AEP Digital Online Classes

AEP Study Circle

FOUNDATION - IX

CHAPTER:4

MATHS: LINES AND ANGLES

LIVE INTERACTIVE SESSIONS FROM :
RANCHI , BOKARO , RAMGARH , MUMBAI , KANPUR, HAZARIBAGH, DELHI NCR

LIVE INTERACTIVE SESSIONS AVAILABLE AT:
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CBSE

THEORY + PROBLEMS

A E P S T U D Y CIRCLE

1. BASIC TERMS AND DEFINITIONS

(*i*) Line-segment, ray and line

A part (or portion) of a line with two end points is called a line-segment and a part of a line with one end point is called a ray. The line segment AB is denoted by AB and its length is denoted by AB. Ray AB is denoted by AB and a line AB is denoted by AB.

However, we do not use these symbols and denoted line segment AB, ray AB, length AB and line AB by the symbol AB. The meaning is made clear from the context. Sometimes small letters l, m, m etc. will be used to denote lines.

(*ii*) Collinear/non-collinear points

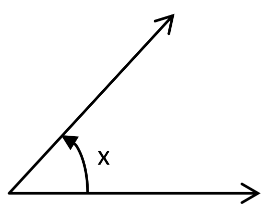
If three or more points lie on the same line, then they are called collinear points; otherwise they are called as non-collinear points.

(*iii*) Angle

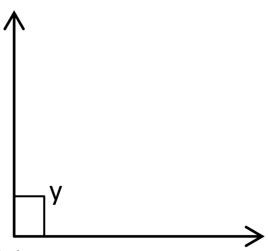
An angle is formed when two rays originate from the same end point. The rays making an angle are called the arms of the angle and the end point is called the vertex of the angle. The angles are of different types such as acute angle, right angle, obtuse angle, straight angle and reflex angle.

(*iv*) Types of angles

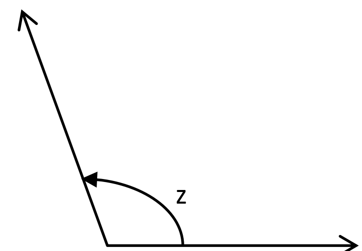
An acute angle measures between 0° and 90° whereas a right ***angle*** is exactly equal to 90°. An angle greater than 90° but less than 180° is called an ***obtuse angle***. Also, a ***straight angle*** is equal to 180°. An angle which is greater than 180° but less than 360° is called a ***reflex angle***. Further, two angles whose sum is 90° are called ***complementary angles*** and those two angles whose sum is 180° are called ***supplementary angles***.



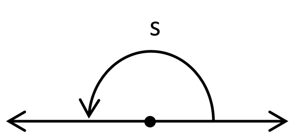
(i) Acute angle: 0° < x < 90°



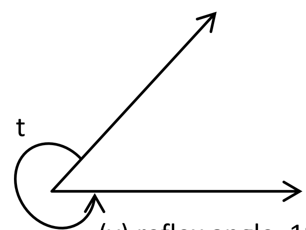
(ii) right angle : y = 90°



(iii) obtuse angle : 90° < z < 180°



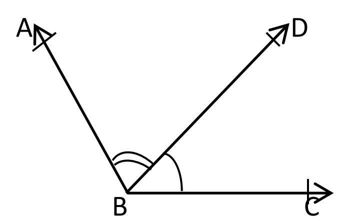
(iv) Straight angle : x = 180°



(v) reflex angle: 180° < t < 360°

(*v*) *Adjacent angles*

Two angles are called adjacent, if they have a common vertex, a common arm and their non-common arms are on the different sides of the common arm. In the adjacent figure ∠ ABD and ∠ DBC are adjacent angles. Ray BD is their common arm and point B is their common vertex. Ray BA and ray BC are non-common arms. Moreover, when two angles are adjacent, then their sum is always equal to the angle formed by the two common arms. So, we can write

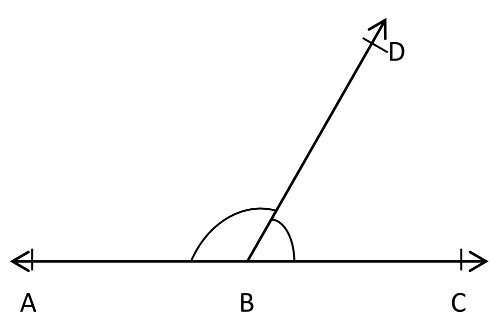


$\angle ABC = \angle ABD + \angle DBC$

◙: ∠ABC and ∠ABD are not adjacent angles because their non-common arms BD and BC lie on the same side of the common arm BA.

Linear pair of Angles

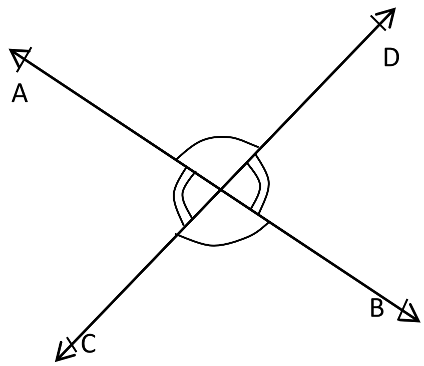
If the non-common arms BA and BC form a line, then in this case ∠ABD and ∠DBC are called linear pair of angles.



Linear pair of angles

(*vi*) *Vertically opposite angles*

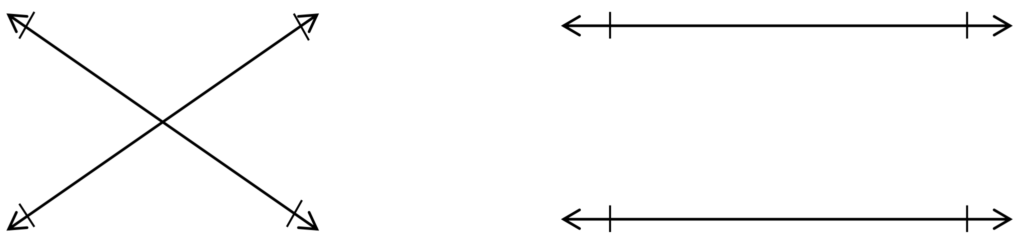
Vertically opposite angles are formed when two lines, say AB and CD, intersect each other say at the point O, There are two pairs of vertically opposite angles formed. One pair is ∠AOD and ∠BOC. The other pair is ∠AOC and ∠BOD.



Vertically opposite angles

2. INTERSECTING LINES AND NON – INTERSECTING ANGLES

We know that a line extends indefinitely on both the directions. We can draw two different lines PQ and RS on a paper in two different ways as shown ahead:



(i) Intersecting lines (ii) Non-intersecting (parallel) lines

Different ways of drawing two lines

In figure (i), lines PQ and RS are intersecting lines and in figure (ii), lines PQ and RS are parallel lines.

Note: The lengths of the common perpendiculars at different point on parallel lines are the same. This equal length is called the distance between two parallel lines.

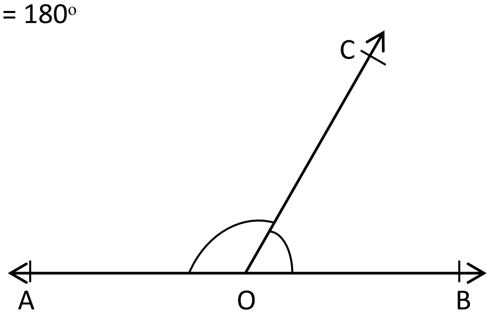
3. LINEAR PAIR OF ANGLES

Let a ray OC stand on a line AB. Then, angles formed at the point O are ∠AOC, ∠COB and ∠AOB.

We may write that ∠AOC + ∠BOC = ∠AOB [∵ ∠AOC and ∠BOC are adjacent angles

But ∠AOB = 180° [∵ A straight angle = 180°

∴ ∠AOC + ∠BOC = 180°



Linear pair of angles

This result leads us to an axiom given below: 3

Axiom 1: If a ray stands on a line, then the sum of two adjacent angles so formed is 180°.

When the sum of two adjacent angles is 180° , then they are called a linear pair of angles.

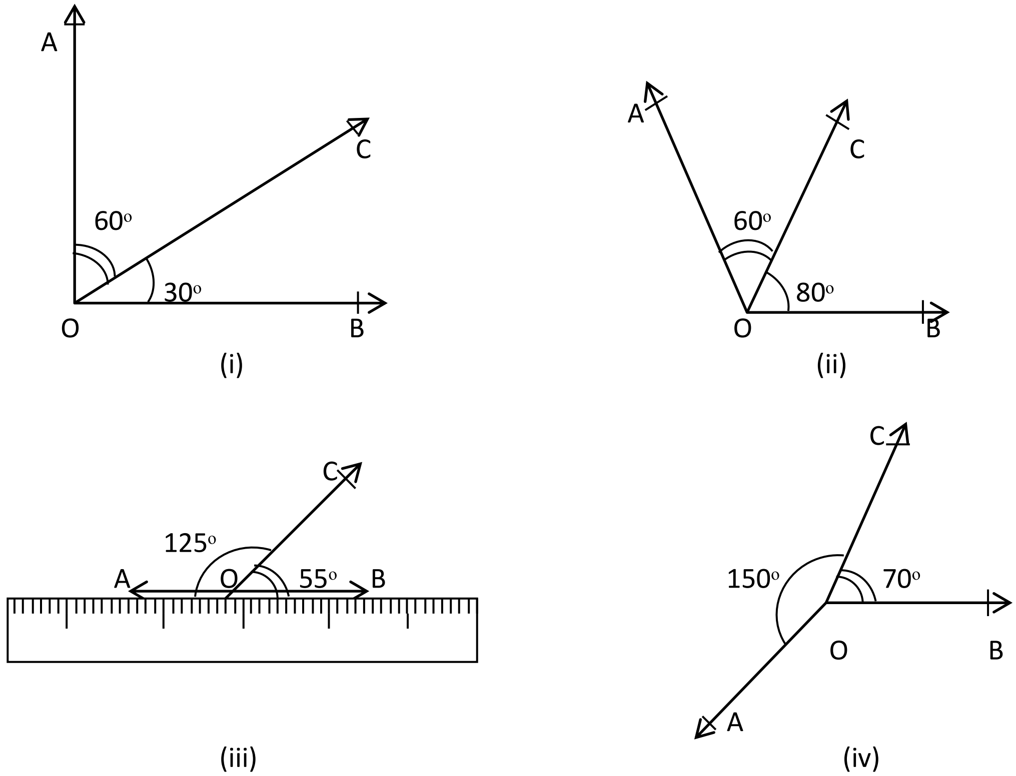
Conversely

Axiom 2: If the sum of two adjacent angles is 180°, then the non-common arms of the angle form a line.

The two axioms above together are called the **Linear Pair Axiom**.

Check: Draw adjacent angles of different measures as shown below. Keep the ruler along one of the non-common arms in each case. We see that the other non-common arms also lies along the ruler only in figure (iii), i.e., both the non-common arms lie along the ruler, that is, points A, O and B lie on the same line and ray OC stands on it. Also, we see that ∠AOC + ∠COB = 125° + 55° = 180°.

From this, we conclude that the above statement is true.



Adjacent angles with different measures

4. THEOREM: If two lines intersect each other, then the vertically opposite angles are equal:

Given: Two lines AB and CD intersecting O

They lead to two pairs of vertically opposite angles, namely,

(i) ∠AOC and ∠BOD

(ii) ∠AOD and ∠BOC

To prove: (i) ∠AOC = ∠BOD

(ii) ∠AOD = ∠BOC

Proof: ∵ Ray OA stands on line CD

∴ ∠AOC + ∠AOD = 180° ... (i) [Linear Pair Axiom]

∵ Ray OD stands on line AB

∴ ∠AOD + ∠BOD = 180° ... (ii) [Linear Pair Axiom]

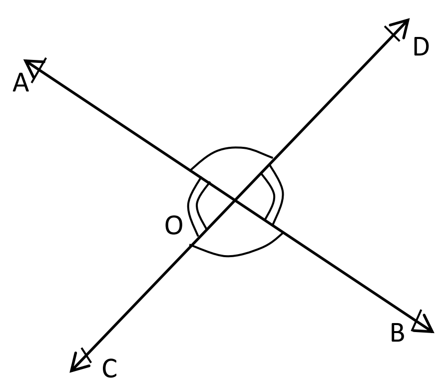
From (1) and (2), we can write that

∠AOC + ∠AOD = ∠AOD + ∠BOD

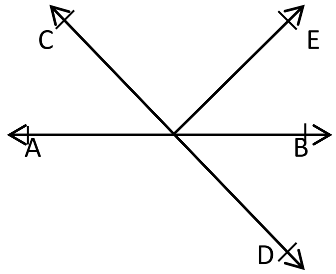
=> ∠AOC = ∠BOD

Similarly, we can prove that

∠AOD = ∠BOC



Q. 1. In figure, lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.



Sol. ∵ Lines AB and CD intersect at O

∴ ∠AOC = ∠BOD [Vertically Opposite Angles]

But ∠BOD = 40° ... (i) [Given]

∴ ∠AOC = 40° ... (ii)

Now, ∠AOC + ∠BOE = 70°

=> 40° + ∠BOE = 70°

=> ∠BOE = 70° – 40°

=> ∠BOE = 30°

Again, Reflex ∠COE = ∠COD + ∠BOD + ∠BOE

= ∠COD + 40° + 30° [Using (1) and (2)]

= 180° + 40° + 30°

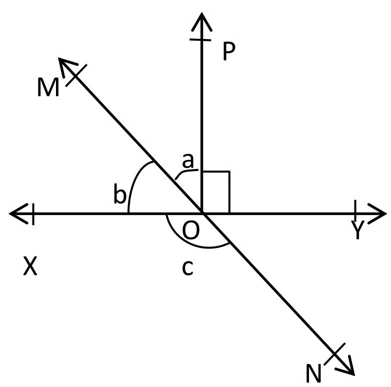
= 250°

∵ Ray OA stands on line CD

∴ ∠AOC + ∠AOD = 180° [Linear Pair Axiom]

=> ∠COD = 180°

Q. 2. In figure, lines XY and MN intersect at O. If ∠POY = 90° and a : b = 2 : 3, find c.



Sol. ∵ Ray OP stands on line XY [Linear Pair Axiom]

∴ ∠POX + ∠POY = 180° [∵ ∠POY = 90° (Given)]

=> ∠POX + 90° = 180°

=> ∠POX = 180° – 90°

=> ∠POX = 90°

=> ∠POM + ∠XOM = 90°

=> a + b = 90° ... (i)

a : b = 2 : 3

=> $\frac{a}{b} = \frac{2}{3}$

=> $\frac{a}{2} = \frac{b}{3}$ = k (say)

=> a = 2k

b = 3k

Putting the values of a and b in (1), we get

2k + 3k = 90°

=> 5k = 90° => $k = \frac{90°}{5}$

=> k = 18°

∴ a = 2k = 2 (18°) = 36°
b = 3k = 3 (18°) = 54° ... (2)

∵ Ray OX stands on line MN

∴ $\angle XOM + \angle XON = 180^\circ$ [Linear Pair Axiom]

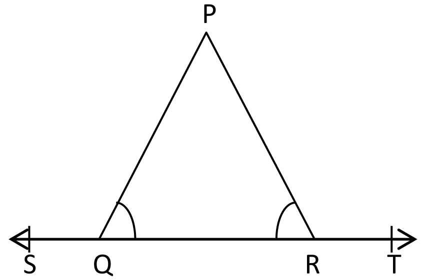
=> $b + c = 180^\circ$

=> $54^\circ + c = 180^\circ$ [Using (2)]

=> $c = 180^\circ - 54^\circ$

=> $C = 126^\circ$

Q. 3. In figure, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT



Sol. ∵ Ray QP stands on line ST

∴ $\angle PQS + \angle PQR = 180^\circ$ ... (1) [Linear Pair Axiom]

∵ Ray RP stands on line ST

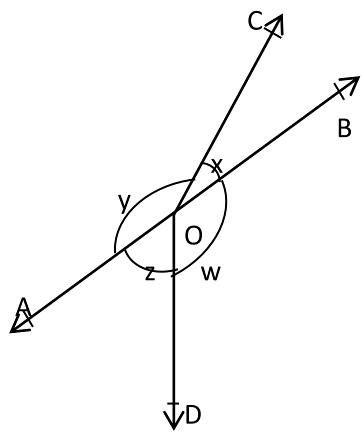
∴ $\angle PRQ + \angle PRT = 180^\circ$ ... (2) [Linear Pair Axiom]

From (1) and (2), we obtain

$\angle PQS + \angle PQR = \angle PRQ + \angle PRT$

=> $\angle PQS = \angle PRT$ [∵ ∠PQR = ∠PRQ [Given]

Q. 4. In figure, if x + y = w + z, then prove that AOB is a line.



Sol. $x + y = w + z$

∴ The sum of all the angles round a point is equal to 360°

∴ $x + y + w + z = 360^\circ$

=> $x + y + x + y = 360^\circ$

=> $2(x + y) = 360^\circ$

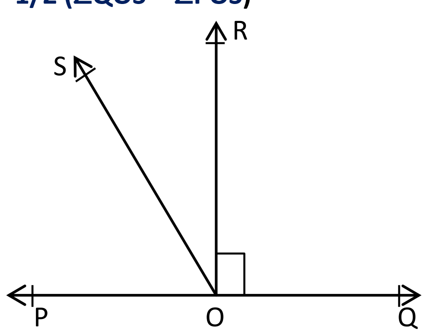
=> $x + y = \frac{360^\circ}{2}$

=> $x + y = 180^\circ$

∴ AOB is a line. [If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line]

Q. 5. In the figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray living between rays OP and OR. Prove that: ∠ROS = 1/2 (∠QOS – ∠POS)

Sol.



∵ Ray OR is perpendicular to line PQ

∴ $\angle QOR = \angle POR = 90^\circ$ ... (1)

$\angle POS = \angle QOR + \angle ROS$ ... (2)

$\angle POS = \angle POR - \angle ROS$ ... (3)

From (2) and (3),

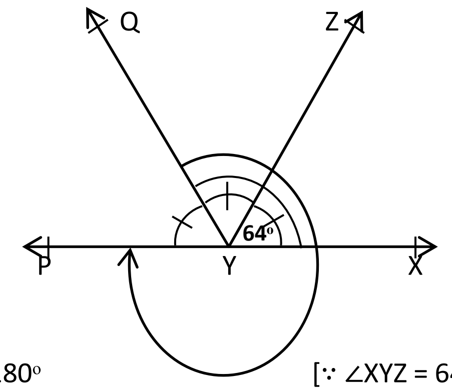
∴ $\angle QOS - \angle POS = (\angle QOR - \angle POR) + 2\angle ROS = 2\angle ROS$ [Using (1)]

=> $\angle ROS = ½ (\angle QOS - \angle POS)$

Q. 6. It is given that $\angle XYZ = 64^\circ$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Sol. ∵ Ray YZ stands on line PX

∴ $\angle XYZ + \angle ZYP = 180^\circ$ [Linear Pair Axiom]



=> $64^\circ + \angle ZYP = 180^\circ$ [∵ $\angle XYZ = 64^\circ$ (Given)]

=> $\angle ZYP = 180^\circ - 64^\circ$

=> $\angle ZYP = 116^\circ$ ... (1)

∵ Ray YQ bisects $\angle ZYP$

∴ $\angle PYQ = \angle ZYQ = ½ \angle ZYP = ½ (116^\circ)$ [Using (1)]

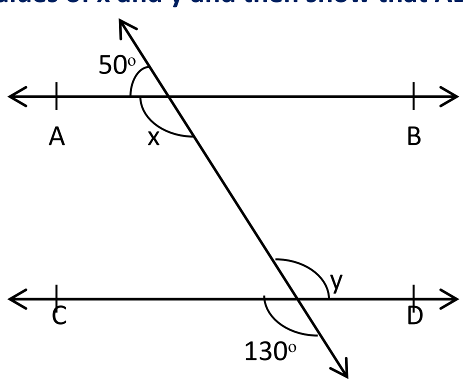
∴ Reflex $\angle QYP = 360^\circ - 58^\circ = 302^\circ$ [∵ The sum of all the angles round a point is equal to 360°]

Again, $\angle XYQ = \angle XYZ + \angle ZYO$

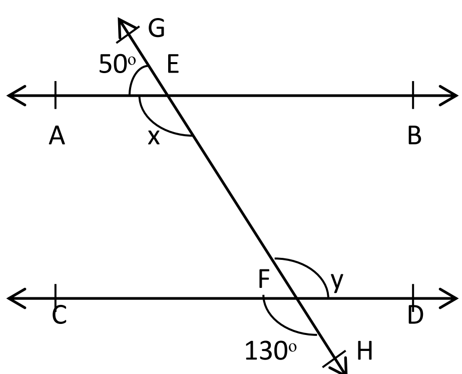
$= 64^\circ + 58^\circ$ [∵ $\angle XYZ = 64^\circ$ (Given) and $\angle ZYQ = 58^\circ$ [From (2)]

$= 122^\circ$

Q. 7. In figure, find the values of x and y and then show that AB ∥ CD.



Sol.



∵ Ray AE stands on line GH

∴ $\angle AEG + \angle AEH = 180^\circ$ [Linear Pair Axiom]

=> $50^\circ + x = 180^\circ$

=> $x = 180^\circ - 50^\circ = 130^\circ$ ... (1)

$y = 130^\circ$ ... (2) [Vertically Opposite Angles]

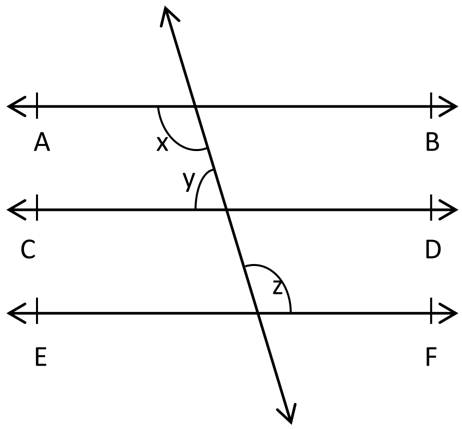
From (1) and (2), we conclude that

$x = y$

But these are alternate interior angles and they are equal.

So, we can say that AB || CD.

Q. 8. In figure, if AB || CD, CD || EF and y : z = 3 : 7, find x.



Sol. ∵ AB || CD

And CD || EF

∴ AB || EF [Lines parallel to the same line are parallel to each other

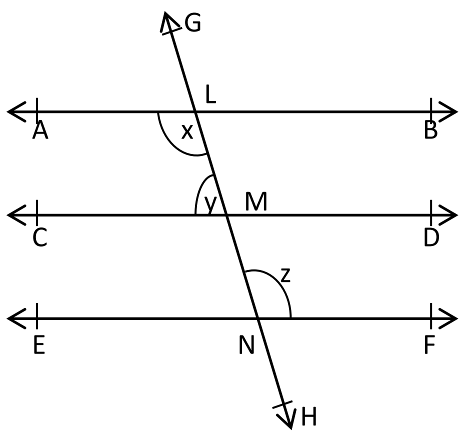
∴ $x = z$ ... (1) [Alternate Interior Angles]

$x + y = 180^o$ ... (2) [Consecutive interior angles on the same side of the transversal GH to parallel lines AB and CD]

From (1) and (2),

$z + y = 180^o$

$y : z = 3 : 7$

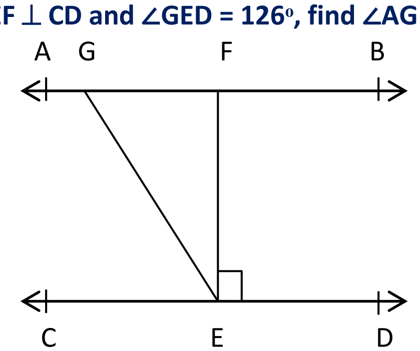


Sum of the ratios = 3 + 7 = 10

∴ $y = \frac{3}{10} \times 180^o = 54^o$ and $z = \frac{7}{10} \times 180^o = 126^o$

∴ $x = z = 126^o$

Q. 9. In figure, if AB || CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.



Sol. (i) $\angle AGE = \angle GED = 126^o$

(ii) $\angle GED = 126^o$ [Alternate Interior Angles]

=> $\angle GEF + \angle FED = 126^o$

=> $\angle GEF + 90^o = 126^o$ [∵ EF ⊥ CD ∴ $\angle FED = 90^o$]

=> $\angle GEF = 126^o - 90^o = 36^o$

(iii) $\angle GEC + \angle GEF + \angle FED = 180^\circ$ [

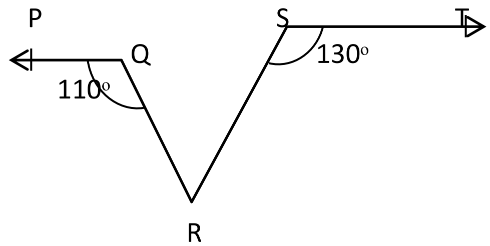
=> $\angle GEC + 36^\circ + 90^\circ = 180^\circ$

=> $\angle GEC = 180^\circ - 126^\circ = 54^\circ$

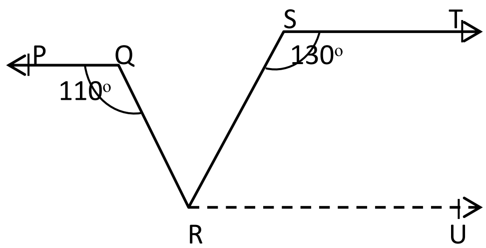
Now, $\angle FGE = \angle GEC = 54^\circ$ [Alternate Interior Angles]

Q. 10. In figure, if PQ || ST, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$, find $\angle QRS$.

[Hint: Draw a line parallel to ST through point R.]



Sol. Construction: Draw a line RU parallel to ST through point R.



$\angle RST + \angle SRU = 180^\circ$ [Sum of the consecutive interior angles on the same side of the transversal is 180°]

=> $130^\circ + \angle SRU = 180^\circ$

=> $\angle SRU = 180^\circ - 130^\circ = 50^\circ$ ... (1)

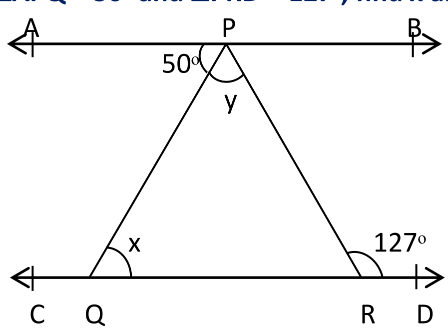
$\angle QRU = \angle PQR = 110^\circ$ [Alternate Interior Angles]

=> $\angle QRS + \angle SRU = 110^\circ$

=> $\angle QRS + 50^\circ = 110^\circ$ [Using (1)]

=> $\angle QRS = 110^\circ - 50^\circ = 60^\circ$

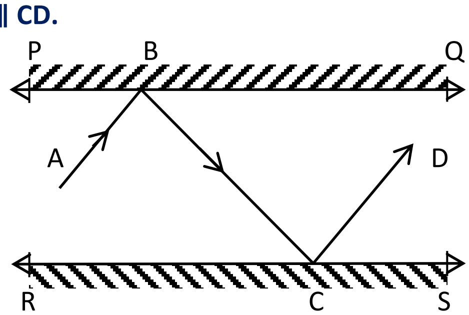
Q. 11. In figure, if AB || CD, $\angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$, find x and y.



Sol. $x = \angle APQ = 50^\circ$ [Alternate Interior Angles]

$Y = \angle PRD = 127^\circ$ [Alternate Interior Angles]

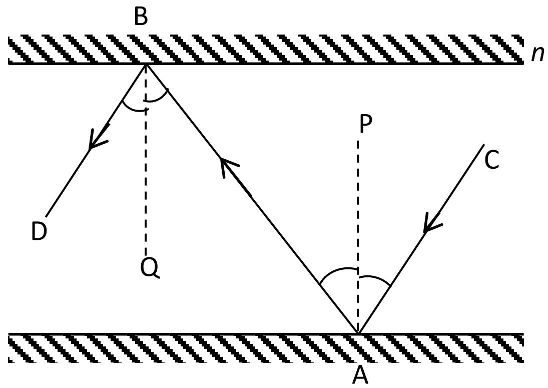
Q. 12. In figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.



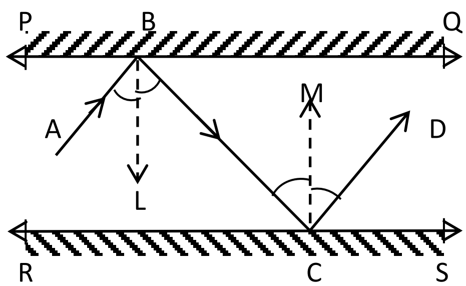
Or

In figure, m and n are two plane mirrors parallel to each other. Prove that incident ray CA is parallel to reflected ray BD.

[Hint: Draw perpendicular (normals) at A and B to the two plane mirrors. Recall that angle of incidence is equal to angle of reflection.]



Sol. <u>Construction</u>: Draw ray BL ⊥ PQ and ray CM ⊥ RS.



***Proof*:** ∴ BL ⊥ PQ, CM ⊥ RS and PQ ⊥ RS

∴ BL || CM

∠LBC = ∠MCB ... (1) [Alternate Interior Angles]

∠ABL = ∠LBC ... (2) [∵ Angle of incidence = Angle of reflection]

∠MCB = ∠MCD ... (3) [∵ Angle of incidence = Angle of reflection]

From (1), (2) and (3), we get

∠ABL = ∠MCD ... (4)

Adding (1) and (4), we get

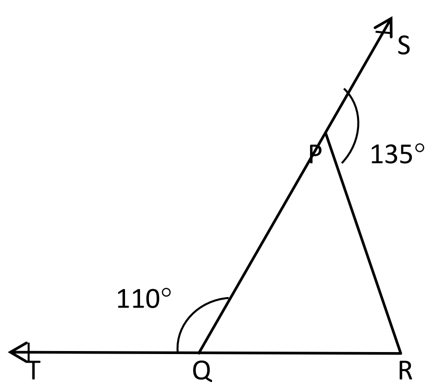
∠LBC + ∠ABL = ∠MCB + ∠MCD

=> ∠ABC = ∠BCD

But these are alternate interior angles and they are equal.

So, AB || CD

Q. 13. In figure, sides QP and RQ of Δ PQR are produced to points S and T respectively. If ∠SPR = 135° and ∠PQT = 110°, find ∠PRQ.



Sol. ∵ TR is a line

∴ ∠PQT + ∠BQR = 180°

=> 110° + ∠PQR = 180°

=> ∠PQR = 180° - 110° = 70° ... (1)

∵ QS is a line

∴ ∠SPR + ∠QPR = 180°

=> 135° + ∠QPR = 180°

=> ∠QPR = 180° – 135° = 45° ... (2)

In ΔPQR

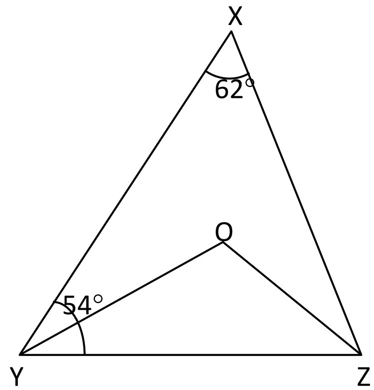
∠PQR + ∠QPR + ∠PRQ = 180° [∵ The sum of all the angles of a triangle is 180°]

=> 70° + 45° + ∠PRQ = 180° [Using (1) & (2)]

=> 115° + ∠PRQ = 185°

=> ∠PRQ = 180° - 115° = 65°

Q. 14. In figure, ∠X = 62°, ∠XYZ = 54°. If YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively of ΔXYZ, find ∠OZY and ∠YOZ.



Sol. In ΔXYZ,

∠XYZ + ∠YZX + ∠ZXY = 180° [∵ The sum of all the angles of a triangle is 180°]

=> 54° + ∠YZX + 62° = 180°

=> 116° + ∠YZX = 180°

=> ∠YZX = 180° = 116° = 64° ... (1)

∵ YO is the bisector of XYZ

∴ ∠XYO = ∠OYZ = ½ ∠XYZ = ½ (54°) = 27° ... (2)

∵ ZO is the bisector of ∠YZX

∴ ∠XZO = ∠OZY = ½ ∠YZX = ½ (64°) = 32° ... (3) [Using (1)]

In ΔOYZ,

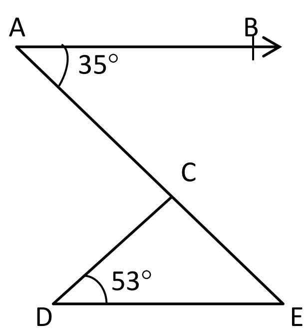
∠OYZ + ∠OZY + ∠YOZ = 180° [∵ The sum of all the angles of a triangle is 180°]

=> 27° + 32° + ∠YOZ = 180° [Using (2) and (3)]

=> 59° + ∠YOZ = 180°

=> ∠YOZ = 180° – 59° = 121°

Q. 15. In figure, if AB ∥ DE, ∠BAC = 35° and ∠CDE = 35°, find ∠DCE.



Sol. ∠DEC = ∠BAC = 35° ... (1) [Alternate Interior Angles]

∠CDE = 53° ... (2) [Given]

In Δ CDE,

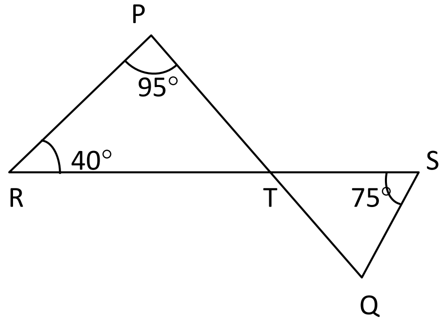
∠CDE + ∠DEC + ∠DCE = 180°

=> 53° + 35° + ∠DCE = 180° [Using (1) and (2)]

=> 88° + ∠DCE = 180°

=> ∠DCE = 180° – 88° = 92°

Q. 16. In figure, if lines PQ and RS intersect at point T, such that ∠PRT = 40°, ∠RPT = 95° and ∠TSQ = 75°, find ∠SQT.



Sol. In ΔPRT,

$\angle PTR + \angle PRT + \angle RPT = 180°$ [∵ The sum of all the angles of a triangle is 180°]

=> $\angle PTR + 40° + 95° = 180°$

=> $\angle PTR + 135° = 180°$

=> $\angle PTR = 45°$

=> $\angle QTS = \angle PTR = 45°$ [Vertically Opposite Angles]

In ΔTSQ,

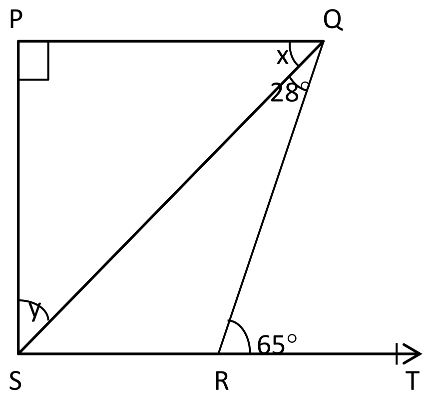
$\angle QTS + \angle TSQ + \angle SQT = 180°$ [∵ The sum of all the angles of a triangle is 180°]

=> $45° + 75° + \angle SQT = 180°$

=> $120° + \angle SQT = 180°$

=> $\angle SQT = 180° – 120° = 60°$

Q. 17. In figure, if PQ ⊥ PS, PQ ∥ SR, ∠SQR = 28° and ∠QRT = 65°, then find the values of x and y.



Sol. $\angle QRT = \angle RQS + \angle QSR$ [∵ The exterior angle is equal to the sum of the two interior opposite angles]

=> $65° = 28° + \angle QSR$

=> $\angle QSR = 65° – 28° = 37°$

∵ $PQ \perp SP$

∴ $\angle QPS = 90°$

∵ $PQ \parallel SR$

∴ $\angle QPS + \angle PSR = 180°$ [∵ The sum of consecutive interior angles on the same side of the transversal is 180°]

=> $90° + \angle PSR = 180°$

=> $\angle PSR + \angle QSR = 90°$

=> $y + 37° = 90°$

=> $y = 90° – 37° = 53°$

In PQS,

$\angle PQS + \angle QSP + \angle QPS = 180°$ [∵ The sum of all the angles of a triangle is 180°]

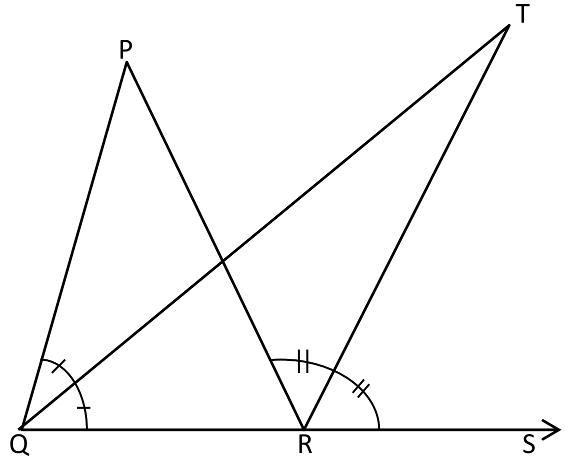
=> $x + y + 90° = 180°$

=> $x + 53° + 90° = 180°$

=> $x + 143° = 180°$

=> $x = 180° - 143° = 37°$

Q. 18. In figure, the side QR of ΔPQR is produced to a point S. If the bisectors of ∠PQR and ∠PRS meet at point T, then prove that ∠QTR = ½ ∠QPR. **12**



Sol. ∵ ∠TRS is an exterior angle of ΔTQR

∴ ∠TRS = ∠TQR + ∠QTR ... (1) [∵ The exterior angle is equal to sum of the two interior opposite angles]

∵ PRS is an exterior angle of ΔPQR

∴ ∠PRS = ∠PQR + ∠QPR ... (2) [∵ The exterior angle is equal to the sum of the two interior opposite angles]

=> 2∠TRS = 2∠TQR + ∠QPR [∵ QT is the bisector of ∠PQR and RT is the bisector of ∠PRS]

=> 2(∠TRS – ∠TQR) = ∠QPR ... (3)

From (1),

∠TRS – ∠TQR = ∠QTR ... (4)

From (3) and (4), we obtain

2∠QTR = ∠QPR

=> ∠QTR = ½ ∠QPR

... END.