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MATHEMATICS
LINEAR INEQUALITIES

BASIC CONCEPTS

Inequality

The real numbers or two algebraic expressions related by the symbol '<', '>', '≤' or '≥' form an inequality.

Linear Inequation in one Variable

Let a be a non-zero real number and x be a variable. Then inequations of the form $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$ and $ax + b \geq 0$ are known as linear inequation in one variable x.

Linear Inequations in two Variables

Let a, b be a non-zero real numbers and x, y be variables then inequations of the form $ax + by < c$, $ax + by \leq c$, $ax + by > c$ and $ax + by \geq c$ are known as linear inequations in two variables x and y.

Quadratic Inequation

Let a be a non-zero real number. Then an equations of the form $ax^2 + bx + c < 0$, $ax^2 + bx + c \leq 0$, $ax^2 + bx + c > 0$ and $ax^2 + bx + c \geq 0$ are known as a quadratic inequation.

Solution of an Inequation

A solution of an inequation is the value(s) of the variable(s) that makes it true statement.

Solving an Inequation

The process of obtaining all possible solutions of an inequation.

Solution Set

The set of all possible solutions of an equation is known as its solution set.

Note

In order to solve linear inequations, we follow the following rules:

(*i*) Equal numbers may be added to (or subtracted from) both sides of an equation without changing the sign of inequality.

(*ii*) Both sides of an equation may be multiplied (or divided)by same non-zero number. However sign of inequation are multiplied or divided by a non-negative number.

(*iii*) Any term of an inequation may be taken to other side with its sign changed without affecting the sign of inequality.

We follow the following algorithm to solve a linear equation in one variable:

Step I: Write all terms involving the variable on one side of the inequation and constant term on the other side.

Step II: Simplify both sides of inequality in their simplest form to reduce the inequation in the form

$$ax < b \text{ or } ax \leq b \text{ or } ax > b \text{ or } ax \geq b$$

Step III: Solve the inequation obtained in step II by dividing both sides of the inequation by the coefficient of the variable.

Step IV: Write the solution set obtained in step III in the form of an interval on real line.

Equation of the Form

$$\frac{ax+b}{cx+d} > k \text{ or } \frac{ax+b}{cx+d} \geq k \text{ or } \frac{ax+b}{cx+d} < k \text{ or } \frac{ax+b}{cx+d} \leq k$$

In order to solve this type of inequation we use the following algorithm:

Step I: Write the inequation.

Step II: Bring all terms in LHS.

Step III: Simplify LHS of the inequation obtained in step II to obtain the inequality of the form

$$\frac{px+q}{rx+s} > 0 \text{ or } \frac{px+q}{rx+s} \geq 0 \text{ or } \frac{px+q}{rx+s} < 0 \quad \text{or} \quad \frac{px+q}{rx+s} \leq 0$$

Step IV: Make the coefficient of x positive in numerator and denominator if they are not.

Step V: Equate numerator and denominator separately to zero and obtain the values of x. These values of x are generally called critical points.

Step VI: Plot the critical points obtained in step V on real line. These points will divide the real line in three regions.

Step VII: In the right most region the expression on LHS of the inequation obtained in step IV will be positive and in other regions it will be alternatively negative and positive. So mark positive sign in the right most region and then mark alternatively negative and positive signs in other regions.

Step VIII: Select appropriate regions on the basis of the sign of inequation obtained in step IV.

Write these regions in the form of intervals to obtain the desired solution sets of the given inequation.

Solution of System of Linear Inequations in One Variable

Step I: Obtain the system of linear inequations.

Step II: Solve each inequation and obtain their solution on sets. Also, represent them on real line.

Step III: Find the intersection of the solution sets obtained in step II by taking the help of the graphical representation of the solution sets in the step II.

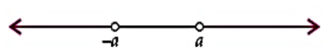
Step IV: The set obtained in step III is the required solution set of the given system of equations.

Some Important Results

Result 1

If a is a positive real number, then

(*i*) $|x| < a \;\Rightarrow\; -a < x < a$, *i.e.*, $x \in (-a, a)$

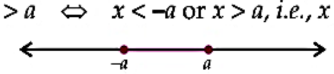


(*ii*) $|x| \leq a \;\Rightarrow\; -a \leq x \leq a$, *i.e.*, $x \in [-a, a]$

Result 2

If a is a real number, then

(*i*) $|x| > a \;\Leftrightarrow\; x < -a$ or $x > a$, *i.e.*, $x \in (-\infty, -a) \cup (a, \infty)$



(*ii*) $|x| \geq a \;\Leftrightarrow\; x \leq -a$ or $x \geq a$, *i.e.*, $x \in (-\infty, -a] \cup [a, \infty)$

Result 3

Let r be a positive real number and a be a fixed real number, then

(*i*) $|x-a| < r \Leftrightarrow a-r < x < a+r$; *i.e.*, $x \in (a-r, a+r)$

(*ii*) $|x-a| \le r \Leftrightarrow a-r \le x \le a+r$, *i.e.*, $x \in [a-r, a+r]$

(*iii*) $|x-a| > r \Leftrightarrow x < a-r$ or $x > a+r$, *i.e.*, $x \in (-\infty, a-r) \cup (a+r, \infty)$

(*iv*) $|x-a| \ge r \Leftrightarrow x \le a-r$ or $x \ge a+r$, *i.e.*, $x \in (-\infty, a-r] \cup [a+r, \infty)$

Result 4

Let a, b be positive real numbers, then

(*i*) $a < |x| < b \Leftrightarrow x \in (-b, -a) \cup (a, b)$

(*ii*) $a \le |x| \le b \Leftrightarrow x \in [-b, -a] \cup [a, b]$

(*iii*) $a < |x-c| < b \Leftrightarrow x \in (-b+c, -a+c) \cup (a+c, b+c)$

(*iv*) $a \le |x-c| \le b \Leftrightarrow x \in [-b+c, -a+c] \cup [a+c, b+c]$

Graphical Solution of Linear Inequations in Two Variables

In order to find the solution set of a linear inequation in two variables, we follow the following algorithm:

Algorithm

Step I: Convert the given inequation, say $ax + by \le c$ into the equation $ax + by = c$ which represents a straight line in xy-plane.

Step II: Put $y = 0$ in the equation obtained in step I to get the point where the line meets with the x-axis. Similarly put $x = 0$ to obtain a point where the line meets with y-axis.

Step III: Join the points in step II to obtain the graph of the line obtained from given inequation. In case strict inequality *i.e.* $ax + by < c$ or $ax + by > c$ draw a dotted line.

Step IV: Choose a point if possible (0, 0) not lying on this line; substitute its co-ordinates in the inequation. If the equation is satisfied then shade the portion of plane which contains the chosen point; otherwise shade the portion which does not contain the chosen point.

Step V: The shaded region obtained in step IV represent the desired solution set.

Remark

In case of inequalities $ax + by \le c$ and $ax + by \ge c$ the points on the line are also part of the shaded region while in case of inequalities $ax + by < c$ or $ax + by > c$ points on the line $ax + by = c$ are not in the shaded region.

SELECTED NCERT QUESTIONS

1. Solve $24x < 100$ when

(*i*) x is a natural number; **(*ii*) x is an integer.**

Sol. Here $24x < 100$

Dividing both sides by 24, we have

$$x < \frac{100}{24} \Rightarrow x < \frac{25}{6}$$

(*i*) When x is a natural number then values of x that make the statement true are 1, 2, 3, 4. The solution set of inequality is {1, 2, 3, 4}.

(*ii*) When x is an integer then values of x that make the statement true are …, –2, –1, 0, 1, 2, 3, 4.
The solution set of inequality is {…, –2, –1, 0, 1, 2, 3, 4}.

2. Solve $-12x > 30$ when

(*i*) x is a natural number (*ii*) x is an integer

Sol. Here $-12x > 30$

Dividing both sides by –12, we have

$$\frac{-12x}{-12} < \frac{30}{-12} \Rightarrow x < \frac{-5}{2}$$

(*i*) When x is a natural number then no values of x make the statement true. So solution set = ϕ

(*ii*) When x is an integer then values of x, that make the statement true are …, –5, –4, –3. The solution set of inequality is {…, –5, –4, –3}.

Solve the Inequalities in questions 3 to 6 for real x:

3. $3(x-1) \leq 2(x-3)$

Sol. Here $3(x-1) \leq 2(x-3)$

$\Rightarrow 3x - 3 \leq 2x - 6 \Rightarrow 3x - 2x \leq -6 + 3 \Rightarrow x \leq -3$

Thus, the solution set is $(-\infty, -3]$.

4. $x + \frac{x}{2} + \frac{x}{3} < 11$

Sol. Here $x + \frac{x}{2} + \frac{x}{3} < 11$

$$\Rightarrow \frac{6x + 3x + 2x}{6} < 11 \Rightarrow \frac{11x}{6} < 11$$

Multiplying both sides by 6, we have

$11x < 66$

Dividing both sides by 11, we have

$x < 6$

Thus, the solution set is $(-\infty, 6)$.

5. $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

Sol. Here $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3} \Rightarrow \frac{3x-6}{5} \leq \frac{10-5x}{3}$

$$\Rightarrow \frac{3x}{5} - \frac{6}{5} \leq \frac{10}{3} - \frac{5x}{3} \Rightarrow \frac{3x}{5} + \frac{5x}{3} \leq \frac{10}{3} + \frac{6}{5}$$

$$\Rightarrow \frac{9x + 25x}{15} \leq \frac{50 + 18}{15} \Rightarrow \frac{34x}{15} \leq \frac{68}{15}$$

Multiplying both sides by 15, we have

$34x \leq 68$

Dividing both sides by 34, we have

$x \leq 2$

Thus, the solution set is $(-\infty, 2]$.

6. $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

Sol. Here $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

$\Rightarrow \frac{2x}{3} - \frac{1}{3} \geq \frac{3x}{4} - \frac{2}{4} - \frac{2}{5} + \frac{x}{5} \quad \Rightarrow \frac{2x}{3} - \frac{3x}{4} - \frac{x}{5} \geq \frac{-2}{4} - \frac{2}{5} + \frac{1}{3}$

$\Rightarrow \frac{40x - 45x - 12x}{60} \geq \frac{-30 - 24 + 20}{60} \quad \Rightarrow \frac{-17x}{60} \geq \frac{-34}{60}$

Multiplying both sides by 60, we have $-17x \geq -34$

Dividing both sides by –17, we have

$\frac{-17x}{-17} \leq \frac{-34}{-17} \quad \Rightarrow \quad x \leq 2$

Thus, the solution set is $(-\infty, 2]$.

Solve the Inequalities in questions 7 to 8 and show the graph of the solution in each case on number line.

7. $3(1-x) < 2(x+4)$

Sol. Here $3(1-x) < 2(x+4)$

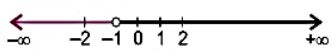
$\Rightarrow 3 - 3x < 2x + 8 \Rightarrow -3x - 2x < 8 - 3 \Rightarrow -5x < 5$

Dividing both sides by –5, we have

$x > -1$

The solution set is $(-1, \infty)$.

The representation of the solution set on the line is



8. $\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

Sol. Here $\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

$\Rightarrow \frac{x}{2} \geq \frac{5x}{3} - \frac{2}{3} - \frac{7x}{5} + \frac{3}{5} \Rightarrow \frac{x}{2} - \frac{5x}{3} + \frac{7x}{5} \geq \frac{-2}{3} + \frac{3}{5}$

$\Rightarrow \frac{15x - 50x + 42x}{30} \geq \frac{-10 + 9}{15} \Rightarrow \frac{7x}{30} \geq \frac{-1}{15}$

Multiplying both sides by 30, we have

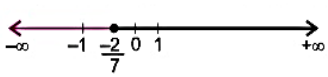
$7x \geq -2$

Dividing both sides by 7, we have

$x \geq \frac{-2}{7}$

The solution set is $\left[\frac{-2}{7}, \infty\right)$.

The representation of the solution set on the number line is



9. Solve: $-5 \le \dfrac{5-3x}{2} \le 8$

Sol. We have $-5 \le \dfrac{5-3x}{2} \le 8$

or $-10 \le 5 - 3x \le 16$ or $-15 \le -3x \le 11$

or $5 \ge x \ge -\dfrac{11}{3}$

which can be written as $\dfrac{-11}{3} \le x \le 5$.

10. Solve the system of inequalities:

$3x - 7 < 5 + x$ ...(*i*)

$11 - 5x \le 1$ ...(*ii*)

and represent the solution on the number line.

Sol. From inequality (*i*), we have

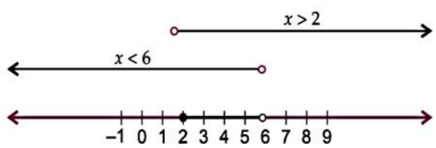
$3x - 7 < 5 + x \Rightarrow 3x - x < 5 + 7 \Rightarrow 2x < 12$

or $x < 6$ ...(*iii*)

Also, from inequality (*ii*), we have

$11 - 5x \le 1$ or $-5x \le -10$ *i.e.*, $x \ge 2$ ...(*iv*)

If we draw the graph of inequalities (*iii*) and (*iv*) on the number line, we see that the values of x, which are common to both, are shown by bold line in Fig.



Thus, solution of the system are real numbers x lying between 2 and 6 including 2,

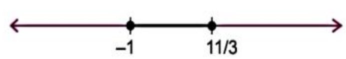
i.e., $2 \le x < 6$.

11. Solve: $7 \le \dfrac{(3x+11)}{2} \le 11$

Sol. We have, $7 \le \dfrac{(3x+11)}{2} \le 11$

$\Rightarrow \quad 14 \le 3x + 11 \le 22 \quad \Rightarrow \quad 3 \le 3x \le 11$

$\Rightarrow \quad 1 \le x \le \dfrac{11}{3}$



12. Solve the system of inequalities:

$5(2x - 7) - 3(2x + 3) \le 0,\ 2x + 19 \le 6x + 47$

Sol. We have

$5(2x - 7) - 3(2x + 3) \le 0$ and $2x + 19 \le 6x + 47$

$\Rightarrow \quad 10x - 35 - 6x - 9 \le 0$ and $-4x \le 28$

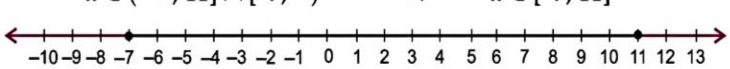
$\Rightarrow \quad 4x - 44 \le 0$ and $x \ge -7$

$\Rightarrow \quad 4x \le 44$ and $x \ge -7$

$\Rightarrow \quad x \le 11$ and $x \ge -7$

$\Rightarrow \quad x \in (-\infty, 11]$ and $x \in [-7, \infty)$

$\Rightarrow \quad x \in (-\infty, 11] \cap [-7, \infty) \quad \Rightarrow \quad x \in [-7, 11]$



13. Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Sol. Let x and $x + 2$ be two consecutive odd positive integers.

Then $\quad x + 2 < 10$ and $x + x + 2 > 11$

$\Rightarrow \quad x < 8$ and $2x + 2 > 11 \quad \Rightarrow \quad x < 8$ and $2x > 11 - 2$

$\Rightarrow \quad x < 8$ and $2x > 9$

$\Rightarrow \quad x < 8$ and $x > \frac{9}{2} \quad \Rightarrow \quad \frac{9}{2} < x < 8$

$\Rightarrow \quad x = 5$ and 7

Thus required pairs of odd positive integers are (5, 7) and (7, 9).

14. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Sol. Let x litres of water be added to 1125 litres of 45% acid solution.

Then total quantity of mixture = $(1125 + x)$ litres

Percentage of acid content = $1125 \times \frac{45}{100} \times \frac{100}{(1125 + x)} = \frac{2025 \times 100}{4(1125 + x)}$

It is given that the resulting mixture must be more than 25% but less than 30% acid content.

$$\frac{25}{100} \times 100 \le \frac{2025 \times 100}{4(1125 + x)} \le \frac{30}{100} \times 100$$

$\Rightarrow \quad 25 \le \frac{50625}{1125 + x} \le 30$

$\Rightarrow \quad 25 \le \frac{50625}{1125 + x}$ and $\frac{50625}{1125 + x} \le 30$

$\Rightarrow \quad 28125 + 25x \le 50625$ and $50625 \le 33750 + 30x$

$\Rightarrow \quad 25x \le 22500$ and $30x \ge 16875$

$\Rightarrow \quad x \le 900$ and $x \ge 562.5$

$\Rightarrow \quad 562.5 \le x \le 900$

Thus minimum 562.5 litres and maximum 900 litres of water need to be added.

15. Solve the inequality $y + 8 \ge 2x$ graphically in two dimensional plane.

Sol. The given inequality is

$y + 8 \ge 2x$ *i.e.*, $2x - y \le 8$

Draw the graph of the line $2x - y = 8$

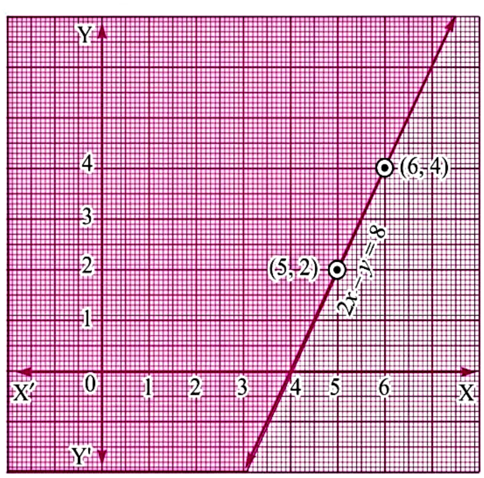
Table of values satisfying the equation $2x - y = 8$

x	5	6
y	2	4

Putting (0, 0) in the given inequality,

we have $2 \times 10 - 0 \le 8 \Rightarrow 0 \le 8$, which is true.

$\therefore$ Half plane of $2x - y \le 8$ is towards origin.



Solve the following system of inequalities graphically Q(16-18):

16. $x + y \le 9,\ y > x,\ x \ge 0$

Sol. The given inequality is $x + y \le 9$. Draw the graph of the line $x + y = 9$.

Putting (0, 0) in the given inequality, we have

$0 + 0 \le 9 \quad \Rightarrow \quad 0 \le 9$, which is true.

$\therefore$ Half plane of $x + y \le 9$ is towards origin.

Also the given inequality is $x - y < 0$.

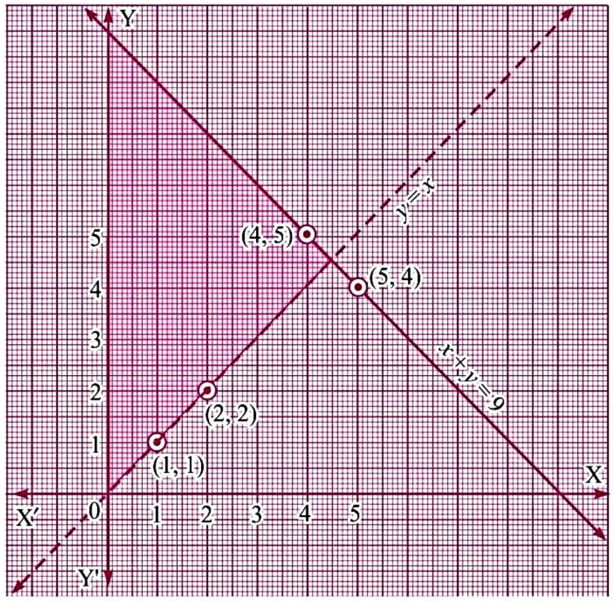
Draw the graph of the line $x - y = 0$.

Putting (0, 3) in the given inequality, we have

$0 - 3 < 0 \quad \Rightarrow -3 < 0$, which is true.

$\therefore$ Half plane of $x - y < 0$ contain the point (0, 3).

$\therefore$ Solution set is shaded region.



17. $4x + 3y \le 60,\ y \ge 2x,\ x \ge 3,\ y \ge 0$

Sol. The given inequality is $4x + 3y \ge 60$. Draw the graph of the line $4x + 3y = 60$.

Putting (0, 0) in the given inequality, we have

$4 \times 0 + 3 \times 0 \le 60, \quad \Rightarrow \quad 0 \le 60$, which is true.

$\therefore$ Half plane of $4x + 3y \le 60$ is towards origin.

Also the given inequality is $2x - y \le 0$.

Draw the graph of the line $2x - y = 0$.

Putting (10, 0) in the given inequality, we have

$2 \times 10 - 0 \le 0 \quad \Rightarrow \quad x \le 0$ which is false.

$\therefore$ Half plane of $2x - y \le 0$ does not contain (10, 0).

The given inequality is $x \ge 3$.

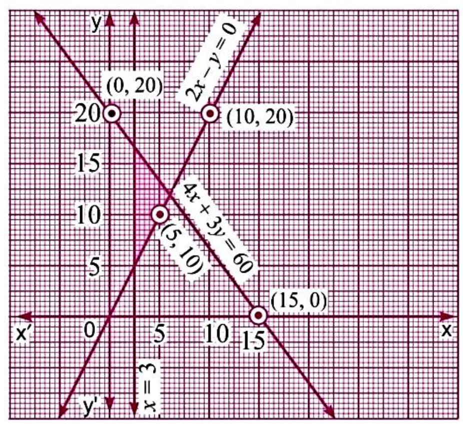
Draw the graph of the line $x = 3$.

Putting (0, 0) in the given inequality, we have

$0 \ge 3$, which is false.

$\therefore$ Half plane of $4x + 3y \le 60$ is away from origin.

$\therefore$ Solution set is shaded region.



18. $\dfrac{1}{2}\left(\dfrac{3x}{5} + 4\right) \ge \dfrac{1}{3}(x - 6)$

Sol. Here $\quad \dfrac{1}{2}\left(\dfrac{3x}{5} + 4\right) \ge \dfrac{1}{3}(x - 6)$

$\Rightarrow \quad \dfrac{3x}{10} + 2 \ge \dfrac{x}{3} - 2 \qquad \Rightarrow \quad \dfrac{3x}{10} - \dfrac{x}{3} \ge -2 - 2$

$\Rightarrow \quad \dfrac{9x - 10x}{30} \ge -4 \qquad \Rightarrow \quad \dfrac{-x}{30} \ge -4$

Multiplying both sides by 30, we have

$-x \ge -120$

Dividing both sides by –1, we have

$$x \le 120$$

Thus, the solution set is $(-\infty, 120]$

19. Solve: $3x + 8 > 2$ when

(*i*) x is an integer **(*ii*) x is a real number**

Sol. Here $3x + 8 > 2$

$$3x > 2 - 8 \Rightarrow 3x > -6$$

Dividing both sides by 3, we have

$$x > -2$$

(*i*) When x is an integer then values of x that make the statement true are –1, 0, 1, 2, 3, ...
The solution set of inequality is {–1, 0, 1, 2, 3, ...}.

(*ii*) When x is a real number, the solution set of inequality is $x \in (-2, \infty)$.

20. Ravi obtained 70 and 75 marks in first two unit tests. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

Sol. Let the marks obtained by Ravi in third test be x.

Then average of three tests $= \dfrac{70 + 75 + x}{3}$.

Now, $\dfrac{70 + 75 + x}{3} \ge 60 \Rightarrow \dfrac{145 + x}{3} \ge 60$

Multiplying both sides by 3, we have

$$145 + x \ge 180$$

$$\Rightarrow x \ge 180 - 145 \Rightarrow x \ge 35$$

Thus, the minimum marks needed to be obtained by Ravi = 35.

21. To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find the minimum marks that Sunita must obtain in fifth examination to get Grade 'A' in the course.

Sol. Let the marks obtained by Sunita in fifth examination be x.

Then average of five examinations $= \dfrac{87 + 92 + 94 + 95 + x}{5} \ge 90 \Rightarrow \dfrac{368 + x}{5} \ge 90$

Multiplying both sides by 5, we have

$$368 + x \ge 450 \Rightarrow x \ge 450 - 368 \Rightarrow x \ge 82$$

Thus, the minimum marks needed to be obtained by Sunita = 82.

22. The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm. Find the minimum length of the shortest side.

Sol. Let the length of the shortest side be x cm.

Then length of longest side = $3x$ cm

Length of third side = $(3x - 2)$ cm.

Perimeter of triangle = $x + 3x + 3x - 2 = (7x - 2)$ cm

Now from question, $7x - 2 \ge 61$

$$\Rightarrow 7x \ge 61 + 2 \Rightarrow 7x \ge 63 \Rightarrow x \ge 9$$

Thus, the minimum length of shortest side = 9 cm.

23. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest side and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5 cm longer than the second?

Sol. Let the length of the shortest board be x cm.

Then length of the second board = $(x + 3)$ cm

Length of the third board $2x$ cm

Now from question, $x + (x + 3) + 2x \leq 91$ and $2x \geq (x + 3) + 5$

$\Rightarrow$ $4x + 3 \leq 91$ and $2x - (x + 3) \geq 5$

$\Rightarrow$ $4x \leq 91 - 3$ and $2x - x - 3 \geq 5$

$\Rightarrow$ $4x \leq 88$ and $x \geq 5 + 3$

$\Rightarrow$ $x \leq 22$ and $x \geq 8$

Thus, minimum length of shortest board is 8 cm and maximum length is 22 cm.

Solve the following inequalities graphically in two dimensional plane Q(24-32):

24. $x + y < 5$

Sol. The given inequality is $x + y < 5$.

Draw the graph of the line $x + y = 5$.

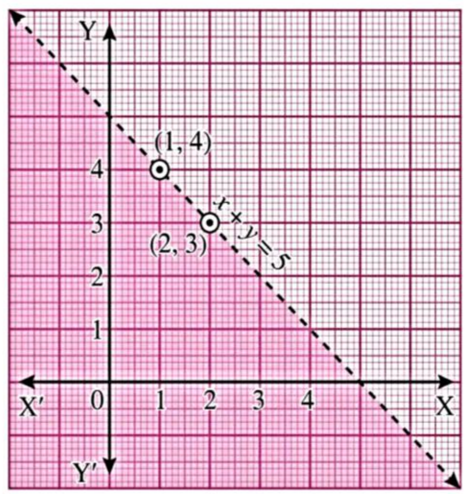
Table of values satisfying the equation $x + y = 5$.

x	1	2
y	4	3

Putting (0, 0) in the given inequality, we have

$0 + 0 < 5 \Rightarrow 0 < 5$ which is true.

$\therefore$ Solution set is half plane of $x + y < 5$ is towards origin.



25. $2x + y \geq 6$

Sol. The given inequality is $2x + y \geq 6$

Draw the graph of line $2x + y = 6$.

Table of values satisfying the equation

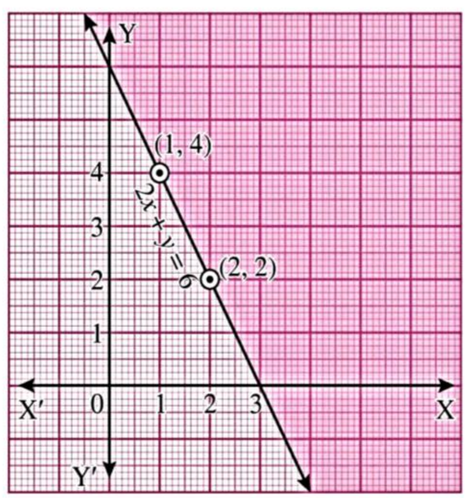
$$2x + y = 6$$

x	1	2
y	4	2

Putting (0, 0) in the given inequality, we have

$2 \times 0 + 0 \geq 6 \Rightarrow 0 \geq 6$ which is false.

$\therefore$ Solution set is half plane of $2x + y \geq 6$ is away from origin.



26. $3x + 4y \le 12$

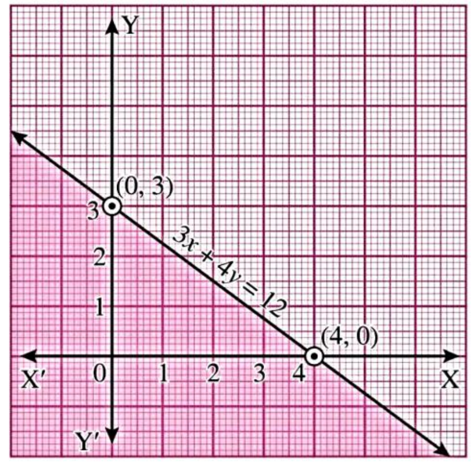
Sol. The given inequality is $3x + 4y \le 12$.

Draw the graph of the line $3x + 4y = 12$.

Table of values satisfying the equation

$3x + 4y = 12$

x	0	4
y	3	0



Putting (0, 0) in the given inequality, we have

$3 \times 0 + 4 \times 0 \le 12 \Rightarrow 0 \le 12$ which is true.

$\therefore$ Solution set is half plane of $3x + 4y \le 12$ is towards origin.

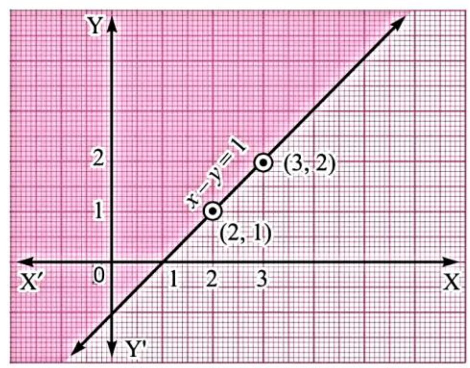
27. $x - y \le 1$

Sol. The given inequality is $x - y \le 1$.

Draw the graph of the line $x - y = 1$.

Table of values satisfying the equation $x - y = 1$.

x	2	3
y	1	2



Putting (0, 0) in the given inequality, we have

$0 - 0 \le 1 \Rightarrow 0 \le 1$ which is true.

$\therefore$ Solution set is half plane of $x - y \le 1$ is towards origin.

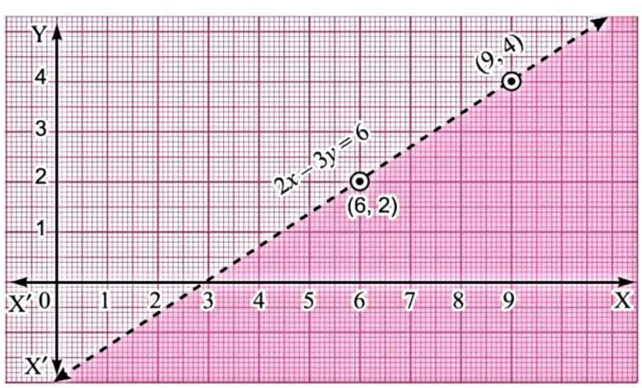
28. $2x - 3y > 6$

Sol. The given inequality is $2x - 3y > 6$.

Draw the graph of line $2x - 3y = 6$.

Table of values satisfying the equation $2x - 3y = 6$.

x	6	9
y	2	4



Putting (0, 0) in the given inequality, we have

$2 \times 0 - 3 \times 0 > 6 \Rightarrow 0 > 6$ which is false.

$\therefore$ Solution set is half plane of $2x - 3y > 6$ is away from origin.

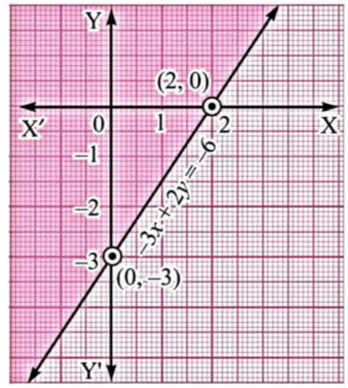
29. $-3x + 2y \geq -6$

Sol. The given inequality is $-3x + 2y \geq -6$.

Draw the graph of the line $-3x + 2y = -6$.

Table of values satisfying the equation $-3x + 2y = -6$.

x	2	0
y	0	–3



Putting (0, 0) in the given inequality, we have

$-3 \times 0 + 2 \times 0 \geq -6 \Rightarrow 0 \geq -6$, which is true.

∴ Solution set is half plane of $-3x + 2y \geq -6$ is towards origin.

30. $3y - 5x < 30$

Sol. The given inequality is $3y - 5x < 30$.

Draw the graph of line $3y - 5x = 30$.

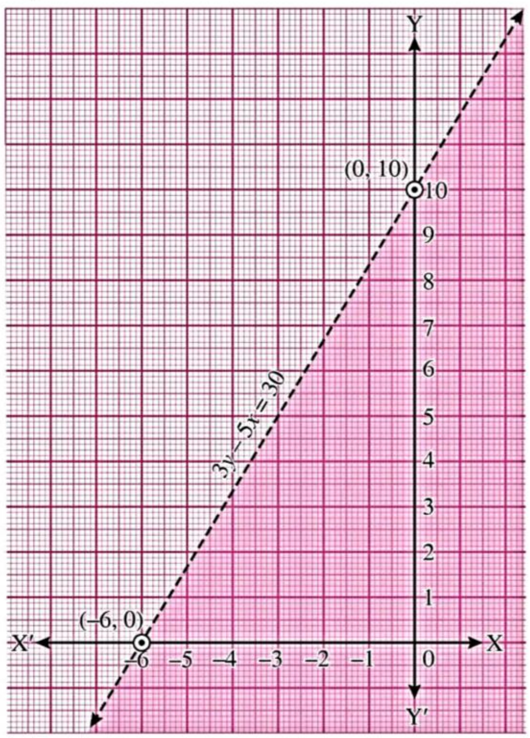


Table of values satisfying the equation $3y - 5x = 30$.

x	–6	0
y	0	10

Putting (0, 0) in the given inequality, we have

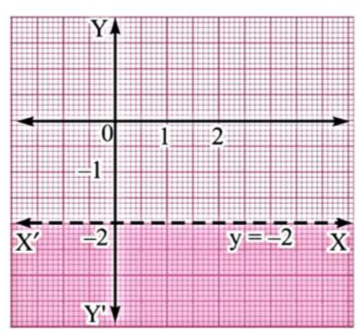
$3 \times 0 - 5 \times 0 < 30 \Rightarrow 0 < 30$ which is true.

∴ Solution set is half plane of $3y - 5x < 0$ is towards origin.

31. $y < -2$

Sol. The given inequality is $y < -2$.

Draw the graph of the line $y = -2$.



Putting (0, 0) in the given inequality, we have

$0 < -2$ which is false.

∴ Solution set is half plane of $y < -2$ is away from origin.

32. $x > -3$

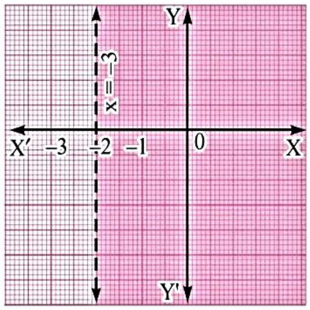
Sol. The given inequality is $x > -3$.

Draw the graph of the line $x = -3$.

Putting (0, 0) in the given inequality, we have

$0 > -3$ which is true.

$\therefore$ Solution set is half plane of $x > -3$ is towards origin.



Solve the following system of inequalities graphically Q(33-45):

33. $x \geq 3, y \geq 2$

Sol. The given inequality is $x \geq 3$.

Draw the graph of the line $x = 3$.

Putting (0, 0) in the given inequality, we have

$0 \geq 3$ which is false.

$\therefore$ Half plane of $x \geq 3$ is away from origin.

Also the given inequality is $y \geq 2$.

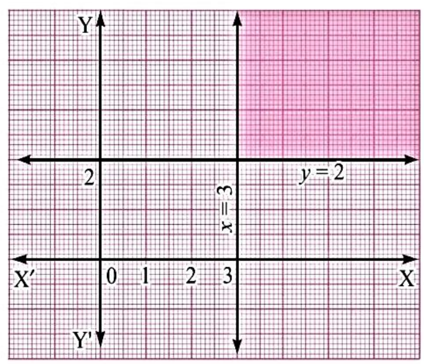
Draw the graph of the line $y = 2$.

Putting (0, 0) in the given inequality, we have

$0 \geq 2$ which is false.

$\therefore$ Half plane of $y \geq 2$ is away from origin.

Hence solution set is shaded region.



34. $3x + 2y \leq 12, x \geq 1, y \geq 2$

Sol. The given inequality is $3x + 2y \leq 12$

Draw the graph of the line $3x + 2y = 12$

Putting (0, 0) in the given inequality, we have

$3 \times 0 + 2 \times 0 \leq 12 \Rightarrow 0 \leq 12$, which is true.

$\therefore$ Half plane of $3x + 2y \leq 12$ is towards origin.

Also the given inequality is $x \geq 1$.

Draw the graph of the line $x = 1$.

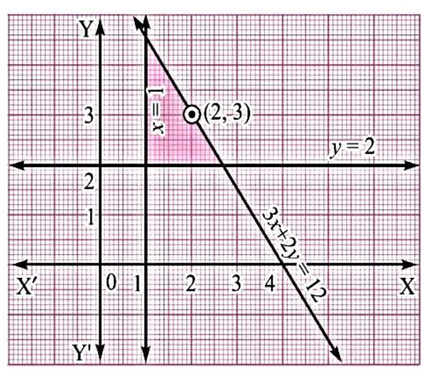
Putting (0, 0) in the given inequality, we have

$0 \geq 1$, which is false.

$\therefore$ Half plane of $x \geq 1$ is away from origin.

The given inequality is $y \geq 2$.

Draw the graph of the line $y = 2$.



Putting (0, 0) in the given inequality, we have

$0 \geq 2$, which is false.

$\therefore$ Half plane of $y \geq 2$ is away from origin.

Hence solution set is shaded region.

35. $\mathbf{2x + y \geq 6, 3x + 4y \leq 12.}$

Sol. The given inequality is $2x + y \geq 6$.

Draw the graph of the line $2x + y = 6$.

Putting (0, 0) in the given inequality, we have

$2 \times 0 + 0 \geq 6 \Rightarrow 0 \geq 6$, which is false.

$\therefore$ Half plane of $2x + y \geq 6$ is away from origin.

Also the given inequality is $3x + 4y \leq 12$.

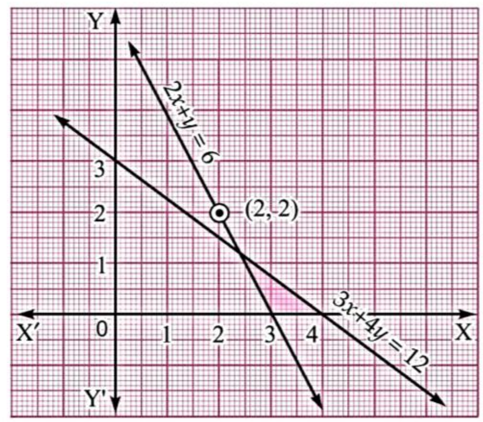
Draw the graph of the line $3x + 4y = 12$.

Putting (0, 0) in the given inequality, we have

$3 \times 0 + 4 \times 0 \leq 12 \Rightarrow 0 \leq 12$, which is true.

$\therefore$ Half plane of $3x + 4y \leq 12$ is towards origin.

Hence solution set is shaded region.



36. $\mathbf{x + y \geq 4, 2x - y > 0.}$

Sol. The given inequality is $x + y \geq 4$.

Draw the graph of the line $x + y = 4$.

Putting (0, 0) in the given inequality, we have

$0 + 0 \geq 4 \Rightarrow 0 \geq 4$, which is false.

$\therefore$ Half plane of $x + y \geq 4$ is away from origin.

Also the given inequality is $2x - y > 0$.

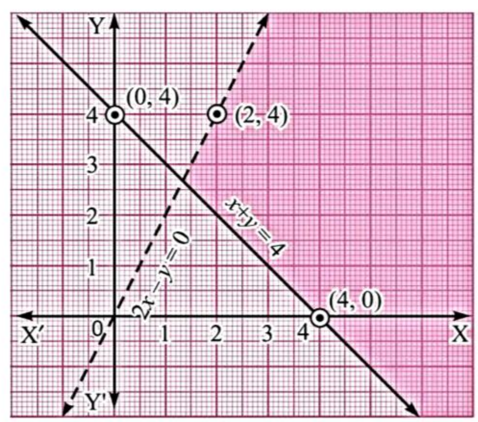
Draw the graph of the line $2x - y = 0$.

Putting (3, 0) in the given inequality, we have

$2 \times 3 - 0 > 0 \Rightarrow 6 > 0$, which is true.

$\therefore$ Half plane of contain (3, 0).

Hence solution set is shaded region.



37. $\mathbf{2x - y > 1, x - 2y < -1.}$

Sol. The given inequality is $2x - y > 1$.

Draw the graph of the line $2x - y = 1$.

Putting (0, 0) in the given inequality, we have

$2 \times 0 - 0 > 1 \Rightarrow 0 > 1$, which is false.

$\therefore$ Half plane of $2x - y > 1$ is away from origin.

Also the given inequality is $x - 2y < -1$.

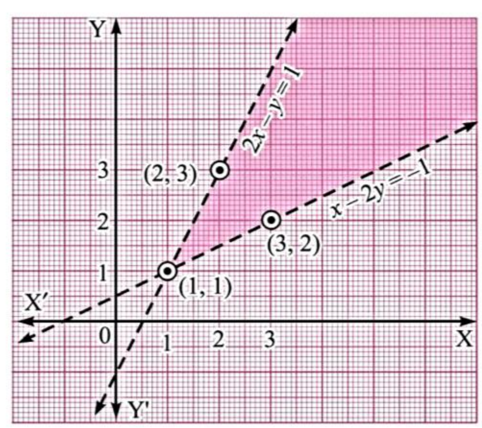
Draw the graph of the line $x - 2y = -1$.

Putting (0, 0) in the given inequality, we have

$0 - 2 \times 0 < -1 \Rightarrow 0 < -1$, which is false.

$\therefore$ Half plane of $x - 2y < -1$ is away from origin.

Hence solution set is shaded region.



38. $x + y \le 6, x + y \ge 4.$

Sol. The given inequality is

$$x + y \le 6$$

Draw the graph of the line $x + y = 6$.

Putting (0, 0) in the given inequality, we have

$0 + 0 \le 6 \Rightarrow 0 \le 6$, which is true.

$\therefore$ Half plane of $x + y \le 6$ is towards origin.

Also the given inequality is $x + y \ge 4$.

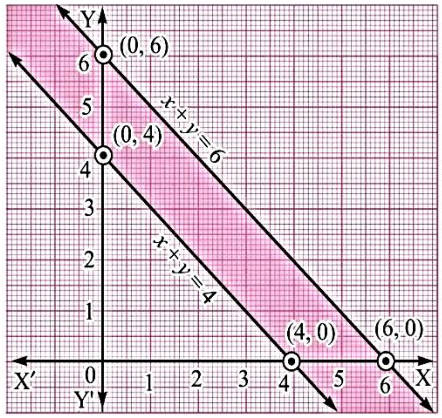
Draw the graph of the line $x + y = 4$.

Putting (0, 0) in the given inequality, we have

$0 + 0 \ge 4 \Rightarrow 0 \ge 4$, which is false.

$\therefore$ Half plane of $x + y \ge 4$ is away from origin.

Hence solution set is shaded region.



39. $2x + y \ge 8, x + 2y \ge 10.$

Sol. The given inequality is $2x + y \ge 8$.

Draw the graph of the line $2x + y = 8$.

Putting (0, 0) in the given inequality, we have

$2 \times 0 + 0 \ge 8 \Rightarrow 0 \ge 8$, which is false

$\therefore$ Half plane of $2x + y \ge 8$ is away from origin.

Also the given inequality is $x + 2y \ge 10$.

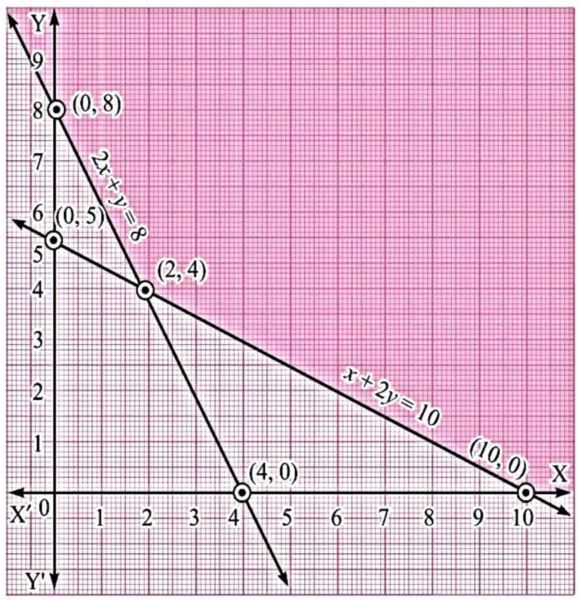
Draw the graph of the line $x + 2y = 10$.

Putting (0, 0) in the given inequality, we have

$0 + 2 \times 0 \ge 10 \Rightarrow 0 \ge 10$, which is false.

$\therefore$ Half plane of $x + 2y \ge 10$ is away from origin.

Hence solution set is shaded region.



40. $5x + 4y \le 20, x \ge 1, y \ge 2.$

Sol. The given inequality is $5x + 4y \le 20$.

Draw the graph of the line $5x + 4y = 20$.

Putting (0, 0) in the given inequality, we have

$5 \times 0 + 4 \times 0 \le 20 \Rightarrow 0 \le 20$, which is true.

$\therefore$ Half plane of $5x + 4y \le 20$ is towards origin.

Also the given inequality is $x \ge 1$.

Draw the graph of the line $x = 1$.

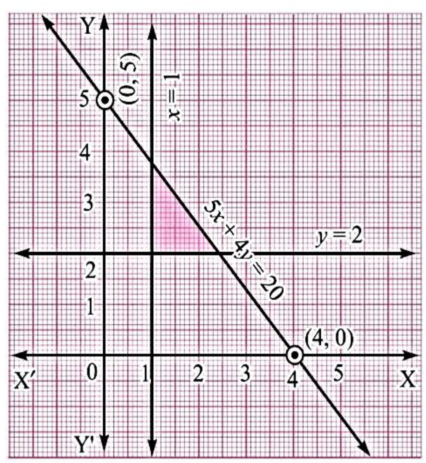
Putting (0, 0) in the given inequality, we have

$0 \ge 1$, which is false.

$\therefore$ Half plane of $x \ge 1$ is away from origin.

The given inequality is $y \ge 2$.

Draw the graph of line $y = 2$.



Putting (0, 0) in the given inequality, we have

$0 \geq 2$, which is false.

$\therefore$ Half plane $y \geq 2$ is away from origin.

Hence solution set is shaded region.

41. $3x + 4y \leq 60, x + 3y \leq 30, x \geq 0, y \geq 0$.

Sol. The given inequality is $3x + 4y \leq 60$.

Draw the graph of the line $3x + 4y = 60$.

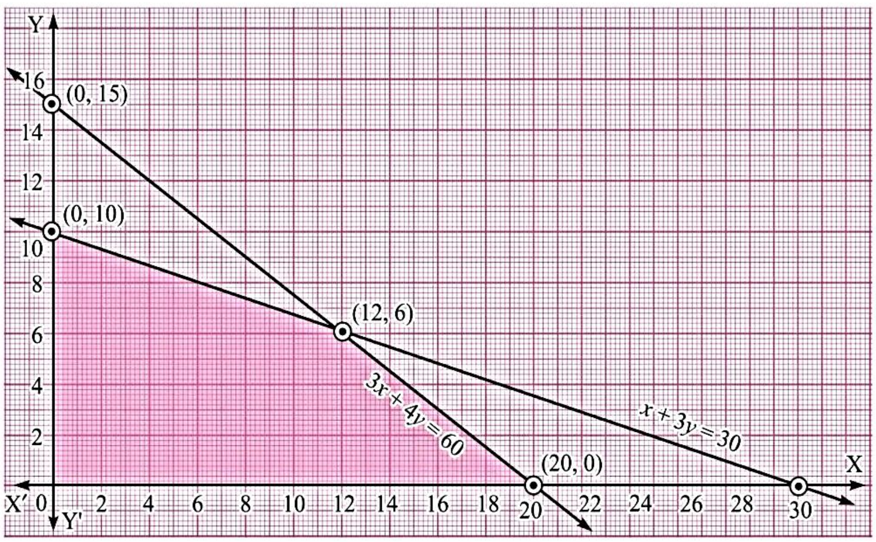
Putting (0, 0) in the given inequality, we have

$3 \times 0 + 4 \times 0 \leq 60 \Rightarrow 0 \leq 60$, which is true.

$\therefore$ Half plane of $3x + 4y \leq 60$ is towards origin.

Also the given inequality is $x + 3y \leq 30$.

Draw the graph of the line $x + 3y = 30$.



Putting (0, 0) in the given inequality, we have

$0 + 3 \times 0 \leq 30 \Rightarrow 0 \leq 30$, which is true.

$\therefore$ Half plane of $x + 3y \leq 30$ is towards origin.

Hence solution set is shaded region.

42. $2x + y \geq 4, x + y \leq 3, 2x - 3y \leq 6$.

Sol. The given inequality is $2x + y \geq 4$.

Draw the graph of the line $2x + y = 4$.

Putting (0, 0) in the given inequality, we have

$2 \times 0 + 0 \geq 0 \Rightarrow 0 \geq 4$, which is false.

$\therefore$ Half plane of $2x + y \geq 4$ is away from origin.

Also the given inequality is $x + y \leq 3$.

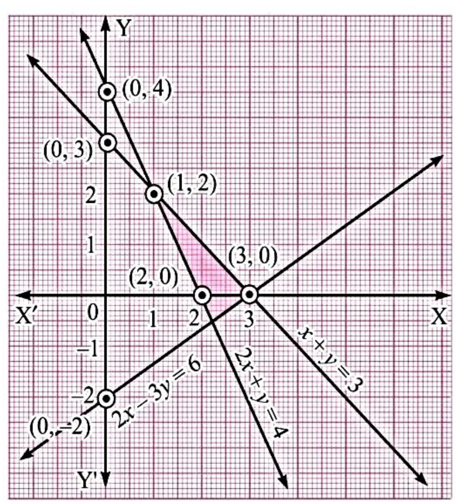
Draw the graph of the line $x + y = 3$.

Putting (0, 0) in the given inequality, we have

$0 + 0 \leq 3 \Rightarrow 0 \leq 3$, which is true.

$\therefore$ Half plane of $x + y \leq 3$ is towards origin.

The given inequality is $2x - 3y \leq 6$.



Draw the graph of the line $2x - 3y = 6$.

Putting (0, 0) in the given inequality, we have

$2 \times 0 - 3 \times 0 \le 6 \Rightarrow 0 \le 6$, which is true.

$\therefore$ Half plane of $2x - 3y \le 6$ is towards origin.

Hence solution set is shaded region.

43. $x - 2y \le 3, 3x + 4y \ge 12, x \ge 0, y \ge 1$.

Sol. The given inequality is $x - 2y \le 3$.

Draw the graph of the line $x - 2y = 3$.

Putting (0, 0) in the given inequality, we have

$0 - 2 \times 0 \le 3 \Rightarrow 0 \le 3$, which is true.

$\therefore$ Half plane of $x - 2y \le 3$ is towards origin.

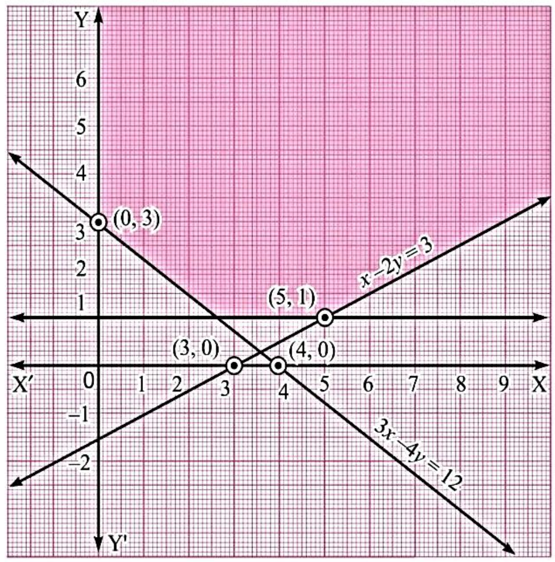
Also the given inequality is $3x + 4y \ge 12$.

Draw the graph of the line $3x + 4y = 12$.

Putting (0, 0) in the given inequality, we have

$3 \times 0 + 4 \times 0 \ge 12 \Rightarrow 0 \ge 12$, which is false.

$\therefore$ Half plane of $3x + 4y \ge 12$ is away from origin.



The given inequality is $y \ge 1$.

Draw the graph of the line $y = 1$.

Putting (0, 0) in the given inequality, we have $0 \ge 1$, which is false.

$\therefore$ Half plane of $y \ge 1$ is away from origin.

Hence solution set is shaded region.

44. $3x + 2y \le 150, x + 4y \le 80, x \le 15, y \ge 0, x \ge 0$.

Sol. The given inequality is $3x + 2y \le 150$.

Draw the graph of the line $3x + 2y = 150$.

Putting (0, 0) in the given inequality, we have

$3 \times 0 + 2 \times 0 \le 150 \Rightarrow 0 \le 150$, which is true.

$\therefore$ Half plane of $3x + 2y \le 150$ is towards origin.

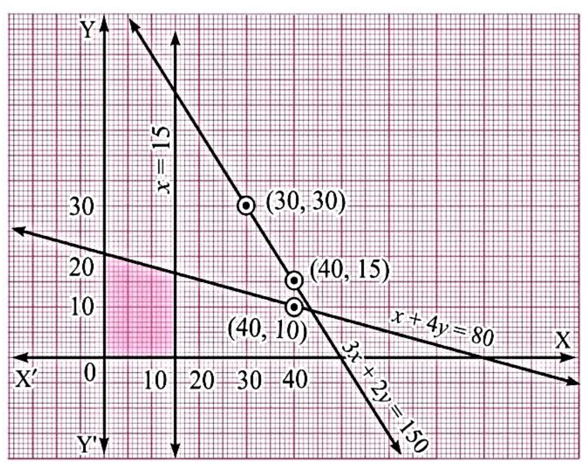
Also the given inequality is $x + 4y \le 80$.

Draw the graph of line $x + 4y = 80$.

Putting (0, 0) in the given inequality, we have

$0 + 4 \times 0 \le 80 \Rightarrow 0 \le 80$, which is true.

$\therefore$ Half plane of $x + 4y \le 80$ is towards origin.



The given inequality is $x \le 15$.

Draw the graph of the line $x = 15$.

Putting (0, 0) in the given inequality, we have $0 \le 15$, which is true.

$\therefore$ Half plane of $x \le 15$ is towards origin.

Hence solution set is shaded region.

45. $x + 2y \le 10, x + y \ge 1, x - y \le 0, x \ge 0, x \ge 0.$

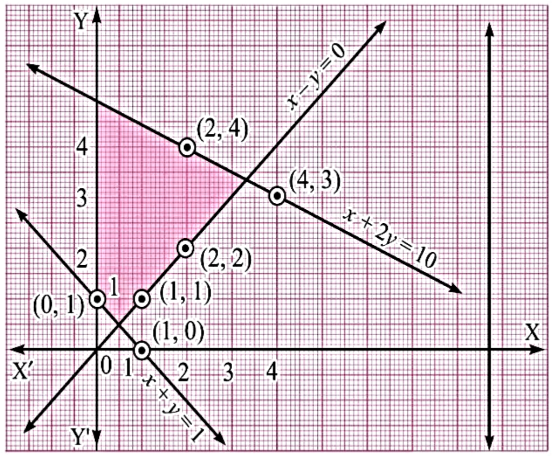
Sol. The given inequality is $x + 2y \le 10$.

Draw the graph of the line $x + 2y = 10$.

Putting (0, 0) in the given inequality, we have

$0 + 2 \times 0 \le 10 \Rightarrow 0 \le 10$, which is true.

$\therefore$ Half plane of $x + 2y \le 10$ is towards origin.



Also the given inequality is $x + y \ge 1$.

Draw the graph of the line $x + y = 1$.

Putting (0, 0) in the given inequality, we have

$0 + 0 \ge 1 \Rightarrow 0 \ge 1$, which is false.

$\therefore$ Half plane of $x + y \ge 1$ is away from origin.

Also the given inequality is $x - y \le 0$.

Draw the graph of the line $x - y = 0$.

Putting (2, 0) in the given inequality, we have

$2 - 0 \le 0 \Rightarrow 2 \le 0$, which is false.

$\therefore$ Half plane of $x - y \le 0$ does not contain the point (2, 0).

Hence solution set is shaded region.

SHORT ANSWER QUESTIONS-I and II [2 and 3 marks]

1. Solve for x, the inequality :

$$\frac{4}{x+1} \le 3 \le \frac{6}{x+1} \quad (x > 0)$$

Sol. Consider first two inequalities,

$$\frac{4}{x+1} \le 3 \quad \Rightarrow \quad 4 \le 3(x+1)$$

$\Rightarrow \; 4 \le 3x + 3 \quad \Rightarrow \quad 4 - 3 \le 3x$ [Subtracting 3 from both sides]

$\Rightarrow \; 1 \le 3x \quad \Rightarrow \quad x \ge \frac{1}{3}$

$\therefore \; x \ge \frac{1}{3} \quad \Rightarrow \quad x \in \left[\frac{1}{3}, \infty\right)$ ...(*i*)

Consider last two inequalities,

$$3 \le \frac{6}{x+1}$$

$\Rightarrow \; 3(x+1) \le 6 \quad \Rightarrow \quad 3x + 3 \le 6$

$\Rightarrow \; 3x \le 6 - 3$ [Subtracting 3 on both sides]

$\Rightarrow \; 3x \le 3$

$\Rightarrow \; x \le 1$ [Dividing by 3 on both sides]

$\therefore \; x \le 1 \quad \Rightarrow \quad x \in (-\infty, 1)$ ...(*ii*)

From equations (i) and (ii), we get

$$x \in \left[\frac{1}{3}, \infty\right) \cap (-\infty, 1] \quad \Rightarrow \quad x \in \left[\frac{1}{3}, 1\right]$$

$$\frac{1}{3} \le x \le 1$$

2. Solve for x: $\dfrac{|x-2|-1}{|x-2|-2} \le 0$

Sol. Let $|x-2| = y$

$\Rightarrow \quad \dfrac{y-1}{y-2} \le 0$

Case I: $y-1 \le 0$ and $y-2 > 0$

$\Rightarrow y \le 1$ and $y > 2$

Which is not possible.

Case II: $y-1 \ge 0$ and $y-2 < 0$

$\Rightarrow y \ge 1$ and $y < 2$

$\Rightarrow y \in [1, 2)$

Which is possible.

$\Rightarrow \quad 1 \le y < 2$

$\Rightarrow \quad 1 \le |x-2| < 2$

$\Rightarrow \quad 1 \le |x-2|$ and $|x-2| < 2$

$\Rightarrow \quad x-2 \le -1$ or $x-2 \ge 1$ and $-2 < (x-2) < +2$

$\Rightarrow \quad x \le 1$ or $x \ge 3$ and $-2+2 < x < 2+2$

$\Rightarrow \quad x \in (-\infty, 1] \cup [3, \infty)$ and $0 < x < 4$

$\Rightarrow \quad x \in \{(-\infty, 1] \cup [3, \infty)\} \cap (0, 4)$

$\therefore \quad x \in (0, 1] \cup [3, 4)$

3. Solve for x: $\dfrac{1}{|x|-3} \le \dfrac{1}{2}$

Sol. Given, $\dfrac{1}{|x|-3} \le \dfrac{1}{2}$

$\Rightarrow \quad |x| - 3 \ge 2$ $\qquad \left[\because \dfrac{1}{a} < \dfrac{1}{b} \Rightarrow a > b\right]$

$\Rightarrow \quad |x| - 3 + 3 \ge 2 + 3$ $\qquad$ [Adding 3 on both sides]

$\Rightarrow \quad |x| \ge 5$

$\Rightarrow \quad x \in (-\infty, -5] \cup [5, \infty)$ $\qquad$ ...(i)

But $\quad |x| - 3 \ne 0$

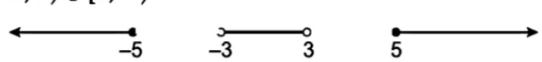
Either $\quad |x| - 3 < 0$ or $|x| - 3 > 0$

$\Rightarrow \quad |x| < 3$ or $|x| > 3$

$\Rightarrow \quad -3 < x < 3$ or $x < -3$ or $x > 3$ $\qquad$ ...(ii)

On combining results of equations (i) and (ii), we get

$$x \in (-\infty, -5] \cup (-3, 3) \cup [5, \infty)$$



4. Solve for x: $|x+1| + |x| > 3$

Sol. LHS $= |x+1| + |x|$

As both the terms contain modulus by equating the expression within modulus to zero,

We get $x = -1, 0$ as critical points.

These critical points divide the real line in three parts as $(-\infty, -1)$, $[-1, 0)$, $[0, \infty)$.

Case I: when $-\infty < x < -1$

$$|x+1| + |x| > 3$$

$$\Rightarrow \quad -x-1-x > 3 \Rightarrow -2x > 1+3 \Rightarrow -2x > 4 \Rightarrow x < -2$$

Case II: when $-1 \le x < 0$

$$|x+1| + |x| > 3$$

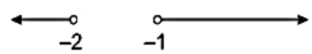
$$\Rightarrow \quad x+1-x > 3 \Rightarrow 1 > 3 \quad \text{(not possible)}$$

Case III: when $0 \le x < \infty$

$$|x+1| + |x| > 3$$

$$\Rightarrow \quad x+1+x > 3 \Rightarrow 2x > 3-1 \Rightarrow 2x > 2 \Rightarrow x > 1$$

Combining the results of cases, we get



$x \in (-\infty, -2) \cup (1, \infty)$

5. The water acidity in a pool is considered normal when the average pH reading of three daily measurement is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, then find the range of pH value for the third reading that will result in the acidity level being normal. [*NCERT Exemplar*]

Sol. Given, first pH value = 8.48

and second pH value = 8.35

Let third pH value be x.

Since, it is given that average pH value lies between 8.2 and 8.5.

$$\therefore \quad 8.2 < \frac{8.48 + 8.35 + x}{3} < 8.5$$

$$\Rightarrow 8.2 \times 3 < 16.83 + x < 8.5 \times 3 \quad \Rightarrow \quad 24.6 < 16.83 + x < 25.5$$

$$24.6 - 16.83 < x < 25.5 - 16.83 \quad \Rightarrow \quad 7.77 < x < 8.67$$

Thus, third pH value lies between 7.77 and 8.67.

6. The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm, then find the minimum length of the shortest side. [*NCERT Exemplar*]

Sol. Let the length of shortest side be x cm.

According to the given information,

Longest side $= 2 \times$ shortest side $= 2x$ cm

and third side $= 2 +$ shortest side $= (2 + x)$ cm

perimeter of triangle $= x + 2x + (x+2) = 4x + 2$

According to the question, perimeter > 166 cm

$$\Rightarrow \quad 4x + 2 > 166 \Rightarrow 4x > 166 - 2 \Rightarrow 4x > 164$$

$$\Rightarrow \quad x > \frac{164}{4} = 41 \text{ cm}$$

Hence, the minimum length of shortest side be 41 cm.

7. Solve $|5-2x| < 1, x \in R$ and represent the solution set on the number line.

Sol. Since $|5-2x| < 1$

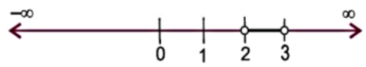
$\Rightarrow \quad -1 < 5-2x < 1$

$\Rightarrow \quad -1 < 5-2x$ and $5-2x < 1 \Rightarrow -1-5 < -2x$ and $-2x < 1-5 \Rightarrow -6 < -2x$ and $-2x < -4$

$\Rightarrow \quad 6 > 2x$ and $-2x < -4 \Rightarrow x < 3$ and $x > 2$

It is represented on number line by the thick line and the open line at 2 and 3 indicates that 2 and 3 are not included in the solution set.

$\therefore$ $2 < x < 3$ is the required solution set.



8. The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160 cm, then find its minimum breadth.

Sol. Let the breadth of the rectangle be x cm.

Given that length of the rectangle is three times the breadth.

$\therefore$ The length of the rectangle = $3x$ cm

Also, the perimeter of the rectangle = $2(3x + x) = 8x$ cm

Given that the minimum perimeter of the rectangle is 160 cm.

$\therefore \quad 8x \geq 160$

$\frac{8x}{8} \geq \frac{160}{8}$ [on dividing both sides by 8]

$x \geq 20$

Hence, the minimum breadth of the rectangle is 20 cm.

9. Solve : $\frac{x-2}{x+5} > 2$

Sol. We have $\frac{x-2}{x+5} > 2$

$\Rightarrow \frac{x-2}{x+5} - 2 > 0$ [Subtracting 2 form each side]

$\Rightarrow \frac{x-2-2(x+5)}{x+5} > 0 \quad \Rightarrow \quad \frac{x-2-2x-10}{x+5} > 0 \quad \Rightarrow \quad \frac{-(x+12)}{x+5} > 0$

$\Rightarrow \frac{x+12}{x+5} < 0$ [Multiplying both sides by –1]

$\Rightarrow x+12 > 0$ and $x+5 < 0$ or $x+12 < 0$ and $x+5 > 0$

$\Rightarrow x > -12$ and $x < -5$ or $x < -12$ and $x > -5$ (not possible)

Therefore, $-12 < x < 5$

i.e., $x \in (-12, 5)$

10. Find the solution set of inequality $\frac{|x-2|}{x-2} \geq 0$.

Sol. $\frac{|x-2|}{x-2} \geq 0$ for $|x-2| \geq 0$ and $x-2 \neq 0$

i.e, $|x-2| > 0$ and $x \neq 2$

$\Rightarrow \quad x-2 > 0$ or $x-2 < 0 \quad \Rightarrow \quad x > 2$ or $x < 2$

i.e., $x \in (2, \infty) \cup (-\infty, 2)$

11. Find solutions of the inequalities comprising a system in variable x are represented on real lines as given below. [*NCERT Exemplar*]

Sol.

$x \in (-\infty, -4] \cup [3, \infty)$ $\qquad$ $x \in (-\infty, -3] \cup [1, \infty)$

$\therefore \quad x \in ((-\infty, -4] \cup [3, \infty)) \cap ((-\infty, -3] \cup [1, \infty))$

i.e., $x \in (-\infty, -4] \cup [3, \infty)$

$\Rightarrow$ Solution set is $(-\infty, -4] \cup [3, \infty)$

12. Find the solution set of inequality $|x+3| \geq 10$, [*NCERT Exampler*]

Sol. $\because \quad |x+3| \geq 10 \Rightarrow x+3 \geq 10$ or $x+3 \leq -10$

$\Rightarrow \quad x \geq 7$ or $x \leq -13$

$\Rightarrow \quad x \in (-\infty, -13] \cup [7, \infty)$

$\Rightarrow$ Solution set is $(-\infty, -13] \cup [7, \infty)$

13. Find the solution set of inequality $5 < |x| < 9$. [*NCERT Exemplar*]

Sol. $5 < |x| < 9 \Rightarrow |x| > 5$ and $|x| < 9$

$\Rightarrow \quad x < -5$ or $x > 5$ and $-9 < x < 9$

$\Rightarrow \quad x \in (-\infty, -5) \cup (5, \infty)$ and $x \in (-9, 9)$

$\Rightarrow \quad x \in \{(-\infty, -5) \cup (5, \infty)\} \cap (-9, 9)$

$\Rightarrow \quad x \in (-9, -5) \cup (5, 9)$

$\Rightarrow$ Solution set is $(-9, -5) \cup (5, 9)$

14. Find solution set of inequality if $1 \leq |x| \leq 5$. [*NCERT Exemplar*]

Sol. We have $1 \leq |x| \leq 5 \Rightarrow |x| \geq 1$ and $|x| \leq 5$

$\Rightarrow \quad x \leq -1$ or $x \geq 1$ $\quad$ and $\quad -5 \leq x \leq 5$

$\Rightarrow \quad x \in ((-\infty, -1] \cup [1, \infty)) \cap [-5, 5]$

$\Rightarrow \quad x \in [-5, -1] \cup [1, 5]$

$\Rightarrow$ Solution set is $[-5, -1] \cup [1, 5]$

15. Find solution set of inequality $1 < |x-2| < 3$.

Sol. $\because \quad 1 < |x-2| < 3$

$\Rightarrow \quad |x-2| > 1$ $\quad$ and $\quad |x-2| < 3$

$\Rightarrow \quad x-2 < -1$ or $x-2 > 1$ $\quad$ and $\quad -3 < x-2 < 3$

$\Rightarrow \quad x < 1$ or $x > 3$ $\quad$ and $\quad -1 < x < 5$

$\Rightarrow \quad x \in (-\infty, 1) \cup (3, \infty)$ $\quad$ and $\quad x \in (-1, 5)$

$\Rightarrow \quad x \in \{(-\infty, 1) \cup (3, \infty)\} \cap (-1, 5)$

$\Rightarrow \quad x \in (-1, 1) \cup (3, 5)$

$\Rightarrow$ Solution set is $(-1, 1) \cup (3, 5)$.

16. Find the solution set of inequality $2 \leq |x-1| \leq 5$.

Sol. $\because \quad 2 \leq |x-1| \leq 5$

$\Rightarrow \quad |x-1| \geq 2$ $\quad$ and $\quad |x-1| \leq 5$

$\Rightarrow \quad x-1 \le -2 \text{ or } x-1 \ge 2 \quad \text{and} \quad -5 \le x-1 \le 5$

$\Rightarrow \quad x \le -1 \text{ or } x \ge 3 \quad \text{and} \quad -4 \le x \le 6$

$\Rightarrow \quad x \in (-\infty, -1] \cup [3, \infty) \quad \text{and} \quad x \in [-4, 6]$

$\Rightarrow \quad x \in \{(-\infty, -1] \cup [3, \infty)\} \cap [-4, 6]$

$\Rightarrow \quad x \in (-4, -1] \cup [3, 6]$

$\Rightarrow$ Solution set is $[-4, -1] \cup [3, 6]$.

17. Find solution set of inequality $|x-2| \ge 3$,

Sol. $|x-2| \ge 3 \Rightarrow x-2 \le -3 \text{ or } x-2 \ge 3$

$\Rightarrow \quad x \le -3+2 \text{ or } x \ge 3+2$

$\Rightarrow \quad x \le -1 \text{ or } x \ge 5$

$\Rightarrow \quad x \in (-\infty, -1] \cup [-5, \infty)$

$\Rightarrow$ Solution set is $(-\infty, -1] \cup [5, \infty)$.

18. Find the solution set of inequality $\left|\frac{2}{x-4}\right| > 1, x \ne 4$

Sol. $\because \quad \left|\frac{2}{x-4}\right| > 1, x \ne 4$

$\Rightarrow \quad \left|\frac{2}{x-4}\right| > 1$

$\Rightarrow \quad \frac{1}{|x-4|} > \frac{1}{2} \quad \Rightarrow \quad |x-4| < 2$

$\Rightarrow \quad -2 < x-4 < 2 \quad \Rightarrow \quad -2+4 < x < 2+4$

$\Rightarrow \quad 2 < x < 6 \quad \Rightarrow \quad x \in (2, 6) \text{ and } x \ne 4$

$\Rightarrow \quad x \in (2, 4) \cup (4, 6)$

$\Rightarrow$ Solution set is $(2, 4) \cup (4, 6)$.

19. Find solution set for inequality $\frac{2x+4}{x-1} \ge 5$

Sol. $\frac{2x+4}{x-1} \ge 5 \Rightarrow \frac{2x+4}{x-1} - 5 \ge 0$

$\Rightarrow \quad \frac{2x+4-5x+5}{x-1} \ge 0 \quad \Rightarrow \quad \frac{-3x+9}{x-1} \ge 0$

$\Rightarrow \quad \frac{3x-9}{x-1} \le 0 \Rightarrow \frac{3(x-3)}{x-1} \le 0 \Rightarrow \frac{x-3}{x-1} \le 0$

$\Rightarrow \quad x-3 \le 0 \text{ and } x-1 > 0 \quad \text{or} \quad x-3 \ge 0 \text{ and } x-1 < 0$

$\Rightarrow \quad x \le 3 \text{ and } x > 1 \quad \text{or} \quad x \ge 3 \text{ and } x < 1$

$\Rightarrow \quad x \in (-\infty, 3] \cap [1, \infty) \quad \text{or} \quad x \in (-\infty, 1) \cap [3, \infty) = \phi$

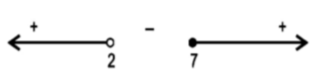
$\Rightarrow \quad x \in (1, 3] \text{ or } x \in \phi$

$\Rightarrow \quad x \in (1, 3]$

20. Find solution set for inequality $\frac{x+3}{x-2} \le 2$

Sol. $\frac{x+3}{x-2} \le 2 \Rightarrow \frac{x+3}{x-2} - 2 \le 0 \Rightarrow \frac{x+3-2x+4}{x-2} \le 0$

$\Rightarrow \frac{-x+7}{x-2} \le 0 \Rightarrow \frac{x-7}{x-2} \ge 0$



$\therefore \; x \in (-\infty, 2) \cup [7, \infty)$

21. Find the solution set of inequality $|x-2| \le 3$,

Sol. $|x-2| \le 3$

$\Rightarrow \quad -3 \le x-2 \le 3 \quad \Rightarrow \quad -3+2 \le x \le 3+2$

$\Rightarrow \quad -1 \le x \le 5 \quad \Rightarrow \quad x \in [-1, 5]$

22. Solve the inequality $\frac{x-1}{x-2} \le 1$.

Sol. We have $\frac{x-1}{x-2} \le 1$

$\Rightarrow \quad \frac{x-1}{x-2} - 1 \le 0 \quad \Rightarrow \quad \frac{(x-1)-(x-2)}{x-2} \le 0$

$\Rightarrow \quad \frac{1}{x-2} \le 0 \quad \Rightarrow \quad x-2<0 \quad \Rightarrow \quad x<2 \quad \Rightarrow \quad x \in (-\infty, 2)$

$\Rightarrow$ Solution set is $(-\infty, 2)$ $\qquad \left[\because \frac{a}{b} \le 0 \text{ and } a>0 \Rightarrow b<0\right]$

23. Solve the inequality $\frac{x+3}{x+7} > 3$.

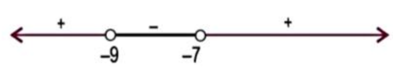
Sol. We have $\frac{x+3}{x+7} > 3$

$\Rightarrow \quad \frac{x+3}{x+7} - 3 > 0 \quad \Rightarrow \quad \frac{x+3-3x-21}{x+7} > 0$

$\Rightarrow \quad \frac{-2x-18}{x+7} > 0 \quad \Rightarrow \quad \frac{-2(x+9)}{x+7} > 0$

$\Rightarrow \quad \frac{x+9}{x+7} < 0 \qquad \ldots(i)$

Equating $x + 9$ and $x + 7$ equal to 0, we get $x = -9$ and -7. Plot these points on number line as shown in fig. The number line is divided in three parts and signs of LHS are marked. Since the inequality in (*i*) possesses less than (<) sign which means LHS of (*i*) is negative. So, the solution set of (*i*) is union of the parts obtaining negative sign in figure.



Hence, the solution set of given inequality is $(-9, -7)$.

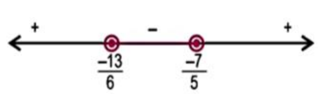
24. Solve the inequality $\frac{3x-5}{5x+7} \le 3$.

Sol. We have

$\frac{3x-5}{5x+7} \le 3 \quad \Rightarrow \quad \frac{3x-5}{5x+7} - 3 \le 0$

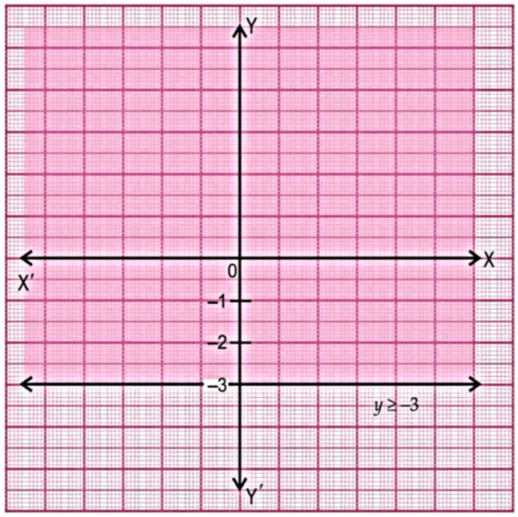
$\Rightarrow \quad \frac{3x-5-15x-21}{5x+7} \le 0 \quad \Rightarrow \quad \frac{-12x-26}{5x+7} \le 0$

$\Rightarrow \quad \frac{-2(6x+13)}{5x+7} \le 0 \quad \Rightarrow \quad \frac{6x+13}{5x+7} \ge 0$



$\Rightarrow \quad x \in \left(-\infty, -\frac{13}{6}\right) \cup \left(-\frac{7}{5}, \infty\right)$

25. Solve the inequality $y \geq -3$ graphically in two dimensional plane.



Sol. Given inequality is $y \geq -3$.

We consider the following equation:

$$y = -3$$

This line divides the xy-plane into two half planes.

We select (0, 0) which does not lie on the line.

Since, (0, 0) satisfies the inequality.

So, the half plane containing the point (0, 0) is the solution region of given inequality.

Solve for x Q(26-29):

26. $\dfrac{4}{x+1} \leq 3 \leq \dfrac{6}{x+1}$ *[NCERT Exemplar]*

Sol. $\dfrac{4}{x+1} \leq 3 \leq \dfrac{6}{x+1}, (x > 0)$

$\Rightarrow \quad \dfrac{4}{x+1} \leq 3$ and $3 \leq \dfrac{6}{x+1}, \ x > 0$

$\Rightarrow \quad 4 \leq 3(x+1)$ and $3(x+1) \leq 6, x > 0$

$\Rightarrow \quad 4 \leq 3x + 3$ and $3x + 3 \leq 6 \quad \Rightarrow \quad 1 \leq 3x$ and $3x \leq 3$

$\Rightarrow \quad \dfrac{1}{3} \leq x$ and $x \leq 1, x > 0 \quad \Rightarrow \quad \dfrac{1}{3} \leq x \leq 1$

27. $|x-1| \leq 5, \ |x| \geq 2$ *[NCERT Exemplar]*

Sol. $|x-1| \leq 5$

$\Rightarrow \quad -5 \leq x - 1 \leq 5$

$\Rightarrow \quad -4 \leq x \leq 6$ ...(*i*)

And $\quad |x| \geq 2$

$x \leq -2$ or $x \geq 2$

$\Rightarrow \quad x \in (-\infty, -2] \cup [2, \infty)$ ...(*ii*)

On combining (*i*) and (*ii*), we get

$$x \in (-4, -2] \cup [2, 6)$$

28. $-5 \le \frac{2-3x}{4} \le 9$ [*NCERT Exemplar*]

Sol. We have $-5 \le \frac{2-3x}{4} \le 9$

Now, $-5 \le \frac{2-3x}{4}$

$\Rightarrow \quad -20 \le 2 - 3x \quad \Rightarrow \quad 3x \le 2 + 20 \quad \Rightarrow \quad x \le \frac{22}{3} \quad \Rightarrow \quad x \in \left(-\infty, \frac{22}{3}\right]$

And $\frac{2-3x}{4} \le 9$

$\Rightarrow \quad 2 - 3x \le 36 \quad \Rightarrow \quad 3x \ge 2 - 36 \quad \Rightarrow \quad x \ge \frac{-34}{3} \quad \Rightarrow \quad x \in \left[\frac{-34}{3}, \infty\right)$

$\therefore \quad x \in \left(-\infty, \frac{22}{3}\right] \cap \left[\frac{-34}{3}, \infty\right)$

$\Rightarrow \quad x \in \left[\frac{-34}{3}, \frac{22}{3}\right]$

29. $4x + 3 \ge 2x + 17, 3x - 5 < -2$ [*NCERT Exemplar*]

Sol. We have $4x + 3 \ge 2x + 17$

$\Rightarrow \quad 4x - 2x \ge 17 - 3 \quad \Rightarrow \quad 2x \ge 14$

$\Rightarrow \quad x \ge 7 \quad \Rightarrow \quad x \in [7, \infty) \quad$ ...(*i*)

Also, we have $3x - 5 < -2$

$\Rightarrow \quad 3x < -2 + 5 \quad \Rightarrow \quad 3x < 3$

$\Rightarrow \quad x < 1 \quad \Rightarrow \quad x \in (-\infty, 1) \quad$ ...(*ii*)

From (*i*) and (*ii*), no value of x is possible.

$\therefore$ Solution set is empty set.

The given inequality has no solution.

30. A company manufactures cassettes. Its cost and revenue functions are $C(x) = 26000 + 30x$ and $R(x) = 43x$, respectively, where x is the number of cassettes produced and sold in a week. How many cassettes must be sold by the company to realise some profit? [*NCERT Exemplar*]

Sol. **Cost function:** $C(x) = 26000 + 30x$

Revenue function: $R(x) = 43x$

For profit, $R(x) > C(x)$

$\Rightarrow \quad 43x > 26000 + 30x$

$\Rightarrow \quad 43x - 30x > 26000$

$\Rightarrow \quad 13x > 26000$

$\Rightarrow \quad x > 2000$

Hence, more than 2000 cassettes must be produced to get profit.

31. In drilling world's deepest hole it was found that the temperature T in degree Celsius, x km below the earth's surface was given by $T = 30 + 25(x - 3)$, $3 \le x \le 15$. At what depth will the temperature be between 155°C and 205°C? [*NCERT Exemplar*]

Sol. We have, $T = 30 + 25(x - 3), 3 \le x \le 15$

Now given that, $155 < T < 205$

$\Rightarrow \quad 155 < 30 + 25(x-3) < 205 \quad \Rightarrow \quad 155 - 30 < 25(x-3) < 205 - 30$

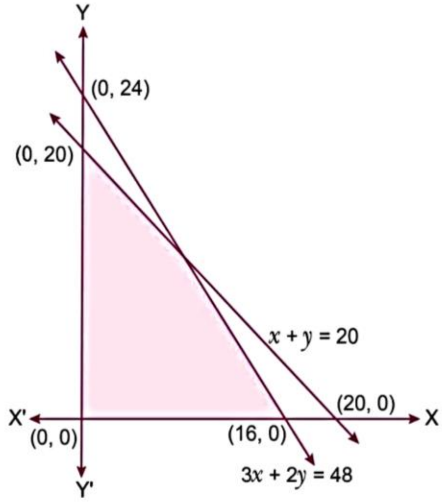
$\Rightarrow \quad 125 < 25(x-3) < 175 \quad \Rightarrow \quad \frac{125}{25} < x - 3 < \frac{175}{25}$

$\Rightarrow \quad 5 < x - 3 < 7 \quad\quad 5 + 3 < x < 7 + 3$

$\Rightarrow \quad 8 < x < 10$

Hence, at the depth 8 to 10 km, temperature lies between 155° to 205°C.

32. Find the linear inequalities for which the shaded region in the given figure is the solution set. [*NCERT Exemplar*]



Sol. We observe that the shaded region and the origin are on the same side of the line $3x + 2y = 48$.

For (0, 0), we have $3(0) + 2(0) - 48 < 0$. So, the shaded region satisfies the inequality $3x + 2y \leq 48$.

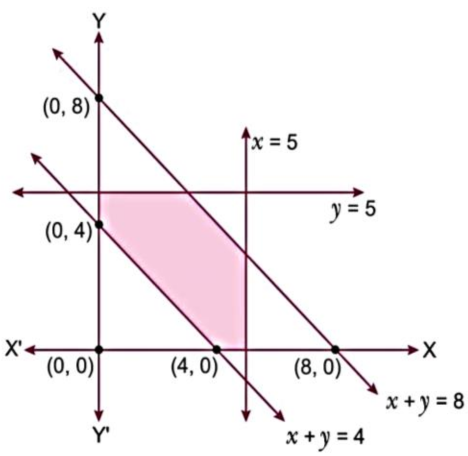
Also, the shaded region and the origin are on the same side of the line $x + y = 20$.

For (0, 0), we have $0 + 0 - 20 < 0$. So, the shaded region satisfies the inequality $x + y \leq 20$.

Also, the shaded region lies in the first quadrant. So, $x \geq 0, y \geq 0$.

Thus, the linear inequation corresponding to the given solution set are $3x + 2y \leq 48$, $x + y \leq 20$ and $x \geq 0, y \geq 0$.

33. Find the linear inequalities for which the shaded region in the given figure is the solution set. [*NCERT Exemplar*]



Sol. We observe that the shaded region and the origin are on the same side of the line $x + y = 8$.

For (0, 0), we have $0 + 0 - 8 < 0$. So, the shaded region satisfies the inequality $x + y \leq 8$.

The shaded region and the origin are on the opposite side of the line $x + y = 4$.

For (0, 0), we have $0 + 0 - 4 < 0$. So, the shaded region satisfies the inequality $x + y \geq 4$.

Further, the shaded region and the origin are on the same side of the lines $x = 5$ and $y = 5$.

So, it satisfies the inequality $x \leq 5$ and $y \leq 5$.

Also, the shaded region lies in the first quadrant. So, $x > 0, y > 0$.

Thus, the linear inequation comprising the given solution set are:

$x + y \geq 4;$

$x + y \leq 8;$

$x \leq 5, y < 5;$

$x \geq 0$ and $y \geq 0$.

LONG ANSWER QUESTIONS [5 marks]

1. Solve the following system of inequalities:

$$\frac{2x+1}{7x-1} > 5, \quad \frac{x+7}{x-8} > 2$$

Sol. The given system of inequations are

$$\frac{2x+1}{7x-1} > 5 \quad \text{and} \quad \frac{x+7}{x-8} > 2$$

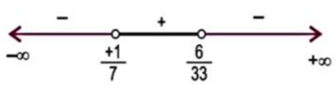
$$\Rightarrow \frac{2x+1}{7x-1} - 5 > 0 \quad \text{and} \quad \frac{x+7}{x-8} - 2 > 0$$

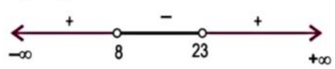
$$\Rightarrow \frac{2x+1-5(7x-1)}{7x-1} > 0 \quad \text{and} \quad \frac{x+7-2(x-8)}{x-8} > 0$$

$$\Rightarrow \frac{2x+1-35x+5}{7x-1} > 0 \quad \text{and} \quad \frac{x+7-2x+16}{x-8} > 0$$

$$\Rightarrow \frac{-33x+6}{7x-1} > 0 \quad \text{and} \quad \frac{-x+23}{x-8} > 0$$

$$x \in \left(\frac{1}{7}, \frac{6}{33}\right) \quad ...(i) \qquad \frac{x-23}{x-8} < 0 \quad \Rightarrow \quad x \in (8, 23) \quad ...(ii)$$





Using the equations (*i*) and (*ii*) we get the null set.

Hence, the given system of equation has no solution.

2. Solve for x, $\frac{|x+3|+x}{x+2} > 1$.

Sol. We have $\frac{|x+3|+x}{x+2} > 1$

$$\Rightarrow \frac{|x+3|+x}{x+2} - 1 > 0 \quad \Rightarrow \quad \frac{|x+3|+x-x-2}{x-2} > 0 \quad \Rightarrow \quad \frac{|x+3|-2}{x-2} > 0$$

Now two cases arise:

Case I when $x + 3 \geq 0$, *i.e.*, $x \geq -3$. Then

$$\frac{|x+3|-2}{x+2} > 0 \quad \Rightarrow \quad \frac{x+3-2}{x+2} > 0 \quad \Rightarrow \quad \frac{x+1}{x+2} > 0$$

$\Rightarrow \quad \{(x+1) > 0 \text{ and } x+2 > 0\}$ or $\{x+1 < 0 \text{ and } x+2 < 0\}$

$\Rightarrow \quad \{x > -1 \text{ and } x > -2\}$ or $\{x < -1 \text{ and } x < -2\}$

$\Rightarrow \quad x > -1$ or $x < -2$

$\Rightarrow \quad x \in (-1, \infty)$ or $x \in (-\infty, -2)$

$\Rightarrow \quad x \in (-3, -2) \cup (-1, \infty)$ [Since $x \geq -3$] ...(*i*)

Case II When $x + 3 < 0$, *i.e.*, $x < -3$

$\Rightarrow \quad \dfrac{|x+3|-2}{x+2} > 0 \quad \Rightarrow \quad \dfrac{-x-3-2}{x+2} > 0 \quad \Rightarrow \quad \dfrac{-x-5}{x+2} > 0$

$\Rightarrow \quad \dfrac{-(x+5)}{x+2} > 0 \quad \Rightarrow \quad \dfrac{x+5}{x+2} < 0$

$\Rightarrow \quad (x + 5 < 0$ and $x + 2 > 0)$ or $(x + 5 > 0$ and $x + 2 < 0)$

$\Rightarrow \quad (x < -5$ and $x > -2)$ or $(x > -5$ and $x < -2)$

It is not possible. It is possible.

$\therefore \quad x \in (-5, -2)$ ...(*ii*)

On combining (*i*) and (*ii*), we get

$x \in (-5, -2) \cup (-1, \infty)$

3. Show that the following system of linear inequalities has no solution.

$x + 2y \leq 3$, $3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$

Sol. Consider the inequation $x + 2y \leq 3$ as an equation.

We have $x + 2y = 3$

$\Rightarrow \quad x = 3 - 2y \quad \Rightarrow \quad 2y = 3 - x$

x	3	1	0
y	0	1	1.5

Now (0, 0) satisfy the inequation $x + 2y \leq 3$

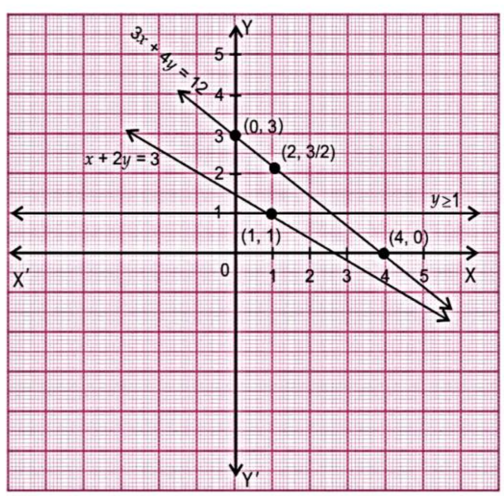
So, half plane contains (0, 0) as the solution and the line $x + 2y = 3$ intersect the coordinate axis at (3, 0) and $(0, \frac{3}{2})$.

Consider the inequation $3x + 4y \geq 12$ as an equation,

we have $3x + 4y = 12$.

$4y = 12 - 3x$

x	0	4	2
y	3	0	$\frac{3}{2}$



Now, (0, 0) does not satisfy the inequation

$3x + 4y = 12$.

$\therefore$ Half plane of the solution does not contain (0, 0).

Consider the inequation $y \geq 1$ as an equation,

We have $y = 1$

It represents a straight line parallel to x-axis passing through point (0, 1).

Now (0, 0) does not satisfy the inequation $y \geq 1$.

It is clear from the graph that shaded portions do not have common region.

So, solution set is null set.

4. Solve the following system of linear inequalities:

$$3x + 2y \geq 24,\ 3x + y \leq 15,\ x \geq 4$$

Sol. Consider the equation $3x + 2y \geq 24$ as an equation,

we have, $3x + 2y = 24 \Rightarrow 2y = 24 - 3x \Rightarrow y = \dfrac{24 - 3x}{2}$

x	0	8	4
y	12	0	6

Now, (0, 0) does not satisfy the inequation $3x + 2y \geq 24$.

$\therefore$ Half plane of the solution set does not contains (0, 0).

Consider the inequation $3x + y \leq 15$ as an equation,

we have, $3x + y = 15 \Rightarrow y = 15 - 3x$

x	0	5	3
y	15	0	6

Now, (0, 0) satisfy the inequation $3x + y \leq 15$.

Therefore, the half plane of the solution contain origin.

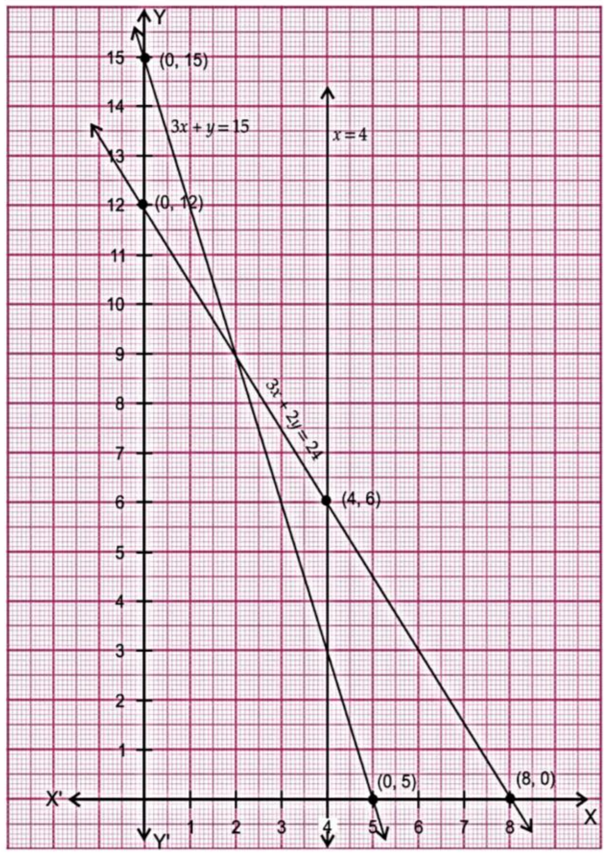
Consider the inequality $x \geq 4$

as an equation, we have $x = 4$.

It represents a straight line parallel to y-axis passing through (4, 0).

Now, point (0, 0) does not satisfy the inequation $x \geq 4$.

Therefore, half plane does not contains (0, 0).



The graph of the above inequations is given aside:

It is clear from the graph that there is no common region corresponding to these inequality.

Hence, the given system of inequalities have no solution.

5. Show that the solution set of the following system of linear inequalities is an unbounded region:

$$2x + y \geq 8,\ x + 2y \geq 10,\ x \geq 0,\ y \geq 0$$

Sol. Consider the inequation $2x + y \geq 8$ as an equation

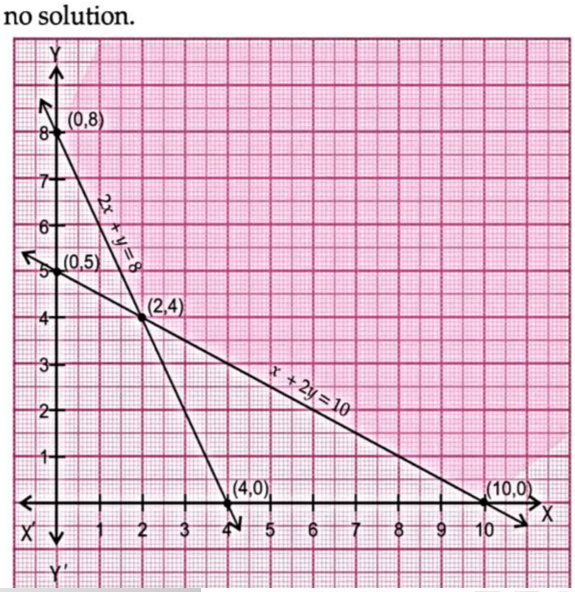
we have $2x + y = 8 \Rightarrow y = 8 - 2x$

x	0	4	3
y	8	0	2

Now, point (0, 0) does not satisfy the inequation $2x + y \geq 8$.

Therefore, half plane does not contain origin.

Consider the inequation $x + 2y \geq 10$, as an equation



We have $x + 2y = 10 \Rightarrow x = 10 - 2y$

x	10	0	6
y	0	5	2

Now, point (0, 0) does not satisfy the inequation.

Therefore, half plane does not contain (0, 0).

Consider the inequation $x \geq 0$ and $y \geq 0$ represents I quadrant.

It is clear from the graph that common shaded portion is unbounded.

6. A solution is to be kept between 68° F and 77° F. What is the range of temperature in degree Celsius (C) if the Celsius/Farhenheit (F) conversion formula is given by $F = \frac{9}{5}C + 32$?

Sol. It is given that $68° < F < 77°$

Putting $F = \frac{9}{5}C + 32$

$$68° < \frac{9}{5}C + 32 < 77° \Rightarrow 36° < \frac{9}{5}C < 45°$$

$$\Rightarrow 180° < 9C < 225° \Rightarrow 20° < C < 25°$$

Thus, the range of temperature is between 20° C and 25° C.

7. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

Sol. Let x litres of 2% boric acid solution be added to 640 litres of 8% boric acid solution. Then

Total quantity of mixture = $(640 + x)$ litres

Total boric acid in $(640 + x)$ litres of mixtures $= \frac{2x}{100} + \frac{8}{100} \times 640 = \frac{x}{50} + \frac{256}{5}$

It is given that the resulting mixture must be more than 4% but less than 6% boric acid.

$$\therefore \quad \frac{4}{100}(640 + x) < \frac{x}{50} + \frac{256}{5} < \frac{6}{100}(640 + x)$$

$$\Rightarrow \quad \frac{640 + x}{25} < \frac{x + 2560}{50} < \frac{1920 + 3x}{50}$$

$$\Rightarrow \quad 1280 + 2x < x + 2560 < 1920 + 3x$$

$$\Rightarrow \quad 1280 + 2x < x + 2560 \quad \text{and} \quad x + 2560 < 1920 + 3x$$

$$\Rightarrow \quad x < 1280 \quad \text{and} \quad -2x < -640$$

$$\Rightarrow \quad x < 1280 \quad \text{and} \quad x > 320$$

$$\Rightarrow \quad 320 < x < 1280$$

Thus, 2% boric acid solution must be more than 320 litres but less than 1280 litres.

8. IQ of a person is given by the formula

$$IQ = \frac{MA}{CA} \times 100$$

where MA is mental age and CA is chronological age. If $80 \leq IQ \leq 140$ for a group of 12 years old children, find the range of their mental age.

Sol. It is given that $80 \leq IQ \leq 140$ and $CA = 12$.

We have $IQ = \frac{MA}{CA} \times 100$

$\therefore \quad 80 \le \frac{MA}{CA} \times 100 \le 140 \quad \Rightarrow \quad 80 \le \frac{MA}{12} \times 100 \le 140$

$\Rightarrow \quad 960 \le MA \times 100 \le 1680 \quad \Rightarrow \quad 9.6 \le MA \le 16.8$

Thus, minimum MA is 9.6 and maximum 16.8.

9. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 litres of the 9% solution, how many litres of 3% solution will have to be added? [*NCERT Exemplar*]

Sol. Let x L of 3% solution be added to 460 L of 9% solution of acid.

Then, total quantity of mixture = $(460 + x)$ L

Total acid content in the $(460 + x)$ L of mixture = $\left(460 \times \frac{9}{100} + x \times \frac{3}{100}\right)$

It is given that acid content in the resulting mixture must be more than 5% but less than 7% acid.

$\therefore \quad 5\% \text{ of } (460 + x) < 460 \times \frac{9}{100} + \frac{3x}{100} < 7\% \text{ of } (460 + x)$

$\Rightarrow \quad \frac{5}{100} \times (460 + x) < 460 \times \frac{9}{100} + \frac{3x}{100} < \frac{7}{100} \times (460 + x)$

$\Rightarrow \quad 5 \times (460 + x) < 460 \times 9 + 3x < 7 \times (460 + x)$

$\Rightarrow \quad 2300 + 5x < 4140 + 3x < 3220 + 7x$

$\Rightarrow \quad 5x < 1840 + 3x < 920 + 7x \quad \Rightarrow \quad 2x < 1840 < 920 + 4x$

$\Rightarrow \quad 2x < 1840 \text{ and } 1840 < 920 + 4x \quad \Rightarrow \quad x < 920 \text{ and } 920 < 4x$

$\Rightarrow \quad x < 920 \text{ and } 230 < x \quad \Rightarrow \quad 230 < x < 920$

Hence, the number of litres of the 3% solution of acid must be more than 230 and less than 920.

10. A solution is to be kept between 40°C and 45°C. What is the range of temperature in degree Fahrenheit, if the conversion formula is $F = \frac{9}{5}C + 32$? [*NCERT Exemplar*]

Sol. Let the required temperature be $x°$ F

Also given that, $F = \frac{9}{5}C + 32 \quad \Rightarrow \quad 5F = 9C + 32 \times 5 \quad \Rightarrow \quad 9C = 5F - 160$

$\therefore \quad C = \frac{5F - 160}{9}$

Since temperature in degree Celsius lies between 40°C to 45°C, we get

$40 < \frac{5F - 160}{9} < 45 \quad \Rightarrow \quad 40 \times 9 < 5x - 160 < 45 \times 9$

$\Rightarrow \quad 360 < 5x - 160 < 405 \quad \Rightarrow \quad 520 < 5x < 565$

$\Rightarrow \quad \frac{520}{5} < x < \frac{565}{5} \quad \Rightarrow \quad 104 < x < 113$

Hence, the range of temperature in degree Fahrenheit is 104°F to 113°F.

11. Show that the following system of linear inequalities has no solution: $x + 2y \le 3$, $3x + 4y \ge 12$, $x \ge 0$, $y \ge 1$. [*NCERT Exemplar*]

Sol. We have $x + 2y \le 3$, $3x + 4y \ge 12$, $x \ge 0$, $y \ge 1$

Now let's plot lines $x + 2y = 3$, $3x + 4y = 12$, $x = 0$ and $y = 1$ in coordinate plane.

Line $x + 2y = 3$ passes through the points $\left(0, \frac{3}{2}\right)$ and (3, 0).

Line $3x + 4y = 12$ passes through points (4, 0) and (0, 3).

For (0, 0), $0 + 2(0) - 3 < 0$.

Therefore, the region satisfying the inequality $x + 2y \leq 3$ and (0, 0) lie on the same side of the line $x + 2y = 3$.

For (0, 0), $3(0) + 4(0) - 12 \geq 0$.

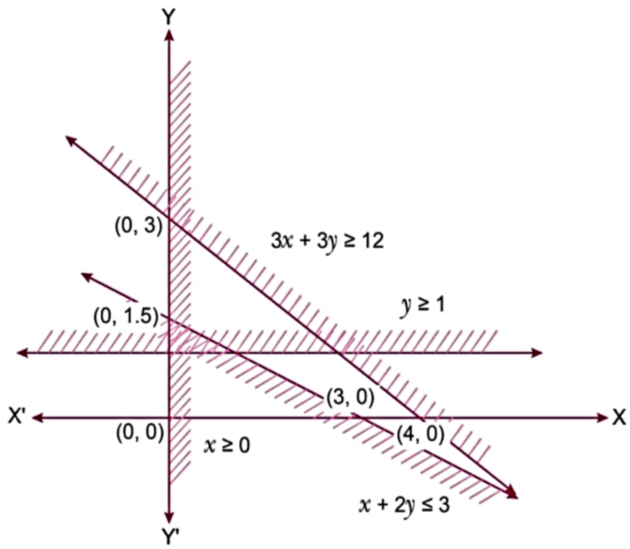
Which is false.

Therefore, the region satisfying the inequality $3x + 4y \geq 12$ and (0, 0) lie on the opposite side of the line $3x + 4y = 12$.

The region satisfying $x > 0$ lies to the right hand side of the y-axis.

The region satisfying $y > 1$ lies above the line $y = 1$.

These regions are plotted as shown in the following figure.



It is clear from the graph that the shaded portions do not have common region. So, solution set is null set.

i.e., It has no solution.

12. Solve the following system of linear inequalities:

$$3x + 2y \geq 24,\ 3x + y \leq 15,\ x \geq 4$$ *[NCERT Exemplar]*

Sol. We have, $3x + 2y \geq 24$, $3x + y \leq 15$, $x \geq 4$

Now let's plot lines $3x + 2y = 24$, $3x + y = 15$ and $x = 4$ on the coordinate plane.

Line $3x + 2y = 24$ passes through the points (0, 12) and (8, 0).

Line $3x + y = 15$ passes through points (5, 0) and (0, 15).

Also line $x = 4$ is passing through the point (4, 0) and vertical.

For (0, 0), $3(0) + 2(0) - 24 < 0$.

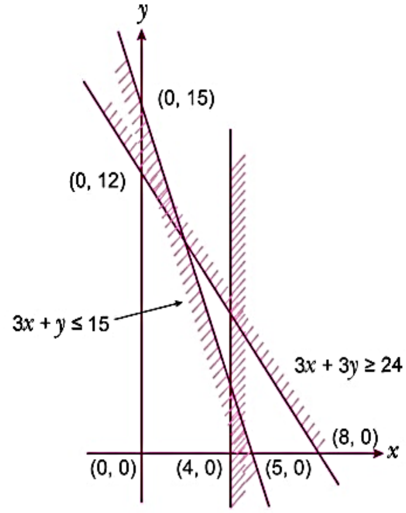
Therefore, the region satisfying the inequality $3x + 2y \geq 24$ and (0, 0) lie on the opposite of the line $3x + 2y = 24$.

For (0, 0), $3(0) + (0) - 15 \leq 0$.

Therefore, the region satisfying the inequality $3x + y \leq 15$ and (0,0) lie on the same side of the line $3x + y = 15$.

The region satisfying $x \geq 4$ lies to the right hand side of the line $x = 4$.

These regions are plotted as shown in the following figure



It is clear from the graph that there is no common region corresponding to these inequalities.

Hence, the given system of inequalities has no solution.

13. Show that the solution set of the following system of linear inequalities is an unbounded region:

$2x + y \geq 8, x + 2y \geq 10, x \geq 0, y \geq 0$ *[NCERT Exemplar]*

Sol. We have $2x + y \geq 8, x + 2y \geq 10, x \geq 0, y \geq 0$

Line $2x + y = 8$ passes through the points (0, 8) and (4, 0).

Line $x + 2y = 10$ passes through points (10, 0) and (0, 5).

For (0, 0), $2(0) + (0) - 8 < 0$.

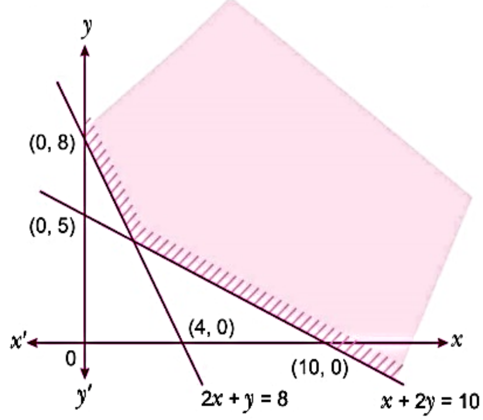
Therefore, the region satisfying the inequality $2x + y \geq 8$ and (0, 0) lie on the opposite side of the line $2x + y = 8$.

For (0, 0), $(0) + 2(0) - 10 \geq 0$ which is false.

Therefore, the region satisfying the inequality $x + 2y \geq 10$ and (0, 0) lie on the opposite side of the line $x + 2y = 10$.

Also, for $x \geq 0, y \geq 0$, region lies in the first quadrant.

The common region is plotted as shown in the following figure.



It is clear from the graph that common shaded, is solution set.

QUESTIONS FOR PRACTICE

1. Solve the inequalities in problems (*i*) to (*iv*) for real x:

   (*i*) $4x + 3 < 5x + 7$ (*ii*) $3(2 - x) \geq 2(1 - x)$

   (*iii*) $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x - 6)$ (*iv*) $\frac{x}{3} > \frac{x}{2} + 1$

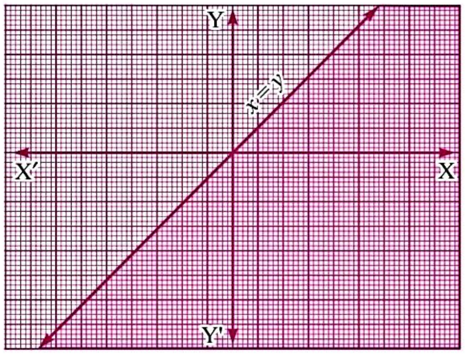
2. Solve $\frac{4x - 10}{2} \leq \frac{5x - 12}{3}$, $x \in R$.
3. Solve $3x - 2(x + 1) > 5(2x - 3) + 9$ for $x \in \mathbb{R}$.
4. Solve $y \leq x$ graphically.
5. Solve $x \leq y$ graphically.
6. In the first four examinations, each of 100 marks, Hamid got 94, 73, 72, 84 marks. If a final average of greater than or equal to 80 and less than 90 is needed to obtain a final grade *B* in a course, what range of marks in the fifth (last) examination will result in Hamid receiving '*B*' in the course?
7. Find all pairs of consecutive even positive integers, both of which are larger than 5, such that their sum is less than 23.
8. A shunt resistor of *R* ohms is to be added to a resistance of 600 ohms in order to reduce the resistance to a value strictly between 540 and 550 ohms. After the shunt is added, the resistance is given by $600R/(600 + R)$. Find the possible value of *R*. (Assume that $R < 0$).
9. Solve the following systems $x \geq 3$, $y \geq 2$ inequalities graphically.
10. Solve the following systems $2x - y > 1$, $x - 2y < -1$ inequalities graphically.
11. A technician determines that an electronic circuit fails to operate because the resistance between points A and B, 1,200 Ohms, exceeds the specifications, which call for a resistance of neither less than 400 Ohms and nor greater than 900 ohms. The circuits can be made to satisfy the specifications by adding a shunt resistor of *R* Ohms, $R > 0$. After adding the shunt resistor, the resistance between points *A* and *B* will be $\frac{1200R}{1200 + R}$ Ohms. What area the possible value of *R*?
12. Psychologists define the intelligence quotient (*IQ*) of a person to be 100 times the ratio of the person's mental age to his or her chronological age. A psychologist is studying a group of 13-years old who have an *IQ* range between 90 and 120, inclusive. What is the corresponding range of mental ages?
13. A loupe is a small magnifying lens set in an eye piece and used by jewellers, watchmakers and hobbyists. If the local length of the lens is *f* centimeters, then an object viewed through the lens at a distance of *p* centimeters from the lens will appear to be magnified by a factor $m = \frac{f}{f - p}$, provided that $p < f$. If $f = 5$ centimeters, what range of values of p will result in a magnification factor of between 2 and 5 inclusive?
14. Solve graphically the inequalities: $5x + 4y \leq 20$, $x \geq 1$, $y \geq 2$
15. Solve graphically the inequalities: $3x + 4y \leq 60$, $x + 3y \leq 30$, $x \geq 0$, $y \geq 0$
16. Solve graphically the inequalities: $x - 2y \leq 3$, $3x + 4y \geq 14$, $x \geq 0$, $y \geq 1$
17. Solve graphically the inequalities: $3x + 2y \leq 150$, $x + 4y \leq 80$, $x \leq 15$, $y \geq 0$, $x \geq 0$
18. Solve graphically the inequalities: $x + 2y \leq 10$, $x + y \geq 1$, $x - y \leq 0$, $x \geq 0$, $y \geq 0$

Answers

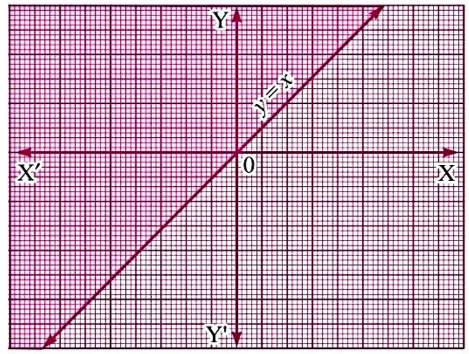
1. (*i*) $(-4, \infty)$ (*ii*) $(-\infty, 4]$ (*iii*) $(-\infty, 120]$ (*iv*) $(-\infty, -6)$

2. $x \in (-\infty, 3]$ **3.** $x < \frac{4}{9}$

4.



5.

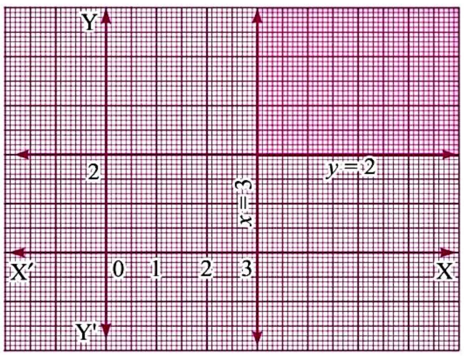


6. 77 or more

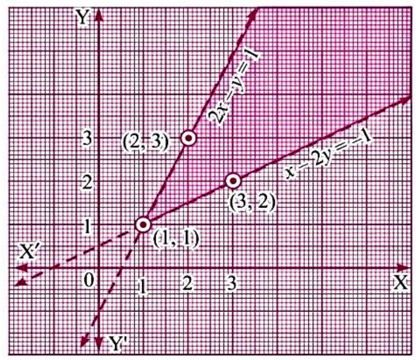
7. (6, 8), (8, 10)

8. $5400 < R < 6600$

9.



10.

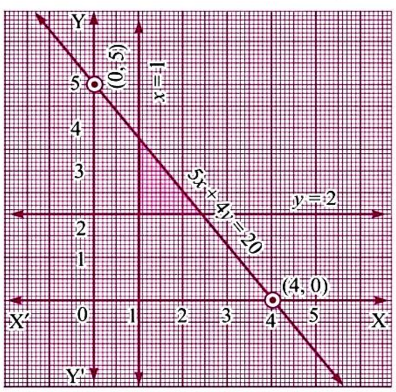


11. More than 600, less than or equal to 3,600

12. [11.7, 15.6]

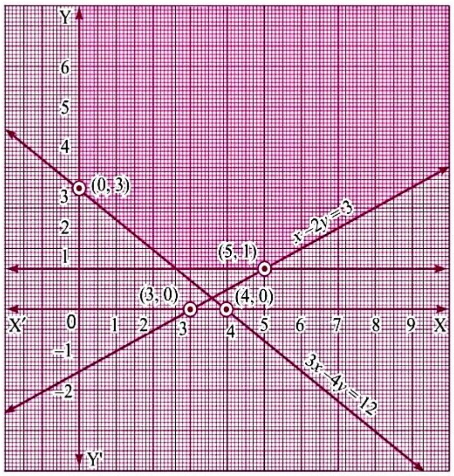
13. $\frac{5}{2} \leq p \leq 4$

14.

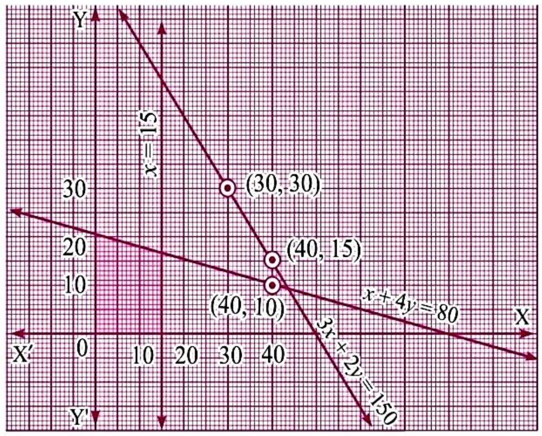


15.

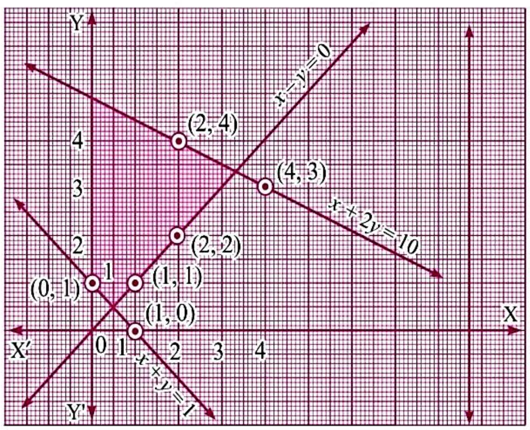
16.


Y
(0, 3)
x – 2y = 3
(5, 1)
(3, 0)
(4, 0)
X′
X
3x – 4y = 12
Y′

17.


Y
x = 15
(30, 30)
(40, 15)
(40, 10)
x + 4y = 80
3x + 2y = 150
X′
X
Y′

18.


Y
x – y = 0
(2, 4)
(4, 3)
x + 2y = 10
(2, 2)
(0, 1)
(1, 1)
(1, 0)
x + y = 1
X′
X
Y′