} \text{ cm}^2$$

8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (Fig. 11.13). Find

- (i) the area of that part of the field in which the horse can graze;  
 (ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use  $\pi = 3.14$ )

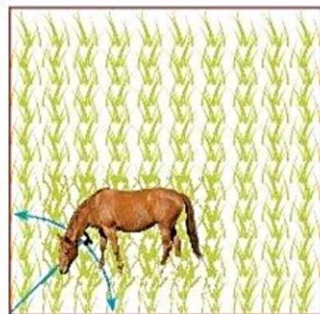


Fig. 11.13

Sol. Let the horse be tied at point  $O$  and the length of the rope is  $OH$  (Fig. 11.14). Thus,

- (i) The area of the part of the field in which the horse can graze = area of the quadrant of a circle ( $OAHB$ )

$$= \frac{\pi r^2}{4} = \frac{1}{4} \times 3.14 \times 5 \times 5 = \frac{78.5}{4} = 19.625 \text{ m}^2$$

- (ii) Now  $r = 10$  m and (Fig. 11.15)

$$\therefore \text{Required area} = \frac{\pi r^2}{4}$$

$$= \frac{3.14 \times (10)^2}{4} = \frac{3.14 \times 100}{4}$$

$$= \frac{314}{4} = 78.5 \text{ m}^2$$

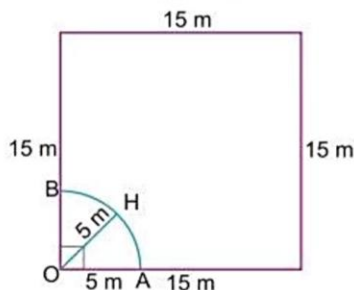


Fig. 11.14

Increase in the grazing area

$$= (78.5 - 19.625) \text{ m}^2$$

$$= 58.875 \text{ m}^2$$

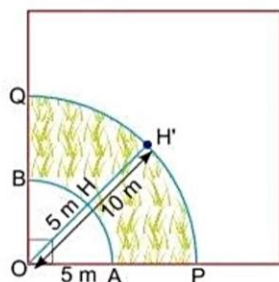


Fig. 11.15

9. An umbrella has 8 ribs which are equally spaced (Fig. 11.16). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

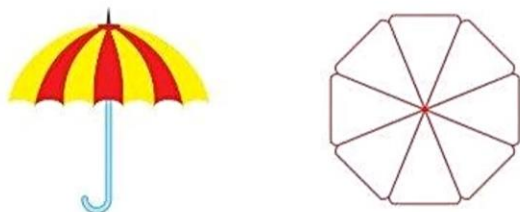


Fig. 11.16

**Sol.** We have,  $r = 45$  cm

$$\therefore \text{Area between two consecutive ribs} = \frac{1}{8} \times \pi r^2$$

$$= \frac{1}{8} \times \frac{22}{7} \times 45 \times 45 = \frac{11 \times 45 \times 45}{4 \times 7}$$

$$= \frac{22275}{28} = 795.54 \text{ cm}^2$$

10. Find the area of the shaded region in Fig. 11.17, where a circular arc of radius 6 cm has been drawn with vertex  $O$  of an equilateral triangle  $OAB$  of side 12 cm as centre. [CBSE (F) 2016]

**Sol.** We have, radius of circular region = 6 cm and each side of  $\triangle OAB = 12$  cm.

$$\therefore \text{Area of the circular portion}$$

$$= \text{area of circle} - \text{area of the sector}$$

$$= \pi r^2 - \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \pi r^2 \left(1 - \frac{\theta}{360^\circ}\right) = \frac{22}{7} \times (6)^2 \left(1 - \frac{60^\circ}{360^\circ}\right)$$

$$= \frac{22}{7} \times 36 \times \frac{5}{6} = \frac{22 \times 30}{7} = \frac{660}{7} \text{ cm}^2$$

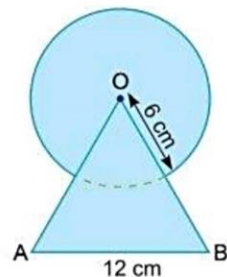


Fig. 11.17

Now, area of the equilateral triangle  $OAB$

$$= \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times (12)^2 = \frac{\sqrt{3}}{4} \times 144 = 36\sqrt{3} \text{ cm}^2$$

$\therefore$  Area of shaded region = area of circular portion + area of equilateral triangle  $OAB$

$$= \left(\frac{660}{7} + 36\sqrt{3}\right) \text{ cm}^2 = \frac{12}{7}(55 + 21\sqrt{3}) \text{ cm}^2$$

$$\begin{aligned} \therefore \text{Area of the design} &= \text{area of the circle} - \text{area of } \triangle ABC \\ &= \left( \frac{22528}{7} - 768\sqrt{3} \right) \\ &= (3218.28 - 1330.176) = 1888.1 \text{ cm}^2 \end{aligned}$$

13. Fig. 11.20, depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:
- the distance around the track along its inner edge.
  - the area of the track.

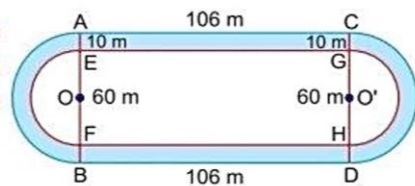


Fig. 11.20

Sol.

Here, we have

$$OE = O'G = 30 \text{ m}$$

$$AE = CG = 10 \text{ m}$$

$$OA = O'C = (30+10) \text{ m} = 40 \text{ m}$$

$$AC = EG = FH = BD = 106 \text{ m}$$

- (i) The distance around the track along its inner edge

$$\begin{aligned} &= EG + FH + 2 \times (\text{circumference of the semicircle of radius } OE = 30\text{cm}) \\ &= 106 + 106 + 2 \left( \frac{1}{2} \times 2\pi \times 30 \right) = 212 + 60\pi \\ &= 212 + 60 \times \frac{22}{7} = \left( 212 + \frac{1320}{7} \right) = \left( \frac{1484 + 1320}{7} \right) = \frac{2804}{7} = 400 \frac{4}{7} \text{ m} \end{aligned}$$

- (ii) Area of the track = area of the shaded region

= area of rectangle  $AEGC$  + area of rectangle  $BFHD$  + 2 (area of the semicircle of radius 40 m - area of the semicircle with radius 30 m)

$$\begin{aligned} &= [(10 \times 106) + (10 \times 106)] + 2 \left\{ \frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2 \right\} \\ &= 1060 + 1060 + \frac{22}{7} [(40)^2 - (30)^2] \\ &= 2120 + \frac{22}{7} \times 700 = 2120 + 2200 = 4320 \text{ m}^2 \end{aligned}$$

14. In Fig. 11.21,  $AB$  and  $CD$  are two diameters of a circle (with centre  $O$ ) perpendicular to each other and  $OD$  is the diameter of the smaller circle. If  $OA = 7$  cm, find the area of the shaded region.

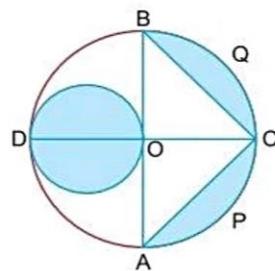


Fig. 11.21

Sol. Here, area of sector  $OBQC$  =  $\frac{90^\circ}{360^\circ} \times \pi \times (7)^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2} \text{ cm}^2$$

and, area of  $\triangle OBC$  =  $\frac{1}{2} \times OC \times OB = \frac{1}{2} \times 7 \times 7 = \frac{49}{2} \text{ cm}^2$

$\therefore$  Area of the segment  $BQC$  = area of sector  $OBQC$  - area of  $\triangle OBC$

$$= \frac{77}{2} - \frac{49}{2} = \frac{28}{2} = 14 \text{ cm}^2$$

Similarly, area of the segment  $APC$  =  $14 \text{ cm}^2$

Now, the area of the circle with  $OD$  as diameter =  $\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \text{ cm}^2$



$$\begin{aligned} \text{Hence, the total area of the shaded region} &= \left(14 + 14 + \frac{77}{2}\right) \text{cm}^2 = \left(28 + \frac{77}{2}\right) \text{cm}^2 \\ &= \left(\frac{56 + 77}{2}\right) \text{cm}^2 = \frac{133}{2} \text{cm}^2 = 66.5 \text{cm}^2 \end{aligned}$$

15. The area of an equilateral triangle  $ABC$  is  $17320.5 \text{ cm}^2$ . With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig. 11.22). Find the area of the shaded region. (Use  $\pi = 3.14$  and  $\sqrt{3} = 1.73205$ )

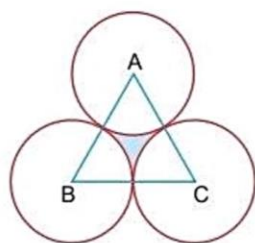


Fig. 11.22

- Sol.** Let each side of the equilateral triangle be  $x \text{ cm}$ . Then,  
 Area of equilateral triangle  $ABC = 17320.5 \text{ cm}^2$  (Given)

$$\Rightarrow \frac{\sqrt{3}}{4} x^2 = 17320.5 \Rightarrow \frac{1.73205}{4} x^2 = 17320.5$$

$$\Rightarrow x^2 = \frac{4 \times 17320.5}{1.73205} \Rightarrow x^2 = 40000$$

$$\therefore x = 200 \text{ cm}$$

$$\text{Thus, radius of each circle} = \frac{200}{2} \text{ cm} = 100 \text{ cm}$$

$$\begin{aligned} \text{Now, area of shaded region} &= \text{area of } \triangle ABC - 3 \times \text{area of a sector of angle } 60^\circ \text{ and radius } 100 \text{ cm} \\ &= 17320.5 - 3 \times \frac{60^\circ}{360^\circ} \times \pi \times (100)^2 \\ &= 17320.5 - \frac{1}{2} \times \pi \times 100 \times 100 \\ &= 17320.5 - 3.14 \times 5000 \\ &= 17320.5 - 15700 = 1620.5 \text{ cm}^2 \end{aligned}$$

16. On a square handkerchief, nine circular designs, each of radius  $7 \text{ cm}$  are made (see Fig. 11.23). Find the area of the remaining portion of the handkerchief.

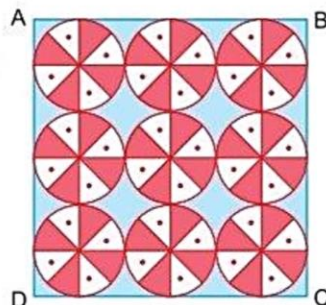


Fig. 11.23

- Sol.** Total area of circular design =  $9 \times$  area of one circular design  
 $= 9 \times \pi \times (7)^2$   
 $= 9 \times \frac{22}{7} \times 7 \times 7 = 1386 \text{ cm}^2$

$$\begin{aligned} \text{Now, each side of square } ABCD &= 3 \times \text{diameter of circular design} \\ &= 3 \times 14 = 42 \text{ cm} \end{aligned}$$

$$\therefore \text{Area of square } ABCD = (42)^2 = 1764 \text{ cm}^2$$

$$\begin{aligned} \therefore \text{Area of the remaining portion of handkerchief} &= \text{area of square } ABCD - \text{total area of circular design} \\ &= (1764 - 1386) \text{ cm}^2 = 378 \text{ cm}^2 \end{aligned}$$

17. In Fig. 11.24,  $OACB$  is a quadrant of a circle with centre  $O$  and radius  $3.5 \text{ cm}$ . If  $OD = 2 \text{ cm}$ , find the area of the (i) quadrant  $OACB$ , (ii) shaded region.

- Sol.** (i) Area of quadrant  $OACB = \frac{1}{4} \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times (3.5)^2$   
 $= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{8} \text{ cm}^2 = 9\frac{5}{8} \text{ cm}^2$

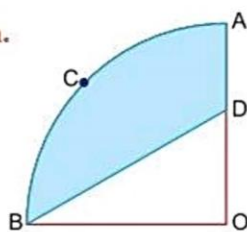


Fig. 11.24

(ii) Now, we find area of  $\triangle OBD$

$$\begin{aligned}\text{We have area of } \triangle OBD &= \frac{1}{2} \times OB \times OD \\ &= \frac{1}{2} \times 3.5 \times 2 = 3.5 \text{ cm}^2 = \frac{7}{2} \text{ cm}^2\end{aligned}$$

Hence, area of shaded region = area of quadrant  $OACB$  - area of  $\triangle OBD$

$$= \left(\frac{77}{8} - \frac{7}{2}\right) \text{cm}^2 = \left(\frac{77-28}{8}\right) \text{cm}^2 = \frac{49}{8} \text{cm}^2 = 6\frac{1}{8} \text{cm}^2$$

18. In Fig. 11.25  $ABPC$  is a quadrant of a circle of radius 14 cm and a semicircle is drawn with  $BC$  as diameter. Find the area of the shaded region.

Sol. In  $\triangle ABC$ , we have

$$\begin{aligned}BC &= \sqrt{(AC)^2 + (AB)^2} \text{ (By Pythagoras Theorem)} \\ &= \sqrt{(14)^2 + (14)^2} = \sqrt{196+196} = \sqrt{392} = 14\sqrt{2} \text{ cm}\end{aligned}$$

$$\text{Now, area of sector } ABPC = \frac{90^\circ}{360^\circ} \times \pi \times (14)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154 \text{ cm}^2$$

$$\text{and, area of } \triangle ABC = \frac{1}{2} \times AC \times AB = \frac{1}{2} \times 14 \times 14 = 98 \text{ cm}^2$$

$$\begin{aligned}\therefore \text{Area of segment } BPC &= \text{area of sector } ABPC - \text{area of } \triangle ABC \\ &= (154 - 98) \text{ cm}^2 = 56 \text{ cm}^2\end{aligned}$$

$$\text{Now, we have radius of semi-circle } BQC = \frac{14\sqrt{2}}{2} \text{ cm} = 7\sqrt{2} \text{ cm}$$

$$\therefore \text{Area of semi-circle} = \frac{1}{2} \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2} = 154 \text{ cm}^2$$

$$\begin{aligned}\text{Hence, area of the shaded region} &= \text{area of the semi-circle } BQC - \text{area of the segment } BPC \\ &= (154 - 56) \text{ cm}^2 = 98 \text{ cm}^2\end{aligned}$$

19. Calculate the area of the designed region in Fig. 11.26, which is common between the two quadrants of circles of radius, 8 cm each. [Competency Based Question]

Sol. Here, radius of each quadrant  $ABPD$  and  $BQDC$  = 8 cm

$$\begin{aligned}\text{Sum of areas of quadrants} &= 2 \times \frac{1}{4} \pi r^2 \\ &= \frac{1}{2} \times \frac{22}{7} \times (8)^2 = \frac{11}{7} \times 64 = \frac{704}{7} \text{ cm}^2\end{aligned}$$

$$\text{Now, area of the square } ABCD = 8 \times 8 = 64 \text{ cm}^2$$

Hence, area of designed region = area of shaded region

$$\begin{aligned}&= \text{sum of areas of quadrants} - \text{area of the square } ABCD \\ &= \frac{704}{7} - 64 = \frac{704 - 448}{7} = \frac{256}{7} = 36.57 \text{ cm}^2\end{aligned}$$

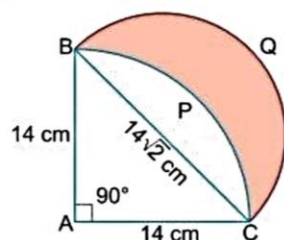


Fig. 11.25

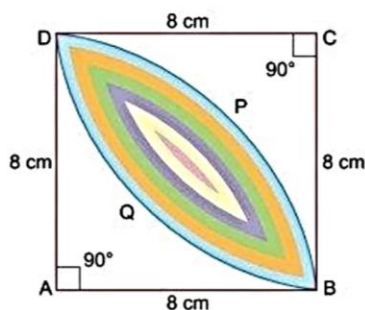


Fig. 11.26

## Multiple Choice Questions

Choose and write the correct option in the following questions.

1. David draws a circle with diameter 6 units. He draws another circle by increasing the radius of the previously drawn circle by 4 units. What would be the quotient if he divides the circumference of the newly formed circle by its diameter?

(a)  $8\pi$  (b)  $12\pi$  (c)  $\pi$  (d)  $2\pi$

2. If the area of a circle is  $154 \text{ cm}^2$ , then its perimeter is [NCERT Exemplar]

(a) 11 cm (b) 22 cm (c) 44 cm (d) 55 cm

3. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is [NCERT Exemplar]

(a) 31 cm (b) 25 cm (c) 62 cm (d) 50 cm

4. Savita has a lamp placed at the centre of her square yard, each side measuring 20 m. The light of lamp covers a circle of radius 10 m on yard. What area of the yard is not lit by the lamp?

[CBSE Question Bank]

(a)  $400\pi \text{ sq. m}$  (b)  $100\pi \text{ sq. m}$  (c)  $(40 - 10\pi) \text{ sq. m}$  (d)  $(400 - 100\pi) \text{ sq. m}$

5. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is [NCERT Exemplar]

(a) 56 cm (b) 42 cm (c) 28 cm (d) 16 cm

6. A fountain is enclosed by a circular fence of circumference 11 m and is surrounded by a circular path. The circumference of the outer boundary of the path is 16 m. A gardener increased the width of the pathway by decreasing the area enclosed by the fence such that the length of the fence is decreased by 3 m. The path is to be covered by the bricks which cost ₹ 125 per  $\text{m}^2$ . What will be the total cost, to the nearest whole number, required to cover the area by the bricks? (Use  $\pi = \frac{22}{7}$ )

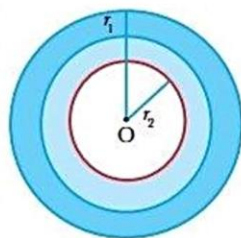


Fig. 11.27

(a) ₹ 1,910 (b) ₹ 9,878 (c) ₹ 39,772 (d) ₹ 79,545

7. The area of the square that can be inscribed in a circle of radius 8 cm is [NCERT Exemplar]

(a)  $256 \text{ cm}^2$  (b)  $128 \text{ cm}^2$  (c)  $64\sqrt{2} \text{ cm}^2$  (d)  $64 \text{ cm}^2$

8. Observe the figure below:

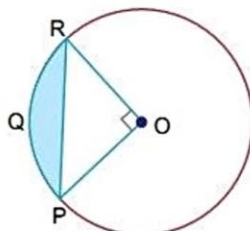


Fig. 11.28

What is the area of the segment PQR, if the radius of the circle is 7 cm? (Use  $\pi = \frac{22}{7}$ )

(a)  $14 \text{ cm}^2$  (b)  $17.3 \text{ cm}^2$  (c)  $28 \text{ cm}^2$  (d)  $91 \text{ cm}^2$

9. The area of the circle that can be inscribed in a square of side 6 cm is [NCERT Exemplar]

(a)  $36\pi \text{ cm}^2$  (b)  $18\pi \text{ cm}^2$  (c)  $12\pi \text{ cm}^2$  (d)  $9\pi \text{ cm}^2$

10. Which of these is equivalent to  $\pi$ ?
- (a)  $\frac{\text{Circumference}}{\text{Radius}}$  (b)  $\frac{\text{Circumference}}{\text{Diameter}}$   
 (c) Circumference  $\times$  Diameter (d) Circumference  $\times$  Radius
11. Area of the largest triangle that can be inscribed in a semi-circle of radius  $r$  units is [NCERT Exemplar]
- (a)  $r^2$ sq. units (b)  $\frac{1}{2}r^2$ sq. units (c)  $2r^2$ sq. units (d)  $\sqrt{2}r^2$ sq. units
12. An arc of a circle of radius 14 cm, subtends an angle of  $45^\circ$  at the centre as shown:

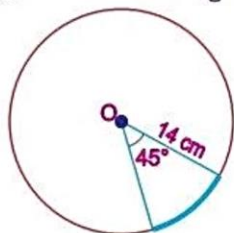


Fig. 11.29

Which of these options is correct?

- (a) The arc shown is a minor arc and its length is 5.5 cm.  
 (b) The arc shown is a major arc and its length is 77 cm.  
 (c) The arc shown is a major arc and its length is 38.5 cm.  
 (d) The arc shown is a minor arc and its length is 11 cm.
13. The perimeter of a circle is equal to that of a square, then the ratio of their areas is [NCERT Exemplar]
- (a) 22 : 7 (b) 14 : 11 (c) 7 : 22 (d) 11 : 14
14. In the figure below, the square  $JKLM$  is inscribed within a circle and  $\Delta JMN$  is a right-angled isosceles triangle. The point marked  $O$  is the centre of the circle.

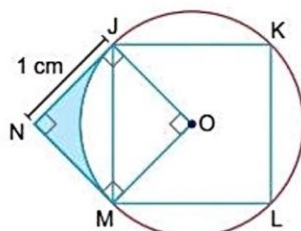


Fig. 11.30

What is the area of the shaded part of the figure?

[Competency Based Question]

- (a)  $\left(\frac{\pi}{4} - \frac{1}{2}\right) \text{ cm}^2$  (b)  $\left(\pi - \frac{1}{2}\right) \text{ cm}^2$  (c)  $\left(1 - \frac{\pi}{4}\right) \text{ cm}^2$  (d)  $(1 - \pi) \text{ cm}^2$
15. The diameter of a wheel is 1 m. The number of revolutions it will make to travel a distance of 22 km will be
- (a) 2,800 (b) 4,000 (c) 5,500 (d) 7,000

**Answers**

1. (c) 2. (c) 3. (d) 4. (d) 5. (c) 6. (a) 7. (b)  
 8. (a) 9. (d) 10. (b) 11. (a) 12. (d) 13. (b) 14. (c)  
 15. (d)

## Very Short Answer Questions

Each of the following questions are of 1 mark.

1. A thin wire is in the shape of a circle of radius 77 cm, it is bent into a square. Find the side of the square. (Use  $\pi = \frac{22}{7}$ )

Sol. Let side of square be  $x$  cm.

$$\therefore \text{Perimeter of the circle} = \text{Perimeter of the square}$$

$$2\pi r = 4x$$

$$\Rightarrow 2 \times \frac{22}{7} \times 77 = 4x \Rightarrow x = \frac{2 \times 22 \times 11}{4}$$

$$\Rightarrow x = 121 \text{ cm}$$

$$\therefore \text{Length of the side of the square} = 121 \text{ cm.}$$

2. If circumference and the area of a circle are numerically equal, find the diameter of the circle.

Sol. Given,  $2\pi r = \pi r^2$

$$\Rightarrow 2r = r^2 \Rightarrow r^2 - 2r = 0$$

$$\Rightarrow r(r-2) = 0 \quad \text{or} \quad r = 2$$

$$\text{i.e.,} \quad d = 4 \text{ units}$$

3. If the perimeter of a semi-circular protractor is 36 cm, find its diameter.

Sol. Perimeter of a semicircular protractor = perimeter of a semicircle  
 $= (2r + \pi r)$  cm

$$\text{Given,} \quad 2r + \pi r = 36$$

$$\Rightarrow r \left( 2 + \frac{22}{7} \right) = 36 \Rightarrow r \left( \frac{36}{7} \right) = 36 \Rightarrow r = 7 \text{ cm}$$

$$\text{Diameter} = 2r = 2 \times 7 = 14 \text{ cm.}$$

4. In Fig. 11.32, there is a sector of circle of radius 10.5 cm. Find the perimeter of the sector. (Take  $\pi = \frac{22}{7}$ ) [CBSE 2020 (30/2/1)]

Sol. Given, radius of the sector of the circle,  $r = 10.5$  cm.

$$\therefore \text{Perimeter of the sector} = \frac{\theta}{360^\circ} \times 2\pi r + 2r$$

$$= \frac{60^\circ}{360^\circ} \times \frac{2 \times 22}{7} \times 10.5 + 2 \times 10.5$$

$$= \frac{1}{6} \times 44 \times 1.5 + 21$$

$$= 11 + 21 = 32 \text{ cm}$$

$$\therefore \text{Perimeter of the sector} = 32 \text{ cm}$$

5. Find the area of a sector of a circle whose radius is  $r$  and length of the arc is  $l$ .

Sol. Area of a sector of a circle with radius  $r$

$$= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{\theta}{360^\circ} \times 2\pi r \frac{r}{2} = \frac{1}{2} lr \text{ sq. units} \quad (\because l = \frac{2\pi r\theta}{360^\circ})$$

6. Find the area of the circle inscribed in a square of side  $a$  cm.

Sol. Diameter of the circle =  $a$  cm

$$\Rightarrow \text{Radius} = \frac{a}{2} \text{ cm}$$

$$\Rightarrow \text{Area} = \pi \left( \frac{a}{2} \right)^2 = \frac{\pi a^2}{4} \text{ cm}^2$$

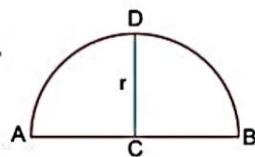


Fig. 11.31

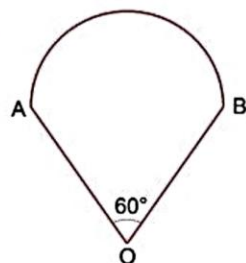


Fig. 11.32

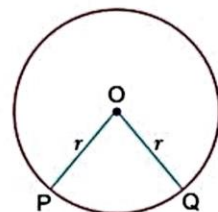


Fig. 11.33

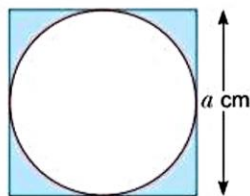


Fig. 11.34

7. What is the angle subtended at the centre of a circle of radius 10 cm by an arc of length  $5\pi$  cm?

Sol. Arc length of a circle of radius  $r = \frac{\theta}{360^\circ} \times 2\pi r$

$$\Rightarrow 5\pi = \frac{\theta}{360^\circ} \times 2\pi \times 10 \quad \text{or} \quad \frac{\theta}{360^\circ} = \frac{5\pi}{20\pi} = \frac{1}{4}$$

$$\Rightarrow \theta = \frac{360^\circ}{4} = 90^\circ$$

8. The radius of a wheel is 0.25 m. Find the number of revolutions it will make to travel a distance of 11 km.

Sol. Number of revolutions =  $\frac{11 \times 1000}{2 \times \frac{22}{7} \times 0.25} = \frac{11 \times 1000 \times 7}{11} = 7000$ .

### Short Answer Questions-I

Each of the following questions are of 2 marks.

1. A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of  $60^\circ$  at its centre. Find the radius of the circle. (Use  $\pi = \frac{22}{7}$ ) [CBSE 2020 (30/1/1)]

Sol. Let  $r$  be the radius and  $\theta$  be the angle subtended by the arc at the centre of the circle.

$\therefore$  Length of arc = length of piece of wire

$$\frac{\theta}{360} \times 2\pi r = 22$$

$$\Rightarrow \frac{60}{360} \times 2\pi r = 22 \quad \Rightarrow \quad \frac{\pi r}{3} = 22$$

$$\Rightarrow r = \frac{3 \times 22}{\pi} = \frac{3 \times 22}{\frac{22}{7}} = 21 \text{ cm}$$

$\therefore$  Radius = 21 cm



Fig. 11.35

2. A race track is in the form of a ring whose inner circumference is 352 m, and the outer circumference is 396 m. Find the width of the track.

Sol. Let the outer and inner radii of the ring be  $R$  m and  $r$  m respectively. Then,  $2\pi R = 396$  and  $2\pi r = 352$

$$\Rightarrow 2 \times \frac{22}{7} \times R = 396 \quad \text{and} \quad 2 \times \frac{22}{7} \times r = 352$$

$$\Rightarrow R = 396 \times \frac{7}{22} \times \frac{1}{2} \quad \text{and} \quad r = 352 \times \frac{7}{22} \times \frac{1}{2}$$

$$\Rightarrow R = 63 \text{ m} \quad \text{and} \quad r = 56 \text{ m}$$

Hence, width of the track =  $(R - r) = (63 - 56) = 7$  m

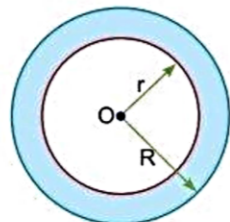


Fig. 11.36

3. In the Fig. 11.37,  $ABCD$  is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region. [Competency Based Question]

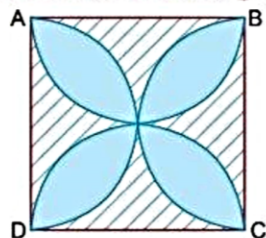


Fig. 11.37

Sol. Area of 1 segment = area of sector - area of triangle

$$\begin{aligned} &= \left(\frac{90^\circ}{360^\circ}\right)\pi r^2 - \frac{1}{2} \times 7 \times 7 \\ &= \frac{1}{4} \times \frac{22}{7} \times 7^2 - \frac{1}{2} \times 7 \times 7 \\ &= 14 \text{ cm}^2 \end{aligned}$$

Area of 8 segments =  $8 \times 14 = 112 \text{ cm}^2$

Area of the shaded region =  $14 \times 14 - 112$   
=  $196 - 112 = 84 \text{ cm}^2$

(each petal is divided into 2 segments)

[CBSE Marking Scheme 2021]

4. In Fig. 11.38, a square  $OABC$  is inscribed in a quadrant  $OPBQ$ . If  $OA = 15 \text{ cm}$ , find the area of the shaded region. (Use  $\pi = 3.14$ )  
[CBSE 2019, (30/2/1)]

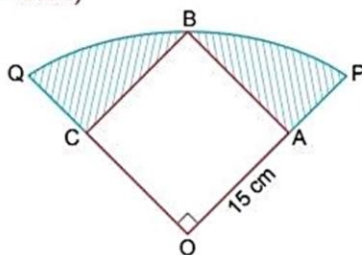


Fig. 11.38

Sol. Radius of quadrant =  $OB = \sqrt{15^2 + 15^2} = 15\sqrt{2} \text{ cm}$ .

Shaded area = Area of quadrant - Area of square

$$\begin{aligned} &= \frac{1}{4}(3.14)(15\sqrt{2})^2 - (15)^2 \\ &= (15)^2 (1.57 - 1) = 128.25 \text{ cm}^2 \end{aligned}$$

[CBSE Marking Scheme 2019 (30/2/1)]

### Short Answer Questions-II

Each of the following questions are of 3 marks.

1. In Fig. 11.39, there are shown two arcs  $PAQ$  and  $PBQ$ . Arc  $PAQ$  is a part of circle with centre  $O$  and radius  $OP$  while arc  $PBQ$  is a semi-circle drawn on  $PQ$  as diameter with centre  $M$ . If  $OP = PQ = 10 \text{ cm}$  show that area of shaded region is  $25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{ cm}^2$ .  
[CBSE (Delhi) 2016]

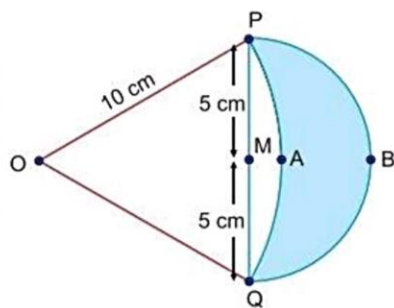


Fig. 11.39

Sol. Since  $OP = PQ = QO$

$\Rightarrow \Delta POQ$  is an equilateral triangle.

$\therefore \angle POQ = 60^\circ$

Area of segment  $PAQM$

$$\begin{aligned} &= \frac{\theta}{360^\circ} \pi r^2 - \frac{\sqrt{3}}{4} a^2 = \frac{60^\circ}{360^\circ} \pi \times 10^2 - \frac{\sqrt{3}}{4} \times 10^2 \\ &= \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right) \text{ cm}^2 = \left(\frac{50\pi}{3} - 25\sqrt{3}\right) \text{ cm}^2 \end{aligned}$$

Area of semicircle with  $M$  as centre =  $\frac{\pi}{2} (5)^2 = \frac{25\pi}{2} \text{ cm}^2$

$$\begin{aligned} \text{Area of shaded region} &= \frac{25\pi}{2} - \left( \frac{50\pi}{3} - 25\sqrt{3} \right) = \frac{25}{2}\pi - \frac{50}{3}\pi + 25\sqrt{3} \\ &= \frac{-25}{6}\pi + 25\sqrt{3} = 25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{cm}^2 \quad \text{Hence Proved.} \end{aligned}$$

2. In Fig. 11.40,  $O$  is the centre of a circle such that diameter  $AB = 13$  cm and  $AC = 12$  cm.  $BC$  is joined. Find the area of the shaded region. (Take  $\pi = 3.14$ )  
[CBSE 2016 (30/2)]

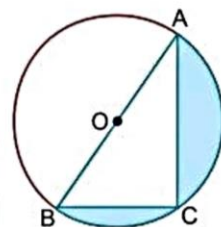


Fig. 11.40

Sol.

13) Q10 -

Radius of semicircle  $ACB = 13$  cm.  
Area of semicircle =  $\frac{1}{2} \pi r^2$

Its area =  $\frac{1 \times 3.14 \times 13 \times 13}{2} \text{cm}^2$

$$\frac{3.14 \times 169}{2} \text{cm}^2 = 530.66 \text{cm}^2$$

Semicircle subtend  $90^\circ$  at circle,  $\angle ACB = 90^\circ$

In  $\Delta ABC$  -  
 $AC^2 + BC^2 = AB^2 \Rightarrow 12^2 + BC^2 = 169 \text{cm}^2$   
 $\Rightarrow BC^2 = (169 - 144) \text{cm}^2 \Rightarrow BC^2 = 25 \text{cm}$   
 $BC = 5 \text{cm}$

Area of  $\Delta = \frac{1}{2} \times \text{Base} \times \text{Height}$

Area of  $\Delta ABC = \frac{1}{2} \times AC \times BC = \frac{1}{2} \times 12 \times 5 = 30 \text{cm}^2$

Area of shaded region =  $\frac{530.66 \text{cm}^2}{8} - 30 \text{cm}^2$

$$\Rightarrow \frac{(66.3325 - 30) \text{cm}^2}{8}$$

$$\Rightarrow \frac{36.3325 \text{cm}^2}{8}$$

[Topper's Answer 2016]

3. A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle  $120^\circ$ . Find the total area cleaned at each sweep of the blades. (Take  $\pi = \frac{22}{7}$ )  
[CBSE 2019 (30/3/1)]

Sol. Total area cleaned =  $2 \times$  area of sector

$$\begin{aligned} &= 2 \times \frac{\pi r^2 \theta}{360^\circ} && 1 \\ &= 2 \times \frac{22}{7} \times 21 \times 21 \times \frac{120^\circ}{360^\circ} && 1 \\ &= 924 \text{cm}^2 && \text{[CBSE Marking Scheme 2019 (30/3/1)]} \end{aligned}$$



4. Find the area of the shaded region in Fig. 11.41, where arcs drawn with centres  $A, B, C$  and  $D$  intersect in pairs at mid-points  $P, Q, R$  and  $S$  of the sides  $AB, BC, CD$  and  $DA$  respectively of a square  $ABCD$  of side 12 cm. [Use  $\pi = 3.14$ ] [NCERT Exemplar, CBSE 2018 (30/1/1)]

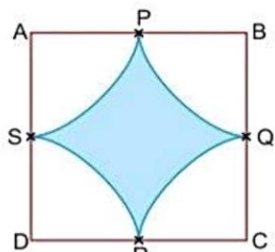


Fig. 11.41

Sol.

20) Given: side of square  $ABCD = 12$  cm.  
To find: shaded area.  
Shaded area + Area of 4 quadrants = Area of square.  
Area of square =  $s^2$  sq. units  
 $= 12^2 = 144$   $\text{cm}^2$   
Area of quadrant =  $\frac{1}{4} \times \pi r^2$  sq. units  
 $= \frac{1}{4} \times 3.14 \times \frac{12}{2} \times \frac{12}{2}$   
 $= 9 \times 3.14 = 28.26$   $\text{cm}^2$   
 $\Rightarrow$  Shaded area = Area of square - 4(Area of quadrant) sq. units  
 $= 144 - 4(28.26)$  sq. cm  
 $= 144 - 113.04$   
 $= 30.96$   $\text{cm}^2$ .

The area of the shaded region is  $30.96$   $\text{cm}^2$ . [Topper's Answer 2018]

5. In Fig. 11.42, three sectors of a circle of radius 7 cm, making angles of  $60^\circ, 80^\circ$  and  $40^\circ$  at the centre are shaded. Find the area of the shaded region. [CBSE 2019 (30/5/1)]

Sol. We have radius of circle = 7 cm

Area of sector

$$= \frac{\theta}{360^\circ} \times \pi r^2$$

Area of sector containing  $60^\circ$  angle

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (7)^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 49$$

$$= \frac{11 \times 7}{3} = \frac{77}{3} \text{ cm}^2$$

Also, area of sector containing  $80^\circ$  angle

$$= \frac{80^\circ}{360^\circ} \times \frac{22}{7} \times (7)^2$$

$$= \frac{2}{9} \times 22 \times 7 = \frac{308}{9} \text{ cm}^2$$

Again, area of sector containing  $40^\circ$  angle

$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times (7)^2$$

$$= \frac{1}{9} \times 22 \times 7 = \frac{154}{9} \text{ cm}^2$$

Area of total shaded region

$$= \frac{77}{3} + \frac{308}{9} + \frac{154}{9}$$

$$= \frac{231 + 308 + 154}{9}$$

$$= \frac{693}{9} = 77 \text{ cm}^2$$

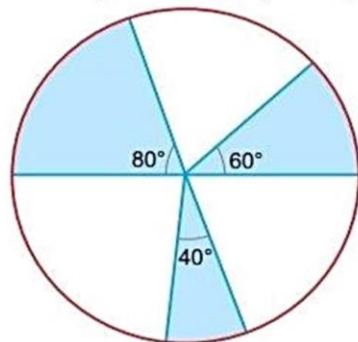


Fig. 11.42

6. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.

[CBSE 2017 (30/3)]

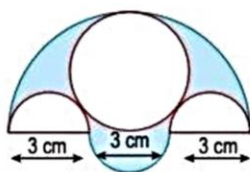


Fig. 11.43

Sol.

13. Area of shaded region = Area of semicircle with  $r = 4.5$  cm  
 + Area of semicircle with  $d = 3$  cm  
 - 2 × area of semicircle with  $d = 3$  cm  
 - area of circle with  $d = 4.5$  cm.

$$= \frac{1}{2} \times \pi \times (4.5)^2 + \left( \frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2 \right) - 2 \times \left( \frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2 \right) - \frac{\pi}{4} (4.5)^2$$

$$= \frac{2 \times 1}{4} \times \pi \times 20.25 - \frac{\pi \times 9}{4} - \pi \times 20.25$$

$$= \frac{\pi}{4} [ 2 \times 20.25 - 9 - 20.25 ]$$

$$= \frac{\pi}{4} [ 40.5 - 4.5 - 20.25 ]$$

$$= \frac{\pi}{4} [ 20.25 - 4.5 ]$$

$$= \frac{\pi}{4} ( 15.75 )$$

$$= \frac{22}{7} \times 2.25 \times 15.75$$

$$= \frac{22 \times 2.25}{2} \times 15.75$$

$$= \frac{24.75}{2}$$

$$= 12.375 \text{ cm}^2$$

∴ area of shaded region is 12.375 cm<sup>2</sup> [Topper's Answer 2017]

7. In Fig. 11.44, ABCD is a trapezium of area 24.5 sq. cm. In it, AD || BC, ∠DAB = 90°, AD = 10 cm and BC = 4 cm. If ABE is a quadrant of a circle, find the area of the shaded region.

(Take  $\pi = \frac{22}{7}$ )

[CBSE (AI) 2014]

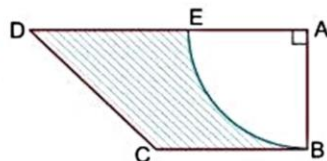


Fig. 11.44

Sol. Area of trapezium = 24.5 cm<sup>2</sup>

$$\frac{1}{2} [AD + BC] \times AB = 24.5$$

$$\frac{1}{2} [10 + 4] \times AB = 24.5$$

$$AB = 3.5 \text{ cm} \Rightarrow r = 3.5 \text{ cm}$$

$$\begin{aligned} \text{Area of quadrant} &= \frac{1}{4} \pi r^2 \\ &= 0.25 \times \frac{22}{7} \times 3.5 \times 3.5 = 9.625 \text{ cm}^2 \end{aligned}$$

$$\text{The area of shaded region} = 24.5 - 9.625 = 14.875 \text{ cm}^2$$

8. Two circles touch internally. The sum of their areas is  $116\pi \text{ cm}^2$  and distance between their centres is 6 cm. Find the radii of the circles. [CBSE (F) 2017, HOTS]

Sol. Let  $R$  and  $r$  be the radii of the circles [Fig. 11.45].

Then, according to question,

$$\begin{aligned} \Rightarrow \pi R^2 + \pi r^2 &= 116\pi \\ \Rightarrow R^2 + r^2 &= 116 \end{aligned} \quad \dots(i)$$

Distance between the centres = 6 cm

$$\begin{aligned} \Rightarrow OO' &= 6 \text{ cm} \\ \Rightarrow R - r &= 6 \end{aligned} \quad \dots(ii)$$

$$\text{Now, } (R + r)^2 + (R - r)^2 = 2(R^2 + r^2)$$

Using the equation (i) and (ii), we get

$$\begin{aligned} (R + r)^2 + 36 &= 2 \times 116 \\ \Rightarrow (R + r)^2 &= (2 \times 116 - 36) = 196 \\ \Rightarrow R + r &= 14 \end{aligned} \quad \dots(iii)$$

Solving (ii) and (iii), we get  $R = 10$  and  $r = 4$

Hence, radii of the given circles are 10 cm and 4 cm respectively.

9. In the given Fig. 11.46, the side of square is 28 cm and radius of each circle is half of the length of the side of the square where  $O$  and  $O'$  are centres of the circles. Find the area of shaded region. [CBSE Delhi 2017, HOTS]

Sol. Area of shaded region

= area of square + area of 2 major sectors having angle  $270^\circ$  at centre

$$\begin{aligned} &= \text{side} \times \text{side} + 2 \times \frac{\pi r^2 \theta}{360^\circ} \\ &= \left[ 28 \times 28 + 2 \times \frac{22}{7} \times 14 \times 14 \times \frac{270^\circ}{360^\circ} \right] \end{aligned}$$

$$= 28 \times 28 \left( 1 + \frac{11}{7} \times \frac{3}{4} \right)$$

$$= 28 \times 28 \left( 1 + \frac{33}{28} \right) = 1708 \text{ cm}^2$$

10. The area of a circular play ground is  $22176 \text{ cm}^2$ . Find the cost of fencing this ground at the rate of ₹ 50 per metre. [CBSE 2020(30/2/1)]

Sol. Given, Area of circular playground =  $22176 \text{ cm}^2$

$$\Rightarrow \pi r^2 = 22176$$

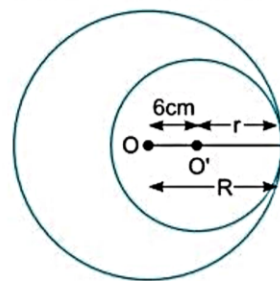


Fig. 11.45

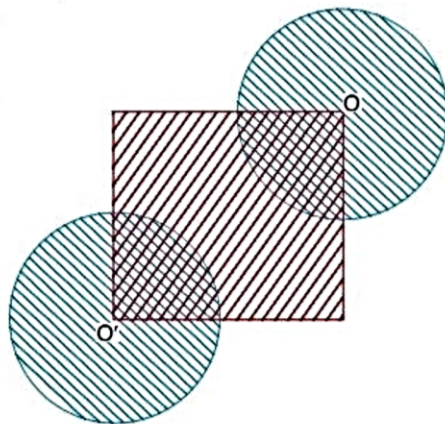


Fig. 11.46

$$\Rightarrow \frac{22}{7} \times r^2 = 22176$$

$$\Rightarrow r^2 = \frac{22176 \times 7}{22} = 7056$$

$$\Rightarrow r^2 = 7056 \Rightarrow r = \sqrt{7056} = 84$$

$$\therefore r = 84 \text{ cm}$$

Now, circumference of the circular ground =  $2\pi r$

$$= 2 \times \frac{22}{7} \times 84 = 2 \times 22 \times 12 = 528 \text{ cm}$$

$$\therefore \text{Total cost of fencing the ground} = \frac{528 \times 50}{100} = ₹ 264$$

11. In Fig. 11.47, a square  $OPQR$  is inscribed in a quadrant  $OAQB$  of a circle. If the radius of circle is  $6\sqrt{2}$  cm, find the area of the shaded region. [CBSE 2020(30/4/1)]

Sol. Given,  $OPQR$  is a square and  $OAQB$  is the quadrant of a circle of radius  $6\sqrt{2}$  cm.

$$\therefore OQ = 6\sqrt{2} \text{ cm}$$

We know that diagonal of square =  $\sqrt{2}a$ , where  $a$  is the side of square.

$$\Rightarrow 6\sqrt{2} = \sqrt{2}a \Rightarrow a = 6 \text{ cm}$$

$$\therefore \text{Area of square } OPQR = (6)^2 = 36 \text{ cm}^2$$

$$\text{Also, area of quadrant } OAQB = \frac{1}{4} \times \pi \times (6\sqrt{2})^2 = \frac{1}{4} \times \pi \times 72 = 18\pi \text{ cm}^2$$

$$\therefore \text{Area of shaded region} = \text{area of quadrant of circle } OAQB - \text{area of square } OPQR$$

$$= (18\pi - 36) \text{ cm}^2 = 18(\pi - 2) \text{ cm}^2$$

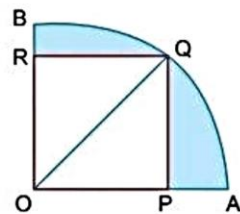


Fig. 11.47

## Long Answer Questions

Each of the following questions are of 5 marks.

1. An elastic belt is placed around the rim of a pulley of radius 5 cm. (Fig. 11.48). From one point  $C$  on the belt, the elastic belt is pulled directly away from the centre  $O$  of the pulley until it is at  $P$ , 10 cm from the point  $O$ . Find the length of the belt that is still in contact with the pulley.

Also find the shaded area. (Use  $\pi = 3.14$  and  $\sqrt{3} = 1.73$ )

[CBSE Delhi 2016]

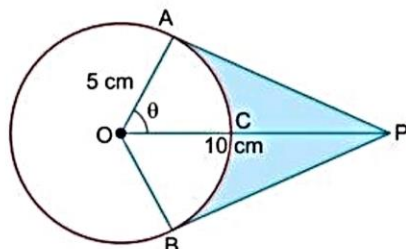


Fig. 11.48

Sol. In  $\triangle AOP$ ,  $\cos \theta = \frac{5}{10}$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

$$\Rightarrow \text{Reflex } \angle AOB = 360^\circ - 2 \times 60^\circ = 240^\circ$$

$$\therefore \text{Length of belt in contact with pulley} = \frac{\theta}{360^\circ} \times 2\pi r = \frac{2 \times 3.14 \times 5 \times 240}{360} = 20.93 \text{ cm}$$

Now,  $\frac{AP}{OA} = \tan 60^\circ$

$$PA = 5\sqrt{3} \text{ cm} = BP \quad (\text{Tangents from an external point are equal})$$

$$\text{Area } (\Delta OAP + \Delta OBP) = 2 \left( \frac{1}{2} \times 5 \times 5\sqrt{3} \right) = 25\sqrt{3} = 43.25 \text{ cm}^2$$

$$\text{Area of sector } OACB = \frac{\theta}{360^\circ} \pi r^2 = \frac{25 \times 3.14 \times 120}{360} = 26.17 \text{ cm}^2$$

$$\text{Shaded area} = 43.25 - 26.17 = 17.08 \text{ cm}^2$$

2. In the given figure,  $O$  is the centre of the circle with  $AC = 24$  cm,  $AB = 7$  cm and  $\angle BOD = 90^\circ$ . Find the area of the shaded region. [CBSE 2017(30/3/1)]

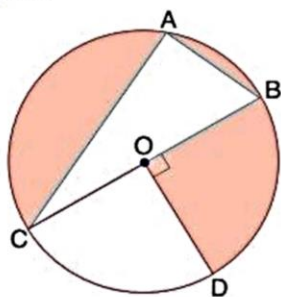
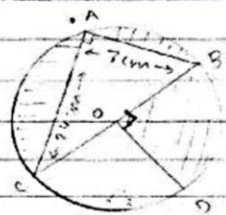


Fig. 11.49

Sol.



$\angle CAB = 90^\circ =$  angle subtended by diameter.

in right  $\Delta CAB$ ,

by pythagoras theorem,

$$AC^2 + AB^2 = BC^2$$

$$24^2 + 7^2 = BC^2$$

$$576 + 49 = BC^2$$

$$625 = BC^2 \quad \text{---(ignoring -ve value)}$$

$$\therefore BC = 25 \text{ cm.} = \text{diameter.}$$

$$\therefore \text{radius} = 12.5 \text{ cm or } \frac{25}{2} \text{ cm.}$$

∴ area of shaded region = area of semicircle + area of quadrant - area of  $\Delta ACB$

$$= \frac{1}{2} \times \pi r^2 + \frac{1}{4} \times \pi r^2 - \frac{1}{2} \times AB \times AC$$

$$= \frac{3}{4} \pi r^2 - \frac{1}{2} \times 7 \times 24$$

$$= \frac{3}{4} \times \frac{11}{7} \times \frac{625}{4} - 7 \times 12$$

$$= 368.3035 - 84$$

$$= 284.3035$$

$$\approx 284.3 \text{ cm}^2$$

∴ The area of shaded region is

$$284.3035 \text{ cm}^2$$

[Topper's Answer 2017]

3. In Fig. 11.50, a sector  $OAP$  of a circle with centre  $O$ , containing angle  $\theta$ .  $AB$  is perpendicular to the radius  $OA$  and meets  $OP$  produced at  $B$ . Prove that the perimeter of shaded region is  $r \left[ \tan \theta + \sec \theta + \frac{\pi \theta}{180^\circ} - 1 \right]$ . [CBSE (AI) 2016]

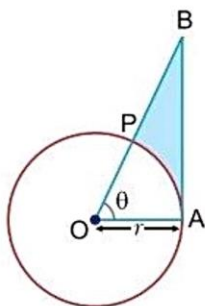


Fig. 11.50

Sol.

25) Given -  $OAP$  is sector of circle with centre  $O$ ,  $\angle POA = \theta$  and  $OA \perp AB$

To prove -  
 Perimeter of shaded region =  $r \left[ \tan \theta + \sec \theta + \frac{\pi \theta}{180} - 1 \right]$

Proof -  
 Perimeter of shaded region =  $BP + AB + \text{arc } \widehat{AP}$  — (IV)

Now -  
 $\tan \theta = \frac{AB}{r} \Rightarrow r \tan \theta = AB$  — (1)

$\sec \theta = \frac{OB}{r} \Rightarrow r \sec \theta = OB$

$OB - OP = BP \Rightarrow r \sec \theta - r = BP$  — (2)

Length of arc  $AP = \frac{\theta \times 2\pi r}{360} = \frac{\theta \times 2\pi r}{360} = \frac{\theta \pi r}{180}$  — (3)

Putting value from eq (1), (2), (3) in eq IV

$\Rightarrow$  Perimeter of shaded region  
 $= r \tan \theta + r \sec \theta - r + \frac{\theta \pi r}{180}$

$= r \left[ \tan \theta + \sec \theta + \frac{\theta \pi}{180} - 1 \right]$

Hence proved. [Topper's Answer 2016]

4. A chord  $PQ$  of a circle of radius 10 cm subtends an angle of  $60^\circ$  at the centre of circle. Find the area of major and minor segments of the circle. [CBSE Delhi 2017]

Sol. Area of minor segment

= area of minor sector having angle  $60^\circ$  at centre - area of equilateral  $\triangle OPQ$

$$= \frac{22}{7} \times 10 \times 10 \times \frac{60^\circ}{360^\circ} - \frac{\sqrt{3}}{4} \times 10 \times 10$$

$$= 10 \times 10 \left[ \frac{22}{7} \times \frac{1}{6} - \frac{\sqrt{3}}{4} \right]$$

$$= \frac{100}{84} (44 - 21\sqrt{3}) \text{ cm}^2 \quad \text{or} \quad \frac{25}{21} (44 - 21\sqrt{3}) \text{ cm}^2$$

Area of major segment = area of circle - area of minor segment

$$= \left[ \frac{22}{7} \times 10 \times 10 - \frac{25}{21} (44 - 21\sqrt{3}) \right]$$

$$= \frac{2200}{7} - \frac{25}{21} (44 - 21\sqrt{3}) = \frac{6600 - 1100 + 25 \times 21\sqrt{3}}{21}$$

$$= \frac{5500 + 25 \times 21\sqrt{3}}{21} = \frac{25}{21} (220 + 21\sqrt{3}) \text{ cm}^2$$

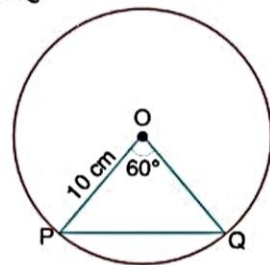


Fig. 11.51

5. In Fig 11.52, a circle is inscribed in an equilateral triangle  $ABC$  of side 12 cm. Find the radius of inscribed circle and the area of the shaded region. (Use  $\pi = 3.14$  and  $\sqrt{3} = 1.73$ )

[CBSE Delhi 2014]

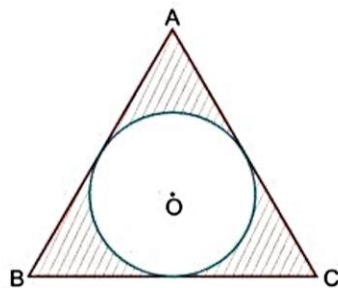


Fig. 11.52

Sol. **Construction:** Join  $OA$ ,  $OB$  and  $OC$

Draw  $OZ \perp BC$ ,  $OX \perp AB$  and  $OY \perp AC$ . (Fig 11.53)

Let the radius of the circle be  $r$  cm.

Area of  $\triangle ABC$  = area of  $\triangle AOB$  + area of  $\triangle BOC$  + area of  $\triangle AOC$

$$\frac{\sqrt{3}}{4} (\text{side})^2 = \frac{1}{2} \times AB \times OX + \frac{1}{2} \times BC \times OZ + \frac{1}{2} \times AC \times OY$$

$$\frac{\sqrt{3}}{4} (12)^2 = \frac{1}{2} \times 12 \times r + \frac{1}{2} \times 12 \times r + \frac{1}{2} \times 12 \times r$$

$$\frac{\sqrt{3}}{4} \times 12 \times 12 = 3 \times \frac{1}{2} \times 12 \times r$$

$$r = 2\sqrt{3} \text{ cm}$$

Area of shaded region = area of  $\triangle ABC$  - area of inscribed circle

$$= \left[ \frac{\sqrt{3}}{4} (12)^2 - \pi (2\sqrt{3})^2 \right] \text{ cm}^2$$

$$= \frac{\sqrt{3}}{4} \times 12 \times 12 - 3.14 \times 4 \times 3$$

$$= 1.73 \times 3 \times 12 - 3.14 \times 4 \times 3$$

$$= 62.28 - 37.68 = 24.6 \text{ cm}^2$$

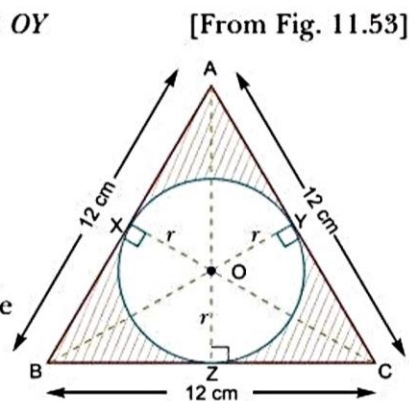


Fig. 11.53

[From Fig. 11.53]

6. In Fig. 11.54, from a rectangular region  $ABCD$  with  $AB = 20$  cm, a right triangle  $AED$  with  $AE = 9$  cm and  $DE = 12$  cm, is cut off. On the other end, taking  $BC$  as diameter, a semicircle is added on outside the region. Find the area of the shaded region. [Use  $\pi = 3.14$ ]  
 [CBSE (F) 2014, HOTS]

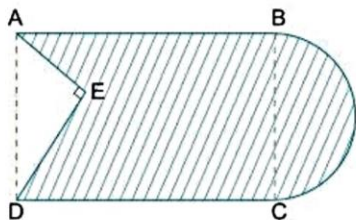


Fig. 11.54

Sol. Area of shaded region

= area of rectangle – area of triangle + area of semicircle.

In right  $\triangle ADE$

$$AD^2 = AE^2 + DE^2$$

$$AD = \sqrt{9^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225} = 15 \text{ cm}$$

$$\text{Area of } \triangle AED = \frac{1}{2} \times DE \times AE = \frac{1}{2} \times 12 \times 9 = 54 \text{ cm}^2$$

In semicircle at  $BC$ , diameter =  $BC = 15$  cm.

$$\text{Radius of semicircle} = \frac{1}{2} \times 15 = 7.5 \text{ cm}$$

$$\text{Area of semicircle} = \frac{\pi r^2}{2} = \frac{3.14 \times 7.5 \times 7.5}{2} = 88.31 \text{ cm}^2$$

$$\text{Area of rectangle} = AB \times BC = 20 \times 15 = 300 \text{ cm}^2$$

$$\text{Area of shaded region} = 300 + 88.31 - 54 = 334.31 \text{ cm}^2$$

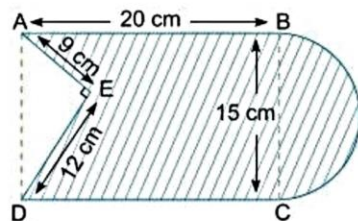


Fig. 11.55

## Case Study-based Questions

Each of the following questions are of 4 marks.

1. Read the following and answer any four questions from (i) to (v).

A brooch is a small piece of jewellery which has a pin at the back so it can be fastened on a dress, blouse or coat.

Designs of some brooches are shown below. Observe them carefully.

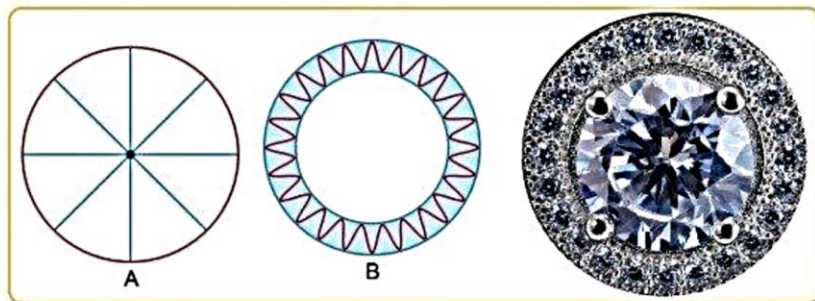


Fig. 11.56

**Design A:** Brooch A is made with silver wire in the form of a circle with diameter 28 mm. A wire used for making 4 diameters which divide the circle into 8 equal parts.

**Design B:** Brooch B is made of two colours gold and silver. Outer part is made with gold. The circumference of silver part is 44 mm and the gold part is 3 mm wide everywhere.

[CBSE Question Bank]



Refer to design A

(i) The total length of silver wire required is

- (a) 180 mm      (b) 200 mm      (c) 250 mm      (d) 280 mm

(ii) The area of each sector of the brooch is

- (a)  $44 \text{ mm}^2$       (b)  $52 \text{ mm}^2$       (c)  $77 \text{ mm}^2$       (d)  $68 \text{ mm}^2$

Refer to design B

(iii) The circumference of outer part (golden) is

- (a) 48.49 mm      (b) 82.2 mm      (c) 72.50 mm      (d) 62.86 mm

(iv) The difference of areas of golden and silver parts is

- (a)  $18\pi$       (b)  $44\pi$       (c)  $51\pi$       (d)  $64\pi$

(v) A boy is playing with brooch B. He makes revolution with it along its edge. How many complete revolutions must it take to cover  $80\pi \text{ mm}$  ?

- (a) 2      (b) 3      (c) 4      (d) 5

**Sol.** (i) We have diameter of the wire = 28 mm

$$\Rightarrow d = 28 \text{ mm}$$

$$\therefore \text{Radius } (r) = 14 \text{ mm}$$

$$\begin{aligned} \text{Total length of silver wire required} &= \text{circumference of circle} + 4 \times \text{diameter of the circle} \\ &= 2\pi r + 4 \times d \\ &= 2 \times \frac{22}{7} \times 14 + 4 \times 28 \\ &= 88 + 112 = 200 \text{ mm} \end{aligned}$$

$\therefore$  Option (b) is correct.

(ii) Angle of the each sector =  $\frac{360^\circ}{8} = 45^\circ$

$$\begin{aligned} \therefore \text{Area of each sector of the brooch} &= \frac{\theta}{360^\circ} \times \pi r^2 \\ &= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14 \\ &= \frac{1}{8} \times 22 \times 2 \times 14 = 77 \text{ mm}^2 \end{aligned}$$

$\therefore$  Option (c) is correct.

(iii) Now, refer to design B, we have

Circumference of silver part = 44 mm

$$2\pi r = 44 \quad (\text{where } r \text{ is the radius of inner circle})$$

$$\Rightarrow r = \frac{44}{2\pi}$$

$$\Rightarrow r = \frac{44}{2 \times \frac{22}{7}} = 7 \text{ mm}$$

$$\begin{aligned} \therefore \text{Radius of outer part } (R) &= (7 + 3) \\ &= 10 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Circumference of outer part} &= 2\pi R \\ &= 2\pi \times 10 = 20\pi \\ &= 20 \times \frac{22}{7} \\ &= \frac{440}{7} = 62.86 \text{ mm} \end{aligned}$$

∴ Option (d) is correct.

$$\begin{aligned} \text{(iv) Difference of areas of golden and silver part} &= \pi R^2 - \pi r^2 \\ &= \pi [(10)^2 - (7)^2] \\ &= \pi(100 - 49) \\ &= 51\pi \text{ mm}^2 \end{aligned}$$

∴ Option (c) is correct.

$$\begin{aligned} \text{(v) Circumference of outer part(circular)} &= 2\pi R \\ &= 2\pi \times 10 = 20\pi \end{aligned}$$

$$\therefore \text{Number of revolution} = \frac{80\pi}{20\pi} = 4.$$

∴ Option (c) is correct.

2. Gauri got her wall painted in a different manner. The whole wall was painted pink, leaving a circular portion of diameter 4.2 m. In this circle, she asked the painter to paint a beautiful scenery in one half of it by drawing a full size triangle possible (as shown in the figure). In the other half of the circle, she drew the largest circle possible and pasted some of her pictures. The remaining part of the big circle was filled with dotted design.

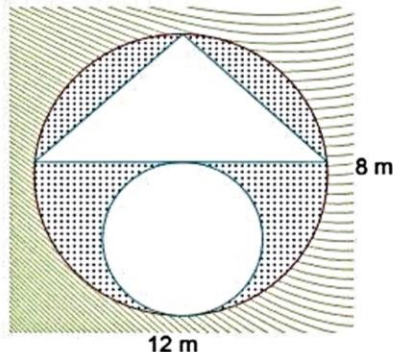


Fig. 11.57

Based on above information answer the following questions.

- (i) (a) What is the radius of the circle allotted for her pictures?  
 (b) What is the area of the wall that is not painted pink?  
 (ii) What is the area of the dotted design?

Sol. (i) (a) Radius of the circle allotted for pictures =  $\frac{2.1}{2} = 1.05 \text{ m}$   
 (b) Area of the wall not painted pink =  $12 \times 8 - \pi(2.1)^2 = (96 - 4.41\pi) \text{ m}^2$   
 (ii) Area of dotted design =  $\pi(2.1)^2 - \text{Area of triangle} - \text{Area of small circle}$   

$$= \pi(2.1)^2 - 4.41 - \pi\left(\frac{2.1}{2}\right)^2 = \pi\left[4.41 - \frac{4.41}{4}\right] - 4.41$$
  

$$= 4.41\left[\pi\left(1 - \frac{1}{4}\right) - 1\right] = 4.41\left(\frac{3}{4} \times \frac{22}{7} - 1\right)$$
  

$$= 4.41 \times \frac{19}{14} = 6 \text{ m}^2 \text{ (approx.)}$$

**PROFICIENCY EXERCISE**

■ Objective Type Questions:

[1 mark each]

1. Choose and write the correct option in each of the following questions.

- (i) If the perimeter of a semicircular protractor is 36 cm then its diameter is  
 (a) 14 cm            (b) 16 cm            (c) 18 cm            (d) 12 cm
- (ii) In the figure given below,  $O$  is the centre of the circle.  $PR$  and  $RQ$  are chords of the circle. The radius of the circle is 5 cm.  $PR = 8$  cm,  $QR = 6$  cm and  $\angle PRQ = 90^\circ$ .

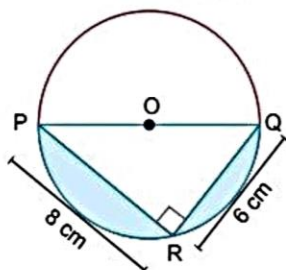


Fig. 11.58

What is the approximate area of the shaded region?

[CBSE Question Bank]

- (a)  $\left(\frac{25}{4}\pi - 24\right) \text{cm}^2$     (b)  $\left(\frac{25}{2}\pi - 24\right) \text{cm}^2$     (c)  $\left(\frac{25}{4}\pi\right) \text{cm}^2$

- (d)  $\left(\frac{25}{2}\pi\right) \text{cm}^2$

- (iii) In the figure below  $RT = 1$  cm and  $OQ = 3$  cm.

[Competency Based Question]

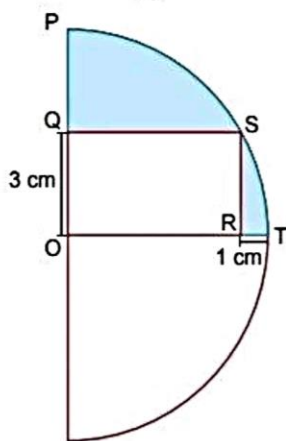


Fig. 11.59

What is the area of the shaded region?

[CBSE Question Bank]

- (a)  $(12.5\pi - 12) \text{cm}^2$     (b)  $(6.25\pi - 12) \text{cm}^2$     (c)  $(12.5\pi - 15) \text{cm}^2$     (d)  $(6.25\pi - 15) \text{cm}^2$

- (iv) The area of a sector of a circle with radius 14 cm and central angle  $45^\circ$  is

- (a)  $76 \text{cm}^2$             (b)  $77 \text{cm}^2$             (c)  $66 \text{cm}^2$             (d)  $55 \text{cm}^2$

- (v) Which of these is equivalent to the sum of the lengths of arc corresponding to the minor and major segment of a circle of radius 12 cm?

- (a)  $24\pi$  cm            (b)  $48\pi$  cm            (c)  $12\pi$  cm            (d)  $144\pi$  cm

■ Very Short Answer Questions:

[1 mark each]

2. What is the diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm?

[NCERT Exemplar]

- The area of a circle is  $220 \text{ cm}^2$ . What will be the area of a square inscribed in it?
- The circumference of a circle is 50 cm. What will be the side of a square that can be inscribed in the circle?
- If the area of a circle increases from  $9\pi$  to  $16\pi$ , then what will be the ratio of the circumference of the first circle to the second circle?
- A wire can be bent in the form of a circle of radius 35 cm. If it is bent in the form of a square, then what will be its area?

■ **Short Answer Questions-I:**

[2 marks each]

- What is the ratio of areas of two circles whose circumferences are in the ratio 3 : 4?
- If the area of a sector of a circle is  $\frac{5}{18}$ th of the area of that circle, then find the central angle of the sector.
- If a circle is inscribed in a square, what is the ratio of the area of the circle and the square?
- What is the length of an arc in terms of  $\pi$  that subtends an angle of  $72^\circ$  at the centre of a circle of radius 10 cm?
- In a circle of radius 8 cm, an arc subtends an angle of  $108^\circ$  at the centre. What is the area of the sector in terms of  $\pi$ ?
- Find the perimeter of a square circumscribing a circle of radius  $a$  cm.
- What is the angle subtended at the centre of a circle of radius 5 cm by an arc length  $4\pi$  cm?
- Find the area of a quadrant of a circle whose circumference is 616 cm.
- Find the radius of a semicircular protractor if its perimeter is 36 cm.

■ **Short Answer Questions-II:**

[3 marks each]

- The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 48 hours. [CBSE 2018, (C) (30/1)]
- The side of a square is 10 cm. Find the area between inscribed and circumscribed circles of the square. [CBSE 2018, (C) (30/1)]
- Find the area of the shaded region in Fig. 11.60, if  $ABCD$  is a rectangle with sides 8 cm and 6 cm and  $O$  is the centre of circle. (Take  $\pi = 3.14$ ) [CBSE 2019, (30/1/1)]

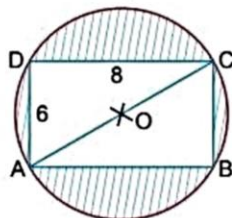


Fig. 11.60

- Find the area of the segment shown in Fig. 11.61, if radius of the circle is 21 cm and  $\angle AOB = 120^\circ$  (Use  $\pi = \frac{22}{7}$ ) [CBSE 2019, (30/1/2)]

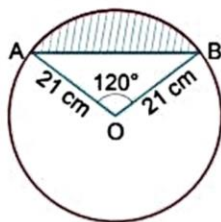


Fig. 11.61

20. In Fig. 11.62,  $ABCD$  is a square with side  $2\sqrt{2}$  cm and inscribed in a circle. Find the area of the shaded region. (Use  $\pi = 3.14$ ) [CBSE 2019, (30/2/1)]

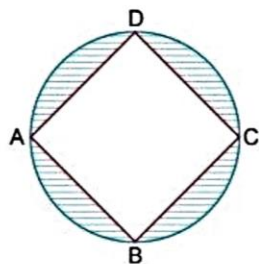


Fig. 11.62

21. A chord of a circle of radius 14 cm subtends an angle of  $60^\circ$  at the centre. Find the area of the corresponding minor segment of the circle. (Use  $\pi = \frac{22}{7}$  and  $\sqrt{3} = 1.73$ ) [CBSE 2019, (30/3/3)]
22. In Fig. 11.63, two concentric circles with centre  $O$ , have radii 21 cm and 42 cm. If  $\angle AOB = 60^\circ$ , find the area of the shaded region. [CBSE 2019, (30/4/2)]

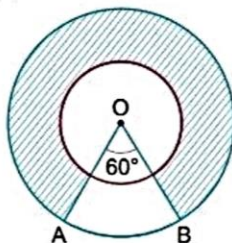


Fig. 11.63

23. In Fig. 11.64, find the area of the shaded region, where  $ABCD$  is a square of side 14 cm in which four semi-circles of same radii are drawn as shown. (Take  $\pi = 3.14$ ) [CBSE 2019, (C) (30/1/2)]

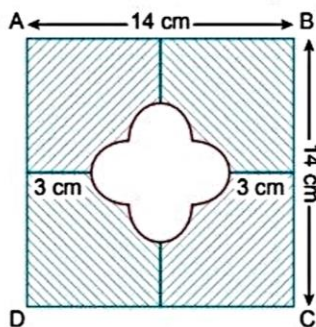


Fig. 11.64

24. The area of an equilateral triangle is  $49\sqrt{3}$  cm<sup>2</sup>. Taking each vertex as centre, circles are described with radius equal to half the length of the side of the triangle. Find the area of the part of the triangle not included in the circles. (Take  $\sqrt{3} = 1.73$ ,  $\pi = \frac{22}{7}$ )
25. In Fig. 11.65, the boundary of shaded region consists of four semicircular arcs, two smallest being equal. If diameter of the largest is 14 cm and that of the smallest is 3.5 cm, calculate the area of the shaded region. (Use  $\pi = \frac{22}{7}$ )

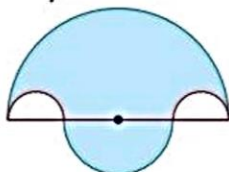


Fig. 11.65

26. The inner circumference of a circular track is 132 m. The track is 2.5 m wide everywhere. Calculate the cost of putting up a fence along the outer circle at the rate of ₹3.50 per metre.
27. A race track is in the form of a ring whose inner and outer circumferences are 44 cm and 66 cm respectively. Find the width of the track.
28. A circular park is surrounded by a road 28 m wide. Find the area of the road if the circumference of the park is 880 m.
29. Find the area of the flower bed (with semicircular ends.) in Fig. 11.66.

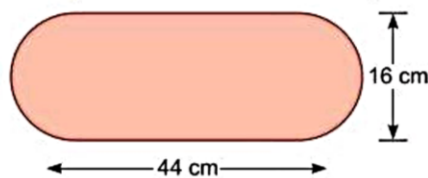


Fig. 11.66

30. Find the area of the shaded field shown in Fig. 11.67.

[NCERT Exemplar]

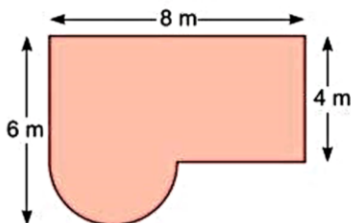


Fig. 11.67

31. In Fig. 11.68, arcs are drawn by taking vertices  $A, B,$  and  $C$  of an equilateral triangle of side 10 cm to intersect the sides  $BC, CA$  and  $AB$  at their respective mid-points  $D, E$  and  $F$ . Find the area of the shaded region. (Use  $\pi = 3.14$ )

[NCERT Exemplar]

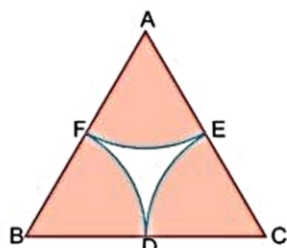


Fig. 11.68

32. Prove that the area of a circular path of uniform width  $h$  surrounding a circular region of radius  $r$  is  $\pi h(2r + h)$ .
33. In Fig. 11.69,  $ABCD$  is a rectangle with  $AB = 14$  cm and  $BC = 7$  cm. Taking  $DC, BC$  and  $AD$  as diameter, three semicircles are drawn. Find the area of the shaded region.

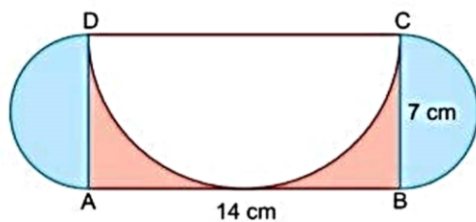


Fig. 11.69

34. In Fig. 11.70, three semicircles  $A, B$  and  $C$  are drawn having diameters 3 cm each and a circle  $D$  is drawn with diameter 4.5 cm. Calculate (i) the area of the shaded region. (ii) the cost of painting the shaded region at the rate of 25 paise per  $\text{cm}^2$ .

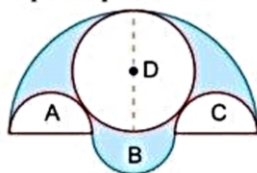


Fig. 11.70

■ Long Answer Questions:

[5 marks each]

35. Three circles each of radius 7 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between the circles.
36. The area of a circular playground is  $88704 \text{ m}^2$ . Find the cost of fencing this ground at the rate of ₹65 per metre.
37. The diameter of front and rear wheels of a tractor are 80 cm and 2 m respectively. Find the number of revolutions that rear wheel will make in covering a distance in which the front wheel makes 1400 revolutions. [NCERT Exemplar]
38. Find the area of the segment of a circle of radius 12 cm whose corresponding sector has a central angle of  $60^\circ$ . (Use  $\pi = 3.14$ )
39. Find the difference of the area of a sector of angle  $90^\circ$  and its corresponding major sector of a circle of radius 9.8 cm.
40. Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending an angle of  $90^\circ$  at the centre.
41. On a square cardboard sheet of area  $784 \text{ cm}^2$ , four congruent circular plates of maximum size are placed such that each circular plate touches the other two plates and each side of the square sheet is tangent to two circular plates. Find the area of the square sheet not covered by the circular plates.
42. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if area of the circle is  $2464 \text{ cm}^2$ . (Hint: radius of circle =  $\frac{1}{2}$  diagonal)
43. Find the number of revolutions made by a circular wheel of area  $6.16 \text{ m}^2$  in rolling a distance of 572 m.
44. With the vertices  $A, B$  and  $C$  of a triangle  $ABC$  as centres, arcs are drawn with radii 6 cm each in Fig. 11.71. If  $AB = 20 \text{ cm}$ ,  $BC = 48 \text{ cm}$  and  $CA = 52 \text{ cm}$ , then find the area of the shaded region. (Use  $\pi = 3.14$ )

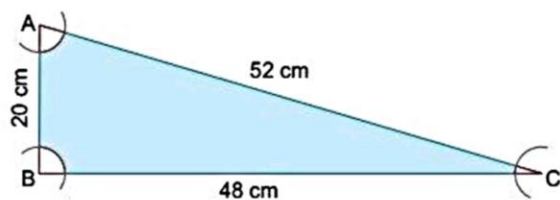


Fig. 11.71

45. In Fig. 11.72,  $PQRS$  is a square lawn with side  $PQ = 42$  metres. Two circular flower beds are there on the sides  $PS$  and  $QR$  with centre at  $O$ , the intersection of its diagonals. Find the total area of the two flower beds (shaded parts). [CBSE (AI) 2015]

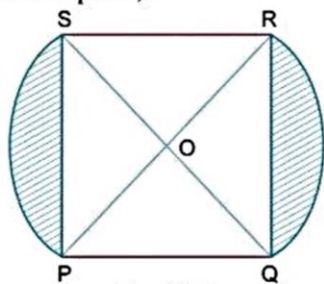


Fig. 11.72

### Answers

1. (i) (a)    (ii) (b)    (iii) (b)    (iv) (b)    (v) (a)
2. 50 cm    3.  $140 \text{ cm}^2$     4.  $\frac{25\sqrt{2}}{\pi} \text{ cm}$     5. 3 : 4    6.  $3025 \text{ cm}^2$     7. 9 : 16
8.  $100^\circ$     9.  $\pi : 2$     10.  $4\pi \text{ cm}$     11.  $19.2\pi \text{ cm}^2$     12.  $8a \text{ cm}$     13.  $144^\circ$
14.  $7546 \text{ cm}^2$     15. 7 cm    16.  $608\pi \text{ cm}$     17.  $25\pi \text{ cm}^2$     18.  $30.5 \text{ cm}^2$
19.  $271.3 \text{ cm}^2$  (Approx)    20.  $4.56 \text{ cm}^2$     21.  $17.90 \text{ cm}^2$  (Approx)    22.  $3465 \text{ cm}^2$
23.  $154.88 \text{ cm}^2$     24.  $7.77 \text{ cm}^2$     25.  $86.625 \text{ cm}^2$     26. ₹129.15
27. 3.5 cm    28.  $27104 \text{ m}^2$     29.  $(704 + 64\pi) \text{ cm}^2$  or  $905.14 \text{ cm}^2$     30.  $(32 + 2\pi)\text{m}^2$
31.  $39.25 \text{ cm}^2$     33.  $59.5 \text{ cm}^2$     34. (i)  $12.375 \text{ cm}^2$  (ii) ₹ 3.10
35.  $7.86 \text{ cm}^2$     36. ₹ 68640    37. 560    38.  $13.08 \text{ cm}^2$     39.  $150.92 \text{ cm}^2$
40.  $\left(\frac{25\pi}{4} + \frac{25}{2}\right) \text{ cm}^2$     41.  $168 \text{ cm}^2$     42.  $1568 \text{ cm}^2$     43. 65    44.  $423.48 \text{ cm}^2$
45.  $504 \text{ m}^2$

### Self-Assessment

Time allowed: 1 hour

Max. marks: 40

#### SECTION A

1. Choose and write the correct option in the following questions. (3 × 1 = 3)
- (i) If the sum of the circumferences of two circles with radii  $R_1$  and  $R_2$  is equal to the circumference of a circle of radius  $R$ , then [NCERT Exemplar]
- (a)  $R_1 + R_2 = R$     (b)  $R_1 + R_2 > R$     (c)  $R_1 + R_2 < R$     (d) None of these
- (ii) The ratio of the areas of a circle and an equilateral triangle whose diameter and a side are respectively equal is
- (a)  $\pi : \sqrt{2}$     (b)  $\pi : \sqrt{3}$     (c)  $\sqrt{3} : \pi$     (d)  $\sqrt{2} : \pi$
- (iii) The area of a quadrant of a circle whose circumference is 616 cm will be
- (a)  $7546 \text{ cm}^2$     (b)  $7500 \text{ cm}^2$     (c)  $7456 \text{ cm}^2$     (d)  $7564 \text{ cm}^2$
2. Solve the following questions. (2 × 1 = 2)
- (i) Find the area of a square inscribed in a circle of diameter  $p$  cm.
- (ii) If the diameter of a semicircular protractor is 14 cm, then find its perimeter.

#### SECTION B

- Solve the following questions. (4 × 2 = 8)
3. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.
4. The area of a circular playground is  $22176 \text{ m}^2$ . Find the cost of fencing this ground at the rate of ₹50 per m.



- Difference between the circumference and radius of a circle is 37 cm. Find the area of circle.
- Find the area of the shaded region in Fig. 11.73, if radii of the two concentric circles with centre  $O$  are 7 cm and 14 cm respectively and  $\angle AOC = 40^\circ$ .

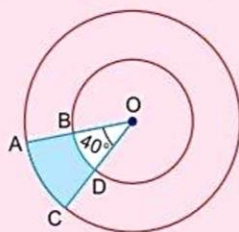


Fig. 11.73

■ Solve the following questions.

(4 × 3 = 12)

- A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60 cm, calculate the speed per hour which the boy is cycling.
- In Fig. 11.74, a square  $OABC$  is inscribed in a quadrant  $OPBQ$ . If  $OA = 20$  cm, find the area of the shaded region. (Use  $\pi = 3.14$ ). [CBSE Delhi 2014]

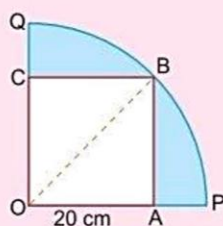


Fig. 11.74

- Find the area of the shaded region in Fig. 11.75, where  $ABCD$  is a square of side 14 cm each.

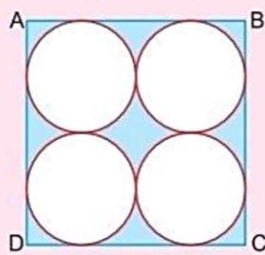


Fig. 11.75

- A square park has each side of 100 m. At each corner of the park, there is a flower bed in the form of a quadrant of radius 14 m as shown in Fig. 11.76. Find the area of the remaining part of the park.

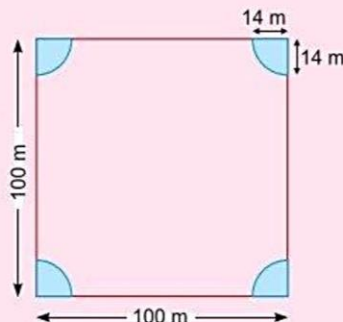


Fig. 11.76

■ Solve the following questions.

(3 × 5 = 15)

11. In Fig. 11.77, a chord  $AB$  of a circle, with centre  $O$  and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment  $AQBP$ . Hence find the area of major segment  $ALBQA$ . (Use  $\pi = 3.14$ )

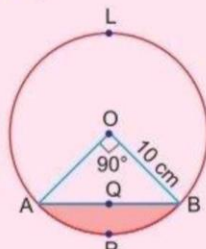


Fig. 11.77

12. A park is in the shape of a circle of diameter 7 m. It is surrounded by a path of width of 0.7 m. Find the expenditure of cementing the path, if its cost is ₹110 per sq. m.
13. In Fig. 11.78,  $ABCD$  is a trapezium with  $AB \parallel DC$ ,  $AB = 18$  cm,  $DC = 32$  cm and distance between  $AB$  and  $DC$  is 14 cm. If arcs of equal radii 7 cm with centres  $A, B, C$  and  $D$  have been drawn, then find the area of the shaded region of the figure. [NCERT Exemplar]

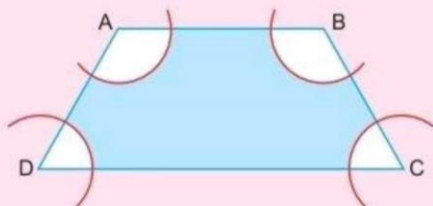


Fig. 11.78

**Answers**

1. (i) (a)      (ii) (b)      (iii) (a)
2. (i)  $\frac{p^2}{2}$  cm<sup>2</sup>      (ii) 36 cm
3. 3.92 cm    4. ₹26400    5. 154 cm<sup>2</sup>    6. 51.33 cm<sup>2</sup>    7. 15.84 km/h
8. 228 cm<sup>2</sup>    9. 42 cm<sup>2</sup>    10. 9384 cm<sup>2</sup>    11. 28.5 cm<sup>2</sup>, 285.5 cm<sup>2</sup>
12. ₹1863.40    13. 196 cm<sup>2</sup>

