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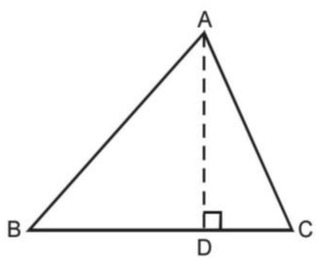
HERON'S FORMULA

Syllabus Reference

- **Perimeter:** The perimeter of a plane figure is the length of its boundary.
- **Area:** The area of a plane figure is the measure of the surface enclosed by its boundary.

AREA OF A TRIANGLE WITH GIVEN BASE AND CORRESPONDING ALTITUDE

Consider a ΔABC with base BC and corres-ponding altitude AD.

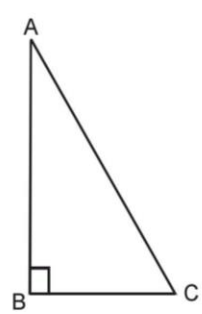


Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ corresponding altitude

$= \frac{1}{2} \times BC \times AD$

AREA OF A RIGHT TRIANGLE

Consider a right triangle ABC, right-angled at B. Using the two sides containing the right angle (BC and AB) as base and corresponding altitude, we have



Area of ΔABC

$= \frac{1}{2} \times$ product of two sides containing right angle

$= \frac{1}{2} \times BC \times AB$

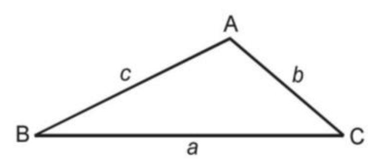
AREA OF A TRIANGLE BY HERON'S FORMULA (AREA OF A SCALENE TRIANGLE)

Consider a scalene triangle ABC, with AB = c units, BC = a units and CA = b units, then

Semi-perimeter of $\Delta ABC = \frac{a+b+c}{2}$

or $\quad s = \frac{a+b+c}{2}$ units

Now, area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ square units.



Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$.

Area of an isosceles triangle $= \frac{a\sqrt{4b^2 - a^2}}{4}$,

where b is each equal side and a is the base.

Area of a parallelogram = base $\times$ corresponding altitude.

Area of a rhombus $= \frac{1}{2} \times$ product of two diagonals.

Area of a trapezium $= \frac{1}{2} \times$ (sum of parallel sides) $\times$ distance between them.

Area of a regular hexagon of side 'a' $= 6 \times \frac{\sqrt{3}}{4} \times a^2$.

NCERT & BOARD QUESTIONS CORNER
(Remembering & Understanding Based Questions)

Very Short Answer Type Questions

1. *In ΔABC, if AB = 7 cm, BC = 8 cm and CA = 5 cm, then find the area of ΔABC.*

Sol. Here, let a = 8 cm, b = 5 cm and c = 7 cm

$$\therefore \quad s = \frac{8+5+7}{2} = 10 \text{ cm}$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{10(10-8)(10-5)(10-7)}$$

$$= \sqrt{10\times2\times5\times3} = 10\sqrt{3} \text{ cm}^2$$

2. *Find the area of a triangle whose two sides are 18 cm, 10 cm and the perimeter is 42 cm.*

Sol. Third side of the triangle = 42 – 18 – 10 = 14 cm

$$\text{Semi-perimeter}(s) = \frac{42}{2} = 21 \text{ cm}$$

Now, area of triangle

$$= \sqrt{21(21-18)(21-14)(21-10)}$$

$$= \sqrt{21\times3\times7\times11} = 21\sqrt{11} \text{ cm}^2$$

3. *If every side of a triangle is doubled, then find the percent increase in area of triangle so formed.*

Sol. Let the sides of the given triangle be a units, b units and c units.

$$\therefore \quad \text{Its area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ sq. units}$$

Now, new sides of the triangle are $2a$ units, $2b$ units and $2c$ units.

$$\text{Thus, its area} = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$$

$$= 4\sqrt{s(s-a)(s-b)(s-c)} \text{ sq. units}$$

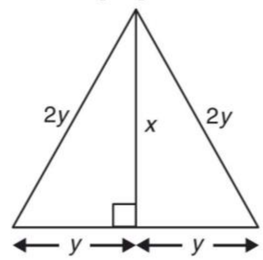
Total increase in area

$$= 3\sqrt{s(s-a)(s-b)(s-c)} \text{ sq. units}$$

Hence, percent increase = 300%

4. *If the length of a median of an equilateral triangle is x cm, then find its area.*

Sol. Let each equal side of given equilateral triangle be $2y$. We know that median is also perpendicular bisector.



$$\therefore \quad y^2 + x^2 = 4y^2$$

$$\Rightarrow \quad x^2 = 3y^2$$

$$\Rightarrow \quad x = \sqrt{3}y$$

$$\text{or} \quad y = \frac{x}{\sqrt{3}}$$

Now, area of given triangle

$$= \frac{1}{2} \times 2y \times x = y \times x = \frac{x}{\sqrt{3}} \times x$$

$$= \frac{x^2}{\sqrt{3}} = \sqrt{3}\,\frac{x^2}{3} \text{cm}^2$$

5. *Find the area of an equilateral triangle whose perimeter is 18 cm, using Heron's formula. (Use $\sqrt{3}$ = 1.73)*

Sol. Perimeter of an equilateral triangle = 18 cm

$$3 \times \text{side} = 18$$

$$\text{side} = 6 \text{ cm}$$

$$\text{Here, } s = \frac{a+b+c}{2} = \frac{18}{2} = 9 \text{ cm}$$

$$\text{Now, required area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{9(9-6)(9-6)(9-6)}$$

$$= \sqrt{9\times3\times3\times3}$$

$$= 9\sqrt{3} = 9 \times 1.73$$

$$= 15.57 \text{ cm}^2$$

6. *An advertisement board is of the form of an equilateral triangle of perimeter 240 cm. Find the area of the board using Heron's formula. (Use $\sqrt{3}$ = 1.73)*

Sol. Perimeter of an equilateral triangular board = 240 cm

$$3 \times \text{side} = 240$$

$$\text{side} = 80 \text{ cm}$$

Also, s = semi-perimeter $= \frac{240}{2} = 120$ cm

$\therefore$ Area of advertisement board

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{120(120-80)(120-80)(120-80)}$

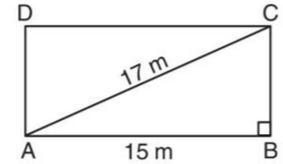
$= \sqrt{120 \times 40 \times 40 \times 40}$

$= 40 \times 40\sqrt{3}$

$= 1600 \times 1.73 = 2768 \text{ cm}^2$

7. *Length of a rectangular field is 15 m and diagonal is of length 17 m. Find its area and the perimeter.*

Sol. In rt. $\angle$ed ΔABC, by Pythagoras Theorem, we have



$BC = \sqrt{AC^2 - AB^2}$

$= \sqrt{17^2 - 15^2}$

$= \sqrt{289-225} = \sqrt{64} = 8\text{m}$

Now, area of rectangular field $= l \times b$

$= 15 \times 8$

$= 120 \text{ m}^2$

Perimeter of rectangular field $= 2(l + b)$

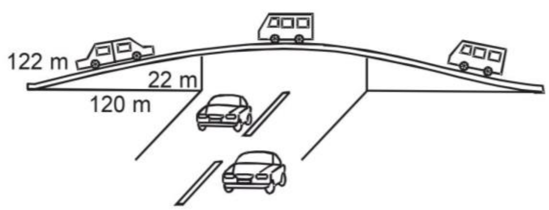
$= 2(15 + 8)$

$= 2 \times 23$

$= 46$ m

Short Answer Type-II Questions

8. *The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig.). The advertisements yield an earning of ₹5000 per m² per year. A company hired one of its walls for 3 months. How much rent did it pay?*



Sol. The lengths of the sides of the triangular walls are 122 m, 22 m and 120 m.

$\therefore \quad s = \frac{122+22+120}{2}$

$= \frac{264}{2} = 132$ m

Area of one triangular wall by using Heron's formula $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{132(132-122)(132-22)(132-120)}$

$= \sqrt{132 \times 10 \times 110 \times 12}$

$= 1320 \text{ m}^2$

Now, yearly rent = ₹ 5000 per m²

$\therefore$ Monthly rent = ₹ $5000 \times \frac{1}{12}$ per m²

Company hired one of its walls for 3 months.

Thus, rent paid by the company for 3 months

= ₹ $1320 \times \frac{5000}{12} \times 3$ = ₹ 1650000

9. *The semi-perimeter of a triangular ground is 450 units and its sides are in the ratio 3 : 5 : 4. Using Heron's formula, find the area of the ground.*

Sol. Let the sides of the triangular ground be $3x$ units, $5x$ units and $4x$ units.

The perimeter of triangular ground

$= 2 \times$ semi-perimeter

$= 2 \times 450$

$= 900$ units

$\Rightarrow 3x + 5x + 4x = 900$

$\Rightarrow \quad 12x = 900$

$\Rightarrow \quad x = \frac{900}{12} = 75$

Sides of triangular park are

$3 \times 75, 5 \times 75, 4 \times 75$

i.e., 225 units, 375 units, 300 units

Let a = 225 units, b = 375 units and

c = 300 units

s = 450 units [given]

$\therefore$ Area of triangular park

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{450(450-225)(450-375)(450-300)}$

$= \sqrt{450 \times 225 \times 75 \times 150}$

$= \sqrt{2 \times 225 \times 225 \times 75 \times 2 \times 75}$

$= 2 \times 75 \times 225 = 33750$ sq. units

10. *Find the area of a triangle whose sides are 11 m, 60 m and 61 m.*

Sol. Here, $a = 11$ m, $b = 60$ m and $c = 61$ m

$\therefore \quad s = \dfrac{a+b+c}{2} = \dfrac{11+60+61}{2}$

$= \dfrac{132}{2} = 66$ m

Now, Area of triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{66(66-11)(66-60)(66-61)}$

$= \sqrt{66 \times 55 \times 6 \times 5}$

$= \sqrt{2 \times 3 \times 11 \times 5 \times 11 \times 2 \times 3 \times 5}$

$= 2 \times 3 \times 5 \times 11$

$= 330 \text{ m}^2$

11. *The sides of a triangle are x, x + 1, 2x – 1 and its area is $x\sqrt{10}$. What is the value of x?*

Sol. Let the sides of triangle be x units, $(x + 1)$ units and $(2x - 1)$ units.

Now, semi-perimeter(s)

$= \dfrac{x+x+1+2x-1}{2}$

$= \dfrac{4x}{2} = 2x$ units

By using Heron's formula, we have

Area of triangle

$= \sqrt{2x(2x-x)(2x-x-1)(2x-2x+1)}$

$= \sqrt{2x(x)(x-1)(1)}$

$= x\sqrt{2(x-1)}$ sq. units.

But it is given that area of triangle $= x\sqrt{10}$ sq. units.

$\Rightarrow \quad x\sqrt{2(x-1)} = x\sqrt{10}$

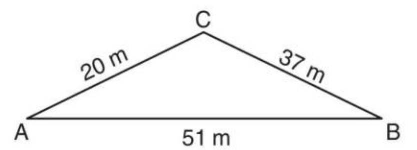
$\Rightarrow \quad 2(x-1) = 10$

$\Rightarrow \quad x - 1 = 5$

$\Rightarrow \quad x = 6$ units

12. *The sides of a triangular field are 51 m, 37 m and 20 m. Find the number of rose beds that can be prepared in the field if each rose bed occupies a space of 6 sq. m.*

Sol. Let ABC be the given triangular field in which AB = 51 m, BC = 37 m and AC = 20 m.



In ΔABC, we have

$s = \dfrac{51+37+20}{2} = \dfrac{108}{2} = 54$ m

Area of ΔABC

$= \sqrt{s(s-a)(s-b)(s-c)}$ (Heron's Formula)

$= \sqrt{54(54-51)(54-37)(54-20)}$

$= \sqrt{54(3)(17)(34)}$

$= \sqrt{3 \times 3 \times 3 \times 2 \times 3 \times 17 \times 2 \times 17}$

$= 3 \times 3 \times 2 \times 17 = 306 \text{ m}^2$

Since each rose bed occupies a space of 6 sq. m.

$\therefore$ Required number of rose beds in the field

$= \dfrac{306}{6} = 51$

13. *An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.*

Sol. Here, each equal side of an isosceles triangle is 12 cm.

And perimeter of the given triangle is 30 cm

$\therefore$ Third side of the triangle

$= 30 - 12 - 12 = 6$ cm

Now, $s = \dfrac{12+12+6}{2} = \dfrac{30}{2} = 15$ cm

Area of the triangle by using Heron's formula

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{15\,(15-12)(15-12)(15-6)}$$

$$= \sqrt{15\times3\times3\times9} = 9\sqrt{15}\ \text{cm}^2$$

14. ***A triangular park has sides 120 m, 80 m and 50 m. A gardener has to put a fence all around it and also plant grass inside. How much area does he need to plant? Find the cost of fencing it with barbed wire at the rate of ₹ 20 per metre leaving a space 3 m wide for a gate on one side.***

Sol. Here, $a = 120$ m, $b = 80$ m, $c = 50$ m

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{120+80+50}{2}$$

$$= \frac{250}{2} = 125 \text{ m}$$

Now, area of triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{125(125-120)(125-80)(125-50)}$$

$$= \sqrt{125\times5\times45\times75}$$

$$= \sqrt{5\times5\times5\times5\times3\times3\times5\times3\times5\times5}$$

$$= 3 \times 5 \times 5 \times 5\sqrt{3\times5}$$

$$= 375\sqrt{15}\ \text{m}^2$$

Perimeter of triangular park leaving a space 3 m wide for a gate = 250 – 3 = 247 m

Total cost of fencing = ₹ 247 × 20
= ₹ 4940

15. ***Using Heron's formula, find the area of an equilateral triangle with side 10 cm.***

Sol. Side of an equilateral triangle = 10 cm

$\therefore a = b = c = 10$ cm

$$s = \frac{a+b+c}{2} = \frac{10+10+10}{2} = 15 \text{ cm}$$

Area of triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{15(15-10)(15-10)(15-10)}$$

$$= \sqrt{15\times5\times5\times5} = \sqrt{3\times5\times5\times5\times5}$$

$$= 5 \times 5 \times \sqrt{3} = 25\sqrt{3}\ \text{cm}^2$$

16. ***The unequal side of an isosceles triangle measures 24 cm and its area is 60 cm². Find the perimeter of the given isosceles triangle.***

Sol. Let a be the base of isosceles triangle and each equal side be b

$\therefore \quad a = 24$ cm

$$\text{Area of triangle} = \frac{a}{4}\sqrt{4b^2 - a^2}$$

$$\Rightarrow \quad 60 = \frac{24}{4}\sqrt{4b^2 - a^2}$$

$$60 = 6\sqrt{4b^2 - 576}$$

$$\Rightarrow \quad 10 = \sqrt{4b^2 - 576}$$

On squaring both sides, we have

$$4b^2 - 576 = 100$$

$$4b^2 = 100 + 576$$

$$\Rightarrow \quad 4b^2 = 676$$

$$\Rightarrow \quad b^2 = \frac{676}{4} = 169$$

$$\Rightarrow \quad b = \sqrt{169} = 13 \text{ cm}$$

$\therefore$ Each equal side of isosceles triangle is 13 cm

Now, perimeter of the triangle

= 24 + 13 + 13
= 50 cm

17. ***The base (non-equal side) of an isosceles triangle is 8 cm. Find its area, if the perimeter of the triangle is 32 cm.***

Sol. Here, perimeter of an isosceles triangle = 32 cm

Base or non-equal side of the triangle = 8 cm

Let each equal side of the isosceles triangle be x cm.

$$\therefore \quad x + x + 8 = 32$$

$$\Rightarrow \quad 2x = 32 - 8 = 24$$

$$\Rightarrow \quad x = 12 \text{ cm}$$

Now, $s = \dfrac{12+12+8}{2} = \dfrac{32}{2} = 16$ cm

Area of the triangle by using Heron's formula

$= \sqrt{s(s-a)(s-b)(s-c)}$

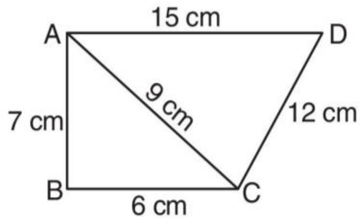
$= \sqrt{16(16-12)(16-12)(16-8)}$

$= \sqrt{16 \times 4 \times 4 \times 8}$

$= 32\sqrt{2}\ \text{cm}^2$

18. *Find the area of a quadrilateral ABCD, where AB = 7 cm, DA = 15 cm, AC = 9 cm, BC = 6 cm and CD = 12 cm.*

Sol. For ΔABC,



$$s = \frac{7+6+9}{2} = \frac{22}{2} = 11 \text{ cm}$$

∴ Area of ΔABC

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{11(11-7)\,(11-6)\,(11-9)}$

$= \sqrt{11(4)\,(5)\,(2)} = 2\sqrt{110}$

$= 20.97\ \text{cm}^2$

For ΔACD,

$$s = \frac{9+12+15}{2} = 18 \text{ cm}$$

∴ Area of ΔACD

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18\,(18-9)\,(18-12)\,(18-15)}$

$= \sqrt{18(9)\,(6)\,(3)}$

$= \sqrt{2 \times 9 \times 9 \times 2 \times 3 \times 3}$

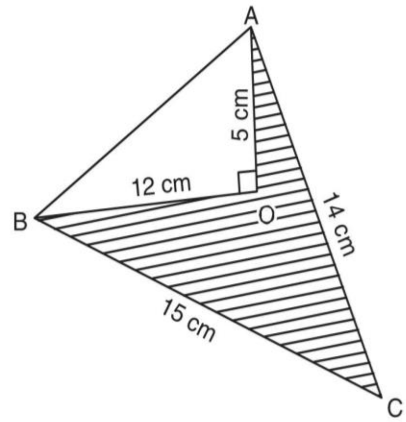
$= 2 \times 3 \times 9$

$= 54\ \text{cm}^2$

Hence, area of quadrilateral ABCD $= 20.97 + 54$

$= 74.97\ \text{cm}^2$.

19. *Calculate the area of the shaded region.*



Sol. Area of ΔAOB $= \frac{1}{2} \times \text{OA} \times \text{OB}$

$= \frac{1}{2} \times 5 \times 12 = 30\ \text{cm}^2$

Also, $\text{AB}^2 = \text{OA}^2 + \text{OB}^2$

$= 5^2 + 12^2$

$= 25 + 144 = 169$

$\Rightarrow \quad \text{AB} = \sqrt{169} = 13 \text{ cm}$

Now, in ΔABC, we have

$a = \text{BC} = 15$ cm, $b = \text{CA} = 14$ cm,

$c = \text{AB} = 13$ cm

$$s = \frac{a+b+c}{2} = \frac{15+14+13}{2}$$

$$= \frac{42}{2} = 21 \text{ cm}$$

Area of ΔABC

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-15)(21-14)(21-13)}$

$= \sqrt{21 \times 6 \times 7 \times 8}$

$= \sqrt{3 \times 7 \times 2 \times 3 \times 7 \times 2 \times 2 \times 2}$

$= 2 \times 2 \times 3 \times 7 = 84\ \text{cm}^2$

Area of shaded region

= Area of ΔABC – Area of ΔAOB

$= 84\ \text{cm}^2 - 30\ \text{cm}^2 = 54\ \text{cm}^2$

20. *Find the area of a triangular park, whose sides are 120 m, 100 m and 60 m.*

Sol. Here the lengths of the sides of the triangular park are 120 m, 100 m and 60 m

$\therefore \quad s = \dfrac{120+100+60}{2} = \dfrac{280}{2} = 140$ m

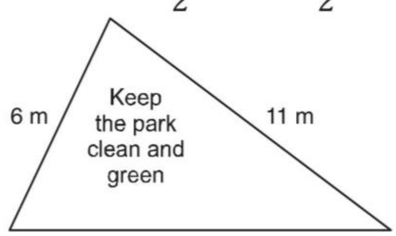
Now, area of triangular park by using Heron's formula

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{140(140-120)(140-100)(140-60)}$$
$$= \sqrt{140(20)(40)(80)} = \sqrt{8960000}$$
$$= 100 \times 8\sqrt{14} = 800\sqrt{14} \text{ m}^2$$
$$= 800 \times 3.742 \text{ m}^2$$
$$= 2993.3 \text{ m}^2$$

21. ***There is a slide in a park. One of its side walls has been painted in some colour with a message "Keep the park clean and green". If sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.***

Sol. Here the sides of the triangular wall are 15 m, 11 m and 6 m.

$\therefore \quad s = \dfrac{15+11+6}{2} = \dfrac{32}{2} = 16$ m



Area of the wall = Area to be painted in colour

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{16(16-15)(16-11)(16-6)}$$
$$= \sqrt{16(1)(5)(10)}$$
$$= 20\sqrt{2} \text{ m}^2$$

22. ***Find the area of a triangle whose two sides are 8 cm and 11 cm and the perimeter is 32 cm.***

Sol. We have perimeter of given triangle as 32 cm and the two sides are 8 cm and 11 cm.

Now, the third side of the triangle

$$= 32 - 8 - 11 = 13 \text{ cm}$$

Semi-perimeter(s) $= \dfrac{32}{2} = 16$ cm

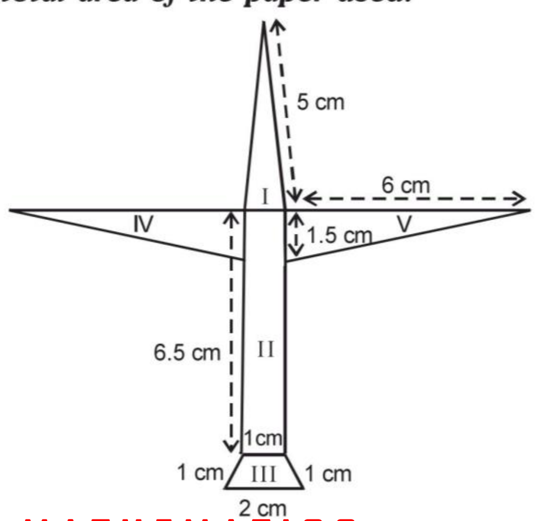
Now, area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ [Heron's formula]

$$= \sqrt{16(16-8)(16-11)(16-13)}$$
$$= \sqrt{16 \times 8 \times 5 \times 3}$$
$$= 8\sqrt{30} \text{ cm}^2$$

$\therefore$ Required area of triangle is $8\sqrt{30}$ cm^2.

Long Answer Type Questions

23. ***Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used.***



Sol. For Part I,

It is a triangle with sides 5 cm, 5 cm and 1 cm.

$\therefore \quad s = \dfrac{5+5+1}{2} = \dfrac{11}{2}$ cm

$\therefore$ Area of part I $= \sqrt{s(s-a)(s-b)(s-c)}$

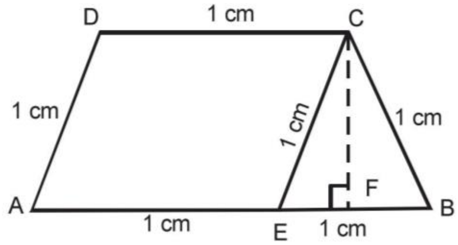
$$= \sqrt{\frac{11}{2}\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-1\right)}$$
$$= \sqrt{\frac{11}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{9}{2}}$$
$$= \frac{3}{4}\sqrt{11} = \frac{3}{4} \times 3.316$$
$$= 2.487 \text{ cm}^2$$

For Part II,

It is a rectangle with sides 6.5 cm and 1 cm.

$\therefore$ Area of part II= $6.5 \times 1 = 6.5$ cm^2

For Part III,



It is a trapezium ABCD.

Draw CE || DA.

ΔEBC is an equilateral with side 1 cm.

$$\therefore \quad \frac{1}{2} \times EB \times CF = \frac{\sqrt{3}}{4} \times (1)^2$$

$$\frac{1}{2} \times 1 \times CF = \frac{\sqrt{3}}{4}$$

$$\Rightarrow \quad CF = \frac{\sqrt{3}}{2} \text{ cm}$$

Now, area of trapezium

$$ABCD = \frac{1}{2}(2+1)\frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} \times 3 \times 1.732$$

$$= 1.299 \text{ cm}^2$$

For Part IV and V,

Each is a right triangle with sides (other than hypotenuse) 6 cm and 1.5 cm.

$\therefore$ Area of part IV and V

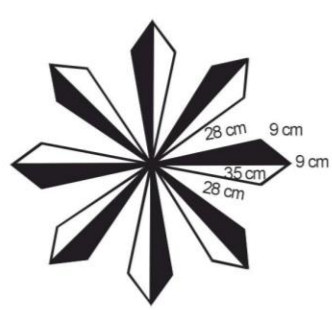
$$= 2 \times \frac{1}{2} \times 6 \times 1.5 = 9 \text{ cm}^2$$

$\therefore$ Total area of paper used

$$= (2.487 + 6.5 + 1.299 + 9) \text{ cm}^2$$

$$= 19.286 \text{ cm}^2 \text{ or } 19.3 \text{ cm}^2$$

24. ***A floral design on a floor is made-up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see fig.). Find the cost of polishing the tiles at the rate of 50 p per cm^2.***



Sol. For each triangular tile, we have

$$s = \frac{9+28+35}{2} = \frac{72}{2} = 36 \text{ cm}$$

$\therefore$ Area of each triangular tile

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{36\,(36-9)(36-28)(36-35)}$$

$$= \sqrt{36 \times 27 \times 8 \times 1}$$

$$= 36\sqrt{6} \text{ cm}^2$$

Total area of 16 such tiles

$$= 16 \times 36\sqrt{6}$$

$$= 16 \times 36 \times 2.45$$

$$= 1411.20 \text{ cm}^2$$

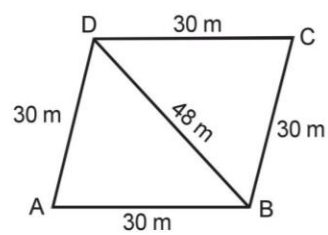
Total cost of polishing the tiles at the rate of 50 paise per cm^2

$$= ₹\ \frac{50}{100} \times 1411.20$$

$$= ₹\ 705.60$$

25. ***A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?***

Sol. Area of the rhombus ABCD = 2 area of ΔABD



For ΔABD,

$$s = \frac{30+30+48}{2} = \frac{108}{2} = 54 \text{ m}$$

$\therefore$ Area of ΔABD

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{54\,(54-30)(54-30)(54-48)}$$

$$= \sqrt{54 \times 24 \times 24 \times 6}$$

$$= 432 \text{ m}^2$$

$\therefore$ Area of the rhombus = $2 \times 432 = 864$ m^2

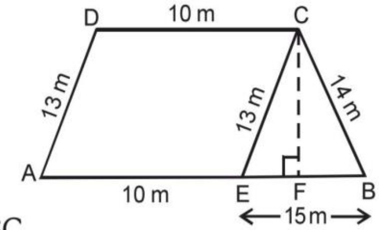
Number of cows = 18

$\therefore$ Area of grass field per cow $= \frac{864}{18} = 48$ m^2

26. ***A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.***

Sol. Here, ABCD is the required trapezium with AB || DC.

Through C, draw CE || DA and CF ⊥ AB



For ΔEBC,

$$s = \frac{15+14+13}{2} = \frac{42}{2} = 21 \text{ m}$$

$\therefore$ Area of $\Delta EBC = \sqrt{s(s-a)(s-b)(s-c)}$

$$\Rightarrow \frac{1}{2} \times EB \times CF = \sqrt{21(21-15)(21-14)(21-13)}$$

$$\frac{1}{2} \times 15 \times CF = \sqrt{21 \times 6 \times 7 \times 8}$$

$$\frac{15}{2} \times CF = 84$$

$$CF = 84 \times \frac{2}{15} = 11.2 \text{ m}$$

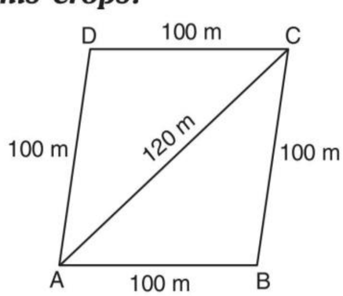
Now, area of the trapezium ABCD

$$= \frac{1}{2}(25 + 10) \times 11.2$$

$$= \frac{1}{2} \times 35 \times 11.2 = 196 \text{ m}^2$$

27. ***Suman has a piece of land, which is in the shape of a rhombus. She wants her two sons to work on the land and produce different crops. She divides the land in two equal parts by drawing a diagonal. If its perimeter is 400 m and one of the diagonals is of length 120 m, how much area each of them will get for his crops?***

Sol. Here, perimeter of rhombus ABCD = 400 m



$$\therefore \quad 4 \times \text{side} = 400$$

$$\Rightarrow \quad \text{side} = 100 \text{ m}$$

One diagonal AC = 120 m

Clearly, diagonal AC, divides the rhombus ABCD into two triangles, ΔABC and ΔADC.

Let a = 100 m, b = 100 m and c = 120 m

$$s = \frac{100+100+120}{2} = \frac{320}{2} = 160 \text{ m}$$

Now, ar(ΔABC)

$$= \sqrt{160(160-100)(160-100)(160-120)}$$

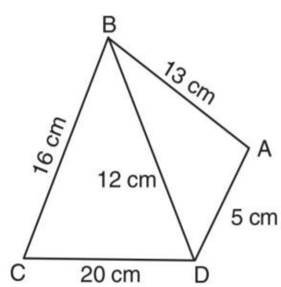
$$= \sqrt{160 \times 60 \times 60 \times 40}$$

$$= 60 \times 40 \times 2 = 4800 \text{ m}^2$$

Hence, area of the land each of them will get is 4800 m^2.

28. ***The sides of a quadrilateral ABCD are AB = 13 cm, BC = 16 cm, CD = 20 cm and DA = 5 cm. If BD = 12 cm, find the area of the quadrilateral using Heron's formula.***

Sol. Here, given quadrilateral ABCD is divided into two triangles, ΔABD and ΔBCD.



For ΔABD,

Let a = 13 cm, b = 12 cm and c = 5 cm

$$s = \frac{13+12+5}{2} = \frac{30}{2} = 15 \text{ cm}$$

$$\text{Now, ar}(\Delta ABD) = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{15(15-13)(15-12)(15-5)}$$

$$= \sqrt{15 \times 2 \times 3 \times 10}$$

$$= \sqrt{30 \times 30} = 30 \text{cm}^2$$

For ΔBCD,

Let a = 16 cm, b = 20 cm, c = 12 cm

$$\therefore s = \frac{16+20+12}{2} = \frac{48}{2} = 24 \text{ cm}$$

$$\text{ar}(\Delta BCD) = \sqrt{24(24-16)(24-20)(24-12)}$$
$$= \sqrt{24\times8\times4\times12}$$
$$= \sqrt{3\times8\times8\times4\times4\times3}$$
$$= 8 \times 4 \times 3$$
$$= 96 \text{ cm}^2$$

Now, ar(quad. ABCD)

$$= \text{ar}(\Delta ABD) + \text{ar}(\Delta BCD)$$
$$= 30 + 96 = 126 \text{ cm}^2$$

29. ***Find the area of the triangle whose perimeter is 180 cm and two of its sides are of length 80 cm and 18 cm. Also, calculate the altitude of the triangle corresponding to the shortest side.***

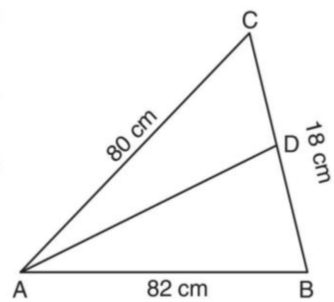
Sol. Here, perimeter of the triangle = 180 cm

$\therefore$ 80 + 18 + Third side = 180

Third side = 180 – 98 = 82 cm

s = semi-perimeter

$$= \frac{180}{2} = 90 \text{ cm}$$



$$\text{ar}(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{90\times(90-80)(90-18)(90-82)}$$
$$= \sqrt{90\times10\times72\times8}$$
$$= \sqrt{900\times576} = 30 \times 24$$
$$= 720 \text{ cm}^2$$

Also, $\frac{1}{2}\times BC\times AD = \text{ar}(\Delta ABC)$

$$\frac{1}{2}\times 18 \times AD = 720$$

$$AD = \frac{720}{9} = 80 \text{ cm}$$

Hence, area of triangle is 720 cm^2 and length of the altitude of the triangle corresponding to the shortest side is 80 cm.

30. ***An umbrella is made by stitching ten triangular pieces of cloth, each measuring 50 cm, 50 cm and 20 cm. Find the area of the cloth required for the umbrella.***

Sol. Here, each triangular piece is an isosceles triangle with sides 50 cm, 50 cm, 20 cm and number of triangular pieces of cloth are 10.

$$\therefore s = \frac{50+50+20}{2} = \frac{120}{2} = 60 \text{ cm}$$

Area of each triangular piece

$$= \sqrt{60(60-50)(60-50)(60-20)}$$
$$= \sqrt{60\times10\times10\times40}$$
$$= \sqrt{6\times4\times10\times10\times10\times10}$$
$$= 200\sqrt{6} \text{ cm}^2$$

Area of the cloth required for the umbrella

$$= 10 \times 200\sqrt{6}$$
$$= 2000\sqrt{6} \text{ cm}^2$$

APPLICATION BASED QUESTIONS (Solved)

1. ***The height of an equilateral triangle is 9 cm. Find the area of the triangle.***

Sol. Let each side of an equilateral triangle be a.

Then, height of an equilateral triangle $= \left(\frac{\sqrt{3}}{2}\times a\right)$

$$\Rightarrow \quad \frac{\sqrt{3}}{2}\times a = 9$$

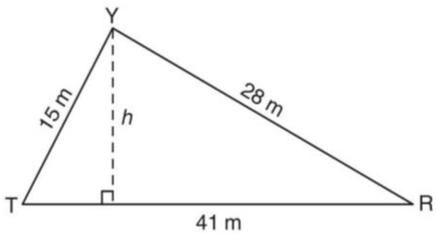
$$\Rightarrow \quad a = \frac{9\times2}{\sqrt{3}} = \frac{18}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$
$$= 6\sqrt{3} \text{ cm}$$

Now, area of an equilateral triangle

$$= \left(\frac{\sqrt{3}}{4}\times6\sqrt{3}\times6\sqrt{3}\right)$$
$$= 27\sqrt{3} = 46.77 \text{ cm}^2$$

2. The lengths of the sides of a triangle are 28 m, 41 m and 15 m. Find the length of the perpendicular from the opposite vertex to the side whose length is 41 m.

Sol. Here, let the sides of the ΔTRY are TR = 41 m, RY = 28 m, YT = 15 m.



Let h be the length of the perpendicular from the opposite vertex to the side whose length is 41 m.

$$s = \frac{41+28+15}{2} = \frac{84}{2} = 42 \text{ m}$$

$$\text{Area of } \Delta TRY = \sqrt{42(42-41)(42-28)(42-15)}$$

$$= \sqrt{42 \times 1 \times 14 \times 27}$$

$$= \sqrt{15876} = 126 \text{ m}^2$$

Now, $\frac{1}{2} \times 41 \times h = 126$

$$h = \frac{126 \times 2}{41} = \frac{252}{41} = 6.15 \text{ m}$$

Hence, the length of the perpendicular is 6.15 m.

3. The sides of a triangle are *p*, *q* and *r*. If *p* + *q* = 45, *q* + *r* = 40 and *p* + *r* = 35, then find the area of the triangle.

Sol. We have $p + q = 45$ ...(*i*)

$q + r = 40$ ...(*ii*)

and $r + p = 35$ ...(*iii*)

Adding (*i*), (*ii*) and (*iii*), we have

$2(p + q + r) = 120$

$\Rightarrow p + q + r = 60$ ...(*iv*)

On subtracting equations (*i*), (*ii*) and (*iii*) one by one from eq. (*iv*)

$\Rightarrow r = 15, p = 20$ and $q = 25$

Now, semi-perimeter of the triangle of sides p, q,

$$r = \frac{60}{2} = 30$$

Here, area of triangle $= \sqrt{s(s-p)(s-q)(s-r)}$

$$= \sqrt{30(10)(5)(15)}$$

$$= \sqrt{10 \times 10 \times 3 \times 3 \times 5 \times 5}$$

$= 150$ sq. units

Hence, required area of triangle is 150 sq. units.

4. A circle, a square and an equilateral triangle have same perimeter. Show that area of the triangle is lesser than the area of a square.

Sol. Let the radius of a circle be r and the side of equilateral triangle be x.

Then using the statement of the question, we have

Perimeter of a circle = Perimeter of equilateral triangle

Circumference of a circle = Perimeter of equilateral triangle

$$2\pi r = 3x \Rightarrow x = \frac{2}{3}\pi r$$

Also, let the side of square be a.

Circumference of a circle = Perimeter of square

$\Rightarrow \quad 2\pi r = 4a$

$\Rightarrow \quad a = \frac{\pi r}{2}$

Now, area of equilateral triangle

$$= \frac{\sqrt{3}}{4}x^2 = \frac{\sqrt{3}}{4}\left(\frac{2}{3}\pi r\right)^2$$

$$= \frac{\sqrt{3}}{4} \times \frac{4}{9}\pi^2 r^2 = \frac{\sqrt{3}}{9}\pi^2 r^2 \quad ...(i)$$

$$\text{Area of a square} = \left(\frac{\pi r}{2}\right)^2 = \frac{\pi^2 r^2}{4} \quad ...(ii)$$

Now, from eqn. (*i*) and (*ii*), we have

$$\frac{\sqrt{3}}{9} < \frac{1}{4}$$

$\Rightarrow$ For the given perimeter, area of equilateral triangle is lesser than area of a square.

ANALYZING, EVALUATING & CREATING TYPE QUESTIONS (Solved)

1. ***A traffic sign board is an equilateral triangle with the length 6 cm each. Find the area of sign board by 'Heron's formula'. If the inner triangle is also an equilateral triangle with perimeter 9 cm, find the area of inner triangle by direct formula.***

Sol. Here, outer triangular piece is an equilateral triangle with sides 6 cm, 6 cm, 6 cm.

$$\Rightarrow \quad s = \frac{6+6+6}{2} = 9 \text{ cm.}$$

Area of triangular sign board

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{9(9-6)(9-6)(9-6)}$$
$$= \sqrt{9(3)(3)(3)}$$
$$= 9\sqrt{3} \text{ cm}^2$$
$$= 15.6 \text{ cm}^2$$

Now, area of inner triangle with perimeter 9 cm.

Each side of equilateral triangle

$$= \frac{9}{3} = 3 \text{ cm.}$$

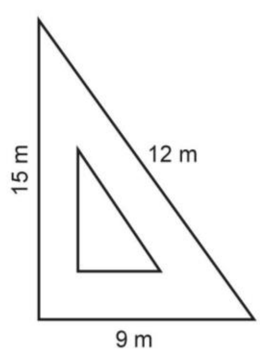
We know, area of equilateral triangle with side x

$$= \frac{\sqrt{3}}{4}x^2$$

$$\therefore \text{ Area of inner triangle } = \frac{\sqrt{3}}{4} \times (3)^2$$

$$= \frac{9}{4}\sqrt{3} \text{ cm}^2 = 3.90 \text{ cm}^2$$

2. ***In the centre of a triangular plot of land of sides 15 m, 12 m and 9 m, a triangular portion is to be covered with trees and the portion in between the two triangles is covered with grass. If the triangular portion inside the plot has perimeter 18 m and its sides are in the ratio 4 : 3 : 2. Find the area to be planted with trees and area to be planted with grass.***



Sol. Here, the lengths of the sides of the triangular plot are 15 m, 12 m and 9 m.

$$\therefore \quad s = \frac{15+12+9}{2} = \frac{36}{2} = 18 \text{ m}$$

Now, area of triangular plot by using Heron's formula $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{18(18-15)(18-12)(18-9)}$$
$$= \sqrt{18(3)(6)(9)}$$
$$= 54 \text{ m}^2$$

We have, perimeter = 18 m with sides of triangular portion where trees is to be planted are in the ratio of 4 : 3 : 2.

Let the sides be $4x$, $3x$ and $2x$.

Now, $4x + 3x + 2x = 18$

$$\Rightarrow \quad 9x = 18 \Rightarrow x = 2$$

So, the sides of triangular portion are 8 m, 6 m and 4 m.

$$\therefore \quad s = \frac{18}{2} = 9 \text{ m}$$

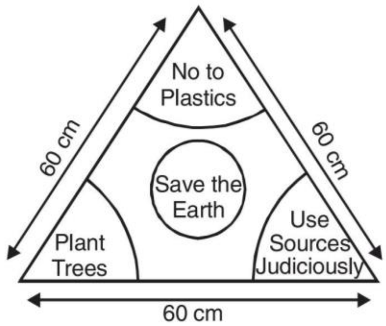
Area of triangular portion where trees are planted

$$= \sqrt{9(9-8)(9-6)(9-4)}$$
$$= \sqrt{9(1)(3)(5)}$$
$$= 3 \times 3.87$$
$$= 11.62 \text{ m}^2$$

Area between the triangles where grass is planted

$$= 54 - 11.62$$
$$= 42.38 \text{ m}^2$$

3. ***Save earth to save our environment for coming generations. A child prepares a poster on 'Save Earth' on a triangular sheet whose each side measures 60 cm. At each corner of the sheet, she draws an arc of radius 14 cm in which she shows the ways to save earth. At the centre, she draws a circle of diameter 14 cm, where she writes a slogan in it. Find the area of remaining sheet.***



Sol. Here, each equal side of equilateral triangle = 60 cm

$$s = \frac{60+60+60}{2} = 90 \text{ cm}$$

Now, area of the given triangle by using Heron's formula

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{90(90-60)(90-60)(90-60)}$$

$$= \sqrt{90(30)(30)(30)} = 900\sqrt{3} \text{ cm}^2$$

$$= 1558.85 \text{ cm}^2$$

Area of triangle covered by each arc

$$= \frac{\pi r^2 \times 60^\circ}{360^\circ} = \frac{22}{7} \times \frac{14\times 14}{6}$$

$$= 102.67 \text{ cm}^2$$

Area of triangle covered by 3 arcs

$$= 3 \times 102.67 = 308 \text{ cm}^2$$

Area of circle $= \pi r^2 = \frac{22}{7}\times 7\times 7 = 154 \text{ cm}^2$

Area of the remaining sheet = Area of triangle – area of triangle covered by 3 arcs – area of circle

$= 1558.85 \text{ cm}^2 - 308 \text{ cm}^2 - 154 \text{ cm}^2$

$= 1096.85 \text{ cm}^2$

4. ***In the given figure, six equilateral triangles representing different values with side 18 cm are joined to make a toy. Find the total area of Honesty and Non-violence used for making the toy.***

Sol. Here, six equilateral triangles are joined to make a toy and side of each is 18 cm.

∴ Area of each equilateral triangle

$$= \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times 18 \times 18$$

$$= \sqrt{3} \times 81 = 140.30 \text{ cm}^2$$

Total area of Honesty and Non-violence

= Area of two equilateral triangles

$= 2 \times 140.30 \text{ cm}^2 = 280.60 \text{ cm}^2$

5. ***To serve huminity, we should respect women, educate women, save women, empower women and give them equal opportunity. In an exhibition, an umbrella is made by stiching 8 triangular pieces of cloth with same message written on two triangular pieces. If each piece of cloth measures 26 cm, 26 cm and 12 cm, find how much cloth is required for each message.***

Sol. Here, each triangular piece is an isosceles triangle with sides 26 cm, 26 cm and 12 cm.

$$\therefore \quad s = \frac{26+26+12}{2} = 32 \text{ cm}$$

∴ Area of each triangular piece

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{32(32-26)(32-26)(32-12)}$$

$$= \sqrt{32(6)(6)(20)}$$

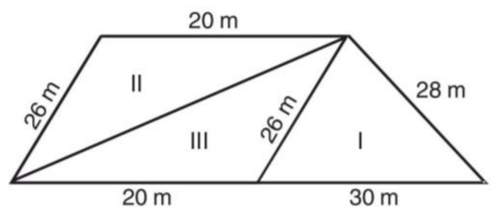
$$= 48\sqrt{10} \text{ cm}^2 = 151.79 \text{ cm}^2$$

Now, there are 2 triangular pieces with same message.

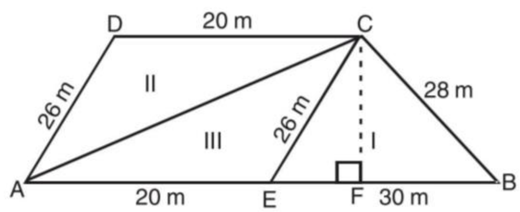
∴ Total area of cloth for each message

$= 2 \times 151.79 = 303.58 \text{ cm}^2$

6. ***Sister Nivedita has trapezium shaped plot which she divided into three triangular portion for different purposes. I-for providing free education for orphan children, II-for providing dispensary for the needy villagers and III- for the library for villagers. Find the area of trapezium plot given in the figure.***



Sol. Here, ABCD is the trapezium with AB || DC.



Through C, Draw CF ⊥ AB

For - I: For area of ΔEBC

$$s = \frac{26+28+30}{2}$$

$$= \frac{84}{2} = 42 \text{ m}$$

Now, area of ΔEBC $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{42(42-26)(42-28)(42-30)}$$

$$= \sqrt{42(16)(14)(12)} = 336 \text{ m}^2$$

But, we know, Area of ΔEBC

$$= \frac{1}{2}(\text{base} \times \text{height})$$

$$\Rightarrow \frac{1}{2} \times 30 \times CF = 336$$

$$15 \times CF = 336$$

$$\Rightarrow \quad CF = \frac{336}{15}$$

$$= 22.4 \text{ m}$$

Now, area of trapezium shaped plot

$$= \frac{1}{2}(20 + 50)(22.4)$$

$$= 35 \times 22.4 \text{ m}^2$$

$$= 784 \text{ m}^2$$

7. ***Planting tree help in reducing pollution and make the environment green and clean. In the centre of a triangular plot of land of sides 15 m, 12 m and 9 m, a triangular portion is to be covered with trees and the portion in between the two triangles is covered with grass. If the triangular protion inside the plot has perimeter 18 m and its sides are in the ratio 4 : 3 : 2. Find the area to be planted with trees and area to be planted with grass.***

Sol. Here, the lengths of the sides of the triangular plot are 15 m, 12 m and 9 m.

$$\therefore \quad s = \frac{15+12+9}{2} = \frac{36}{2} = 18 \text{ m}$$

Now, area of triangular plot by using Heron's formula

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{18(18-15)(18-12)(18-9)}$$

$$= \sqrt{18(3)(6)(9)} = 54 \text{ m}^2$$

We have, perimeter = 18 m with sides of triangular portion where trees is to be planted are in the ratio of 4 : 3 : 2.

Let the sides be $4x$, $3x$ and $2x$.

Now, $4x + 3x + 2x = 18$

$\Rightarrow \quad 9x = 18 \Rightarrow x = 2$

So, the sides of triangular portion are 8 m, 6 m and 4 m.

$$\therefore \quad s = \frac{18}{2} = 9 \text{ m}$$

Area of triangular portion where trees are planted

$$= \sqrt{9(9-8)(9-6)(9-4)}$$

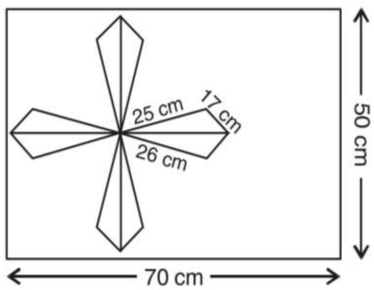
$$= \sqrt{9(1)(3)(5)}$$

$$= 3 \times 3.87 = 11.62 \text{ m}^2$$

Area between the triangles where grass is planted

$$= 54 - 11.62 = 42.38 \text{ m}^2$$

8. ***A design is made on a rectangular tile of dimensions 50 cm by 70 cm as shown in the given figure. The design shows eight triangles, each of side 26 cm, 17 cm and 25 cm. Find the total area of the design and the remaining area of the tile.***



Sol. Here, length of tile = 70 cm

and breadth of tile = 50 cm

then, the area of tile = length × breadth

$= 50 \times 70 = 3500 \text{ cm}^2$

Since the design contains 8 triangles with the sides $a = 26$ cm, $b = 17$ cm and $c = 25$ cm

$$\Rightarrow \quad s = \frac{26+17+25}{2} = \frac{68}{2} = 34 \text{ cm}$$

Hence, area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{34(8)(17)(9)}$$

$$= \sqrt{2\times17\times2^3\times17\times3^2}$$

$$= \sqrt{2^4\times3^2\times17^2}$$

$$= 204 \text{ cm}^2$$

The area of design = 8 × area of triangle

$= 8 \times 204$

$= 1632 \text{ cm}^2$

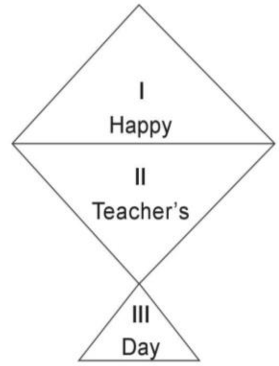
Hence, the remaining area of tile

= Area of tile – Area of design

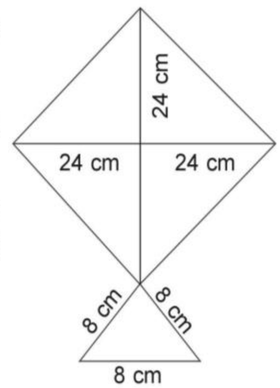
$= (3500 - 1632) \text{ cm}^2$

$= 1868 \text{ cm}^2$

9. ***On the occasion of 'Teacher's Day', Ashwani made a kite in the shape of a square with diagonal 48cm and an equilateral triangle of base 8 cm as shown in figure. How much paper of each shade has been used by Ashwani?***



Sol. Since the kite is in the shape of a square.



For Part I: Each diagonal of the square = 48 cm. The diagonals of square bisect each other at right angles.

∴ Area of part I

$$= \frac{1}{2} \times 48 \times 24$$

$$= 576 \text{ cm}^2$$

For Part II:

Area of Part II $= \frac{1}{2} \times 48 \times 24 = 576 \text{ cm}^2$

For Part III: It is an equilateral triangle with side 8 cm.

$$\Rightarrow \quad s = \frac{8+8+8}{2} = 12 \text{ cm}$$

Area of Part III $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{12(12-8)(12-8)(12-8)}$$

$$= \sqrt{12(4)(4)(4)}$$

$$= 16\sqrt{3} \text{ cm}^2$$

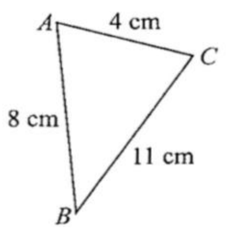
$$= 27.71 \text{ cm}^2$$

FOUNDATION : IIT-NDA-KVPY-OLYM

Heron's Formula

MATHEMATICAL REASONING

1. In the given figure, the area of the ΔABC is



(a) $13.24\, cm^2$ (b) $12.28\, cm^2$
(c) $11.32\, cm^2$ (d) $15.37\, cm^2$

2. The difference between the semi-perimeter and the sides of a ΔABC are 8 cm, 7 cm and 5 cm respectively. The area of the triangle is____.
(a) $20\sqrt{7}\, cm^2$ (b) $10\sqrt{14}\, cm^2$
(c) $20\sqrt{14}\, cm^2$ (d) $140\, cm^2$

3. The perimeter of a triangle is 540 m and its sides are in the ratio $25:17:12$. Find its area.
(a) $9100\, m^2$ (b) $9000\, m^2$
(c) $9200\, m^2$ (d) $9500\, m^2$

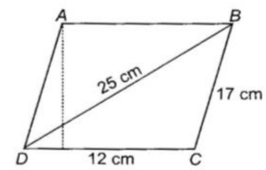
4. The perimeter of an isosceles triangle is 32 cm. The ratio of one of the equal sides to its base is $3:2$. Find the area of the triangle.
(a) $48\, cm^2$ (b) $28\sqrt{3}\, cm^2$
(c) $32\sqrt{2}\, cm^2$ (d) $44\, cm^2$

5. If each side of the rhombus is 40 m and its longer diagonal is 48 m, then the area of rhombus is____.
(a) $1536\, m^2$ (b) $1636\, m^2$
(c) $1236\, m^2$ (d) $1336\, m^2$

6. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.
(a) 15 cm (b) 14 cm
(c) 12 cm (d) 13 cm

7. The area of a parallelogram ABCD in which AB = 12 cm, BC = 9 cm and diagonal AC = 15 cm is k cm2. Find the value of $\frac{k-100}{4}$.
(a) 3 (b) 4
(c) 2 (d) 5

8. In the given parallelogram, find the length of the altitude from vertex A on the side DC.

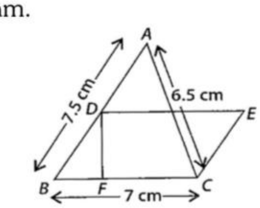


(a) 18 cm (b) 12 cm
(c) 15 cm (d) 25 cm

9. A rhombus shaped sheet with perimeter 40 cm and one diagonal 12 cm, is painted on both sides at the rate of ₹ 5 per cm^2. Find the cost of painting.
(a) ₹ 880 (b) ₹ 1020
(c) ₹ 960 (d) ₹ 980

10. The area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm is $k\sqrt{30}\, cm^2$. Find the value of k.
(a) 8 (b) 6
(c) 7 (d) 9

11. In the given figure, AABC has sides AB = 7.5 cm, AC = 6.5 cm and BC = 7 cm. On base BC a parallelogram DBCE of same area as that of ΔABC is constructed. Find the height DF of the parallelogram.



(a) 3 cm (b) 6 cm
(c) 4 cm (d) 2 cm

12. The sides of a triangle are 11 cm, 15 err, and 16 cm. The altitude to the largest side is____.
(a) $30\sqrt{7}\, cm$ (b) $\frac{15\sqrt{7}}{2}\, cm$
(c) $\frac{15\sqrt{7}}{4}\, cm$ (d) $30\, cm$

13. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

(a) 196 cm^2 (b) 186 cm^2
(c) 169 cm^2 (d) 199 cm^2

14. Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of ₹ 7 per m^2.

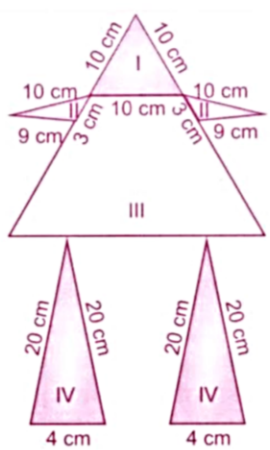
(a) ₹ 9500 (b) ₹ 11000
(c) ₹ 10500 (d) ₹ 12500

15. The base of an isosceles triangle measures 24 cm and its area is 192 cm^2. Find its perimeter.

(a) 64 cm (b) 46 cm
(c) 84 cm (d) 54 cm

EVERYDAY MATHEMATICS

16. Suman made a picture with some white paper and a single coloured paper as shown in figure. White paper is available at her home and free of cost. The cost of coloured paper used is at the rate of 10p per cm^2. Find the total cost of the coloured paper used. (Take $\sqrt{3} = 1.732$ and $\sqrt{11} = 3.31$)

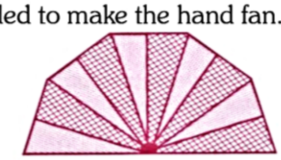


(a) ₹ 14.92 (b) ₹ 14
(c) ₹ 16 (d) ₹ 13

17. An umbrella is made by stiching 12 triangular pieces of cloth of two different coloured as shown in given figure. Each piece measuring 40 cm, 40 cm and 18 cm. How much cloth of each colour is required for the umbrella?

(a) 2104.56 cm^2, 2104.56 cm^2
(b) 4209.22 cm^2, 2104.56 cm^2
(c) 1204.61 cm^2, 1204.61 cm^2
(d) 2014.61 cm^2, 1204.61 cm^2

18. A hand fan is made by stitching 10 equal size triangular strips of two different types of paper as shown below. The dimensions of equal strips are 25 cm, 25 cm and 14 cm. Find the total area of paper needed to make the hand fan.

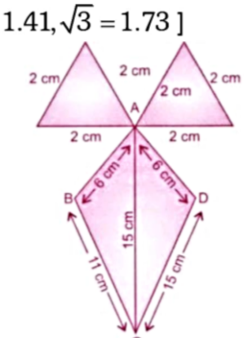


(a) 840 cm^2 (b) 1680 cm^2
(c) 480 cm^2 (d) 7844 cm^2

19. The perimeter of a field in the form of an equilateral triangle is 36 cm, then its area is given by

(a) $98\sqrt{3}\,cm^2$ (b) $8\sqrt{3}\,cm^2$
(c) $42\sqrt{3}\,cm^2$ (d) $36\sqrt{3}\,cm^2$

20. Tanya joined four triangles of cardboard to create a mask of Joker as shown in the given figure. Find the total area of the mask.
(Given $\sqrt{2} = 1.41, \sqrt{3} = 1.73$]



(a) 59.86 cm^2 (b) 50 cm^2
(c) 59 cm^2 (d) 53 cm^2

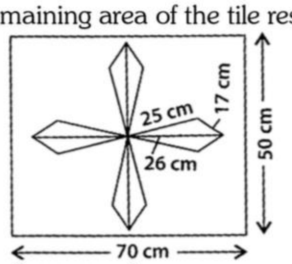
ACHIEVERS SECTION (HOTS)

21. ABC is an equilateral triangle of side $4\sqrt{3}$ cm. P, Q and R are mid-points of AB, CA and BC respectively. Find the area of triangle PQR is ____.

(a) $3\sqrt{3}\,cm^2$ (b) $2\sqrt{3}\,cm^2$

(c) $\frac{\sqrt{3}}{2}cm^2$ (d) $\frac{\sqrt{3}}{4}cm^2$

22. A design is made on a rectangular tile of dimensions $50\,cm \times 70\,cm$ as shown in figure. The design shows 8 triangles, each of sides 26 cm, 17 cm and 25 cm. Find the total area of the design and the remaining area of the tile respectively.



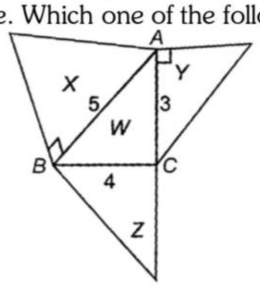
(a) $1632\,cm^2, 1886\,cm^2$

(b) $1538\,cm^2, 1632\,cm^2$

(c) $1632\,cm^2, 1868\,cm^2$

(d) $1538\,cm^2, 1632\,cm^2$

23. Right isosceles triangles are constructed on the sides of right angled ΔABC with sides 3, 4, 5 units, as shown. A capital letter indicates area of each triangle. Which one of the following is true?



(a) $X+Z=Y+W$ (b) $W+X=Z$

(c) $Y+Z=X$ (d) $X+W=\frac{1}{2}(Y+Z)$

24. State T for true and 'F' for false.

(i) The lengths of the three sides of a triangular field are 40 m, 24 m and 32 m respectively. The area of the triangle is 384 m^2.

(ii) The area of a quadrilateral ABCD in which B = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm is 18 cm^2.

(iii) An advertisement board is in the form of an isosceles triangle with its sides equal to 12 m, 10 m and 10 m. The cost of painting it at ₹ 2.25 per m^2 is 18 cm^2. 112.

(iv) Heron's formula cannot be used to calculate area of quadrilaterals.

	(i)	(ii)	(iii)	(iv)
(a)	T	F	F	T
(b)	F	T	F	F
(c)	T	F	T	F
(d)	T	F	F	F

25. Find the area of quadrilateral ABCD in which AB = 9 cm, 6C = 40 cm, CD = 28 cm, DA = 15 cm and $\angle ABC = 90°$.

(a) $300\,cm^2$ (b) $180\,cm^2$

(c) $126\,cm^2$ (d) $306\,cm^2$

HINTS & EXPLANATIONS

1. (b) : Here a = 1 cm, b = 4 cm, c = 8 cm

$\therefore\ s = \frac{11+4+8}{2} = \frac{23}{2} = 11.5\,cm$

Area

$= \sqrt{11.5\times(11.5-11)\times(11.5-4)\times(11.5-8)}$

$= \sqrt{11.5\times 0.5\times 7.5\times 3.5}$

$= \sqrt{150.94} = 12.28\,cm^2$

2. (c) : Let the sides of ΔABC be a, b, c

Then,

$(s-a)=8, (s-b)=7$ and $(s-c)=5$

$\Rightarrow (s-a)+(s-b)+(s-c)=20$

$\Rightarrow (s-a)+(s-b)+(s-c)=20$

$\Rightarrow 3s-(a+b+c)=20$

$\Rightarrow 3s-2s=20$

$\Rightarrow s=20$ $\left[\because s=\frac{a+b+c}{2}\right]$

$\therefore$ Area of $\Delta ABC = \sqrt{20\times 8\times 7\times 5}$

$= \sqrt{5600}\,cm^2 = 20\sqrt{14}\,cm^2$

3. (b) : Let the sides of the triangle be

$a=25x, b=17x, c=12x$

Perimeter of triangle = 540cm

$\Rightarrow 25x+17x+12x=540$

$\Rightarrow 54x+540 \Rightarrow x=10$

$\therefore a=25\times 10=250m, b=17\times 10=170m,$

$c=12\times 10=120m.$

Now, $s=\frac{540}{2}=270m$

$\therefore$ Area of triangle

$= \sqrt{270(270-250)(270-170)(270-120)}$

$= \sqrt{270\times 20\times 100\times 150}\, m^2 = 9000 m^2.$

4. (c) : Perimeter = 32 cm

Let one of the equal sides be $3x$ and other be $2x$

$\therefore\ 3x+3x+2x=32$

$\Rightarrow\ 8x=32 \Rightarrow x=4$

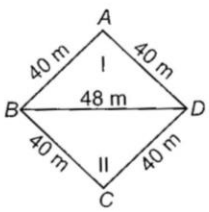
Sides of isosceles triangles are

12 cm, 12cm, 8 cm

$\therefore\ s=\dfrac{32}{2}=16 cm$

Area $= \sqrt{16\times 4\times 4\times 8} = 32\sqrt{2}\, cm^2$

5. (a) : Here, each side of rhombus = 40 cm. One of the diagonal = 48 m



a = 40, b = 40, c = 48

$s=\dfrac{a+b+c}{2}=\dfrac{40+40+48}{2}=\dfrac{128}{2}=64 m$

Area of triangle I

$=\sqrt{64(64-40)(64-40)(64-48)}$

$=\sqrt{64(24)(24)(16)}$

$=768 m^2$

Similarly, area of

triangle $|| = 768 m^2$

So,

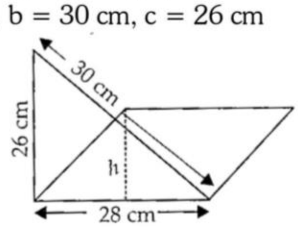
So, area of rhombus

$=768 m^2 + 768 m^2$

$=15.36 m^2$

6. (c) : For the given triangle, we have

a = 28 cm, b = 30 cm, c = 26 cm



So, $s=\dfrac{a+b+c}{2}=\dfrac{28+30+26}{2}$

$s=\dfrac{a+b+c}{2}=\dfrac{28+30+26}{2}$

$=\dfrac{84}{2}=42 cm$

Area of the triangle

$=\sqrt{42(42-28)(42-30)(42-26)}\, cm^2$

$=\sqrt{42\times 14\times 12\times 16}\, cm^2$

$=\sqrt{112896}\, cm^2 = 336 cm^2$

Area of the parallelogram

= Area of the triangle

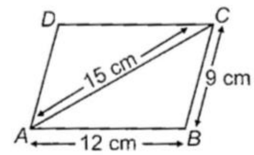
$\therefore$ Area of the parallelogram $= 336 cm^2$

$\Rightarrow\ base\times height = 336 \Rightarrow 28\times h = 336$

$\Rightarrow\ h=\dfrac{336}{28} cm = 12 cm$

Thus, the height of the parallelogram = 12 cm

7. (c) : In ΔABC,



$a = 9 cm, b = 15 cm,$

$c = 12 cm$

$s=\dfrac{a+b+c}{2}$

$=\dfrac{12+9+15}{2}=\dfrac{36}{2}=18$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{18(18-12)(18-9)(18-15)}$

$=\sqrt{18\times 6\times 9\times 3} = 54 cm^2$

Area of parallelogram ABCD

$= 2(\text{Area of } \Delta ABC)$

$= 2\times 54 = 108 cm^2 = k\, cm^2$ (given)

$\Rightarrow\ k = 108$

$\therefore$ The value of $\dfrac{k-100}{4}=\dfrac{108-100}{4}=2$

8. (c) : In $\Delta ABCD$ let a = 12cm, b = 17 cm and c = 25 cm.

$\therefore$ Semi-perimeter of $\Delta ABCD$.

$S=\left(\dfrac{12+17+25}{2}\right)cm=\dfrac{54}{2}cm=27 cm$

$\therefore$ Area of ΔBCD

$=\sqrt{27(27-12)(27-17)(27-25)}\, cm^2$

$=\sqrt{27\times 15\times 10\times 2}\, cm^2 = 90 cm^2$

Now, area of parallelogram ABCD

$= 2\times$ Area of ΔBCD

$= (2\times 90) cm^2 = 180 cm^2$ ...(i)

Let altitude of parallelogram ABCD from vertex A be h cm.

Also, area of parallelogram = Base × Altitude

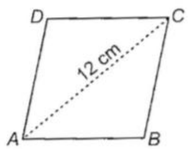
$\Rightarrow\ 180 = DC\times h$ [From (i)]

$\Rightarrow\ 180 = 12\times h$

$\therefore \quad h = \frac{180}{12} = 15$

Required length of the altitude is 15 cm.

9. (c) : Let ABCD be a rhombus having sides A = BC = CD = DA = $x\,cm$



Perimeter of rhombus

$= 40\,cm$ [Given]

$\Rightarrow x + x + x + x = 40$

$\Rightarrow 4x = 40$

$\Rightarrow x = 10$

In ΔABC, let $a = 10\,cm, b = 12\,cm$ and $c = 10\,cm$

Now, semi-perimeter of $\Delta ABC, s = \frac{a+b+c}{2}$

$= \left(\frac{10+10+12}{2}\right)cm = \frac{32}{2}cm = 16\,cm$

$\therefore$ Area of ΔABC

$= \sqrt{16(16-10)(16-10)(16-12)}\,cm^2$

$= \sqrt{16\times6\times6\times4}\,cm^2 = 48\,cm^2$

$= \sqrt{16\times6\times6\times4}\,cm^2 = 48\,cm^2$

Now, area of the rhombus ABCD

$= 2(\text{Area of }\Delta ABC) = (2\times48)\,cm^2 = 96\,cm^2$

$\because$ Cost of painting the sheet of area $1\,cm^2$ = ₹ 5

$\therefore$ Cost of painting the sheet of area $96\,cm^2$ = ₹ (96×5) = ₹ 480

Thus, the cost of painting the sheet on both sides = ₹ (2×480) = ₹ 960

10. (a) : Here we have perimeter of the triangle = 32 cm

Let a = 8 cm and b = 11 cm

Third side, $c = 32-(8+11) = 13\,cm$

$\therefore \quad s = \frac{32}{2} = 16\,cm$

Therefore, area of the triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16\times8\times5\times3}\,cm^2$

$= 80\sqrt{30}\,cm^2 = k\sqrt{30}\,cm^2$

$\therefore k\sqrt{30} = 8\sqrt{30} \Rightarrow k = 8$

11. (a) : In ΔABC, a = 7 cm, b = 6.5 cm and c = 7.5 cm

$\therefore \; s = \left(\frac{7.5+7+6.5}{2}\right)cm = \frac{21}{2}cm = 10.5\,cm$

$\therefore$ Area of ΔABC

$= \sqrt{10.5(10.5-7.5)(10.5-7)(10.5-6.5)}\,cm^2$

$= \sqrt{10.5\times3\times3.5\times4}\,cm^2 = \sqrt{441}\,cm^2$

$= 21\,cm^2$

Since, Area of ΔABC

= Area of parallelogram BCED

$\therefore 21 = BC \times DF$

$\Rightarrow 21 = 7\times DF$

$\Rightarrow DF = \frac{21}{7} = 3\,cm$

12. (c) : We have, sides of triangle 11 cm, 15 cm and 16 cm.

$s = \frac{11+15+16}{2} = 21$

$\therefore$ Area of triangle

$= \sqrt{21(21-11)(21-15)(21-16)}$

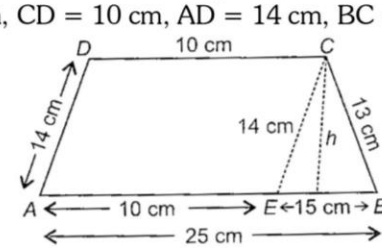
$= 30\sqrt{7}\,cm^2$

Let altitude to the largest side be h cm

$\therefore \quad \frac{1}{2}\times16\times h = 30\sqrt{7} \Rightarrow 8h = 30\sqrt{7}$

$\Rightarrow \quad h = \frac{15\sqrt{7}}{4}\,cm$

13. (a) : Let ABCD be the trapezium with sides AB = 25 cm, CD = 10 cm, AD = 14 cm, BC = 14 cm.



We draw $CE \parallel AD$

$\therefore$ Area of trapezium ABCD

= area of parallelogram AECD + area of ΔECB

Now, In ΔECB

$s = \frac{14+13+15}{2} = 21$

$\therefore$ Area of ΔECB

$= \sqrt{21(21-14)(21-13)(21-15)}$

$= 84\,cm^2$

Also, Area of $\Delta EBC = \frac{1}{2}\times BE\times h$

$\Rightarrow \frac{1}{2}\times15\times h = 84$

$\therefore h = 11.2cm$

$\therefore$ Area of parallelogram AECD = $AE \times h$

$= 10 \times 11.2$

$= 112 cm^2$

Hence, Area of trapezium ABCD

$= (112 + 84)\text{cm}^2 = 196 cm^2$

14. (c) : Let sides of triangular field be a = 50 m, b = 65 m and c = 65 m

Semi-perimeter of triangular field. $s = \frac{a+b+c}{2}$

$= \left(\frac{50+65+65}{2}\right)m = \frac{180}{2}m = 90m$

Area of triangular field

$= \sqrt{90(90-50)(90-65)(90-65)}\, m^2$

$= \sqrt{90 \times 40 \times 25}\, m^2 = 1500 m^2$

$\because$ Cost of laying grass in $1m^2$ area = ₹ 7

$\therefore$ Cost of laying grass in $1500m^2$ area

= ₹ $(7 \times 1500) =$ ₹ 10500

15. (a) : Let the other two equal sides of an isosceles triangle be a cm.

Then, $s = \frac{a+a+24}{2} = (a+12)cm$

Area of triangle $= 192 cm^2 = 192 cm^2$

$\Rightarrow \sqrt{(a+12)(a+12-a)(a+12-a)(a+12-24)} = 192$

$\Rightarrow 144(a^2 - 144) = (192)^2$

$\Rightarrow a^2 = 400 \Rightarrow a = 20cm$

$\therefore$ Perimeter = 20 + 20 + 24 = 64 cm

16. (a) : (I) $s = \frac{10+10+10}{2} = 15cm$

$\therefore$ Area of $I = \sqrt{15 \times 5 \times 5 \times 5} = 25\sqrt{3}\, cm^2$

$= 43.3 cm^2$

(II) $s = \frac{10+9+3}{2} = 11cm$

$\therefore$ Area of $II = 2\sqrt{11 \times 1 \times 2 \times 8} = 8\sqrt{11}\, cm^2$

$= 26.48 cm^2$

(IV) $s = \frac{20+20+4}{2} = 22cm$

$\therefore$ Area of $IV = 2\sqrt{22 \times 2 \times 2 \times 18}$

$= 24\sqrt{11}cm^2 = 79.44 cm^2$

$\therefore$ Total area of coloured paper used

$= (43.3 + 26.48 + 79.44)cm^2 = 149.22 cm^2$

Cost of coloured paper used $= \frac{10}{100} \times 149.22$

= ₹ 14.92

17. (a) : We have, a = 40 cm, b = 40 cm and c = 18 cm

$s = \frac{40+40+18}{2} = 49cm$

$\therefore$ Area of one triangular piece

$= \sqrt{49 \times 9 \times 9 \times 31} = 350.76 cm^2$

$\therefore$ Area of 6 triangular piece 350.76×6

$= 21.04.56 cm^2$

Similarly, area of another 6 triangular piece $= 21.04.56 cm^2$

18. (b) : $s = \frac{25+25+14}{2} = 32cm$

$\therefore$ Area of 1 triangular piece

$= \sqrt{32 \times 7 \times 7 \times 18} = 168 cm^2$

$\therefore$ Total area of paper needed to make the hand $fan = (168 \times 10)cm^2 = 1680 cm^2$

19. (d) : Since. All the sides are equal in an equilateral triangle.

So, perimeter = a + a + a, where a is the side of equilateral triangle.

$\Rightarrow 3a = 36 \Rightarrow a = 12cm$

Area $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(12)^2 = \frac{\sqrt{3}}{4} \times 44$

$= 36\sqrt{3}\, cm^2$

20. (a) : Area of I & II part

$= 2 \times \frac{\sqrt{3}}{4} \times (2)^2 = 2\sqrt{3}\, cm^2 = 3.46 cm^2$

Since, $s = \frac{6+11+15}{2} = 16cm$

[For III & IV part]

$\therefore$ Area of III & IV part $= 2 \times \sqrt{16 \times 10 \times 15 \times 1}$

$= 56.4 cm^2$

Hence, total area of the mask

$= (3.46 + 56.4)cm^2 = 59.86 cm^2$

21. (a) :

22. (c) : Area of rectangular tile

$= (50 \times 70)\text{cm}^2 = 3500 cm^2$

We have,

$a = 25cm, b = 17cm$ and $c = 26cm$

$\therefore s = \frac{a+b+c}{2} = \left(\frac{25+17+6}{2}\right)cm$

= 34 cm

$\therefore$ Area of 1 triangular tile