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01 CBSE TRIGONOMETRICAL IDENTITIES

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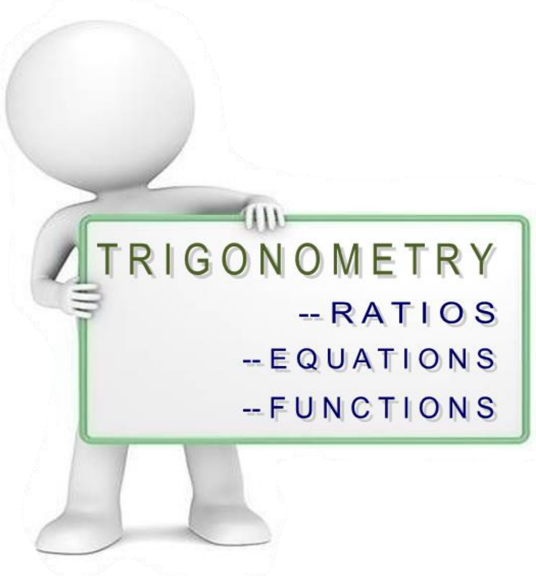
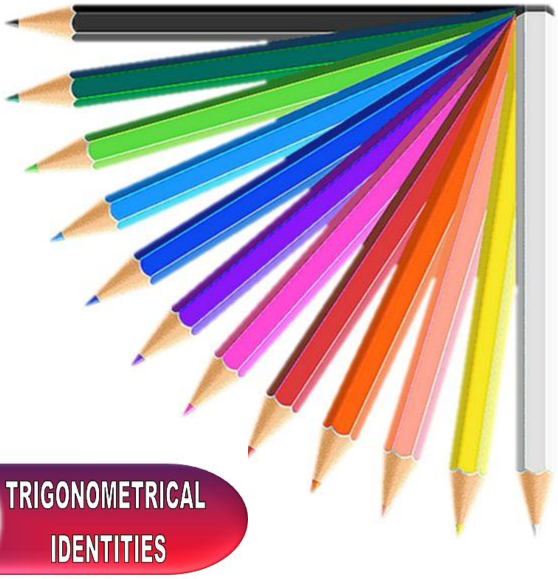
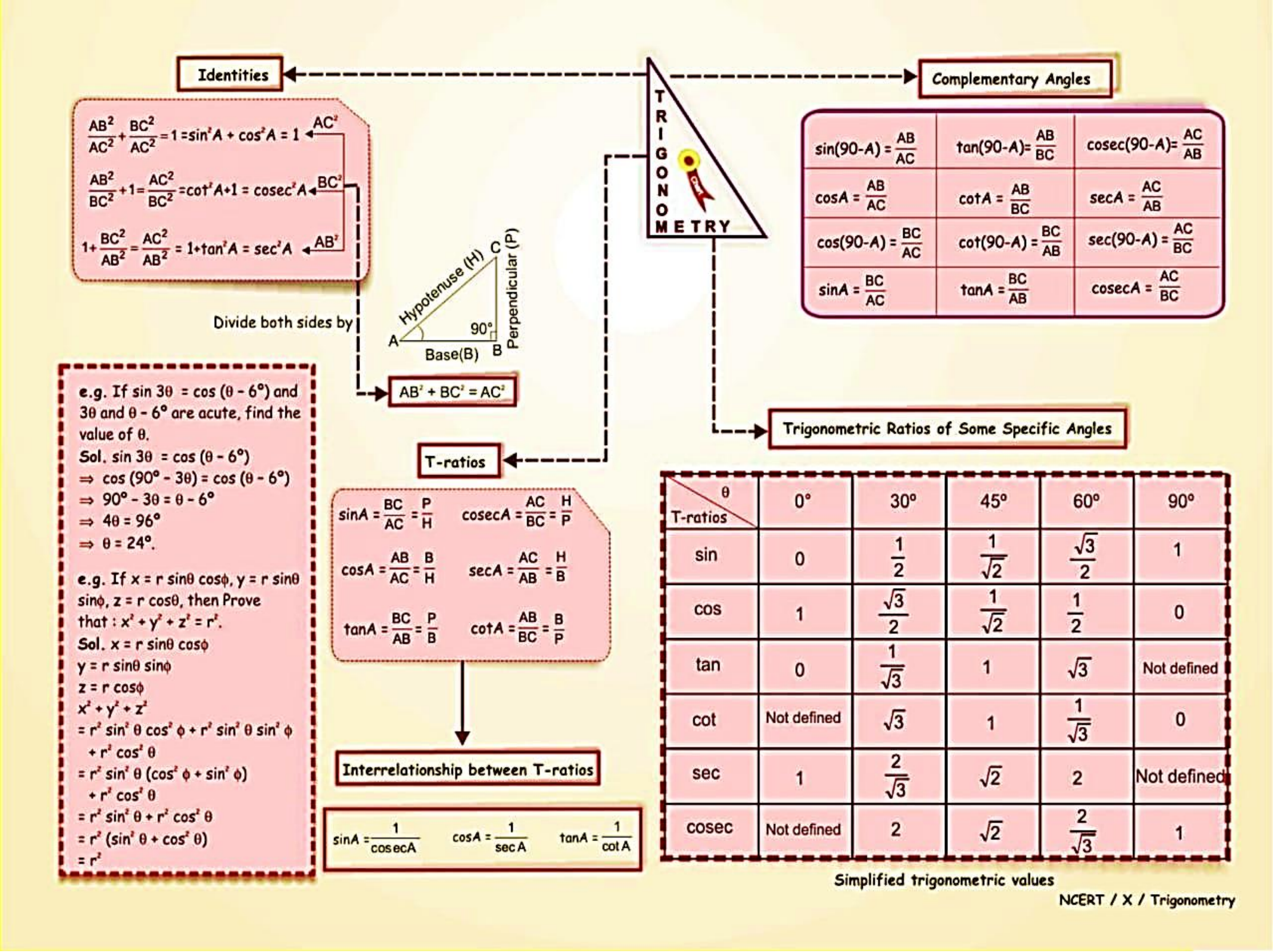




01 CBSE TRIGONOMETRICAL IDENTITIES



Identities

$$\frac{AB^2}{AC^2}+\frac{BC^2}{AC^2}=1=\sin^2A+\cos^2A=1 \leftarrow AC^2$$

$$\frac{AB^2}{BC^2}+1=\frac{AC^2}{BC^2}=\cot^2A+1=\text{cosec}^2A \leftarrow BC^2$$

$$1+\frac{BC^2}{AB^2}=\frac{AC^2}{AB^2}=1+\tan^2A=\sec^2A \leftarrow AB^2$$

Divide both sides by

$AB^2 + BC^2 = AC^2$

Complementary Angles

$\sin(90-A)=\frac{AB}{AC}$	$\tan(90-A)=\frac{AB}{BC}$	$\text{cosec}(90-A)=\frac{AC}{AB}$
$\cos A=\frac{AB}{AC}$	$\cot A=\frac{AB}{BC}$	$\sec A=\frac{AC}{AB}$
$\cos(90-A)=\frac{BC}{AC}$	$\cot(90-A)=\frac{BC}{AB}$	$\sec(90-A)=\frac{AC}{BC}$
$\sin A=\frac{BC}{AC}$	$\tan A=\frac{BC}{AB}$	$\text{cosec}A=\frac{AC}{BC}$

e.g. If sin 3θ = cos (θ - 6°) and 3θ and θ - 6° are acute, find the value of θ.

Sol. $\sin 3\theta = \cos(\theta - 6°)$

$\Rightarrow \cos(90° - 3\theta) = \cos(\theta - 6°)$

$\Rightarrow 90° - 3\theta = \theta - 6°$

$\Rightarrow 4\theta = 96°$

$\Rightarrow \theta = 24°$.

e.g. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, then Prove that : $x^2 + y^2 + z^2 = r^2$.

Sol. $x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\phi$

$x^2 + y^2 + z^2$

$= r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta$

$= r^2\sin^2\theta(\cos^2\phi + \sin^2\phi) + r^2\cos^2\theta$

$= r^2\sin^2\theta + r^2\cos^2\theta$

$= r^2(\sin^2\theta + \cos^2\theta)$

$= r^2$

T-ratios

$\sin A=\frac{BC}{AC}=\frac{P}{H}$ $\quad \text{cosec}A=\frac{AC}{BC}=\frac{H}{P}$

$\cos A=\frac{AB}{AC}=\frac{B}{H}$ $\quad \sec A=\frac{AC}{AB}=\frac{H}{B}$

$\tan A=\frac{BC}{AB}=\frac{P}{B}$ $\quad \cot A=\frac{AB}{BC}=\frac{B}{P}$

Interrelationship between T-ratios

$\sin A=\frac{1}{\text{cosec}A}$ $\quad \cos A=\frac{1}{\sec A}$ $\quad \tan A=\frac{1}{\cot A}$

Trigonometric Ratios of Some Specific Angles

θ / T-ratios	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cosec	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Simplified trigonometric values

NCERT / X / Trigonometry

Reciprocal and Quotient Identities	$\sin\theta=\frac{1}{\csc\theta}$ $\csc\theta=\frac{1}{\sin\theta}$ $\cos\theta=\frac{1}{\sec\theta}$ $\sec\theta=\frac{1}{\cos\theta}$ **2** $\tan\theta=\frac{1}{\cot\theta}=\frac{\sin\theta}{\cos\theta}$ $\cot\theta=\frac{1}{\tan\theta}=\frac{\cos\theta}{\sin\theta}$
Pythagorean Identities	$\sin^2\theta+\cos^2\theta=1$ $\tan^2\theta+1=\sec^2\theta$ $\cot^2\theta+1=\csc^2\theta$
Sum and Difference Identities	$\sin(A+B)=\sin A\cos B+\cos A\sin B$ $\cos(A+B)=\cos A\cos B-\sin A\sin B$ $\sin(A-B)=\sin A\cos B-\cos A\sin B$ $\cos(A-B)=\cos A\cos B+\sin A\sin B$ $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$
Double Angle Identities	$\sin(2A)=2\sin A\cos A$ $\cos(2A)=\cos^2A-\sin^2A$ $=1-2\sin^2A$ $=2\cos^2A-1$ $\tan(2A)=\frac{2\tan A}{1-\tan^2A}$
Half Angle Identities	$\sin\left(\frac{A}{2}\right)=\pm\sqrt{\frac{1-\cos A}{2}}$ $\cos\left(\frac{A}{2}\right)=\pm\sqrt{\frac{1+\cos A}{2}}$ $\tan\left(\frac{A}{2}\right)=\frac{\sin A}{1+\cos A}=\frac{1-\cos A}{\sin A}$

TRIGONOMETRIC IDENTITIES

Complementary angles

$\sin\theta=\cos(90°-\theta)$ → $\sin 40°=\cos 50°$

$\cos\theta=\sin(90°-\theta)$ → $\cos 15°=\sin 75°$

$\tan\theta=\cot(90°-\theta)$ → $\tan 30°=\cot 60°$

$\sec\theta=\frac{1}{\cos\theta}$

$\text{cosec}\,\theta=\frac{1}{\sin\theta}$

$\cot\theta=\frac{1}{\tan\theta}$

$\tan\theta=\frac{\sin\theta}{\cos\theta}$

$\cot\theta=\frac{\cos\theta}{\sin\theta}$

$\sin^2\theta+\cos^2\theta=1$

$\tan^2\theta+1=\sec^2\theta$

$1+\cot^2\theta=\text{cosec}^2\theta$

$\sin^2\theta+\cos^2\theta=1$

$\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}$ $(\div\cos^2\theta)$

$\tan^2\theta+1=\sec^2\theta$

$\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$ $(\div\sin^2\theta)$

$1+\cot^2\theta=\text{cosec}^2\theta$

α	0°	30°	45°	60°	90°	120°	180°	270°	360°
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	0	1
$\tan\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	0	∞	0
$\cot\alpha$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	∞	0	∞
$\sec\alpha$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-2	-1	∞	1
$\text{cosec}\,\alpha$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	∞	-1	∞

Trigonometric Identities and Equation

Some Standard. Formula

1. $\sin^2 A + \cos^2 A = 1$

2. $1 + \tan^2 A = \sec^2 A$ or, $\sec^2 A - \tan^2 A = 1$

or, $\sec A + \tan A = \dfrac{1}{\sec A - \tan A}$, where $A \neq n\pi + \dfrac{\pi}{2}$, $n \in Z$.

3. $1 + \cot^2 A = \operatorname{cosec}^2 A$ or, $\operatorname{cosec}^2 A - \cot^2 A = 1$ or, $\operatorname{cosec} A + \cot A = \dfrac{1}{\operatorname{cosec} A - \cot A}$, where $A \neq n\pi.\ n \in Z$

4. $\sin(A + B) = \sin A \cos B + \cos A \sin B$

5. $\sin(A - B) = \sin A \cos B - \cos A \sin B$

6. $\cos(A + B) = \cos A \cos B - \sin A \sin B$

7. $\cos(A - B) = \cos A \cos B + \sin A \sin B$

8. $\left.\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right\}$, where $A \neq n\pi + \dfrac{\pi}{2}$, $B \neq n\pi + \dfrac{\pi}{2}$

9. $\left.\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}\right\}$ and , $A \pm B \neq m\pi + \dfrac{\pi}{2}$

10. $$\left.\begin{aligned}\cot(A + B) &= \frac{\cot A \cot B - 1}{\cot A + \cot B}\\ \cot(A - B) &= \frac{\cot A \cot B + 1}{\cot B - \cot A}\end{aligned}\right\}$$, where $A \neq n\pi$, $B \neq n\pi$ and, $A \pm B \neq n\pi$

11. $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cot^2 B - \cos^2 A$

12. $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

13. $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1 + \tan^2\theta}$

14. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$

15. $1 + \cos 2\theta = 2\cos^2\theta$, $1 - \cos 2\theta = 2\sin^2\theta$

or, $\dfrac{1 + \cos 2\theta}{2} = \cos^2\theta$, $\dfrac{1 - \cos 2\theta}{2} = \sin^2\theta$

16. $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$, where $\theta \neq (2n + 1)\dfrac{\pi}{4}$

17. $\dfrac{1 - \cos\theta}{\sin\theta} = \tan\dfrac{\theta}{2}$, where $\theta \neq 2n\pi$

18. $\dfrac{1 + \cos\theta}{\sin\theta} = \cot\dfrac{\theta}{2}$, where $\theta \neq (2n + 1)\pi$

19. $\dfrac{1 - \cos\pi}{1 + \cos\theta} = \tan^2\dfrac{\theta}{2}$, where $\theta \neq (2n + 1)\pi$

20. $\dfrac{1 + \cos\theta}{1 - \cos\theta} = \cot^2\dfrac{\theta}{2}$, where $\theta \neq 2n\pi$

21. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

22. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

23. $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

24. $\cos A \cos 2A \cos 2^2 A \ldots \cos 2^{n-1} A = \dfrac{\sin 2^n A}{2^n \sin A}$

25. $\sin\theta\sin(60^\circ - \theta)\sin(60^\circ + \theta) = \dfrac{1}{4}\sin 3\theta$

26. $\cos\theta\cos(60^\circ - \theta)\cos(60^\circ + \theta) = \dfrac{1}{4}\cos 3\theta$

27. $\tan\theta\tan(60^\circ - \theta)\tan(60^\circ + \theta) = \tan 3\theta$

28. $\sin A + \sin B = 2\sin\left(\dfrac{A + B}{2}\right)\cos\left(\dfrac{A - B}{2}\right)$

29. $\sin A - \sin B = 2\sin\left(\dfrac{A - B}{2}\right)\cos\left(\dfrac{A + B}{2}\right)$

30. $\cos A + \cos B = 2\cos\left(\dfrac{A + B}{2}\right)\cos\left(\dfrac{A - B}{2}\right)$

31. $\cos A - \cos B = -2\sin\left(\dfrac{A + B}{2}\right)\sin\left(\dfrac{A - B}{2}\right)$

32. $\left.\tan A + \tan B = \dfrac{\sin(A + B)}{\cos A \cos B}\right\}$, where $A, B \neq n\pi + \dfrac{\pi}{2}$

33. $\left.\tan A - \tan B = \dfrac{\sin(A - B)}{\cos A \cos B}\right\}$, $n \in Z$

34. $\left.\cot A + \cot B = \dfrac{\sin(A + B)}{\sin A \sin B}\right\}$, where $A, B \neq n\pi$, $n \in Z$

35. $\left.\cot A - \cot B = \dfrac{\sin(B - A)}{\sin A \sin B}\right\}$, where $A, B \neq n\pi$, $n \in Z$.

36. $2\sin A \cos B = \sin(A + B) + \sin(A - B)$

37. $2\cos A \sin B = \sin(A + B) - \sin(A - B)$

38. $2\cos A \cos B = \cos(A + B) + \cos(A - B)$

39. $2\sin A \sin B = \cos(A - B) - \cos(A + B)$

40. $\sin(A + B + C) = \sin A \cos B \cos C + \cos A \sin B \cos C$

$+ \cos A \cos B \sin C - \sin A \sin B \sin C$

or,

$\sin(A + B + C)$

$= \cos A \cos B \cos C(\tan A + \tan B + \tan C$

$- \tan A \tan B \tan$

41. $\cos(A + B + C) = \cos A \cos B \cos C - \sin A \sin B \cos C$

$- \sin A \cos B \sin C - \cos A \sin B \sin C$ or,

$$\sin C = \frac{\textit{side opposite to angle } C}{\textit{hypotenuse}} = \frac{p}{h}$$

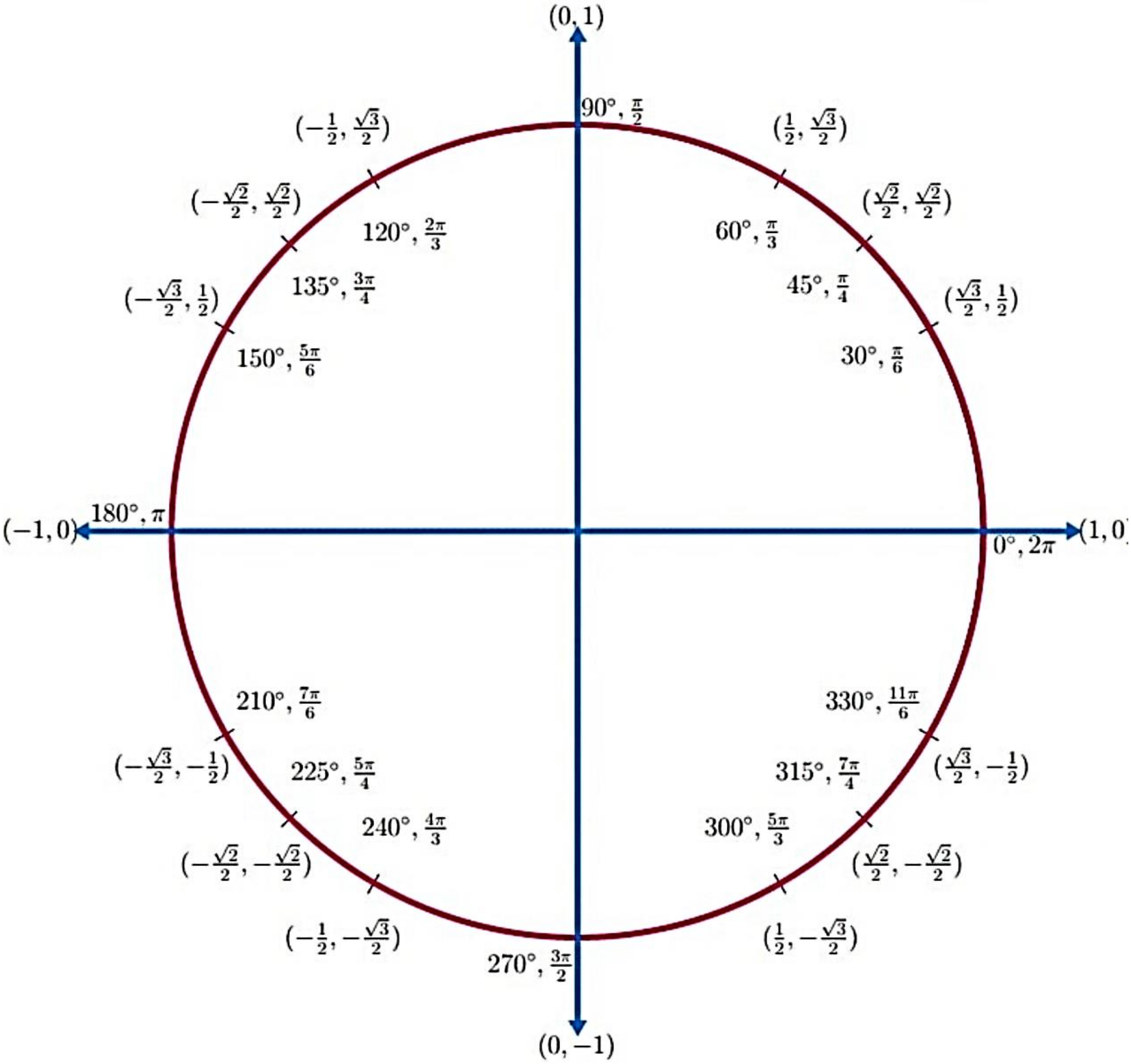
$$\cos C = \frac{\textit{side adjacent to angle } C}{\textit{hypotenuse}} = \frac{b}{h}$$

$$\tan C = \frac{\textit{side opposite to angle } C}{\textit{side adjacent to angle } C} = \frac{p}{b}$$

$$cosec\,\theta = \frac{1}{\sin\theta}; \sec\theta = \frac{1}{\cos\theta}; \cot\theta = \frac{1}{\tan\theta}; \tan\theta = \frac{\sin\theta}{\cos\theta}$$

∠A	**0°**	**30°**	**45°**	**60°**	**90°**
Sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
Cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
Cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

UNIT CIRCLE


$(0,1)$
$90°, \frac{\pi}{2}$
$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$
$(\frac{1}{2}, \frac{\sqrt{3}}{2})$
$(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
$120°, \frac{2\pi}{3}$
$60°, \frac{\pi}{3}$
$135°, \frac{3\pi}{4}$
$45°, \frac{\pi}{4}$
$(-\frac{\sqrt{3}}{2}, \frac{1}{2})$
$(\frac{\sqrt{3}}{2}, \frac{1}{2})$
$150°, \frac{5\pi}{6}$
$30°, \frac{\pi}{6}$
$180°, \pi$
$(-1,0)$
$(1,0$
$0°, 2\pi$
$210°, \frac{7\pi}{6}$
$330°, \frac{11\pi}{6}$
$(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$
$(\frac{\sqrt{3}}{2}, -\frac{1}{2})$
$225°, \frac{5\pi}{4}$
$315°, \frac{7\pi}{4}$
$240°, \frac{4\pi}{3}$
$300°, \frac{5\pi}{3}$
$(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
$(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
$(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$
$(\frac{1}{2}, -\frac{\sqrt{3}}{2})$
$270°, \frac{3\pi}{2}$
$(0,-1)$

INTRODUCTION TO TRIGONOMETRY

BASIC CONCEPTS AND FORMULAE

1. Trigonometry is the branch of mathematics in which we deal measurement of the sides and angles of the triangles.

Trigonometric Ratio's:

Let ΔABC be a right-triangle right angled at B. Let ∠CAB = θ,

Then,

$$\sin\theta = \frac{BC}{AC} \qquad \cos\theta = \frac{AB}{AC} \qquad \tan\theta = \frac{BC}{AB}$$

$$\cot\theta = \frac{AB}{BC} \qquad \sec\theta = \frac{AC}{AB} \qquad \text{cosec}\,\theta = \frac{AC}{BC}$$

Relation among t-functions:

(i) Reciprocal Relations:

$$\sin\theta = \frac{1}{\text{cosec}\,\theta} \Rightarrow \qquad \text{cosec}\,\theta = \frac{1}{\sin\theta} \Rightarrow \qquad \sin\theta \,.\, \text{cosec}\,\theta = 1$$

$$\cos\theta = \frac{1}{\sec\theta} \Rightarrow \qquad \sec\theta = \frac{1}{\cos\theta} \Rightarrow \qquad \sec\theta \,.\, \cos\theta = 1$$

$$\tan\theta = \frac{1}{\cot\theta} \Rightarrow \qquad \cot\theta = \frac{1}{\tan\theta} \Rightarrow \qquad \tan\theta \,.\, \cot\theta = 1$$

(ii) Square Relations

(i) $\sin^2\theta + \cos^2\theta = 1$, $\cos^2\theta = 1 - \sin^2\theta$, $\sin^2\theta = 1 - \cos^2\theta$

(ii) $\sec^2\theta - \tan^2\theta = 1$ $\sec^2\theta = 1 + \tan^2\theta$, $\tan^2\theta = \sec^2\theta - 1$

(iii) $\text{cosec}^2\theta - \cot^2\theta = 1$ $\text{cosec}^2\theta = 1 + \cot^2\theta$, $\cot^2\theta = \text{cosect}^2\theta - 1$

(iii) Quotient Relations

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{and} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

2. An expression having equal sign (=) is called an equation.
3. An equation which is true for all values of the variable involved, is called an identity.
4. An equation which involves trigonometric ratios of an angle and is true for all values of angle is called a trigonometric identity. Thus, above relations among t-functions are known as fundamental trigonometric identities.
5. While proving the identities following points can be useful:

(i) Try to simplify LHS to RHS if RHS is more complicated.

(ii) Try to simplify RHS to LHS, if LHS is more complicated.

(iii) You may try to make both sides equivalent to a third expression.

(iv) Sometimes changing tan x, cot x, sec x and cosec x into sin x and cos x may help.

(v) If the numerator and denominator of a fraction contains a factor of (1 + sin x) or (1 – sin x), multiply and divide by (1 – sin x) or (1 + sin x) whichever is applicable. This is then replaced by cos x. The same applies to a factor of (1 ± cos x), (sec x ± 1), (cosec x ± 1), (sec x ± tan x) and (cosec x ± cot x).

(vi) Always keep it mind the RHS, if you are dealing with LHS or vice-versa, to be sure that you are approaching in right direction.

□□□ These guidelines are only a helping aid and there is no hard and fast rule to solve the particular problems. Only practice can make you perfect in proving the identities. Moreover, you have different ways to approach a result. Always try to find the shortest route.

6. Two acute angles α and β are compliments of each other if $\alpha + \beta = 90°$. In a right angled triangle, the two angles that are not right angles are complements of each other. If one angles is θ, then other is 90° – θ
7. Two angles α and β are said to be supplementary, if $\alpha + \beta = 180°$

	0°	30°	45°	60°	90°
sin θ	0	1/2	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	Not defined (∞)
cot θ	Not defined (∞)	√3	1	1/√3	0
sec θ	1	2/√3	√2	2	Not defined (∞)
cosec θ	Not defined (∞)	2	√2	2/√3	1

□□□: As we can observe that:

(i) Values of cosec θ is the corresponding reciprocal values of sin θ.

(ii) Values of sec θ is the corresponding reciprocal values of cos θ.

(iii) Values of cot θ is the corresponding reciprocal values of tan θ.

□□□ There is an easy way to remember the values of sin θ for θ = 0°, 30°, 45°, 60°, and 90°0

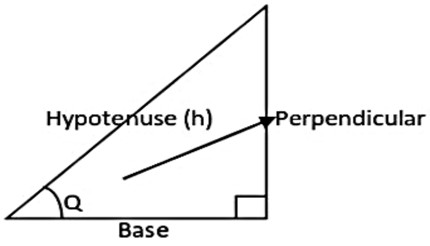
In brief:

	0	0°	0°	5°	0°	0°	
s in θ	Write the five numbers in the sequence of 0, 1, 2, 3, 4. Divide by 4 and take their square root.	0		2	3		Incr easing order
c os θ	Write the values of sin θ in reverse order	1	3	2			Dec reasing order
t an θ	Dividing values of sin θ by cos θ i.e., $\tan\theta = \frac{\sin\theta}{\cos\theta}$	0	3		3	ot defi-ned	Incr easing order

□□□: (i) The values of sin θ increases from 0 to 1 as θ increases from 0° to 90° and value of cos θ decreases from 1 to 0 as θ increases from 0 to 90°. The value of tan θ also increases from 0 to a big number as θ increases from 0° to 90°.

(ii) If A and B are acute angles such that A > B, then sin A > sin B, cos A < cos B, tan A > tan B and cosec A < cosec B, sec A > sec B , cot A < cot B.

◘ Fundamental



=> $\sin\theta = \frac{p}{h}$

=> $\cos\theta = \frac{b}{h}$

=> $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{p/\cancel{h}}{b/\cancel{h}} = \frac{p}{b}$

=> $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{b/\cancel{h}}{p/\cancel{h}} = \frac{b}{p}$

=> $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{b}{h}} = \frac{h}{b}$

=> $\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{p}{h}} - \frac{h}{p}$

◘ 3 × 3 Relations

[A] $\sin^2\theta + \cos^2\theta = 1$

(i) $\sin^2\theta = 1 - \cos^2\theta$ => $\sin\theta = \sqrt{1 - \cos^2\theta}$

(ii) $\cos^2\theta = 1 - \sin^2\theta$ => $\cos\theta = \sqrt{1 - \sin^2\theta}$

[B] $\text{Sec}^2\theta - \tan^2\theta = 1$

(i) $\sec^2\theta = 1 + \tan^2\theta$ => $\sec\theta = \sqrt{1 + \tan^2\theta}$

(ii) $\sec^2\theta = 1 + \tan^2\theta$ => $\tan\theta = \sqrt{\sec^2\theta - 1}$

[C] $\text{cosec}^2\theta - \cot^2\theta = 1$

(i) $\text{cosec}^2\theta = 1 + \cot^2\theta$ => $\text{cosec}\,\theta = \sqrt{1 + \cot^2\theta}$

(ii) $\cot^2\theta = \text{cosec}^2\theta - 1$ => $\cot\theta = \sqrt{\text{cosec}^2\theta - 1}$

◘ Some Important Relations

(i) $\sin\theta = \frac{1}{\text{cosec}\,\theta}$ or $\tan\theta = \frac{\sin\theta}{\cos\theta}$

(ii) $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ or $\cot\theta = \frac{\cos\theta}{\sin\theta}$

(iii) $\cos\theta = \frac{1}{\sec\theta}$

(iv) $\sec\theta = \frac{1}{\cos\theta}$

(v) $\tan\theta = \frac{1}{\cot\theta}$

(vi) $\cot\theta = \frac{1}{\tan\theta}$

◘ Complementary angle Relation

(i) $\sin(90-\theta) = \cos\theta$

(ii) $\cos(90-\theta) = \sin\theta$

(iii) $\tan(90-\theta) = \cot\theta$

(iv) $\cot(90-\theta) = \tan\theta$

(v) $\text{Sec}(90-\theta) = \operatorname{cosec}\theta$

(vi) $\text{coses}(90-\theta) = \sec\theta$

◘ Trigonometry Table

	°	30°	5°	6 0°	9 0°
		1		3	4
	/4	1/4	/4	3 /4	4 /4
	0	√1/ 4	1/2	√ 3/4	√ 1
sin θ		1/2	/√2	√ 3/2	1
cos θ		√3/ 2	/√2	1 /2	0
tan θ		1/√ 3		√ 3	∞
cot θ		√3		1 /√3	0
sec θ		2/√ 3	2	2	∞
cosec θ		2	2	2 /√3	1

SET I [PATTERN 1ST]

Prove the following Identities:

Q. 1. $1-\cos^2\theta-\sin^2\theta = 0$

Sol. LHS

$\Rightarrow \quad 1-\cos^2\theta-\sin^2\theta = 0$

$\Rightarrow \quad 1-(\sin^2\theta+\cos^2\theta) = 0$

$\Rightarrow \quad 1-1 = 0$

$= 0$ LHS = RHS

Q. 2. $\frac{1-\cos^2\theta}{\sin^2\theta} = 1$

Sol. LHS

$= \frac{1-\cos^2\theta}{\sin^2\theta}$

$= \frac{\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}}$

$= 1$, LHS = RHS

Q. 3. $(1-\cos^2\theta)\times\operatorname{cosec}^2\theta = 1$

Sol. LHS

$= (1-\cos^2\theta)\operatorname{cosec}^2\theta$

$= \sin^2\theta\times\operatorname{cosec}^2\theta$

$= \frac{1}{\cancel{\operatorname{cosec}^2\theta}}\times\cancel{\operatorname{cosec}^2\theta}$

$= 1$

LHS = RHS

Q. 4. $(1-\sin^2 A)\sec^2 A = 1$

Sol. LHS

$= (1-\sin^2 A)\sec^2 A$

$= \cos^2 A\times\sec^2 A$

$= \frac{1}{\sec^2 A}\times\sec^2 A$

$= 1$

LHS = RHS

Q. 5. $(1+\tan^2 A)\cos^2 A = 1$

Sol. LHS

$= (1+\tan^2 A)\cos^2 A$

$= \sec^2 A\times\cos^2 A$

$= \frac{d}{\cos^2 A}\times\cos^2 A$

$= 1$ LHS = RHS

Q. 6. $(1-\cos^2 A)\sec^2 A = \tan^2 A$

Sol. LHS

$= (1-\cos^2 A)\sec^2 A$

$= \sin^2 A\times\frac{1}{\cos^2 A}$

$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A = \text{RHS}$

Q. 7. $\cot^2 A\,(1-\cos^2 A) = \cos^2 A$

Sol. LHS

$= \cot^2 A\,(1-\cos^2 A)$

$= \cot^2 A \times \sin^2 A$

$= \frac{\cos^2 A}{\cancel{\sin^2 A}} \times \cancel{\sin^2 A}$

$= \cos^2 A =$ RHS

Q. 8. $\frac{1}{\cos^2 A} - \tan^2 A = 1$

Sol. LHS

$= \frac{1}{\cos^2 A} - \tan^2 A$

$= \sec^2 A - \tan^2 A$

$= 1 =$ RHS

Q. 9. $(\text{cosec}^2 A - 1)\tan^2 A = 1$

Sol. LHS

$= (\text{cosec}^2 A - 1)\tan^2 A$

$= \cot^2 A \times \tan^2 A$

$= \frac{\cancel{\cos^2 A}}{\cancel{\sin^2 A}} \times \frac{\cancel{\sin^2 A}}{\cancel{\cos^2 A}}$

$= 1 =$ RHS

Q. 10. $(1-\cot^2\theta)\sin^2\theta = 1$

Sol. LHS

$= (1+\cot^2\theta)\sin^2\theta$

$= \text{cosec}^2\theta \times \sin^2\theta$

$= \cancel{\text{cosec}^2\theta} \times \frac{1}{\cancel{\text{cosec}^2\theta}}$

$= 1 =$ RHS

Q. 11. $(\text{cosec}^2 A - 1)\tan^2 A = 1$

Sol. LHS

$= \cot^2 A \times \tan^2 A$

$= \frac{\cos^2 A}{\sin^2 A} \times \frac{\sin^2 A}{\cos^2 A}$

$= 1 =$ RHS

SET II

Q. 1. $\cot^2\theta - \frac{1}{\sin^2\theta} = -1$

Sol. LHS

$= \cot^2\theta - \text{cosec}^2\theta = -1$

$= -(\text{cosec}^2\theta - \cot^2\theta) = -1$

$\Rightarrow -1 = -1$

LHS = RHS

Q. 2. $(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta) = 1$

Sol. LHS

$= (1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta)$

$= (1+\tan^2\theta)(1-\sin^2\theta)$

$= \sec^2\theta \times \cos^2\theta$

$= \frac{1}{\cancel{\cos^2\theta}} \times \cancel{\cos^2\theta}$

$= 1 =$ RHS

Q. 3. $(1+\cot^2\theta)(1-\cos\theta)(1+\cos\theta)$

Sol. LHS $= (1+\cot^2\theta)(1-\cos\theta)(1-\cos^2\theta)$

$= (1+\cot^2\theta)(1-\cos^2\theta)$

$= \text{cosec}^2\theta \times \sin^2\theta$

$= \frac{1}{\cancel{\sin^2\theta}} \times \cancel{\sin^2\theta}$

$= 1 =$ RHS

Q. 4. $\tan^2\theta - \frac{1}{\cos^2\theta} = -1$

Sol. LHS $= \tan^2\theta - \frac{1}{\cos^2\theta}$

$= \tan^2\theta - \sec^2\theta$

$= -1$

= RHS

Q. 5. $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$

Sol. LHS $= (\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$

$= \tan^2\theta \times \cot^2\theta$

$= \frac{1}{\cancel{\cot^2\theta}} \times \cancel{\cot^2\theta}$

$= 1 =$ RHS

Q. 6. $\tan^2\theta \times \cos^2\theta = 1 - \cos^2\theta$

Sol. LHS $= \tan^2\theta \times \cos^2\theta$

$= \frac{\sin^2\theta}{\cancel{\cos^2\theta}} \times \cancel{\cos^2\theta}$

$= \sin^2\theta$

$= 1 - \cos^2\theta$

= RHS.

Q. 7. $\tan\theta + \frac{1}{\tan\theta} = \sec\theta \times \text{cosec}\,\theta$

Sol. LHS $= \tan\theta + \frac{1}{\tan\theta}$

$= \frac{\sin\theta}{\cos\theta} + \frac{1}{\frac{\sin\theta}{\cos\theta}}$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta + \cos\theta}$

$= \frac{1}{\sin\theta\,.\,\cos\theta}$

$= \frac{1}{\sin\theta}\,.\,\frac{1}{\cos\theta}$

$= \text{cosec}\,\theta \times \sec\theta$

LHS = RHS

Q. 8. $\sin^2 A + \frac{1}{1+\tan^2 A} = 1$

Sol. LHS $= \sin^2 A + \frac{1}{1+\tan^2 A}$

$= \sin^2 A + \frac{1}{\sec^2\theta}$

$= \sin^2 A + \cos^2\theta$

$= 1 =$ RHS

Q. 9. $\sin^2 A + (\sin^2 A \times \tan^2 A) = \tan^2 A$

Sol. LHS $= \sin^2 A\,\sin^2 A \times \tan^2 A$

$= \sin^2 A\,(1+\tan^2 A)$

$= \sin^2 A \times \sec^2 A$

$= \sin^2 A \times \frac{1}{\cos^2 A}$

$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 =$ RHS

Q. 10. $1 + \tan^2\theta = \sec^2\theta$

Sol. LHS = $1 + \tan^2\theta$

$= 1 + \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta}$

$= \sec^2\theta$ = RHS

SET III

Q. 1. $\left(\frac{\cos^2\theta + 1}{\sin^2\theta}\right) \tan^2 = \frac{1}{\cos^2\theta}$

Sol. LHS = $\left(\frac{\cos^2\theta}{\sin^2\theta} + 1\right) \tan^2\theta$

$= (\cot^2\theta + 1) + \tan^2\theta$

$= \mathrm{cosec}^2\theta \times \tan^2\theta$

$= \frac{1}{\cancel{\sin^2\theta}} \times \frac{\cancel{\sin^2\theta}}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta}$

= RHS

Q. 2. *cos A × tan A = sin A*

Sol. LHS = cos A × tan A

$= \cos A \times \frac{\sin A}{\cos A}$

= sin A

= RHS

Q. 3. *cot θ + tan θ = cosec θ × sec θ*

Sol. LHS = cos A × tan A

$= \cos A \times \frac{\sin A}{\cos A}$

= sin A

= RHS

Q. 4. $\tan^2\theta - \sin^2\theta = \tan^2\theta \times \sin^2\theta$

Sol. LHS = $\tan^2\theta - \sin^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$

$= \frac{\sin^2\theta - \sin^2\theta \,.\, \cos^2\theta}{\cos^2\theta}$

$= \frac{\sin^2\theta\,(1 - \cos^2\theta)}{\cos^2\theta}$

$= \tan^2\theta \times \sin^2\theta$

LHS = RHS

Q. 5. $(\sec\theta + \cos\theta)(\sec^2\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$

Sol. LHS = $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta)$

$= \sec^2\theta - \cos^2\theta$

$= (\tan^2\theta + 1) - (1 - \sin^2\theta)$

$= \cancel{\tan^2}\theta + 1 - 1 + \sin^2\theta$

$= \tan^2\theta + \sin^2\theta$

LHS = RHS

Q. 6. *Sec A (1 − sin A) (sec A + tan A) = 1*

Sol. LHS = sec A (1 − sin A) (sec A + tan A)

$= \sec A\,(1 - \sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$

$= \sec A\,(1 - \sin A)\left(\frac{1 + \sin A}{\cos A}\right)$

$= \sec A\left(\frac{1 - \sin^2 A}{\cos A}\right)$

$= \frac{1}{\cos A} \times \frac{\cos^2 A}{\cos A} = 1$ LHS = RHS

Q. 7. $\sin^2 A \times \cot^2 A + \cos^2 A \times \tan^2 A = 1$

Sol. LHS = $\cancel{\sin^2 A} \times \frac{\cos^2 A}{\cancel{\sin^2 A}} + \cancel{\cos^2 A} \times \frac{\sin^2 A}{\cancel{\cos^2 A}}$

$= \cos^2 A + \sin^2 A$

= 1 LHS = RHS

Q. 8. $(\cot\theta - \tan\theta) = \frac{2\cos^2\theta - 1}{\sin\theta \times \cos\theta}$

Sol. LHS = $\cot\theta - \tan\theta$

$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta \times \cos\theta}$

$= \frac{\cos^2\theta - (1 - \cos^2\theta)}{\sin\theta \times \cos\theta}$

$= \frac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta + \cos\theta} = \frac{2\cos^2\theta - 1}{\sin\theta \times \cos\theta}$

Q. 9. *sin A (1 + tan A) + cos A (1 + cot A) = sec A + cosec A*

Sol. LHS = sin A (1 + tan A) + cos A (1 + cot A)

$= \sin A\left(1 + \frac{\sin A}{\cos A}\right) + \cos A\left(1 + \frac{\cos A}{\sin A}\right)$

$= \sin A\left(\frac{\cos A + \sin A}{\cos A}\right) + \cos A\left(\frac{\sin A + \cos A}{\sin A}\right)$

$= \sin A + \cos A\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$

$= \sin A + \cos A\left(\frac{\sin^2 A + \cos^2 A}{\sin A + \cos A}\right)$

$= \sin A + \cos A\left(\frac{1}{\sin A \times \cos A}\right)$

$= \sin A + \cos A \times \frac{1}{\sin A} \times \frac{1}{\cos A} = \frac{\sin A + \cos A}{\sin A \times \cos A} = \frac{\cancel{\sin A}}{\cancel{\sin A} \times \cos A} + \frac{\cancel{\cos A}}{\sin A \times \cancel{\cos A}} =$ sec A + cosec

Q. 10. $\cos^2\theta - \mathrm{cosec}\,\theta + \sin\theta = 0$

Sol. LHS = $\frac{\cos^2\theta - \mathrm{cosec}\,\theta \times \sin\theta + \sin^2\theta}{\sin\theta}$

$= \frac{\cos^2\theta + \sin^2\theta - \frac{1}{\cancel{\sin\theta}} \times \cancel{\sin\theta}}{\sin\theta}$

$= \frac{1 - 1}{\sin\theta}$

$= \frac{0}{\sin\theta} = 0$

= LHS = RHS

Q. 11. $(\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta) = -1$

Sol. LHS = $(\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta)$

$= \tan^2\theta\,[-(\operatorname{cosec}^2\theta - 1)]$

$= \tan^2\theta \times \cot^2\theta$

$= -\dfrac{1}{\cot^2\theta} \times \cot^2\theta$

$= -1 \quad = \text{RHS}$

Q. 12. $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$

Sol. LHS = $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$

$= (1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$

$= \left(1 + \dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta}\right)\left(1 + \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}\right)$

$= \left(\dfrac{1\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\dfrac{\cos^2\theta + \sin\theta + 1}{\cos\theta}\right)$

$= \dfrac{\sin^2\theta + \cos^2\theta + 2\sin\theta\,.\cos\theta - 1}{\sin\theta\,.\cos\theta}$

$= \dfrac{\cancel{1} + 2\sin\theta\,.\cos\theta - \cancel{1}}{\sin\theta\,.\cos\theta} \quad = \quad 2 \;= \text{RHS}$

Q. 13. $\operatorname{cosec}\theta(1 + \cos\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

Sol. LHS = $\operatorname{cosec}\theta(1 + \cos\theta)(\operatorname{cosec}\theta - \cot\theta)$

$= \dfrac{1}{\sin\theta}(1 + \cos\theta)(\operatorname{cosec}\theta - \cot\theta)$

$= \dfrac{1 + \cos\theta}{\sin\theta}(\operatorname{cosec}\theta - \cot\theta)$

$= \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}\;(\operatorname{cosec}\theta - \cot\theta)$

$= (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)$

$= \operatorname{cosec}^2\theta - \cot^2\theta \quad = 1 \quad = \text{RHS}$

Q. 14. $\sec\theta(1 + \sin\theta)(\sec\theta - \tan\theta) = 1$

Sol. LHS = $(\sec\theta + \sec\theta \times \sin\theta)(\sec\theta - \tan\theta)$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}\;(\sec\theta - \tan\theta)$

$= (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$= \sec^2\theta - \tan^2\theta$

$= \sec^2\theta - \tan^2\theta$

$= 1 \quad = \text{RHS}$

Q. 15. $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$

sol. LHS = $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta)$

$= \sec^2\theta - \cos^2\theta$

$= 1 + \tan^2\theta - (1 - \sin^2\theta)$

$= 1 + \tan^2\theta - 1 + \sin^2\theta$

$= \tan^2\theta + \sin^2\theta$

= RHS

Q. 16. $(\sin A - \operatorname{cosec} A)(\cos A - \sec A)(\tan A + \cot A) = 1$

Sol. LHS = $\left(\sin A - \dfrac{1}{\sin A}\right)\left(\cos A - \dfrac{1}{\cos A}\right)\left(\sin\ A - \dfrac{1}{\tan A}\right)$

$= \left(\dfrac{\sin^2 A - 1}{\sin A}\right)\left(\dfrac{\cos^2 A - 1}{\cos A}\right)\left(\dfrac{\tan^2 A + 1}{\tan A}\right)$

$= \left(\dfrac{-\cos^2 A}{\sin A}\right) \times \left(\dfrac{-\sin^2 A}{\cos A}\right) \times \sec^2 A \times \dfrac{\cos A}{\sin A}$

$= \dfrac{\cancel{(-\cos^2 A)}}{\cancel{\sin A}} \times \dfrac{\cancel{(-\sin^2 A)}}{\cancel{\cos A}} \times \dfrac{1}{\cancel{\cos^2 A}} \times \dfrac{\cancel{\cos A}}{\cancel{\sin A}}$

$= 1 \quad = \text{RHS}$

Q. 17. $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \times \operatorname{cosec}^2\theta$

Sol. LHS = $\sec^2\theta + \operatorname{cosec}^2\theta$

$= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta \times \cos^2\theta}$

$= \dfrac{1}{\sin^2\theta \times \cos^2\theta} \quad = \quad \sec^2\theta \times \operatorname{cosec}^2\theta$

Q. 18. $(\sec^2 A - 1)(\operatorname{cosec}^2 A - 1) = 1$

Sol. LHS = $(\sec^2 A - 1)(\operatorname{cosec}^2 A - 1)$

$= \tan^2 A \times \cot^2 A$

$= \dfrac{1}{\cot^2 A} \times \cot^2 A$

$= 1 = \text{RHS}$

Q. 19. $(\sin A + \cos A)(\tan A + \cot A) = \sec A + \operatorname{cosec}\theta$

sol. LHS = $(\sin A + \cos A)(\tan A + \cot A)$

$= (\sin A + \cos A)\left(\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}\right)$

$= (\sin A + \cos A)\left(\dfrac{\sin^2 A + \cos^2 A}{\sin A \times \cos A}\right)$

$= (\sin A + \cos A)\left(\dfrac{1}{\sin A \times \cos A}\right)$

$= \dfrac{\sin A + \cos A}{\sin A \times \cos A}$

$= \dfrac{\cancel{\sin A}}{\cancel{\sin A \times}\cos A} + \dfrac{\cancel{\cos A}}{\sin A \cancel{\times \cos A}} \quad = \quad \sec A + \operatorname{cosec} A$

SET IV

Q. 1. $\tan^2\theta + \cot^2\theta + 2 = \sec^2\theta + \operatorname{cosec}^2\theta$

Sol. LHS = $\tan^2\theta + \cot^2\theta + 2$

$= \dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} + 2$

$= \dfrac{\sin^2\theta}{\cos^2\theta} + 1 \quad + \dfrac{\cos^2\theta}{\sin^2\theta} + 1$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} + \dfrac{\cos^2 + \sin^2 + \theta}{\sin^2\theta}$

$= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$

$= \sec^2\theta + \operatorname{cosec}^2\theta$

= RHS

Q. 2. $\dfrac{1 - \tan^2\theta}{\cot^2\theta - 1} = \tan^2\theta$

Sol. LHS = $\dfrac{1 - \tan^2\theta}{\cot^2\theta - 1}$

$= \dfrac{1 - \dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta}{\sin^2\theta} - 1}$

$= \dfrac{\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}} = \dfrac{\cancel{(\cos^2\theta - \sin^2\theta)}}{\cancel{(\cos^2\theta - \sin^2\theta)}} \times \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta = \text{RHS}$

Q. 3. **$(1 + \tan^2\theta)(1 - \sin^2\theta) = 1$**

Sol. LHS $= (1 + \tan^2\theta)(1 - \sin^2\theta)$

$= \sec^2\theta \times \cos^2\theta$

$= \dfrac{1}{\cos^2\theta} \times \cos^2\theta$

$= 1$

$=$ RHS

Q. 5. $\dfrac{\cos\theta}{\operatorname{cosec}\theta + 1} + \dfrac{\cos\theta}{\operatorname{cosec}\theta - 1} = 2\tan\theta$

Sol. LHS $= \dfrac{\cos\theta}{\operatorname{cosec}\theta + 1} + \dfrac{\cos\theta}{\operatorname{cosec}\theta + 1}$

$= \cos\theta\left(\dfrac{1}{\operatorname{cosec}\theta + 1} + \dfrac{1}{\operatorname{cosec}\theta - 1}\right)$

$= \cos\theta\left(\dfrac{\operatorname{cosec}\theta - \cancel{1} + \operatorname{cosec}\theta + \cancel{1}}{\operatorname{cosec}^2\theta - 1}\right)$

$= \cos\theta\left(\dfrac{2\operatorname{cosec}\theta}{\cot\theta}\right)$

$= \cos\theta \times 2 \times \dfrac{\dfrac{1}{\sin\theta}}{\dfrac{\cos^2\theta}{\sin\theta}}$

$= \cancel{\cos\theta} \times \dfrac{2}{\cancel{\sin\theta}} \times \dfrac{\sin\cancel{^2}\theta}{\cos\cancel{^2}\theta}$

Q. 7. ***$(1 + \tan\theta)^2 + (1 - \tan\theta)^2 = 2\sec^2\theta$***

Sol. LHS $= (1 + \tan\theta)^2 + (1 - \tan\theta)^2$

$= 1 + \tan^2\theta + 2\tan\theta + 1 + \tan^2\theta - 2\tan\theta$

$= 2 + 2\tan^2\theta$

$= 2(1 + \tan^2\theta)$

$= 2\sec^2\theta =$ RHS

Q. 9. $\dfrac{3 - 4\sin^2\theta}{\cos^2\theta} = 3 - \tan^2\theta$

Utilization of 1

Sol. LHS $= \dfrac{3 - 4\sin^2\theta}{\cos^2\theta}$

$= \dfrac{3}{\cos^2\theta} - \dfrac{4\sin^2\theta}{\cos^2\theta}$

$= \dfrac{3}{\cos^2\theta} - 4\tan^2\theta$

$= \dfrac{3}{1/\sec^2\theta} - 4\tan^2\theta$

$= 3\sec^2\theta - 4\tan^2\theta$

$= 3\sec^2\theta - 3\tan^2\theta - \tan^2\theta$

$= 3(\sec^2\theta - \tan^2\theta) - \tan^2\theta$

$= 3 \times 1 - \tan^2 + \theta$

$= 3 - \tan^2\theta =$ RHS

Q. 10. ***$(\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta) = -1$***

Sol. LHS $= (\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta)$

$= \tan^2\theta \times -(\operatorname{cosec}^2\theta - 1)$

$= -\tan^2\theta \times \cot^2\theta$

$= \cancel{-\tan^2\theta} \times \dfrac{1}{\cancel{\tan^2\theta}}$

$= -1 =$ RHS

Q. 4. $\dfrac{\operatorname{cosec}\theta}{\cot\theta + \tan\theta} = \cos\theta$

Sol. LHS $= \dfrac{\operatorname{cosec}\theta}{\cot\theta + \tan\theta}$

$= \dfrac{\dfrac{1}{\sin\theta}}{\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}}$

$= \dfrac{\dfrac{1}{\sin\theta}}{\dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta.\cos\theta}}$

$= \dfrac{\dfrac{1}{\sin\theta}}{\dfrac{1}{\sin\theta.\cos\theta}}$

$= \dfrac{\sin\theta \times \cos\theta}{\sin\theta} = \cos\theta$ LHS = RHS

Q. 6. $\dfrac{1 + \cos\theta - \sin^2\theta}{\sin\theta(1 + \cos\theta)} = \cot\theta$

Sol. LHS $= \dfrac{1 + \cos\theta - \sin^2\theta}{\sin\theta(1 + \cos\theta)}$

$= \dfrac{\cos\theta + \cos^2\theta}{\sin\theta(1 + \cos\theta)}$

$= \dfrac{\cos\theta\cancel{(1 + \cos\theta)}}{\sin\theta\cancel{(1 + \cos\theta)}}$

$= \cot\theta$

$=$ RHS

Q. 8. $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \cot^2\theta + \tan^2\theta$

Sol:LHS $= (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta.\operatorname{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta.\sec\theta$

$= \sin^2\theta + \cos^2\theta + \operatorname{cosec}^2\theta + \sec^2\theta + 2 + 2$

$= 1 + 4 + \operatorname{cosec}^2\theta + \sec^2\theta$

$= 5 + 1 + \cot^2 + \theta + \sec^2\theta$

$= 6 + \cot^2\theta + \sec^2\theta$

$= 6 + \cot^2\theta + 1 + \tan^2 + \theta = 7 + \cot^2\theta + \tan^2\theta$

Aliter

$\dfrac{3 - 4\sin^2\theta}{\cos^2\theta}$

$= \dfrac{3 \times 1 - 4\sin^2\theta}{\cos^2\theta}$

$= \dfrac{3(\sin^2\theta + \cos^2\theta) - 4\sin^2\theta}{\cos^2\theta}$

$= \dfrac{3\sin^2\theta + 3\cos^2\theta - 4\sin^2\theta}{\cos^2\theta}$

$= \dfrac{3\cos^2\theta - \sin^2\theta}{\cos^2\theta}$

$= \dfrac{3\cancel{\cos^2\theta}}{\cancel{\cos^2\theta}} - \dfrac{\sin^2\theta}{\cos^2\theta} = 3 - \tan^2\theta$

Q. 11. $\dfrac{(1 + \tan^2\theta) \times \cot\theta}{\operatorname{cosec}^2\theta}$

Sol. LHS $= \dfrac{(1 + \tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta}$

$= \dfrac{\sec^2\theta \times \cot\theta}{\operatorname{cosec}^2\theta}$

$= \dfrac{1}{\cos^2\theta} \times \dfrac{\cos\theta}{\sin\theta}$

$$= \frac{\frac{1}{\sin\theta \times \cos\theta}}{\frac{1}{\sin^2\theta}}$$

$$= \frac{\overset{\sin^2\theta}{\cancel{\sin^2\theta}}}{\cancel{\sin\theta} \times \cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} \quad = \quad \tan\theta \quad = \text{RHS}$$

Q. 12. $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

Sol. $\text{LHS} = \dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$$

$$= \tan\theta \times \left(\frac{\sin^2\theta + \cos^2\theta - 2\sin^2\theta}{2\cos^2 - (\sin^2\theta + \cos^2\theta)}\right)$$

$$= \tan\theta \times \left(\frac{\cancel{=\sin^2\theta + \cos^2\theta}}{\cancel{\cos^2\theta - \sin^2\theta}}\right)$$

$$= \tan\theta \quad = \text{RHS}$$

Q. 13. $(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 = 2$

Sol. $\text{LHS} = (\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2$

$$= \cos^2\theta + \sin^2\theta + \cancel{2\sin\theta\,.\,\cos\theta} + \cos^2\theta + \sin^2\theta \cancel{-2\sin\theta\,.\,\cos\theta}$$

$$= (\sin^2\theta + \cos^2\theta) + (\sin^2 \ + \cos^2\theta)$$

$$= 1 + 1$$

$$= 2 \quad = \text{RHS}$$

Q. 14. $(1 + \tan^2 A)\sin A\cos A = \tan A$

Sol. $\text{LHS} = (1 + \tan^2 A)\sin A \times \cos A$

$$= \sec^2 A \times \sin A \times \cos A$$

$$= \frac{1}{\cos^{\cancel{2}} A} \times \sin A \times \cancel{\cos A}$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A \quad = \text{RHS}$$

Q. 15. $\dfrac{\tan\theta}{\dfrac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta} = 1$

Sol. $\text{LHS} = \dfrac{\tan\theta}{\dfrac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta}$

$$= \frac{\tan\theta}{\sin\theta\ \dfrac{\sin^2\theta}{\cos\theta} + \cos\theta}$$

$$= \frac{\tan\theta}{\sin\theta\left(\dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta}\right)} \quad = \quad \frac{\tan\theta}{\sin\theta/\cos\theta}$$

$$= \frac{\cancel{\tan\theta}^{\,1}}{\cancel{\tan\theta}} \quad = \quad 1 \quad = \text{RHS}$$

Q. 16. $1 + \dfrac{\dfrac{\cos^2\theta}{\sin^2\theta}}{1 + \dfrac{1}{\sin\theta}} = \dfrac{1}{\sin\theta}$

Sol. $\text{LHS} = \dfrac{1 + \dfrac{\cos^2\theta}{\sin^2\theta}}{1 + \dfrac{1}{\sin\theta}}$

$$= \frac{1 + \dfrac{\cos^2\theta}{\sin^2\theta}}{\dfrac{\sin\theta + 1}{\sin\theta}}$$

$$= 1 + \frac{\cos^2\theta}{\sin^{\cancel{2}}\theta} \times \frac{\sin\theta}{\cancel{\sin\theta+}1}$$

$$= 1 + \frac{\cos^2\theta}{(\sin\theta + 1)\sin\theta}$$

$$= \frac{\sin\theta\,(1 + \sin\theta) + \cos^2\theta}{\sin\theta\,(\sin\theta + 1)}$$

$$= \frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\sin\theta\,(\sin\theta + 1)}$$

$$= \frac{\cancel{(\sin\theta + 1)}}{\sin\theta\,\cancel{(\sin\theta + 1)}}$$

$$= \frac{1}{\sin\theta} \qquad = \text{RHS}$$

Q. 17. $\dfrac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \dfrac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2$

Sol. $\text{LHS} = \dfrac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \dfrac{\cos^3 - \sin^3\theta}{\cos\theta - \sin\theta}$

$$= \frac{\cancel{(\cos\theta + \sin\theta)}(\cos^2\theta - \cos\theta\,.\,\sin\theta + \sin^2\theta)}{\cancel{(\cos\theta + \sin\theta)}} + \frac{\cancel{(\cos\theta - \sin\theta)}(\cos^2\theta + \cos\theta\,.\,\sin\theta + \sin^2\theta)}{\cancel{(\cos\theta\,.\,\sin\theta)}}$$

$$= \cos^2\theta \cancel{-\cos\theta\,.\,\sin\theta} + \sin^2\theta + \cos^2\theta + \cos\theta\,.\,\sin\theta + \sin^2\theta$$

$$= (\sin^2\theta + \cos^2\theta) + (\sin^2\theta + \cos^2\theta)$$

$$= 1 + 1$$

$$= \text{RHS}$$

Q. 18. $(1 - \tan\theta)^2 + (1 - \cot^2\theta) = (\sec\theta - \operatorname{cosec}\theta)^2$

Sol. $(1 - \tan^2\theta) + (1 - \cot^2\theta)$

$$= 1 + \tan^2\theta - 2\tan\theta + 1 + \cot^2\theta - 2\tan\theta$$

$$= \sec^2\theta - 2\tan\theta + \operatorname{cosec}^2\theta - 2\cot\theta$$

$$= \sec^2\theta + \operatorname{cosec}^2\theta - 2\tan\theta - 2\cot\theta$$

$$= \sec^2\theta + \operatorname{cosec}^2\theta - 2\left(\tan\theta + \frac{1}{\tan\theta}\right)$$

$$= \sec^2\theta + \operatorname{cosec}^2\theta - 2\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \sec^2\theta + \operatorname{cosec}^2\theta - 2 \times \left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\,.\,\cos\theta}\right)$$

$$= \sec^2\theta + \operatorname{cosec}^2\theta - 2 \times \frac{1}{\sin\theta\,.\,\cos\theta}$$

$$= \sec^2\theta + \operatorname{cosec}^2\theta - 2\sec\theta \times \operatorname{cosec}\theta$$

$$= (\sec\theta - \operatorname{cosec}^2\theta)^2 \qquad = \text{RHS}$$

Q. 19. $\dfrac{1}{1 - \sin A} + \dfrac{1}{1 + \sin A} = 2\sec^2 A$

Q. 20. $\dfrac{1 + \cos A}{\sin A} + \dfrac{\sin A}{1 + \cos A} = 2\operatorname{cosec} A$

Sol. LHS = $\frac{1}{1-\sin A} + \frac{1}{1+\sin A}$

= $\frac{1+\cancel{\sin A}+1-\cancel{\sin A}}{1-\sin^2 A}$

= $\frac{2}{\cos^2 A}$

= $2\sec^2 A$ = RHS

Q. 21. $\frac{1+\sin A}{\cos A} + \frac{\cos A}{1+\sin A} = 2\sec A$

Sol. LHS = $\frac{1+\sin A}{\cos A} + \frac{\cos A}{1+\sin A}$

= $\frac{1+\sin^2 A+2\sin A+\cos^2 A}{\cos A\,(1+\sin A)}$

= $\frac{1+1+2\sin A}{\cos A\,(1+\sin A)}$

= $\frac{2\,\cancel{(1+\sin A)}}{\cos A\,\cancel{(1+\sin A)}}$ = 2 sec A= RHS

Q. 23. $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \sec\theta\times\text{cosec}\,\theta + 1$

Sol. LHS = $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

= $\frac{\tan\theta}{1-1/\tan\theta} + \frac{\tan\theta}{1-\tan\theta}$

= $\frac{\tan\theta}{\frac{\tan\theta-1}{\tan\theta}} + \frac{1}{\tan\theta}\times\frac{1}{1-\tan\theta}$

= $\frac{\tan^2\theta}{\tan\theta-1} + \frac{1}{\tan\theta\,(1-\tan\theta)}$

= $\frac{\tan^2\theta}{\tan\theta-1} + \frac{1}{-\tan\theta\,(\tan\theta-1)}$

= $\frac{\tan^2\theta}{\tan\theta-1} - \frac{1}{\tan\theta\,(\tan\theta-1)}$

= $\frac{\tan^3\theta-1}{\tan\theta\,(\tan\theta-1)}$

= $\frac{\cancel{(\tan\theta-1)}\,(\tan^2+\tan\theta+1)}{\tan\theta\,\cancel{(\tan\theta-1)}}$

= $\frac{\tan^2\theta}{\tan\theta} + \frac{\tan\theta}{\tan\theta} + \frac{1}{\tan\theta}$

= $\tan\theta + \cot\theta + 1$

= $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} + 1$

= $\frac{\sin^2\theta+\cos^2\theta+\sin\theta\,.\cos\theta}{\sin\theta\,.\cos\theta}$

= $\frac{1+\sin\theta\,.\cos\theta}{\sin\theta\,.\cos\theta}$

= $\frac{1}{\sin\theta\,.\cos\theta} + \frac{\cancel{\sin\theta\,.\cos\theta}}{\cancel{\sin\theta\,.\cos\theta}}\;^{1}$

= $\frac{1}{\sin\theta}\times\frac{1}{\cos\theta} + 1$

= $\sec\theta\times\sec\theta + 1$ = RHS

Sol. LHS = $\frac{1+\cos A}{\sin A} + \frac{\sin A}{1+\cos A}$

= $\frac{1+\cos^2 A+2\cos A+\sin^2 A}{\sin A\,(1+\cos A)}$

= $\frac{1+1+2\cos A}{\sin A\,(1+\cos A)}$

= $\frac{2\,(1+\cancel{\cos A})}{\sin A\,(1+\cancel{\cos A})}$

= 2 cosec A = RHS

Q. 22. $\frac{\sin\theta}{1+\cos\theta} + \frac{\sin\theta}{1-\cos\theta} = \frac{2}{\sin\theta}$

Sol. LHS = $\frac{\sin\theta}{1+\cos\theta} + \frac{\sin\theta}{1-\cos\theta}$

= $\frac{\sin\theta - \cancel{\sin\theta\,.\cos\theta} + \sin\theta + \cancel{\sin\theta\,.\cos\theta}}{1-\cos\theta}$

= $\frac{2\sin\theta}{\sin^2\theta}$

= $\frac{2}{\sin\theta}$ = RHS

Q. 24. $\frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A-\sin A} = \cos A+\sin A$

Sol. LHS = $\frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A-\sin A}$

= $\frac{\cos A}{1-\frac{\sin A}{\cos A}} - \frac{\sin^2 A}{\cos A-\sin A}$

= $\frac{\cos A}{\frac{\cos A-\sin A}{\cos A}} - \frac{\sin^2 A}{\cos A-\sin A}$

= $\frac{\cos^2 A}{\cos A-\sin A} - \frac{\sin^2 A}{\cos A-\sin A}$

= $\frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$

= $\frac{(\cos A+\sin A)\,(\cos A-\sin A)}{(\cos A-\sin A)}$

= $\cos A+\sin A$

= RHS

Q. 25. $\frac{\sin A}{1-\frac{1}{\sin A}} + \frac{1}{\frac{\sin A}{1-\sin A}} = 1+\sin A+\frac{1}{\sin A}$

Sol. LHS = $\frac{\sin A}{1-\frac{1}{\sin A}} + \frac{1}{\frac{\sin A}{1-\sin A}}$

= $\frac{\sin^2 A}{\sin A-1} + \frac{1}{\sin A\,(1-\sin A)}$

= $\frac{\sin^2 A}{\sin A-1} - \frac{1}{\sin A\,(\sin A-1)}$

= $\frac{\sin^3 A-1}{\sin A\,(\sin A-1)}$

= $\frac{\cancel{(\sin A-1)}\,(\sin^2 A+\sin A+1)}{\sin A\,\cancel{(\sin A-1)}}$

= $\frac{\sin^2 A}{\sin A} + \frac{\sin A}{\sin A} + \frac{1}{\sin A}$

= $1+\sin A+\frac{1}{\sin A}$ = RHS

Q. 26. $\left(\tan\theta+\frac{1}{\cos\theta}\right)^2 + \left(\tan\theta-\frac{1}{\cos\theta}\right)^2 = 2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$

Sol. LHS = $\left(\tan\theta+\frac{1}{\cos\theta}\right)^2 + \left(\tan\theta-\frac{1}{\cos\theta}\right)^2$

= $(\tan\theta+\sec\theta)^2 + (\tan\theta-\sec)^2$ = $\tan^2\theta+\sec^2\theta+2\,\cancel{\tan\theta\,.\sec\theta}+\tan^2\theta+\sec^2\theta-2\,\cancel{\tan\theta\,.\sec\theta}$ $=2\tan^2\theta+2\sec^2\theta$

= $2\left(\frac{\sin^2\theta}{\cos^2\theta}+\frac{1}{\cos^2\theta}\right)$ = $2\;\frac{\sin^2\theta+1}{\cos^2\theta}$ = $2\left(\frac{1+\sin^2\theta}{1-\text{Sin}^2\theta}\right)$ = RHS

Q. 27. $\frac{\tan^2\theta}{\tan^2\theta - 1} + \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta} = \frac{1}{\sin^2\theta - \cos^2\theta}$

Sol. LHS = $\frac{\tan^2\theta}{\tan^2\theta - 1} + \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta}$

$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta - 1}{\cos^2\theta}} + \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta}$

$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta - \cos^2\theta}{\cos^2\theta}} + \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta} + \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta}$

$= \frac{\sin^2\theta}{\sin^2\theta - \cos^2\theta} + \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta}$

$= \frac{1}{\sin^2\theta - \cos^2\theta}$

= RHS

Q. 28. $\frac{\cos A}{1 - \tan A} + \frac{\sin^2 A}{\sin A - \cos A} = \sin A + \cos A$

Sol. LHS = $\frac{\cos A}{1 - \tan A} + \frac{\sin^2 A}{\sin A - \cos A}$

$= \frac{\cos A}{1 - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$

$= \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin^2 A}{\sin A - \cos A}$

$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$

$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$

$= \frac{(\cos A + \sin A)\cancel{(\cos A - \sin A)}}{\cancel{(\cos A - \sin A)}}$

$= \cos A + \sin A$

= RHS

Q. 29. $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$

Sol. LHS = $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$

$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\sin A + \sin B)(\cos A + \cos B)}$

$= \frac{\sin^2 A + \cos^2 A - (\sin^2 B + \cos^2 B)}{(\sin A + \sin B)(\cos A + \cos B)}$

$= \frac{1 - 1}{(\sin A + \sin B)(\cos A + \cos B)}$

$= \frac{0}{(\sin A + \sin B)(\cos A + \cos B)}$

= 0 RHS

Q. 30. $\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right) = \frac{1}{\tan\theta + \cot\theta}$

Sol. LHS = $\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right)$

$= \left(\frac{1 - \cos^2\theta}{\cos\theta}\right)\left(\frac{1 - \sin^2\theta}{\sin\theta}\right)$

$= \frac{\sin^{\cancel{2}}\theta}{\cancel{\cos\theta}} \times \frac{\cos^{\cancel{2}}\theta}{\cancel{\sin\theta}}$

$= \frac{\sin^2\theta \times \cos^2\theta}{1}$

$= \frac{\sin\theta \times \cos\theta}{\sin^2\theta + \cos^2\theta}$

$= \frac{1}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta \times \cos\theta}}$

$= \frac{1}{\frac{\sin^{\cancel{2}}\theta}{\cancel{\sin\theta \times}\cos\theta} + \frac{\cos^{\cancel{2}}\theta}{\sin\theta \times \cancel{\cos\theta}}}$

$= \frac{1}{\tan\theta + \cot\theta}$

Q. 31. $\left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\operatorname{cosec}^2\theta - \sin^2\theta}\right)\sin^2\theta . \cos^2\theta = \frac{1 - \sin^2\theta\cos^2\theta}{2 + \sin^2\theta\cos^2\theta}$

Sol. LHS = $\left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\operatorname{cosec}^2\theta - \sin^2\theta}\right)\sin^2\theta . \cos^2\theta$

$= \left(\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right)\sin^2\theta . \cos^2\theta$

$= \left(\frac{\cos^2\theta}{1 - \cos^2\theta} + \frac{\sin^2\theta}{1 - \sin^2\theta}\right)\sin^2\theta . \cos^2\theta$

$= \left(\frac{\cos^2\theta}{(1)^2 - (\cos^2\theta)^2} + \frac{\sin^2\theta}{(1)^2 - (\sin^2\theta)^2}\right)\sin^2\theta . \cos^2\theta$

$= \left(\frac{\cos^2\theta}{(1 + \cos^2\theta)(1 - \cos^2\theta)} + \frac{\sin^2\theta}{(1 - \sin^2\theta)\ 1 + \cos^2\theta}\right)\sin^2\theta . \cos^2\theta$

$= \left(\frac{\cos^4\theta(1 + \sin^2\theta) + \sin^4\theta(1 + \cos^2\theta)}{\cancel{(\sin^2\theta)}(1 + \cos^2\theta)\cancel{\cos^2\theta}(1 + \sin^2\theta)}\right)\cancel{\sin^2\theta . \cos^2\theta}$

$= \frac{\cos^4\theta + \sin^4\theta + \sin^2\theta \times \cos^2\theta + \cos^4\theta \times \sin^2\theta}{1 + (\sin^2\theta + \cos^2\theta) + \sin^2\theta \times \cos^2\theta}$

$= \frac{(\cos^2\theta)^2 + (\sin^2\theta)^2 + \sin^2\theta . \cos^2\theta(\sin^2\theta + \cos^2\theta)}{1 + 1 + \sin^2\theta . \cos^2\theta}$

$= \frac{(\cos^2 + \sin^2\theta)^2 = 2\sin^2\theta . \cos^2\theta + \sin^2\theta + \cot^2\theta) \times 1}{2 + \sin^2 . \cos^2\theta}$

$= \frac{1 - \sin^2\theta . \cos^2\theta}{2 + \sin^2\theta . \cos^2\theta} = \frac{1 - \sin^2\theta . \cos^2\theta}{2 + \sin^2\theta . \cos^2\theta} =$ RHS

Q. 32. $\frac{\cos\theta}{1-\sec\theta}+\frac{\sec\theta}{1-\cos\theta}=1+\sec\theta+\cos\theta$

Sol. LHS = $\frac{\cos\theta}{1-\sec\theta}+\frac{\sec\theta}{1-\cos\theta}$

$$=\frac{\cos\theta}{1-\frac{1}{\cos\theta}}+\frac{\frac{1}{\cos\theta}}{1-\cos\theta}$$

$$=\frac{\cos^2\theta}{\cos\theta-1}+\frac{1}{\cos\theta\,(1-\cos\theta)}$$

$$=\frac{\cos^2\theta}{\cos\theta-1}-\frac{1}{\cos\theta\,(\cos\theta-1)}$$

$$=\frac{\cos^3\theta-1}{\cos\theta\,(\cos\theta-1)}$$

$$=\frac{\cancel{(\cos\theta-1)}\,(\cos^2\theta+\cos\theta+1)}{\cos\theta\,\cancel{(\cos\theta-1)}}$$

$$=\frac{\cos^{\cancel{2}}\theta}{\cancel{\cos\theta}}+\frac{\cancel{\cos\theta}}{\cancel{\cos\theta}}\cancel{+}\frac{1}{\cos\theta}$$

$$=1+\cos\theta+\sec\theta \quad = \quad \text{RHS}$$

Q. 33. $(\text{cosec}\,\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

Sol. LHS = $(\text{cosec}\,\theta-\cot\theta)^2$

$$=\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2$$

$$=\left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$=\frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$=\frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$=\frac{(1-\cos\theta)\,(1-\cos\theta)}{(1+\cos\theta)\,(1-\cos\theta)}$$

$$=\frac{1-\cos\theta}{1+\cos\theta}$$

= RHS

Q. 34. $\sec^4\theta-\sec^2\theta=\tan^4\theta+\tan^2\theta$

Sol. LHS = $\sec^4\theta-\sec^2\theta$

$=\sec^2\theta\,(\sec^2\theta-1)$

$=\sec^2\theta\times\tan^2\theta$

$=(1+\tan^2\theta)\tan^2\theta$

$=\tan^2\theta+\tan^4\theta$

= RHS

Q. 35. $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\frac{2}{\sin^2\theta-\cos^2\theta}$

Sol. LHS = $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

$$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\,.\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\,.\cos\theta}{\sin^2\theta-\cos^2\theta}$$

$$=\frac{1+1}{\sin^2\theta-\cos^2\theta}$$

$$=\frac{2}{\sin^2\theta-\cos^2\theta}=\quad\text{RHS}$$

Q. 36. $\frac{\tan\theta-\cot\theta}{\sin\theta\times\cos\theta}=\sec^2\theta-\text{cosec}\,\theta=\tan^2\theta-\cot^2\theta$

Sol. LHS = $\frac{\tan\theta-\cot\theta}{\sin\theta\,.\cos\theta}$

$$=\frac{\frac{\sin\theta}{\cos\theta}-\frac{\cos\theta}{\sin\theta}}{\sin\theta.\cos\theta}$$

$$=\frac{\frac{\sin^2\theta-\cos^2\theta}{\sin\theta\,.\cos\theta}}{\sin\theta\,.\cos\theta}$$

$$=\frac{\sin^2\theta-\cos^2\theta}{\sin^2\theta\,.\cos^2\theta}$$

$$=\frac{\sin^2\theta}{\sin^2\cancel{\theta\,.\cos}^2\theta}-\frac{\cancel{\cos^2\theta}}{\sin^2\theta-\cancel{\cos^2\theta}}$$

$$=\cancel{\sec^2\theta}-\text{cosec}^2\theta=\text{RHS}$$

Again, LHS = $\sec^2\theta-\text{cosec}^2\theta$

$=1+\tan^2\theta\,(1+\cot^2\theta)$

$=\cancel{1}+\tan^2\theta-\cancel{1}-\cot^2\theta$

$=\tan^2\theta-\cot^2\theta \quad = \quad$ RHS

Q. 37. $\cos^4\theta-\cos^2\theta=\sin^4\theta-\sin^2\theta$

Sol. LHS = $\cos^4\theta-\cos^2\theta=\sin^4\theta-\sin^2\theta$

$=\cos^2\theta\,(\cos^2\theta-1)$

$=(1\cancel{-}\sin^2\theta)\,(1\cancel{-}\sin^2\theta-1)$

$=-\sin^2\theta\,(1-\sin^2\theta)$

$=-\sin^2\theta+\sin^4\theta$

$=\sin^4\theta-\sin^2\theta$

= RHS

Q. 38. $\cot^4\theta-1=\text{cosec}^4\theta-2\,\text{cosec}^2\theta$

Sol. LHS = $\cot^4-1$

$=(\cot^2\theta-1)\,(\cot^2\theta+1)$

$=(\cot^2\theta-1)\,\text{cosec}^2\theta$

$=(\text{cosec}^2\theta-1-1)\,\text{cosec}^2\theta$

$=(\text{cosec}^2-2)\,\text{cosec}^2\theta$

$=\text{cosec}^4\theta-2\,\text{cosec}^2\theta \quad = \quad$ RHS

Q. 39. $\sin^4\theta+\cos^4\theta=1-2\sin^2\theta\cos^2\theta$

Sol. LHS = $\sin^4+\cos^4\theta$

$=(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\,.\cos^2\theta$

$=1-2\sin^2\theta\,.\cos^2\theta$

= RHS

Q. 40. $\sin^4-\cos^4\theta=1-2\sin^2\theta$

Sol. LHS = $\sin^4-\cos^4\theta$

$=(\sin^2\theta+\cos^2\theta)\,(\sin^2\theta-\sin^2\theta)$

$=1\times\sin^2\theta-\cos^2\theta$

$=1-\cos^2\theta-\cos^2\theta$

$=1-2\cos^2\theta$

= RHS

Q. 41. $\sin^6\theta+\cos^6\theta=1-3\sin^2\theta\,.\cos^2\theta$

Sol. LHS = $(\sin^2\theta)^3+(\cos^2\theta)^3$

$=(\sin^2\theta+\cos^2)\,(\sin^4\theta-\sin^2\theta\,.\cos^2\theta+\cos^4\theta)$

$=\sin^4\theta+\cos^4-\sin^2\theta\,.\cos^2\theta$

$=(\sin^2\theta)^2+(\cos^2\theta)^2-\sin^2\theta\,.\cos^2\theta$

$=(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\,.\cos^2\theta-\sin^2\theta\,.\cos^2\theta$

$=1-3\sin^2\theta\,.\cos^2\theta$

= RHS

Q. 42. $\sin^8 \theta - \cos^8 \theta = (\sin^2 \theta - \cos^2 \theta)(1 - 2\sin^2 \theta . \cos^2 \theta)$

Sol. LHS $= \sin^8 \theta = \cos^8 \theta$

$= (\sin^4 \theta)^2 - (\cos^4 \theta)^2$

$= (\sin^4 \theta + \cos^4 \theta)(\sin^4 \theta - \sin^4 \theta)$

$= (\sin^4 \theta + \cos^4 \theta)(\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta)$

$= (\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta . \cos^2 \theta (\sin^2 \theta - \cos^2 \theta)$

$= (1 - 2\sin^2 \theta . \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta)$

$=$ RHS

Q. 43. $\sin^6 \theta + \cos^6 \theta + 3\sin^2 \theta \cos^2 \theta = 1$

Sol. LHS $= \sin^6 \theta + \cos^6 \theta + 3\sin^2 \theta \cos^2 \theta$

$= (\sin^2 \theta + \cos^2)^{2+} - 3\sin^2 \theta . \cos^2 \theta$

$(\sin^2 \theta + \cos^2 \theta + 3\sin^2 \theta . \cos^2 \theta$

$= 1 - \sin^2 \theta . \cos^2 \theta \times 1 + 3\sin^2 \theta . \cos^2 \theta$

$= 1 \quad =$ RHS

Q. 44. $2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1 = 0$

Sol. LHS $= 2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1$

$= 2[(\sin^2 \theta + \cos^2 \theta)^2 - 3\sin^2 \theta . \cos^2 \theta (\sin^2 \theta + \cos^2 \theta)] - 3(\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta . \cos^2 \theta) + 1$

$= 2[1 - 3\sin^2 \theta . \cos^2 \theta] - 3[1 - 2\sin^2 . \cos^2 \theta] + 1$

$= 2[1 - 3\sin^2 \theta . \cos^2 \theta] - 3 + 6\sin^2 \theta . \cos^2 \theta + 1$

$= 2 - 6\sin^2 \theta . \cos^2 \theta - 3 + 6\sin^2 \theta . \cos^2 \theta + 1$

$= 2 - 3 + 1$

$= -1 + 1 = 0 \quad =$ RHS

Q. 45. $(1 + \tan A \tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A \sec^2 B$

Sol. LHS $= (1 + \tan A . \tan B)^2 + (\tan A - \tan B)^2$

$= 1 + \tan^2 A . \tan^2 B + \cancel{2\tan A . \tan B} + \tan^2 A + \tan^2 B - \cancel{2\tan A \tan B}$

$= 1 + \tan^2 A . \tan^2 B + \tan^2 A + \tan^2 B$

$= 1 + \tan^2 A . \tan^2 B + \tan^2 A + \tan^2 B$

$= \cancel{1} + \tan^2 A . \tan^2 B + \sec^2 A - 1 + \sec^2 B - \cancel{1}$

$= \tan^2 A . \tan^2 B + \sec^2 A - 1 + \sec^2 B$

$= (\sec^2 A - 1)(\sec^2 B - 1) + \sec^2 A - 1 + \sec^2 B$

$= \sec^2 A \sec^2 B - \cancel{\sec^2 A} - \cancel{\sec^2 B} + 1 + \cancel{\sec^2 A} - 1 + \cancel{\sec^2 B}$

$= \sec^2 A \sec^2 A \quad =$ RHS

Q. 46. $(\tan A + \operatorname{cosec} B)^2 - (\cot B - \sec A)^2 = 2\tan A . \cot B (\operatorname{cosec} A + \sec B)$

Sol. LHS $= (\tan A + \operatorname{cosec} B)^2 - (\cot B - \sec A)^2$

$= (\tan^2 A + \operatorname{cosec}^2 B + 2\tan A . \operatorname{cosec} B) - (\cot^2 B + \sec^2 A - 2\cot B . \sec A)$

$= (\tan^2 A + \operatorname{cosec}^2 B) + 2\tan \theta . \operatorname{cosec} \theta) - \cot^2 B - \sec^2 A + 2\cot B . \sec A$

$= (\operatorname{cosec}^2 B - \cot^2 B) - (\sec^2 A - \tan^2 A) + 2\tan A . \operatorname{cosec} B + 2\cot B - \sec A$

$= 1 - 1 + 2\left(\frac{\sin A}{\cos A} \times \frac{1}{\sin B} + \frac{\cos B}{\sin B} \times \frac{1}{\cos A}\right)$

$= 2\left(\frac{\sin A}{\cos A . \sin B} + \frac{\cos B}{\sin B . \cos A}\right)$

Multiplying numerator and denominator by tan A × cot B

$= \tan A \times \cot B = \dfrac{\dfrac{\sin A + \cos B}{\cos A . \sin B}}{\tan A \times \tan B}$

$= 2\tan A \times \cot B = \left(\dfrac{\sin A + \cos B}{\cancel{\cos A} \times \cancel{\sin D} \times \dfrac{\sin A}{\cancel{\cos A}} \times \dfrac{\cos B}{\sin B}}\right)$

$= 2\tan A \times \cot B \left(\dfrac{\sin A + \cos B}{\sin A \times \cos B}\right)$

$= 2\tan A \times \cot B \dfrac{\sin A + \cancel{\cos B}}{\cancel{\sin A} \cancel{\times \cos} B \times \sin A \times \cancel{\cos B}}$

$= 2\tan A \times \cot B \times (\sec B + \operatorname{cosec} A) \quad =$ RHS

Q. 47. $\dfrac{\cos A}{1 - \sin A} + \dfrac{\sin A}{1 - \cos A} + 1 = \dfrac{\sin A \times \cos A}{(1 - \sin A)(1 - \cos A)}$

Sol. LHS $= \dfrac{\cos A}{1 - \sin A} + \dfrac{\sin A}{1 - \cos A} + 1$

$= \dfrac{\cos A - \cos^2 A + \sin A - \sin^2 A + (1 - \sin A)(1 - \cos A)}{(1 - \sin A)(1 - \cos A)}$

$= \dfrac{(\cos A + \sin A) - (\sin^2 A + \cos^2 A) + (1 - \sin A)(1 - \cos A)}{(1 - \sin A)(1 - \cos A)}$

$= \dfrac{-(\cos A + \sin A) + (1 - \sin A)(1 - \cos A)}{(1 - \sin A)(1 - \cos A)}$

$= \dfrac{-\cancel{\cos A} \cancel{+ \sin A} + (1 \cancel{- \cos A} \cancel{- \sin A} + \sin A \times \cos A)}{(1 - \sin A)(1 - \cos A)}$

$= \dfrac{-1 + 1 + \sin^2 A \times \cos^2 A}{(1 - \sin A)(1 - \cos A)} \quad =$ RHS

Q. 48. $$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec+1}=\frac{1+\sin\theta}{\cos\theta}$$

Sol.
$$\text{LHS}=\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$$
$$=\frac{\tan\theta+\sec\theta-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$$
$$=\frac{(\tan\theta+\sec\theta)-(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{\tan\theta-\sec\theta+1}$$
$$=\frac{(\tan\theta+\sec\theta)(1-\sec\theta+\tan\theta)}{(\tan\theta-\sec\theta+1)}$$
$$=\tan\theta+\sec\theta$$
$$=\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}=\frac{1+\sin\theta}{\cos\theta}=\quad\text{RHS}$$

Q. 49. $$\cot^2\theta\left(\frac{\sec\theta-1}{1+\sin\theta}\right)+\sec^2\theta\left(\frac{\sin\theta-1}{1+\sec\theta}\right)=0$$

Sol.
$$\text{LHS}=\cot^2\theta\left(\frac{\sec\theta-1}{1+\sin\theta}\right)+\sec^2\theta\left(\frac{\sin\theta-1}{1+\sec\theta}\right)$$
$$=\frac{\cot^2\theta(\sec\theta-1)}{1+\sin\theta}+\frac{\sec^2\theta(\sin\theta-1)}{1+\sec\theta}$$
$$=\frac{\cot^2\theta(\sec\theta-1)(1+\sec\theta)+\sec^2\theta(\sin\theta-1)(1+\sin\theta)}{(1+\sin\theta)(1+\sec\theta)}$$
$$=\frac{\cot^2\theta(\sec^2\theta-1)+\sec^2\theta(\sin^2\theta-1)}{(1+\sin^2\theta)(1+\sec\theta)}$$
$$=\frac{\cot^2\theta\times\tan^2\theta+\sec^2\theta\times(-\cos^2\theta)}{(1+\sin\theta)(1+\sec\theta)}$$
$$=\frac{\cot^2\theta\times\frac{1}{\cos^2\theta}+\frac{1}{\cos^2\theta}(\cos^2\theta)}{(1+\sin\theta)(1+\sec\theta)}$$
$$=\frac{1-1}{(1+\sin\theta)(1+\sec\theta)}$$
$$=0\qquad=\text{RHS}$$

Q. 50. $$\frac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}=\frac{1+\cos\theta}{\sin\theta}$$

Sol.
$$\text{LHS}=\frac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}$$
$$=\frac{\cot\theta+\operatorname{cosec}\theta-(\operatorname{cosec}^2\theta-\cot^2\theta)}{\cot\theta-\operatorname{cosec}\theta+1}$$
$$=\frac{\cot\theta+\operatorname{cosec}\theta-(\operatorname{cosec}\ (\operatorname{cosec}\theta+\cot\theta)(\operatorname{cosec}\theta-\cot\theta)}{\cot\theta-\operatorname{cosec}\ \theta+1}$$
$$=\frac{(\cot\theta+\operatorname{cosec}\theta)\cancel{(1-\operatorname{cosec}\theta+\cot\theta)}}{\cancel{(\cot\theta-\operatorname{cosec}\theta+1)}}$$
$$=\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}$$
$$=\frac{\cos\theta+1}{\sin\theta}=\frac{1+\cos\theta}{\sin\theta}=\text{RHS}$$

Q. 55. $$\left(1+\frac{1}{\tan^2\theta}\right)\left(1+\frac{1}{\cot^2\theta}\right)=\frac{1}{\sin^2\theta-\sin^4\theta}$$

Sol.
$$\text{LHS}=\left(1+\frac{1}{\tan^2\theta}\right)\left(1+\frac{1}{\cot^2\theta}\right)$$
$$=\left(\frac{\tan^2\theta+1}{\tan^2\theta}\right)\left(\frac{\cot^2\theta+1}{\cot^2\theta}\right)$$
$$=\left(\frac{\sec^2\theta}{\tan^2\theta}\right)\times\frac{\operatorname{cosec}^2\theta}{\cot^2\theta}$$
$$=\frac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\times\frac{\frac{1}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}}$$
$$=\frac{\cancel{\cos^2\theta}}{\cancel{\sin^2\theta}\,.\cos^2\theta}\times\frac{\cancel{\sin^2\theta}}{\sin^2\theta\,.\cancel{\cos^2\theta}}$$
$$=\frac{1}{\cos^2\theta}\times\frac{1}{\sin^2\theta}$$
$$=\frac{1}{1-\sin^2\theta}\times\frac{1}{\sin^2\theta}$$

Q. 51. $$1+\frac{\cot^2\theta}{1+\operatorname{cosec}\theta}=\operatorname{cosec}\theta$$

Sol.
$$\text{LHS}=1+\frac{\cot^2\theta}{1+\operatorname{cosec}\theta}$$
$$=\frac{1+\operatorname{cosec}\theta+\cot^2\theta}{1+\operatorname{cosec}\theta}$$
$$=\frac{\operatorname{cosec}\theta+\operatorname{cosec}^2\theta}{1+\operatorname{cosec}^2\theta}$$
$$=\frac{\operatorname{cosec}\theta(1+\operatorname{cosec}\theta)}{(1+\operatorname{cosec}\theta)}$$
$$=\operatorname{cosec}\theta$$
$$=\text{RHS}$$

Q. 52. $$1+\frac{\tan^2\theta}{1+\sec\theta}=\sec\theta$$

Sol.
$$\text{LHS}=1+\frac{\tan^2\theta}{1+\sec\theta}$$
$$=\frac{1+\sec\theta+\tan^2\theta}{1+\sec\theta}$$
$$=\frac{\sec\theta+\sec^2\theta}{1+\sec\theta}$$
$$=\frac{\sec\theta\,\cancel{(1+\sec\theta)}}{(1+\sec\theta)}$$
$$=\sec\theta$$
$$=\text{RHS}$$

Q. 53. $$\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{\cot^2\theta}{1+\cot^2\theta}=1$$

Sol.
$$\text{LHS}=\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{\cot^2\theta}{1+\cot^2\theta}$$
$$=\frac{\tan^2\theta+\cot^2\theta}{\sec^2\theta\quad\operatorname{cosec}^2\theta}$$
$$=\frac{\sin^2\theta\ \cancel{\cos^2\theta}}{\cancel{\cos^2\theta}}+\frac{\cos^2\theta\ \cancel{\sin^2\theta}}{\cancel{\sin^2\theta}}$$
$$=\sin^2\theta+\cos^2\theta$$
$$=1$$
$$=\text{RHS}$$

Q. 54. $$\frac{(1+\tan^2\theta)}{\operatorname{cosec}^2\theta}\times\cot\theta=\tan\theta$$

Sol.
$$\text{LHS}=\left(\frac{1+\tan^2\theta}{\operatorname{cosec}^2\theta}\right)\times\cot\theta$$
$$=\frac{\sec^2\theta}{\operatorname{cosec}^2\theta}\times\cot\theta$$
$$=\frac{\sin^{\cancel{2}}\theta}{\cos^{\cancel{2}}\theta}\times\frac{\cancel{\cos\theta}}{\cancel{\sin\theta}}=\tan\theta$$

Q. 58. $$\frac{\cos^2\theta}{1-\tan\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}=1+\sin\theta\times\cos\theta$$

Sol.
$$\text{LHS}=\frac{\cos^2\theta}{1-\tan\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}$$
$$=\frac{\cos^2\theta}{1-\frac{\sin\theta}{\cos\theta}}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}$$
$$=\frac{\cos^3\theta}{\cos\theta-\sin\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}$$
$$=\frac{\cos^3\theta}{\cos\theta-\sin\theta}-\frac{\sin^3\theta}{\cos\theta-\sin\theta}$$
$$=\frac{\cos^3\theta-\sin\theta}{\cos\theta-\sin\theta}$$
$$=\frac{\cancel{(\cos\theta-\sin\theta)}(\cos^2\theta+\sin^2\theta+\sin\theta\,.\cos\theta)}{\cancel{(\cos\theta-\sin\theta)}}$$
$$=(\sin^2\theta+\cos^2\theta)+\sin\theta\,.\cos\theta$$
$$=1+\sin\theta\,.\cos\theta$$
$$=\text{RHS}$$

$$= \frac{1}{\sin^2\theta\,(1-\sin^2\theta)}$$

$$= \frac{1}{\sin^2\theta-\sin^4\theta} \qquad = \text{RHS}$$

Q. 56. $(1+\tan^2\theta)\ \ 1+\left(\frac{1}{\tan^2\theta}\right) = \frac{1}{\sin^2\theta-\sin^2\theta}$

Sol. $\text{LHS} = (1+\tan^2\theta)\left(1+\frac{1}{\tan^2\theta}\right)$

$$= (1+\tan^2\theta)\left(\frac{\tan^2\theta+1}{\tan^2\theta}\right)$$

$$= (1+\tan^2\theta)\ \frac{\sec^2\theta}{\tan^2\theta}$$

$$= 1+\frac{\sin^2\theta}{\cos^2\theta}\ \ \frac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \left(\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}\right)\ \frac{\cancel{\cos^2\theta}}{\sin^2\theta\,.\,\cancel{\cos^2\theta}}$$

$$= \frac{1}{\cos^2\theta}\times\frac{1}{\sin^2\theta}$$

$$= \frac{1}{(1-\sin^2\theta)\times\sin^2\theta}$$

$$= \frac{1}{\sin^2\theta-\sin^4\theta} \qquad = \ \text{RHS}$$

Q. 57. $\frac{\tan\theta}{(1+\tan^2\theta)^2}+\frac{\cot\theta}{(1+\cot^2\theta)^2} = \sin\theta\,.\,\cos\theta$

Sol. $\text{LHS} = \frac{\tan\theta}{(1+\tan^2\theta)^2}+\frac{\cot\theta}{(1+\cot^2\theta)^2}$

$$= \frac{\tan\theta}{(\sec^2\theta)}+\frac{\cot^2\theta}{\text{cosec}^2\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^4\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin^4\theta}}$$

$$= \frac{\sin\theta\times\cos^{\overset{3}{\cancel{4}}}\theta}{\cancel{\cos\theta}}+\frac{\cos\theta\times\sin^{\overset{3}{\cancel{4}}}\theta}{\cancel{\sin\theta}}$$

$$= \sin\theta\times\cos^3\theta+\cos\theta\times\sin^3\theta$$

$$= \sin\theta\,.\,\cos\theta\,(\cos^2\theta\,.\,\tan^2\theta)$$

$$= \sin\theta\,.\,\cos\theta \qquad = \qquad \text{RHS}$$

Q. 59. $\left(\frac{\tan\theta}{\sec\theta-1}\right)+\left(\frac{\tan\theta}{\sec\theta+1}\right) = 2\ \text{cosec}\ \theta$

Sol. $\text{LHS} = \frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}$

$$= \frac{\tan\theta\,(\sec\theta+1)+\tan\theta\,(\sec\theta-1)}{\sec^2\theta-1}$$

$$= \frac{\cancel{\tan\theta}(\sec\theta+\cancel{1}+\sec\theta-\cancel{1})}{\tan^{\cancel{2}}\theta}$$

$$= \frac{2\sec\theta}{\tan\theta}$$

$$= \frac{2\times\ 1/\cos\theta}{\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{2}{\cancel{2\cos\theta}}\times\frac{\cancel{\cos\theta}}{\sin\theta}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\ \text{cosec}\ \theta$$

$$= \text{RHS}$$

Q. 60. $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta-\sin^2\theta$

Sol. $\text{LHS} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}}$$

$$= \cos^2\theta-\sin^2\theta$$

$$= \text{RHS}$$

Q. 61. $\frac{\sin\theta}{\cot\theta+\text{cosec}\,\theta} = 2+\frac{\sin\theta}{\cot\theta-\text{cosec}\,\theta}$

Sol. $\text{LHS} = \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}}$

$$= \frac{\sin\theta}{\frac{\cos\theta+1}{\sin\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta+1}$$

$$= \frac{1-\cos^2\theta}{\cos\theta+1}$$

$$= \frac{(1+\cos\theta)\,(1-\cos\theta)}{(1+\cos\theta)}$$

$$= 1-\cos\theta$$

$\text{RHS} = 2+\frac{\sin\theta}{\cot\theta - \text{cosec}\,\theta}$

$$= 2+\frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}\ \frac{1}{\sin\theta}}$$

$$= 2+\frac{\sin^2\theta}{\cos\theta-1}$$

$$= 2+\frac{1-\cos^2\theta}{\cos\theta-1}$$

$$= 2+\frac{(1+\cos\theta)\,(1-\cos\theta)}{(\cos\theta-1)}$$

$$= 2-\frac{(1+\cos\theta)\,(1-\cos\theta)}{(1-\cos\theta)}$$

$= 2-1-\cos\theta \qquad = \qquad 1-\cos\theta$; **LHS = RHS**

Q. 62. $\frac{1}{\cos\theta+\sin\theta-1}+\frac{1}{\cos\theta+\sin\theta+1} = \text{cosec}\,\theta+\sec\theta$

Sol. $\text{LHS} = \frac{1}{\cos\theta+\sin\theta-1}+\frac{1}{\cos\theta+\sin\theta+1}$

$$= \frac{\cos\theta+\sin\theta+\cancel{1}+\cos\theta+\sin\theta-\cancel{1}}{(\cos\theta+\sin\theta-1)\,(\cos\theta+\sin\theta+1)}$$

$$= \frac{2\,(\sin\theta+\cos\theta)}{(\cos\theta+\sin\theta)^2-1}$$

$$= \frac{2\,(\sin\theta+\cos\theta)}{\sin^2\theta+\cos^2\theta+2\sin\theta\,.\,\cos\theta-1}$$

$$= \frac{2\,(\sin\theta+\cos\theta)}{\cancel{1}+2\sin\theta\,.\,\cos\theta\cancel{-1}}$$

$$= \frac{2\,(\sin\theta+\cos\theta)}{2\sin\theta\,.\,\cos\theta} = \frac{\cancel{\sin\theta}}{\cancel{\sin\theta}\,.\,\cos\theta}+\frac{\cancel{\cos\theta}}{\sin\theta\,.\,\cancel{\cos\theta}}$$

$$= \sec\theta+\text{cosec}\,\theta \qquad = \qquad \text{RHS}$$

Q. 63. $$\frac{\sin\theta}{\sec\theta+\tan\theta-1}+\frac{\cos\theta}{\mathrm{cosec}\,\theta+\cot-1}=1$$

Sol. $$\text{LHS}=\frac{\sin\theta}{\sec\theta+\tan\theta-1}+\frac{\cos\theta}{\mathrm{cosec}\,\theta+\cot-1}$$

$$=\frac{\sin\theta}{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}-1}+\frac{\cos\theta}{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}-1}$$

$$=\frac{\sin\theta}{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}-\cos\theta}+\frac{\cos\theta}{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}-\sin\theta}$$

$$=\frac{\sin\theta\,.\cos\theta}{1+\sin\theta-\cos}+\frac{\cos\theta\,.\sin\theta}{1+\cos\theta-\sin\theta}$$

$$=\frac{\sin\theta\,.\cos\theta\,(1+\cos\theta-\sin\theta)+\cos\theta\,.\sin\theta\,(1+\sin\theta-\cos\theta)}{(1+\sin\theta-\cos\theta)\,(1+\cos\theta-\sin\theta)}$$

$$=\frac{\sin\theta\,.\cos\theta+\cancel{\sin\theta\,.\cos^2\theta}-\cancel{\sin^2\theta\,.\cos\theta}+\sin\theta-\cos\theta\cancel{+\sin^2\theta\,.\cos}\,\theta\cancel{=\cos^2\theta\,.\sin}\,\theta}{(1+\sin\theta-\cos\theta)\,(1+\cos\theta-\sin\theta)}$$

$$=\frac{2\sin\theta\,.\cos\theta}{1+\cancel{\cos\theta}-\cancel{\sin\theta}+\cancel{\sin\theta}+\sin\theta\,.\cos\theta-\sin^2\theta-\cancel{\cos\theta}-\cos^2\theta+\cos\theta\,.\sin^2\theta}$$

$$=\frac{2\sin\theta\,.\cos\theta}{1-(\sin^2\theta+\cos^2\theta)+2\sin\theta\,.\cos\theta}=\frac{2\sin\theta\,.\cos\theta}{2\sin\theta\,.\cos\theta}$$

Q. 64. $\sec^4\theta\,(1-\sin^4\theta)-2\tan^2\theta=1$

Sol. $\text{LHS}=\sec^4\theta\,(1-\sin^2\theta)-2\tan^2\theta$

$=\sec^4\theta-\sec^4\theta\sin^2\theta-2\tan^2\theta$

$=(1+\tan^2\theta)^2-\dfrac{\sin^4\theta}{\cos^4\theta}-2\tan^2\theta$

$=1+\tan^4\theta+2\tan^2\theta-\tan^4\theta-2\tan^2\theta$

$=1+2\tan^2\theta-2\tan\theta$

$=1$

$=$ RHS

Q. 65. $\dfrac{\mathrm{cosec}\,\theta\times\cos\theta-\sin\theta\sec\theta}{\cos\theta+\sin\theta}=\mathrm{cosec}\,\theta-\sec\theta$

Sol. $$\text{LHS}=\frac{\mathrm{cosec}\,\theta\times\cos\theta-\sin\theta\sec\theta}{\cos\theta+\sin\theta}$$

$$=\frac{\frac{1}{\sin\theta}\times\cos\theta-\sin\theta\times\frac{1}{\cos\theta}}{\cos\theta+\sin\theta}$$

$$=\frac{\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}}{\cos\theta+\sin\theta}$$

$$=\frac{\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\,.\cos\theta}}{\cos\theta+\sin\theta}$$

$$=\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\,.\cos\theta\,(\cos\theta+\sin\theta)}$$

$$=\frac{(\cancel{\cos\theta+\sin\theta})\,(\cos\theta-\sin^2\theta)}{\sin\theta\,.\cos\theta\,(\cancel{\cos\theta-\sin\theta})}$$

$$=\frac{\cancel{\cos\theta}}{\sin\theta\,.\cancel{\cos\theta}}\quad\frac{\cancel{\sin\theta}}{\cos\theta\,.\cancel{\sin\theta}}$$

$=\mathrm{cosec}\,\theta-\sec\theta$

$=$ RHS

Q. 66. $(1+\tan\theta+\cot\theta)\sin\theta-\cos\theta=\dfrac{\sec\theta}{\mathrm{cosec}^2\,\theta}-\dfrac{\mathrm{cosec}\,\theta}{\sec^2\theta}$

Sol. $$\text{RHS}=\frac{\sec\theta}{\mathrm{cosec}^2\,\theta}-\frac{\mathrm{cosec}\,\theta}{\sec^2\theta}$$

$$=\frac{\sec^3\theta-\mathrm{cosec}^3\,\theta}{\sec^2\theta\,.\,\mathrm{cosec}^2\,\theta}$$

$$=\frac{\frac{1}{\cos^3\theta}-\frac{1}{\sin^3\theta}}{\frac{1}{\cos^2\theta}-\frac{1}{\sin^2\theta}}$$

Q. 67. $\tan^2\theta-\tan^2 B=\dfrac{\sin^2 A-\sin^2 B}{\cos^2 A\times\cos^2 B}$

Sol. $\text{LHS}=\tan^2 A-\tan^2 B$

$$=\frac{\sin^2 A}{\cos^2 A}-\frac{\sin^2 B}{\cos^2 B}$$

$$=\frac{\sin^2 A\times\cos^2 B-\sin^2 B\times\cos^2 A}{\cos^2 A\times\cos^2 B}$$

$$=\frac{\sin^2 A\,(1-\sin^2 B)-\sin^2 B\,(1-\sin^2 A)}{\cos^2 A\times\cot^2 B}$$

$$=\frac{\sin^2 A-\cancel{\sin^2 A\times\sin^2 B}-\sin^2 B+\cancel{\sin^2 B\times\sin^2 A}}{\cos^2 A\times\cos^2 B}$$

$$=\frac{\sin^2 A-\sin^2 B}{\cos^2 A\times\cos^2 B}$$

$$=\frac{\sin^2 A-\sin^2 B}{\cos^2 A\,.\cos^2 B}$$

$=$ RHS

Q. 68. $\tan^2 A\times\sec^2 B-\sec^2 A\times\tan^2 B=\tan^2 A-\tan^2 B$

Sol. $\text{LHS}=\tan^2 A\times\sec^2 B-\sec^2 A\times\tan^2 B$

$=\tan^2 A\,(1+\tan^2 B)-(1+\tan^2 A)\tan^2 B$

$=\tan^2 A+\cancel{\tan^2 A\times\tan^2 B}-\tan^2 B-\cancel{\tan^2 A\times\tan^2 B}$

$=\tan^2 A-\tan^2 B$

$=$ RHS

Q. 69. $\dfrac{1}{\mathrm{cosec}\,A-\cot A}-\dfrac{1}{\sin A}=\dfrac{1}{\sin A}-\dfrac{1}{\mathrm{cosec}\,A+\cot A}$

Sol. $$\text{LHS}=\frac{1}{\mathrm{cosec}\,A-\cot A}=\frac{1}{\sin A}$$

$$=\frac{\mathrm{cosec}^2\,A-\cot^2 A}{\mathrm{cosec}\,A-\cot A}-\mathrm{cosec}\,A$$

$$=\frac{(\mathrm{cosec}\,A+\cot A)\,(\cancel{\mathrm{cosec}\,A-\cot A})}{(\cancel{\mathrm{cosec}\,A-\cot A})}-\mathrm{cosec}\,A$$

$=\cancel{\mathrm{cosec}\,A}+\cot A-\cancel{\mathrm{cosec}\,A}$

$=\cot A$

$$\text{RHS}=\frac{1}{\sin A}-\frac{1}{\mathrm{cosec}\,A+\cot A}$$

$$=\mathrm{cosec}\,A-\frac{\mathrm{cosec}^2\,A-\cot^2 A}{\mathrm{cosec}\,A+\cot A}$$

$$=\mathrm{cosec}\,A-\frac{(\cancel{\mathrm{cosec}\,A+\cot A})\,(\mathrm{cosec}\,A-\cot A)}{(\cancel{\mathrm{cosec}\,A+\cot A})}$$

$=\mathrm{cosec}\,A-\mathrm{cosec}\,A+\cot A$

$=\cot A$

$$= \frac{\frac{\sin^3\theta - \cos^3\theta}{\sin^3\theta \,.\, \cos^3\theta}}{\frac{1}{\sin^2\theta \,.\, \cos^2\theta}}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta \,.\, \cos\theta}$$

$$= \frac{(\sin\theta - \cos\theta)\,(\sin^2\theta + \cos^2\theta + \sin^2\theta \,.\, \cos\theta)}{\sin\theta \,.\, \cos\theta}$$

$$= (\sin\theta - \cos\theta)\ \frac{\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}\,.\cos\theta} + \frac{\cos^{\cancel{2}}\theta}{\sin\theta\,.\,\cancel{\cos\theta}} + \frac{\sin\theta\,.\,\cos\theta}{\sin\theta\,.\,\cos\theta}$$

$$= (\sin\theta - \cos\theta)\,(\tan\theta + \cot\theta + 1) \qquad = \qquad \text{LHS}$$

Q. 70. $\dfrac{1}{\sec A - \tan A} - \dfrac{1}{\cos A} = \dfrac{1}{\cos A} - \dfrac{1}{\sec A + \tan A}$

Sol. $\text{LHS} = \dfrac{1}{\sec A - \tan A} = \dfrac{1}{\cos A}$

$$= \frac{1}{\sec A + \tan A} - \sec A$$

$$= \frac{\sec^2 A - \tan^2 A}{\sec A + \tan A} - \sec A$$

$$= \frac{\cancel{(\sec A + \tan A)}\,(\sec A - \tan A)}{\cancel{(\sec A + \tan A)}} - \sec A$$

$$= \cancel{\sec A} - \tan A - \cancel{\sec A}$$

$$= -\tan A$$

$$\text{RHS} = \frac{1}{\cos A} - \frac{1}{\sec A + \tan A}$$

$$= \sec A - \frac{(\sec^2 A - \tan^2 A)}{(\sec A - \tan A)}$$

$$= \sec A - \frac{(\sec A \cancel{+ \tan A)\,(\sec} A - \tan A)}{\cancel{(\sec A - \tan A)}}$$

$$= \sec A - \sec A - \tan A$$

$$= -\tan A$$

LHS = RHS

RATIONALISING FACTOR

Q. 1. $\dfrac{1}{1 - \cos\theta} = \dfrac{1 + \cos\theta}{\sin^2\theta}$

Sol. $\text{LHS} = \dfrac{1}{1 - \cos\theta}$

$$= \frac{1}{1 - \cos\theta} \times \frac{1 + \cos\theta}{1 + \cos\theta}$$

= RHS

$$= \frac{1 + \cos\theta}{1 - \cos^2\theta}$$

$$= \frac{1 + \cos\theta}{\sin^2\theta} \qquad = \qquad \text{RHS}$$

Q. 3. $\dfrac{\cos\theta}{1 + \sin\theta} = \dfrac{1 - \sin\theta}{\cos\theta}$

Sol. $\text{LHS} = \dfrac{\cos\theta}{1 + \sin\theta}$

$$= \frac{\cos\theta}{1 + \sin\theta} \times \frac{1 - \sin\theta}{1 - \sin\theta}$$

$$= \frac{\cos\theta\,(1 - \sin\theta)}{1 - \sin^2\theta}$$

$$= \frac{\cancel{\cos\theta}\,(1 - \sin\theta)}{\cos^{\cancel{2}}\theta}$$

$$= \frac{1 - \sin\theta}{\cos\theta}$$

= RHS

Q. 2. $\dfrac{\cos\theta}{1 - \sin\theta} = \dfrac{1 + \sin\theta}{\cos\theta}$

Sol. $\text{LHS} = \dfrac{\cos\theta}{1 - \sin^2\theta}$

$$= \frac{\cos\theta}{1 - \sin\theta} \times \frac{1 + \sin\theta}{1 + \sin\theta} = \frac{\cos\theta\,(1 + \sin\theta)}{1 - \sin^2\theta}$$

$$= \frac{\sin\theta\,(1 + \sin\theta)}{\cos^2\theta}$$

$$= \frac{1 + \sin\theta}{\cos\theta} = \text{RHS}$$

Q. 4. $\dfrac{1 + \sec\theta}{\sec A} = \dfrac{\sin^2\theta}{1 - \cos\theta}$

Sol. $\text{LHS} = \dfrac{1 + \sec\theta}{\sec\theta}$

$$= \frac{1 + 1/\cos\theta}{1/\cos\theta}$$

$$= \frac{\frac{\cos\theta + 1}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\cancel{\cos\theta}\,(\cos\theta + 1)}{\cancel{\cos\theta}}$$

$$= (\cos\theta + 1) \times \frac{1 - \cos\theta}{1 - \cos\theta}$$

$$= \frac{1 - \cos^2\theta}{1 - \cos\theta} = \frac{\sin^2\theta}{1 - \cos\theta} = \text{RHS}$$

Q. 5. $(\text{cosec} + \cot\theta)^2 = \frac{1+\cos\theta}{1-\cos\theta}$

Sol. LHS = $(\text{cosec}\,\theta + \cot\theta)^2$

$$= \frac{1+\cos\theta}{\sin\theta}^{2}$$

$$= \frac{(1+\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1+\cos\theta)\,(1+\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{\cancel{(1+\cos^2\theta)}\,(1+\cos\theta)}{\cancel{(1+\cos^2\theta)}\,(1-\cos\theta)}$$

$$= \frac{1+\cos\theta}{1-\cos\theta} = \text{RHS}$$

Q. 6. $\frac{1+\sin\theta}{1-\sin\theta} = (\tan\theta + \sec\theta)^2$

Sol. LHS = $\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$

$$= \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{(1+\sin\theta)^2}{(\cos\theta)^2}$$

$$= \left(\frac{1}{\cos^2\theta} + \frac{\sin\theta}{\cos^2\theta}\right)^2$$

$$= (\sec\theta + \tan\theta)^2$$

= RHS

Q. 7. $\frac{1-\cos\theta}{1+\cos\theta} = (\text{cosec}\,\theta - \cot\theta)^2$

Sol. LHS = $\frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos)^2}{\sin^2\theta}$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = (\text{cosec}\,\theta - \cot\theta)^2 = \text{RHS}$$

Q. 8. $\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$

Sol. LHS = $\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1-\sin^2\theta)^2}{(\cos\theta)^2}$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= (\sec\theta - \tan\theta)^2$$

= RHS

Q. 9. $\frac{1}{\sec\theta + \tan\theta} = \frac{1-\sin\theta}{\cos\theta}$

Sol. LHS = $\frac{1}{\sec\theta + \tan\theta}$

$$= \frac{1}{\sec\theta + \tan\theta} \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta}$$

$$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$$= \frac{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{1} = \frac{1-\sin\theta}{\cos\theta}$$

Q. 10. $\frac{1}{\sec\theta + \tan\theta} = \sec\theta + \tan\theta$

Sol. LHS $\frac{1}{\sec\theta + \tan\theta}$

$$= \frac{1}{\sec\theta + \tan\theta} \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$$

$$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$$= \frac{\sec\theta + \tan\theta}{1} = \sec\theta + \tan\theta$$

Q. 11. $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = (\sec\theta + \tan\theta)^2$

Sol. LHS = $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta}$

$$= \frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$$

$$= \frac{(\sec\theta + \tan\theta)^2}{\sec^2\theta - \tan\theta}$$

$$= (\sec\theta + \tan\theta)^2$$

= RHS

Q. 12. $\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = \frac{\cos^2\theta}{(1+\sin\theta)^2}$

Sol. LHS = $\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}$

$$= \frac{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{1-\sin\theta}{\cancel{\cos\theta}}}{\frac{1+\sin\theta}{\cancel{\cos\theta}}}$$

$$= \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta} = \frac{1-\sin^2\theta}{(1+\sin\theta)^2}$$

$$= \frac{\cos^2\theta}{(1+\sin\theta)^2} = \text{RHS}$$

Q. 13. $\frac{\text{cosec}\,\theta + \cot\theta}{\text{cosec}\,\theta - \cot\theta} = (\text{cosec}\,\theta + \cot\theta)^2$

Sol. LHS = $\frac{\text{cosec}\,\theta + \cot\theta}{\text{cosec}\,\theta - \cot\theta} \times \frac{\text{cosec} + \cot\theta}{\text{cosec}\,\theta + \cot\theta}$

$$= \frac{(\text{cosec}\,\theta + \cot\theta)^2}{\text{cosec}^2\theta - \cot^2\theta}$$

$$= (\text{cosec}\,\theta + \cot\theta)^2$$

= RHS

Q. 14. $\frac{1+\cos\theta}{1-\cos\theta} = \frac{\tan^2\theta}{(\sec^2\theta - 1)^2}$

Sol. LHS = $\frac{1+\cos\theta}{1-\cos\theta}$

$$= \frac{1 + 1/\sec\theta}{1 - 1/\sec\theta}$$

$$= \frac{\frac{\sec\theta + 1}{\cancel{\sec\theta}}}{\frac{\sec\theta - 1}{\cancel{\sec\theta}}}$$

$$= \frac{\sec\theta + 1}{\sec\theta - 1} \times \frac{\sec\theta - 1}{\sec\theta - 1} = \frac{\sec^2\theta - 1}{(\sec\theta - 1)^2} = \frac{\tan^2\theta}{(\sec\theta - 1)^2} = \text{RHS}$$

Q. 15. $\frac{1+\sin\theta}{1-\sin\theta} = 1 + \frac{2\tan\theta}{\cos\theta} + 2\tan^2\theta$

Sol. $\text{LHS} = \frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$

$= \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$

$= \frac{(1+\sin\theta)^2}{\cos^2\theta}$

$= \frac{1+\sin^2\theta+2\sin\theta}{\cos^2\theta}$

$= \frac{\sin^2\theta+\cos^2\theta+\sin^2\theta+2\sin\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta+2\sin^2\theta+2\sin\theta}{\cos^2\theta}$

$= \frac{\cancel{\cos^2\theta}}{\cancel{\cos^2\theta}} + \frac{2\sin^2\theta}{\cos^{\cancel{2}}\theta} + \frac{2\sin\theta}{\cos\times\cancel{\cos\theta}}$

$= 1 + 2\tan^2\theta + \frac{2\tan\theta}{\cos\theta}$

Q. 16. $\frac{1+\cos\theta}{1-\cos\theta} = \frac{2-\sin^2\theta+2\cos\theta}{\sin^2\theta}$

Sol. $\text{LHS} = \frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos^2\theta}{1+\cos\theta}$

$= \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$

$= \frac{1+\cos^2\theta+2\cos\theta}{\sin^2\theta}$

$= \frac{1+1-\sin^2\theta+2\cos\theta}{\sin^2\theta}$

$= \frac{2-\sin^2\theta+2\cos\theta}{\sin^2\theta}$

= RHS

Q. 17. $\frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$

Sol. $\text{LHS} = \frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta}$

Dividing Numerator and Denominator by cos θ

$\frac{\frac{\sin\theta+1-\cos^2\theta}{\cos\theta}}{\frac{\cos\theta-1+\sin\theta}{\cos\theta}}$

$= \frac{\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}-\frac{\cos\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}-\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}$

$= \frac{\tan\theta+\sec\theta-1}{1-\sec\theta+\tan\theta}$

$= \frac{\tan\theta+\sec\theta-(\sec^2\theta-\tan^2\theta)}{1-\sec\theta+\tan\theta}$

$= \frac{(\tan\theta+\sec\theta)\cancel{(1-\sec\theta+\tan\theta)}}{\cancel{(1-\sec\theta+\tan\theta)}}$

$= \frac{\sec\theta}{\cos\theta}+\frac{1}{\cos\theta} \quad = \quad \frac{1+\sin\theta}{\cos\theta} = \text{RHS}$

Q. 18. $\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$

Sol. $\text{LHS} = \frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$

Dividing Numerator and Denominator cos θ

$= \frac{\frac{1}{\cos\theta}+\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}+\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\sec\theta+1+\tan\theta}{\sec\theta+1-\tan\theta}$

$= \frac{(\sec\theta+\tan\theta)+(\sec^2\theta-\tan^2\theta)}{(\sec\theta+1-\tan\theta)}$

$= \frac{(\sec\theta+\tan\theta)\cancel{(1+\sec\theta-\tan\theta)}}{\cancel{(\sec\theta+1-\tan\theta)}}$

$= \frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}$

$= \frac{1+\sin\theta}{\cos\theta} = \quad \text{RHS}$

PROBLEMS BASED ON SQUARE ROOT

Q. 1. $\sqrt{\frac{1-\cos^2\theta}{1-\sin^2\theta}} = \tan\theta$

Sol. $\text{LHS} = \sqrt{\frac{1-\cos^2\theta}{1-\sin^2\theta}}$

$= \frac{\sqrt{1-\cos^2\theta}\times\sqrt{1+\sin^2\theta}}{\sqrt{1-\sin^2\theta}\ \sqrt{1+\sin^2\theta}}$

$= \frac{\sqrt{\sin^2\theta}\times\sqrt{1+\sin^2\theta}}{\sqrt{\cos^2\theta}\times\sqrt{1-\sin^2\theta}}$

$= \frac{\sin\theta\times\sqrt{1+\cancel{\sin^2\theta}}}{\cos\theta\ \sqrt{1+\cancel{\sin^2\theta}}} \quad = \quad \tan\theta$

Q. 2. $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta-\tan\theta$

Sol. $\text{LHS} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$= \frac{\sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta}} \times \frac{\sqrt{1-\sin\theta}}{\sqrt{1-\sin\theta}}$

$= \frac{\sqrt{(1-\sin\theta)^2}}{\sqrt{1-\sin^2\theta}}$

$= \frac{1-\sin\theta}{\sqrt{\cos^2\theta}} \quad = \quad \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta} \quad =\sec\theta-\tan\theta$

Q. 3. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta+\tan\theta$

Sol. $\text{LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} \Rightarrow \frac{\sqrt{1+\sin\theta}\times\sqrt{1+\sin\theta}}{\sqrt{1+\sin\theta}\ \sqrt{1+\sin\theta}}$

$= \frac{\sqrt{(1+\sin\theta)^2}}{\sqrt{1-\sin^2\theta}} \Rightarrow \frac{1+\sin\theta}{\sqrt{\cos^2\theta}} \Rightarrow \frac{1+\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta} \Rightarrow \sec\theta+\tan\theta$

Q. 4. $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta-\cot\theta$

Sol. $\text{LHS} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sqrt{1+\cos\theta}\times\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}\ \sqrt{1-\cos\theta}}$

$= \frac{\sqrt{(1-\cos\theta)^2}}{\sqrt{1-\cos^2\theta}} = \frac{1-\cos\theta}{\sqrt{\sin^2\theta}} \Rightarrow \frac{1-\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta} = \operatorname{cosec}\theta-\cot\theta$

Q. 5. $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$

Sol. LHS = $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$ => $\frac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}} \times \frac{\sqrt{1+\cos\theta}}{\sqrt{1+\cos\theta}}$

= $\frac{\sqrt{(1+\cos\theta)^2}}{\sqrt{1-\cos^2\theta}}$ => $\frac{1+\cos\theta}{\sqrt{\sin^2\theta}}$ => $\frac{1+\cos\theta}{\sin\theta}$

= $\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$ => $\operatorname{cosec}\theta + \cot\theta$

Q. 6. $\sqrt{\frac{1+\sin^2\theta\times\sec^2\theta}{1+\cos^2\theta\times\operatorname{cosec}^2\theta}} = \tan\theta$

Sol. LHS = $\sqrt{\frac{1+\sin^2\theta\times\sec^2\theta}{1+\cos^2\theta\times\sec^2\theta}}$

= $\frac{\sqrt{1+\sin^2\theta\times 1/\cos^2\theta}}{\sqrt{1+\cos^2\theta\times 1/\sin^2\theta}}$

= $\frac{\sqrt{1+\tan^2\theta}}{\sqrt{1+\cot^2\theta}} \times \frac{\cancel{\sqrt{1-\cot^2\theta}}}{\cancel{\sqrt{1-\cot^2\theta}}}$

= $\frac{\sqrt{\sec^2\theta}}{\sqrt{\operatorname{cosec}^2\theta}}$ => $\frac{\sec\theta}{\operatorname{cosec}\theta}$ => $\frac{1}{\cos\theta} \times \sin\theta$ $= \tan\theta$

Q. 7. $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{\tan\theta+\sin\theta}{\tan\theta\times\sin\theta}$

Sol. LHS = $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$ => $\frac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}} \times \frac{\sqrt{1+\cos\theta}}{\sqrt{1+\cos\theta}}$

= $\frac{\sqrt{(1+\cos\theta)^2}}{\sqrt{(1-\cos\theta)^2}}$ => $\frac{1+\cos\theta}{\sqrt{\sin^2\theta}}$ => $\frac{1+\cos\theta}{\sin\theta}$

= $\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$

= $\frac{1}{\sin\theta} + \cot\theta$

= $\frac{\tan\theta+\sin\theta}{\tan\theta\,.\,\sin\theta}$ LHS = RHS

Q. 8. $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$

Sol. LHS = $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}$

= $\sqrt{1+\tan^2\theta+1+\cot^2\theta}$

= $\sqrt{\tan^2\theta+\cot^2\theta+2}$

= $\sqrt{(\tan\theta+\cot\theta)^2}$

= $\tan\theta+\cot\theta$

∴ LHS = RHS

Q. 9. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \frac{\sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta}} = 2\sec\theta$

Sol. LHS = $\frac{\sqrt{1+\sin\theta}}{\sqrt{1-\sin\theta}} + \frac{\sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta}}$

= $\frac{\sqrt{(1-\sin\theta)^2}+\sqrt{(1-\sin\theta)^2}}{\sqrt{1-\sin^2\theta}}$

= $\frac{1+\sin\theta+1-\sin\theta}{\sqrt{\cos^2\theta}}$

= $\frac{2}{\cos\theta}$ => $2\sec\theta$

LHS = RHS

Q. 10. $\frac{\sqrt{\sec\theta-1}}{\sqrt{\sec\theta+1}} + \frac{\sqrt{\sec\theta+1}}{\sqrt{\sec\theta-1}} = 2\operatorname{cosec}\theta$

Sol. LHS = $\frac{\sqrt{\sec\theta-1}}{\sqrt{\sec\theta+1}} + \frac{\sqrt{\sec\theta+1}}{\sqrt{\sec\theta-1}}$

= $\frac{\sqrt{(\sec\theta-1)^2}+\sqrt{(\sec\theta+1)^2}}{\sqrt{\sec^2\theta-1}}$

= $\frac{\sec\theta\cancel{-1}+\sec\theta\cancel{+1}}{\sqrt{\tan^2\theta}}$

= $\frac{2\sec\theta}{\tan\theta}$

= $2\,.\,\frac{1}{\cos\theta}\times\frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$ ∴ LHS = RHS

Q. 11. $\operatorname{Cosec}\theta\,.\,\sqrt{1-\cos^2\theta} = 1$

Sol. LHS = $\operatorname{cosec}\theta\sqrt{1-\cos^2\theta}$

= $\operatorname{cosec}\theta\,.\,\sqrt{\sin^2\theta}$

= $\cancel{\operatorname{cosec}\theta}\,.\,\frac{1}{\cancel{\operatorname{cosec}\theta}} = 1$

∴ LHS = RHS

Q. 12. $\sec\theta[\sqrt{1-\sin^2\theta}] = 1$

Sol. LHS = $\sec\theta\sqrt{\cos^2\theta}$

= $\frac{1}{\cos\theta}\times\cos\theta$

= $\cancel{1}$ ∴ LHS = RHS

Q. 13. $\sqrt{\operatorname{cosec}^2\theta-1} = \cos\theta\times\operatorname{cosec}\theta$

Sol. LHS = $\sqrt{\operatorname{cosec}^2\theta-1}$

= $\sqrt{\cot^2}$

= $\cot\theta$

= $\frac{\cos\theta}{\sin\theta}$

Q. 14. ***$\frac{\sqrt{1+\sin\theta}}{\sqrt{1-\sin\theta}}=\frac{\cos\theta}{1-\sin\theta}$***

Sol. LHS = $\frac{\sqrt{1+\sin\theta}}{\sqrt{1-\sin\theta}}\times\frac{\sqrt{1-\sin\theta}}{\sqrt{1-\sin\theta}}$

= $\frac{\sqrt{(1-\sin^2\theta}}{\sqrt{(1-\sin^2\theta}}$ => $\frac{\sqrt{\cos\theta}}{1-\sin\theta}$

= $\frac{\cos\theta}{1-\sin\theta}$

∴ LHS = RHS

Q. 15. ***$\frac{\sqrt{1+\sin\theta}}{\sqrt{1+\sin\theta}}=\frac{\sin\theta}{1+\sin\theta}$***

Sol. LHS = $\frac{\sqrt{1+\cos\theta}}{\sqrt{1+\cos\theta}}\times\frac{\sqrt{1+\cos\theta}}{\sqrt{1+\cos\theta}}$

= $\frac{\sqrt{1-\cos^2\theta}}{\sqrt{(1+\cos\theta)^2}}$

= $\frac{\sqrt{\sin^2\theta}}{1+\cos\theta}$ => $\frac{\sin\theta}{1+\cos\theta}$

∴ LHS = RHS

Q. 16. ***$\sqrt{\cot^2\theta-\cot^2\theta}=\cos\theta\times\cot\theta$***

Sol. LHS = $\sqrt{\cot^2\theta-\cos^2\theta}$

= $\sqrt{\frac{\cos^2\theta}{\sin^2\theta}-\cos^2\theta}$

= $\sqrt{\frac{\cos^2\theta}{\sin^2\theta}-\cos^2\theta}$

= $\sqrt{\frac{\cos^2\theta-\cos^2\theta-\sin^2\theta}{\sin^2\theta}}$

= $\sqrt{\frac{\cos^2\theta}{\sin^2\theta}\ (1-\sin^2\theta)}$

= $\sqrt{\cot^2\theta\,.\,\cos^2\theta}$

= $\cot\theta\,.\,\cos^2\theta$ ∴ LHS = RHS

Q. 17. ***$\sin\theta=\sqrt{2}\cos\theta-\cos\theta$***

Sol. LHS = $\sin\theta=\cos\theta\,(\sqrt{2}-1)$

=> $\frac{\sin\theta}{(\sqrt{2}-1)}=\cos\theta$

=> $\frac{\sin\theta}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}=\cos\theta$

=> $\frac{\sqrt{2}\sin+\sin\theta}{2-1}=\cos\theta$

=> $\sqrt{2}\sin\theta+\sin\theta=\cos\theta$

=> $\sqrt{2}\sin\theta=\cos\theta-\sin\theta$

∴ LHS = RHS

SET VII

Q. 1. ***If $x=r\sin\theta\cos\phi$, $y=r\sin\theta\,.\,\sin\phi$ and $z=r\cos\theta$. Then prove that $x^2+y^2+z^2=r^2$***

Sol. LHS = $x^2+y^2+z^2$

= $r^2\sin^2\theta\cos^2\phi+r^2\sin^2\theta\sin^2\phi+r^2\cos^2\theta$ = $r^2\sin^2\theta(\cos^2\phi+\sin^2\phi)+r^2\cos^2\theta$

= $r^2\sin^2\theta+r^2\cos^2\theta$ = $r^2(\sin^2\theta+\cos^2\theta)$

= $r^2\times1$ = r^2 = RHS

Q. 2. ***If $\tan\theta+\sin\theta=m$ and $\tan\theta-\sin\theta=n$. Show that $m^2-n^2=4\sqrt{mn}$***

Sol. LHS = m^2-n^2

= $(\tan\theta+\sin\theta)^2-(\tan\theta-\sin\theta)^2$

= $\cancel{\tan^2\theta}+\cancel{\sin^2\theta}+2\tan\theta\,.\,\sin\theta-\cancel{\tan^2\theta}-\cancel{\sin^2\theta}+2\tan\theta\,.\,\sin\theta$ = $4\tan\theta\,.\,\sin\theta$

RHS = $4\sqrt{mn}$

= $4\sqrt{(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)}$

= $4\sqrt{\tan^2\theta-\sin^2\theta}$

= $4\sqrt{\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta}$

= $4\sqrt{\sin^2\theta\left(\frac{1}{\cos^2\theta}-1\right)}$

= $4\sqrt{\sin^2\theta\times\frac{1-\sin^2\theta}{\cos^2\theta}}$

= $4\sin\theta\sqrt{\frac{\sin^2\theta}{\cos\theta}}$ = $4\sin\theta\sqrt{\tan^2\theta}$ = $4\sin\theta\times\tan\theta$

Q. 3. ***If $\tan\theta+\sin\theta=m$ and $\tan\theta-\sin\theta=n$ then prove that $(m^2-n^2)^2=16\,mn$.***

Sol. LHS = $(m^2-n^2)^2$

= $[(\tan\theta+\sin\theta)^2-(\tan\theta-\sin^2\theta)^2]^2$

= $[\cancel{\tan^2\theta}+\cancel{\sin^2\theta}+2\tan\theta\,.\,\sin\theta-\cancel{\tan^2\theta}-\cancel{\sin^2\theta}+2\tan\theta\,.\,\sin\theta]^2$

= $[4\tan\theta\,.\,\sin\theta]^2$

= $16\tan^2\theta\,.\,\sin^2\theta$

RHS = 16 mn

= $16(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)$ = $16(\tan^2\theta-\sin^2\theta)$

= $16\left(\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta\right)=16\left(\frac{\sin^2\theta-\sin^2\theta\,.\,\cos^2\theta}{\cos^2\theta}\right)$

= $16\,\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}=$ $16\,\frac{\sin^2\theta}{\cos^2\theta}\times(1-\cos^2\theta)$ = $16\tan^2\theta\times\sin^2\theta$ ∴ LHS = RHS

Q. 4. ***If $\sin\theta+\cos\theta=p=0\sec\theta+\mathrm{cosec}\,\theta=q$, Show that $q(p^2-1)=2p$***

Sol. LHS = $q(p^2-1)$

= $(\sec\theta+\mathrm{cosec}\,\theta)[(\sin\theta+\cos\theta)^2-1]$

= $\left(\frac{1}{\cos\theta}+\frac{1}{\sin\theta}\right)(\sin^2\theta+\cos^2\theta+2\sin^2\theta\,.\,\cos\theta-1)$

= $\left(\frac{\sin\theta+\cos\theta}{\sin\theta\,.\,\cos\theta}\right)(1+2\sin\theta\,.\,\cos\theta-1)$

= $\frac{\sin\theta+\cos\theta}{\sin\theta\,.\,\cos\theta}\times2\sin\theta\,\cancel{.\cos\theta}$ = $2\times p$ = $2\,p$ = RHS

Q. 5. ***If $x = a \sin\theta$, $y = b \tan\theta$ then prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.***

Sol. LHS $= \frac{a^2}{x^2} - \frac{b^2}{y^2}$

$= \frac{a^2}{a^2 \sin\theta} - \frac{b^2}{b^2 \tan^2\theta} = \operatorname{cosec}^2\theta - \cot^2\theta = 1 = \text{RHS}$

Q. 6. ***If $\sec\theta + \tan\theta = p$. Show that $\frac{P^2 - 1}{p^2 + 1} = \sin\theta$***

Sol. LHS $= \frac{p^2 - 1}{p^2 + 1}$

$= \frac{(\sec\theta + \tan\theta)^2 - 1}{(\sec\theta + \tan\theta)^2 + 1}$

$= \frac{\sec^2\theta + \tan^2\theta + 2\sec\theta\,.\,\tan\theta - 1}{\sec^2\theta + \tan^2\theta + 2\sec\theta\,.\,\tan\theta + 1}$

$= \frac{\tan^2\theta + \tan^2\theta + 2\sec\theta\,.\,\tan\theta}{\sec^2\theta + \sec^2\theta + 2\sec\,.\,\tan\theta}$

$= \frac{2\tan^2\theta + 2\sec\theta\,.\,\tan\theta}{2\sec^2\theta + 2\sec\theta\,.\,\tan\theta}$

$= \frac{\cancel{2}\tan\theta\,\cancel{(\tan\theta + \sec\theta)}}{\cancel{2}\sec\theta\,\cancel{(\sec\theta + \tan\theta)}}$

$= \frac{\tan\theta}{\sec\theta}$

$\frac{\frac{\sin\theta}{\cancel{\cos\theta}}}{\frac{1}{\cancel{\cos\theta}}}$

$=$ RHS

Q. 7. ***If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\beta}{\sin\beta} = n$,***

show that $(m^2 + n^2)\cos^2\beta = n^2$

Sol. LHS $= (m^2 + n^2)\cos^2\beta$

$= \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\beta}{\sin^2\beta}\right)\cos^2\beta$

$= \cos^2\alpha\left(\frac{\sin^2\beta + \cos^2\beta}{\cancel{\cos^2\beta}\sin^2\beta°}\right)\cancel{\cos^2\beta}$

$= \cos^2\alpha \times \frac{1}{\sin^2\beta}$

$= \frac{\cos^2\alpha}{\sin^2\beta}$

$= \left(\frac{\cos\alpha}{\sin\beta}\right)^2$

$= n^2$

$=$ RHS

Q. 8. ***If $\operatorname{cosec}\theta - \sin\theta = l$ and $\sec\theta - \cos\theta = m$, Prove that $l^2 m^2 (l^2 m^2 + 3) = 1$***

Sol. LHS $= l^2 m^2 (l^2 + m^2 + 3)$

$= (\operatorname{cosec}\theta - \sin\theta)^2 (\sec\theta - \cos\theta)^2 [(\operatorname{cosec}\theta - \sin\theta)^2 + (\sec\theta - \cos\theta)^2 + 3]$

$= \left(\frac{1}{\sin\theta} - \sin\theta\right)^2 \left(\frac{1}{\cos\theta} - \cos\theta\right)^2 \left[\left(\frac{1}{\sin\theta} - \sin\theta\right)^2 + \left(\frac{1}{\cos\theta} - \cos\theta\right)^2 + 3\right]$

$= \left(\frac{1 - \sin^2\theta}{\sin\theta}\right)^2 \left(\frac{1 - \cos^2\theta}{\cos\theta}\right)^2 \left[\left(\frac{1 - \sin^2\theta}{\sin\theta}\right)^2 + \left(\frac{1 - \cos^2\theta}{\cos\theta}\right)^2 + 3\right]$

$= \frac{\cos^{\cancel{4}\;2}\theta}{\sin^2\theta} \times \frac{\sin^{\cancel{4}\;2}\theta}{\cos^2\theta}\left[\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4\theta}{\cos^2\theta} + 3\right]$

$= \cancel{\sin^2\theta} \times \cancel{\cos^2\theta}\left[\frac{\cos^6\theta + \sin^6\theta + 3\sin^2\theta\,.\,\cos^2\theta}{\cancel{\sin^2\theta}\,.\,\cancel{\cos^2\theta}}\right]$

$= \cos^6\theta + \sin^6\theta + 3\sin^2\theta\,.\,\cos^2\theta = [(\cos^6\theta)^3 + (\sin^2\theta)^3] + 3\sin^2\theta\,.\,\cos^2\theta$

$= [\cos^2\theta + \sin^2\theta]^3 - 3\cos^2\theta\,.\,\sin^2\theta(\cos^2\theta + \sin^2\theta) + 3\sin^2\theta\,.\,\cos^2\theta)$

$= 1 - 3\cos^2\theta\,.\,\sin^2\theta \times 1 + 3\sin^2\theta\,.\,\cos^2\theta = 1 = \text{RHS}$

Q. 9. ***If $\sin\theta + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^2\theta = 1$***

Sol. We have,

$\sin\theta + \sin^2\theta = 1$

$\sin\theta = 1 - \sin^2\theta$

$\sin\theta = \cos^2\theta$

LHS $= \cos^4\theta + \cos^2\theta$

$= (\cos^2\theta)^2 + \cos^2\theta$

$= (\sin\theta)^2 + \cos^2\theta$

$= \sin^2\theta + \cos^2\theta$

$=$ 1 RHS

Q. 10. ***If $x\sin^2\theta + y\cos^3\theta = \sin\theta\,.\,\cos\theta$ and $x\sin\theta = y\cos\theta$. Prove that $x^2 + y^2 = 1$***

Sol. We have,

$x\sin^3\theta + y\cos^3\theta = \sin\theta\,.\,\cos\theta$

$= x\sin\theta(\sin^2\theta) + y\cos\theta(\cos^2\theta) = \sin\theta\,.\,\cos\theta$

$= x\sin\theta(\sin^2\theta) + x\sin\theta(\cos^2\theta) = \sin\theta\,.\,\cos\theta$

$= x\cancel{\sin\theta}(\sin^2\theta + \cos^2\theta) = \cancel{\sin\theta}\,.\,\cos\theta$

$= x = \cos\theta$ ... (i)

Again,

$x\sin\theta = y\cos\theta$

$\cancel{\cos\theta} \times \sin\theta = y\,\cancel{\cos\theta}$

$y = \sin\theta$

Now, RHS $= x^2 + y^2$

$= (\cos^2\theta + \sin^2\theta)$

$=$ 1 RHS

Q. 11. ***If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$ then prove that $x^2 - y^2 = a^2 - b^2$.***

Sol. LHS $= x^2 - y^2$

$= (a^2\sec^2\theta + b\tan\theta)^2 - (a\tan\theta + b\sec\theta)^2$

$= a^2\sec^2\theta + b^2\tan^2\theta + 2\,ab\sec\theta\tan\theta - (a^2\tan^2\theta + b^2\sec^2\theta + 2\,ab\sec\theta\,.\,\tan\theta)$

$= a^2\sec^2\theta + b^2\tan^2\theta + \cancel{2\,ab\,.\sec\theta + \tan\theta} - a^2\tan^2\theta - b^2\sec^2\theta - \cancel{2ab\,.\sec\theta\,.\tan\theta}$

$= (a^2\sec^2\theta - a^2\tan^2\theta) + (b^2\tan^2\theta - b^2\sec^2\theta)$

$= a^2(\sec^2\theta - \tan^2\theta) + b^2(\tan^2\theta - \sec^2\theta)$

$= a^2 \times 1 + b^2 \times \{-1\}$

$= a^2 - b^2 = \text{RHS}$

Q. 12. *If $x = a\cos^3\theta$ and $y = b\sin^3\theta$ then prove that $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$*

Sol. LHS = $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3}$

$= \left(\frac{a\cos^3\theta}{a}\right)^{2/3} + \left(\frac{b\sin^3\theta}{b}\right)^{2/3}$

$= (\cos^3\theta)^{2/3} + (\sin^3\theta)^{2/3}$

$= \cos^{\cancel{3}\times 2/\cancel{3}}\theta + \sin^{\cancel{3}\times 2/\cancel{3}}\theta \quad = \quad \cos^2\theta + \sin^2\theta = 1$ RHS

Q. 13. *If $\sec\theta + \tan\theta = m$ and $\sec\theta - \tan\theta = n$ then prove that $mn = 1$*

Sol. LHS = mn

$= (\sec\theta + \tan\theta)(\sec\theta + \tan\theta) \quad = \quad \sec^2\theta - \tan^2\theta = 1$ RHS

Q. 14. *If $\sin\theta + \cos\theta = a$ and $\sin\theta - \cos\theta = b$ then prove that $a^2 + b^2 = 2$*

Sol. LHS = $a^2 + b^2$

$= (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

$= \sin^2\theta + \cos^2\theta + \cancel{2\sin\theta.\cos\theta} + \sin^2\theta + \cos^2\theta - \cancel{2\sin\theta.\cos\theta} \quad = \quad 1 + 1 \quad = \quad 2$ RHS

Q. 15. *If $\cot\theta + \tan\theta = x$ and $\sec\theta - \cos\theta = y$ then prove that $(x^2y)^{2/3} - (xy^2)^{2/3} = 1$*

Sol. We have,

$x = \cot\theta + \tan\theta$

$x = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$

$x = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta}$

$x = \frac{1}{\sin\theta.\cos\theta}$

Also, $y = \sec\theta - \cos\theta$

$y = \frac{1}{\cos\theta} - \cos\theta$

$y = \frac{1 - \cos^2\theta}{\cos\theta}$

$y = \frac{\sin^2\theta}{\cos\theta}$

Now, LHS = $(x^2y)^{2/3} - (xy^2)^{2/3}$

$= \left[\left(\frac{1}{\sin\theta.\cos\theta}\right)^2\left(\frac{\sin^2\theta}{\cos\theta}\right)\right]^{2/3} - \left[\left(\frac{\sin^2\theta}{\cos\theta}\right)^2\left(\frac{1}{\sin\theta.\cos\theta}\right)\right]^{2/3}$

$= \left[\left(\frac{1}{\sin\theta.\cos\theta} \times \frac{\cancel{\sin^2\theta}}{\cos\theta}\right)\right]^{2/3} - \left[\frac{\sin^{\cancel{4}\ 3}\theta}{\cos^2\theta} \times \frac{1}{\cancel{\sin\theta}.\cos\theta}\right]^{2/3}$

$= \left(\frac{1}{\cos^3\theta}\right)^{2/3} - \left(\frac{\sin^3\theta}{\cos^3\theta}\right)^{2/3}$

$= (\sec^3\theta)^{2/3} - (\tan^3\theta)^{2/3}$

$= \sec^{\cancel{3}\times 2/\cancel{3}}\theta - \tan^{\cancel{3}\times 2/\cancel{3}}\theta$

$= \sec^2\theta - \tan^2\theta$

$= 1$ RHS

Q. 16. *If $\mathrm{cosec}\,\theta - \sin\theta = m$ and $\sec\theta - \cos\theta = x$ then prove that $(m^2n)^{2/3} + (mn^2)^{2/3} = 1$*

Sol. Where,

$m = \frac{1}{\sin\theta} - \sin\theta$

$m = \frac{1 - \sin^2\theta}{\sin\theta}$

$m = \frac{\cos^2\theta}{\sin\theta}$

Also, $n = \frac{1}{\cos\theta} - \cos\theta$

$n = \frac{1 - \cos^2\theta}{\cos\theta}$

$n = \frac{\sin^2\theta}{\cos\theta}$

Now,

LHS = $(m^2n)^{2/3} + (mn^2)^{2/3}$

$= \left[\left(\frac{\cos^2\theta}{\sin\theta}\right)^2 \times \frac{\sin^2\theta}{\cos\theta}\right]^{2/3} + \left[\frac{\cos^2\theta}{\sin\theta} \times \left(\frac{\sin^2\theta}{\cos\theta}\right)^2\right]^{2/3}$

$= \left[\frac{\cos^{\cancel{4}\ 3}\theta}{\cancel{\sin^2\theta}} \times \frac{\cancel{\sin^2\theta}}{\cos\theta}\right]^{2/3} + \left[\frac{\cancel{\cos^2\theta}}{\cancel{\sin^2\theta}} \times \frac{\sin^{\cancel{4}\ 3}\theta}{\cancel{\cos^2\theta}}\right]^{2/3}$

$= (\cos^3\theta)^{2/3} + (\sin^3\theta)^{2/3} \quad = \quad \cos^{\cancel{3}\times 2/\cancel{3}}\theta + \sin^{\cancel{3}\times 2/\cancel{3}}\theta = \cos^2\theta + \sin^2\theta \quad = \quad 1$ RHS

Q. 17. *If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$ then prove that $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$*

Sol. $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ ... (i)

=> $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$ ... (ii)

Adding equation (i) and (ii), we get

=> $\left[\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right] + \left[\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right] = (1)^2 + (1)^2$

Ag. On squaring

=> $\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + \cancel{2\frac{xy}{ab}\sin\theta\ \cos\theta} + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - \cancel{2\frac{xy}{ab}\sin\theta\ \cos\theta} = 1 + 1$

=> $\frac{x^2}{a^2}(\cos^2\theta + \sin^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 2$

=> $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

Q. 18. ***If cosec $\theta - \sin\theta = a^3$ and sec $\theta - \cos\theta = b^3$ then prove that $a^2 b^2 (a^2 + b^2) = 1$***

Sol. We have, $a^3 = \operatorname{cosec}\theta - \sin\theta$

$$a = \left[\frac{1}{\sin\theta} - \sin\theta\right]^{1/3}$$

$$a = \left[\frac{1 - \sin^2\theta}{\sin\theta}\right]^{1/3}$$

Squaring both sides

$$a^2 = \left[\frac{\cos^2\theta}{\sin\theta}\right]^{2/3}$$

Also, $b^3 = \sec\theta - \cos\theta$

$$b = \frac{1}{\cos\theta} - \cos\theta^{\,1/3}$$

$$b = \left[\frac{1 - \cos^2\theta}{\cos\theta}\right]^{1/3}$$

Squaring both sides

$$b^2 = \left[\frac{\sin^2\theta}{\cos\theta}\right]^{2/3}$$

LHS = $a^2 b^2 (a^2 + b^2)$

$$= \frac{\cos^{4/3}\theta}{\sin^{2/3}\theta} \times \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\left[\frac{\cos^{4/3}\theta}{\sin^{2/3}\theta} + \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\right]$$

$$= \cos^{4/3-2/3} \times \sin^{4/3-2/3}\theta\left[\frac{\cos^{4/3+2/3}\theta + \sin^{4/3+2/3}\theta}{\sin^{2/3}\theta\cos^{2/3}\theta}\right]$$

$$= \cos^{2/3}\theta \times \sin^{2/3}\theta \times \frac{\cos^2\theta + \sin^2\theta}{\sin^{2/3}\theta\cos^{2/3}\theta} = \quad \cos^2\theta + \sin^2\theta = \quad 1 \qquad \text{RHS}$$

COMPLEMENTARY ANGLES

Q. 1. ***Without using below show that:***

(i) tan 7° tan 23° tan 60° tan 67° tan 83° = √3

(ii) $\dfrac{\cos^2 20° + \cos^2 70°}{\sin^2 20° + \sin^2 70°} + \sin^2 64° + \cos 64°.\ \sin 26° = 2$

Sol. (i) LHS = tan 7° tan 23° tan 60° tan 67° tan 83°

= tan 7° tan 23° tan 60° tan (90° – 23)° tan (90° – 7)°

= tan 7° tan 23° tan 60° cot 23° cot 7°

$= \tan 7° \tan 23° \tan 60° .\ \dfrac{1}{\tan 23°} .\ \dfrac{1}{\tan 7°}$ $\quad\left[\because \tan\theta = \dfrac{1}{\cot\theta}\right]$

$= \left(\tan 7° .\ \dfrac{1}{\tan 7°}\right)\left(\tan 23° .\ \dfrac{1}{\tan 23°}\right).\sqrt{3}$ $\quad(\because \tan 60° = \sqrt{3})$

$= 1.1.\ \sqrt{3} = \sqrt{3}$ = RHS

(ii) LHS $= \dfrac{\cos^2 20° + \cos^2 70°}{\sin^2 20° + \sin^2 70°} + \sin^2 64° + \cos 64° \sin 26°$

$= \dfrac{\cos^2 20° + \cos^2 (90-20)°}{\sin^2 20° + \sin^2 (90-20)°} + \sin^2 64° + \cos 64° \sin (90° - 64°)$

$= \dfrac{\cos^2 20° + \sin^2 20°}{\sin^2 20° + \cos^2 20°} + \sin^2 64° + \cos 64 \cos 64°$

$= \dfrac{1}{1} + (\sin^2 64° + \cos^2 64°) = 1 + 1 = 2$ = RHS

Q. 2. ***Without using trigonometrical tables, find the value of:***

tan 5° tan 25° tan 30° tan 65° tan 85°

Sol. We know that tan (90° – θ) = cot θ

∴ tan 5° tan 25° tan 30° tan 65° tan 85° = tan (90° – 85°) tan (90° – 65°) tan 30° . tan 65° tan 85°

= cot 85° cot 65° tan 30° tan 65° tan 85°

$= \dfrac{1}{\tan 85°} .\ \dfrac{1}{\tan 65°} \tan 30° \tan 65° \tan 85°$

$= \left(\dfrac{1}{\tan 85°} .\ \tan 85°\right)\left(\dfrac{1}{\tan 65°} .\ \tan 45°\right).\ \tan 30° = 1.1.\ \tan 30° = \dfrac{1}{\sqrt{3}}$

Q. 3. ***Without using tables, evaluate the following:***

$\cos (40° + \theta) - \sin (50° - \theta) + \dfrac{\cos^2 40° + \cos^2 50°}{\sin^2 40° + \sin^2 50°}$

Sol. We have $\cos (40° + \theta) - \sin (50° - \theta) + \dfrac{\cos^2 40° + \cos^2 50°}{\sin^2 40° + \sin^2 50°}$

$= \cos (40° + \theta) - \sin \{90° - (40° + \theta)\} + \dfrac{\cos^2 40° + \cos^2 (90° - 40°)}{\sin^2 40° + \sin^2 (90° - 40°)}$

$= \cos (40° + \theta) - \cos (40° + \theta) + \dfrac{\cos^2 40° + \sin^2 40°}{\sin^2 40° + \cos^2 40°} = \dfrac{1}{1} = 1$

Q. 4. ***Without using tables, evaluate:***

$$\frac{\sin^2 20^\circ + \sin^2 70^\circ}{\cos^2 20^\circ + \cos^2 70^\circ} + \left[\frac{\sin(90^\circ - \theta) . \sin\theta}{\tan\theta} + \frac{\cos(90^\circ - \theta) . \cos\theta}{\cot\theta}\right]$$

Sol. We have,

$$\frac{\sin^2 20^\circ + \sin^2 70^\circ}{\cos^2 20^\circ + \cos^2 70^\circ} + \left[\frac{\sin(90^\circ - \theta) . \sin\theta}{\tan\theta} + \frac{\cos(90^\circ - \theta) . \cos\theta}{\cot\theta}\right]$$

$$= \frac{\sin^2 20^\circ + \sin^2 (90^\circ - 20^\circ)}{\cos^2 20^\circ + \cos^2 (90^\circ - 20^\circ)} + \left[\frac{\cos\theta . \sin\theta}{\tan\theta} + \frac{\cos\theta . \sin\theta}{\cot\theta}\right]$$

$$= \frac{\sin^2 20^\circ + \cos^2 20^\circ}{\cos^2 20^\circ + \sin^2 20^\circ} + \left[\frac{\cos\theta . \sin\theta}{\frac{\sin\theta}{\cos\theta}} + \frac{\cos\theta . \sin\theta}{\frac{\cos\theta}{\sin\theta}}\right]$$

$$= \frac{1}{1} + [\cos^2\theta + \sin^2\theta] = 1 + 1 = 2$$

Q. 4. ***Without using tables, evaluate:***

$$\frac{2\cos 67^\circ}{\sin 23^\circ} - \frac{\tan 40^\circ}{\cot 50^\circ} - \cos 0^\circ + \tan 15^\circ . \tan 25^\circ . \tan 60^\circ . \tan 65^\circ . \tan 75^\circ$$

Sol. We have

$$\frac{2\cos 67^\circ}{\sin 23^\circ} - \frac{\tan 40^\circ}{\cot 50^\circ} - \cos 0^\circ + \tan 15^\circ . \tan 25^\circ . \tan 60^\circ . \tan 65^\circ . \tan 75^\circ$$

$$= 2\frac{\cos(90^\circ - 23^\circ)}{\sin 23^\circ} - \frac{\tan 40^\circ}{\cot(90^\circ - 50^\circ)} - \cos 0^\circ + \tan 15^\circ . \tan 25^\circ . \tan 60^\circ . \tan 65^\circ . \tan 75^\circ$$

$$= 2\frac{\sin 23^\circ}{\sin 23^\circ} - \frac{\tan 40^\circ}{\tan 40^\circ} - \cos 0^\circ + \tan 15^\circ . \tan 25^\circ . \tan 60^\circ . \tan(90^\circ - 25^\circ) . \tan(90^\circ - 15^\circ)$$

$$= 2\frac{\sin 23^\circ}{\sin 23^\circ} - \frac{\tan 40^\circ}{\tan 40^\circ} - \cos 0^\circ + \tan 15^\circ . \tan 60^\circ . \tan 25^\circ . \cot 25^\circ . \cot 15^\circ$$

$$= 2\frac{\sin 23^\circ}{\sin 23^\circ} - \frac{\tan 40^\circ}{\tan 40^\circ} - \cos 0^\circ + (\tan 15^\circ . \tan 25^\circ) . \tan 60^\circ . (\tan 25^\circ \cot 25^\circ)$$

$$= 2 . 1 - \frac{1}{1} - 1 + 1.1. \sqrt{3} = 2 - 2 + 1 . \sqrt{3} = \sqrt{3}$$

Q. 5. ***Without using tables, evaluate the following:***

$$3\cot 68^\circ . \operatorname{cosec} 22^\circ - ½ \tan 43^\circ . \tan 47^\circ . \tan 12^\circ . \tan 60^\circ . \tan 78^\circ$$

Sol. We have, $3\cot 68^\circ . \operatorname{cosec} 22^\circ - ½ \tan 43^\circ . \tan 47^\circ . \tan 12^\circ . \tan 60^\circ . \tan 78^\circ$

$$= 3\cos(90^\circ - 22^\circ) . \operatorname{cosec} 22^\circ - ½ . \{\tan 43^\circ . \tan(90^\circ - 43^\circ)\} . \{\tan 12^\circ . \tan(90^\circ - 12^\circ) . \tan 60^\circ\}$$

$$= 3\sin 22^\circ . \operatorname{cosec} 22^\circ - ½ (\tan 43^\circ . \cot 43^\circ) . (\tan 12^\circ . \cot 12^\circ) . \tan 60^\circ$$

$$= 3.1 - ½ \times 1 \times 1 \times \sqrt{3} = 3 - \frac{\sqrt{3}}{2} = \frac{6 - \sqrt{3}}{2}$$

Q. 6. ***Without using tables, evaluate the following:***

$$\frac{2\sin 68^\circ}{\cos 22^\circ} - \frac{2\cot 15^\circ}{5\tan 75^\circ} - \frac{3\tan 45^\circ . \tan 20^\circ . \tan 40^\circ . \tan 50^\circ . \tan 70^\circ}{5}$$

Sol. We have,

$$\frac{2\sin 68^\circ}{\cos 22^\circ} - \frac{2\cot 15^\circ}{5\tan 75^\circ} - \frac{3\tan 45^\circ . \tan 20^\circ . \tan 40^\circ . \tan 50^\circ . \tan 70^\circ}{5}$$

$$= \frac{2\sin(90^\circ - 22^\circ)}{\cos 22^\circ} - \frac{2\cot 15^\circ}{5\tan(90^\circ - 15^\circ)} - \frac{3\tan 45^\circ . \tan 20^\circ . \tan 40^\circ . \tan(90^\circ - 40^\circ) . \tan(90^\circ - 20^\circ)}{5}$$

$$= \frac{2\cos 22^\circ}{\cos 22^\circ} - \frac{2\cot 15^\circ}{5\cot 15^\circ} - \frac{3\tan 45^\circ . \tan 20^\circ . \tan 40^\circ . \cot 40^\circ . \cot 20^\circ}{5}$$

$$= \frac{2\cos 22^\circ}{\cot 22^\circ} - \frac{2\cot 15^\circ}{5\cot 15^\circ} - \frac{3\tan 45^\circ . (\tan 20^\circ . \cot 20^\circ) . (\tan 40^\circ - \cot 40^\circ)}{5}$$

$$= 2 . 1 - \frac{2}{5} - \frac{3}{5} . 1 . 1 . 1 = 2 - \frac{2}{5} - \frac{3}{5} = 2 - 1 = 1$$

Q. 7. ***Evaluate:***

$$\frac{\sec\theta . \operatorname{cosec}(90^\circ - \theta) - \tan\theta . \cot(90^\circ - \theta) + \sin^2 55^\circ + \sin^2 35^\circ}{\tan 10^\circ . \tan 20^\circ . \tan 60^\circ . \tan 70^\circ . \tan 80^\circ}$$

Sol. We have,

$$\frac{\sec\theta . \operatorname{cosec}(90^\circ - \theta) - \tan\theta . \cot(90^\circ - \theta) + \sin^2 55^\circ + \sin^2 35^\circ}{\tan 10^\circ . \tan 20^\circ . \tan 60^\circ . \tan 70^\circ . \tan 80^\circ}$$

$$= \frac{\sec\theta . \sec\theta - \tan\theta . \tan\theta + \sin^2 55^\circ + \sin^2 (90^\circ - 55^\circ)}{\tan 10^\circ . \tan 20^\circ . \tan 60^\circ . \tan(90^\circ - 20^\circ) . \tan(90^\circ - 10^\circ)}$$

$$= \frac{\sec^2\theta - \tan^2\theta + \sin^2 55^\circ + \cos^2 55^\circ}{\tan 10^\circ . \tan 20^\circ . \tan 60^\circ . \cot 20^\circ . \cot 10^\circ} = \frac{(\sec^2\theta - \tan^2\theta) + \sin^2 55^\circ + \cos^2 55^\circ}{(\tan 10^\circ . \cot 10^\circ) . (\tan 20^\circ . \cot 20^\circ) . \tan 60^\circ}$$

$$= \frac{1 + 1}{(1) . (1) . \sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Q. 8. ***Without using tables, evaluate the following:***

$$\frac{\sec^2 54^\circ - \cot^2 36^\circ}{\operatorname{cosec}^2 57^\circ - \tan^2 33^\circ} + 2\sin^2 38^\circ . \sec^2 52^\circ - \sin^2 45^\circ$$

Sol. We have $\dfrac{\sec^2 54° - \cot^2 36°}{\text{cosec}^2 57° - \tan^2 33°} + 2 \sin^2 38° . \sec^2 52° - \sin^2 45°$

$= \dfrac{\sec^2 (90° - 36°) - \cot^2 36°}{\text{cosec}^2 (90° - 33°) - \tan^2 33°} + 2 \sin^2 38° . \sec^2 (90° - 38°) - \sin^2 45°$

$= \dfrac{\text{cosec}^2 36° - \cot^2 36°}{\sec^2 33° - \tan^2 33°} + 2 \sin^2 38° . \text{cosec}^2 38 - \left(\dfrac{1}{\sqrt{2}}\right)^2$

$= \dfrac{1}{1} + 2 . 1 - \dfrac{1}{2} = 3 - \dfrac{1}{2} = \dfrac{5}{2}$

Q. 9. ***Without using trigonometric tables, evaluate the following:***

$\dfrac{\cot (90° - \theta) . \sin (90° - \theta)}{\sin \theta} + \dfrac{\cot 40°}{\tan 50°} - (\cos^2 20° + \cos^2 70°)$

Sol. We have $\dfrac{\cot (90° - \theta) . \sin (90° - \theta)}{\sin \theta} + \dfrac{\cot 40°}{\tan 50°} - (\cos^2 20° + \cos^2 70°)$

$= \dfrac{\tan \theta . \cos \theta}{\sin \theta} + \dfrac{\cot 40°}{\tan (90° - 40°)} - \{\cos^2 20° + \cos^2 (90° - 20°)\}$

$= \dfrac{\dfrac{\sin \theta}{\cos \theta} . \cos \theta}{\sin \theta} + \dfrac{\cot 40°}{\cot 40°} - \{\cos^2 20° + \sin^2 20°\} = 1 + 1 - 1 = 1$

Q. 10. ***Without using trigonometric tables, prove that:***

$\dfrac{\sec^2 \theta - \cot^2 (90° - \theta)}{\text{cosec}^2 67° - \tan^2 23°} + (\sin^2 40° + \sin^2 50°) = 2$

Sol. We have,

LHS $= \dfrac{\sec^2 \theta - \cot^2 (90° - \theta)}{\text{cosec}^2 67° - \tan^2 23°} + (\sin^2 40° + \sin^2 50°)$

$= \dfrac{\sec^2 \theta - \tan^2 \theta}{\text{cosec}^2 (90° - 23°) - \tan^2 23°} + \{\sin^2 40° + \sin^2 (90° - 40°)\}$

$= \dfrac{\sec^2 \theta - \tan^2 \theta}{\sec^2 23° - \tan^2 23°} + (\sin^2 40° + \cos^2 40°)$

$= \dfrac{1}{1} + 1 = 1 + 1 = 2 =$ RHS

Q. 11. ***Without using trigonometric tables, evaluate the following:***

$\dfrac{\cos^2 20° + \cos^2 70°}{\sec^2 50° - \cot^2 40°} + 2 \text{cosec}^2 58° - 2 \cot 58° \tan 32° - 4 \tan 13° \tan 37° \tan 45° \tan 53° \tan 77°$

Sol. We have, $\dfrac{\cos^2 20° + \cos^2 70°}{\sec^2 50° - \cot^2 40°} + 2 \text{cosec}^2 58° - 2 \cot 58° \tan 32° - 4 \tan 13° \tan 37° \tan 45° \tan 53° \tan 77°$

$= \cos^2 (90° - 70°) + \cos^2 70° + 2 \text{cosec}^2 (90° - 32°) - 2 \cot (90° - 32°) \tan 32°$
$- 4 \tan (90° - 77°) . \tan (90° - 53°) . 1 . \tan 53° . \tan 77°$

$= \dfrac{\sin^2 70° + \cos^2 70°}{\text{cosec}^2 70° - \cot^2 40°} + 2 \sec^2 32° - 2 \tan 32° . \tan 32° - 4 \cot 77° . \cot 53° . \tan 53° . \tan 77°$

$= 1 + 2 \sec^2 32° - 2 \tan^2 32° - 4 \times \dfrac{1}{\tan 77°} \times \dfrac{1}{\tan 53°} \times \tan 53° \times \tan 77°$

$= 1 + 2 (\sec^2 32° - \tan^2 32°) - 4$

$= 1 + 2 \times 1 - 4 \quad => \quad 1 + 2 - 4$

$= 3 - 4 = - 1$

Q. 12. ***Prove that:*** $\sqrt{\dfrac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\dfrac{\sec \theta + 1}{\sec \theta - 1}} = 2 \text{ cosec } \theta$

Sol. LHS $= \sqrt{\dfrac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\dfrac{\sec \theta + 1}{\sec \theta - 1}}$

$= \sqrt{\dfrac{(\sec \theta - 1)(\sec \theta - 1)}{(\sec \theta + 1)(\sec \theta - 1)}} + \sqrt{\dfrac{(\sec \theta + 1)(\sec \theta + 1)}{(\sec \theta - 1)(\sec \theta + 1)}}$

$= \sqrt{\dfrac{(\sec \theta - 1)^2}{\sec^2 \theta - 1}} + \sqrt{\dfrac{(\sec \theta + 1)^2}{\sec^2 \theta - 1}}$

$= \sqrt{\dfrac{(\sec \theta - 1)^2}{\tan^2 \theta}} + \sqrt{\dfrac{(\sec \theta + 1)^2}{\tan^2 \theta}} = \dfrac{\sec \theta - 1}{\tan \theta} + \dfrac{\sec \theta + 1}{\tan \theta}$

$= \dfrac{\sec \theta - 1 + \sec \theta + 1}{\tan \theta} = \dfrac{2 \sec \theta}{\tan \theta} = 2 \times \dfrac{\dfrac{1}{\cos \theta}}{\sin \theta / \cos \theta}$

$= \dfrac{2}{\sin \theta} = 2 \text{ cosec } \theta =$ RHS

Q. 13. $$\frac{3\cos 55°}{7\sin 35°} - \frac{4\,(\cos 70°\,.\,\mathrm{cosec}\,20°)}{7\,(\tan 5°\,.\,\tan 25°\,.\,\tan 45°\,.\,\tan 65°\,.\,\tan 85°)}$$

Sol. We have,

$$\frac{3\cos 55°}{7\sin 35°} - \frac{4\,(\cos 70°\,.\,\mathrm{cosec}\,20°)}{7\,(\tan 5°\,.\,\tan 25°\,.\,\tan 45°\,.\,\tan 65°\,.\,\tan 85°)}$$

$$= \frac{3\cos(90° - 35°)}{7\sin 35°} - \frac{4\cos(90° - 20°)\,.\,\mathrm{cosec}\,20°}{7\,(\tan(90° - 85°)\,.\,\tan(90° - 65°)\,.\,1\,.\,\tan 65°\,.\,\tan 85°)}$$

$$= \frac{3\sin 35°}{7\sin 35°} - \frac{4\sin 20°\,.\,\mathrm{cosec}\,20°}{7\cot 85°\,.\,\cot 65°\,.\,\tan 65°\,.\,\tan 85°}$$

$$= \frac{3}{7} - \frac{4}{7} = -\frac{1}{7}$$

Q. 14. ***If sin (A – B) = ½, cos (A + B) = ½, 0° < A + B ≤ 90°, A > B, find A and B.***

Sol. Since, sin (A – B) = ½, therefore, A – B = 30° ... (i)

Also, since cos (A + B) = ½, therefore, A + B = 60° ... (ii)

Solving (i) and (ii), we get A = 45° and B = 15°

Q. 15. ***If tan A = n tan B and sin A = m sin B, prove that*** $\cos^2 A = \dfrac{m^2 - 1}{n^2 - 1}$.

Sol. We have to find $\cos^2 A$ in terms of m and n. This means that the angle B is to be eliminated from the given relations.

Now, $\tan A = n\tan B \Rightarrow \tan B = \dfrac{1}{n}\tan A \Rightarrow \cot B = \dfrac{n}{\tan A}$

and $\sin A = m\sin B \Rightarrow \sin B = \dfrac{1}{m}\sin A \Rightarrow \mathrm{cosec}\,B = \dfrac{m}{\sin A}$

Substituting the values of cot B and cosec B in $\mathrm{cosec}^2 B - \cot^2 B = 1$, we get

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$$

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2\cos^2 A}{\sin^2 A} = 1$$

$$\Rightarrow \frac{m^2 - n^2\cos^2 A}{\sin^2 A} = 1$$

$$\Rightarrow m^2 - n^2\cos^2 A = \sin^2 A \Rightarrow m^2 - n^2\cos^2 A = 1 - \cos^2 A$$

$$\Rightarrow m^2 - 1 = n^2\cos^2 A - \cos^2 A \Rightarrow m^2 - 1 = (n^2 - 1)\cos^2 A$$

$$\Rightarrow \frac{m^2 - 1}{n^2 - 1} = \cos^2 A$$

Q. 16. ***If*** $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ ***and*** $x\sin\theta = y\cos\theta$, ***prove*** $x^2 + y^2 = 1$

Sol. We have,

$$x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$\Rightarrow (x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$$

$$\Rightarrow x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta \qquad [\because x\sin\theta = y\cos\theta]$$

$$x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$$

$$\Rightarrow x\sin\theta = \sin\theta\cos\theta$$

$$\Rightarrow x = \cos\theta$$

Now, putting $x\sin\theta = y\cos\theta$ in (i)

$$\Rightarrow \cos\theta\sin\theta = y\cos\theta \qquad [\because x = \cos\theta]$$

$$\Rightarrow y = \sin\theta$$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

Q. 17. ***If cosec A = √2, find the value of*** $\dfrac{2\sin^2 A + 3\cot^2 A}{4\tan^2 A - \cos^2 A}$.

Sol. We have

$$\mathrm{cosec}\,A = \sqrt{2} \Rightarrow \frac{1}{\sin A} = \sqrt{2} \Rightarrow \sin A = \frac{1}{\sqrt{2}}$$

Now, $$\cos A = \sqrt{1 - \sin^2 A} \Rightarrow \cos A = \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{\sqrt{2}}$$

$$\therefore \tan A = \frac{\sin A}{\cos A} \Rightarrow \tan A = \frac{1/\sqrt{2}}{1/\sqrt{2}} = 1$$

$$\therefore \cot A = 1$$

Hence, $$\frac{2\sin^2 A + 3\cot^2 A}{4\tan^2 A - \cos^2 A} = \frac{2 \times (1/\sqrt{2})^2 + 3\,(1)^2}{4\,(1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{2 \times ½ + 3}{4 - ½} = \frac{1 + 3}{7/2} = \frac{8}{7}$$

Q. 18. ***If tan A + sin A = m and tan A – sin A = n, prove that*** $(m^2 - n^2)^2 = 16\,mn$

Sol. We have, LHS $= (m^2 - n^2)^2$

$$= \{(\tan A + \sin A)^2 - (\tan A - \sin A)^2\}^2 = \{(\tan^2 A + \sin^2 A + 2\tan A\sin A) - (\tan^2 A + \sin^2 A - 2\tan A\sin A)$$

$$= (4\tan A\sin A)^2 = 16\tan^2 A\sin^2 A \qquad \text{... (i)}$$

And, RHS = 16 mn

$= 16 (\tan A + \sin A) (\tan A - \sin A)$

$= 16 (\tan^2 A - \sin^2 A)$

$= 16 \left(\frac{\sin^2 A}{\cos^2 A}\right) - \sin^2 A \quad = 16 \left(\frac{\sin^2 A - \cos^2 A \sin^2 A}{\cos^2 A}\right)$

$= 16 \frac{\sin^2 A (1 - \cos^2 A)}{\cos^2 A}$

$= 16 \frac{\sin^2 A}{\cos^2 A} \sin^2 A \quad = 16 \tan^2 A \sin^2 A$ ... (ii)

From (i) and (ii) it follows that LHS = RHS i.e., $(m^2 - n^2)^2 = 16$ mn

Q. 19. ***If sin 3A = cos (A – 26°), where 3 A is an acute angle, find the value of A.***

Sol. We are given that sin 3A = cos (A – 26°)

Since sin 3A = cos (90° – 3A), we can write (i) as

cos (90° – 3A) = cos (A – 26°)

Since 90° – 3A and A – 26° are both acute angles, therefore,

90° – 3A = A – 26°

Which gives A = 29°

Q. 20. ***Express the ratios cos A, tan A and sec A in terms of sin A.***

Sol. Since $\cos^2 A + \sin^2 A = 1$, therefore,

$\cos^2 A = 1 - \sin^2 A$, i.e., $\cos A = \pm\sqrt{1 - \sin^2 A}$

This gives $\cos A = \sqrt{1 - \sin^2 A}$

Hence, $\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1 - \sin^2 A}}$ and $\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1 - \sin^2 A}}$

Q. 21. ***Prove that sec A (1 – sin A) (sec A + tan A) = 1***

Sol. LHS $= \sec A (1 - \sin A) (\sec A + \tan A) = \left(\frac{1}{\cos A}\right) (1 - \sin A) \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$

$= \frac{(1 - \sin A)(1 + \sin A)}{\cos^2 A} = \frac{1 - \sin^2 A}{\cos^2 A}$

$= \frac{\cos^2 A}{\cos^2 A} = 1 =$ RHS

Q. 22. ***Evaluate the following:***

(i) sin 60° cos 30° + sin 30° cos 60°

(ii) $2 \tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

(iv) $\frac{5 \cos^2 60° + 4 \sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Sol. (i) sin 60° cos 30° + sin 30° cos 60°

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

(ii) $2 \tan^2 45° + \cos^2 30° - \sin^2 60°$

$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2 + \frac{3}{4} - \frac{3}{4} = 2$

(iii) $\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3} - 4}{4 + 3\sqrt{3}} = \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{3\sqrt{3} - 4}{3\sqrt{3} - 4}$$ [On rationalising]

$$= \frac{(3\sqrt{3} - 4)^2}{(3\sqrt{3})^2 - (4)^2} = \frac{27 + 16 - 24\sqrt{3}}{27 - 16}$$

$$= \frac{43 - 24\sqrt{3}}{11}$$

(iv) $\frac{5 \cos^2 60° + 4 \sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

$$= \frac{5 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - 1}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{5/4 + 6/13 - 1}{4/4} = \frac{15 + 64 - 12}{12} = \frac{67}{12}$$

Q. 23. Choose the correct option and justify your choice:

(i) $\dfrac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} =$

(a) sin 60° ***(b) cos 60°*** ***(c) tan 60°*** ***(d) sin 30°***

(ii) $\dfrac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$

(a) tan 90° ***(b) 1*** ***(c) sin 45°*** ***(d) 0***

(iii) sin 2A = 2 sin A is true when A =

(a) 0° ***(b) 30°*** ***(c) 45°*** ***(d) 60°***

(iv) $\dfrac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$

(a) cos 60° ***(b) sin 60°*** ***(c) tan 60°*** ***(d) sin 30°***

Sol. (i) (a)

∵ $$\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = 2 \times \frac{\frac{1}{\sqrt{3}}}{1 + \left[\frac{1}{\sqrt{3}}\right]^2} = 2 \times \frac{\frac{1}{\sqrt{3}}}{1 + \frac{1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

(ii) (d)

∵ $$\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \frac{1 - (1)^2}{1 + (1)^2} = \frac{0}{2} = 0$$

(iii) (a)

∵ When A = 0°, sin 2A = sin 2 × 0 = sin 0 = 0

and 2 sin A = 2 sin 0 = 2 × 0 = 0

=> sin 2A = 2 sin A when A = 0

∵ $$\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \times 1/\sqrt{3}}{1 - [1/\sqrt{3}]^2} = \frac{2 \times 1/\sqrt{3}}{1 - 1/3} = \frac{2/\sqrt{3}}{2/3}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3} = \tan 60^\circ$$

Q. 24. Show that:

(i) tan 48° tan 23° tan 42° tan 67° = 1

(ii) cos 38° cos 52° – sin 38° sin 52° = 0

Sol. (i) LHS = tan 48° . tan 23° . tan 42° . tan 67°

= tan (90° – 42°) . tan (90° – 67°) . tan 42° . tan 67°

= cot 42° . cot 67° . tan 42° . tan 67°

$= \dfrac{1}{\tan 42^\circ} \cdot \dfrac{1}{\tan 67^\circ} \cdot \tan 42^\circ \cdot \tan 67^\circ$

= 1

(ii) LHS = cos 38° . cos 52° – sin 38° . sin 52°

= cos (90° – 52°) . cos (90° – 38°) – sin 38° . sin 52°

= sin 52° . sin 38° – sin 38° . sin 52°

= 0

Q. 25. If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.

Sol. We have, tan 2A = cot (A – 18)

=> cot (90° – 2A) = cot (A – 18)

∴ 90° – 2A = A – 18

=> 90° + 18° = 2A + A

=> 108° = 3A

∴ $A = \dfrac{180^\circ}{3} = 36^\circ$

Q. 26. Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

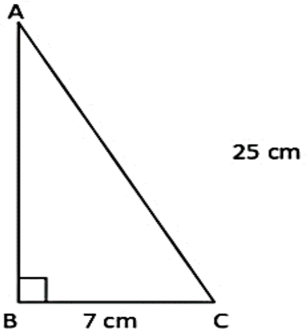
Sol. sin 67° + cos 75°

= sin (90° – 23°) + cos (90° – 15°)

= cos 23° + sin 15°

NCERT EXERCISES

Q. 1. ***In Δ ABC, right-angles at B, AB = 24 cm, BC = 7 cm. Determine:*** **1**
(i) sin A , cos A ***(ii) sin C, cos C***



Sol. We have,

AB = 24 cm and BC = 7 cm

Now, by Pythagoras theorem, we have

$(AC)^2 = (AB)^2 + (BC)^2$

=> $(AC)^2 = (24)^2 + (7)^2$

=> $(AC)^2 = 576 + 49 = 625$

∴ $AC = \sqrt{625} = 25$ cm

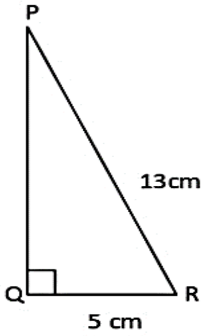
(i) $\text{Sin } A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$

$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$

(ii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$

$\text{Cos } C = \frac{BC}{AC} = \frac{7}{25}$

Q. 2. ***In figure, find tan P – cot R.***



Sol. Using Pythagoras theorem, we have

$(PR)^2 = (PQ)^2 + (QR)^2$

=> $(13)^2 = (12)^2 + (QR)^2$

=> $169 = 144 + (QR)^2$

=> $(QR)^2 = 169 - 144 = 25$

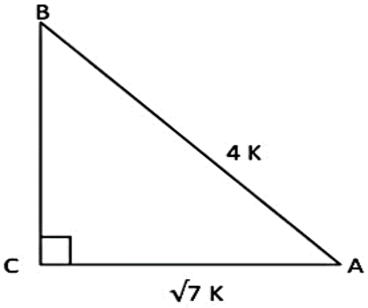
=> $QR = 5$ cm

Now, $\tan P = \frac{QR}{PQ} = \frac{5}{12}$

$\cot R = \frac{QR}{PQ} = \frac{5}{12}$

∴ $\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

Q. 3. ***If sin A = ¾, calculate cos A and tan A.***



Sol. Let us first draw a right ΔABC in which ∠C = 90°

Now, we know that

$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AB} = \frac{3}{4}$

Let BC = 3K and AB = 4K

Then, by Pythagoras Theorem, we have

$(AB)^2 = (BC)^2 + (AC)^2$

=> $(4K)^2 = (3K)^2 + (AC)^2$

=> $16K^2 - 9K^2 = (AC)^2$

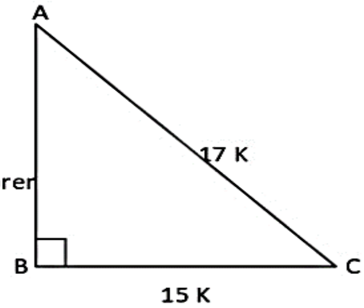
=> $7K^2 = (AC)^2$

∴ $AC = \sqrt{7}\ K$

∴ $\cos A = \frac{AC}{AB} = \frac{\sqrt{7}K}{4K} = \frac{\sqrt{7}}{4}$

and $\tan A = \frac{BC}{AC} = \frac{3K}{\sqrt{7}K} = \frac{3}{\sqrt{7}}$

Q. 4. ***given 15 cot A = 8, find sin A and sec A.***



Sol. Let us find draw a right Δ ABC, in which ∠B = 90°

Now, we have

15 cot A= B

∴ $\cot A = \frac{8}{15} = \frac{AB}{BC} = \frac{\text{Base}}{\text{Perpendicular}}$

Let AB = 8K and BC = 15 K

Then, $AC = \sqrt{(AB)^2 + (BC)^2}$ (By Pythagoras theore

$= \sqrt{(8K)^2 + (15\ K)^2}$

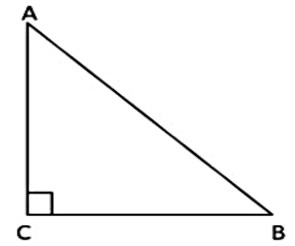
$= \sqrt{64\ K^2 + 225K^2} = \sqrt{289\ K^2} = 17\ K$

∴ $\sin A = \frac{BC}{AC} = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{15\ K}{17\ K} = \frac{15}{17}$

and, $\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB} = \frac{17\ K}{8K} = \frac{17}{8}$

Q. 5. ***If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.***

Sol. In right-angled ΔACB, in which $\angle C = 90°$



We have,

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AC}{AB}$$

and $$\cos B = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AB}$$

We have $\cos A = \cos B$ [given]

=> $\frac{AC}{AB} = \frac{BC}{AC}$

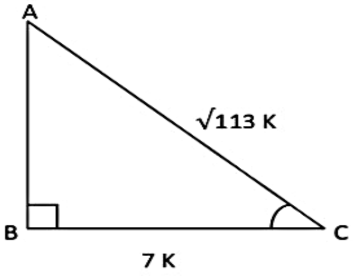
=> $AC = BC$

=> $\angle B = \angle A$ [angles opposite to equal sides are equal]

=> $\angle A = \angle B$

Q. 6. ***If*** $\cot \theta = \frac{7}{8}$***, evaluate: (i)*** $\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$***, (ii)*** $\cot^2 \theta$

Sol. Let us draw a right angle ΔABC in which $\angle B = 90°$ and $\angle C = \theta°$



We have

$$\cot \theta = \frac{7}{8} = \frac{\text{Base}}{\text{Perpendicular}} = \frac{BC}{AB} \quad \text{[given]}$$

Let BC = 7K and AB = 8K

Therefore, by Pythagoras Theorem

$$(AC)^2 = (AB)^2 + (BC)^2$$
$$= (8K)^2 + (7K)^2 = 64K^2 + 49K^2$$

∴ $(AC)^2 = 113\ K^2$

∴ $$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{8K}{\sqrt{113}\ K} = \frac{8}{\sqrt{113}}$$

and $$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7\ K}{\sqrt{113}\ K} = \frac{7}{\sqrt{113}}$$

(i) ∴ $$\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta} = \frac{1 - [8/\sqrt{113}]^2}{1 - [7/\sqrt{113}]^2}$$

$$= \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}} = \frac{\frac{113 - 64}{113}}{\frac{113 - 49}{113}} = \frac{49}{64}$$

(ii) $$\cot^2 \theta = (7/8)^2 = \frac{49}{64}$$

Q. 7. ***If 3 cot A = 4, check whether*** $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ ***or not.***

Sol. Let us consider a right triangle ABC in which $\angle B = 90°$

Now, $$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{4}{3}$$

Let AB = 4K and BC = 3 K

∴ By Pythagoras Theorem

$$(AC)^2 = (AB)^2 + (BC)^2$$

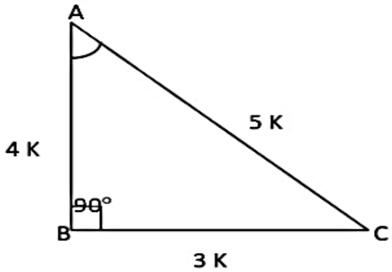
=> $$(AC)^2 = (4K)^2 + (3K)^2$$
$$= 16\ K^2 + 9\ K^2$$
$$(AC)^2 = 25\ K^2$$

∴ $AC = 5\ K$

Therefore, $$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{3\ K}{4\ K} = \frac{3}{4}$$

and, $$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3K}{5\ K} = \frac{3}{5}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{4K}{5K} = \frac{4}{5}$$



Now, $$\text{LHS} = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - (\tfrac{3}{4})^2}{1 + (\tfrac{3}{4})^2} = \frac{1 - 9/16}{1 + 9/16} = \frac{16 - 9}{16 + 9} = \frac{7}{25}$$

Now, $$\text{RHS} = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Hence, $$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

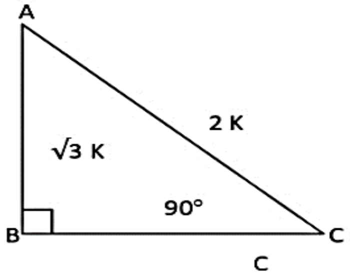
Q. 8. ***In triangle ABC, right-angled at B, if*** $\tan A = \frac{1}{\sqrt{3}}$***, find the value of:***

(i) sin A cos C + cos A sin C ***(ii) cos A cos C – sin A sin C***

Sol. We have a right-angled ΔABC in which $\angle B = 90°$

and, $\tan A = \frac{1}{\sqrt{3}}$ 3

Now, $\tan A = \frac{1}{\sqrt{3}} = \frac{BC}{AB}$



Let BC = K and AB = $\sqrt{3}$ K

∴ By Pythagoras Theorem, we have

$(AC)^2 = (AB)^2 + (BC)^2$

=> $(AC)^2 = (\sqrt{3}K)^2 + (K)^2$

$= 3K^2 + K^2$

$(AC)^2 = 4K^2$

∴ $AC = 2K$

Now, $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{K}{2K} = \frac{1}{2}$

$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{3}K}{2K} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\sqrt{3}K}{2K} = \frac{\sqrt{3}}{2}$

$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{K}{2K} = \frac{1}{2}$

(i) ∴ $\sin A . \cos C + \cos A . \sin C$

$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

(ii) $\cos A. \cos C - \sin A . \sin C$

$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

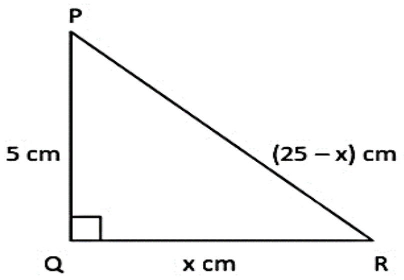
Q. 9. ***In Δ PQR, right-angled at Q PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.***

Sol. We have a right-angled ΔPQR in which ∠Q = 90°

Let QR = x cm

Therefore, PR = (25 – x) cm

By Pythagoras Theorem, we have



$(PR)^2 = (PQ)^2 + (QR)^2$

$(25 - x)^2 = (5)^2 + x^2$

=> $(25 - x)^2 - x^2 = (5)^2$

=> $(25 - x - x)(25 - x + x) = 25$

=> $(25 - 2x)\, 25 = 25$

=> $25 - 2x = 1$

$25 - 1 = 2x$

=> $24 = 2x$

∴ $x = 12$ cm

Hence, QR = 12 cm

PR = (25 – x) cm = 25 – 12 = 13 cm

PQ = 5 cm

∴ $\sin P = \frac{QR}{PR} = \frac{12}{13}$

$\cos P = \frac{PQ}{PR} = \frac{5}{13}$

$\tan P = \frac{QR}{PQ} = \frac{12}{5}$

Q. 10. ***State whether the following are true or false. Justify your answer.***

(i) The value of tan A is always less than 1.

(ii) sec A = $\frac{12}{5}$ for some value of angle A.

(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) sin θ = $\frac{4}{3}$ for some angle θ.

Sol. (i) False, because tan A have any value. (ii) True, because sec A is always greater than
(iii) True (iv) False (v) False because sin θ can never be greater than 1.

Q. 11. ***Evaluate the following:***

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2 \tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\frac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°}$

(iv) $\frac{5 \cos^2 60° + 4 \sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Sol. (i) sin 60° cos 30° + sin 30° cos 60° 4

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) $2 \tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3} - 4}{4 + 3\sqrt{3}} = \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{3\sqrt{3} - 4}{3\sqrt{3} - 4}$$ [On rationalising]

$$= \frac{(3\sqrt{3} - 4)^2}{(3\sqrt{3})^3 - (4)^2} = \frac{27 + 16 - 24\sqrt{3}}{27 - 16}$$

$$= \frac{43 - 24\sqrt{3}}{11}$$

(iv) $\dfrac{5 \cos^2 60° + 4 \sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

$$= \frac{5 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - 1}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{5/4 + 16/3 - 1}{4/4} = \frac{15 + 64 - 12}{12} = \frac{67}{12}$$

Q. 12. ***Choose the correct option and justify your choice:***

(i) $\dfrac{2 \tan 30°}{1 \tan^2 30°} =$

(a) sin 60° *(b) cos 60°* *(c) tan 60°* *(d) sin 30°*

(ii) $\dfrac{1 - \tan^2 45°}{1 + \tan^2 45°} =$

(a) tan 90° *(b) 1* *(c) sin 45°* *(d) 0*

(iii) sin 2A = 2 sin A is true when A =

(a) 0° *(b) 30°* *(c) 45°* *(d) 60°*

(iv) $\dfrac{2 \tan 30°}{1 - \tan^2 30°} =$

(a) cos 60° *(b) sin 60°* *(c) tan 60°* *(d) sin 30°*

Sol. (i)

∵ $$\frac{2 \tan 30°}{1 + \tan^2 30°} = \frac{2 \times 1/\sqrt{3}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2 \times 1/\sqrt{3}}{1 + 1/3}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2} = \sin 60°$$

∴ Correct option is (a) sin 60°.

(ii)

∵ $$\frac{1 - \tan^2 45°}{1 + \tan^2 45°} = \frac{1 - (1)^2}{1 + (1)^2} = \frac{0}{2} = 0$$

∴ Correct option is (d) 0.

(iii)

∵ When A = 0°, sin 2A = sin 2 × 0 = sin 0 = 0

and 2 sin A = 2 sin 0 = 2 × 0 = 0

=> sin 2A = 2 sin A ∴ Correct option is (a) 0

(iv)

∵ $$\frac{2 \tan 30°}{1 - \tan^2 30°} = \frac{2 \times 1/\sqrt{3}}{1 - [1/\sqrt{3}]^2} = \frac{2 \times 1/\sqrt{3}}{1 - 1/3} = \frac{2/\sqrt{3}}{2/3} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3} = \tan 60°$$

∴ Correct option is (c) tan 60°.

Q. 3. ***If tan (A + B) = √3 and tan (A – B) = 1/√3; 0° < A + B ≤ 90°; A > B, find A and B.***

Sol. We have, tan (A + B) = √3

=> tan (A + B) = tan 60°

∴ tan (A + B) = 60° ... (i)

Again, tan (A – B) = 1/√3

=> tan (A – B) = tan 30°

∴ A – B = 30° ... (ii)

Adding (i) and (ii), we have 5

$2A = 90°$ => $A = 45°$

Putting the value of A in (i), we have

$45° + B = 60°$

∴ $B = 60° - 45° = 15°$

Hence, $A = 45°$; $B = 15°$

Q. 4. ***State whether the following are true or false. Justify your answer.***

(i) sin (A + B) = sin A + sin B ***(ii) The value of sin θ increases as θ increases.***

(iii) The value of cos θ increases as θ increases. ***(iv) sin θ = cos θ for all values of θ.***

(v) cot A is not defined for A = 0°

Sol. (i) False, because when A = 30 and B = 60.

Then $\sin(A + B) = \sin(30 + 60) = \sin 90 = 1$

and $\sin A + \sin B = \sin 30 + \sin 60 = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$ Hence, $\sin(A + B) \neq \sin A + \sin B$

(ii) True, because we can see that when

$\theta = 0°, 30°, 45°, 60°, 90°$ respectively.

$\sin\theta = 0, \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, 1$ respectively.

i.e., as θ increases sin θ increases.

(iii) False, because as we can see that when

$\theta = 0°, 30°, 45°, 60°, 90°$ respectively.

$\cos\theta = 1, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}, 0$ respectively.

i.e., as θ increases cos θ decreases.

(iv) False, because it is only true when

$\theta = 45°$

i.e, $\sin = 45° = \cos 45° = \frac{1}{\sqrt{2}}$

(v) True, because when A = 0°

$\cot A = \cot 0 = \frac{1}{\tan 0} = \frac{1}{0}$ = undefined

Q. 5. ***Evaluate:***

(i) $\frac{\sin 18°}{\cos 72°}$ ***(ii)*** $\frac{\tan 26°}{\cot 64°}$

(iii) cos 48° – sin 42° ***(iv) cosec 31° – sec 59°***

Sol. (i) $\frac{\sin 18°}{\cos 72°} = \frac{\sin(90° - 72°)}{\cos 72°} = \frac{\cos 72°}{\cos 72°} = 1$ (ii) $\frac{\tan 26°}{\cot 64°} = \frac{\tan(90° - 64°)}{\cot 64°} = \frac{\cot 64°}{\cot 64°} = 1$

(iii) $\cos 48° - \sin 42° = \cos(90° - 42°) - \sin 42° = \sin 42° - \sin 42° = 0$

(iv) $\operatorname{cosec} 31° - \sec 59° = \operatorname{cosec}(90° - 59°) - \sec 59°$

$= \sec 59° - \sec 59° = 0$

Q. 6. ***Show that:***

(i) tan 48° tan 23° tan 42° tan 67° = 1

(ii) cos 38° cos 52° – sin 38° sin 52° = 0

Sol. (i) LHS $= \tan 48° . \tan 23° . \tan 42° . \tan 67°$

$= (\tan 48° \tan 42°)(\tan 23° \tan 67°)$

$= (\tan(90° - 42°) \tan 42)(\tan(90° - 67°) \tan 67°)$

$= (\cot 42° \tan 42°)(\cot 67° \tan 67°)$

$= \left(\frac{1}{\tan 42°} . \tan 42°\right)\left(\frac{1}{\tan 67°} \tan 67°\right)$

$= 1 =$ RHS

(ii) LHS $= \cos 38° . \cos 52° - \sin 38° . \sin 52°$

$= \cos(90° - 52°) . \cos(90° - 38°) - \sin 38° . \sin 52°$

$= \sin 52° . \sin 38° - \sin 38° . \sin 52°$ $= 0 =$ RHS

Q. 7. ***If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.***

Sol. We have, $\tan 2A = \cot(A - 18)$

=> $\cot(90° - 2A) = \cot(A - 18)$

∴ $90° - 2A = A - 18$

=> $90° + 18° = 2A + A$

=> $108° = 3A$

∴ $A = \frac{180°}{3} = 36°$

Q. 8. ***If tan A = cot B, prove that A + B = 90°***

Sol. We have, $\tan A = \cot B$

=> $\tan A = \tan(90° - B)$

$A = 90° - B$ => $A + B = 90°$

Q. 9. ***If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A.*** ***6***

Sol. We have, $\sec 4A = \text{cosec}\,(A - 20^\circ)$

=> $\text{cosec}\,(90^\circ - 4A) = \text{cosec}\,(A - 20^\circ)$

∴ $90^\circ - 4A = A - 20^\circ$

=> $90^\circ + 20^\circ = A + 4A$

=> $110 = 5A$

∴ $A = \frac{110}{5} = 22^\circ$

Q. 10. ***If A, B and C are interior angles of a triangles ABC, then show that***

$$\sin\left(\frac{B + C}{2}\right) = \cos\frac{A}{2}$$

Sol. Since A, B and C are the interior angles of a ΔABC,

Therefore, $A + B + C = 180^\circ$

=> $\frac{A + B + C}{2} = \frac{180^\circ}{2}$

$\frac{A}{2} + \frac{(B + C)}{2} = 90^\circ$

=> $\frac{B + C}{2} = 90^\circ - \frac{A}{2}$

Now, taking sin on both sides, we have

$$\sin\left(\frac{B + C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$$

$$\sin\frac{B + C}{2} = \cos\frac{A}{2}$$

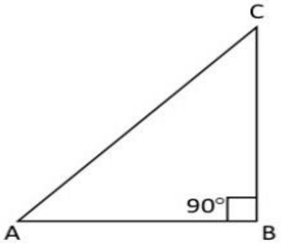
Q. 11. ***Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.***

Sol. $\sin 67^\circ + \cos 75^\circ$

$= \sin(90^\circ - 23^\circ) + \cos(90^\circ - 15^\circ) = \cos 23^\circ + \sin 15^\circ$

Q. 12. ***Express the trigonometric ratios sin A, sec A and tan A in terms of cot A,***

Sol. Let us consider a right-angled ΔABC in which ∠B = 90°



For ∠A, we have

Base = AB

Perpendicular = BC

and Hypotenuse = AC

∴ $\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC}$

=> $\frac{\cot A}{1} = \frac{AB}{BC}$ => $AB = BC \cot A$

Let $BC = k$

$AB = k \cot A$

then by Pythagoras Theorem, we have

$(AC)^2 = (AB)^2 + (BC)^2$

=> $(AC)^2 = k^2 \cot^2 A + k^2$

∴ $AC = \sqrt{k^2 (1 + \cot^2 A)} = k\sqrt{1 + \cot^2 A}$

∴ $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{k\sqrt{1 + \cot^2 A}} = \frac{1}{\sqrt{1 + \cot^2 A}}$

$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB} = \frac{k\sqrt{1 + \cot^2 A}}{k \cot A} = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$

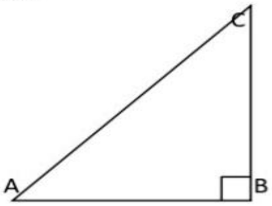
and $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{k}{k \cot A} = \frac{1}{\cot A}$

Q. 13. ***Write all the other trigonometric ratios of ∠A in terms of sec A.***

Sol. Let us consider a right angled Δ ABC, in which ∠B = 90°

For ∠A, we have

Base AB, Perpendicular = BC and Hypotenuse = AC



∴ $\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB}$

=> $\frac{\sec A}{1} = \frac{AC}{AB}$

$AC = AB \sec A$

Let $AB = k$

$Ac = k \sec A$

∴ By Pythagoras Theorem, we have

$(AC)^2 = (AB)^2 + (BC)^2$

$k^2 \sec^2 A = k^2 + (BC)^2$

∴ $(BC)^2 = k^2 \sec^2 A - k^2$ => $BC = k\sqrt{\sec^2 A - 1}$

∴ $$\sin A = \frac{BC}{AC} = \frac{k\sqrt{\sec^2 A - 1}}{k \sec A} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\cos A = \frac{AB}{AC} = \frac{k}{k \sec A} = \frac{1}{\sec A}$$

$$\tan A = \frac{BC}{AB} = \frac{k\sqrt{\sec^2 A - 1}}{k} = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\text{cosec } A = \frac{AC}{BC} = \frac{k \sec A}{k\sqrt{\sec^2 A}} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Q. 14. ***Evaluate:***

(i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$ *(ii)* $\sin 25° \cos 65° + \cos 25° \sin 65°$

Sol. (i) $$\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°} = \frac{\sin^2 (90° - 27°) + \sin^2 27°}{\cos^2 (90° - 73°) + \cos^2 73°}$$

$$= \frac{\cos^2 27° + \sin^2 27°}{\sin^2 73° + \cos^2 73°} = \frac{1}{1} = 1$$

(ii) $\sin 25° . \cos 65° + \cos 25° . \sin 65°$

$$= \sin (90° - 65°) . \cos 65° + \cos (90° - 65°) . \sin 65°$$
$$= \cos 65° . \cos 65° + \sin 65° . \sin 65°$$
$$= \cos^2 65° + \sin^2 65°$$
$$= 1$$

Q. 15. ***Choose the correct option. Justify your choice.***

(i) $9 \sec^2 A - 9 \tan^2 A =$

(a) 1 *(b)* 9 *(c)* 8 *(d)* 0

(ii) $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \text{cosec } \theta) =$

(a) 0 *(b)* 1 *(c)* 2 *(d)* $\cos A$

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(a) $\sec A$ *(b)* $\sin A$ *(c)* $\text{cosec } A$ *(d)* $\cos A$

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} =$

(a) $\sec^2 A$ *(b)* -1 *(c)* $\cot^2 A$ *(d)* $\tan^2 A$

Sol. (i)

∵ $$9 \sec^2 A - 9 \tan^2 A = 9 (\sec^2 A - \tan^2 A)$$
$$= 9 \times 1 = 9$$

(ii)

∵ $$(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \text{cosec } \theta)$$

$$= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right)\left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right)$$

$$= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right)\left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right)$$

$$= \frac{(\cos \theta + \sin \theta)^2 - 1}{\sin \theta . \cos \theta} = \frac{\cos^2 \theta + \sin^2 \theta + 2 \sin \theta . \cos \theta - 1}{\sin \theta . \cos \theta}$$

$$= \frac{1 + 2 \sin \theta . \cos \theta - 1}{\sin \theta . \cos \theta} = \frac{2 \sin \theta . \cos \theta}{\sin \theta . \cos \theta} = 1$$

∴ Correct the option is (b) 1.

(iii)

∵ $$(\sec A + \tan A)(1 - \sin A)$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$$

∴ Correct option is (d) cos A.

(iv)

∵ $$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\text{cosec}^2 A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

∴ Correct option is (d) $\tan^2 A$.

Q. 16. ***Prove the following identities, where the angles involved are acute angles for which the expressions are defined.***

(i) $(\text{cosec } \theta - \cot \theta)^2 = \dfrac{1 - \cos \theta}{1 + \cos \theta}$

(ii) $\dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A} = 2 \sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$ 8

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$, using the identify $\operatorname{cosec}^2 A = 1 + \cot^2 A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

Sol. (i) LHS $= (\operatorname{cosec}\theta - \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta} = \text{RHS}$$

(ii) LHS $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A} = \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{(\cos^2 A + \sin^2 A) + 1 + 2\sin A}{(1+\sin A)\cos A} = \frac{1+1+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)\cos A}$$

$$= \frac{2}{\cos A} = 2\sec A = \text{RHS}$$

(iii) LHS $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} \quad \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta-\cos\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\,.\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\,.\cos\theta)}{\sin\theta\,.\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1+\sin\theta\cos\theta}{\sin\theta\,.\cos\theta} = \frac{1}{\sin\theta\,.\cos\theta} + \frac{\sin\theta\,.\cos\theta}{\sin\theta\,.\cos\theta}$$

(iv) LHS $= \frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}} \quad \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$

$$= \frac{1+\cos A}{\cos A} \times \frac{\cos A}{1} = 1+\cos A$$

$$= 1+\cos\theta \times \frac{1-\cos A}{1-\cos A} = \frac{1-\cos^2 A}{1-\cos A} = \frac{\sin^2 A}{1-\cos A} = \text{RHS}$$

(v) LHS $= \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{\frac{\cos A - \sin A + 1}{\sin A}}{\frac{\cos A + \sin A - 1}{\sin A}}$

$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cos A + 1 - \operatorname{cosec} A}$ 9

$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1}$ [∵ $\operatorname{cosec}^2 A - \cot^2 A = 1$]

$= \frac{(\cot A + \operatorname{cosec} A) - [(\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)]}{\cot A - \operatorname{cosec} A + 1}$

$= \frac{(\operatorname{cosec} A + \cot A)(1 - \operatorname{cosec} A + \cot A)}{(\cot A - \operatorname{cosec} A - 1)}$

$= \operatorname{cosec} A + \cot A = RHS$

(vi) LHS $= \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sqrt{\frac{1 + \sin A}{1 - \sin A} \times \frac{1 + \sin A}{1 + \sin A}}$

$= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} = \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} = \frac{1 + \sin A}{\cos A}$

$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A \; 9 \tan A = RHS$

(vii) LHS $= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$

$= \tan\theta\left(\frac{1 - 2\sin^2\theta}{2 - 2\sin^2\theta - 1}\right) = \tan\theta\left(\frac{1 - 2\sin^2\theta}{1 - 2\sin^2\theta}\right)$

$= \tan\theta = RHS$

(viii) LHS $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A . \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \cos A . \sec A$

$= (\sin^2 A + \operatorname{cosec}^2 A + 2) + (\cos^2 A + \sec^2 A + 2)$ ($\sin A . \operatorname{cosec} A = 1$)

$= (\sin^2 A + \cos^2 A) + (\operatorname{cosec}^2 A + \sec^2 A) + 4$ ($\cos A . \sec A = 1$)

$= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 4$

$= 7 + \tan^2 A + \cot^2 A = RHS$

(ix) LHS $= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \frac{1 - \sin^2 A}{\sin A} \times \frac{1 - \cos^2 A}{\cos A}$

$= \sin A . \cos A = \frac{\sin A . \cos A}{\sin^2 A + \cos^2 A}$

$\frac{\frac{\sin A . \cos A}{\sin A . \cos A}}{\frac{\sin^2 A + \cos^2 A}{\sin A . \cos A}}$ [divide numerator and denominator by sin A. cos A]

$= \frac{1}{\tan A + \cot A} = RHS$

(x) LHS $= \left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$

$\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$

RHS $= \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \left(\frac{1 - \tan A}{1 - \frac{1}{\tan A}}\right)^2$

$= \left(\frac{1 - \tan A}{\frac{\tan A - 1}{\tan A}}\right)^2 = \left(\frac{1 - \tan A \times \tan A}{\frac{\tan A - 1}{\tan A}}\right)^2 = (-\tan A)^2 = \tan^2 A$

LHS = RHS

…….END