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CLASSIFICATION – MATTER and EARTH'S MAGNETISM

"The phenomenon of attraction small pieces of iron , steel, nickel etc towards the one (magnitude) is called magnetism"
The ore of iron i.e., magnetic is called normal magnets. (Fe_3O_4).

► **Magnets and magnetism**: A magnet is a material that has both attractive and directive properties. It attracts small pieces or iron, nickel, cobalt etc. This property of attraction is called magnetism.
► When suspended freely, a thin long piece of magnet comes to rest nearly in the geographical north-south direction.

ARTIFICIAL MAGNETS

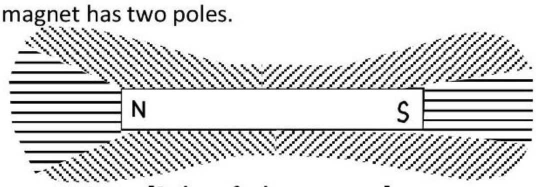
Artificial magnets: Generally, the natural magnets are not strong enough magnetically and have inconvenient shapes. The pieces of iron and other magnetic materials can be made to acquire the properties of natural magnets. Such magnets are called artificial magnets.

- The main advantage of these magnets is that they can be made much stronger than the natural magnets and also of any convenient shape and size.
- General forms:

1. **Bar magnet**: It is a bar of circular of rectangular cross-section.
2. **Magnetic needle**: It is a thin magnetised steel needle having pointed ends and is pivoted at its centre so that it is free to rotate in a horizontal plane.
3. **Horse shoe magnet**: It has the shape of a horse-shoe and thus it has been named so.
4. **Ball-ended magnet**: It is a thin bar of circular cross-section ending in two spherical balls.

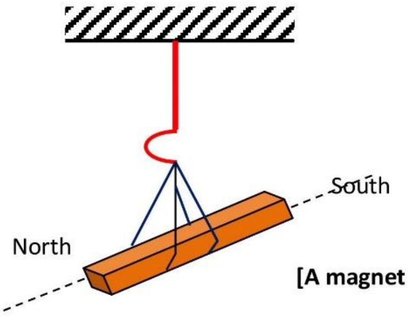
BASIC PROPERTIES OF MAGNETS:

1. Attractive property: A magnet attracts small pieces of iron, cobalt, nickel. etc. When a magnet is brought near a heap of iron filings, the ends of the magnet show the greatest attraction. These ends, where the magnetic attraction is the maximum, are called poles of the magnet. Thus every magnet has two poles.



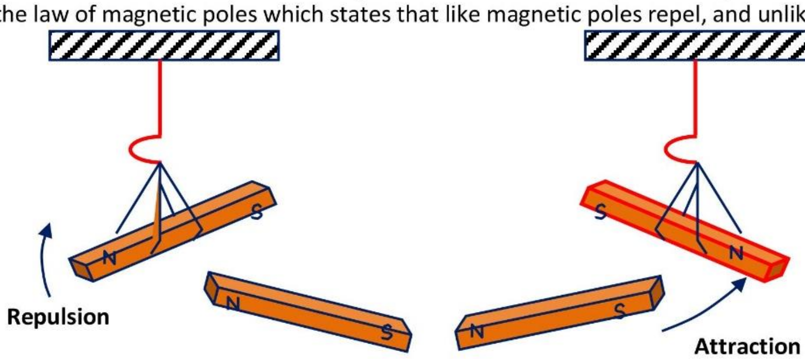
[Poles of a bar magnet]

2. Directive property: When a magnet is suspended or pivoted freely, it aligns itself in the geographical north-south direction. The pole of the magnet which points towards the geographical north is called the north-seeking or north (N) pole. The other pole which points towards the geographical south is called the south-seeking or south (S) pole of the magnet.



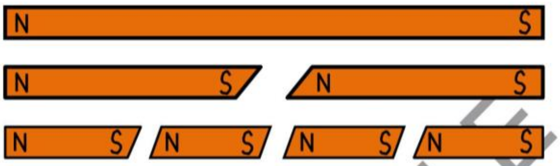
[A magnet points north-south when freely suspended]

3. Like poles repel and unlike poles attract: if the N-pole of a magnet is brought near the N-pole of a suspended magnet, the poles are found to repel each other. Two S-poles also repel each other. N-and S-poles always attract each other. This action can be described by the law of magnetic poles which states that like magnetic poles repel, and unlike magnetic poles attract each other.



[Like poles repel and unlike poles attract]

4. Magnetic poles always exist in pairs: If we try is isolated the two poles of a magnet from each other by breaking the magnet in the middle, each broken part is found to be a magnet with N-and S-poles at its ends. If we break these parts further, each part again is found to be a magnet. So unlike electrci charges, magnetic monopoles do not exist. Every magnet exists as a dipole.



[Poles always exist in pairs]

5. Magnetic induction: A magnet induces magnetism in a magnetic substance placed near it. This phenomenon is called magnetic induction. When N-pole of a powerful magnet is placed close to a soft iron bar, the closer and of the bar becomes S-pole and the farther end N-pole. As a result, the magnet attracts the iron bar. Thus induction precedes attraction.

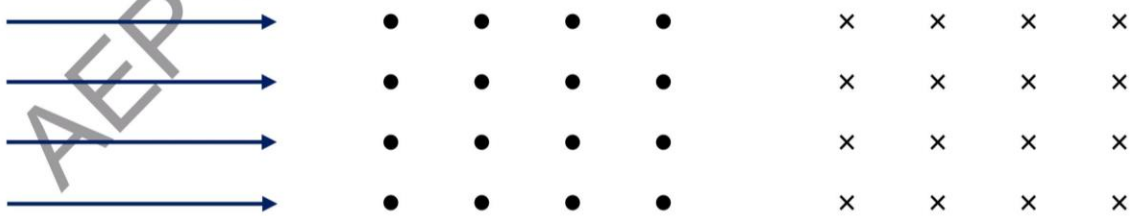
In electricity:- **fundamental object** **– Point change**
In magnetism :- **fundamental object** **– magnetic dipole**

SOME IMPORTANT TERMS CONNECTED WITH MAGNETISM

1. Magnetic field: The space around a magnet within which its influence can be experienced is called its magnetic field.

2. Uniform magnetic field: A magnetic field in a region is said to be uniform if it has same magnitude and direction at all points of that region. At a given place, the magnetic field of the earth can be considered uniform. The field due to a bar magnet is not uniform.

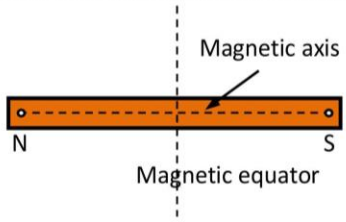
■ A uniform magnetic field acting in the plane of paper is represented by equidistant parallel lines. A uniform magnetic field acting perpendicular to the paper and directed outwards is represented to the paper and directed outwards is represented by dots. A uniform magnetic field acting perpendicular to the plane of paper and directed inwards is represented by crosses.



(a) (b) (c)

[Representations of a uniform magnetic field]

3. Magnetic poles: These are the regions of apparently concentrated magnetic strength in a magnet where the magnetic attraction is maximum. The poles of a magnet lie somewhat inside the magnet and hot at its geometrical ends.



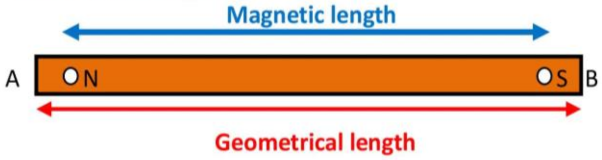
[A bar magnet]

4. Magnetic axis: The line passing through the poles of a magnet is called the magnetic axis of the magnet.

5. Magnetic equator: The line passing through the centre of the magnet and at right angles to the magnetic axis is called the magnetic equator of the magnet.

6. Magnetic length: The distance between the two poles of a magnet is called the magnetic length of the magnet. It is slightly less than the geometrical length of the magnet. It is found that

$$\frac{\text{Magnetic length}}{\text{Geometrical length}} = 6/7 = 0.84$$



[Magnetic and geometrical lengths of a magnet]

► *Poles N & S are situated a little inwards from the geometrical ends A and B magnetic length (NS) = 6/7 geographical length (AB)*

COULOMB'S LAW OF MAGNETIC FORCE

" This law states that the force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them".

If q_{m1} and q_{m2} are the pole strengths of the two magnetic poles which are distance r apart, then the force between them is given by

$$F \propto \frac{q_{m1}\, q_{m2}}{r^2}$$

or $$F = k.\frac{q_{m1} q_{m2}}{r^2}$$

[Where k is a proportionality constant which depends on the nature of the medium as well as on the system of units chosen.]

► **For SI units and for poles in vacuum,**

$$F = \frac{\mu_0}{4\pi} \cdot \frac{qm_1 qm_2}{r^2}$$

Where μ_0 ***is the permeability of free space and is equal to*** $4\pi \times 10^{-7}$ ***henry/metre***.

If $q_{m1} = q_{m2} = 1$ unit; r = 1 m, then

$$F = \frac{\mu_0}{4\pi} = 10^{-7} \text{ N.}$$

Hence ***a unit magnetic pole may be defined as that pole which when placed in vacuum at a distance of one metre from an identical pole repels it with a force of*** 10^{-7} ***Newton***.

MAGNETIC DIPOLE AND MAGNETIC DIPOLE MOMENT

Magnetic dipole: In electricity, the fundamental or simplest structure that can exist is a point charge.
Here two equal an opposite charges separated by a small distance constitute an electric dipole, which is described by an electric dipole moment $\vec{p}$. In magnetism, isolated magnetic poles do not exist.

- ***An arrangement of two equal and opposite magnetic poles separated by a small distance is called a magnetic dipole.***
- ► Every bar magnet is a magnetic dipole.
- ► A current carrying loop behaves as a magnetic dipole. Even an atom acts as a magnetic dipole due to the circulatory motion of the electrons around its nucleus.

Magnetic dipole moment: ***The magnetic dipole moment of a magnetic dipole is defined as the product of its pole strength and magnetic length.***

► It is a vector quantity, directed form S-pole to N-pole.

$$\vec{m} = q_m \times 2\vec{l}$$

Where q_m is the pole strength and $2\vec{l}$ is the magnetic length of the dipole measured in the direction S – to N – pole.

► SI unit of magnetic dipole moment is ampere metre^2 (Am^2) or joule per tesla (JT^{-1}).

Examples based on Coulomb's Law and Dipole Moment of a Magnet

- **Formulae Used**: **1. Magnetic dipole moment, $m = q_m \times 2l$.**
  2. Coulomb's law, $F = \frac{\mu_0}{4\pi} \cdot \frac{qm_1\, qm_2}{r^2}$
- **Units Used** : **Pole strength is in Am, force if Newton, distance in metre**
- **Constant Used** : $\mu_0 = 4\pi \times 10^{-7}$ **TmA^{-1}**

Q. 1. ***Two magnetic poles, one of which is four times stronger than the other, exert a force of 5 g f on each other when placed at a distance of 10 cm. Find the strength of each pole.***

Sol. Let the pole strengths of the two dipoles be q_m and 4_{qm}.

Here $F = 5$ g f $= 5 \times 10^{-3}$ kg f $= 5 \times 10^{-3}$ kg f $= 5 \times 10^{-3} \times 9.8$ N, r = 10 cm = 0.1 m

Using Coulomb's law of magnetism,

$$F = \frac{\mu_0}{4\pi} \cdot \frac{q_{m1}\, q_{m2}}{r^2}$$

$\therefore$ $$5 \times 10^{-3} \times 9.8 = \frac{10^{-7} \times q_m \times 4q_m}{(0.1)^2}$$

or $$q_m^2 = \frac{5 \times 9.8 \times (0.1)^2 \times 10^4}{4} = 25 \times 49$$

or $q_m = 5 \times 7 = 35$ Am and, $4q_m = 4 \times 35 = 140$ Am.

Q. 2. ***Two similar magnetic poles, having pole strengths in the ratio 1 : 2 are placed 1 m apart. Find the point where a unit pole experiences no net force due to the two poles.***

Sol. Let the pole strengths of the two magnetic poles be q_m and $2q_m$. Suppose the required point is located at distance x from the first pole. Then at this point,

Force on unit pole due to first pole = Force on unit pole due to second pole

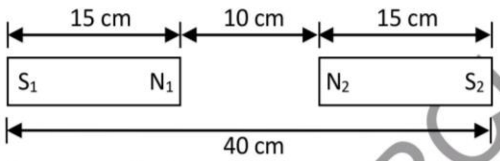
or $$\frac{\mu_0}{4\pi} = \frac{q_m \times 1}{x^2} = \frac{\mu_0}{4\pi} \cdot \frac{2\, q_m \times 1}{(1-x)^2}$$

or $2x^2 = (1-x)^2$ or $\sqrt{2}x = 1 - x$ 4

or $x = \dfrac{1}{1+\sqrt{2}} = 0.414$ m.

Q. 3. ***Calculate the force acting between two magnets of length 15 cm each and pole strength 80 Am each when the separation between their north poles is 10 cm and that between south poles is 40 cm.***

Sol. The situation is shown in Fig. Here, $q_{m1} = q_{m2} = 80$ Am



Force of repulsion between poles N_1 and N_2 is

$$F_1 = \frac{\mu_0}{4\pi} \cdot \frac{q_{m1}\, q_{m2}}{r^2} = \frac{10^{-7} \times 80 \times 80}{(0.10)^2} = 0.064 \text{ N}$$

Force of repulsion between poles S_1 and S_2 is

$$F_2 = \frac{10^{-7} \times 80 \times 80}{(0.40)^2} = 0.004 \text{ N}$$

Force of attraction between N_1 and S_2 is

$$F_3 = \frac{10^{-7} \times 80 \times 80}{(0.25)^2} = 0.010 \text{ N}$$

Force of attraction between N_2 and S_1 is

$$F = F_1 + F_2 + F_3 + F_4 = 0.064 + 0.004 - 0.010 - 0.010 \quad = 0.040 \text{ N (repulsive).}$$

Q. 4. ***A magnetic dipole of length 10 cm has pole strength of 20 Am. Find the magnetic moment of the dipole.***

Sol. Here $2l = 10$ cm $= 0.10$ m, $q_m = 20$ Am

∴ Magnetic moment,

$m = q_m \times 2l = 20 \times 0.10 \text{ Am}^2 = 2.0 \text{ Am}^2$

Q. 5. ***A bar magnet of magnetic moment 5.0 Am² has poles 20 cm apart. Calculate the pole strength.***

Sol. Here $m = 5.0 \text{ Am}^2$, $2l = 20$ cm $= 0.20$ m

As $m = q_m \times 2l$

∴ Pole strength, $q_m = \dfrac{m}{2l} = \dfrac{5.0}{0.20} = 25$ Am.

Q. 6. ***A steel wire of length l has a magnetic moment m. It is bent into a semicircular arc. What is the new magnetic moment?***

Sol. Pole strength, $q_m = \dfrac{m}{l}$

When the wire is bent into a semicircular arc, the separation between the poles changes from l to $2r$, where r is the radius of the semicircular arc. Thus

$l = \pi r$ or $r = \dfrac{l}{\pi}$

New magnetic moment $= q_m \times 2r = \dfrac{m}{l} \times \dfrac{2l}{\pi} = \dfrac{2m}{\pi}$

MAGNETIC FIELD LINES.

Magnetic lines of force: Michael Faraday, the celebrated physicist of London (1791 – 1867) introduced the concept of the magnetic lines of force to represent a magnetic field visually. Magnetic lines of force do not really exist but they are quite useful in describing many different magnetic phenomena.

A magnetic line of force may be defined as the curve the tangent to which at any point gives the direction of the magnetic field at that point. It may also be defined as the path along which a unit north pole would tend to move if free to do so.

Properties of lines of force:

1. Magnetic lines of force are closed curves which start in air from the N-pole and end at the S-pole and then return to the N-pole through the interior of the magnet.

2. The lines of force never cross each other. If they do so, that would mean there are two directions of the magnetic field at the point of intersection, which is impossible.

3. They start from and end on the surface of the magnet normally.

4. The lines of force have a tendency to contract length wise and expand side wise. This explains attraction between unlike poles and repulsion between like poles.

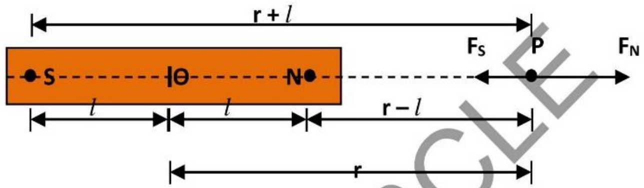
5. The relative closeness of the lines of force gives a measure of the strength of the magnetic field which is maximum at the poles.

- Electric field lines of force do not exist within the charged conductor whereas magnetic lines exist within the of magnet.
- Electric field lines of force do not exist within the charged conductor the electric field inside the shaped conductor zero but magnetic field is never zero inside a magnet.

MAGNETIC FIELD OF A BAR MAGNET AT AN AXIAL POINT

Expression for the magnetic field intensity at a point on the axis of a bar magnet: **5**

Let NS be a bar magnet of length $2l$ and of pole strength q_m. Suppose the magnetic field is to be determined at a point P which lies on the axis of the magnet at a distance r from its centre, as shown in Fig.



[Magnetic field of a bar magnet at an axial point]

Imagine a unit north pole placed at point P. Then from Coulomb's law of magnetic forces, the force exerted by the N-pole of strength q_m on unit north pole will be

$$F_N = \frac{\mu_0}{4\pi} \frac{q_m}{(r-l)^2}, \quad \text{along } \overrightarrow{NP}$$

Similarly, the force exerted by S-pole on unit north pole is

$$F_S = \frac{\mu_0}{4\pi} \frac{q_m}{(r+l)^2}, \quad \text{along } \overrightarrow{PS}$$

Therefore, the strength of the magnetic field $\vec{B}$ at point P is

B_{axial} = Force experienced by a unit North-pole at point P

$$= F_N - F_S = \frac{\mu_0 q m}{4\pi}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right] = \frac{\mu_0 q m}{4\pi} \cdot \frac{4rl}{(r^2 - l^2)^2}$$

But, **$q_m \cdot 2l = m$**, is the **magnetic dipole moment,** so

$$\mathbf{B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}, \text{ along } \overrightarrow{NP}} \quad \text{.................... (1)}$$

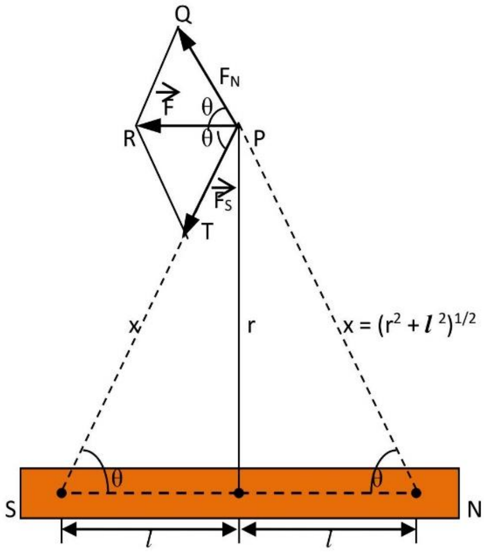
Clearly, the magnetic field at any axial point of magnetic dipole is in the same direction as that of its magnetic dipole moment i.e., from S-pole to N-pole, so we can write

$$\vec{B}_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{m}}{r^3}$$

MAGNETIC FIELD OF A BAR MAGNET AT AN EQUATORIAL POINT

Expression for the magnetic field intensity at a point on the equatorial line of a bar magnet.

Consider a bar magnet NS of length $2\,l$ and of pole strength qm. Suppose the magnetic field is to be determined at a point P lying on the equatorial line of the magnet NS at a distance r from its centre.



[Magnetic field of a bar magnet at an equatorial point]

Imagine a unit north-pole placed at point P. Then from Coulomb's law of magnetic forces, the force exerted by the N-pole of the magnet on unit north-pole is

$$F_N = \frac{\mu_0}{4\pi} \cdot \frac{q_m}{x^2}, \text{ along NP}$$

Similarly, the force exerted by the S-pole of the magnet on unit north-pole is

$$F_S = \frac{\mu_0}{4\pi} \cdot \frac{q_m}{x^2}, \text{ along PS}$$

As the magnitudes of F_N and F_S are equal, so their **vertical components get cancelled** while the **horizontal components add up along PR.**

Hence the magnetic field at the equatorial point P is

B_{equa} = Net force on a unit N-pole placed at point P

$= F_N \cos\theta + F_S \cos\theta$

$= 2\, F_N \cos\theta$ $\qquad [\because F_N = F_S]$

$= 2 \cdot \mu_0 \cdot q_m \cdot \dfrac{l}{x}$ $\qquad \left[\because \cos\theta = \dfrac{l}{x}\right]$

or $\quad B_{equa} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{m}{(r^2 + l^2)^{3/2}}$ $\qquad [\because x = (r^2 + l^2)^{1/2}]$

Where $m = q_m \cdot 2l$, is the magnetic dipole moment.

Again for a short magnet, $l << r$, so we have

$$\mathbf{B_{equa} = -\frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}}\text{, along PR} \qquad \text{... (2)}$$

Clearly, the magnetic field at any equatorial point of a magnetic dipole is in the direction opposite to that of its magnetic dipole moment i.e., from N-pole ot S-pole. So we can write

$$\vec{B}_{equa} = -\frac{\mu_0 \vec{m}}{4\pi r^3}$$

► On comparing equations (1) and (2), ***the magnetic field at a point at a certain distance on the axial line of a short magnet is twice of that at the same distance on its equatorial line.***

Examples based on Magnetic Field of a Bar Magnet

- **Formulae Used**

Magnetic field of bar magnet of length 2l and dipole moment m at a distance r from its centre,

1. $B_{axial} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{2mr}{(r^2 - l^2)^2}$ (on the axial line)
2. $B_{equa} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{m}{(r^2 - l^2)^{3/2}}$ (on the equatorial line)

For a short magnet, $l << r$, so

3. $B_{axial} = = \dfrac{\mu_0}{4\pi} \cdot \dfrac{2m}{r^3}$ (on the axial line)
4. $B_{equa} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{m}{r^3}$ (on the equatorial line)

- **Units Used**: Magnetic field B is in tesla, distances r and l in metre and magnetic moment in JT^{-1} or Am^2.

Q. 1. ***What is the magnitude of the equatorial and axial fields due to a bar magnet of length 5 cm at a distance of 50 cm from the midpoint? The magnetic moment of the bar magnet is 0.40 Am^2.***

Sol. Here $\quad m = 0.40\ Am^2 \quad r = 50\ cm = 0.50\ m, \quad 2l = 5.0\ cm$

Clearly, the magnet is a short magnet ($l << r$)

(i) $B_{equa} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{m}{r^3} = \dfrac{10^{-7} \times 0.4}{(0.5)^3} = 3.2 \times 10^{-7}\ T$

(ii) $B_{axial} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{2m}{r^3} = 6.4 \times 10^{-7}\ T$

Q. 2. ***A bar magnet of length 10 cm has a pole strength of 10 Am. Calculate the magnetic field at a distance of 0.2 m from its centre at a point on its (i) axial line and (ii) equatorial line.***

Sol. Here $2l = 10$ cm or $l = 5$ cm $= 0.05$ m

$q_m = 10$ Am, $r = 0.2$ m

Magnetic field on axial line is

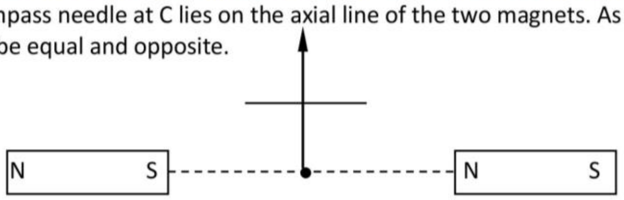
(i) Magnetic field on axial line is

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2mr}{(r^2 - l^2)^2} = \frac{10^{-7} \times 2 \times 1 \times 0.2}{(0.2^2 - 0.05^2)^2}$$

$$= \frac{10^{-7} \times 0.4}{(0.0375)^2}\ T \quad = 284 \times 10^{-5}\ T$$

Q. 3. ***Two small magnets are placed horizontally, perpendicular to the magnetic meridian. Their north poles at 30 cm west from a campus needle. If the compass needle remains undeflected, compare the magnetic moments of the magnets.***

Sol. The compass needle at C lies on the axial line of the two magnets. As it remains undeflected, the fields of the two magnets at C must be equal and opposite.

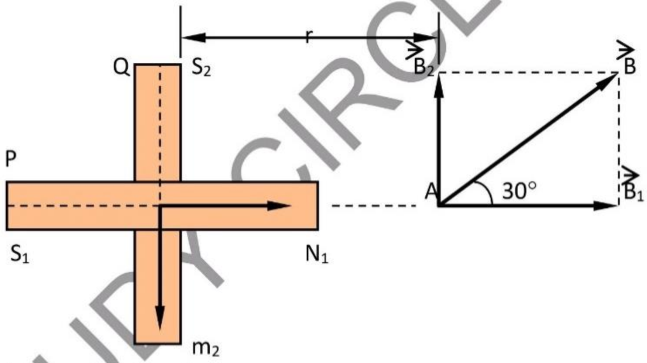


∴ $B_1 = B_2$ or $\frac{\mu_0}{4\pi} \cdot \frac{2m_1}{r_1^3} = \frac{\mu_0}{4\pi} \cdot \frac{2m_2}{r_2^3}$

or $\frac{m_1}{m_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{20}{30}\right)^3 = \frac{8}{27} = 8 : 27.$

Q. 4. ***Two short magnets P and Q are placed one over another with their magnetic axes mutually perpendicular to each other. It is found that the resultant field at a point on the prolongation of the magnetic axis of P is inclined at 30° with this axis. Compare the magnetic moments.***

Sol. Let A be any point on the prolongation of the axis of magnet P. Let B_1 and B_2 be the fields of the magnets P and Q respectively at the point A. Let m_1 and m_2 be the magnetic moments of the two magnets.



As point A lies on the axial line of P, therefore,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\,m_1}{r^3}$$

The point A lies on the broad-side-on position of Q, therefore,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{m_2}{r^3} \qquad \therefore \qquad \frac{B_2}{B_1} = \frac{m_2}{2\,m_1}$$

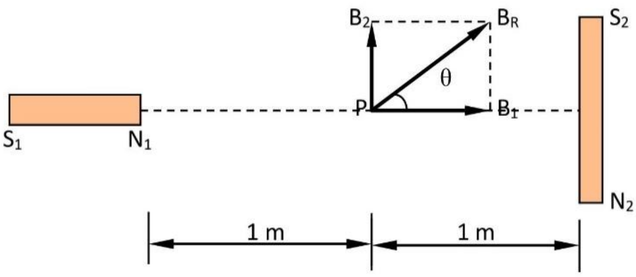
But the resultant field B_1 is inclined at 30° with B_1, so

$$\frac{B_2}{B_1} = \tan 30° = \frac{1}{\sqrt{3}}$$

Hence, $\frac{1}{\sqrt{3}} = \frac{m_2}{2m_1}$ or $\frac{m_1}{m_2} = \frac{\sqrt{3}}{2}$

Q. 5. ***Two identical magnetic field dipoles of magnetic moments 1.0 Am² each are placed at a separation of 2 m with their axes perpendicular to each other. What is the resultant magnetic field at a point mid-way between the dipoles?***

Sol.



The magnetic field of the two magnets at the mid point P are

$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} = \frac{10^{-7} \times 2 \times 1}{1^3} = 2 \times 10^{-7}$ T [In horizontal direction]

$B_2 = \frac{\mu_0}{4\pi} \frac{m}{r^3} = 10^{-7}$ T [in vertical direction]

∴ $B_R = \sqrt{B_1^2 + B_2^2} = \sqrt{5} \times 10^{-7}$ T

If the resultant field B_R makes angle θ with B_1, then

$$\tan\theta = \frac{B_2}{B_1} = \frac{10^{-7}}{2 \times 10^{-7}} = 0.5 \qquad \therefore \qquad \theta = 26.57°.$$

Q. 6. **The intensities of magnetic field at two points on the axis of a bar magnet at distances of 10 cm and 20 cm from the middle point are in the ratio 18:1. Find the distance between the poles of the magnet.**

Sol. Let the magnetic length of the bar magnet be 2l cm

As $B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2mr}{(r^2 - l^2)^2}$

So the field of the two magnets at the given points are

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2m \times 10 \times 10^{-2}}{(10^2 - l^2)^2 \times 10^{-8}}$$

and $B_2 = \frac{\mu_0}{4\pi} = \frac{2m \times 20 \times 10^{-2}}{(20^2 - l^2)^2 \times 10^{-8}}$ 8

∴ $\frac{B_1}{B_2} = \frac{(20^2 - l^2)^2}{2\,(10^2 - l^2)^2}$

But $\frac{B_1}{B_2} = \frac{18}{1}$

∴ $\frac{18}{1} = \frac{(20^2 - l^2)^2}{2\,(10^2 - l^2)^2}$ or $36 = \frac{(20^2 - l^2)^2}{(10^2 - l^2)^2}$

or $6 = \frac{400 - l^2}{100 - l^2}$ or $600 - 6l^2 = 400 - l^2$

or $5l^2 = 200$ or $l^2 = 40$ or $l = \sqrt{40} = 6.325$ cm ∴ Length of magnet = 2l = 2 × 6.325 = 12.65 cm

TORQUE ON A MAGNETIC DIPOLE IN A MAGNETIC FIELD

Expression for the torque on a magnetic dipole placed in a uniform magnetic field.

Torque on a magnetic dipole in a uniform magnetic field. Consider a bar magnet NS of length 2l placed in a uniform magnetic field $\vec{B}$. Let q_m be the pole strength of its each pole. Let the magnetic axis of the bar magnet make an angle θ with the field $\vec{B}$, as shown in Fig.

Force on N-pole = $q_m B$; along $\vec{B}$

Force on S-pole = $q_m B$, opposite to B

The forces on the two poles are equal and opposite. They form a couple. Moment of couple or torque is given by

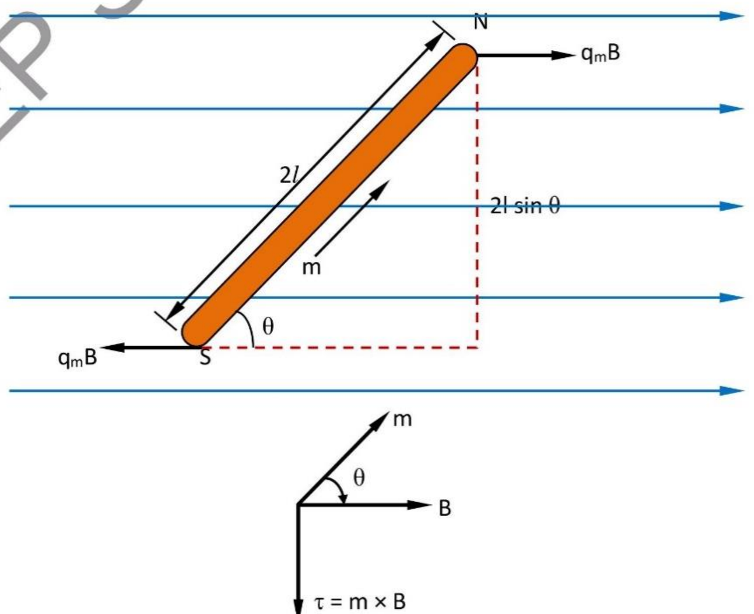
τ = Force × perpendicular distance

$= q_m B \times 2l \sin\theta = (q_m \times 2l)\, B \sin\theta$

or $\tau = mB \sin\theta$ ... (1)

where $m = q_m \times 2l$, is the magnetic dipole moment of the bar magnet. In vector notation,

$\vec{\tau} = \vec{m} \times \vec{B}$ ... (2)



[(a) Torque on a bar magnet in a magnetic field. (b) Relation between the directions of $\vec{\tau}$, $\vec{m}$ and $\vec{B}$]

The direction of the torque τ is given by the right hand screw rule as indicated in Fig. The effect of the torque $\vec{\tau}$ is to make the magnet align itself parallel to the field B. That is why a freely suspended magnet aligns itself in the north-south direction because the earth has its own magnetic field which exerts a torque on the magnet tending it to align along the field.

➢ **Special cases:**

Case 1: When the magnet lies perpendicular to the direction of the field,

$\theta = 0°$, $\sin\theta = 0$, $\tau = 0$,

Thus the torque is minimum.

Case 2: When the magnet lies perpendicular to the direction of the field,

$\theta = 90°$, $\sin\theta = 1$, $\tau = mB$

Thus the torque is maximum

$\tau_{max} = mB$

Definition of magnetic dipole moment. If in Eq. (1), B = 1, θ = 90°,

then $\tau = m$

Hence ***the magnetic dipole moment may be defined as the torque acting on a magnetic dipole placed perpendicular to a]] uniform magnetic field of unit strength***.

SI unit of magnetic moment. As 9

$$m = \frac{\tau}{B \sin\theta}$$

∴ SI units of $m = \frac{1\ Nm}{1T.\ 1}$

$= NmT^{-1}$ or JT^{-1} or Am^2

POTENTIAL ENERGY OF A MAGNETIC DIPOLE IN A MAGNETIC FIELD

Expression for the potential energy of a dipole placed in a uniform magnetic field at an angle θ with it.

Potential energy of a magnetic dipole: when a magnetic dipole is placed in a uniform magnetic field B at angle θ with it, it experiences a torque

$$\tau = mB \sin\theta$$

This torque tends to align the dipole in the direction of $\vec{B}$.

▶ ***If the dipole is rotated against the action of this torque, work has to be done. This work is stored as potential energy of the dipole.***

The work done in turning the dipole through a small angle dθ is

$$dW = \tau\, d\theta = mB \sin\theta\, d\theta$$

If the dipole is rotated from an initial position $\theta = \theta_1$ to the final position $\theta = \theta_2$, then the total work done will be

$$W = \int_{\theta_1}^{\theta_2} dW = \int_{\theta_1}^{\theta_2} mB \sin\theta\, d\theta = mB\,[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= -mB(\cos\theta_2 - \cos\theta_1)$$

This work done is stored as the potential energy U of the dipole.

∴ $U = -mB(\cos\theta_2 - \cos\theta_1)$

The potential energy of the dipole is zero when $\vec{m} \perp \vec{B}$. So potential energy of the dipole in any orientation θ can be obtained by putting $\theta_1 = 90°$ and $\theta_2 = \theta$ in the above equation.

∴ $U = -mB(\cos\theta - \cos 90°)$

or $\mathbf{U = -mB\cos\theta = -\vec{m}\,.\,\vec{B}}$.

➢ **Special cases**:

Case 1: When θ = 0°, U = – mB cos 0° = – mB

*Thus the potential energy of a dipole is minimum when $\vec{m}$ is parallel to $\vec{B}$. In this state, the magnetic dipole is in **stable equilibrium.***

Case 2. When θ = 90°, U = – mB cos 90° = 0

Case 3. When θ = 180°, U = – mB cos 180° = + mB.

Thus the potential energy of a dipole is maximum when $\vec{m}$ is antiparallel to $\vec{B}$. In this state, the magnetic dipole is in unstable equilibrium.

CURRENT LOOP AS A MAGNETIC DIPOLE

Current loop as a magnetic dipole: We know that the magnetic field produced at a large distance r from the centre of a circular loop (of radius 'a') along its axis is given by

$$B = \frac{\mu_0 Ia^2}{2r^3}$$

or
$$B = \frac{\mu_0}{4\pi} \cdot \frac{2IA}{r^3} \quad \ldots (1)$$

Where I is the current in the loop and $A = \pi a^2$ is its area. On the other hand, the electric field of and electric dipole at an axial point lying far away from it is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3} \quad \ldots (2)$$

Where p is the electric dipole moment of the electric dipole.

On comparing equations (1) and (2), we note that both B and E have same distance dependence $\frac{1}{r^3}$. 10

Moreover, ***they have same direction at any far away point, not just on the axis. This suggests that a circular current loop behaves as a magnetic dipole of magnetic moment,***

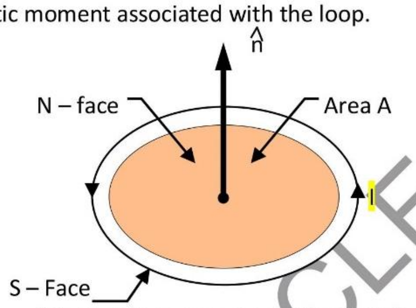
$$m = IA$$

In vector notation,

$$\vec{m} = I\vec{A}$$

▶ *This result is valid for planar current loop of any shape. Thus the magnetic dipole moment of any current loop is equal to the product of the current and its loop area. Its direction is defined to be normal to the plane of the loop in the sense given by right hand thumb rule.*

► **Right hand thumb rule**: If we curl the fingers of the right hand in the direction of current in the loop, then the extended thumb gives the direction of the magnetic moment associated with the loop.



[Current loop as a magnetic dipole]

Conclusion : The upper face of the current loop shown in Fig. has N-polarity and the lower face has S-polarity.

Thus a current loop behaves like a magnetic dipole.

If a current carrying coil consists of N turns, then

$$m = NIA$$

The factor NI is called amperes turns of current loop. So

Magnetic dipole moment of current loop = Ampere turns × loop area

► **Dimensions of magnetic moment = [A] [L²] = [AL²]**

► **SI unit of magnetic dipole moment is Am².** ***It is defined as the magnetic moment associated with one turn loop of area one square metre when a current of one ampere flows through it.***

Physical quantity	Electrostatics	Magnetism
Free space constant	$1/\varepsilon_0$	μ_0
Dipole moment	$\vec{p}$	$\vec{m}$
Axial field	$\frac{1}{4\pi\varepsilon_0} \cdot \frac{2\vec{p}}{r^3}$	$\frac{\mu_0}{4\pi} \cdot \frac{2\vec{m}}{r^3}$
Equatorial field	$-\frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}}{r^3}$	$-\frac{\mu_0}{4\pi} \cdot \frac{\vec{m}}{r^3}$
Torque in external field	$\vec{p} \times \vec{E}$	$\vec{m} \times \vec{B}$
P.E. in external field	$-\vec{p} \cdot \vec{E}$	$-\vec{m} \cdot \vec{B}$

MAGNETIC DIPOLE MOMENT OF A REVOLVING ELECTRON

Expression for the magnetic dipole moment of an electron revolving around a nucleus.

Magnetic dipole moment of a revolving electron: According to **Bohr model** of hydrogen – like atoms, negatively charged electron revolves around the positively charged nucleus. This uniform circular motion of the electron is equivalent to a current loop which possesses a magnetic dipole moment = IA. Consider an electron **revolving anticlockwise** around a nucleus in an orbit of **radius 'r'** with **speed 'v'** and **time period 'T'**.

Equivalent current, $I = \frac{\text{Charge}}{\text{Time}} = \frac{e}{T} = \frac{e}{2\pi r / v} = \frac{ev}{2\pi r}$

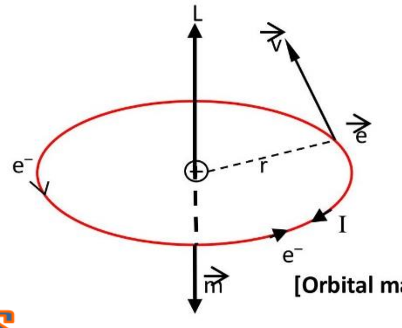
Area of the current loop, $A = \pi r^2$

Therefore, the orbital magnetic moment (magnetic moment due to orbital motion) of the electron is

$$\mu_1 = IA = \frac{ev}{2\pi r} \cdot \pi r^2$$

or $$\mu_1 = \frac{evr}{2} \quad \text{... (1)}$$

As the negatively charged electron is revolving anticlockwise, the associated **current flows clockwise**. According to right hand thumb rule, the direction of the magnetic dipole moment of the revolving electron will be perpendicular to the plane of its orbit and in the downward direction,



[Orbital magnetic moment of a revolving electron]

Also, the angular momentum of the electron due to its orbital motion is 11

$l = m_e v r$ ... (2) [since, ang.momemtum , L = P x r]

The direction of l is normal to the plane of the electron orbit and in the upward direction, as shown in Fig.

Dividing equation (1) by (2), we get

$$\frac{\mu_l}{l} = \frac{evr/2}{m_e vr} = \frac{e}{2m_e}$$

The above ratio is a constant called gyromagnetic ratio. It value is 8.8×10^{10} C kg^{-1}. So

$$\mu_l = \frac{e}{2m_e} l$$

Vectorially, $\vec{\mu_l} = -\frac{e}{2\,m_e} \vec{l}$

► The *negative sign shows that the direction of $\vec{l}$ is opposite to that of $\vec{\mu_l}$. According to Bohr's quantisation condition, the angular momentum of an electron in any permissible orbit is integral multiple of $h/2\pi$, where h is Planck's constant, i.e.,*

$l = \frac{nh}{2\pi}$, where n = 1, 2, 3, ...

$\therefore \quad \mu l = n\left(\frac{eh}{4\pi\,m_e}\right)$

[This equation gives orbital magnetic moment of an electron revolving in nth orbit.]

Bohr magneton: "*It is defined as the magnetic moment associated with an electron due to its orbital motion in the first orbit or hydrogen atom*".

It is the minimum value of μ_1 which can be obtained by putting n = 1 in the above equation. Thus Bohr magneton is given by

$$\mu_B = (\mu_l)_{min} = \frac{eh}{4\pi\,m_e}$$

Putting the values of various constants, we get

$$\mu B = \frac{1.6 \times 10^{-19}\ C \times \mathbf{6.63 \times 10^{-34}\ Js}}{4 \times 3.14 \times 9.11 \times 10^{-31}\ kg}$$

$= 9.27 \times 10^{-24}$ Am^2

Besides the orbital angular momentum $\vec{l}$, an electron has spin angular momentum $\vec{S}$ due to it spinning motion. The magnetic moment possessed by an electron due to its spinning motion is called ***intrinsic magnetic moment or spin magnetic moment***. It is given by $\mu_S = -\frac{e}{m_e}\vec{S}$

The total magnetic moment of the electron is the vector sum of these two momenta. It is given by

$$\vec{\mu} = \vec{\mu_l} + \vec{\mu_S} = -\frac{e}{2m_e}(\vec{l} + 2\vec{S})$$

Examples based on Torque and Potential Energy of a Dipole, and Magnetic Moment of a Current Loop:

- **Formulae Used**

  1. Torque, $\tau = mB \sin\theta$ or $\vec{\tau} = \vec{m} \times \vec{B}$
  2. Work done in turning the dipole of P.E. of a dipole, $W = U = -mB(\cos\theta_2 - \cos\theta_1)$
  3. If initially the dipole is perpendicular to the field,

     $U = -mB\cos\theta$ (i) When $\vec{m}$ is parallel to $\vec{B}$, $\theta = 0°$, U = – m(ii) When m is perpendicular to $\vec{B}$, $\theta = 90°$, U = 0.

     (iii) When m is antiparallel to B, $\theta = 180°$, U = + mB

     Potential energy of the dipole is maximum. It is in a state of unstable equilibrium.
  4. Magnetic moment of a current loop, m = NIA
  5. Orbital magnetic moment of an electron in n th orbit,

     $$\mu_l = (\mu l)min = \left(\frac{eh}{4\,\pi m_e}\right)$$
  6. Bohr magneton is the magnetic moment of an electron in first (n = 1) orbit.

     $$\mu B = (\mu_l)_{min} = \frac{eh}{4\pi m_e}.$$

- **Unit Used**:Torque τ is in Nm, magnetic moment m in JT^{-1} or Am^2, field B in tesla, potential energy U in joule.

Q. 1. ***A magnetised needle of magnetic moment 4.8×10^{-2} JT^{-1} is placed at 30° with the direction of uniform magnetic field of magnetic 3×10^{-2} J T^{-1} is placed at 30° with the direction of uniform magnetic field of magnitude 3×10^{-2} T. What is the torque acting on the needle?***

Sol. Here m = 4.8×10^{-2} J T^{-1}, $\theta = 30°$, B = 3×10^{-2} T

$\therefore$ Torque, $\tau = mB\sin\theta = 4.8 \times 10^{-2} \times \sin 30° = 7.2 \times 10^{-4}$ J.

Q. 2. ***A short bar magnet placed with its axis at 30° to a uniform magnetic field of 0.2 T experiences a torque of 0.06 Nm. (i) Calculate the magnetic moment of the magnet. (ii) Find out what orientation of the magnetic corresponds to its stable equilibrium in the magnetic field.*** **12**

Sol. (i) Here B = 0.2 T, $\theta = 30°$, $\tau = 0.06$ Nm

Magnetic moment,

$$m = \frac{\tau}{B\sin\theta} = \frac{0.06}{0.2\sin 30°} = \frac{0.06}{0.2 \times 0.5}$$

$= 0.6\ Am^2$

(ii) The P.E. of a magnetic dipole in a uniform magnetic field is, $U = -mB\cos\theta$

In stable equilibrium, the P.E. is minimum. So

$\cos\theta = 1$ or $\theta = 0°$

Hence the bar magnet will be in stable equilibrium when its magnetic moment $\vec{m}$ is parallel to the magnetic field $\vec{B}$.

Q. 3. ***In an iron bar (5 cm × 1 cm × 1 cm) the magnetic moment of an atom is 1.8×10^{-23} Am². (i) What will be magnetic moment of the bar in the state of magnetic saturation? (ii) What torque will have to be applied to keep the bar perpendicular to an external magnetic field of 15,000 gauss? Density of iron = 7.8 g cm⁻³, its atomic mass = 56.***

Sol. (i) Mass of iron bar = volume × density

$= 5\ cm^3 \times 7.8\ g\ cm^{-3} = 39\ g$

Number of atoms in 56 g of iron = 6.02×10^{23}

∴ Number of atoms in 39 g or iron

$$= \frac{6.02 \times 10^{23} \times 39}{56} = 4.19 \times 10^{23}$$

Magnetic moment of each atom= 1.8×10^{-23} AM²

Magnetic moment of the iron bar in the state of magnetic saturation is, $m = 1.8 \times 10^{-23} \times 4.19 \times 10^{23} = 7.54\ Am^2$

(ii) Here $\theta = 90°$, B = 15, 000 G = 15000×10^{-4} T.

∴ Required torque, $\tau = mB\sin\theta = 7.54 \times 15000 \times 10^{-4} \times \sin 90° = 11.3$ Nm

Q. 4. ***A planar loop of irregular shape encloses an area of 7.5×10^{-4} m² and carries a current of 12 A. The sense of flow of current appears to be clockwise to an observer. What is the magnitude and direction of the magnetic moment vector associated with the current loop?***

Sol. Here $A = 7.5 \times 10^{-4}\ m^2$, I = 12 A

Magnetic moment associated with the loop is , $m = IA = 12 \times 7.5 \times 10^{-4} = 9.0 \times 10^{-3}\ J\ T^{-1}$.

Applying right hand rule, the direction of magnetic moment is along the normal to the plane of the loop away from the observer.

Q. 5. ***A current of 5 A is flowing through a 10 turn circular coil of radius 7 cm. The coil lies in the x – y plane. What is the magnitude and direction of the magnetic dipole moment associated with it?***
If this coil were to be placed in a uniform external magnetic field directed along the x-axis, in which plane would the coil lie, when in equilibrium?

Sol. Magnetic dipole moment,

$$m = NIA = NI \times \pi r^2 = 10 \times 5 \times \frac{22}{7} \times \left(\frac{7}{100}\right)^2 = 0.77\ Am^2$$

The direction of magnetic dipole moment is perpendicular to the plane of the coil. Hence it is along z-axis.

Torque on the current loop of magnetic moment m is, $\tau = mB\sin\alpha$

where α is angle between m and B. For stable equilibrium torque is zero, so $\alpha = 0°$. For this B should be perpendicular to the plane of the coil. Hence the coil will lie in y – z plane in the condition of stable equilibrium.

Q. 6. ***A bar magnet with poles 25 cm apart and of pole strength 14.4 Am rests with its centre on a friction less pivot. It is held in equilibrium at 60° to a uniform magnetic field of induction 0.25 T by applying a force F, at right angles to its axis, 12 cm from its pivot. Calculate F. What will happen if the force F is removed?***

Sol. Here $m = q_m \times 2l = 14.4 \times 0.25 = 3.6\ Am^2$

$\theta = 60°$, B = 0.25 T, r = 12 cm = 0.12 m

Torque, $\tau = Fr = mB\sin\theta$

$$\therefore F = \frac{mB\sin\theta}{r} = \frac{3.6 \times 0.25 \times \sin 60°}{0.12}$$

$$= \frac{3.6 \times 0.25 \times 0.866}{0.12} = \mathbf{6.5\ N}$$

When the force F is removed, the magnet aligns itself in the direction of field B.

Q. 7. ***An electron in an atom revolves around the nucleus in an orbit of radius 0.5 Å. Calculate the equivalent magnetic moment if the frequency of revolution of the electron is 10^{10} MHz.***

Sol. The electron revolving around the nucleus in the circular orbit is equivalent to a current loop. Its magnetic moment is $m = IA = ev \times \pi r^2$

Here $e = 1.6 \times 10^{-19}$ C,

$v = 10^{10}$ MHz = 10^{16} Hz,

$\therefore m = 1.6 \times 10^{-19} \times 10^{16} \times 3.14 \times (0.5 \times 10^{-10})^2 = \mathbf{1.257 \times 10^{-23}\ Am^2}$.

Q. 8. ***The electron in the hydrogen atom is moving with a speed of 2.3×10^6 ms⁻¹ in an orbit of radius 0.53 Å. Calculate the magnetic moment of the revolving electron.***

Sol. Here $v = 2.3 \times 10^6$ ms – 1

$r = 0.53$ Å $= 0.53 \times 10^{-10}$ m, $e = 1.65 \times 10^{-19}$ C

Orbital magnetic moment of the electron,

$$\mu l = \frac{evr}{2} = \frac{1.6 \times 10^{-19} \times 2.3 \times 10^6 \times 0.53 \times 10^{-10}}{2} = \mathbf{9.75 \times 10^{-24}\ Am^2}.$$

Q. 9. ***An electron moves around the nucleus in a hydrogen atom of radius 0.51 Å, with a velocity of 2×10^6 m/s. Calculate the following: (i) The equivalent current due to orbital motion of electron***
(ii) The magnetic field produced at the centre of the nucleus
(iii) The magnetic moment associated with the electron.

Sol. Here $r = 0.51 \times 10^{-10}$ m, $v = 2 \times 10^5$ ms^{-1}

(i) $I = \frac{e}{T} = \frac{ev}{2\pi r} = \frac{1.6 \times 10^{-19} \times 2 \times 10^5}{2\pi \times 0.51 \times 10^{-10}} = 10^{-4}$ A

(ii) $B = \frac{\mu_0 I}{2r} = \frac{4\pi \times 10^{-7} \times 10^{-4}}{2 \times 0.51 \times 10^{-10}} = 1.23$ T

(iii) $m = IA = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2}$

$$= \frac{1.6 \times 10^{-19} \times 2 \times 10^5 \times 0.51 \times 10^{-10}}{2} = \mathbf{8.16 \times 10^{-25}\ Am^2}$$

Q. 10. ***Two magnets of magnetic moments m and $\sqrt{3}$ m are joined to form a cross (+). The combination is suspended freely in a uniform magnetic field. In equilibrium position, the magnet of magnetic moment m makes an angle θ with field. Find θ***

Sol. When the magnet of moment m makes and θ with the field B, the other magnet of moment $\sqrt{3}$ m will make angle $(90° - \theta)$ with field B. In the equilibrium position,

Torque experienced by first magnet = Torque experienced by second magnet

or $mB \sin\theta = \sqrt{3}\, mB \sin(90° - \theta)$

or $\sin\theta = \sqrt{3}\cos\theta$

or $\frac{\sin\theta}{\cos\theta} = \sqrt{3}$ or $\tan\theta = \sqrt{3}$ $\therefore$ $\boldsymbol{\theta = 60°}$

Q. 11. ***A bar magnet having a magnetic moment of 1.0×10^4 J T^{-1} is free to rotate in a horizontal plane. A horizontal magnetic field of 4×10^{-5} T exists in space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60° from the field.***

Sol. Here $m = 1.0 \times 10^4$ J T^{-1}, $B = 4 \times 10^{-5}$ T, $\theta_1 = 0°$, $\theta_2 = 60°$

Work done, $W = -mB(\cos\theta_2 - \theta_1)$ $= -1.0 \times 10^4 \times 4 \times 10^{-5}(\cos 60° - \cos 0°)$ $= 1.0 \times 10^4 \times 4 \times 10^{-5} \times \frac{1}{2} = 0.2$ J

Q. 12. ***A current 7.0 A is flowing in a plane circular coil of radius 1.0 cm having 100 turns. The coil is placed in a uniform magnetic field of 0.2 Wbm^{-2}. If the coil is free to rotate, what orientation would correspond to its (i) stable equilibrium and (ii) unstable equilibrium? Calculate the potential energy of the coil in these cases.***

Sol. Here N = 100, A = 7.0 A $r = 1.0$ cm $= 1.0 \times 10^{-2}$ m, $B = 0.2$ Wb m^{-2}

Magnetic moment associated with the coil is

$m = NIA = NI \times \pi r^2$

$= 100 \times 7.0 \times \frac{22}{7} \times (1.0 \times 10^{-2})^2 = 0.22$ Am^2

(i) The stable equilibrium corresponds to $\vec{m}$ parallel to $\vec{B}$. The potential energy is then minimum.

$U_{min} = -mB\cos 0° = -0.22 \times 0.2 \times 1 = -0.044$ J

(ii) the unstable equilibrium corresponds to $\vec{m}$ anti parallel to $\vec{B}$. The potential energy is then maximum.

$U_{max} = -mB\cos 180° = -0.22 \times 0.2 \times (-1) = +0.044$ J

Q. 13. ***A short bar magnet placed with its axis at 30° experiences a torque of 0.016 Nm in an external field of 800 G. (a) What is the magnetic moment of the magnet? (b) What is the work done by an external force in moving it from its most stable to most unstable position? (c) What is the work done by the force due to the external magnetic field in the process mentioned in part (b)? (d) The bar magnet is replaced by a solenoid of cross-sectional area 2×10^{-4} m^2 and 1000 turns, but the same magnetic moment. Determine the current flowing through the solenoid.***

Sol. (a) Here $\theta = 30°$, $B = 800$ G $= 800 \times 10^{-4}$ T, $\tau = 0.016$ Nm

Magnetic moment,

$$m = \frac{\tau}{B\sin\theta}$$

$$= \frac{0.016}{800 \times 10^{-4} \times \sin 30°} = 0.40\ \text{Am}^2$$

(b) For most stable position, $\theta = 0°$ and for most unstable position $\theta = 180°$. So the required work done by the external force,

$W = -mB(\cos 180° - \cos 0°) = 2mB$ $= 2 \times 0.40 \times 800 \times 10^{-4}$ $= 0.064$ J.

(c) Here the displacement and the torque due to the magnetic field are in opposition. So the work done by the magnetic field due to the external magnetic field is , $W_B = -0.064$ J

(d) Here $A = 2 \times 10^{-4}$ m^2, N = 1000

Magnetic moment of solenoid, $m_s = m = 0.40$ Am^2

But $m_s = NIA$

∴ Current, $I = \frac{m_s}{NA} = \frac{0.40}{1000 \times 2 \times 10^{-4}} = 2$ A

Q. 14. ***A closely wound solenoid of 1000 turns and area of cross-section 2.0 × 10⁻⁴ m² carries a current of 2.0 A. It is placed with its horizontal axis at 30° with the direction of a uniform horizontal magnetic field of 0.16 T, as shown in Fig.***
(a) What is the torque experienced by the solenoid due to the field?
(b) If the solenoid is free to turn about the vertical direction, specify its orientations of stable and unstable equilibrium. What is the amount of work needed to displace the solenoid from its stable orientation to its unstable orientation?

Sol. The magnitude of magnetic moment of the solenoid is given by

$m = NIA$

Here $N = 1000$, $I = 2.0$ A, $A = 2.0 \times 10^{-4}$ m²

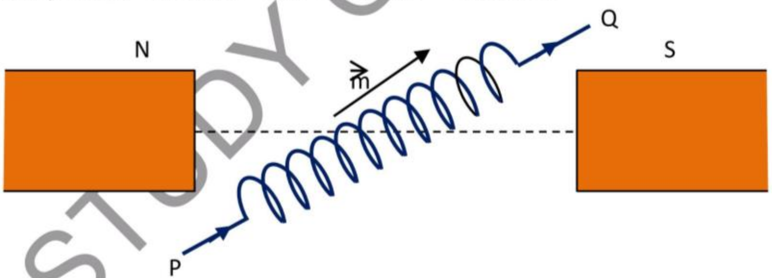
∴ $m = 1000 \times 2.0 \times 2.0 \times 10^{-4} = 0.40$ Am²

or $m = 0.40$ JT⁻¹

The direction of m is along the axis of the solenoid in the direction PQ, as shown in Fig.

(a) Now $m = 0.40$ Am², $B = 0.106$ T, $\theta = 30°$

∴ Magnitude of torque is , $\tau = mB \sin\theta = 0.40 \times 0.16 \times ½ = 0.032$ Nm.



The torque τ acts vertically downwards and tends to bring the axis of the solenoid along the N-S direction of the external field.

(b) The stable orientation of the solenoid corresponds to $\vec{m}$ parallel to $\vec{B}$.

∴ $U_i = -mB \cos 0° = -0.40 \times 0.16 \times 1 = \mathbf{-0.064\ J}$

The unstable orientation of the solenoid corresponds to m antiparallel to $\vec{B}$.

∴ $U_f = -mB \cos 180° = -0.40 \times 0.16 \times (-1) = +0.064$ J

Hence the amount of work needed to displace the solenoid from its stable to unstable orientation is

$W = U_f - U_i = 0.064 - (-0.064) = +0.128$ J

Q. 15. ***A circular coil of 100 turns has an effective radius of 0.05 m and carries a current of 0.1 A. How much work is required to turn it in an external magnetic field of 1.5 Wb m⁻² through 180° about an axis perpendicular to the magnetic field. The plane of the coil is initially perpendicular to the magnetic field.***

Sol. The potential energy of a closed loop of area A and carrying current I in a magnetic field B is

$U = -mB \cos\theta = -NIAB \cos\theta$

where θ is the angle between field B and the normal to the plane of the coil. Initially, when the plane of the coil is perpendicular to the magnetic field, θ = 0°, its initial potential energy is

$U_i = -NIAB \cos 0° = -NIAB$

After the coil is turned through 180°, its final potential energy is

$U_f = -NIAB \cos 180° = NIAB$

∴ Required work done

$= U_f - U_i = NIAB - (-NIAB) = 2\,NIAB = 2\,NI\pi r^2 B = 2 \times 100 \times 0.1 \times 3.14 \times (0.05)^2 \times 1.5$ J$= 0.2356$ J

Probelms for Practice

Q. 1. ***A short bar magnet of magnetic moment 0.9 JT⁻¹, is placed with its axis at 45° to a uniform magnetic field. If it experiences a torque of 0.063 J, (i) Calculate the magnitude of the magnetic field and (ii) what orientation of the bar magnet corresponds to the stable equilibrium in the magnetic field?***

Sol. Here $m = 0.9$ J T⁻¹, $\theta = 45°$, $\tau = 0.063$ J

(i) $B = \frac{\tau}{m \sin\theta} = \frac{0.063}{0.9 \times \sin 45°} = \frac{0.063 \times \sqrt{2}}{0.9 \times 1} = 0.099$ T

(ii) The bar magnet will be in stable equilibrium when its magnetic moment m is parallel to the magnetic field B i.e., θ = 0°.

Q. 2. ***A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.16 t experiences a torque of magnitude 0.032 J.(a) Estimate the magnetic moment of the magnet. (b) If the bar were free to rotate, which orientations would correspond to its (i) stable, and (ii) unstable equilibrium?***
What is its potential energy in the field for cases (i) and (ii)?

Sol. (a) Here $\theta = 30°$, $B = 0.16$ T, $\tau = 0.032$ J

Magnetic moment,

$m = \frac{\tau}{B \sin\theta} = \frac{0.032}{0.16 \times \sin 30°} = 0.40$ J T⁻¹

(b) Potential energy of the dipole in a magnetic field $\vec{B}$ is given by

$U = -\vec{m}.\vec{B} = -mB \sin\theta$

(i) The bar will be in stable equilibrium when its magnetic moment $\vec{m}$ is parallel to $\vec{B}$ ($\theta = 0°$). Its potential energy is then minimum and is given by, $U_{min} = -mB\cos 0° = -mB = -0.40 \times 0.16 = -0.064$ J

(ii) The bar will be in unstable equilibrium when m is antiparallel to B ($\theta = 180°$). Its potential energy is then maximum and is given by, $U_{max} = -\vec{m}\vec{B}\cos 180° = +\vec{m}B = +0.064$ J

Q. 3. ***A bar magnet of magnetic moment 2.5 Am² is free to rotate about a vertical axis through its centre. The magnet is released from rest from the east-west direction. Find the kinetic energy of the magnet as it aligns itself in the north-south direction. The horizontal component of earth's magnetic field is 0.3 G***

Sol. Here $\theta_1 = 90°$, $\theta_2 = 0°$, $m = 2.5$ Am²,

$B = 0.3$ G $= 0.3 \times 10^{-4}$ t

Kinetic energy = Loss in P.E. $= U_i - U_f = -mB\cos 90° + mB\cos 0° = 2.5 \times 0.3 \times 10^{-4}$ J $= 75$ μJ.

Q. 4. ***An electron is an atom revolves around the nucleus in an orbit of radius 0.53 Å. Calculate the equivalent magnetic moment if the frequency of revolution of the electron is 6.8 × 10⁹ MHz.***

Sol. Here $e = 1.6 \times 10^{-19}$ C,

$v = 6.8 \times 10^9$ MHz $= 6.8 \times 10^{15}$ Hz,

$r = 0.53 \times 10^{-10}$ m ,,$m = IA = ev \times \pi r^2 = 1.6 \times 10^{-19} \times 6.8 \times 10^{15} \times 3.14 \times (0.53 \times 10^{-10})^2 = 0.96 \times 10^{-23}$ Am²

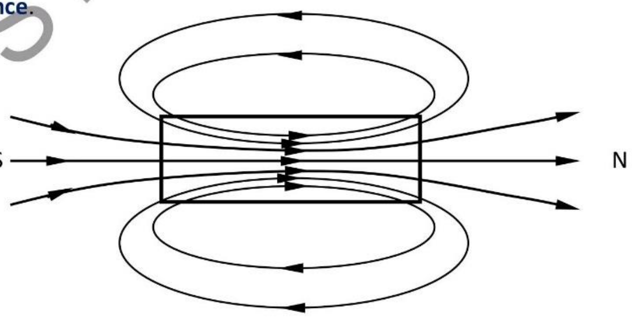
BAR MAGNET AS AN EQUIVALENT SOLENOID

Similarities between a current carrying solenoid and a bar magnet: When a current is passed through a solenoid, it behaves like a bar magnet. Similar behaviour are:

1. **A current carrying solenoid suspended freely always comes to rest in north-south direction.**

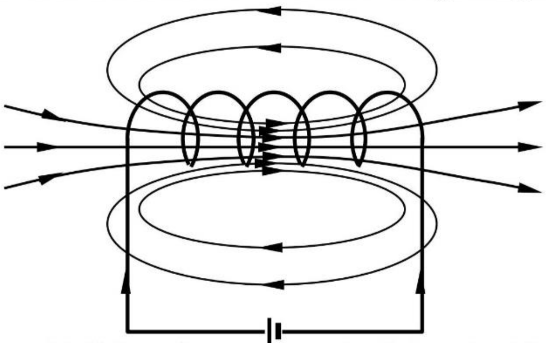
2. **Two current-carrying solenoids exhibit mutual attraction and repulsion when brought closer to one another. This shows that their end faces act as N-and S-poles like that of a bar magnet.**

3. **Fig. shows the lines of force of a bar magnet while Fig. shows the lines of force of a finite solenoid. The two patterns have a striking resemblance.**



[Field lines of a bar magnet]

If we move a small compass needle in the neighbourhood of the bar magnet and the current carrying finite solenoid, we shall find that deflections of the needle are similar in the two cases. This again supports the similarity between the two fields.



[Field lines of a current carrying finite solenoid]

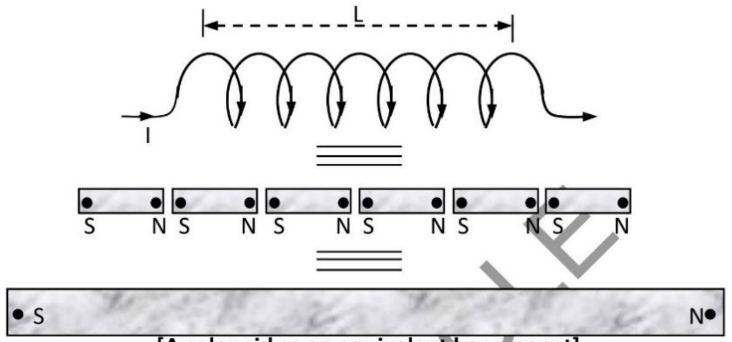
4. The magnetic fields of both the bar magnet and current carrying solenoid at any far away axial point are given by the same expression:

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

Thus a bar magnet and a solenoid produce similar magnetic fields.

► ***A current carrying solenoid equivalent to a bar magnet*** **:**

A solenoid can be regarded as a combination of circular loops placed side by side. Each turn of the solenoid can be regarded as a small magnetic dipole of dipole moment IA. Then the solenoid becomes an arrangement of small magnetic dipoles placed in the line with each other. The number of such dipoles is equal to the number of turns in the solenoid. The north pole of one touches the south of the adjacent one. The opposite poles neutralise each other except at the ends. Thus, a current carrying solenoid can be replaced by just a single south pole and s single north pole, separated by a distance equal to the length of the solenoid. Hence a current carrying solenoid is equal to a bar magnet as shown in Fig. (c).

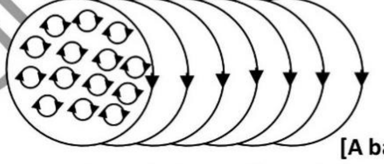


[A solenoid as an equivalent bar magnet]

► ***A bar magnet equivalent to a current carrying solenoid.***

According to **Ampere's hypothesis**, all magnetic effects are produced by current-loops. The electrons in an atom keep on revolving around its nucleus and hence set up electric currents. These atomic currents are equivalent to small circular current-loops. In a magnet, these current-loops are arranged parallel to each other and have currents in the same sense.

Fig. shows the atomic current loops in a cross-section of a cylindrical bar magnet. At any point inside the magnet, the currents from the adjacent loops cancel each other and hence the net current is zero. But there is a net current on the surface. Due to this surface current, the bar magnet is equivalent to a closely wound, current carrying solenoid. Hence a bar magnet produces a magnetic field similar to the solenoid.


[A bar magnet as an equivalent solenoid]

■ At the ends of the magnet, the current loops behave differently from those inside the magnet. As a result, the magnetic poles are located slightly inside the bar magnet. That is why the magnetic length of a bar magnet is slightly less than its geometrical length.

GAUSS'S LAW IN MAGNETISM

Gauss's law in magnetism: ***Gauss's law in electrostatics states that the surface integral of the electrostatic field E over a closed surface S is equal to $1/\varepsilon_0$ times the total charge q enclosed by the surface S, i.e.***,

$$\oint_S E.\,dS = \frac{q}{\varepsilon_0}$$

Suppose that the closed surface S ***encloses an electric dipole*** which consists of two equal and opposite charges. Then the total charge enclosed by S is zero so that the surface integral of the electrostatic field of a dipole over the closed surface is also zero, i.e.,

$$\oint_S E_{dipole}\,.\,dS = 0$$

Now a **magnetic field is produced only by a magnetic dipole because isolated magnetic poles do not exist**, so the above equation for a magnetic field can be written as

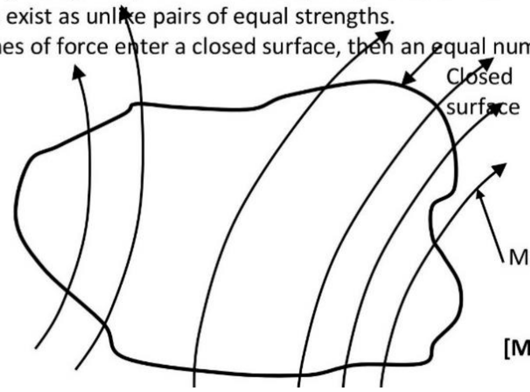
$$\oint_S \vec{B}.\,\vec{dS} = 0$$

This is Gauss's law in magnetism which states ***that the surface integral of a magnetic field over a closed surface is always zero. But the surface integral of a magnetic field over a surface gives magnetic flux through that surface***.

■ So, Gauss's law in magnetismcan also be stated as follows: The net magnetic flux through a closed surface is zero.

♋ **Consequences of Gauss's law:**

1. Gauss's law indicates that there are no sources or sinks of magnetic field inside a closed surface. So there is not point at which the field lines start or there is no point at which the field lines terminate. In other words, *there are no free magnetic charges. Hence isolated magnetic poles (also called monopoles) do not exist*.
2. The magnetic poles always exist as unlike pairs of equal strengths.
3. If a number of magnetic lines of force enter a closed surface, then an equal number of lines of force must leave that surface.



[Magnetic field lines never terminate]

- Gauss's law of magnetism formally expresses the fact that magnetic monopoles do not exist. *Hence the most elementary magnetic element is a magnetic dipole or a current loop. All magnetic phenomena can be explained in terms of an arrangement of magnetic dipole and/or current loops.* 17
- ***Basic difference between electric and magnetic lines of force***: An important consequence of the fact that magnetic monopoles do not exist is that magnetic lines of force are continuous and form closed loops. They do not start or end at a point. In contrast, the electric lines of force start from a positive charge and end on a negative charge or they fade out at infinity in case of isolated charges.

MAGNETIC FIELD OF THE EARTH

► ***Experimental evidence which support the existence of earth's magnetic field.***

Magnetic field of the earth: Earth is a powerful natural magnet. Its magnetic field is present everywhere near the earth's surface. This field can be approximated to the field of a magnetic dipole of dipole moment **8.0×10^{22} Am^2** assumed to be located at the centre of the earth.

- The axis of the dipole makes an angle of about 20° with the axis of rotation of the earth. The magnetic north pole N_m of the earth lies somewhere near the geographic south pole S_g while the magnetic south pole S_m lies somewhere near the geographic north pole N_g.
- The magnitude of the magnetic field on the earth's surface is typically about 10^{-4} T which is equal to 1 gauss (G). A gauss is also often called an Oersted. Thus the earth's magnetic field is of the order of 1 oersted.
- The branch of physics that deals with the study of earth's magnetism is called ***terrestrial magnetism*** or ***geomagnetism***.

Experimental evidences in support of earth's magnetism:

1. A freely suspended magnetic needle comes to rest roughly in north-south direction. This suggests that the earth behaves as a huge magnet with its south pole lying somewhere near the geographic north pole and its north pole lying somewhere near the geographic south pole.
2. An iron bar buried in the earth becomes weak magnet after some time. The magnetism is induced by earth's magnetic field.
3. Existence of neutral points near a bar magnet indicates the presence of earth's magnetic field. At these points, the magnetic field of the magnet is cancelled by the earth's magnetic field.

ORIGIN OF EARTH'S MAGNETIC FIELD

● ***Theories regarding the source of earth's magnetism.***

Origin of earth's magnetic field: The magnetic field of the earth is approximately like that of a giant bar magnet embedded deep inside the earth. Many theories have been proposed about the cause of earth's magnetism from time to time

1. In 1600, William Gilbert in his book 'De Magnete' first suggested that the earth behaves as a bar magnet and its magnetism is ***due to the presence of magnetic material at its centre, which could be a permanent magnet***. However, the core of the earth is so hot that a permanent magnet cannot exist there.

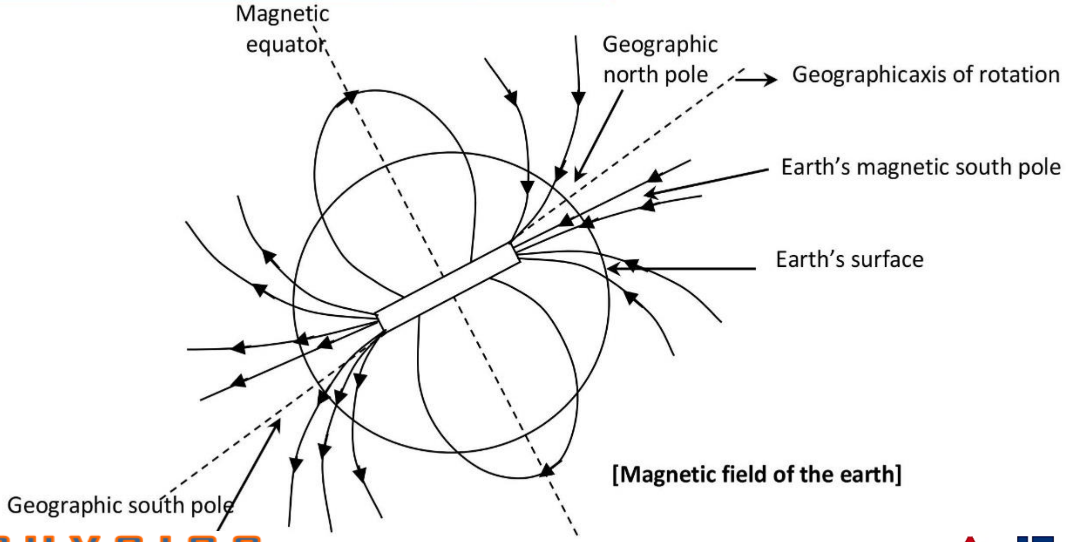
2. Prof. Blackett suggested that the ***earth's magnetism is due to the rotation of the earth about its own axis***. Every substance is made of charged particles such as protons and electrons. As these particles rotate along with the earth, they cause circulating currents which, in turn, magnetise the earth.

3. ***Cosmic rays cause the ionisation of gases in the earth's atmosphere***. As the earth rotates, strong electric currents are set up due to the movement of the charged ions. These currents may be the source of earth's magnetism.

4. According to Sir E. Bullard (U.K.) and W.M. Elaster (U. S. A.), ***there are large deposits of ferromagnetic materials like iron, nickel, etc. in the core of the earth. The core of the earth is very hot and molten***. The circulating ions in the highly conducting liquid region of the earth's core from current loop and hence produce a magnetic field. At present, this hypothesis seems most probable because our moon, which has no molten core, has no magnetic field. Venus, which has a slower rate of rotation, has a weaker magnetic field while Jupiter, with a faster rate of rotation has a stronger magnetic field.

The changes in the earth's magnetic field are so complicated and irregular that the exact cause of earth's magnetism is yet to be known.

TERMS ASSOCIATED WITH EARTH'S MAGNETISM



[Magnetic field of the earth]

1. Geographic axis: The straight line passing through the geographical north and south poles of the earth is called its geographical axis. It is the axis of rotation of the earth. 18

2. Magnetic axis: The straight line passing through the magnetic north and south poles of the earth is called its magnetic axis.

The magnetic axis of the earth makes an angle of nearly 20° with the geographic axis. At present, the magnetic south pole S_m is located at a point in Northern Canada at a latitude of 70.5° N and a longitude of 96° W. The magnetic north pole N_m is located diametrically opposite to S_m i.e., at a latitude of 70.5° S and a longitude of 84° E. The magnetic poles are nearly 2000 km away from the geographic poles. The magnetic equator intersects the geographic equator at longitudes of 6° W and 174° E.

3. Magnetic equator: It is the great circle on the earth perpendicular to the magnetic axis.

4. Magnetic meridian: The vertical plane passing through the magnetic axis of a freely suspended small magnet is called magnetic meridian. The earth's magnetic field acts in the direction of the magnetic meridian.

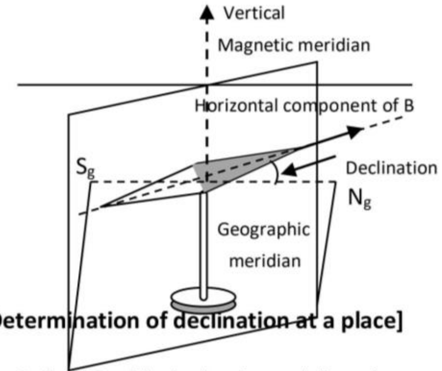
5. Geographic meridian: The vertical plane passing through the geographic north and south poles is called geographic meridian.

ELEMENTS OF EARTH'S MAGNETIC FIELD

Elements of earth's magnetic field: The earth's magnetic field at a place can be completely described by three parameters which are called elements of earth's magnetic field. They are declination, dip and horizontal component of earth's magnetic field.

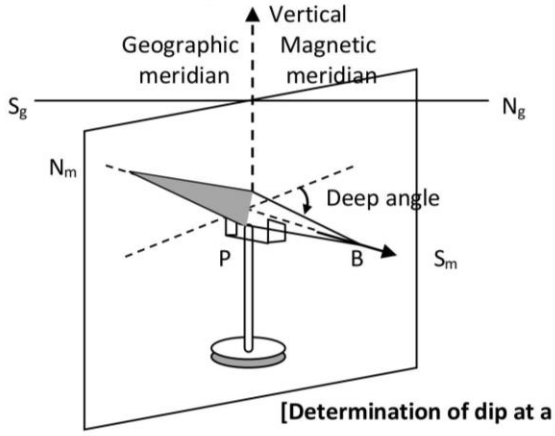
1. Magnetic declination: The angle between the geographical meridian and the magnetic meridian at a place is called the magnetic declination (α) at that place.

Magnetic declination arises because the magnetic axis of the earth does not coincide with its geographic axis. To determine magnetic declination at a place, set up a compass needle that is free to rotate in a horizontal plane about a vertical axis, as shown in Fig. the angle α that this needle makes with the geographic north-south ($N_g - S_g$) direction is the magnetic declination. By knowing declination, we can determine the vertical plane in which the earth's magnetic field lies. In India, the value of α is small. It is 0°41'E for Delhi and 0°58' W from Mumbai. This means that the N-pole of a compass needle almost points in the direction of geographic north.



[Determination of declination at a place]

2. Angle of dip or magnetic inclination: The angle made by the earth's total magnetic field B with the horizontal direction in the magnetic meridian is called angle of dip (δ) at any place.



[Determination of dip at a place]

● ***The angle of dip is different places on the surface of the earth.***

Consider a dip needle, which is just another compass needle but pivoted horizontally so that it is free to rotate in a vertical plane coinciding with the magnetic meridian. It orients itself so that its N-pole finally points exactly in the direction of the earth's total magnetic field $\vec{B}$. the angle between the horizontal and the final direction of the dip needle given the angle of dip at a given location.

At the magnetic equator, the dip needle rests horizontally so that the angle of dip is zero at a magnetic equator.

- The dip needle rests vertically at the magnetic poles so that the angle of dip is 90° at the magnetic poles.
- At all other places, the dip angle lies between 0° and 90°.

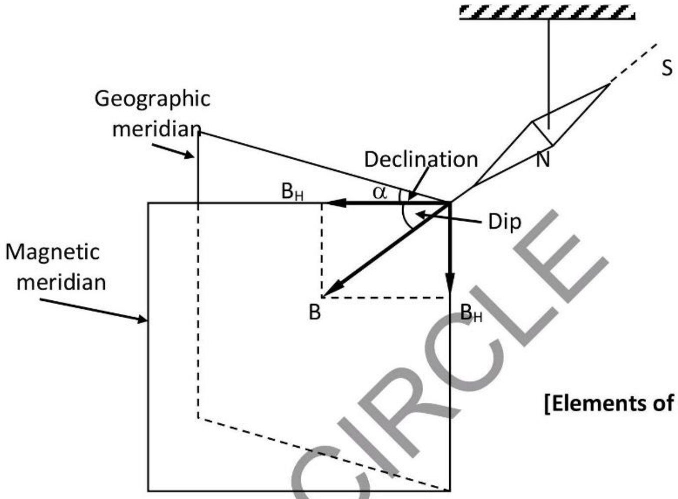
3. Horizontal component of earth's magnetic field: ***It is the component of the earth's total magnetic field B in the horizoı direction in the magnetic meridian.*** If δ is the angle of dip at any place, then the horizontal component of earth's field B at that place is given by $B_H = B\cos\delta$

At the magnetic equator, $\delta = 0°$, $B_H = B\cos 0° = B$

At the magnetic poles, $\delta = 90°$, $B_H = B\cos 90° = 0$

Thus the value of B_H is different at different places on the surface of the earth.

Relations between elements of earth's magnetic field: Fig. shows the three elements of earth's magnetic field. If δ is the angle of dip at any place, then the horizontal and vertical components of earth's magnetic field B at that place will be



[Elements of earth's magnetic field]

$$B_H = B\cos\delta \quad \text{... (1)}$$

and $$B_V = B\sin\delta$$

$$\therefore \quad \frac{B_V}{B_H} = \frac{B\sin\delta}{B\cos\delta}$$

or $$\frac{B_V}{B_H} = \tan\delta \quad \text{... (2)}$$

Also $$B_H^2 + B_V^2 = B^2(\cos^2\delta + \sin^2\delta) = B^2$$

or $$B = \sqrt{B_H^2 + B_V^2} \quad \text{... (3)}$$

Equations (1), (2) and (3) are the different relations between the elements of earth's magnetic field. By knowing the three elements, we can determine the magnitude and direction of the earth's magnetic field at any place.

Magnetic maps: these are the detailed charts which indicate on the world map the lines passing through all such placed where on e of the three magnetic elements has the same value. Three types of lines are drawn on such maps. These are:

1. **Isogonic lines:** ***The lines joining the places of equal declination are called isogonic lines**. The line of zero declination is called agonic line.*

2. **Isoclinical lines:** ***The lines joining the places of equal dip or inclination are called isoclinical lines***. The line of zero dip is called **aclinic line** or magnetic equator. The points of 90° dip are called magnetic poles. ***The magnetic equator crosses the geographic equator twice once in Atlantic and then in Pacific ocean.***

3. **Isodynamic lines:** ***The lines joining the places having the same value of the horizontal component of earth's magnetic field are called isodynamic lines***. *The horizontal component is zero at poles and maximum at the magnetic equator*.

GLOBAL VARIATIONS IN THE EARTH'S MAGNETIC FIELD

Global variations in the earth's magnetic field: Earth's magnetic field changes both in magnitude and direction from place to place. Global variations are as:

► 1. The magnitude of the magnetic field on earth's surface is small, nearly 4×10^{-5} T.

► 2. Still smaller is the background field of our own galaxy, the milky Way, being about 2 pT i.e., 2×10^{-12} T.

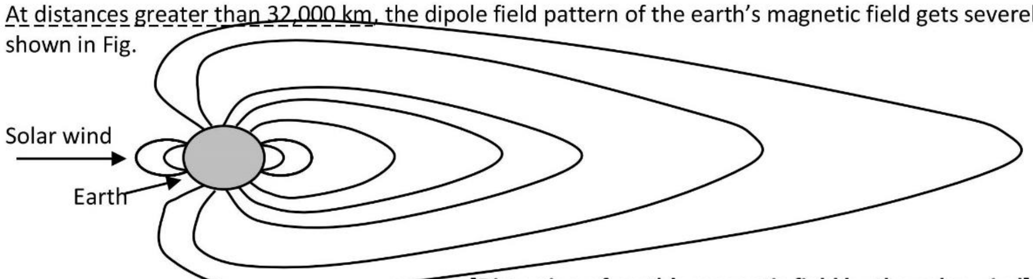
► 3. If we assume that the earth's field is due to dipole of 8.0×10^{22} Am^2 located at its centre, then the earth's magnetic field will be less than 1 μT (10^{-6} T) at a distance of 5 times the radius of the earth i.e., at about 32,000 km. Upto this distance, the magnetic field is entirely governed by the earth.

► 4. At distances greater than 32,000 km, the pattern of the earth's magnetic field gets severely distorted by the solar wind.

► 5. Solar wind causes ionisation of atmosphere near the magnetic poles of the earth. This in turn causes ***beautiful displays of colours high up in the sky and is known as aurora.***

SOLAR WIND:: ***The solar wind is a stream of hot charged ions, composed of equal numbers of protons and electrons continuously flowing radially outward from the sun with a speed of approximately 400 km/s.*** A long magneto tail stretches out for several thousand earth diameters in a direction away from the sun.

At distances greater than 32,000 km, the dipole field pattern of the earth's magnetic field gets severely disorted by the solar wind, as shown in Fig.



[Distortion of earth's magnetic field by the solar wind]

AURORA BOREALIS AND AURORA AUSTRALIS

Aurora borealis and aurora australis: This is a spectacular display of light seen in the night sky at high altitudes, occurring most frequently near the earth's magnetic poles. The displays of aurora appear as giant curtains high up in the atmosphere. The aurora is caused when the charged particles of the solar wind get attracted by the magnetic poles of the earth and there they ionise the atmospheric atoms or molecules. The aurora in the northern hemisphere is called ***aurora borealis*** or ***northern lights*** and the aurora in southern hemisphere is called ***aurora australis*** or ***southern lights***.

TEMPORAL VARIATIONS IN THE EARTH'S MAGNETIC FIELD

Temporal variations in the earth's magnetic field: The earth's magnetic field changes both in magnitude and direction as time passes. These changes are of two types:

► **(i) Short term changes:** The positions of the magnetic poles of the earth keep shifting slowly with the passage of time. In a period of 240 years, from 1580 to 1820, the magnetic declination at London has changed by 35°. The magnetic south pole in the northern Arctic region of Canada has been found to shift in the north-west direction at the rate of 10 km per year in recent times.

► **(ii) Long term changes:** The changes in earth's magnetic field over long term or geological time scales are interesting. The studies of basalt reveal that earth's magnetic field reverses its direction every million years or so. This means that once in a million years or so, the currents in earth's core cool down, come to a halt and then pick up speed in the opposite direction.

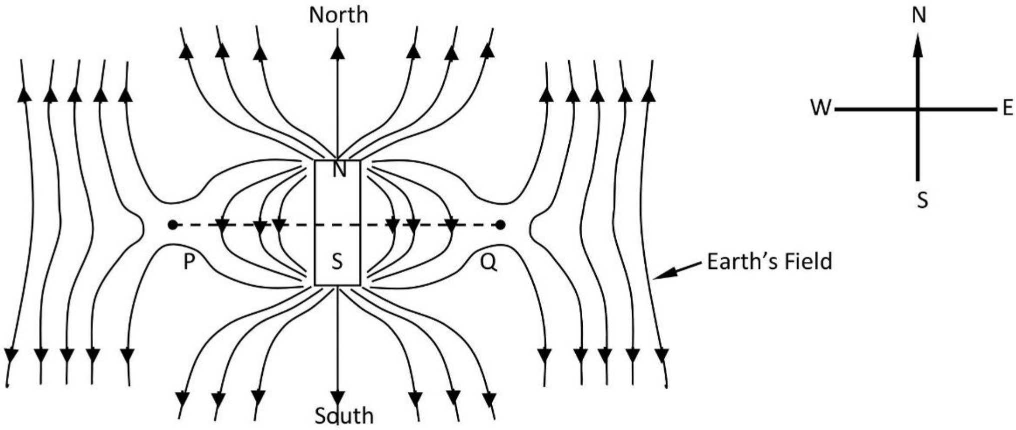
► **Besalt which contains iron**, is emitted during volcanic activity on the ocean floor. As it cools, it solidifies and provides a picture of earth's magnetic field. Its age can be determined by other means.

NEUTRAL POINT

Neutral point: ***It is the point where the magnetic field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field.***

◘The resultant magnetic field at the neutral point is zero. If a compass needle is placed at such a point, it can stay in any position.

(i) Magnet placed in the magnet meridian with its north pole pointing north: The magnetic lines of force of a bar magnet placed in the magnetic meridian with its north-pole pointing towards the geographic north of the earth. The fields due to the magnet and are in opposite directions at points on the equatorial line. So the resultant field is stronger at axial points and weaker at equatorial points. ***The two neutral point P and Q lie on the equatorial line.***



[Field lines of a bar magnet with its N-pole towards north]

Let r = distance of each neutral point from the centre of the magnet
2l = length of the magnet
m = dipole moment of the magnet

Then magnetic field strength at each neutral point is

$$B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)^{3/2}}$$

For a short magnet, l << r, therefore,

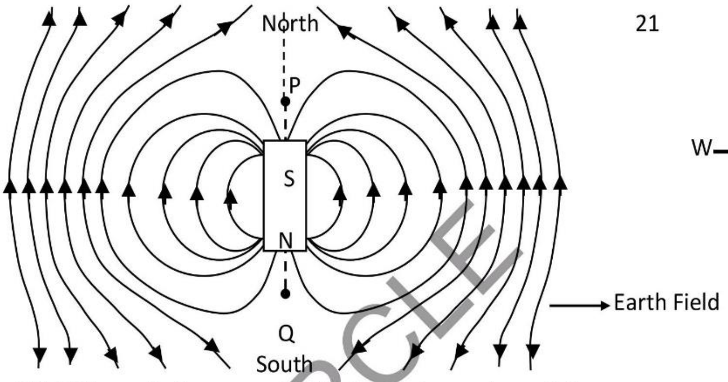
$$B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$$

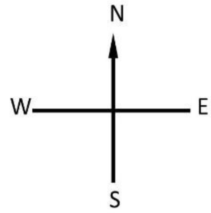
At the neutral point, the field of the magnet is balanced by the horizontal component B_H of the earth's magnetic field so that

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$$

Knowing r and B_H, the value of the magnetic dipole moment m can be determined.

(ii) Magnet placed in the magnetic meridian with its south-pole pole pointing north: Fig. shows the magnetic lines of force of a bar magnet placed in the magnetic meridian with its south-pole pointing towards the geographic north of the earth. *Here the fields due to the magnet and the earth are in the same direction at points on the equatorial line and are in opposite direction at points on the axial line of the magnet*. So the ***resultant field is weaker at axial points and is stronger at equatorial points***. In this case the two neutral point P and Q lie on the axial line near the ends of the magnet.





[Field lines of a bar magnet with S-pole towards north]

Suppose r be the distance of each neutral point from the centre of the magnet. Let 2l be the length of the magnet. Then magnitude of the magnetic field at either of the neutral points will be

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2mr}{(r^2 - l^2)^2}$$

For a short magnet, l << r, therefore

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

Again, at the neutral point, the field of the magnet is balanced by the horizontal component B_H of the earth's magnetic field, so we have

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

Knowing the values of r and B_H, the magnetic dipole moment m of the magnet can be determined.

Examples based on Earth's Magnetism and Neutral Points:

- **Formulae Used**

  1. Declination (α) = Angle between geographic meridian and magnetic meridian.
  2. Relations between elements of earth's magnetic fields are

  $B_H = B \cos \delta$ and $B_V = B \sin \delta$

  $B_V = \tan \delta$ and $B = \sqrt{B_H^2 + B_V^2}$

  3. For a magnet placed with its N-pole pointing north, neutral point lie at its equatorial line.

  $$B_H = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)^{3/2}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} \quad \text{[For a short magnet]}$$

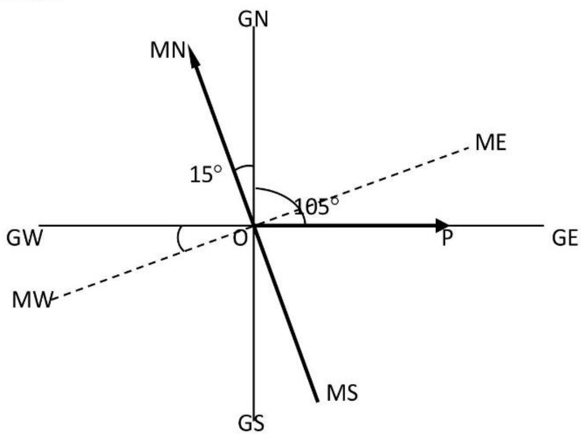
  4. For a magnet placed with its N-pole pointing south, neutral points lie on its axial line.

  $$B_H = \frac{\mu_0}{4\pi} \cdot \frac{2m}{(r^2 - l^2)^{3/2}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} \quad \text{[For a short magnet]}$$

- **Units Used**: Magnetic fields B, B_H and B_V are in tesla, distances r and l in metre, magnetic moment m in J T^{-1} or Am^2, angle α and δ are in degrees.
- **Formulae Used**: $\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1}

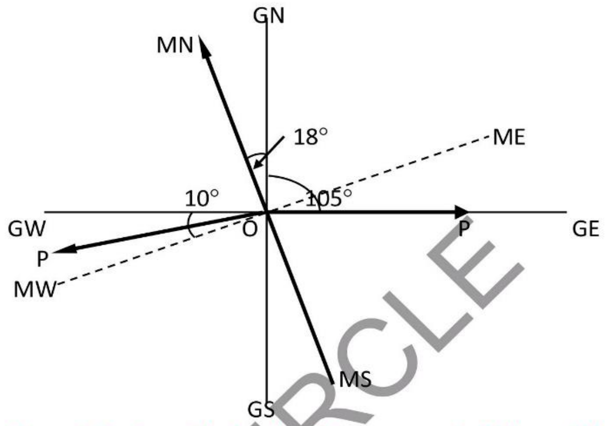
Q. 1. The declination at a place is 15° west of north. In which direction should a ship be steered so that it reaches a place due east?

Sol. As the ship is to reach a place due east i.e., along OP (fig), so its should be steered at angle of 15° + 90° = 105° with the direction of the compass needle.



Q. 2. A ship is to reach a place 10° south of west. In which direction should it be steered if the declination at the place is 18° west of north.

Sol. As the ship is to reach a place 10° south of west i.e., along OP[fig], so it should be steered west of magnetic north at angle of 90 – 18 + 10 = 82°.



Q. 3. ***In the magnetic meridian of a certain place, the horizontal component of the earth's magnetic field is 0.26 G and the dip angle is 60°. What is the magnetic field of the earth in this location?***

Sol. Here $B_H = 0.26$ G, $\delta = 60°$

As $B_H = B\cos\delta$

$\therefore$ $B = \dfrac{B_H}{\cos\delta} = \dfrac{0.26\text{ G}}{\cos 60°} = \dfrac{0.26\text{ G}}{0.5} = 0.52$ G

Q. 4. ***A compass needle whose magnetic moment is 60 Am² pointing geographical north at a certain place where the horizontal component of earth's magnetic field is 40 μ Wb/m² experiences a torque of 1.2 × 10⁻³ Nm. What is the declination of the place?***

Sol. In stable equilibrium, a compass needle points along magnetic north and experiences no torque. When it is turned declination α, it points along geographic north and experiences torque, $\tau = mB\sin\alpha$

$\therefore$ $\sin\alpha = \dfrac{\tau}{mB} = \dfrac{1.2\times10^{-3}}{60\times40\times10^{-6}} = \dfrac{1}{2}$ or $\alpha = 30°$

Q. 5. ***The horizontal and vertical components of earth's field at a place are 0.22 gauss and 0.38 gauss respectively. Calculate the angle of dip and resultant intensity of earth's field.***

Sol. Here $B_H = 0.22$ G, $B_V = 0.38$ G

Now $\tan\delta = \dfrac{B_V}{B_H} = \dfrac{0.38}{0.22} = 1.7272$

$\therefore$ Angle of dip, $\delta = 59°\ 56'$

Resultant magnetic field of the earth is $B = \sqrt{B_H^2 + B_V^2} = \sqrt{0.22^2 + 0.38^2} = 0.427$ G

Q. 6. ***If the horizontal component of earth's magnetic field at a place where the angle of dip is 60° is 0.4 × 10⁻⁴ T, calculate the vertical component and the resultant magnetic field at that place.***

Sol. Here $\delta = 60°$, $B_H = 0.4\times10^{-4}$ T

$B_V = B_H\tan\delta = 0.4\times10^{-4}\tan 60°$

$= 0.4\times10^{-4}\times\sqrt{3} = 0.69$ T

Resultant magnetic field, $B = \dfrac{B_H}{\cos\delta} = \dfrac{0.4\times10^{-4}}{\cos 60°} = 0.8\times10^{-4}$ T

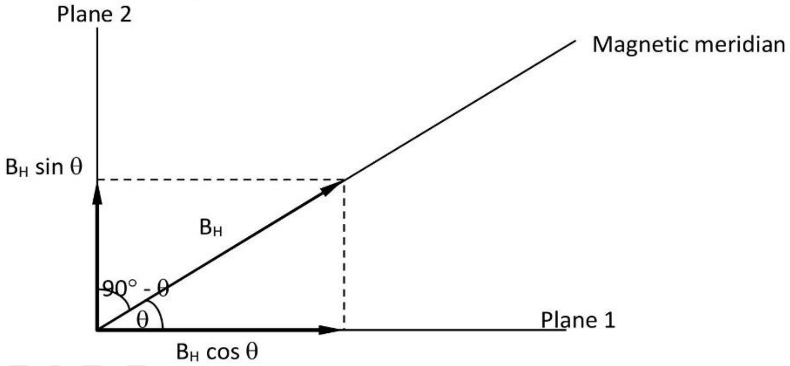
Q. 7. ***If δ_1 and δ_2 be the angles of dip observed in two vertical planes at right angles to each other and δ is the true angle of dip, prove that $\cot^2\delta_1 + \cot^2\delta_2 = \cot^2\delta$***

Sol. Let B_H and B_V be the horizontal and vertical components of earth's magnetic field $\vec{B}$. Since δ is the true angle of dip, therefore

$\tan\delta = \dfrac{B_V}{B_H}$ or $\cot\delta = \dfrac{B_H}{B_V}$ ... (1)

As shown in Fig. suppose planes 1 and 2 are two mutually perpendicular planes and respectively make angle θ and $90° - \theta$ with the magnetic meridian. The vertical components of earth's magnetic field remain same in the two planes but the effective horizontal components in the two planes will be

$B_1 = B_H\cos\theta$ and $B_2 = B_H\sin\theta$



The angles of dip δ_1 and δ_2 in the two planes are given by

$$\tan\delta_1 = \frac{B_V}{B_1} = \frac{B_V}{B_H\cos\theta}$$

or $$\cot\delta_1 = \frac{B_H\cos\theta}{B_V} \quad \ldots(2)$$

$$\tan\delta_2 = \frac{B_V}{B_2} = \frac{B_V}{B_H\sin\theta}$$

or $$\cot\delta_2 = \frac{B_H\sin\theta}{B_V} \quad \ldots(3)$$

From equations (2) and (3), we have, $\cot^2\delta_1 + \cot^2\delta_2 = \cot^2\delta$ [Using equation (1)]

Q. 8. ***A dip circle shows an apparent dip of 60° at a place where the true dip is 45°. If the dip circle is rotated through 90°, what apparent dip will it show?***

Sol. Here $\delta_1 = 60°$ and $\delta = 45°$

As $\cot^2\delta = \cot^2\delta_1 + \cot^2\delta^2$

∴ $\cot^2 45° = \cot^2 60° + \cot^2\delta_2$

or $\cot^2\delta_2 = (1)^2 - \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{2}{3}$ or $\cot\delta_2 = 0.816$ ∴ $\delta_2 = 51°$

Q. 9. ***True value of dip at a place is 45°. The plane of the dip circle is turned through 60° from the magnetic meridian. Find the apparent value of dip.***

Sol. Here $\delta = 45°$, $\theta = 60°$, $\delta' = ?$

$$\tan\delta' = \frac{B_V}{B_H} = \frac{B_H\tan\delta}{B_H\cot\theta}$$

$$= \frac{\tan\delta}{\cos\theta} = \frac{\tan 45°}{\cos 60°} = 2$$

∴ Apparent dip, $\delta' = 63.4°$.

Q. 10. ***A bar magnet of length 10 cm is placed in the magnetic meridian with its north pole pointing towards the geographical north. A neutral point is obtained at a distance of 12 cm from the centre of the magnet. Find the magnetic moment of the magnet, if B_H = 0.34 G.***

Sol. Here, $2l = 10$ cm, $l = 5$ cm $= 5 \times 10^{-2}$ m

$r = 12$ cm $= 12 \times 10^{-2}$ m

$B_H = 0.34$ G $= 0.34 \times 10^{-4}$ T

In this case, the neutral points lie on the equatorial line of the magnet so that at any neutral point,

$$B_{equa} = B_H \quad \text{or} \quad \frac{\mu_0}{4\pi}\cdot\frac{m}{(r^2+l^2)^{3/2}} = B_H$$

∴ Magnetic moment

$$m = B_H \cdot \frac{4\pi}{\mu_0}\cdot(r^2+l^2)^{3/2}$$

$$= 0.34 \times 10^{-4} \times \frac{1}{10^{-7}}[5^2+12^2]^{3/2}(10^{-4})^{3/2} = 0.747\ JT^{-1}$$

Q. 11. ***The magnetic moment of a short bar magnet is 1.6 × Am². It is placed in the magnetic meridian with north pole pointing south. The neutral point is obtained at distance of 20 cm from the centre of the magnet. Find the horizontal component of earth's magnetic field. If the magnet be reversed, i.e., north pole pointing north, find the position of neutral point.***

Sol. Here $m = 1.6$ Am², $r = 20$ cm $= 0.20$ m

When N-pole of the magnet points south, the neutral points lie on the axial line of the magnet.

Hence at the neutral point,

$$B_{axial} = B_H = \frac{\mu_0}{4\pi}\cdot\frac{2m}{r^3}$$

or $$B_H = \frac{10^{-7}\times 2\times 1.6}{(0.20)^2} = 4\times 10^{-5}\ T$$

When the magnet is reversed, its north pole points north. The neutral points will lie on the equatorial line of the magnet.

Hence $$B_{equa} = B_H = \frac{\mu_0}{4\pi}\cdot\frac{2m}{r^3}$$

or $$r^3 = \frac{\mu_0}{4\pi}\cdot\frac{m}{B_H} = \frac{10^{-7}\times 1.6}{4\times 10^{-5}} = 4.0\times 10^{-3}\ m^3$$

∴ $r = (4.0\times 10^{-3})^{1/3} = 1.6\times 10^{-1}$ m $= 16$ cm

Q. 12. ***A magnet placed in the magnetic meridian with its north pole pointing north of the earth produces a neutral point at a distance of 0.15 m from either pole. It is then broken into two equal pieces and one such piece is placed in a similar position. Find the position of the neutral point.***

Sol. Here the neutral points lie on the equatorial line of the magnet at distance x from each of the two poles.

∴ $$B_H = B_{equa} = \frac{\mu_0}{4\pi}\cdot\frac{m}{x^3}$$

When the magnet is broken into two parts, its pole strength remains unchanged.

Original magnetic moment, $m = q_m \times 2l$

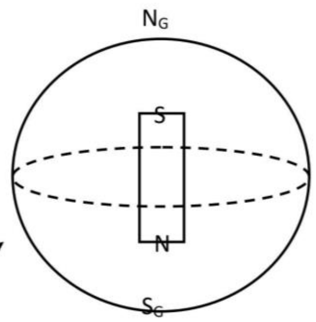
Magnetic moment of each part, $m' = q_m \times \frac{2l}{2} = \frac{m}{2}$

$\therefore \quad B_H = \frac{\mu_0}{4\pi} \cdot \frac{m/2}{x'^3}$

Hence, $\frac{\mu_0}{4\pi} \cdot \frac{m}{x^3} = \frac{\mu_0}{4\pi} \cdot \frac{m}{2x'^3}$

or $\quad 2x'^3 = x^3$ or $\quad x' = \frac{x}{2^{1/3}}$

or $\quad x' = \frac{0.15}{1.26} = 0.119$ m, from each pole.



Q. 13. The magnetic field at a point on the magnetic equator is 3.1×10^{-5} T. Taking the radius of the earth equal to 6400 km, find the magnetic moment of the assumed dipole at the earth's centre.

Sol. Any point on the magnetic equator lies in the broad side on position of the assumed magnetic dipole. Hence

$$B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{R^3}$$

or $\quad m = \frac{4\pi}{\mu_0} \cdot B_{equa} R^3 = 10^7 \times 3.1 \times 10^{-5} \times (6400 \times 10^3)^3 = 8.1 \times 10^{22}$ Am^2.

Q. 14. The earth's magnetic field at the equator is approximately 0.4 G. Estimate the earth's dipole moment. Radius of the earth = 6400 km.

Sol. Here $\quad B_H = B_{equa} = \frac{\mu_0 m}{4\pi r^3}$

or $\quad 0.4 \times 10^{-4} = \frac{10^{-7} \times m}{(6.4 \times 10^6)^3}$ or $\quad m = 1.04 \times 10^{23}$ Am^2

Q. 15. A short bar magnet is placed in a horizontal plane with its axis in the magnetic meridian. Null points are found on its equatorial line (i.e., it normal bisector) at 12.5 cm from the centre of the magnet. The earth's magnetic field at the place is 0.38 G and the angle of dip is zero.

(i) What is the total magnetic field at points on the axis of the magnet located at the same distance (12.5 cm) as the null-points from the centre?

(ii) Locate the null points when the magnet is turned around by 180°.

Assume that the length of the magnet is negligible as compared to the distance of the null-points from the centre of the magnet.

Sol. (a) At the neutral point on the equatorial line of a short magnet, we have

$$B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} = B_H$$

Magnetic field of the magnet on its axial line at the same distance will be

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} = 2B_H = 2 \times 0.38 = 0.76 \text{ G}$$

At any point on the axial line, B_H and B_{axial} are in the same direction. So total magnetic field,

$$B = B_{axial} + B_H = 0.76 + 0.38 = 1.14 \text{ G}.$$

(b) When the magnet is turned through 180°, the neutral points lie on the axial line.

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{x^3} = B_H$$

But $\quad B_H = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} \quad \therefore \quad \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{x^3}$

or $\quad x^3 = 2r^3$

or $\quad x = (2)^{1/3} r = 1.26 \times 12.5$ cm

$\quad = 15.75$ cm

CLASSIFICATION OF MAGNETIC MATERIALS

Some Important term used in Magnetism:- (To describe the magnetic properties of materials)

1. Magnetic Permeability :- "(μ) *Magnetic permeability is the ability of a material to permit the passage of magnetic lines of force through it*".

2. Relative Magnetic Permeability :- (μ_r) "*The degree of extent to which magnetic field can penetrate (or permeate) a material is called Relative magnetic permeability of the material*".

Or

"*μ_r of material is also defined as the ratio of the no. of lines of magnetic induction per unit area (flux density) in that medium to the no. of magnetic lines per unit are that would be present if the medium were replaced by vaccum*".

$\therefore \quad \mu_r = \dfrac{\vec{B}}{\vec{B_0}}$ [flux density (Medium)] / [flux density (vaccum)] or, $B = \mu_r B_0$

"*μ r of a material is also defined as the Ratio of magnetic permeability of the material (μ) and magnetic permeability of free space (μ_0)*"

$\therefore \quad \boxed{\mu_r = \mu/\mu_0}$ [∵as $\mu_0 = B/B_0 = \mu H/\mu_0 H$ [where , H = Intensity of Mag. Field]

$= \mu/\mu_0$

■ S.I Unit: **Henry/meter** $[Hm^{-1}] = wb\ A^{-1} = T\ m^2 A^{-1} m^{-1} = Tm\ A^{-1}$

3. Magnetic Induction :- (*Flux density*) $\vec{B}$:

When a ***positive test charge*** q_0 is fired with a ***velocity*** $\vec{v}$ through a point P and the moving charge experience a side way **force** $\vec{F}$, we can say that a magnetic field is present at P.

∴ ***From Lorentz force***, $\vec{F} = q_0 (\vec{v} \times \vec{B})$

$|\vec{F}| = q_0 v B \sin\theta$

Therefore, "*Magnetic induction of a magnetic field being equal to the force experienced by a unit positive charge moving with unit velocity in a direction ⊥ar to the magnetic field.*"

i.e., $F = B \times 1 \times \sin 90^0 \times 1$

∴ $\boxed{B = F}$

S.I Unit Of $\vec{B}$ = wb/m^2 or **Tesla (T)**

4. Magnetic Intensity (Or Magnetizing force):- (H)

"*The degree or extent to which the magnetizing field can magnetize a Substance is known as the intensity of magnetizing filed*".

Consider a toroidal solenoid with ***n turns/unit length*** carrying **current I** wound round a ring of magnetic material.

∴ Mag. field induction produced in the material of the solenoid , $B = \mu n I$

▶ The ***product of 'n I'*** is called **Magnetic intensity, H**

∴ $B = \mu H$

"*Magnitude of magnetizing force is defined as the no. of Ampere turns flowing round unit length of toroidal solenoid to produced the magnetic induction B in the solenoid*".

▶ Inside to toroidal solenoid there is free space, then magnetic induction.

$\boxed{B_0 = \mu_0 H}$ → Magnetic permeability of free space

▶ **S.I Unit** :- $H = \dfrac{B_0}{\mu_0} = \dfrac{F/q_0 v}{q_0} = \dfrac{F}{\mu_0 v \mu_0} = \dfrac{N}{C\ (m/s)\ Tm\ A^{-1}}$

∴ (i) $\boxed{H = Nm^{-2} T^{-1}}$

(ii) $H = N/m^2T = N/wb = N\ wb^{-1} = J\ m^{-1} wb^{-1}$

5. Intensity of Magnetization:- "*Intensity of magnetization of magnetic material is defined as the Magnetic moment per unit volume of the material*".

i.e., $\boxed{I = \dfrac{M}{V}}$ → Magnetic moment, → Volume

Also, $\boxed{I = \dfrac{m \times 2l}{a\ (area) \times 2l} = \dfrac{m}{a}}$ → Cross sectional area

∴ "*Intensity of magnetization of a magnetic is also defined as the pole strength per unit area of cross section of the material*".

▶ **S.I unit** $I = \dfrac{Amp\ .\ m}{m^2} = A\ m^{-1}$

6 Magnetic flux (ϕ) *"Magnetic flux through a surface is defined as the no. of magnetic lines of force passing normally the space."*

$\therefore \quad \phi = \vec{B} . \triangle \vec{S}$ → surface area; $\vec{B}$ → Field Induction

► **Unit :-** w b

7. Magnetic Susceptibility :- (χ m) (χ m helps us to determine how easily a specimen can be magnetized when placed in magnetizing filed).

"The Magnetic susceptibility of a magnetic material is defined as the ratio of intensity of magnetization to the magnetic intensity.

$\therefore \chi_m = I/H$ (No units)

► χ m is usually ***called volume susceptibility*** of the material because ***'I is magnetic moment per unit volume'***.

Relation between Magnetic Permeability (μ) and Magnetic Susceptibility (X m):-

When a magnetic material is placed in a Magnetizing filed of intensity $\vec{H}$, the material gets magnetized . The total magnetic induction $\vec{B}$ in the material is the sum of Magnetic induction $\vec{B_0}$ in vaccum produced by magnetic intensity & Magnetic induction $\vec{Bm}$ due to magnetization of the material.

i.e., $\vec{B} = \vec{B_0} + \vec{B_m}$

$= \mu_0 \vec{H} + \mu_0 \vec{I}$ [∵ $B_0 = \mu H$ & $Bm = \mu_0 I$]

$\vec{B} = \mu_0 [\, H + I \,]$

Dividing both side by H, we get

$B/H = \mu_0 (H/H + I/H)$

$B/H = \mu_0 (1 + I/H)$

$\mu = \mu_0 (1 + \chi_m)$ (as $\mu = B/H$ & $\chi_m = I/H$)

Or, $\mu / \mu_0 = (1 + \chi_m)$

$\mu_r = 1 + \chi_m$ (as $\mu/\mu_0 = \mu_r$) (relative. permeability)

● **CLASSIFICATION OF MAGNETIC MATERIALS**: Faraday classified the various substances into three categories[*On the basis of their behaviour in external magnetic fields*]:

1. DIAMAGNETIC SUBSTANCE: Diamagnetic substances are those which develop feeble magnetisation in the opposite direction of the magnetising field.

► Such substances are feebly repelled by magnets and tend to move from stronger to weaker parts of a magnetic field.

Examples: Bismuth, copper, lead, zinc, tin, gold, silicon, nitrogen (at STP), water, sodium chloride, etc.

Thus, *"The material which are weakly magnetized in a direction opposite to the direction of applied magnetic field are known as Diamagnetic Substance".*

- ► Diamagnetic material are those in which the individual atoms/molecule/ ions do not posses any Net Magnetic moment in their own.
- ► When a diamagnetic material is placed in external magnetic filed (B) a small magnetic moment is produced in each atom/ molecule / ions proportional to B, but pointing in opposite direction.
- ► The diamagnetic effect are too weak to be detected unless the applied magnetic field is strong.
- ► Magnetic Behaviour of Diamagnetic material normally does not depends upon change in temp[3]. (They do not obey curic's law)

2. PARAMAGNETIC SUBSTANCES: Paramagnetic substances are those which develop feeble magnetisation in the direction of the magnetising field.

► Such substances are feebly attracted by magnets and tend to move from weaker to stronger parts of a magnetic field.

Thus, *"Paramagnetic substance are those substance which are weakly magnetized in the direction of applied magnetic field".*

Ex:- Aluminium, Chromium , Manganese, Oxygen, Platinum, alkali, alkaline earth metal etc.

- ► Paramagnetic substance are those in which each atom/ molecule/ ion has a net non zero magnetic of its own.
- ► When a paramagnetic substance are placed in an external magnetic field of induction B, it tries to align the individual dipole moment in the direction of the field.
- ► For strong field induction, there is a net average magnetic dipole moment density in the direction of B.

3. Ferromagnetic substances: Ferromagnetic substances are those which develop strong magnetisation in the direction of the magnetising field.

► They are strongly attracted by magnets and tend to move from weaker to stronger parts of a magnetic field.

Examples: Iron, cobalt, nickel, gadolinium and alloys like alnico.

"The substance which are strongly magnified in the direction of the applied magnetic field are known as ferromagnetic materials".

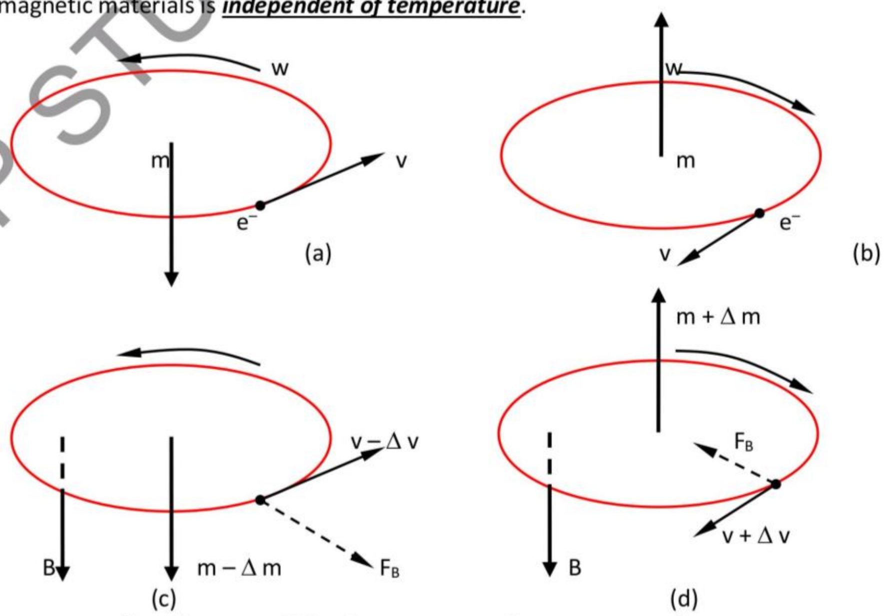
- ► Ferromagnetic substance behave just like paramagnetic materials (but it develops strong induced magnetism).
- ► F.M surface are those in which each individual atom/molecule/ion has a non zero magnetic moment.
- ► The individual magnetic moment interact with one another in such a way as to align themselves spontaneously in a common direction over microscopic Volume(domains).
- ► *When a F.M substance is placed inside a magnetic filed, the field inside the ferromagnetic substance get greatly enhanced. Magnitude moments of different domains are aligned and the material gets strongly magnetized in the direction of applied magnetic field. d*

ORIGIN OF DIAMAGNETISM

In atoms of some materials like Bi, Cu, Pb, the magnetic moments due to different electrons cancel out. In such atoms, electrons occur in pairs with one of them revolving clockwise and other anticlockwise around the nucleus. Net magnetic moment of an atom is zero, [shown in Fig. (a) and (b)].

When such an atom is placed in a magnetic field $\vec{B}$, the speed of revolution of one electron increases and that of other decreases. The magnetic moment of the former electron increases to $\vec{m} + \Delta\vec{m}$ and that of the latter electron decreases to $m - \Delta m$. So each electron pair gains a net magnetic moment $2\Delta\vec{m}$ which is proportional to the field $\vec{B}$ but points in its opposite direction [shown in Fig. (c) and (d)]. A sufficient magnetic moment is induced in the diamagnetic sample in the opposite direction of $\vec{B}$. This sample moves from stronger to the weaker parts of the field B, i.e., **a diamagnetic substance is repelled by a magnet**.

- The behaviour of diamagnetic materials is ***independent of temperature***.



[An electron orbiting in an atom produces a moment]

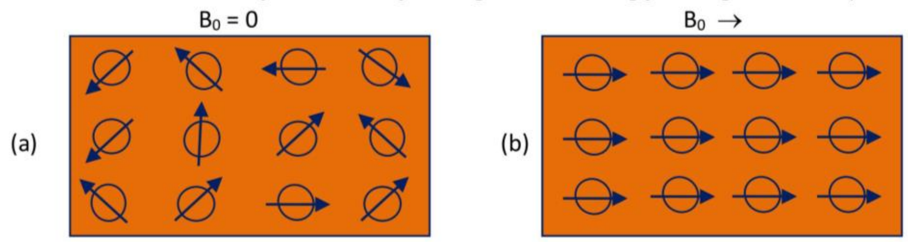
ORIGIN OF PARAMAGNETISM

According to Langevin, ***the atoms or molecules of a paramagnetic material possess a permanent magnetic moment either due to the presence of some unpaired electron or due to the non-cancellation of the spins of two electrons. In the absence of an external magnetic field, the atomic dipoles are randomly oriented due to their ceaseless random motion***, [as shown in Fig. (a)].

∴ There is no net magnetisation.

► When a strong enough field $\vec{B}_0$ is applied and the temperature is low enough, the field B_0 tends to align the atomic dipoles in its own direction, producing a ***weak magnetic moment*** in the direction of $\vec{B}_0$.

► The material tends to ***move from a weak field region to a strong field region***. This is paramagnetism.



[(a) Randomly distributed atomic dipoles in a paramagnetic material in the absence of magnetic field.
(b) Alignment of dipoles in the presence of magnetic field.

►At very high magnetic field or at very low temperatures, the magnetisation approaches its maximum value when all the atomic dipole moments get aligned. **This is called the saturation magnetisation value I_S.** 29

Curie's law: From experiments, it is found that the intensity of magnetisation (I) of a paramagnetic material is

(i) ***Directly proportional to the magnetising field intensity H,*** because the latter tends to align the atomic dipole moments.

(ii) ***Inversely proportional to the absolute temperature T,*** because the latter tends to oppose the alignment of the atomic dipole moments.

Therefore at low H/T values, we have

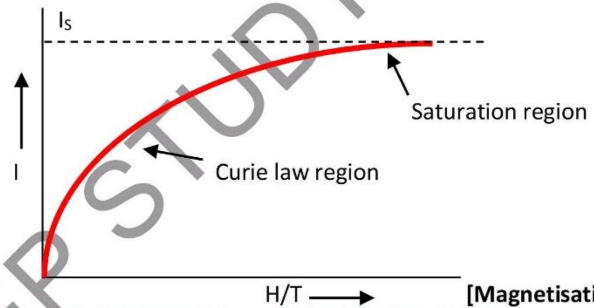
$$I \propto \frac{H}{T}$$

or $$I = C.\frac{H}{T}$$

or $$\frac{I}{H} = \frac{C}{T} \quad \text{or} \quad \chi_m = \frac{C}{T}$$

Here ***C is curie constant*** and ***χ_m is the susceptibility of the material***. The above relation is called Curie's law. ***This law states that far away from saturation, the susceptibility of a paramagnetic material is inversely proportional to the absolute temperature.***

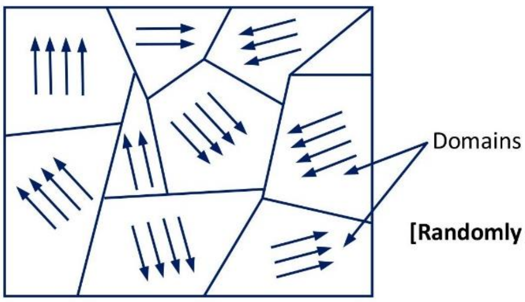
Fig. shows the variation of intensity of magnetisation I as a function of H/T. Beyond the saturation value I, Curie law is not valid.



[Magnetisation I as a function of H/T]

ORIGIN OF FERROMAGNETISM: DOMAIN THEORY

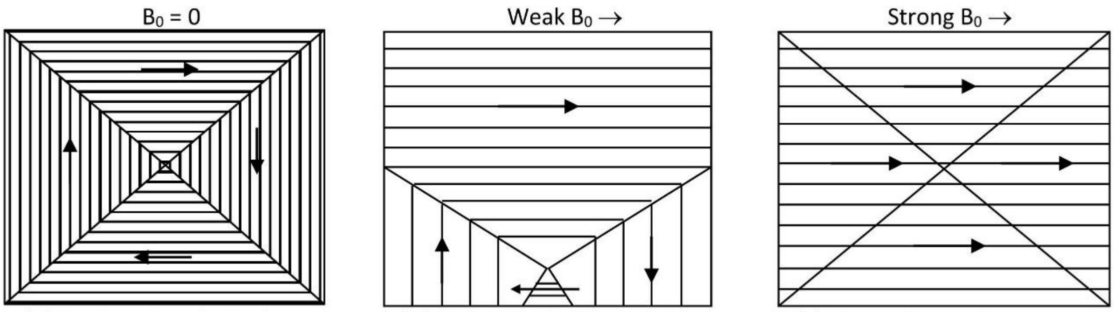
Weiss explained ferromagnetism on the basis of his domain theory. In materials like Fe, Ni, Co, the individual atoms are associated with large magnetic moments. The magnetic moments of neighbouring atoms intersect with each other and align themselves spontaneously in a common direction over macroscopic regions called domains. Each domain has a typical size of about 1 mm and contains about 10^{11} atoms. So each domain possesses a strong magnetic moment. In the absence of any external magnetic field, these domains are randomly distributed so that the ***net magnetic moment is zero***.



[Randomly oriented domains in a ferromagnetic substance]

►When a ferromagnetic material is placed in a magnetic field, all the domains align themselves along the direction of the field leading to the strong magnetisation of the material along the direction of the field. That is why the ferromagnetic substances are strongly attracted by magnets. The alignment of domains may occur in either of the following two ways:

1. By displacement of the boundaries of domains: When the external field B_0 is weak, the domains aligned in the direction of B_0 grow in size while those oppositely directed decrease in size,[as shown in Fig.]



(a) Unmagnetised sample (b) Magnetisation by growing of domains (c) Magnetisation by rotation of domains

[Magnetisation of a ferromagnetic sample]

2. By rotation of domains: When the external field B_0 is strong, the domains rotate till their magnetic moments get aligned in the direction of B_0 as shown in Fig. (c).

3. Quantum Mechanical effect (Exchange interaction) :- In the atoms of elements iron, cobalt, Nickel, Gadolinium & dysprosium (all F.M material) there are vacancies in the inner shell so, the electron in these shells are not paired off with anti parallel spins and equal & opposite orbital magnetic moments. Moreover, an unpaired electron in one atom interact strongly with the unpaired electron in the adjacent atom consequently the magnetic moment gets aligned in the same direction. This is called ***exchange interaction***.

- **Since iron (Ferrum) is the most representative substance in this category therefore these are called ferromagnetic substance.**
- The magnetic moment of the all the atoms in a domain are parallel to each other & hence a domain possesses a small value of net magnetic dipole moment.
- In the absence of external magnetic field, the direction of magnetic moment in different domains are obtained randomly in different direction there by cancels the effect of the other, So that the Net M of the material is zero. But within a domains, the adjacent dipole moments are bound together by strong force which give rise to a quantum interaction called exchange interaction.
- When external magnetic filed is applied, the domains having magnetic moment || to the direction of Ext. B starts growing in size at the cost of other domains.

Modified Curie's law for ferromagnetic substance: *When a ferromagnetic sample is heated, its magnetisation decreases due to the increase in the randomisation of its domains.*

- ***At a sufficiently high temperature, the domain structure disintegrates and the ferromagnetic substance becomes paramagnetic. The temperature at which a ferromagnetic substance becomes paramagnetic is called Curie temperature or Curie point T_C.***

Above the Curie point i.e., in the paramagnetic phase, the susceptibility varies with temperature as

$$\chi_m = \frac{C'}{T - T_C} \qquad [T > T_C]$$

Where C' is a constant. This is modified Curie's law for a ferromagnetic material above the Curie temperature. It is also known as Curie-Weiss law.

- This law states that the susceptibility of a ferromagnetic substance above its Curie temperature is inversely proportional to the excess of temperature above the Curie temperature.

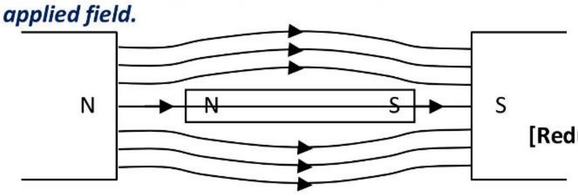
☑Table: Curie Temperatures of some Ferromagnetic Materials:

Material	T_C (K)
Cobalt	1394
Iron	1043
Fe_2O_3	893
Nickel	631
Gadolinium	317

PROPERTIES OF MAGNETIC MATERIALS :

PROPERTIES OF DIAMAGNETIC SUBSTANCES:

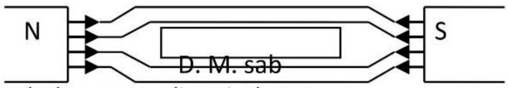
1. *When placed in an external magnetic field, a diamagnetic substance develops* ***feeble magnetisation*** *in the* ***opposite direction of the applied field.***



[Reduction fo lines of force in a diamagnetic rod]

2. When a rod of a diamagnetic material is placed in a magnetic field, poles are induced on it in a direction opposite to that of the inducing field.

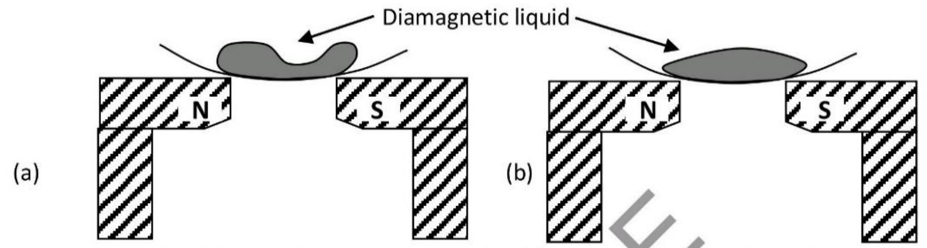
Explanation:- Poles are induced in the diamagnetic in a direction opp. to the direction of magnetic field.
Therefore, Field within the sample is decreased from B to smaller value. **Since B < H** or **B/H <1** or **μ < 1**
Hence, **permeability of DM. substance is always less than 1.**



- So the lines of force prefer to pass through the surrounding air then to pass through the surrounding air than to pass through the material itself i.e., the lines of force get expelled or repelled, as shown in Fig.
- Consequently, ***the magnetic induction B inside the material becomes less than the magnetising field, $B_0 = \mu_0 H$.*** The reduction is very small, about 1 part in 10^5.

3. When placed in a non-uniform magnetic field, a diamagnetic substance moves from stronger to the weaker parts of the field.

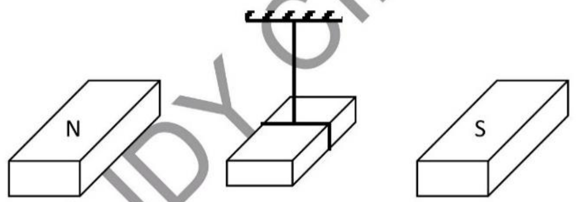
When a watch glass containing a diamagnetic liquid is placed over two closely lying (3 – 4 mm apart) pole pieces of a magnet, the liquid is found to move towards the poles causing a depression in the middle. This indicates that the field is stronger in the middle than that near the poles. Now if the poles are moved apart sufficiently, the magnetic field at the middle becomes weaker than that near the poles. Consequently, the liquid accumulates in the middle and thins out near the poles.



[Effect of non-uniform magnetic field on a diamagnetic liquid when (a) poles are quite close to each other, (b) poles are sufficiently apart.]

✱ **Explanation:-** Potential energy of the dipole placed in uniform magnetic field is – 'MB Cos θ' .The value of θ is 180^0 for a diamagnetic substance. So the PE is MB. Since every system in nature tends to have least PE .Therefore the diamagnetic substance move to a region of minimum (b).

○ 4. ***When a rod of a diamagnetic material is suspended freely in a uniform magnetic field, it aligns itself perpendicular to the magnetising field.***



[A freely suspended diamagnetic rod in a uniform field]

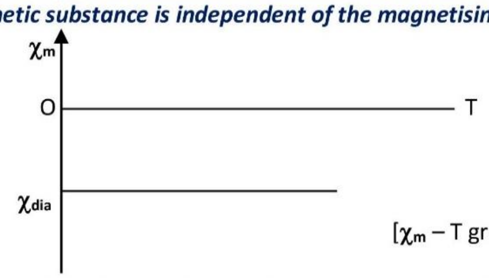
✱ **Explanation:** Potential energy unit volume is the case of diamagnetic substance is MB/V or $\frac{M}{V}$ B i.e., IB, clearly, the diamagnetic rod will have minimum energy it length is $\perp^{ar}$ to the direction of Magnetic field.

○ 5. ***As a diamagnetic substance develops a weak magnetisation in a opposite direction of the magnetising field, the susceptibility (χ_m = M/H) of diamagnetic materials is small and negative. For bismuth, χ_m = – 0.00015.***

- The magnetic susceptibility of a D.M substance has a small Negative value as $\mu < 1$
  Therefore, From the Relation, $\mu_r = (1 + \chi_m)$
  χ_m = ive value.

○ 6. **The relative permeability μ_r (= $1 + \chi_m$) is positive *but less than 1 for a diamagnetic material*.**

○ 7. ***The susceptibility of diamagnetic substance is independent of the magnetising field and the temperature***, as shown in Fig.



[χ_m – T graph for diamagnetic material]

○ **8. The magnetisation of a diamagnetic substance lasts so long as the magnetising field is applied**.

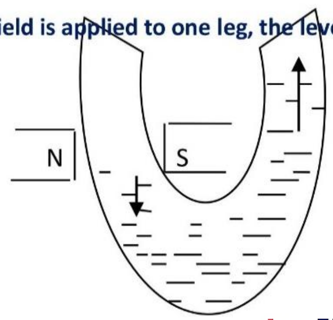
■ ***Diamagnetism in superconducting metals***: When a metal is cooled to a temperature below its critical temperature in a magnetic field, it attains both superconductivity and perfect diamagnetism. The magnetic lines of force get completely expelled from it and it repels a magnet. For this material, χ = – 1 and μ_r = 0. This phenomenon of diamagnetism in super conductors is called **Meissner effect**. This effect forms the basis for running magnetically levitated superfast trains.

○ **9. Diamagnetic substance can be compared to non – polar dielectric.**

○ **10. Flux density (B) inside a diamagnetic material is less tham in air.**

○ **11. Intensity of Magnetisation (I) has small Negative value.**

○ **12. When a sample of diamagnetic liquid is put in a U- shaped tube & magnetic field is applied to one leg, the level falls in that leg i.e., from stronger to weaker magnetic field.**

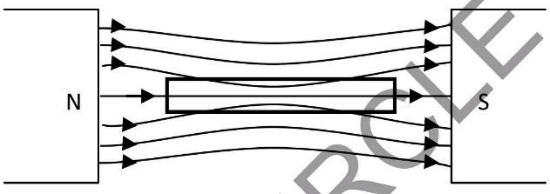


PROPERTIES OF PARAMAGNETIC SUBSTANCES:

1. When placed in an external magnetic field, a paramagnetic substance develops feeble magnetisation in the direction of the applied field.

2. When a rod of paramagnetic material is placed in a magnetic field, the lines of force prefer to pass through it than through the surrounding air i.e., the lines of force get slightly more concentrated inside the material, as shown in Fig.

The magnetic induction B becomes **slightly greater than the magnetising field, $B_0 = \mu_0 H$.** The increase is very small, about 1 part in 10^5.



[Concentration of lines of force in a paramagnetic rod]

Explanation : We know that U = - MB Cos θ

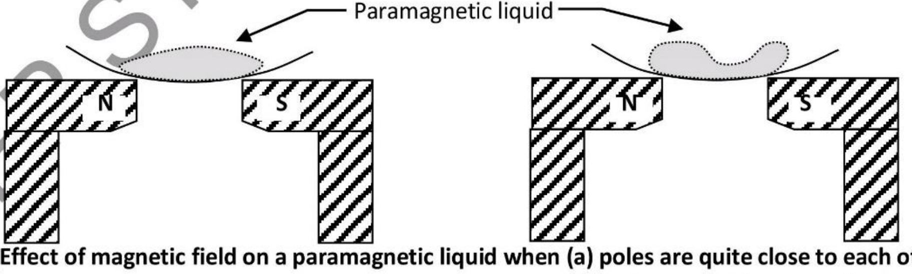
In case of paramagnetic Substance , θ = 0^0

∴ U = - M

Clearly, potential energy will be mini. is in a region of maximum B

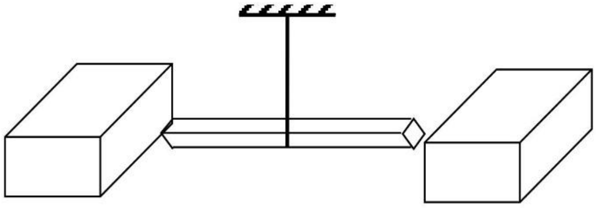
3. When placed in a non-uniform magnetic field, a paramagnetic substance moves from weaker to the stronger parts of the field.

When a watch glass containing a paramagnetic liquid is placed over two closely lying pole pieces of a magnet, the liquid accumulates and elevates in the middle and thins out near the poles [Fig. (a)]. This is because the field in the centre is the strongest. When the poles are moved apart, the field at the poles becomes stronger than that at the centre and the liquid moves towards the poles (b).



[Effect of magnetic field on a paramagnetic liquid when (a) poles are quite close to each other, (b) poles are farther apart.]

4. When a rod of paramagnetic material is suspended freely in a uniform magnetic field, it aligns itself parallel to the magnetic field.



[A free suspended paramagnetic rod in a uniform magnetic field]

Explanation : We know that U = - MB Cos θ

∴ Potential energy per unit volume – M/V B Cos θ = - IB Cos θ

In case of paramagnetic substance θ = 0^0

∴ Potential = - IB Cos θ = - IB

Clearly, this will be minimum volume is maximum this will be possible only if the length of the rod is in the direction of B.

This behavior indicates that –

i) Field within the sample is more than the magnetic intensity i.e. **μ is more than unity (B > H) or (B/H > 1 or μ> 1)**

ii) **Flux density (B) inside a paramagnetic material is larger than in air.**

iii) **Since μ> 1 , therefore susceptibility of a paramagnetic substance is positive (small)**

iv) **Intensity has a small positive value.**

v) **Paramagnetic substance are badly effected with the rise in temperature due to rise in temperature, they lose magnetic property.**

vi) **When a sample of paramagnetic liquid is put in a U – tube & magnetic field is applied to one limb i.e., from weaker to stronger magnetic field.**

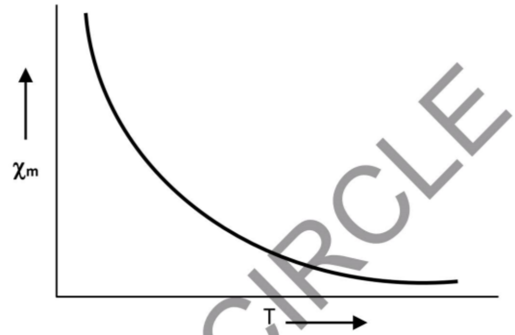
5. A paramagnetic material develops small magnetisation in the direction of the magnetising field, so its susceptibility has small but positive value. For aluminium, $\chi = 1.8 \times 10^{-6}$.

6. The relative permeability ($\mu_r = 1 + \chi_m$) for a paramagnetic material has a value slightly greater than.

7. The magnetic susceptibility of a paramagnetic material varies inversely as the absolute temperature, i.e.,

$$\chi_m \, \chi \, \frac{1}{T} \quad \text{or} \quad \chi_m = \frac{C}{T}$$

Where C is a constant called the Curie constant and this equation is known as Curie's law.



[χ_m – T graph for a paramagnetic material]

8. For a given temperature, the intensity of magnetisation is proportional to the magnetising field, so the susceptibility and permeability do not show any variation with the field $\vec{B}_0$.

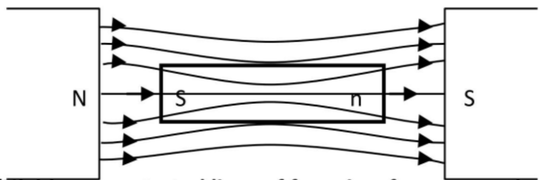
9. As soon as the magnetising field is removed, a paramagnetic substance loses its magnetism.

PROPERTIES OF FERROMAGNETIC SUBSTANCES

Ferromagnetic substance exhibit properties similar to those of paramagnetic substances but in the highly **dominant manner**.

1. When placed in an external magnetic field, a ferromagnetic material develops strong magnetisation in the direction of the applied field.

2. When a ferromagnetic substance is placed in a magnetic field, the lines of force concentrate greatly into the material so that the magnetic induction B becomes much more than the magnetising field B_0.



[Highly concentrated lines of force in a ferromagnetic rod]

3. When a ferromagnetic substance is placed in non-uniform magnetic field, it moves from weaker to the stronger parts of the field.

4. When a rod of a ferromagnetic material is suspended freely in f uniform magnetic field, it quickly aligns itself parallel to the magnetic field.

5. The intensity of magnetisation M is proportional to the magnetising field intensity H for its smaller values. For moderate values of H, M increases rapidly and then finally attains constant value for large H. This indicates the attainment of the saturation stage of magnetisation.

6. The susceptibility of a ferromagnetic material has a large positive value. This is because

$$\chi_m = \frac{M}{H}$$

and M > > H for a ferromagnetic material. It is of the order of several thousands.

7. The relative permeability ($\mu_r = 1 + \chi_m$) of a ferromagnetic material has a large positive value. It is of the order of several thousands. For iron, $\mu_r = 1000$.

8. The susceptibility of ferromagnetic material decreases with temperature in accordance with Curie-Weiss law:

$$\chi_m = \frac{C'}{T - T_C} \qquad (T > T_C)$$

9. At a certain temperature called the Curie point, the susceptibility suddenly falls and the ferromagnetic substance becomes paramagnetic.

10. The magnetisation developed depends not only on the value of magnetising field but also on the past magnetic and mechanical history of the material.

11. A ferromagnetic substance retains magnetism even after the magnetising field is removed.

KNOWLEDGE plus.................

- In the presence of an external magnetic field, magnetic moments are induced in all materials. Hence diamagnetism is universal. But paramagnetism and ferromagnetism are much stronger than diamagnetism, so it is difficult to detect diamagnetism in para- and ferro-magnetic substances.
- Magnetic materials are broadly classified as diamagnetic, paramagnetic and ferromagnetic. However, there exist some other types of magnetic materials with mysterious properties. These include ferromagnetic, anti-ferromagnetic, anti-ferromagnetic, spin glass, etc.]
- A very small variation in the value of χ_m may lead to an altogether different magnetic behaviour: diamagnetic versus paramagnetic. For diamagnetic materials, $\chi_m \simeq -10^{-5}$ where $\chi_m = +10^{-5}$ for paramagnetic materials.

Comparatively study of the properties of dia-, para- and ferromagnetic substances.

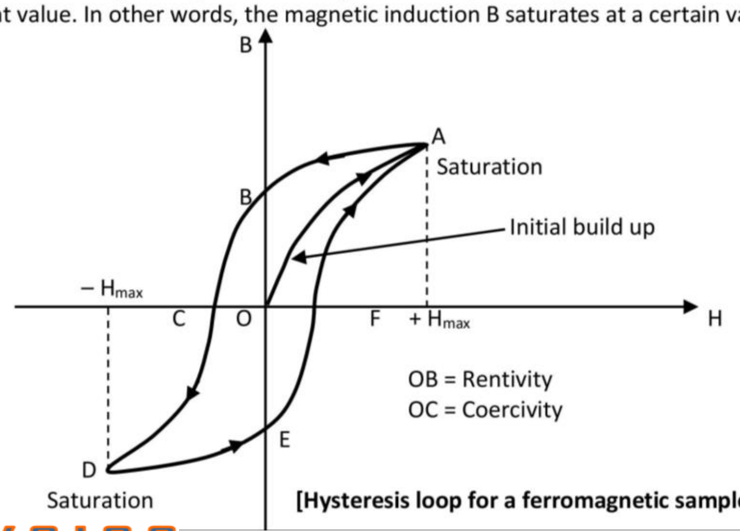
Property	Diamagnetic substance	Paramagnetic substance	Ferromagnetic substance
1. Effect of magnets	They are feebly repelled by magnets.	The are feebly attracted by magnets.	They are strongly attracted by magnets.
2. In external magnetic field	Acquire feeble magnetisation in the opposite direction of the magnetising field.	Acquire feeble magnetisation in the direction of the magnetising field.	Acquire strong magnetisation in the direction of the magnetising field.
3. In a non-uniform magnetic field	Tend to move slowly from stronger to weaker parts of the field.	Tend to move slowly from weaker to stronger parts of the field.	Tend to move quickly from weaker to stronger parts of the field.
4. In a uniform magnetic field	A freely suspended diamagnetic rod aligns itself perpendicular to the field.	A freely suspended paramagnetic rod aligns itself parallel to the field	A freely suspended ferromagnetic rod aligns itself parallel to the field.
5. Susceptibility value (χ_m)	Susceptibility is small and negative. $-1 \le \chi_m < 0$	Susceptibility is small and positive. $0 < \chi_m < \varepsilon$, where ε is small number	Susceptibility is very large and positive. $\chi_m > 1000$
6. Relative permeability value (μ_r)	Slightly less than 1 $0 \le \mu_r < 1$	Slightly greater than 1 $1 < \mu_r < 1 + \varepsilon$	Of the order of thousands $\mu_r > 1000$
7. Permeability value (μ)	$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$
8. Effect of temperature	Susceptibility is independent of temperature	Susceptibility varies inversely as temperature: $\chi_m \propto 1/T$	Susceptibility decreases with temperature in a complex manner. $\chi_m \propto \frac{1}{T - T_C}$ $(T > T_C)$
9. Removal of magnetising field	Magnetisation lasts as long as the magnetising field is applied.	As soon as the magnetising field is removed, magnetisation is lost.	Magnetisation is retained even after the magnetising field is removed.
10. Variation of M with H	M changes linearly with H.	M changes linearly with H and attains saturation at low temperature and in very strong fields.	M changes with H non-linearly and ultimately attains saturation.
11. Hysteresis effect	B-vector shows no hysteresis.	B-vector shows no hysteresis.	B-vector shows hysteresis.
12. Physical state of the material	Solid, liquid or gas	Solid, liquid or gas.	Normally solids only.
13. Examples	Bi, Cu, Pb, Si, N_2 (at STP), H_2O, NaCl.	Al, Na, Ca, O_2 (at STP), $CuCl_2$.	Fe, Ni, Co, Gd, Fe_2O_3, Alnico.

HYSTERESIS

Hysteresis: When a ferromagnetic sample is placed in a magnetising field, ***the sample gets magnetised by induction***.

- As the **magnetising field intensity H varies, the magnetic induction B does not vary linearly with H,** i.e., the permeability $\mu (= B/H)$ is not constant but varies with H. In fact, it also depends on the past history of the sample.

The variation of magnetic induction B with magnetising field intensity H. Point O represents the initial unmagnetised state of a ferromagnetic sample. As the magnetising field intensity H increases, the magnetic induction B first gradually increases and then attains a constant value. In other words, the magnetic induction B saturates at a certain value $+H_{max}$.



[Hysteresis loop for a ferromagnetic sample]

- **If the magnetic field intensity H is gradually decreases to zero,** ***B decreases but along a new path AB***. 35

It is found that the magnetic induction B does not become zero even when the magnetising field H is zero, i.e., the sample is not demagnetised even when the magnetising field has been removed. The magnetic induction (= OB) left behind in the sample after the magnetising field has been removed is called **residual magnetism** or **retentivity or remanence.**

To reduce the magnetism to zero, the field H is gradually increased in the reverse direction, the induction B decreases and becomes zero at a value of H = OC. The value of reverse magnetising field intensity H required for the residual magnetism of a sample to become zero **called Coercivity of the sample**.

On further increasing H in the reverse direction to a value $-H_{max}$, we reach the saturation point D located symmetrically to point A. Now if H is decreased gradually, the point A is reached after going through the path DEFA.

- The **closed curve ABCDEFA which represents a cycle of magnetisation of a ferromagnetic sample is called its hysteresis loop**. Throughout the cycle, the magnetic field B lags behind the magnetising field intensity H, i.e., the value of B when H is decreasing is always more than when H is increasing.
- **The phenomenon of the lagging of magnetic induction behind the magnetising field is called hysteresis**[meaning 'delayed'].

- **REMANENCE (OR RETENTIVITY OR RESIDUAL MAGNETISM)** :- ***"The value of intensity of magnetization of a material when the magnetizing filed is reduced to zero, is called retentivity (or Remanence or Residual magnetism)"***

=> After the specimen has been magnetized to saturation ,a reversed magnetizing field is required to reduce the retentivity to zero thus.

- **COERCIVITY:-** (H_C) ***" The value of magnetizing field required to reduce residual magnetism (or retentivity) to zero is called corecivity or coexive force".***

Conclusion:- When a specimen of a magnetic material is taken through a cycle of magnetization, the intensity of magnetization (I) and magnetic induction (B) lags behind the magnetizing field (H) therefore, even if H is made zero, the value of I and B do not reduce to zero.

=>For a given value of H :- the value of B & I is not unique but depends upon the previous history of the specimen.

Now, $BH = \frac{B(B)}{\mu} = \frac{B^2}{\mu_0\mu_0}$ has the dimension of energy per unit volume therefore area within the BH loop represents energy dissipated per unit volume.

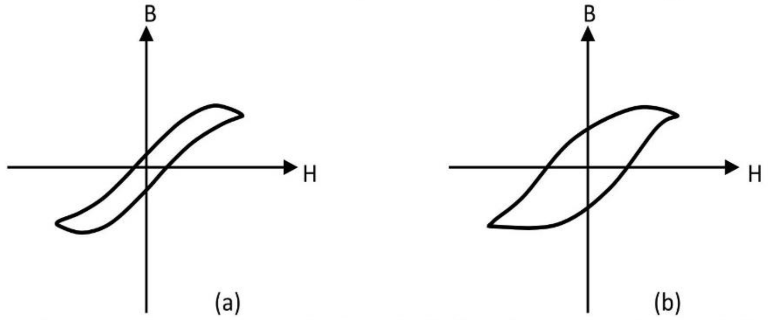
- **Significance of the area of hysteresis loop**: The product $BH = B\left(\frac{B}{\mu}\right) = \frac{B^2}{\mu_0 \mu_r}$, has the dimensions of energy per unit volume. Hence the area within the B – H loops represents the energy dissipated per unit cycle of magnetisation. The source is the source of emf used in magnetising the material and the sink is the hysteretic heat loss in the magnetic material.

- **Practical importance of hysteresis loops**: A study of hysteresis loop provides us information about retentivity, Coercivity and hysteresis loss of a magnetic material. ***This helps in proper selection of materials for designing cores of transformers and electromagnets and in making permanent magnets***.

- **Difference between soft and hard ferromagnetic materials.**

Types of ferromagnetic materials: Ferromagnetic materials can be divided into two categories:

1. Soft ferromagnetic materials or soft ferromagnets: These are the ferromagnetic materials in which the magnetisation disappears on the removal of the external magnetising field. Such materials have narrow hysteresis loop, as shown in Fig. (a). Consequently, they have low retentivity, low Coercivity, and low hysteresis loss. But they have high relative magnetic permeability. They are used as cores of solenoid and transformers. Examples: Soft iron, mu metal, stalloy, etc.



[Magnetic hysteresis loop for (a) soft, (b) hard ferromagnetic material]

2. Hard ferromagnetic materials or hard ferromagnets: These are the ferromagnetic materials which retain magnetisation even after removal of the external magnetising field. Such materials have wide hysteresis loop, as shown in Fig. (b). Consequently, they have high retentivity, high Coercivity and large hysteresis loss. They are used for making permanent magnets.

Example: Steel, alnico, lodestone, ticonal, etc.

Examples based on Magnetic Properties of Materials:

Q. 1. ***A magnet of magnetic moment 2.5 Am² weighs 66 g. If the density of the material of the magnet is 7500 kg m⁻³, find the intensity of magnetisation.***

Sol. Volume,

$$V = \frac{\text{Mass}}{\text{Density}} = \frac{66 \times 10^{-3}\ \text{kg}}{7500\ \text{kg m}^{-3}} = \frac{66 \times 10^{-5}}{75}\ \text{m}^3$$

Magnetisation,

$$M = \frac{m}{V} = \frac{2.5}{\frac{66 \times 10^{-5}}{75}} = \frac{2.5 \times 75 \times 10^5}{66} = 2.84 \times 10^5\ \text{Am}^{-1}$$

Q. 2. ***Obtain the earth's magnetisation. Assume that the earth's field can be approximated by a giant bar magnet of magnetic moment 8.0 × 10²² Am². The earth's radius is 6400 km.***

Sol. Her magnetic moment, $m = 8.0 \times 10^{22}$ Am²

Radius of the earth, $R = 6400$ km $= 6.4 \times 10^6$ m

Magnetisation,

$$M = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^3} = \frac{8.0 \times 10^{22} \times 3}{4 \times 3.14 \times (6.4 \times 10^6)^3} = 72.9\ \text{Am}^{-1}$$

Q. 3. ***A domain in ferromagnetic iron is in the form of a cube of side length 1 μm. Estimate the number of iron atoms in the domain and the maximum possible dipole moment and magnetisation of the domain. The molecular mass of iron is 55 g/mole and its density is 7.9 g/cm³. Assume that each iron atom has a dipole moment of 9.27 × 10⁻²⁴ Am².***

Sol. Each side of cubic domain, $l = 1\ \mu\text{m} = 10^{-6}$ m

Volume of the domain,

$$V = l^3 = (10^{-6}\ \text{m})^3 = 10^{-18}\ \text{m}^3 = 10^{-12}\ \text{cm}^3$$

Mass of domain = Volume × density

$$= 10^{-12}\ \text{cm}^3 \times 7.9\ \text{g cm}^{-3}$$

$$= 7.9 \times 10^{-12}\ \text{g}$$

Number of atoms in 55 g iron

$$= 1\ \text{mole} = 6.023 \times 10^{23}$$

∴ Number of atoms in 7.9×10^{-12} g iron

$$= \frac{6.023 \times 10^{23} \times 7.9 \times 10^{-12}}{55}$$

or $N = 8.65 \times 10^{10}$ atoms.

Dipole moment of each iron atom,

$$m = 9.27 \times 10^{-24}\ \text{Am}^2$$

The dipole moment of the domain will be maximum when all its atomic dipoles get perfectly aligned. Its value will be

$$m_{max} = mN = 9.27 \times 10^{-24}\ \text{Am}^2 \times 8.65 \times 10^{10}$$

$$= 8.0 \times 10^{-13}\ \text{Am}^2$$

The maximum possible magnetisation of the domain,

$$M = \frac{m_{max}}{V} = \frac{8.0 \times 10^{-13}\ \text{Am}^2}{10^{-18}\ \text{m}^3}$$

$$= 8.0 \times 10^5\ \text{Am}^{-1}.$$

Q. 4. ***A magnetising field of 1500 A/m produces a magnetic flux of 2.4 × 10⁻⁵ weber in a bar of iron of cross-section 0.5 cm². Calculate permeability and susceptibility of the iron-bar used.***

Sol. Here $H = 1500\ \text{Am}^{-1}$,

$\phi = 2.4 \times 10^{-5}$ Wb,

$A = 0.5 \times 10^{-4}$ m²

Magnetic induction,

$$B = \frac{\phi}{A} = \frac{2.4 \times 10^{-5}}{0.5 \times 10^{-4}} = 0.48\ \text{Wb m}^{-2}$$

Permeability, $\mu = B = 0.48 = 3.2 \times 10^{-4}\ \text{TmA}^{-1}$

As $\mu = \mu_0 (1 + \chi_m)$

∴ Susceptibility, $\chi_m = \frac{\mu}{\mu_0} - 1 = \frac{3.2 \times 10^{-4}}{4 \times 3.14 \times 10^{-7}}$

$$= 254.77 - 1 = 253.77$$

Q. 5. ***Assume that each iron atom has a permanent magnetic moment equal to 2 Bohr magnetons (1 Bohr magneton = 9.27 × 10⁻²⁴ Am²). the number density of atoms in iron is 8.52 × 10²⁸ m⁻³. (i) Find the maximum magnetisation M in a long iron bar. (ii) Find the maximum magnetic induction B in the bar.***

Sol. (i) Number of atoms per unit volume, $n = 8.52 \times 10^{28}\ m^{-3}$

Magnetic moment of each iron atom

$= 2\ \mu_B = 2 \times 9.27 \times 10^{-24}\ Am^2$

As magnetisation M is the magnetic moment per unit volume, so the maximum value of magnetisation is

$M_{max} = n \times 2\mu_B$ (when all the dipoles get aligned)

$= 8.52 \times 10^{28} \times 2 \times 9.27 \times 10^{-24} = 1.58 \times 10^6\ Am^{-1}$

(ii) Magnetic induction, $B = \mu_0 (H + M)$

As no magnetising field is applied, so H = 0. Hence

$B = \mu_0 M = 4\pi \times 10^{-7} \times 1.58 \times 10^6 = 1.985$ T

Q. 6. ***A solenoid of 500 turns/m is carrying a current of 3 A. Its core is made of iron which has a relative permeability of 5000. Determine the magnitudes of the magnetic intensity, magnetisation and the magnetic field inside the core.***

Sol. Here n = 500 turns/m, I = 3A, $\mu_r = 5000$

Magnetic intensity,

$H = nI = 500\ m^{-1} \times 3\ A = 1500\ Am^{-1}$.

As $\mu_r = 1 + \chi_m$

∴ $\chi_m = \mu_r - 1 = 5000 - 1 = 4999 \simeq 5000$

Also, $\mu_r = \dfrac{\mu}{\mu_0} = 5000$ or $\mu = 5000\ \mu_0$

Magnetisation,

$M = \chi_m H = 5000 \times 1500 = 7.5 \times 10^6\ Am^{-1}$

Magnetic field inside the core,

$B = \mu H = 5000\ \mu_0 H = 5000 \times 4\pi \times 10^{-7} \times 1500 = 3\pi = 9.4$ T.

Q. 7. ***An iron rod of volume $10^{-4}\ m^3$ and relative permeability 1000 is placed inside a long solenoid wound with 5 turns per cm. If a current of 0.5 A is passed through the solenoid, find the magnetic moment of the rod.***

Sol. The relation between the magnetic induction B, magnetising field intensity H and the magnetisation M is given by

$B = \mu_0 (H + M)$

∴ $M = \dfrac{B}{\mu_0} - H = \dfrac{\mu H}{\mu_0} - H$ [∵ $B = \mu H$]

$= \mu_r H - H = (\mu_r - 1) H$ $\left(\mu_r = \dfrac{\mu}{\mu_0}\right)$

But for a long solenoid, we have

H = nI

where n is the number of turns per metre.

∴ $M = (\mu_r - 1)\ nI$

Here $\mu_r = 1000$, I = 0.5 A

$n = \dfrac{5}{0.01}$ turns/m = 500 turns/m

∴ $M = (1000 - 1) \times 500 \times 0.5 = 2.5 \times 10^5\ Am^{-1}$

Magnetic moment, $m = M \times V = 2.5 \times 10^5 \times 10^{-4}\ Am^2 = 25\ Am^2$.

Q. 8. ***The hysteresis loss for a specimen of iron weighing 12 kg is equivalent to 300 Jm^{-3} cycle^{-1}. Find the loss of energy per hour at 50 cycle s^{-1}. Find the loss of energy per hour at 50 cycle s^{-1}. Density of iron is 7500 kg m^{-3}.***

Sol. The relation between the magnetic induction B, magnetising field intensity H and the magnetisation M is given by

Let Q be the energy dissipated per unit volume per hysteresis cycle in the given sample. Then the total energy lost by the volume V of the sample in time t will be

$W = Q \times V \times v \times t$

where v is the number of hysteresis cycles per second.

Here $Q = 300\ Jm^{-3}$ cycle^{-1}, $v = 50$ cycle s^{-1}, t = 1 h = 3600 s

Volume, $V = \dfrac{\text{Mass}}{\text{Density}} = \dfrac{12}{7500}\ m^3$

∴ Hysteresis loss,

$W = 300 \times \dfrac{12}{7500} \times 50 \times 3600$ J = 86400 J

Q. 9. ***The Coercivity of a certain permanent magnet is $4.0 \times 10^4\ Am^{-1}$. This magnet is placed inside a solenoid 15 cm long and having 600 turns and a current is passed in the solenoid to demangnetise it completely. Find the current.***

Sol. The coercivity of $4 \times 10^4\ Am^{-1}$ of the permanent magnet implies that a magnetic intensity $H = 4 \times 10^4\ Am^{-1}$ is required to be applied in opposite direction to demagnetise the magnet.

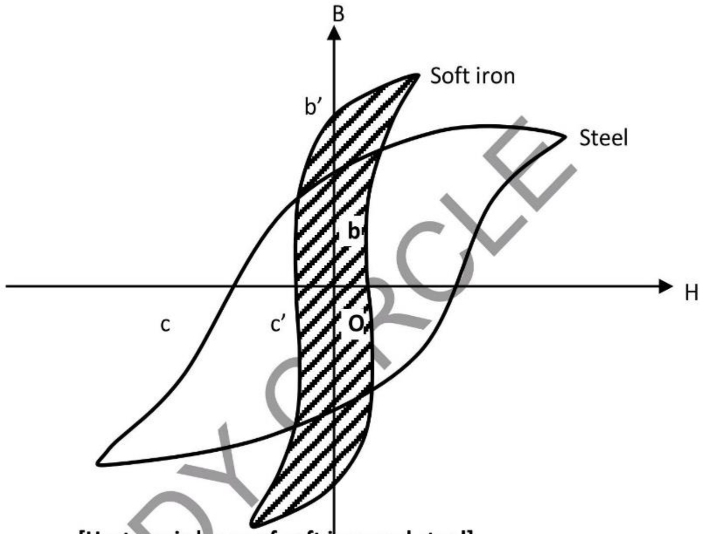
Here $n = \dfrac{600}{15\ cm} = \dfrac{600}{15 \times 10^{-2}\ m} = 4000$ turns/m

As H = nI

∴ Current, $I = \dfrac{H}{n} = \dfrac{4 \times 10^4}{4000} = 10$ A.

PERMANENT MAGNETS AND ELECTROMAGNETS

Comparison of the magnet properties of soft iron and steel: Fig. shows the hysteresis loops for soft iron and steel.



[Hysteresis loops of soft iron and steel]

A study of these B – H loops reveals the following information:

1. Permeability: For a given H, B is more for soft iron than steel. So soft iron has a greater permeability ($\mu = B/H$) than steel.

2. Susceptibility: As permeability of soft iron is greater than steel, so soft iron is greater than steel, so soft iron has a greater susceptibility ($\chi m = \mu_r - 1$) than steel.

3. Retentivity: the retentivity of soft iron (Ob') is greater than the retentivity (Ob) of steel.

4. Coercivity: The coercivity of soft iron (Oc') is less than the coercivity (Oc) of steel.

5. Hysteresis loss: As the area of the hysteresis loop of soft iron is much smaller than that of steel, so the hysteresis loss per unit volume per cycle is less for soft iron than or steel.

1. Permeability
2. Susceptibility
3. Retentivity

] **are greater for soft iron than for steel**

4. Coercivity
5. Hysteresis loss

] **are less for soft iron than for steel**

SELECTION OF MAGNETIC MATERIALS: The choice of magnetic materials for making permanent magnets, electromagnets and cores of transformers is decided from the hysteresis loop of the material.

- **A. Permanent magnets:** The materials used for making permanent magnets must have the following characteristics:

1. High retentivity so that it produces a strong magnetic field.
2. High coercivity so that its magnetisation is not destroyed by stray magnetic fields, temperature variations or minor mechanical damage.
3. High permeability.

Inspite of its slightly smaller retentivity than soft iron, steel is favoured for making permanent magnets. Steel has much higher coercivity than soft iron. the magnetisation of steel is not easily destroyed by stray fields. Once magnetised under a strong field, it retains magnetisation for a long duration. Other suitable materials for making permanent magnets are

Cobalt steel (52 % Fe, 36 % Co, 7 % W, 3.5 % Cr, 0.5 % Mn, 0.7 % C)
Carbon steel (98 % Fe, 0.86 % C, 0.9 % Mn)
Alnico (55 % Fe, 10 % Al, 17 % Ni, 12 % Co, 6 % Cu)
Ticonal (42 % Co, 26.5 Fe, 14 % Ni, 8 % Al, 6.5 Ti, 3 % Cu)

- **B. Electromagnets:** The material used for making cores of electromagnets must have the following characteristics:

1. High initial permeability so that magnetisation is large even for a small magnetising field.
2. Low retentivity so that the magnetisation is lost as the magnetising current is switched off.

So soft iron is more suitable than steel for cores of electromagnets.

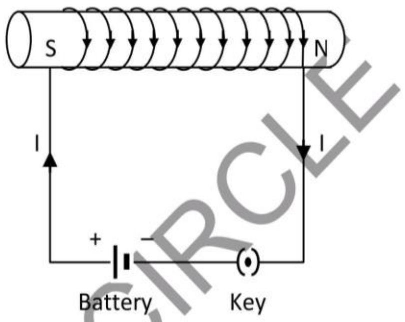
- **C. Transformer cores**: The material used for making cores of transformers must have the following characteristics:

1. High initial permeability so that the magnetic flux is large even for low magnetising fields.
2. Low hysteresis loss as the materials are subjected to alternating magnetising fields of high frequency.
3. Low resistivity to reduce losses due to eddy currents.

Soft iron is preferred for making transformer cores and telephone diaphragms.

Methods for making permanent magnets: A hard ferromagnetic material like steel can be converted into a permanent magnet by
1. By holding the steel rod in north-south direction and hammering it repeatedly.
2. Hold a steel rod and stroke it with one end of a bar magnet a number of times, always in the same sense to make a permanent magnet.
3. The most efficient way of making a permanent magnet is to place a steel rod in a solenoid and pass a strong current. The rod gets magnetised due to the magnetic field of the solenoid.

Electromagnet: As shown in Fig., take a soft iron rod and wind a large number of turns of insulated copper wire over it.
When we pass a current through the solenoid, a magnetic field is set up in the space within the solenoid. The high permeability of soft iron increases the field one thousand times. The end of the solenoid at which the current in the solenoid seems to flow anticlockwise acts as N-pole and other one as S-pole. When the current in the solenoid is switched off, the soft iron rod loses its magnetism almost completely due to its low retentivity.



[An electromagnet]

¶☑ **Uses of electromagnets:**

1. Electromagnets are used in electric bells, loudspeakers and telephone diaphragms.
2. Large electromagnets are used in cranes to lift heavy machinery, and bulk quantities of iron and steel.
3. In hospitals, electromagnets are used to remove iron or steel bullets from the human body.

OSCILLATIONS OF A FREELY SUSPENDED MAGNET

Oscillations of a freely suspended magnet in a magnetic field: Suspended magnet in a magnetic field: Suspended a small bar magnet of magnetic moment $\vec{m}$ in a uniform magnetic field $\vec{B}$ so that it is free to vibrate in a horizontal plane about a vertical axis through its centre of mass. In a position of equilibrium, it lies along $\vec{B}$. When it is slightly rotated from this position and released, it begins to vibrate about the field direction. If at any instant the magnet makes an angle θ with $\vec{B}$, then restoring torque on the market will be

$$\tau = -mB\sin\theta$$

The negative sign indicates that the direction of the torque τ is such so as to decrease θ. For small angular displacement θ,

$$\sin\theta \simeq \theta$$

$$\therefore \quad \tau = -mB\theta$$

Now the deflecting torque on the magnet is

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$$

where I is the moment of inertia of the magnet about the axis of rotation and $d^2\theta$ is the angular acceleration. In the equilibrium condition,

Deflecting torque = Restoring torque

i.e., $$I\frac{d^2\theta}{dt^2} = -mB\theta$$

or $$\frac{d^2\theta}{dt^2} = -\frac{mB}{I}\theta = -\omega^2\theta$$

i.e., angular acceleration $d^2\theta \propto$ angular displacement θ.

Hence the oscillations of a freely suspended magnet in a uniform magnetic field are simple harmonic. The period of vibration is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mB}}$$

By knowing T, I and B; the magnetic moment m of the magnet can be determined.