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MATHEMATICS
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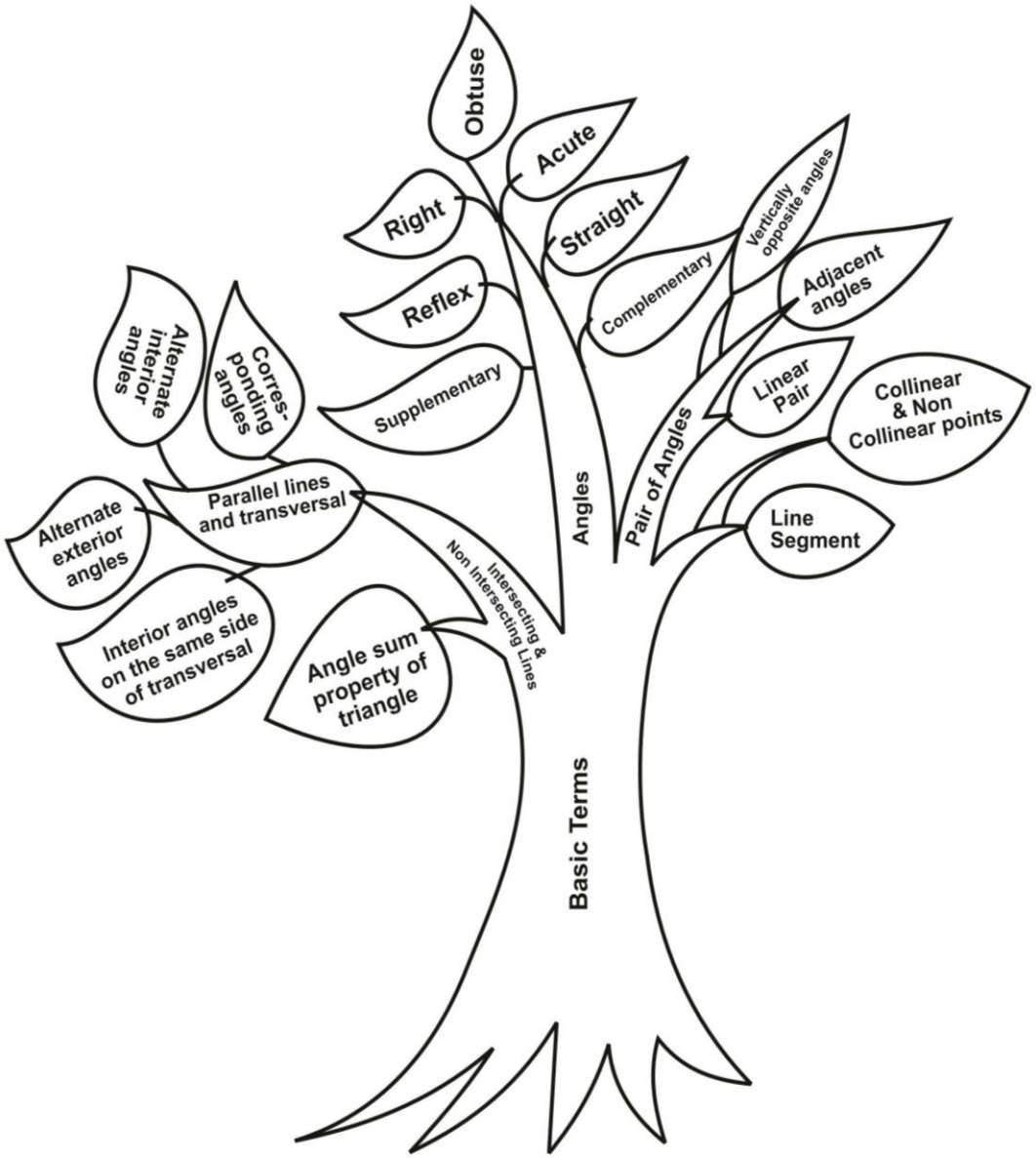
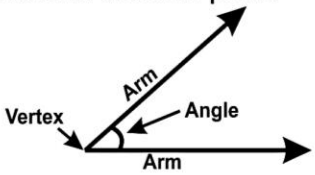
MIND MAP

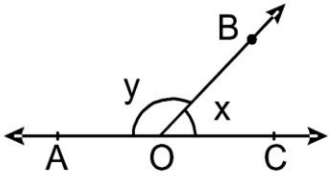


KEY POINTS

- Line is a collection of points which has only length, neither breadth nor thickness.
- **Line Segment :** A part or portion of a line with two end points.
- **Ray :** A part of a line with one end point.
- **Collinear points :** Three or more points lying on the same line.
- **Non-Collinear Points:** Three or more points which donot lie on same line.
- **Angle :** An angle is formed when two rays originate from the same end point. The rays making an angle are called the arms and the end point is the vertex.

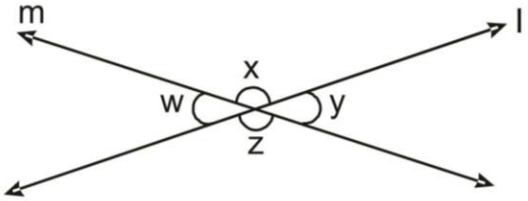


- **Acute angle :** An angle measure between 0° and 90°
- **Right angle :** Angle exactly equal to 90°
- **Obtuse angle :** An angle greater than 90° but less than 180°
- **Straight angle :** An angle exactly equal to 180°
- **Reflex Angle :** An angle greater than 180° but less than 360°
- **Complimentary Angles :** A pair of angles whose sum is 90°
- **Supplementary angle :** A pair of angles whose sum is 180°
- **Complete Angle :** An angle whose measure is 360°.
- **Adjacent angles :** Two angles are adjacent if
  (i) They have a common vertex.
  (ii) a common arm
  (iii) Their non common arms are on opposite sides of common arm.
- **Linear pair of angle :** A pair of adjacent angles whose sum is 180°



$\angle AOB$ & $\angle COB$ are forming linear pair.

- **Vertically opposite angles :** Angles formed by two intersecting lines on opposite side of the point of intersection.



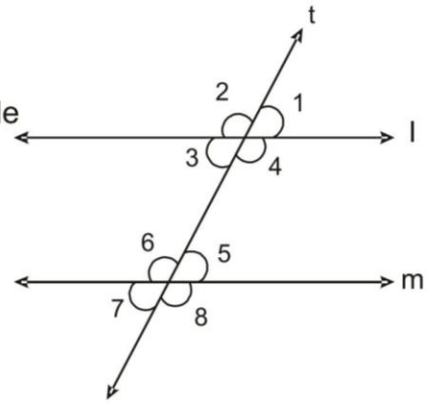
$\angle x = \angle z$
$\angle y = \angle w$

- **Intersecting lines:** Two lines are said to be intersecting when the perpendicular distance between the two lines is not same every where. They intersect at some point.

- **Non Intersecting lines :** Two lines are said to be non-intersecting lines when the perpendicular distance between them is same every where. They do not intersect. If these lines are in the same plane these are known as **Parallel lines.**

- **Transversal line :** In the given figure l || m and t is transversal then

(a) $\left.\begin{aligned}\angle 1 &= \angle 3\\ \angle 2 &= \angle 4\\ \angle 5 &= \angle 7\\ \angle 6 &= \angle 8\end{aligned}\right]$ Vertically opposite angle



(b) $\left.\begin{aligned}\angle 1 &= \angle 5\\ \angle 2 &= \angle 6\\ \angle 3 &= \angle 7\\ \angle 4 &= \angle 8\end{aligned}\right]$ Corresponding angle

(c) $\left.\begin{aligned}\angle 3 &= \angle 5\\ \angle 4 &= \angle 6\end{aligned}\right]$ Alternate Interior angle

(d) $\left.\begin{aligned}\angle 2 &= \angle 8\\ \angle 1 &= \angle 7\end{aligned}\right]$ Alternate Exterior Angle

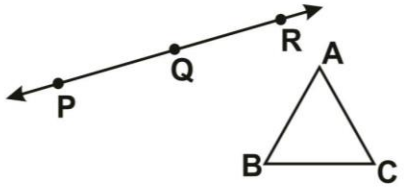
(e) $\left.\begin{aligned}\angle 3 + \angle 6 &= 180^\circ\\ \angle 4 + \angle 5 &= 180^\circ\end{aligned}\right]$ Angles on the same sides of a transversal are supplementary.

$\angle 3, \angle 6$ and $\angle 4, \angle 5$ are called co-interior angles or allied angles or consecutive interior angles.

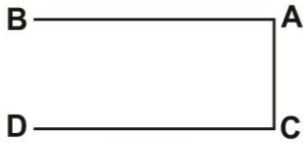
- Sum of all interior angles of a triangle is 180°.
- Two lines which are parallel to the third line are also parallel to each other.

Very Very Short Answer Type Questions (1 Marks)

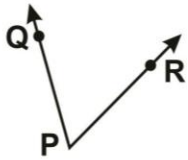
1. In the given figure, identify group of collinear points from (P,Q,R) and (A,B,C,)



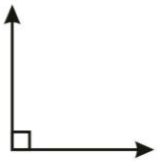
2. In the given figure, write the name of line segment whose one end point is B.



3. In the given figure, name the vertex of the angle.

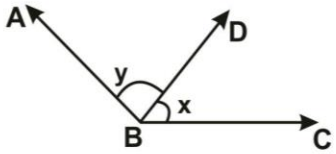


4. In the figure given in Q.3, name the two arms of the angle.
5. Which type of angle is formed in the given figure

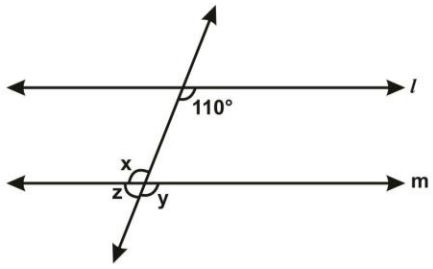


Fill in the blanks-

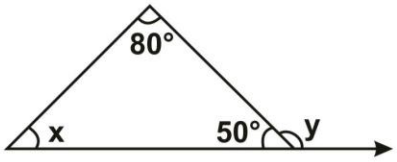
6. A ray has only _____ end point.
7. A line segment has a _____ length.
8. If two lines are non-intersecting, then they will be _____
9. An angle whose measure is more than 0° but less than 90°, is called an _____ angle.
10. A straight angle has ___ right angles.
11. An angle whose measure is more than 180° but less than 360° is called a _____ angle.
12. If an angle is equal to its complement, then its measure is _____.
13. If two angle are complements to each other, then what is the type of each angle?
14. In the give figure, for what value of x+y, will ABC be a line?



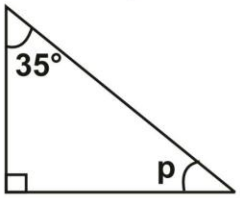
15. If two angles of a triangle are complementary then what type of triangle will be formed?
16. Two lines l and m are perpendicular to the same line n, Now l and m will be intersecting or parallel lines?
17. What is common between the three angles of a triangle and a linear pair?
18. In the given figure, find the value of x where $l \| m$.



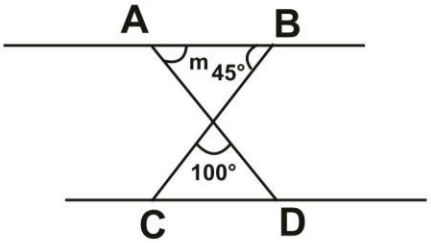
19. In the figure given in Q.18, find the value of y.
20. In the figure given in Q.18, find the value of z.
21. One of the angle of a linear pair is 65°. What will be the measure of the other angle?
22. In the given figure, find the value of x.



23. In the figure given in Q.22, find the value of y.
24. In the given figure find the value of p.



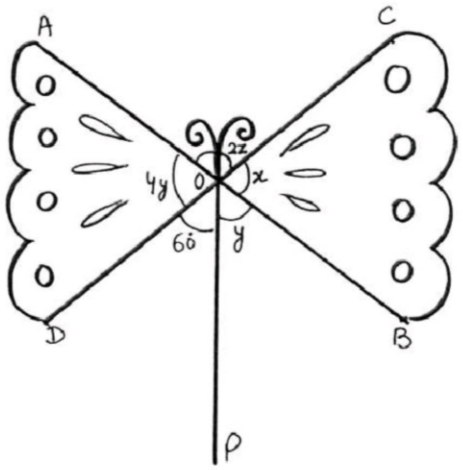
25. In the given figure find the value of m.



Case Study Based Questions (4 Marks)

26. Rashmi was making a toy butterfly with sticks for her younger sister. She arranged the sticks as shown in figure. AB and CD are two sticks intersecting (Joined) at O and a third stick OP is also joined to hold the toy butterfly.

Based on the above information, answer the following question-



(i) At what angle does Rashmi inclined the two sticks AB and CD?

(a) 86° (b) 96°

(c) 76° (d) 67°

(ii) Find the value of y in the figure-

(a) 24° (b) 42°

(c) 62° (d) 26°

(iii) Find the value of Z in the figure-

(a) 24° (b) 42°

(c) 62° (d) 26°

(iv) If the two sticks of lengths 10cm each are joined at the mid point, then find the value of OA-

(a) 5cm (b) 10cm

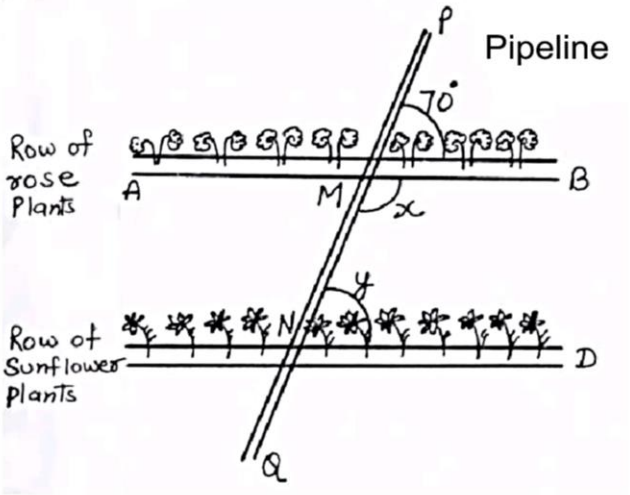
(c) 7.5cm (d)15cm

(v) What will be the sum of interior angles of both the wings of the butterfly-

(a) 180° (b) 270°

(c) 192° (d) 520°

27. Once four students of class IX are selected in Eco club of the school for plantation work. They are Shreya, Khushi, Vaibhav and Sushant. Shreya and Vaibhav planted a row of rose plants as shown in figure with line AB. Now Khushi and Sushant want to plant another row of sunflower plants parallel to rose plants row. Also there is a pipeline PQ passing through AB & CD. Based on the above information, answer the following questions-



(i) At what angle with PQ should Khushi and Sushant plant the row CD to make it parallel to row AB.

(a) 60° (b) 65°

(c) 70° (d) 75°

(ii) What will be the sum of angles between AB and CD marked as x and y.

(a) 90° (b) 180°

(c) 70° (d) 140°

(iii) What will be the value of x?

(a) 110° (b) 120°

(c) 90° (d) 70°

(iv) What will be the value of y?

(a) 110° (b) 120°

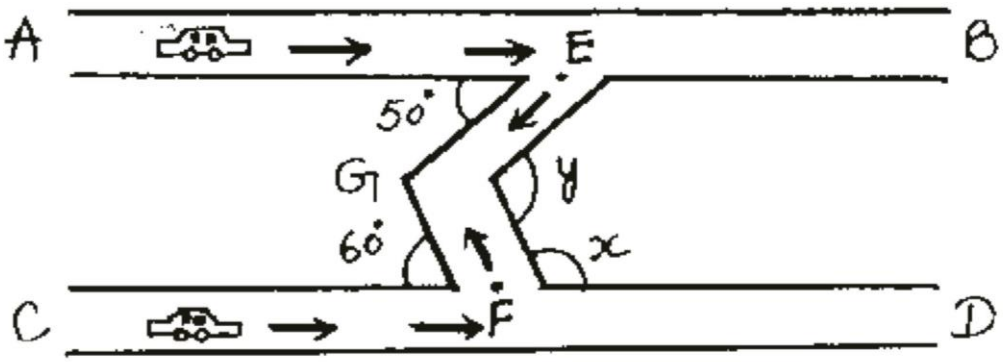
(c) 90° (d) 70°

(v) What will be the value of 2x+y=?

(a) 290° (b) 210°

(c) 220° (d) 100°

28. Two cars are moving on two parallel roads represented as AB and CD respectively in the given figure. First car reaches at point E and takes a turn towards its right at an angle of 50°. At the same time, second car reaches at point F and takes a turn towards its left at an angle of 60°. They both meet at a point G. Based on the above information and given figure, answer the following question (without considering the width of the roads)



(i) What will be the measure of angle x marked in the figure?

(a) 60° (b) 80°

(c) 100° (d) 120°

(ii) What will be the measure of ∠EGF marked as y?

(a) 50° (b) 100°

(c) 150° (d) 250°

(iii) What will be the measure of reflex ∠EGF?

(a) 90° (b) 100°

(c) 110° (d) 120°

(iv) If EF is joined, what type of triangle will EGF made?

(a) Scalene (b) Isosceles

(c) Right angled (d) None of these.

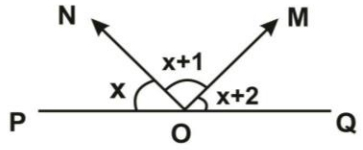
(v) If ΔEGF would have been an isosceles triangle with EG=GF, then what would be the measure of ∠GFC considering ∠AEG as 50°?

(a) 50° (b) 60°

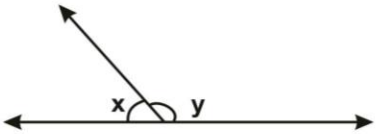
(c) 70° (d) 80°

Very short Answer type Questions (2 Marks)

29. In the given figure POQ is a straight line and OM and ON are two rays. The three adjacent angles so formed are consecutive numbers. Find the value of x.

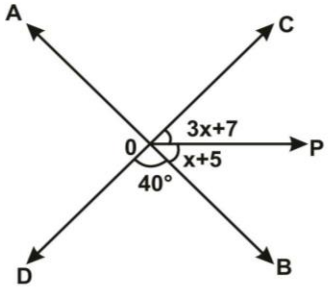


30. If x and y are linear pair and twice of x is 30° less than y, then find the value of x and y.



31. One of the angles of a pair of supplementary angles is 2° more than its supplement. Find the angles.

32. In the given figure AB and CD are two straight lines intessecting at O and OP is a ray. What is the measure of ∠AOD? Also find the value of x.

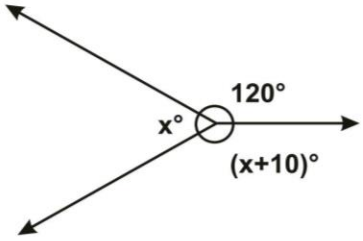


33. If the difference between two supplementary angles is 40°, then find smaller angle.

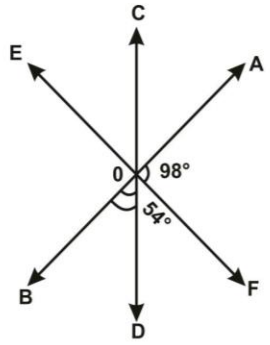
34. Find the angle which is four times more than its complement.

35. An exterior angle of a triangle is 100° and its two interior opposite angles are equal. Find the measure of these interior angles.

36. Find the value of x in the given figure.

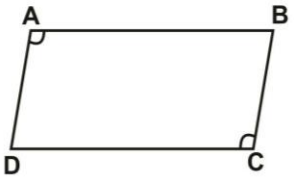


37. In the given figure AB, CD and EF are three straight lines intersecting at O. Find the measure of $\angle BOC$.

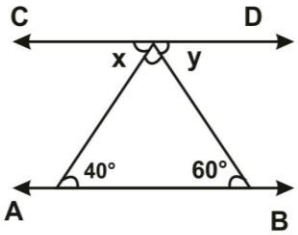


38. $\angle x$ and $\angle y$ forms a linear pair and $x-2y=30°$. Find the value of x.

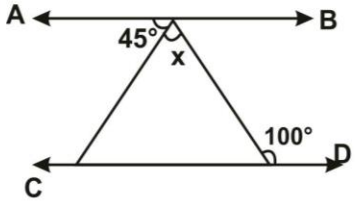
39. In the given figure, AB||DC and AD||BC. Prove that $\angle DAB=\angle DCB$.



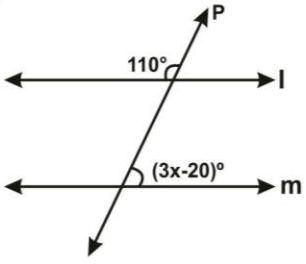
40. In the given figure AB||CD. what is the value of x+y.



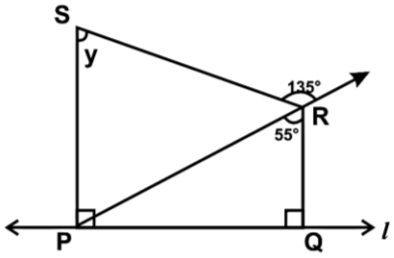
41. In the given figure, find the value of x if AB || CD.



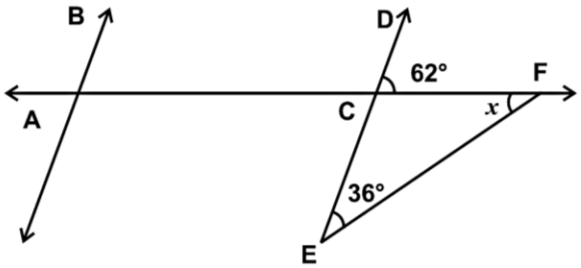
42. In the given figure, if l || m then what is the value of x.



43. In the given figure, SP $\perp l$ and RQ $\perp l$. Find the measure of $\angle y$.

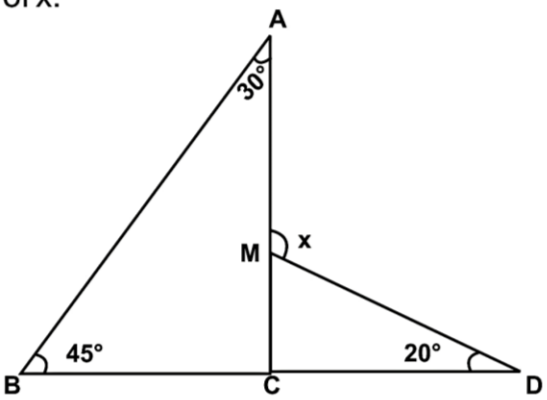


44. In the given figure if AB || ED, then find the value of x.



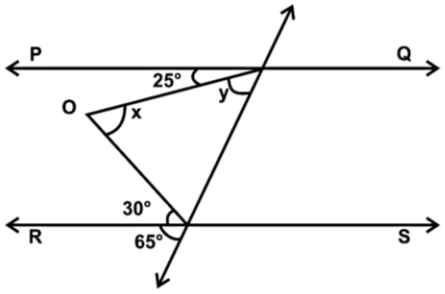
45. If the angles of a triangle are in the ratio 5:3:7 then show that the triangle is an acute angled triangle.

46. Find the value of x.



Short Answer Type Question (3 Marks)

47. In the adjoining figure *PQ* || *RS* find *x and y.*



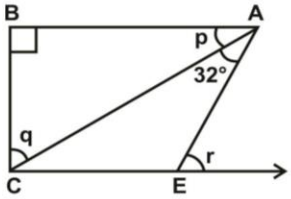
48. By contributing money, 5 friends bought pizza. They want to divide it equally among themselves. But one of them was given double share, as he was very hungry. Find the angle of the piece of pizza each one received.

49. ABC is a triangle in which AB and AC are produced.

    BO and *CO* are bisectors of exterior $\angle B$ *and* $\angle C$ intersecting at O. If $\angle A=60°$, $\angle ABC = 70°$, Find $\angle BOC$.

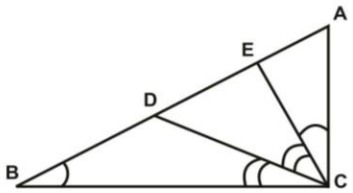
50. In the figure ,

    If $p : q = 11 : 19$, $AB \parallel CE$, then find the values of p, q and r.

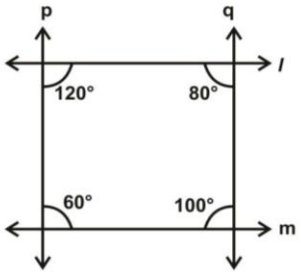


51. Prove that if two lines intersect then vertically opposite angles are equal.

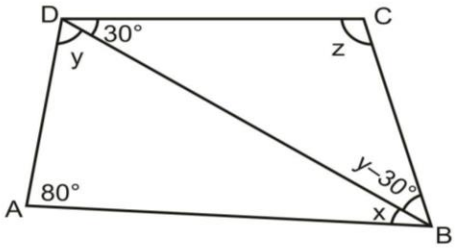
52. In the figure, *CD* is the angle bisector of $\angle ECB$, $\angle B = \angle ACE$. Prove that $\angle ADC = \angle ACD$.



53. In the figure, choose the pair of lines which are parallel. Give reasons also.

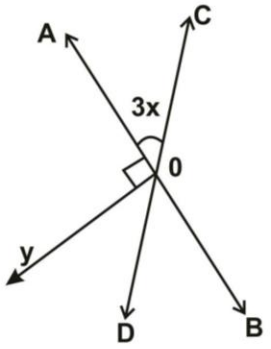


54. In the given figure if AB || DC and $\angle BDC=30°$ $\angle BAD=80°$ find $\angle x$, $\angle y$, $\angle z$.

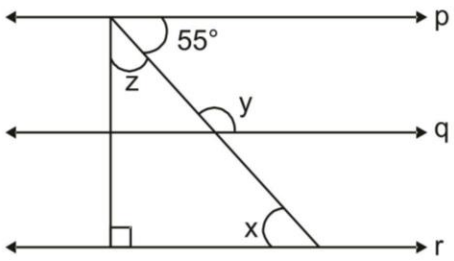


55. If one of the angle of two intersecting lines is right angle then prove that other three angles will also be right angles.

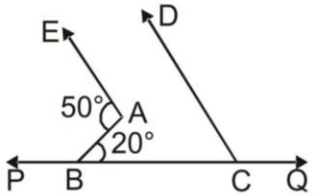
56. AB and CD are intersecting lines. OD is bisector of ∠BOY. Find x.



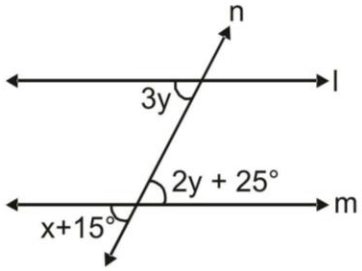
57. If p || q || r, find x, y, z from given figure.



58. In the given figure find ∠DCB if AE || CD

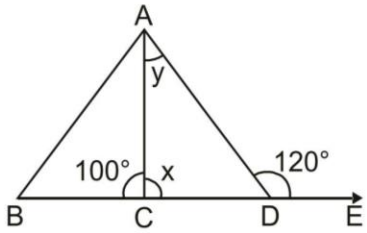


59. In the given figure l || m and n is the transversal, find x.



60. Two lines are respectively perpendicular to two parallel lines show that they are parallel to each other.

61. As shown in the figure find x & y if $\angle ACB=100°$, $\angle ADE=120°$.

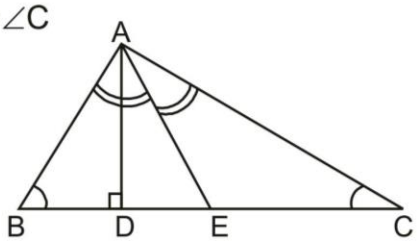


62. Prove that the bisectors of the angles of a linear pair are at right angle.
63. If two complementary angles are such that two times the measure of one is equal to three times the measure of the other. Find the measure of larger angle.
64. Prove that the sum of all exterior angles of a triangle is 360°.
65. If the bisectors of $\angle Q$ and $\angle R$ of a triangle ΔPQR meet at point S, then prove that

$$\angle QSR = 90° + \frac{1}{2}\angle P$$

66. Show that if sum of the two angles of a triangle is equal to the third angle then the triangle is right angled triangle.
67. If a transversal intersects two parallel lines prove that internal bisectors of the angle on the same side of a transversal meet at right angles.
68. In the figure AE is the bisector of $\angle A$, $AD \perp BC$. Show that

$2(\angle ADE - \angle EAC) = \angle B + \angle C$



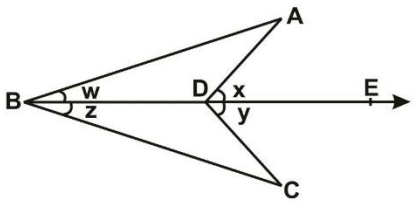
69. Prove that quadrilateral formed by the intersection of bisectors of interior angles made by a transversal on two parallel lines is a rectangle.
70. In right angled ΔABC right angled at B, $\angle BCA=2\angle CAB$. Show that hypotenuse AC=2BC.

Long Anser Type Questions (5 Marks)

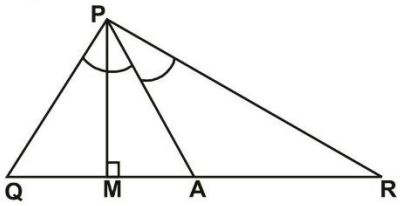
71. Two lines are respectively perpendicular to two parallel lines. Show that they are parallel to each other.

72. In the given figure, prove that

$\angle ADC = \angle A + \angle B + \angle C$



73. In the given figure, $\angle Q > \angle R$, PA is the bisector of $\angle QPR$ and $PM \perp QR$. Prove that $\angle APM = \frac{1}{2}(\angle Q - \angle R)$



74. If the arms of one angle are respectively parallel to the arms of another angle, then show that the two angels are either equal or supplementary.

75. If the bisectors fo the base angles of a triangle enclose an angle of 135°, then prove that the triangle is a right angled triangle.

ANSWERS

1. (P,Q,R)
2. $\overline{BA}$
3. P
4. PQ, PR
5. Right Angle
6. One
7. definite
8. parallel
9. acute
10. two
11. reflex
12. 45°
13. acute
14. 180°
15. Right angled
16. Parallel
17. Sum of angles of a triangle and linear pair is 180°
18. 110°
19. 110°
20. 70°
21. 115°
22. 50°
23. 113°
24. 55°
25. 35°
26. (i) (b) 96°
    (ii) (a) 24°
    (iii) (b) 42°
    (iv) (a) 5cm
    (v) (c) 192°
27. (i) (c) 70°
    (ii) (b) 180°
    (iii) (a) 110°
    (iv) (d) 70°
    (v) (a) 290°
28. (i) (d) 120°
    (ii) (d) 250°
    (iii) (c) 110°
    (iv) (a) scalene
    (v) (a) 50°
29. 59°
30. x=50°, y=130°
31. 89°, 91°
32. 140°, x=32°
33. 70°
34. 72°
35. 50°
36. 115°
37. 152°
38. 130°
39. Hint: Use the property that sum of interior angles on the same side of transversal are supplementary.
40. 100°
41. 55°
42. 30°
43. 80°
44. 26°

45. Let the angles are 5x, 3x, 7x then $5x+3x+7x=180°$

$x=12°$

$\therefore$ Angles are $60°, 36°, 84°$

$\therefore$ triangle is acute angled triangle.

46. $95°$

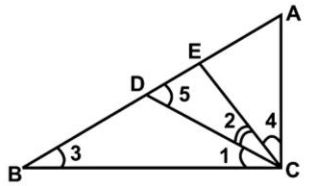
47. $x=55°, y=40°$

48. 4 equal pieces$=60°$, one double piece$=120°$

49. $60°$

50. $33°, 57°, 65°$

52.



$\angle 1=\angle 2$

$\angle 3=\angle 4$

$\angle 1+\angle 3=\angle 2+\angle 4$

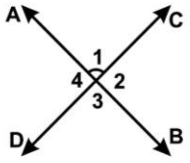
$\angle 5=\angle 2+\angle 4$

$\angle ADC=\angle ACD$

53. l || m because sum of interior angles on the same side of transversal is $180°$

54. $x=30°, y=70°, z=110°$

55.



Given :- $\angle 1=90°$

To prove :- $\angle 2=\angle 3=\angle 4=90°$

$\angle 1+\angle 2=180°$ [Linear Pair]

$90°+\angle 2=180°$

$\angle 2=90°$

$\angle 3=\angle 1=90°$ (Vertically Opp. $\angle$s)

$\angle 4=\angle 2=90°$ (Vertically opp. $\angle$s)

56. $x=15°$

57. $x=55°, y=125°, z=35°$

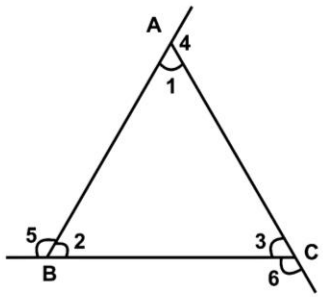
58. $30°$

59. $60°$

61. $x=80°, y=40°$

63. $60°$

64.



To prove: $\angle 4+\angle 5+\angle 6=360°$

$\angle 1+\angle 2+\angle 3=180°$

$\angle 4=180-\angle 1$

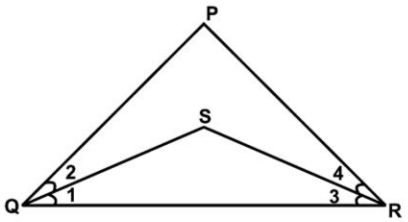
$\angle 5=180-\angle 2$

$\angle 6=180-\angle 3$

$\angle 4+\angle 5+\angle 6=3(180)-(\angle 1+\angle 2+\angle 3)$

$=540-180=360°$

65.



Given:- ΔPQR, QS is bisector of $\angle Q$, RS is bisector of $\angle R$.

To prove : $\angle QSR=90+\frac{1}{2}\angle P$

Prof : $\angle QSR+\angle 1+\angle 3=180°$—①

In ΔPQR

$\angle P + \angle PQR + \angle PRQ = 180°$

$\angle P + \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

$\angle P + 2\angle 1 + 2\angle 3 = 180°$

$\frac{1}{2}\angle P + \angle 1 + \angle 3 = 90°$

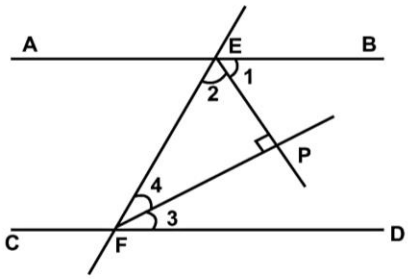
$\angle 1 + \angle 3 = 90° - \frac{1}{2}\angle P$

Putting the value of $\angle 1 + \angle 3$ in ①

$\angle QSR + 90° - \frac{1}{2}\angle P = 180°$

$\angle QSR = 90° + \frac{1}{2}\angle P$

67.



Given: AB || CD, EF is transversal

EP is bisector of $\angle BEF$

FP is bisector of $\angle DFE$

$\therefore \quad \angle 1 = \angle 2, \angle 3 = \angle 4$

To prove: $\angle EPF = 90°$

Proof: $\angle BEF + \angle DFE = 180°$

(Angles on the same side of transversal are supplementary)

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

$2\angle 2 + 2\angle 4 = 180°$

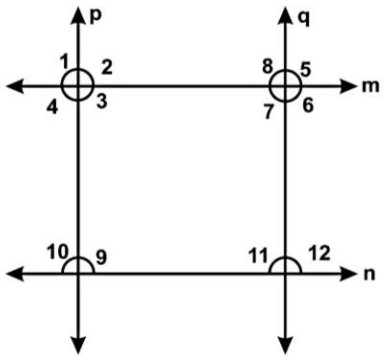
$\angle 2 + \angle 4 = 90°$

Also $\angle 2 + \angle 4 + \angle EPF = 180°$

$\angle EPF = 180 - 90 = 90°$

69. Hint: Take help of solution 67.

71.



given: n || m, p⊥ m, p⊥n, q⊥m, q⊥n

To prove: p || q

$\angle 1 = \angle 10 = 90°$

$\angle 2 = \angle 9 = 90°$

$\angle 4 = \angle 9 = 90°$

$\angle 3 = \angle 10 = 90°$

$\angle 7 = 90°$ & $\angle 11 = 90°$

$\angle 3 + \angle 7 = 90° + 90° = 180°$

$\therefore$ p ||q $\therefore$ If a transversal intersect two lines such that a pair of interior angles on the same side of transversal is supplementary then two lines are parallel.

72. Hint: use exterior angle property in ΔABD & ΔCBD.

73. In figure

PA is bisector of $\angle QPR$

$\angle QPR = \angle APR$ — (i)

In ΔPQM, $\angle PQM + \angle PMQ + \angle QPM = 180°$

$\angle PQM = 90 - \angle QPM$—(ii)

In ΔPMR

$\angle PMR + \angle PRM + \angle RPM = 180°$

$90° + \angle PRM + \angle RPM = 180°$

$\angle PRM = 90° - \angle RPM$ —(iii)

Subracting eq (iii) from (ii)

$\angle Q - \angle R = (90° - \angle QPM) - (90° - \angle RPM)$

$\angle Q - \angle R = \angle RPM - \angle QPM$

$\angle Q - \angle R = \angle RPA - \angle APRm - [\angle QPA - \angle APM]$

$\angle Q - \angle R = \angle QPA + \angle APM - \angle QPA + \angle APM$

$\angle Q - \angle R = 2\angle APM$

$\angle APM = \frac{1}{2}(\angle Q - \angle R)$