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ROCKET PROPULSION

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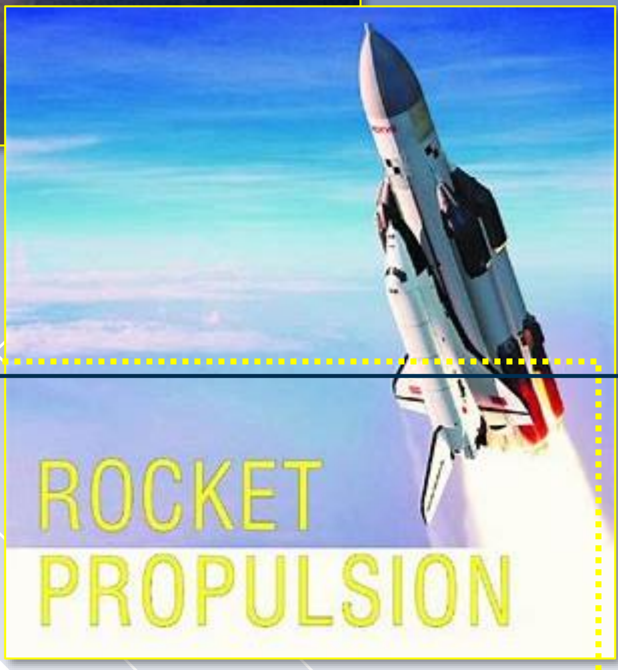

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ROCKET PROPULSION

CHANGING MASS IN MOTION:

Principle. *The propulsion of a rocket is based on the principle of conservation of linear momentum.* Before a rocket is fired, the total linear momentum of rocket plus fuel is zero. Since the system is essentially an isolated system, the linear momentum of the system remains the same. When rocket is fired, fuel is burnt and very hot gases are formed. These gases are expelled from the back of the rocket. Since linear momentum acquired by the gases is directed toward the rear, the rocket must acquire an equal linear momentum in the opposite direction (forward) in order to conserve linear momentum. This is how a rocket is propelled.

THEORY : Consider a Rocket moving vertically Upwards in free space with no external forces. Thus the system (rocket + fuel) continues as isolated system so that LINEAR MOMENTUM remains the same.

Suppose ,

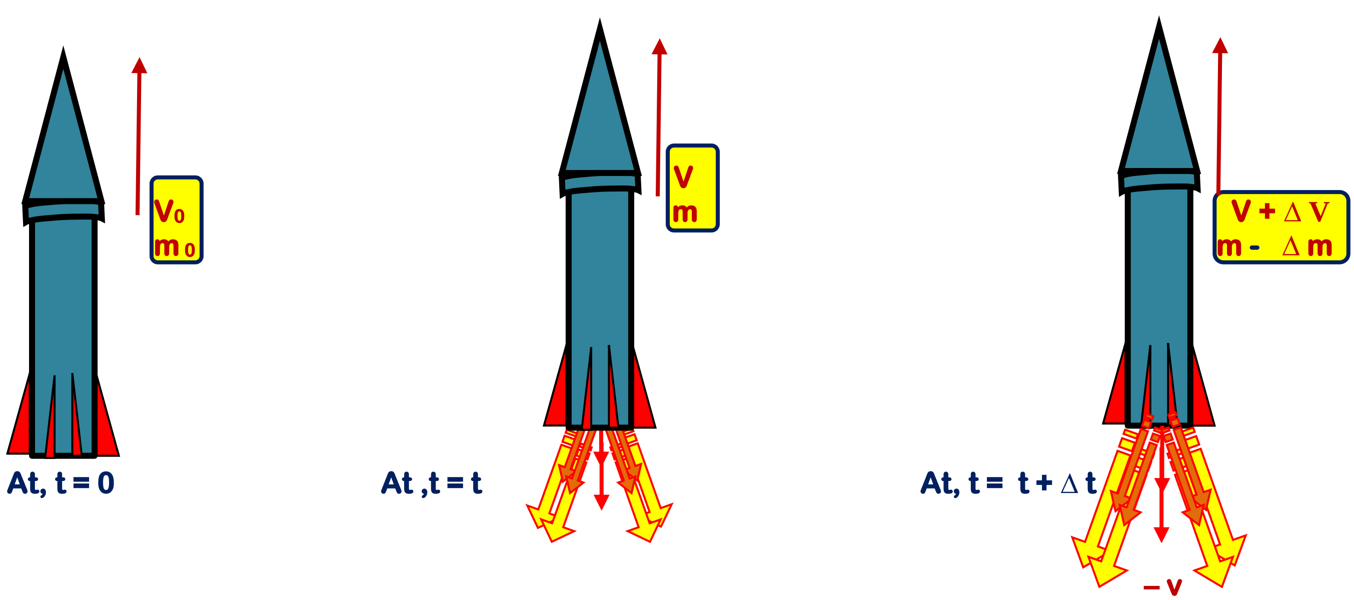
At $t = 0$, the mass of the rocket (container + fuel) = m_0
At $t = 0$, the Velocity of the rocket w.r.t the earth = V_0

Let , At $t = t$, the mass of the rocket (container + fuel) = m
At $t = t$, the Velocity of the rocket w.r.t the earth = V

As, the fuel is burnt , the velocity of rocket increases and its mass decreases. And, therefore,

$$\boxed{m_0 < m} \quad \text{and,} \quad \boxed{V_0 < V}$$

In time Δt , the rocket will eject a mass of Δm (burnt gases) with velocity '$-v$' w.r.t the rocket However, the velocity of of the ejected gases w.r.t the earth is ($V - v$) . At the end time interval Δt , the mass of the rocket had decreased to ($m - \Delta m$) and its velocity has increased to ($V + \Delta V$)



At, t = 0   At ,t = t   At, t = t + Δ t

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Applying the law of conservation of linear momentum to the system ,

Initial momentum = Final momentum

(i.e., at the start of time interval Δt) (i.e., At the end time interval , Δt)

$$m V = (m - \Delta m)(V + \Delta V) + \Delta m (V - v)$$
$$m V = m V - V \Delta m + m \Delta V - \Delta m \Delta V + V \Delta m - v \Delta m$$
$$0 = m \Delta V - \Delta m \Delta V - V \Delta$$

The term $\Delta m \ \Delta V$ is the product of two small quantities and, therefore , can be neglected.

$$0 = m \Delta V - v \Delta m$$

The quantity of fuel ejected (Δm) is equal to the loss of mass of the rocket ($-\Delta m$) . The negative sign indicate a decrease in mass m.

$$0 = m \Delta V - v(-\Delta m)$$

$$\Delta V = -\frac{\Delta m}{m} v$$

Taking limit of $\Delta t \rightarrow 0$, then ΔV and Δm can be replaced by dV and dm respectively,

$$dV = -v\frac{dm}{m} \quad \text{----------[1]}$$

At $t = 0$; $V = V_0$ and $m = m_0$

At $t = t$; $V = V$ and $m = m$

$$\int_{V_0}^{V} dV = -\int_{m_0}^{m} v\frac{dm}{m}$$

**** The velocity of exhaust gases w.r.t rocket is assumed to be constant**

$$\int_{V_0}^{V} dV = -v\int_{m_0}^{m} \frac{dm}{m}$$

$$|V|_{V_0}^{V} = -v|{}^{*}\log_e m|_{m_0}^{m}$$

$$V - V_0 = -v\,[\log_e m - \log_e m_0] = v\,[\log_e m_0 - \log_e m]$$

$$V = V_0 + v\log_e\frac{m_0}{m} \quad \text{----------[2]}$$

Equation {2} gives the velocity of the rocket at any time t when the mass of the rocket is m .
If the initial velocity of the rocket is zero ($V_0 = 0$) then,

$$V = v\log_e\frac{m_0}{m}$$

Thus , the velocity of the Rocket at any instant is directly proportional to –

[1] Exhaust speed of the ejecting gases w.r.t rocket.

[2] natural log of the ratio of initial mass of the rocket to its mass at that instant