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03 KIRCHOFF'S LAWS

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CURRENT LAW [KCL]

KIRCHOFF'S LAWS

VOLTAGE LAW [KVL]

German physicist Kirchhoff extended Ohm's to law to complicated circuits and gave two laws, which **enable us to determine current in any part of such a circuit.**

Current and voltages in a simple electrical circuit can be found by using Ohm's law.

But for complex circuit (containing more than one source of emf, resistor, Capacitor etc), K's law applied to find the current and voltage in different parts of the circuit.

Before understanding these laws, we first define a few terms.

1. Electric network: The term electric network is used for a complicated system of electrical conductors.

2. Junction: Any point in an electric circuit where two or more conductors are joined together is a junction.

3. Loop or Mesh: Any closed conducting path in an electric network is called a loop or mesh.

4. Branch: A branch is any part of the network that lies between two junctions.

Kirchhoff's first law of junction rule: In an electric circuit, the algebraic sum of currents at any junction is zero.

Or, the sum of currents entering a junction is equal to the sum of currents leaving that junction.

Mathematically, this law may be expressed as

$$\sum I = 0$$

SIGN CONVENTION FOR APPLYING JUNCTION RULE:

☞ **1. THE CURRENTS FLOWING TOWARDS THE JUNCTION ARE TAKEN AS POSITIVE.**

☞ **2. THE CURRENTS FLOWING AWAY FROM THE JUNCTION ARE TAKEN AS NEGATIVE.**

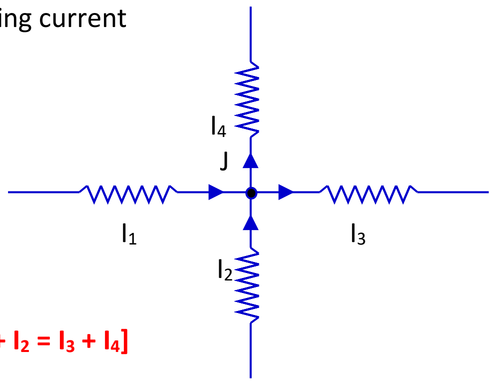
Fig. represents a junction J in a circuit where four currents meet. The currents I_1 and I_2 flowing towards the junction are positive, while the current I_3 and I_4 flowing away from the junction are negative, therefore, by junction rule:

$\sum I = 0$

or $I_1 + I_2 - I_3 - I_4 = 0$

or $I_1 + I_2 = I_3 + I_4$

i.e., Incoming current = Outgoing current



[Junction rule: $I_1 + I_2 = I_3 + I_4$]

➤ K'S 1ST LAW SUPPORT THE LAW OF CONSERVATION OF CHARGE .

Explanation: *This is due to the fact that a point (or junction) cannot acts as a source or sink of charge.*

Consider a point or junction O in an electrical circuit at which a no. of conductors meeting and carrying current I_1, I_2, I_3, I_4 & I_5.

First law is also called Kirchhoff's current law (KCL). It is based on the law of conservation of charge. When currents in a circuit are steady, charges cannot accumulate or originate at any point of the circuit. So whatever charge flows towards the junction in any time interval, an equal charge must flow away from that junction in the same time interval.

Applying K's **Current** law, $+ I_1 + I_2 + (-I_3) + (-I_4) + (-I_5) = 0$

$$I_1 + I_2 = I_3 + I_4 + I_5 \quad \text{——————} \quad \{1\}$$

i.e., $\sum I = 0$

From (1), Let equation (1) be multiplied by small time interval 't', then

$$I_1 t + I_2 t = I_3 t + I_4 t + I_5 t$$

$$Q_1 + Q_2 = Q_3 + Q_4 + Q_5$$

i.e., total charge entering the junction during given time is equal to total charge leaving the junction during the same time interval. (*This* is in accordance with the conservation of charge which is the basis of K's current rule.)

▣ KIRCHHOFF'S SECOND LAW OR LOOP RULE: Around any closed loop of a network, the algebraic sum of changes in potential must be zero. Or, the algebraic sum of the emfs in any loop of a circuit is equal to the sum of the products of currents and resistances in it.

: "It states that the algebraic sum of product of current (I) and resistance (R) of any closed path (loop) of an electrical circuit is equal to the algebraic sum of emf's acting in that part (loop) of the circuit".

i.e., $\sum IR = \sum E$

or, "It states that in any closed path of an electrical circuit (or closed loop), the algebraic sum of potential is zero".

Mathematically, the loop rule may be expressed as

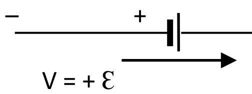
$\Sigma \Delta V = 0$ **OR** $\Sigma \mathcal{E} = \Sigma IR$

➢ **The loop law is based on the law of conservation of energy. Charges circulates around the closed loop in a particular direction because of electrostatic force (conservative force). Work done by the electrostatic force in the closed path is zero i.e., net change in the energy of a charge is zero when it completes its path along a closed loop. (Otherwise, there will be continuous gain in energy around a closed loop in a particular direction, which is not possible.)**

Sign convention for applying loop rule:

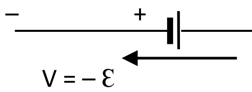
☞1. We can take any direction (clockwise or anticlockwise) as the direction of traversal.

☞2. The emf of cell is taken as positive if the direction of traversal is from its negative to the positive terminal (through the electrolyte).



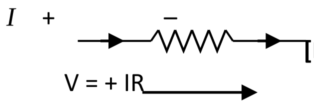
[Positive emf]

☞3. The emf of a cell is taken as negative if the direction of transversal is from its positive to the negative terminal.



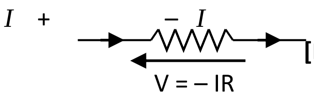
[Negative emf]

☞4. The current-resistance (IR) product is taken as positive if the resistor is traversed in the same direction of assumed current.



[Positive potential drop across a resistor]

☞5. The IR product is taken as negative if the resistor is traversed in the opposite direction of assumed current.



[Negative potential drop across a resistor]

✪ **Sign Conventions :** —— In applying K's 2[nd] law, we have to keep in mind that

◆——►potential falls along the direction of current (convention) in closed path.

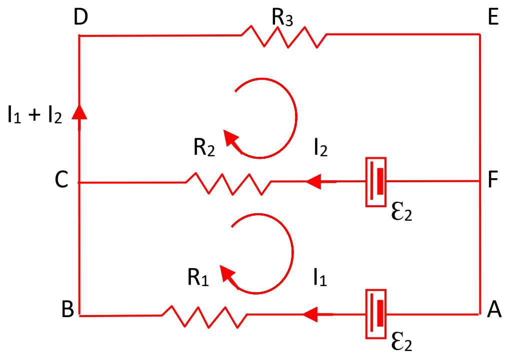
◆——►Potential rises along a path from negative terminal to positive terminal.

[I] The emf of a cell is taken ***NEGATIVE***, if one moves in the direction of increasing potential ***(i.e., from negative pole to positive pole through the cell)*** And emf is taken ***POSITIVE***, if one moves in the direction of decreasing potential ***(i.e., from positive to negative pole of the cell.)***

While traveling in a closed loop, if negative pole of the cell is encountered first, its emf is NEGATIVE otherwise POSITIVE.

[II] The *product of resistance and current (IR) in* an arm of the circuit is taken **positive** if the direction of current in that arm is in the same sense as one move in a closed path. And is taken **negative** if the direction of current in that arm is opposite to the sense as one moves in closed path.

ILLUSTRATION: Let us consider the circuit shown in Fig.



[An electrical circuit]

In fig., traversing in the clockwise direction around the loop ABCFA, we find that:

Algebraic sum of current resistance products

$= I_1R_1 - I_2R_2$

Algebraic sum of emfs $= \mathcal{E}_1 - \mathcal{E}_2$

Applying Kirchhoff's loop rule to closed path ABCFA, we get

$$\mathcal{E}_1 - \mathcal{E}_2 = I_1R_1 - I_2R_2$$

Similarly, applying Kirchhoff's second rule to mesh CDEFC, we get

$$\mathcal{E}_2 = I_2R_2 - (I_1 + I_2) R_3$$

Second law is also called **Kirchhoff's voltage law (KVL)**. It is based on the law of conservation of energy. As the electrostatic force is a conservative force, so the work done by it along any closed path must be zero.

Examples based on Kirchhoff's Laws

- **Formulae Used**

  1. $\sum I = 0$ [Junction rule]

     or Total incoming current = Total outgoing current

  2. $\sum \mathcal{E} = \sum IR$ [Loop rule]

- **Units Used** Current I is in ampere, resistance R in ohm and emf Ɛ in volt.

Q. 1. Network PQRS (Fig.) is made as under: PQ has a battery of 4 V and negligible resistance with positive terminal connected to P, QR has a resistance of 60 Ω. PS has a battery of 5 V and negligible resistance with positive terminal connected to P, RS has a resistance of 200 Ω. If a milliammeter, of 20 Ω resistance is connected between P and R, calculate the reading of the milliammeter.

Sol. Applying Kirchhoff's second law to the loop PRQP, we get

$$20I_1 + 60I = 4 \quad \text{... (i)}$$

Similarly, from the loop PSRP, we get

$$200(I - I_1) - 20I_1 = -5$$

Or $40I - 44I_1 = -1$ ... (ii)

Multiplying (i) by 2 and (ii) by 3, we get

$$120I + 40I_1 = 8 \quad \text{... (iii)}$$

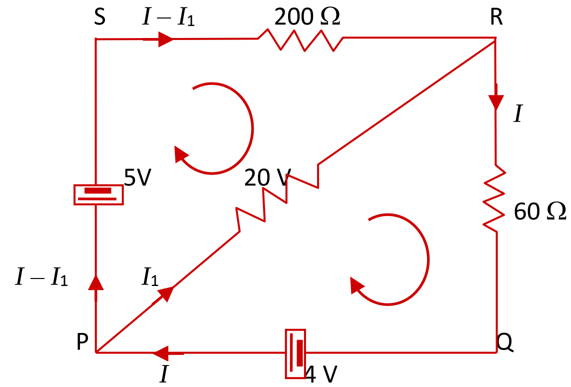
And $120I - 132I_1 = -3$ ... (iv)

Subtracting (iv) from (iii), we get

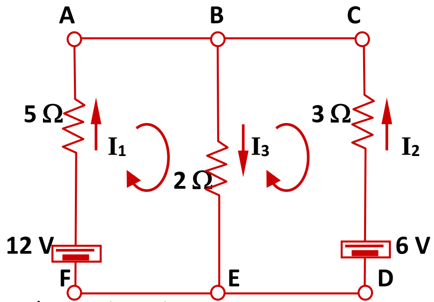
$$172I_1 = 11$$

Or $I = \dfrac{11}{172} = 0.064$ A.

Thus, the milliammeter of 20 Ω will read 0.064 A.



Q. 2. ***Using Kirchhoff's laws in the electrical network shown in Fig. 3.156, calculate the values of I_1, I_2 and I_3.***



Sol. Applying Kirchhoff's first law at junction B,

$I_1 + I_2 = I_3$ ... (1)

Applying Kirchhoff's second law to loops ABEFA and BCDEB, we get

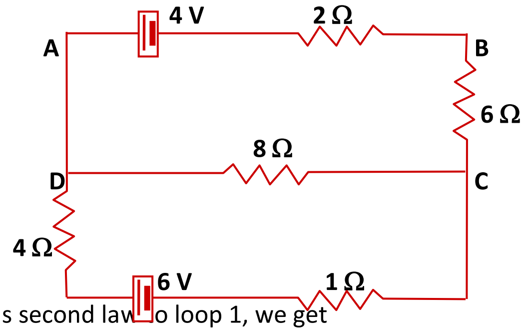
$2I_3 + 5I_1 = 12$ ... (2)

$-2I_3 - 3I_2 = -6$ ... (3)

Solving equations (1), (2) and (3), we get

$I_1 = \frac{48}{31}$ A, $I_2 = \frac{18}{31}$ A, $I_3 = \frac{66}{31}$ A

Q. 3. ***Using Kirchhoff's laws, calculate the potential difference across the 8 ohm resistor.***



Sol. Applying Kirchhoff's second law to loop 1, we get

$(2 + 6)\, I_1 + 8\,(I_1 + I_2) = 4$

$8I_1 + 4I_2 = 2$ ... (1)

From loop 2, we get

$(1 + 4)I_2 + 8\,(I_1 + I_2) = 6$

$8I_1 + 13I_2 = 6$ ... (2)

Subtracting (1) from (2), we get

$9I_2 = 4$

Or $I_2 = \frac{4}{9}$ A

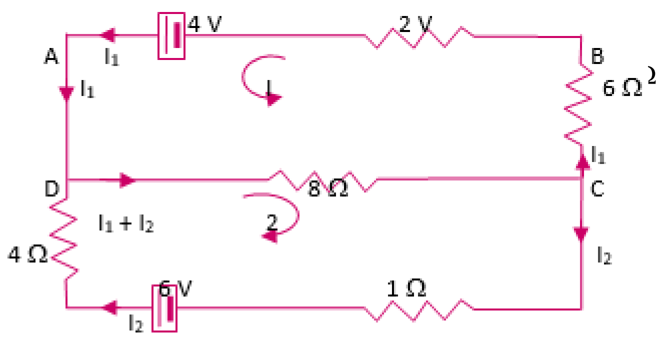
$8I_1 + 13 \times \frac{4}{9} = 6$

Or $I_1 = \frac{2}{8 \times 9} = \frac{1}{36}$ A

∴ $I_1 + I_2 = \frac{1}{36} + \frac{4}{9} = \frac{17}{36}$ A

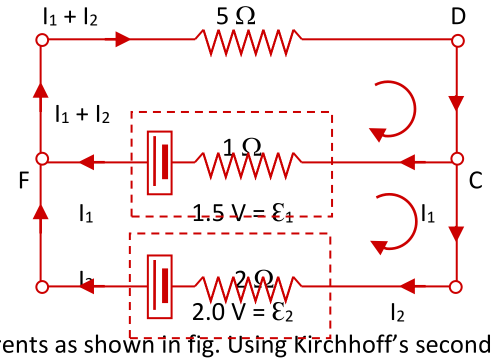
P.D. across 8 Ω resistor

$= 8 \times \frac{17}{36} = 4.25$ A



Q. 4. ***Two cells of e.m.f.s. 1.5 V and 2.0 V and internal resistance 1 Ω and 2 Ω respectively are connected in parallel so as to send current in the same direction through an external resistance of 5 Ω.***
(i) Draw the circuit diagram (ii) p.d. across the 5 Ω resistance.

Sol. (i) The circuit diagram is shown in fig.



(ii) (a) Let $I_1 = I_2$ be the currents as shown in fig. Using Kirchhoff's second law for the loop AFCBA, we get

$$2I_2 - 1I_1 = \mathcal{E}_2 - \mathcal{E}_1 = 2 - 1.5$$

Or $2I_2 - I_1 = 0.5$ ... (1)

For loop CFEDC, we have

$$1I_1 + 5(I_1 + I_2) = \mathcal{E}_1 = 1.5$$

Or $5I_2 + 6I_1 = 1.5$ ... (2)

Solving equations (1) and (2), we get

$$I_1 = \frac{1}{34}\text{ A},\quad I_2 = \frac{9}{34}\text{ A}$$

∴ Current through branch BA,

$$I_1 = \frac{1}{34}\text{ A}$$

Current through branch CF,

$$I_2 = \frac{9}{34}\text{ A}$$

Current through branch DE,

$$I_1 + I_2 = \frac{10}{34}\text{ A}$$

(b) P.D. across the 5 Ω resistance

$$= (I_1 + I_2) \times 5$$

$$= \frac{10}{34} \times 5\text{ V} = 1.47\text{ V}$$

Q. 5. ***Find the current flowing through each cell in the circuit shown in Fig. Also calculate the potential difference across the terminals of each cell.***

Sol. Suppose the current in the cells $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3$ are respectively I_1, I_2, I_3. Applying Kirchhoff's first law at the junction B, we get

$$I_1 + I_2 + I_3 = 0 \quad \text{... (1)}$$

Applying Kirchhoff's second law to the loop AE_1BE_2A, we have

$$I_1 \times 1 - I_2 \times 2 = (10 - 4) \quad \Rightarrow \quad I_1 - 2I_2 = 6 \quad \text{... (2)}$$

Similarly, from the closed loop AE_2BE_3A, we have

$$I_2 \times 2 - I_3 - 1 = 4 - 13 \text{ or } 2I_2 - I_3 = -9$$

But from equation (1), we have

$$I_3 = -(I_1 + I_2)$$

∴ $2I_2 + (I_1 + I_2) = -9$ or $I_1 + 3I_2 = -9$ ... (3)

Solving equations (2) and (3), we get

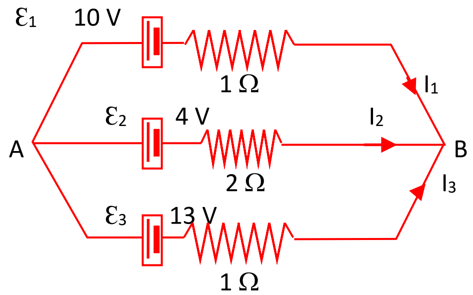
$$I_1 = 0 \text{ and } I_2 = -3\text{ A}$$

I_2 is negative. It means that the direction of I_2 is opposite to the direction indicated in Fig. 3.160.

From (1), $I_3 = -(I_1 + I_2) = 3$ A

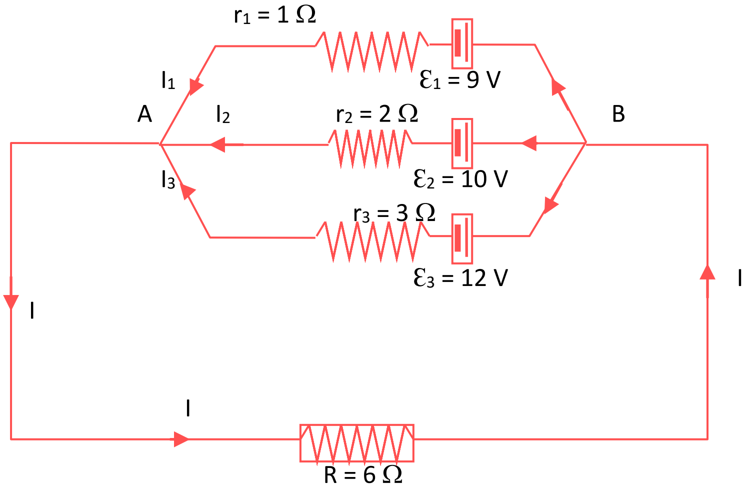
Thus, the current in the 10 V cell is zero. The current given by the 13 V cell to the circuit is 3 A, and the current taken by the 4 V cell from the circuit is 3 A.

As there is not current in the 10 V cell, so the potential difference across its ends is equal to its emf. i.e., 10 V. Since all the three cells are in parallel, the potential difference across the terminals of each is 10 V.



Q. 6. ***Three batteries of emfs 9, 10 and 12 V and internal resistance of 1, 2 and 3 Ω respectively are connected in parallel with each other. The combination sends current through an external resistance of 6 Ω. Find the current through 6 Ω resistances and through each battery.***

Sol. The circuit arrangement is shown in Fig. 3.161. Let I_1, I_2 and I_3 be the currents drawn from the three batteries. Applying Kirchhoff's first law at junction A, we get $I_1 + I_2 + I_3 = I$



Applying Kirchhoff's second law to the loop AE_1 BRA, we get

$I_1 \times 1 + 6 \times I = 9$

Or $I_1 + 6I = 9$ ... (1)

From the loop AE_2 BRA, we have

$I_2 \times 2 + 6 \times I = 10$

Or $I_2 + 3I = 5$ ... (2)

Similarly, from the loop AE_3 BRA, we have

$I_3 \times 3 + 6 \times I = 12$

$I_3 + 2I = 4$ ... (3)

Adding equations (1), (2) and (3), we get

$(I_1 + I_2 + I_3) + 11I = 18$ or $I + 11I = 18$

Or $12I = 18$

∴ $I = \frac{18}{12} = 1.5$ A

= Current through 6 Ω resistance.

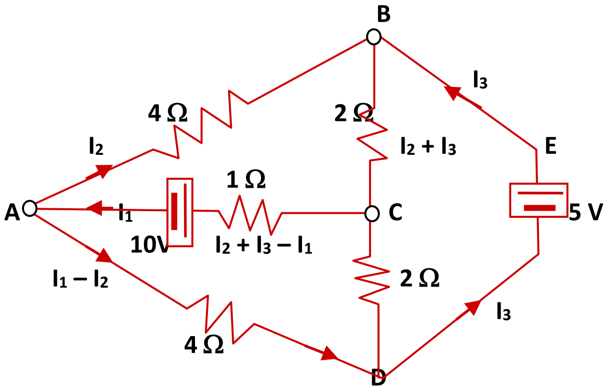
Currents through different batteries are

$I_1 = 9 - 6I = 9 - 6 \times 1.5 = 0$

$I_2 = 5 - 3I = 5 - 3 \times 1.5 = 0.5$ A

And $I_3 = 4 - 2I = 4 - 2 \times 1.5 = 1.0$ A

Q. 7. Determine the current in each branch of the network shown in the fig.



Sol. Let I_1, I_2 and I_3 be the currents as shown in Fig. Kirchhoff's second rule for the closed loop ADCA gives

$10 - 4(I_1 - I_2) + 2(I_2 + I_3 - I_1) - I_1 = 0$

Or $7I_1 - 6I_2 - 2I_3$ ... (1)

For the closed loop ABCA, we get

$10 - 4I_2 - 2(I_2 + I_3) - I_1 = 0$

$I_1 + 6I_2 + 2I_3 = 10$ ... (2)

For the closed loop BCDEB, we get

$5 - 2(I_2 + I_3) - 2(I_2 + I_3 - I_1) = 0$

Or $2I_1 - 4I_2 - 4I_3 = -5$ ... (3)

On solving equations (1), (2) and (3), we get

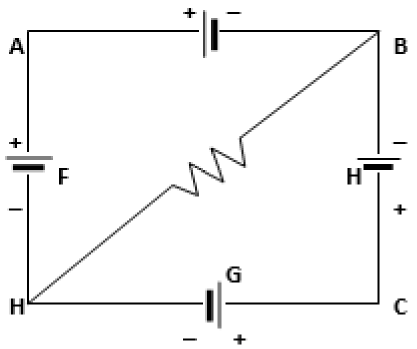
$I_1 = 2.5$ A, $I_2 = \frac{5}{8}$ A, $I_3 = 1\frac{7}{8}$ A

The currents in the various branches of the network are:

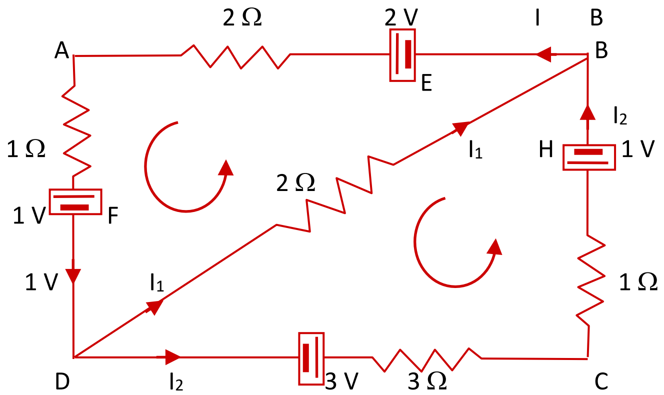
$I_{AB} = \frac{5}{8}$ A; $I_{CA} = 2\frac{1}{2}$ A; $I_{DEB} = 1\frac{7}{8}$ A

$I_{AD} = 1\frac{7}{8}$ A; $I_{CD} = 0$; $I_{BC} = 2\frac{1}{2}$ A

Q. 8. In the circuit shown in Fig. E, F, G and H are cells of emf 2 V, 1 V, 3 V and 1 V, and their internal resistance are 2 Ω, 1 Ω, 3 Ω and 1 Ω, respectively. Calculate (i) the potential difference between B and D and (ii) the potential difference across the terminals of each of the cell's G and H.



Sol. In Fig. the network has been redrawn showing the emfs and internal resistance of the cells explicitly.



(i) Applying Kirchhoff's first law at junction D, we get

$I = I_1 + I_2$ … (i)

Applying Kirchhoff's second law to loop ADBA, we get

$2I + I + 2I_1 = = 2 - 1$

Or $3I + 2I_1 = 1$ … (ii)

On solving equations (i), (ii) and (iii) we get

$I_1 = -\frac{1}{13}$ A, $I_2 = \frac{6}{13}$ A and $I = \frac{5}{13}$ A

P.D. between the points B and D,

$V_1 = I_1 \times 2 = \frac{2}{13}$ V

(ii) P.D. between the terminals of G (giving current),

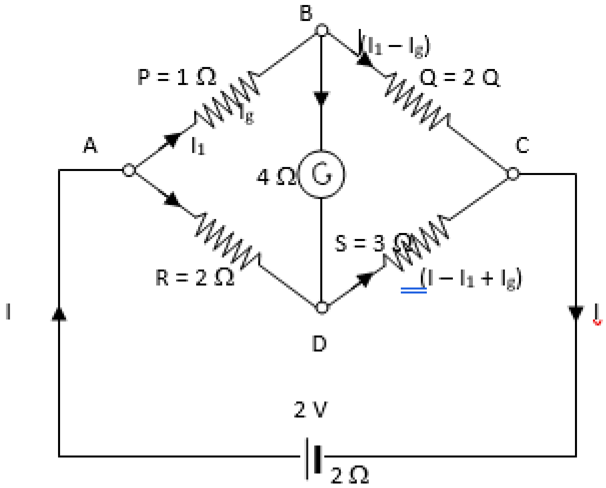
$V_2 = \mathcal{E} - I_2 r = 3 - \frac{6}{13} \times 3 = 1.615$ V

P.D. between the terminals of H (taking current),

$V_3 = \mathcal{E}' + I_2 r' = 1 + \frac{6}{13} \times 1 = 1.46$ V

Q. 9. In a Wheatstone bridge, P = 1 Ω, Q = 2 Ω, R = 2 Ω, S = 3 Ω and R_g = 4 Ω. Find the current through the galvanometer in the unbalanced position of the bridge, when a battery of 2 V and internal resistance 2 Ω is used.

Sol. The circuit for the given Wheatstone bridge is shown in Fig. Let I, I_1 and I_g be the currents as shown.



Applying Kirchhoff's second law to loop ABDA,

We get, $I_1 \times 1 + I_g \times 4 - (I - I_1) \times 2 = 0$

Or $3I_1 - 2I + 4I_g = 0$ ... (1)

Applying Kirchhoff's second law to loop BCDB,

We get $(I_1 - I_g) \times 2 - (I - I_1 + I_g) \times 3 - I_g \times 4 = 0$

$5I_1 - 3I - 9I_g = 0$ ... (2)

Applying Kirchhoff's second law to loop ADCEA, we get

$2(I - I_1) + 3(I - I_1 + I_g) + 2I = 2$

Or $-5I_1 + 7I + 3I_g = 2$ ... (3)

Adding (2) and (3), $4I - 6I_g = 2$ ... (4)

Multiplying (1) by 5 and (2) by 3 and subtracting we get

$-I + 47 I_g = 0$ or $I = 47 I_g$

From (4)

$4 \times 47I_g - 6I_g = 2$ or $182 I_g = 2$

∴ $I_g = \frac{2}{182} = \frac{1}{91}$ A

Q. 10. The four arms of a Wheatstone bridge have the following resistances:

AB = 100 Ω, BC = 10 Ω, CD = 5 Ω and DA = 60 Ω

A galvanometer of 15 Ω resistance is connected across BD, Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC.

Sol. Applying Kirchhoff's second law to loop BADB, we get

$100 I_1 + 15 I_g - 60 I_2 = 0$

Or $20I_1 + 3I_g - 12I_2 = 0$ ... (1)

Considering the loop BCDB, we get

$10 (I - I_g) - 15I_g - 5 (I_2 + I_g) = 0$

$10I_1 - 30I_g - 5I_2 = 0$

$2I_1 - 6I_g - I_2 = 0$ ... (2)

Considering the loop ADCEA, we get

$60I_2 + 5 (I_2 + I_g) = 10$

$65I_2 + 5I_g = 10$

$13I_2 + I_g = 2$ ... (3)

Multiplying Eq. (2) by 10, we get

$20I_1 - 60I_g - 10I_2 = 0$ ... (4)

From equations (1) and (4), we get

$63I_g - 2I_2 = 0$

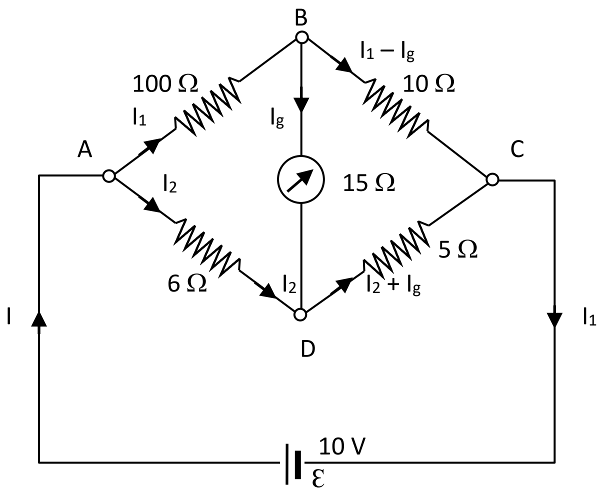
$I_2 = \frac{63}{2} I_g = 31.51 g$

Substituting the value of I_2 in Eq. (3), we get

$13 (31.51g) + I_g = 2$

Or $410.5I_g = 2$

Or $I_g = \frac{2}{410.5}$ A $= 4.87$ mA.



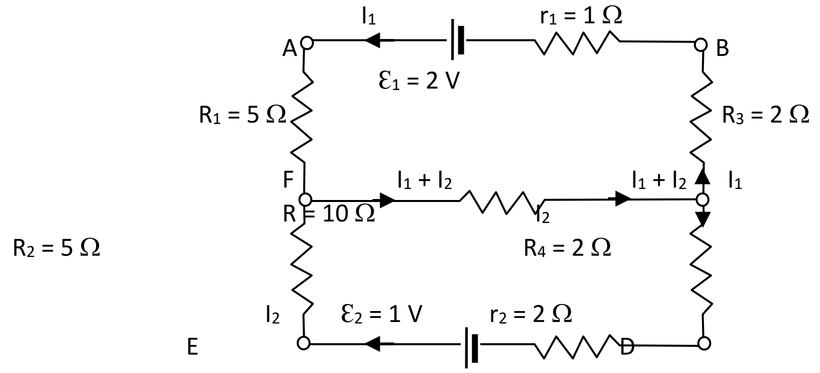
Q. 11. ***Two cells of emfs 1.5 V and internal resistances 2 Ω and 1 Ω respectively have their negative terminals joined by a wire of 6 Ω and positive terminals by a wire of 4 Ω resistance. A third resistance wire of 8 Ω connects middle points of these wires. Draw the circuit diagram. Using Kirchhoff's laws, find the potential difference at the end of this third wire.***

Sol. As shown in Fig. the positive terminals of cells $\mathcal{E}_1$ and $\mathcal{E}_2$ are connected to the wire AE of resistances 4 Ω and negative terminals to the wire BD of resistance 6 Ω. The 8 Ω coil is connected between the middle points F and C of the wires AE and BD respectively.

∴ $R_1 = R_2 = \frac{4}{2} = 2\ \Omega$

And $R_3 = R_4 = \frac{6}{2} = 3\ \Omega$

The distribution of current in various branches is shown in Fig.



Applying Kirchhoff's second law to the loop ABCFA, we get

$I_1 \times r_1 + I_1 \times R_1 + (I_1 + I_2) R + I_1 \times R_3 = \mathcal{E}_1$

$I_1 \times 2 + I_1 \times 2 + (I_1 + I_2) \times 8 + I_1 \times 3 = 1.5$

$15 I_1 + 8I_2 = 1.5$ ... (i)

Applying Kirchhoff's second law to the loop CDEFC, we get

$I_2 \times r_2 + I_2 \times R_2 + (I_1 + I_2) R + I_2 \times R_4 = \mathcal{E}_2$

$I_2 \times 1 + I_2 \times 2 + (I_1 + I_2) \times 8 + I_2 \times 3 = 2$

$8I_1 + 14I_2 = 2$

Or $4I_1 + 7I_2 = 1$ ... (ii)

On solving equations (i) and (ii), we get

$I_1 = \frac{5}{146}$ A and $I_2 = \frac{18}{146}$ A

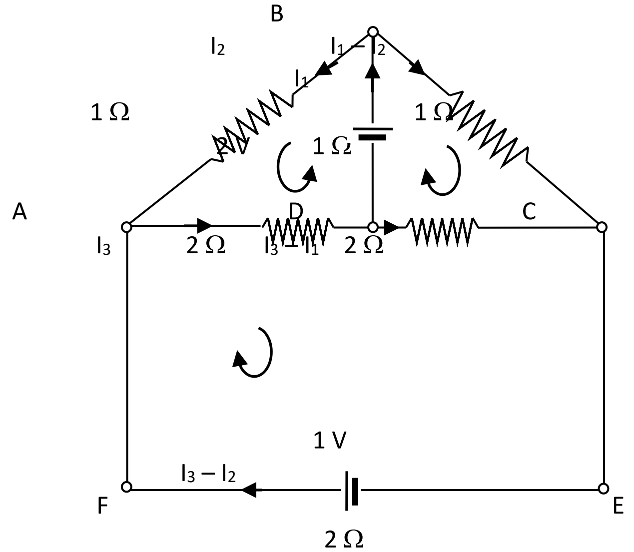
Current through the 8 Ω resistance wire is

$I_1 + I_2 = \frac{5}{146} + \frac{18}{146} = \frac{23}{146}$ A

P.D. across the ends of 8 Ω resistance wire $= \frac{23}{146} \times 8 = 1.26$ V.

Q. 12. ***AB, BC, CD and DA are resistors of 1, 1, 2 and 2 Ω respectively connected in series. Between A and C is a 1 volt cell of resistance 2 Ω, A being positive. Between B and D is a 2 V cell of 1 Ω resistance, B being positive. Find the current in each branch of the circuit.***

Sol. The circuit arrangement and current distribution is shown in Fig.



Applying Kirchhoff's second law to loops BADB, BCDB and ADCEFA, we get

$1 \cdot I_2 + 2 \cdot I_3 + 1 \cdot I_1 = 2$

Or $I_1 + I_2 + 2\,I_3 = 2$ ... (1)

Or $1\,(I_1 - I_2) - 2\,(I_3 - I_1) + I_1 = 2$

Or $4\,I_1 - I_2 - 2\,I_3 = 2$ ... (2)

And $2\,I_3 + 2\,(I_3 - I_1) + 2\,(I_3 - I_2) = 1$

Or $-2\,I_1 - 2\,I_2 + 6\,I_3 = 1$ ... (3)

Solving equations (1), (2) and (3), we get

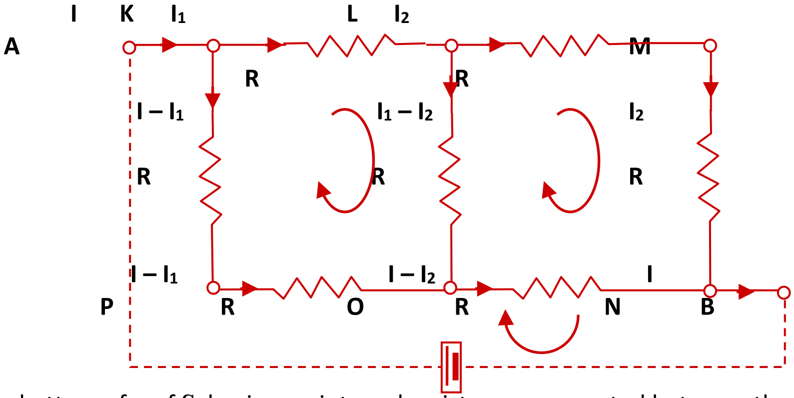
$I_1 = 0.8$ A, $I_2 = 0.2$ A and $I_3 = 0.5$ A

Currents in different branches are

$I_{AB} = I_2 = 0.2$ A; $I_{BC} = I_1 - I_2 = 0.6$ A;

$I_{CD} = I_1 - I_3 = 0.3$ A; $I_{AD} = I_3 = 0.5$ A; $I_{EF} = I_3 - I_2 = 0.3$ A;

Q. 13.Find the equivalent resistance between the terminals A and B in the network shown in Fig. Given each resistor R is of 10 Ω.



Sol. Imagine a battery of emf ℰ, having no internal resistance, connected between the points A and B. The 9 distribution of current through various branches is as shown in Fig. Applying Kirchhoff's second law to loop KLOPK, we get

$I_1R + (I_1 - I_2)R - 2\,(I - I_1)\,R = 0$

Or $4I_1$ d $I_2 = 2\,I$ ... (1)

Similarly, from the loop LMNOL, we have

$2I_2R - (I - I_2)\,R - (I_1 - I_2)\,R = 0$

Or $-I_1 + 4\,I_2 = I$ ... (2)

From the loop AKPONBEA, we have

$2\,(I - I_1)\,R + (I - I_2)\,R = \mathcal{E}$ ... (3)

Solving equations (1) and (2), we get

$I_1 = \frac{3}{5}I$ and $I_2 = \frac{2}{5}I$

Substituting these values in equation (3), we get

$2\left(I - \frac{3}{5}I\right)R + \left(I - \frac{2}{5}I\right)R = \mathcal{E}$

$\frac{7}{5}\,IR = \mathcal{E}$ ... (4)

If R' is the equivalent resistance between A and B, then

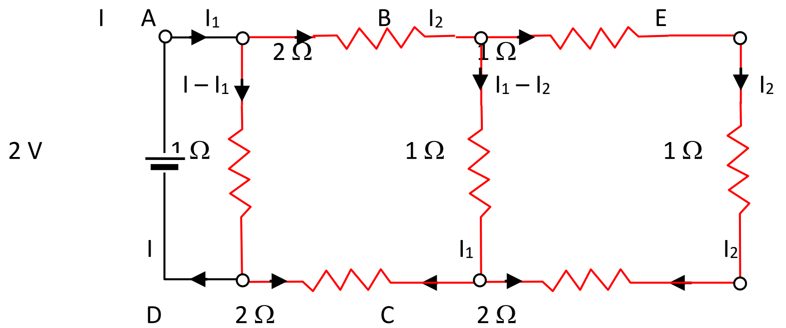
$IR' = \mathcal{E}$ ... (5)

From (4) and (5),

$IR' = \frac{7}{5}\,IR$ or $R' = \frac{7}{5}R = \frac{7}{5} \times 10 = 14\ \Omega$

Q. 14. Two squares ABCD and BEFC have the side BC in common. The sides are of conducting wires with resistances as follows: AB, BE, FC and CD each 2 Ω; AD, BC, EF each 1 Ω. A cell of emf 2 V and internal resistance 2 Ω is joined across AD. Find the currents in various branches of the circuit.

Sol. The current distribution in various branches of the circuit is shown in Fig.



Applying Kirchhoff's second law to the loop containing the cell and AD, we get

$2 \times I + 1 \times (I - I_1) = 2$ or $3\,I - I_1 = 2$ ... (1)

From the loop ABCDA, we get

$2 \times I_1 + 1 \times (I_1 - I_2) + 2 \times I_1 - 1 \times (I - I_1) = 0$

Or $\quad -I + 6 I_1 - I_2 = 0$ ... (2)

Similarly, from the loop BEFCB we get

$2 \times I_2 + 1 \times I_2 + 2 \times I_2 - 1 \times (I_1 - I_2) = 0$

Or $\quad -I_1 + 6 I_2 = 0$ ... (3)

Solving equations (1), (2) and (3), we get

$I = \frac{70}{99}$ A, $\quad I_1 = \frac{12}{99}$ A, $\quad I_2 = \frac{2}{99}$ A

Currents in different branches are

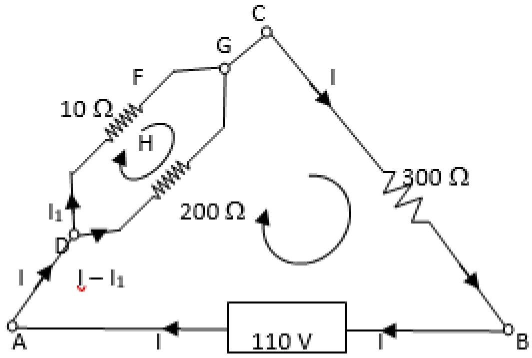
$I_{AB} = I_{CD} = I_1 = \frac{12}{99}$ A, $\quad I_{BE} = I_{EF} = I_{CF} = I_2 = \frac{2}{99}$ A

$I_{AD} = I - I_1 = \frac{58}{99}$ A, $\quad I_{BC} = I_1 - I_2 = \frac{10}{99}$ A

Current through the cell = $I = \frac{70}{99}$ A

Q. 15. *Two points A and B are maintained at a constant potential difference of 110 V. A third point is connected to A by two resistance of 100 and 200 Ω in parallel, and to B by a single resistance of 300 Ω. Find the current in each resistance and the potential difference between A and C and between C and B.*

Sol. The circuit arrangement and the current distribution is shown in Fig.



Applying Kirchhoff's second law to the loop DEFGHID, we get

$I_1 \times 100 - (I - I_1) \times 200 = 0$ 10

Or $\quad 300 I_1 - 200 I = 0$ ... (1)

Similarly, from loop ADIHCGBA, we get

$(I - I_1) 200 + I \times 300 = 110$

Or $\quad 500 I - 200 I_1 = 110$ ... (2)

Solving equations (1) and (2), we get

$I = \frac{3}{10}$ A $\quad$ and $\quad I_1 = \frac{1}{5}$ A

∴ Current through 100 Ω resistance = $I_1 = \frac{1}{5}$ A

Current through 200 Ω resistance = $I - I_1 = \frac{1}{10}$ A

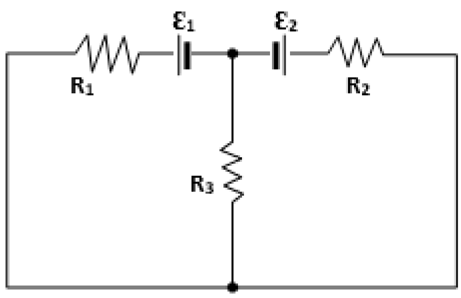
Current through 300 Ω resistance = $I = \frac{3}{10}$ A

P.D. between A and C = P.D. across 100 Ω resistor= $I_1 \times 100 = \frac{1}{5} \times 100 = 20$ V

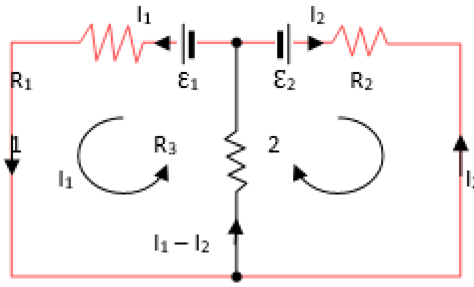
P.D. between C and B = P.D. across 300 Ω resistor

$= I \times 300 = \frac{3}{10} \times 300 = 90$ V

Q. 16. Determine the currents through the resistors R_1, R_2 and R_3 shown in Fig.



Sol. Suppose currents I_1 and I_2 start from the cells of emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ respectively, as shown in fig 2. Then currents through the three resistors R_1, R_2 and R_3 will be I_1, I_2 and $I_1 + I_2$ respectively.



[Fig 2]]

Applying Kirchhoff's second law to the loops 1 and 2, we get

$R_1I_1 + R_3(I_1 + I_2) = \mathcal{E}_1$ or $(R_1 + R_3) I_1 + R_3 I_2 = \mathcal{E}_1$ ... (1)

And $R_2I_2 + R_3(I_1 + I_2) = \mathcal{E}_2$ or $(R_2 + R_3) I_2 + R_3I_1 = \mathcal{E}_2$ ... (2)

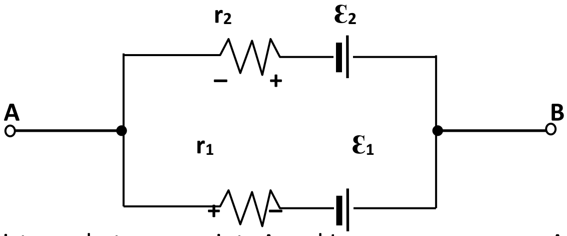
To eliminate I_2, multiply (1) multiply (1) by $(R_2 + R_3)$ and (2) by R_3 and subtract. Then

$$(R_1 + R_3)(R_2 + R_3) I_1 - R_3^2 I_1 = \mathcal{E}_1(R_2 + R_3) - \mathcal{E}_2R_3$$

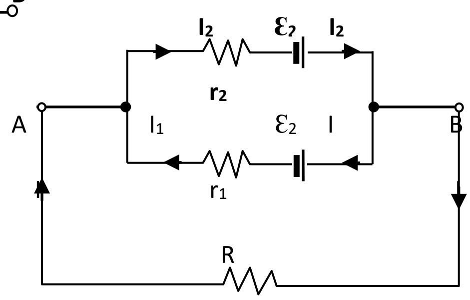
Or $$I_1 = \frac{\mathcal{E}_1(R_2 + R_3) - \mathcal{E}_2 R_3}{(R_1 + R_3)(R_2 + R_3) - R_3^2} = \frac{\mathcal{E}_1(R_2 + R_3) - \mathcal{E}_2 R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

∴ $$I_1 + I_2 = \frac{\mathcal{E}_1 R_2 + \mathcal{E}_2R_1}{R_1R_2 + R_2R_3 + R_3R_1}$$

Q. 17. *Find the emf ($\mathcal{E}_0$) and internal resistance (r_0) of a battery which is equivalent to a parallel combination of two batteries of emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ and internal resistance r_1 and r_2 respectively, with polarities as shown in Fig.*



Sol. Supposing we connect a resistance between points A and B. Then the circuit will be of the form as shown in Fig.



Applying Kirchhoff's first law at junction A,

$I = I_1 - I_2$ ... (1)

Applying Kirchhoff's second law

$\mathcal{E}_1 = I_1r_1 + IR$

Or $IR = \mathcal{E}_1 - I_1R_1$ ... (2)

And $\mathcal{E}_2 = Ir_2 - IR$

Or $IR = -\mathcal{E}_2 + I_2 r_2$ ... (3)

From (1) and (3), $IR = -\mathcal{E}_2 + (I_1 - I) r_2$

Or $I(R + r) = -\mathcal{E}_2 + I_1 r_2$ ... (4)

Multiplying (2) by r_2 and (4) by r_1,

$IR r_2 + I(R + r_2) r_1 = \mathcal{E}_1r_2 - \mathcal{E}_2r_1$

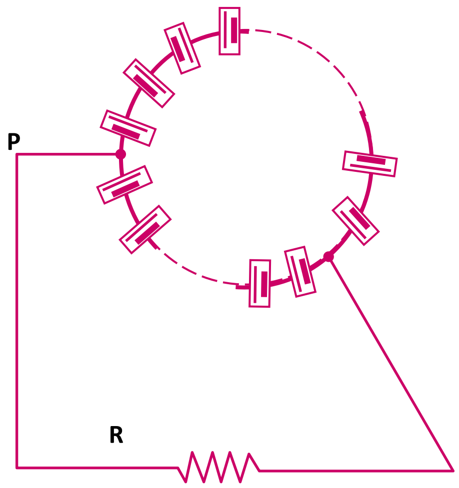
Or $$I = \frac{\mathcal{E}_1r_2 - \mathcal{E}_2r_1}{R(r_1 + r_2) + r_1r_2} = \frac{(\mathcal{E}_1r_2 - \mathcal{E}_2r_1)/(r_1 + r_2)}{R + \frac{r_1r_2}{r_1 + r_2}} = \frac{\mathcal{E}_0}{R + r_0}$$

Clearly, $\mathcal{E}_0 = \frac{\mathcal{E}_1r_2 - \mathcal{E}_2r_1}{r_1 + r_2}$ = emf of the battery required

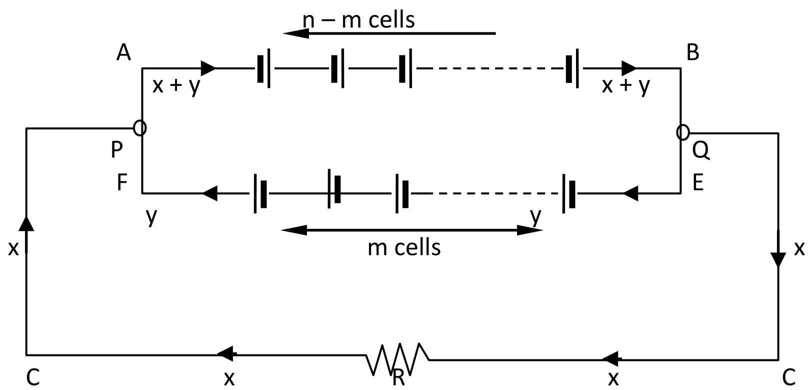
and $r_0 = \frac{r_1 r_2}{r_1 + r_2}$

= internal resistance of the battery required.

Q. 18. ***'n' cells, each of emf 'e' and internal resistance 'r' are connected in a closed circuit so that the positive terminal of a cell is joined to the negative terminal of the next, as shown in Fig. Any two points of the circuit are connected by an external resistance R. Find the current in R.***



Sol. Suppose m cells are connected on one side of R and remaining (n – m) cells on the other side of R. Then the equivalent circuit will be of the form shown in Fig. The distribution of current is shown.



Applying Kirchhoff's second law to the loop ABQCDPA, we get

$(n - m)\,e = r\,(n - m)\,(x + y) + xR$ ... (1)

Similarly, from the loop QEFPDCQ, we get

$me = - mry - xR$ or $mry = me + xR$

or $y = \dfrac{me + xR}{mr}$

Substituting this value in (1), we get

$(n - m)\,e = r\,(n - m)\left(x + \dfrac{me + xR}{Mr}\right) + xR$

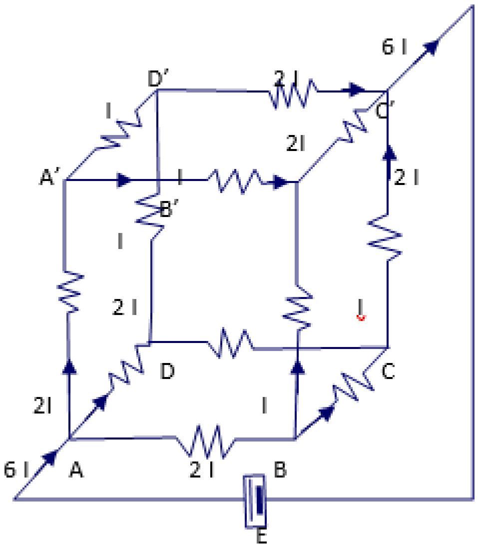
On simplifying, we get,

$x\,(mrn - rm^2 + nR) = 0$ $\therefore$ $x = 0$

i.e., Current through R is zero.

Q. 19 ***A battery of 10 V and negligible internal resistance is connected across the diagonally opposite corners of a cubical network consisting of 12 resistance of the network and the current along each edge of the cube.***

Sol. Let 6I be the current through the cell. Since the paths AA', AD and AB are symmetrically placed, current through each of them IS same, i.e., 2I. At the junctions A', B and D the incoming current 2I splits equally into the two outgoing branches, the current through each branch is I, as shown in Fig. At the junctions B' C and D', these currents reunite and the currents along B'C', D'C' and CC' are 2I each. The total current at junction C' is 6I again.



Applying Kirchhoff's second law to the loop ABCC'EA, we get

$-2\,IR - IR - 2IR + \mathcal{E} = 0$ or $\mathcal{E} = 5\,IR$

Where R is the resistance of each edge and $\mathcal{E}$ is the emf of the battery.

∴ The equivalent resistance of the network is

$$R' = \frac{\text{Total emf}}{\text{Total current}} = \frac{\mathcal{E}}{6\,I} = \frac{5\,IR}{6\,I} = \frac{5}{6}R$$

But $R = 1\,\Omega$ ∴ $R' = \frac{5}{6}\,\Omega$

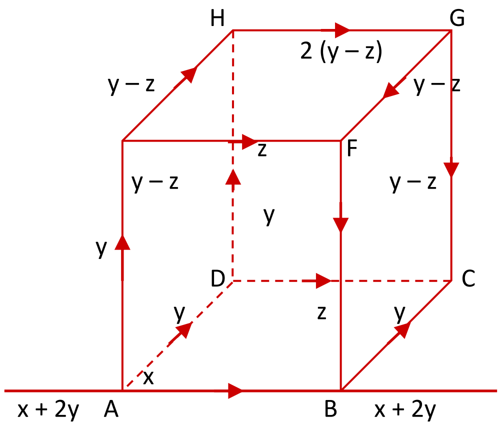
Total current in the network is

$6I = \frac{\mathcal{E}}{R'} = \frac{10}{\frac{5}{6}} = 12$ A or $I = 2$A

The current flowing in each branch can be read off easily.

Q. 20. Twelve wire each having a resistance of r Ω are connected to form a skeleton cube; find the resistance of the cube between the two corners of the same edge.

Sol. Let a current x + 2y enter the junction A of the cube ABCDEFGH. From the symmetry of the parallel paths, current distribution will be as shown in Fig.



Applying Kirchhoff's second law to the loop DHGCD, we get

$(y - z)\,r + 2\,(y - z)\,r + (y - z)r + (y - z)\,r - z\,r = 0$

Or $4y\,r - 5z\,r = 0$ or $5z = 4y$ or $z = \frac{4}{5}y$

Applying Kirchhoff's second law to the loop ABCDA, we get

$xr - yr = zr - yr = 0$ or $x - 2y - z = 0$

or $x - \left(2y - \frac{4}{5}y\right) = 0$ ∵ $z = \frac{4}{5}y$

Or $x = \frac{14}{5}y$; $y = \frac{5}{14}x$

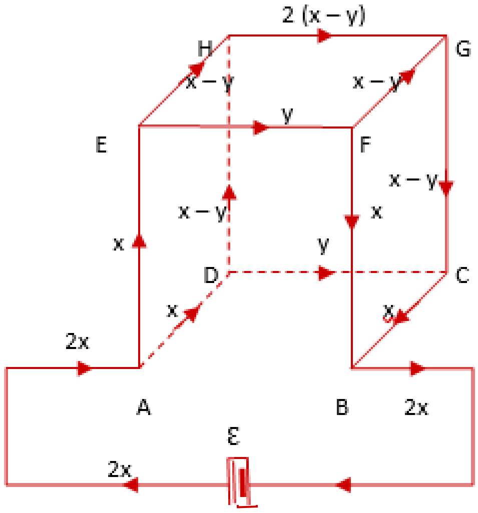
Let R be the resistance across AB. Then

P.D. across AB = xr i.e., $(x + 2y)\,R = xr$

Or $\left(x + \frac{10}{14}x\right) R = xr$ or $\frac{12}{7}R = r$ Hence, $R = \frac{7}{12}\,r\,\Omega$

Q. 21. Eleven equal wires each of resistance r form the edges of an incomplete cube. Find the total resistance from on end of the vacant edge of the cube to the other.

Sol. Let A and B be the vacant edges of the cube. Let an emf $\mathcal{E}$ applied across AB send a current 2x in the circuit. Since the paths AD and AE are symmetrical, the current 2x in the circuit. Since the paths AD and AE are symmetrical, the current 2x at A is divided into two equal parts x and x. At other points, the current is divided as shown in Fig., so that again the currents combine at B to given current 2x. Let R be the total resistance of the cube between A and B.



Applying Kirchhoff's second law to the loop, ABCDA, we get

$xr + yr + xr = \mathcal{E}$

From Ohm's law, $\mathcal{E} = 2x \cdot R$

$\therefore \quad 1xr + yr = 2xR$

Applying Kirchhoff's second law to the loop EFGHE, we get

$yr - (x - y)\,r - 2\,(x - y)\,r - (x - y)\,r = 0$

or $\quad y - x + y - 2x + 2y - x + y = 0$

or $\quad -4x + 5y = 0$

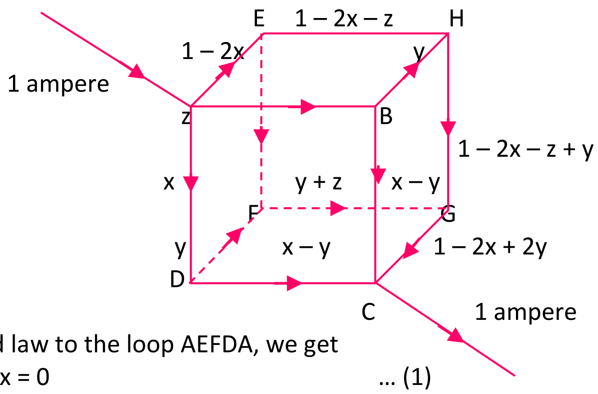
or $\quad y = \frac{4}{5}x$

$\therefore \quad 2xr + \frac{4}{5}xr = 2x\,R \quad$ or $\quad \frac{14r}{5} = 2\,R$

Hence, $\quad R = 1.4\,r\,\Omega$

Q. 22. Twelve wires each having a resistance of 1 Ω are connected to form a cube. Find the resistance of the cube between two corners of a diagonal of one face of a cube.

Sol. Imagine a battery connected between points A and C so that a current of 1 A enters junction A. This current is divided equally along AB and AD. The distribution of current in various branches is shown in Fig. These currents finally add so that a current of 1 A flows out of junction C.



Applying Kirchhoff's second law to the loop AEFDA, we get

$-(1 - 2x) - z + y + x = 0 \quad \ldots (1)$

Similarly, from the loop BHGCB, we have

$-y - (1 - 2x - z + y) - (1 - 2x + 2y) + (x - y) = 0 \quad \ldots (2)$

Again, from the loop FGCDF, we have

$-(y + z) - (1 - 2x + 2y) + (x - y) - y = 0 \quad \ldots (3)$

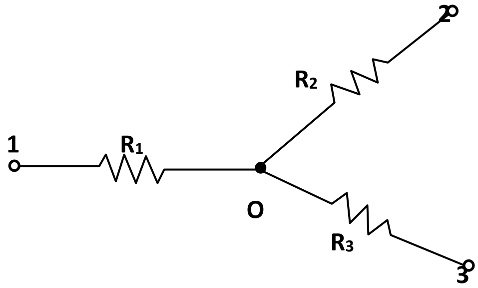
On solving equations (1), (2) and (3), we get

$x = \frac{3}{3}\,A, \quad y = 0, \quad z = \frac{1}{8}\,A$

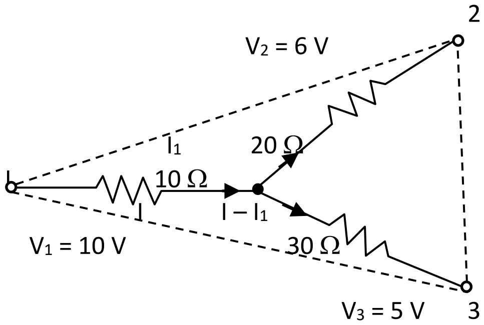
Now, $V_{AC} = V_{AB} + V_{BC} = 1 \times \frac{3}{8} + 1 \times \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$ V

∴ Equivalent resistance between A and C, $R = \frac{V_{AC}}{1} = \frac{3/4}{1} = \frac{3}{4}\ \Omega$

Q. 23 ***Find the current flowing through the resistance R_1 of the circuit shown in Fig. Given $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$ and $R_3 = 30\ \Omega$ and the potentials of points 1, 2 and 3 are $V_1 = 10$ V, $V_2 = 6$ V and $V_3 = 5$ V.***



Sol. The distribution of current for the given network is shown in Fig.



Applying Kirchhoff's second law to the loop 1021, we get

$10I + 20I_1 = 10 - 6$

Or $10I + 20I_1 = 4$ ... (i)

Similarly, from loop 0320, we get

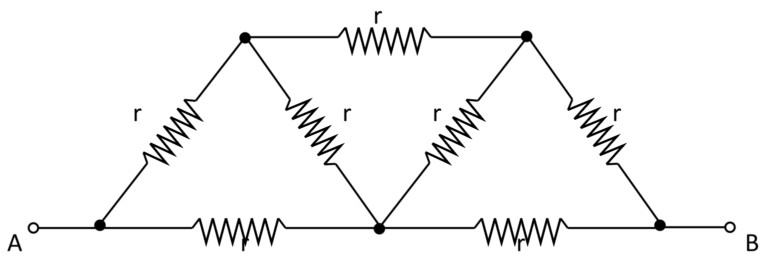
$3(I - I_1) - 20I_1 = 6 - 5$

Or $30I - 50I_1 = 1$ ... (ii)

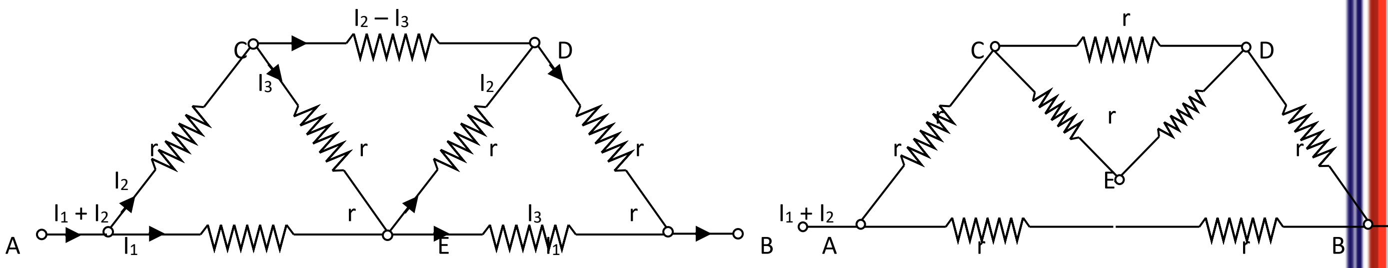
On solving equations (i) and (ii), we get $I = 0.2$ A

Thus, the current flowing through the resistance R_1 is 0.2 A.

Q. 24. ***In the network as shown in Fig. each resistance r is of 2 Ω. Find the effective resistance between points A and B.***



Sol. The distribution of current is shown in Fig. (a). By symmetry, current in arm AE = current in arm EB. As the current in arm CE is equal to the current in arm ED, so the resistance of the network will not be affected if the wire CED is disconnected from the wire AEB at the point E, as shown in Fig. (b).



Resistance of wire ABD $= r + r = 2r$

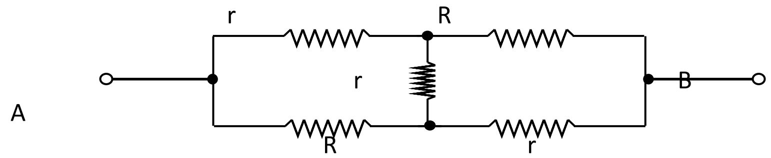
Resistance of wire ACDEB $= r + \frac{2r \times r}{2r + r} + r = \frac{8r}{3}$

As these two resistances are in parallel, so the equivalent resistance R between points A and B is given by

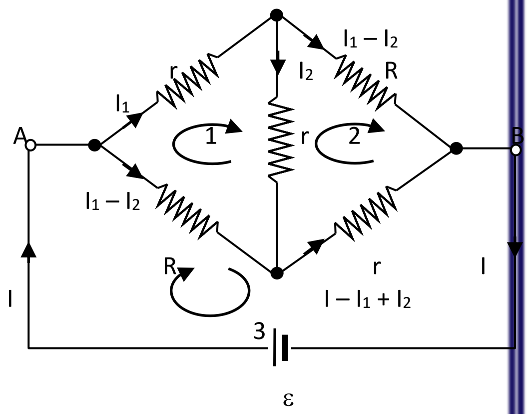
$$\frac{1}{R} = \frac{1}{2r} + \frac{3}{8r} = \frac{7}{8r} \quad \text{or} \quad R = \frac{8r}{7}$$

Given r = 2 Ω, therefore, $R = \frac{8 \times 2}{7} = \frac{16}{7}\,\Omega$

Q. 25. ***Calculate the equivalent resistance between the points A and B in the network shown in Fig.***



Sol. Suppose a cell of emf ℰ is connected between A and B. Then the given circuit can be represented by an unbalanced Wheatstone bridge as shown in Fig. The distribution of current is also shown.



Applying Kirchhoff's second law to the loop1

We get $I_1 r + I_2 r - (I - I_1) R = 0$

or $I_1 (r + R) + I_2 r - IR = 0$ ... (1)

From the loop 2, we have

$(I_1 - I_2) R - (I - I_1 + I_2) r - I_2 r = 0$

or $I_1 (R + r) - I_2 (R + 2r) - Ir = 0$ ... (2)

Solving equation (1) and (2), we get

$$I_1 = \frac{R + r}{R + 3r} I \quad \text{and} \quad I_2 = \frac{R - r}{R + 3r} I \quad \text{... (3)}$$

Similarly, from the loop 3, we have

$(I - I_1) R + (I - I_1 + I_2) r = \mathcal{E}$

or $- I_1 (R + r) + I_2 r + I (R + r) = \mathcal{E}$

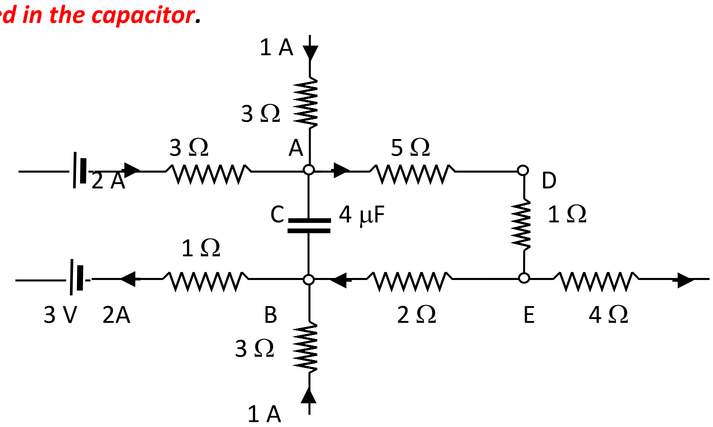
Substituting the values of I_1 and I_2 from equation (3), we get

$$-\frac{(R + r)^2}{R + 3r} I + \frac{(R - r)\,r}{R + 3r} I + (R + r) I = \mathcal{E}$$

or $$\frac{3r R + r^2}{R + 3r} I = \mathcal{E}$$

Equivalent resistance between A and B, $R' = \frac{\mathcal{E}}{I} = \frac{3r R + r^2}{R + 3r} = \frac{r (3R + r)}{(R + 3r)}$

Q. 26. ***A part of the circuit in a steady state along with the currents flowing in the branches, the values of resistance, etc. is shown in the Fig. Calculate the energy stored in the capacitor.***



Sol. In the steady state, when the capacitor is fully charged, there is no current in the arm AB.

Applying Kirchhoff's first law at the junction A, the current in arm AD

= 1 + 2 = 3 A = current in arm DE

Applying Kirchhoff's first law at the junction A, the current in arm EB

= 2 – 1 = 1 A

Now $V_{AD} = 5 \times 3 = 15$ V, $V_{DE} = 1 \times 3 = 3$ V,

$V_{EB} = 2 \times 1 = 2$ V

∴ P.D. across the ends A and B of the capacitor is

$V = V_{AD} + V_{DE} + V_{EB} = 15 + 3 + 2 = 20$ V

Energy stored in the capacitor is

$U = \frac{1}{2} CV^2 = \frac{1}{2} \times (4 \times 10^{-6}) \times (20)^2 = 8 \times 10^{-4}$ J

POTENTIOMETER

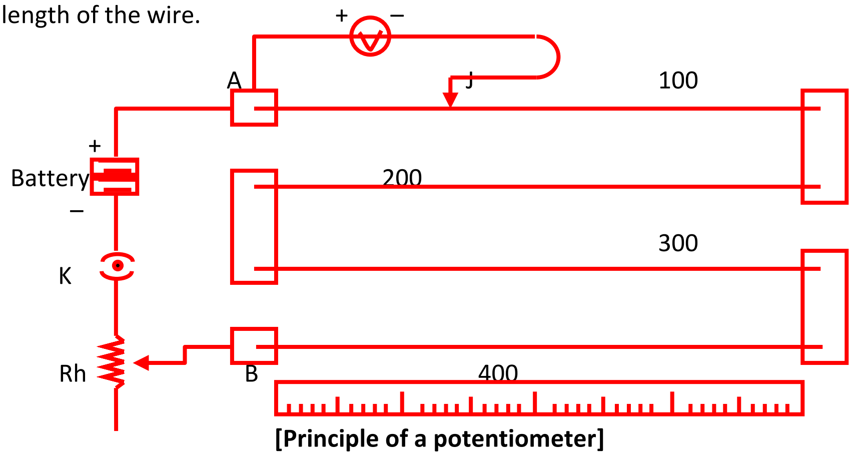
Potentiometer: ***An ideal voltmeter which does not change the original potential difference needs to have infinite resistance***.
But a voltmeter cannot be designed to have an infinite resistance.

☞ **Potentiometer is one such device which does not draw any current from the circuit and still measures the potential difference. So it acts as an ideal voltmeter.**

A potentiometer is a device used to measure an unknown emf or potential difference accurately.

☞ **"Potentiometer is a device commonly used to compare e.m.fs of two cell and to measure internal resistance of cells."**

Construction: A potentiometer consists of a long wire AB of uniform cross-section, usually 4 to 10 m long, of material having high resistivity and low temperature coefficient such as constantan or manganin. Usually, 1 m long separate pieces of wireS are fixed on a wooden board parallel to each other. The wires are joined in series by thick copper strips. A metre scale is fixed parallel to the wires. The ends A and B are connected to a strong battery, a plug key K and rheostat Rh. This circuit, called driving or auxiliary circuit, sends a constant current I through the wire AB. Thus, the potential gradually falls from A to B.
A jockey can slide along the length of the wire.



[Principle of a potentiometer]

☞ The current is kept **constant using by Rheostats (Rh).**

Principle: The basic principle of a potentiometer is that when a constant current flow through a wire of uniforms cross-sectional area and composition, the potential drop across any length of the wire is directly proportional to that length.

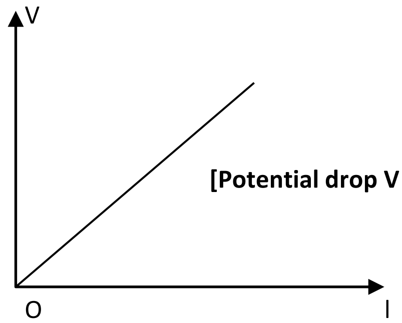
Theory: In Fig. if we connect a voltmeter between the end A and the jockey J, it reads the potential difference V across the length l of the wire AJ. By Ohm's law,

$$V = IR = I.\frac{\rho l}{A} \qquad \left(\because \ R = \rho\frac{l}{A}\right)$$

For the wire of uniform cross-section and uniform composition, resistivity ρ and area of cross-section A are constants. Therefore, when a steady current I flows through the wire.

$$\frac{I\rho}{A} = \text{a constant, } k$$

Hence $V = k\,l$ or $V \propto l$



[Potential drop V ∝ length l.]

This is the principle of a potentiometer.
☞ A graph drawn between V and I will be a straight line passing through the origin O, as shown in Fig.
Potential gradient: The potential drop per unit length of the potentiometer wire is known as potential gradient. It is given by

$$k = \frac{V}{l}$$

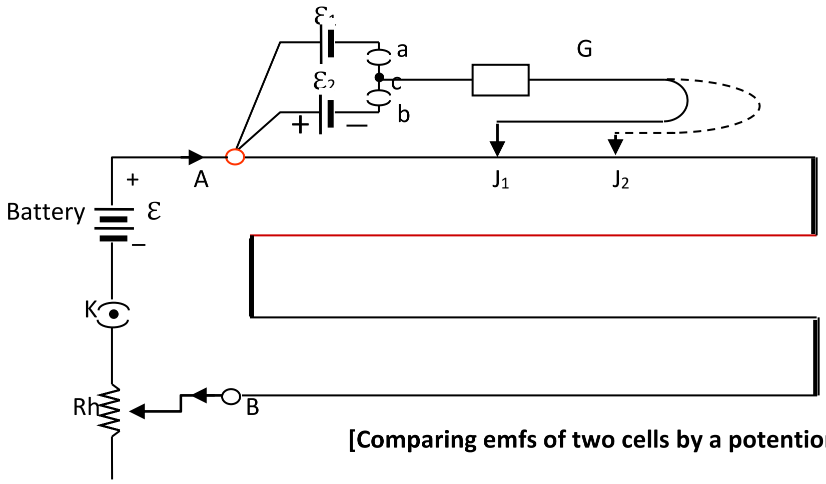
SI unit of potential gradient = Vm^{-1}
Practical unit of potential gradient = $V\ cm^{-1}$

APPLICATIONS OF A POTENTIOMETER

- **A potentiometer can be used to compare the emfs of two primary cells.**
- **A potentiometer can be used to measure the internal resistance of a primary cell.**

[A] COMPARISON OF EMFS OF TWO PRIMARY CELLS:

A constant current is maintained in the potentiometer wire AB by means of a battery of emf Ɛ through a key K and rheostat Rh. Let $Ɛ_1$ and $Ɛ_2$ be the emfs of the two primary cells which are to be compared. The positive terminals of these cells are connected to the end A of the potentiometer wire and their negative terminals are connected to a high resistance box R.B. a galvanometer G and a jockey J through a two-way key.
A high resistance R is inserted in the circuit from resistance box R.B. to prevent excessive currents flowing through the galvanometer.



[Comparing emfs of two cells by a potentiometer]

As the plug is inserted between a and c, the cell $Ɛ_1$ gets introduced in the circuit, The jockey J is moved along the wire AB till the galvanometer shows no deflection. Let the position of the jockey be J_1 and length of wire $AJ_1 = l_1$. If k is the potential gradient along the wire AB, then at null point, $Ɛ_1 = kl_1$

By inserting the plug between b and c, the null point Is again obtained for cell $Ɛ_2$. Let the balancing length be $AJ_2 = l_2$. Then

$$Ɛ_2 = k\, l_2$$

Hence $$\frac{Ɛ_2}{Ɛ_1} = \frac{l_2}{l_1}$$

If one of the two cells is a standard cell of known emf, then emf of the other cell can be determined.

$$Ɛ_2 = \frac{l_2}{l_1} . Ɛ_1$$

In order to get the null point on the potentiometer wire, it is necessary that the emf, Ɛ of the auxiliary battery must be greater than both $Ɛ_1$ and $Ɛ_2$.

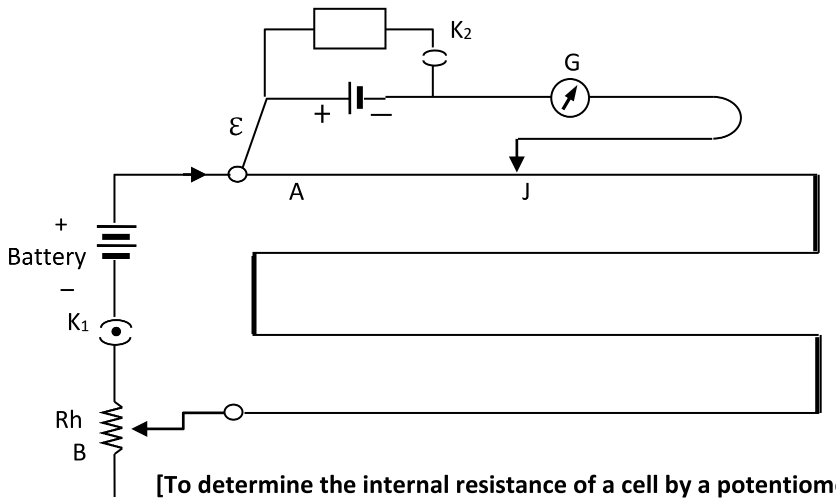
[B] INTERNAL RESISTANCE OF A PRIMARY CELL BY A POTENTIOMETER:

The +ve terminal of the cell whose emf Ɛ is to be measured is connected to the end A of the potentiometer wire and its negative terminal to a galvanometer G and jockey J. A resistance box R.B. is connected across the cell through a key K_2.
Close the key K_1. A constant current flow through the potentiometer wire. With key K_2 kept open, move the jockey along AB till it balances the emf Ɛ of the cell. Let l_1 be the balancing length of the wire. If k is the potential gradient, then emf of the cell will be

$$Ɛ = kl_1$$

With the help of resistance box R.B., introduce a resistance R and close key K_2. Find the balance point for the terminal potential difference V of the cell. If l_2 is the balancing length, then

$$V = kl_2$$



[To determine the internal resistance of a cell by a potentiometer]

∴ $\frac{\varepsilon}{V} = \frac{l_1}{l_2}$

Let r be the internal resistance of the cell. If current I flow through cell when it is shunted with resistance R, then from Ohm's law we get

$\varepsilon = I(R + r)$

and $V = IR$

∴ $\frac{\varepsilon}{V} = \frac{R + r}{R} = \frac{l_1}{l_2}$

$1 + \frac{r}{R} = \frac{l_1}{l_2}$

or $\frac{r}{R} = \frac{l_1 - l_2}{l_2}$

∴ Internal resistance, $r = R\left(\frac{l_1 - l_2}{l_2}\right)$

◙ **A potentiometer preferred over a voltmeter for measuring the emf of a cell?**

Superiority of a potentiometer to a voltmeter: Potentiometer is a null method device. At null point, it does not draw any current from the cell and thus there is no potential drop due to the internal resistance of the cell. It measures the p.d. in an open circuit which is equal to the actual emf of the cell.

On the other hand, a voltmeter draws a small current from the cell for its operation. So it measured the terminal p.d. in a closed circuit which is less than the emf of a cell. That is why a potentiometer is preferred over a voltmeter for measuring the emf of a cell.

SENSITIVENESS OF A POTENTIOMETER

A potentiometer is sensitive if (i) It is capable of measuring very small potential differences, and (ii) It shows a significant change in balancing length for a small change in the potential difference being measured.

The sensitivity of a potentiometer depends on the potential gradient along its wire. Smaller the potential gradient, grater will be the sensitivity of the potentiometer.

The sensitivity of a potentiometer can be increased by reducing the potential gradient. This can be done in two ways:

(i) For a given potential difference, the sensitivity can be increased by increasing the length of the potentiometer wire.

(ii) For a potentiometer wire of fixed length, the potential gradient can be decreased by reducing the current in the circuit with the help of a rheostat.

KNOWLEDGE+..........................

☑ A potentiometer can be regarded as an *ideal voltmeter with infinite resistance* because it does not draw any current from the source of emf at the null point.

☑ The principle of potentiometer requires that (i) the potentiometer wire should be of uniform cross-section and (ii) the current through the wire should remain constant.

☑ The emf of the auxiliary battery must be greater than the emf of the cell to be measured.

☑ The balance point cannot be obtained on the potentiometer if the fall of potential along the potentiometer if the fall of potential along the potentiometer wire due to the auxiliary battery is less than the emf of the cell to be measured.

☑ The positive terminals of the auxiliary of the auxiliary battery and the cell whose emf is to be determined must be connected to the zero end of the potentiometer.

☑ Other uses of a potentiometer: Any physical quantity that can produce or control an potential difference can be measured using a potential difference can be measured using a potentiometer. Thus, a potentiometer can be used to measure and control stress, temperature, radiation, pH, frequency, etc.

Examples based on Potentiometer

♦ **Formulae Used**

1. For comparing e.m.f.s of two cells, $\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{l_2}{l_1}$

2. For measuring internal resistance of a cell,

$$r = \frac{l_1 - l_2}{l_2} \times R$$

3. Potential gradient of the potentiometer wire,

$$k = \frac{V}{l}$$

4. Unknown emf balanced against length l, $\mathcal{E} = kl$

♦ **Units Used**

The emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ are in volt, lengths l_1 and l_2 of potentiometer wire in metre.

Q. 1. ***A potentiometer wire is 10 m long and has a resistance of 18 Ω. It is connected to a battery of emf 5 V and internal resistance 2 Ω. Calculate the potential gradient along the wire.***

Sol. Here $l = 10$ m, $R = 18\ \Omega$, $\mathcal{E} = 5$ V, $r = 2\ \Omega$

Current through the potentiometer wire,

$$I = \frac{\mathcal{E}}{R + r} = \frac{5}{18 + 2} = \frac{5}{20} = \frac{1}{4}\text{ A}$$

∴ Potential gradient $= \frac{IR}{l} = \frac{1}{4} \times \frac{18}{10} = 0.45\ \text{Vm}^{-1}$

Q. 2. ***A potentiometer wire is supplied a constant voltage of 3 V. A cell o emf 1.08 V is balanced by the voltage drop across 216 cm of the wire. Find the total length of the potentiometer wire.***

Sol. Here $\mathcal{E} = 3$ V, $\mathcal{E}_1 = 1.08$ V, $l_1 = 216$ cm, $l = ?$

As $\frac{\mathcal{E}}{\mathcal{E}_1} = \frac{l}{l_1}$ ∴ $l = \frac{\mathcal{E}}{\mathcal{E}_1} \times l_1 = \frac{3 \times 216}{1.08} = 600$ cm

Q. 3. ***In a potentiometer arrangement, a cell of emf 1.20 volt gives a balance point at 30 cm length of the wire. This cell is now replaced by another cell of unknown emf. If the ratio of the emfs of the two cells is 1.5, calculate the difference in the balancing length of the potentiometer wire in the two cases.***

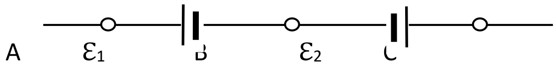
Sol. Here $\mathcal{E}_1 = 1.20$ V, $l_1 = 30$ cm

Also, $\frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{l_1}{l_2} = 1.5$ ∴ $l_2 = \frac{l_1}{1.5} = \frac{30}{1.5} = 20$ cm

Difference in the balancing lengths,

$l_1 - l_2 = 30 - 20 = 10$ cm

Q. 4. ***Two cells of emf $\mathcal{E}_1$ and $\mathcal{E}_2$ ($\mathcal{E}_1 > \mathcal{E}_2$) are connected as shown in the Fig.***



When a potentiometer is connected between A and B, the balancing length of the potentiometer wire is 300 cm. On connecting the same potentiometer between A and C, the balancing length is 100 cm. Calculate the ration of $\mathcal{E}_1$ and $\mathcal{E}_2$.

Sol. As emf ∝ balancing length of the potentiometer wire

∴ When the potentiometer is connected between A and B, $\mathcal{E}_1 \propto 300$

When potentiometer is connected between A and C, $\mathcal{E}_1 - \mathcal{E}_2 \propto 100$

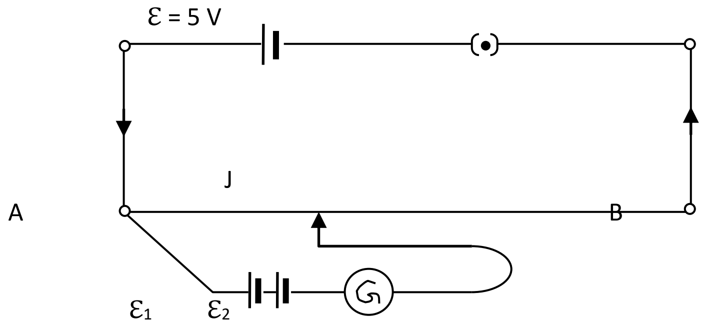
Hence $\frac{\mathcal{E}_1 - \mathcal{E}_2}{\mathcal{E}_1} = \frac{100}{300}$ or $1 - \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{100}{300}$

or $\frac{\mathcal{E}_2}{\mathcal{E}_1} = 1 - \frac{1}{3} = \frac{2}{3}$ or $\frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{3}{2} = 3:2$

Q. 5. ***In a potentiometer, a standard cell of emf 5 V and of negligible resistance maintains a steady current through the potentiometer wire of length 5 m. Two primary cells of emf $\mathcal{E}_1$ and $\mathcal{E}_2$ are joined in series with (i) same polarity, and (ii) opposite polarity. The combination is connected through a galvanometer and a jockey to the potentiometer. The balancing lengths in the two cases are found to be 350 cm and 50 cm respectively.***

(i) Draw the necessary circuit diagram. (ii) Find the value of the emfs of the two cells.

Sol. (i) The circuit diagram is shown in Fig.



(ii) Here $k = \frac{5\text{ V}}{5\text{ m}} = \frac{5\text{ V}}{500\text{ cm}} = \frac{1}{100}\text{ V cm}^{-1}$

In first case, $\mathcal{E}_1 + \mathcal{E}_2 = kl_1 = \frac{1}{100} \times 350$

or $\mathcal{E}_1 + \mathcal{E}_2 = 3.50\text{ V}$ ... (i)

In second case,

$\mathcal{E}_1 - \mathcal{E}_2 = kl_2 = \frac{1}{100} \times 50 = 0.50\text{ V}$ ... (ii)

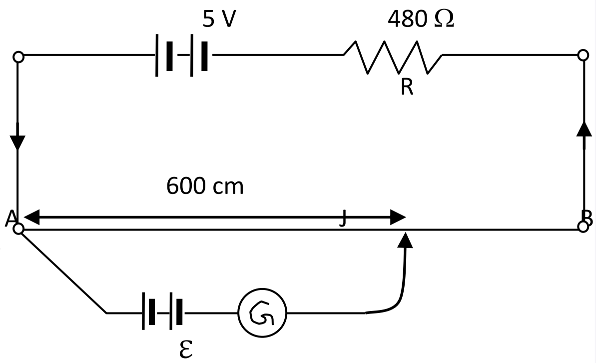
On solving (i) and (ii), we get

$\mathcal{E}_1 = 2.0\text{ V}$ and $\mathcal{E}_2 = 1.50\text{ V}$

Q. 6. A 10-metre-long wire of uniform cross-section of 20 Ω resistance is used as a potentiometer wire. This wire is connected in series with a battery of 5 V, along with an external resistance of 480 Ω. If an unknown emf Ɛ is balanced at 600 cm of this wire, calculate (i) the potential gradient of the potentiometer wire and (ii) the value of the unknown emf Ɛ.

Sol. Current in the circuit or through the potentiometer wire is

$I = \frac{V}{R_{AB} + R} = \frac{5\text{ V}}{(20 + 480)\,\Omega} = 0.01\text{ A}$



Resistance of potential wire, $R_{AB} = 20\,\Omega$

∴ P.D. across the wie,

$V = IR_{AB} = 0.01 \times 20 = 0.2\text{ V}$

Length of potentiometer wire,

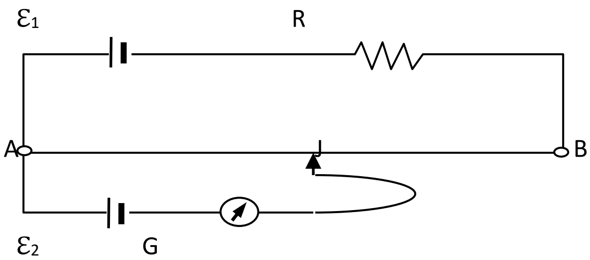
$l = 10\text{ m} = 1{,}000\text{ cm}$

∴ Potential gradient, $k = \frac{V}{l} = \frac{0.2\text{ V}}{1,000\text{ cm}} = 0.0002\text{ V cm}^{-1}$

Unknown emf balanced against 600 cm length is

$\mathcal{E} = kl = 0.0002 \times 600 = 0.12\text{ V}$

Q. 7. ***In the circuit diagram given below, AB is a uniform wire of resistance 15 ohm and length one metre. It is connected to a series arrangement of cell $\mathcal{E}_1$ of emf 2.0 V and negligible internal resistance and a resistor R. Terminal A is also connected to an electrochemical cell $\mathcal{E}_2$ of emf 75 mV and a galvanometer G. In this set-up, a balancing point is obtained at 30 cm mark from A. Calculate the resistance of R. If $\mathcal{E}_2$ were to have an emf of 300 mV, where will you expect the balancing point to be?***



Sol. Current through the potentiometer wire,

$I = \frac{\mathcal{E}_1}{R + R_{AB}} = \frac{2}{R + 15}$

Resistance of the 30 cm length of wire, which balances the emf $\mathcal{E}_2$, is

$$R' = \frac{15}{100} \times 30 = 4.5\ \Omega$$

Now, $\mathcal{E}_2$ = Potential drop across R'

$$\therefore \quad 75 \times 10^{-3} = \frac{2}{R + 15} \times 4.5$$

$$\text{or} \quad R = \frac{2 \times 4.5}{75 \times 10^{-3}} - 15 = 120 - 15 = 105\ \Omega$$

For $\mathcal{E}_2$ = 300 mV, the balancing length is given by

$$l_2 = \frac{\mathcal{E}_2}{\mathcal{E}_1} \cdot l_1 = \frac{300}{} \times 30 = 120 \text{ cm}$$

As the length of the potentiometer wire is just 100 cm, so this balance point cannot be obtained on the wire.

Q. 8. The length of a potentiometer wire is 5 m. It is connected to a battery of constant emf. For a given Leclanche cell, the position of zero galvanometer deflection is obtained at 100 cm. If the length of the potentiometer wire be made 8 m instead of 5 m, calculate the length of wire for zero deflection in the galvanometer for the same cell.

Sol. Here $l = 5$ m, $l_1 = 100$ cm = 1 m, $l' = 8$ m, $l'_1 = ?$

Let $\mathcal{E}$ be the emf of the Leclanche cell.

In first case, $\mathcal{E} = \frac{IR\, l_1}{l}$ ... (1)

In second case, $\mathcal{E} = \frac{IR\, l_1'}{}$ ... (2)

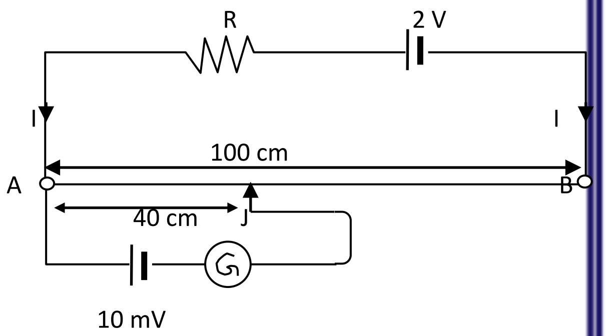
Comparing equations (1) and (2),

$$\frac{l_1'}{l'} = \frac{l_1}{l}$$

$$\text{or} \quad l_1' = \frac{l_1}{l} \times l' = \frac{1}{5} \times 8 = 1.6 \text{ m}$$

Q. 9. A potentiometer wire of length 100 cm has a resistance of 100 Ω. It is connected in series with a resistance and a battery of emf 2 V and of negligible internal resistance. A source of emf 10 mV is balanced against a length of 40 cm of the potentiometer wire. What is the value of the external resistance?

Sol. Fig. shows a potentiometer wire of length 100 cm connected in series to a cell of emf 2 V and an unknown resistance R. The cell of emf 10 mV balances length AJ = 40 cm of the wire.



Resistance of wire AJ = $\frac{10}{100} \times 40 = 4\ \Omega$

Current through wire AJ,

$$I = \frac{10 \text{ mV}}{4\ \Omega} = \frac{10 \times 10^{-3} \text{ V}}{4\ \Omega} = 2.5 \times 10^{-3} \text{ A}$$

The same current flows through the potentiometer wire and through the external resistance R.

Total resistance = (R + 10) Ω

$$\therefore \quad 2.5 \times 10^{-3} \text{ A} = \frac{2\text{V}}{(R + 10)\ \Omega}$$

$$\text{or} \quad R + 10 = \frac{2}{2.5 \times 10^{-3}} = 800$$

R = 800 – 10 = 790 Ω

Q. 10. A cell gives a balance with 85 cm of a potentiometer wire. When the terminals of the cell are shorted through a resistance of 7.5 Ω, the balance is obtained at 75 cm. Find the internal resistance of the cell.

Sol. Here $l_1 = 85$ cm, $l_2 = 75$ cm, R = 7.5 Ω

Internal resistance,

$$r = R\,\frac{(l_1 - l_2)}{l_2}$$

$$= 7.5\ \frac{85 - 75}{75} = 1\ \Omega$$

Q 11. When a resistor of 5 Ω is connected across cell, its terminal p.d. is balanced by 150 cm of potentiometer wire and when a resistor of 10 Ω resistance is connected across the cell, the terminal p.d. is balanced by 175 cm of the potentiometer wire, find the internal resistance of the cell.

Sol. in the first case r, = $R_1 \left(\frac{l - l_1}{l_1}\right)$

∴ $r\dfrac{l_1}{R_1} = l - l_1$ … (1)

In the second case, $r = R_2\dfrac{l - l_2}{l_2}$

∴ $r\dfrac{l_2}{R_2} = l - l_2$ … (2)

Subtracting (2) from (1),

$$r\left(\frac{l_1}{R_1} - \frac{l_2}{R_2}\right) = l - l_1 - l + l_2$$

∴ $r = \dfrac{l_2 - l_1}{\dfrac{l_1}{R_1} - \dfrac{l_2}{R_2}} = \dfrac{175 - 150}{\dfrac{150}{5} - \dfrac{175}{10}} = \dfrac{25}{12.5} = 2\ \Omega$

Q. 12. The length of a potentiometer wire is 600 cm and it carries 40 mA current. For a cell of emf 2 volt and internal resistance 10 ohms, the null-point is found at 500 cm. If a voltmeter is connected across the cell, the balancing length of the wire is decreased by 10 cm. Find (i) The resistance of the whole wire, (ii) reading of voltmeter, (iii) resistance of voltmeter.

Sol. (i) Let I be the current in the potentiometer wire AB and ρ the resistance per cm of AB. Then the potential gradient is

$k = I\rho$

Balancing length, AC = 500 cm

P.D. between A and C, $V = k\,l = I\,\rho\,l$

In no-deflection condition, the p.d. V equals the emf $\mathcal{E}$ of the cell.

∴ $\mathcal{E} = I\,\rho\,l$

or $\rho = \dfrac{\mathcal{E}}{Il} = \dfrac{2\text{ V}}{(40 \times 10^{-3}\text{ A}) \times 500\text{ cm}}$

$= 0.1\ \Omega\ \text{cm}^{-1}$

∴ Total resistance of the wire AB

$= \rho \times AB = 0.1 \times 600 = 60\ \Omega.$

(ii) When voltmeter is connected across the cell, the null-point is C′, where AC′ = l′ = 490 cm.

P.D. between A and C, $V' = k\,l' = I\,\rho\,l'$

$= 40 \times 10^{-3} \times 0.1 \times 490 = 1.96$ V

As there is no current in the galvanometer G, the same (1.96 V) is the p.d. across the cell and this is the reading of the voltmeter.

(iii) The cell is sending current in the voltmeter. Let R be the resistance of the voltmeter and r the internal resistance of the cell. The current supplied by the cell is

$I = \dfrac{\mathcal{E}}{R + r} = \dfrac{2}{R + 10}$ A

∴ Reading of voltmeter,

$V' = E - I'r$

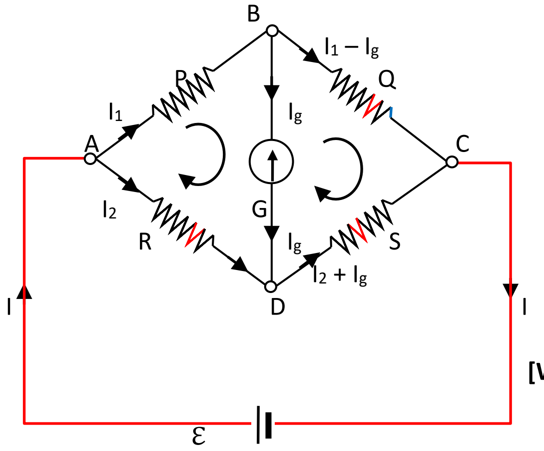
or $1.96 = 2 - \dfrac{2 \times 10}{R + 10}$

On solving, we get

$R = 490\ \Omega$

WHEATSTONE BRIDGE

It is an arrangement of four resistances used to determine one of these resistances quickly and accurately in terms of the remaining three resistances. This method was first suggested by a British Physicist Sir Charles F. Wheatstone in 1843. A Wheatstone bridge consists of four resistances P, Q, R and S; connected to form the arms of quadrilateral ABCD. A battery of emf $\mathcal{E}$ is connected between points A and C and a sensitive galvanometer between B and D, as shown in Fig.



[Wheatstone bridge]

Let S be the resistance to be measured. The resistance R is so adjusted that there is no deflection in the galvanometer. The bridge is said to balance when the potential difference across the galvanometer is zero so that there is no current through the galvanometer. In the balanced condition of the bridge,

$$\frac{P}{Q} = \frac{R}{S}$$

UNKNOWN RESISTANCE, $S = \frac{P}{Q} \cdot R$

Knowing the ratio of resistance P and Q, and the resistance R, we can determine the unknown resistance S. That is why the arms containing the resistance P and Q are called ratio arms, the arm AD containing R standard arm and the arm CD containing S the unknown arm.

Derivation of balance condition from Kirchhoff's laws: In accordance with Kirchhoff's first law, the currents through various branches are as shown in Fig.

Applying Kirchhoff's second law to the loop ABDA, we get

$$I_1 P + I_g G - I_2 R = 0$$

where G is the resistance of the galvanometer. Again, applying Kirchhoff's second law to the loop BCDB, we get

$$(I_1 - I_g) Q - (I_2 + I_g) S - GI_g = 0$$

In the balanced condition of the bridge, $I_g = 0$. The above equation becomes

$$I_1 P - I_2 R = 0$$

or $\quad I_1 P = I_2 R \quad$ ... (i)

and $\quad I_1 Q - I_2 S = 0$

or $\quad I_1 Q = I_2 S \quad$ ... (ii)

On dividing equation (i) by (ii), we get

$$\frac{P}{Q} = \frac{R}{S}$$

This proves the condition for the balanced Wheatstone bridge.

Sensitivity of a Wheatstone bridge: A Wheatstone bridge is said to be sensitive if it shows a large deflection in the galvanometer for a small change of resistance in the resistance arm.

The sensitivity of the Wheatstone bridge depends on two factors:

Relative magnitudes of the resistances in the four arms of the bridge. The bridge is most sensitive when all the four resistances are of the same order.

According to Calender for the greater sensitivity of the Wheatstone bridge, the battery should be so connected that the resistance in series with the resistance to be measured is greater than the resistance in parallel with it.

According to Maxwell for the greater sensitivity of the Wheatstone bridge, out of the battery and the galvanometer, the one having the higher resistance should be connected between the junction of the two highest resistance and the junction of the two lowest resistances.

Advantages of Wheatstone bridge method:

(i) It is a null method. Hence the internal resistance of the cell and the resistance of the galvanometer do not affect the null point.
(ii) As the method does not involve any measurement of current and potential difference, so the resistance of ammeters and voltmeter do not affect the measurements.
(iii) The unknown resistance can be measured to a very high degree of accuracy by increasing the ratio of the resistances in arms P and Q.

Knowledge PLUS......................

☑ When the Wheatstone bridge is balanced, the potential difference between the points B and D is zero.

☑ The Wheatstone bridge is most sensitive when the resistances in the four arms are of the same order.

☑ Wheatstone bridge method is not suitable for the measurement of very low and very high resistances.

☑ In the balanced Wheatstone bridge, the resistance in arm BD is ineffective. The equivalent resistance of the balanced Wheatstone bridge between the points A and C will be

$$R_{eq} = \frac{(P + Q)(R + S)}{P + Q + R + S}$$

☑ If the bridge is balanced, then on interchanging the positions of the galvanometer and the battery there is no effect on the balance of the bridge. That is why the arms BD and AC are called conjugate arms of the bridge.

☑ The Wheatstone bridge is the simplest example of an arrangement, the variants of which are used for a large number of electrical measurements. The important applications of Wheatstone bridge are metre bridge, Carey-Foster's bridge and post office box.

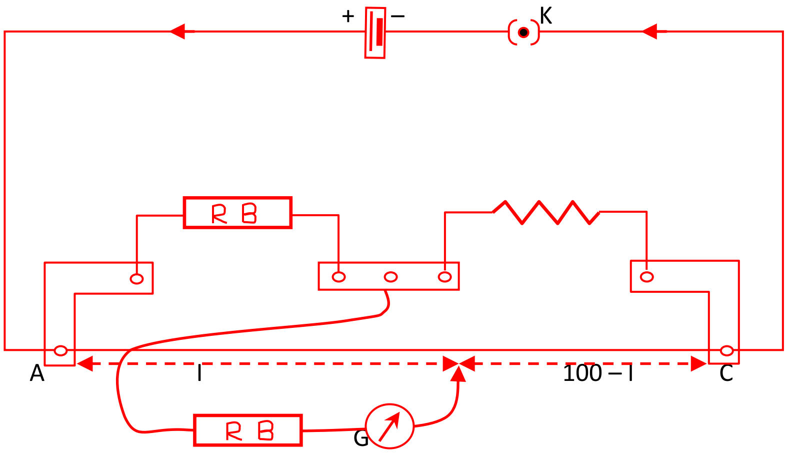
METRE BRIDGE OR SLIDE WIRE BRIDGE

It is the simplest practical application of the Wheatstone bridge that is used to measure an unknown resistance.

Principle: Its working is based on the principle of Wheatstone bridge. When the bridge is balanced,

$$\frac{P}{Q} = \frac{R}{S}$$

Construction: It consists of usually one-metre-long manganin wire of uniform cross-section, stretched along a metre scale fixed over a wooden board and with its two ends soldered to two L-shaped thick copper strip A and C. Between these two copper strips, another copper strip is fixed so as to provide two gaps ab and $a_1 b_1$. A resistance box R.B. is connected in the gap ab and the unknown resistance S is connected in the gap $a_1 b_1$. A source of e.m.f. Ɛ is connected across AC. A movable jockey and a galvanometer are connected across BD, as shown in Fig.



[Measurement of unknown resistance by a metre bridge]

Working: After taking out a suitable resistance R from the resistance box, the jockey is moved along the wire AC till there is no deflection in the galvanometer. This is the balanced condition of the Wheatstone bridge. If P and Q are the resistances of the parts AB and BC of the wire, then for the balanced condition of the bridge, we have

$$\frac{P}{Q} = \frac{R}{S}$$

Let total length of wire AC = 100 cm and AB = l cm, then BC = (100 – l) cm. Since the bridge wire is of uniform cross-section, therefore,

resistance of wire $\propto$ length of wire

or $\frac{P}{Q} = \frac{\text{resistance of AB}}{\text{resistance of BC}} = \frac{\sigma l}{\sigma(100 - l)} = \frac{l}{100 - l}$

where σ is the resistance per unit length of the wire.

Hence $\frac{R}{S} = \frac{l}{100 - l}$

or $S = \frac{R(100 - l)}{l}$

Knowing l and R, unknown resistance S can be determined.

Determination of resistivity: If r is the radius of the wire and l' its length, then resistivity of the its material will be

$$\rho = \frac{SA}{l'} = \frac{S \times \pi r^2}{l'}$$

Examples based on (i) Wheatstone Bridge (ii) Slide Wire Bridge

♦ **Formulae Used**

1. For a balanced Wheatstone bridge, $\frac{P}{Q} = \frac{R}{S}$

If X is the unknown resistance

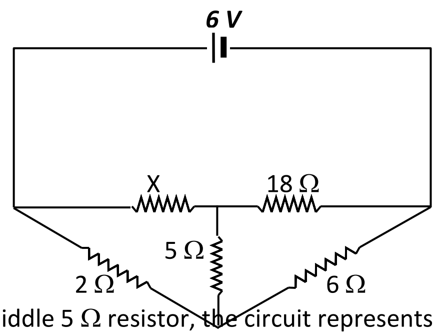
$\frac{P}{Q} = \frac{R}{X}$ or $X = \frac{RQ}{P}$

2. In the slide wire bridge, if balance point is obtained at l cm from the zero end, then

$\frac{P}{Q} = \frac{R}{X} = \frac{l}{100 - l}$ or $X = \left(\frac{100 - l}{l}\right) R$

♦ **Units Used** All resistances are in ohm and distances in cm.

Q. 1. ***Find out the magnitude of resistance X in the circuit shown in Fig., when no current flows through the 5 Ω resistor.***



Sol. As no current flows through the middle 5 Ω resistor, the circuit represents a balanced Wheatstone bridge.

$\therefore$ $\frac{X}{18} = \frac{2}{6}$

or $X = \frac{2}{6} \times 18 = 6\ \Omega$

Q. 2. ***P, Q, R and S are four resistance wires of resistances 2, 2, 2 and 3 ohms respectively. Find out the resistance with which S must be shunted in order than bridge may be balanced.***

Sol. For a balanced Wheatstone bridge,

$\frac{P}{Q} = \frac{R}{S}$

But P = 2 Ω, Q = 2 Ω, R = 2 Ω $\therefore$ $\frac{2}{2} = \frac{2}{S}$

i.e., resistances S must have a total resistance of 2 Ω. In arm S, the resistance of 3 Ω must be shunted with a resistance r so that the combined resistance is of 2 Ω.

i.e., $\frac{1}{r} + \frac{1}{3} = \frac{1}{2}$ or $\frac{1}{r} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

$\therefore$ Required shunt, r = 6 Ω

Q. 3. ***In a Wheatstone bridge arrangement, the ratio arms P and Q are nearly equal. The bridge is balanced when R = 500 Ω. On interchanging P and Q, the value of R for balancing is 505 Ω. Find the value of S and the ratio P/Q.***

Sol. For balanced Wheatstone bridge,

$\frac{P}{Q} = \frac{R}{X}$

In the first case, R = 500 Ω

$\therefore \quad \frac{P}{Q} = \frac{500}{X}$ ... (1)

In the second case when P and Q are interchanged, R = 505 Ω

$\therefore \quad \frac{Q}{P} = \frac{505}{X}$

Multiplying equations (1) and (2), $1 = \frac{500 \times 505}{X^2}$

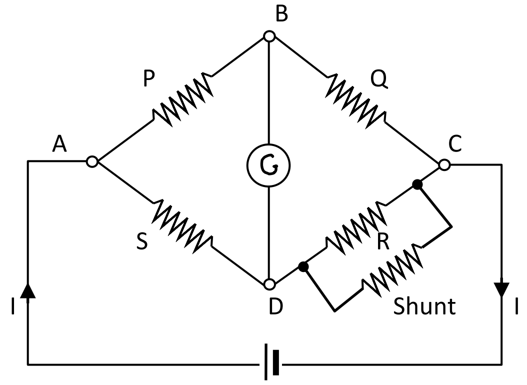
or $\quad X = \sqrt{500 \times 505} = 502.5\ \Omega$

Substituting the value of X in (1), we get

$\frac{P}{Q} = \frac{500}{502.5} = \frac{1}{1.005} = 1 : 1.005$

Q. 4. P, Q, R and S are the resistances taken in cyclic order in a Wheatstone bridge network. P and Q are the ratio coils, S is unknown resistance and R is a 10 Ω coil. A balance is obtained when R is shunted with a resistance of 190 Ω. When P and Q are interchanged, the balance is restored by altering the shunt across R to 265 Ω. Find the resistance of S and the ratio P : Q.

Sol. The four resistances are arranged as shown in Fig. When the bridge is balanced,



Here R = 10 Ω. It is shunted by a resistance of 190 Ω. Effective resistance is,

$R_1 = \frac{10 \times 190}{10 + 190} = \frac{19}{2}\ \Omega$

$\therefore \quad \frac{P}{Q} = \frac{2\,S}{19}$ ... (1)

When P and Q are interchanged, the 10 Ω resistance is shunted by a resistance of 265 Ω. In this case, we have

$\frac{Q}{P} = \frac{S}{R_2}$

But $\quad R_2 = \frac{10 \times 265}{10 + 265} = \frac{106}{11}\ \Omega \qquad \therefore \quad \frac{Q}{P} = \frac{11\,S}{106}$ ... (2)

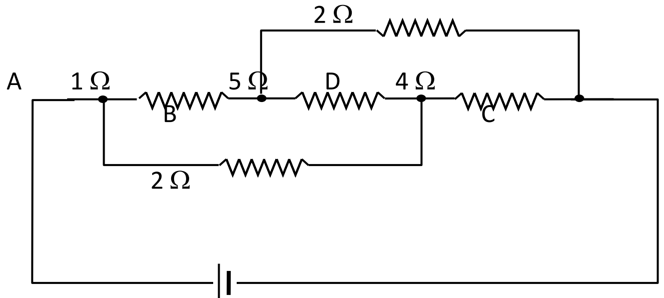
Multiplying equations (1) and (2), we get

$1 = \frac{2\,S}{19} \times \frac{11\,S}{106}$

$S^2 = \frac{19 \times 106}{2 \times 11} = 91.54 \quad$ or $\quad S = 9.57\ \Omega$

From (1), $\frac{P}{Q} = \frac{2 \times 9.57}{19} = \frac{19.14}{19} = 1.007 : 1$

Q. 5. Calculate the current drawn from the battery by the network of resistors shown in Fig.



Sol. The given network is equivalent to the circuit shown in Fig.

Now $\quad \frac{1\ \Omega}{2\ \Omega} = \frac{2\ \Omega}{4\ \Omega} \qquad$ i.e., $\quad \frac{P}{Q} = \frac{R}{S}$

The given circuit is a balanced Wheatstone bridge. The resistance of 5 Ω in arm BD is ineffective. The equivalent circuit reduces to the circuit shown in Fig.

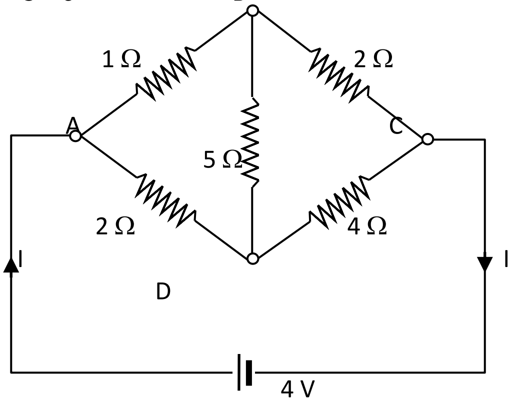
Resistances in AB and BC are in series, their equivalent resistance = 1 + 2 = 3 Ω

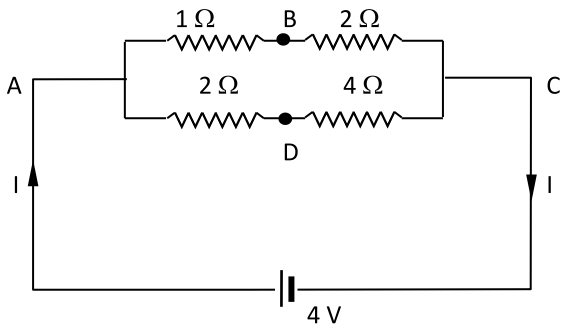
Resistances in AD and DC are in series, their equivalent resistance = 2 + 4 = 6 Ω

The resistance of 3 Ω and 6 Ω are in parallel.

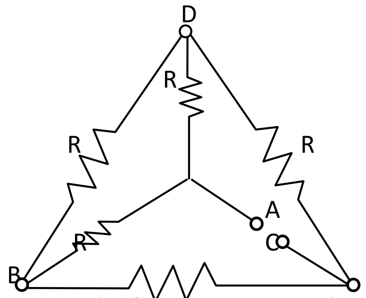
The equivalent resistance R between A and C is

$R = \frac{3 \times 6}{3 + 6} = 2\ \Omega$ ∴ Current, $I = \frac{V}{R} = \frac{4}{2} = 2\ A$

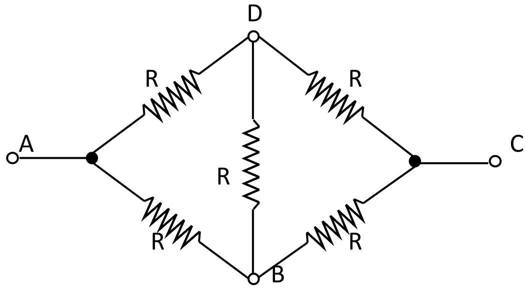




Q. 6. ***Each of the resistance in the network shown in Fig. (a) equals R. Find the resistance between two terminals A and C.***



Sol. The network shown in Fig. (b) is the equivalent network of the given network.



It is a balanced Wheatstone bridge because $\frac{R}{R} = \frac{R}{R}$

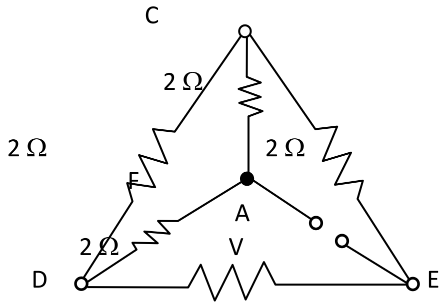
Hence the points B and D must be at the same potential. The resistance R in arm BD in ineffective.

Total resistance along ADC = R + R = 2 R Ω Total resistance along ABC = R + R = 2 R Ω

These two resistances from a parallel combination

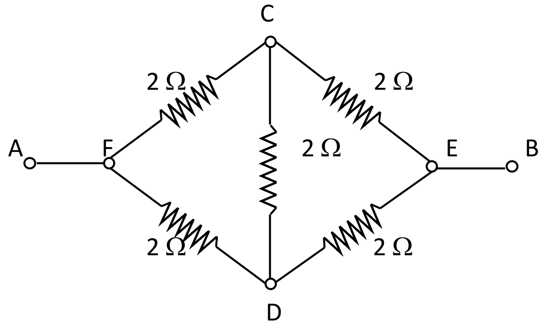
∴ Effective resistance between A and C = $\frac{2R \times 2R}{2R + 2R} = R\ \Omega$

Q. 7. ***A potential difference of 2 V is applied between the points A and B shown in network drawn in Fig. Calculator.***
(i) The equivalent resistance of the network between the points A and B, and
(ii) The magnitudes of currents flowing in the arms AFCEB and AFDEB.



Sol. (i) The equivalent network is shown in Fig. (a). It is a balanced Wheatstone bridge because

$$\frac{2\,\Omega}{2\,\Omega} = \frac{2\,\Omega}{2\,\Omega}$$

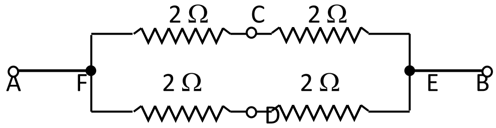


Hence the points C and D are at the same potential. The resistance in arm CD is ineffective. The given network reduces to the equivalent circuit shown in Fig. (b).

Total resistance along FCE = 2 + 2 = 4 Ω

Total resistance along FDE = 2 + 2 = 4 Ω

These two resistances form a parallel combination.



∴ Equivalent resistance between points

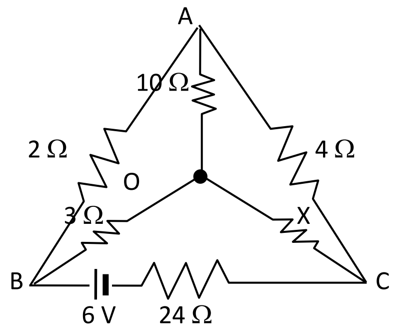
A and B = $\frac{4 \times 4}{4 + 4} = 2\,\Omega$

(ii) Total current in the circuit = $\frac{V}{R} = \frac{2\text{ V}}{2\,\Omega} = 1$ A

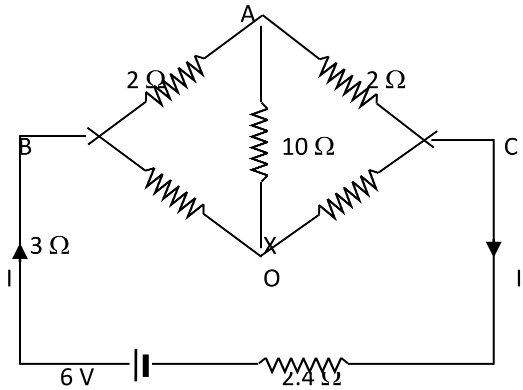
Current through arm AFCEB = current through arm AFDEB

= ½ A = 0.5 A

Q. 8. Find the value of the unknown resistance X, in the following circuit, if no current flows through the section AO. Also calculate the current drawn by the circuit from the battery of emf 6 V and negligible internal resistance.



Sol. The equivalent circuit for the given network is shown below:



As no current flows through the section AO, so the given circuit is a balanced Wheatstone bridge. Hence

$\frac{2}{4} = \frac{3}{X}$ or $X = \frac{3 \times 4}{2} = 6\,\Omega$

The resistance of 10 Ω is section AO is not effective.

Total resistance along BAC = 2 + 4 = 6 Ω

Total resistance along BOC = 3 + 6 = 9 Ω

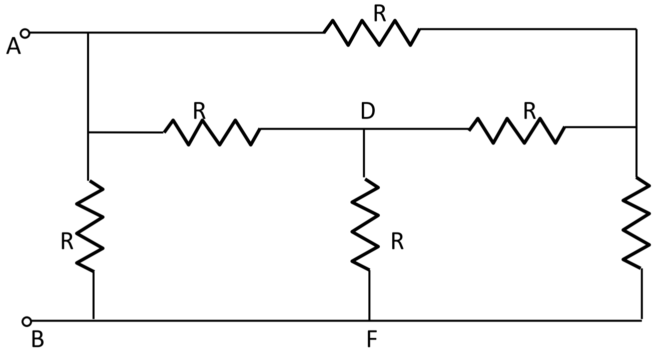
These two resistances form a parallel combination. The effective resistance between B and C is

$R = \frac{6 \times 9}{6 + 9} = \frac{18}{5} = 3.6\,\Omega$

Total resistance in the circuit = 3.6 + 2.4 = 6 Ω

Current, $I = \frac{6\text{ V}}{6\,\Omega} = 1\text{ A}$

Q. 9. ***Six equal resistors, each of value R, are joined together as shown in the given figure. Calculate the equivalent resistance across AB. If a supply of emf E is connected across AB, compute the current through the arms DE and AB.***



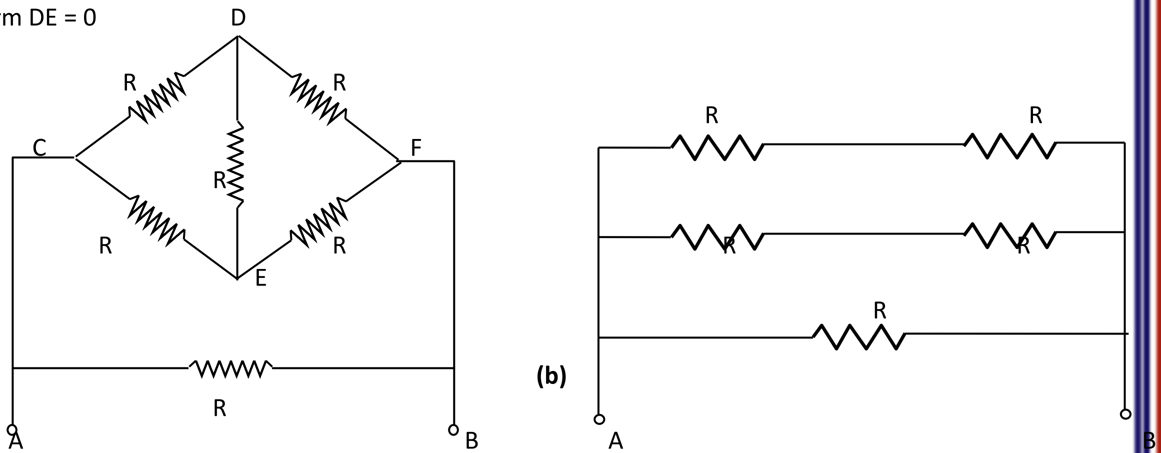
Sol. The equivalent circuits are shown below. The resistance R in arm DE of the balanced Wheatstone bridge is ineffective.

The equivalent resistance R′ across AB is given by

$$\frac{1}{R'} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{R} = \frac{4}{2R} = \frac{2}{R} \quad \text{or} \quad R' = R/2$$

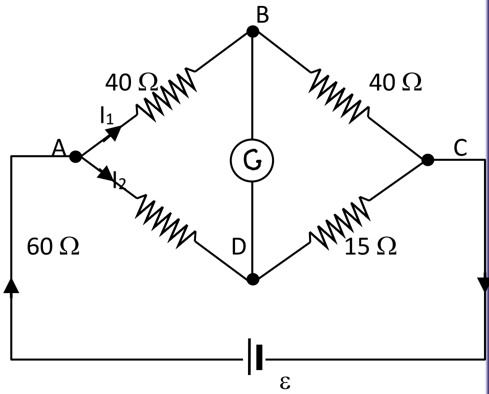
Current through arm AB = $\frac{E}{R'} = \frac{E}{R/2} = \frac{2E}{R}$

Current through arm DE = 0



(a) (b)

Q. 10. ***Calculate the ratio of the heat produced in the four arms of the Wheatstone bridge shown in Fig.***



Sol. As $\frac{40\,\Omega}{10\,\Omega} = \frac{60\,\Omega}{15\,\Omega}$

The bridge is balanced.

∴ P.D. across AB = P.D. across AD

or $40\,I_1 = 60\,I_2$

or $\frac{I_1}{I_2} = \frac{60}{40} = 1.5$

or $I_1 = 1.5\,I_2$

Heats produced in time t in different arms of Wheatstone bridge is

$H_{AB} = I^2{}_1 Rt = (1.5\,I_2)^2 \times 40 \times t = 90\,I^2{}_2\,t$

$H_{BC} = I^2{}_1 \times 10 \times t = (1.5\,I_2)^2 \times 10 \times t = 22.5\,I^2{}_2\,t$

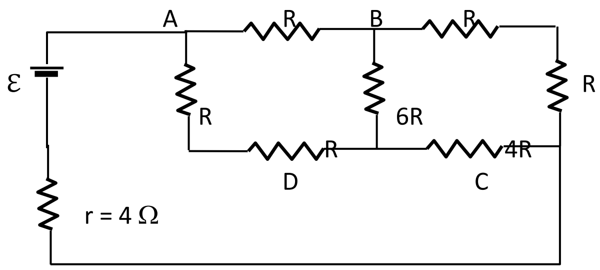
$H_{AD} = 1_2{}^2 \times 60 \times t = 60\,I_2{}^2\,t$

$H_{DC} = I_2{}^2 + 15 \times t = 25\,I_2{}^2\,t$

Hence the ratio of the heats produced in the four arms is

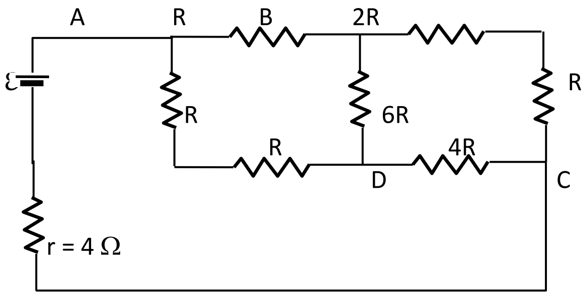
$H_{AB} : H_{BC} : H_{AD} : H_{DC} = 90\,I^2{}_2\,t : 22.5\,I_2{}^2\,t : 60\,I_2{}^2\,t : 15\,I_2{}^2\,t = 90 : 22.5 : 60 : 15 = 6 : 15 : 4 : 1$

Q. 11. ***A battery of internal resistance r (= 4 Ω) is connected to the network of resistances, as shown in Fig. What must be the value or R, so that maximum power?***



Sol. The equivalent circuit for the given network is shown in Fig.

Clearly, $\frac{R}{2R} = \frac{2R}{4R}$



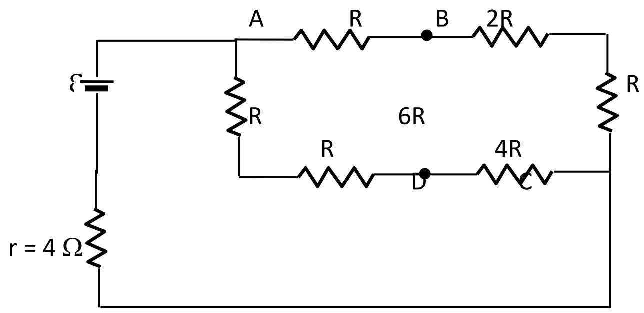
Thus, the part of the circuit on the right side is a balanced Wheatstone bridge. Hence resistance 6 R in arm BD is ineffective. The equivalent circuit then reduces to the circuit shown in Fig.

Effective resistance along ABC = R + 2R = 3R

Effective resistance along ADC = 2R + 4R = 6R

∴ Effective resistance between points A and C,

$$R' = \frac{3R \times 6R}{3R + 6R} = 2R$$



Current drawn from the battery,

$$I = \frac{\varepsilon}{r + R'} = \frac{\varepsilon}{R + 2R}$$

Power delivered to the network,

$$P = I^2 R' = \left(\frac{\varepsilon}{r + 2R}\right)^2 \times 2R$$

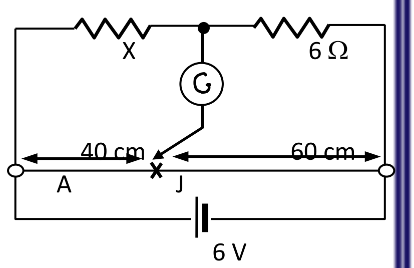
According to maximum power theorem, power delivered to the network will be maximum when

Resistance of the network = Internal resistance of the battery

or $2R = r = 4\ \Omega$ or $R = 2\ \Omega$

Also, $P_{max} = \left(\frac{\varepsilon}{4 + 2 \times 2}\right)^2 \times 2 \times 2 = \frac{\varepsilon^2}{16}$

Q. 12. In the following circuit, a metre bridge is shown in its balanced state. The metre bridge wire has a resistance of 1 ohm/cm. Calculate the value of the unknown resistance X and the current drawn from the battery of negligible internal resistance.



Sol. In balanced condition, no current flows through the galvanometer.

Here P = Resistance of wire AJ = 40 Ω

Q = Resistance of wire BJ = 60 Ω

R = X, S = 6 Ω

In the balanced condition,

$$\frac{P}{Q} = \frac{R}{S} \quad \text{or} \quad \frac{40}{60} = \frac{X}{6} \quad \text{or} \quad X = 4\ \Omega$$

Total resistance of wire AB = 100 Ω

Total resistance of resistance X and 6 Ω connected in series = 4 + 6 = 10 Ω

This series combination is in parallel with wire AB.

∴ Equivalent resistance $= \frac{10 \times 100}{10 + 100} = \frac{100}{11}\ \Omega$

emf of the battery = 6 V

∴ Current drawn from the battery,

$$I = \frac{\text{emf}}{\text{Resistance}} = \frac{6}{100/11} = 0.66\ \text{A}$$

Q. 13. ***In a metre bridge, the length of the wire is 100 cm. At what position will the balance point be obtained if the two resistances are in the ratio 2 : 3.***

Sol. For a balanced metre bridge,

$$\frac{X}{R} = \frac{l}{100 - l}$$

But $\frac{X}{R} = \frac{2}{3}$ ∴ $\frac{2}{3} = \frac{l}{100 - l}$

or $200 - 2l = 3l$ or $l = \frac{200}{5} = 40$ cm

Q. 14. ***With a certain resistance in the left gap of a slide wire metre bridge, the balance point is obtained when a resistance of 10 Ω is taken out from the resistance box. On increasing the resistance from the resistance box by 12.5 Ω, the balance point shifts by 20 cm. Find the unknown resistance.***

Sol. With unknown resistance X in the left gap and known resistance of 10 Ω in the right gap,
Suppose the balance point is obtained at l cm from the zero end. Then

$$\frac{X}{10} = \frac{l}{100 - l} \quad \ldots (1)$$

When the resistance in the right gap is increased by 12.5 Ω, total resistance becomes 22.5 Ω. The balance point shifts towards zero end by 20 cm.

∴ $$\frac{X}{22.5} = \frac{l - 20}{100 - (l - 20)} = \frac{l - 20}{120 - l} \quad \ldots (2)$$

Dividing (1) by (2), $$\frac{22.5}{10} = \frac{l}{100 - l} \times \frac{120 - l}{l - 20}$$

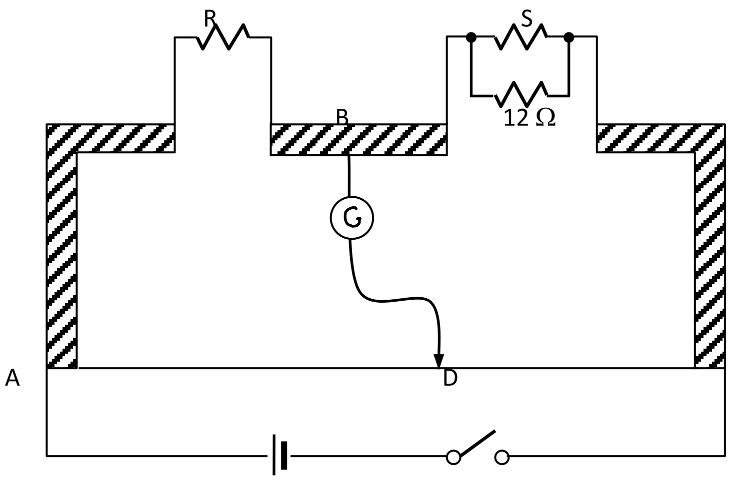
On solving, we get $l^2 - 120\,l + 3600 = 0$

∴ $l = 60$ cm

From (1),

$$\frac{X}{10} = \frac{60}{100 - 60} \quad \text{or} \quad X = \frac{60 \times 10}{40} = 15\ \Omega$$

Q. 15. ***In a metre bridge (Fig.), the null point is found at a distance of 33.7 cm from A. If now a resistance of 12 Ω is connected in parallel with S, the null point occurs at 51.9 cm. Determine the values of R and S.***



Sol. Since the wire is of uniform cross-section, the resistance of the two segments of the wire AD and DC are in the ratio of the lengths of AD and DC. Using the null-point condition of a Wheatstone bridge, we have

$$\frac{R}{S} = \frac{33.7}{66.3} \quad \ldots (1)$$

When S is shunted by a resistance of 12.0 Ω, its net resistance changes to

$$S_{eq} = \frac{12\,Y}{Y + 12}$$

With the new balance condition, we get

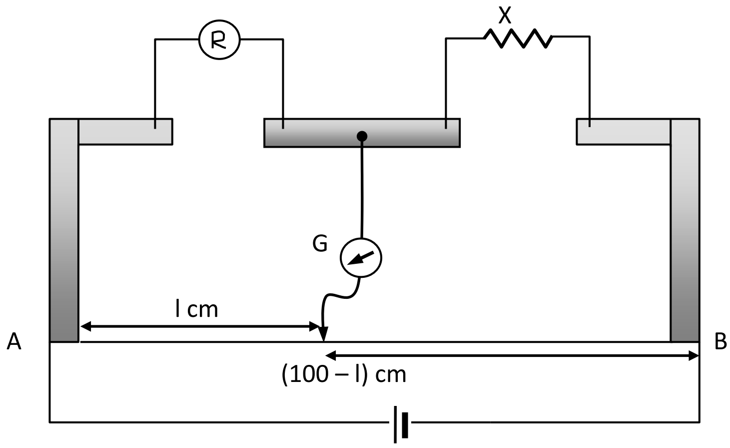
$$\frac{R}{S_{eq}} = \frac{51.9}{48.1} \quad \text{or} \quad \frac{R\,(S + 12)}{12\,S} = \frac{51.9}{48.1}$$

i.e., $$\frac{S + 12}{12} = \frac{51.9}{48.1} \times \frac{S}{R} = \frac{51.9}{48.1} \times \frac{66.3}{33.7} \quad \text{[Using eq. (1)]}$$

or $S = 13.1\ \Omega$

and $R = \frac{33.7}{66.3} \times 13.1 = 6.64\ \Omega$

Q. 16. ***A resistance R = 2 Ω is connected to one of the gaps in a metre bridge, which uses a wire of length 1 m. An unknown resistance X > 2 Ω is connected in the other gap as shown in the figure. The balance point is noticed at 'l' from the positive end of the battery. On interchanging R and X, it is found that the balance point further shifts by 20 cm (away from end A). Neglecting the end correction, calculate the value of unknown resistance X used.***



Sol. In first case, $\frac{R}{X} = \frac{l}{100 - 1}$

In second case, $\frac{X}{R} = \frac{l + 20}{100 - (l + 20)} = \frac{l + 20}{80 - l}$

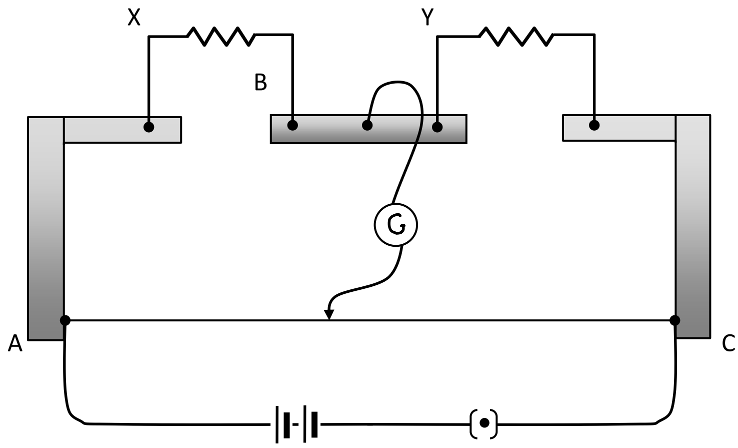
On multiplying the two equations,

$$1 = \frac{l}{100 - 1} \times \frac{l + 20}{80 - 1}$$

or $8000 - 180l + l^2 = l^2 + 20\,l$ or $200\,l = 8000$ or $l = 40$ cm

Now $X = \frac{l + 20}{80 - l} R = \frac{40 + 20}{80 - 40} \times 2 = 3\ \Omega$

Q. 17. ***The given figure shows the experimental setup of a metre bridge. The null point is found to be 60 cm away from the end A with X and Y in position as shown. When a resistance of 15 Ω is connected in series with Y. the null point is found to shift by 10 cm towards the end A of the wire. Find the position of null point if a resistance of 30 Ω were connected in parallel with Y.***



Sol. In first case, $\frac{X}{Y} = \frac{60}{40}$ or $\frac{X}{Y} = \frac{3}{2}$

In second case, $\frac{X}{Y + 15} = \frac{50}{50} = 1$

∴ $\frac{X}{Y} \times \frac{Y + 15}{X} = \frac{3}{2} \times 1$ or $1 + \frac{15}{Y} = \frac{3}{2}$ or $Y = 30\ \Omega$

$$X = \frac{3}{2} Y = \frac{3}{2} \times 30 = 45\ \Omega$$

When a resistance of 30 Ω is connected in parallel with Y, the resistance in the right gap becomes

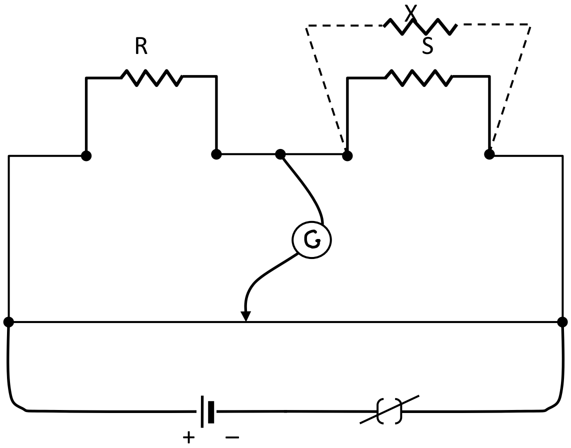
$$Y' = \frac{30\,Y}{30 + Y} = \frac{30 \times 30}{30 + 30} = 15\ \Omega$$

Suppose the null point occurs at l cm from end A. Then

$$\frac{X}{15} = \frac{l}{100 - l} \text{ or } \frac{45}{15} = \frac{l}{100 - l}$$

or $300 - 3l = l$ or $4l = 300$ or $l = 75$ cm

Q. 18. ***When two known resistances, R and S, are connected in the left and right gaps of a metre bridge, the balance point is found at a distance l_1 from the 'zero end' of the metre bridge wire. An unknown resistance X is now connected in parallel to the resistance S and the balance point is now found at a distance l_2 from the zero end of the metre bridge wire. Obtain a formula for X in terms of l_1, l_2 and S.***



Sol. In first case, $\frac{R}{S} = \frac{l_1}{100 - l_1}$ ... (i)

In second case, $\frac{R}{XS/(X+S)} = \frac{l_2}{100 - l_2}$ ... (ii)

Dividing (ii) by (i), we get

$$\frac{X+S}{X} = \frac{l_2}{l_1}\left(\frac{100 - l_1}{100 - l_2}\right) \text{ or } 1 + \frac{S}{X} = \frac{l_2}{l_1}\left(\frac{100 - l_2}{100 - l_2}\right)$$

$$X = \frac{S}{\frac{l_2}{l_2}\left(\frac{100 - l_1}{100 - l_2}\right) - 1}$$

✎ PROBLEMS FOR PRACTICE

Q. 1. ***Four resistances of 15 Ω, 12 Ω, 4 Ω and 10 Ω respectively are connected in cyclic order to form a Wheatstone bridge. Is the network balanced? If not, calculate the resistance to be connected in parallel with the resistance of 10 Ω to balance the network.***

Sol. The four resistances are connected in a cyclic order as shown in Fig.

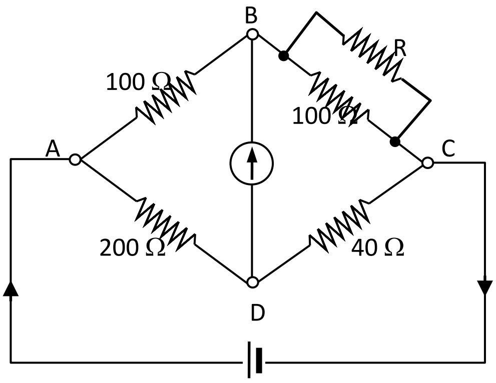
As $\frac{15}{12} \neq \frac{10}{4}$

Thus, Wheatstone bridge is not balanced. To balance the network, suppose resistance R is connected in parallel with 10 Ω resistance. Then

$$\frac{15}{12} = \frac{\frac{10R}{10+R}}{4} \quad \text{or} \quad \frac{10R}{10+R} = 5$$

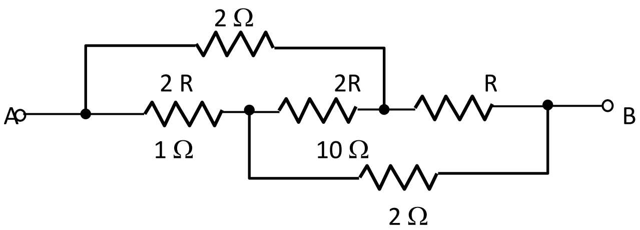
or $R = 10\ \Omega$

Q. 2. ***The Wheatstone's bridge of Fig is showing no deflection in the galvanometer joined between the points B and D. Compute the value of R.***

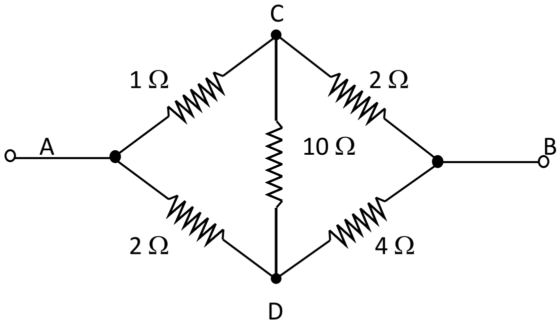


Sol. $\frac{100}{\frac{100R}{100+R}} = \frac{200}{40}$ or $\frac{100R}{100+R} = 20$ ∴ $R = 25\ \Omega$

Q. 3. ***Calculate the equivalent resistance between points A and B of the network shown in Fig.***



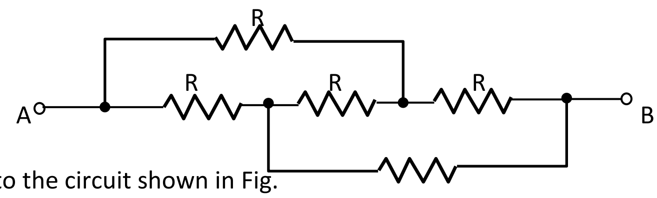
Sol. As $\frac{1}{2} = \frac{2}{4}$ ∴ The given circuit is a balanced Wheatstone bridge as shown in Fig. The resistance of 10 Ω is ineffective.



We have (1 Ω + 2 Ω) and (2 Ω + 4 Ω) combinations in parallel.

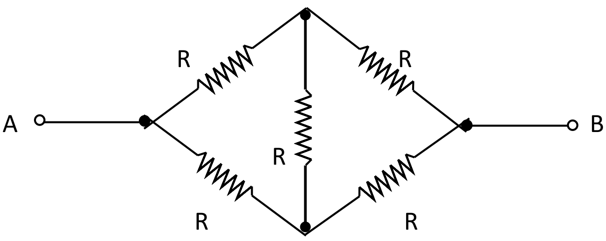
∴ $R = \frac{3 \times 6}{3 + 6} = 2\ \Omega$

Q. 4. ***Calculate the equivalent resistance between the points A and B of the network shown in Fig.***



Sol. The given circuit is equivalent to the circuit shown in Fig.

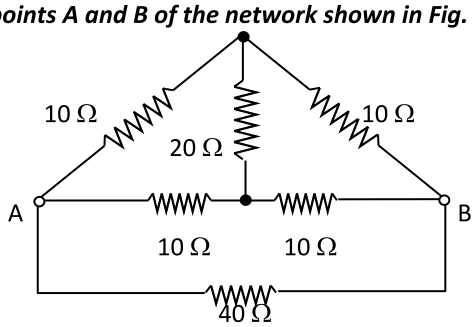
Here $\frac{R}{R} = \frac{R}{R}$



So it is a balanced Wheatstone bridge. We have resistance (R + R) and (R + R) in parallel.

∴ Equivalent resistance $= \frac{2R \times 2R}{2R + 2R} = R$

Q. 5. ***Calculate the resistance between the points A and B of the network shown in Fig.***

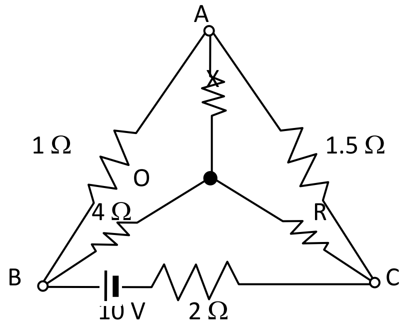


Sol. Here $\frac{10}{10} = \frac{10}{10}$ ∴ Resistance of 20 Ω is ineffective.

We have resistance of (10 Ω + 10 Ω), (10 Ω + 10 Ω) and 40 Ω in parallel.

∴ $\frac{1}{R} = \frac{1}{20} + \frac{1}{20} + \frac{1}{40} = \frac{5}{40}$ or $R = 8\ \Omega$

Q. 6. ***For the network shown in Fig., determine the value of R and the current through it, if the current through the branch AO is zero.***



Sol. As points A and O are at the same potential, therefore

$$\frac{1}{1.5} = \frac{4}{R} \quad \text{or} \quad R = 4 \times 1.5 = 6\ \Omega$$

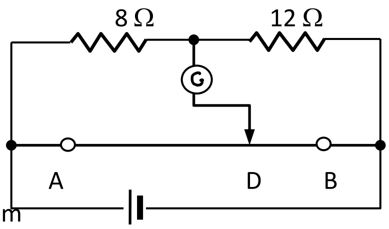
If R′ is the equivalent resistance of the network between B and C, then

$$R' = \frac{2.5 \times 10}{2.5 + 10} + 2 = 4\ \Omega$$

Current in the circuit, $1 = \frac{10}{4} = 2.5$ A

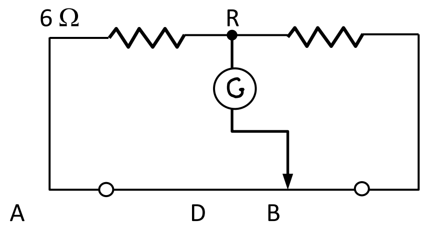
Current through R (= 6 Ω) = $\frac{2.5}{2.5 + 10} \times 2.5 = 0.5$ A

Q. 7. ***The potentiometer wire AB shown in Fig. Is 40 cm long. Where the free end of the galvanometer should be connected on AB so that the galvanometer may show zero deflection?***



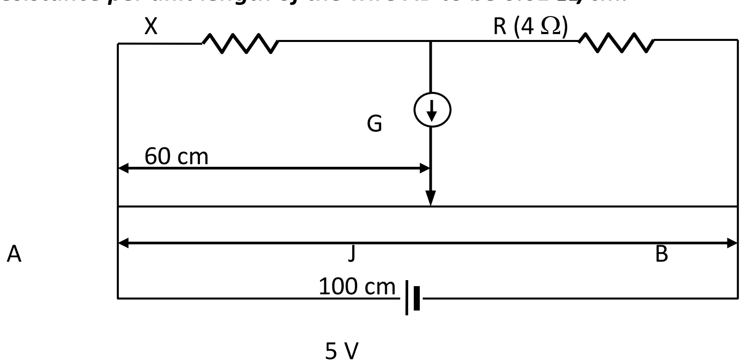
Sol. $\frac{8}{12} = \frac{AD}{DB} = \frac{l}{40 - l}$ ∴ l = 16 cm

Q. 8.The potentiometer wire AB shown in Fig. Is 50 cm long. When AD = 30 cm, no deflection occurs in the galvanometer. Find R.



Sol. $\frac{6}{R} = \frac{AD}{DB} = \frac{30}{50 - 30}$ ∴ R = 4 Ω

Q. 9. ***Calculate the value of unknown resistance X and the current drawn by the circuit, assuming that no current flows through the galvanometer. Assume the resistance per unit length of the wire AB to be 0.01 Ω/cm.***



Sol. Resistance of wire AJ = 60 × 0.01 = 0.60 Ω

Resistance of wire BJ = 40 × 0.01 = 0.40 Ω

When no current flows through the galvanometer,

$$\frac{P}{Q} = \frac{R}{S} \quad \text{or} \quad \frac{0.60}{0.40} = \frac{X}{4}$$

∴ $X = \frac{0.60 \times 4}{0.40} = 6\ \Omega$

Total resistance of X and R in series = 6 + 4 = 10 Ω ; Total resistance of wire AB = 0.60 + 0.40 = 1.0 Ω

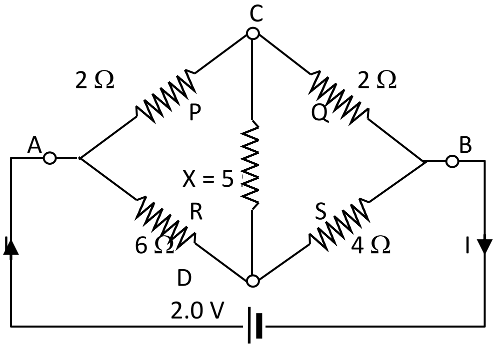
The above two resistances are in parallel. ∴ Total resistance of the circuit = $\frac{10 \times 1}{10 + 1} = \frac{10}{11}\ \Omega$

Current, $I = \frac{EMF}{Resistance} = \frac{5}{10/11} = 5.5$ A

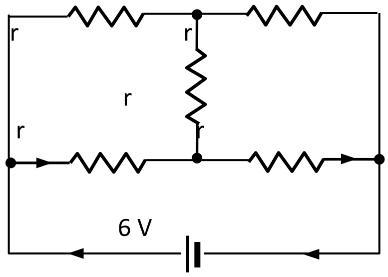
Q. 10. ***In Fig., P = 3 Ω, Q = 2 Ω, R = 6 Ω, S = 4 Ω and X = 5 Ω. Calculate the current I.***

Sol. The circuit is a balanced Wheatstone bridge. Its effective resistance R is given by

$\frac{1}{R} = \frac{1}{3+2} + \frac{1}{6+4} = \frac{3}{10}$ or $R = \frac{10}{3}\ \Omega$ ∴ Current, $I = \frac{V}{R} = \frac{2}{10/3} = 0.6$ A



Q. 11. **Each resistor r shown in Fig. has a resistance of 10 Ω and the battery has an emf of 6 V. Find the current supplied by battery.**

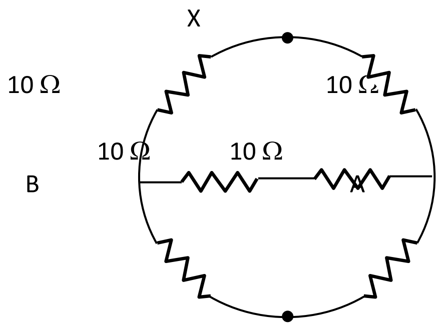


Sol. As r = r, so the given circuit is a balanced Wheatstone bridge and the resistance r in the vertical arm is ineffective. The circuit is then equivalent to two resistances of 2r and 2r connected in parallel.

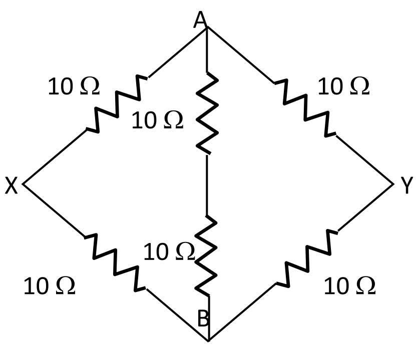
∴ Equivalent resistance, $R = \frac{2r \times 2r}{2r + 2r} = r = 10\ \Omega$

Current supplied by the battery of emf 6 V, $I = \frac{\varepsilon}{R} = \frac{\varepsilon}{10} = 0.6$ A

Q. 12. ***Find the equivalent resistance between the points X and Y of the network shown in fig.***



Sol. The equivalent circuit is shown in Fig.



The resistance in arm AB is ineffective.

$\therefore \quad \frac{1}{R} = \frac{1}{10+10} + \frac{1}{10+10} = \frac{1}{10}$ or $R = 10\ \Omega$

Q. 13. ***The ratio arms of the post office box are 1000: 10. Calculate unknown resistance, if third resistance is 948 Ω?***

Sol. Here ratio arms, $\frac{P}{Q} = \frac{1000}{10} = 100$, $R = 948$, $X = ?$

$X = R \times \frac{Q}{P} = 948 \times \frac{1}{100} = 9.48\ \Omega$

Q. 14. ***An unknown resistance 'X' is placed in the left gap and a known resistance of 60 Ω is placed in the right gap of metre bridge. The null point is obtained at 40 cm from the left end of the bridge. Find the unknown resistance.***

Sol. The unknown resistance X is given by

$\frac{X}{R} = \frac{l}{100 - l}$

Here $R = 60\ \Omega$, $l = 40$ cm, $100 - l = 60$ cm

$\therefore \quad \frac{X}{60} = \frac{40}{60}$ or $X = 40\ \Omega$

Q. 15. ***In a metre-bridge experiment, two resistances P and Q are connected in series in the left gap. When the resistance in series in the left gap. When the resistance in the right gap is 50 Ω, the balance point is at the centre of the slide wire. If P and Q are connected in parallel in the left gap, the resistance in the right gap has to be changed to 12 Ω so as to obtain the balance point at the same position. Find P and Q.***

Sol. When P and Q are connected in series in the left gap,

$\frac{P+Q}{50} = \frac{50}{100-50} \quad \therefore \quad P + Q = 50\ \Omega \qquad \dots (1)$

When P and Q are connected in parallel in the left gap,

$\frac{\frac{PQ}{P+Q}}{12} = \frac{50}{50} = 1$

$\therefore \quad PQ = 12\,(P + Q) = 12 \times 50 = 600$

$(P - Q)^2 = (P + Q)^2 - 4\,PQ = 50^2 - 4 \times 600 = 100$

$\therefore \quad P - Q = 10 \qquad \dots (2)$

Solving (1) and (2), $P = 30\ \Omega$ and $Q = 20\ \Omega$

Q. 16. ***In a metre bridge when the resistance in the left gap is 2 Ω and an unknown resistance in the right gap, the balance point is obtained at 40 cm from the zero end. On shunting the unknown resistance with 2 Ω, find the shift of the balance point on the bridge wire.***

Sol. If X is the unknown resistance, then

$\frac{2}{X} = \frac{40}{100-40}$ or $X = \frac{2 \times 60}{40} = 3\ \Omega$

When resistance X is shunted with 2 Ω resistor, the effective resistance becomes

$X' = \frac{X \times 2}{X+2} = \frac{3 \times 2}{3+2} = 1.2\ \Omega$

Now if the balance point is obtained at distance l' from the left end, then

$\frac{2}{X'} = \frac{l'}{100 - l'}$ or $\frac{2}{1.2} = \frac{l'}{100 - l'}$

$\therefore \quad l' = 62.5$ cm

Shift in the balance point $= l' - l = 62.5 - 40 = 22.5$ cm

Q. 17. ***In a metre-bridge experiment, with a resistance R_1 in the left gap and a resistance X in the right gap, null point is obtained at 40 cm from the left end. With a resistance R_2 in the left gap and the same resistance X in the right gap, null point is obtained at 50 cm from the left end. Where will be the null point if R_1 and R_2 are put in series in the left gap, right gap containing X?***

Sol. In the first case, $\frac{R_1}{X} = \frac{40}{100-40}$

$\therefore \quad X = \frac{60}{40} \times R_1 = \frac{3}{2} R_1 \qquad \dots (1)$

In the second case, $\frac{R_2}{X} = \frac{50}{100-50} = 1$

$\therefore \quad X = R_2 \qquad \dots (2)$

In the third case, when R_1 and R_2 are connected in series in the left gap, suppose the balance point is at l' cm from the left.

Then $\quad \frac{R_1 + R_2}{X} = \frac{l'}{100 - l'}$

or $X = \frac{100 - l'}{l'} \cdot (R_1 + R_2)$ ... (3)

From (2) and (3),

$R_2 = \frac{100 - l'}{l'}(R_1 + R_2)$ or $1 + \frac{R_1}{R_2} = \frac{l'}{100 - l'}$

From (1) and (2), $\frac{3}{2} R_1 = R_2$

or $\frac{R_1}{R_2} = \frac{2}{3}$ ∴ $1 + \frac{2}{3} = \frac{l'}{100 - l'}$

On solving, $l' = 62.5$ cm

VERY SHORT ANSWER CONCEPTUAL PROBLEMS

Q. 1. ***Is electric current a scalar or a vector quantity? Give reason.***

Sol. Electric current is a scalar quantity. The reason is that laws of ordinary algebra are used to add electric currents and laws of vector addition do not apply to the addition of electric currents.

Q. 2. ***Is a wire carrying current charged?***

Sol. No, the current in a wire is due to flow of electrons in a definite direction. At any instant, the wire has as much negative charges as the positive charges.

Q. 3. ***A steady current is flowing in a cylindrical conductor. Is there any electric field within the conductor?***

Sol. Yes, It is under the influence of the electric field only that the free electrons flow inside the conductor and constitute the electric current.

Q. 4. ***How is current kept continuous inside a conductor?***

Sol. By maintaining a potential difference between the two ends of the conductor.

Q. 5. ***For the flow of electricity through gases, they should be better exposed to some high energy radiations like X-rays. Why?***

Sol. High energy radiations cause ionisation of gases: the resulting positive ions and electrons increase the conductivity of gases.

Q. 6. ***Does the emf represent a force or potential energy or work done per unit charge or potential difference? Does emf have electrostatic origin?***

Sol. The term emf of is a misnomer. Literally, emf means the force that pushes the electrons in a circuit. Since emf does not have simple electrostatic origin, so the concept of potential is not strictly applicable. It has the nature of work done per unit charge and not that of force.

Q. 7. ***What is the difference between resistance and resistor?***

Sol. The property, by virtue of which a material opposes the flow of current through it, is called resistance and any material that has some resistance is called a resistor.

Q. 8. ***What is the difference between resistance and resistivity?***

Sol. Resistance of a conductor is the ratio of the potential difference applied across its ends to the current flowing through it. Resistance of a conductor depends on its shape and size and also on the nature of its material. Resistivity of a material. It depends on the nature of the material and on the physical conditions like temperature and pressure.

Q. 9. ***What is the cause of resistance of a conductor?***

Sol. While drifting, the free electrons collide with the ions and atoms of the conductor, i.e., motion of the electrons is opposed during the collisions. This is the basic cause of resistance in a conductor.

Q. 10. ***Do bends in a wire affect its resistance?***

Sol. No, this is because the mean free path free electrons in the wire is less than the radius of curvature of the bend and so they can easily go round the bends. The resistance is not affected as long as the area of cross-section remains same at the bend.

Q. 11. ***Resistivity of copper, constantan and silver are $1.7 \times 10^{-8}\ \Omega$m, $39.1 \times 10^{-8}\ \Omega$m and $1.6 \times 10^{-8}\ \Omega$m, respectively. Which has the best conductivity?***

Sol. Conductivity = $\frac{1}{\text{Resistivity}}$

As silver has the lowest resistivity, so it has the best conductivity.

Q. 12. ***A larger number of free electrons are present in metals. Why is there no current in the absence of electric field across it?***

Sol. In the absence of an electric field, the motion of the electrons in a metal is random. There is no net flow of charge across any section of the conductor. So no current flows in the metal.

Q. 13. ***Two wires of equal lengths, one of copper and the other of manganin have the same resistance. Which wire will be thicker?***

Sol. AS $R = \rho \frac{l}{A}$ ∴ $A = \frac{\rho l}{R}$

For both wires R and l are same and

$\rho_{copper} < \rho_{manganin}$

∴ $A_{copper} < A_{manganin}$

i.e., manganin wire is thicker than copper wire.

Q. 14. ***If a wire is stretched to double its original length without loss of mass, how will the resistivity of the wire be influenced?***

Sol. The resistivity of a wire depends on the nature of its material. The increase in length will not affect its resistivity.

Q. 15. ***Two wires of equal cross-sectional area, one of copper and other of manganin have the same resistance. Which one will be longer?***

Sol. As $R = \rho \frac{l}{A}$ ∴ $A = \frac{RA}{\rho}$

For both wires R and A are same and

$\rho_{copper} < \rho_{manganin}$

∴ $A_{copper} < A_{manganin}$

i.e., manganin wire is thicker than copper wire.

Q. 16. ***There is two conductors A and B of the same material, having lengths l and 2l; and having radii r and r/2 respectively. What is the ratio of their resistances?***

Sol. $\frac{R_1}{R_2} = \frac{l_1}{l_2} \cdot \left(\frac{r_2}{r_1}\right)^2 = \frac{l}{2l}\left(\frac{r/2}{r}\right)^2 = \left(\frac{1}{8}\right) = 1:8$

Q. 17. ***Two wires A and B are of some metal, have the same area of cross-section and have their lengths in the ratio 2 : 1. What will be the ratio of currents flowing through them respectively when the same potential difference is applied across the length of each of them?***

Sol. Current, $I = \frac{V}{R} = \frac{VA}{\rho l}$ $\left(\because R = \rho \frac{l}{A}\right)$

For the two wires ρ, A and V are same, therefore

$I \propto \frac{1}{l}$

Hence $\frac{I_A}{I_B} = \frac{l_B}{l_A} = \frac{1}{2} = 1:2$ $[\because l_A : l_B = 2 : 1]$

Q. 18. ***Two wires A and B of the same metal and of the same length have their area of cross-section in the ratio of 2 : 1. If the same potential difference is applied across each wire in turn, what will be the ratio of the currents flowing in A and B?***

Sol. Current, $I = \frac{V}{R} = \frac{VA}{\rho l}$

For the two wires, ρ, l and V are same, therefore

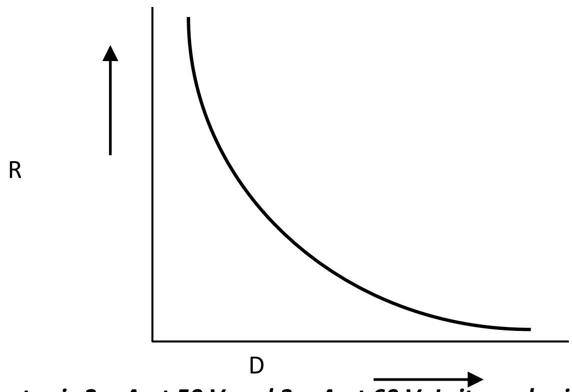
$I \propto A$

Hence $\frac{I_A}{I_B} = \frac{A_A}{A_B} = \frac{2}{1} = 2:1$

Q. 19. ***Draw a graph to show the variation of resistance a metal wire as a function of its diameter, keeping length and temperature constant,***

Sol. $R = \rho \frac{l}{A} = \rho \frac{4l}{\pi D^2}$ i.e., $R \propto \frac{1}{D^2}$

Hence the graph of R versus D is of the form shown in Fig.



Q. 20. ***The current flowing through a conductor is 2 mA at 50 V and 3 mA at 60 V. Is it an ohmic or non-ohmic conductor?***

Sol. The resistance in the two cases are

$R_1 = \frac{V_1}{I_1} = \frac{50}{2 \times 10^{-3}} = 25000\ \Omega$

$R_2 = \frac{V_2}{I_2} = \frac{60}{3 \times 10^{-3}} = 20000\ \Omega$

As the resistance changes with current, so the given conductor is non-ohmic.

Q. 21. ***Why do we use copper wires as connecting wires?***

Sol. This is because copper has a high electrical conductivity.

Q. 22. Does the value of resistance of a conductor depend on the potential difference applied across is or the current passed through it?

Sol. The resistance of conductor is constant provided the potential difference applied or current passed is not large.

Q. 23. Is the formula V = IR true for non-ohmic resistance also?

Sol. Yes, the formula defines resistance and not Ohm's law.

Q. 24. Is V = IR Ohm's law? Give correct form of Ohm's law.

Sol. V = IR is not Ohm's law. It only defines and gives value of resistance R. It is Ohm's relation.
$V \propto I$, is ohm's law.

Q. 25. How does the drift velocity of electrons in a metallic conductor vary with the increase in temperature?

Sol. The drift velocity decreases because of the increase in collision frequency of free electrons at higher temperature.

Q. 26. If the temperature of a good conductor increases, how does the relaxation time of electrons in the conductor change?

Sol. With the increase in temperature, the electrons collide more frequency with positive metal ions. So their relaxation time decreases.

Q. 27. If potential difference V applied across a conductor is increased to 2 V, how will the drift velocity of the electrons change?

Sol. Drift velocity, $v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml}$

Clearly, when V is increased to 2 V, drift velocity also gets obtained.

Q. 28. What happens to the drift velocity v_d of electrons and to the resistance R, if length of a conductor is doubled (keeping potential difference unchanged)?

Sol. Drift velocity,

$$v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml} \qquad \left(\because E = \frac{V}{l}\right)$$

When l is doubled, drift velocity becomes one-half of the initial value and resistance becomes twice the initial value ($R \propto l$).

Q. 29. A certain potential difference V is applied across a conductor. If another conductor is connected in parallel with it, what happens to the drift velocity?

Sol. Drift velocity remains unchanged.

Q. 30. When we switch on an electric bulb, it lights up almost instantaneously though drift velocity of electrons in copper wires is very small. Give reasons.

Sol. When the electric circuit is closed, electric field is set up throughout the circuit almost instantly with the speed of light. Electrons in every part of the circuit begin to drift in the opposite direction of electric field. A current starts flowing in the circuit almost instantaneously.

Q. 31. Of metals and alloys, which have greater value of temperature coefficient of resistance.

Sol. Metals have greater value of temperature coefficient of resistance than alloys.

Q. 32. Write two special characteristics of manganin due to which it is used in making standard resistances.

Sol. (i) The temperature coefficient of resistance for manganin is low. (ii) Manganin has a high value of resistivity.

Q. 33. Name any one material having a small value of temperature coefficient of resistance. Write one use of this value.

Sol. Alloy like nichrome has a very small value of temperature coefficient of resistance ($\alpha = 1.7 \times 10^{-7}\ °C^{-1}$). For this reason, it is used to make standard resistance.

Q. 34. Light from a bathroom bulb gets dimmer for a moment when the geyser is switched on. Why?

Sol. The current drawn by the high powered geyser from the supply initially lowers the voltage for a moment until the voltage gets stabilised by the transmission grid.

Q. 35. When a motor car is started, the car lights become slightly dim, why?

Sol. When a motor car is started, its starter takes a high current from the battery, so a larger potential drop (V = Current × internal resistance) occurs at the terminals of the battery and the bulbs get dim.

Q. 36. A carbon resistor is marked in coloured bands of red, black, orange and silver. What is the resistance and the tolerance value of the resistor?

Sol.

Red	Black	Orange	Silver
↓	↓	↓	↓
2	0	3	± 10 %

$R = 20 \times 10^3\ \Omega$, Tolerance = ± 10 %

Q. 37. A carbon resistor of 47 k Ω is to be marked with rings of difference colours for its identification. Write the sequence of colours.

Sol. Here $R = 47\ k\Omega = 47 \times 10^3\ \Omega$

∴ Colours sequence is : Yellow, Violet, Orange

Q. 38. Is internal resistance a defect of a cell?

Sol. Yes, because a part of the electrical energy obtained from the cell is used in sending current through the interior of the cell itself due to its internal resistance.

Q. 39. Why is the terminal voltage of a cell less than its emf?

Sol. This is because a part of the emf is consumed in doing work against the internal resistance of the cell ($V = \mathcal{E} - Ir$).

Q. 40. ***You are given a primary and a secondary cell of the same emf. From which cell will you be able to draw larger current and why?***

Sol. The internal resistance of a primary cell is more than that of a secondary cell, so a secondary cell will provide a larger current.

Q. 41. ***A car battery has an emf of 12 V. Eight ordinary cells connected in series can also supply 12 V. But we do not such an arrangement of cells in the car. Why?***

Sol. Eight cells arrangement cannot provide the high current needed to start the car engine because it has a high internal resistance of about 10 Ω while the internal resistance of a car battery is just 0.1 Ω.

Q. 42. ***Why does the conductivity of a semiconductor increase with rise of temperature?***

Sol. As temperature increases, covalent bonds begin to break in the semiconductor, setting free more and more electrons. So the conductivity increases.

Q. 43. ***A uniform wire of resistance 20 Ω is cut into two equals parts. These parts are now connected in parallel. What will be the resistance of the combination?***

Sol. Resistance of each part = 20/2 = 10 Ω

Resistance of the parallel combination

$$= \frac{10 \times 10}{10 + 10} = 5\ \Omega$$

Q. 44. ***Why resistance becomes more in series combination?***

Sol. In series combination, the effective length of conductor increases. As $R \propto l$, so resistance increases.

Q. 45. ***Why resistance becomes less in parallel combination?***

Sol. In parallel combination, the effective area of cross-section of the conductor increases. As $R \propto 1/A$, so resistance decreases.

Q. 46. ***A battery of emf 2 V and internal resistance 0.1 Ω is being charged by a current of 5 A. What will be the direction of current inside the battery? What is the potential difference between the terminals of the battery?***

Sol. During charging of a battery, the current inside the battery flows from the positive to the negative terminals of the battery, so the terminal p.d. is greater than the emf.

Q. 47. ***Why is a potentiometer named so?***

Sol. Because it is used to measure potential difference.

Q. 48. ***State the principle of a potentiometer.***

Sol. A potentiometer works on the principle that when a steady current flow through a wire of uniform cross-section and composition. The potential drop across any length of the wire is directly proportional to that length.

Q. 49. ***Why should the potentiometer wire be of uniform cross-section and composition?***

Sol. Only then it will have same resistance per unit length throughout. Then potential difference will be proportional to length of the wire, as required by the principle of potentiometer.

Q. 50. ***Why should the material of the potentiometer wire be of high specific resistance?***

Sol. This makes the resistance of the entire length of the wire sufficiently large and hence for a given current, there is an appreciable potential drop.

Q. 51. ***Why should the material of the potentiometer wire be of low temperature coefficient of resistance?***

Sol. A material having low temperature coefficient of resistance ensures that its resistance does not change appreciably due to heating.

Q. 52. ***Of which material is a potentiometer wire normally made and why?***

Sol. The potentiometer wire is usually made of an alloy such as nichrome or manganin. Such an alloy has high resistivity and low temperature coefficient of resistance.

Q. 53. ***Can we use copper wire as a potentiometer wire?***

Sol. Resistivity of copper is small, so there will not be an appreciable potential drop across the ends of potentiometer wire. Also temperature coefficient of resistance of copper is large.

Q. 54. ***Why should the current be not passed through potentiometer wire for a long time?***

Sol. This will heat up the potentiometer wire and will change its resistance. Potential drop per unit length of the wire will also change.

Q. 55. ***What type of cell should be used in the main circuit of the potentiometer and why?***

Sol. A Leclanche cell should be used in the main circuit of the potentiometer. This is because of the fact that Leclanche cell us useful, when the current is drawn for a short time.

Q. 56. ***The emf of the cell used in the main circuit of the potentiometer should be more than the potential difference to be measure. Why?***

Sol. If it is not so, the balance point will not be obtained on the potentiometer wire.

Q. 57. ***Why should the jockey be not rubbed against the potentiometer wire?***

Sol. Rubbing of jockey against the potentiometer wire affects the uniformity of the cross-sectional area of the wire and hence changes the potential drop across the wire.

Q. 58. ***What is meant by the sensitivity of a potentiometer?***

Sol. A potentiometer is said to be sensitivity if

(i) It can measure very small potential differences, and

(ii) For a small change in potential difference being measured, it shows a large change in balancing length.

Q. 59. ***How can the sensitivity of potentiometer be increased?***

Sol. The sensitivity can be increased by reducing the potential gradient. This can be done by (i) increasing the length of the wire and (ii) by reducing the current in the main circuit.

Q. 60. How can you make a potentiometer of given wire length more sensitive using a resistance box?

Sol. This can be done by introducing some resistance in the circuit through the resistance box. This decreasing the current in the circuit. Consequently, the potential gradient decreases and hence sensitivity of the potentiometer increases.

Q. 61. Why do we prefer a potentiometer with a longer bridge wire?

Sol. A potentiometer with a longer bridge wire has a small potential gradient. Consequently, it is more sensitive and hence preferred.

Q. 62. Why is a ten-wire potentiometer more sensitive than a four-wire one?

Sol. This is because potential gradient for ten-wire potentiometer is smaller than that for a four-wire one.

Q. 63. Why is a Wheatstone bridge so called?

Sol. It is so called because this method was first suggested by a British physicist, Sir Charles F. Wheatstone in 1843. It is called a bridge because the galvanometer circuit forms a kind of bridge by connecting two points having same potential.

Q. 64. When is a Wheatstone bridge said to be balanced?

Sol. A Wheatstone bridge is said to be balanced if no current flows through its galvanometer arm. When the Wheatstone bridge is balanced.

Q. 65. What do you mean by sensitiveness of a Wheatstone bridge?

Sol. A Wheatstone bridge is said to be sensitive if it produces more deflection in the galvanometer for a small change of resistance in resistance arm.

Q. 66. When is Wheatstone bridge most sensitive?

Sol. When all the four resistors P, Q, R and S are nearly of the same magnitude.

Q. 67. Why a slide wire bridge or metre bridge is named so?

Sol. As in it a jockey is made to slide over the bridge wire to get a null point, so its called a slide-wire bridge. As it uses one metre long wire, so it is called a metre bridge.

Q. 68. Why should we get a null point in the middle of the metre bridge wire?

Sol. The metre bridge is most sensitivity when the four resistances forming the Wheatstone bridge are equal. This is possible only if the balanced point is somewhere near the middle of the wire.

Q. 69. What is the end error in a metre bridge?

Sol. The end error in a metre bridge is due to the following reasons:

(i) The zero mark of the scale provided along the wire may not start from the position where the bridge wire leaves the copper strip and 100 cm mark of the scale may not end at position where the wire touches the copper strip.

(ii) The resistance of copper wires and copper strips of metre bridge has not been taken into account.

Q. 70. What are the advantages of a Wheatstone bridge method of measuring resistance over other methods?

Sol. (i) It is a null method; hence the result is free form the effect of extra resistances (cell resistances) of the circuit. (ii) Being null method, it is easier to detect a small change in deflection than to read a deflection directly.

Q. 71. Why are the connecting resistors in a metre bridge made of thick copper strips?

Sol. Thick copper strips offer minimum resistance and hence avoid the error due to end resistance which have not been taken into account in the bridge formula.

Q. 72. Why is Wheatstone bridge (or metre bridge) method considered unsuitable for the measurement of very low resistances?

Sol. For measuring low resistance, all other resistances used should have low value of ensure the sensitivity of the bridge. This requires a galvanometer of very low resistance which itself would be very sensitivity Also, the end-resistances and resistances of connecting wires become comparable to the resistance being measured and introduce error in the result.

Q. 73. Why metre bridge method is considered unsuitable for the measurement of very high resistance?

Sol. For measuring high resistance, all other resistances forming the bridge should also be high so as to ensure the sensitivity of the bridge. But this reduces the current through the galvanometer which becomes insensitive.

Q. 74. Why is Wheatstone bridge method suitable for comparing the resistances of the same order of magnitude?

Sol. Metre bridge is based on the principle of Wheatstone bridge which is most sensitive when all the resistances are nearly of the same magnitude. So metre bridge is suitable for comparing the resistance of the same order of magnitude.

Q. 75. What happens if the galvanometer and cell are interchanged at the balance point of the bridge? Would the galvanometer show any current?

Sol. When galvanometer and cell are interchanged, condition for balance of the bridge remains satisfies. So galvanometer will show no current.

Q. 76. Three resistors have resistances 2, 3 and 4 Ω. If they are connected to the same battery in turn, in which case the power dissipated will be maximum?

Sol. $P = \frac{V^2}{R}$ ∴ For a given V, $P \propto \frac{1}{R}$

Hence power dissipation will be maximum for the 2 Ω resistor.

Q. 77. A constant voltage is applied between the two ends of a uniform metallic wire. Heat Q is developed in it. If another wire, double the radius and twice the length is used, how much heat will be developed in it?

Sol. Heat produced, $Q = \frac{V^2}{R} t = V^2 \cdot \frac{\pi r^2}{\rho l} \cdot t$

When wire of double radius and twice the length is used, heat produced is

$$Q' = V^2 \cdot \frac{\pi (2r)^2}{\rho \cdot 2l} \cdot t = 2Q$$

Q. 78. Two wires A and B of the same material and having same length, have their cross-sectional areas in the ratio 1:4. What would be the ratio of heat produced in these wires when same voltage is applied across each?

Sol. The resistances of the two wires can be written as

$$R_A = \frac{\rho l}{A}, \quad R_B = \frac{\rho l}{4A} \quad \therefore \quad \frac{R_A}{R_B} = 4$$

Ratio of heats produced in the two wires will be

$$\frac{P_A}{P_B} = \frac{V^2/R_A}{V^2/R_B} = \frac{R_B}{R_A} = \frac{1}{4} = 1:4$$

Q. 79. The element of the heater is very hot while the wires carrying current are cold, why?

Sol. Heat produced, $H = I^2 Rt$. Both heater element and conducting wires carry same current. But the heater element becomes very hot due to its high resistance and the conducting wires remain cold due to their low resistance.

Q. 80. Though same current flows through the electric line wires and the bulb filament, yet only the filament glows. Why?

Sol. The bulb filament has a much higher resistance than the electric line wires. Current passed through the high resistance filament produces a large amount of heat ($H = I^2 Rt$) and makes it glow.

Q. 81. The temperature of the filament of an electric bulb is 2700 ℃ when it glows. Why is it not burnt up at such a high temperature?

Sol. This is because the lamp's filament has high melting point and is held in an atmosphere of inert gases which prevent its oxidation.

Q. 82. A toaster produces more heat than a light bulb when connected in parallel to the 220 V mains. Which has a greater resistance?

Sol. The amount of heat produced by an electric current in time t is given by

$$H = I^2 Rt = \frac{V^2 t}{R} \quad \text{i.e.,} \quad H \propto \frac{1}{R}$$

For the parallel combination, potential difference V is same. As toaster produces move heat, so light bulb should have a greater resistance.

Q. 83. Heat is generated continuously in an electric heater but its temperature becomes constant after some time. Why?

Sol. When the temperature of the heater becomes greater than the temperature of its surroundings, it begins to lost heat to surroundings. A stage is reached when the rate of production of heat becomes equal to the rate of loss of heat and the temperature becomes constant.

Q. 84. Twenty electric bulbs are connected in series with the mains of a 220 V supply. After one bulb is fused, the remaining 19 bulbs are again connected in series across the same mains. What will be the effect on illumination?

Sol. The Combined resistance of 19 bulbs is less than the combined resistance of 20 bulbs. Therefore, the current will increase. As heat produced, $H \propto I^2$, therefore, illumination will increase.

Q. 85. A current is passed through a steel wire heating it to red hot. The half of the wire is immersed in cold water. Which half of the wire will heat up more and why?

Sol. The temperature of the wire outside the water is more than that inside the water. As resistance of the wire increases with temperature, the resistance of the part of the wire outside the water will also be higher. Since same current is flowing through the two parts, more heat will be produced in the part outside the water.

Q. 86. The maximum power rating of a 20 Ω resistor is 2 kilowatt. Can it be connected to a 220 V d.c. supply?

Sol. The maximum voltage that can be applied across the resistor is

$$V_{max} = \sqrt{P_{max} R} = \sqrt{2000 \times 20} = 220 \text{ V}$$

So the given resistor cannot be connected to a 220 V d.c. supply.

Q. 87. In an electric kettle, water boils in 20 minutes after the kettle is switched on. With the same supply voltage if the water is to boil in 10 minutes, should the length of the heating element be decreased or increased?

Sol. For a given supply voltage, the rate of production of heat is

$$P = \frac{V^2}{R} = \frac{V^2 A}{\rho l} \quad \therefore \quad P \propto \frac{1}{l}$$

Clearly, in order to double the rate of production of heat, length of the wire should be decreased to half its original length.

Q. 88. Three bulbs 40 W, 60 W and 100 W are connected to 220 V mains. Which bulb will glow brightly, if they are connected in series?

Sol. In the series circuit, the same current flows through each bulb. But the 40 W bulb has the highest resistance ($R = V^2/P$). The 40 W bulb produces maximum heat per second ($P = I^2 R$), so it will glow more brightly than the other two bulbs.

Q. 89. A 100 W and a 500 W bulb are joined in parallel to the mains. Which bulb will glow brighter?

Sol. In parallel, same voltage V is applied to both the bulbs. But 500 W bulb has a smaller resistance [$R = V^2/P$], so it will produce more heat per second [$P = V^2/R$] and will glow brighter that 100 W bulb.

Q. 90. Two heating coils, one of fine wire and the other of thick wire, made of same material and of the same length are connected in turn to a source of emf. Which of the coils will produce more heat?

Sol. As $P = V^2/R$, hence for same V, the thicker wire will produce more heat because of its smaller resistance.

Q. 91. Why an electric bulb becomes dim when an electric heater in parallel circuit is switched on? Why dimness decreases after some time?

Sol. Electric heater with more power has a smaller element resistance. It draws a higher current. Some current from the bulb is diverted into heater. Bulb becomes dim. After some time, heater element becomes hot. Its resistance increases. It takes less current. Current through the bulb increases and its dimness decreases.

Q. 92. Why does a bulb not become dim when another bulb of same power in parallel circuit is switched on?

Sol. As the two bulbs have same power, their filament resistance is also same. When second bulb is switched on a parallel, total resistance decreases to one-half and current becomes double. The double current divides itself equally in the two bulbs. Current through each bulb is same as the original one. So there is no dimness.

Q. 93. Nichrome and copper wires, having same length and same area of cross-section, are connected across a battery in turn. In which case the rate of production of heat will be higher?

Sol. As the resistivity of copper is less than that of nichrome, so resistance of copper wire is less than that of nichrome wire of same length and diameter.

As rate of production of heat, $P = \frac{V^2}{R}$

∴ For given V, $P \propto 1/R$

Hence rate of production of heat in copper wire is higher than that in nichrome wire.

Q. 94. Nichrome and copper wires of same length and area of cross-section are connected in series, current in passed through them. Why does the nichrome wire get heater first?

Sol. In series, the current through both wires is same.

As the rate of production of heat, $P = I^2 R$

∴ For given I, $P \propto R$

As the resistivity of nichrome wire is much more than that of copper, so the resistance of nichrome wire is much larger than that of copper wire of same length and area of cross-section. Hence the rate of production of heat in nichrome wire will be higher or the nichrome wire gets heated first.

Q. 95. A heater joined in series with a 60 W bulb is connected to the mains. If 60 W bulbs is replaced by a 100 W bulb, will the rate of heat produced by the heater be more, less or remain the same?

Sol. The resistance of the 100 W bulb is less than that of 60 W bulb. When 60 W bulb is replaced by 100 bulb, the resistance in the series circuit decreases and current increases. Hence the rate of heat produced by the heater will be more.

Q. 96. A 60 W electric bulb connected in parallel with a room heater is further connected across the mains. If 60 W bulb is replaced by 100 W bulb, will the heat produced by the heater be smaller, remain the same or be larger and why?

Sol. In parallel connection, a part of current from the mains flows through the 60 W bulb and remaining current flows through the heater. When 60 W bulb is replaced by 100 W bulb, the 100 W bulb now draws larger current due to its smaller resistance than 60 W bulb. The current through the heater decreases and it produces smaller heat.

Q. 97. Two 120 V light bulbs, one of 25 W and other of 200 W were connected in series across a 240 V line. One bulb burnt out almost instantaneously. Which one was burnt and why?

Sol. As $R = V^2/P$, so 25 watt bulb has more resistance. In the series circuit, same current flows through both the bulbs. The 25 W bulb develops more heat ($H = I^2 Rt$) and hence burns out almost instantaneously.

Q. 98. Equation $P = I^2 R$ seems to suggest that the rate of joule heating in a resistor is reduced if the resistance is made less, however, the equation $P = V^2/R$ seems to suggest the opposite. How do you reconcile this apparent paradox?

Sol. There is no paradox in the problem. When V is constant and R is reduced, there is an increase in current and consequently an increase in joule heating.

Q. 99. If three electric bulbs of 100 W, 50 W and 25 W connected in the electric fittings of a house are lighted by the electric supply, what will be the total power consumed?

Sol. Total power consumed = 100 + 50 + 25 = 175 W

This is because the bulbs in house fitting are connected in parallel and total power consumed is equal to the sum of the powers of the individual bulbs.

Q. 100. If a battery is short-circuited by means of a heavy copper wire, its temperature rises. Why?

Sol. Rate of production of heat,

$$P = \frac{V^2}{R}$$

As the resistance of heavy copper wire is negligibly small, the rate of production of heat will be quite high, so the temperature rises.

Q. 101. How does the use of a fuse wire protect the electrical installation?

Sol. A fuse wire has high resistance and low melting point. It is connected in series with the electric installations. When the current exceeds the safety limit, the fust wire melts away. The circuits breaks and electrical installations are saved.

SHORT ANSWER TYPE CONCEPTUAL PROBLEMS

Q. 1. ***Name the carriers of electric current in***

(a) A bar made of silver ***(b) A hydrogen discharge tube***

(c) A voltaic cell ***(d) A lead accumulator being charged by an external supply***

(e) A germanium semiconductor ***(f) A wire made of the alloy nichrome*** ***(g) A superconductor.***

In each case relate the motion of the carrier with the direction of current.

Sol. **(a)** Electrons, their addition of motion is opposite to that of the current.

(b) Electrons and positive ions (protons) of hydrogen, produced by the ionisation of hydrogen atoms in the discharge tube. Protons move in the direction of current but electrons move in the opposite direction.

(c) H^+ and SO^{2+}_4 ions. Inside the voltaic cell, the movement of H^+ ions towards the copper electrode and that of SO^{2-}_4 towards the zinc plate is equivalent to the conventional current from zinc side to copper side of the solution.

(d) H^+ and SO^{2+}_4 ions. During charging, H^+ ions move towards cathode and SO^{2-}_4 ions move towards anode, in the electrolyte. The direction of current is from anode to cathode inside the cell which is opposite to that during the discharging process.

(e) Electrons and holes (holes are vacant states from which electrons have been removed and effectively act as positive charge carriers), holes move in the direction of current while electrons move in the opposite direction.

(f) Electrons, electrons move in the opposite direction of current.

(g) Mutually coherent group of electrons, which move in the opposite direction of current.

Q. 2. ***Clarify your elementary notions about current in a metallic conductor by answering the following queries:***

(a) The electrons drift speed is estimated to the only a few mms^{-1} for currents in the range of a few amperes? How then is current established almost the instant a circuit is closed?

(b) The electron drift arises due to the force experienced by electrons in the electric field inside the conductor. But force should cause acceleration. Why then do the electrons acquire a steady average drift speed?

(c) If the electron drift speed is so small, and the electron's charge is small, how can we still obtain larger amounts of current in a conductor?

(d) When electrons drift in a metal from lower to higher potential, does it mean that all 'free' electrons of the metal are moving in the same direction?

(e) Are the paths of electrons straight lines between successive collisions (with the positive ions of the metal) in the (i) absence of electric field, (ii) presence of electric field?

Sol. **(a)** Electrons are present everywhere in an electric circuit. As soon as the circuit is closed, electric field is set up throughout the circuit almost instantly (with the speed of light). Electrons in every part of the circuit begin to drift in the opposite direction of electric field. A current start flowing in the circuit almost instantly.

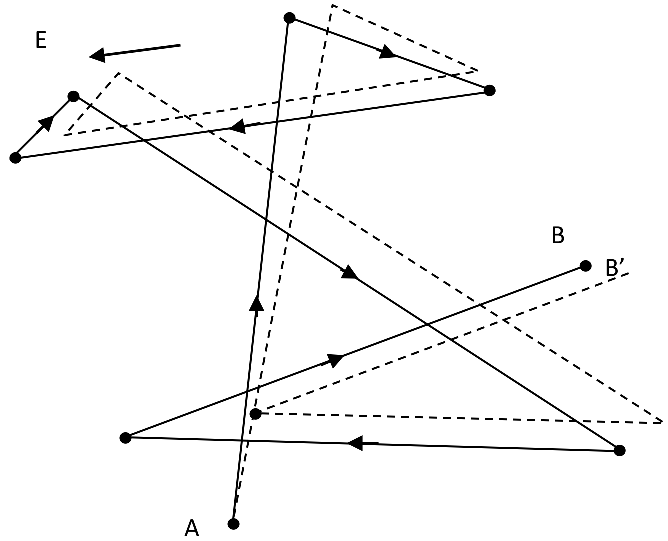
(b) Due to the force exerted by the external electric field, each 'free' electron inside the conductor does accelerate and increase drift speed until it collides with a positive ion of the metal. It loses its drift speed after collision but starts to accelerate and increase its drift speed again only to suffer a collision again and so on. On the average, therefore, the electrons acquire only a drift speed.

(c) The current in a conductor is given by

$$I = e_n Av_d$$

Although the electron's charge e and drift speed v_d are very small quantities, yet we can obtain a large amount of current in a conductor. This is because the free electron density (number of electrons per unit volume) of a conductor is very large $\approx 10^{29}\ m^{-3}$. The drifts of a very larger number of free electrons add up to cause a large current inside the conductor.

(d) No, only the drift velocities of the electrons are superposed over their random (haphazard) thermal velocities. In Fig. the solid line shows the random path followed by a free electron in the absence of an external field; the electron proceeds from A to B, making six collisions on its path. The dotted curve shows how the random motion of the same electron gets modified when an electric field is applied. Clearly, the electron drifts slowly to the right, ending its motion at B' rather than at B.



(e) (i) In the absence of electric field, the paths are straight lines. (ii) In the presence of electric field, the paths are curved.

Q.3. ***Answer the following questions:***

(a) It is easier to confine electric current to definite paths (by the use of electric insulators) than the direct heat flow along definite routes using heat insulators. Why?

(b) It is easier to start a car engine on a warm day than on a chilly day. Why?
(c) In which respect, does a nearly discharged lead acid secondary cell differ mainly from a freshly charged cell in its emf or in its internal resistances?

Sol. **(a)** Typical electrical insulators (e.g., glass) differ in their resistivity from metals enormously, by a factor of the order or 10^{22}. The corresponding factor for thermal insulators versus thermal conductors is only 10^3. So it is easier to send electric currents to definite paths by the use of insulators than to direct heat flow along definite routes using heat insulators.
(b) The internal resistance of a car battery decreases with increase in temperature. Therefore, on a warm day a car battery gives large current which helps in starting the car engine.
(c) It mainly differs in internal resistances but partly in its emf also. A nearly discharged lead acid secondary cell possesses a very high internal resistance.

Q. 4. Choose the correct alternative:
(a) Doping a semiconductor (with small traces of impurity atoms) reduces/increases its resistivity.
(b) The resistance of graphite and most non-metals increases/decreases with increase in temperature.
(c) The resistivity of a semiconductor increases/decreases rapidly with increasing temperature.

Sol. (a) Reduces (b) decreases (c) decreases.

Q. 5. Write dimensional formula of electrical conductivity.

Sol. $\text{Electrical conductivity} = \dfrac{1}{\text{Resistivity}} = \dfrac{\text{Length}}{\text{Resistance} \times \text{Area}}$

$\therefore \quad [\text{Electrical conductivity}] = \dfrac{L}{ML^2T^{-3}A^{-2}L^2} = [M^{-1}L^{-3}T^3A^2]$

Q. 6. Two wires of the same material having lengths in the ratio 1 : 2 and diameters in the ratio 2 : 3 are connected in series with an accumulator. Compute the ratio of P.D. across the two wires.

Sol. Resistance,

$$R = \rho\frac{l}{A} = \rho\frac{l}{\pi\left(\frac{D}{2}\right)^2} = \frac{4\rho l}{\pi D^2}$$

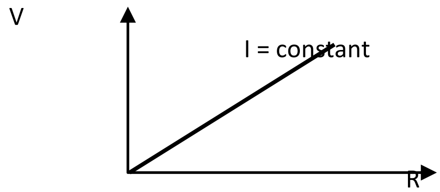
$$\therefore \quad \frac{R_A}{R_B} = \frac{4\rho l_A/\pi D_A^2}{4\rho l_B/\pi D_B^2} = \frac{l_A}{l_B}\cdot\left(\frac{D_B}{D_A}\right)^2$$

$$= \frac{1}{2}\times\left(\frac{3}{2}\right)^2 = \frac{9}{8}$$

$$\frac{\text{P.D. across A}}{\text{P.D. across B}} = \frac{V_A}{V_B} = \frac{I_A R_A}{I_B R_B} = \frac{R_A}{R_B} = \frac{9}{8} = 9 : 8 \qquad [\because \text{For series resistances, } I_A = I_B]$$

Q. 7. If the current supplied to a variable resistor is constant, draw a graph between voltage and resistance.

Sol. According to Ohm's law, For constant I, $V \propto R$
Hence the graph between voltage V and resistance R will be a straight passing through the origin, as shown in Fig.

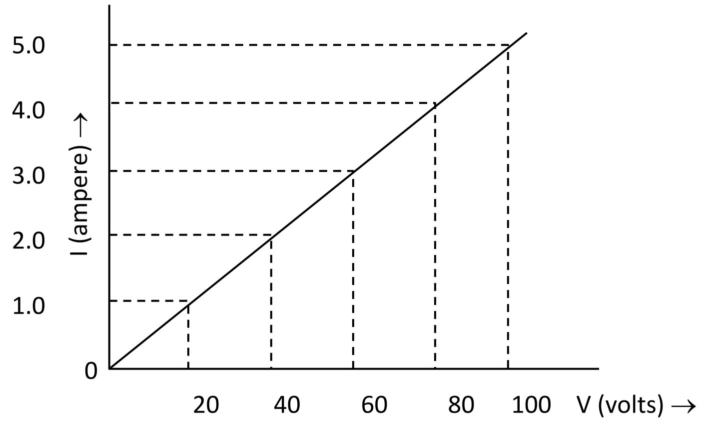


Q. 8. Observation on a given device yielded the following current-voltage data:

Current (Ampere)	Voltage (Volt)	Current (Ampere)	Voltage (Volt)
1.0	19.7	4.0	78.8
2.0	39.4	5.0	98.5
3.0	59.1		

Draw V – I graph using the above data.
what conclusion can you draw from the graph with regard to ohmic or non-ohmic nature of device?

Sol. Fig. Shows the V – I graph for the given data. As the graph is straight line, so the given device is ohmic.

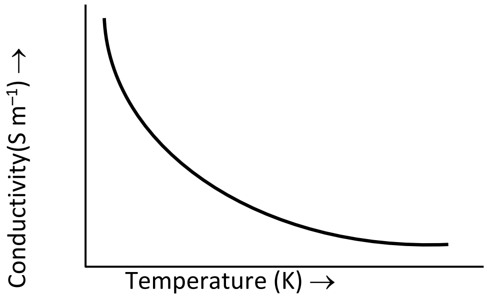


Q. 9. ***Explain, with the help of a graph, the variation of conductivity with temperature for a metallic conductor.***

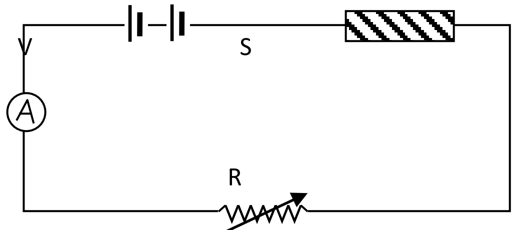
Sol. Conductivity of a metallic conductor is given by

$$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$$

For metals, the number of free electrons is fixed. As the temperature increases, the amplitude of vibrations of the atoms increases. The collisions of electrons with these atoms become more effective and frequent. The relaxation time τ decreases. Hence the conductivity of a metallic conductor decreases with the increase of temperature, as shown below graphically.

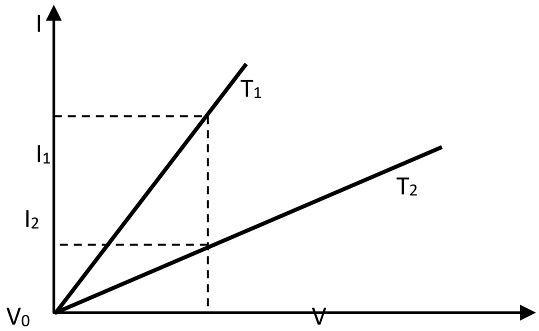


Q. 10. ***The diagram shows a piece of pure semiconductor, S in series with a variable resistor R, and a source of constant voltage V. Would you increase or decrease the value of R to keep the reading of ammeter (A) constant, when semiconductors S is heater? Give reason.***



Sol. The value of R should be increased. As the temperature of semiconductor S increases, its resistivity decreases. As a result, the circuit resistance decreases and current tends to increase. To keep to reading of ammeter (A) constant, the value of R has to be increased.

Q. 11. ***The current voltage graphs for a given metallic wire at different temperature T_1 and T_2 are shown in Fig. Which of the temperature T_1 and T_2 is greater?***



Sol. For the same potential V_0,

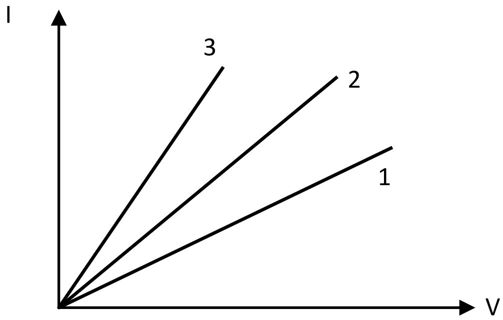
Resistance at T_1, $R_1 = \frac{V_0}{I_1}$

Resistance at T_2, $R_2 = \frac{V_0}{I_2}$

$\because$ $I_2 < I_1$ $\therefore$ $R_2 > R_1$ Since, resistance of a metal increases with temperature, therefore, $T_2 > T_1$.

Q. 12. ***The V – I graphs of two resistors, and their series combination, are shown in Fig. Which one of these graphs represents the series combination of the other two? Give reason for your answer.***

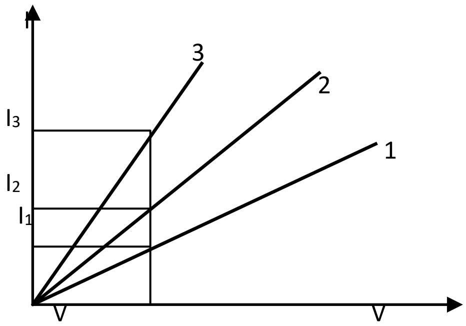
Sol.



Refer to Fig., various resistances are

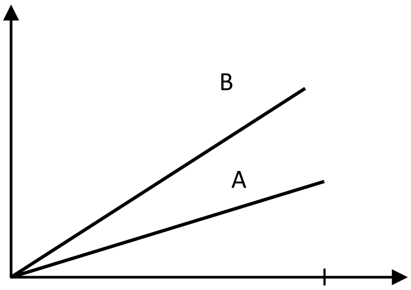
$R_1 = \frac{V}{I_1}$, $R_2 = \frac{V}{I_2}$, $R_3 = \frac{V}{I_3}$ But $I_1 < I_2 < I_3$ $\therefore$ $R_1 > R_2 > R_3$

Hence graph 1 represents the series combination of other two resistors.



Q. 13. ***V – I graphs for parallel and series combination of two metallic resistors are as shown in Fig. Which graph represents parallel combination? Justify your answer.***

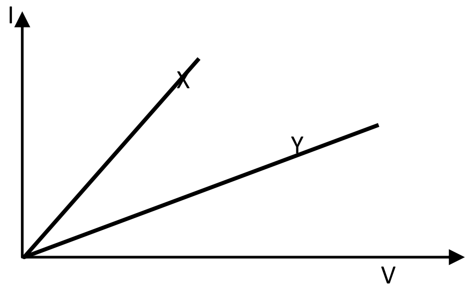
Sol. V



As R = V/I. Clearly, slop of V – I graph gives resistance R. As graph B has a greater slope than A, so graph B represents series combination (higher resistance) and graph A represents parallel combination (lower resistance).

Q. 14. ***The voltage current variation of two metallic wires X and Y at constant temperature are shown in Fig. Assuming that the wires have the same length and the same diameter, explain which of the two wires will have larger resistivity.***

Sol.



Slope of I – V line for wire X > slope of I – V line for wire Y.

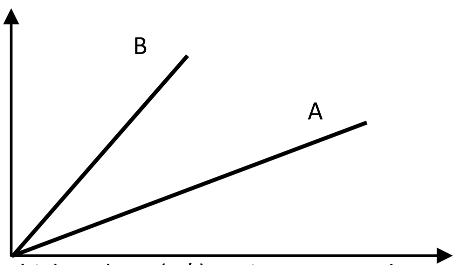
∴ Conductance of wire X > Conductance of wire Y or Resistance of wire X < Resistance of wire Y

or $\rho_X \cdot l/A < \rho_Y \cdot l/A$

or $\rho_X < \rho_Y$ Thus wire Y has a larger resistivity.

Q. 15. ***The voltage-current graphs for two resistors of the same material and the same radii with lengths L_1 and L_2 are shown in Fig. If $L_1 > L_2$, state with reason, which of these graphs represents voltage-current change for L_1.***

Sol. V



I

As $L_1 > L_2$, so $R_1 > R_2$. Now the graph B has a higher slope (V/I), so it corresponds to a greater resistance. Clearly, graph B represents voltage-current change for length L_1.

Q. 16. ***A potential difference V is applied to a conductor of length l, diameter D. How are the electric field E, the drift velocity v_d and the resistance R affected when (i) V is doubled (ii) l is doubled (iii) D is doubled?***

Sol. Electric field, $E = \frac{V}{l}$

Drift velocity, $v_d = \frac{eE}{M}\tau = \frac{eV}{ml}\tau$

Resistance, $R = \rho\frac{l}{A} = \frac{4\rho l}{\pi D^2}$

(i) When V is doubled, E becomes double, v_d becomes double and R remains unchanged.

(ii) When I is doubled, E becomes half, v_d becomes half and R becomes double.
(iii) When D is doubled, E remains unchanged, v_d is also unchanged and R becomes one-fourth.

Q. 17. ***The current I flows through a wire of radius r and the free electrons drift with velocity v_d. What is the drift velocity of electrons through a wire of same material but having double the radius, when a current 2I flows through it?***

Sol. Initial drift velocity, $v_d = \frac{I}{enA} = \frac{I}{en \times \pi r^2}$

For a wire of same material (hence same n) and double the radius, the drift velocity with current 2 I will be

$$v_d = \frac{2I}{e_n \times \pi (2r)^2} = \frac{1}{2} \cdot \frac{I}{en \times \pi r^2} = \frac{1}{2} v_d$$

Q. 18. ***Explain how electron mobility changes for a good conductor when (i) the temperature of the conductor is decreased at constant potential difference and (ii) applied potential difference is doubled at constant temperature.***

Sol. Electron mobility of a conductor,

$$\mu = \frac{e\tau}{m}$$

(i) When the temperature of the conductor decreases, the relaxation time τ of free electrons decreases, so mobility μ decreases.
(ii) Mobility μ is independent of applied potential difference.

Q. 19. ***Two wires X, Y have the same resistivity, but their cross-sectional areas in the ratio 2 : 3 and lengths in the ratio 1 : 2. They are fist connected in series and then in parallel to a d.c. source. Find out the ratio of the drift speeds of the electrons in the two wires for the two cases.***

Sol. Current, $I = e_n A v_d$

(i) In series, current I is constant. So, $v_d \propto 1/A$

$$\frac{v_d (X)}{v_d (Y)} = \frac{A_Y}{A_X} = \frac{3}{2} = 3 : 2$$

(ii) In parallel, V = IR = constant

$\therefore \quad I \propto \frac{1}{R}$

But $\quad R \propto \frac{l}{A}$

$\therefore \quad I \propto \frac{A}{l}$

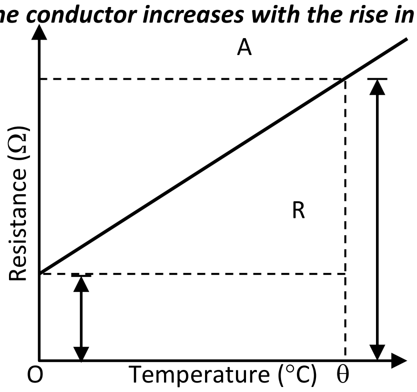
or $\quad \frac{I_X}{I_Y} = \frac{A_X}{A_Y} \cdot \frac{l_Y}{l_X} = \frac{2}{3} \times \frac{2}{1} = \frac{4}{3}$

Also, $\quad \frac{I_X}{I_Y} = \frac{enA_X v_d (X)}{enA_Y v_d (Y)} = \frac{2\, v_d (X)}{3\, v_d (Y)}$

Hence, $\quad \frac{2\, v_d (X)}{2\, v_d (Y)} = \frac{4}{3}$

or $\quad \frac{v_d (X)}{v_d (Y)} = \frac{2}{1} = 2 : 1$

Q. 20. ***The variation of the resistance of a metallic conductor with temperature is shown in Fig.***
(i) Calculate the temperature coefficient of resistance from the graph.
(ii) State why the resistance of the conductor increases with the rise in temperature.



Sol. (i) Let R be the resistance and θ be the temperature of the conductor corresponding to some point A on the graph. Then temperature coefficient of the conductor will be $\quad \alpha = \frac{R - R_0}{R_0 \times \theta}$

(ii) The resistance of a conductor increases with the rise in temperature due to the increase in collision frequency of electrons with the positive metal ions.

Q. 21. ***Write down the simple relationship to show the variation of resistance with temperature. Define the temperature coefficient of resistance.***

Sol. The resistance of a metallic conductor at temperature θ°C is given by

$$R = R_0 (1 + \alpha\theta)$$

where R_0 is the resistance at 0°C and α is the temperature coefficient of resistance. Clearly,

$$\alpha = \frac{R - R_0}{R_0 \times \theta}$$

Thus, the temperature coefficient of a material may be defined as the increase in its resistance per unit resistance at 0°C per degree rise of temperature.

Q. 22. The resistivity of the material of a conductor of uniform cross-section varies along its length as $\rho = \rho_0 (1 + \alpha x)$. Deduce the expression for the resistance of the conductor, if its length is L and area of cross-section is A.

Sol. The resistance of the small length dx of the conductor at distance x from its one end will be

$$dR = \rho \frac{dx}{A} = \rho_0 (1 + \alpha x) \frac{dx}{A}$$

The resistance of the length L of the conductor will be

$$R = \int dR = \frac{\rho_0}{A} \int_0^L (1 + \alpha x)\, dx$$

$$= \frac{\rho_0}{A} = \left[|x|_0^L + \alpha \left[\frac{x^2}{2} \right]_0^L \right]$$

$$= \frac{\rho_0}{A} [(L - 0) + ½ \alpha (L^2 - 0)] = \rho_0 [L + ½ \alpha L^2]$$

Q. 23. Three materials A, B and C have electrical conductivities σ, 2 σ and 2 σ respectively. Their number densities of free electrons are n, 2n and n respectively. For which material, is the average collision time of free electrons maximum?

Sol. Conductivity, $\sigma = \frac{ne^2 \tau}{M}$

∴ Relaxation time, $\tau = \frac{\sigma m}{ne^2}$

For material A, $\tau_A = \frac{\sigma m}{ne^2}$

For material B, $\tau_B = \frac{2\sigma m}{2ne^2} = \frac{\sigma m}{ne^2}$

For material C, $\tau_C = \frac{2\sigma m}{ne^2}$

Clearly, $\tau_C > \tau_B = \tau_A$.

Q. 24. A uniform wire of resistance R is shaped into a regular n sided polygon, where n is even. Find the equivalent resistance between (i) opposite corners of the polygon and (ii) adjacent corners of polygon.

Sol. Resistance of each side of polygon = $\frac{R}{n}$

(i) Equivalent resistance between two opposite corners of polygon. The resistances of the two parts of the polygon will be

$$R_1 = R_2 \times \frac{n}{2} \times \frac{R}{n} = \frac{R}{2}$$

These two parts form a parallel combination. So the equivalent resistance between two opposite corners of the polygon will be

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{R/2 \times R/2}{R/2 + R/2} = \frac{R}{4}$$

(ii) Equivalent resistance between two adjacent corners of the polygon. The resistances of the two parts will be

R_1 = Resistance of one side = $\frac{R}{n}$

R_2 = Resistance of remaining (n – 1) sides connected in series

$= \frac{(n-1)R}{n}$

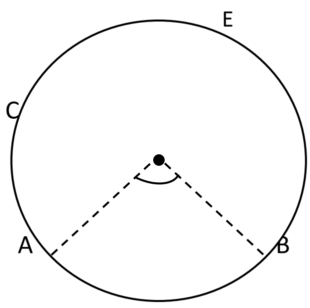
These two parts form a parallel combination. So the equivalent resistance of the polygon between two adjacent corners will be

$$R'' = \frac{R/n \times \frac{(n-1)R}{n}}{R/n + \frac{(n-1)R}{n}} = \frac{(n-1)R}{n^2}$$

Q. 25. As shown in Fig. A and B are two points on a uniform ring of resistance r. If the part AB of the ring subtends an angle θ at the centre C of the ring, find the equivalent resistance of the ring between the points A and B.

Sol. Let r be the radius of the ring.

Then the resistance per unit length of the ring = $\frac{R}{2\pi r}$



Length of the arc ADB = r θ [θ (rad) = Arc/radius]

∴ Resistance of the arc ADB,

$$R_1 = \frac{R}{2\pi r} \times r\,\theta = \frac{R\theta}{2\pi}$$

Length of the arc AED = r (2π – θ)

Resistance of the arc AED,

$$R_2 = \frac{R}{2\pi r} \times r(2\pi - \theta) = \frac{R(2\pi - \theta)}{2\pi}$$

As the two arcs form a parallel combination, so the equivalent resistance of the ring between points A and B is

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{\frac{R\theta}{2\pi} \times \frac{R(2\pi - \theta)}{2\pi}}{\frac{R\theta}{2\pi} + \frac{R(2\pi - \theta)}{2\pi}} = \frac{(2\pi - \theta)\theta R}{4\pi^2}$$

Q. 26. ***A battery of emf $\mathcal{E}$ and internal resistance r sends currents I_1 and I_2, when connected to external resistance R_1 and R_2 respectively. Find the emf and the internal resistance of the battery.***

Sol. Current in first case, $I_1 = \frac{\mathcal{E}}{R_1 + r}$

∴ $\mathcal{E} = I_1 (R_1 + r)$ ... (i)

Current in second case, $I_2 = \frac{\mathcal{E}}{R_2 + r}$

∴ $\mathcal{E} = I_2 (R_2 + r)$ ... (ii)

From equations (i) and (ii), we have

$I_1 (R_1 + r) = I_2 (R_2 + r)$

or $r (I_1 - I_2) = I_2 R_2 - I_1 R_1$

or $r = \frac{I_2 R_2 - I_1 R_1}{I_1 - I_2}$

Putting the value r in equation (i), we get

$$\mathcal{E} = I_1 \left(R_1 + \frac{I_2 R_2 - I_1 R_1}{I_1 - I_2} \right)$$

$$= I_1 \left(\frac{I_1 R_1 - I_2 R_1 + I_2 R_2 - I_1 R_1}{I_1 - I_2} \right)$$

or $\boldsymbol{\mathcal{E} = \frac{I_1 I_2 (R_2 - R_1)}{I_1 - I_2}}$

Q. 27. ***At 0 ℃, the resistance of a conductor B is n times that of the conductor A. The temperature coefficients of A and B are α_1 and 48 α_2 respectively. For the series combination of the two conductors, find (i) the resistance at 0 ℃ and (ii) the temperature coefficient of resistance.***

Sol. Let R_0 be the resistance of the conductor A at 0°C.

Then resistance of conductor B at 0°C = n R_0

Resistance of conductor A at 0°C = nR_0

Resistance of conductor A at θ°C will be

$R = R_0 (1 + \alpha_1 \theta)$ ... (i)

Resistance of conductor B at θ°C will be

$R' = nR_0 (1 + \alpha_2 \theta)$

Thus the resistance of the series combination at θ°C will be

$R_S = R + R' = R_0 (1 + \alpha_1 \theta) + nR_0 (1 + \alpha_2 \theta)$

$= R_0 [(1 + n) + (\alpha_1 + n\,\alpha_2)\,\theta]$

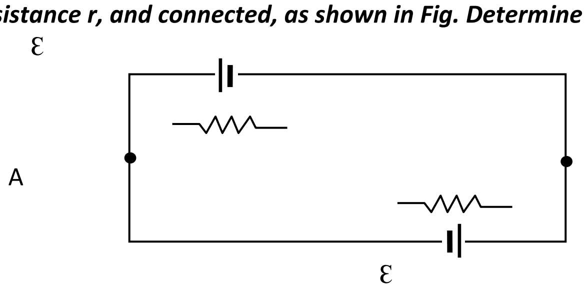
$= (1 + n) R_0 \left(1 + \frac{\alpha_1 + n\,\alpha_2}{1 + n} \theta \right)$ ... (iii)

Comparing equation (ii) with equation (i), we get

(i) Resistance of the series combination at 0°C = (1 + n) R_0.

(ii) Temperature coefficient of resistance of the series combination = $\frac{\alpha_1 + n\alpha_2}{1 + n}$

Q. 28. ***Two identical storage batteries, each having emf $\mathcal{E}$ and internal resistance r, and connected, as shown in Fig. Determine the potential difference set up between the points A and B.***



Sol. Clearly, the two batteries are connected in series.

∴ Total emf the two batteries = $\mathcal{E} + \mathcal{E} = 2\mathcal{E}$

Total resistance of the circuit = r + r = 2r

Current in the circuit, $I = \frac{2\mathcal{E}}{2r} = \frac{\mathcal{E}}{r}$

The potential difference between the points A and B,

$V_A - V_B$ = emf of either cell – p.d. across its internal resistance

$= \mathcal{E} - Ir = \mathcal{E} - \frac{\mathcal{E}}{R} \times r = 0$

Q. 29. ***A battery of emf $\mathcal{E}$ and internal resistance r is connected to a variable external resistance R. Find the value of R for which (i) current in the circuit is maximum and (ii) terminal p.d. across the battery is maximum. Also, find the maximum value of current in case (i) and maximum terminal p.d. in case (ii).***

Sol. (i) Current in the circuit, $I = \frac{\mathcal{E}}{R + r}$

Current in the circuit will be maximum when R = 0

$$I_{max} = \frac{\varepsilon}{0 + r} = \frac{\varepsilon}{r}$$

(ii) Terminal p.d. across the battery,

$$V = \varepsilon - Ir = \varepsilon - \frac{\varepsilon r}{R + r}$$

For V to be maximum, $\varepsilon r/(R + r)$ should be minimum. This requires

$$V_{max} = \varepsilon - \frac{\varepsilon r}{\infty + r} = \varepsilon$$

Q. 30. ***Under what condition will be strength of current in a wire be the same for connections in series and in parallel of identical cells?***

Sol. Let internal resistance of each cell be r and its emf be ε. When n cells are connected in series, the current in the circuit is

$$I_1 = \frac{n\varepsilon}{R + nr}$$

When the n cells are connected in parallel, the current in the circuit is

$$I_2 = \frac{\varepsilon}{R + r/n} = \frac{n\varepsilon}{nR + r}$$

But $I_1 = I_2$

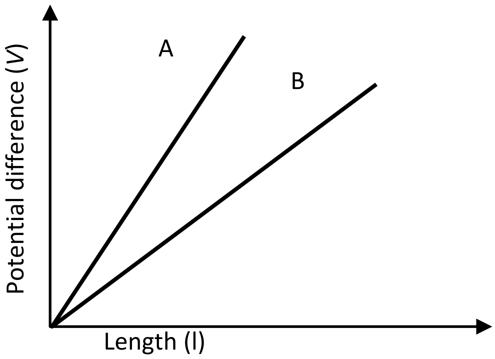
$\therefore$ $$\frac{n\varepsilon}{R + nr} = \frac{n\varepsilon}{nR + r}$$

or $R + nr = nR + r$ or $R = r$

Q. 31. ***Why is the use of a potentiometer preferred over that of a voltmeter for measurement of emf of a cell? <u>Or</u>***
In what respect is the potentiometer better than a moving coil voltmeter in comparing the emfs of two cells.

Sol. Potentiometer is a null method device. No current is drawn from the cell at null point, thus there is no potential drop due to the internal resistance of the cell. So it measures the p.d. in an open circuit. But voltmeter requires a small current for its operation. So voltmeter measures the p.d. in a closed circuit which is less than the actual emf of the cell.

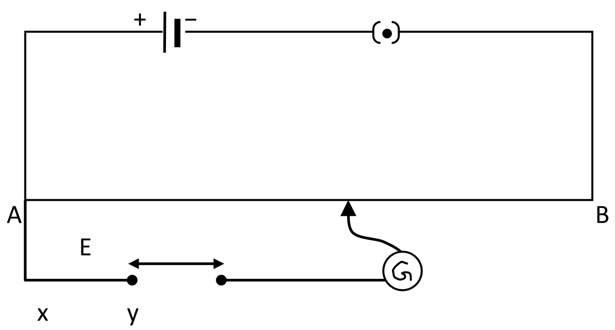
Q. 32. The variation of potential difference with length in case of two potentiometers A and B is shown in Fig. Which of the two is preferred to find e.m.f. of a cell? Give reason for your answer. <u>Or</u>
The variation of potential difference V with length l in case of two potentiometers A and B is as shown. Which of these two will you prefer for comparing emfs of two primary cells?



Sol. Potential gradient = $\frac{dV}{dl}$ = slope of V – l graph.

To measure emf of a cell, or to compare the emfs of two primary cells, the potentiometer b is preferred over A because it has a smaller potentiometer gradient and hence it is more sensitive.

Q. 33. ***For the potentiometer circuit, shown in Fig. Points X and Y represent the two terminals of an unknown emf E. A student observed that when the jockey is moved from the end A to the end B of the potentiometer wire, the deflection in the galvanometer remains in the same direction. What are the two possible faults in the circuit that could result in this observation?***

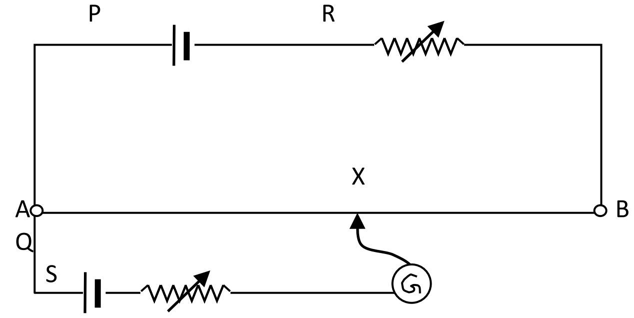


If the galvanometer deflection at the end B is (i) more (ii) less than that at the end A, which of the two faults, listed above, would be there in the circuit? Give reason in support of your answer in each case.

Sol. Two possible faults for one-sided deflection are as follows:

(a) The positive terminals of all the cells are not connected to the point A of the potentiometer.
(b) The e.m.f. of the driving cell is less than the e.m.f. of the cell to be balanced.
(i) In case (a) the deflection at the end B is more than that at end A.
Reason: The two emfs support each other and the resultant emf becomes maximum at the end B.
(ii) In case (b) the deflection at the end B is less than that at A.
Reason: The two emfs oppose each other and the resultant deflection at the end B (where the main emf is maximum) would be less than at the end A.

Q. 34. ***In the potentiometer circuit shown, the balance (null) point is at X. State with reasons, where the balance point will be shifted when***



(i) Resistance R is increased, keeping all parameters unchanged.
(ii) Resistance S is increased, keeping R constant.
(iii) Cell P is replaced by another cell whose e.m.f. is lower than that of cell Q.

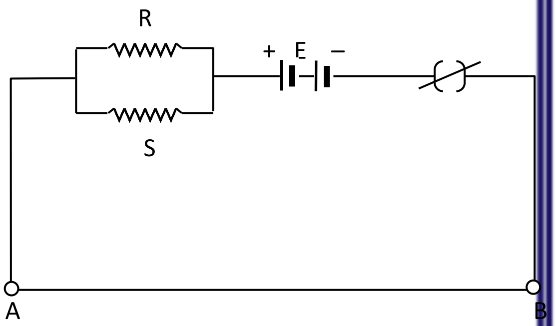
Sol. (i) When resistance R is increased, the potential gradient k decreases.

$$k = \frac{I\,R_{AB}}{l_{AB}}$$

As $V_{AX} = k\,l_{AX}$ $\quad\therefore\quad$ $l_{AX} = \frac{V_{AX}}{k}$

Due to the decrease in k, length l_{AX} will increase i.e., the balance point will shift towards the end B.
(ii) No effect, because at null point no current is drawn from the cell Q.
(iii) Balance point is not found on wire AB because in this situation the potential drop across the wire is less than the emf of the cell Q.

Q. 35. ***A potentiometer wire has a length L and a resistance R_0. It is connected to a battery and a resistance combination as shown. Obtain an expression for the potential drop per unit length of this potentiometer wire.***
What is the maximum emf of a 'test cell' for which one can get a 'balance point' on this potentiometer wire? What precaution should one take, while connecting this 'test cell' in the circuit?



Sol. Total resistance $= R_0 + \frac{RS}{R+S}$

Current in the circuit, $I = \frac{E}{R_0 + \frac{RS}{R+S}}$

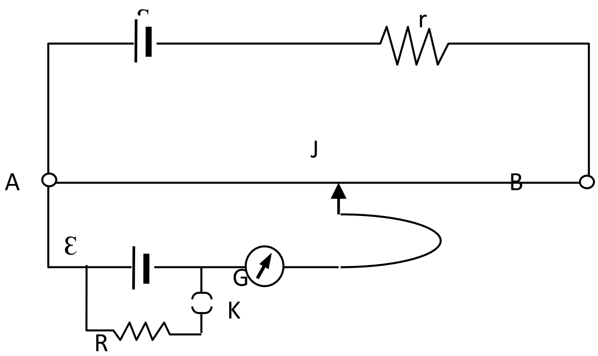
Total P.D. across the ends of wire AB,

$$V = IR_0 = \frac{ER_0}{R_0 + \frac{RS}{R+S}}$$

P.D. per unit length, $k = \frac{V}{L} = \frac{ER_0}{L\left(R_0 + \frac{RS}{R+S}\right)}$

The emf of the test cell must be less than the total P.D. at the ends of wire AB. The positive terminal of the cell should be connected to the end A of the potentiometer wire.

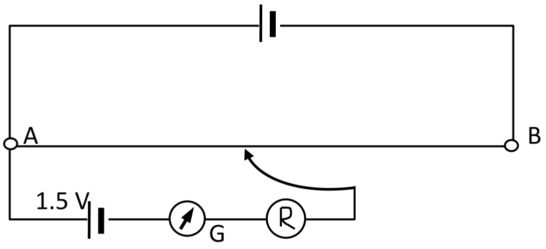
Q. 36. ***The following circuit shows the use of potentiometer to measure the internal resistance of a cell:***



(i) When the key K is open, how does the balance point change, if the current from the driver cell decreases?
(ii) When the key K is closed, how does the balance point change if R is increased, keeping the current from the driver cell constant?

Sol. (i) When the current from the driver cell decreases, the potential gradient across the potentiometer wire decreases, and balance point shifts towards the end B.
(ii) When R is increased and K is kept closed, terminal p.d. of the cell increases and balance point again shifts towards the end B.

Q. 37. A potentiometer wire of length 1 m is connected to a driver cell of emf 3 V as shown in the figure. When a cell of 1.5 V emf is used in the secondary circuit, the balance point is found to be 60 cm. On replacing this cell and using a cell of unknown emf, the balance point shifts to 80 cm.



(i) Calculate unknown emf of the cell.
(ii) Explain with reason, whether the circuit works, if the driver cell is replaced with a cell of emf 1 V.
(iii) Does the high resistance R, used in the secondary circuit affect the balance point? Justify your answer.

Sol. (i) $\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{l_2}{l_1}$ $\therefore$ $\mathcal{E}_2 = \frac{l_2}{l_1} . \mathcal{E}_1 = \frac{80}{60} \times 1.5 = 2.0$ V
(ii) The circuit will not work because with a driver cell of 1 V, the balance point cannot be obtained for a cell of emf 1.5 V on the wire AB.
(iii) No, the balance point is not affected by the high resistance R because no current flows through the cell at the balance point.

Q. 38. Answer the following questions:
(a) What is the order of magnitude of the resistance of a (dry) human body?
(b) If the resistance of our body is so large, why does one experience a strong shock (sometimes even fatal) when one accidently touches the live wire of, say 240 V supply?
(c) There is an impression among many people that a person touching a high power line gets 'stuck' with the line. Is that true? Explain.
(d) Currents of the order of 0.1 A through the human body are fatal. What causes the death: heating of the body due to electric current or something else?
(e) A nerve fibre contains a membrane separating tow conducting 'fluids' maintained at a potential difference. What is the order of this potential difference?

Sol. (a) About 10 k Ω, it is mainly due to skin through which current enters and leaves our body.
(b) It is because our body is sensitive to minute currents even as low as a few mA.
(c) This impression is misleading. There is no special attractive force that keeps a person 'stuck' with a high-power line. Actually, current of the order of 0.05 A or even much less is enough to disorganise our nervous system. The result is that the affected person may temporarily lose his ability to exercise his nervous control to get himself 'free' from the high voltage point.
(d) The cause of death is not heating, though a person may receive burns if the currents are too larger. The cause of death is the interference caused by external currents in our highly sensitive nervous system which is basically electrical in nature. External current cause convulsive actions and especially interface with the nerve processes related to our heart beating. Beyond a certain point, this interference is fatal. (e) About 0.1 V

Q. 39. Alloys of metals have greater resistivity than that of their constituent metals. Why?

Sol. In an alloy like nichrome, an electron sees a disordered arrangement of Ni^{2+} and Cr^{3+} ions and is scattered by them randomly and very frequently. This decreases the relaxation time and increases the resistivity.

Q. 40. Why constantan or manganin are chosen for making standard resistances?

Sol. Constantan or manganin are used for making standard resistances because of their following properties:
1. They possess a high value of resistivity.
2. Their temperature coefficient of resistance is extremely small.
3. They are least affected by atmospheric conditions like air, moisture, etc.
4. Their contact potential with copper is small.

Q. 41. What is a thermistor? Give its two applications.

Sol. A thermistor is semiconducting metal oxide whose resistance varies appreciably with rise of temperature.
Applications of thermistors:
(i) Thermistors with negative temperature coefficients are used to stabilise transistor receiver circuit against undesirable temperature rise.
(ii) Thermistors with positive temperature coefficients are used as temperature-controlled switches.

... End.