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N L M-FRICTION 02

BASIC CONCEPTS

FREE BODY DIAGRAM

SOLVED EXAMPLES

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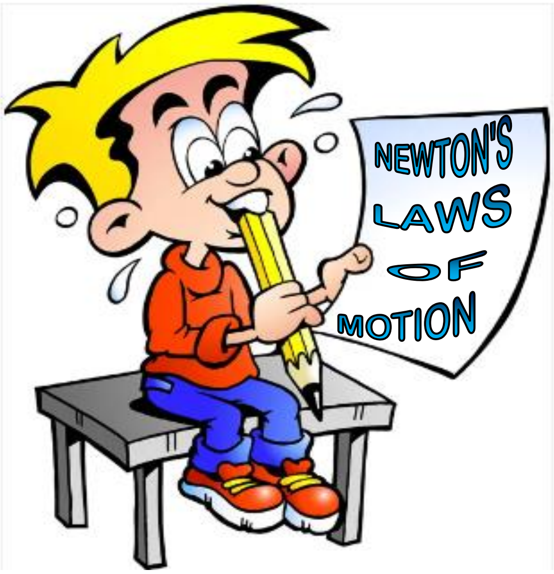


02

NEWTON'S LAWS OF MOTION

REVIEW OF BASIC CONCEPTS

1. Newton's First Law of Motion

Newton's first law of motion states that *every body continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by an external unbalanced force.*

2. Newton's Second Law of Motion

Newton's second law of motion states that *the rate of change of linear momentum of a body is directly proportional to the applied force and the change takes place in the direction in which the force acts.*

Linear Momentum Newton defined linear momentum as the product of the mass and the velocity of a body.

$$\mathbf{p} = m\mathbf{v}$$

where $\mathbf{v}$ is the velocity at a certain instant of time. Differentiating this equation with respect to time, we get

$$\frac{d\mathbf{p}}{dt} = m\frac{d\mathbf{v}}{dt} \qquad (\because m \text{ is constant})$$

$$F = m\mathbf{a}$$

where $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$ is the acceleration produced.

> **Note**
> 1. Force = slope of momentum–time $(p - t)$ graph.
> 2. Change in momentum = area under the force–time $(F - t)$ graph.

EXAMPLE 1 The velocity of a body of mass 2 kg changes from $\mathbf{v}_1 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})\,\text{ms}^{-1}$ to $\mathbf{v}_2 = (-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})\,\text{ms}^{-1}$ in 3 s. Find (a) the magnitude of the change in momentum of the body and (b) the magnitude of the force applied.

SOLUTION

(a) Change in momentum = final momentum – initial momentum

or
$$\Delta\mathbf{p} = m\mathbf{v}_2 - m\mathbf{v}_1$$
$$= m[(-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})]$$
$$= m(-5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}})$$

$\therefore$
$$|\Delta\mathbf{p}| = 2\sqrt{(-5)^2 + (-1)^2 + (4)^2}$$
$$= 2 \times \sqrt{42} = 12.96 \text{ kg ms}^{-1}$$

(b) $$|\mathbf{F}| = \frac{|\Delta\mathbf{p}|}{\Delta t} = \frac{12.96}{3} = 4.32 \simeq 4.3 \text{ N}$$

EXAMPLE 2 A particle of mass 1 g is moving along the positive x-axis under the influence of a force.

$$F = -\frac{k}{x^2}$$

where $k = 10^{-3}$ Nm^2. When the particle is at $x = 1.0$ m, its velocity $v = 0$. Find (a) the magnitude of its velocity when, it reaches $x = 0.5$ m and (b) its position when its speed is 1 ms^{-1}.

SOLUTION

$$F = ma = m\frac{dv}{dt} = m\frac{dv}{dx}\cdot\frac{dx}{dt} = mv\frac{dv}{dx}$$

Given, $F = -\dfrac{k}{x^2}$. Therefore

$$-\frac{k}{x^2} = mv\frac{dv}{dx} \Rightarrow v\,dv = -\frac{k}{mx^2}dx$$

Integrating, we have

$$\int v\,dv = -\frac{k}{m}\int x^{-2}\,dx$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{k}{mx} + c \qquad \text{(i)}$$

where c is the constant of integration. Given $v = 0$ when $x = 1.0$ m. Using this in eq. (i), we get $c = -\frac{k}{m}$. Equation (i) becomes

$$\frac{v^2}{2} = \frac{k}{mx} - \frac{k}{m}$$

$$\Rightarrow \quad v = \left[\frac{2k}{m}\left(\frac{1}{x}-1\right)\right]^{1/2}$$

$$v = \left[\frac{2\times10^{-3}}{10^{-3}}\left(\frac{1}{x}-1\right)\right]^{1/2}$$

$$[\because k = 10^{-3}\ \text{Nm}^2 \text{ and } m = 10^{-3}\ \text{kg}]$$

$$\Rightarrow \quad v = \left[2\left(\frac{1}{x}-1\right)\right]^{1/2}$$

(a) When $x = 0.5$ m, $v = \left[2\left(\frac{1}{0.5}-1\right)\right]^{1/2} = \sqrt{2}\ \text{ms}^{-1}$

(b) When $v = 1\ \text{ms}^{-1}$, $1 = \left[2\left(\frac{1}{x}-1\right)\right]^{1/2} \Rightarrow x = 0.67$ m

3. Newton's Third Law of Motion

Newton's third law of motion states that *whenever one body exerts a force on a second body, the second body exerts an equal and opposite force on the first, or, to every action there is an equal and opposite reaction*. The action and reaction forces act on different body.

4. Law of Conservation of Linear Momentum

The *law of conservation of linear momentum* may be stated as '*when no net external force acts on a system consisting of several particles, the total linear momentum of the system is conserved, the total linear momentum being the vector sum of the linear momentum of each particle in the system*'.

Recoil of a Gun

The gun and the bullet constitute a two-body system. Before the gun is fired, both the gun and the bullet are at rest. Therefore, the total momentum of the gun-bullet system is zero. After the gun is fired, the bullet moves forward and the gun recoils backwards. Let m_b and m_g be the masses of the bullet and the gun. If $\mathbf{v}_b$ and $\mathbf{v}_g$ are their respective velocities after firing, the total momentum of the gun-bullet system after firing is $(m_b\,\mathbf{v}_b + m_g\,\mathbf{v}_g)$. From the law of conservation of momentum, the total momentum after and before the gun is fired must be the same, i.e.

$$m_b\,\mathbf{v}_b + m_g\,\mathbf{v}_g = 0$$

or

$$\mathbf{v}_g = -\frac{m_b\,\mathbf{v}_b}{m_g}$$

The negative sign indicates that the gun recoils in a direction opposite to that of the bullet. In terms of magnitudes, we have

$$v_g = \frac{m_b\,v_b}{m_g}$$

5. Impulse

Consider a collision between two bodies A and B moving in the same straight line. Let Δt be the duration of the collision, i.e. the time for which the bodies were in contact during which time the transfer of momentum took place. We assume that the bodies continue moving in the same straight line after the collision with velocities different from their initial velocities.

Impulse of a force is the product of the average force and the time for which the force acts and it is equal to the change in momentum of the body during that time. Impulse is a vector and is measured in kg m s^{-1} or N s.

$$\mathbf{I} = \mathbf{F}_{av}\,\Delta t = \Delta\mathbf{p}$$

EXAMPLE 3 A ball of mass m is moving with a velocity $\mathbf{v}$ towards a rigid vertical wall. After striking the wall, the ball deflects through an angle θ without change in its speed. Obtain the expression for the impulse imparted to the ball.

SOLUTION Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the initial and final velocities of the ball [Fig. 3.1(a)]

$$\text{Impulse} = \text{change in momentum}$$
$$= m\mathbf{v}_2 - m\mathbf{v}_1 = m(\mathbf{v}_2 - \mathbf{v}_1) = m[\mathbf{v}_2 + (-\mathbf{v}_1)]$$

or

$$\text{Impulse} = m\Delta\mathbf{v}$$

where $\Delta\mathbf{v} = \mathbf{v}_2 + (-\mathbf{v}_1)$ is the resultant of $\mathbf{v}_2$ and $-\mathbf{v}_1$ [Fig. 3.1(b)].

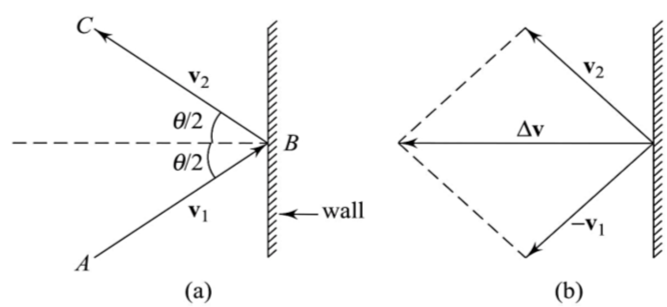


Fig. 3.1

Magnitude of $\Delta\mathbf{v}$ is

$[\because$ magnitude of $\mathbf{v}_1$ = magnitude of $\mathbf{v}_2 = v]$

$$\Delta v = \sqrt{v_1^2 + v_1^2 + 2v_1v_2\cos\theta}$$
$$= \sqrt{v^2 + v^2 + 2v^2\cos\theta} = \sqrt{2v^2(1+\cos\theta)}$$
$$= 2v\cos\left(\frac{\theta}{2}\right)$$

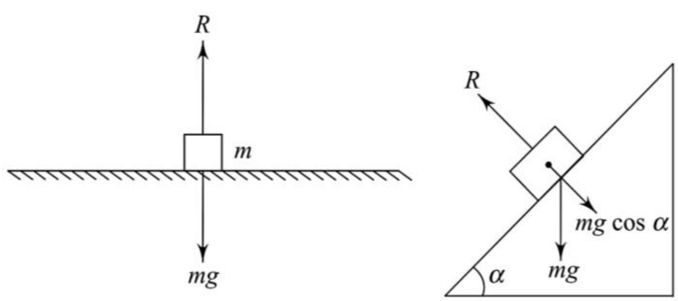
The direction of impulse is perpendicular to the wall and away from it.

6. Contact Forces

(1) *Normal Reaction*

The force exerted by one body when placed on the surface of another body is known as contact force. If the two surfaces in contact are perfectly smooth (i.e., frictionless), then the contact force acts only perpendicular (normal) to their surface of contact and is known as *normal reaction* (R).

If a block of mass m is placed on a horizontal frictionless surface [Fig. 3.2 (a)], the normal reaction $R = mg$. If the block is placed on an inclined plane of inclination α [Fig. 3.2 (b)], the normal reaction $R = mg\cos\alpha$



(a) (b)

Fig. 3.2

If there is friction between the surfaces of contact, then the component of the contact force perpendicular to their surface gives the normal reaction and the other component which acts along the tangent to the surface of contact gives the force of friction. The normal reaction, tension and friction are examples of contact force.

(2) *Tension*

The force in a string is called tension (T). If the string is massless, the tension has the same magnitude at all points of the string. Tension in the string always acts away from the body to which it is attached. If the string passes over a frictionless pulley and its ends are attached to two bodies, the tension in the entire string has the same magnitude and its direction is towards its point of contact with the pulley.

7. Friction

Friction is the force which comes into play when one body slides or rolls over the surface of another body and acts in a direction tangential to the surfaces in contact and opposite to the direction of motion of the body.

The maximum (or limiting) force of friction when a body just begins to slide over the surface of another body is called the *limiting friction*. The force of friction just before one body begins to slide over another is called the *limiting* or *static friction* (f_s). The coefficient of limiting or static friction (μ_s) is defined as

$$\mu_s = \frac{f_s}{R}$$

where R is the normal reaction, i.e. the normal force pressing the two surfaces together.

The force necessary to maintain a body in uniform motion over the surface of another body, after motion has started, is called the *kinetic* or *sliding friction* (f_k). The *coefficient of kinetic friction* (μ_k) is defined as

$$\mu_k = \frac{f_k}{R}$$

Note that μ_k is always less than μ_s.

Angle of Friction Angle of friction is the angle between the resultant of the force of limiting friction (f) and the normal reaction (R). In Fig. 3.3, θ is the angle of friction, which is given by

$$\tan\theta = \frac{f}{R} = \mu$$
$$\theta = \tan^{-1}(\mu)$$

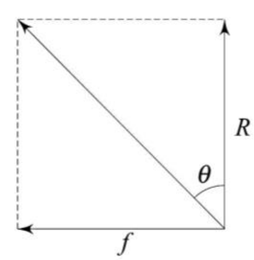


Fig. 3.3

Angle of Repose Suppose a body is placed on an inclined plane. The angle of inclination is gradually increased until the body just begins to slide along the plane. When this happens the angle of inclination α of the inclined surface with the horizontal is called the angle of repose (see Fig. 3.4). It follows from the figure that

Force of limiting friction $(f) = mg\sin\alpha$

Force of normal reaction $(R) = mg\cos\alpha$

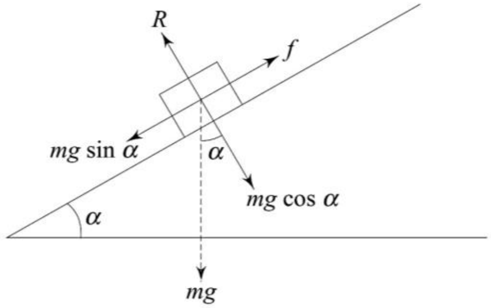


Fig. 3.4

Therefore, $\tan \alpha = \frac{f}{R} = \mu = \tan \theta$

or $\alpha = \theta = \tan^{-1}(\mu)$

8. Solving Problems in Mechanics by Free Body Diagram Method

In mechanics, we often have to handle problems which involve a group of bodies connected to one another by strings, pulleys, springs, etc. They exert forces on one another. Furthermore, there are frictional forces and the force of gravity acting on each body in the group. To solve such complicated problems, it is always convenient to choose one body in the group, find the magnitude and the direction of the forces acting on this body by all the remaining bodies in the group. Then we find the resultant of all the forces acting on the body to obtain the net force exerted on it. We then use the laws of motion to determine the dynamics of the body. We apply this procedure to all other bodies in the group one by one. It is useful to draw a separate diagram for each body, showing the directions of the different forces acting on it. Such a diagram is called the *free body diagram* (F.B.D.) of the body.

(1) Two masses tied to a string going over a frictionless pulley Consider two bodies of masses m_1 and m_2 ($m_1 > m_2$) connected by a string which passes over a pulley, as shown in Fig. 3.5(a). When the bodies are released, the heavier one moves downwards and the lighter one moves up.

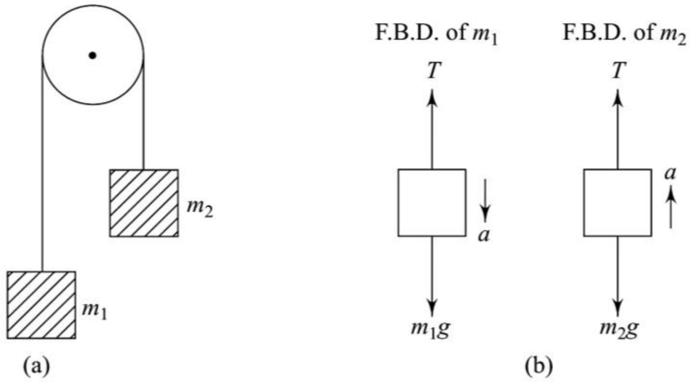


Fig. 3.5

Net force in the direction of motion of m_1 is $m_1g - T$. Therefore, the equation of motion of m_1 is

$$m_1g - T = m_1a \qquad \text{(i)}$$

Net force in the direction of motion of m_2 is $(T - m_2g)$. Therefore, the equation of motion of m_2 is

$$T - m_2g = m_2a \qquad \text{(ii)}$$

From (i) and (ii) we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

and

$$T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$$

(2) Two masses in contact Figure 3.6(a) shows two blocks of masses m_1 and m_2 placed in contact on a horizontal frictionless surface. A force F is applied to mass m_1. As a result, the masses move with a common acceleration a. To find a and the contact force on m_2, we draw the free body diagrams as shown in Figs. 3.6(b) and (c).

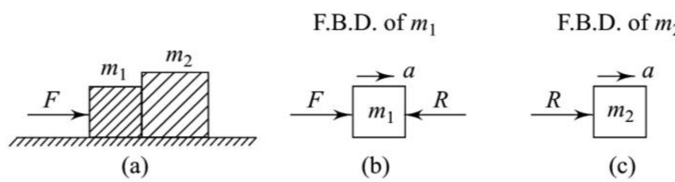


Fig. 3.6

R = normal reaction force between the blocks. From Figs. 3.6(b)and (c), we get

$$F - R = m_1a \qquad \text{(i)}$$

and

$$R = m_2a \qquad \text{(ii)}$$

Adding (i) and (ii) we get

$$F = (m_1 + m_2)a$$

$$\Rightarrow \quad a = \frac{F}{(m_1 + m_2)}$$

Contact force on m_2 is

$$F_2 = m_2\,a = \frac{m_2F}{(m_1 + m_2)}$$

(3) Three masses in contact Figure 3.7(a) shows three blocks of masses m_1, m_2, and m_3 placed in contact on a horizontal frictionless surface. A force F is applied to m_1. As a result, the three masses move with a common acceleration a. To find a and the contact forces on m_2 and m_3, we draw the free body diagrams as shown in Figs. 3.7(b) (c) and (d).

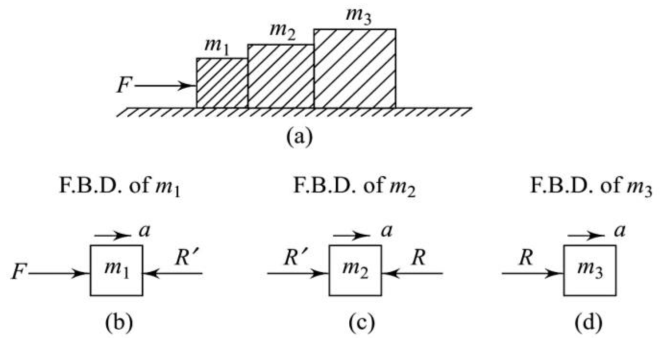


Fig. 3.7

R' = contact force on m_2 = reaction force between m_1 and m_2

R = contact force on m_3 = reaction force between m_2 and m_3

It follows from Figs. 3.7(b), (c) and (d) that

$$F - R' = m_1 a \quad \text{(i)}$$

$$R' - R = m_2 a \quad \text{(ii)}$$

and $$R = m_3 a \quad \text{(iii)}$$

Adding (i), (ii) and (iii) we get

$$a = \frac{F}{(m_1 + m_2 + m_3)}$$

Contact force on m_2 is $F_2 = R'$ which from (ii) is given by

$$F_2 = R' = R + m_2 a$$

Using (iii) we have

$$F_2 = m_3 a + m_2 a$$

$$= (m_2 + m_3)a$$

$$\Rightarrow \quad F_2 = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

Contact force on m_3 is

$$F_3 = R = m_3 a = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

(4) Two masses connnected with a string Figure 3.8(a) shows two blocks of masses m_1 and m_2 connected with a string and lying on a horizontal frictionless surface. A force F is applied to m_2. As a result, the masses move with a common acceleration a. To find a and force exerted on m_1, we draw the free body diagrams as shown in Figs. 3.8(b) and (c). T is the tension in the string.

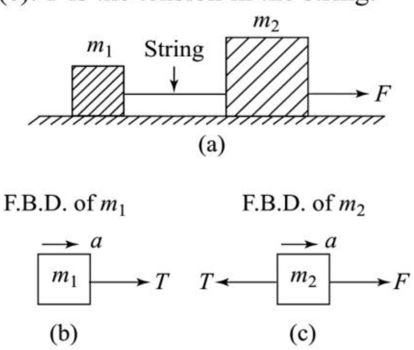


Fig. 3.8

It follows from Figs. 3.8(b) and (c) that

$$T = m_1 a \quad \text{(i)}$$

and $$F - T = m_2 a \quad \text{(ii)}$$

Adding (i) and (ii) we get

$$a = \frac{F}{m_1 + m_2}$$

Tension in the string is

$$T = m_1 a = \frac{m_1 F}{m_1 + m_2}$$

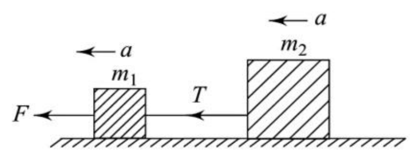


Fig. 3.9

If force F is applied on mass m_1 as shown in Fig. 3.9, then

$$a = \frac{F}{m_1 + m_2}$$

Tension in the string is

$$T = \frac{m_2 F}{m_1 + m_2}$$

(5) Three masses connected by strings Figure 3.10 (a) shows three blocks of masses m_1, m_2 and m_3 connected by two strings and placed on a horizontal frictionless surface. A force F is applied to m_1. As a result, the blocks move with a common acceleration a. To find a and the forces acting on m_2 and m_3, we draw free body diagrams as shown in Fig. 3.10(b) and (c) and (d). T is the tension in the string between m_1 and m_2 and T' is the tension in the string between m_2 and m_3.

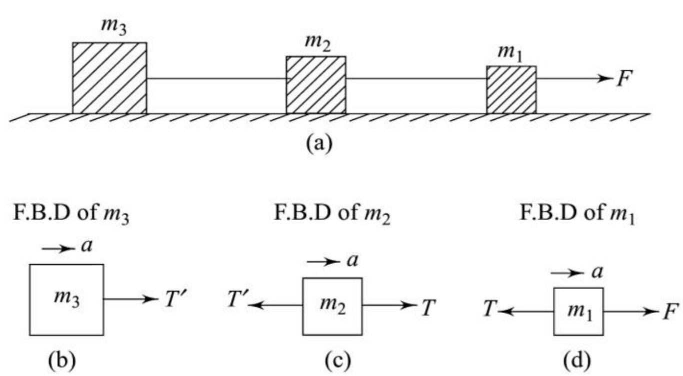


Fig. 3.10

It follows from Figs. 3.10(b), (c) and (d) that

$$T' = m_3 a \quad \text{(i)}$$

$$T - T' = m_2 a \quad \text{(ii)}$$

and $$F - T = m_1 a \quad \text{(iii)}$$

Adding (i), (ii) and (iii), we get

$$a = \frac{F}{(m_1 + m_2 + m_3)}$$

The tension in the string between m_1 and m_2 is T, which is obtained by adding (i) and (ii).

$$T = (m_2 + m_3)a = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

The tension in the string between m_2 and m_3 is T', which from (i) is given by

$$T' = m_3 a = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

(6) Two masses connected by a string and suspended from a support Two blocks of masses m_1 and m_2 are connected by two strings and suspended from a support as shown in Fig. 3.11(a). Mass m_2 is pulled down by a force F. The tension T in the string between m_1 and m_2 and tension T' in the string between m_1 and the support can be found from the free body diagrams as shown in Fig. 3.11(b) and (c).

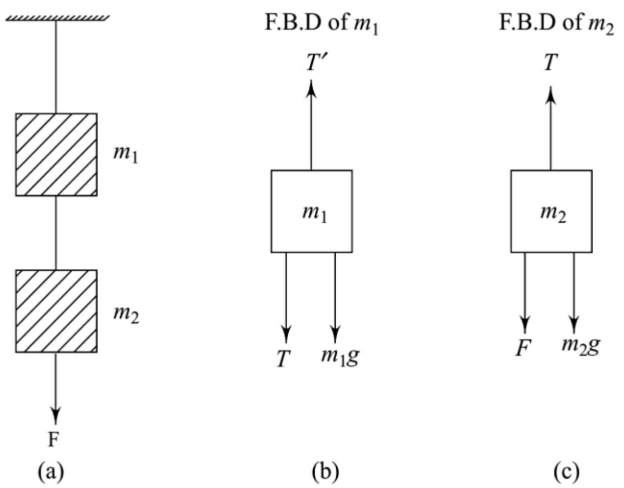


Fig. 3.11

$$T' = T + m_1 g \quad \text{(i)}$$

$$T = F + m_2 g \quad \text{(ii)}$$

Using (ii) and (i), we get

$$T' = F + (m_1 + m_2)g$$

(7) Two blocks connected by a string passing over a frictionless pulley fixed at the edge of a horizontal table Consider a block of mass m_1 lying on a frictionless table connected through a pulley to another block of mass m_2 hanging vertically (Fig. 3.12). When the system is released, let acceleration of the blocks be a.

From free body diagrams, the equations of motion of m_1 and m_2 are

$$T = m_1 a \quad \text{(i)}$$

and

$$m_2 g - T = m_2 a \quad \text{(ii)}$$

Adding (i) and (ii), we get

$$a = \frac{m_2 g}{(m_1 + m_2)}$$

Also

$$T = m_1 a = \frac{m_1 m_2 g}{(m_1 + m_2)}$$

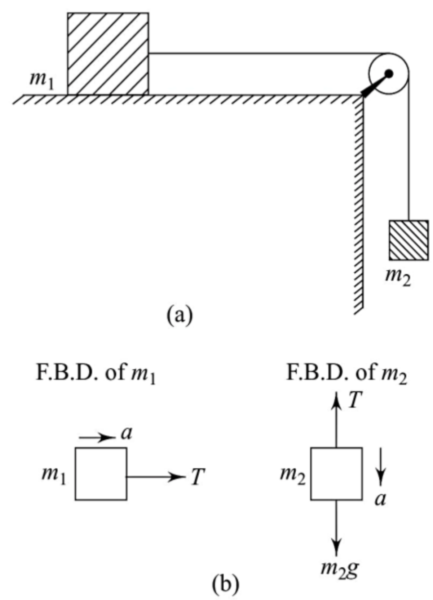


Fig. 3.12

If the table top is frictionless, the blocks will move even if $m_2 < m_1$.

If μ is the coefficient of friction between block m_1 and the table, the force of friction is

$$f = \mu R = \mu m_1 g$$

From F.B.D. of m_1 (Fig. 3.13), Eq. (i) becomes

$$T - f = m_1 a \quad \Rightarrow \quad T - \mu m_1 g = m_1 a$$

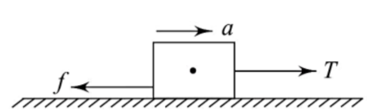


Fig. 3.13

(8) Two blocks connected by a string passing over a frictionless pulley fixed at the top of an inclined plane Let T be the tension in the string. Since the pulley is frictionless, the tension is the same throughout the string (Fig. 3.14). There are the following two cases:

(a) Mass m_1 moving up along the incline with acceleration a [Fig. 3.14]

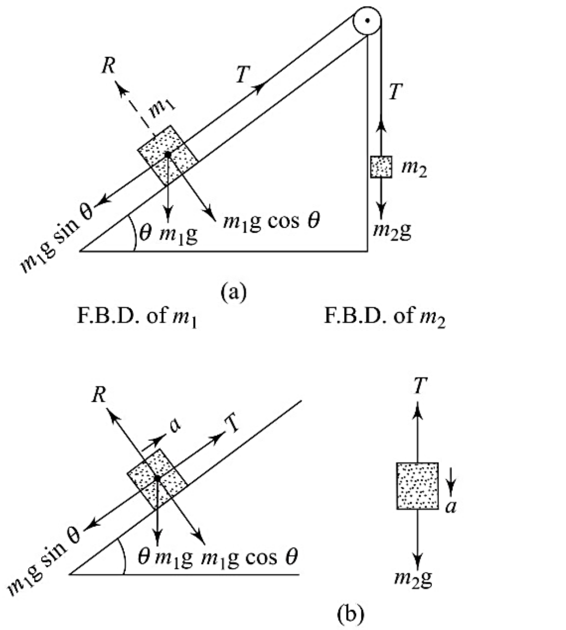


Fig. 3.14

The equations of motion of m_1 and m_2 [see Fig. 3.14(b)]

$$T - m_1g \sin\theta = m_1a \quad \text{(i)}$$

$$m_2g - T = m_2a \quad \text{(ii)}$$

which give
$$a = \frac{(m_2 - m_1 \sin\theta)g}{(m_1 + m_2)}$$

and
$$T = m_2(g - a)$$

If μ is the coefficient of friction between m_1 and the inclined plane, the frictional force $f = \mu R = \mu m_1 g \cos\theta$ will act down the plane because the block m_1 is moving up the plane. In this case, Eq. (i) is replaced by

$$T - m_1g \sin\theta - f = m_1a$$

$$\Rightarrow \quad T - m_1g \sin\theta - \mu m_1g \cos\theta = m_1a$$

(*b*) *Mass m_1 moves down the incline with acceleration a*

In this case, we get $m_1g \sin\theta - T = m_1a$ and $T - m_2g = m_2a$ which give

$$a = \frac{(m_1 \sin\theta - m_2)g}{(m_1 + m_2)}$$

and $\quad T = m_2(g + a)$

(9) Two blocks connected by a string passing over a frictionless pulley fixed at the top of a double inclined plane Let the block of mass m_1 move up along the inclined plane of angle of inclination θ_1, and the block of mass m_2 move down the inclined plane of angle of inclination θ_2 (Fig. 3.15). Let T be the tension in the string. Then, for m_1 and m_2, we have

$$T - m_1g \sin\theta_1 = m_1a$$

and
$$m_2g \sin\theta_2 - T = m_2a$$

Eliminating T, we get

$$a = \frac{(m_2 \sin\theta_2 - m_1 \sin\theta_1)g}{(m_1 + m_2)}$$

Also
$$T = m_1(a + g \sin\theta_1)$$
$$= m_2(g \sin\theta_2 - a)$$

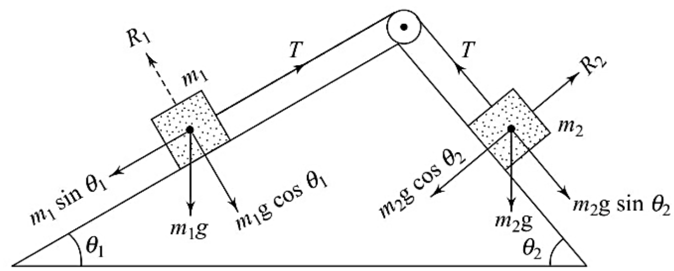


Fig. 3.15

(10) One blocks are placed on top of the another A block of mass m_1 is placed on another block of mass m_2, which is lying on a horizontal frictionless surface. The coefficient of friction between the blocks is μ.

Case 1: The maximum force that can be applied on the lower block so that the upper block does not slip [Fig. 3.16(a)]

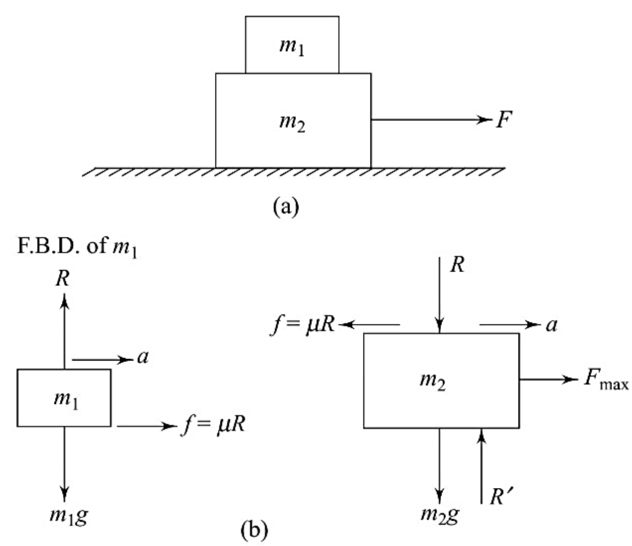


Fig. 3.16

F_{max} = maximum value of force F so that block m_1 does not slip of block m_2

f = frictional force on m_1 due to m_2

$= \mu R = \mu m_1 g$

Due to friction, m_2 will try to drag m_1 to the right. Hence frictional force f acts towards left. From F.B.D. of m_1,

$$f = m_1 a \Rightarrow \mu m_1 g \quad \Rightarrow \mu g = a \qquad \text{(i)}$$

Here, a is the acceleration of each block.

R' = normal reaction on m_2 by the horizontal surface.

From F.B.D. of m_2, we have

$$R + m_2 g = R' \qquad \text{(ii)}$$

and $$F_{max} - f = m_2 a$$

$$\Rightarrow \quad F_{max} - \mu R = m_2 a \qquad \text{(iii)}$$

$$\Rightarrow \quad F_{max} - \mu m_1 g = m_2 a$$

Using (i) in (iii), we get

$$F_{max} = (m_1 + m_2)\,\mu g$$

(a) If $F > F_{max}$, m_1 will begin to slide on m_2 and then their accelerations will be different.

(b) If $F < F_{max}$, m_1 and m_2 move together without any relative motion between them.

Case 2: The maximum force that can be applied on the upper block so that it does not slip on the lower block. [Fig. 3.17(a)]

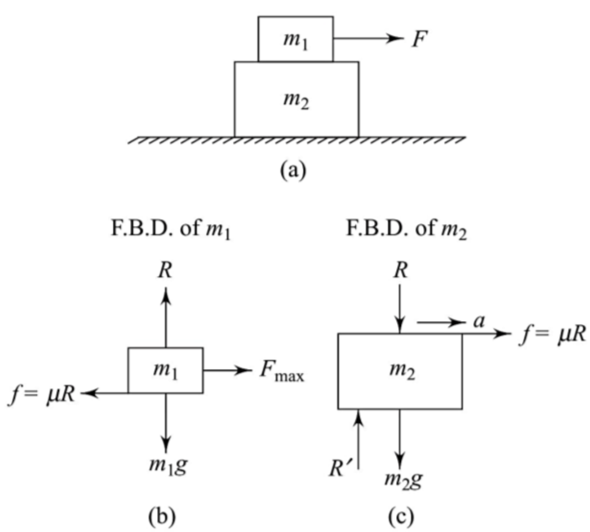


Fig. 3.17

F_{max} = maximum value of force F so that block m_1 just begins to slide on block m_2

Block m_1 tries to drag block m_2 toward right due to frictional force $f = \mu R = \mu m_1 g$. The frictional force exerted by m_1 on m_2 will be towards right.

R' = normal reaction on m_2 by the horizontal surface. If a is the acceleration of blocks towards right, from F.B.D. of m_1 we have

$$F_{max} - \mu R = m_1 a \text{ and } R = m_1 g$$

or $$F_{max} - \mu m_1 g = m_1 a \qquad \text{(i)}$$

From F.B.D. of m_2,

$$f = m_2 a \Rightarrow \mu_1 m_1 g = m_2 a$$

$$\Rightarrow \quad a = \frac{\mu m_1 g}{m_2} \qquad \text{(ii)}$$

Using (ii) in (i), we get

$$F_{max} = \frac{\mu(m_1 + m_2)m_1 g}{m_2}$$

(a) If $F < F_{max}$, the blocks move together without any relative motion.

(b) If $F > F_{max}$, the blocks slide relative to each other and then their accelerations are different.

EXAMPLE 4 A block of mass $m = 1$ kg is pulled by a force $F = 10$ N at an angle $\theta = 60°$ with a horizontal surface as shown in Fig. 3.18. Find the acceleration of the block if

(a) the surface is frictionless and

(b) the coefficient of kinetic friction between the surface and the block is $\mu = 0.2$. Take $g = 10$ ms^{-2}.

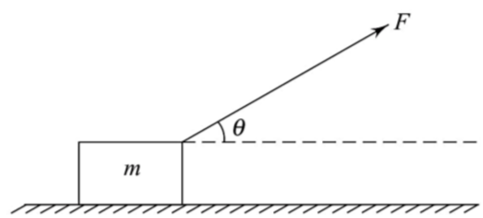
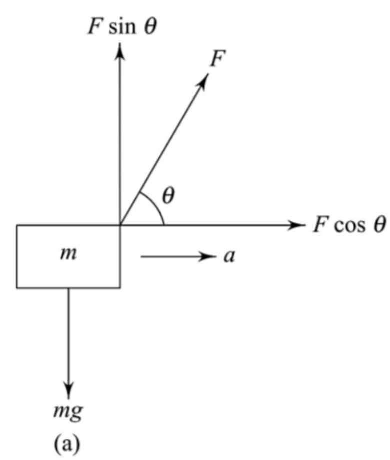
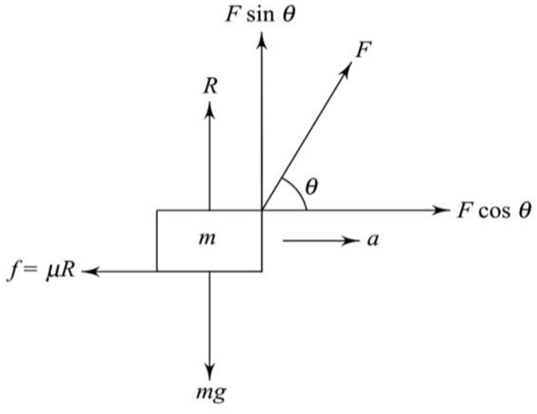


Fig. 3.18

SOLUTION The free body diagrams of the block in the two cases are shown in Fig. 3.19.





(b)

Fig. 3.19

(a) From Fig. 3.19(a)

$$F\cos\theta = ma$$

$$\Rightarrow \quad a = \frac{F\cos\theta}{m} = \frac{10\times\cos 60^\circ}{1} = 5\ \text{ms}^{-2}$$

(b) From Fig. 3.19(b)

$$F\cos\theta - f = ma$$

$$\Rightarrow \quad F\cos\theta - \mu mg = ma$$

$$\Rightarrow \quad a = \frac{F\cos\theta - \mu mg}{m} = \frac{10\cos 60^\circ - 0.2\times 1\times 10}{1} = 3\ \text{ms}^{-2}$$

From Fig. 3.19(b) we also have $F\sin\theta + R = mg$ or $F\sin\theta = mg - R$. Since $F\sin\theta < mg$, the block does not move upwards.

EXAMPLE 5 Two blocks of masses $m_1 = 2$ kg and $m_2 = 3$ kg are suspended from a rigid support by means of strings *AB* and *CD* as shown in Fig. 3.20. String *AB* has negligible mass and string *CD* has mass 0.5 kg/m. Each string has length 50 cm. Find the tension (a) at mid-point *P* of string *AB* and (b) at point *Q* of string *CD* where *CQ* = 20 cm. Take $g = 10\ \text{ms}^{-2}$.

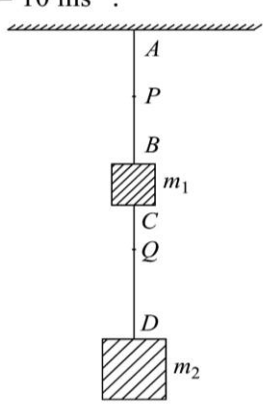


Fig. 3.20

SOLUTION Mass of string *CD* is $m = 0.5 \times 0.5 = 0.25$ kg Since string *AB* is massless, the tension in *AB* is the same at every point.

(a) Total mass below point $P = m_1 + m + m_2$

$$= 2 + 0.25 + 3 = 5.25\ \text{kg}$$

$$\therefore \quad \text{Tension at } P = 5.25 \times 10 = 52.5\ \text{N}$$

(b) Total mass below point Q = mass of length $QD + m_2$

$$= 0.5 \times 0.3 + 3 = 3.15\ \text{kg}$$

$$\therefore \quad \text{Tension at } Q = 3.15 \times 10 = 31.5\ \text{N}$$

EXAMPLE 6 A block of mass $m = 100$ g is placed on an inclined plane of inclination $\theta = 30^\circ$ as shown in Fig. 3.21. There is no friction between the block and the inclined plane. What minimum acceleration *a* should be given to the system to the left so that the block does not slide down the plane?

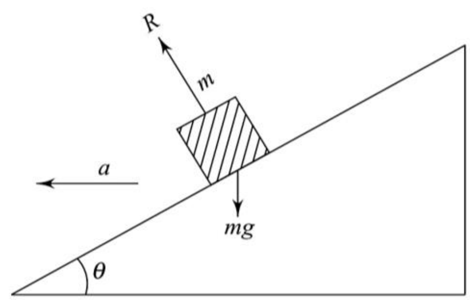


Fig. 3.21

SOLUTION Figure 3.22 shows the forces acting on the block.

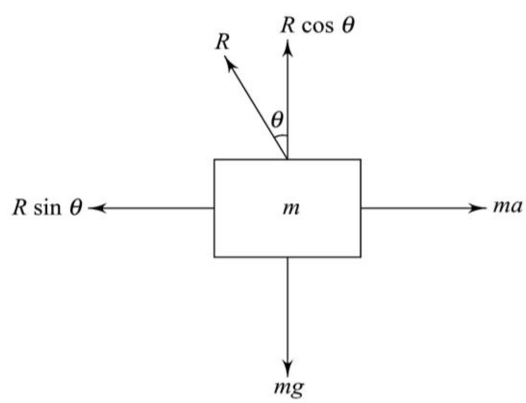


Fig. 3.22

$$R\cos\theta = mg$$

$$R\sin\theta = ma$$

$$\therefore \quad a = g\tan\theta = g\tan 30^\circ = \frac{g}{\sqrt{3}}$$

EXAMPLE 7 A pendulum of bob of mass m = 100 g is suspended from the ceiling of the compartment of a train. If the train has the acceleration a as shown in Fig. 3.23, the string makes an angle $\theta = 60°$ with the vertical. Find the value of a.

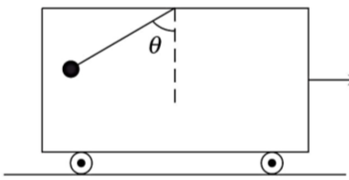


Fig. 3.23

SOLUTION Figure 3.24 shows the free body diagram of the bob.

Force on the bob in the direction of motion of the train = $T \sin \theta$. Hence the equation for horizontal direction is

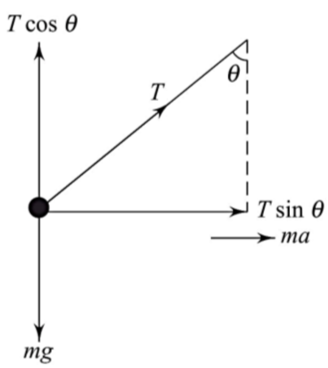


Fig. 3.24

$$T \sin \theta = ma \quad \text{(i)}$$

For equilibrium along the vertical direction

$$T \cos \theta = mg \quad \text{(ii)}$$

Dividing (i) and (ii), we get

$$\tan \theta = \frac{a}{g} \quad \Rightarrow \quad a = g \tan \theta = 9.8 \times \tan 60°$$

$$= 9.8 \times \sqrt{3} \simeq 17\,\text{ms}^{-2}$$

EXAMPLE 8 Two blocks of masses $m_1 = 1.5$ kg and $m_2 = 2$ kg are attached to each other by strings and pulleys as shown in Fig. 3.25. Assume that pulleys are massless and frictionless and strings are massless. The system is released. If the table is frictionless, find the accelerations of m_1 and m_2 and tensions in the strings. Take $g = 10$ ms^{-2}.

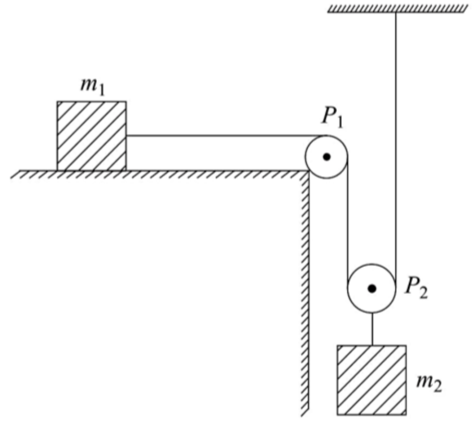


Fig. 3.25

SOLUTION Let a_1 and a_2 be the acceleration of m_1 and m_2 respectively. Let x_1 and x_2 be the distances moved by m_1 and m_2 in a time t. Since the total length of the string remains unchanged, it follows that if m_1 moves a distance x_1 to the right, m_2 will descend by a distance $x_2 = \frac{x_1}{2}$ or $x_1 = 2x_2$. Differentiating twice w.r.t. time we get

$$\frac{d^2x_1}{dt^2} = 2\frac{d^2x_2}{dt^2} \Rightarrow a_1 = 2a_2 \quad \text{(i)}$$

Figure 3.26 shows the free body diagrams of m_1 and m_2. Here T_1 = tension in string attached to m_1 and T_2 = tension in string attached to m_2.

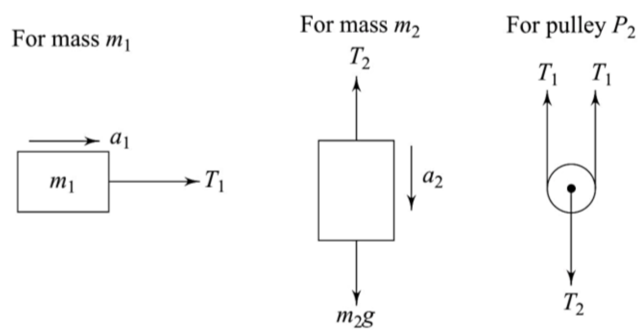


Fig. 3.26

For block m_1

$$T_1 = m_1 a_1 \quad \text{(i)}$$

For block m_2

$$m_2 g - T_2 = m_2 a_2 \quad \text{(ii)}$$

Since the pulley is massless and frictionless $T_2 = 2T_1$ (iii)

Also $a_1 = 2a_2$. (iv)

Using (iii) and (iv) in (ii), we have

$$m_2 g - 2T_1 = \frac{m_2 a_1}{2} \qquad \text{(v)}$$

Eliminating T_1 from (i) and (v), we get

$$a_1 = \frac{2m_2 g}{m_2 + 4m_1} = \frac{2\times 2\times 10}{2+4\times 1.5} = 5 \text{ ms}^{-2}$$

$$\therefore \quad a_2 = \frac{a_1}{2} = \frac{5}{2} = 2.5 \text{ ms}^{-2}$$

From eq. (i),

$$T_1 = m_1 a_1 = 1.5 \times 5 = 7.5 \text{ N}$$

$$\therefore \quad T_2 = 2T_1 = 2 \times 7.5 = 15 \text{ N}$$

EXAMPLE 9 Two blocks of masses $m_1 = 100$ g and $m_2 = 5$ kg with m_1 placed on m_2 are connected to a frictionless and massless pulley as shown in Fig. 3.27. The string connecting them is also massless. The coefficient of static friction between m_1 and m_2 is $\mu = 0.5$. There is no friction between m_2 and the horizontal surface. Find the maximum horizontal force F that can be applied on m_1 so that it does not slide on m_2.

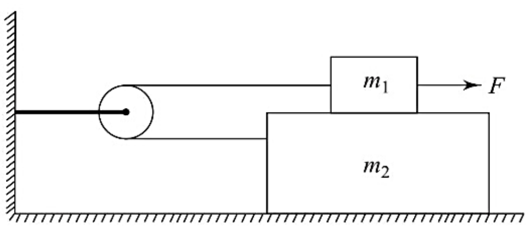


Fig. 3.27

SOLUTION Frictional force between m_1 and m_2 is $f = \mu R_1 = \mu m_1 g$. If T is the tension in the string, the free body diagrams of m_1 and m_2 are as shown in Fig. 3.28.

It follows from Fig. 3.28(a) that

$$R_1 = m_1 g \qquad \text{(i)}$$

and $$F - f - T = 0 \qquad \text{(ii)}$$

From Fig. 3.28(b), we have

$$T - f = 0 \Rightarrow T = f \qquad \text{(iii)}$$

and $$R_2 = R_1 + m_2 g \qquad \text{(iv)}$$

Using (iii) in (ii)

$$F - f - f = 0 \Rightarrow F = 2f = 2\mu m_1 g$$

Block m_1 will not slide on block m_2 if F is less than a maximum value F_{max} given by

$$F_{max} = 2\mu m_1 g$$

$$= 2 \times 0.5 \times 0.1 \times 10 = 1 \text{ N}$$

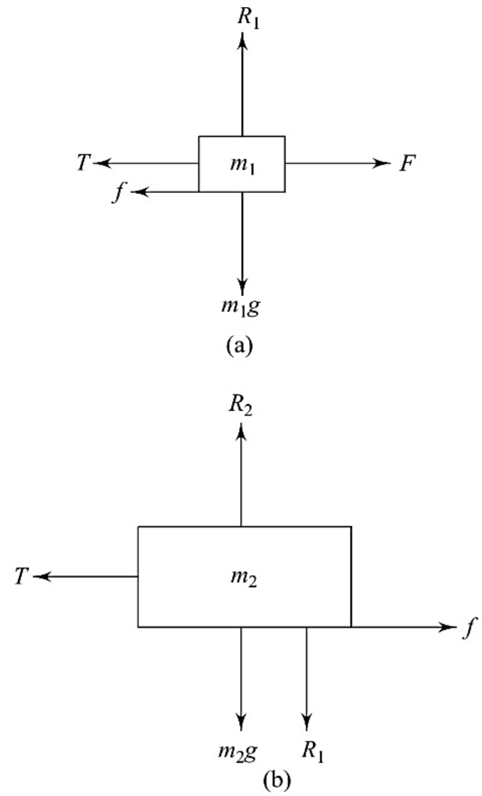


Fig. 3.28

EXAMPLE 10 A boy of mass $m = 50$ kg is standing on a weighing machine placed on the floor of an elevator. What is the weight of the boy when the elevator is (a) at rest, (b) moving up with an acceleration $a = 2.2 \text{ ms}^{-1}$ and (c) moving down with an acceleration 2.2 ms^{-2}.

SOLUTION The weighing machine reads the reaction R exerted by it on the boy. Fig. 3.29 shows the free body diagrams in the three cases.

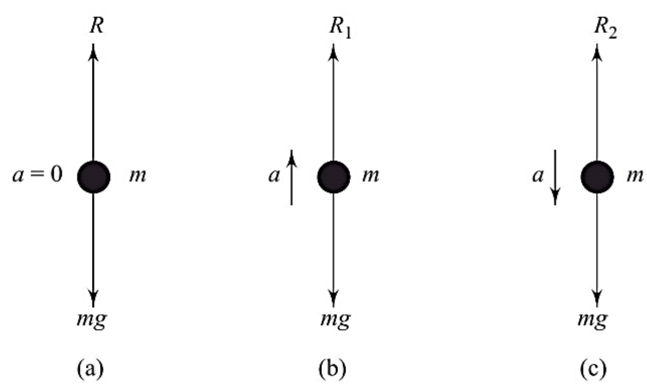


Fig. 3.29

(a) $R = mg = 50 \times 9.8 = 490$ N, the true weight of the boy.

(b) $R_1 - mg = ma$

$\Rightarrow R_1 = m(g + a) = 50 \times (9.8 + 2.2) = 600$ N

(c) $mg - R_2 = ma$

$\Rightarrow R_2 = m(g - a) = 50 \times (9.8 - 2.2) = 380$ N

> **Note** If the elevator is moving up or down with a uniform velocity $a = 0$ then the reading of the machine gives the actual weight.

EXAMPLE 11 A uniform cord AB of mass $M = 2$ kg and length $L = 100$ cm is pulled at ends A and B will forces $F_1 = 4$ N and $F_2 = 3$ N as shown in Fig. 3.30(a). Find the tension at point P at a distance $x = 20$ cm from end A.

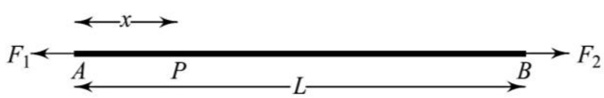


Fig. 3.30(a)

SOLUTION Since $F_1 > F_2$, the cord will accelerate to the left. Let a be the acceleration. To find tension T at P we consider the sections AP and PB of the cord.

$$\text{Mass of part } AP \text{ is } m_1 = \frac{Mx}{L}$$

$$\text{Mass of part } PB \text{ is } m_2 = \frac{M}{L}(L - x)$$

Figure 3.30(b) shows the free body diagrams of parts AP and PB.

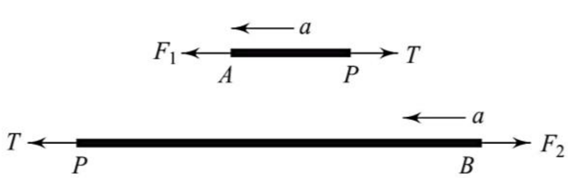


Fig. 3.30(b)

For part AP: $F_1 - T = m_1 a$ (i)

For part PB: $T - F_2 = m_2 a$ (ii)

Dividing (i) and (ii) we get

$$\frac{F_1 - T}{T - F_2} = \frac{m_1}{m_2} = \frac{\frac{Mx}{L}}{\frac{M}{L}(L-x)} = \frac{x}{L - x}$$

which gives $$T = \frac{F_1(L - x) + F_2 x}{L}$$

$$= \frac{4(1 - 0.2) + 3 \times 0.2}{1}$$

$= 3.8$ N

EXAMPLE 12 A monkey of mass $m = 30$ kg is climbing up a rope with an acceleration $a = 5$ ms^{-2} relative to the rope. The rope passes over a frictionless fixed pulley and has a block of mass $M = 15$ kg at the other end as shown in Fig. 3.31(a). Find (a) acceleration of the rope, (b) acceleration of monkey and (c) tension in the rope. Take $g = 10$ ms^{-2}.

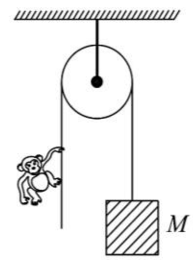


Fig. 3.31(a)

SOLUTION Let A be the acceleration of the block in the upward direction and T be the tension in the rope. The rope will have acceleration A in the downward direction. Hence the monkey will have a net acceleration $(A - a)$ in the downward direction. Figure 3.31(b) shows the free body diagrams of the monkey and the block.

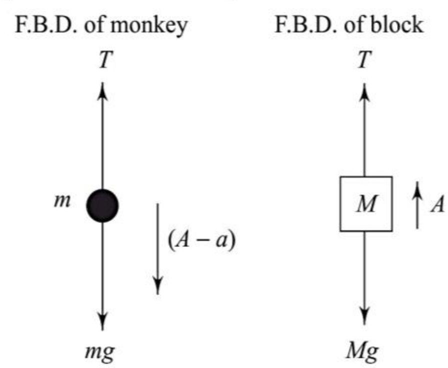


Fig. 3.31(b)

For monkey: $mg - T = m(A - a)$ (i)

For block: $T - Mg = MA$ (ii)

Adding (i) and (ii), we get

$$mg - Mg = m(A - a) + MA$$

$$\Rightarrow \quad A = \frac{m(g + a) - Mg}{M + m}$$

$$= \frac{30(10 + 5) - 15 \times 10}{15 + 30}$$

$= 6.7$ ms^{-2}

Acceleration of monkey $= A - a = 6.7 - 5$

$= 1.7$ ms^{-2}

Tension in the rope $T = M(g + A)$

$= 15 \times (10 + 6.7) \simeq 250$ N

EXAMPLE 13 A block of mass m = 2 kg is held stationary against a wall by applying a horizontal force F on it as shown in Fig. 3.32(a). If the coefficient of friction between the block and the wall is $\mu = 0.25$, find the minimum value of F required to hold the block against the wall. Take $g = 10$ ms^{-2}.

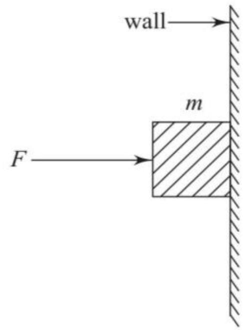


Fig. 3.32(a)

SOLUTION Let R be the normal reaction exerted on the block by the wall and f be the frictional force. Figure 3.32(b) shows the forces on the block.

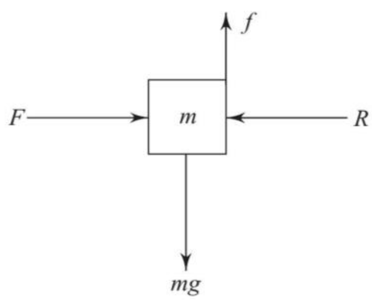


Fig. 3.32(b)

Since the block is held stationary, no net force acts on it. Hence

$$f = mg$$

and $$F = R$$

For no slipping, $f \le \mu R$ or $mg \le \mu F$ or $F \ge \dfrac{mg}{\mu}$

$$\therefore \qquad F_{\text{min}} = \frac{mg}{\mu} = \frac{2 \times 10}{0.25} = 80 \text{ N}$$

EXAMPLE 14 A block of mass m = 2 kg is held in contact with a block of mass M = 10 kg by applying a horizontal force F on it as shown in Fig. 3.33(a). Block M is lying on a horizontal frictionless surface. The coefficient of friction between the blocks is $\mu = 0.4$. Find the minimum value of F required to hold m against M. Take $g = 10$ ms^{-2}.

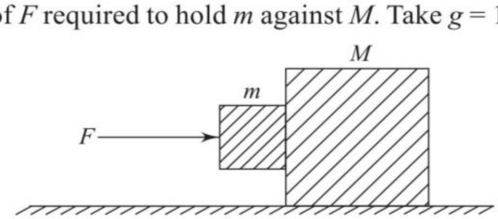


Fig. 3.33(a)

SOLUTION Since the two blocks are always in contact, they will have the same acceleration, say a. Figure 3.33(b) show the free body diagrams of the blocks.

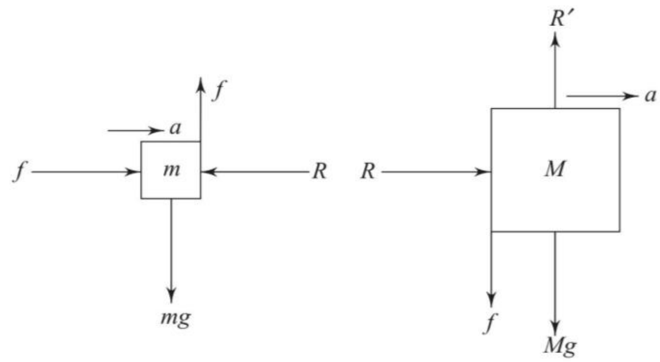


Fig. 3.33(b)

For block m : $\qquad F - R = ma \qquad$ (i)

and $\qquad f = mg \qquad$ (ii)

For block M : $\qquad R = Ma \qquad$ (iii)

and $\qquad Mg + f = R' \qquad$ (iv)

Eliminating a from (i) and (iii), we get

$$R = \frac{MF}{M+m}$$

For no slipping, $f \le \mu R$

or $$mg \le \frac{\mu MF}{M+m}$$

or $$F \ge \frac{mg(M+m)}{\mu M}$$

$$\therefore \qquad F_{\text{min}} = \frac{mg(M+m)}{\mu M}$$

$$= \frac{2 \times 10(10+2)}{0.4 \times 10} = 60 \text{ N}$$

EXAMPLE 15 A block P of mass m = 1 kg is placed over a plank Q of mass M = 6 kg placed over a smooth horizontal surface as shown in Fig. 3.34. Block P is given a velocity $v = 2$ ms^{-1} to the right. If the coefficient of friction between P and Q is $\mu = 0.3$, find the acceleration of Q relative to P.

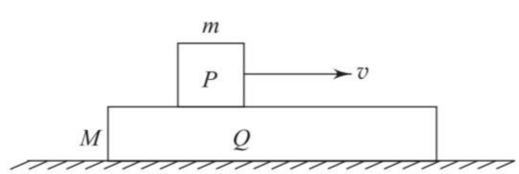


Fig. 3.34

SOLUTION Frictional force between P and Q is $f = \mu mg$ which will retard P and accelerate Q.

Retardation of P is $a_P = -\dfrac{f}{m} = -\dfrac{\mu mg}{m} = -\mu g$

Acceleration of Q is $a_Q = \frac{f}{M} = \frac{\mu mg}{M}$

$\therefore$ Acceleration of Q relative to P is

$$a_{QP} = a_Q - a_P = \frac{\mu mg}{M} - (-\mu g)$$

$$= \mu g\left(1 + \frac{m}{M}\right)$$

$$= 0.3 \times 10\left(1 + \frac{1}{6}\right)$$

$$= 3.5 \text{ ms}^{-2}$$

EXAMPLE 16 A block of mass $m = 500$ g is placed on the top of an inclined of inclination $\theta = 30°$ kept on the floor of a lift which is moving up with an acceleration $a = 2$ ms^{-2}. Find the coefficient of friction between the block and the incline so that the block moves down with a constant velocity.

SOLUTION $g_{eff} = g + a$

The block will move down the plane with a constant velocity if no net force acts on it, i.e.

Force down the plane = frictional force

$$mg_{eff} \sin\theta = \mu mg_{eff} \cos\theta$$

$$\Rightarrow \quad \mu = \tan\theta = \tan 30° \simeq 0.58$$

EXAMPLE 17 A cube of mass $m = 1$ kg is placed on a wedge of mass $M = 2$ kg as shown in Fig. 3.35(a). There is no friction between the cube and the wedge. Find the minimum coefficient of friction between the wedge and the horizontal surface so that the wedge does not move.

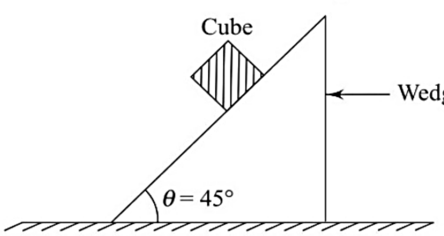


Fig. 3.35(a)

SOLUTION Figure 3.35(b) shows the horizontal force F_x and the vertical force F_y exerted by the cube on the wedge.

$F_x = (mg \cos\theta) \sin\theta$ and $F_y = (mg \cos\theta) \cos\theta$

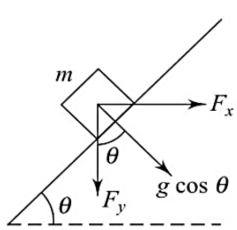


Fig. 3.35(b)

Weight of the wedge = Mg acting vertically down wards. Hence Net horizontal force on the wedge is

$$F = mg \cos\theta \sin\theta$$

Net vertical force on the wedge is

$$N = Mg + mg\cos^2\theta$$

$$\therefore \quad \mu_{min} = \frac{F}{N} = \frac{m\cos\theta\sin\theta}{M + m\cos^2\theta}$$

$$= \frac{1 \times \cos 45° \times \sin 45°}{2 + 1 \times \cos^2 45°} = 0.2$$

EXAMPLE 18 In Example 17 above there is no friction between the cube and the wedge and between the wedge and the horizontal surface below. If the wedge moves towards the right with an acceleration $a = \frac{1}{\sqrt{2}}$ ms^{-2}, find the acceleration of the cube relative to the wedge when the cube is released.

SOLUTION Let A be the acceleration of the cube relative to the wedge as the cube moves down the plane. Its acceleration when the wedge moves to the right with acceleration a is $(A\cos\theta - a)$ directed towards the left. For dynamic equilibrium,

$$m(A\cos\theta - a) = Ma$$

$$\Rightarrow \quad A = \frac{(M + m)a}{m\cos\theta}$$

$$= \frac{(2+1) \times 1/\sqrt{2}}{1 \times \cos 45°}$$

$$= 3 \text{ ms}^{-2}$$

EXAMPLE 19 A block of mass m is lying on the floor of a lift. With what acceleration a should the lift descend so that the block exerts a force $mg/3$ on the floor of the lift?

(a) $\frac{g}{3}$ (b) $\frac{2g}{3}$

(c) g (d) $\frac{3g}{2}$

SOLUTION Refer to Fig. 3.36.

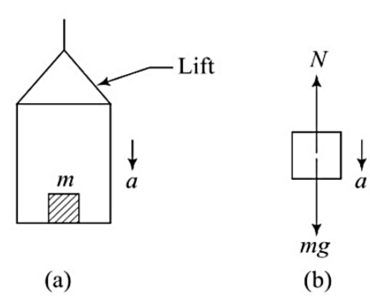


Fig. 3.36

Figure 3.36(b) shows the free body diagram of the block. It is clear that

$$mg - N = ma$$

For $$N = \frac{mg}{3},$$

$$mg - \frac{mg}{3} = ma$$

$$\Rightarrow \quad a = \frac{2g}{3}$$

EXAMPLE 20 A block of mass m is placed on a frictionless inclined plane of inclination θ. The inclined plane has its base fixed on the floor of a lift which is going up with a constant acceleration a. When the block is released, it will slide down the plane with acceleration

(a) $(g + a) \sin \theta$ (b) $(g - a) \sin \theta$
(c) $(g + a) \cos \theta$ (d) $(g - a) \cos \theta$

SOLUTION Refer to Fig. 3.37.

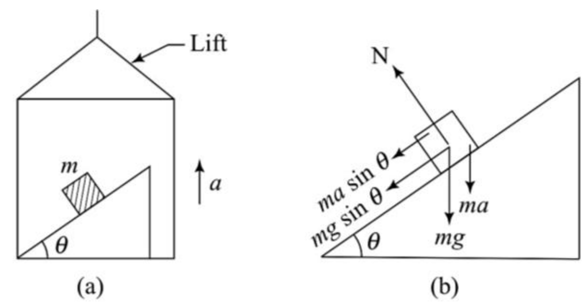


Fig. 3.37

Figure 3.37(b) shows the free body diagram of the block. The forces acting on the block are:

(i) weight mg downwards
(ii) normal reaction N perpendicular to the inclined plane
(iii) reaction force ma downwards (pseudo force).

The components of mg and ma parallel to the inclined plane are $mg \sin \theta$ and $ma \sin \theta$ respectively. The net force on the block sliding down the plane is

$$F = mg \sin \theta + ma \sin \theta = m (g + a) \sin \theta$$

$$\therefore \quad \text{Acceleration of the block} = \frac{F}{m} = (g + a) \sin \theta$$

EXAMPLE 21 In Example 22 above, the normal force acting on the block is

(a) $m(g + a) \sin \theta$ (b) $m (g - a) \sin \theta$
(c) $m (g + a) \cos \theta$ (d) $m (g - a) \cos \theta$

SOLUTION The components of mg and ma perpendicular to the inclined plane are $mg \cos \theta$ and $ma \cos \theta$ respectively [see Fig. 3.37(b)]. Since the block does not move perpendicular to the plane, the total force on it must be zero, i.e.

$$N - mg \cos \theta - ma \cos \theta = 0$$

$$\Rightarrow \quad N = m(g + a) \cos \theta$$

EXAMPLE 22 A block B of mass m placed on a horizontal frictionless surface is tied to a point A on a vertical wall by means of a massless string going over a frictionless moveable pulley P as shown in Fig. 3.38(a). A force F is applied to the pulley as shown. The acceleration of the block is

(a) $\frac{2F}{m}$ (b) $\frac{F}{m}$
(c) $\frac{F}{2m}$ (c) zero

SOLUTION Refer to Fig. 3.38.

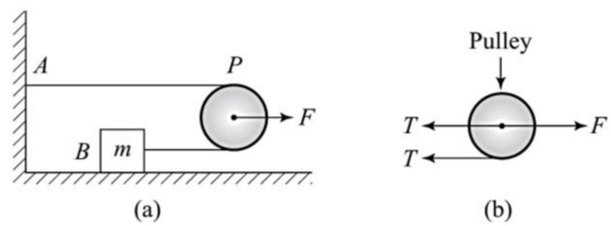


Fig. 3.38

Figure 3.38(b) shows the free body diagram of the pulley. The forces acting on the pulley are:

(i) F towards right
(ii) tension T towards left by portion PA of the string
(iii) tension T towards left by portion PB of the string

Since the mass of the pulley is zero, we have

$$F - T - T = 0$$

$$\Rightarrow \quad T = \frac{F}{2}$$

Now, for the block, the only horizontal force is tension T acting towards right, its acceleration is

$$a = \frac{T}{m} = \frac{F}{2m}$$

EXAMPLE 23 A block of mass m is suspended by strings AB and CB making angles α and β with the horizontal as shown in Fig. 3.39(a). If the strings have negligible mass, the tension in AB is

(a) $\frac{mg \cos \beta}{\sin (\alpha + \beta)}$ (b) $\frac{mg \sin \beta}{\sin (\alpha + \beta)}$

(c) $\frac{mg \cos \alpha}{\cos (\alpha + \beta)}$ (d) $\frac{mg \sin \alpha}{\cos (\alpha + \beta)}$

SOLUTION Refer to Fig. 3.39

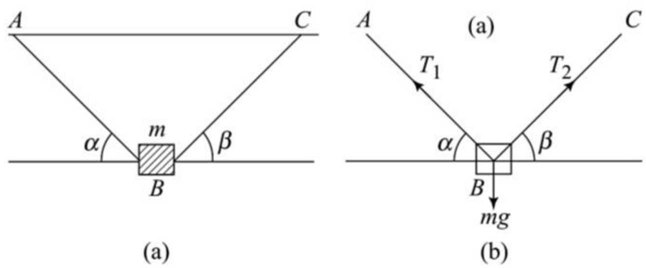


Fig. 3.39

Figure 3.39 (b) shows the free body diagram of the block. The horizontal components of T_1 and T_2 give

$$T_1 \cos \alpha = T_2 \cos \beta \qquad \text{(i)}$$

For the vertical components, we have

$$T_1 \sin \alpha + T_2 \sin \beta = mg \qquad \text{(ii)}$$

Eliminating T_2 from (i) and (ii), we get

$$T_1 = \frac{mg \cos \beta}{\sin (\alpha + \beta)}$$

EXAMPLE 24 Block A of mass m and block B of mass $2m$ are placed on a fixed triangular wedge by means of a massless string and a frictionless pulley as shown in the figure. The coefficient of friction between block A and the wedge is 2/3 and that between block B and the wedge is 1/3. If the blocks are released from rest, find the acceleration of block A.

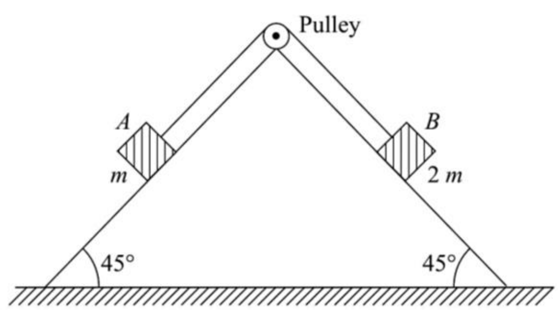


Fig. 3.40(a)

SOLUTION Case (a): Let us assume that block A moves up the plane and block B moves down the plane. The free body diagrams of the blocks are as follows.

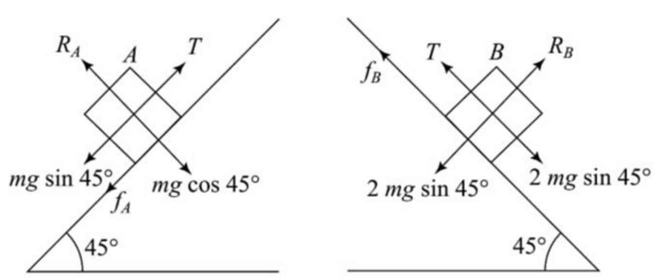


Fig. 3.40(b)

The equations of motion of blocks A and B are $T - mg \sin 45° - \mu_A\, mg \cos 45° = ma$, where $\mu_A = 2/3$ and $2\,\mu g \sin 45° - \mu_B\, 2\,mg \cos 45° - T = 2\,ma$, where $\mu_B = 1/3$.

Adding these equations and solving we get $a = -\dfrac{g}{9\sqrt{2}}$

Case (b): If we assume that block A moves down and block B moves up, we would get $a = -\dfrac{7g}{9\sqrt{2}}$. Thus in both cases, the acceleration has a negative value which implies that the blocks will decelerate. This is not possible because the blocks start from rest. Hence when the blocks are released, they move with zero acceleration. Thus acceleration of block $A = 0$.

9. Solving Problems in Mechanics by Constraint Relation Method

(1) *Constraint Relation for a Moveable Pulley*

Consider two blocks 1 and 2 attached at the ends of a string going over a moveable pulley. Let x_p be the displacement of the pulley and x_1 and x_2 be the displacements of the blocks.

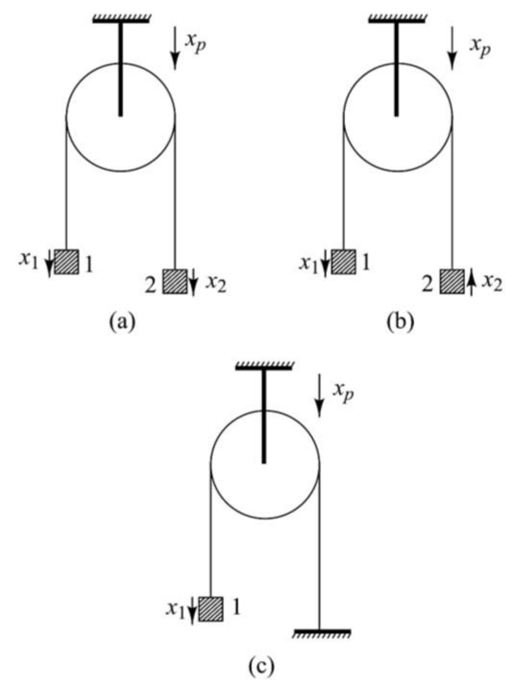


Fig. 3.41

The constraint relation is as follows. The displacement of the pulley = average displacement of the blocks, i.e.

(a) If the blocks are displaced in the direction of the displacement of the pulley, the constraint relation is [Fig. 3.41 (a)]

$$x_p = \frac{x_1 + x_2}{2}$$

(b) If block 1 moves up and block 2 moves down as shown in Fig. 3.41(b), the constraint relation is

$$x_p = \frac{x_1 - x_2}{2}$$

(c) If one ends of the string is connected to a fixed end, as shown in Fig. 3.41(c), then $x_2 = 0$ and the constraint relation is

$$x_p = \frac{x_1 + 0}{2} = \frac{x_1}{2} \quad \text{or} \quad x_1 = 2x_p$$

This result can also be obtained as proved in Example 25 below.

EXAMPLE 25 A block B of mass m placed on a horizontal frictionless surface is tied to a point A on a vertical wall by means of a masslass string going over a frictionless moveable pulley P as shown in Fig. 3.42. Show that if the pulley is displaced by a distance x_p, the block will be displaced by $2x_p$.

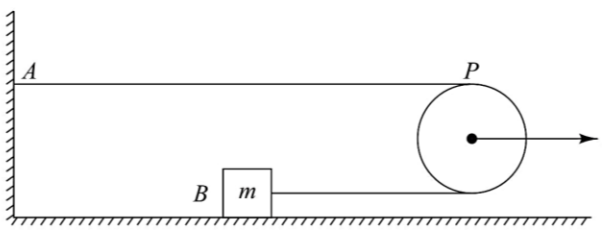


Fig. 3.42

SOLUTION Suppose the pulley is displaced to P' and block to B' (see Fig. 3.43)

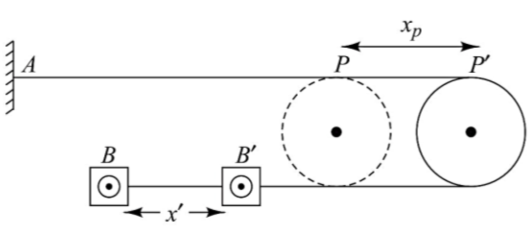


Fig. 3.43

Since the total length of the string remains the same

$$AP + PB = AP + PP' + P'P + PB'$$

Hence $$AP + PB' + B'B = AP + 2PP' + PB'$$

$$B'B = 2PP'$$

$$\Rightarrow \quad x = 2x_p$$

EXAMPLE 26 If a force F is applied to the pulley as shown in Fig. 3.42, find (a) the acceleration of the block and (b) the acceleration of the pulley.

SOLUTION Refer to the solution of Example 22 on page 3.15.

(a) The acceleration of the block is $a = \dfrac{F}{2m}$

(b) Since the displacement of the pulley = half the displacement of the block, the acceleration of the pulley is

$$a_p = \frac{a}{2} = \frac{F}{4m}$$

EXAMPLE 27 Figure 3.44 shows two blocks 1 and 2 connected by means of strings to two pulleys P_1 and P_2. Pulley P_1 is fixed and pulley P_2 is moveable. Show that, if block 1 is moved down by a distance x_1, the block 2 moves up by a distance $x_2 = \dfrac{x_1}{2}$.

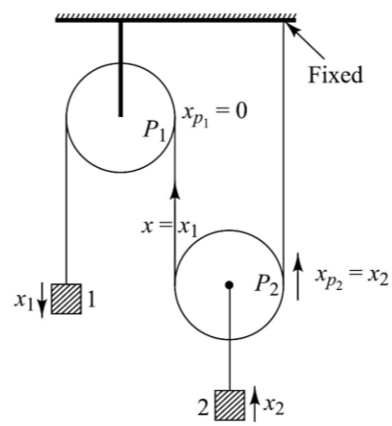


Fig. 3.44

SOLUTION As pulley P_1 is fixed $x_{P_1} = 0$. Therefore, if block 1 is moved down by x_1, the other end of the string must move up by x_1, because

$$x_{P_1} = 0 = \frac{x_1 + x}{2} \quad \Rightarrow \quad x = -x_1$$

Since the other end of the string going over pulley P_2 is fixed,

$$x_{P_2} = \frac{x_1 + 0}{2} = \frac{x_1}{2}$$

which gives $x_1 = 2x_{P_2} \Rightarrow x_1 = 2x_2 \Rightarrow x_2 = \dfrac{x_1}{2}$

EXAMPLE 28 Figure 3.45 shows two blocks 1 and 2 connected by means of strings to two pulleys P_1 and P_2. Pulley P_1 is fixed and pulley P_2 is moveable. Show that, if block 1 is moved up by x_1, the block 2 moves down by $x_2 = 2x_1$.

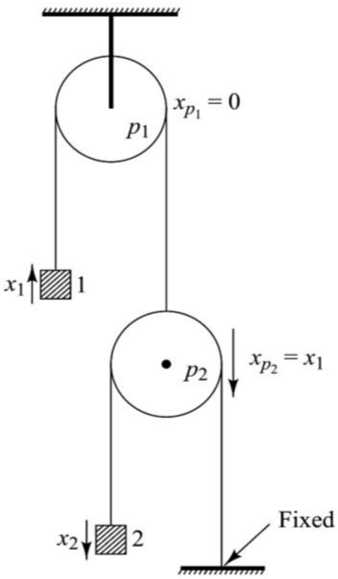


Fig. 3.45

SOLUTION As pulley P_1 is fixed, pulley P_2 will move down by x_1. Hence

$$x_{P_2} = x_1$$

For pulley 2,

$$x_{P_2} = -x_1 = \frac{x_2 + 0}{2} \quad \Rightarrow \quad x_2 = -2x_1.$$

The negative sign shows that if block 1 moves up, block 2 will move down.

(2) *Constraint Relation for a Moveable Wedge (or Inclined Plane)*

Consider a block of mass m moving with a certain velocity on a wedge (inclined plane) of inclination θ as shown in Fig. 3.46(a). The wedge in moved with a velocity u as shown.

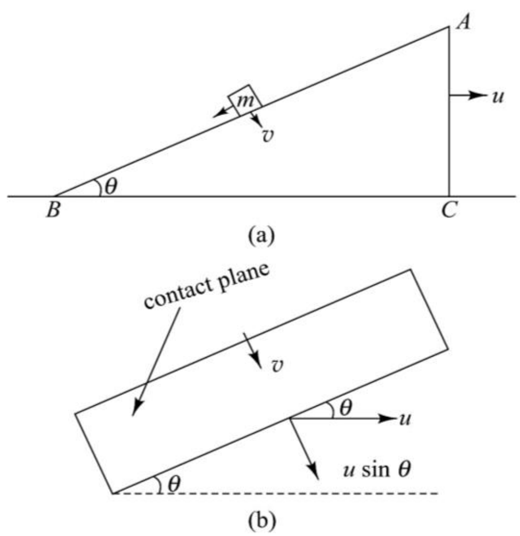


Fig. 3.46

To find the velocity v of the block perpendicular to the contact plane AB, we use the following constraint relation: *The relative velocity of the block (with respect to the wedge) in a direction perpendicular to the contact plane is always zero*. i.e. velocity v of the block perpendicular to AB = component of velocity v of the block perpendicular to AB. From Fig. 3.46 (b) it follows that

$$v = u \sin \theta$$

Note : If the wedge is immoveable or is at rest, $u = 0$ then $v = 0$, i.e. the block cannot move perpendicular to AB. This happens because the component $mg \cos \theta$ of the weight of the block balances with the normal reaction.

(3) *Constraint Relation when the Distance Between Two Points Always Remains Constant*

In cases when the distance between two points always remains constant, we use the following constraint relation. *The relative velocity of one point of an object with respect to any other point on the same object in the direction of the line joining them always remains equal to zero*, i.e. the velocity of one point on the object = the components of velocity of any other point along the line joining them. This is illustrated by the following examples 29 and 30.

EXAMPLE 29 A rod AB of length L is leaning on a wall and the floor at an angle θ as shwon in Fig. 3.47(a). The end A is moved with a constant velocity u to the left. Find the velocity v with which the end B moves downwards.

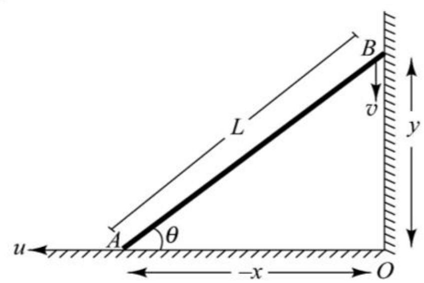


Fig. 3.47(a)

SOLUTION Using the constraint relation, (since the distance between points A and B always remains constant = L), we have [see Fig. 3.47(b)]

Velocity of B along BA = velocity of A along BA.

or $\quad v \sin \theta = u \cos \theta$

$\Rightarrow \quad v = u \cot \theta$

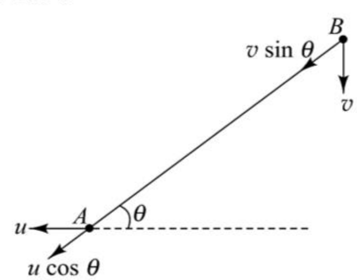


Fig. 3.47(b)

Alternative Method

$$x^2 + y^2 = L^2$$

Differentiating w.r.t. time t, we have (since L = constant)

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

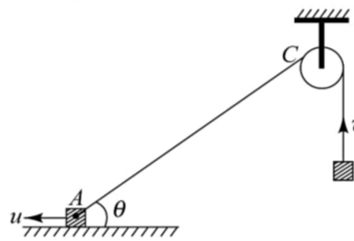
$\Rightarrow$ $$u = -v\,\frac{y}{x} \quad \left(\because \frac{dx}{dt} = u,\ \frac{dy}{dt} = v\right)$$

$\Rightarrow$ $$= +v\,\frac{y}{-x}$$

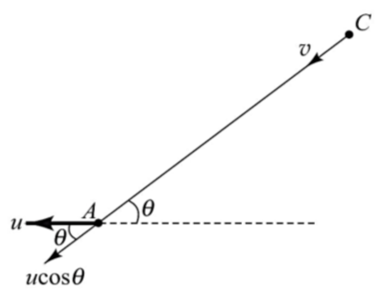
$\Rightarrow$ $u = v\tan\theta$ $\qquad \left(\because \tan\theta = \dfrac{OB}{OA}\right)$

$\Rightarrow$ $v = u\cot\theta$

EXAMPLE 30 A block A placed one horizontal frictionless surface is tied to another block B by means of an inextensible string going over a pulley as shown in Fig. 3.48(a). The block A is moved towards left with a velocity u. Find the velocity with which block B moves upwards.



(a)



(b)

Fig. 3.48

SOLUTION Since the distance between points A and C remains constant [see Fig. 3.48(b)] and the pulley is fixed, velocity of point C along $CA = v$, the velocity of block B upwards.

Velocity of point C along CA = velocity of point A along CA

or $$v = u\cos\theta$$

10. Centripetal Acceleration

If a body moves in a circle at a constant speed, it is said to be in uniform circular motion. In such a motion, the magnitude of the velocity (i.e. speed) is constant but the direction of the velocity vector is continually changing. Thus the velocity is changing with time. Hence the motion of the body is accelerated (see Fig. 3.49). The acceleration is directed towards the centre of the circle and is called centripetal acceleration. The magnitude of the centripetal acceleration is given by

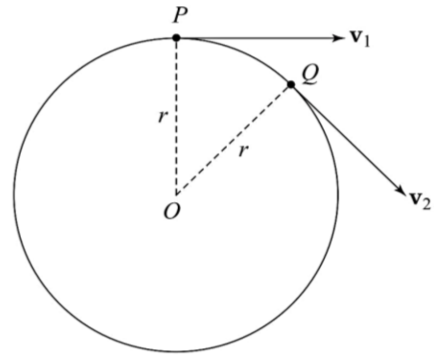


Fig. 3.49

$$a_c = \omega v$$

where ω is the angular velocity (or angular frequency) and v is the speed along the circle. Since $v = r\omega$, we have

$$a_c = \omega v = \omega^2 r = \frac{v^2}{r}$$

where r is the radius of the circular path. The angular frequency is related to time period T and frequency ν as

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

Therefore, centripetal acceleration is also given by (since $v = 2\pi r/T$)

$$a_c = \omega v = \frac{4\pi^2 r}{T^2} = 4\pi^2 r\nu^2$$

11. Banking of Round Tracks

When a car (or some other vehicle) negotiates a curved level road, the centripetal force required to keep the car in motion around the curve is provided by the friction between the road and the tyres. The weight of the car is supported by the normal reaction due to the earth. If the surface of the road is very rough, it provides a large amount of friction and hence the car can successfully negotiate the bend with a fairly high speed. If F is the total frictional force between the tyres and the road, then

$$F = \frac{mv^2}{R}$$

when m is the mass of the car, v its speed around the curve and R is the radius of the curved track. The higher the value of F, the faster is the speed at which the bend can be negotiated. If μ is the coefficient of friction between the tyres and the road, then $F \le \mu N$; N = normal reaction = mg. The maximum speed which friction can sustain is

$$v \le v_{max} = (\mu Rg)^{1/2}$$

The large amount of friction between the tyres and the road would damage the tyres. To minimize the wearing out of tyres the road bed is banked, i.e. the outer part of the road is raised a little so that the road slopes towards the centre of the curved track. Suppose a car of mass m is moving around a banked track in a circular path of radius R as shown in Fig. 3.50. Let N_1 and N_2 be the reaction at each tyre due to the road. Then the total reaction is $N = N_1 + N_2$ acting in the middle of the car. If θ is the angle of the banking, the vertical component $N\cos\theta$ supports the weight mg of the car while the horizontal component $N\sin\theta$ provides the necessary centripetal force.

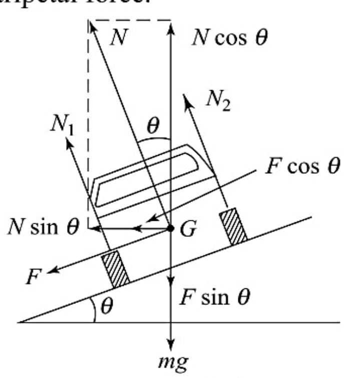


Fig. 3.50 Car on banked curved road

Thus

$$N\sin\theta = \frac{mv^2}{R} - F\cos\theta$$

and $$N\cos\theta = mg + F\sin\theta$$

Also $$F = \mu N$$

where F is the force of friction acting radially inwards on the car. These equations give

$$\tan\theta = \frac{v^2 - \mu Rg}{Rg + \mu v^2} \text{ and } v^2 = Rg\,\frac{\mu + \tan\theta}{1 - \mu\tan\theta}$$

The first equation determines the proper banking angle for given v, R and μ, and the second equation the maximum speed at which the car can successfully negotiate the curve for given R, μ and θ.

For given θ and R, there is an optimum (best) speed for negotiating a banked curve at which there will be the least wear and tear, i.e. when friction is not needed at all (μ = 0). If $\mu = 0$, this speed is

$$v = (Rg\tan\theta)^{1/2}$$

The car will not skid if the angle of banking of the track satisfies the relation

$$\tan\theta = \frac{v^2}{Rg}$$

12. A Cyclist Negotiating a Curved Level Road

While negotiating a curved level (unbanked) road, a cyclist has to lean inwards which provides the necessary centripetal force which prevents him from falling down. Figure 3.51 shows a cyclist leaning at an angle θ with the vertical. N is the normal reaction which is given by

$$N = mg$$

where m is the mass of the cyclist plus the bicycle. The force of friction between the road and the tyres is

$$F = \mu N = \mu mg$$

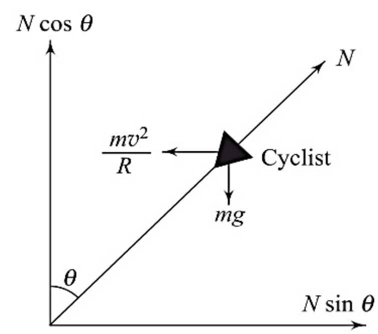


Fig. 3.51

The cyclist will skid if the centripetal force mv^2/R exceeds the frictional force F, i.e. if

$$\frac{mv^2}{R} > \mu mg$$

where R is the radius of the curved road. Thus skidding occurs if

$$v > \sqrt{\mu gR}$$

13. Motion in a Vertical Circle

Figure 3.52 shows an object of mass m whirled with a constant speed v in a vertical circle of centre O with a string of length R. When the object is at top A of the circle, let us say that the tension (force) in the string is T_1. Since the weight mg acts vertically downwards towards the centre O, we have,

Force towards centre, $$F = T_1 + mg = \frac{mv^2}{R}$$

or $$T_1 = \frac{mv^2}{R} - mg \quad \text{(i)}$$

At the point B, where OB is horizontal, the weight mg has no component along OB. Thus, if the tension in the string is T_2 at B, we have

Force towards centre,

$$F = T_2 = \frac{mv^2}{R} \quad \text{(ii)}$$

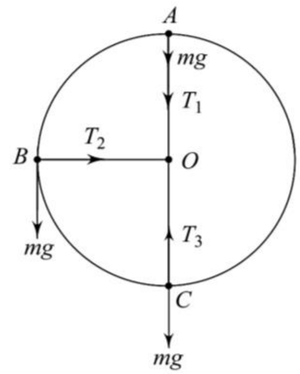


Fig. 3.52

At C, the lowest point of motion, the weight mg acts in the opposite direction to the tension T_3 in the string. Thus at C we have,

Force towards centre, $F = T_3 - mg = \dfrac{mv^2}{R}$

or $$T_3 = \frac{mv^2}{R} + mg$$

From (i), (ii) and (iii), we see that the maximum tension occurs at lowest point C of the motion. Here the tension T_3 must be greater than mg by $\dfrac{mv^2}{R}$ to keep the object in a circular path. The minimum tension is given by (i) when the object is at the highest point A of the motion. Here part of the required centripetal force is provided by the weight and the rest by T_1.

In order to keep a body of mass m in a circular path, the centripetal force, at the highest point A, must at least be equal to the weight of the body. Thus

$$\frac{mv_A^2}{R} = mg \quad \text{or} \quad v_A^2 = Rg \quad \text{or} \quad v_A = \sqrt{Rg}$$

gives the minimum speed the body must have at the highest point so that it can complete the circle. Then the minimum speed v_C the body must have at the lowest point C is given by

$$v_C^2 = v_A^2 + 2 \times 2\,Rg$$

where we have used $v^2 = u^2 + 2gh$, with $h = 2R$. Thus

$\therefore$ $$v_C^2 = Rg + 4\,Rg = 5\,Rg$$

or $$v_C = \sqrt{5Rg}$$

The tension at this point is given by

$$T_1 = m\left(\frac{v_C^2}{R} + g\right) = m\,(5g + g)$$
$$= 6\,mg$$

EXAMPLE 31 A body of mass $m = 20$ g is attached to an elastic spring of length $L = 50$ cm and spring constant $k = 2\ \text{Nm}^{-1}$. The system is revolved in a horizontal plane with a frequency $\nu = 30$ rev/min. Find the radius of the circular motion and the tension in the spring.

SOLUTION Angular velocity

$$\omega = 2\pi\nu = 2\pi \times \frac{30}{60}$$
$$= \pi\ \text{rad s}^{-1}.$$

For an elastic spring force $F = kx$ where x is the extension.

Radius of circular motion $r = L + x$.

Centripetal force $= mr\omega^2 = F$

Or $$m(L + x)\omega^2 = kx$$

$\Rightarrow$ $$x = \frac{mL\omega^2}{k - m\omega^2}$$
$$= \frac{0.02 \times 0.5 \times (3.14)^2}{2 - 0.02 \times (3.14)^2}$$
$$\simeq 0.05\ \text{m}$$

$\therefore$ $$r = L + x = 0.5 + 0.05$$
$$= 0.55\ \text{m}$$

Tension $$F = kx = 2 \times 0.05 \simeq 0.1\ \text{N}$$

EXAMPLE 32 A liquid of mass M and density ρ is filled in a tube AB of length L. The tube is rotated about end A with angular velocity ω. Obtain the expression for the force exerted at the other end B.

SOLUTION

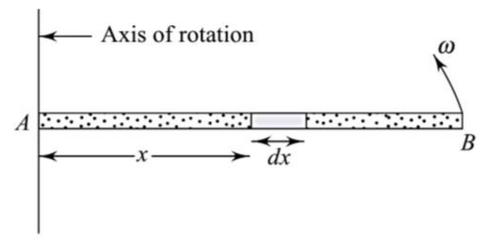


Fig. 3.53

Mass of element of length dx is $m = \dfrac{M}{L}dx$

$$\text{Centripetal force on element} = \int_o^L m\omega^2 x$$
$$= \frac{M\omega^2}{L}\int_o^L x\,dx$$
$$= \frac{1}{2}ML\omega^2$$

EXAMPLE 33 A conical pendulum has a string of length $l = 50$ cm and bob of mass $m = 200$ g. The bob is revolved in a horizontal circle of radius $r = 20$ cm. If the string makes an angle $\theta = 60°$ with the vertical, find (a) the tension in the string and (b) the speed of the bob around the circle. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION Refer to Fig. 3.54,

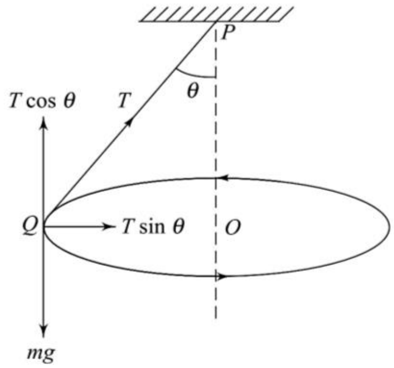


Fig. 3.54

$$OQ = r \text{ and } PQ = l$$

Force towards centre $O = T \sin\theta$

i.e. $$\frac{mv^2}{r} = T\sin\theta \quad \text{(i)}$$

Also $$mg = T\cos\theta \quad \text{(ii)}$$

(a) From (ii) $$T = \frac{mg}{\cos\theta} = \frac{0.2\times 10}{\cos 60^\circ} = 4\text{ N}$$

(b) Dividing (i) by (ii) we have

$$\frac{v^2}{rg} = \tan\theta$$

$$\Rightarrow \quad v = \sqrt{rg\tan\theta}$$

$$= \sqrt{0.2\times 10\times \tan 60^\circ}$$

$$= 1.86\text{ ms}^{-1}$$

EXAMPLE 34 A coin of mass $m = 10$ g is placed at a distance of 30 cm from the centre of a disc. The disc is rotated at 30 rev/min about a vertical axis passing through its centre. What should be the minimum value of the coefficient of friction between the coin and the disc so that the coin does not skid off the disc?

SOLUTION For no slipping, frictional force > centripetal force, i.e.

$$\mu mg > m\omega^2 r$$

$$\Rightarrow \quad \mu > \frac{\omega^2 r}{g}$$

$$\Rightarrow \quad \mu > \frac{(2\pi\nu)^2 r}{g}$$

$$\nu = 30\text{ rev/min} = \frac{30}{60} = 0.5\text{ Hz},$$

$$r = 0.3\text{ m and } g = 9.8\text{ ms}^{-2}.$$

$$\therefore \quad \mu_{\min} = \frac{4\pi^2\nu^2 r}{g}$$

$$= \frac{4\times(3.14)^2\times(0.5)^2\times 0.3}{9.8}$$

$$= 0.3$$

EXAMPLE 35 A small sphere of mass $m = 500$ g moving on the inner surface of a large hemispherical bowl of radius $R = 5$ m describes a horizontal circle at a distance $OC = 2.5$ m below the centre O of the bowl as shown in Fig. 3.55. Find the force exerted by the sphere on the bowl and the time period of revolution of the sphree around the circle. Take $g = 10\text{ ms}^{-2}$.

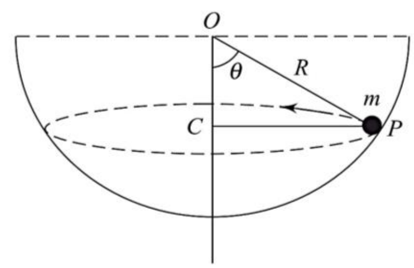


Fig. 3.55

SOLUTION Given $OP = 5$ m and $OC = 2.5$ m. Therefore $\cos\theta = \frac{OC}{OP} = \frac{1}{2} \Rightarrow \theta = 60^\circ$.

Radius of circle is $r = CP = OP\sin\theta = 5\sin 60^\circ = \frac{5\times\sqrt{3}}{2}$ m

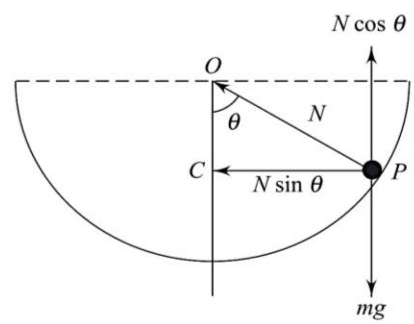


Fig. 3.56

Figure 3.56 shows the forces acting on the sphere, N is the normal reaction.

Net force towards centre C of the circle = $N\sin\theta$

$$\Rightarrow \quad mr\omega^2 = N\sin\theta \quad \text{(i)}$$

Also $$mg = N\cos\theta \quad \text{(ii)}$$

From (ii), $$N = \frac{mg}{\cos\theta} = \frac{0.5\times 10}{\cos 60^\circ} = 10\text{ N}$$

From (i), $$m\omega^2 R\sin\theta = N\sin\theta$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{N}{mR}} = \sqrt{\frac{10}{0.5 \times 5}} = 2 \text{ rad s}^{-1}$$

$$\therefore \text{ Time period} \qquad T = \frac{2\pi}{\omega} = \pi \text{ second} = 3.14 \text{ s}$$

EXAMPLE 36 A steel metre rod of mass $m = 1.5$ kg rests with its upper end against a smooth vertical wall and its lower end on rough horizontal floor. What should be the minimum coefficient of friction between the ground and the rod so that it can be inclined at an angle of 30° with the floor without slipping?

SOLUTION Refer to Fig. 3.57. Length of rod $AB = 1$ m

N_1 = normal reaction of the wall

N_2 = normal reaction of the floor

The frictional force (f) between the rod and the floor acts along AD. The weight mg of the rod acts at its mid-point (centre of mass) C so that $AC = AB/2$.

For translational equilibrium,

$$N_2 = mg \quad \text{and} \quad N_1 = f$$

Taking moments of forces about A, we have for rotational equilibrium

$$N_2 \times 0 - mg \times AD + N_1 \times BE = 0$$

$$\Rightarrow \; 0 - N_2 \times AD + f \times BE = 0 \; (\because mg = N_2, N_1 = f)$$

$$\Rightarrow \qquad \frac{f}{N_2} = \frac{AD}{BE} = \frac{AC\cos 30^\circ}{AB\sin 30^\circ}$$

$$\Rightarrow \qquad \mu = \frac{1}{2} \times \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{2} = 0.87$$

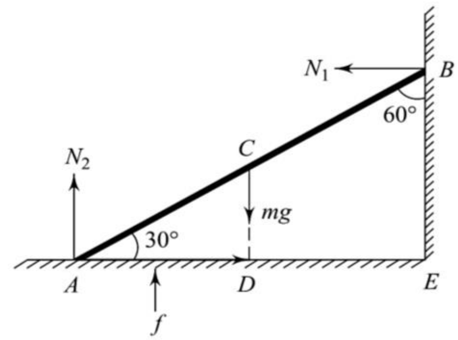


Fig. 3.57

1 SECTION Multiple Choice Questions with One Correct Choice

Level A

1. Two masses m and $2m$ are joined to each other by means of a frictionless pulley as shown in Fig. 3.58. When the mass $2m$ is released, the mass m will ascend with an acceleration of

(a) $\frac{g}{3}$ (b) $\frac{g}{2}$

(c) g (d) $2\,g$

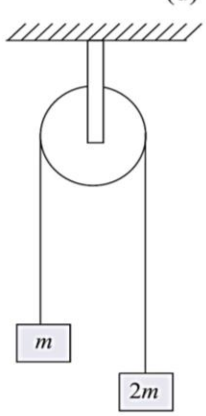


Fig. 3.58

2. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force F is applied at the free end of the rope, the net force exerted on the block will be

(a) $\frac{FM}{(M+m)}$ (b) $\frac{Fm}{(M+m)}$

(c) $\frac{FM}{(M-m)}$ (d) F

3. A block is pulled along a horizontal frictionless surface by a rope. The tension in the rope will be the same at all points on it

(a) if and only if the rope is not accelerated
(b) if and only if the rope is massless
(c) if either the rope is not accelerated or is massless
(d) always

4. A block is placed on the top of a smooth inclined plane of inclination θ kept on the floor of a lift. When the lift is descending with a retardation a, the block is released. The acceleration of the block relative to the incline is

(a) $g \sin\theta$ (b) $a \sin\theta$
(c) $(g-a)\sin\theta$ (d) $(g+a)\sin\theta$

5. A block takes twice as much time to slide down a rough 45° inclined plane as it takes to slide down an identical smooth 45° inclined plane. The coefficient of kinetic friction between the block and the rough inclined plane is
(a) 0.25 (b) 0.50
(c) 0.75 (d) 1.0

6. The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half is rough. A block starting from rest at the top of the plane will again come to rest at the bottom if the coefficient of friction between the block and the lower half of the plane is given by
(a) $\mu = 2\tan\theta$ (b) $\mu = \tan\theta$
(c) $\mu = \dfrac{2}{\tan\theta}$ (d) $\mu = \dfrac{1}{\tan\theta}$

7. A boy of mass m stands on one end of a wooden plank of length L and mass M. The plank is floating on water. If the boy walks from one end of the plank to the other end at a constant speed, the resulting displacement of the plank is given by
(a) $\dfrac{mL}{M}$ (b) $\dfrac{ML}{m}$
(c) $\dfrac{mL}{(M+m)}$ (d) $\dfrac{mL}{(M-m)}$

8. Ball A of mass m_1 moving with a velocity v undergoes a head–on collision with ball B of mass m_2 at rest. After collision, ball A continues moving in its original direction with half its original speed. The speed of ball B after collision will be
(a) $\dfrac{m_1 v}{2m_2}$ (b) $\dfrac{m_2 v}{2m_1}$
(c) $\dfrac{m_1 v}{m_2}$ (d) $\dfrac{m_2 v}{m_1}$

9. A shell of mass $2m$ fired with a speed u at an angle θ to the horizontal explodes at the highest point of its trajectory into two fragments of mass m each. If one fragment falls vertically, the distance at which the other fragment falls from the gun is given by
(a) $\dfrac{u^2 \sin 2\theta}{g}$ (b) $\dfrac{3u^2 \sin 2\theta}{2g}$
(c) $\dfrac{2u^2 \sin 2\theta}{g}$ (d) $\dfrac{3u^2 \sin 2\theta}{g}$

10. A truck carrying sand is moving on a smooth horizontal road with a uniform speed u. If a mass Δm of sand leaks in a time Δt from the bottom of the truck, the force needed to keep the truck moving at its uniform speed u is given by
(a) $\dfrac{\Delta m u}{\Delta t}$ (b) $\dfrac{\Delta m u}{2\Delta t}$
(c) $\dfrac{\Delta m u^2}{\Delta t}$ (d) zero

11. A block of mass m_2 lying on a horizontal frictionless surface is connected to a block of mass m_1 by means of string which passes over a frictionless pulley as shown in Fig. 3.59. If $m_1 > m_2$, the common acceleration of the masses is given by (g is the acceleration due to gravity)

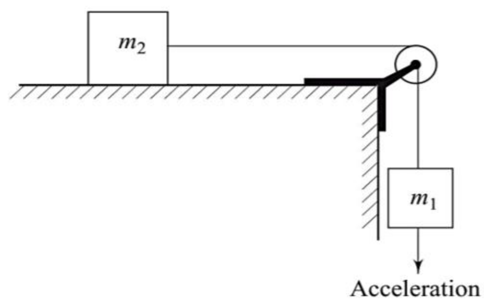


Fig. 3.59

(a) $\dfrac{m_1}{m_2} g$ (b) $\dfrac{m_2}{m_1} g$
(c) $\dfrac{m_1}{(m_1+m_2)} g$ (d) $\dfrac{m_2}{(m_1+m_2)} g$

12. A force F is applied horizontally to block A of mass m_1 which is in contact with a block B of mass m_2, as shown in Fig. 3.60. If the surfaces are frictionless, the force exerted by A on B is given by

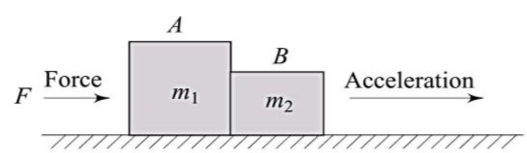


Fig. 3. 60

(a) $\dfrac{m_1}{m_2} F$ (b) $\dfrac{m_2}{m_1} F$
(c) $\dfrac{m_1 F}{(m_1+m_2)}$ (d) $\dfrac{m_2 F}{(m_1+m_2)}$

13. Two skaters A and B of mass 50 kg and 70 kg, respectively, stand facing each other, 6 metres apart on a horizontal smooth surface. They pull a rope

stretched between them. How far has each moved when they meet?
(a) Both have moved 3 m.
(b) A moves 4 m and B moves 2 m.
(c) A moves 2.5 m and B moves 3.5 m.
(d) A moves 3.5 m and B moves 2.5 m.

14. Which of the following statements is correct?
(a) A body has a constant velocity but a varying speed.
(b) A body has a constant speed but a varying acceleration.
(c) A body having a constant speed cannot have an acceleration.
(d) A body having a constant speed can have a varying velocity.

15. A person is sitting facing the engine in a moving train. He tosses a coin. The coin falls behind him. This shows that the train is
(a) moving forward with a finite acceleration
(b) moving forward with a finite retardation
(c) moving backward with a uniform speed
(d) moving forward with a uniform speed.

16. N bullets each of mass m kg are fired with a velocity v ms^{-1}, at the rate of n bullets per second, upon a wall. The reaction offered by the wall to the bullets is given by
(a) $nNmv$ (b) $\frac{Nmv}{n}$
(c) $\frac{nNm}{v}$ (d) $\frac{nNv}{m}$

17. A block A is released from the top of smooth inclined plane and slides down the plane. Another block B is dropped from the same point and falls vertically downwards. Which one of the following statements will be true if the friction offered by air is negligible?
(a) Both blocks will reach the ground at the same time.
(b) Block A reaches the ground earlier than block B.
(c) Both blocks will reach the ground with the same speed.
(d) Block B reaches the ground with a higher speed than block A.

18. A block is released from the top of an inclined plane of height h and angle of inclination θ. The time taken by the block to reach the bottom of the plane is given by
(a) $\sqrt{\frac{2h}{g}}$ (b) $\sin\theta\sqrt{\frac{2h}{g}}$
(c) $\frac{1}{\sin\theta}\cdot\sqrt{\frac{2h}{g}}$ (d) $\frac{1}{\cos\theta}\cdot\sqrt{\frac{2h}{g}}$

19. A shell, initially at rest, suddenly explodes into two equal fragments A and B. Which one of the following is observed?
(a) A and B move in the same direction at the same speed.
(b) A and B move at right angles to each other at the same speed.
(c) A and B move in opposite directions at the same speed.
(d) A and B move in any direction at the same speed.

20. A bomb at rest explodes into a large number of tiny fragments. The total momentum of all the fragments
(a) is zero
(b) depends on the total mass of all the fragments
(c) depends on the speeds of various fragments
(d) is infinity

21. A block of mass M is resting on an inclined plane as shown in Fig. 3.61. The inclination of the plane to the horizontal is gradually increased. It is found that when the angle of inclination is θ the block just begins to the slide down the plane. What is the minimum force F applied parallel to the plane that would just make the block move up the plane?

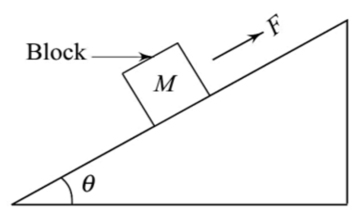


Fig. 3.61

(a) $Mg\sin\theta$ (b) $Mg\cos\theta$
(c) $2Mg\cos\theta$ (d) $2Mg\sin\theta$

22. The driver of a car moving at a speed of 20 ms^{-1} sees a child standing in the middle of the road. He immediately applies brakes to bring the car to rest in 5 s, just in time to save the child. If the mass of the car is 940 kg that of the driver 60 kg, what is the magnitude of the retarding force on the vehicle?
(a) 1000 N (b) 2000 N
(c) 3000 N (d) 4000 N

23. In Q.22, how far was the child from the car when the brakes were applied?
(a) 50 m (b) 75 m
(c) 100 m (d) 125 m

24. A body of mass 200 g is moving with a velocity of 5 ms^{-1} along the positive x–direction. At time $t = 0$, when the body is at $x = 0$, a constant force of 0.4 N

directed along the negative x–direction is applied to the body for 10 s. What is the position (x) of the body at $t = 2.5$ s?

(a) $x = 1.0$ m (b) $x = 1.25$ m
(c) $x = 1.5$ m (d) $x = 1.75$ m

25. In Q.24, what is the velocity of the body at $t = 2.5$ s?

(a) 7.5 ms^{-1} (b) 6.25 ms^{-1}
(c) 5.0 ms^{-1} (d) zero

26. In Q.24, what is the position (x) of the body at $t = 30$ s?

(a) $x = -350$ m (b) $x = -400$ m
(c) $x = -450$ m (d) $x = -500$ m

27. In Q. 24, what is the speed of the body at $t = 30$ s?

(a) 10 ms^{-1} (b) 15 ms^{-1}
(c) 20 ms^{-1} (d) 25 ms^{-1}

28. A train starts from rest with a constant acceleration $a = 2$ ms^{-2}. After 5 s, a stone is dropped from the window of the train. If the window is at a height of 2 m from the ground, what is the magnitude of the velocity of the stone 0.2 s after it was dropped? Take $g = 10$ ms^{-2}.

(a) $4\sqrt{6}$ ms^{-1} (b) 10 ms^{-1}
(c) $2\sqrt{26}$ ms^{-1} (d) 12 ms^{-1}

29. In Q.28, the angle, which the resultant velocity vector makes with the horizontal is given by

(a) $\theta = \tan^{-1}(0.1)$ (b) $\theta = \tan^{-1}(0.2)$
(c) $\theta = \tan^{-1}(0.3)$ (d) $\theta = \tan^{-1}(0.4)$

30. In Q.28, the acceleration of the stone after it is dropped is given by

(a) $g + a$ (b) $\sqrt{g^2 + a^2}$
(c) a (d) g

31. A rope which can withstand a maximum tension of 400 N is hanging from a tree. If a monkey of mass 30 kg climbs on the rope, in which of the following cases will the rope break? Take $g = 10$ ms^{-2} and neglect the mass of the rope.

(a) The monkey climbs up with a uniform speed of 5 ms^{-1}.
(b) The monkey climbs up with a uniform acceleration of 2 ms^{-2}.
(c) The monkey climbs up with a uniform acceleration of 5 ms^{-2}.
(d) The monkey climbs down with a uniform acceleration of 5 ms^{-2}.

32. A stream of a liquid of density ρ flowing horizontally with a speed v gushes out of a tube of radius r and hits at a vertically wall nearly normally. Assuming that the liquid does not rebound from the wall, the force exerted on the wall by the impact of liquid is given by

(a) $\pi r\rho v$ (b) $\pi r\rho v^2$
(c) $\pi r^2\rho v$ (d) $\pi r^2\rho v^2$

33. Two blocks of masses $m_1 = 5$ kg and $m_2 = 6$ kg are connected by a light string passing over a light frictionless pulley as shown in Fig. 3.62. The mass m_1 is at rest on the inclined plane and mass m_2 hangs vertically. If the angle of incline $\theta = 30°$, what is the magnitude and direction of the force of friction on the 5 kg block. Take $g = 10$ ms^{-2}.

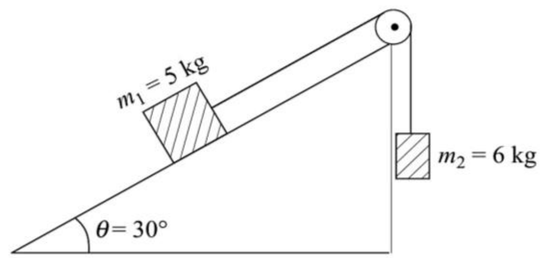


Fig. 3.62

(a) 35 N up the plane (b) 35 N down the plane
(c) 85 N up the plane (d) 85 N down the plane

34. A horizontal force of 300 N pulls two blocks of masses $m_1 = 10$ kg and $m_2 = 20$ kg which are connected by a light inextensible string and lying on a horizontal frictionless surface (Fig. 3.63). What is the acceleration of each mass?

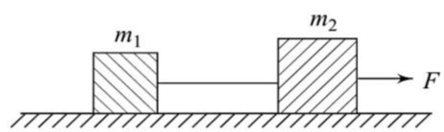


Fig. 3.63

(a) 10 ms^{-2} (b) 15 ms^{-2}
(c) 30 ms^{-2} (d) zero

35. In Q.34, the tension in the string is

(a) 100 N (b) 200 N
(c) 300 N (d) zero

36. In Q.34, the force on mass m_1 is

(a) 100 N (b) 200 N
(c) 300 N (d) zero

37. In Q.34, what will be the tension in the string if the force F is applied to mass m_1 as shown in Fig. 3.64?

(a) 100 N (b) 200 N
(c) 300 N (d) zero

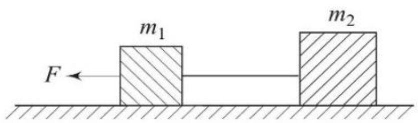


Fig. 3.64

38. In Q.37, the force exerted on mass m_2 is
(a) 100 N (b) 200 N
(c) 300 N (d) zero

Level B

39. A shell explodes into three fragments of equal masses. Two fragments fly off at right angles to each other with speeds of 9 ms^{-1} and 12 ms^{-1}. What is the speed of the third fragment?
(a) 9 ms^{-1} (b) 12 ms^{-1}
(c) 15 ms^{-1} (d) 18 ms^{-1}

40. A cricket ball of mass 150 g is moving with a velocity of 12 ms^{-1} and is hit by a bat so that the ball is turned back with a velocity of 20 ms^{-1}. The force of the blow acts for 0.1 s. What is the average force exerted on the ball by the bat?
(a) 18 N (b) 30 N
(c) 48 N (d) 60 N

41. Two billiard balls each of mass 50 g, moving in opposite directions each with a speed 6 ms^{-1}, collide and rebound with the same speed. The impulse imparted to each ball due to the other is
(a) 0.3 Ns (b) 0.6 Ns
(c) 0.9 Ns (d) 1.2 Ns

42. A ball of mass m is moving towards a batsman at a speed v. The batsman strikes the ball and deflects it by an angle θ without changing its speed. The impulse imparted to the ball is given by
(a) $mv\cos(\theta)$ (b) $mv\sin(\theta)$
(c) $2mv\cos\left(\frac{\theta}{2}\right)$ (d) $2mv\sin\left(\frac{\theta}{2}\right)$

43. Figure 3.65 shows the position–time (x–t) graph of one-dimensional motion of a body of mass 0.4 kg. What is the time interval between consecutive impulses received by the body?
(a) 2 s (b) 4 s
(c) 8 s (d) 16 s

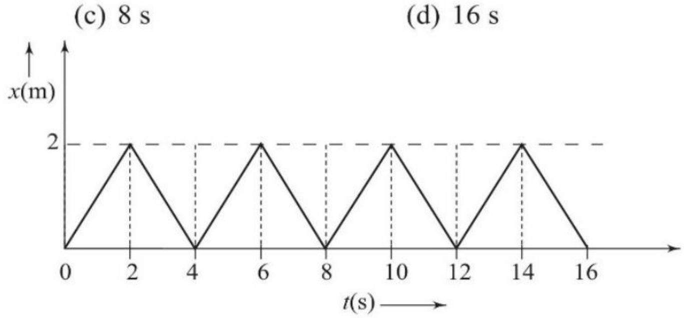


Fig. 3.65

44. In Q.43, what is the magnitude of each impulse?
(a) 0.2 Ns (b) 0.4 Ns
(c) 0.8 Ns (d) 1.6 Ns

45. An aeroplane of mass M requires a speed v for take–off. The length of the runway is s and the coefficient of friction between the tyres and the ground is μ. Assuming that the plane accelerates uniformly during the take–off, the minimum force required by the engine of the plane for take–off is given by

(a) $M\left(\frac{v^2}{2s}+\mu g\right)$ (b) $M\left(\frac{v^2}{2s}-\mu g\right)$

(c) $M\left(\frac{2v^2}{s}+2\mu g\right)$ (d) $M\left(\frac{2v^2}{s}-2\mu g\right)$

46. A block of mass m is projected up an inclined plane of inclination θ with an initial velocity u. If the coefficient of kinetic friction between the block and the plane is μ, the distance up to which the block will rise up the plane, before coming to rest, is given by

(a) $\frac{u^2\mu}{2g\sin\theta}$ (b) $\frac{u^2\mu}{2g\cos\theta}$

(c) $\frac{u^2}{4g\sin\theta}$ (d) $\frac{u^2}{4g\cos\theta}$

47. A block of mass 5 kg is resting on an inclined plane. The inclination of the plane to the horizontal direction is gradually increased. It is found that, when the angle of inclination is 30°, the block just begins to slide down the plane. The coefficient of sliding friction μ_s between the block and the plane is
(a) $\mu_s = \sin 30°$ (b) $\mu_s = \cos 30°$
(c) $\mu_s = \tan 30°$ (d) $\mu_s = \cot 30°$

48. In Q.47, what minimum force must be applied parallel to the plane that would just make the block move up the plane? Take $g = 10$ ms^{-2}.
(a) 25 N (b) 50 N
(c) $25\sqrt{3}$ N (d) $50\sqrt{3}$ N

49. A block of mass 10 kg is placed at a distance of 5 m from the rear end of a long trolley as shown in Fig. 3.66. The coefficient of friction between the block and the surface below is 0.2. Starting from rest, the trolley is given a uniform acceleration of 3 ms^{-2}. At what distance from the starting point will the block fall off the trolley? Take $g = 10$ ms^{-2}.
(a) 15 m (b) 20 m
(c) 25 m (d) 30 m

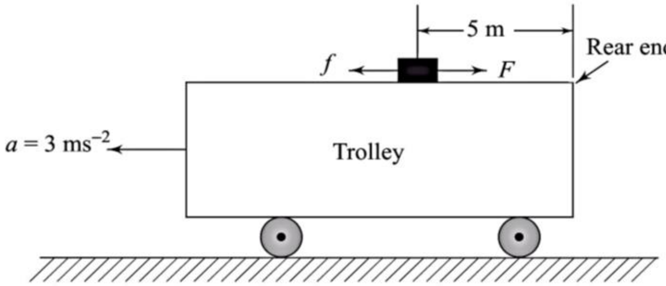


Fig. 3.66

50. Two blocks of equal masses $m_1 = m_2 = 3$ kg, connected by a light string, are placed on a horizontal surface which is not frictionless. If a force of 20 N is applied in the horizontal direction on a block, the acceleration of each block is 0.5 ms^{-2}. Assuming that the frictional forces on the two blocks are equal, the tension in the string will be
(a) 10 N (b) 20 N
(c) 40 N (d) 60 N
51. In Q.50, the frictional force on each block is
(a) 6.5 N (b) 7.5 N
(c) 8.5 N (d) 9.5 N
52. A body slides down an inclined plane of inclination θ. The coefficient of friction down the plane varies in direct proportion to the distance moved down the plane ($\mu = k\,x$). The body will move down the plane with a
(a) constant acceleration = $g \sin \theta$
(b) constant acceleration = $(g \sin \theta - \mu\, g \cos \theta)$
(c) constant retardation = $(\mu\, g \cos \theta - g \sin \theta)$
(d) variable acceleration that first decreases from $g \sin \theta$ to zero and then becomes negative.
53. A jet of water with a cross-sectional area a is striking against a wall at an angle θ to the normal and rebounds elastically. If the velocity of water in the jet is v, the normal force acting on the wall is,
(a) $2\, a\, v^2 \rho \cos \theta$ (b) $a\, v^2 \rho \cos \theta$
(c) $2\, a\, v\, \rho \cos \theta$ (d) $a\, v\, \rho \cos \theta$
54. A given object takes n times as much time to slide down a 45° rough incline as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is given by
(a) $\mu_k = 1/(1 - n^2)$ (b) $\mu_k = 1 - 1/n^2$
(c) $\mu_k = \sqrt{1/\left(1-n^2\right)}$ (d) $\sqrt{\left(1-1/n^2\right)}$
55. A body is sliding down a rough inclined plane of angle of inclination θ for which the coefficient of friction varies with distance x as $\mu(x) = kx$, where k is a constant. Here x is the distance moved by the body down the plane. The net force on the body will be zero at a distance x_0 given by
(a) $\dfrac{\tan \theta}{k}$ (b) $k \tan \theta$
(c) $\dfrac{\cot \theta}{k}$ (d) $k \cot \theta$
56. A body is moving down a long inclined plane of angle of inclination θ. The coefficient of friction between the body and the plane varies as $\mu = 0.5\, x$, where x is the distance moved down the plane. The body will have the maximum velocity when it has travelled a distance x given by
(a) $x = 2 \tan \theta$ (b) $x = \dfrac{2}{\tan \theta}$
(c) $x = \sqrt{2}\, \cot \theta$ (d) $x = \dfrac{\sqrt{2}}{\cot \theta}$
57. An object is kept on a smooth inclined plane of 1 in l. The horizontal acceleration to be imparted to the inclined plane so that the object is stationary relative to the incline is given by
(a) $g\sqrt{l^2 - 1}$ (b) $g\,(l^2 - 1)$
(c) $\dfrac{g}{\sqrt{l^2 - 1}}$ (d) $\dfrac{g}{l^2 - 1}$
58. An insect is crawling up a hemispherical bowl of radius R. If the coefficient of friction is 1/3, the insect will be able to go up to height h equal to (take $3/\sqrt{10} = 0.95$)
(a) $\dfrac{R}{5}$ (b) $\dfrac{R}{10}$
(c) $\dfrac{R}{20}$ (d) $\dfrac{R}{30}$
59. Two blocks of masses 5 kg and 3 kg are placed in contact on a horizontal frictionless surface as shown in Fig. 3.67. A force of 4 N is applied mass 5 kg as shown. The acceleration of the mass 3 kg will be
(a) $\dfrac{4}{5}$ ms^{-2} (b) $\dfrac{4}{3}$ ms^{-2}
(c) 2 ms^{-2} (d) 0.5 ms^{-2}

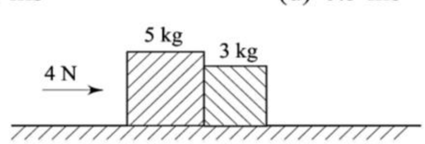


Fig. 3.67

60. In Q.59, what force is exerted on mass 3 kg?
(a) 0.5 N (b) 1.0 N
(c) 1.5 N (d) 4 N

61. In Q.59, if a force of 4 N is applied on mass 3 kg, the acceleration of mass 5 kg will be
(a) $\frac{4}{3}$ ms^{-2} (b) $\frac{4}{5}$ ms^{-2}
(c) 2 ms^{-2} (d) 0.5 ms^{-2}

62. In Q.61, what force is exerted on mass 5 kg?
(a) 2.5 N (b) 3 N
(c) 3.5 N (d) 4 N

63. Three blocks of masses $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg are placed in contact on a horizontal frictionless surface as shown in Fig. 3.68. A force $F = 12$ N is applied to mass m_1 as shown. The acceleration of the system is

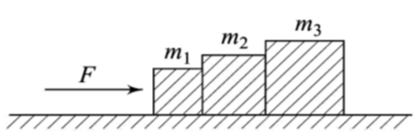


Fig. 3.68

(a) 12 ms^{-2} (b) 6 ms^{-2}
(c) 4 ms^{-2} (d) 2 ms^{-2}

64. In Q. 63, the contact force acting on mass m_2 is
(a) 12 N (b) 10 N
(c) 8 N (d) 6 N

65. In Q.63, the contact force acting on mass m_3 is
(a) 12 N (b) 10 N
(c) 8 N (d) 6 N

66. Three blocks of masses $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg are connected by massless strings and placed on a horizontal frictionless surface as shown in Fig. 3.69. A force $F = 12$ N is applied to mass m_1 as shown. The acceleration of the system is
(a) 12 ms^{-2} (b) 6 ms^{-2}
(c) 4 ms^{-2} (d) 2 ms^{-2}

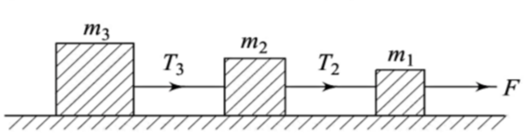


Fig. 3.69

67. In Q.66, what is tension T_2 between masses m_1 and m_2?
(a) 12 N (b) 10 N
(c) 8 N (d) 6 N

68. In Q.66, what is tension T_3 between masses m_2 and m_3?
(a) 12 N (b) 10 N
(c) 8 N (d) 6 N

69. A ball of mass m is connected to a ball of mass M by means of a massless spring. The balls are pressed so that the spring is compressed. When released, ball of mass m moves with acceleration a. The magnitude the acceleration of mass M will be
(a) $\frac{ma}{(M+m)}$ (b) $\frac{Ma}{(M+m)}$
(c) $\frac{ma}{M}$ (d) $\frac{Ma}{m}$

70. A boy, while catching a ball, experiences an impulse of 6 Ns. If the mass of the ball is 200 g, what was the speed of the ball before it was caught?
(a) 10 ms^{-1} (b) 20 ms^{-1}
(c) 30 ms^{-1} (d) 40 ms^{-1}

71. A car moving at a speed v is stopped by a retarding force F in a distance s. If the speed of the car were $3v$, the force needed to stop it within the same distance s will be
(a) 3 F (b) 6 F
(c) 9 F (d) 12 F

72. A car moving at a speed v is stopped by a retarding force F in a distance s. If the retarding force were $3F$, the car will be stopped in a distance
(a) $\frac{s}{3}$ (b) $\frac{s}{6}$
(c) $\frac{s}{9}$ (d) $\frac{s}{12}$

73. Two blocks of masses m and M are placed on a horizontal frictionless table connected by a spring as shown in Fig. 3.70. Mass M is pulled to the right with a force F. If the acceleration of mass m is a, the acceleration of mass M will be
(a) $\frac{(F-ma)}{M}$ (b) $\frac{(F+ma)}{M}$
(c) $\frac{F}{M}$ (d) $\frac{am}{M}$

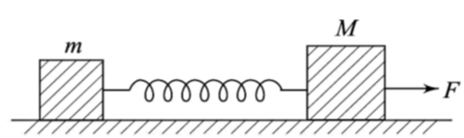


Fig. 3.70

74. A boy wants to climb down a rope. The rope can withstand a maximum tension equal to two–thirds the weight of the boy. If g is the acceleration due to

gravity, the minimum acceleration with which the boy should climb down the rope should be

(a) $\frac{g}{3}$ (b) $\frac{2g}{3}$

(c) g (d) zero

75. A mass m is suspended from a rigid support P by means of a massless string as shown in Fig. 3.71. A horizontal force F is applied at point O of the string. The system is in equilibrium when the string makes an angle θ with the vertical. Then the relation between the tension T, force F and angle θ is

(a) $F = T \sin\theta$ (b) $F = T\cos\theta$

(c) $F = \frac{T}{\sin\theta}$ (d) $F = \frac{T}{\cos\theta}$

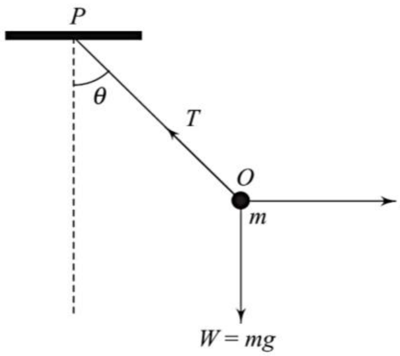


Fig. 3.71

76. In Q. 75, what is the relation between the weight $W = mg$, T and θ?

(a) $W = T\sin\theta$ (b) $W = T\cos\theta$

(c) $W = \frac{T}{\sin\theta}$ (d) $W = \frac{T}{\cos\theta}$

77. In Q. 75, the relation between T, W and F is

(a) $T^2 = WF$ (b) $T = \frac{W^2}{F}$

(c) $T = \frac{F^2}{W}$ (d) $T^2 = W^2 + F^2$

78. In Q. 75, the relation between F, W and θ is

(a) $F = W\tan\theta$ (b) $F = W\cot\theta$

(c) $F = W\sin\theta$ (d) $F = W\cos\theta$

79. Two masses $m_1 = 6$ kg and $m_2 = 4$ kg are connected by means of a string which passes over a frictionless pulley as shown in Fig. 3.72. When the masses are released, what is the acceleration of the masses? Take $g = 10\text{ ms}^{-2}$.

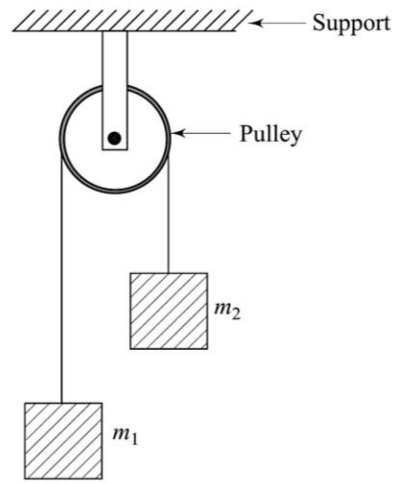


Fig. 3.72

(a) 2 ms^{-2} (b) 4 ms^{-2}

(c) 8 ms^{-2} (d) 10 ms^{-2}

80. In Q.79, what is the tension in the string?

(a) 40 N (b) 48 N

(c) 56 N (d) 60 N

81. The support of the pulley in Q.79 is attached to the ceiling of a lift. What will be the tension in the string if the lift starts ascending with a constant acceleration $a = 5\text{ ms}^{-2}$?

(a) 24 N (b) 48 N

(c) 60 N (d) 72 N

82. In Q. 81, what will be the tension in the string if the lift starts descending with a constant acceleration of $a = 5\text{ ms}^{-2}$?

(a) 24 N (b) 48 N

(c) 60 N (d) 72 N

83. The support of the pulley in Q. 79 is attached to the ceiling of a compartment of a train. What will be the tension in the string if the train moves in the horizontal direction with a constant acceleration of 5 ms^{-2}?

(a) 24 N (b) 48 N

(c) 60 N (d) 72 N

84. A block, released from rest from the top of a smooth inclined plane of angle of inclination θ_1, reaches the bottom in time t_1. The same block, released from rest from the top of another smooth inclined plane of angle of inclination θ_2, reaches the bottom in time t_2. If the two inclined planes have the same height, the relation between t_1 and t_2 is

(a) $\frac{t_2}{t_1}=\left(\frac{\sin\theta_1}{\sin\theta_2}\right)^{1/2}$ (b) $\frac{t_2}{t_1}=\frac{\sin^2\theta_1}{\sin^2\theta_2}$

(c) $\frac{t_2}{t_1}=\frac{\sin\theta_1}{\sin\theta_2}$ (d) $\frac{t_2}{t_1}=1$

85. A thick uniform rope of mass 6 kg and length 3 m is hanging vertically from a rigid support. The tension in the rope at a point 1 m from the support will be (Take $g = 10\ ms^{-2}$)

(a) 20 N (b) 30 N
(c) 40 N (d) 60 N

86. A block released from rest from the top of a smooth inclined plane of inclination 45° takes t seconds to reach the bottom. The same block released from rest from top of a rough inclined plane of the same inclination of 45° takes $2t$ seconds to reach the bottom. The coefficient of friction is

(a) $\sqrt{0.5}$ (b) $\sqrt{0.75}$
(c) 0.5 (d) 0.75

87. A block, released from rest from the top of a smooth inclined plane of inclination θ, has a speed v when it reaches the bottom. The same block, released from the top of a rough inclined plane of the same inclination θ, has a speed v/n on reaching the bottom, where n is a number greater than unity. The coefficient of friction is given by

(a) $\mu=\left(1-\frac{1}{n^2}\right)\tan\theta$

(b) $\mu=\left(1-\frac{1}{n^2}\right)\cot\theta$

(c) $\mu=\left(1-\frac{1}{n^2}\right)^{1/2}\tan\theta$

(d) $\mu=\left(1-\frac{1}{n^2}\right)^{1/2}\cot\theta$

88. Two blocks of masses $M = 5$ kg and $m = 3$ kg are placed on a horizontal surface as shown in Fig. 3.73. The coefficient of friction between the blocks is 0.5 and that between the block M and the horizontal surface is 0.7. What is the maximum horizontal force F that can be applied to block M so that the two blocks move without slipping? Take $g = 10\ ms^{-2}$.

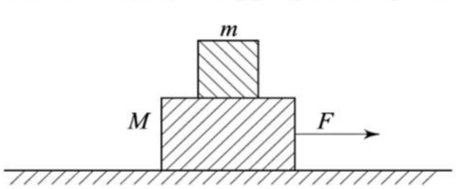


Fig. 3.73

(a) 4 N (b) 16 N
(c) 24 N (d) 96 N

89. A box is gently placed on a horizontal conveyer belt moving with a speed of 4 ms^{-1}. If the coefficient of friction between the box and the belt is 0.8, through what distance will the block slide without slipping? Take $g = 10\ ms^{-2}$

(a) 0.6 m (b) 0.8 m
(c) 1.0 m (d) 1.2 m

90. Two blocks of masses $m_1 = 4$ kg and $m_2 = 6$ kg are connected by a string of negligible mass passing over a frictionless pulley as shown in Fig. 3.74. The coefficient of friction between block m_1 and the horizontal surface is 0.4. When the system is released, the masses m_1 and m_2 start accelerating. What additional mass m should be placed over mass m_1 so that the masses $(m_1 + m)$ slide with a uniform speed?

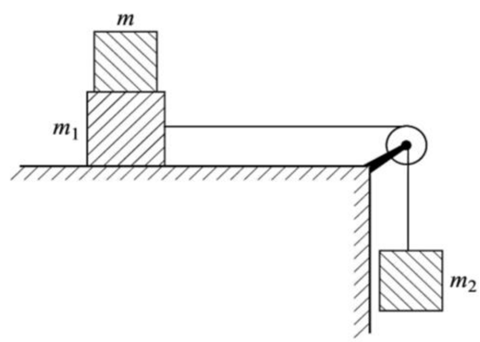


Fig. 3.74

(a) 9 kg (b) 10 kg
(c) 11 kg (d) 12 kg

91. Two blocks of masses $m_1 = m_2 = m$, are connected by a string of negligible mass which passes over a frictionless pulley fixed on the top of an inclined plane as shown in Fig. 3.75. When the angle of inclination $\theta = 30°$, the mass m_1 just begins to move up the inclined plane. What is the coefficient of friction between block m_1 and the inclined plane?

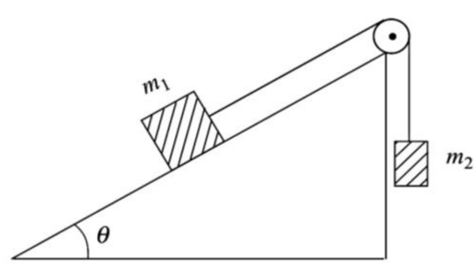


Fig. 3.75

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2\sqrt{3}}$

92. In Q. 91 above, if $\mu = 1/2\sqrt{3}$ and $\theta = 30°$, what should be the ratio m_1/m_2 of the masses so that mass m_1 just begins to slide down the plane?
(a) 4 (b) 3
(c) 2 (d) 1

93. A block is resting on an inclined plane. The angle of inclination is gradually increased. The block just begins to slide down the plane when the angle of inclination is 30°. What is the coefficient of friction between the inclined surface and the block?
(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{1}{2\sqrt{3}}$ (d) $\frac{1}{3\sqrt{3}}$

94. In Q. 93, if $\theta = 30°$ and $\mu = 1/\sqrt{3}$, the downward acceleration of the block will be
(a) zero (b) $1/\sqrt{3}$ ms^{-2}
(c) $1/2\sqrt{3}$ ms^{-2} (d) $1/3\sqrt{3}$ ms^{-2}

95. A boy of mass m is sliding down a vertical pole by pressing it with a horizontal force f. If μ is the coefficient of friction between his palms and the pole, the acceleration with which he slides down will be
(a) g (b) $\frac{\mu f}{m}$
(c) $g + \frac{\mu f}{m}$ (d) $g - \frac{\mu f}{m}$

96. A boy of mass 40 kg is climbing a vertical pole at a constant speed. If the coefficient of friction between his palms and the pole is 0.8 and $g = 10$ ms^{-2}, the force that he is applying on the pole is
(a) 300 N (b) 400 N
(c) 500 N (d) 600 N

97. A block of mass m is lying on a another block of mass M, lying on a horizontal frictionless surface as shown in Fig. 3.76. If the coefficient static friction between the two blocks is μ_s, the minimum horizontal force F that must be applied to block of mass m so that it moves over block of mass M is
(a) zero (b) mg
(c) mg/μ_s (d) $\mu_s mg$

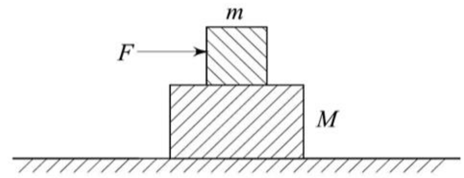


Fig 3.76

98. In Q. 97, if a force F (> μ_s mg) is applied to block m, the acceleration with which block M will move on the horizontal surface is given by (here μ_k is the coefficient of kinetic friction between block M and the horizontal surface).
(a) $\frac{\mu_k mg}{M}$ (b) $\frac{\mu_s Mg}{m}$
(c) $\frac{\mu_s}{\mu_k}\frac{mg}{M}$ (d) $\frac{\mu_s}{\mu_k}\frac{Mg}{m}$

99. A block of mass 5 kg is lying on a rough horizontal surface. The coefficients of static and kinetic friction between the block and the surface respectively are 0.7 and 0.5. A horizontal force just sufficient to move the block is applied to it. If the force continues to act even after the block has started moving, the acceleration of block will be (take $g = 10$ ms^{-2})
(a) 1 ms^{-2} (b) 2 ms^{-2}
(c) 3 ms^{-2} (d) 4 ms^{-2}

100. A body projected along an inclined plane of angle of inclination 30° stops after covering a distance x_1. The same body projected with the same speed stops after covering a distance x_2, if the angle of inclination of the inclined plane is increased to 60°. The ratio x_1/x_2 is
(a) 1 (b) $\sqrt{2}$
(c) $\sqrt{3}$ (d) 2

101. A spring is compressed between two blocks of masses m_1 and m_2 placed on a horizontal frictionless surface as shown in Fig. 3.77. When the blocks are released, they have initial velocity of v_1 and v_2 as shown. The blocks travel distances x_1 and x_2 respectively before coming to rest. The ratio x_1/x_2 is

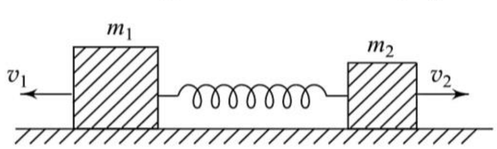


Fig. 3.77

(a) $\frac{m_1}{m_2}$ (b) $\frac{m_2}{m_1}$
(c) $\sqrt{\frac{m_1}{m_2}}$ (d) $\sqrt{\frac{m_2}{m_1}}$

102. A smooth inclined plane of angle of inclination 30° is placed on the floor of a compartment of a train moving with a constant acceleration a. When a block is placed on the inclined plane, it does not slide down or up the plane. The acceleration a must be

(a) g (b) $\frac{g}{2}$

(c) $\frac{g}{\sqrt{2}}$ (d) $\frac{g}{\sqrt{3}}$

103. A block released on a rough inclined plane of inclination $\theta = 30°$ slides down the plane with an acceleration $g/4$, where g is the acceleration due to gravity. What is the coefficient of friction between the block and the inclined plane?

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{2\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

104. A block of mass m placed on a rough inclined plane of inclination $\theta = 30°$ can be just prevented from sliding down by applying a force F_1 up the plane and it can be made to just slide up the plane by applying a force F_2 up the plane. If the coefficient of friction between the block and the inclined plane is $1/2\sqrt{3}$, the relation between F_1 and F_2 is

(a) $F_2 = F_1$ (b) $F_2 = 2F_1$

(c) $F_2 = 3F_1$ (d) $F_2 = 4F_1$

105. A person standing in a stationary lift drops a coin from a certain height h. It takes time t to reach the floor of the lift. If the lift is rising up with a uniform acceleration a, the time taken by the coin, dropped from the same height h, to reach the floor will be

(a) t (b) $t\sqrt{\frac{a}{g}}$

(c) $t\left(1+\frac{a}{g}\right)^{1/2}$ (d) $t\left(1-\frac{a}{g}\right)^{1/2}$

106. In Q. 105, if the lift is descending with an acceleration a, the time taken by the coin to reach the floor will be

(a) t (b) $t\sqrt{\frac{a}{g}}$

(c) $t\left(1+\frac{a}{g}\right)^{1/2}$ (d) $t\left(1-\frac{a}{g}\right)^{1/2}$

107. A block is lying on a horizontal frictionless surface. One end of a uniform rope is fixed to the block which is pulled in the horizontal direction by applying a force F at the other end. If the mass of the rope is half the mass of the block, the tension in the middle of the rope will be

(a) F (b) $\frac{2F}{3}$

(c) $\frac{3F}{5}$ (d) $\frac{5F}{6}$

108. When a force F acts on a body of mass m, the acceleration produced in the body is a. If three equal forces $F_1 = F_2 = F_3 = F$ act on the same body as shown in Fig. 3.78 the acceleration produced is

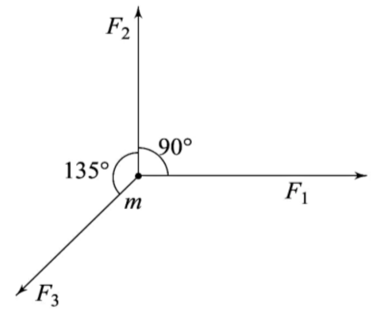


Fig. 3.78

(a) $(\sqrt{2}-1)\ a$ (b) $(\sqrt{2}+1)\ a$

(c) $\sqrt{2}\ a$ (d) a

109. A shell of mass m is at rest initially. It explodes into three fragments having masses in the ratio 2 : 2 : 1. The fragments having equal masses fly off along mutually perpendicular directions with speed v. What will be the speed of the third (lighter) fragment?

(a) v (b) $\sqrt{2}\ v$

(c) $2\sqrt{2}\ v$ (d) $3\sqrt{2}\ v$

110. The linear momentum p of a body of mass 2 kg varies with time t as

$$p = 3t^2 + 4$$

where p and t are in SI units. It follows that the body is moving with a

(a) constant speed
(b) constant acceleration
(c) variable acceleration
(d) variable retardation.

111. In Q. 110, what is the acceleration of the body at $t = 2$s?

(a) 3 ms^{-2} (b) 6 ms^{-2}

(c) 9 ms^{-2} (d) 12 ms^{-2}

112. In Q. 110, the force acting on the body varies with time t as

(a) $t^{1/2}$ (b) t

(c) $t^{3/2}$ (d) t^2

113. An elastic spring has a length l_1 when it is stretched with a force of 2 N and a length l_2 when it is stretched with a force of 3 N. What will be the length of the spring if it is stretched with force of 5N?

(a) $l_1 + l_2$ (b) $\frac{1}{2}(l_1 + l_2)$

(c) $3l_2 - 2l_1$ (d) $3l_1 - 2l_2$

114. A shell is fired from a cannon with a speed of $100\,ms^{-1}$ at an angle 30° with the vertical (y-direction). At the highest point of its trajectory, the shell explodes into two fragments of masses in the ratio 1 : 2. The lighter fragment moves vertically upwards with an initial speed of 200 ms^{-1}. What is the speed of the heavier fragment at the time of explosion?

(a) 125 ms^{-1} (b) 150 ms^{-1}

(c) 175 ms^{-1} (d) 200 ms^{-1}

115. A balloon of mass M is rising up with an acceleration a. If a mass m is removed from the balloon, its acceleration becomes

(a) $\frac{Ma + mg}{M - m}$ (b) $\frac{Ma + mg}{M + m}$

(c) $\frac{ma + Mg}{M - m}$ (d) $\frac{ma + Mg}{M + m}$

116. A uniform rope is hanging vertically from the ceiling such that its free end just touches the horizontal floor of a room. The upper end of the rope is then released. At any instant during the fall of the rope, the total force exerted by it on the floor is n times the weight of that part of the rope which is on the floor at that time. What is the value of n?

(a) 1 (b) 2

(c) 3 (d) 4

117. The distance x (in metre) travelled by a body in time t (in second) is given by

$$\frac{d^2x}{dt^2} = 2t - t^2$$

How much distance does the body travel before reversing its direction of motion?

(a) $\frac{3}{4}$ m (b) $\frac{(3)^2}{4}$ m

(c) $\frac{(3)^3}{4}$ m (d) $\frac{(3)^4}{4}$ m

118. A long horizontal rod has a bead which can slide along its length, and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is

(a) $\sqrt{\frac{\mu}{\alpha}}$ (b) $\frac{\mu}{\sqrt{\alpha}}$

(c) $\frac{1}{\sqrt{\mu\alpha}}$ (d) infinitesimal

119. A force vector $\mathbf{F} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$ newton applied to a body accelerates it by 1 ms^{-2}. What is the mass of the body?

(a) $10\sqrt{2}$ kg (b) $2\sqrt{10}$ kg

(c) 10 kg (d) 20 kg

120. A uniform chain of length L is lying on the horizontal surface of a table. If the coefficient of friction between the chain and the table top is μ, what is the maximum length of the chain that can hang over the edge of the table without disturbing the rest of the chain on the table?

(a) $\frac{L}{1+\mu}$ (b) $\frac{\mu L}{1+\mu}$

(c) $\frac{L}{1-\mu}$ (d) $\frac{\mu L}{1-\mu}$

121. In Q. 120 above, if $\mu = 0.25$, what is the maximum percentage of the length of the chain that can hang over the edge of the table without disturbing the rest of the chain on the table?

(a) 8% (b) $\frac{40}{3}$ %

(c) 20% (d) 25%

122. A block of mass m is lying on a horizontal surface of coefficient of friction μ. A force F is applied to the block at an angle θ with the horizontal. The block will move with a minimum force F if

(a) $\mu = \tan\theta$ (b) $\mu = \cot\theta$

(c) $\mu = \sin\theta$ (d) $\mu = \cos\theta$

123. In Q. 122 above, the minimum F is given by

(a) $\frac{\mu mg}{\mu - 1}$ (b) $\frac{\mu mg}{\mu + 1}$

(c) $\frac{\mu^2 mg}{\sqrt{1-\mu^2}}$ (d) $\frac{\mu mg}{\sqrt{1+\mu^2}}$

124. The coefficients of static and kinetic friction between a body and the surface are 0.75 and 0.5 respectively. A force is applied to the body to make it just slide with a constant acceleration which is

(a) $\frac{g}{4}$ (b) $\frac{g}{2}$

(c) $\frac{3g}{4}$ (d) g

125. Two blocks A and B are connected to each other by a string and a spring of force constant k, as shown in

Fig. 3.79. The string passes over a frictionless pulley as shown. The block B slides over the horizontal top surface of a stationary block C and the block A slides along the vertical side of C both with the same uniform speed. The coefficient of friction between the surfaces of the blocks B and C is μ. If the mass of block A is m, what is the mass of block B?

(a) $\frac{m}{\sqrt{\mu}}$ (b) $\frac{m}{\mu}$

(c) $\sqrt{\mu}m$ (d) μm

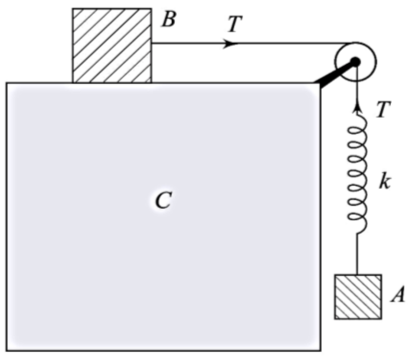


Fig. 3.79

126. In Q. 125 above, the energy stored in the spring is

(a) $\frac{m^2g^2}{2k}$ (b) $\frac{m^2g^2}{k}$

(c) $\frac{\mu m^2g^2}{2k}$ (d) μmgk

127. A trolley of mass M is attached to a block of mass m by a string passing over a frictionless pulley as shown in Fig. 3.80. If the coefficient of friction between the trolley and the surface below is μ, what is the acceleration of the trolley and the block system, when they are released?

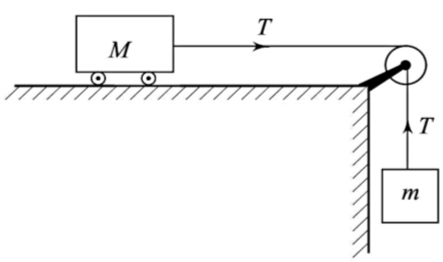


Fig. 3.80

(a) $\left(\frac{m-M}{m+M}\right)g$ (b) $\frac{m}{M}g$

(c) $\left(\frac{\mu m-M}{m+M}\right)g$ (d) $\left(\frac{m-\mu M}{m+M}\right)g$

128. In Q. 127 above, the tension T in the string is

(a) $\frac{mMg\,(1+\mu)}{m+M}$ (b) $\frac{mMg\,(1-\mu)}{M-m}$

(c) μmg (d) $(m+\mu M)\,g$

129. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are:

(a) up the incline while ascending and down the incline while descending

(b) up the incline while ascending as well as descending

(c) down the incline while ascending and up the incline while descending

(d) down the incline while ascending as well as descending

130. An insect crawls up a hemispherical surface very slowly (see Fig. 3.81). The coefficient of friction between the insect and the surface is 1/3. If the line joining the center of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by

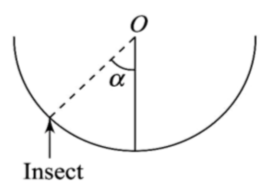


Fig. 3.81

(a) $\cot\alpha = 3$ (b) $\tan\alpha = 3$

(c) $\sec\alpha = 3$ (d) $\operatorname{cosec}\alpha = 3$

131. The pulleys and strings shown in Fig. 3.82 are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be

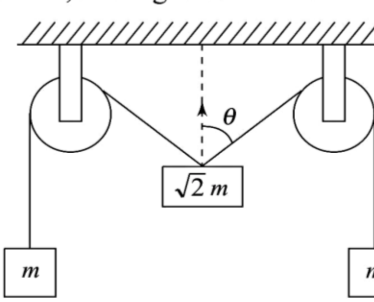


Fig. 3.82

(a) 0° (b) 30°

(c) 45° (d) 60°

132. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as

shown in Fig. 3.83. The force on the pulley by the clamp is given by

(a) $\sqrt{2}\ Mg$ (b) $\sqrt{2}\ mg$

(c) $\sqrt{(M+m)^2+m^2}\ g$ (d) $\sqrt{(M+m)^2+M^2}\ g$

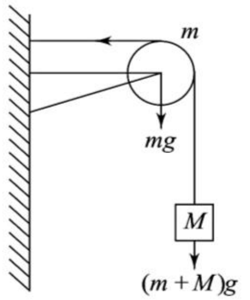


Fig. 3.83

133. A block of weight 200 N is pulled along a rough horizontal surface at a constant speed by a force of 100 N acting at an angle of 30° above the horizontal. The coefficient of friction between the block and the surface is

(a) 0.43 (b) 0.58
(c) 0.75 (d) 0.85

134. Bullets of mass 0.03 kg each hit a plate at the rate of 200 bullets per second with a velocity of 50 ms^{-1} and reflect back with a velocity of 30 ms^{-1}. The average force (in newton) acting on the plate is

(a) 120 (b) 180
(c) 300 (d) 480

135. A body of mass M kg is on the top point of a smooth hemisphere of radius 5 m. It is released to slide down the surface of the hemisphere. It leaves the surface when its velocity is 5 m/s. At this instant the angle made by the radius vector of the body with the vertical is: (Acceleration due to gravity = 10 ms^{-2})

(a) 30° (b) 45°
(c) 60° (d) 90°

136. A stationary body of mass 3 kg explodes into three equal pieces. Two of the pieces fly off at right angles to each other, one with a velocity $2\hat{\mathbf{i}}$ m/s and the other with a velocity $3\hat{\mathbf{j}}$ m/s. If the explosion takes place in 10^{-5} sec, the average force acting on the third piece in newton is:

(a) $(2\hat{\mathbf{i}}+3\hat{\mathbf{j}})\times10^{-5}$ (b) $-(2\hat{\mathbf{i}}+3\hat{\mathbf{j}})\times10^{5}$

(c) $(3\hat{\mathbf{j}}-2\hat{\mathbf{i}})\times10^{5}$ (d) $(2\hat{\mathbf{i}}-3\hat{\mathbf{j}})\times10^{-5}$

137. A string can withstand a tension of 25 N. What is the greatest speed at which a body of mass 1 kg can be whirled in a horizontal circle using a 1 m length of a string?

(a) 2.5 ms^{-1} (b) 5.0 ms^{-1}
(c) 7.5 ms^{-1} (d) 10 ms^{-1}

138. A body of mass 0.5 kg is whirled in a vertical circle at an angular frequency of 10 rad s^{-1}. If the radius of the circle is 0.5 m, what is the tension in the string when the body is at the top of the circle? Take $g = 10\ ms^{-2}$.

(a) 10 N (b) 20 N
(c) 30 N (d) 40 N

139. In Q.138, what will be the tension in the string when the body is at the bottom of the circle?

(a) 10 N (b) 20 N
(c) 30 N (d) 40 N

140. A cyclist is moving with a speed of 6 ms^{-1}. As he approaches a circular turn on the road of radius 120 m, he applies brakes and reduces his speed at a constant rate of 0.4 ms^{-2}. The magnitude of the net acceleration of the cyclist on the circular turn is

(a) 0.5 ms^{-2} (b) 1.0 ms^{-2}
(c) 2.0 ms^{-2} (d) 4.0 ms^{-2}

141. A simple pendulum of bob mass m swings with an angular amplitude of 40°. When its angular displacement is 20°, the tension in the string is

(a) mg cos 20°
(b) mg sin 20°
(c) greater than mg cos 20°
(d) greater than mg sin 20°

142. A car moves at a speed of 36 km h^{-1} on a level road. The coefficient of friction between the tyres and the road is 0.8. The car negotiates a curve of radius R. If g = 10 ms^{-2}, the car will skid (or slip) while negotiating the curve if value of R is

(a) 10 m (b) 13 m
(c) 14 m (d) 16 m

143. A train has to negotiate a curve of radius 200 m. By how much should the outer rails be raised with respect to the inner rails for a speed of 36 km h^{-1}. The distance between the rails is 1.5 m. Take $g = 10\ ms^{-2}$.

(a) 7.5 cm (b) 10 cm
(c) 12.5 cm (d) 15 cm

144. A train rounds an unbanked circular bend of radius 50 m at a speed of 54 km h^{-1}. If g = 10 ms^{-2}, the angle of banking required to prevent wearing out of rails is given by

(a) $\theta = \tan^{-1}(0.15)$ (b) $\theta = \tan^{-1}(0.25)$
(c) $\theta = \tan^{-1}(0.35)$ (d) $\theta = \tan^{-1}(0.45)$

145. One end of a string of length R is tied to a stone of mass m and the other end to a small pivot on a frictionless vertical board. The stone is whirled in a vertical circle with the pivot as the centre. The minimum speed the stone must have, when it is at the topmost point on the circle, so that the string does not slack is given by

(a) $\sqrt{gR}$ (b) $\sqrt{mgR}$
(c) $\sqrt{2gR}$ (d) $\sqrt{2mgR}$

146. In Q.145, the minimum speed the stone must have, when it is at the lowermost point on the circle, so that the stone can complete the circle is given by

(a) $\sqrt{4gR}$ (b) $\sqrt{4mgR}$
(c) $\sqrt{5gR}$ (d) $\sqrt{5mgR}$

147. The pilot of an aircraft, who is not tied to his seat, can loop a vertical circle in air without falling out at the top of the loop. What is the minimum speed required so that he can successfully negotiate a loop or radius 4 km? Take $g = 10$ ms^{-2}.

(a) 100 ms^{-1} (b) 200 ms^{-1}
(c) 300 ms^{-1} (d) 400 ms^{-1}

148. A disc of radius $r = 20$ cm is rotating about its axis with an angular speed of 20 rad s^{-1}. It is gently placed on a horizontal surface which is perfectly frictionless (Fig. 3.84). What is the linear speed of point A on the disc?

(a) 1 ms^{-1} (b) 2 ms^{-1}
(c) 3 ms^{-1} (d) 4 ms^{-1}

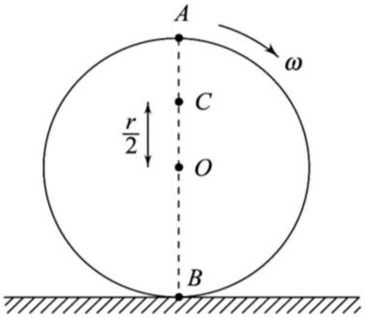


Fig. 3.84

149. In Q.148, the linear speed of point B on the disc will be

(a) 1 ms^{-1} (b) 2 ms^{-1}
(c) 3 ms^{-1} (d) 4 ms^{-1}

150. In Q.148, what is the linear speed of point C on the disc?

(a) 1 ms^{-1} (b) 2 ms^{-1}
(c) 3 ms^{-1} (d) 4 ms^{-1}

151. The combined mass of a rider and a motorcycle is 200 kg. What is the necessary frictional force if the rider is to negotiate a curve of 80 m radius at a speed of 72 km h^{-1}? Take $g = 10$ ms^{-2}.

(a) 500 N (b) 750 N
(c) 1000 N (d) 1250 N

152. In Q.151, the angle at which rider must lean to avoid falling is given by

(a) $\theta = \tan^{-1}(0.2)$ (b) $\theta = \tan^{-1}(0.3)$
(c) $\theta = \tan^{-1}(0.4)$ (d) $\theta = \tan^{-1}(0.5)$

153. A car, moving at a speed of 54 km h^{-1}, is to go round a curved road of radius 30 m. If the curved road is not banked, what must be the coefficient of friction between the tyres and the road for the car to negotiate the curve? Take $g = 10$ ms^{-2}.

(a) zero (b) 0.25
(c) 0.50 (d) 0.75

154. A string of length $L = 1$ m is fixed at one end and carries a mass of 100 g at the other end. The string makes $\sqrt{5}/\pi$ revolutions per second about a vertical axis passing through its second end. What is the angle of inclination of the string with the vertical? Take $g = 10$ ms^{-2}.

(a) 30° (b) 45°
(c) 60° (d) 75°

155. In Q.154, the tension in the string is

(a) 2 N (b) 3 N
(c) 4 N (d) 5 N

156. In Q.154, what is the linear speed of the mass?

(a) $\sqrt{10}$ ms^{-1} (b) $\sqrt{15}$ ms^{-1}
(c) $2\sqrt{5}$ ms^{-1} (d) 5 ms^{-1}

157. A simple pendulum of length $r = 1$ m and bob mass 100 g is swinging with an angular amplitude of 60°. What is the tension in the string when the bob passes through the equilibrium position? Take $g = 10$ ms^{-2}.

(a) 1 N (b) 2 N
(c) 3 N (d) 4 N

158. A spring which obey's Hooke's law extends by 1 cm when a mass is hung on it. It extends by a further 3 cm when the attached mass is moved in a horizontal circle making 2 revolutions per second. What is the length of the unstretched spring? Take $g = \pi^2$ ms^{-2}.

(a) 18 cm (b) 19 cm
(c) 20 cm (d) 21 cm

159. In Q.158, the angle of inclination of the spring to the vertical is given by

(a) $\theta = \cos^{-1}\left(\frac{1}{4}\right)$ (b) $\theta = \cos^{-1}\left(\frac{1}{3}\right)$

(c) $\theta = \cos^{-1}\left(\frac{1}{2}\right)$ (d) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

160. A boy is seated on top of a hemispherical mound of ice of radius R. He is given a little push and he starts sliding down the ice. If ice is frictionless, the boy will leave the ice at a point whose height is

(a) $\frac{3R}{4}$ (b) $\frac{2R}{\sqrt{3}}$

(c) $\frac{2R}{3}$ (d) $\frac{R}{3}$

161. A boy whirls a stone in a horizontal circle 2 m above the ground by means of a string 1.25 m long. The string breaks and the stone flies off horizontally, striking the ground 10 m away. What is the magnitude of the centripetal acceleration during circular motion? Take $g = 10\ ms^{-2}$?

(a) $100\ ms^{-2}$ (b) $200\ ms^{-2}$

(c) $300\ ms^{-2}$ (d) $400\ ms^{-2}$

162. Refer again to Q.145. If $v_B = \sqrt{5gr}$, what will be the kinetic energy of the stone when it is at the topmost point of the circle? B is the lowermost point of the circle.

(a) $\frac{1}{2}\ mgR$ (b) mgR

(c) $\frac{3}{2}\ mgR$ (d) $\frac{5}{2}\ mgR$

163. In Q.145, if $v_A = \sqrt{gR}$, what will be the kinetic energy of the stone when the string becomes horizontal? A is the topmost point of the circle.

(a) $\frac{1}{2}\ mgR$ (b) mgR

(c) $\frac{3}{2}\ mgR$ (d) $2\ mgR$

164. A body is resting on top of a hemispherical mound of ice of radius R. If ice is frictionless, what minimum horizontal velocity must be imparted to the body so that it leaves the mound without sliding over it?

(a) $\sqrt{\frac{gR}{2}}$ (b) $\sqrt{gR}$

(c) $\sqrt{2gR}$ (d) $2\sqrt{gR}$

165. A car is negotiating a curved road of radius r. If the coefficient of friction between the tyres and the road is μ, the car will skid if its speed exceeds

(a) $\sqrt{\mu rg}$ (b) $\sqrt{2\mu rg}$

(c) $\sqrt{3\mu rg}$ (d) $2\sqrt{\mu rg}$

166. The over-bridge of a river is in the form of a circular arc of radius of curvature 10 m. If $g = 10\ ms^{-2}$, what is the highest speed at which a motor cyclist can cross the bridge without leaving the ground?

(a) $10\ ms^{-1}$ (b) $10\sqrt{2}\ ms^{-1}$

(c) $10\sqrt{3}\ ms^{-1}$ (d) $20\ ms^{-1}$

167. The over-bridge of a river is in the form of a circular arc of radius of curvature R. If m is the combined mass of the motorcycle and the rider crossing the bridge at a speed v, the thrust on the bridge at the highest point will be

(a) $\frac{mv^2}{R}$ (b) mg

(c) $\frac{mv^2}{R} - mg$ (d) $\frac{mv^2}{R} + mg$

168. The blocks A and B of masses 2 m and m are connected as shown in Fig. 3.85. The spring has negligible mass. The string is suddenly cut. The magnitudes of accelerations of masses 2 m and m at that instant are

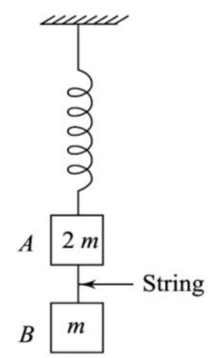


Fig. 3.85

(a) g, g

(b) $g, \frac{g}{2}$

(c) $\frac{g}{2}, g$

(d) $\frac{g}{2}, \frac{g}{2}$

169. A force F hits a block of mass m = 35 g initially at rest. The duration of the impact is 5 ms. It the force varies with time as shown in Fig. 3.86, the speed of the block immediately after impact is

(a) $1\ ms^{-1}$ (b) $2\ ms^{-1}$

(c) $3\ ms^{-1}$ (d) $4\ ms^{-1}$

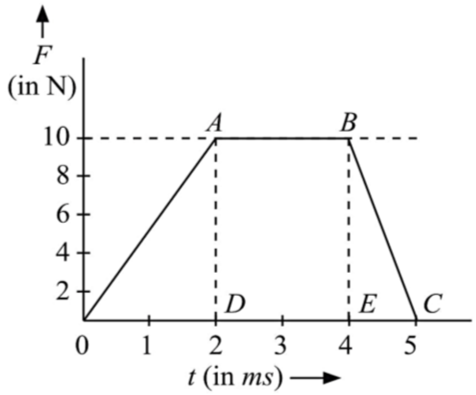


Fig. 3.86

170. A rope of mass m is attached to a block of mass M lying on a horizontal surface. The block is pulled along the surface by applying a force F on the free end of the rope. If μ is the coefficient of friction between block and the surface, the force exerted by the rope on the block is

(a) $\frac{m}{(M+m)}(F-\mu mg)$ (b) $\frac{M}{(M+m)}(F-\mu Mg)$

(c) $\frac{m}{M}(F+\mu Mg)$ (d) $\frac{M}{m}(F+\mu Mg)$

171. Figure 3.87 shows an elevator and blocks A and B. The block A has mass m and block B has mass M. If the elevator is descending will an acceleration a, the force exerted by block A or block B is

(a) $\frac{m}{(M+m)}(g-a)$

(b) $\frac{M}{(M+m)}(g+a)$

(c) $m(g-a)$

(d) $M(g+a)$

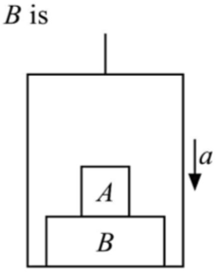


Fig. 3.87

172. Three blocks A, B and C of masses m_1, m_2 and m_2 are connected by a string passing over a fixed frictionless pulley as shown in Fig. 3.88. If $(m_2+m_3) > m_1$, and $m_1+m_2+m_3 = M$, the tension in the string connecting blocks B and C is

(a) $\frac{2m_1m_2\,g}{M}$

(b) $\frac{2m_1m_3\,g}{M}$

(c) $\frac{2m_2m_3g}{M}$

(d) zero

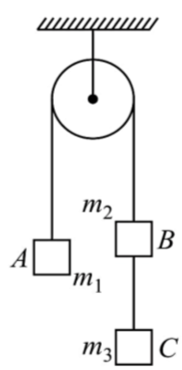


Fig. 3.88

173. A man of mass M stands on the floor of a box of mass m as shown in Fig. 3.89. He raises himself and the box with an acceleration $a = g/3$ by means of a rope going over a fixed frictionless pulley. If the mass of the rope is negligible compared to $(M + m)$ and if $M = 2m$, the tension in the rope will be

(a) 2 mg

(b) $\frac{2mg}{3}$

(c) mg

(d) $\frac{4mg}{3}$

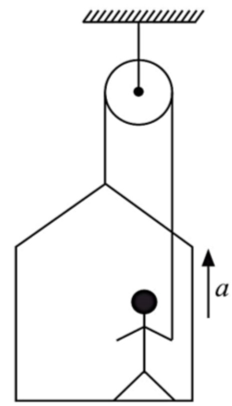


Fig. 3.89

174. In Q. 173 above, the normal reaction (or contact force) between the man and the box is

(a) $\frac{mg}{3}$ (b) $\frac{2mg}{3}$

(c) $\frac{4mg}{3}$ (d) zero

175. Figure 3.90 shows a block of mass m placed on a horizontal surface. The coefficient of static friction between the block and the surface is μ. The maximum force F that can be applied at point O such that the block does not slip on the surface is

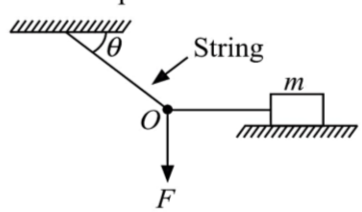


Fig. 3.90

(a) $\mu\,mg \sin\theta$ (b) $\mu\,mg\cos\theta$

(c) $\mu\,mg\tan\theta$ (d) $\mu\,mg$

Answers

(Level A)

1. (a)	2. (a)	3. (c)	4. (d)
5. (c)	6. (a)	7. (c)	8. (a)
9. (b)	10. (d)	11. (c)	12. (d)
13. (d)	14. (d)	15. (a)	16. (a)

17. (c)	18. (c)	19. (c)	20. (a)
21. (d)	22. (d)	23. (a)	24. (b)
25. (d)	26. (a)	27. (b)	28. (c)
29. (b)	30. (d)	31. (c)	32. (d)
33. (b)	34. (a)	35. (a)	36. (b)
37. (b)	38. (a)		

Level B

39.(c)	40. (c)	41. (b)	42. (c)
43. (a)	44. (c)	45. (a)	46. (c)
47. (c)	48. (b)	49. (a)	50. (a)
51. (c)	52. (d)	53. (a)	54. (b)
55. (a)	56. (a)	57. (c)	58. (c)
59. (d)	60. (c)	61. (d)	62. (a)
63. (d)	64. (b)	65. (d)	66. (d)
67. (b)	68. (d)	69. (c)	70. (c)
71. (c)	72. (a)	73. (a)	74. (a)
75. (a)	76. (b)	77. (d)	78. (a)
79. (a)	80. (b)	81. (d)	82. (a)
83. (b)	84. (c)	85. (c)	86. (d)
87. (a)	88. (d)	89. (c)	90. (c)
91. (c)	92. (a)	93. (b)	94. (a)
95. (d)	96. (c)	97. (d)	98. (a)
99. (b)	100. (c)	101. (b)	102. (d)
103. (c)	104. (c)	105. (c)	106. (d)
107. (d)	108. (a)	109. (c)	110. (c)
111. (b)	112. (b)	113. (c)	114. (a)
115. (a)	116. (c)	117. (b)	118. (a)
119. (a)	120. (b)	121. (c)	122. (a)
123. (d)	124. (a)	125. (b)	126. (a)
127. (d)	128. (a)	129. (b)	130. (a)
131. (c)	132. (d)	133. (b)	134. (d)
135. (c)	136. (b)	137. (b)	138. (b)
139. (c)	140. (a)	141. (c)	142. (a)
143. (a)	144. (d)	145. (a)	146. (c)
147. (b)	148. (d)	149. (d)	150. (b)
151. (c)	152. (d)	153. (d)	154. (c)
155. (a)	156. (b)	157. (b)	158. (d)
159. (a)	160. (c)	161. (b)	162. (a)
163. (c)	164. (b)	165. (a)	166. (a)
167. (d)	168. (c)	169. (a)	170. (a)
171. (c)	172. (c)	173. (b)	174. (c)

Solutions

1. The net force acting on mass $m = \dfrac{2mg - mg}{3} = \dfrac{mg}{3}$.

   Therefore, the acceleration of mass $m = \dfrac{mg}{3m} = \dfrac{g}{3}$.

   Hence the correct choice is (a).

2. The total mass of the block-rope system = $M + m$.

   Therefore, the acceleration of the block–rope system $= \dfrac{F}{M+m}$. Thus, the net force acting on the block = acceleration × mass = $\dfrac{FM}{M+m}$. Hence the correct choice is (a).

3. The tension in the rope depends on the acceleration of the block–rope system and the mass of the rope. The tension will be the same at all points on the rope if either the acceleration of the rope is zero or the mass of the rope is negligible compared to the mass of the block. Hence the correct choice is (c).

4. When the lift is descending with a retardation (negative acceleration) a, the effective value of g is $g_{eff} = g + a$. The component of this acceleration along the inclined plane is $g_{eff} \sin\theta = (g + a)\sin\theta$. Hence the correct choice is (d).

5. The accelerations of the block sliding down a smooth and rough 45° inclined planes respectively are

   $$a_1 = g \sin 45° = \frac{g}{\sqrt{2}}$$

   and $a_2 = g(\sin 45° - \mu_k \cos 45°) = \dfrac{g}{\sqrt{2}}(1 - \mu_k)$

   where μ_k is the coefficient of kinetic friction. Now, we know that the square of the time of slide is inversely proportional to the acceleration. Therefore

   $$\frac{t_2^2}{t_1^2} = \frac{a_1}{a_2} = \frac{1}{1-\mu_k}$$

   Since $t_2 = 2t_1$, we have $4 = \dfrac{1}{1-\mu_k}$ or $\mu_k = 0.75$.

   Hence the correct answer is (c).

6. The acceleration of the block while it is sliding down the upper half of the inclined plane is $g \sin\theta$. If μ is the coefficient of kinetic friction between the block and the lower half of the plane, the retardation of the block while it is sliding down the lower half $= -(g \sin\theta - \mu g \cos\theta)$. For the block to come to rest at the bottom of the inclined plane, the acceleration

in the first half must be equal to the retardation in the second half, i.e.

$$g \sin \theta = -(g \sin \theta - \mu g \cos \theta)$$

or $$\mu \cos \theta = 2 \sin \theta$$

$$\mu = 2 \tan \theta$$

Hence the correct choice is (a).

7. Before the boy starts walking on the plank, both the boy and the plank are at rest. Therefore, the total momentum of the boy–plank system is zero. If the boy walks with a speed v on the plank and as a result if the speed of the plank in the opposite direction is V, then the total momentum of the system is $mv - (M + m)V$. From the principle of conservation of momentum, we have

$$mv - (M+m)V = 0$$

or $$\frac{V}{v} = \frac{m}{(M+m)}$$

Since the distance moved is proportional to speed, the displacement L' of the plank is given by

$$\frac{L'}{L} = \frac{V}{v} = \frac{m}{(M+m)}$$

or $$L' = \frac{mL}{(M+m)}$$

Hence the correct choice is (c).

8. The law of conservation of momentum gives

$$m_1 v = m_1 \frac{v}{2} + m_2 v'$$

where v' is the speed of ball B after collision. Thus

$$v' = \frac{m_1 v}{2m_2}$$

Hence the correct choice is (a).

9. At the highest point of trajectory, the projectile has only a horizontal velocity which is $u \cos \theta$. After explosion, the fragment falling downwards has no horizontal velocity. If u' is the horizontal velocity of the other fragment, the law of conservation of momentum gives

$$(2m)\, u \cos \theta = m \times 0 + mu'$$

which gives $$u' = 2u \cos \theta$$

Now, the time taken to reach the highest point (as well as the time taken to fall down from this point) is $\frac{u \sin \theta}{g}$. Therefore, the horizontal distance travelled by the other fragment is

$$u \cos \theta \times \frac{u \sin \theta}{g} + 2\, u \cos \theta \times \frac{u \sin \theta}{g}$$

$$= \frac{u^2 \sin 2\theta}{2g} + \frac{u^2 \sin 2\theta}{g} = \frac{3u^2 \sin 2\theta}{2g}$$

Hence the correct choice is (b).

10. The force exerted by the leaking sand on the truck

$$= \text{rate of change of momentum}$$

$$= \frac{\Delta mu}{\Delta t}$$

The sand falling vertically downward will exert this force on the truck in vertically upward direction. This perpendicular force can do no work on the truck. Since friction is absent, no force is needed to keep the truck moving at a constant speed in the horizontal direction. Hence the correct choice is (d).

11. For masses m_1 and m_2, we have

$$T = m_2 a$$

$$m_1 g - T = m_1 a$$

Adding the two equations, we get

$$a = \frac{m_1 g}{(m_1 + m_2)}$$

Hence, the correct choice is (c).

12. The contact force exerted by A on B is $F_2 = \frac{m_2 F}{(m_1 + m_2)}$, which is choice (d).

13. Let m_A and m_B be the masses of skaters A and B and a_A and a_B their respective accelerations, when they pull at each other. From Newton's third law, action and reaction forces are equal in magnitude, i.e.

$$m_A a_A = m_B a_B$$

or $$m_A \frac{v_A}{t} = m_B \frac{v_B}{t}$$

or $$m_A v_A = m_B v_B$$

or $$m_A^2 v_A^2 = m_B^2 v_B^2 \quad \text{(i)}$$

where v_A and v_B are their respective speeds and t is the time taken for them to meet. Let s_A and s_B be the distances travelled by them when they meet, we have,

$$2a_A s_A = v_A^2 \quad \text{and} \quad 2a_B s_B = v_B^2$$

Using these equations in Eq. (i), noting that $m_A a_A = m_B a_B$, we get $\frac{s_A}{s_B} = \frac{m_B}{m_A} = \frac{70}{50} = \frac{7}{5}$. Since $s_A + s_B = 6$ m; $s_A = 3.5$ m and $s_B = 2.5$ m. Hence, the correct choice is (d).

14. If a body has a constant velocity, both its speed and its direction of motion are constant. Hence, choice (a) is incorrect. A body having a constant speed can have an acceleration, called centripetal acceleration ($= v^2/R$) which is not variable. Hence, choices (b) and (c) are also incorrect. A body having a constant speed can have a varying velocity, e.g. a body moving in a circle with a constant speed; its direction of motion and hence its velocity is continuously changing with time. Hence, the only correct choice is (d).

15. As long as the coin is in the hand of the person, it shares the acceleration of the train; it has the inertia of motion. When he tosses the coin, it falls behind him opposite to the direction of accelerated motion but now it no longer shares the acceleration of the train. Hence the correct choice is (a).

16. The reaction force offered by the wall to the bullets = the force exerted by bullets on the wall (third law of motion) = the rate of change of momentum of bullets (second law of motion). Now, total mass of n bullets $= Nm$. Momentum of n bullets $= Nmv$. If n bullets are fired per second, the change of momentum per second $= nNmv$. Hence, the correct choice is (a).

17. Since $v = \sqrt{2gh}$, the correct choice is (c).

18. The correct choice is (c).

19. Since the shell is at rest, the initial momentum is zero. After it explodes, the total momentum of the equal fragments A and B must be zero, which is possible only if they fly off in opposite directions with equal speeds. Hence the correct choice is (c).

20. Since the bomb is at rest, its momentum is zero. From the principle of conservation of momentum, it follows that, after explosion, the total momentum of all the fragments must be zero. Hence the correct choice is (a).

21. The component of weight Mg of the block along the inclined plane $= Mg \sin\theta$. The minimum frictional force to be overcome is also $Mg \sin\theta$. To make the block just move up the plane the minimum force applied must overcome the component $Mg \sin\theta$ of gravitational force as well as the frictional force $Mg \sin\theta$. Hence the correct choice is (d).

22. Now, $u = 20$ ms^{-1}, $t = 5$s and $v = 0$ (since the car is brought to rest). The acceleration of the car is

$$a = \frac{v-u}{t} = \frac{0-20}{5} = -4 \text{ ms}^{-2}$$

The negative sign indicates retardation and its magnitude is 4 ms^{-2}.

$\therefore$ Retardation force = mass × acceleration

$$= (940 + 60) \times 4 = 4000 \text{ N}$$

Hence the correct choice is (d).

23. Now $2as = v^2 - u^2$. Therefore

$$s = \frac{v^2 - u^2}{2a} = \frac{0-(20)^2}{2\times(-4)} = 50 \text{ m}$$

Hence the correct choice is (a).

24. Given $u = +5$ ms^{-1} along positive x–direction

$F = -0.4$ N along negative x–direction

$m = 200$ g $= 0.2$ kg

The acceleration $a = \frac{F}{m} = \frac{-0.4}{0.2} = -2$ ms^{-2}. The negative sign shows that the motion is retarded. The position of the body at time t is given by

$$x = x_0 + ut + \frac{1}{2}at^2$$

At $t = 0$, the body is at $x = 0$. Therefore, $x_0 = 0$. Hence

$$x = ut + \frac{1}{2}at^2$$

Since the force acts during the time interval from $t = 0$ to $t = 10$ s, the motion is decelerated only between $t = 0$ and $t = 10$ s. The position of the body at $t = 2.5$ s is given by

$$x = 5 \times 2.5 + \frac{1}{2} \times (-2) \times (2.5)^2$$
$$= 1.25 \text{ m}$$

Hence the correct choice is (b).

25. The velocity of the body at $t = 2.5$ s is

$$v = u + at = 5 + (-2) \times (2.5)$$
$$= 5 - 5 = 0$$

Thus, the correct choice is (d).

26. During the first ten seconds (i.e. from $t = 0$ to $t = 10$ s) the motion is decelerated. During this time $a = -2$ ms^{-2}. Putting $u = 5$ ms^{-1}, $a = -2$ ms^{-2} and $t = 10$ s in equation $x = ut + \frac{1}{2}at^2$. We have

$$x_1 = 5\times10+\frac{1}{2}\times(-2)\times(10)^2$$
$$= -50 \text{ m} \qquad \text{(i)}$$

The velocity of the body at $t = 10$ s is

$$v = u + at = 5 + (-2)\times 10$$
$$= -15 \text{ ms}^{-1}$$

During the remaining 20 seconds, i.e. from $t = 10$ s to $t = 30$ s, the acceleration $a = 0$, because the force ceases to act after $t = 10$ s. The velocity of the body remains constant at -15 ms^{-1} during the last 20 seconds. The distance covered by the body during the last 20 seconds is

$$x_2 = -15\times 20 = -300 \text{ m}$$

∴ Position of the body at $t = 30$ s is

$$x = x_1 + x_2 = -50 - 300 = -350 \text{ m}$$

Thus, the correct choice is (a).

27. The magnitude of the velocity (i.e. speed) of the body at $t = 30$ s is 15 ms^{-1}. Hence, the correct choice is (b).

28. Given $u = 0$, $a = 2$ ms^{-2}. Since the stone is located in the train, the acceleration of the stone is $a = 2$ ms^{-2}. At time $t = 5$s, the velocity of the stone is $v = u + at = 0 + 2\times5 = 10$ ms^{-1}. Before the stone is dropped, its motion is accelerated with the train. But, the moment it is dropped, its acceleration due to the motion of the train ceases. Therefore, after the stone is dropped, it has the following two motions:

(a) a uniform motion with velocity 10 ms^{-1} parallel to the ground, i.e.

$$v_x = 10 \text{ ms}^{-1} \text{ (the horizontal velocity)}$$

(b) an accelerated motion vertically downwards due to gravity. In time $t = 0.2$ s, the vertical velocity of the stone is $v_y = 0 + gt = 10\times0.2 = 2$ ms^{-2}.

The resultant velocity of stone at $t = 0.2$ s is

$$v = \sqrt{v_x^2+v_y^2} = \sqrt{(10)^2+(2)^2}$$
$$= = \sqrt{104} = 2\sqrt{26} \text{ ms}^{-1}$$

Hence the correct choice is (c).

29. The angle, which the resultant velocity vector, makes with the horizontal is given by

$$\tan\theta = \frac{v_y}{v_x} = \frac{2}{10} = 0.2$$

Hence, the correct choice is (b).

30. After the stone is dropped, the horizontal velocity v_x remains unchanged because the acceleration is zero along the horizontal direction. The only acceleration of the stone is the acceleration due to gravity. Hence the correct choice is (d).

31. Mass of monkey (m) = 30 kg, g = 10 ms^{-2}. If the monkey climbs up the rope with a uniform acceleration a, the tension in the rope is $T = m(g + a)$. If he climbs down with a uniform acceleration a, the tension is $T = m(g - a)$. In choice (a), since the speed is uniform acceleration $a = 0$. Therefore

$$T = mg = 30\times10 = 300 \text{ N}$$

In case (b), the tension is

$$T = m(g + a) = 30\times(10+2)$$
$$= 360 \text{ N}$$

In case (c), the tension is

$$T = m(g + a) = 30\times(10+5)$$
$$= 450 \text{ N}$$

In case (d), the tension is

$$T = m(g - a) = 30\times(10-5)$$
$$= 150 \text{ N}$$

Since the rope can withstand a maximum tension of 400 N, the rope will break only in case (c). Hence, the correct choice is (c).

32. Cross–sectional area of tube (A) = πr^2. Since the speed of the liquid is v, the volume of liquid flowing out per second = $Av = \pi r^2 v$. Mass of liquid flowing out per second = $\pi r^2 v\rho$. Therefore,

Initial momentum of liquid per second

= mass of liquid flowing per second × speed of liquid

$$= \pi r^2 \rho v^2$$

This is the rate at which momentum is imparted to wall on impact. Since the liquid does not rebound after impact, the momentum after impact is zero. Hence, the rate of change of momentum = $\pi r^2\rho v^2$. From Newton's second law, the force exerted on the wall = rate of change of momentum = $\pi r^2\rho v^2$. Hence, the correct choice is (d).

33. Weight of mass $m_2 = 6\times10 = 60$ N. The weight of m_2 provides the tension. Thus

$$T = 60 \text{ N}$$

Opposing this force along the plane is the component $F_1 = m_1 g \sin\theta$ of the force $m_1 g$. Now $F_1 = m_1 g\sin\theta = 5\times10\times\sin30° = 25$ N. Since F_1 is less than T and is, therefore, insufficient to balance T (see Fig. 3.91), the force of friction (f) down the plane is necessary to keep block m_1 at rest. Thus, f must act down the plane. Since mass m_1 is at rest, the net force on m_1 along the plane must be zero. Thus

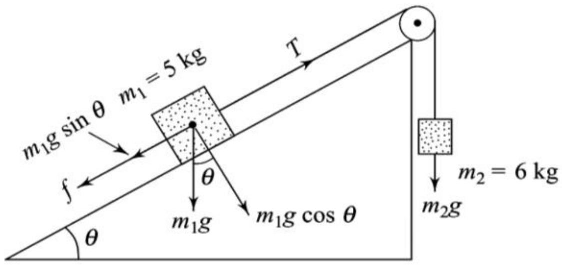


Fig. 3.91

$$T - m_1g \sin 30° - f = 0$$

or
$$f = T - m_1g \sin 30°$$
$$= 60 - 5 \times 10 \times \sin 30°$$
$$= 60 - 25 = 35 \text{ N}$$

Hence, the correct choice is (b).

34. The acceleration of each mass is

$$a = \frac{F}{m_1 + m_2} = \frac{300}{10 + 20} = 10 \text{ ms}^{-2}$$

Hence, the correct choice is (a).

35. The tension in the string is

$$T = m_1 a = 10 \times 10 = 100 \text{ N}$$

Thus, the correct choice is (a).

36. The force on mass m_1 is

$$F_1 = \frac{m_2 F}{m_1 + m_2} = \frac{20 \times 300}{10 + 20} = 200 \text{ N}$$

Hence, the correct choice is (b).

37. If the force is applied to mass m_1, the acceleration remains the same; it does not depend on whether force F is applied to m_2 or m_1. Thus $a = 10$ ms^{-2}. The tension in the string, in this case, is

$$T = m_2 a = 20 \times 10 = 200 \text{ N}$$

Hence, the correct choice is (b).

38. The force exerted on m_2 is

$$F_2 = \frac{m_1 F}{m_1 + m_2} = \frac{10 \times 300}{10 + 20} = 100 \text{ N}$$

Hence, the correct choice is (a).

(Level B)

39. The momentum of the shell before explosion is zero. The total momentum of the three fragments after explosion must also be zero. If m is the mass of each fragment and $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ their velocities, then their momenta are

$$\mathbf{p}_1 = m\,\mathbf{v}_1$$
$$\mathbf{p}_2 = m\,\mathbf{v}_2$$
$$\mathbf{p}_3 = m\,\mathbf{v}_3$$

where $\mathbf{v}_1 = 9$ ms^{-1} along, say, the x–direction and $\mathbf{v}_2 = 12$ ms^{-1} along the y-direction (Fig. 3.92). The resultant of $\mathbf{p}_1$ and $\mathbf{p}_2$ has a magnitude given by

$$p = (p_1^2 + p_2^2)^{1/2} = m(v_1^2 + v_2^2)^{1/2}$$

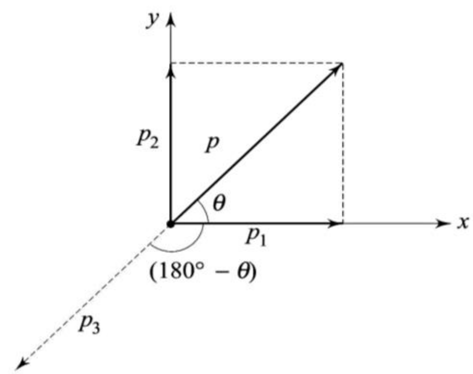


Fig. 3.92

The direction of the resultant vector $\mathbf{p}$ is at an angle θ with the x-axis. Since the total momentum is zero, we have $\mathbf{p}_3 + \mathbf{p} = 0$ or $\mathbf{p}_3 = -\mathbf{p}$. Therefore, the magnitude of $\mathbf{p}_3$ is equal to that of $\mathbf{p}$ but its direction is opposite $(180° - \theta)$ with x–axis. Therefore, magnitude of $\mathbf{p}_3$ is

$$p_3 = \text{magnitude of } \mathbf{p}$$
$$= m(v_1^2 + v_2^2)^{1/2}$$
$$= m \times (9^2 + 12^2)^{1/2}$$
$$= 15\, m \text{ kg ms}^{-1}$$

$\therefore$ Speed of the third fragme]nt $= \dfrac{p_3}{m} = \dfrac{15\,m}{m} =$ 15 ms^{-1}. Hence, the correct choice is (c).

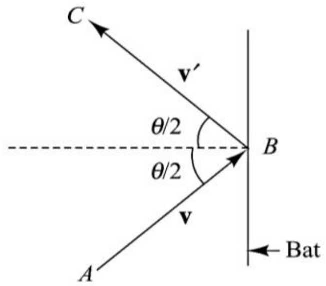
40. Mass of the ball = 0.15 kg. Initial momentum of the ball = 0.15 kg × 12 ms^{-1} = 1.8 kg ms^{-1}. Final momentum of the ball = 0.15 kg × (– 20 ms^{-1}) = – 3 kg ms^{-1}. Change in momentum = 1.8 – (– 3) = 4.8 kg ms^{-1}. This is the impulse of the force exerted by the bat. Now, impulse = average force × time of impact.

$\therefore$ Average force $= \dfrac{\text{impulse}}{\text{time}} = \dfrac{4.8}{0.1} = 48$ N. Hence, the correct choice is (c).

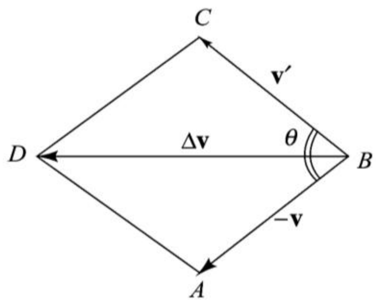
41. Mass of a ball (m) = 0.05 kg, initial velocity each ball (u) = 6 ms^{-1} and final velocity of each ball = – 6 ms^{-1}. Change in momentum of each ball = $mu - mv = m\,(u - v) = 0.05 \times \{6 - (-6)\} = 0.6$ kg ms^{-1}.

Now, impulse = change in momentum = 0.6 kg ms^{-1} = 0.6 Ns. Hence, the correct choice is (b).

42. The ball moving along AB with velocity **v** is deflected along BC with velocity **v′**. The magnitude of vectors **v** and **v′** is the same = v [see Fig. 3.93 (a)].



(a)



(b)

Fig. 3.93

Change in velocity is $\Delta \mathbf{v} = \mathbf{v}' - \mathbf{v} = \mathbf{v}' + (-\mathbf{v})$, i.e. $\Delta \mathbf{v}$ is the resultant of vectors $\mathbf{v}'$ and $-\mathbf{v}$. As shown in Fig. 3.89 (b), the magnitude of $\Delta \mathbf{v}$ is given by

$$\Delta v = \sqrt{v^2 + v^2 + 2v^2 \cos\theta}$$
$$= \sqrt{2v^2(1+\cos\theta)}$$
$$= 2\, v \cos\left(\frac{\theta}{2}\right)$$

∴ Change in momentum = $2\, mv \cos\left(\frac{\theta}{2}\right)$. Hence the correct choice is (c)

43. The slope of the graph between $t = 0$ and $t = 2$ s is constant and positive. Therefore, the body moves from position $x = 0$ to $x = 2$ m during the time interval from $t = 0$ to 2 s. Between $t = 2$ s and $t = 4$ s, the slope of the graph is constant but negative. This implies that at $t = 2$s, the velocity of the body is reversed and it retraces its path and returns to $x = 0$ at $t = 4$ s; and so on. Thus, the body receives impulses at $t = 0$, 2 s, 4 s, ..., etc. Therefore, the interval between two consecutive impulses is 2 s. Hence, the correct choice is (a).

44. Between $t = 0$ and $t = 2$s, the speed of the body is v = slope of the (x–t) graph between $t = 0$ and $t = 2$s, i.e.

$$v = \frac{(2-0)\text{m}}{(2-0)\text{s}} = 1 \text{ ms}^{-1}$$

At $t = 2$ s, the velocity of the body is reversed and it moves in the opposite direction with a speed = – 1 ms^{-1}. Therefore,

$$\text{Impulse} = \text{change in momentum}$$
$$= mv - (-mv) = 2\,mv$$
$$= 2 \times 0.4 \text{ kg} \times 1 \text{ ms}^{-1}$$
$$= 0.8 \text{ kg ms}^{-1} = 0.8 \text{ Ns}$$

Hence, the correct choice is (c).

45. The required force is to (i) accelerate the plane from rest to a speed v over a distance s and (ii) to overcome the force of friction (= $\mu R = \mu Mg$). The acceleration a required to impart a speed v in a distance s is given by $v^2 - u^2 = 2as$. Since, $u = 0$, we have $v^2 = 2\,as$ or $a = v^2/2$s. The force needed to produce this acceleration is

$$F_1 = \text{mass} \times \text{acceleration} = \frac{Mv^2}{2s}$$

The force needed to overcome the force of friction is

$$F_2 = \mu Mg$$

$$\therefore \text{ Total force needed} = F_1 + F_2 = M\left(\frac{v^2}{2s} + \mu g\right)$$

Hence, the correct choice is (a).

46. Refer to Fig. 3.94. Since the block is projected upwards, it rises after overcoming two forces: (i) the component $mg \sin\theta$ of the weight mg and (ii) the force of friction $F = mg \sin\theta$, both acting downwards. Therefore, the total downward acceleration is

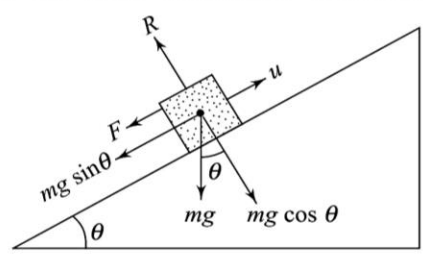


Fig. 3.94

$a = -g \sin\theta - g \sin\theta = -2g \sin\theta$

Let s be the distance moved up the plane before the block comes to rest. Then, from $v^2 - u^2 = 2as$, we have

$$0 - u^2 = 2 \times (-2g \sin\theta) \times s$$

or
$$s = \frac{u^2}{4g\sin\theta}$$

Hence, the correct choice is (c).

47. The weight $W = mg$ of the block can be resolved into two rectangular components: one along the plane ($W \sin \theta$) and the other perpendicular to it ($W \cos \theta$). Let R be the magnitude of the normal reaction and f_s be the force of sliding friction (see Fig. 3.95).

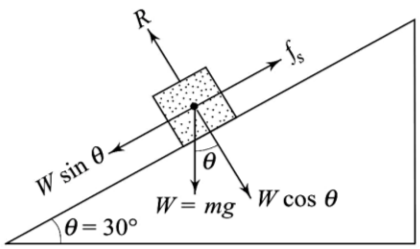


Fig. 3.95

When these forces are in equilibrium, the block just begins to slide, i.e.

$$f_s = W \sin \theta$$

Also $$R = W \cos \theta$$

$\therefore$ Coefficient of sliding friction is $\mu_s = \dfrac{f_s}{R} = \dfrac{W \sin \theta}{W \cos \theta}$

$$= \tan \theta = \tan 30°.$$

Hence, the correct choice is (c).

48. The minimum force F, applied parallel to the plane, that would just make the block move up the plane, must not only overcome the force of friction f_s but also the component $mg \sin \theta$ of the weight along the plane. Therefore,

$$F = f_s + mg \sin \theta$$

But $f_s = mg \sin \theta$. Therefore, $F = 2\, mg \sin \theta = 2 \times 5 \times 10 \times \sin 30° = 50$ N.

Hence the correct choice is (b).

49. Since the block is placed on the trolley, the acceleration of the block = acceleration of the trolley $a = 3$ ms^{-2}. Therefore, the force acting on the block is

$$F = ma = 10 \times 3 = 30 \text{ N}$$

The weight mg of the block is balanced by the normal reaction R. As the trolley accelerates in the forward direction, it exerts a reaction force $F = 30$ N on the block in the direction, as shown in the Fig. 3.63 on page 3.28. The force of friction will oppose this force and will act in a direction opposite to that of F. The force of limiting friction f is given by

$$\mu = \frac{f}{R} = \frac{f}{mg}$$

or $$f = \mu mg = 0.2 \times 10 \times 10 = 20 \text{ N}$$

Thus, the block is acted upon by two forces-force $F = 30$ N towards the right and frictional force $f = 20$ N towards the left. The net force on the block towards the right, i.e. towards the rear end of the trolley is

$$F' = F - f = 30 - 20 = 10 \text{ N}$$

Due to this force, the block experiences an acceleration towards the rear end which is given by

$$a' = \frac{F'}{m} = \frac{10}{10} = 1 \text{ ms}^{-2}$$

Let t be the time taken for the block to fall from the rear end of the trolley. Clearly, the block has to travel a distance $S' = 5$ m to fall off the trolley. Since the trolley starts from rest, initial velocity $u = 0$. Now t can be obtained from the relation

$$s = ut + \frac{1}{2} at^2$$

Putting $s = 5$ m, $u = 0$ and $a = a' = 1$ ms^{-2}, we get $t = \sqrt{10}$ s.

The distance covered by the trolley in time $t = \sqrt{10}$ s is ($\because u = 0$)

$$s' = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} \times 3 \times 10 = 15 \text{ m}$$

Hence, the correct choice is (a).

50. Let f be the frictional force on each block.

$$m_1 a = T - f \qquad \text{(i)}$$

and $$m_2 a = F - T - f \qquad \text{(ii)}$$

Subtracting the two equations, we have

$$(m_1 - m_2)\, a = 2T - F$$

Since $m_1 = m_2$, we get $0 = 2T - F$ or $T = \dfrac{F}{2} = \dfrac{20}{2} = 10$ N

Hence, the correct choice is (a).

51. Putting $T = 10$ N in Eq. (i) above, we have

$$f = T - m_1 a$$
$$= 10 - 3 \times 0.5 = 8.5 \text{ N}$$

Hence, the correct choice is (c).

52. Force of friction is

$$f = \mu R = \mu\, mg \cos \theta$$

When the body slides down, the downward force along the plane = component $mg \sin \theta$ of the weight mg. Since the force of friction acts upwards along the plane, the effective downward force = $mg \sin \theta - \mu mg \cos \theta = mg(\sin \theta - \mu \cos \theta)$

$\therefore$ Acceleration = force/mass = $g(\sin \theta - \mu \cos \theta) = g(\sin \theta - kx \cos \theta)$. Hence, the acceleration varies with x and decreases as x increases. Thus, the correct choice is (d).

53. The mass of water stream striking against the wall in 1 second = $av\rho$. Hence, the change in its momentum per second is $(av\rho)v - (-av\rho)v = 2a\rho v^2$. The normal component of the rate of change of momentum and, therefore, force is $2a\rho v^2 \cos\theta$. Hence the correct choice is (a).

54. The square of the time of slide is inversely proportional to the acceleration. The accelerations in the two cases are

$$a_1 = g \sin 45° = \frac{g}{\sqrt{2}} \text{ and}$$

$$a_2 = (g \sin 45° - \mu_k g \cos 45°) = \frac{g}{\sqrt{2}}(1 - \mu_k)$$

$$\therefore \quad \frac{t_2^2}{t_1^2} = n^2 = \frac{a_1}{a_2} = \frac{1}{1-\mu_k}$$

or $$\mu_k = 1 - \frac{1}{n^2}.$$

Hence, the correct choice is (b).

55. Refer to Fig. 3.96. The net downward force on the body at a distance x is

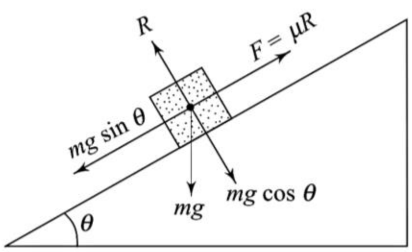


Fig. 3.96

$$f(x) = mg \sin\theta - \mu mg \cos\theta$$
$$= mg(\sin\theta - \mu\cos\theta)$$
$$= mg(\sin\theta - kx\cos\theta)$$

$\therefore f(x) = 0$ at a value of $x = x_0$ given by

$$\sin\theta - kx_0\cos\theta = 0$$

which gives $$x_0 = \frac{\tan\theta}{k}$$

Thus, the correct choice is (a).

56. The acceleration of the body down the plane is $g\sin\theta - \mu g\cos\theta = g(\sin\theta - \mu\cos\theta) = g(\sin\theta - 0.5x\cos\theta)$. Therefore, the body will first accelerate up to $x < 2\tan\theta$. The velocity will be maximum at $x = 2\tan\theta$, because for $x > 2\tan\theta$, the body starts decelerating. Hence, the correct choice is (a).

57. Refer to Fig. 3.97. Given $AB = 1$, $AC = l$, so that $BC = \sqrt{l^2-1}$. Thus $\tan\theta = AB/BC = 1/\sqrt{l^2-1}$. A horizontal acceleration a imparted to the inclined plane has a component $a\cos\theta$ down the plane. If this equals the component $g\sin\theta$ of the g along the plane, the object will appear stationary relative to the incline, i.e. if

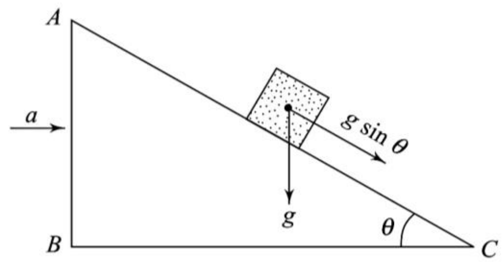


Fig. 3.97

$$a\cos\theta = g\sin\theta$$

or $$a = g\tan\theta = \frac{g}{\sqrt{l^2-1}}$$

Hence, the correct choice is (c).

58. Refer to Fig. 3.98.

Let A be the position of the insect when it has reached the maximum height h. Now $OA = OC = R$. The insect will crawl up the bowl until the component $mg\sin\theta$ of his weight down the plane equals the force $F = \mu mg\cos\theta$ of limiting friction (the insect will slip down if $mg\sin\theta$ exceeds $\mu mg\cos\theta$). Thus $mg\sin\theta = \mu mg\cos\theta$ or $\tan\theta = \mu = 1/3$.

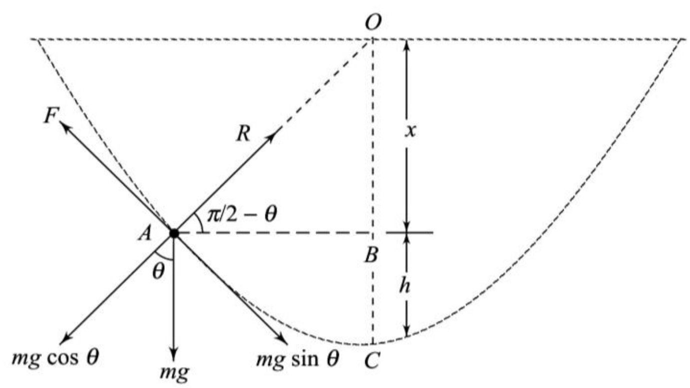


Fig. 3.98

Now in triangle OAB, $\tan\left(\frac{\pi}{2} - \theta\right) = \frac{OB}{AB}$.

Let $OB = x$, then $AB = \sqrt{R^2 - x^2}$. Thus

$$\cot\theta = \frac{x}{\sqrt{R^2-x^2}}$$

or $3 = \dfrac{x}{\sqrt{R^2 - x^2}}$ $\left(\because \tan\theta = \dfrac{1}{3}\right)$

which gives $x = \dfrac{3R}{\sqrt{10}} = 0.95\,R$. Therefore, $h = R - x = R - 0.95\,R = 0.05\,R$. Hence, the correct choice is (c).

59. The acceleration a is

$$a = \frac{F}{m_1 + m_2} = \frac{4}{5+3} = 0.5\ \text{ms}^{-2}$$

which is the same for both masses. Hence, the correct choice is (d).

60. The force on mass of 3 kg = $m_2 a = 3\ \text{kg} \times 0.5\ \text{ms}^{-2} = 1.5$ N. Hence the correct choice is (c).

61. The acceleration a, which is the same for both masses will still be 0.5 ms^{-2}. Hence the correct choice is (d).

62. Force exerted on mass 5 kg = 5 kg × 0.5 ms^{-2} = 2.5 N. Hence the correct choice is (a).

63. The acceleration of the system is

$$a = \frac{\text{net force}}{\text{total mass}} = \frac{12}{1+2+3} = 2\ \text{ms}^{-2}$$

Hence the correct choice is (d).

64. The contact force on mass m_2 is

$$F_2 = (m_2 + m_3)a = (2 + 3) \times 2 = 10\ \text{N}$$

Hence the correct choice is (b).

65. The contact force on mass m_3 is

$$F_3 = m_3 a = 3 \times 2 = 6\ \text{N}$$

Hence the correct choice is (d).

66. The acceleration of the system is

$$a = \frac{\text{net force}}{\text{total mass}} = \frac{12}{1+2+3} = 2\ \text{ms}^{-2}$$

Hence the correct choice is (d).

67. Tension T_2 between m_1 and m_2 is

$$T_2 = (m_2 + m_3)a = (2 + 3) \times 2 = 10\ \text{N}$$

Hence the correct choice is (b).

68. Tension T_3 between m_2 and m_3 is

$$T_2 = m_3 a = 3 \times 2 = 6\ \text{N}$$

Hence the correct choice is (d).

69. When the balls are released, the force experienced by mass m is F = its mass × its acceleration = ma. This is the force exerted by mass M on mass m. From Newton's third law, the mass m will exert an equal force F on mass M. Thus, force on $m_2 = F = ma$. Therefore, the acceleration of M is

$$a' = \frac{F}{M} = \frac{ma}{M}$$

Hence the correct choice is (c).

70. Impulse = change in momentum. Let v be the speed of the ball before it was caught and m its mass. Since the ball is brought to rest after it is caught, the change in momentum = mv. Now m = 0.2 kg. Therefore, impulse is 0.2 v. Given, impulse = 6 Ns. Hence

$$v = \frac{6}{0.2} = 30\ \text{ms}^{-1}$$

Thus, the correct choice is (c).

71. Let m be the mass of the car and a be the deceleration produced by force F, then $F = ma$, where a is given by

$$2as = v^2 \text{ or } a = \frac{v^2}{2s}$$

Therefore, $F = \dfrac{mv^2}{2s}$. Thus $F \propto v^2$. If v is increased by 3 times, F will increase by 9 times. Hence, the correct choice is (c).

72. Now, $F = \dfrac{mv^2}{2s}$, which implies that $s \propto \dfrac{1}{F}$, i.e. s is inversely proportional to F. Thus, the correct choice is (a).

73. Force acting on mass m is $f = ma$. Mass m will pull mass M to the left with a force $f = ma$. Hence, the net acting on mass $M = F - f = F - ma$. Therefore, acceleration of mass $M = \dfrac{F - ma}{M}$. Hence the correct choice is (a).

74. Let a be the minimum acceleration with which the boy must climb down the rope. Then $mg - T = ma$ or $T = mg - ma$ is the maximum tension. Now, $T = \dfrac{2}{3}\,mg$. Therefore,

$$\frac{2}{3}\,mg = mg - ma$$

which gives $a = g/3$. Hence the correct choice is (a).

75. The mass is in equilibrium at point O under the action of the concurrent forces F, T and $W = mg$. Therefore, as shown in Fig. 3.99, The horizontal component $T \sin\theta$ of tension T must balance with force F and

the vertical component $T\cos\theta$ must balance with weight $W = mg$. Thus

$$F = T\sin\theta \quad \text{(i)}$$

and $$W = T\cos\theta \quad \text{(ii)}$$

Thus, the correct choice is (a).

76. From Eq. (ii) above, it follows that the correct choice is (b).

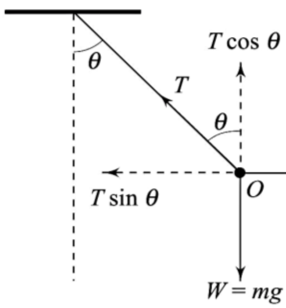


Fig. 3.99

77. Squaring Eqs. (i) and (ii) and adding we get $T^2 = W^2 + F^2$. Hence the correct choice is (d).

78. Dividing (i) by (ii) we get $F = W\tan\theta$, which is choice (a).

79. The common acceleration of the masses is

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} = \frac{(6-4)\times 10}{(6+4)} = 2\text{ ms}^{-2}$$

Hence the correct choice is (a).

80. The tension in the string is

$$T = \frac{2m_1m_2g}{(m_1+m_2)} = \frac{2\times 6\times 4\times 10}{(6+4)} = 48\text{ N}$$

Hence the correct choice is (b).

81. If the lift rises upwards with an acceleration a, the effective value of g is $g_{eff} = g + a$. Hence

$$T = \frac{2m_1m_2}{(m_1+m_2)}\times(g+a) = \frac{2\times 6\times 4}{10}\times(10+5)$$
$$= 72\text{ N}$$

Thus, the correct choice is (d).

82. In this case, $g_{eff} = g - a$. Hence

$$T = \frac{2m_1m_2}{(m_1+m_2)}\times(g-a) = 24\text{ N}$$

Thus, the correct choice is (a).

83. Since the acceleration of the train is perpendicular to the acceleration due to gravity, the acceleration vector of the train has no component along the vertical direction. Hence, in this case, $g_{eff} = g$. Thus, the correct choice is (b).

84. Let h be the height of each inclined plane. Then, the distances along the plane are $s_1 = \dfrac{h}{\sin\theta_1}$ and $s_2 = \dfrac{h}{\sin\theta_2}$ respectively. The accelerations of the block are $a_1 = g\sin\theta_1$ and $a_2 = g\sin\theta_2$ respectively. Now, since the block is released from rest, the velocity of the block when it reaches the bottom of the planes is $v_1^2 = 2a_1s_1$ and $v_2^2 = 2a_2s_2$ respectively. But $v_1 = a_1t_1$ and $v_2 = a_2t_2$ or $a_1^2t_1^2 = 2a_1s_1$ and $a_2^2t_2^2 = 2a_2s_2$. These equations give

$$\frac{t_2^2}{t_1^2} = \frac{a_1}{a_2}\cdot\frac{s_2}{s_1} = \frac{g\sin\theta_1}{g\sin\theta_2}\cdot\frac{h}{\sin\theta_2}\cdot\frac{\sin\theta_1}{h}$$
$$= \frac{\sin^2\theta_1}{\sin^2\theta_2}$$
$$\Rightarrow \quad \frac{t_2}{t_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

Hence, the correct choice is (c).

85. Let m be the mass of the rope and l its length. The tension T at a distance x from the support = weight of length $(l - x)$ of the rope = $\dfrac{mg}{l}\times(l-x)$ or

$$T = \frac{6\times 10}{3}\times(3-1) = 40\text{ N}$$

Hence the correct choice is (c).

86. The acceleration of the block sliding down the smooth inclined plane is $a_1 = g\sin\theta$ and down the rough inclined plane is $a_2 = g\sin\theta - \mu g\cos\theta$. Given $t_1 = t$ and $t_2 = 2t$. If the length of the inclined plane is s, we have

$$s = \frac{1}{2}a_1t_1^2 = \frac{1}{2}a_2t_2^2$$

or $$a_1t_1^2 = a_2t_2^2$$

or $$g\sin\theta\times t^2 = (g\sin\theta - \mu g\cos\theta)\times(2t)^2$$

or $$\sin\theta = 4(\sin\theta - \mu\cos\theta)$$

which gives $$\mu = \frac{3}{4}\tan\theta = \frac{3}{4} \qquad (\because\ \theta = 45°)$$

Hence the correct choice is (d).

87. We use the relation $v^2 - u^2 = 2as$. Since $u = 0$, we have $v^2 = 2as$.

Now $v_1^2 = 2a_1s$ or $v^2 = 2g\sin\theta\times s$

and $v_2^2 = 2a_2s$ or $\dfrac{v^2}{n^2} = 2(g\sin\theta - \mu g\cos\theta)\times s$

Dividing, we get or $n^2(\sin\theta - \mu\cos\theta) = \sin\theta$

which gives $\mu = \left(1 - \dfrac{1}{n^2}\right)\tan\theta$, which is choice (a).

88. The force of friction between block m and block $M = \mu_1 mg$, where μ_1 is the coefficient of friction between the two blocks. Now, the force of friction between block M (with block m on top of it) and the horizontal surface $= \mu_2 (M + m)g$, where μ_2 is the coefficient of friction between block M and the surface. The maximum force F applied to block M must be enough to overcome this force of friction and the force due to acceleration of the system. If the acceleration of the system is a then this force $= (M + m)a$. Thus

$$F = (M + m)a + \mu_2 (M + m)g \quad \text{(i)}$$

Now, since the force on block m is $\mu_1 mg$, its acceleration is

$$a = \frac{\text{force on mass } m}{\text{mass } m} = \frac{\mu_1 mg}{m} = \mu_1 g \quad \text{(ii)}$$

Using (ii) in (i) we get

$$\begin{aligned} F &= \mu_1 (M + m)g + \mu_2 (M + m)g \\ &= (\mu_1 + \mu_2)(M + m)g \\ &= (0.5 + 0.7) \times (5 + 3) \times 10 \\ &= 96 \text{ N} \end{aligned}$$

Hence the correct choice is (d).

89. The force of friction acting on the box $= \mu mg$, where m is the mass of the box. This force produces an acceleration $a = \mu mg/m = \mu g$ in the box. The box will slide on the belt, without slipping, till it attains a speed (v) = the speed of the belt. The distance s moved by the box is given by

$$v^2 - u^2 = 2as$$

Since $u = 0$, $s = \dfrac{v^2}{2a} = \dfrac{v^2}{2\mu g} = \dfrac{4 \times 4}{2 \times 0.8 \times 10} = 1$ m.

Hence the correct choice is (c).

90. When the masses are accelerating, there is a tension in the string. When a mass m is added to m_1 such that the acceleration is zero, the system of masses $(m_1 + m)$ will slide on the surface with a uniform speed and then there is no tension in the string. This will happen if the downward force $m_2 g$ equals the force of friction $\mu(m_1 + m)g$ on blocks m_1 and m, i.e. if $\mu(m_1 + m)g = m_2 g$ or $m = \dfrac{m_2}{\mu} - m_1 = \dfrac{6}{0.4} - 4 =$ 11 kg. Hence the correct choice is (c).

91. The block m_1 will just begin to move up the plane if the downward force $m_2 g$ due to mass m_2 trying to pull the mass m_1 up the plane just equals the force $(m_1 g \sin\theta + \mu m_1 g \cos\theta)$ trying to push the mass m_1 down the plane, i.e. when

$$m_2 g = m_1 g (\sin\theta + \mu \cos\theta)$$

Now, it is given that $m_1 = m_2 = m$ and $\theta = 30°$. Therefore, we have

$$1 = \sin 30° + \mu \cos 30°$$

which gives $\mu = \dfrac{1}{\sqrt{3}}$. Hence the correct choice is (c).

92. The block m_1 will just begin to move down the plane if the downward force $(m_1 g \sin\theta - \mu m_1 g \cos\theta)$ on m_1 just equals the upward force $m_2 g$ acting on m_1 due to m_2, i.e. if

$$m_2 g = m_1 g (\sin\theta - \mu \cos\theta)$$

or $$\frac{m_1}{m_2} = \frac{1}{\sin\theta - \mu\cos\theta} = \frac{1}{\sin 30° - \frac{1}{2\sqrt{3}}\cos 30°}$$

$= 4$, which is choice (a).

93. The block will just begin to slide when the force of limiting friction ($mg \sin\theta$) = force of normal reaction $\mu mg \cos\theta$, i.e. if $mg \sin\theta = \mu mg \cos\theta$ or $\mu = \tan\theta$. Therefore, $\mu = \tan 30° = 1/\sqrt{3}$. Hence the correct choice is (b).

94. Downward acceleration is

$$\begin{aligned} a &= g(\sin\theta - \mu\cos\theta) \\ &= g \times \left(\sin 30° - \frac{1}{\sqrt{3}} \times \cos 30°\right) \\ &= g \times \left(\frac{1}{2} - \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}\right) = 0 \end{aligned}$$

Hence the correct choice is (a).

95. Normal reaction $R = f$. Therefore, force of friction $= \mu R = \mu f$. The net downward force $F = mg - \mu f$. Hence, the acceleration $a = \dfrac{F}{m} = \dfrac{mg - \mu f}{m} = g - \dfrac{\mu f}{m}$. Hence the correct choice is (d).

96. As the boy is climbing the pole at a constant speed (no acceleration), the force of friction must be just balanced by his weight, i.e. $\mu R = mg$ or $R = \dfrac{mg}{\mu} = \dfrac{40 \times 10}{0.8} = 500$ N. Hence the correct choice is (c).

97. Block m will move over block M if the force F applied to m exceeds the force of static friction between the two blocks. The minimum F must be just enough to overcome static friction. Thus F_{min} = coefficient of friction × normal reaction of M on $m = \mu_s mg$.
Hence the correct choice is (d).

98. If $F > \mu_s mg$, the block m will start moving on block M. It will, therefore, exert a force on block M due to kinetic friction between m and M. This force $f_k = \mu_k mg$. Thus, the acceleration of block $M = \frac{f_k}{M} = \mu_k \frac{mg}{M}$. Hence the correct choice is (a).

99. Given m = 5 kg, μ_s = 0.7 and μ_k = 0.5. The force applied to the block sufficient to move it = force of static friction, i.e. $F = \mu_s mg = 0.7 \times 5 \times 10 = 35$ N. Force responsible for producing acceleration of the block is

$$\begin{aligned} f &= \text{applied force} - \text{force of dynamic friction} \\ &= F - \mu_k mg \\ &= 35 - 0.5 \times 5 \times 10 \\ &= 35 - 25 = 10 \text{ N} \end{aligned}$$

$\therefore$ Acceleration $a = \frac{f}{m} = \frac{10}{5} = 2 \text{ ms}^{-2}$

Hence the correct choice is (b).

100. Let the initial speed be u. Final speed $v = 0$ in both cases. The retardation for $\theta_1 = 30°$ is $a_1 = g \sin \theta_1$ and for $\theta_2 = 60°$ is $a_2 = g \sin \theta_2$. Now, using $v^2 - u^2 = 2ax$, we have

$$u^2 = 2a_1x_1 = 2a_2x_2$$

Thus $$\frac{x_1}{x_2} = \frac{a_2}{a_1} = \frac{g\sin\theta_2}{g\sin\theta_1} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$$

Hence the correct choice is (c).

101. From the principle of conservation of momentum, we have

$$m_1v_1 = m_2v_2 \text{ or } \frac{v_1}{v_2} = \frac{m_2}{m_1} \quad \text{(i)}$$

When the spring is released, it exerts an equal and opposite force F on each block. Let a_1 and a_2 be the accelerations of blocks m_1 and m_2 respectively. Then

$$F = m_1a_1 = m_2a_2 \text{ or } \frac{a_2}{a_1} = \frac{m_1}{m_2} \quad \text{(ii)}$$

Also $v_1^2 = 2a_1x_1$ and $v_2^2 = 2a_2x_2$, which give

$$\frac{x_1}{x_2} = \frac{v_1^2}{v_2^2}.\frac{a_2}{a_1} = \left(\frac{m_2}{m_1}\right)^2 \times \left(\frac{m_1}{m_2}\right) = \frac{m_2}{m_1}$$

[Use Eqs. (i) and (ii)]

Hence the correct choice is (b).

102. Refer to Fig. 3.100. The component of acceleration vector **a** along the plane is $a \cos \theta$. The component of acceleration due to gravity **g** along the plane is $g \sin \theta$. The block will stay at rest if $a \cos \theta = g \sin \theta$ or $a = g \tan \theta$

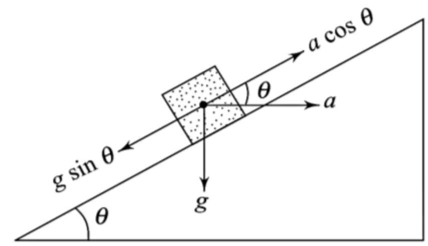


Fig. 3.100

Now $\theta = 30°$. Therefore, $a = g \tan 30° = \frac{g}{\sqrt{3}}$. Hence the correct choice is (d).

103. The acceleration of block moving down the inclined plane is

$$a = g \sin \theta - \mu g \cos \theta$$

Putting $a = \frac{g}{4}$ and $\theta = 30°$, we get $\mu = \frac{1}{2\sqrt{3}}$, which is choice (c).

104. The force F_1 required to prevent the block from sliding down is

$$F_1 = mg \sin \theta - \mu\, mg \cos \theta \quad \text{(i)}$$

The force F_2 required to make the block move up the plane is

$$F_2 = mg \sin \theta + \mu\, mg \cos \theta \quad \text{(ii)}$$

From Eqs. (i) and (ii) we get

$$F_2 + F_1 = 2\, mg \sin \theta$$

and $$F_2 - F_1 = 2\, \mu mg \cos \theta$$

Dividing the two equations, we get

$$\frac{F_2 + F_1}{F_2 - F_1} = \frac{\tan \theta}{\mu} = \frac{\tan 30°}{1/2\sqrt{3}} = 2$$

which gives $F_2 = 3\, F_1$. Hence the correct choice is (c).

105. We have, $h = \frac{1}{2} gt^2$. When the lift is rising up with an acceleration a, the effective acceleration is $g' = g + a$ and t' is given by $h = \frac{1}{2} g't'^2$. Thus

$$\frac{1}{2} g't'^2 = \frac{1}{2} gt^2$$

or $$(g + a)t'^2 = gt^2$$

or $$t' = t\left(1 + \frac{a}{g}\right)^{1/2}$$ which is choice (c).

106. In this case, the effective acceleration of the coin is $g' = g - a$. Thus the correct choice is (d).

107. Let M be the mass of the block and m that of the rope. The acceleration of the block–rope system is

$$a = \frac{F}{(M+m)}$$

Therefore, the tension at the middle point of the rope will be

$$T = \left(M + \frac{m}{2}\right)a = \frac{\left(M + \frac{m}{2}\right)F}{(M+m)}$$

Given, $m = \frac{M}{2}$. Therefore, $T = \frac{5F}{6}$. Hence the correct choice is (d).

108. The acceleration $a = \frac{F}{m}$. The resultant of F_1 and F_2 has magnitude F' given by (see Fig. 3.101).

$F' = (F_1^2 + F_2^2)^{1/2} = \sqrt{2}\,F \; (\because F_1 = F_2 = F)$

The direction $\mathbf{F}'$ is opposite to that of $\boldsymbol{F}_3$.

$\therefore$ Net force on body $= F' - F_3$

$$= \sqrt{2}\;F - F$$

$$= (\sqrt{2} - 1)\,F$$

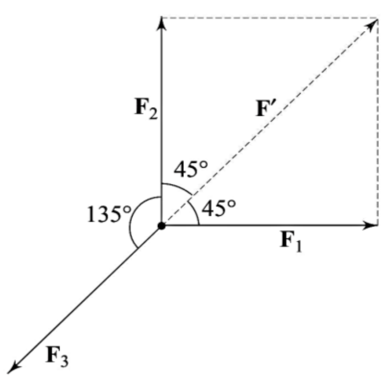


Fig. 3.101

$\therefore$ Acceleration $= (\sqrt{2} - 1)\,\frac{F}{m} = (\sqrt{2} - 1)a$. Hence the correct choice is (a).

109. The mass of two fragments of equal masses $= \frac{2m}{5}$ each. The mass of the lighter fragment $= \frac{m}{5}$. The momenta of heavier fragments are $p = \frac{2mv}{5}$. The resultant of momenta p and p is

$$p' = (p^2 + p^2)^{1/2} = \sqrt{2}\;p$$

From the principle of conservation of momentum, the momentum of the third (lighter) fragment of mass $\frac{m}{5}$ must be $\sqrt{2}\,p$ but opposite in direction. Thus, if V is the speed of the lighter fragment, we have

$$\frac{mV}{5} = \sqrt{2}p = \sqrt{2}\,\frac{2mv}{5}$$

or $$V = 2\sqrt{2}\;v$$

Hence the correct choice is (c).

110. Now, $mv = 3t^2 + 4$. Since $m = 2$ kg, $v = \frac{3}{2}\,t^2 + 2$. The acceleration is $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{3}{2}t^2 + 2\right) = 3t$. Thus, the acceleration of the body is increasing with time. Hence the correct choice is (c).

111. At $t = 2$s, acceleration $a = 3 \times 2 = 6$ ms^{-2}. Hence the correct choice is (b).

112. We know from Newton's second law that force F is given by

$$F = \frac{dp}{dt} = \frac{d}{dt}\left(3t^2 + 4\right) = 6t$$

Hence the correct choice is (b).

113. For an elastic spring, the relation between force F and extension x is

$$F = kx$$

where k is the force constant of the spring. Let l_0 be the original length of the spring, then $F = k(l - l_0)$ where l is the spring length when stretched by a force F. We are given that

$$2 = k(l_1 - l_0) \quad \text{(i)}$$

and $$3 = k(l_2 - l_0) \quad \text{(ii)}$$

Dividing (ii) by (i) we have

$$\frac{3}{2} = \frac{l_2 - l_0}{l_1 - l_0}$$

Which gives $l_0 = 3l_1 - 2l_2$. Using this value of l_0 in either (i) or (ii) we get $k = \frac{1}{l_2 - l_1}$.

When a stretching force of 5 N is applied, let l_3 be the length of the spring. Then

$$5 = k(l_3 - l_0)$$

Substituting the values of l_0 and k, and solving we get

$$l_3 = 3l_2 - 2l_1$$

Hence the correct choice is (c).

114. Refer to Fig. 3.102. The velocity of the shell at the highest point is $v = u \sin\theta = 100 \times \sin 30° = 50 \text{ ms}^{-1}$ parallel to the positive x-direction. Let m be the mass of the shell. Then the mass of the lighter fragment is $m_1 = \frac{m}{3}$ and its momentum is $p_1 = m_1 v_1$; where $v_1 = 200 \text{ ms}^{-1}$. The direction of p_1 is vertically upwards. The mass of the heavier fragment is $m_2 = \frac{2m}{3}$ and its momentum is $p_2 = m_2 v_2$, where v_2 is the speed of the heavier fragment at the time of explosion. Let momentum vector p_2 substend an angle α with the x-direction as shown. From the law of conservation of momentum, the component $p_2 \sin\alpha$ of p_2 along y-direction must balance with p_1 and the component $p_2 \cos\alpha$ must balance with p, i.e.

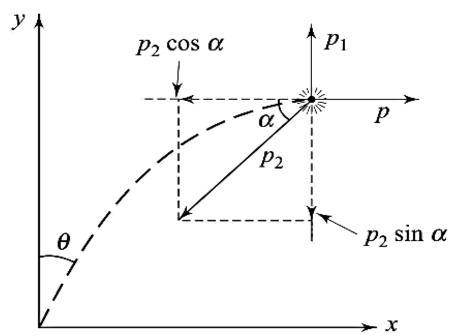


Fig. 3.102

$p_2 \sin\alpha = p_1$ or $m_2 v_2 \sin\alpha = m_1 v_1$

or $$\frac{2mv_2}{3}\sin\alpha = \frac{mv_1}{3}$$

or $$2v_2 \sin\alpha = v_1 \quad \text{(i)}$$

and $p_2 \cos\alpha = p$ or $m_2 v_2 \cos\alpha = mv$

or $$\frac{2mv_2}{3}\cos\alpha = mv \text{ or}$$

$$2\,v_2 \cos\alpha = 3v \quad \text{(ii)}$$

Squaring and adding (i) and (ii) we have

$$4v_2^2 = v_1^2 + 9v^2$$

or $$2v_2 = (v_1^2 + 9v^2)^{1/2}$$

Now $v = 50 \text{ ms}^{-1}$ and $v_1 = 200 \text{ ms}^{-1}$ (given). Using these values, we get $v_2 = 125 \text{ ms}^{-1}$. Hence the correct choice is (a).

115. The forces acting on the balloon are its weight acting downwards and upthrust F acting upwards. Thus

$$F - Mg = Ma \quad \text{(i)}$$

When mass m is removed, we have

$$F - (M - m)\,g = (M - m)\,a' \quad \text{(ii)}$$

where a' is the new acceleration. Eliminating F from (i) and (ii) and simplifying we get

$$a' = \frac{Ma + mg}{M - m}$$

which is choice (a).

116. Let m be the mass per unit length of the rope. Let x be the part of the rope on the floor at time t. The weight of this part is

$$F_1 = mgx$$

Now, if a small part dx falls on the floor in time dt, the force exerted by it is

$$F_2 = \text{rate of change of momentum}$$

$$= \frac{(m\,dx)v}{dt}$$

Now $\frac{dx}{dt} = v$, where v is the velocity of that part of the rope at that instant. But $v^2 = 2gx$. Hence $F_2 = mv^2 = m \times (2gx) = 2mgx$. Total force $F = F_1 + F_2 = mgx + 2\,mgx = 3mgx = 3F_1$

Hence the correct choice is (c).

117. Given $\frac{d^2x}{dt^2} = 2t - t^2$ or $\frac{dv}{dt} = 2t - t^2$. Integrating

we get $$v = t^2 - \frac{t^3}{3} = t^2\left(1 - \frac{t}{3}\right)$$

The body will reverse its direction of motion at a time t when $v = 0$, i.e. at $t = 3$ s. Now, since $v = \frac{dx}{dt}$, we have $\frac{dx}{dt} = t^2 - \frac{t^3}{3}$

Integrating, we have

$$x = \frac{t^3}{3} - \frac{t^4}{12} = \frac{t^3}{3}\left(1 - \frac{t}{4}\right)$$

$$\therefore \quad x\,(\text{at } t = 3\text{ s}) = \frac{(3)^3}{3}\left(1 - \frac{3}{4}\right) = \frac{(3)^2}{4} \text{ m}$$

Hence the correct choice is (b).

118. The radius of the circular motion of the bead is $r = L$. The linear acceleration of the bead is $a = \alpha r = \alpha L$. If m is the mass of the bead, then

Force acting on the bead $= ma = m\,\alpha L$

$\therefore$ Reaction force acting on the bead is $R = m\,\alpha L$

The bead starts slipping when frictional force between the bead and the rod becomes equal to centrifugal force acting on the bead, i.e.

$$\mu R = \frac{mv^2}{r}$$

or $\mu\, m\, \alpha\, L = mr\omega^2 = mL\omega^2 \quad (\because v = r\omega)$

or $\mu\alpha = \omega^2 = (\alpha t)^2 \quad (\because \omega = \alpha t)$

or $\mu\alpha = \alpha^2 t^2$ or $t = \sqrt{\dfrac{\mu}{\alpha}}$,

which is choice (a).

119. The magnitude of the force is

$$F = \sqrt{\mathbf{F}\cdot\mathbf{F}} = \left[\left(6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}\right)\cdot\left(6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}\right)\right]^{1/2}$$

$$= \{(6)^2 + (8)^2 + (10)^2\}^{1/2} = (200)^{1/2} = 10\sqrt{2}\ \text{N}$$

$\therefore$ Mass $= \dfrac{F}{a} = \dfrac{10\sqrt{2}\text{N}}{1\,\text{ms}^{-2}} = 10\sqrt{2}$ kg. Hence the correct choice is (a).

120. let M be the mass of the chain and L its length. If a length l hangs over the edge of the table, the force pulling the chain down is $\dfrac{Ml}{L}g$. The force of friction between the rest of the chain of length $(L - l)$ and the table is $\dfrac{\mu M(L-l)}{L}g$.

For equilibrium, the two forces must be equal, i.e.

$$\frac{Ml}{L}g = \frac{\mu M(L-l)}{L}g$$

or $l = \mu(L - l)$

or $l = \dfrac{\mu L}{1+\mu}$

Thus, the correct choice is (b).

121. $\dfrac{l}{L} = \dfrac{\mu}{1+\mu} = \dfrac{0.25}{1+0.25} = \dfrac{1}{5}$ or 20%, which is choice (c).

122. Refer to Fig. 3.103. Vertical component of F is $F\sin\theta$ and the horizontal component is $F\cos\theta$. Thus

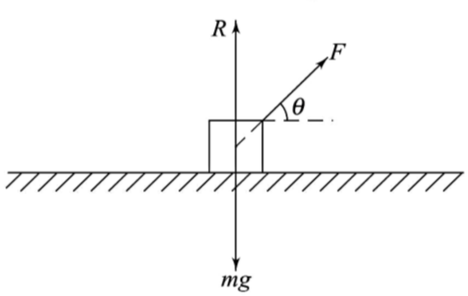


Fig. 3.103

$R + F\sin\theta = mg$

or $R = mg - F\sin\theta$

Frictional force $\mu R = \mu(mg - F\sin\theta)$. Also

$\mu(mg - F\sin\theta) = F\cos\theta$

or $F = \dfrac{\mu mg}{(\mu\sin\theta + \cos\theta)}$ (i)

F will be minimum if the denominator is maximum, i.e. if

$$\frac{d}{d\theta}(\mu\sin\theta + \cos\theta) = 0$$

or $\mu\cos\theta - \sin\theta = 0$ or $\mu = \tan\theta$, which is choice (a).

123. Now $\tan\theta = \mu$. Therefore, $\cos\theta = \dfrac{1}{\sqrt{1+\mu^2}}$ and $\sin\theta = \dfrac{\mu}{\sqrt{1+\mu^2}}$

Using these in Eq. (i) above and simplifying, we get

$$F = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

Hence the correct choice is (d).

124. Force required to accelerate the body of mass m is $F = (\mu_s - \mu_k)\, mg = (0.75 - 0.5)\, mg = 0.25\, mg$

$\therefore$ Acceleration $= \dfrac{F}{m} = 0.25\, g$, which is choice (a).

125. Since the blocks slide at the same uniform speed, no net force acts on them. If M is the mass of block B, then the tension in the string is $T = \mu M g$. Also $T = mg$. Equating the two, we get $\mu M = m$ or $M = \dfrac{m}{\mu}$, which is choice (b).

126. Extension in the spring $x = \dfrac{F}{k} = \dfrac{mg}{k}$. Therefore, potential energy stored in the spring is

$$\text{PE} = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{m^2g^2}{2k}$$

Hence the correct choice is (a).

127. If the acceleration of the block and trolley system is a, then we have

$$mg - T = ma \quad \text{(i)}$$

and $T - \mu M g = Ma$ (ii)

Eliminating T, we get

$$a = \left(\frac{m - \mu M}{m + M}\right)g, \text{ which is choice (d).}$$

128. Dividing Eq. (i) by Eq. (ii) and simplifying we find that the correct choice is (a).

129. When a cylinder rolls up or down an inclined plane, its angular acceleration is always directed down the plane. Hence the frictional force acts up the inclined plate when the cylinder rolls up or down the plane. Thus, the correct choice is (b).

130. As shown in Fig. 3.104, the insect will crawl without slipping if the value of α is not greater than that given by the condition: force of friction $f = mg \sin \alpha$. Now $f = \mu N$, where N is the normal reaction. Thus

$$\mu N = mg \sin \alpha$$

or $$\mu\, mg \cos \alpha = mg \sin \alpha$$

or $$\cot \alpha = \frac{1}{\mu} = 3,$$ which is choice (a).

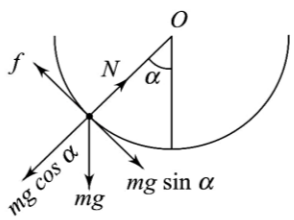


Fig. 3.104

131. Let T be the tension in the string. When the system is in equilibrium, then for the two equal masses m, we have

$$T = mg \quad \text{(i)}$$

and for the mass $\sqrt{2}\, m$, we have

$$2T \cos \theta = \sqrt{2}\; mg \quad \text{(ii)}$$

Dividing (ii) by (i), we get $\cos \theta = \frac{1}{\sqrt{2}}$ or $\theta = 45°$, which is choice (c).

132. The force F on the pulley by the clamp is given by the resultant of two forces: tension $T = Mg$ acting horizontally and a force $(m + M)g$ acting vertically downwards. Thus

$$F = \sqrt{(Mg)^2 + \{(m+M)g\}^2} = [M^2 + (m+M)^2]^{1/2} g$$

which is choice (d).

133. Refer to Fig. 3.105. Since the block moves with a constant velocity, no net force acts on it. Therefore, the horizontal component $F \cos \theta$ of force F must balance with the friction force, i.e. $f_r = F \cos \theta$. Also

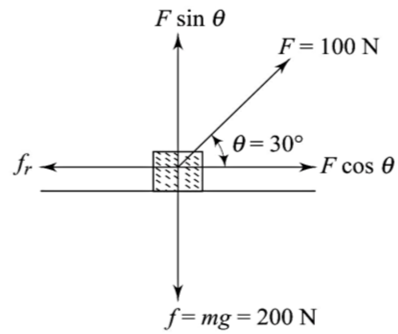


Fig. 3.105

$$f_r = \mu\,(mg - F \sin \theta)$$
$$= \mu\,(f - F \sin \theta)$$

$\therefore$ $$\mu\,(f - F \sin \theta) = F \cos \theta$$

or $$\mu\,(200 - 100 \sin 30°) = 100 \cos 30°$$

or $$\mu \left(200 - 100 \times \frac{1}{2}\right) = 100 \times 0.866 = 86.6$$

or $$\mu = \frac{86.6}{150} = 0.58,$$

which is choice (b).

134. Change of momentum of one bullet

$$= m\,(v - u)$$
$$= 0.03 \times \{50 - (-30)\}$$
$$= 2.4 \text{ kg ms}^{-1}$$

Average force = rate of change of momentum of 200 bullets

$$= 200 \times 2.4 = 480 \text{ N},$$

which is choice (d).

135. Let the body leave the surface at point B as shown in Fig. 3.106. When the body is between points A and B, we have

$$Mg \cos \theta - N = \frac{Mv^2}{r}$$

When the body leaves the surface at point B, the normal reaction N becomes zero. Thus

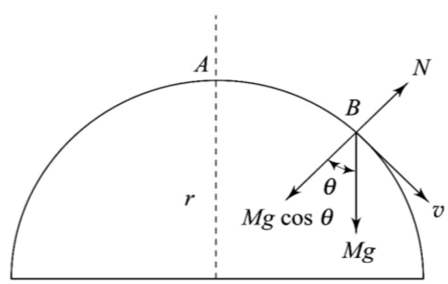


Fig. 3.106

$$Mg \cos \theta = \frac{Mv^2}{r}$$

or $$\cos \theta = \frac{v^2}{rg} = \frac{(5)^2}{5 \times 10} = \frac{1}{2} \quad \text{or} \quad \theta = 60°$$

Hence the correct choice is (c).

136. Mass of each piece $(m) = 1$ kg. Initial momentum = 0. Final momentum = $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3$. From the principle of conservation of momentum, we have

$$\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$$

or
$$\mathbf{p}_3 = -(\mathbf{p}_1 + \mathbf{p}_2) = -(m\mathbf{v}_1 + m\mathbf{v}_2)$$
$$= -m(\mathbf{v}_1 + \mathbf{v}_2)$$
$$= -1 \text{ kg} \times (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \text{ ms}^{-1}$$
$$= -(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \text{ kg ms}^{-1}$$

Force
$$F = \frac{\mathbf{p}_3}{t} = \frac{-(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \text{ kgms}^{-1}}{10^{-5}\text{s}}$$
$$= -(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times 10^5 \text{ newton}$$

Hence the correct choice is (b).

137. Since the body is whirled in a horizontal circle, the gravity, acting vertically downwards, has no effect on the motion. If v is the greatest speed with which the body can be whirled, the maximum centripetal force (or tension) in the string is mv^2/R, which must balance a force of 25 N. Thus

$$25 = \frac{mv^2}{R} = \frac{1 \times v^2}{1}$$

which gives $v = 5$ ms^{-1}, which is choice (b).

138. It is clear from Fig. 3.107, that when the body at the top point A of the circle, its weight mg and tension T_1 in the string act downwards towards the centre O of the circle and the sum of the two provides the necessary centripetal force. Thus

$$T_1 + mg = mR\omega^2$$

or
$$T_1 = m(R\omega^2 - g)$$
$$= 0.5 \times (0.5 \times 10^2 - 10)$$
$$= 20 \text{ N}$$

Thus, the correct choice is (b)

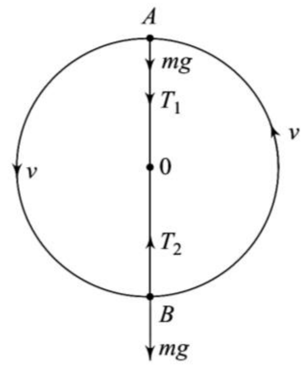


Fig. 3.107

139. When the body is at the bottom of the circle (point B in Fig. 3.107), the tension T_2 is opposite to weight mg and the difference $(T_2 - mg)$ provides the necessary centripetal force. Therefore, we have

$$T_2 = m(R\omega^2 + g)$$
$$= 0.5 \times (0.5 \times 10^2 + 10) = 30 \text{ N}$$

Thus, the correct choice is (c).

140. Referring to Fig. 3.108, the cyclist is moving on a straight road from A to B with a velocity $v = 6$ ms^{-1}.

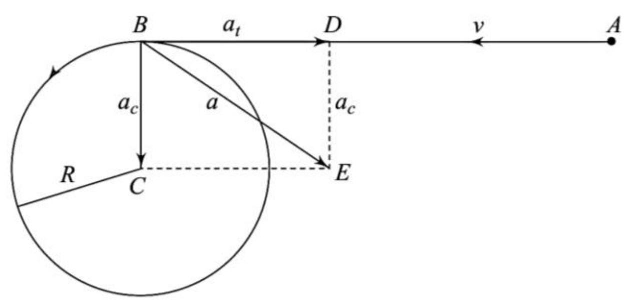


Fig. 3.108

As he approaches the circular turn, he decelerates at rate $\mathbf{a}_t$, represented by vector **BD**. The magnitude of declaration is $a_t = 0.4$ ms^{-2}. At point B, two accelerations $\mathbf{a}_t$ and $\mathbf{a}_c$, the centripetal acceleration directed towards the centre C act on the cyclist. Now $a_c = \frac{v^2}{R} = \frac{(6)^2}{120} = 0.3$ ms^{-2}. Using the law of parallelogram of vector addition, vector **B E** gives the resultant acceleration a whose magnitude is ($\because$ **DE** = $\mathbf{a}_c$)

$$a = (a_t^2 + a_c^2)^{1/2} = \{(0.4)^2 + (0.3)^2\}^{1/2} = 0.5 \text{ ms}^{-2}$$

Hence the correct choice is (a).

141. The tension in the string is given by

$$T = mg\cos\theta + \frac{mv^2}{r}$$

where r is the length of the string and v, the velocity of the bob when its angular displacement is θ. When the angular displacement is maximum, i.e. when $\theta = 40°$, $v = 0$. Tension at $\theta = 20°$ is given by

$$T = mg\cos 20° + \frac{mv^2}{r}$$

where v is the velocity when $\theta = 20°$. Hence the correct choice is (c).

142. Speed of car $(v) = 36\text{ km h}^{-1} = 10\text{ ms}^{-1}$. The maximum centripetal force that friction can provide is

$$f_{\max} = \mu mg = \frac{mv^2}{R}$$

or $$R_{\min} = \frac{v^2}{\mu g} = \frac{10\times 10}{0.8\times 10} = 12.5\text{ m}$$

This is the minimum radius the curve must have for the car to negotiate it without sliding at a speed of 10 ms^{-1}. Hence the correct choice is (a).

143. Speed of train $(v) = 36\text{ km h}^{-1} = 10\text{ ms}^{-1}$

Radius of the curve $(R) = 200$ m

Distance between rails $(x) = 1.5$ m

Let the outer rails be raised by a height h with respect to the inner rails so that the angle of banking is θ (Fig. 3.109).

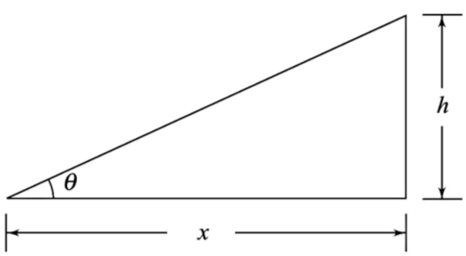


Fig. 3.109

Then $$\tan\theta = \frac{h}{x} = \frac{v^2}{Rg}$$

or $$h = \frac{xv^2}{Rg} = \frac{1.5\times(10)^2}{200\times 10}$$

$$= 0.075\text{ m} = 7.5\text{ cm}$$

Thus, the correct choice is (a).

144. Now $v = 54\text{ km h}^{-1} = 15\text{ ms}^{-1}$, $R = 50$ m. The required angle of banking is given by

$$\tan\theta = \frac{v^2}{Rg} = \frac{15\times 15}{50\times 10} = 0.45$$

Thus, the correct choice is (d)

145. Refering to Fig. 3.110, when the stone is at the topmost point A, the net force towards the centre is

$$T_A + mg = \frac{mv_A^2}{R} \quad \text{(i)}$$

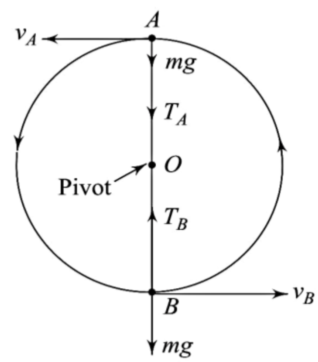


Fig. 3.110

When the stone is at the lowermost point B, the net force towards the centre is

$$T_B - mg = \frac{mv_B^2}{R} \quad \text{(ii)}$$

The relation between v_A and v_B can be obtained from the principle of conservation of energy. Let the gravitational potential energy be zero at the lowermost point B. Then

KE at B + PE at B = KE at A + PE at A

or $$\frac{1}{2}mv_B^2 + 0 = \frac{1}{2}mv_A^2 + mg\times AB$$

$$= \frac{1}{2}mv_A^2 + mg\times 2R$$

or $$v_B^2 = v_A^2 + 4gR \quad \text{(iii)}$$

Now, when the stone is at A, the string will not slack if the whole of centripetal force is provided by the weight mg, i.e. $T_A = 0$. Putting $T_A = 0$, we have

$$mg = \frac{mv_A^2}{R} \text{ or } v_A = \sqrt{gR}$$

Hence, the correct choice is (a).

146. The minimum speed of the stone when it is at its lowermost position B, so that the stone can complete the circle is obtained from Eq. (iii) above by putting $v_A = \sqrt{gR}$ which gives

$$v_B = \sqrt{5gR}$$

Hence, the correct choice is (c).

147. The pilot does not drop down when he is at the top of the loop because his weight mg is less than the centripetal force $m\,v^2/R$ required to keep him in the

loop. The rest of the centripetal force is balanced by the reaction of the seat. Hence, he is stuck to the seat without being tied to it. If the speed of the aircraft is reduced so that $mg > mv^2/R$, he will fall off from his seat. Therefore, the minimum speed v_{min} required to successfully negotiate the vertical loop is given by

$$mg = \frac{mv_{min}^2}{R}$$

or $$v_{min} = \sqrt{gR} = \sqrt{10 \times 4000} = 200 \text{ ms}^{-1}$$

Thus, the correct choice is (b).

148. Since the surface is perfectly frictionless, the disc will not roll on the surface; it will simply keep on rotating at point B where it is placed. Now

Linear speed = distance from centre × angular speed

Given, $r = 20$ cm = 0.2 m and $\omega = 20$ rad s^{-1}.

$\therefore$ Linear speed at point $A = OA \times \omega = r\omega = 0.2 \times 20 = 4$ ms^{-1}. Thus, the correct choice is (d).

149. Linear speed at point $B = BO \times \omega = r\omega = 0.2 \times 20 = 4$ ms^{-1}. Hence, the correct choice is (d).

150. Linear speed at point $C = CO \times \omega = \frac{1}{2} r\omega = 2$ ms^{-1}, which is choice (b).

151. Given, $m = 200$ kg, $R = 80$ m and $v = 72$ km h^{-1} = 20 ms^{-1}. The necessary frictional force is given by

$$F = \frac{mv^2}{R} = \frac{200 \times (20)^2}{80} = 1000 \text{ N}$$

Thus, the correct choice is (c).

152. The angle at which the rider must lean is given by

$$\tan\theta = \frac{v^2}{Rg} = \frac{(20)^2}{80 \times 10} = 0.5$$

Hence, the correct choice is (d).

153. The maximum centripetal force that the friction can provide is

$$F = \mu mg = \frac{mv^2}{R}$$

or $$\mu = \frac{v^2}{Rg} = \frac{15 \times 15}{30 \times 10} = 0.75 \ (\because 54 \text{ km h}^{-1} = 15 \text{ ms}^{-1})$$

Hence, the correct choice is (d).

154. Given, $m = 100$ g = 0.1 kg, $n = \frac{\sqrt{5}}{\pi}$ Hz and $L = 1$ m.

Referring to Fig. 3.111, tension T can be resolved into two perpendicular components: $T\sin\theta$ and $T\cos\theta$. The horizontal component $T\sin\theta$ provides the centripetal force for circular motion and the vertical component $T\cos\theta$ balances with the weight mg. Thus, since

$R = CB = L\sin\theta$,

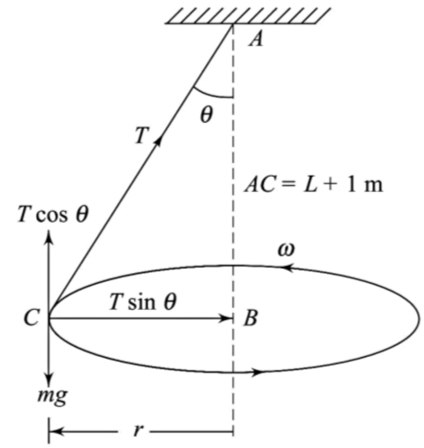


Fig. 3.111

$$T\sin\theta = \frac{mv^2}{R} = m\omega^2 R = m\omega^2 L\sin\theta \quad \text{(i)}$$

and $$T\cos\theta = mg \quad \text{(ii)}$$

From (i) we have $T = m\omega^2 L = m\omega^2$ $(\because L = 1 \text{ m})$

Using this in Eq. (ii) we get,

$$m\omega^2 \cos\theta = mg$$

or $$\cos\theta = \frac{g}{\omega^2} = \frac{g}{4\pi^2 n^2} = \frac{10}{4\pi^2 \times \frac{5}{\pi^2}} = 0.5$$

which gives $\theta = 60°$. Thus the correct choice is (c).

155. $T = m\omega^2 = 0.1 \times 4\pi^2 \times \frac{5}{\pi^2} = 2$ N, which is choice (a).

156. Linear speed $v = \omega R = 2\pi n L\sin\theta$

$$= 2\pi \times \frac{\sqrt{5}}{\pi} \times 1 \times \sqrt{1 - \cos^2 60°} = \sqrt{15} \text{ ms}^{-1}$$

Hence, the correct choice is (b).

157. When the bob passes through the equilibrium position O, the tension in the string is given by

$$T = mg + \frac{mv^2}{r}$$

where v is the speed of the bob at O. Now, the potential energy of the bob at the extreme position $A = mgh$ (see Fig. 3.112) which is converted into kinetic energy $\frac{1}{2}mv^2$ when it reaches O. Therefore

$$\frac{1}{2}mv^2 = mgh$$

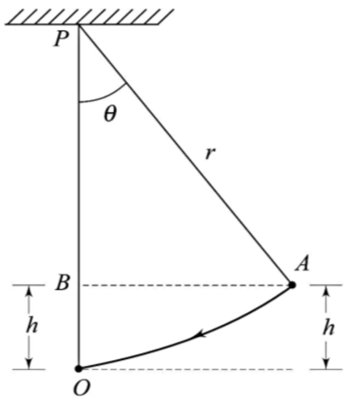


Fig. 3.112

or $v^2 = 2gh$. Using this we get $T = mg + \frac{2mgh}{r}$. $PB = r\cos\theta$. Therefore, $h = PO - PB = r - r\cos\theta = r(1 - \cos\theta)$. Using this value of h, the tension is

$$T = mg + 2\,mg\,(1 - \cos\theta)$$
$$= mg\,\{1 + 2\,(1 - \cos\theta)\}$$

Given, $m = 100$ g $= 0.1$ kg, $r = 1$ m and $\theta = 60°$. Putting these values, we get $T = 2$ N. Hence the correct choice is (b).

158. According to Hookes' law, the stretching force $F = kx$, where k is the force constant and x, the extension of the spring. A force mg stretches the spring by 1 cm. When the mass is describing the horizontal circle, total stretching $= 1 + 3 = 4$ cm. Hence

$$T = 4\,mg$$

Referring to Fig. 3.113, the horizontal component $T\sin\theta$ provides the necessary centripetal force for circular motion, i.e.

$$T\sin\theta = \frac{mv^2}{r} = m\omega^2 r$$

Given, $\omega = 2\pi n = 2\pi \times 2 = 4\pi$ rad s^{-1}. Let L cm be the length of the unstretched spring. Then $AC = (L + 4)$ cm and $r = (L + 4)\sin\theta$.

$$\therefore \quad T\sin\theta = m \times (4\pi)^2 \times (L + 4)\sin\theta$$

or $L + 4 = \frac{T}{16\pi^2 m} = \frac{4mg}{16\pi^2 m}$ $\quad (\because T = 4\,mg)$

$$= \frac{g}{4\pi^2} = \frac{\pi^2}{4\pi^2} = 0.25 \text{ m} = 25 \text{ cm}$$

or $L = 25 - 4 = 21$ cm. Hence the correct choice is (d).

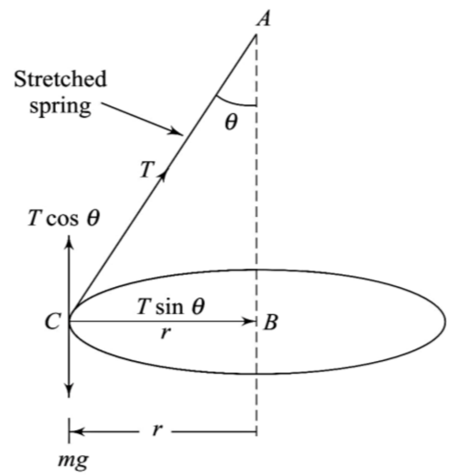


Fig. 3.113

159. We have seen above that $T = 4mg$. Referring to Fig. 3.101, we have

$$T\cos\theta = mg$$

or $$\cos\theta = \frac{mg}{T} = \frac{mg}{4mg} = \frac{1}{4}$$

Hence the correct choice is (a).

160. As the boy is given a little push, his initial speed can be taken to be zero. Suppose he leaves the mound at point P at a height h (Fig. 3.114). The forces acting on him are his weight mg and the normal reaction F. It is clear that at point P

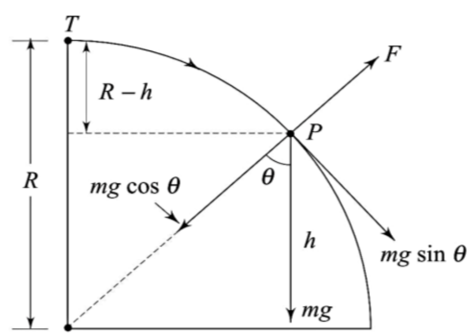


Fig. 3.114

$$mg\cos\theta - F = \frac{mv^2}{R} \qquad \text{(i)}$$

where v is the speed of the boy at P. This is the speed with which he leaves the ice. Energy conservation gives

$$\frac{1}{2} mv^2 = mg(R-h) \qquad \text{(ii)}$$

Using Eqs (i) and (ii) we get,

$$mg\cos\theta - F = 2mg\left(\frac{R-h}{R}\right) \qquad \text{(iii)}$$

As the boy leaves the mound, the normal reaction F vanishes. Thus, putting $F = 0$ in Eq. (iii), we have

$$\cos\theta = 2\left(\frac{R-h}{R}\right)$$

But $\cos\theta = \frac{h}{R}$. Therefore, we get $\frac{h}{R} = 2 - \frac{2h}{R}$ or $h = \frac{2}{3}R$. Hence, the correct choice is (c).

161. Given, $h = 2$ m, $R = 1.25$ m and horizontal distance $s = 10$ m. When the string breaks, the stone is projected in the horizontal direction, which means that there is no initial vertical velocity.

From $s = ut + \frac{1}{2}gt^2$, we have ($\because u = 0$),

$$h = \frac{1}{2}gt^2 \qquad \text{(i)}$$

The horizontal distance travelled in time t is

$$s = vt \qquad \text{(ii)}$$

where v is the velocity of the stone in the horizontal direction which is the same as its velocity in circular motion.

Eliminating t from (i) and (ii) we get

$$v^2 = \frac{gs^2}{2h}$$

Now, centripetal acceleration is

$$a_c = \frac{v^2}{R} = \frac{gs^2}{2hR} = \frac{10\times 100}{2\times 2\times 1.25}$$

$$= 200 \text{ ms}^{-2}$$

Thus, the correct choice is (b).

162. Refer again to the solution to Q. 161. We have

$$v_B^2 = v_A^2 + 4gR$$

Putting $v_B = \sqrt{5gR}$, we get $v_A = \sqrt{gR}$. Therefore, kinetic energy at $A = \frac{1}{2}mv_A^2 = \frac{1}{2}mgR$. Hence the correct choice is (a).

163. Kinetic energy at $A = \frac{1}{2}mv_A^2 = \frac{1}{2}mgR$. When the string becomes horizontal, the stone falls through a height R and gains kinetic energy = potential energy lost in falling through height $R = mgR$. Hence, the kinetic energy when the string is horizontal = $\frac{1}{2}mgR + mgR = \frac{3}{2}mgR$. Thus, the correct choice is (c).

164. Refer also to the solution of Q.160 again. The horizontal velocity v must be such that the centripetal force equals the weight of the body, i.e. $\frac{mv^2}{R} = mg$ or $v = \sqrt{gR}$.

165. The car will skid if the normal reaction $F = \mu mg$ is less than the centripetal force mv^2/r or if $\frac{mv^2}{r} > \mu mg$ or $v > \sqrt{\mu rg}$. Hence, the correct choice is (a).

166. The motor cyclist can leave the ground only at the highest point on the bridge. At this point, the centripetal force is mv^2/R. He will not leave the ground if the centripetal force equals the weight mg. Thus $\frac{mv^2}{R} = mg$ or $v = \sqrt{gR} = \sqrt{10\times 10} = 10 \text{ ms}^{-1}$.

Hence, the correct choice is (a)

167. Thrust at the highest point = centripetal force + weight

$$= \frac{mv^2}{R} + mg$$

Hence, the correct choice is (d).

168. When the system is in equilibrium, the spring force $= 3\,mg$. When the string is cut, the net force on block $A = 3\,mg - 2\,mg = mg$. Hence the acceleration of this block at this instant is

$$a = \frac{\text{force on block A}}{\text{mass of block A}} = \frac{mg}{2m} = \frac{g}{2}$$

When the string is cut, the block B falls freely with an acceleration equal to g. Hence the correct choice is (c).

169. Impulse I = area under the $F - t$ graph

$$= AD\times\frac{1}{2}(AB + OC)$$

$$= 10\text{N}\times\frac{1}{2}(2+5)\text{ms}$$

$$= 35\times 10^{-3} \text{ Ns}$$

Impulse = change in momentum, *i.e.*

$$I = mv - mu = mv \ (\because u = 0)$$

$$\Rightarrow \qquad v = \frac{I}{m} = \frac{35\times10^{-3}\,\text{Ns}}{35\times10^{-3}\,\text{kg}} = 1\,\text{ms}^{-1}$$

So the correct choice is (*a*)

170. The free - body diagrams of the block and the rope are shown in Fig. 3.115.

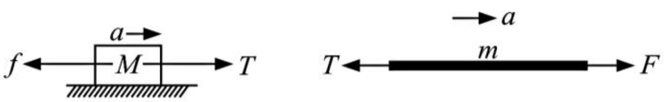


Fig. 3.115

For block : $T - f = Ma$ where $f = \mu Mg$

$$\therefore\ T - \mu Mg = Ma \qquad (1)$$

For rope : $F - T = ma$ (2)

Dividing (1) by (2) and simplifying, we find that the correct choice is (a).

171. Let R be the force exerted by A or B. The free-body diagrams are (since B applies an equal and opposite force R or A) as shown in Fig. 3.116

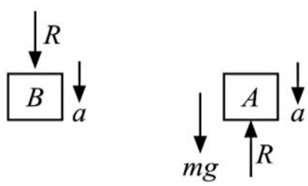


Fig. 3.116

For block A : $mg - R = ma$

$$\Rightarrow \qquad R = m\,(g - a)$$

So the correct choice is (c).

172. Since $(m_2 + m_3) > m_1$, blocks B and C will move down will an acceleration and block A will move up with the same acceleration. Let T_1 be the tension in the string connecting A and B and T_3 be the tension in the string connecting B and C. The free body diagrams of A, B and C are shown in Fig. 3.117.

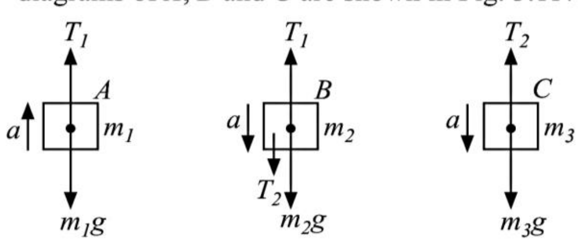


Fig. 3.117

For block A : $T_1 - m_1 g = m_1 a$ (1)

For block B : $T_2 + m_2 g - T_1 = m_2 a$ (2)

For block C : $m_3 g - T_2 = m_3 a$ (3)

Adding (1) and (2) and simplifying, we get

$$a = \frac{T_2 + (m_1 - m_2)g}{(m_1 + m_2)} \qquad (4)$$

Using (4) in (3) and simplifying, we get

$$T_2 = \frac{2m_2 m_3 g}{(m_1 + m_2 + m_3)} = \frac{2m_2 m_3 g}{M}$$

So the correct choice is (c).

173. Let R be the normal reaction exerted downward by man on the floor of the box. The box, in turn, will exert a force R upward on the man. The free body diagrams of the box and the man are shown in Fig. 3.118. Let T be the tension in the rope.

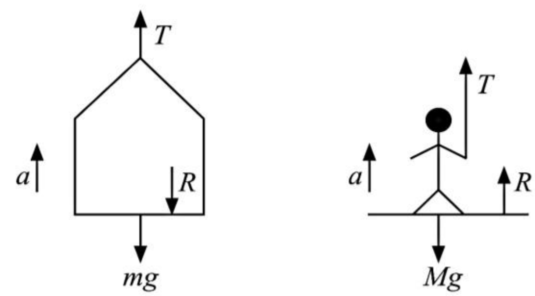


Fig. 3.118

For box : $T - R - mg = ma$ (1)

For man : $T + R - Mg = Ma$ (2)

Adding (1) and (2) we get

$$T = \frac{1}{2}(m + M)(g + a)$$

Putting $M = 2m$ and $a = g/3$, we get $T = \dfrac{2mg}{3}$. The correct choice is (a).

174. Subtracting (1) and (2) we get.

$$R = \frac{1}{2}(M - m)(g + a)$$

$$= \frac{1}{2}(2m - m)\left(g + \frac{g}{3}\right)$$

$$= \frac{2mg}{3}$$

So the correct choice in (b).

175. Let T_1 and T_2 be the tensions in the string as shown in Fig. 3.119.

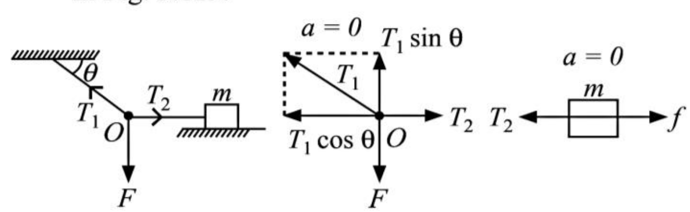


Fig. 3.119

Here frictional force $f = \mu mg$. Since the system is in static equilibrium, no net force acts at point O and on block m. Hence, acceleration of O and m is zero.

For point O : $T_2 = T_1 \cos\theta$ and

: $F = T_1 \sin\theta$ (1)

For block : $T_2 = f$

$\Rightarrow$: $T_1 \cos\theta = \mu mg$ (2)

Dividing (1) and (2) we get

$$F = \mu mg \tan\theta \quad (3)$$

If force F exceeds the value given by (3), the block will begin to slide on the surface. So the correct choice is (c).

SECTION 2 Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage.

Passage I

A block of masses m is initially at rest on a frictionless horizontal surface. A time-dependent force $F = at - bt^2$ acts on the body, where a and b are positive constants.

1. The magnitude of the force is maximum at time t_1 given by

(a) $\frac{a}{b}$ (b) $\frac{2a}{b}$

(c) $\frac{a}{2b}$ (d) $\frac{a}{\sqrt{2}b}$

2. The maximum force F_{max} is given by

(a) $\frac{a^2}{2b}$ (b) $\frac{a^2}{4b}$

(c) $\frac{2a^2}{b}$ (d) $\frac{4a^2}{b}$

3. The maximum impulse I_{max} imparted to the block is given by

(a) $\frac{a^3}{3b^2}$ (b) $\frac{a^3}{6b^2}$

(c) $\frac{a^3}{9b^2}$ (d) $\frac{a^3}{12b^2}$

4. The maximum velocity v_{max} attained by the block is

(a) $\frac{a^3}{4mb^2}$ (b) $\frac{a^3}{8mb^2}$

(c) $\frac{a^3}{12mb^2}$ (d) $\frac{a^3}{16mb^2}$

Solutions

1. The force is maximum when $\frac{dF}{dt} = 0$ and $\frac{d^2F}{dt^2} < 0$.

Now $\frac{dF}{dt} = \frac{d}{dt}(at - bt^2) = a - 2bt$

Putting $\frac{dF}{dt} = 0$ and $t = t_1$, we get

$0 = a - 2\,bt_1 \Rightarrow t_1 = \frac{a}{2b}$

Also $\frac{d^2F}{dt^2} = \frac{d}{dt}(a - 2\,bt) = -2\,b$, which is negative.

Hence the correct choice is (c).

2. $F_{max} = at_1 - bt_1^2 = a \times \frac{a}{2b} - b \times \left(\frac{a}{2b}\right)^2 = \frac{a^2}{4b}$. Hence the correct choice is (b).

3. Maximum impulse is given by

$$I_{max} = \int_0^{t_1} F dt$$
$$= \int_0^{t_1} (at - bt^2) dt$$
$$= \frac{at_1^2}{2} - \frac{bt_1^3}{3}$$
$$= \frac{a}{2}\left(\frac{a}{2b}\right)^2 - \frac{b}{3}\left(\frac{a}{2b}\right)^3 = \frac{a^3}{12b^2}$$

Hence the correct choice is (d).

4. Now impulse = change in momentum = $mv - 0 = mv$

$\therefore v_{max} = \frac{I_{max}}{m} = \frac{a^3}{12mb^2}$, which is choice (c).

Questions 5 to 8 are based on the following passage.

Passage II

A body of mass m is initially at rest. A periodic force $F = a\cos(bt + c)$ is applied to it, where a, b and c are constants.

5. The time period T of the force is

(a) $\frac{1}{b}$ (b) $\frac{2\pi}{b}$

(c) $2\pi\sqrt{\frac{a}{b}}$ (d) $2\pi\sqrt{\frac{b}{a}}$

6. The maximum velocity of the body is

(a) $\frac{a}{mb}$ (b) $\frac{b}{mc}$

(c) $\frac{c}{ma}$ (d) $\frac{b+c}{ma}$

7. The smallest value of t after $t = 0$ when the velocity of the body becomes zero is given by

(a) $t_1 = \frac{\pi}{a}$ (b) $t_1 = \frac{\pi - a}{c}$

(c) $t_1 = \frac{\pi - c}{b}$ (d) $t_1 = \frac{\pi}{b}$

8. The distance travelled by the body from time $t = 0$ to $t = t_1$ is given by

(a) $\frac{a}{mb^2}\cos c$ (b) $\frac{2a}{mb^2}\sin c$

(c) $\frac{a^2}{mb}\cos c$ (d) $\frac{2a^2}{mb}\sin c$

Solutions

5. F will repeat itself at values of t given by $\cos(bt + c) = +1$, i.e.

$$bt + c = 0, 2\pi, 4\pi, \cdots$$

$$\Rightarrow \quad t = -\frac{c}{b}, \frac{2\pi - c}{b}, \frac{4\pi - c}{b}, \cdots$$

The smallest time interval is $T = \frac{2\pi}{b}$. Hence the correct choice is (b).

6. From Newton's second law of motion,

$$F = \frac{dp}{dt} = m\frac{dv}{dt}$$

Thus $m\frac{dv}{dt} = a\cos(bt + c)$

$$\Rightarrow \quad dv = \frac{a}{m}\cos(bt + c)dt$$

$$\therefore v = \frac{a}{m}\int_0^t \cos(bt + c)dt = \frac{a}{mb}\sin(bt + c) \qquad \text{(i)}$$

Since the maximum value of $\sin(bt + c) = 1$, $v_{max} = \frac{a}{mb}$. Hence the correct choice is (a).

7. From Eq (i) it follows that $v = 0$ at values of t given by $\sin(bt + c) = 0$ or $(bt + c) = 0, \pi, 2\pi, \ldots$ or $t = -\frac{c}{b}, \frac{\pi - c}{b}, \frac{2\pi - c}{b}$. Therefore,

$t_1 = \frac{\pi - c}{b} - \left(-\frac{c}{b}\right) = \frac{\pi}{b}$, which is choice (d).

8. Now $v = \frac{dx}{dt} \Rightarrow dx = v\,dt$. Therefore, the distance moved between $t = 0$ and $t = t_1$ is

$$x = \int_0^{t_1} v\,dt = \frac{a}{mb}\int_0^{t_1}\sin(bt + c)dt$$

$$= -\frac{a}{mb^2}\cos(bt_1 + c)$$

$$= -\frac{a}{mb^2}\cos\left[b \times \frac{\pi}{b} + c\right]$$

$$= -\frac{a}{mb^2}\cos(\pi + c) = \frac{a\cos c}{mb^2}$$

Hence the correct choice is (a).

Questions 9 to 11 are based on the following passage.

Passage III

Three masses $m_1 = m$, $m_2 = 2m$ and $m_3 = 3m$ are hung on a string passing over a frictionless pulley as shown in Fig. 3.120. The mass of the string is negligible. The system is then released.

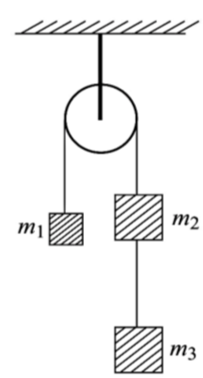
9. If a_1, a_2 and a_3 are the accelerations of masses m_1, m_2 and m_3 respectively, then

(a) $a_1 < a_2 < a_3$

(b) $a_1 > a_2 > a_3$

(c) $a_1 > a_2 = a_3$

(d) $a_1 = a_2 = a_3$



3. 120

10. The tension in the string between masses m_2 and m_3 is

(a) mg

(b) $3\,mg$

(c) $4\,mg$

(d) $\frac{5mg}{3}$

11. The tension in the string between masses m_1 and m_2 is

(a) 4 mg (b) $\frac{2mg}{3}$

(c) $\frac{5mg}{3}$ (d) 2 mg

Solutions

9. When the masses are released, mass m_1 moves upward and masses m_2 and m_3 move downward with a common acceleration given by

$$a = \frac{(m_2 + m_3 - m_1)g}{(m_1 + m_2 + m_3)} = \frac{(2m + 3m - m)g}{(m + 2m + 3m)} = \frac{2g}{3}$$

The correct choice is (d).

10. Let T be the tension in the string between m_1 and m_2 and T' be the tension in the string between m_2 and m_3 [see Fig. 3.121 (a)]. Figure 3.121 (b) shows the free-body diagram of mass m_3.

$$m_3 g - T' = m_3 a$$

$$\Rightarrow \quad T' = m_3(g - a) = 3\,m \times \left(g - \frac{2g}{3}\right) = mg$$

Hence the correct choice is (a).

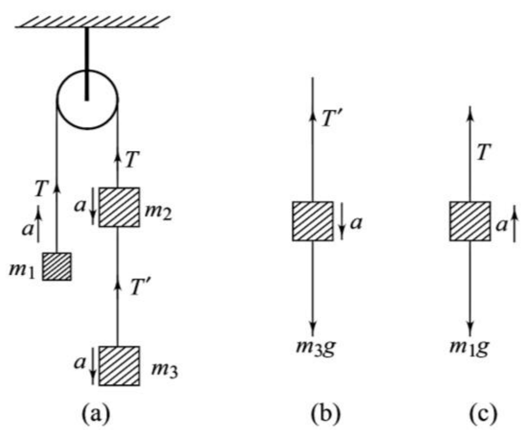


Fig. 3.121

11. Figure 3.113(c) shows the free-body diagram of mass m_1.

$$T - m_1 g = m_1 a$$

$$\Rightarrow \quad T = m_1(g + a) = m \times \left(g + \frac{2g}{3}\right) = \frac{5mg}{3}$$

Hence the correct choice is (c).

Questions 12 to 14 are based on the following passage.

Passage IV

Two blocks of masses $m_1 = 3\,m$ and $m_2 = 2\,m$ are suspended from a rigid support by two inextensible uniform wires A and B. Wire A has negligible mass and wire B has a mass $m_3 = m$, as shown in Fig. 3.122. The whole system of blocks, wires and the support have an upward acceleration a.

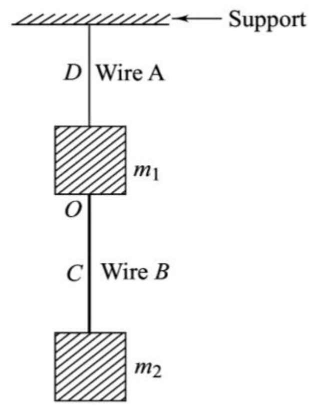
12. The tension at the mid-point C of wire B is

(a) $\frac{1}{2}\ m(g + a)$

(b) $\frac{3}{2}\ m(g - a)$

(c) $\frac{3}{2}\ m(g + a)$

(d) $\frac{5}{2}\ m(g + a)$



3. 122

13. The tension at point O of wire B is

(a) $3m(g + a)$

(b) $3m(g - a)$

(c) $2m(g + a)$

(d) $2m(g - a)$

14. The tension at the mid-point D of wire A is

(a) $2m(g + a)$ (b) $4m(g - a)$

(c) $6m(g + a)$ (d) $8m(g - a)$

Solutions

12. Refer to Fig. 3.123. Let T be the tension at the mid-point C of wire B. Then

$$T - \left(m_2 + \frac{m_3}{2}\right)g = \left(m_2 + \frac{m_3}{2}\right)a$$

$$\Rightarrow \quad T = \left(m_2 + \frac{m_3}{2}\right)(g + a)$$

$$= \left(2m + \frac{m}{2}\right)(g + a)$$

$$= \frac{5}{2}\ m(g + a),$$

which is choice (d).

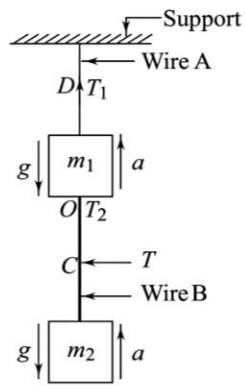


Fig. 3.123

13. Let T_1 be the tension in wire A. Since this wire has negligible mass, the tension is the same ($= T_1$) at every point on this wire. Let T_2 be the tension at point O of wire B. Then, we have for wire A.

$$T_1 - T_2 - m_1 g = m_1 a \quad \text{(i)}$$

where T_2 is given by

$$T_2 - (m_2 + m_3)g = (m_2 + m_3)a$$

$$\Rightarrow \quad T_2 = (m_2 + m_3)(g + a)$$

$$= (2m + m)(g + a) = 3m(g + a)$$

Hence the correct choice is (a).

14. Putting $T_2 = 3m(g + a)$ in Eq. (i), we get

$$T_1 = 6\,m(g + a).$$

Hence the correct choice is (c).

3 SECTION Assertion-Reason Type Questions

In the following questions, **Statement-1 (Assertion)** is followed by **Statement-2 (Reason)**. Each question has four choices out of which only **one** choice is correct

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true but Statement-2 is not the correct explanation for Statement-1.

(c) Statement-1 is true; Statement-2 is false.

(d) Statement-1 is false; Statement-2 is true.

1. **Statement-1**

A block is pulled along a horizontal frictionless surface by a thick rope. The tension in the rope will not always be the same at all points on it.

Statement-2

The tension in the rope depends on the acceleration of the block-rope system and the mass of the rope.

2. **Statement-1**

A truck moving on a horizontal surface with a uniform speed u is carrying sand. If a mass Δm of the sand 'leaks' from the truck in a time Δt, the force needed to keep the truck moving at its uniform speed is $u\,\Delta m/\Delta t$.

Statement-2

Force = rate of change of momentum.

3. **Statement-1**

Two blocks of masses m and M are placed on a horizontal surface as shown in Fig. 3.124. The coefficient of friction between the two blocks is μ_1 and that between the block M and the horizontal surface is μ_2. The maximum force that can be applied to block M so that the two blocks move without slipping is $F = (\mu_1 + \mu_2)(M + m)g$.

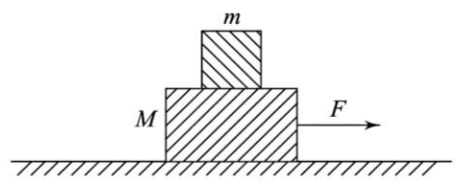


Fig. 3.124

Statement-2

Maximum force = total mass × maximum acceleration.

4. **Statement-1**

A shell of mass m is at rest initially. It explodes into three fragments having masses in the ratio 2:2:1. The fragments having equal masses fly off along mutually perpendicular directions with speed v. The speed of the third (lighter) fragment will be $2\sqrt{2}\,v$.

Statement-2

The momentum of a system of particles is conserved if no external force acts on it.

5. **Statement-1**

The maximum value of force F such that the block shown in Fig. 3.125 does not move is $\mu\,mg/\cos\theta$, where μ is the coefficient of friction between the block and the horizontal surface.

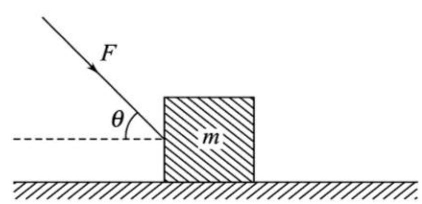


Fig. 3.125

Statement-2

Frictional force = coefficient of friction × normal reaction.

6. **Statement-1**

   A ball of mass m is moving towards a batsman at a speed v. The batsman strikes the ball and deflects it by an angle θ without changing its speed. The impulse imparted to the ball is zero.

   Statement-2

   Impulse = change in momentum

7. **Statement-1**

   A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.

   Statement-2

   For every action there is an equal and opposite reaction.

8. **Statement-1**

   When a ball dropped from a certain height hits the floor, it exerts a force equal to the rate of change of momentum.

   Statement-2

   The floor does not move because the action and reaction forces, being equal and opposite, cancel each other.

Solutions

1. The correct choice is (a).
2. The correct choice is (d). The force exerted by the leaking sand on the truck = rate of change of momentum = $u\ \Delta m/\Delta t$. The sand falling vertically downwards will exert this force on the truck in the vertically upward direction. This perpendicular force can do no work on the truck. Since the truck moves with a uniform velocity, the force exerted just overcomes the frictional force.
3. The correct choice is (c). The force of friction between block m and block $M = \mu_1 mg$, where μ_1 is the coefficient of friction between the two blocks. Now, the force of friction between block M (with block m on top of it) and the horizontal surface = $\mu_2(M+m)g$, where μ_2 is the coefficient of friction between block M and surface. The maximum force F applied to block M must be enough to overcome this force of friction and the force due to acceleration of the system. If the acceleration of the system is a then this force = $(M+m)a$. Thus

   $$F = (M+m)a + m_2(M+m)g \qquad \text{(i)}$$

   Now, since the force on block m is $\mu_1 mg$, its acceleration a is

   $$a = \frac{\text{force on mass } m}{\text{mass } m} = \frac{\mu_1 mg}{m} = \mu_1 g \qquad \text{(ii)}$$

   Using (ii) in (i) we get

   $$F = \mu_1(M+m)g + \mu_2(M+m)g$$
   $$= (\mu_1+\mu_2)(M+m)g$$

4. The correct choice is (a). The mass of two fragments of equal masses = $\frac{2m}{5}$ each. The mass of the lighter fragment = $\frac{m}{5}$. The momenta of heavier fragments are $p = \frac{2mv}{5}$. The resultant of momenta p and p is

   $$p' = (p^2+p^2)^{1/2} = \sqrt{2}\,p$$

   From the principle of conservation of momentum, the momentum of the third (lighter) fragment of mass $\frac{m}{5}$ must be $\sqrt{2}\,p$ but opposite in direction. Thus, if V is the speed of the lighter fragment, we have

   $$\frac{mV}{5} = \sqrt{2}\,p = \sqrt{2}\,\frac{2mv}{5}$$

   or $$V = 2\sqrt{2}\,v$$

5. The correct choice is (a). The component of F parallel to the horizontal surface is $F\cos\theta$. F will be maximum when $F\cos\theta$ just overcomes the frictional force $f = \mu mg$. Thus

   $$F_{\max}\cos\theta = \mu mg \Rightarrow F_{\max} = \frac{\mu mg}{\cos\theta}$$

6. The correct choice is (d). Refer to the solution of Q.44 of section I.
7. Statement-1 follows the Newton's first law of motion also called the law of inertia. The dishes are not dislodged even when the cloth is suddenly pulled because the dishes have the inertia of rest. Statement-2 is Newton's third law of motion, it does not explain statement-1. Hence the correct choice is (b).
8. The assertion is true but the reason is not correct because action and reaction forces do not act on the same body and hence do not cancel each other. Hence the correct choice is (c).

4 SECTION Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

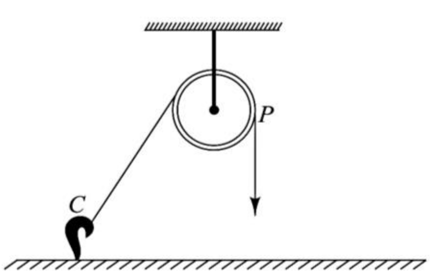
1. The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is
(a) 60 (b) 30
(c) 15 (d) 25 **[2002]**

2. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The accelerations of the ball as observed by the man in the lift and man standing stationary on the ground respectively are
(a) g, g (b) $(g-a), (g-a)$
(c) $(g-a), g$ (d) a, g **[2002]**

3. When forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ are acting on a particle of mass m such that $\vec{F}_2$ and $\vec{F}_3$ are mutually perpendicular, then the particle remains stationary. If force $\vec{F}_1$ is now removed, the magnitude of acceleration of the particle will be
(a) $\frac{F_1}{m}$ (b) $\frac{F_2F_3}{mF_1}$
(c) $\frac{F_2-F_3}{m}$ (d) $\frac{F_2}{m}$ **[2002]**

4. The speeds of two identical cars are u and $4u$ at a given instant. The ratio of the respective distances at which the two cars are stopped from that instant is
(a) 1:1 (b) 1:4
(c) 1:8 (d) 1:16 **[2001]**

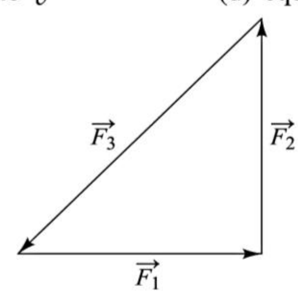
5. One end of a massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. The rope can bear a maximum tension of 360 N. With that maximum accleration (in ms^{-2}) can a man of mass 60 kg climb on the rope?
(a) 16 (b) 6
(c) 4 (d) 8 **[2002]**



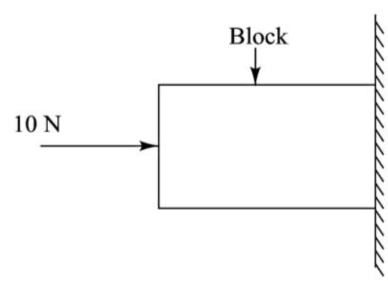
6. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $\frac{g}{8}$, then the ratio of masses is
(a) 8:1 (b) 9:7
(c) 4:3 (d) 5:3 **[2002]**

7. A spring balance is attached to the ceilling of a lift. A man hangs his bag on the spring and the balance reads 49 N, when the lift is stationary. If the lift moves downwards with an acceleration 5 ms^{-2}, the reading of the balance will be
(a) 24 N (b) 74 N
(c) 15 N (d) 49 N **[2003]**

8. Three forces start acting simultaneously on a particle moving with a velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangles as shown in the figure. The particle will now move with velocity
(a) less then $\vec{v}$ (b) greater $\vec{v}$
(c) equal to $\vec{v}$ (d) equal to zero **[2003]**



9. A horizontal force of 10 N is necessary to just hold a block stationary against a wall as shown in the figure. If the coefficient of friction between the block and the wall is 0.2, the weight of the block is
(a) 20 N (b) 50 N
(c) 100 N (d) 2 N **[2003]**



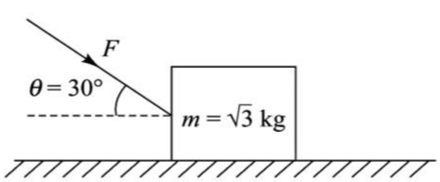
10. A marble block of mass 2 kg lying on ice when given a velocity of 6 ms^{-1} is stopped by friction in 10 s. The coefficient of friction between the block and ice is
(a) 0.02 (b) 0.03
(c) 0.06 (d) 0.01 **[2003]**

11. A block of mass M is pulled along a horizontal frictionless surface by a rope is mass m. If a force F is applied at the free end of the rope, the force exerted by the rope on the block is
(a) $\frac{mF}{M+m}$ (b) $\frac{mF}{M-m}$
(c) F (d) $\frac{MF}{M+m}$ **[2003]**

12. A light spring balance B_2 hangs from the hook of another light spring balance B_1. A block of mass M kg is hung from B_2. Choose the correct statement from the following.
(a) Both B_1 and B_2 read M kg each
(b) B_2 reads M kg and B_1 reads zero
(c) The readings of B_1 and B_2 can have any values but the sum of the readings will be M kg
(d) Both B_1 and B_2 will read $\frac{M}{2}$ kg. **[2003]**

13. A rocket with a of mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 ms^{-2}. The initial thrust of the blast is
(a) 3.5×10^5 N (b) 7.0×10^5 N
(c) 14.0×10^5N (d) 1.75×10^5 N **[2003]**

14. What is the maximum value of the force F such that the block shown in the arrangement does not move? The coefficient of friction between the block and the horizontal surface is 0.5. (Take $g = 10$ ms^{-2})
(a) 20 N (b) 10 N
(c) 12 N (d) 15 N **[2003]**



15. A car is moving in a circular path of radius 500 m with a speed of 30 ms^{-1}. If the speed is increased at the rate of 2ms^{-2}, the resultant acceleration will be
(a) 2 ms^{-2} (b) 2.5 ms^{-2}
(c) 2.7 ms^{-2} (d) 4 ms^{-2} **[2003]**

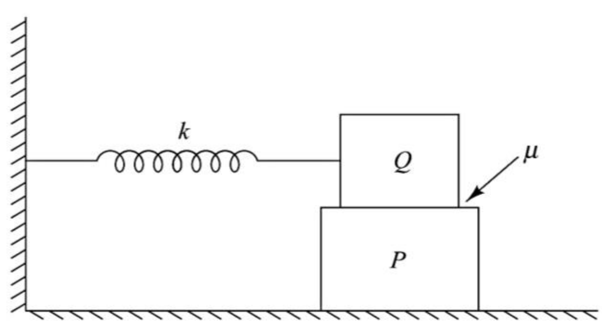
16. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most?
(a) one (b) two
(c) three (d) four **[2004]**

17. A string passing over a light frictionless pulley carries two masses m_1 = 5 kg and m_2 = 4.8 kg at its ends hanging vertically. When the masses are released, their acceleration (in ms^{-2}) will be
(a) 0.2 (b) 9.8
(c) 5 (d) 4.8 **[2004]**

18. A block rests on a rough inclined plane making an angle of 30º with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is (take g = 10 ms^{-2})
(a) 2.0 (b) 4.0
(c) 1.6 (d) 2.5 **[2004]**

19. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes in a plane. It follows that
(a) Its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a straight line **[2004]**

20. A block P of mass m is placed on a horizontal frictionless surface. Another block Q of same mass is kept on P and connected to a rigid wall by means of a spring of spring constant k as shown in the figure. The two blocks move together, without slipping, performing simple harmonic motion of amplitude A. If μ is the coefficient of static friction between blocks P and Q, the maximum value of the force of friction between P and Q is
(a) mg (b) $\frac{kA}{2}$
(c) kA (d) zero **[2004]**



21. A smooth block is released from rest on a 45º rough incline and then slides a distance d. The time taken to slide is n times the time it takes to slide the same distance on a perfectly smooth 45º incline. the coefficient of kinetic friction is

(a) $\mu_k = 1 - \frac{1}{n^2}$ (b) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$

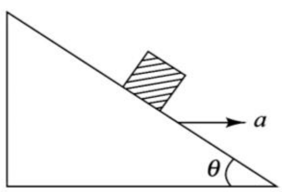
(c) $\mu_k = \sqrt{\frac{1}{1 + \frac{1}{n^2}}}$ (d) $\mu_k = \frac{1}{1 + n^2}$ **[2005]**

22. The upper half of an inclined plane with inclination θ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction of the lower half is

(a) $2 \sin \theta$ (b) $2 \cos \theta$
(c) $2 \tan \theta$ (d) $\tan \theta$ **[2005]**

23. A block is kept on a frictionless inclined surface of inclination θ as shown in the figure. The incline is given an acceleration a to keep the block stationary. Then a is

(a) $\frac{g}{\tan \theta}$ (b) $g \operatorname{cosec} \theta$
(c) g (d) $g \tan \theta$ **[2005]**



24. A particle of mass 0.3 kg is subjected to force $F = -kx$ where $k = 15$ N m^{-1}. What will be its initial acceleration (in ms^{-2}) if it is released from point 20 cm away from the origin?

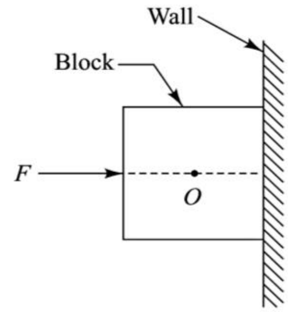
(a) 3 (b) 15
(c) 5 (d) 10 **[2005]**

25. A car is moving on a straight road with a speed of 100 ms^{-1}. If the coefficient of friction between the types and road is 0.5, the distance at which the car can be stopped is

(a) 800 m (b) 1000 m
(c) 100 m (d) 400 m **[2005]**

26. A block of mass m is held stationary against a wall by applying a horizontal force F on the block. Which of the following statement is false?

(a) The frictional force acting on the block is $f = mg$
(b) The normal reaction force acting on the block is $N = F$
(c) No net torque acts on the block
(d) N does not produce any torque. **[2005]**



27. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to

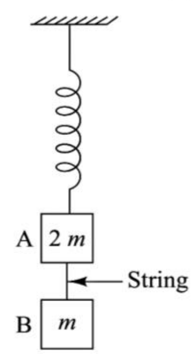
(a) 30 N (b) 300 N
(c) 150 N (d) 3 N **[2006]**

28. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes up to 2 m height further, find the magnitude of the force. Consider $g = 10$ m/s^2.

(a) 20 N (b) 22 N
(c) 4 N (d) 16 N **[2006]**

29. The blocks A and B of masses 2 m and m are connected as shown in the figure. The spring has negligible mass. The string is suddenly cut. The magnitudes of accelerations of masses 2 m and m at that instant are

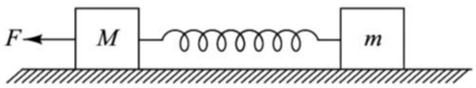
(a) g, g (b) $g, \frac{g}{2}$
(c) $\frac{g}{2}, g$ (d) $\frac{g}{2}, \frac{g}{2}$ **[2006]**



30. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k as shown in the figure. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m.

(a) $\frac{mF}{M}$ (b) $\frac{(M+m)F}{M}$

(c) $\frac{mF}{(m+M)}$ (d) $\frac{MF}{(m+M)}$ **[2007]**



31. A particle moves in the $x-y$ plane under the influence of a force such that its linear momentum is

$$\vec{p}(t) = A[\hat{\mathbf{i}}\ \cos\ (kt) - \hat{\mathbf{j}}\sin(kt)]$$

Where A and K are constants. The angle between the force and momentum is

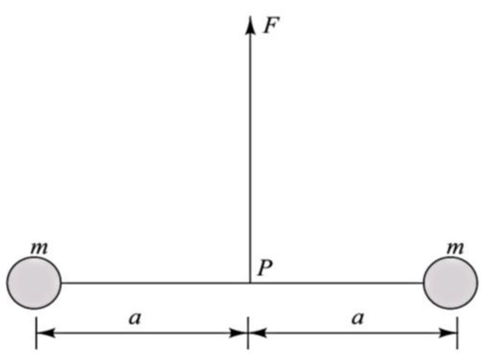
(a) 0º (b) 30º
(c) 45º (d) 90 **[2007]**

32. Two particles of mass m each are tied at the ends of a light string of length $2a$. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance 'a' from the centre P as shown in the figure. Now the mid-point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes $2x$, is

(a) $\dfrac{F}{2m}\dfrac{a}{\sqrt{a^2-x^2}}$ (b) $\dfrac{F}{2m}\dfrac{x}{\sqrt{a^2-x^2}}$

(c) $\dfrac{F}{2m}\dfrac{x}{a}$ (d) $\dfrac{F}{2m}\dfrac{\sqrt{a^2-x^2}}{x}$

[2007]



33. A body of mass m = 3.513 kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The megnitude of its momentum (in kg ms^{-1}) is recorded as

(a) 17.6 (b) 17.565
(c) 17.56 (d) 17.57 **[2008]**

34. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then

(a) at θ =30º, the block will start sliding down the plane
(b) the block will remain at rest on the plane up to certain θ and then it will topple
(c) at θ = 60º, the block will start sliding down the plane and continue to do so at higher angles
(d) at θ = 60º, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ. **[2009]**

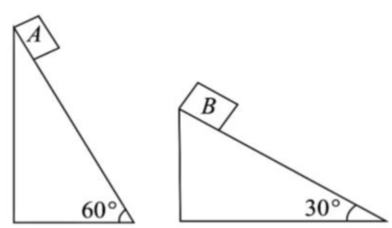
35. A piece of wire is bent in the shape of parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now acclerated parallel to the x-axis with a constant accleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is

(a) $\dfrac{a}{gk}$ (b) $\dfrac{a}{2gk}$

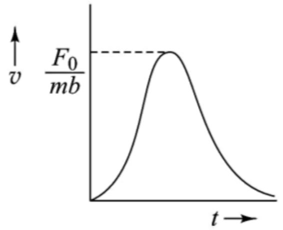
(c) $\dfrac{2a}{gk}$ (d) $\dfrac{a}{4gk}$ **[2009]**

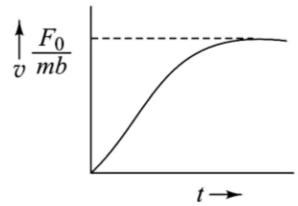
36. Two fixed frictionless inclined planes making an angle 30º and 60º with horizontal are shown in the figure. Two block A and B are place on the two planes, What is the relative vertical acceleration of A with respect to B?

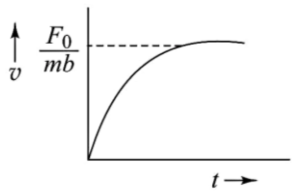
(a) 4.9 ms^{-2} in horizontal direction
(b) 9.8 ms^{-2} in vertical direction
(c) zero
(d) 4.9 ms^{-2} in vertical direction **[2010]**

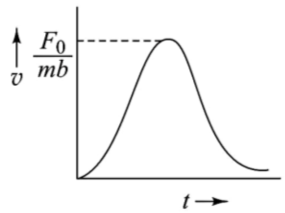


37. A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force. $F(t) = F_0e^{-bt}$ in the x direction where F_0 and b are constants. Which of the following graphs represents the variation of the velocity (v) of the particle with time t?

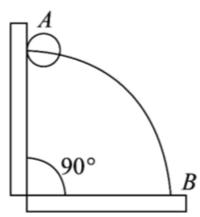








38. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is



(a) always radially outwards
(b) always radially inwards
(c) radially outwards initially and radially inwards later.
(d) radially inwards initially and radially outwards later. **[2014]**

Answers

1. (b)	2. (c)	3. (a)	4. (d)
5. (c)	6. (b)	7. (a)	8. (d)
9. (d)	10. (c)	11. (d)	12. (a)
13. (a)	14. (b)	15. (c)	16. (c)
17. (a)	18. (a)	19. (c)	20. (b)
21. (a)	22. (c)	23. (d)	24. (d)
25. (b)	26. (d)	27. (a)	28. (b)
29. (c)	30. (c)	31. (d)	32. (b)
33. (a)	34. (b)	35. (b)	36. (d)
37. (c)	38. (d)		

Solutions

1. $v = \sqrt{\mu R g} = \sqrt{0.6 \times 150 \times 10} = 30\ \text{ms}^{-1}$

2. Apparent weight in the lift = $m(g - a)$. Therefore, apparent accleration inside the lift is $(g - a)$. If the man is standing stationary on the ground, the acceleration of the falling ball is g. So the correct choice is (c).

3. Since the particle remains stationary, the resultant of $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ is zero, i.e.

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$\Rightarrow \quad \vec{F}_1 = -(\vec{F}_2 + \vec{F}_3)$$

Thus magnitude of $\vec{F}_1$ is equal to the magnitude of $(\vec{F}_2 + \vec{F}_3)$ but opposite in direction. Hence if $\vec{F}_1$ is removed, the magnitude of the force on the particle = magnitude of $(\vec{F}_2 + \vec{F}_3)$ = – magnitude of $\vec{F}_1$.

$$\therefore \quad \text{Acceleration } \vec{a} = -\frac{\vec{F}_1}{m}$$

Magnitude of acceleration = $-\dfrac{F_1}{m}$ but its direction is opposite to $\vec{F}_1$.

4. Since the cars are identical, their retardation a due to frictional force will be the same. From $v^2 - u^2 = 2as$, we have

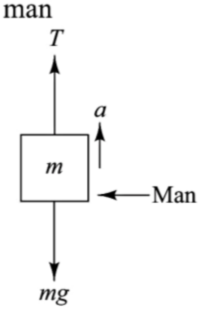
$$0 - u^2 = -2as_1 \Rightarrow s_1 = \frac{u^2}{2a}$$

and $$0 - (4u)^2 = -2as_2 \Rightarrow s_2 = \frac{16u^2}{2a}$$

$$\therefore \quad \frac{s_1}{s_2} = \frac{1}{16}$$

5. From the free body diagram of the man

$$T - mg = ma$$



$$\Rightarrow \quad a = \frac{T - mg}{m}$$

$$\therefore \quad a = \frac{T_{max} - mg}{m}$$

$$= \frac{360 - 60 \times 10}{60} = -4\ \text{ms}^{-2}$$

Negative acceleration implies climbing up the rope. The rope will break even if the man does not climb up the rope. However, choice (c) will be correct if the man were to climb down the rope.

6. Refer to Fig. 3.5 on page 3.4 of this chapter. The acceleration of the blocks is

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g \quad \text{...(i)}$$

Given $a = \frac{g}{8}$. Using this in Eq (i), we get

$$\frac{m_1}{m_2} = \frac{9}{7}$$

7. Mass of bag is $m = \frac{49}{9.8} = 5$ kg

Aparent weight of the bag when the lift moves down an acceleration a is

$$W' = m(g - a) = 5 \times (9.8 - 5) = 24 \text{ N}$$

8. The resultant of three forces which can be represented in magnitude and direction by the three sides of triangle taken in the same order is zero. This follows from the triangle law of vector addition. From Newton's first law, if no net force acts on a particle, its velocity remains unchanged in magnitude and direction. Hence the correct choice is (d)

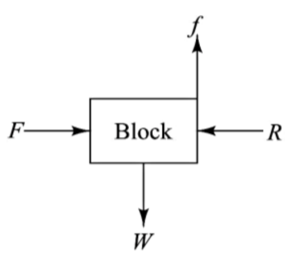
9. It follows from the free body diagram shown in the figure that

$$W = f$$

and $F = R$

But $f = \mu R$. Therefore

$$W = \mu R = \mu F$$
$$= 0.2 \times 10$$
$$= 2 \text{ N}$$



10. Frictional force $f = \mu mg$. Therefore retardation is

$$a = -\frac{f}{m} = -mg$$

From $v = u + at$, we have

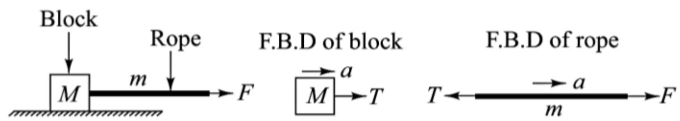
$$0 = 6 - a \times 10 \Rightarrow a = \frac{6}{10}$$

$$\therefore f = m \times \frac{6}{10}$$

or $\mu mg = m \times \frac{6}{10}$

$$\therefore \mu = \frac{6}{10g} = \frac{6}{10 \times 10} = 0.06$$

11. Refer to the following figure.



Acceleration of block-rope system is

$$a = \frac{F}{M + m} \quad \text{...(i)}$$

From free body diagrams of block and rope, we have

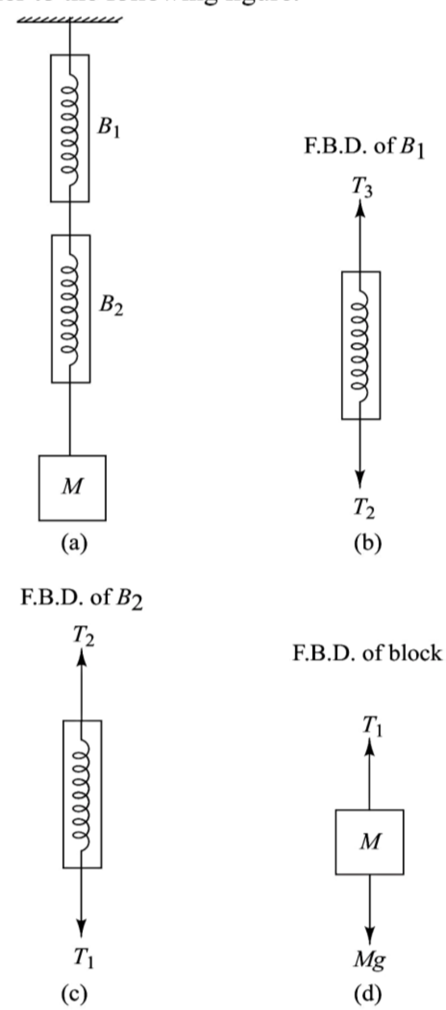
$$T = Ma$$

and $F - T = ma$ ...(ii)

Using (i) in (ii) we have

$$T = \frac{MF}{M + m}, \text{ which is choice } (d)$$

12. Refer to the following figure.



T_1 = reading of B_3 and T_2 = reading of B_1.

For equilibrium

$$T_1 = Mg$$
$$T_2 = T_1$$
and $$T_3 = T_2$$

Hence $T_1 = T_2 = Mg$. So both B_1 and B_2 will read M kg.

13. Initial thrust = ma

$$= (3.5 \times 10^4) \times 10 = 3.5 \times 10^5 \text{ N}$$

14. The horizontal component of F parallel to the surface is $F \sin \theta$. Hence maximum value of F is given by

$$F \sin \theta = \mu mg$$

or $$F \sin 60^\circ = 0.5 \times \sqrt{3} \times 10$$

or $$F \frac{\sqrt{3}}{2} = 0.5 \times \sqrt{3} \times 10$$

Which gives $F = 10$ N.

15. Centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{(30)^2}{500} = 1.8 \text{ ms}^{-2}$$

Tangential acceleration is

$a_t = 2\text{ms}^{-2}$

Resultant acceleration a = $\sqrt{a_c^2 + a_t^2}$

$$= \sqrt{(1.8)^2 + (2)^2} = 2.7 \text{ ms}^{-2}$$

16. Momentum of one bullet = mv. If n bullets are fired per second, the momentum imparted to the gun per second = nmv. From Newton's second law, force = rate of change of momentum, i.e.

$$F = nmv$$

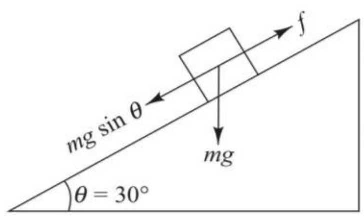
$$\Rightarrow \quad n = \frac{F}{mv} = \frac{144}{0.04 \times 1200} = 3$$

17. Refer to Fig 3.5 on page 3.4 of this chapter. The common acceleration of the masses is

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

$$= \left(\frac{5 - 4.8}{5 + 4.8}\right) \times 9.8 = 0.2 \text{ ms}^{-2}$$

18. Angle of repose is $\alpha = \tan^{-1}(\mu) = \tan-1\ (0.8) = 39.7^\circ$. Since $\theta < \alpha$, the frictional force f on the block is less than the limiting friction and is given by ($\because$ the block is at rest)



$$f = mg \sin \theta$$

$$\Rightarrow 10 = m \times 10 \times \sin 30^\circ \Rightarrow m = 2.0 \text{ kg}$$

19. Power $P = \vec{F} \cdot \vec{v} = Fv \cos \theta$. Since $\theta = 90^\circ$, $P = 0$, i.e. $\frac{dW}{dt} = 0$. Hence the work done by the force is constant. Therefore, kinetic energy of the particle remains constant.

20. Let a be the acceleration at a time t of the blocks executing SHM. The force on the blocks due to acceleration is

$$F = (m + m)\, a = 2\, ma$$

$$\therefore \quad F_{max} = 2\, m\, a_{max} \quad (1)$$

Now, the acceleration is maximum when the blocks are at the extreme position of maximum displacement, i.e.

$$F_{max} = kA$$

Equating (1) and (2), we get

$$a_{max} = \frac{kA}{2m}$$

$\therefore$ Maximum force of friction = ma_{max}

$$= m \times \frac{kA}{2m} \times \frac{kA}{2} \quad (2)$$

21. The acceleration of the block sliding on a smooth inclined plane of inclination θ is

$$a_1 = g \sin \theta \quad \text{(i)}$$

$\therefore$ Distance moved in time t_1 is

$$S_1 = \frac{1}{2} a_1 t_1^2 \quad \text{(ii)}$$

The acceleration on a rough inclined plane is

$$a_2 = g \sin \theta - \mu_k g \cos \theta \quad \text{(iii)}$$

Distance moved in time t_2 is

$$s_2 = \frac{1}{2} a_2 t_2^2 \quad \text{(iv)}$$

Given $s_1 = s_2 = d$ and $t_2 = nt_1$. Using these in (ii) and (iv), we have

$$a_1 t_1^2 = a_2 (nt_1)^2$$

$$\Rightarrow \quad a_1 = n^2 a_2 \Rightarrow \frac{a_1}{a_2} = n^2 \quad \text{(v)}$$

Dividing (i) and (iii)

$$\frac{a_1}{a_2} = \frac{g \sin \theta}{g \sin \theta - \mu_k g \cos \theta}$$

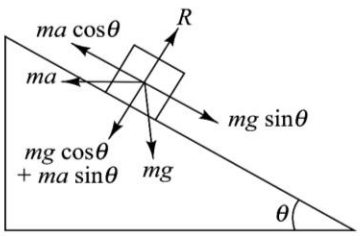
$$\Rightarrow \quad n^2 = \frac{\sin 45^\circ}{\sin 45^\circ - \mu_k \cos 45^\circ} = \frac{1}{1 - \mu_k}$$

$$\Rightarrow \quad \mu_k = 1 - \frac{1}{n^2}$$

22. Refer to the solution of Question 6 on page 3.21. The correct choice is (c).

23. If the inclined plane is given an acceleration a to the right, the block experiences a force ma to the left, where m is the mass of the block.

The following figure shows the forces acting on the block.



The block with remain stationary if no net force acts on it down the plane, i.e. if

$mg\sin\theta - ma\cos\theta = 0$

which gives $a = g\tan\theta$.

24. Acceleration $a = \dfrac{F}{m} = \dfrac{-kx}{m}$

$$= \frac{-15\times 0.2}{0.3} = -10\ \text{ms}^{-2}$$

25. Magnitude of acceleration $= 10\ \text{ms}^{-2}$

Frictional force $f = \mu mg$. Therefore retardation is

$$a = -\frac{f}{m} = -\mu g = -0.5\times 10 = -5\ \text{ms}^{-2}$$

Using $v^2 - u^2 = 2as$, we have

$$0-(100)^2 = 2\times -5\times s \Rightarrow s = 1000\ \text{m}$$

26. Since the block is held stationary, it is in translational as well as rotational equilibrium. Hence no net force and no net torque acts on the block. No net force will act on the block if $f = mg$ and $N = F$. No net torque will act on the block, if torque by frictional force f about centre O = counter torque by normal reaction N about centre O. Hence choice (d) is false.

27. Given $m = 150\times 10^{-3}$ kg, $= 20\ \text{ms}^{-1}$ and $t = 0.1$ s. Let F be the force of the impact. Now, impulse = force × time of impact $= F\times t$. Also impulse = change in momentum $= mv$. Equating them we have $F\times t = mv$

or $$F = \frac{m\times v}{t} = \frac{(150\times 10^{-3})\times 20}{0.1} = 30\ \text{N}$$

28. A velocity of the ball just after it is released from the hand is

$$v = \sqrt{2gh} = \sqrt{2\times 10\times 2} = \sqrt{40}\ \text{ms}^{-1}$$

Let a be the acceleration imparted to the ball during the time the hand was moving. During this time, the distance moved is $s = 0.2$ m. The value of a is given by

$$v^2 - u^2 = 2as$$

or $$40 - 0 = 2\times a\times 0.2$$

which gives $a = 100\ \text{ms}^{-2}$. If F is the magnitude of the applied force, then $F - mg = ma$ or $F = m(g+a)$ $= 0.2\times(10+100) = 22$ N.

29. When the system is in equilibrium, the spring force $= 3\,mg$. When the string is cut, the net force on block $A = 3\,mg - 2\,mg = 1\,mg$. Hence the acceleration of this block at this instant is

$$a = \frac{\text{force on block A}}{\text{mass of block A}} = \frac{mg}{2m} = \frac{g}{2}$$

When the string is cut, the block B falls freely with an acceleration equal to g.

30. The common acceleration of the blocks is

$$a = \frac{F}{(m+M)}$$

$\therefore$ Force on block of mass $m = ma = \dfrac{mF}{(m+M)}$

31. Force $\vec{F} = \dfrac{d\vec{p}}{dt}$

$$= \frac{d}{dt}[A\{\hat{i}\cos(kt) - \hat{j}\sin(kt)]$$

$$= Ak[-\hat{i}\sin(kt) - \hat{j}\cos(kt)]$$

Now, $\vec{F}\cdot\vec{p} = Ak[-\hat{i}\sin(kt) - \hat{j}\cos kt]$.

$$A[\hat{i}\cos(kt) - \hat{j}\sin(kt)].$$

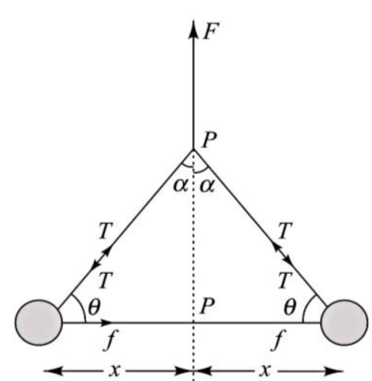
$$= A^2K[-\sin(kt)\cos(kt) + \cos(kt)\sin kt]$$

$$= 0\,(\because \hat{i}.\hat{i} = \hat{j}.\hat{j} = 1 \text{ and } \hat{i}.\hat{j} = 0)$$

Hence the angle between $\vec{F}$ and $\vec{p}$ is 90º.

32. Refer to the figure. Let f be the force producing the acceleration of each mass. It follows from the figure that $F = T\cos\alpha + T\cos\alpha = 2T\cos\alpha = 2T\sin\theta$ ($\because\ \alpha = 90º - \theta$)

$$\Rightarrow \quad T = \frac{F}{2\sin\theta} \qquad \text{(i)}$$



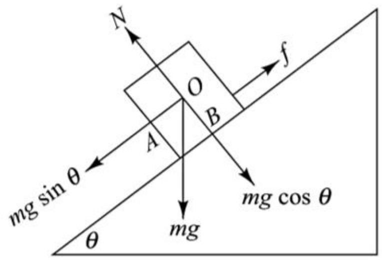
Also $T\cos\theta = f = ma$ (2)

Using (1) and (2), we get

$$a = \frac{F\cos\theta}{2m\sin\theta} = \frac{F}{2m\tan\theta}$$
$$= \frac{Fx}{2m\sqrt{a^2 - x^2}}$$

33. $p = mv = 3.513 \times 5.00 = 17.565$ kg ms^{-1}. Since the speed $v = 5.00$ ms^{-1} has 3 significant figures, The result of multiplication must have 3 significant figures. Hence the correct choice is (a).

34.



The block will just begin to slide if the downward force $mg \sin\theta$ just overcomes the frictional force, i.e. if $mg\sin\theta = \mu N = \mu\, mg\cos\theta \Rightarrow \tan\theta = \mu = \sqrt{3}$ $\Rightarrow \theta = 60°$

The block will topple if the torque due to normal reaction N about O just exceeds the torque due to $mg \sin\theta$ about O, i.e.

$$N \times OA = mg\sin\theta \times OB$$
$$\Rightarrow \quad mg\cos\theta \times 5\text{ cm} = mg\sin\theta \times \frac{15}{2}\text{ cm}$$
$$\Rightarrow \quad \tan\theta = \frac{2}{3} \Rightarrow \theta = 34°.$$

Since θ for toppling is less than θ for sliding, the correct choice is (b).

35. For the bead to stay at rest,

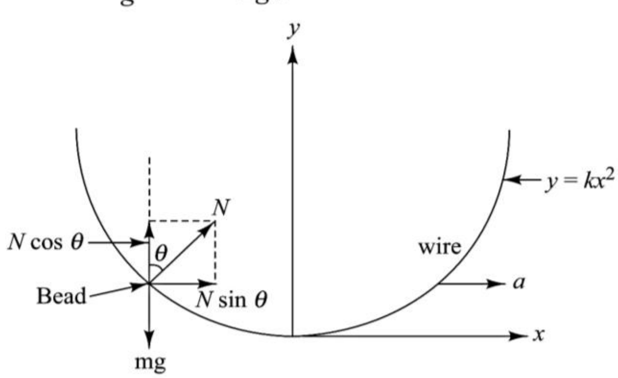
$N\cos\theta = mg$

$N\sin\theta = ma$

which gives $\tan\theta = \dfrac{a}{g}$. Now

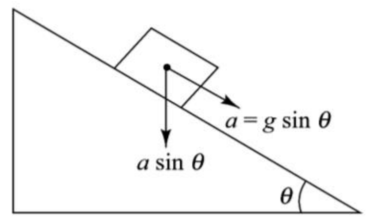
$\tan\theta$ = slope of the curve $= \dfrac{dy}{dx} = \dfrac{d}{dx}(kx^2) = 2kx$

$\therefore\ 2kx = \dfrac{a}{g} \Rightarrow x = \dfrac{a}{2gk}$



36. Accleration along the inclined plane is $a = g\sin\theta$.

Vertical component of a is $a\cos(90° - \theta) = a\sin\theta$ $= g\sin^2\theta$.



For block A, vertical acceleration is is $g\sin^2(60°) = \dfrac{3g}{4}$ and for block B the vertical acceleration is $g\sin^2(30°) = \dfrac{g}{4}$. Therefore, the relative vertical acceleration of A with respect to $B = \dfrac{3g}{4} - \dfrac{g}{4} = \dfrac{g}{2}$ $= 4.9$ ms^{-2}.

37. $F = F_0 e^{-bt} \Rightarrow ma = F_0 e^{-bt}$

$$\therefore \quad m\frac{dv}{dt} = F_0 e^{-bt}$$
$$\Rightarrow \quad dv = \frac{F_0}{m} e^{-bt}\, dt$$

Integrating, we get

$$v = \frac{F_0}{m}\left|\frac{e^{-bt}}{-b}\right|_0^t$$
$$\Rightarrow \quad v = \frac{F_0}{mb}(1 - e^{-bt})$$

It is clear that $v = 0$ at $t = 0$ and $\dfrac{F_0}{mb}$ as $t \to \infty$. So the correct graph is (c).

38. Initially the bead exerts an inward radial force (centripetal force) on the wire and the wire exerts a normal reaction N radially outwards. At a certain instant during the motion, the normal reaction N becomes zero. After that instant, the normal reaction N will act radially outwards. So the correct choice is (d).