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# FRICTION

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## UNIT – III CHAP – 02

NEWTON'S LAWS OF MOTION

CBSE – XI – PHYSICS – FRICTION – (NLM)

Physics CBSE

UNIT – III

CHAP – 02

FRICTION


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1. **Friction**: Whenever a body moves or tends to move over the surface of another body, a force comes into play which acts parallel to the surface of contact and opposes the relative motion. This opposing force is called friction.
2. **Static friction**: The force of friction which comes into play between two bodies before one body actually starts moving over the other is called static friction ($f_s$): Static friction is a self-adjusting force.
3. **Limiting friction**: The maximum force is static friction which comes into play when a body just starts moving over the surface of another body is called limiting friction ($f_s^{max}$).
4. **Kinetic friction**: The force of friction which comes into play when a body is in a steady motion over the surface of another body is called kinetic or dynamic friction ($f_k$), kinetic friction is less than limiting friction.
5. **Laws of limiting friction:** (i) The force is limiting friction depends upon the nature of the two surfaces in contact and their state of roughness.
(ii) The force of limiting friction acts tangential to the two surfaces in contact and in a direction opposite to that of the applied force.
(iii) The force of limiting friction between any two surfaces is independent of the shape or area of the surfaces in contact so long as the normal reaction remains the same.
(iv) The force of limiting friction between two given surfaces is directly proportional to the normal reaction between the two surfaces.
$f \propto R$ or $f = \mu_s R$ where the constant of proportionality $\mu_s$ is called the coefficient of limiting friction.
6. **Coefficient of limiting friction**: It is the ratio of limiting friction to the normal reaction.
$$\mu_s = \frac{f_s^{max}}{R} = \frac{\text{Limiting friction}}{\text{Normal reaction}}$$
7. **Coefficient of kinetic friction**: It is the ratio of kinetic friction ($f_k$) to the normal reaction.
$$\mu_k = \frac{f_k}{R} = \frac{\text{Kinetic friction}}{\text{Normal reaction}}$$
As $f_k < f_s^{max}$ or $\mu_k R < \mu_s R$ $\therefore$ $\mu_k < \mu_s$
8. **Angle of friction**: It is the angle which the resultant of the limiting friction and the normal reaction makes with the normal reaction. If $\theta$ is the angle of friction, then $\tan\theta = \mu_s$.
9. **Angle of repose**: It is the minimum angle that an inclined plane makes with the horizontal when a body placed on it just begins to slide down. If $\phi$ is the angle of repose, then $\tan\phi = \mu_s$.
10. **Motion along a rough horizontal surface**: If a body of mass m is moved over a rough horizontal surface through distance s, then
Force of friction, $f = \mu R = \mu mg$
Retardation produced, $a = \frac{f}{m} = \mu g$
Work done against friction, $W = f \times s = \mu mg s$
Power $= f \times v = \mu mg v$
11. **Motion along a rough inclined plane**: (i) When a body moves down an inclined plane with uniform velocity (a = 0), net downward force needed is
$F = mg\sin\theta - f = mg(\sin\theta - \mu\cos\theta)$
Work done, $W = F_s = mg(\sin\theta - \mu\cos\theta)s$
(ii) When a body moves up an inclined plane with uniform velocity (a = 0), net upward force needed is
$\vec{F} = mg\sin\theta + f = mg(\sin\theta + \mu\cos\theta)$
$W = Fs = mg(\sin\theta + \mu\cos\theta)\, s$
(iii) When a body moves up an inclined plane with acceleration a, net upward force needed is
$F = ma + mg\sin\theta + f = m(a + g\sin\theta + \mu g\cos\theta)$
$W = m(a + g\sin\theta + \mu g\cos\theta)\, s$
12. **Acceleration of body sliding down a rough inclined plane**: When the angle of inclination is greater than the angle of repose, the acceleration produced is
$a = g(\sin\theta - \mu_k\cos\theta)$
13. **Centripetal force**: It is the force required to make a body move along a circular path with a uniform speed. It always acts along the radius and towards the centre of the circular path. The centripetal force required to move a body of mass m along a circular path of radius r with speed v is given by
$$F = \frac{mv^2}{r} = mr\omega^2 = mr(2\pi v)^2 = mr\left(\frac{2\pi}{T}\right)^2$$
14. **Centrifugal force**: It is a fictitious force acting radially outwards on a particle moving in a circle and is equal in magnitude to the centripetal force.
15. **A vehicle taking circular turn on a level road**: If $\mu$ is the coefficient of friction between tyres and road, then the maximum velocity with which the vehicle can safely take a circular turn of radius r is given by
$v = \sqrt{\mu r g}$

**16.** **Banking of tracks (roads):** The maximum angle with which a vehicle (in the absence of friction) can negotiate a circular turn of radius r and banked at angle θ is given by

$$v = \sqrt{rg \tan\theta}$$

When the frictional forces are also taken into account, the maximum safe velocity is given by

$$v = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$$

**17.** **Bending of a cyclist:** In order to take a circular turn of radius r with speed v, the cyclist should bend himself through an angle θ from the vertical such that

$$\tan\theta = \frac{v^2}{rg}$$

**18.** **Motion in a vertical circle:** As shown in Fig. Consider a body of mass m tied at the end of a string and rotating in a vertical circle of radius r. Then

(i) Velocity at any point P at a height h from the lowest point L,

$$v = \sqrt{u^2 - 2gh}$$

(ii) Tension in the string at any point P,

$$T = m(u^2 - 3gh + gr)$$

(iii) Tension at the lowest point (h = 0),

$$T_L = \frac{m}{r}(u^2 + gr)$$

(iv) Tension at the highest point H (h = 2r),

$$T_H = \frac{m}{r}(u^2 - 5gr)$$

(v) Difference in tensions at the highest and lowest points,

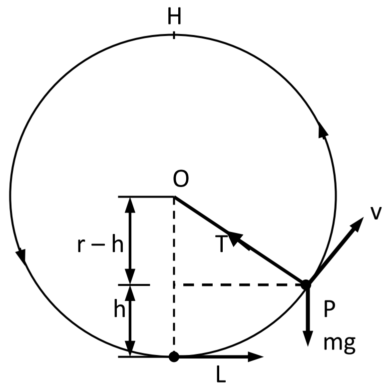
$$T_L - T_H = 6\,mg$$

(vi) Minimum velocity at the lowest point L for looping the loop,

$$V_L = \sqrt{5gr}$$

(vii) Velocity at the highest point for looping the loop,

$$V_H = \sqrt{gr}$$

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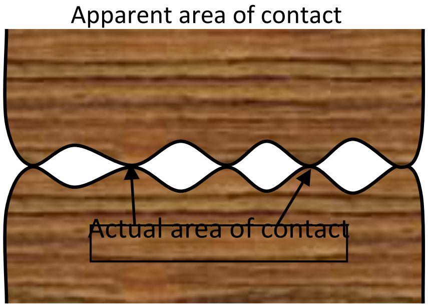
***"Friction is the force which opposes the relative motion (or the tendency of relative motion) between two the bodies in contact and acts tangentially along the surface of contact".***

In other words, **"Friction is an opposing force that comes into play when one body moves (slides or rolls) or even tends to move over the surface of another body".**

**Origin of friction:** The force of friction is due to the atomic or molecular force of attraction between the two surfaces at the points of actual contacts. Fig. shows two surfaces in contact, as seen through a powerful microscope. Due to the surface irregularities, the actual area of contact is much smaller than the apparent area of contact is much smaller than the apparent area of contact. The pressure at the points of contacts is very large. Molecular bonds are formed at these points. When one body is pulled over the other, the bonds break, the material is deformed and new bonds are formed. The local deformation and new bonds are formed. The local deformation sends vibration waves into the bodies. These vibrations finally damp out and energy appears as heat. Hence a force is needed to start or maintain the motion.




**[Cause of sliding friction]**

◘ **Friction will be independent of apparent area of contact but depends upon actual area of contact.**
When the surface in contact are extra smooth, then force of adhesion (or cohesion) between them will increase tremendously resulting in the increase of friction between them.

◘ **Force of friction always acts in a direction opposite to that of the motion of the object.**

◘ **Force of friction is non conservative force.**

▶ **CONSERVATIVE FORCES** --- *If the amount of work done against a force depends only on the initial and final position of the body moved and not on the path followed by the body, then such force is called a conservative force.*

▶ **NON-CONSERVATIVE FORCES**--- **If the amount of work done against a force depends on the path followed by the body , then such a force is called non – conservative force.**

▶ CLASSIFICATION **OF FRICTION**: - Friction can be classified into two categories---
........[A] External friction .........[B] Internal force {viscosity} Contact friction

◙ **EXTERNAL FRICTION** :- **External friction, arises when two bodies in contact with each other try to move or there is actual relative motion between the two.**

◙ **INTERNAL FRICTION** :-- **Internal friction, arises when there is a relative motion between every two layer of a liquid.**

## [A] EXTERNAL FRICTION

[I] Static friction | [II] Limiting friction | [III] Kinetic friction

## STATIC, LIMITING AND KINETIC FRICTIONS

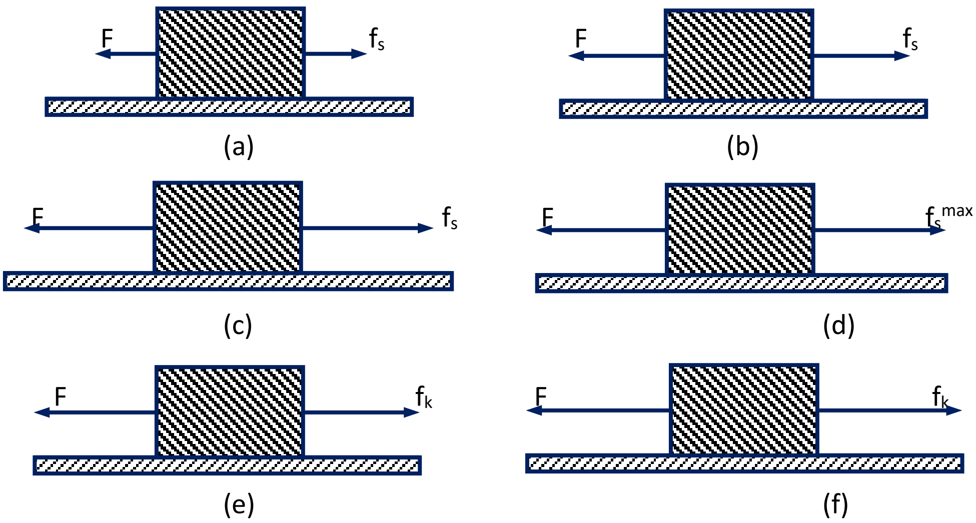
**Static, limiting and kinetic frictions:** **static friction is a self-adjusting force**: Consider a wooden block placed over a horizontal table. Apply a small force F on it [Fig. (a)]. The block does not move. The force of friction f comes into action which balances the applied force F.

◙**The force of friction which comes into play between two bodies before one body actually starts moving over the other is called static friction ($f_s$).**

Static friction opposes the impending motion i.e., the motion that would take place (but does not take place) under the applied force, if friction were absent.

As the applied force on the block is increased, the static friction $f_s$ also increases [Fig. (b) and (c)] to balanced the applied force and the block does not move.

Once the applied force is increased beyond a certain limit, the block just begins to move. At this stage static friction is maximum [Fig (d)].



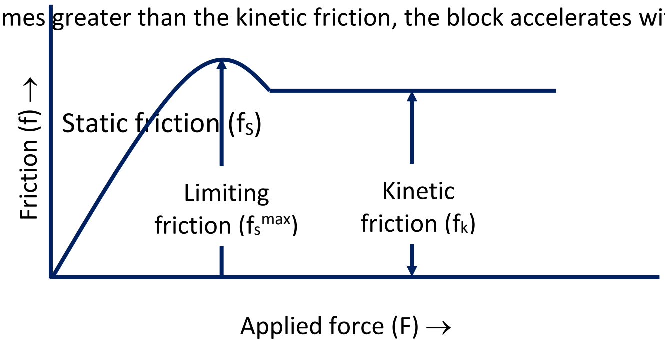

**[Types of friction]**

◙**The maximum force of static friction ($f_s^{max}$) which comes into play when a body just starts moving over the surface of another body is called limiting friction.**

Clearly $f_s \le f_s^{max}$.

◙**Once the motion has begun, the force of friction decreases. A smaller force is now necessary to maintain uniform motion. The force of friction which comes into play when a body is in a state of steady motion over the surface of another body is called kinetic or dynamic friction ($f_k$)**

. When $F = f_k$, the body moves with a constant velocity v.
When the applied force becomes greater than the kinetic friction, the block accelerates with acceleration equal to $(F - f_k)/m$




**[Variation of the force of friction with the applied force]**

Fig. shows the variation of the force of friction f with the applied force F. Obviously, as the applied force F increases, the static friction $f_s$ increases accordingly to balance it. This shows that static friction is a self-adjusting force. The kinetic friction is always less than the limiting friction $f_s^{max}$ i.e., $f_k < f_s^{max}$.

Thus,

**[I] STATIC FRICTION**: ----*"The force of friction that comes into play when one body tends to move over the surface of another but the actual motion has yet not started is called static friction".*

In other words, **"The force of friction which exactly counter balance the applied force during the stationary state of body is called static friction".**

- **The body remains at rest as long as $f = f_s$**
- **Magnitude of static friction is not constant. It always adjust itself so as be equal to applied force.**

**[II] LIMITING FRICTION**:**"The maximum value of static frictional force which comes into play when a body is just going to starts sliding over the surface of another body is called limiting force friction".**

## LAWS OF LIMITING FRICTION ------

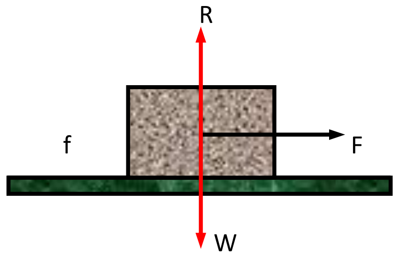
☻ [I] **The direction of limiting friction is always opposite to the direction in which the motion takes place or tends to takes place or tends to takes place.**

☻ [II] **The force of friction acts tangentially along the surface of contact of two bodies.**

☻ [III] **The magnitude of the limiting friction of force (F) is directly proportional of the reaction (R)between the two surface i.e. $F \alpha R$**

**i.e., $f_s^{max} \propto R$ or $f_s^{max} = \mu_s R$**

**or $\mu_s = \dfrac{f_{s(max)}}{R} = \dfrac{\text{Limiting friction}}{\text{Normal reaction}}$**




**The proportionality constant $\mu_s$ is called coefficient of static friction: If is defined as the ratio of limiting friction to the normal reaction.**

☻ [IV] **Force of limiting of friction is independent of the area of the surface of contact of two bodies as long as normal reaction in contact.**

☻ [V] **The force of limiting friction depends on the nature of the surfaces in contact.**

[B] KINETIC FRICTION -- **"The force of friction which comes into play when body in contact with another body has actual relative motion w.r.t the $2^{nd}$ body, is called dynamic or kinetic friction".**

In the above-mentioned example, if we increase the applied force slightly beyond limiting friction, the actual motion starts. This means that the applied force is now greater than the force of limiting friction. The force of friction at this stage is called kinetic friction.

## Types of Kinetic fricyion : SLIDING AND ROLLING FRICTIONS

**Kinetic friction is of two types:**

*(i)* **Sliding friction: The force of friction that comes into play when a body slides over the surface of another body is called sliding friction. When a wooden block is pulled or pushed over a horizontal surface, sliding friction comes into play.**

- *SLIDING FRICTION* –– **"The force of friction which comes into play when one body slides over the surface of another body is called sliding friction".**

EXAMPLE: - When a brick is pulled or pushed over the surface of the ground, then sliding friction comes into play.

### LAW OF SLIDING FRICTION -------

[ I] *The sliding friction opposes the applied force and has a constant value, depending upon the nature of the two surfaces in relative motion.*

[II] *The force of sliding friction is the directly proportional to the normal reaction R.*

The value of kinetic friction $f_k$ is directly proportional to the normal reaction R between the two surfaces.

i.e., $f_k \propto R$ or $f_k = \mu_k R$

or $\mu_k = \frac{f_k}{R} = \frac{\text{Kinetic friction}}{\text{Normal reaction}}$

The proportionality constant $\mu_k$ is called coefficient of kinetic friction. If is defined as the ratio of kinetic friction to the normal section.

As $f_k < f_s^{max}$ or $\mu_k R < \mu_s R$ $\therefore$ $\mu_k < \mu_s$

Thus, the coefficient of kinetic friction is less than the coefficient of static friction.

[III] *The sliding FRICTION does is not depends upon the velocity, provided the velocity is neither too large nor too low.*

[IV] *The sliding frictional force is independent of the area of the contact between the two surface so along as the normal reactions the same.*

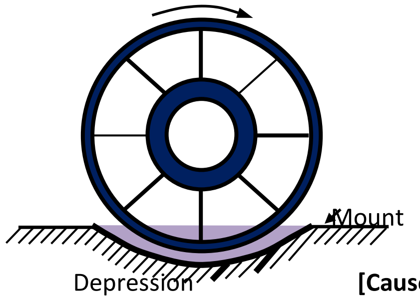
**(ii) Rolling friction: The force of friction that comes into play when a body rolls over the surface of another body is called rolling friction.**

** When a wheel rolls over a road, rolling friction comes into play.

** For the same magnitude of normal reaction, rolling friction is always much smaller than the sliding friction.

**Rolling friction is smaller than sliding friction: When a wheel rolls without slipping over a horizontal plane, the surfaces at contact do not rub each other. The relative velocity of the point of contact of the slipping. There is no sliding or static friction in such an ideal situation. We need to overcome rolling friction only which is much smaller than sliding friction. For this reason, wheel has been considered as one of the greatest inventions.**

**Cause of rolling friction:** Consider a wheel rolling along a road. As the wheel rolls, it exerts a large pressure(Weight/area) due to its small area. This causes a slight depression of the road below and a small elevation or mount in front of it. In addition to this, the rolling wheel has to continuously detach itself from the surface on which it rolls. This is opposed by the adhesive force between the two surfaces in contact. On account of both of these factors, a force originates which retards the rolling motion. This retarding force is known as rolling friction.




**[Cause of rolling friction]**

**Laws of rolling friction:** Experiments show that (i) Rolling friction is directly i.e.,

$f_r \propto R$

**(ii)** Rolling friction is inversely proportional to the radius of the rolling cylinder or wheel i.e.,

$f_r \propto \frac{1}{r}$

Combining the two laws, we get

$f_r \propto \frac{R}{r}$

or $$f_r = \mu_r \frac{R}{r}$$

Here $\mu_r$ is the coefficient of rolling friction. Unlike $\mu_r$ or $\mu_k$ (which is a pure ratio and has no dimensions), $\mu_r$ has the dimensions of length and its SI unit is metre. The above equation is applicable only when there is rolling without slipping.

✱**Coefficient of static friction** ----- The limiting friction { i.e maximum static friction fs [max] } is directly proportional to the normal reaction (R) between the two surfaces in contact.

i.e. fs (max) α R Or, fs (max) = $\mu_s$ R or $$\mu_s = fs\,(max)/R$$ where $\mu_s$ = constant of proportionality and is called coefficient of static friction.

"**COEFFICIENT OF LIMITING** FRICTION between any two surfaces in contact is defined as the ratio of the force limiting friction and normal reaction between them".

✱ **Coefficient of kinetic friction**: --- The kinetic friction {$f_k$} is directly proportional to the normal reaction (R) between the two surfaces in contact.

That is, $f_k$ α R , or, $f_k = \mu_k$ R $$\mu_k = f_k/R$$

**Therefore,** *"coefficient of kinetic friction between any two surfaces in contact is defined as the ratio of the kinetic frictional force and normal reaction between them".*

Coefficient of friction μ has no units as it is ratio of forces.

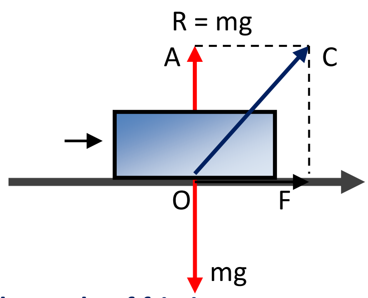
Since, $f_k < f_s$ and therefore $\mu_k < \mu_s$

**Angle of Friction:** - *The angle between the normal reaction and the resultant of limiting friction and normal reaction is called angle of friction.*

R = mg

∟AOC = θ = angle of friction

Now, tan θ = F/R but, F/R = $\mu_s$ Therefore, **θ = $\mu_s$**




**Coefficient of static friction is numerically equal to the tangent of the angle of friction.**

## ANGLE OF REPOSE

**Angle of repose: It is the minimum angle that an inclined plane makes with the horizontal when a body placed on its just begins to slide down.**

*"Angle of repose is the angle that an inclined plane makes with the horizontal when a body placed on it just starts sliding down".*

**Relation between angle of repose and coefficient of friction:** Consider a body of mass m placed on an inclined plane. The angle of inclination $\phi$ of the inclined plane is so adjusted that a body placed on its just begins to slide down. Thus $\phi$ is the angle of repose.

**Various forces acting on the body are:**

(i) Weight mg of the body acting vertically downwards.

(ii) The limiting position $f_s^{max}$ in upward direction along the inclined plane. It balances the component mg sin $\phi$ of the weight mg along the inclined plane. Thus

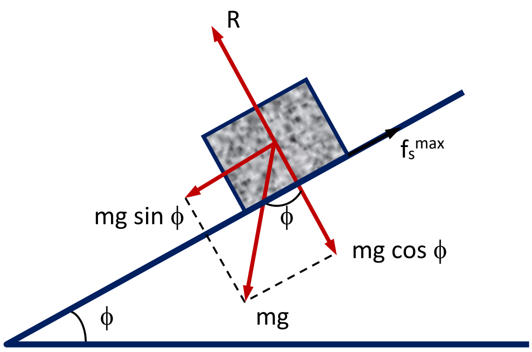
$$f_s^{max} = mg \sin \phi \qquad \ldots (1)$$

(iii) The normal reaction R perpendicular to the inclined plane. It balances the component mg cos $\phi$ of the weight mg perpendicular to the inclined plane. Thus

$$R = mg \cos \phi \qquad \ldots (2)$$

Dividing equation (1) by (2), we get

$$\frac{f_s^{max}}{R} = \frac{mg \sin \phi}{mg \cos \phi} \quad \text{or} \quad \mu_s = \tan \phi$$




**[Angle of repose]**

**Thus, the coefficient of static friction is equal to the angle of repose.**

**As $\mu_s = \tan \theta = \tan \phi$ $\quad \therefore$ $\theta = \phi$**

**Thus, the angle of repose is equal to the angle of friction.**

## Examples based on Coefficient of Friction and Angle of Friction

**✖Formulae Used**

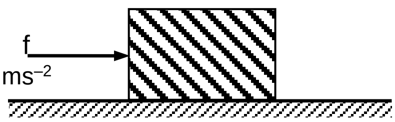
1. **Coefficient of limiting friction = $\frac{\text{Limiting friction}}{\text{Normal reaction}}$ or $\mu_s = \frac{f_s^{max}}{R}$ or $f_s^{max} = \mu_s R$**
2. **Coefficient of kinetic friction = $\frac{\text{Kinetic friction}}{\text{Normal reaction}}$ or $\mu_k = f_k$ or $f_k = \mu_k R$**
3. **For a body placed on horizontal surface, R = mg**
   **$\therefore$ $f_s^{max} = \mu_s . mg$ and $f_k = \mu_k . mg$**
4. **Static friction, $f_s \le f_s^{max}$ or $f_s \le \mu_s R$**
5. **Kinetic friction $f_k < f_s^{max}$**
6. **If $\theta$ is the angle of friction, then $\mu_s = \tan \theta$**
7. **If $\phi$ is the angle of repose, then $\mu_s = \tan \phi$**
8. **Angle of repose = Angle of friction i.e., $\theta = \phi$**
9. **For a body moving on a rough horizontal surface with retardation a, $\mu = \frac{f}{R} = \frac{ma}{mg} = \frac{a}{g}$**
10. **$f_r = \mu_r . \frac{R}{r}$ and $\mu_r < \mu_k < \mu_s$**

**Where $\mu_r$ is the coefficient of rolling friction, $f_r$ is the rolling friction and r is the radius of the rolling body.**

**✖Units Used**

Force of friction f and normal reaction R are in newton (N), coefficient of friction $\mu$ has no units.

**Q. 1. A block of mass 2 kg is placed on the floor. The coefficient of static friction is 0.4. A force of friction of 2.5 N is applied on the block as shown in Fig. Calculate the force of friction between the block and the floor.**




**Sol.** Here m = 2 kg, $\mu_s = 0.4$, g = 9.8 $ms^{-2}$

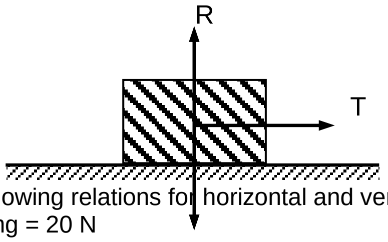
The value of limiting friction,

$$f_s^{max} = \mu_s R = \mu_s . mg = 0.4 \times 2 \times 9.8 = 7.84 \text{ N}$$

As the applied force of 2.5 N is less than the limiting friction (7.84 N), so the block does not move. In this situation,

Force of friction = Applied force = 2.5 N

**Q. 2. A block of weight 20 N is placed on a horizontal table and a tension T, which can be increase to 8 N before the block begins to slide, is applied at the block as shown in Fig. A force of 4 N keeps the block moving at constant speed once it has been set in motion. Find the coefficient of static and kinetic friction.**




**Sol.** **For static friction:** We have the following relations for horizontal and vertical component of forces:

$R - mg = 0$ or $R = mg = 20$ N

and $T - f_s^{max} = 0$ or $f_s^{max} = T = 8$ N

∴ Coefficient static friction,

$\mu_s = \frac{f_s^{max}}{R} = \frac{8}{20} = 0.40$

For kinetic friction: We can write the following relations:

$R - mg = 0$ or $R = mg = 20$ N

and $T - f_k = 0$ or $f_k = T = 4$ N

∴ Coefficient of kinetic friction,

$\mu_k = \frac{f_k}{R} = \frac{4}{20} = 0.20$

**Q. 3. A force of 49 N is just sufficient to pull a block of wood weighing 10 kg on a rough horizontal surface. Calculate the coefficient of friction and angle of friction.**

**7**

**Sol.** Here P = applied force = 49 N,

$m = 10$ kg, $g = 9.8$ ms$^{-2}$

Coefficient of friction,

$\mu = \frac{f}{R} = \frac{P}{mg} = \frac{49}{10 \times 9.8} = 0.5$

As $\tan\theta = \mu = 0.5$

∴ $\theta = \tan^{-1}(0.5) = 26° 34'$.

**Q. 4. A cubical block rests on an inclined plne of $\mu = 1/\sqrt{3}$, determine the angle of inclination when the block just slides down the inclined plane.**

**Sol.** When the block just slides down the inclined plane, the angle of inclination is equal to angle of repose ($\alpha$).

∴ $\tan\alpha = \mu = \frac{1}{\sqrt{3}}$ or $\alpha = 30°$

**Q. 5. A mass of 4 kg rests on a horizontal plane. The plane is gradually inclined until at an angle $\theta = 15°$ with the horizontal, the mass just begins to slide. What is the coefficient of static friction between the block and the surface?**

**Sol.** Here $\theta = 15°$ is the angle of repose.

∴ Coefficient of friction,

$\mu = \tan\theta = \tan 15° = 0.27$

**Q. 6. A body rolled on ice with a velocity of 8 ms$^{-1}$ comes to rest after travelling 4 m. Compute the coefficient of friction. Given. g = 9.8 ms$^{-2}$.**

**Sol.** Here $u = 8$ ms$^{-1}$, $v = 0$, $s = 4$ m, $a = ?$

As $v^2 - u^2 = 2as$ ∴ $0^2 - 8^2 = 2a \times 4$

or $a = -\frac{64}{8} = -8$ ms$^{-2}$

Negative sign indicates retardation.

Now $\mu = \frac{f}{R} = \frac{ma}{mg} = \frac{a}{g} = \frac{8}{9.8} = 0.8164$

**Q. 7. The coefficient of friction between the ground and the wheels of a car moving on a horizontal road is 0.5. If the starts from rest, what is the minimum distance in which it can acquire a speed of 72 kmh$^{-1}$? Take g = 10 ms$^{-2}$.**

**Sol.** Here $\mu = 0.5$, $u = 0$, $v = 72$ kmh$^{-1}$ $= 72 \times \frac{5}{18} = 20$ ms$^{-1}$

As $\mu = \frac{f}{R} = \frac{ma}{mg} = \frac{a}{g}$

∴ $a = \mu g = 0.5 \times 10 = 5$ ms$^{-2}$

As $v^2 - u^2 = 2as$ ∴ $(20)^2 - (0)^2 = 2 \times 5 \times s$

or $s = \frac{400}{10} = 40$ m

**Q. 8. Determine the maximum acceleration of the train in which a box lying on its floor will remain stationary, given that the coefficient of static friction between the box and the train's floor is 0.15. Take g = 10 ms$^{-2}$.**

**Sol.** As the acceleration of the box is due to static friction, so

$ma = f \le \mu_s R = \mu_s mg$

∴ $a \le \mu_s g$

or $a_{max} = \mu_s g = 0.15 = 10 = 1.5$ ms$^{-2}$

**Q. 9.** **A bullet of mass 0.01 kg is fired horizontally into a 4 kg wooden block at rest on a horizontal surface. The coefficient of the kinetic friction between the block and the surface is 0.25. The bullet gets embedded in the block and the combination moves 20 m before coming to rest. With what speed did the bullet strike the block?**

**Sol.** Mass of block, M = 4 kg
Mass of bullet, m = 0.01 kg
After the bullet gets embedded in the block, the force of kinetic friction is

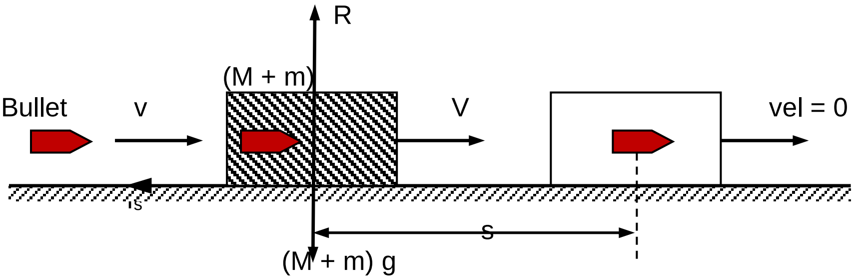
$f_k = \mu_k R = \mu_k (M + m) g$

If the kinetic friction produces retardation a in the system, then

$f_k = (M + m) a$

or $a = \mu_k \times g = 0.25 \times 9.8 = 2.45 \text{ ms}^{-2}$

After the bullet enters the block, suppose the system attains velocity V. Now the system comes to rest after covering a distance, s = 20 m, as shown in Fig.




As $v^2 - u^2 = 2as$

∴ $0 - v^2 = 2 \times (-2.45) \times 20$

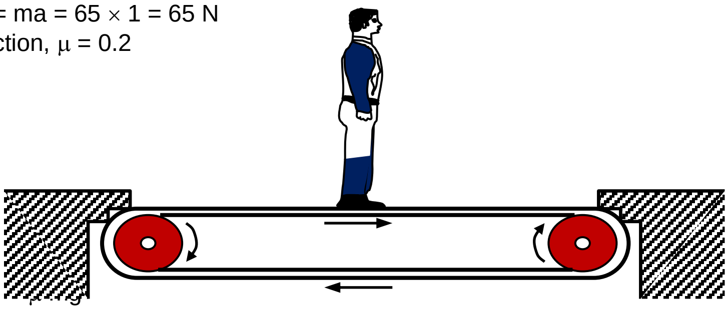
or $V = \sqrt{98} = 9.8995 \text{ ms}^{-1}$

If v is the velocity with which the bullet struck the block, then applying the law of conservation of momentum, we get
Mv= (M + m) V

or $v = \frac{M + m}{m} \cdot V = \frac{4 + 0.01}{0.01} \times 9.8995 = 401 \times 9.8995 = 3969.7 \text{ ms}^{-1}$

**Q. 10.** **Fig. shows a man standing stationary with respect to a horizontal conveyor belt that is accelerating with 1 ms$^{-2}$. What is the net force on the man? If the coefficient of static friction between the man's shoes and belt is 0.2, upto what acceleration of the belt can the man continue to be stationary relative to the belt? Mass of the man = 65 kg.**

**Sol.** As the man is standing stationary w.r.t. the belt,
∴ Acceleration of the man = Acceleration of the belt = a = 1 ms$^{-2}$
Mass of the man, m = 65 kg
Net force on the man = ma = 65 × 1 = 65 N
Given coefficient of friction, μ = 0.2



∴ Limiting friction, f = μR = μmg
If the man remains stationary w.r.t. the maximum acceleration a' of the belt, then

$ma' = f = \mu mg$

∴ $a' = \mu g = 0.2 \times 9.8 = 1.96 \text{ ms}^{-2}$

**Q. 11.** **A block of mass 15 kg is placed on a long trolley. The coefficient of friction between the block and the trolley is 0.18. The trolley accelerates from rest with 0.5 ms$^{-2}$ for 20 s and then moves with uniform velocity. Discuss the motion of the block as viewed by (i) a stationary observer on the ground (ii) and observer moving with the trolley.**

**Sol.** Mass of block, m = 15 kg  $\mu_s = 0.18$, a = 0.5 ms$^{-2}$, t = 20 s
Maximum value of static friction,

$f_{ms} = \mu_s R = \mu_s mg = 0.18 \times 15 \times 9.8 = 26.46 \text{ N}$

Force acting on the block during the accelerated motion,

$F = ma = 15 \times 0.5 = 7.5 \text{ N}$

As $f_{ms} > F$, so the block does not move. It remains at rest w.r.t. the trolley, even when it is accelerated. When the trolley moves with uniform velocity, acceleration is zero and hence no force is acting on the trolley.
(i) The stationary observer will see the accelerated and the uniform motions.
(ii) When the observer is in the trolley, he is in an accelerated or non-inertial frame. The laws of motion are not applicable. But during uniform motion he will see that the block is at rest w.r.t. him.

**Q. 12. The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in Fig. The coefficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with 2 ms$^{-2}$. At what distance from the starting point does the box fall off the truck? Ignore the mass of the box.**

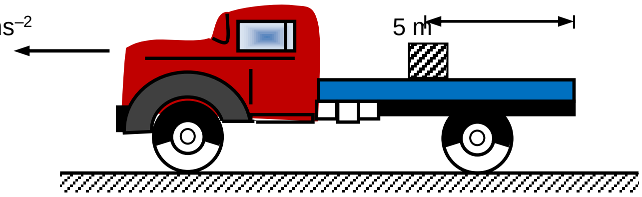
***Sol.*** Mass of box, $m = 40$ kg ; Acceleration of truck, $a = 2$ ms$^{-2}$

Distance of the box from the rear end, $s = 5$ m ; Coefficient of friction, $\mu = 0.15$

As the box is in an accelerated frame, it experiences a backward force,

$F = ma$

Motion of the box is opposed by the frictional force,




$f = \mu R = \mu mg$

∴ Net force on the box in the backward direction is

$F' = F - f = ma - \mu mg = m(a = \mu g)$

$= 40 (2 - 0.15 \times 9.8) = 21.2$ N

Acceleration produced in the box in the backward direction,

$a' = \frac{F'}{m} = \frac{21.2}{40} = 0.53$ ms$^{-2}$

If the box takes time t to fall off the truck, then

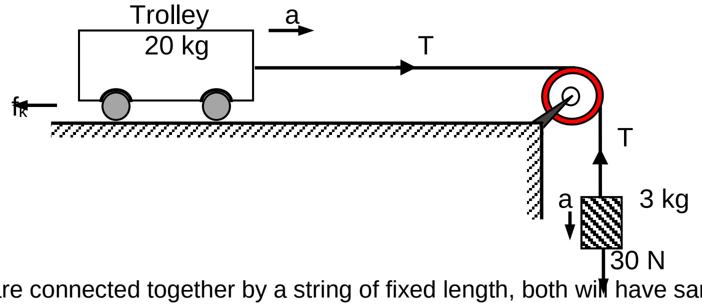
$s = ut + \frac{1}{2} a' t^2$ or $5 = 0 \times t + \frac{1}{2} \times 0.53 \times t^2$

or $t^2 = \frac{5 \times 2}{0.53} = \frac{10}{0.53}$

The distance covered by the truck accelerating at 2 ms$^{-2}$ during this time is

$s' = \frac{1}{2} at^2 = \frac{1}{2} \times 2 \times \frac{10}{0.53} = 18.57$

**Q. 13. What is the acceleration of the block and the trolley system shown in fig., if the coefficient of kinetic friction between the trolley and the surface is 0.04? What is the tension in the string? Take g = 10 ms$^{-2}$ Neglect the mass of the string.**




***Sol.*** As the block and the trolley are connected together by a string of fixed length, both will have same acceleration a.

Applying Newton's second law to the motion of the block,

$30 - T = 3a$ ... (i)

Applying second law to the motion of the trolley,

$T - f_k = 20$ a

But $f_k = \mu_k R = \mu_k mg = 0.04 \times 20 \times 10 = 8$ N

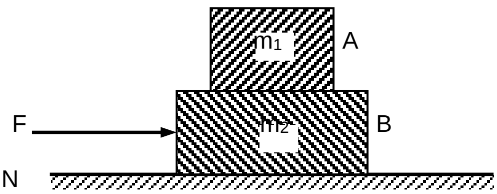
∴ $T - 8 = 20$ a ... (ii)

Adding (i) and (ii), we get

$22 = 23$ a or $a = \frac{22}{23} = 0.96$ ms$^{-2}$

From (i), $30 - T = 3 \times 0.96$ or $T = 30 - 2.88 = 27.12$ N

**Q. 14. A block A of mass 4 kg is placed on another block B of mass 5 kg, and the block B rests on a smooth horizontal table. For sliding the block A on B, a horizontal force of 12 N is required to be applied on it. How much maximum horizontal force can be applied on B so that both A and B more together? Also find out the acceleration produced by this force.**




**Sol.** Here $m_1 = 4$ kg, $m_2 = 5$ kg

Force applied on block A = 12 N

This force must at least be equal to the kinetic friction applied on A by B.

∴ $12 = f_k = \mu_k R = \mu_k m_1 g$

or $12 = \mu_k \times 4g$ or $\mu_k = \frac{12}{4g} = \frac{3}{g}$

The block B is on a smooth surface. Hence to move A and B together, the (maximum) force F that can be applied on B is equal to the frictional forces applied on A by B and applied on B on A.

$F = \mu_k m_1 g + \mu_k m_2 g = \mu_k (m_1 + m_2) g = \frac{3}{g} (4 + 5) g = 27 g$

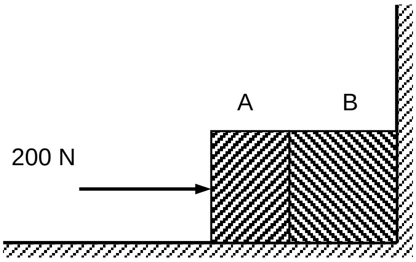
As this force moves both the blocks together on a smooth table, so the acceleration produced is

$a = \frac{F}{m_1 + m_2} = \frac{27}{4 + 5} = 3$ ms$^{-2}$

**Q. 15 Two bodies A and B of masses 5 kg and 10 kg in contact with each other rest on a table against a rigid partition. The coefficient of friction between the bodies and the table is 0.15. A force of 200 N is applied horizontally at A. What are (i) the reaction of the partition (ii) the action-reaction forces between A and B? What happens when a partition is removed? Does answers to (ii) change, when the bodies are in motion? Ignore difference between $\mu_s$ and $\mu_k$.**

**Sol.** Mass of body A, $m_A = 5$ kg
Mass of body B, $m_B = 10$ kg
Coefficient of friction, $\mu = 0.15$
Applied force, P = 200 N
(i) Force of limiting friction is

$$f = \mu R = \mu (m_1 + m_2) g$$
$$= 0.15 \times (5 + 10) \times 9.8 = 22.05 \text{ N (towards left)}$$




When a force of 200 N is applied, the net force exerted on the partition is

$$P' = P - f = 200 - 22.05 = 177.95 \text{ N (towards right)}$$

Reaction of the partition = 177.95 N (towards left)
(ii) Force of limiting friction on body A is

$$f_1 = \mu m_1 g = 0.15 \times 5 \times 9.8 = 7.35 \text{ N}$$

Net force exerted by body A on body B is

$$P_1 = P - f_1 = 200 - 7.35 = 192.65 \text{ N} \quad \text{(towards right)}$$

**When the partition is removed:** The system of the two bodies moves under the action of the net force,

$$P' = 177.95 \text{ N}$$

Acceleration produced in the system,

$$a = \frac{P'}{m_1 + m_2} = \frac{177.95}{5 + 10} = 11.86 \text{ ms}^{-2}$$

Force producing motion in the body A

$$= m_1 a = 5 \times 11.86 = 59.3 \text{ N}$$

Net force exerted by A on B after the removal of partition

$$= P_1 - 59.3 = 192.65 - 59.3 = 133.35 \text{ N} \quad \text{[towards right]}$$

Reaction of the body B on A = 133.5 N [towards left]

**Q. 16. An engine of 100 H.P. draws a train of mass 200 metric ton with a velocity by 36 kmh$^{-1}$. Find the coefficient of friction.**

**Sol.** Power of engine,

$$P = 100 \text{ H.P.} = 100 \times 746 = 74600 \text{ W}$$

Velocity, v = 36 kmh$^{-1}$ = 10 ms$^{-1}$
If the frictional force overcome by the engine is F, then

$$P = F \times v \quad \text{or} \quad F = \frac{P}{v} = \frac{74600}{10} = 7460 \text{ N}$$

Normal reaction, R = mg = 200 × 1000 × 9.8 N
Coefficient of friction,

$$\mu = \frac{F}{R} = \frac{7460}{200 \times 100 \times 9.8} = 0.0038$$

## Problems For Practice

**Q. 1. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6. If the acceleration of the truck is 5 ms$^{-2}$, calculate the frictional force acting on the block.**

**Sol.** Limiting friction, $f = \mu mg = 0.6 \times 1 \times 9.8 = 5.88$ N
Applied force, $F = ma = 1 \times 5 = 5$ N
As F < f, so force of friction = 5 N

**Q. 2. A body weighing 20 kg just slides down a rough inclined plane that rises 5 m in every 13 m. What is the coefficient of friction?**

**Sol.** Here $\sin\theta = \frac{5}{13}$

$$\therefore \quad \cos\theta = \sqrt{1 - \left(\frac{5}{13}\right)^2} = \frac{12}{13}$$

$$\mu = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5}{12} = 0.4167$$

**Q. 3. A scooter weighs 120 kg f. Brakes are applied so that wheels stop rolling and start skidding. Find the force of friction if the coefficient of friction is 0.4.**

**Sol.** Here R = weight of scooter = 120 kg f, $\mu = 0.4$

$$\therefore \quad f = \mu R = 0.4 \times 120 = 48 \text{ kg f}$$

**Q. 4. An automobile is moving on a horizontal road with a speed v. If the coefficient of friction between the tyres and road is $\mu$, show that the shortest distance in which the automobile can be stopped is $v^2/2\mu g$.**

**Sol.** Here u = v, v = 0, $a = -\mu g$, s = ?
As $v^2 - u^2 = 2as$ $\quad \therefore \quad 0 - v^2 = 2(-\mu g)s$
or $s = \frac{v^2}{2g}$

**Q. 5. Find the distance travelled by a body before coming to rest, if it is moving with a speed of 10 $ms^{-1}$ and the coefficient of friction between the ground the body is 0.4.**

**Sol.** Here $a = \mu g = 0.4 \times 9.8 = 3.92\ ms^{-2}$
Now $u = 10\ ms^{-1}$, $v = 0$, $s = ?$ $a = -3.92\ ms^{-2}$
$\therefore \quad 0^2 - 10^2 = 2 \times (-3.92) \times s$ or $s = 12.75$ m

**Q. 6. A motor car running at the rate of 7 $ms^{-1}$ can be stopped by applying brakes in 10 m. Show that total resistance to the motion, when brakes are on, is one fourth of the weight of the car.**

**Sol.** Here $u = 7\ ms^{-1}$, $v = 0$, $s = 10$ m
As $v^2 - u^2 = 2as \quad \therefore \quad 0 - 7^2 = 2a \times 10$
or $a = -2.45\ ms^{-2} = \frac{9.8}{4}\ ms^{-2} = -\frac{g}{4}$ Total resistance to motion = – ma = ¼ × mg = ¼ = weight of the car.

**Q. 7. Find the power of an engine which can maintain a speed of 50 $ms^{-1}$ for a train of mass $3 \times 10^5$ kg on a rough line. The coefficient of friction is 0.05. Take g = 10 $ms^{-2}$.**

**Sol.** Here $v = 50\ ms^{-1}$, $\mu = 0.05$, $m = 3 \times 10^5$ kg
Friction, $f = \mu R = \mu mg = 0.05 \times 3 \times 10^5 \times 10 = 1.5 \times 10^5$ N
Power, $P = f \times v = 1.5 \times 10^5 \times 50 = 75 \times 10^5$ W = 7500 kW.

**Q. 8. A train weighing 1000 quintals is running on a level road with a uniform speed of 72 km $h^{-1}$. If the frictional resistance amounts to 50 g wt per quintal, find power in watt. Take g = 9.8 $ms^{-2}$.**

**Sol.** Here m = 1000 quintals, $v = 72\ kmh^{-1} = 20\ ms^{-1}$
Total friction resistance, $f = 50 \times 1000$ g wt = 50 kg wt = $50 \times 9.8$ N
Power, {= $f \times v = 50 \times 9.8 \times 20 = 9800$ W

**Q. 9. An automobile of mass m starts from rest and accelerates at a maximum rate possible without slipping on a road with $\mu_s = 0.5$. If only the rear wheels are driven and half the weight of the automobile is supported on these wheels, how much time is required to reach a speed of 100 $kmh^{-1}$.**

**Sol.** Normal load on rear wheel = ½ Mg = R (Normal reaction)
Maximum sustainable frictional force, $f_{s max} = \mu_s R = ½ \mu_s Mg$
Acceleration of the automobile = $\frac{F}{M} = \frac{f_s^{max}}{M} = ½ \mu_s g$

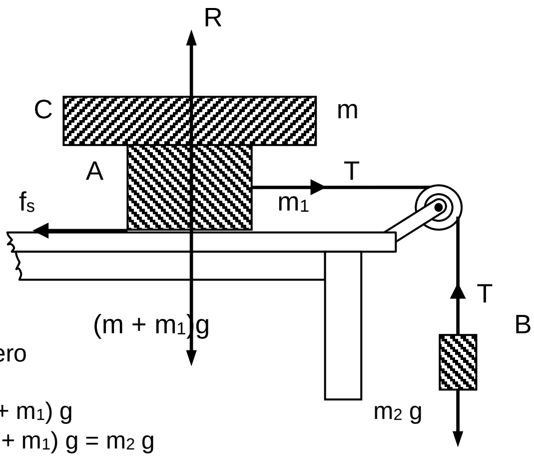
**Q. 11. A truck moving at 72 $kmh^{-1}$ carries a steel girder which rests on its wooden floor. What is the minimum time in which the truck can come to stop without the girder moving forward? Coefficient of static friction between steel and wood is 0.5.**

**Sol.** Here $u = 3\ ms^{-1}$, $v = 0$, $a = -\mu g = -0.5 \times 9.8\ ms^{-2}$
As $v^2 - u^2 = 2as \quad \therefore \quad 0 - 3^2 = -2 \times 0.5 \times 9.8 \times s$
or $s = \frac{9}{9.8} = 0.92$ m

**Q. 12. A bullet of mass 10 g is fired horizontally into a 5 kg wooden block, at rest on a horizontal surface. The coefficient of kinetic friction between the block and the surface is 0.1. Calculate speed of the bullet striking the block, if the combination moves 20 m before coming to rest.**

**Sol.** Here $u = 72\ kmh^{-1} = 20\ ms^{-1}$, $\mu_s = 0.5$
$f_s^{max} = \mu_s R = \mu_s mg = 0.5$ mg
Retardation = $\frac{f_s^{max}}{m} = \frac{0.5\ mg}{m} = 0.05$ g
As $v = u + at \quad \therefore \quad 0 = 20 - 0.5\ gt$
or $t = \frac{20}{0.5\ g} = \frac{20}{0.5 \times 9.8} = 4.08$ s

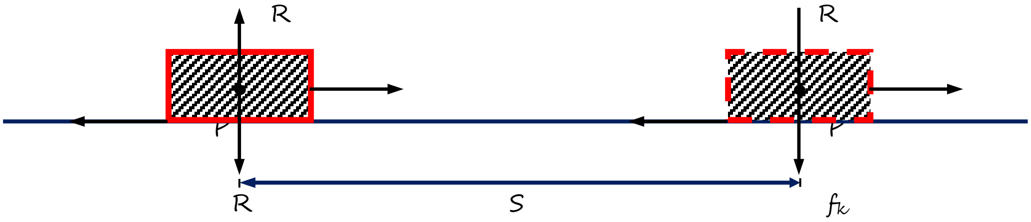
**Q. 13. In Fig. the masses of A and B are 10 kg and 5 kg respectively. Calculate the minimum mass of C which may stop A from slipping. Coefficient of static friction between A and the table.**




***Sol.*** Net force on each body is zero
For A : $R = (m + m_1)$ g
and $T = f_s = \mu R = \mu (m + m_1)$ g
For B: $T = m_2$ g or $\mu (m + m_1) g = m_2 g$
$\therefore \quad m = \frac{m_2 - \mu m_1}{\mu} = \frac{m_2}{\mu} - m_1$
$= \frac{5}{0.2} - 10 = 25 - 10 = 15$ kg

## WORK DONE AGAINST FRICTION

***Case – I :*** **Work done in sliding a body over a horizontal surface:** Consider a body of weight mg resting on a rough horizontal surface, as shown in Fig. The weight mg is balanced by the normal reaction R of the horizontal surface.




**[Work done in sliding a body over horizontal surface]**

Suppose a force P is applied horizontally so that the body just begins to slide. Let $f_k$ be the kinetic friction. Work done against friction in moving the body through distance S will be

$W = f_k \times S$

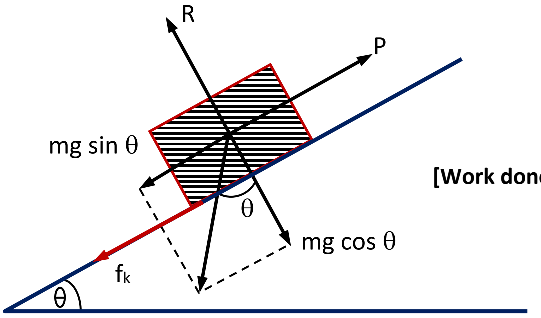
But $f_k = \mu_k R = \mu_k . mg$ ∴ $\mathbf{W = \mu_k . mg\ S}$

**Case – II : Work done in moving a body up an inclined plane:** Suppose a body of weight mg is placed on an inclined plane, as shown in Fig. Let θ be the angle of inclination. A force P is applied on the body so that it begins to slide up the inclined plane.

The weight mg of the body has two components:

(i) mg cos θ perpendicular to the inclined plane. It balances the normal reaction R. Thus R = mg cos θ

(ii) mg sin θ down the inclined plane.




**[Work done in moving a body up an inclined plane]**

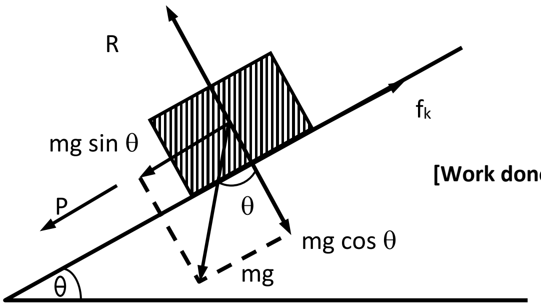
If $f_k$ is the kinetic friction, then the force needed to be applied upwards to just move the body up the inclined plane must be $P = mg \sin\theta + f_k$

But $f_k = \mu_k R = \mu_k mg \cos\theta$

∴ $P = mg \sin\theta + \mu_k mg \cos\theta$ $\mathbf{= mg (\sin\theta + \mu_k \cos\theta)}$

Work done in pulling the body through distance S up the inclined plane is, W $\mathbf{= P \times S = mg (\sin\theta + \mu_k \cos\theta) S.}$

**Case – III : Work done in moving a body down an inclined plane:** Suppose a body of weight mg is placed on an inclined plane, as shown in Fig. Suppose the angle of inclination θ be less than the angle of repose. A force P is applied to just slide the body down the inclined plane.




**[Work done in moving a body down an inclined plane]**

The weight mg of the body has two components:

(i) mg cos θ perpendicular to the inclined plane. It balances the normal reaction R. Thus

$R = mg \cos\theta$

(ii) mg sin θ down the inclined plane.

The force of friction $f_k$ acts up the inclined plane. The applied force needed to just move the body down the inclined plane must be $P = f_k - mg \sin\theta$

But $f_k = \mu_k R = \mu_k mg \cos\theta$

∴ $P = \mu_k mg \cos\theta - mg \sin\theta$

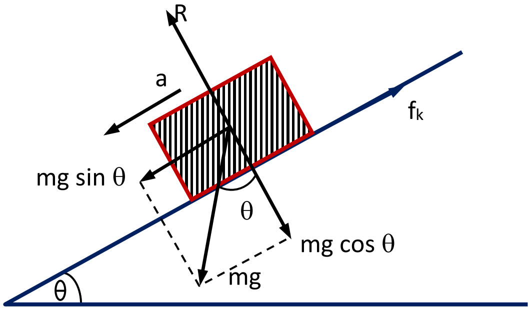
$= mg (\mu_k \cos\theta) - \sin\theta$

The work done in sliding the body through distance S down the inclined plane is

**$W = P \times S = mg (\mu_k \cos\theta - \sin\theta) S$**

## ACCELERATION OF A BODY SLIDING DOWN A ROUGH INCLINED PLANE

**Acceleration of a body sliding down an inclined plane:** As shown in Fig., consider a body of weight mg placed on an inclined plane. Suppose the angle of inclination θ be greater than the angle of repose. Let a be the acceleration with which the body slides down the inclined plane.




**[Acceleration of a body down an inclined plane]**

The weight mg has two rectangular components:

(i) mg cos θ perpendicular to the inclined plane. It balances the normal reaction R. Thus

$R = mg \cos\theta$

(ii) mg sin θ down the inclined plane.

If $f_k$ is the kinetic friction, then the net force acting down the plane is

$F = mg \sin\theta - f_k$

But $f_k = \mu_k R = \mu_k mg \cos\theta$

∴ $ma = mg \sin\theta - \mu_k mg \cos\pi$

Hence **$a = g (\sin\theta - \mu_k \cos\theta)$**

## Examples based on Motion along Rough Inclined Plane

✖ Formulae Used

1. For a body placed on an inclined plane of inclination θ, Normal reaction, $R = mg \cos\theta$
   Friction, $f = \mu R = \mu\, mg \cos\theta$
2. When a body moves down an inclined plane without any acceleration, net downward force needed is
   $F = mg \sin\theta - f = mg (\sin\theta - \mu \cos\theta)$
   Work done, $W = Fs = mg (\sin\theta - \mu \cos\theta) s$
3. When a body moves up an inclined plane without acceleration, net upward force needed is
   $F = mg \sin\theta + f = mg (\sin\theta + \mu \cos\theta)$
4. When a body moves up an inclined Plane, with acceleration a, net upward force needed is
   $F = ma + mg \sin\theta + f = m (a + g \sin\theta + \mu g \cos\theta)$
   $W = m (a + g \sin\theta + \mu g \cos\theta) s$

✖ Units Used

Forces F, f, R and mg are in newton, angle θ in degree, distance s in metre, work done W in joule, coefficient of friction μ has no units.

**Q. 1. A box of mass 4 kg is placed on a wooden plank of length 1.5 m which is lying on the ground. The plank is lifted from one end along its length so that it becomes inclined. It is noted that when the vertical height of the top end of the plank from the ground becomes 0.5 m, the box begins to slide. Find the coefficient of friction between the box and the plank.**

**Sol.** Here m = 4 kg, AB = 1.5 m, BC = 0.5 m

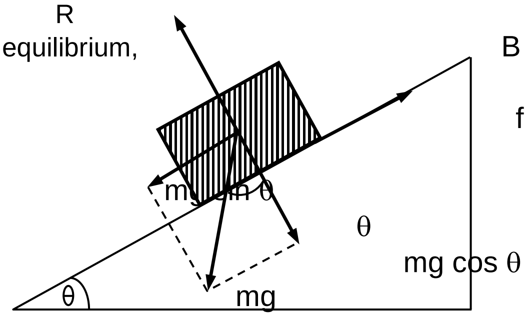
Various forces acting on the box are as shown in Fig. In equilibrium,

$R = mg \cos\theta$ and $f = mg \sin\theta$

∴ $\mu = \dfrac{f}{R} = \dfrac{mg \sin\theta}{mg \cos\theta} = \tan\theta = \dfrac{BC}{AC}$

$= \dfrac{BC}{\sqrt{AB^2 - BC^2}} = \dfrac{0.5}{\sqrt{(1.5)^2 - (0.5)^2}}$

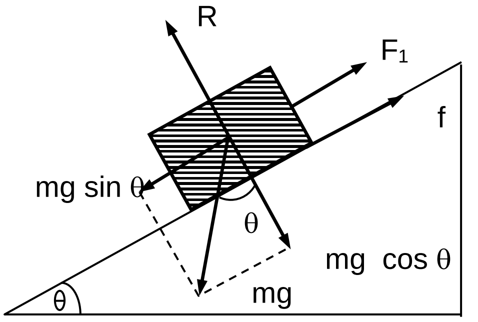
$= \dfrac{0.5}{\sqrt{2}} \simeq 0.7 \times 0.5 = 0.35$



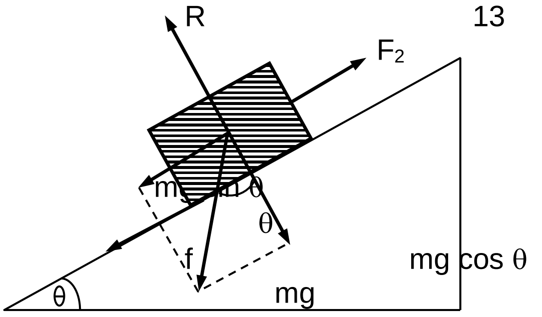
**Q. 2.** **A mass of 200 kg is resting on a rough inclined plane of 30°. If the coefficient of friction is $1/\sqrt{3}$, find the least and the greatest forces acting parallel to the plane to keep the mass in equilibrium.**

***Sol.*** Here $m = 200$ kg, $\theta = 30°$, $\mu = 1/\sqrt{3}$






(a)




(b)

From the above figures, force of friction is

$f = \mu R = \mu\, mg \cos\theta = 1/\sqrt{3} \times 200 \times 9.8 \cos 30°$

$= 1/\sqrt{3} \times 200 \times 9.8 \times \sqrt{3}/2 = 980$ N

Component of the weight acting down the inclined plane

$= mg \sin\theta = 200 \times 9.8 \sin 30°$

$= 200 \times 9.8 \times 0.5 = 980$ N

(i) From Fig. (a), the least force required to prevent the mass from sliding down (when friction f acts upwards) is

$F_1 = mg \sin\theta - f = 980 - 980 = 0$

(ii) From Fig. (b), the greatest force required to prevent the mass from sliding up (when friction f acts downwards) is

$F_2 = mg \sin\theta + f = 980 + 980 = 1960$ N

**Q. 3.** **A box of mass 4 kg rests upon an inclined plane. The inclination of the plane to the horizontal is gradually increased. It is found that when the slope of the plane is 1 in 3, the box starts sliding down the plane. Given $g = 9.8$ ms$^{-2}$. (i) Find the coefficient of friction between the box and the plane.**
**(ii) What force applied to the box parallel to the plane will just make is move up plane.**

**Sol.** Here m = 4 kg

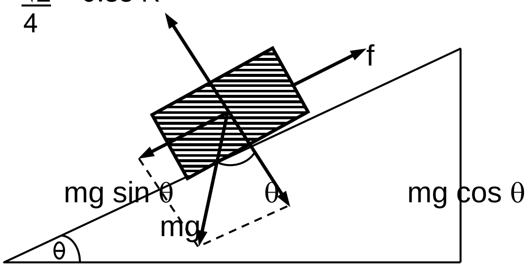
$\sin\theta = 1/3, \quad g = 9.8$ ms$^{-2}$

(i) Various forces acting on the box are shown in Fig. When the box just begins to slide, the forces are in equilibrium.

$\therefore \quad f = mg \sin\theta \quad$ and $\quad R = mg \cos\theta$

$$\mu = \frac{f}{R} = \frac{mg \sin\theta}{mg \cos\theta} = \tan\theta$$

$$= \frac{1}{\sqrt{3^2 - 1^2}} = \frac{1}{\sqrt{8}} = \frac{\sqrt{2}}{4} = 0.35 \text{ R}$$




(ii) When the block moves up the inclined plane, friction f acts down the plane. So minimum force needed to just move the box up the inclined plane is

$F = mg \sin\theta + f = mg \sin\theta + \mu R = mg(\sin\theta + \mu \cos\theta)$ [$\because R = mg \cos\theta$]

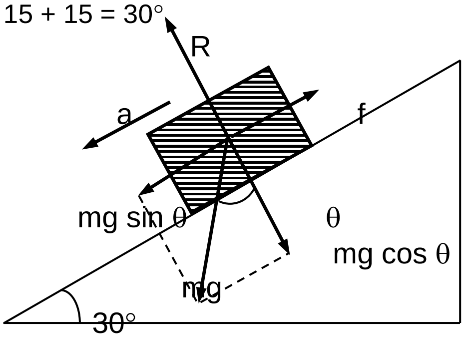
$$= 4 \times 9.8\left(\frac{1}{3} + \frac{1}{\sqrt{8}}\right) \cdot \frac{\sqrt{8}}{3} = 4 \times 9.8 \times \frac{2}{3} = 26.13$$

**Q. 4.** **A block of metal of mass 50 g when placed over an inclined plane at an angle of 15° slides down without acceleration. If the inclination is increased by 15°, what would be the acceleration of the block?**

**Sol.** Here $m = 50$ g $= 0.05$ kg

Angle of repose, $\alpha = 15°$ $\quad\therefore\quad$ $\mu = \tan\alpha = \tan 15° = 0.2679$

New angle of inclination = 15 + 15 = 30°




Let a be the downward acceleration produced in the block. Net downward force on the block is

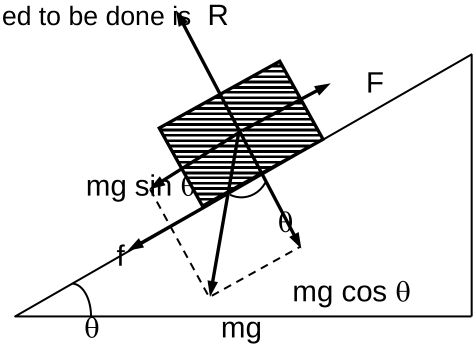
$F = mg \sin\theta - f$

$ma = mg \sin\theta - \mu\, mg \cos\theta$ $\quad [\because f = \mu R = \mu\, mg \cos\theta]$

$\therefore a = g(\sin\theta - \mu \cos\theta) = 9.8 (\sin 30° - 0.2679 \cos 30°) = 9.8 (0.5 - 0.2679 \times 0.866) = 9.8 \times 0.2680 = 2.6\ ms^{-2}$

**Q. 5.** **When an automobile moving with a speed of 36 $kmh^{-1}$ reaches an upward inclined road of angle involved is 0.1, how much distance will the automobile move before coming to rest? Take g = 10 $ms^{-2}$.**

**Sol.** As shown in Fig., when a body moves up an inclined plane, force of friction f acts down the plane. So the force against which work is needed to be done is




$F = mg \sin\theta + f = mg \sin\theta + \mu\, mg \cos\theta = mg (\sin\theta + \mu \cos\theta)$

If m is the mass of the automobile, retardation produced in it will be

$a = F = g(\sin\theta + \mu \cos\theta) = 10 (\sin 30° + 0.1 \cos 30°) = 10 (0.5 + 0.1 \times 0.866) = 5.866\ ms^{-2}$

Now $u = 36\ kmh^{-1} = 10\ ms^{-1}$,

$v = 0, \quad a = -5.866\ ms^{-2}, s = ?$

As $v^2 - u^2 = 2as$ $\quad \therefore \quad 0^2 - 10^2 = 2 \times (-5.866)\, s$ or $s = 8.52$ m

**Q. 6.** **Find the force required to move a train of 2000 quintals up an incline of 1 in 50, with an acceleration of 2 $ms^{-2}$, the force of friction being 0.5 newton per quintal.**

**Sol.** Here m = 2000 quintals = $2 \times 10^5$ kg, $\sin\theta = 1/50$, $a = 2\ ms^{-2}$.

Force of friction = 0.5 newton per quintal

$\therefore$ Total force of friction = 0.5 × 2000 = 1,000 N

Force required against gravity in moving the train up the inclined plane

$= mg \sin\theta = 2 \times 10^5 \times 9.8 \times \frac{1}{50} = 39{,}200$ N

Force required to produce an acceleration of 2 $ms^{-2}$ = ma = $2 \times 10^5 \times 2$ = 400,000 N

$\therefore$ Total force required = 1,000 + 39,200 + 400,000 = 440,200 N

**Q. 7.** **An engine of mass 6.5 metric ton is going upon incline of 5 in 13 at the rate of 9 $kmh^{-1}$. Calculate the power of the engine if $\mu$ = 1/12 and g = 9.8 $ms^{-2}$.**

***Sol.*** Here m = 6.5 metric ton

$= 6500\ kg,\ g = 9.8\ ms^{-2},\ v = 9\ kmh^{-1} = 9 \times \frac{5}{18} = 2.5\ ms^{-1}, \quad \sin\theta = \frac{5}{13}$

$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$

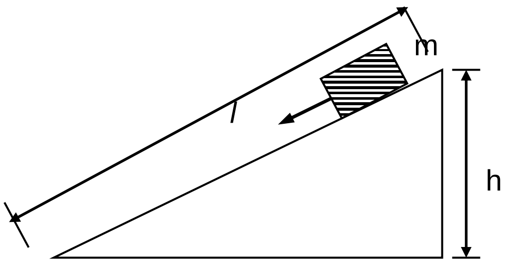
Total force required against which the engine needs to work. $F = mg \sin\theta + f = mg \sin\theta + \mu\, mg \cos\theta$

$= mg (\sin\theta + \mu \cos\theta)$

$= 6500 \times 9.8 \left(\frac{5}{13} + \frac{1}{12} \times \frac{12}{13}\right) = 29400$ N

Power of the engine = Fv = 39400 × 2.5 = 73500 W = 73.5 kW.

**Q. 8.** **A body of mass m is released from the top of a rough inclined plane as shown in Fig. If the force of friction be f, then prove that the body will reach the bottom with a velocity given by** $v = \sqrt{\frac{2}{M}(mgh - fl)}$




***Sol.*** P.E. lost by the body in reaching the bottom = mgh ; K.E. gained by the body in reaching the bottom = $\frac{1}{2}mv^2$

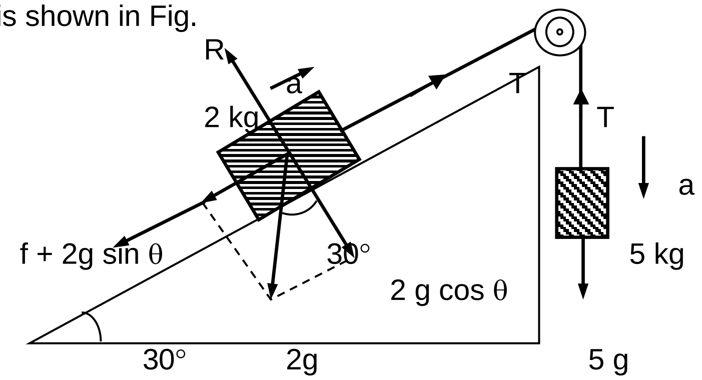
∴ Net loss in energy = $mgh - \frac{1}{2}mv^2$

Work done against friction = fl Hence $mgh - \frac{1}{2}mv^2 = fl$

or $\frac{1}{2}mv^2 = mgh - fl$ or $v = \sqrt{\frac{2(mgh - fl)}{m}}$

***Q. 9. Two blocks of mass 2 kg and 5 kg are connected by an ideal string passing over a pulley. The block of mass 2 kg is free to slide on a surface inclined at an angle of 30° with the horizontal whereas 5 kg block hangs freely. Find the acceleration of the system and the tension in the string. Give $\mu = 0.30$***

***Sol.*** The situation is shown in Fig.




Let a be the acceleration of system in the direction as shown and T the tension in the string.

Equations of motion for 5 kg and 2 kg blocks can respectively be written as

$5g - T = 5a$ ... (1)

and $T - 2g\sin\theta - f = 2a$ ... (2)

where f is the limiting friction and is given by

$f = \mu R = \mu\, mg\cos\theta = 0.3 \times 2g\cos 30°$

Adding (1) and (2), we get

$5g - 2g\sin\theta - f = 7a$

or $5g - 2g\sin 30° - 0.6\,g\cos 30° = 7a$

or $g(5 - 2 \times 0.5 - 0.6 \times 0.866) = 7a$

or $a = \frac{9.8 \times 3.4804}{7} = 4.87\ ms^{-2}$

From (1), $T = 5g - 5a = 5(9.8 - 4.87) = 5 \times 4.93 = 24.65$ N

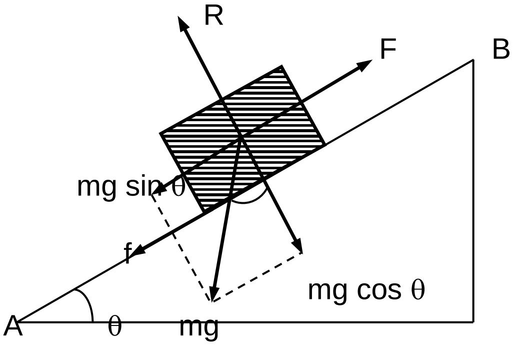
**Q. 10.** **A truck tows a trailer of mass 1200 kg at a speed of 10 $ms^{-1}$ on a level road. The tension in the coupling is 1000 N. (i) What is the power expanded on the trailer? (ii) Find the tension in the coupling when the truck ascends a road having an inclination of 1 in 6. Assume that the frictional resistance of the incline is the same as that on the level road.**

**Sol.** On the level road, force applied by the truck is equal to the friction overcome.

∴ f = 1000 N

Speed of truck, v = 10 $ms^{-1}$

Power expanded on the Trailor, P = fv = 1000 × 10 = $10^4$ W




As shown in fig., when the truck ascends a road of inclination 1 in 6 (i.e., sin θ = 1/6), it overcomes not only the force of friction f but also the component mg sin θ of the weight of the trailor. So the tension in the coupling is

$F = f + mg \sin\theta = 1000 + 1200 \times 9.8 \times 1/6$
$= 1000 + 1960 = 2960$ N

***Problems For Practice***

**Q. 1. A block of mass 2 kg rests on a plane inclined at an angle of 30° with the horizontal. The coefficient of friction between the block and the surface is 0.7. What will be the frictional force acting on the block?**

**Sol.** Here $f = \mu R = \mu\, mg \cos\theta = 0.7 \times 2 \times 9.8 \cos 30°$
$= 0.7 \times 9.8 \times 0.866 = 11.9$ N

**Q. 2. A block of mass 10 kg is sliding on a surface inclined at an angle of 30° with the horizontal. If the coefficient of friction between the block and the surface is 0.5, find the acceleration produced in the block.**

**Sol.** Acceleration,
$a = g(\sin\theta - \mu_k \cos\theta) = 9.8(\sin 30° - 0.5 \cos 30°)$
$= 9.8(0.5 - 0.5 \times 0.866) = 0.657\ ms^{-2}$

**Q. 3. Find the force required to move a train of mass $10^5$ kg up an incline 1 in 50 with an acceleration of 2 $ms^{-2}$. Coefficient of friction between the train and the rails is 0.005. Take g = 10 $ms^{-2}$.**

**Sol. Here m = $10^5$ kg, a = 2 $ms^{-2}$, μ = 0.005, g = 10 $ms^{-2}$**
sin θ = 1/50
∴ $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1 - \frac{1}{2500}} = \sqrt{\frac{2499}{2500}} = 1$
$F = ma + mg(\sin\theta + \mu\cos\theta)$
$= 10^5 \times 2 + 10^5 \times 10 \left(\frac{1}{50} + 0.005 \times 1\right)$
$= 2 \times 10^5 + 0.25 \times 10^5 = 2.25 \times 10^5$ N

**Q. 4. A block slides down an inclined of 30° with an acceleration equal to g/4. Find the coefficient of kinetic friction.**

**Sol.** Here $mg \sin\theta - f_k = ma$ or $mg \sin 30° - f_k = mg/4$
∴ $f_k = mg/2 - mg/4 = mg/4$
and $R = mg \cos\theta = mg \cos 30° - f_k = mg/4$
Hence $\mu_k = \frac{f_k}{R} = \frac{mg/4}{mg(\sqrt{3}/2)} = \frac{1}{2\sqrt{3}}$

**Q. 5. A 10 kg block slides without acceleration down a rough inclined plane making an angle of 20° with the horizontal. Calculate the acceleration when the inclination of the plane is increased to 30° and the work done over a distance of 1.2 m. Take g = 9.8 $ms^{-2}$.**

***Sol.*** Here μ = tan 20° = 0.3647
When the block slides down the inclined plane,
$a = g(\sin\theta - \mu\cos\theta) = 9.8(\sin 30° - 0.3647 \cos 30°)$
$= 9.8(0.5 - 0.3647 \times 0.866) = 1.8\ ms^{-2}$.
$W = F_s = mas = 10 \times 1.8 \times 1.2 = 21.6$ J

**Q. 6. A railway engine weighing 40 metric ton is travelling along a level track at a speed of 54 $kmh^{-1}$. What additional power is required to maintain the same speed up an incline rising 1 in 49. Given μ = 0.1, g = 9.8 $ms^{-2}$. 16**

**Sol.** Here m = 40 metric ton = 40 × $10^3$ kg,
v = 54 $kmh^{-1}$ = 15 $ms^{-1}$

$\sin\theta = \frac{1}{49}$, $\mu = 0.1$, $g = 9.8\ ms^{-2}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{1}{49}\right)} = \sqrt{\frac{2400}{2401}} \simeq 1$

Power required on level track, $P_1 = f \times v = \mu mg \times v$

Power required up the incline, $P_2 = (mg\sin\theta + \mu mg\cos\theta)\, v$

Additional power required $= P_2 - P_1 = [\mu\, mg \sin\theta + \mu\, mg\cos\theta - \mu\, mg]\, v$

or $P = (mg\sin\theta + \mu mg \times 1 - \mu\, mg)\, v = mg\sin\theta \times v$

$= 40 \times 10^3 \times 9.8 \times 1/49 \times 15 = 120 \times 10^3\ W = 120\ kW$

**Q. 7. A metal block of mass 0.5 kg is placed on a plane inclined to the horizontal at an angle of 30° . If the coefficient of friction is 0.2, what force must be applied (i) to just prevent the block from sliding down the inclined plane (ii) to just move the block up the inclined plane and (iii) to move it up the inclined plane with an acceleration of 20 cms$^{-2}$?**

**Sol.** Here $\theta = 30°$, $\mu = 0.2$, $m = 0.5$ kg

(i) Force needed to just prevent the block from sliding down the inclined plane,

$F_1 = mg\sin\theta - f = mg\sin\theta - \mu\, mg\cos\theta$ $[\therefore f = \mu R = \mu mg\cos\theta]$

$= mg(\sin\theta - \mu\cos\theta) = 0.5 \times 9.8\ (\sin 30° - 0.2\cos 30°) = 1.6\ N$

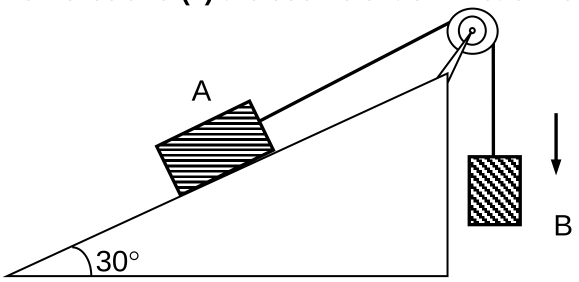
(ii) Force needed to just move the block up the inclined plane,

$F_2 = mg(\sin\theta + \mu\cos\theta) = 0.5 \times 9.8\ (\sin 30° + 0.2 \times \cos 30°) = 3.299\ N$

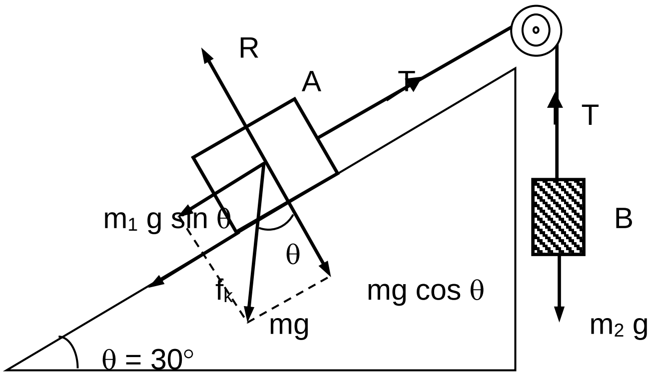
(iii) Force needed to move the block up the inclined plane with an acceleration of 20 cms$^{-2}$

$F_3 = F_2 + ma = 3.299 + 0.5 \times 0.20 = 3.299 + 0.1 = 3.399$

**Q. 8. A block 'A' of mass 14 kg moves along a plane that makes an angle of 30° with the horizontal [Fig.]. Block A is connected to another block B of mass 14 kg by a taut massless string that runs around a frictionless, massless pulley. The block B moves downward with constant velocity. What is (i) the magnitude of the frictional force and (ii) the coefficient of kinetic friction?**




***Sol.*** (i) Various forces acting on the bodies A and B are shown in Fig.




Let T be the tension in the chord. As the block B moves downward with a constant velocity, so net force on it zero

$\therefore$ $T - m_2 g = 0$ or $T_2 = m_2 g$ ... (i)

Net force on block A is also zero.

$\therefore$ $T - f_k - m_1 g\sin\theta = 0$

or $f_k = T - m_1 g\sin\theta = m_2 g - m_1 g\sin\theta = 14 \times 9.8 - 14 \times 9.8 \times ½ = 68.6\ N$

(ii) $\mu_k = \frac{f_k}{R} = \frac{68.6}{m_1 g\sin 30°} = \frac{68.6 \times 2}{14 \times 9.8 \times \sqrt{3}} = 0.58$

**Q. 9. A wooden block of mass 100 kg rests on a flat wooden floor, the coefficient of friction between the two being 0.4. The block is pulled by a rope making an angle of 30° with the horizontal. What is the minimum tension along the rope that just makes the block sliding?**

**Sol.** Various forces acting on the wooden block are as shown in Fig. Let T be the minimum tension that just makes the block sliding. When the block just slides,

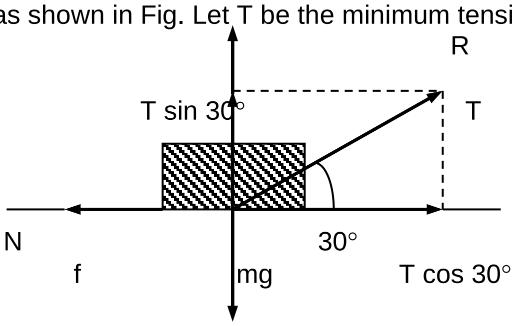
$R = mg - T\sin 30°$

and $T\cos 30° = f = \mu R = \mu(mg - T\sin 30°)$

∴ $T(\cos 30° + \mu \sin 30°) = \mu mg$

or $T = \dfrac{\mu mg}{\cos 30° + \mu \sin 30°}$

$= \dfrac{0.4 \times 100 \times 9.8}{0.866 + 0.4 \times 0.5} = \dfrac{392}{1.060} = 367.7$ N




## FRICTION IS A NECESSARY EVIL

**Friction is a necessary:** (Advantages of friction):

(i) It is due to friction between the ground and the feet that we are able to walk.

(ii) The brakes of a vehicle cannot work without friction.

► **Friction is necessary evil**: - Friction plays a vital role in our day-to-day life. In same cases. Friction is desirable i.e is useful to us and we want to keep (or increase it) But in other cases friction is harmful and we want to reduce it.

► Friction **is useful (necessary)**:-

- i) It is due to friction that we are able to walk, a run or drive car.
- ii) Friction enables us to write on paper.
- iii) Spikes are provided in the shoes of player and athletes to increase friction and prevent slipping
- iv) Without friction no knot could be made and no women or knitted cloth could be hold together.
- v) Without friction a chalk could not write on the black board.
- vi) Without friction, belts could not drive machine.
- vii) Without friction we cannot light a match stick in all the above cases friction is desirable.

► **Friction is harmful:** - Friction opposes motion between surfaces in contact. Friction is particularly harmful to those machines which have moving parts in them. Friction is harmful to machine as :-

i) Friction reduces the efficiency of machine.

ii) Friction produces heat which could damage the machine.

iii) Friction wears out the rubbing machine parts gradually.

Thus, we find that although friction has a disadvantage in many cases and we would like to reduce it to the maximum. It is at the same times a necessity as our life on the earth becomes impossible without friction.

**And therefore, friction is necessary evil.**

■ **Methods of reducing friction**: - Friction is due to the irregularities or roughness of the surface of the body and Therefore, any process which makes the two-surface smooth will reduces friction. Following are the methods by which can reduce friction –

i) By polishing surfaces.

ii) Using proper lubricants (like grease or oil ) to surfaces .

iii) By converts sliding friction into rolling friction like using ball bearing or roller bearing.

iv) By stream lining or streamlined shape of car, aero plane and ships reduce air and water friction .

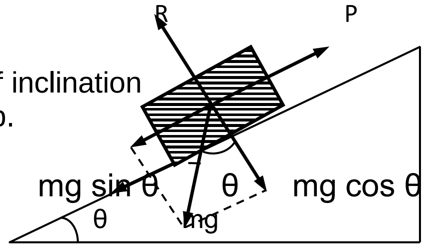
■ **Methods of increasing friction**: -

i) By making both the surfaces very rough.

ii) By making both the surfaces highly smooth, so that there is a very large force of attraction between the molecules and hence the force of friction is also increased.

## ◘ WORK DONE IN MOVING A BODY UP IN A INCLINED PLANE :-

Consider a body of mass M placed on a inclined plane having angle of inclination equal to θ. Let P = applied force so that the body just begins to slide up.




.'. Components of mg are ---

(i) **mg Cos θ opposite to R** (ii) **mg Sin θ acts downward**

As the body is just sliding therefore,

Applied force, P = mg Sin θ + force of friction

$= mg\sin\theta + F = mg\sin\theta\ \mu_k R = mg\sin\theta + \mu_k mg\cos\theta = mg(\sin\theta + \mu_k \cos\theta)$

◙ If the body moves to distance S up an inclined plane, then, work done, **W = P.S = mg (Sin θ + $\mu_k$ cos θ) S**

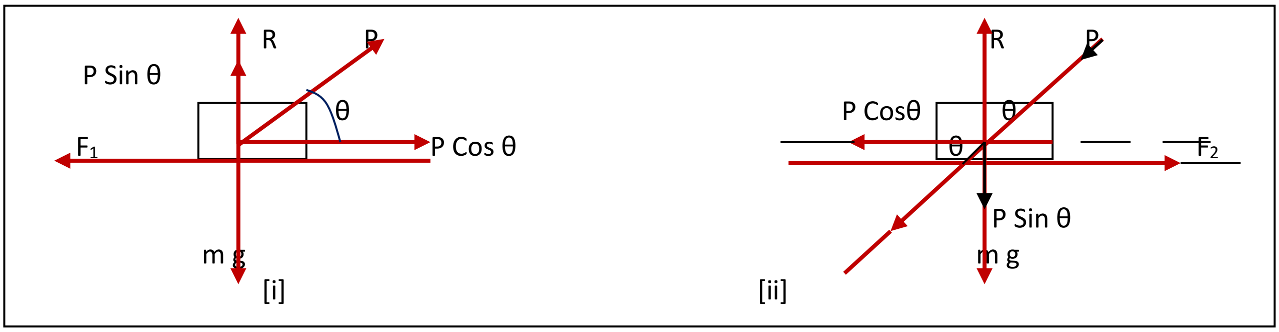
◙ If the body moves down the inclined plane, then work done, **W = mg (Sin θ – $\mu_k$ cos θ) S**

✱ **In moving a body over a rough horizontal surface, work has to be done only against the force of friction.**

❋ In moving a body up on an inclined plane work has to be done against the force of friction as well as a component of weight in downward direction i.e mg sin θ.

◙ ☻ It is easier to pull than to push a body: -

Explanation --- In first case: Let P = applied force to pull a body of weight mg {fig i}




Rectangular component of applied force – (i) P Cos θ & (ii) P Sin θ

And, R = mg – P Sin θ also, **$F_1 = \mu_k R = \mu_k (mg - P \sin\theta)$** ......................{A}

**In 2nd case**: - Let P = applied force to push a body of weight mg {fig ii}

Then, R = mg + P Sin θ

∴ kinetic friction $F_2 = \mu_k R = \mu_k (mg + P \sin\theta)$ ................................ {B}

From {A} & {B} $F_2 > F_1$

**i.e., Force of friction in case of push is more than in case of pull. Hence, it is easier to pull than to push the body.**

## ◘ Dynamics of uniform circular motion [ Concept of CENTRIPETAL FORCE]

*"Centripetal force is the force required to move a body uniformly in a circle".*

In other words, "The force which deviates a particle from it's linear path to move is along a circular path and is directed radially inwards is called centripetal force".

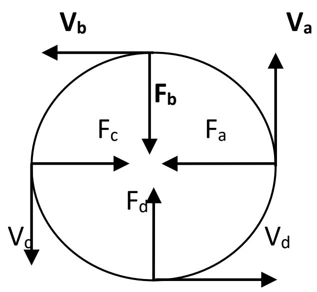
☻ **Centripetal force acts along the radius and towards the centre of the circle.**

EXPLANATION – When a body moves in circle, it's direction of motion at any instant is along the tangent to the circle at that instant. In this way the direction of motion of the body moving in a circle goes on a changing continuously.

☻ A/N's 1st law of motion, a body cannot change its direction of motion by itself. An external force is required for this purpose. It is this external force which is called the centripetal force.

**Nature of external force (Centripetal force)** –The direction of the external force is such that only the direction of the velocity vector changes. The magnitude of the velocity remains unchanged.

**This is possible only if the force always Acts at right angles of the velocity vector. If the force acts in any other direction, it will have a component which would change the speed of the particle.**




Expression for Centripetal force: -

We know that, centripetal acceleration, $a = v^2/r = r^2\omega^2/r = r\omega^2$

where, v = linear velocity

= Angular velocity

But F = ma {from N's 2nd law of motion}

i.e., Centripetal force = mass × centripetal acceleration

∴ **$F = m v^2 / r$ ................................................ (i)**

**$F = m r \omega^2$ ................................................ (ii)** [ since, $a = r\omega^2$]

**$F = m r (2\pi)^2 / T$** { since $\omega = 2\pi/T$}

∴ $F = 4\pi^2 mr / T^2$ ................................................(iii)

Or, $F = 4\pi^2 m r\, 1/T^2$

$F = 4\pi^2 m r \upsilon^2$ ............................................ (iv) [ since frequency, $\upsilon = 1/T$]

………. (i), (ii), (iii) & (iv) are different forms of centripetal force.

Example – (i) The electrostatic force of attraction between the nucleus and an electron provides the necessary centripetal force to move the electron around the nucleus.

(ii) The gravitational force of attraction between the sun and the earth provides the necessary centripetal force to move the earth around the sun.

(iii)When a stone tie to a string is revolved in a circle, the tension in the string suppliers the necessary centripetal force.

- **When the centripetal force ceases to act the particle would move in straight line along the tangent to the circular path at that point the force has case acts.**
- **In the absence of the centripetal force, the particle has a natural tendency to move in a straight line in accordance with the Newton's first law of motion.**

## CENTRIFUGAL FORCE: -

► **Centrifugal force is not a force of reaction. It is a fictitious force which has a concept only in a rotating frame of reference.** We know that the natural tendency of the body is to move uniformly along a straight line. Also, a body is always under the action of centripetal force while moving uniformly a long circular path (inertia of direction) . This inertia of direction gives rises to a force called centrifugal force.

The body exerts an equal and opposite force on the agent that supplies the centripetal force. The centrifugal force acting on the agent supplying the centripetal force along the radius of the circular path and away from the center.

Thus, "**Centripetal force is a force that arises when a body is moving along a circular path, by mean of tendency of the body to regain its natural straight-line path**".

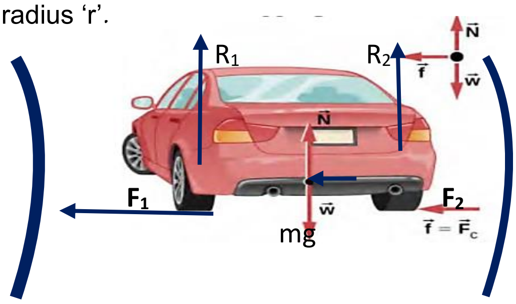
☻ ***Centripetal force can be regarded as the reaction of centripetal force.***

**Therefore, Magnitude of centrifugal force, $F = m V^2 / r = m r \omega^2 = 4\pi^2 mr / T^2 = 4\pi^2 m r \upsilon^2$**

## ◘ Application of centripetal & centrifugal force ...........

[i] **Car on a level curved road** ---When a vehicle goes round a curved road, it requires some centripetal force. While round the curve the wheels of the vehicles have a tendency to leave the curved path and regain the straight line path. Force of friction between the wheels and the road opposes this tendency of the wheels.

This force of friction acts towards the centre of the circular tracks and provides the necessary centripetal force. Consider a car of mass 'm' moving with constant speed 'v' on a level road of radius 'r'.




$R_1$ & $R_2$ are the force of normal reaction on the wheels

∴ **$R_1 + R_2 = mg$ ------------------------------(i)**

$F_1$ & $F_2$ are the forces of friction between the tyres and the road, directed towards the centre of curved track.

∴ $F_1 = \mu R_1$ & $F_2 = \mu R_2$

**Total fictional force, $F = \mu [R_1 + R_2] = \mu (mg)$ ......... from (i)**

Also, required centripetal force, $= m v^2/r$

Since, this force is to be provided only by the force of friction, therefore,

$m v^2/r \le \mu [mg]$ or, $v = \sqrt{\mu r g}$

Therefore, **maximum speed, $v_{max}$ , then the car will skid and go off the road in a circle of radius > r.**

**◘ Banking of Road** -- The maximum a velocity [ $V_{max} = \sqrt{\mu r g}$] with which a vehicle can go round a level curved road depends on.

**The value of μ Decreases when ------ [i] Road is smooth --- [ii] Tyres of the vehicle are worn out. ------ [iii] The road is wet etc.**

In the above-mentioned case, force of friction is not reliable source for providing the required centripetal force. To overcome this problem the road is banked that is the outer edge of the road is raised a little above the inner edge. By doing so, a component of normal reaction of the road be spared to provide the centripetal force. Thus,

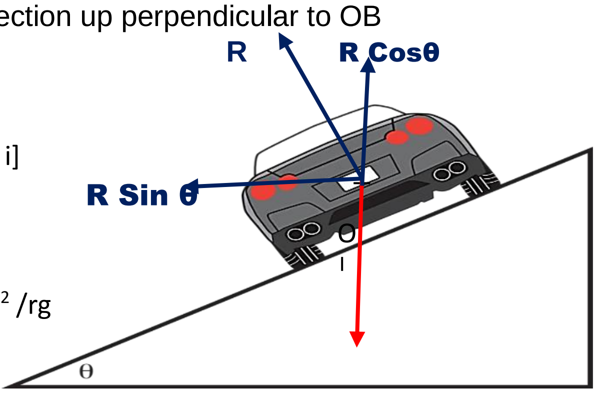
**"The phenomenon of raising outer edge of the curved road above the inner edge is called banking of road".**

## Case – I. [ neglecting μ]

Consider a car of mass 'm' moving on a banked road of radius 'r'.

∴ ∟ **AOB = θ = ANGLE OF BANKING.**

Now, weight [mg] of the vehicle going over the banked road acts vertically downwards.
Also, Normal reaction [R] of the banked road acts upwards in direction up perpendicular to OB




Rectangular components of R are --- {I} R Cos θ
--- {ii} R Sin θ

Since, R Cos θ balances the weight [mg] , ∴ **R Cos θ = mg** ---------[ i]

and, R Sin θ provides the necessary centripetal force,

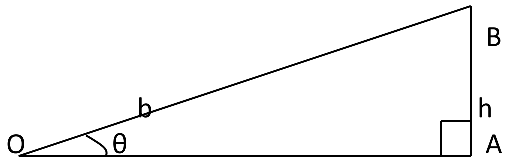
∴ **R Sin θ = m $v^2$/r** ..........................................[2]

Dividing [2] by [1], we get R Sin θ / R Cos θ = m $v^2$/r / mg => tan θ= $v^2$ /rg

**Safe speed,** $$V = \sqrt{\tan\theta\, r\, g}$$

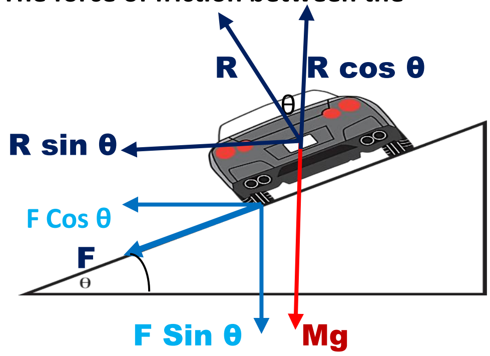
again, In Δ AOB, let OB = breadth of the road = b
AB = height of the outer edge of the road from the inner edge = h
tan θ = AB/OA = h /$\sqrt{b^2 - h^2}$

also, **tan θ = $v^2$/rg = h/b** [ since , h << b, therefore, $h^2$ can be neglected ]




## Case II: -- [ taking μ into account]

**Weight [mg] acts vertically downwards and R of the road acts on the car. The force of friction between the tyres and the road = F**




Rectangular component of R .... **[i] R Cos θ and [ii] R Sin θ**

| |y, Rectangular component of F .... [i] F Cos θ and [ii] F Sin θ

For equilibrium, mg + F Sin θ = R Cos θ

∴ **mg = R Cos θ - F Sin θ** .............................................(i)

Also, R Sin θ + F Cos θ provides necessary centripetal force

∴ **R Sin θ + F Cos θ = m $v^2$/r**

or, m $v^2$/r = R Sin θ + F Cos θ ......................................(ii)

Dividing [ii] by [i] $$\frac{m v^2}{r\, mg} = \frac{R \sin\theta + F \cos\theta}{R \cos\theta - F \sin\theta}$$

$$\frac{v^2}{rg} = \frac{R \sin\theta + \mu R \cos\theta}{R \cos\theta - \mu R \sin\theta}$$

$$\frac{v^2}{rg} = \frac{R [\sin\theta + \mu \cos\theta]}{R [\cos\theta - \mu \sin\theta]}$$

Dividing both numerator and Denominator by Cos θ , we get

$$\frac{v^2}{rg} = \frac{[\sin\theta/\cos\theta + \mu \cos\theta/\cos\theta]}{[\cos\theta/\cos\theta - \mu \sin\theta/\cos\theta]}$$

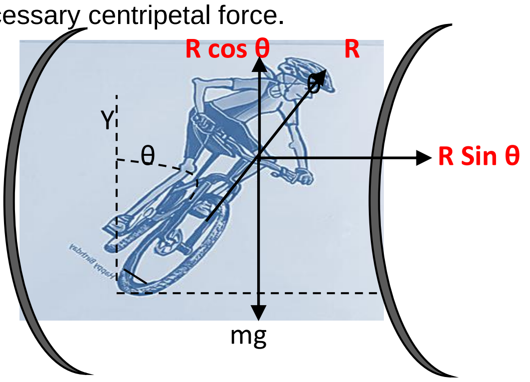
$$\frac{v^2}{rg} = \frac{[\tan\theta + \mu]}{[1 - \mu \tan\theta]}$$

$$\mathbf{v = \sqrt{\frac{[\tan\theta + \mu]\, rg}{[1 - \mu \tan\theta]}}}$$

**When the actual speed of the vehicle exceeds greatly the safe speed limit chances of over turning of the vehicle increases.**

**Bending of a cyclist**: When a cyclist negotiates a curve road, he bends slightly from his vertical position towards the inner side and of the curve.

Explanation ---- If the cyclist keeps himself vertical while running the weight is balanced by the normal reaction of the ground and he has to depend upon the force of friction between the tyres and the road for centripetal force. As the force of friction is small, therefore dependence on it is not safe.
For obtaining centripetal force the cyclist has still bend a little towards from his vertical position, while turning down. So, A component of R in the horizontal direction provides the necessary centripetal force.




Let, Mass of the cyclist [ cycle + man] = m
Velocity of the cyclist = v
Radius of the circular path = r
And, Angle of bending with vertical = θ
Weight [mg] Acts vertically downward.
Normal reaction = R acts at an angle θ with the vertical
Rectangular component of R ---------- [i] R Cos θ ....... [ii] R Sin θ
R Cos θ balances the weight of the cyclist.

∴ R Cos θ = mg

R Cos θ provides the centripetal force,

∴ $R \sin\theta = m v^2/r$

Now, $R \sin\theta / R \cos\theta = mv^2/ mg\, r$

$\tan\theta = v^2/ rg$ $\quad$ $\boldsymbol{\theta = \tan^{-1} v^2/ rg}$

**For safe turn, θ should be small for which v should be small and r should large i.e. turning should be at a also speed and track of larger radius.**

**1. Find the angle through which a cyclist bends when he covers a circular path 34.3 m long in √22 sec. Given g = 9.8 $ms^{-2}$.**

**Sol.** Here, θ =?

If r is radius of circular path, then length of path = 2πr = 34.3 m

$r = \frac{34.3}{2\pi}$ m

Time taken, t = √22 m

Therefore, velocity, $v = \frac{\text{length of path}}{\text{time}}$

$= \frac{34.3}{\sqrt{22}}$ $ms^{-1}$

As $\tan\theta = \frac{v^2}{rg}$

Therefore, $\tan\theta = \left(\frac{34.3}{\sqrt{22}}\right)^2 \times \frac{2\pi}{34.3 \times 9.8}$

$\tan\theta = \frac{34.3 \times 34.3}{22} \times \frac{2 \times 22}{7 \times 34.3 \times 9.8}$

$= \frac{34.3 \times 2}{68.6} = 1$  ∴ $\theta = 45^0$

**2. A curved road of diameter 1.8 km is banked so that no friction is required at a speed of 30 $ms^{-1}$. What is the banking sangle?**

**Sol.** Here, Diameter, 2r = 1.8 km = 1800 m

r = 1800/2 = 900 m

speed, v = 30 $ms^{-1}$ θ =?

As $\tan\theta = \frac{v^2}{rg}$

Therefore, $\tan\theta = \frac{30 \times 30}{900 \times 9.8} = 0.102$

$\theta = 6^0$.

**3. A bend in a level road has a radius of 80 m. Find the maximum speed which a car turning this bend may have without Skidding, if the coefficient of friction between the car tyres and the road is 0.25.**

**Sol.** Here, r = 80 m,

Maximum speed, v =?

μ = 0.25

On a level road, there will be ni skidding, when force of friction is enough to provide the necessary centripetal force

i.e. $\frac{mv^2}{r} = F = \mu R = \mu mg$

$v = \sqrt{\mu rg} = \sqrt{0.25 \times 80 \times 9.8}$

$v = \sqrt{196} = 14$ $ms^{-1}$

**3. A car travels on a flat, circular track of radius 200 m at 30 $ms^{-1}$ and has a centripetal acceleration = 4.5 $ms^{-2}$. (a) If the mass of the car is 1000 kg, what frictional force is required to provide the acceleration? (b) If the coefficient of static friction is 0.8, what is the maximum speed at which are the car can circle the track?**

**Sol.** Here, r = 200 m, v = 30 $ms^{-1}$

Centripetal acc., a = 4.5 $ms^{-2}$; m = 1000 kg

Frictional force required, F =?

As, F = Accelerating force

= m a = 1000 × 4.5 = 4500 N

(b) μ = 0.8, v =?  $v = \sqrt{\mu rg} = \sqrt{0.8 \times 200 \times 9.8}$

v = 39.6 $ms^{-1}$

**4. An aircraft executes a horizontal loop at a speed of 720 km/h with its wings banked at $15^0$. What is the radius of the loop?**

**Sol.** Here, $\theta = 15^0$

$v = 720 \text{ km/h} = \frac{720 \times 1000}{60 \times 60} = 200$ $ms^{-1}$; g = 9.8 $ms^{-2}$

From $\tan\theta = \frac{v^2}{rg}$

$v^2 = rg\tan\theta$

∴ $r = \frac{v^2}{g\tan\theta} = \frac{(200)^2}{9.8 \times \tan 15^0}$ = 15232 m = 15.232 km.

**5. A train rounds an unbanked circular bend of radius 30 m at a speed of 54 km/h. The mass of the train is $10^6$ kg. What provides the centripetal force required for this purpose? The engine or the rails? The outer or the inner rails? Which rail will wear out faster, the outer or the inner rail? What is the angle of banking required? to prevent wearing out the rails?**

**So*l*.** The centripetal force is provided by the lateral thrust exerted by the rails on the wheels. By Newton's 3rd law, the train exerts an equal and opposite thrust on the rails causing its wear and tear.

Obviously, the outer rail will wear out faster due to the larger force exerted by the train on it.

Here, v = 54 km/h

$$= \frac{54 \times 1000}{60 \times 60} = 15 \text{ m/s.}$$

$$g = 9.8 \text{ ms}^{-2}$$

$$\text{As } \tan\theta = \frac{v^2}{rg} = \frac{15 \times 15}{30 \times 9.8} = 0.76$$

$$\therefore \quad \theta = \tan^{-1} 0.76 = 37.40.$$

**6. A long-playing record revolves with a speed of 33.1/3 rev./min. and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the co-efficient of friction between the coins snd the record is 0.15, which of the two coins will revolve with the record?**

**So*l*.** The coin revolves with the record in the case when the force of friction is enough to provide the necessary centripetal force. If this force is not sufficient to provide centripetal force, the coin slips on the record.

Now the frictional force is μ R where R is the normal reaction, and R = mg

Hence force of friction $= \mu mg$

and centripetal force required is $\frac{mv^2}{r}$ or $m r \omega^2$

μ, ω are same for both the coins and we have different values of r for the twp coins.

So to prevent slipping i.e., causing coins to rotate $\mu mg \geq m r \omega^2$

or $\mu g > r\omega^2$ ........................ (i)

For 1st coin

$$r = 4 \text{ cm} = \frac{4}{100} \text{ m}$$

$$n = 33.\frac{1}{3} \text{ rev./min.}$$

$$= \frac{100}{3 \times 60} \text{ rev./sec.}$$

$$\omega = 2\pi n = 2\pi \times \frac{100}{180} = 3.49 \text{ s}^{-1}$$

$$\therefore \quad r\omega^2 = \frac{4}{100} \times (3.49)^2 = 0.49 \text{ ms}^{-2}$$

and $\mu g = 0.15 \times 9.8 = 1.47 \text{ ms}^{-2}$

As $\mu g > r\omega^2$, therefore, this coin will revolve with the record.

**For 2nd coin**

$$r = 14 \text{ cm} = \frac{14}{100} \text{ m;}$$

$$\omega = 3.49 \text{ s}^{-1}$$

$$r\omega^2 = \frac{14}{100} \times (3.49)^2 = 1.705 \text{ ms}^{-2}$$

and $\mu g = 1.47 \text{ ms}^{-2}$

Here, $\mu g \geq r\omega^2$ is not satisfied, so this coin will not revolve with the record.

- We have nothing to do with the radius of the record (= 15 cm)

**7. A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev./min. in a horizontal plane. What is the tension in the string? What is the maximum speed with which the? stone can be whirled around if the string can withstand a maximum tension of 200 N?**

**So*l*.** Here, m = 0.25 kg, r = 1.5 m.

$$n = 40 \text{ rpm} = \frac{40}{60} \text{ rps} = \frac{2}{3} \text{ rps, T =?}$$

$$T = m r \omega^2 = m r (2\pi n)^2 = 4\pi^2 m r n^2$$

$$T = 4 \times \frac{22}{7} \times \frac{22}{7} \times 0.25 \times (2/3)^2 = 6.6 \text{ N}$$

If $T_{max} = 200$ N, then from

$$T_{max} = \frac{mv^2_{max}}{r}$$

$$v^2_{max} = \frac{T_{max} \times r}{m}$$

$$= \frac{200 \times 1.5}{0.25} = 1200$$

$v_{max} = \sqrt{1200} = 34.6$ m/s.

**8. One end of a string of length 1.5 m is tied to the stone of mass 0.4 kg and the other and to a small pivot on a smooth vertical board. What is the minimum speed of the stone required at its lowermost point so that the string does not slack at any point in its motion along the vertical circle?**

***Sol.*** Here, r = 1.5 mm = 0.4 kg;

$g = 9.8$ m/s$^2$

The minimum speed at the lowest point of the vertical circle is

$V_L = \sqrt{5 g r} = \sqrt{5 \times 9.8 \times 1.5}$

$V_L = 8.6$ ms$^{-1}$.

**9. A motor cyclist loops the loop whose diameter is 8 m. From what minimum height must be start in order to roll down and go around the loop?**

***Sol.*** Here, diameter of loop,

2r = 8 m; r = 4 m

height h = ?

As $h = \frac{5}{2}$ $= \frac{5}{2} \times 4$ 10 m

**10. Find the minimum speed of a bike at the highest point of a globe of radius 9.8 m.**

***Sol.*** Here, $V_H$ =? r = 9.8 m ; g = 9.8 ms$^{-2}$

At the highest point, vH = $\sqrt{r g} = 9.8 \times 9.8 = 9.8$ ms$^{-1}$

**11. A body of mass 0.5 kg is whirled in a vertical circle by a string 1 m long. Calculate (i) minimum speed it must have at the bottom of the circle so that the string may not slack when the body reaches the top. (ii) In that case, will the velocity at the top of the circle be zero?**

***Sol.*** Here, m = 0.5 kg, r = 1 m.

Minimum speed at the bottom, vL = ?

As $v_L = \sqrt{5 r g}$

Therefore, vL = $\sqrt{5 \times 1\ 9.8}$

$= \sqrt{49} = 7$ ms$^{-1}$

(ii) In that event, velocity at the top of the circle shall not be zero, Its value will be $v_H = \sqrt{r g} = \sqrt{1 \times 9.8} = 3.1$ ms$^{-1}$.

**12. A stone of mass 0.3 kg tied to the end of a string in a horizontal plane is whirled around in a circle of radius 1 m with a speed of 40 rev./min. What is the tension in the string? What is the max. Speed with which the stone can be whirled around if the string can string can withstand and maximum tension of 200 N ?**

***Sol.*** Here, m = 0.3kg ; r = 1 m

T = Time for 1 revolution

$$= \frac{1}{40} \text{ min} = \frac{60}{40} \text{ sec.} = \frac{3}{2} \text{ sec}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3/2} = \frac{4}{3}\pi \text{ rad.s}^{-1}$$

Tension in the string = Centripetal force = m r $\omega^2$

Therefore, T = (0.3) (1) $(4\pi/3)^2$ = 5.26 N

(ii) Maximum tension T = 200 N

From $\frac{mv^2}{r} = T$

$$v^2 = \frac{T \times r}{m} = \frac{200 \times 1}{0.3}$$

$v = \sqrt{200/0.3} = 25.8$ ms$^{-1}$ .

**13. A baby is sitting on the horizontal platform of a joy wheel at a distance of 5 m from the centre. The joy wheel begins to rotate and when the angular speed exceeds 10 r.p.m, the boy just slips. What is the coefficient of friction between the boy and the platform?**

***Sol.*** Here, r = 5 m

Angular speed, $\omega$ = 10 revol/min.

$$= \frac{10}{60} \times 2\pi \text{ radian/sec}$$

$$= \frac{\pi}{3} \text{ radian/sec}$$

The boy would slip when centrifugal force ( $m r \omega^2$) exceeds the force of friction.

$$m r \omega^2 = F = \mu R = \mu mg$$

$$\mu = \frac{r \omega^2}{g} = 5 (\pi/3)^2 \times \frac{1}{9.8}$$

$$\mu = \frac{5\pi^2}{9 \times 9.8} = 0.56$$

**14. A bucket containing water is tied to one end of a rope 2.45 m long and rotated about the other end in a vertical circle. Find the minimum velocity at the highest and lowest points in order that water in the bucket may not spill.**

***Sol.*** Here, $r = 2.45$ m.

Minimum velocity at highest point

$$v_H = \sqrt{r g} = \sqrt{2.45 \times 9.8}$$

$$v_H = \sqrt{9.8/4 \times 9.8}$$

$$= \frac{9.8}{2} = 4.9 \text{ ms}^{-1}$$

Minimum velocity at the lowest point,

$$v_L = \sqrt{5 r g} = \sqrt{5 \times 2.45 \times 9.8} = 10.78 \text{ ms}^{-1}.$$