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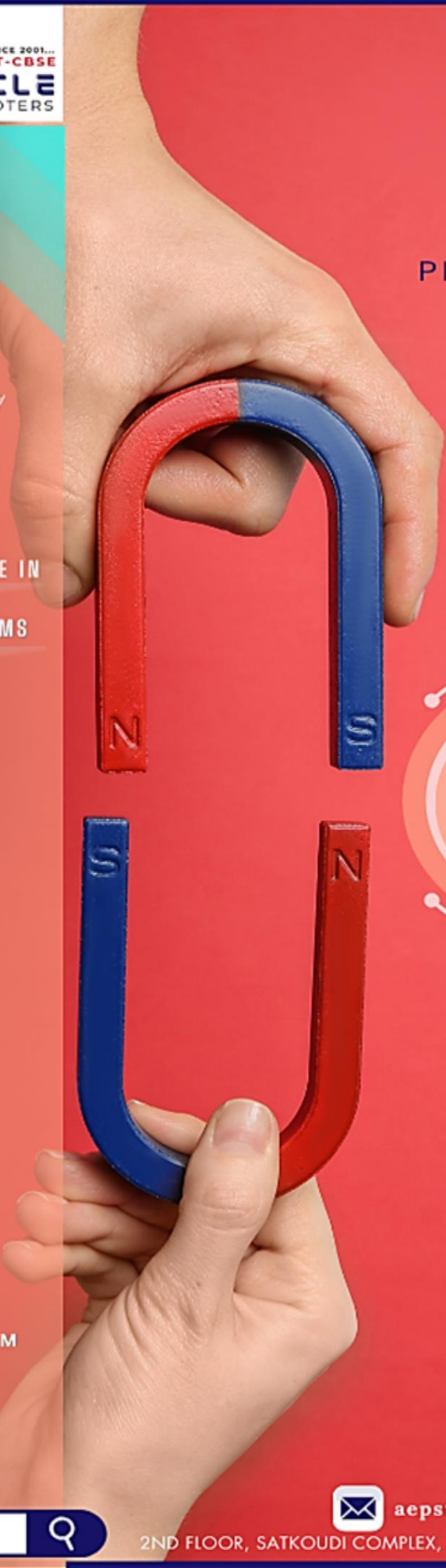

IIT-JEE
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MAGNETISM 02

BASIC CONCEPTS

FREE BODY DIAGRAM

SOLVED EXAMPLES

PRACTICE SET

PREVIOUS YEAR PROBLEM

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MAGNETIC EFFECTS OF CURRENT AND MAGNETISM

REVIEW OF BASIC CONCEPTS

1. Biot-Savart Law

According to Biot-Savart law, the magnetic field $\overrightarrow{dB}$ at a point whose position vector with respect to a current element $\overrightarrow{dl}$ is $\vec{r}$ is given by

$$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi} \frac{(\overrightarrow{dl} \times \vec{r})}{r^3} \quad (1)$$

where $\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$

2. Ampere's Circuital Law

The loop or line circuit integral of the magnetic field along a closed curve is proportional to the current threading or passing through the closed circuit i.e.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

where μ_0 is the permeability of free space.

Biot-Savart Law and Ampere's Circuital Law are used to find the magnetic field due to current carrying conductors.

3. Magnetic Field Due to Current Carrying Conductors

(i) *Magnetic field at point P due to an infinitely long wire carrying a current I* (Fig. 13.1)

$B = \dfrac{\mu_0 I}{2\pi r}$ directed into the page (away from the reader) if the I is upwards and towards the reader if I is downwards. At points Q or S, $B = 0$.

(ii) *Magnetic field at the centre of a circular loop of radius r* (Fig. 13.2)

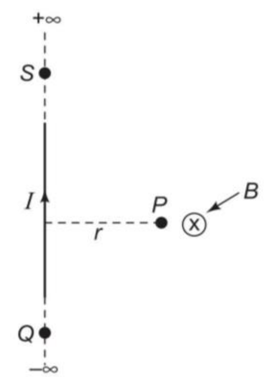


Fig. 13.1

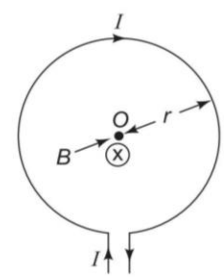


Fig. 13.2

$B = \dfrac{\mu_0 I}{2r}$ directed into the page if I is clockwise and outside the page if I is anticlockwise.

For a coil of N turns.

$$B = \frac{\mu_0 NI}{2r}$$

(iii) *Magnetic field at the centre of a curved element* (Fig. 13.3).

$$B = \frac{\mu_0 I}{2r} \times \frac{\theta}{2\pi}$$

directed into the page. Here θ is in radian.

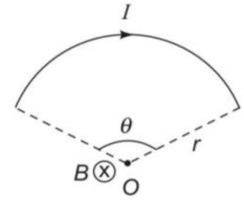


Fig. 13.3

For a semi-circular element ($\theta = \pi$)

$$B = \frac{\mu_0 I}{4r}$$

(iv) *Magnetic field at point P due to a straight wire XY of finite length* (Fig. 13.4)

$$B = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$$

directed into the page.

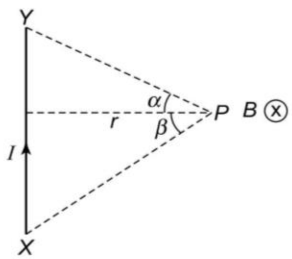


Fig. 13.4

Special Cases

(a) If the conductor XY is of infinite length and point P lies near the centre of the conductor (as in Fig. 13.1), $\alpha = \beta = 90°$ so

$$B = \frac{\mu_0 I}{4\pi r}(\sin 90° + \sin 90°)$$

$$= \frac{\mu_0 I}{2\pi r}$$

(b) If the conductor XY is of infinite length but point P lies near the end X or Y as shown in Fig. 13.5, then $\alpha = 90°$ and $\beta = 0°$, then

$$B = \frac{\mu_0 I}{4\pi r}(\sin 90° + \sin 0°)$$

$$= \frac{\mu_0 I}{4\pi r}$$

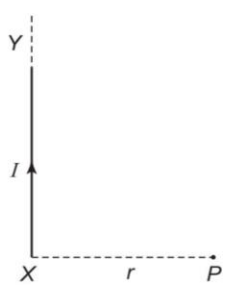


Fig. 13.5

Note For an infinitely long straight conductor carrying a current, the magnetic field near its centre is twice that near one of its ends.

(c) If the conductor XY has a finite length L and point P lies on the right bisector of the conductor, as shown in Fig. 13.6, then

$\alpha = \beta$

and $\sin\alpha = \sin\beta = \dfrac{L/2}{x}$

$$= \frac{L/2}{\sqrt{r^2 + \left(\frac{L}{2}\right)^2}}$$

$$= \frac{L}{\sqrt{4r^2 + L^2}}$$

So $\quad B = \dfrac{\mu_0 I}{2\pi r} \cdot \dfrac{L}{\sqrt{4r^2 + L^2}}$

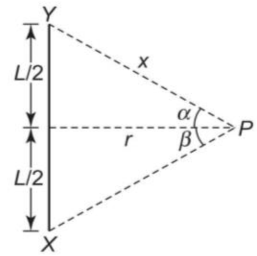


Fig. 13.6

(d) If the point P lies on the straight conductor or on its axis, then $\vec{dl}$ and $\vec{r}$ for each element of the straight conductor are parallel. Therefore, $\vec{dl} \times \vec{r} = 0$. Hence $\vec{B} = 0$ at point P.

(v) *Magnetic field at centre due to a wire PQRST* (Fig. 13.7)

Magnetic field at O due to straight portions PQ and ST is zero and due to semicircular part QRS is

$$B = \frac{\mu_0 I}{4r} \text{ directed into the page.}$$

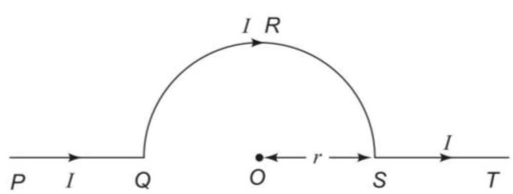


Fig. 13.7

If the current is anticlockwise, B is directed towards the reader.

(vi) *Magnetic field at Centre O of a rectangular coil (Fig. 13.8)*

$$B = \frac{2\mu_0 I}{\pi} \times \frac{\sqrt{a^2 + b^2}}{ab} \text{ directed into the page.}$$

For a square coil ($b = a$)

$$B = \frac{2\sqrt{2}\,\mu_0 I}{\pi a}$$

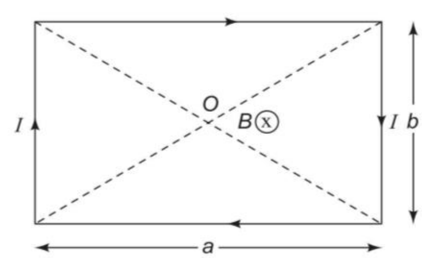


Fig. 13.8

(vii) *Magnetic field due to a hollow metal pipe of radius R carrying current in its walls* (Fig. 13.9)

At point P, $B = \dfrac{\mu_0 I}{2\pi r}$

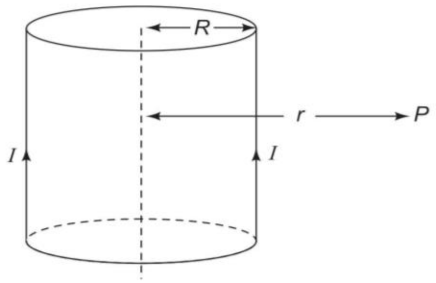


Fig. 13.9

For a solid pipe of radius R (Fig. 13.9)

Inside the pipe at a point at a distance r from the axis,

$$B = \frac{\mu_0 I r}{2\pi R^2}$$

Outside the pipe at a point P at a distance r from the axis,

$$B = \frac{\mu_0 I}{2\pi r}$$

(viii) *Magnetic field on the axis of a circular coil of radius R* (Fig. 13.10)

At point P, $B = \dfrac{\mu_0}{4\pi}\dfrac{2M}{(R^2 + r^2)^{3/2}}$

where $M = IA = I \times \pi R^2$ is the magnetic moment. If current I is anticlockwise $\vec{B}$ is directed from O to P. For clockwise current $\vec{B}$ is from P to O.

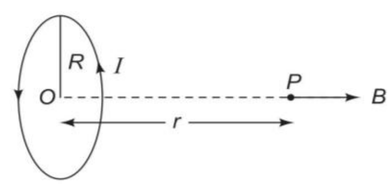


Fig. 13.10

(ix) *Magnetic field due to an electron (charge e) revolving in a circular orbit of radius r with speed v and frequency n* (Fig. 13.11)

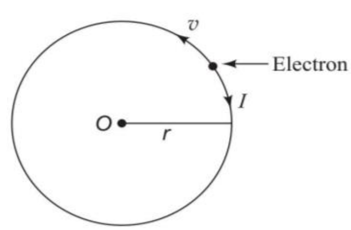


Fig. 13.11

Current along orbit $I = \dfrac{e}{T} = en = \dfrac{ev}{2\pi r}$

The direction of I is opposite to the direction of motion of the electron. Magnetic field at O is

$$B = \frac{\mu_0 I}{4r}$$

where $I = \dfrac{ev}{2\pi r}$ and is directed into the page.

Magnetic moment $M = IA = I \times \pi r^2 = \dfrac{evr}{2}$

(x) *Magnetic field due to a current carrying straight solenoid*

In the middle region $B = \mu_0 n I; n =$ no. of turns per unit length.

At the ends of solenoid, $B = \dfrac{\mu_0 n I}{2}$

For a toroid of radius R, $B = \mu_0 n I$, where $n = \dfrac{N}{L} = \dfrac{N}{2\pi R}$; $N =$ total no. of turns. Outside the solenoid, $B = 0$.

EXAMPLE 1 Figure 13.12 shows two stationary and infinitely long bent wires PQR and STU lying in the x-y plane and each carrying a current I as shown. Find the magnitude and direction of the magnetic field at origin O. Given $OQ = OT = a$

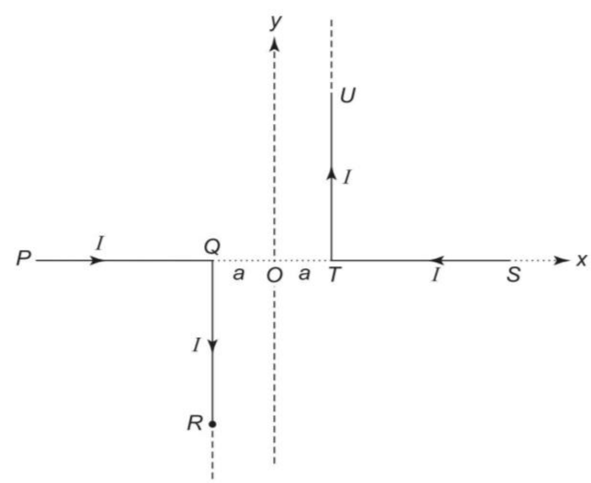


Fig. 13.12

SOLUTION As point O is along the line segments PQ and ST, the magnetic field at O due to PQ and ST is zero. The magnetic field at O due to wires QR and TU respectively are

$$\mathbf{B}_1 = \frac{\mu_0 I\,(\hat{\mathbf{k}})}{4\pi(OQ)} \text{ and } \mathbf{B}_2 = \frac{\mu_0 I\,(\hat{\mathbf{k}})}{4\pi(OT)}$$

both directed along the positive z-axis. The resultant field at O is ($\because OQ = OT = a$)

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = 2 \times \frac{\mu_0 I}{4\pi a}(\hat{\mathbf{k}}) = \frac{\mu_0 I}{2\pi a}\hat{\mathbf{k}}$$

EXAMPLE 2 Two infinitely long wires carrying equal current I in the opposite direction are placed perpendicular to the x-y plane. One wire is located at point P (0, a, 0) and the other wire at Q (0, $-a$, 0). Find the magnitude and direction of at point $A(x, 0, 0)$.

SOLUTION Refer to Fig. 13.13. Wire 1 carries a current I along the positive z-direction and wire 2 carries a current I along the negative z-direction.

$$OP = OQ = a,\ OA = x,\ PA = QA = r.$$

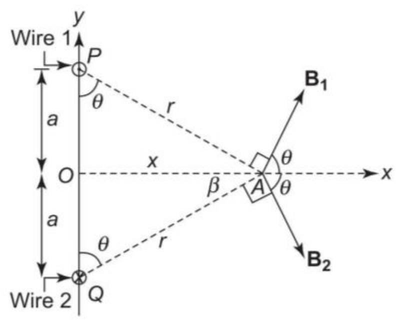


Fig. 13.13

Magnetic field at A due to wire 1 is

$$B_1 = \frac{\mu_0 I}{2\pi(PA)} = \frac{\mu_0 I}{2\pi\sqrt{a^2 + x^2}}$$

According to Biot-Savart law, $\mathbf{B}_1$ is perpendicular to both PA and wire 1 and therefore in the x-y plane. Similarly, magnetic field at A due to wire 2 is

$$B_2 = \frac{\mu_0 I}{2\pi\sqrt{a^2 + x^2}}$$

The y-components of $\mathbf{B}_1$ and $\mathbf{B}_2$ cancel each other but the x-components add up. These components are $B_1 \cos\theta$ and $B_2 \cos\theta$ both along the positive x-direction. Therefore, the resultant magnetic field at A is

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = (B_1\cos\theta + B_2\cos\theta)\,\hat{\mathbf{i}}$$

$$= \frac{\mu_0 I}{\pi\sqrt{(a^2 + x^2)}} \times \frac{a}{\sqrt{a^2 + x^2}} \times \hat{\mathbf{i}}$$

$$\left(\because \cos\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2 + x^2}}\right)$$

$$= \frac{\mu_0 I a}{\pi(a^2 + x^2)}\,\hat{\mathbf{i}}$$

EXAMPLE 3 Two wires A and B have the same length L and carry equal currents I. Wire A is bent into a circle and wire B is bent into a square. (a) Obtain expression for the magnitude of the magnetic field at (i) the centre of the circular loop and (ii) the centre of the square. Which wire produces a greater magnetic field at the centre?

SOLUTION

(a) (i) Radius r of wire A when it is bent into a circle is given by

$$2\pi r = L \Rightarrow r = \frac{L}{2\pi}$$

Magnetic field at the centre of the circular loop is

$$B_1 = \frac{\mu_0 I}{2r} = \frac{\mu_0 I \pi}{L} \qquad (1)$$

(ii) Refer to Fig. 13.14. The magnetic field at O due to wire PQ is ($OT = a$)

$$B_{PQ} = \frac{\mu_0 I}{4\pi a}(\sin 45° + \sin 45°)$$

$$= \frac{\mu_0 I}{2\sqrt{2}\,\pi a}$$

$$= \frac{4\,\mu_0 I}{\sqrt{2}\,\pi L} \qquad \left(\because a = \frac{L}{8}\right)$$

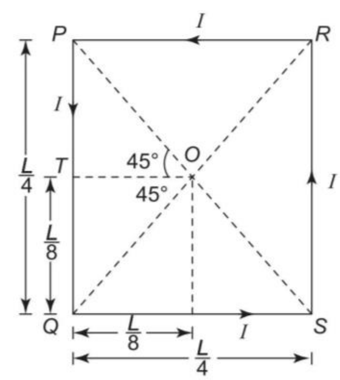


Fig. 13.14

Since centre O of the square is at the same distance from each side of the square and each arm carries the same current, the magnetic field due to each side of the square is of the same and in the same direction. Hence the total magnetic field at O is

$$B_2 = 4B_{PQ} = \frac{16\,\mu_0 I}{\sqrt{2}\,\pi L} = \frac{8\sqrt{2}\,\mu_0 I}{\pi L} \qquad (2)$$

Dividing (2) by (1) we get

$$\frac{B_2}{B_1} = \frac{8\sqrt{2}}{\pi^2} = \frac{8 \times 1.41}{(3.14)^2} = 1.16$$

Hence $B_2 > B_1$. The magnetic field at the centre due to the square loop will be greater than that due to the circular loop.

EXAMPLE 4 Figure 13.15 shows a wire loop *ABCDEA* carrying a current I as shown.

Given $AE = ED = a$ and $AB = CD = a/2$.

Find the magnitude and direction of the magnetic field at point F where $BF = CF = a/2$.

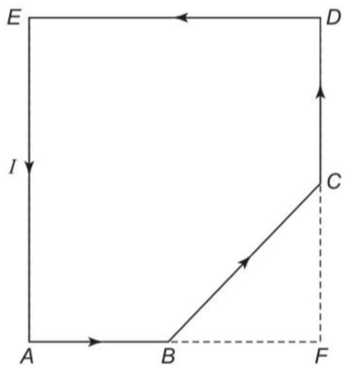


Fig. 13.15

SOLUTION Magnetic field at F is

$$\mathbf{B} = \mathbf{B}_{AB} + \mathbf{B}_{BC} + \mathbf{B}_{CD} + \mathbf{B}_{DE} + \mathbf{B}_{EA}$$

Since point F lies in line with current elements AB and CD, $B_{AB} = B_{CD} = 0$

Also $B_{DE} = B_{EA} = \dfrac{\mu_0 I}{4\pi a}(\sin 0° + \sin 45°) = \dfrac{\mu_0 I}{4\sqrt{2}\,\pi a}$

directed out of the page and towards the reader.

$$B_{BC} = \frac{\mu_0 I}{4\pi\, BC/2}(\sin 45° + \sin 45°)$$

directed into the page and away from the reader. Now

$$BC = \sqrt{BF^2 + FC^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}}$$

$$\therefore \quad B_{BC} = \frac{\mu_0 I}{4\pi\left(\frac{a}{2\sqrt{2}}\right)}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I}{\pi a}$$

directed into the page.

Now $B_{DE} + B_{EA} = \dfrac{\mu_0 I}{2\sqrt{2}\pi a}$ directed out of the page.

Since $B_{BC} > B_{DE} + B_{EA}$ the net field, **B** is directed into the page and has a magnitude

$$B = \frac{\mu_0 I}{\pi a} - \frac{\mu_0 I}{2\sqrt{2}\,\pi a} = \frac{\mu_0 I}{\pi a}\left(1 - \frac{1}{2\sqrt{2}}\right)$$

EXAMPLE 5 A wire *ABCDE* is bent as shown in Fig. 13.16. The wire carries a current I and the radius of the bent coil *BCD* is r. Find the magnitude and direction of the magnetic field at centre O.

SOLUTION The straight line segments AB and DE are collinear with O. Hence the magnetic field due to AB and DE at O is zero. Angle subtended at O by arc $BCD = 2\pi - \theta$. The magnetic field due to BCD at O is

$$B = \frac{\mu_0 I}{2r}\left(\frac{2\pi - \theta}{2\pi}\right)$$

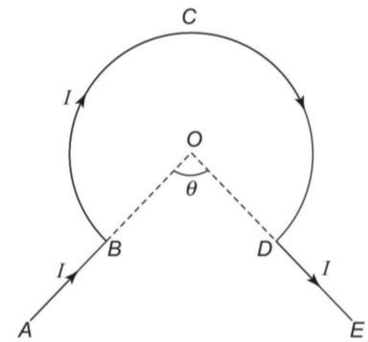


Fig. 13.16

The current through BCD is clockwise. Therefore, the direction of the magnetic field at O is into the page and away from the reader.

EXAMPLE 6 A wire *ABCDEFA* is bent as shown in Fig. 13.17 and caries a current I. The radius of the smaller arc ABC is $r_1 = r$ and that of the bigger arc is $r_2 = 2r$. Find the magnitude of the magnetic field at centre O.

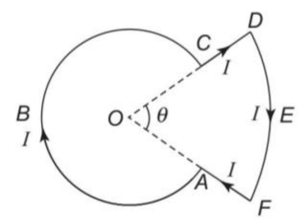


Fig. 13.17

SOLUTION Magnetic field due to arc ABC at O is

$$B_1 = \frac{\mu_0 I}{2r_1}\left(\frac{2\pi - \theta}{2\pi}\right)$$

Magnetic field due to arc DEF at O is

$$B_2 = \frac{\mu_0 I}{2r_2}\cdot\frac{\theta}{2\pi}$$

Since B_1 and B_2 are both directed into the page, the total magnetic field at O is

$$B = B_1 + B_2 = \frac{\mu_0 I}{2r_1}\left(\frac{2\pi - \theta}{2\pi}\right) + \frac{\mu_0 I}{2r_2}\,\frac{\theta}{2\pi}$$

Putting $r_1 = r$ and $r_2 = 2r$, we get

$$B = \frac{\mu_0 I}{2r}\left(1 - \frac{\theta}{4\pi}\right)$$

EXAMPLE 7 A long straight cylinder of radius R carries a current I which is uniformly distributed across its cross-section. Find the magnetic field at a point at a distance r from the axis of the cylinder in cases (a) $r > R$ and (b) $r < R$.

SOLUTION Figure 13.18 shows the cross-sectional view of the cylinder.

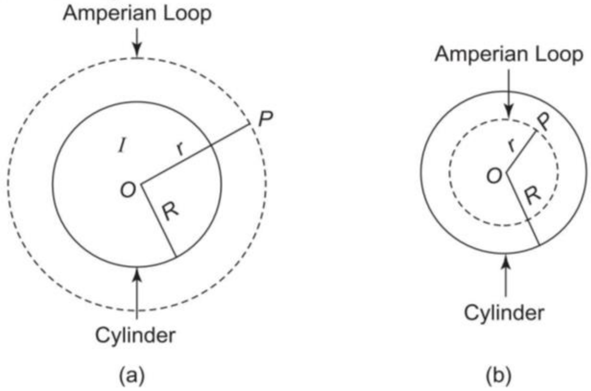


Fig. 13.18

Case (a) $r > R$. For this case, the Amperian loop is a circle of radius r concentric with the cross-section [Fig. 13.18 (a)]. For this loop, $L = 2\pi r$ and the current threading the loop is $i = I$. From Ampere's circuital law.

$$BL = \mu_0 i \Rightarrow B \times 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

Case (b) $r < R$. For this case, $L = 2\pi r$ [Fig. 13.18 (b)] and the current threading the loop is

i = current per unit cross-sectional area of the cylinder × cross-sectional area of the Amperian loop

$$= \frac{I}{\pi R^2} \times \pi r^2 = \frac{Ir^2}{R^2}$$

From Ampere's law,

$$B \times 2\pi r = \mu_0 i = \frac{\mu_0 I r^2}{R^2}$$

$$\Rightarrow \qquad B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r$$

Note

(1) For $r < R$, $B \propto r$ and for $r > R$, $B \propto \frac{1}{r}$. Figure 13.19 shows the variation of B with r.

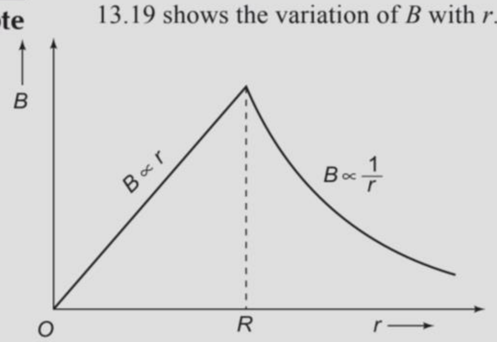


Fig. 13.19

(2) For a hollow cylinder, the current flows along its walls. Therefore, in the case $r < R$ [Fig. 13.18 (b)], no current threads the Amperian loop. Hence $B = 0$ for points inside a hollow cylinder.

EXAMPLE 8 The current density J (current per unit area) in a solid cylinder of radius R varies with distance r from its axis as $J = kr$ where k is a constant. Find the magnetic field at a point P where (a) $r > R$ and (b) $r < R$.

SOLUTION

$$\text{Current } I = \int J dA = \int kr \times (2\pi r dr)$$

Case (a) We take the Amperian loop of radius $r > R$. Since the loop is outside the cylinder, the current through the loop is

$$I = \int_0^R kr \times (2\pi r dr) = 2\pi k \int_0^R r^2 dr = \frac{2\pi\, kR^3}{3}$$

$$\therefore \qquad B \times 2\pi r = \mu_0 I = \frac{2\pi\, \mu_0 k R^3}{3} \Rightarrow B = \frac{\mu_0 k R^3}{3r}$$

Case (b) For $r < R$, the current through the Amperian loop is

$$I = \int_0^r kr(2\pi r dr) = \frac{2\pi k r^3}{3}$$

$$\therefore \qquad B \times 2\pi r = \mu_0 I = \frac{\mu_0 \times 2\pi k r^3}{3} \Rightarrow B = \frac{\mu_0 k r^2}{3}$$

EXAMPLE 9 Two long wires 1 and 2 carrying equal currents $I_1 = I_2 = 9$ A are held parallel to each other 6 cm apart as shown in Fig. 13.20. Find the magnetic field at (a) point P, (b) point Q and (c) point R.

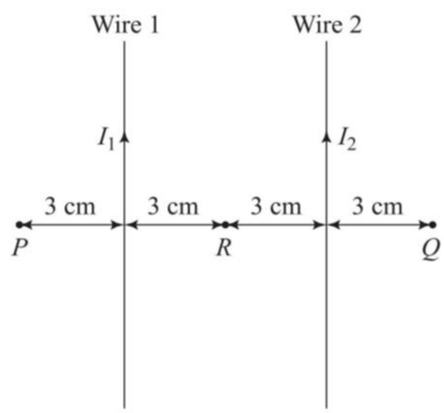


Fig. 13.20

SOLUTION (a) Magnitude of magnetic field at P due to wire 1 is

$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 9}{2\pi \times 3 \times 10^{-2}} = 6 \times 10^{-5}\ \text{T}$$

The direction of the field is perpendicular to plane of the page and towards the reader. The magnitude of magnetic field at P due to wire 2 is

$$B_2 = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 9}{2\pi \times 9 \times 10^{-2}} = 2 \times 10^{-5}\ \text{T}$$

The direction of this field is the same that of B_1. Thus, the net field at P is

$$B_P = B_1 + B_2 = 8 \times 10^{-5}\ \text{T}$$

and its direction is towards the reader.

(b) Similarly, the net field at Q will be 8×10^{-5} T and its direction is perpendicular to the page and away from the reader.

(c) At point R, the magnetic field due to wires 1 and 2 will have equal magnitude but opposite directions. Hence the net magnetic field at R will be zero.

EXAMPLE 10 Two long wires 1 and 2 are kept 8 cm apart and carry currents of $I_1 = 2$ A and $I_2 = 10$ A in opposite directions. At what distance from wire 1 will the resultant magnetic be zero?

(a) 1 cm (b) 2 cm

(c) 3 cm (d) 4 cm

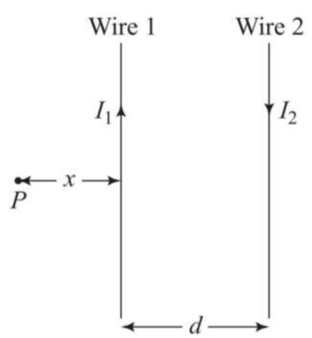


Fig. 13.21

SOLUTION Since the currents are in opposite directions, the resultant magnetic field cannot be zero at any point between the wires; it can be zero at a point to the left of wire 1 or to the right of wire 2. Let the net magnetic field be zero at point P at a distance x from wire 1 (Fig. 13.21). The magnetic field at P due to wire 1 is

$$B_1 = \frac{\mu_0 I_1}{2\pi x} \text{ directed towards the reader.}$$

The magnetic field at P due to wire 2 is

$$B_2 = \frac{\mu_0 I_2}{2\pi (d + x)} \text{ directed away from the reader.}$$

The resultant magnetic field will be zero if $B_1 = B_2$, i.e. if

$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi (d + x)}$$

$$\Rightarrow \quad \frac{I_1}{x} = \frac{I_2}{d + x}$$

$$\Rightarrow \quad \frac{2}{x} = \frac{10}{8 + x}$$

$$\Rightarrow \quad x = 2 \text{ cm}$$

EXAMPLE 11 Figure 13.22(a) shows a straight wire AB of length L carrying a current I. The magnitude of magnetic field at point P which is at a perpendicular distance $r = L$ from one end of wire is

(a) $\frac{\mu_0 I}{\sqrt{2}\,\pi L}$ (b) $\frac{\mu_0 I}{2\pi L}$

(c) $\frac{\mu_0 I}{2\sqrt{2}\,\pi L}$ (d) $\frac{\mu_0 I}{4\sqrt{2}\,\pi L}$

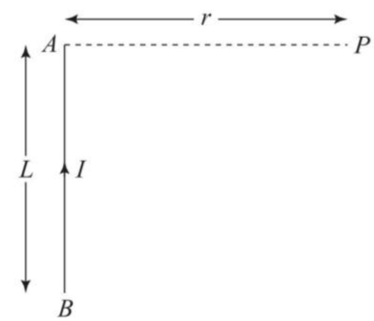


Fig. 13.22(a)

SOLUTION Refer to Fig. 13.22(b).

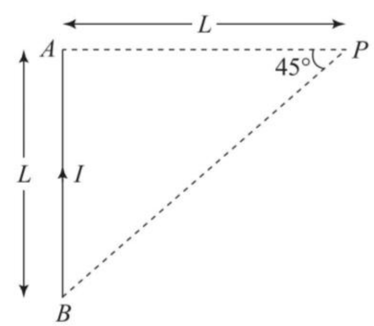


Fig. 13.22(b)

$$B = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$$

Here $\alpha = 45°$ and $\beta = 0°$ and $r = L$.

$$\therefore \quad B = \frac{\mu_0 I}{4\pi L}(\sin 45° + \sin 0°)$$

$$= \frac{\mu_0 I}{4\pi L} \times \frac{1}{\sqrt{2}}$$

So the correct choice is (d).

EXAMPLE 12 Figure 13.23(a) shows a straight wire AB of length L carrying a current I. The magnitude of magnetic field at point P on the perpendicular bisector of the wire at a distance $r = \frac{L}{2}$ is

(a) $\frac{\sqrt{2}\,\mu_0 I}{\pi L}$ (b) $\frac{\mu_0 I}{\sqrt{2}\,\pi L}$

(c) $\frac{\mu_0 I}{2\sqrt{2}\,\pi L}$ (d) zero

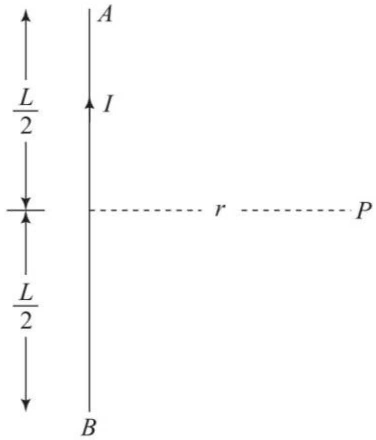


Fig. 13.23(a)

SOLUTION Refer to Fig. 13.23(b).

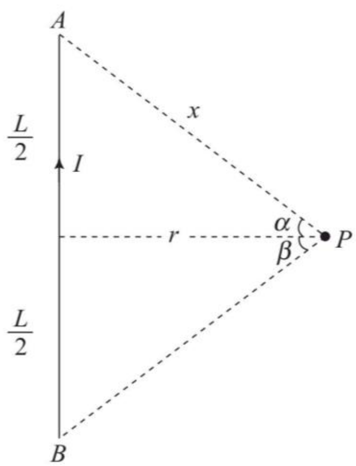


Fig. 13.23(b)

$$B = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$$

Here $\alpha = \beta$,

and $\sin\alpha = \sin\beta = \frac{L/2}{x}$

$$= \frac{L/2}{\sqrt{r^2 + \left(\frac{L}{2}\right)^2}}$$

$$= \frac{L}{\sqrt{4r^2 + L^2}}$$

$$= \frac{1}{\sqrt{2}} \qquad \left(\because\ r = \frac{L}{2}\right)$$

$$\therefore \qquad B = \frac{\mu_0 I}{4\pi \times \frac{L}{2}} \times \frac{2}{\sqrt{2}}$$

$$= \frac{\mu_0 I}{\sqrt{2}\,\pi L}, \text{ which is choice (b).}$$

EXAMPLE 13 A uniform straight wire of length L is turned into a circular wire loop of radius r. The diametrically opposite points P and Q are connected to a battery as shown in Fig. 13.24(a). If I is the current flowing through the battery, the magnetic field at centre O will be

(a) $\frac{\mu_0 I \pi}{L}$ (b) $\frac{\mu_0 I}{\pi L}$

(c) $\frac{\mu_0 I}{2\pi L}$ (d) zero

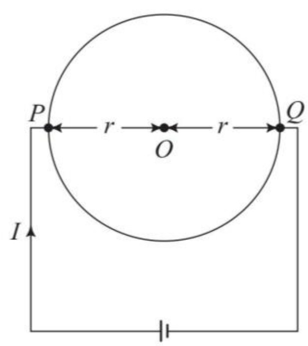


Fig. 13.24(a)

SOLUTION As shown in Fig. 13.24(b), current divides into two equal parts $I_1 = I_2 = I/2$, flowing along semicircular wires PRQ and PSQ. These equal currents produce equal and opposite magnetic fields at centre O. Hence the resultant magnetic field at O will be zero.

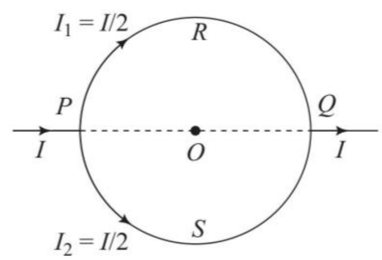


Fig. 13.24(b)

EXAMPLE 14 A uniform straight wire of length L is turned into a square. The points P and R are connected to a battery as shown in Fig. 13.25(a). If current I flows through the battery, the magnetic field at centre O of the square will be

(a) $\frac{8\sqrt{2}\mu_0 I}{\pi L}$ (b) $\frac{4\sqrt{2}\mu_0 I}{\pi L}$

(c) $\frac{2\sqrt{2}\mu_0 I}{\pi L}$ (d) zero

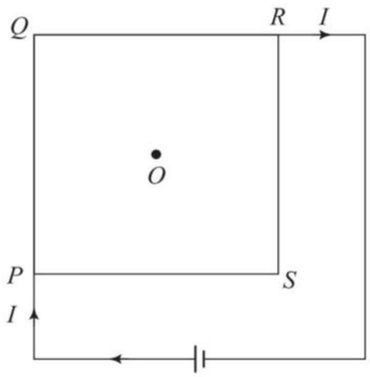


Fig. 13.25(a)

SOLUTION The current divides equally at P and R so that current $I/2$ flows in branch PQR and $I/2$ in branch PSR. Magnetic fields at O due to currents in QR and PS will be equal and opposite and will cancel each other. Similarly currents in PQ and SR will produce equal and oposite fields at O which will cancel each other. Hence the net magnetic field at O will be zero.

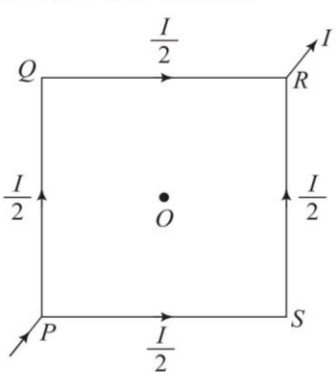


Fig. 13.25(b)

4. Force on a Moving Charge in a Magnetic Field

The force on a charge q moving with a velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ is given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

The direction of $\mathbf{F}$ is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$. The magnitude F of vector $\mathbf{F}$ is given by

$$F = qvB \sin\theta$$

where θ is the angle between vectors $\mathbf{v}$ and $\mathbf{B}$.

(1) $\mathbf{F} = 0$ if $\mathbf{v} = 0$, i.e. a charge at rest does not experience any magnetic force.

(2) $\mathbf{F} = 0$ if $\theta = 0$ or 180°, i.e. the magnetic force vanishes if $\mathbf{v}$ is either parallel or antiparallel to the direction of $\mathbf{B}$.

(3) F is maximum $= F_{max}$ if $\theta = 90°$, i.e. if $\mathbf{v}$ is perpendicular to $\mathbf{B}$, the magnetic force has a maximum value given by

$$F_{max} = qvB$$

The direction of the force when $\mathbf{v} \perp \mathbf{B}$ is given by Fleming's left hand rule.

(4) If $\mathbf{v}$ is perpendicular to both $\mathbf{E}$ and $\mathbf{B}$ and $\mathbf{E}$ is perpendicular to $\mathbf{B}$, then $\mathbf{F} = 0$ if $v = \frac{E}{B}$.

5. Motion of a Charged Particle in a Magnetic Field

Case (a): If $\mathbf{v}$ is perpendicular to $\mathbf{B}$, the particle describes a circle in the region of the magnetic field because $\mathbf{F} \perp \mathbf{v}$.

(i) The speed along the circular path is constant.

(ii) The kinetic energy is constant.

(iii) Velocity and momentum continually change.

(iv) The radius r of the circular path is given by

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where m = mass of particle and K = kinetic energy. If the particle is accelerated through a potential difference V, then $K = qV$.

(v) Time period of revolution is $T = \frac{2\pi m}{qB}$

(vi) Frequency of revolution is $\nu = \frac{qB}{2\pi m}$ which is independent of both v and r.

Case (b): If $\mathbf{v}$ is inclined to $\mathbf{B}$ at an angle θ, the particle moves in a helical path. The radius of helix is $r = \frac{mv\sin\theta}{qB}$, time period $T = \frac{2\pi m}{qB}$ and pitch of the helix $= v\cos\theta \times T$

Applications

(i) *The particle moving hori-zontally and entering a region of magnetic field B is as shown in Fig. 13.26. The particle describes a semi-circle of radius.*

$$r = \frac{mv}{qB}$$

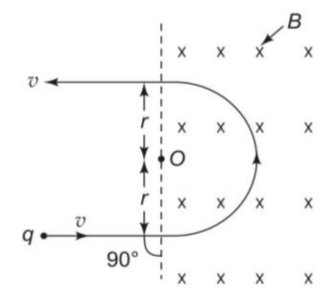


Fig. 13.26

Time spend in the region of magnetic field is

$$t = \frac{T}{2} = \frac{\pi m}{qB}$$

(ii) *If the particle enters the region of magnetic field as shown in Fig. 13.27, then*

$$r = \frac{mv}{qB}$$

and $$t = \frac{2\theta m}{qB}$$

where θ is in radian.

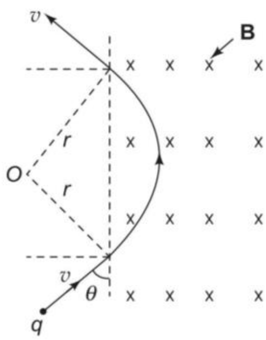


Fig. 13.27

(iii) *In Fig. 13.28, the particle will not be able to hit the wall if $d > r$, i.e.*

$$d > \frac{mv}{qB} \Rightarrow B > \frac{mv}{qd}$$

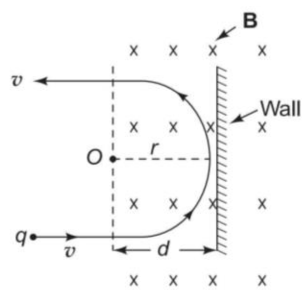


Fig. 13.28

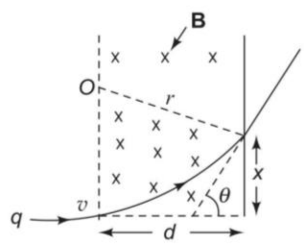


Fig. 13.29

(iv) *If $d < r$, as shown in Fig. 13.29, the deflection θ when the particle leaves the field is given by*

$$\sin\theta = \frac{d}{r} = \frac{qBd}{mv}$$

Linear defection $x = r(1 - \cos\theta)$

EXAMPLE 15 An electron emitted from a hot filament is accelerated through a potential difference of 18 kV and enters a region of a uniform magnetic field of 0.1 T with a certain initial velocity. What is the trajectory of the electron if the magnetic field (a) is transverse to the initial velocity and (b) makes an angle of 30° with the initial velocity? Mass of electron = 9×10^{-31} kg.

SOLUTION

$V = 18 \times 10^3$ V

$$\frac{1}{2}mv^2 = eV \Rightarrow v = \left(\frac{2eV}{m}\right)^{1/2}$$

$$= \left[\frac{2 \times (1.6 \times 10^{-19}) \times (18 \times 10^3)}{9 \times 10^{-31}}\right]^{1/2} = 8 \times 10^7 \text{ ms}^{-1}$$

(a) Since v is $\perp$ to **B**, $\theta = 90°$, the trajectory of the electron is circular having a radius

$$r = \frac{mv}{eB} = \frac{(9 \times 10^{-31}) \times (8 \times 10^7)}{(1.6 \times 10^{-19}) \times 0.1}$$

$$= 4.5 \times 10^{-3} \text{ m} = 4.5 \text{ mm}$$

(b) The trajectory of the electron is helical. The radius of heix is

$$r = \frac{mv_\perp}{eB} = \frac{mv}{eB} \times \sin\theta$$

$$= 4.5 \text{ mm} \times \sin 30° = 2.25 \text{ mm}$$

EXAMPLE 16 A long straight wire lying along the y-axis carries a current of 10 A along the positive y-direction. A proton moving with a velocity of 10^7 ms^{-1} is at a distance 5 cm from the wire at a certain instant. Find the magnitude and direction of the force acting on the proton at that instant if its velocity is directed

(a) along the negative x-direction

(b) along the positive y-direction and

(c) along the positive z-direction

SOLUTION Refer to Fig. 13.30.

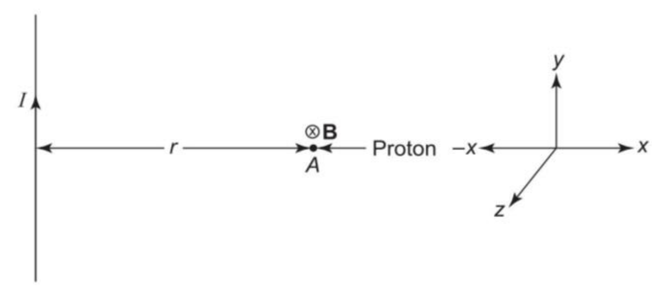


Fig. 13.30

Magnetic field at A is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.05} = 4 \times 10^{-5} \text{ T}$$

directed inwards along the negative z-direction

(a) $\theta = 90°$. Therefore, force on proton is

$F = qvB \sin\theta$

$= (1.6 \times 10^{-19}) \times 10^7 \times (4 \times 10^{-5}) \times \sin 90°$

$= 6.4 \times 10^{-17}$ N

According to Fleming's L.H. rule, the direction of the force is parallel to the wire and opposite to the direction of current I, i.e. **F** is along the negative y-direction

(b) $\theta = 90°$, $F = qvB = 6.4 \times 10^{-17}$ N. The force is directed towards the wire, i.e. along negative x-direction

(c) $\theta = 180°$. $F = qvB \sin 180° = 0$

EXAMPLE 17 A proton and an α-particle move perpendicular to a uniform magnetic field. The mass of an α-particle is four times that of a proton and its charge is twice that of a proton. Find the ratio of radii of the circular path followed by them if both

(a) have equal velocities,
(b) have equal kinetic energies and
(c) have equal linear momenta,
(d) are accelerated through the same potential difference.

SOLUTION Given $\frac{m_\alpha}{m_p} = 4$ and $\frac{q_\alpha}{q_p} = 2$

(a) $r = \frac{mv}{qB} \Rightarrow r_p = \frac{m_p v}{q_p B}$ and $r_\alpha = \frac{m_\alpha v}{q_\alpha B}$

$$\therefore \frac{r_p}{r_\alpha} = \frac{m_p}{m_\alpha} \times \frac{q_\alpha}{q_p} = \frac{1}{4} \times 2 = \frac{1}{2}$$

(b) $r = \frac{mv}{qB}$

Kinetic energy $K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$

$$\therefore r = \frac{m}{qB} \times \sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2\,mK}$$

$$\therefore \frac{r_p}{r_\alpha} = \frac{q_\alpha}{q_p} \times \sqrt{\frac{m_p}{m_\alpha}} = 2 \times \sqrt{\frac{1}{4}} = 1$$

(c) $r = \frac{mv}{qB} = \frac{p}{qB}$

$$\therefore \frac{r_p}{r_\alpha} = \frac{q_\alpha}{q_p} = 2$$

(d) $K = qV$. Therefore,

$$r = \frac{1}{qB}\sqrt{2mqV} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$\therefore \frac{r_p}{r_\alpha} = \sqrt{\frac{m_p}{m_\alpha} \times \frac{q_\alpha}{q_p}} = \sqrt{\frac{1}{4} \times 2} = \frac{1}{\sqrt{2}}$$

6. Force on a Current Carrying Conductor in a Magnetic Field

(i) *Force on a straight current carrying conductor in a magnetic field*

If a straight wire of length L carrying a current I is placed in a uniform magnetic field **B**, the force on it is given by (Fig. 13.31)

$$\mathbf{F} = I(\mathbf{L} \times \mathbf{B})$$

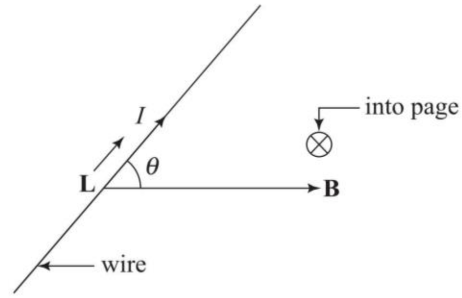


Fig. 13.31

where **L** is a vector whose magnitude is equal to the length of the wire and the direction is the same that of the current. The magnitude of ***F*** is

$$F = I\,L\,B \sin\theta$$

where θ is the angle between vectors **L** and **B**. The direction of **F** is given by the right hand screw rule. In the special case when **L** is perpendiculer to **B**, F is maximum = BIL.

In this case, the direction of **F** can be easily obtained by Fleming's Left Hand rule.

(ii) *Force on a straight conductor placed perpendicular to magnetic field (Fig. 13.32).*

$F = BIL$ upwards if current I is from left to right and downwards if I is from right to left (given by Fleming's Left Hand rule)

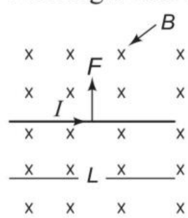


Fig. 13.32

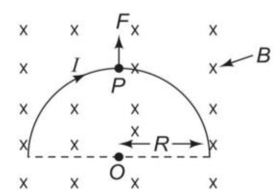


Fig. 13.33

(iii) *Force at point P on a semicircular wire of radius R (Fig. 13.33)*

$F = BI(2R) = 2BIR$ vertically upward for clockwise current and downward for anticlockwise current.

(iv) *Force on a circular wire of radius R (Fig. 13.34)*

Net force $F = F_1 - F_2 = 0$

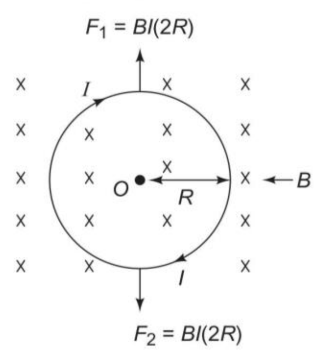


Fig. 13.34

(v) *Force per unit length between two long straight and parallel wires carrying currents I_1 and I_2 and separated by distance r is given by*

$$f = \frac{\mu_0 I_1 I_2}{2\pi r}$$

and is attractive if I_1 and I_2 are in the same direction and repulsive if I_1 and I_2 are in opposite directions. Force on a segment of length l of either wire is $F = f \times l$.

(vi) *Force on a rod carrying a current I_1 placed at a distance x from an infinitely long wire carrying a current I_2 as shown in Fig. 13.35.*

$$F = \frac{\mu_0 I_1 I_2}{2\pi r}\log_e\left(1+\frac{L}{x}\right) \text{ vertically upwards.}$$

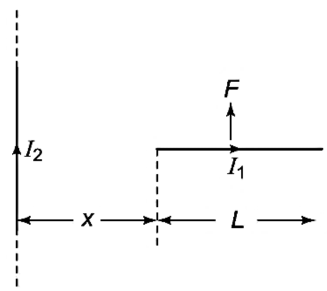


Fig. 13.35

(vii) *Force on a rectangular coil carrying a current I_1 placed at a distance x from an infinitely long wire carrying a current I_2 as shown in Fig. 13.36.*

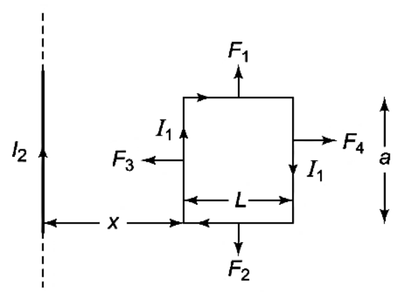


Fig. 13.36

Force F_1 and F_2 being equal and opposite cancel and F_3 and F_4 are given by expression above. Net force on coil is

$$F = F_3 - F_4 = \frac{\mu_0 I_1 I_2 a}{2\pi x} - \frac{\mu_0 I_1 I_2 a}{2\pi(x+L)}$$

$$\Rightarrow \quad F = \frac{\mu_0 I_1 I_2 a L}{2\pi x(x+L)}$$

directed towards the wire (attractive).

EXAMPLE 18 A uniform wire of length l is shaped into an equilateral triangle PQR which carries a current I as shown in Fig. 13.37. A uniform magnetic field **B** exists parallel to side QR. The magnitude of the force on wire PR is

(a) $\frac{\sqrt{3}BIL}{2}$ (b) $\frac{3BIL}{2\sqrt{2}}$

(c) $\frac{BIL}{2\sqrt{3}}$ (d) $\frac{BIL}{6}$

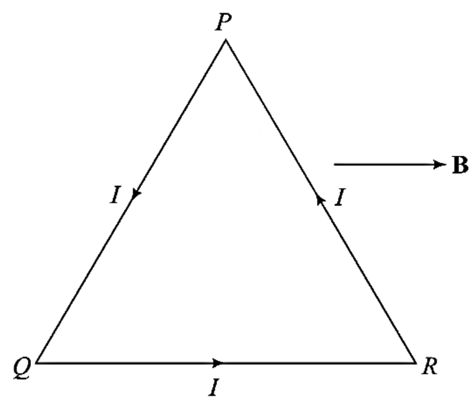


Fig. 13.37

SOLUTION $\mathbf{F} = I(\mathbf{l} \times \mathbf{B}) = I\,l\,B \sin\theta$.

Here $l = \frac{L}{3}$ and $\theta = 120°$. Therefore,

$$\therefore \quad F = I \times \frac{L}{3} \times B \times \sin 120°$$

$$= \frac{BIL}{2\sqrt{3}}$$

EXAMPLE 19 In Example 18 above, find the magnitude of the force on (a) wire PQ and (b) wire QR.

SOLUTION (a) For wire PQ, $\theta = 120°$. Therefore,

$$F = I \times \frac{L}{3} \times B \times \sin 60° = \frac{BIL}{2\sqrt{3}}$$

(b) For wire QR, $\theta = 0°$. Therefore, $F = 0$

EXAMPLE 20 A long wire carries a current of 10 A. A particle of charge q = 2.0 μC travels at a velocity of 5×10^5 ms^{-1} at a perpendicular distance 20 cm from the wire in a direction opposite to the direction of the current in the wire. Find the magnitude and direction of the force experienced by the particle.

SOLUTION Refer to Fig. 13.38. Given I = 10 A, r = 20 cm = 0.2 m, $v = 5 \times 10^5$ ms^{-1} and q = 2.0 μC = 2.0×10^{-6} C.

The magnetic field due to current I at the site of the charged particle is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}) \times 10}{2\pi \times 0.2}$$

$$= 1.0 \times 10^{-5} \text{ T}$$

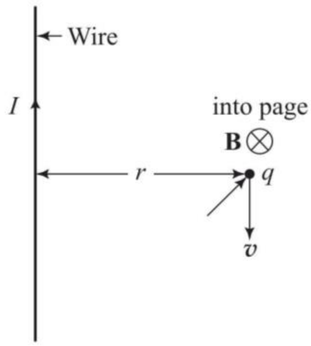


Fig. 13.38

According to Right Hand Thumb rule, the direction of the field is into the page. Hence the charged particle is moving perpendicular to the field, i.e. $\theta = 90°$. The force experienced by the particle is

$$F = q\,v\,B \sin\theta$$
$$= (2.0\times10^{-6})\times(5\times10^{5})\times(1.0\times10^{-5})\times\sin 90°$$
$$= 1.0\times10^{-5}\text{ N}$$

According to Fleming's Left Hand rule, the direction of the force is to the right, i.e. perpendicular to the wire and away from it.

EXAMPLE 21 Two thick and straight conductors *AB* and *CD* are placed horizontally and parallel to each other at a separation of 20 cm. They are connected to battery as shown in Fig. 13.39(a). A straight wire *PQ* of mass 150 g can slide on *AB* and *CD*. If the current I = 2 A, g = 10 ms^{-2} and a magnetic field **B** = 1.5 T is applied as shown in the figure, the minimum coefficient of friction between the wire and the conductors so that the wire is prevented from sliding is

(a) 0.1 (b) 0.2
(c) 0.3 (d) 0.4

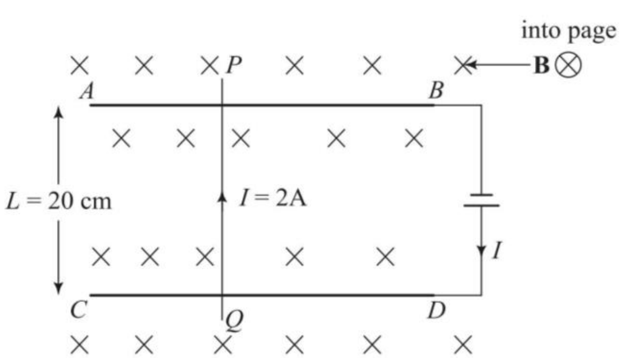


Fig. 13.39(a)

SOLUTION The magnitude of force on wire *PQ* due to the magnetic field is

$$F = B\,I\,L$$

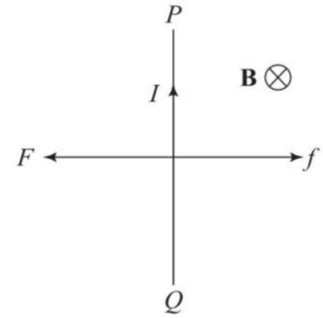


Fig. 13.39(b)

From Fleming's Left Hand rule, the direction of this force is to the left [Fig. 13.39(b)]. If the wire is just prevented from sliding on the conductor, the force of friction (f) which acts to the right must be equal to F. If $\mu_{\min}$ is the minimum coefficient of friction, then $f = \mu_{\min}\,mg$. Thus,

$$\mu_{\min}\,mg = B\,I\,L$$

$$\Rightarrow \quad \mu_{\min} = \frac{B\,I\,L}{mg} = \frac{1.5\times2\times0.2}{0.150\times10} = 0.4$$

EXAMPLE 22 In Example 21 above, if the coefficient of friction were one-fourth of $\mu_{\min}$, then wire *PQ* will

(a) stay at rest
(b) move to the left with an acceleration of 3.0 ms^{-2}
(c) move to the right will an acceleration of 1.5 ms^{-2}
(d) execute simple harmonic motion.

SOLUTION Given $\mu = \frac{1}{4}\mu_{\min} = \frac{0.4}{4} = 0.1.$

Referring to Fig. 13.39(b), the net force on wire *PQ* will be to the left.

$$F_{\text{net}} = B\,I\,L - \mu\,mg$$
$$= 1.5\times2\times0.2 - 0.1\times0.15\times10$$
$$= 0.6 - 0.15 = 0.45\text{ N}$$

$\therefore$ Acceleration $a = \frac{F_{\text{net}}}{m} = \frac{0.45}{0.15} = 3.0\text{ ms}^{-2}$. So, the correct choice is (b).

EXAMPLE 23 A particle of charge q is revolving in a circle of radius r with a constant speed v. The ratio of the magnitudes of magnetic moment and angular momentum of the particle is

(a) $\frac{q}{\sqrt{2}\,m}$ (b) $\frac{q}{m}$

(c) $\frac{q}{2m}$ (d) $\frac{\sqrt{2}\,q}{m}$

SOLUTION Time period $T = \dfrac{2\pi r}{v}$. The particle passes through a given point on the circle after one complete revolution. Hence the current round the circle is

$$I = \frac{q}{T} = \frac{qv}{2\pi r}$$

Magnetic moment M = current × area enclosed by the circular current

or $$M = I \times \pi r^2 = \frac{qv}{2\pi r} \times \pi r^2 = \frac{qvr}{2}$$

The direction of **M** is perpendicular to the plane of the circular loop.

Angular momentum $L = mvr$

The direcstion of **L** is the same as that of **M** if q is positive.

$$\therefore \quad \frac{M}{L} = \frac{qvr}{2} \times \frac{1}{mvr} = \frac{q}{2m}$$

So the correct choice is (c).

EXAMPLE 24 The battery of a car is connected to the motor by 50 cm long wires which are 1.0 cm apart. If the current in the wires is 200 A, find the force between the wires. Is the force attractive or repulsive.

SOLUTION Force per unit length is

$$f = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{(4\pi \times 10^{-7}) \times 200 \times 200}{2\pi \times (1.0 \times 10^{-2})} = 0.8 \text{ Nm}^{-1}$$

$$\therefore \quad F = f \times l = 0.8 \times 0.5 = 0.4 \text{ N}$$

Since the currents in the wires are in opposite direction, the force is repulsive.

EXAMPLE 25 A small rectangular loop $ABCD$ of sides 5 cm and 3 cm carries a current of 5 A. It is placed with its longer side parallel to a long straight conductor PQ of length 5 m at a distance of 2 cm from it as shown in Fig. 13.40. If the current in PQ is 20 A, find the net force on the loop. Is the loop attracted towards PQ or repelled away from it?

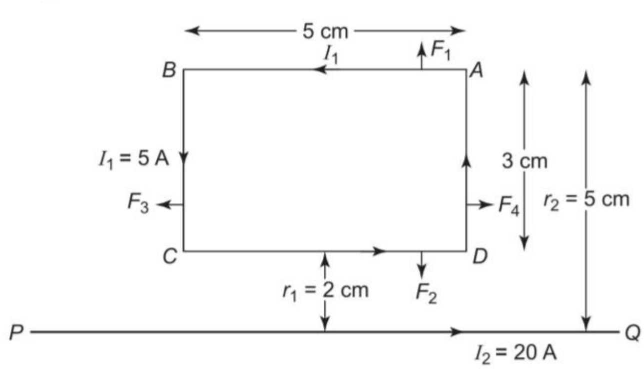


Fig. 13.40

SOLUTION Force exerted by PQ on AB is

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi r_1} \times AB$$

$$= \frac{(4\pi \times 10^{-7}) \times 5 \times 20 \times 5 \times 10^{-2}}{2\pi \times 5 \times 10^{-2}}$$

$= 2 \times 10^{-5}$ N (repulsive since I_1 and I_2 are in opposite directions)

Force exerted by PQ on CD is

$$F_2 = \frac{\mu_0 I_1 I_2}{2\pi r_2} \times CD$$

$$= \frac{(4\pi \times 10^{-7}) \times 5 \times 20 \times 5 \times 10^{-2}}{2\pi \times 2 \times 10^{-2}}$$

$= 5 \times 10^{-5}$ N (attractive since I_1 and I_2 are in the same direction)

From Fleming's L.H. rule, the magnetic field due to current in PQ is directed outwards (towards the reader) and perpendicular to the plane of the coil. Therefore, forces F_3 and F_4 on BC and AD are equal and opposite and hence cancel each other. Therefore, the net force on coil $ABCD$ is

$$F = F_2 - F_1 = 5 \times 10^{-5} - 2 \times 10^{-5} = 3 \times 10^{-5} \text{ N}$$

(attractive). Hence coil is attracted towards PQ.

EXAMPLE 26 A particle of charge q and mass m moving in region I with a velocity v enters normally a region II of width d where a uniform magnetic field B (directed inwards) exists as shown in Fig. 13.41. There is no magnetic field in regions I and III.

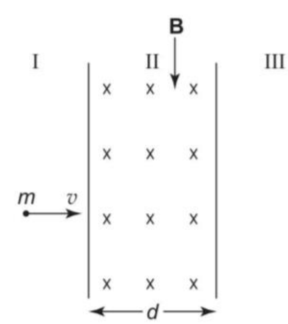


Fig. 13.41

(a) What is the maximum speed (v_{max}) of the particle so that it returns back in region I?

(b) What will happen if $v = \sqrt{2}\, v_{max}$?

SOLUTION

(a) Refer to Fig. 13.42(a). The particle describes a circular path of radius $r = mv/qB$ in region II. It will return to region I if it describes a semicircle in region II. This happens if

$$r < d \Rightarrow \frac{mv}{qB} < d \text{ or } v < \frac{qBd}{m}$$

$$\therefore v_{max} = \frac{qBd}{m}$$

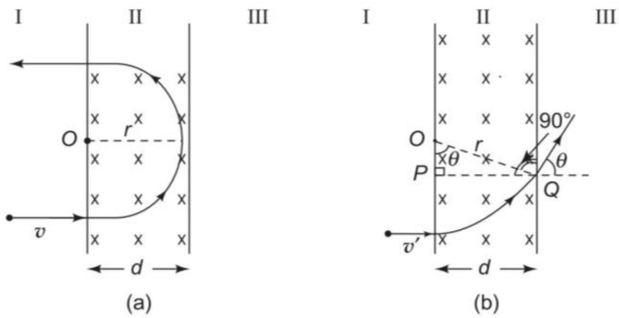


Fig. 13.42

(b) Refer to Fig. 13.32 (b). If $v > v_{max}$, the particle is cross over to region III after describing a circular trajectory in region II with O as the centre. In region III, the particle is moved along the tangent at Q. The particle will suffer a deviation θ.

In triangle OPQ

$$\sin\theta = \frac{PQ}{OQ} = \frac{d}{r}$$

$$\Rightarrow \quad \sin\theta = \frac{qBd}{mv} = \frac{v_{max}}{v}$$

If $v = \sqrt{2}\, v_{max}$, then $\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

EXAMPLE 27 A straight horizontal conducting rod of length 50 cm and mass 60 g is suspended by two vertical wires at its ends. A current of 5 A set up in the rod.

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero?

(b) What will be the tension in the wires if the direction of the current is reversed, keeping the magnetic field the same?

Ignore the mass of the wires and take $g = 10\ \text{m/s}^{-2}$.

SOLUTION Refer to Fig. 13.43.

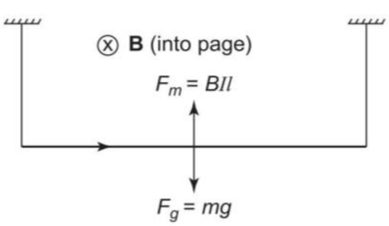


Fig. 13.43

(a) Tension in the wires will be zero if

$$F_m = F_g$$

$$\Rightarrow \quad BIl = mg$$

$$\Rightarrow \quad B = \frac{mg}{Il} = \frac{(60\times10^{-3})\times10}{5\times(50\times10^{-2})} = 0.24\ \text{T}$$

(b) If current I is reversed, force F_m acts downwards. Hence

Tension $T = BIl + mg$

$= 0.24 \times 5 \times 0.5 + 60 \times 10^{-3} \times 10 = 1.2$ N

7. Torque on a Current Carrying Coil in a Magnetic Field

The torque on a coil of N turns, area A carrying a current I in a magnetic field B is given by

$$\vec{\tau} = \mathbf{M} \times \mathbf{B}$$

Magnitude of torque is $\tau = MB\sin\theta = NIAB\sin\theta$ where $M = NIA$ is the magnetic moment and θ is the angle between the normal to the plane of coil and magnetic field.

The magnitude of torque on a coil in radial magnetic field in moving coil galvanometer is

$$\tau = k\alpha$$

where k is the restoring couple per unit twist and α is the deflection of the coil. For radial magnetic field, $\alpha = 90°$. Then

$$NIAB = k\alpha \Rightarrow I = \frac{k\alpha}{NAB} \quad \text{or } I \propto \alpha$$

Current sensitivity of the galvanometer is

$$C_s = \frac{\alpha}{I} = \frac{NAB}{k}$$

8. Torque on a Bar Magnet in a Magnetic Field

The magnetic dipole moment of a bar magnet of pole strength q and length $(2a)$ is defined as

$$\mathbf{M} = q(2\mathbf{a})$$

It is a vector pointing from the south to the north pole of a magnet.

Force on north pole N of magnet $= q\mathbf{B}$ (in the direction of $\mathbf{B}$)

Force on south pole S of the magnet $= -q\mathbf{B}$ (opposite to $\mathbf{B}$)

Thus the magnetic field exerts two equal, parallel and opposite forces on the magnet. The two forces, therefore, constitute a coupe which tends to rotate the magnet in the clockwise direction. The arm of the couple is 2 **a**. The torque is given by

$$\tau = \text{arm of the couple} \times \text{force}$$
$$= 2\mathbf{a} \times q\mathbf{B} = q(2\mathbf{a}) \times \mathbf{B}$$

or $$\tau = \mathbf{M} \times \mathbf{B}$$

where $\mathbf{M} = q(2\mathbf{a})$ is called the magnetic moment of the bar magnet. The direction of τ is perpendicular to both **M** and **B**. If **M** and **B** are both in the plane of the paper then the torque τ will be perpendicular to the plane of the paper and directed into it away from the reader. The magnitude of the torque is

$$\tau = MB \sin\theta$$

where θ is the angle between **M** and **B**.
The SI unit of **M** is Nm T^{-1} or JT^{-1} (joule per tesla).

9. Potential Energy of a Magnetic Dipole

The magnetic potential energy of a magnetic dipole in any orientation θ with an external uniform magnetic field **B** is defined as the work that an external agent must do to turn the dipole from its zero energy position ($\theta = 90°$) to the given position. θ.

$$U = -MB\cos\theta$$

In vector notation,

$$\mathbf{U} = -(\mathbf{M}\cdot\mathbf{B})$$

For stable equilibrium U is minimum. Hence $\theta = 0$ and $\tau = 0$. For unstable equilibrium, U is maximum i.e. $\theta = 180°$. Hence $\tau = 0$

10. Some Useful Tips

1. Magnetic dipole moment of a bar magnet is $M = m \times l$, where m is pole strength and l is the length of the magnet. The value of M depends on the volume of the magnet.
   (a) If a magnet is cut into two equal parts by cutting it by a plane along its length, its volume is halved, Hence the magnetic dipole moment of a piece is halved $= M/2$. The pole strength $m = \dfrac{M}{l}$ is also halved as length l remains the same.

   (b) If a magnet is cut into two equal parts by cutting it by a plane transverse to its length, the volume and length are both halved. Hence the magnetic moment becomes $M/2$ but pole strength m remains the same.

   (c) If a wire of magnetic dipole moment M and length l is bent as shown in Fig. 13.44, the distance between the pole becomes $\dfrac{1}{\sqrt{2}}$ and magnetic moment becomes

   $$M' = m \times \frac{l}{\sqrt{2}} = \frac{M}{\sqrt{2}}$$

   If the wire is bent as shown in Fig. 13.45, the magnetic moment becomes

   $$M' = m \times \frac{l}{\sqrt{2}} = \frac{M}{\sqrt{2}}$$

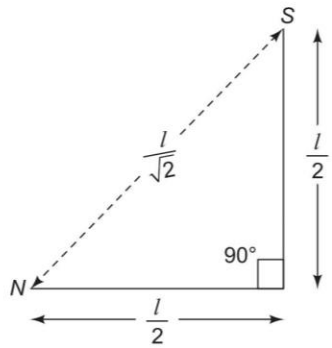


Fig. 13.44

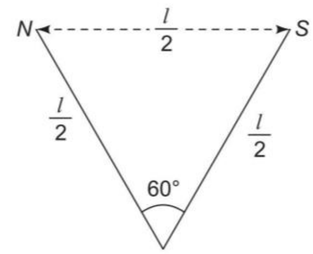


Fig. 13.45

> **Note**: Magnetic dipole moment M is a vector quantity directed from south pole to north pole.

2. A bar magnet placed in a uniform magnetic field experiences no net force but experiences a torque $\vec{\tau} = \boldsymbol{M} \times \boldsymbol{B}$. The magnitude of $\vec{\tau}$ is $\tau = MB\sin\theta$ where θ is the angle between $\boldsymbol{M}$ and $\boldsymbol{B}$.
   (a) when $\theta = 90°$, $\tau_{max} = MB$
   (b) when $\theta = 0°$, $\tau_{min} = 0$ (stable equilibrium)
   (c) when $\theta = 180°$, $\tau_{min} = 0$ (unstable equilibrium)
   Potential energy is $U = -\boldsymbol{M}.\boldsymbol{B} = -MB\cos\theta$. When $\theta = 0°$, P.E is minimum $U_{min} = -\boldsymbol{MB}$. U is max $= U_{max} = MB$ when $\theta = 180°$
   (d) Work done in rotating the magnet from θ_1 to θ_2 is
   $W = MB(\cos\theta_1 - \cos\theta_2)$
   (e) In a non-uniform magnetic field, a bar magnet experiences a force as well as a torque.

3. The time period of a bar magnet oscillating in a uniform magnetic field is $T = 2\pi\sqrt{\dfrac{I}{MB}}$, where I is the moment of inertia of the bar magnet $= \dfrac{ml^2}{12}$, m = mass of magnet and l = length of magnet.
   (a) If a bar magnet is cut into two equal parts by cutting along its length, then each part has $M' = M/2$ and $I' = I/2$. Hence $T' = T$.
   (b) If a bar magnet is cut into two equal parts by cutting perpendicular to its length, then each part has $M' = M/2$ and $I' = I/8$. Hence $T' = T/2$.
   (c) If two bar magnets of magnetic moments M_1 and M_2 are placed one on top of the other as shown in Fig. 13.46, then time period is given by (since $I = I_1 + I_2$ and $M = M_1 + M_2$)

$$T_1 = 2\pi\sqrt{\frac{(I_1 + I_2)}{(M_1 - M_2)B}}$$

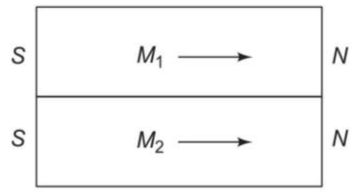


Fig. 13.46

If the magnets are placed as shown in Fig. 13.47, then $I = I_1 + I_2$ but $M = M_1 - M_2$ and

$$T_2 = 2\pi\sqrt{\frac{(I_1 + I_2)}{(M_1 - M_2)B}}$$

T_1 and T_2 are related as $\dfrac{M_1}{M_2} = \dfrac{(T_2^2 + T_1^2)}{(T_2^2 - T_1^2)}$

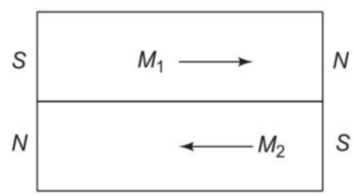


Fig. 13.47

4. Magnetic field due to a bar magnet
   (a) At a point P on axial line (Fig. 13.48)

$$B_a = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2} \text{ parallel to } \vec{M} = m \times 2\vec{l}\,.$$

For a very short magnet ($l << r$), $B_a = \dfrac{\mu_0}{4\pi}\dfrac{2M}{r^3}$

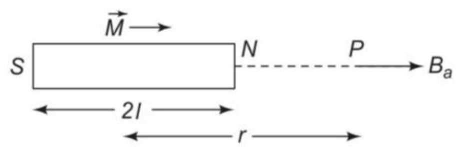


Fig. 13.48

   (b) At a point Q on the equatorial line (Fig. 13.49)

$$B_e = \frac{\mu_0}{4\pi}\frac{M}{(r^2 + l^2)^{3/2}} \text{ antiparallel to } \mathbf{M}$$

For $l << r$,

$$B_e = \frac{\mu_0 M}{4\pi r^3}$$

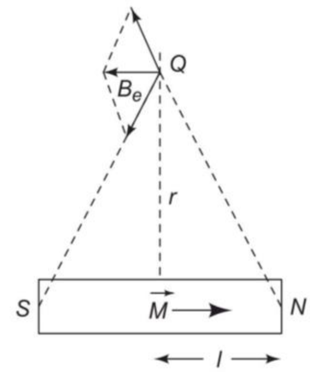


Fig. 13.49

EXAMPLE 28 A closely wound solenoid of 1000 turns and area of cross-section 5 cm^2 carries a current of 3 A. It is suspended through its centre (a) what is the magnetic moment? (b) What is the force and torque acting on the solenoid if a uniform magnetic field of 8×10^{-2} T is set up at an angle of 30° with the axis of the solenoid?

SOLUTION

(a) Magnetic moment $M = NIA = 1000 \times 3 \times (5 \times 10^{-4})$ $= 1.5\ \text{JT}^{-1}$ or Am^2

(b) Since the magnetic field is uniform, the force acting on the solenoid is zero

$$\text{Torque } \tau = MB \sin\theta = 1.5 \times (8 \times 10^{-2}) \times \sin 30° = 6 \times 10^{-2}\ \text{J}$$

EXAMPLE 29 In a hydrogen atom, the electron moves in a circular orbit of radius 0.5 Å making 10^{16} revolutions per second. Calculate the magnetic moment associated with the orbital motion of electron.

SOLUTION $M = \pi e v r^2 = 3.14 \times (1.6 \times 10^{-19}) \times 10^{16} \times (0.5 \times 10^{-10})^2 = 1.26 \times 10^{-23}$ Am^2

EXAMPLE 30 A bar magnet is suspended at a place where it is acted upon by two magnetic fields which are inclined to each other at an angle of 75°. One of the fields has a magnitude $\sqrt{2} \times 10^{-2}$ T. The magnet attains stable equilibrium at an angle of 30° with this field. Find the magnitude of the other field.

SOLUTION Magnetic field $\mathbf{B}_1$ exerts anticlockwise torque τ_1 to orient **M** along itself and magnetic field $\mathbf{B}_2$ exerts a stockwise torque τ_2 to orient **M** along itself (Fig. 13.50). For equilibrium,

$$\tau_1 = \tau_2$$

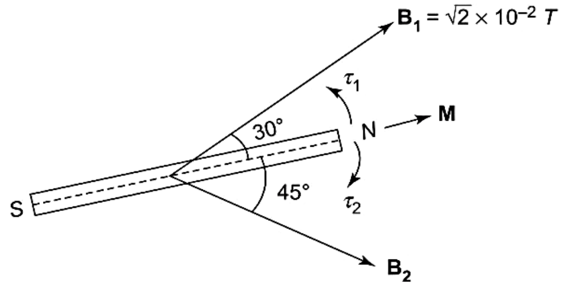


Fig. 13.50

$$MB_1 \sin\theta_1 = MB_2 \sin\theta_2$$

$$\Rightarrow \quad B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2} = \frac{\sqrt{2}\times 10^{-2} \times \sin 30°}{\sin 45°} = 10^{-2}\ \text{T}$$

EXAMPLE 31 A uniform wire is bent into the shape of an equilateral triangle of side a. It is suspended from a vertex at a place where a uniform magnetic field B exists parallel to its plane. Find the magnitude of the torque acting on the coil when a current I is passed through it.

SOLUTION Area of the coil is ($AB = a$, $BD = a/2$)

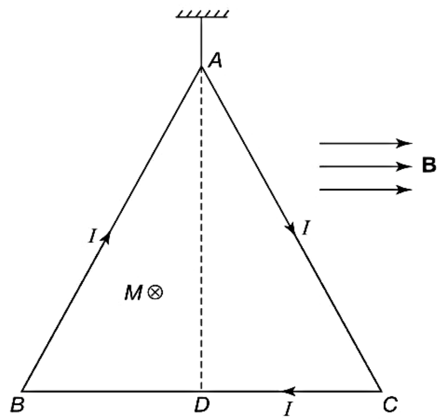


Fig. 13.51

$$A = 2 \times \text{area of triangle } ABD$$

$$= 2 \times \left(\frac{1}{2} \times AD \times BD\right)$$

$$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} a \times \frac{a}{2}$$

$$= \frac{\sqrt{3}}{4} a^2$$

Magnetic moment of the loop is

$$M = IA = I \times \frac{\sqrt{3}}{4} a^2$$

Since the current is clockwise, the direction of vector **M** is perpendicular to the plane of the coil directed inwards as shown in Fig. 13.51. Hence $\theta = 90°$. The magnitude of the torque acting on the coil is

$$\tau = MB \sin\theta = \frac{\sqrt{3}}{4} Ia^2 B \sin 90° = \frac{\sqrt{3}}{4} Ia^2 B$$

EXAMPLE 32 A wire loop $ABCD$ carrying a current I_2 is placed on a frictionless horizontal table as shown in Fig. 13.52. A long straight wire PQ carrying a current I_1 is placed at a distance a from side $AB = l$. Find the work done by the magnetic field in shifting the wire from position PQ to position $P'Q'$.

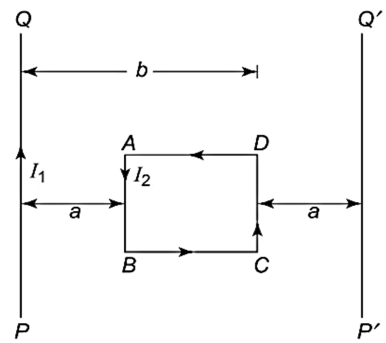


Fig. 13.52

SOLUTION

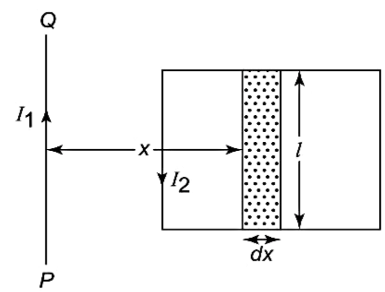


Fig. 13.53

Divide the loop into a large number of elements each of a very small width dx. Consider one such element at a distance x from PQ [see Fig. 13.53]. The magnetic field at any point on the element is

$$B = \frac{\mu_0 I_1}{2\pi x} \text{ directed inwards}$$

Magnetic moment of the element is

$$dM = I_2 \times \text{area of element}$$

$$= I_2 \times l dx$$

Since the current in the coil is anticlockwise, d**M** is directed outwards. Hence angle between vectors **B** and d**M** is $\theta = 180°$.

Potential energy when the wire is a distance x from the elements is

$$dU = -d\mathbf{M}\cdot\mathbf{B} = -dM \times B\cos\theta$$

$$= -I_2\, ldx \times \frac{\mu_0 I_1}{2\pi x} \cos 180°$$

$$= \frac{\mu_0 I_1 I_2 l}{2\pi} \frac{dx}{x}$$

∴ Potential energy of the system when the wire is at position PQ is

$$U_{PQ} = \frac{\mu_0 I_1 I_2 l}{2\pi} \int_a^b \frac{dx}{x} = \frac{\mu_0 I_1 I_2 l}{2\pi} \ln\left(\frac{b}{a}\right)$$

By symmetry, the potential energy of the system when the wire is shifted to position $P'Q'$ is

$$U_{P'Q'} = -U_{PQ}$$

∴ Work done in shifting the wire from position PQ to $P'Q'$ is

$$W = -(U_{P'Q'} - U_{PQ})$$

$$= 2U_{PQ} = \frac{\mu_0 I_1 I_2 l}{\pi} \ln\left(\frac{b}{a}\right)$$

11. Earth's Magnetic Field

The magnetic field of the earth is described in terms of the following three parameters.

(1) Magnetic Declination

The angle between the magnetic meridian at a place and the geographic meridian at that place is called magnetic declination. The magnetic meridian at a place is a vertical plane containing the magnetic axis of a freely suspended small magnet when it has settled in the earth's field. The geographic meridian at a place is a plane containing the place and the earth's axis of rotation.

The magnetic declination is different at different places on the surface of the earth.

(2) Angle of Dip

The angle between the horizontal component and the total magnetic field of the earth is called the angle of dip. This angle is also different at different places on the surface of the earth. Naturally, on the magnetic equator the angle of dip is zero, because the dip needle rests horizontally at the magnetic equator. The needle rests vertically at the two magnetic poles, i.e. the angle of dip is 90° at the magnetic poles. At other places, its value lies between 0° and 90°.

(3) Horizontal Component

The total magnetic field strength B at a place has a horizontal component B_H given by

$$B_H = B \cos \theta$$

where θ is the dip angle at that place. At the magnetic poles, $B_H = B \cos 90° = 0$ and at the magnetic equator $B_H = B \cos 0° = B$. The value of B_H thus differs from place to place on the surface of the earth.

12. Neutral Points

A neutral point is a point in space where the field due to the magnet is completely neutralized by the horizontal component of the earth's magnetic field. If $\mathbf{B}_m$ is the magnetic field due to the magnet and $\mathbf{B}_H$ the horizontal component of the earth's magnetic field, then at the neutral point, $\mathbf{B}_m$ and $\mathbf{B}_H$ are equal in magnitude but opposite in direction.

The magnetic field due to a short dipole at a distance r on the equatorial line is given by

$$B_m = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$$

where m is the magnetic moment of the magnet. On the axial line,

$$B_m = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

13. Relation between B and H

Magnetic induction field **B** and magnetic field intensity **H** are related as

$$\mathbf{B} = \mu \mathbf{H}$$

The constant μ is called the *permeability* of the material. If the material is removed leaving a vacuum inside the solenoid, the magnetic field reduces to B_0 given by

$$B_0 = \mu_0 H$$

where μ_0 is the permeability of vacuum.

The ratio μ/μ_0 is called the relative permeability of the material:

$$\mu_r = \frac{\mu}{\mu_0}$$

14. Relation between M and H

Intensity of magnetisation **I** and magnetising field intensity **H** are related as

$$\mathbf{I} = \chi \mathbf{H}$$

where χ is the susceptibility of the material.

Also $\mu_r = 1 + \chi$

15. Three Kinds of Magnetic Materials

Different magnetic materials have different χ values which may vary over a very wide range. Depending on the value and sign of χ, magnetic materials are divided into the following three categories.

(1) Diamagnetic Materials

Materials for which χ is small and negative are called *diamagnetic*. In such materials, the individual magnetic moments of the various atoms tend to cancel out completely. So, in the normal state, the atoms of such substances (in the absence of external field) have no magnetic moment at all. They have a small and negative susceptibility χ. For bismuth $\chi = -0.00015$ and $\mu_r (= 1 + \chi)$ is slightly less than unity. Bismuth, copper, lead, water and sodium chloride are all diamagnetic.

When a sample of a diamagnetic material is placed in a magnetic field **B**, a net magnetic moment is induced in it which is proportional to **B** but opposite in direction. If **B** is non–uniform, the sample tends to move from the region where the field is strong to the region where it is weak.

(2) Paramagnetic Materials

Paramagnetic materials are those whose atoms, in the normal state, already have an intrinsic non-zero magnetic moment, even in the absence of an external magnetic field. Examples are aluminium, sodium, copper chloride and liquid oxygen. Paramagnetics have a small but positive value of χ . The susceptibility χ satisfies the relation

$$\chi = \frac{C}{T}$$

where C is a constant called the *Curie constant* and the above relation is called *Curie's law.*

(3) Ferromagnetic Materials

Materials having a very high (of the other of 1000 or more) and positive susceptibility are called ferromagnetic. Examples are iron, nickel, cobalt and numerous alloys. In such materials there are strong interactions between the individual magnetic moments of the various atoms. Above a certain temperature called the *Curie point,* ferromagnetics become paramagnetics.

SECTION 1 Multiple Choice Questions with One Correct Choice

Level A

1. A wire of length L carrying a current I is bent into a circle. The magnitude of the magnetic field at the centre of the circle is

(a) $\frac{\pi \mu_0 I}{L}$ (b) $\frac{\mu_0 I}{2L}$

(c) $\frac{2\pi \mu_0 I}{L}$ (d) $\frac{\mu_0 I}{2\pi L}$

2. A part of a long wire carrying a current I is bent into a circle of radius r as shown in Fig. 13.54. The net magnetic field at the centre O of the circular loop is

(a) $\frac{\mu_0 I}{4r}$ (b) $\frac{\mu_0 I}{2r}$

(c) $\frac{\mu_0 I}{2\pi r}(\pi + 1)$ (d) $\frac{\mu_0 I}{2\pi r}(\pi - 1)$

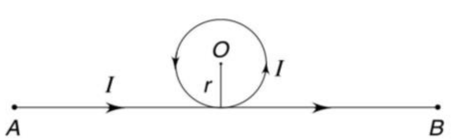


Fig. 13.54

3. A wire is bent into a circular loop of radius R and carries a current I. The magnetic field at the centre of the loop is B. The same wire is now bent into a double loop. If both loops carry the same current I in the same direction, the magnetic field at the centre of the double loop will be

(a) zero (b) 2 B

(c) 4 B (d) 8 B

4. In Q. 3, if the currents I in the two loops are in opposite directions, the magnetic field at the centre of the double loop will be

(a) zero (b) B

(c) $\frac{B}{4}$ (d) 4 B

5. The magnetic field at the centre of a circular coil of radius r and carrying a current I is B. What is the magnetic field at a distance $x = \sqrt{3}\ r$ from the centre, on the axis of the coil?

(a) B (b) 2 B

(c) 4 B (d) 8 B

6. A coil of 50 turns and 10 cm diameter has a resistance of 10 Ω. What must be the potential difference across the coil so as to nullify the earth's magnetic field H = 0.314 G at the centre of the coil?

(a) 0.5 V (b) 1.0 V

(c) 1.5 V (d) 2.0 V

7. The magnetic field at the point of intersection of diagonals of a square loop of side L carrying a current I is

(a) $\frac{\mu_0 I}{\pi L}$ (b) $\frac{2\mu_0 I}{\pi L}$

(c) $\frac{\sqrt{2}\mu_0 I}{\pi L}$ (d) $\frac{2\sqrt{2}\mu_0 I}{\pi L}$

8. Two identical coils carry equal currents and have a common centre, but their planes are at right angles to each other. What is the magnitude of the resultant magnetic field at the centre if the field due to one coil alone is B?

(a) zero (b) $B/\sqrt{2}$

(c) $\sqrt{2}\ B$ (d) $2\ B$

9. The direction of the force experienced by a charged particle moving with a velocity **v** in a uniform magnetic field **B** is

(a) parallel to **v** and perpendicular to **B**
(b) parallel to **B** and perpendicular to **v**
(c) parallel to both **v** and **B**
(d) perpendicular to both **v** and **B**.

10. When a charged particle moves perpendicular to a uniform magnetic field, its

(a) energy and momentum both change
(b) energy changes but momentum remains unchanged
(c) momentum changes but energy remains unchanged
(d) energy and momentum both do not change.

11. A particle is projected into a uniform magnetic field acting perpendicular to the plane of the paper. The field points into the paper, indicated by × which represents the tail of the field vector. The trajectory shown could be that of a (see Fig. 13.55)

(a) neutron (b) proton

(c) alpha particle (d) electron

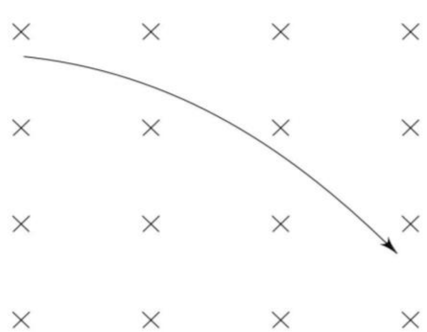


Fig. 13.55

12. A proton with kinetic energy K describes a circle of radius r in a uniform magnetic. An α–particle with kinetic energy K moving in the same magnetic field will describe a circle of radius

(a) $\frac{r}{2}$ (b) r

(c) $2\ r$ (d) $4\ r$

13. An α–particle moving with a velocity v in a uniform magnetic field is moving in a circular path at frequency ν called the cyclotron frequency. The cyclotron frequency of a proton moving with a speed $2v$ in the same magnetic field will be

(a) $\frac{\nu}{4}$ (b) $\frac{\nu}{2}$

(c) ν (d) $2\ \nu$

14. In the region around a charge at rest, there is

(a) electric field only
(b) magnetic field only
(c) neither electric nor magnetic field
(d) electric as well as magnetic field.

15. In the region around a moving charge, there is

(a) electric field only
(b) magnetic field only
(c) neither electric nor magnetic field
(d) electric as well as magnetic field.

16. An electron is accelerated to a high speed down the axis of a cathode ray tube by the application of a potential difference of V volts between the cathode and the anode. The particle then passes through a uniform transverse magnetic field in which it experiences a force F. If the potential difference between the anode and the cathode is increased to 2 V, the electron will now experience a force

(a) $F/\sqrt{2}$ (b) $F/2$

(c) $\sqrt{2}\ F$ (d) $2\ F$

17. A magnetic needle is kept in a non-uniform magnetic field. It experiences

(a) a force as well as a torque
(b) a force but no torque
(c) a torque but no force
(d) neither a force nor a torque

18. A conducting circular loop of radius r carries a constant i. It is placed in a uniform magnetic field B such that B is perpendicular to the plane of the loop. The magnetic force acting on the loop is

(a) $i\ r\ B$ (b) $2\ \pi\ i\ r\ B$

(c) zero (d) $\pi\ i\ r\ B$

19. Two long parallel wires separated by a distance R have equal current I flowing in each. The magnetic field of

one exerts a force F on the other. The distance R is increased to $2R$ and the current in each wire is reduced from I to $I/2$. What is the force between them now?

(a) $4F$ (b) F

(c) $F/4$ (d) $F/8$

20. A straight horizontal conducting rod of length 0.5 m and mass 50 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires. What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero? Ignore the mass of the wires and take $g = 10\ ms^{-2}$.

(a) 0.1 T (b) 0.2 T

(c) 0.3 T (d) 0.4 T

21. In the hydrogen atom the electron moves around the proton with a speed of $2.0 \times 10^6\ ms^{-1}$ in a circular orbit of radius 5.0×10^{-11} m. What is the equivalent dipole moment?

(a) $2 \times 10^{-24}\ Am^2$ (b) $4 \times 10^{-24}\ Am^2$

(c) $8 \times 10^{-24}\ Am^2$ (d) $16 \times 10^{-24}\ Am^2$

22. In Q. 21, what is the strength of the magnetic field at the site of the proton (i.e. at the centre of the orbit)?

(a) 12.8 T (b) 6.4 T

(c) 3.2 T (d) 1.6 T

23. In a hydrogen atom, an electron of mass m and charge e is in an orbit of radius r making n revolutions per second. If the mass of the hydrogen nucleus is M, the magnetic moment associated with the orbital motion of the electron is

(a) $\dfrac{\pi n e r^2 m}{M}$ (b) $\dfrac{\pi n e r^2 M}{m}$

(c) $\dfrac{\pi n e r^2 m}{(M+m)}$ (d) $\pi n e r^2$

24. A wire of length l metres carrying a current I amperes is bent in the form of a circle. The magnitude of the magnetic moment is

(a) $\dfrac{l I^2}{2\pi}$ (b) $\dfrac{l I^2}{4\pi}$

(c) $\dfrac{l^2 I}{2\pi}$ (d) $\dfrac{l^2 I}{4\pi}$

25. An electric field of magnitude 5000 V m^{-1} and a magnetic field of magnitude of 0.01 T exist at right angles to each other in a region of space. With what speed must electrons be projected at right angles to both the fields so that they experience no net force?

(a) $5 \times 10^5\ ms^{-1}$ (b) $3 \times 10^6\ ms^{-1}$

(c) $5 \times 10^7\ ms^{-1}$ (d) $3 \times 10^8\ ms^{-1}$

26. Two circular current carrying coils of radii 3 cm and 6 cm are each equivalent to a magnetic dipole having equal magnetic moments. The currents through the coils are in the ratio of

(a) $\sqrt{2} : 1$ (b) $2 : 1$

(c) $2\sqrt{2} : 1$ (d) $4 : 1$

27. A proton (mass = 1.7×10^{-27} kg) moves with a speed of $5 \times 10^5\ ms^{-1}$ in a direction perpendicular to a magnetic field of 0.17 T. The acceleration of the proton is

(a) zero (b) $2 \times 10^{12}\ ms^{-2}$

(c) $4 \times 10^{12}\ ms^{-2}$ (d) $8 \times 10^{12}\ ms^{-2}$

28. An electron of charge e moves in a circular orbit of radius r around the nucleus at a frequency ν. The magnetic moment associated with the orbital motion of the electron is

(a) $\pi \nu e r^2$ (b) $\dfrac{\pi \nu r^2}{e}$

(c) $\dfrac{\pi \nu e}{r^2}$ (d) $\dfrac{\pi e r^2}{\nu}$

29. An electron of charge e moves in a circular orbit of radius r around a nucleus. The magnetic field due to orbital motion of the electron at the site of the nucleus is B. The angular velocity ω of the electron is

(a) $\omega = \dfrac{2\mu_0 e B}{4\pi r}$ (b) $\omega = \dfrac{\mu_0 e B}{\pi r}$

(c) $\omega = \dfrac{4\pi r B}{\mu_0 e}$ (d) $\omega = \dfrac{2\pi r B}{\mu_0 e}$

30. The frequency of the charged particle circulating at right angles to a uniform magnetic field does not depend upon the

(a) speed of the particle (b) mass of the particle

(c) charge of the particle (d) magnetic field

31. The vertical component of the earth's magnetic field is zero at the

(a) magnetic poles (b) magnetic equator

(c) geographic poles (d) 45° latitude

32. The angle of dip is 90° at the

(a) magnetic poles (b) magnetic equator

(c) geographic poles (d) 90° latitude

33. A dip needle free to move in a vertical plane perpendicular to the magnetic meridian will remain

(a) vertical
(b) horizontal
(c) at an angle of 60° to the vertical
(d) at an angle of 45° to the horizontal

34. A sensitive magnetic instrument can be shielded very effectively from outside fields by placing it inside a box of
(a) teak wood
(b) plastic material
(c) a metal of low magnetic permeability
(d) a metal of high magnetic permeability

35. When a material is subjected to a small magnetic field *H*, the intensity of magnetisation is proportional to
(a) $H^{1/2}$ (b) H
(c) H^2 (d) $H^{-1/2}$

36. The magnetic permeability of a paramagnetic substance is
(a) small and positive
(b) small and negative
(c) large and positive
(d) large and negative.

37. Which of the following has the highest magnetic permeability?
(a) Paramagnetic substances
(b) Diamagnetic substances
(c) Ferromagnetic substances
(d) Vacuum

38. For a paramagnetic material, the dependence of the magnetic susceptibility χ on the absolute temperature *T* is given by (*C* is a constant)
(a) $\chi = CT$ (b) $\chi = C/T$
(c) $\chi = CT^2$ (d) $\chi = CT^{-2}$

39. When a ferromagnetic substance is heated to a temperature above its Curie temperature, it
(a) behaves like a paramagnetic substance
(b) behaves like a diamagnetic substance
(c) remains ferromagnetic
(d) is permanently magnetised.

40. The relative permeability of iron is of the order of
(a) zero (b) 10^{-4}
(c) 1 (d) 10^3

41. The relative permeability of a substance *A* is slightly greater than unity while that of a substance *B* is slightly less than unity. Then
(a) *A* is ferromagnetic and *B* paramagnetic
(b) *A* is diamagnetic and *B* paramagnetic
(c) *A* is paramagnetic and *B* diamagnetic
(d) *A* and *B* are both ferromagnetic

42. A closely wound solenoid of 1000 turns and area of cross-section 1.5×10^{-4} m^2, carrying a current of 2A, is suspended through its centre allowing it to turn in a horizontal plane. What is the magnetic moment associated with the solenoid?
(a) 0.3 Am^2 (b) 0.5 Am^2
(c) 0.75 Am^2 (d) 1.5 Am^2

43. A bar magnet of magnetic moment 2.0 JT^{-1} lies aligned with the direction of a uniform magnetic field of 0.25 T. What is the amount of work required to turn the magnet so as to align its magnetic moment normal to the field direction?
(a) 0.125 J (b) 0.25 J
(c) 0.5 J (d) 1.0 J

44. In Q. 43, what is the work done to turn the magnet so as to align its magnetic moment opposite to the field direction?
(a) 0.25 J (b) 0.5 J
(c) 0.75 J (d) 1.0 J

45. A magnetic dipole is acted upon by two magnetic fields which are inclined to each other at an angle of 75°. One of the fields has a magnitude of $\sqrt{2} \times 10^{-2}$ T. The dipole attains stable equilibrium at an angle of 30° with this field. What is the magnitude of the other field?
(a) 0.01 T (b) 0.02 T
(c) 0.03 T (d) 0.04 T

46. At a certain place on earth a magnetic needle is placed along the magnetic meridian at an angle of 60° to the horizontal. If the horizontal component of the earth's field at the place is 0.20×10^{-4} T, what is the magnitude of the total earth's field at that place?
(a) 0.2×10^{-4} T (b) 0.4×10^{-4} T
(c) 0.8×10^{-4} T (d) 1.6×10^{-4} T

47. A short bar magnet of length 4 cm has a magnetic moment of 4 JT^{-1}. What is the magnitude of the magnetic field at a distance of 2 m from the centre of the magnet on its equatorial line?
(a) 10^{-7} T (b) 5×10^{-6} T
(c) 10^{-6} T (d) 5×10^{-5} T

48. In Q. 47, what is the magnitude of the magnetic field at a distance of 2 m from the centre of the magnet on its axial line?
(a) 10^{-4} T (b) 10^{-5} T
(c) 10^{-6} T (d) 10^{-7} T

49. A toroidal solenoid has 3000 turns and a mean radius of 10 cm. It has a soft iron core of relative permeability 2000. What is the magnitude of the magnetic field in the core when a current of 1 A is passed through the solenoid?
(a) 0.012 T (b) 0.12 T
(c) 1.2 T (d) 12 T

50. A sample of paramagnetic salt contains 2×10^{24} atomic dipoles, each of dipole moment 1.5×10^{-23} JT^{-1}. The sample is placed in a magnetic field of 0.6 T and cooled to a temperature of 4 K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment of the sample for a magnetic field of 0.9 T and a temperature of 3 K?
(a) $3\ J\ T^{-1}$ (b) $6\ J\ T^{-1}$
(c) $9\ J\ T^{-1}$ (d) $12\ J\ T^{-1}$

51. A bar magnet of pole strength q and magnetic moment m is divided into two equal pieces by cutting it perpendicular to its length. Then
(a) q is halved and m is doubled
(b) q and m both are halved
(c) q is halved but m remains the same
(d) q remains the same but m is halved

52. A bar magnet of pole strength q and magnetic moment m is divided into two equal pieces by cutting it along its length. Then
(a) q is halved and m is doubled
(b) q and m both are halved
(c) q is halved but m remains the same
(d) q remains the same but m is halved

53. A bar magnet of magnetic moment m is placed along the magnetic meridian. If the earth's magnetic field is B, the work required to turn the magnet through an angle θ is
(a) $m\,B\sin\theta$ (b) $m\,B\cos\theta$
(c) $m\,B\,(1-\cos\theta)$ (d) $m\,B\,(1+\cos\theta)$

54. A small piece of a material is repelled by a strong magnet. The material is
(a) paramagnetic (b) ferromagnetic
(c) diamagnetic (d) non-magnetic

55. Choose the correct statement. There will be no force experienced, if
(a) two parallel wires carry currents in the same direction
(b) two parallel wires carry currents in the opposite direction
(c) a positive charge is projected between the pole pieces of a bar magnet
(d) a positive charge is projected along the axis of a solenoid carrying current

Level B

56. Choose the WRONG statement. The sensitivity of a moving coil galvanometer can be increased by
(a) increasing the number of turns in the coil
(b) inserting a soft iron cylinder inside the coil
(c) increasing the strength of the magnetic field
(d) using a suspension fibre of a higher restoring couple per unit twist.

57. A potential difference of 600 V is applied across the plates of a parallel plate copacitor. The separation between the plates is 3 mm. An electron projected vertically, parallel to the plates, with a velocity of $2 \times 10^6\ m\ s^{-1}$ moves undeflected between the plates. What is the magnitude of the magnetic field between the capacitor plates?
(a) 0.1 T (b) 0.2 T
(c) 0.3 T (d) 0.4 T

58. Two straight infinitely long and thin parallel wires are held 0.1 m apart and carry a current of 5A each in the same direction. The magnitude of the magnetic field at a point distant 0.1 m from both wires is
(a) 10^{-5} T (b) $\sqrt{2} \times 10^{-5}$ T
(c) $\sqrt{3} \times 10^{-5}$ T (d) 2×10^{-5} T

59. When a long wire carrying a steady current is bent into a circular coil of one turn, the magnetic field at its centre is B. When the same wire carrying the same current is bent to form a circular coil of n turns of a smaller radius, the magnetic field at the centre will be
(a) $\frac{B}{n}$ (b) nB
(c) $\frac{B}{n^2}$ (d) $n^2 B$

60. A pair of stationary and infinitely long bent wires are placed in the x-y plane as shown in Fig. 13.56. The wires carry a current $I = 10$ A each as shown.

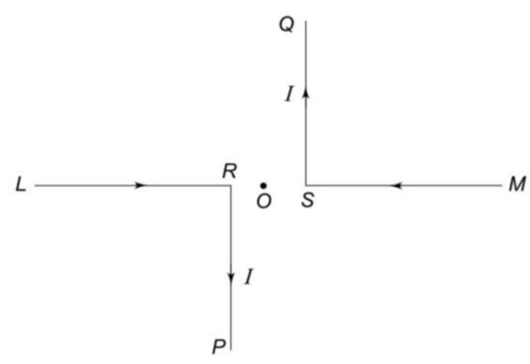


Fig. 13.56

The segments LR and SM are along the x-axis. The segments PR and QS are along the y-axis, such that $OS = OR = 0.02$ m. What is the magnitude and direction of the magnetic field at the origin O?
(a) 100 Wb m^{-2} vertically upward
(b) 10^{-4} Wb m^{-2} vertically downward
(c) 10^{-4} Wb m^{-2} vertically upward
(d) 10^{-2} Wb m^{-2} vertically downward

61. Two long parallel wires P and Q are held perpendicular to the plane of the paper at a separation of 5 m. If P and Q carry currents of 2.5 A and 5 A respectively in the same direction, then the magnetic field at a point mid-way between P and Q is

(a) $\frac{\mu_0}{\pi}$ (b) $\frac{\sqrt{3}\mu_0}{\pi}$

(c) $\frac{\mu_0}{2\pi}$ (d) $\frac{3\mu_0}{2\pi}$

62. A proton of mass m and charge $+e$ is moving in a circular orbit a magnetic field with energy 1 MeV. What should be the energy of an α-particle (mass $4m$ and charge $+2e$) so that it revolves in a circular orbit of the same radius in the same magnetic field?

(a) 1 MeV (b) 2 MeV

(c) 4 MeV (d) 0.5 MeV

63. At a certain place, the horizontal component of earth's magnetic field is $\sqrt{3}$ times the vertical component. The angle of dip at that place is

(a) 30° (b) 45°

(c) 60° (d) 90°

64. A proton, a deuteron and an alpha particle having the same kinetic energy are moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_α denote respectively the radii of trajectories of these particles, then

(a) $r_\alpha = r_p < r_d$ (b) $r_\alpha > r_d > r_p$

(c) $r_\alpha = r_d > r_p$ (d) $r_p = r_d = r_\alpha$

65. Two particles each of mass m and charge q, are attached to the two ends of a light rigid rod of length $2l$. The rod is rotated at a constant angular speed about a perpendicular axis passing through its centre. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of the rod is

(a) $\frac{q}{2m}$ (b) $\frac{q}{m}$

(c) $\frac{2q}{m}$ (d) $\frac{q}{\pi m}$

66. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The particle will move in a

(a) straight line (b) circle

(c) helix (d) cycloid

67. A loosely wound helix made of stiff wire is mounted vertically with the lower end just touching a dish of mercury. When a current from the battery is started in the coil through the mercury

(a) the wire oscillates

(b) the wire continues making contact

(c) the wire breaks contact just when the current is passed

(d) the mercury will expand by heating due to passage of current

68. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic field at the centre is

(a) $\frac{\mu_0 NI}{b}$ (b) $\frac{2\mu_0 NI}{a}$

(c) $\frac{\mu_0 NI}{2(b-a)}\ln\frac{b}{a}$ (d) $\frac{\mu_0 NI}{2(b-a)}\ln\frac{a}{b}$

69. A particle of mass m and charge q moves with a constant velocity v along the positive x direction. It enters a region containing a uniform magnetic field B directed along the negative z direction, extending from $x = a$ to $x = b$. The minimum value of v required so that the particle can just enter the region $x > b$ is

(a) qbB/m (b) $q(b-a)B/m$

(c) qaB/m (d) $q(b+a)B/2m$

70. A magnet of length 10 cm and magnetic moment 1 Am^2, is placed along the side AB of an equilateral triangle ABC. If the length of side AB is 10 cm, the magnetic field at point C is

(a) 10^{-9} T (b) 10^{-7} T

(c) 10^{-5} T (d) 10^{-4} T

71. A magnetized wire of magnetic moment M is bent into an arc of a circle that subtends an angle of 60° at the centre. The equivalent magnetic moment is

(a) $\frac{M}{\pi}$ (b) $\frac{2M}{\pi}$

(c) $\frac{3M}{\pi}$ (d) $\frac{4M}{\pi}$

72. Two poles of the same strength attract each other with a force of magnitude F when placed at two corners of an equilateral triangle. If a north pole of the same strength is placed at the third vertex, it experiences a force of magnitude

(a) $\sqrt{3}\ F$ (b) F

(c) $\sqrt{2}\ F$ (d) $2\ F$

73. A circular coil of radius r having number of turns n and carrying a current I, produces magnetic field of magnitude B at its centre. B can be doubled by

(a) keeping n unchanged and changing I to $\frac{I}{2}$
(b) changing n to $\frac{n}{2}$ and keeping I unchanged
(c) changing n to $2n$ and I to $2I$
(d) keeping I unchanged and changing n to $2n$

74. An electron moves with a speed of 2×10^5 ms^{-1} along the positive x-direction in a magnetic field $\mathbf{B} = (\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$ tesla. The magnitude of the force (in newton) experienced by the electron is (the charge on electron = 1.6×10^{-19} C)
(a) 1.18×10^{-13} (b) 1.28×10^{-13}
(c) 1.6×10^{-13} (d) 1.72×10^{-13}

75. A single charged ion has a mass of 1.13×10^{-23} g. It is accelerated through a potential difference of 500 V and then enters a magnetic field of 0.4 T, moving perpendicular to the field. The radius of its path in the field is
(a) 2.1 cm (b) 2.1 mm
(c) 1.17 m (d) 2.0 m

76. A proton of velocity $(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ ms^{-1} enters a magnetic field of $(2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ tesla. The acceleration produced in the proton is (charge to mass ratio of proton = 0.96×10^8 C kg^{-1})
(a) $2.88 \times 10^8\ (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$
(b) $2.88 \times 10^8\ (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
(c) $2.88 \times 10^8\ (2\hat{\mathbf{i}} + 3\hat{\mathbf{k}})$
(d) $2.88 \times 10^8\ (\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

77. Two short bar magnets of magnetic moments 'M' each are arranged at the opposite corners of a square of side 'd', such that their centres coincide with the corners and their axes are parallel. If the like poles are in the same direction, the magnetic field at any of the other corners of the square is
(a) $\frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}$ (b) $\frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3}$
(c) $\frac{\mu_0}{4\pi} \cdot \frac{M\sqrt{5}}{d^3}$ (d) $\frac{\mu_0}{4\pi} \cdot \frac{3M}{d^3}$

78. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then
(a) the magnetic field at all points inside the pipe is the same but not zero.
(b) the magnetic field at any point inside the pipe is zero.
(c) the magnetic field is zero only on the axis of the pipe.
(d) the magnetic field is different at different points inside the pipe.

79. A power line lies along the east-west direction and carries a current of 10 A. The force per metre due to the earth's magnetic field of 10^{-4} T is
(a) 10^{-5} Nm^{-1} (b) 10^{-4} Nm^{-1}
(c) 10^{-3} Nm^{-1} (d) 10^{-2} Nm^{-1}

80. Two straight and long conductors AOB and COD are perpendicular to each other and carry currents of I_1 and I_2. The magnitude of the magnetic field at a point P at a distance a from point O in a direction perpendicular to the plane $ABCD$ is
(a) $\frac{\mu_0}{2\pi a}(I_1 + I_2)$ (b) $\frac{\mu_0}{2\pi a}(I_1 - I_2)$
(c) $\frac{\mu_0}{2\pi a}(I_1^2 + I_2^2)^{1/2}$ (d) $\frac{\mu_0}{2\pi a}\frac{I_1 I_2}{(I_1 + I_2)}$

81. A 2 MeV proton is moving perpendicular to a uniform magnetic field of 2.5 T. The force on the proton is
(a) 2.5×10^{-10} N (b) 8×10^{-11} N
(c) 2.5×10^{-11} N (d) 8×10^{-12} N

82. A straight section PQ of a circuit lies along the x-axis from $x = -\frac{a}{2}$ to $x = \frac{a}{2}$ and carries a current I. The magnetic field due to the section PQ at point $x = +a$ will be
(a) proportional to a (b) proportional to a^2
(c) proportional to $\frac{1}{a}$ (d) equal to zero

83. Two charged particles M and N enter a region of uniform magnetic field with velocities perpendicular to the field. The paths of particles are shown in Fig. 13.57. The possible reason is
(a) The charge of M is greater than that of N
(b) The momentum of M is greater than that of N
(c) The charge to mass ratio of M is greater than that of N
(d) The speed of M is greater than that of N

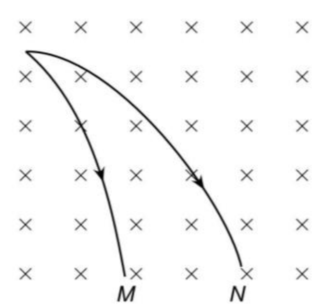


Fig. 13.57

84. The monoenergetic beam of electrons moving along + y direction enters a region of uniform electric and magnetic fields. If the beam goes straight undeflected, then fields **B** and **E** are directed respectively along
(a) $-y$ axis and $-z$ axis
(b) $+z$ axis and $+x$ axis
(c) $+x$ axis and $+z$ axis
(d) $-x$ axis and $-y$ axis

85. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on its axis at a distance of 4 cm from the centre is 54 μT. The magnetic field (in μT) at the centre of the loop will be
(a) 250 (b) 150
(c) 125 (d) 72

86. A wire *ABCDEF* (with each side of length *L*) bent as shown in Fig. 13.58 and carrying a current *I* is placed in a uniform magnetic field *B* parallel to the positive *y*–direction. What is the magnitude and direction of the force experienced by the wire?
(a) *BIL* along positive *z*-direction
(b) BI^2/L along positive *z*-direction
(c) *BIL* along negative *z*-direction
(d) BL/I along negative *z*-direction

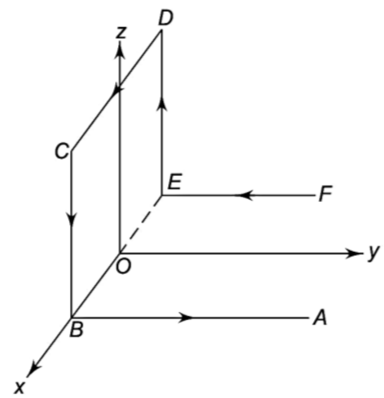


Fig. 13.58

87. A particle of charge q moves with a velocity $\mathbf{v} = a\,\hat{\mathbf{i}}$ in a magnetic field $\mathbf{B} = b\,\hat{\mathbf{j}} + c\,\hat{\mathbf{k}}$ where a, b and c are constants. The magnitude of the force experienced by the particle is
(a) zero (b) $qa(b + c)$
(c) $qa(b^2 - c^2)^{1/2}$ (d) $qa(b^2 + c^2)^{1/2}$

88. A current I is flowing through the sides of an equilateral triangle of side a. The magnitude of the magnetic field at the centroid of the triangle is
(a) $\dfrac{3\mu_0 I}{2\pi a}$ (b) $\dfrac{9\mu_0 I}{2\pi a}$
(c) $\dfrac{3\sqrt{3}\mu_0 I}{2\pi a}$ (d) zero

89. A particle of mass m and charge q, accelerated by a potential difference V enters a region of a uniform transverse magnetic field B. If d is the thickness of the region of B, the angle θ through which the particle deviates from the initial direction on leaving the region is given by
(a) $\sin\theta = Bd\left(\dfrac{q}{2mV}\right)^{1/2}$
(b) $\cos\theta = Bd\left(\dfrac{q}{2mV}\right)^{1/2}$
(c) $\tan\theta = Bd\left(\dfrac{q}{2mV}\right)^{1/2}$
(d) $\cot\theta = Bd\left(\dfrac{q}{2mV}\right)^{1/2}$

90. A metal wire of mass m slides without friction on two rails spaced at a distance d apart. The track lies in a vertical uniform magnetic field B. A constant current I flows along one rail, across the wire and back down the other rail. If the wire is initially at rest, the time taken by it to move through a distance x along the track is
(a) $t = \sqrt{\dfrac{BId}{2xm}}$ (b) $t = \sqrt{\dfrac{2xm}{BId}}$
(c) $t = \sqrt{\dfrac{BIdm}{2x}}$ (d) $t = \sqrt{\dfrac{2dm}{BIx}}$

91. A particle of charge q and mass m is released from the origin with a velocity $\mathbf{v} = a\,\hat{\mathbf{i}}$ in a uniform magnetic field $\mathbf{B} = b\,\hat{\mathbf{k}}$. The particle will cross the y-axis at a point whose y-coordinate is
(a) $y = \dfrac{ma}{qb}$ (b) $y = \dfrac{2ma}{qb}$
(c) $y = -\dfrac{ma}{qb}$ (d) $y = -\dfrac{2ma}{qb}$

92. A U-shaped wire *PQRS* of mass m carrying a current I is stationary above the surface of the earth in the region of a uniform magnetic field **B** directed into the page as shown in Fig. 13.59. Then

(a) $I = \dfrac{mg}{l_2 B}$ (b) $I = \dfrac{mg}{2l_2 B}$

(c) $I = \dfrac{mg}{(2l_1 + l_2)B}$ (d) $I = \dfrac{mg}{(2l_1 - l_2)B}$

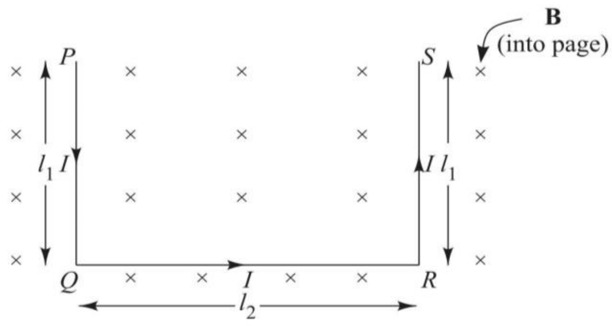


Fig. 13.59

93. A rectangular coil $PQRS$ of length $PQ = a$ and breadth $SP = b$ is suspended with is plane parallel to a horizontal magnetic field **B** as shown in Fig. 13.60.

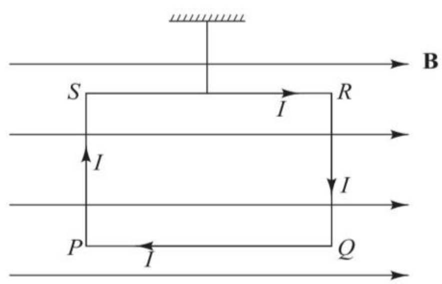


Fig. 13.60

When a current I is passed in the coil as shown, the coil will experience

(a) no net force and no net torque

(b) a net force $F = 2BIa$ will be move it is the direction of **B** but no net torque

(c) a net torque $\tau = BIab$ which will rotate it in the clockwise sense but no net force

(d) a net torque $\tau = BI(a^2 + b^2)$ which will rotate it in the anticlockwise sense and also a net force $F = BI(a + b)$ which will be parallel to **B**.

94. A wire is bent into a form $PQRST$ and carries a current I as shown in Fig. 13.61. Straight segment PQ and ST are of equal length l and the semi-circular segment QRS has a radius r. The frame is placed in a region of uniform magnetic field **B** directed as shown in the Figure. The magnetic force exerted on the wire frame is

(a) $2BI(l + r)$ (b) $2BI(l + \pi r)$

(c) $BI(2l + r)$ (d) $BI(2l + \pi r)$

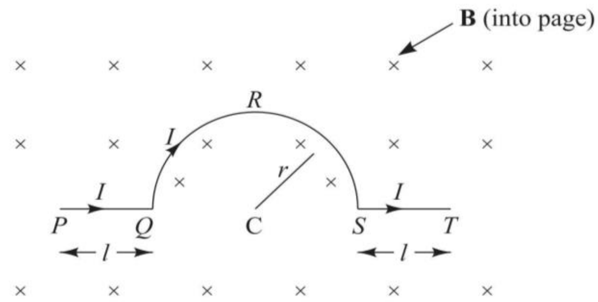


Fig. 13.61

95. A particle of charge $-q$ moves with a velocity **v** in a magnetic field **B** as shown in Fig. 13.62.

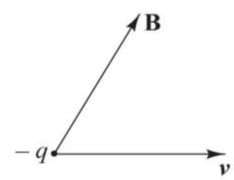


Fig. 13.62

The direction of the magnetic field on the particle will be

(a) to the left

(b) downward in the plane of the page

(c) out of the plane of the page

(d) into the plane of the page

96. Choose the only correct choice out of the four choices (a), (b), (c) and (d) given after the following three statements.

I. The magnetic field lines due to a current carrying wire radiate away from the wire.

II. The kinetic energy of a charged particle can be increased by a magnetic field.

III. A charged particle can move through a region containing only magnetic field without feeling any force.

(a) Only Statement I is true.

(b) Statements II and II are both true

(c) Statements II and III are both true

(d) Only Statement III is true

97. A particle of change q and mass m moves in a circular orbit of radius r in a region of uniform magnetic field **B**. The magnitude the angular momentum of the particle about the centre of the circle is

(a) qBr^2 (b) $mqBr$

(c) qBr (d) $mqBr^2$

98. Two long parallel wires 1 and 2 separated by a distance d carry equal currents I as shown in Fig. 13.63. The magnetic field at the point p which is exactly mid-way between the two wires is

(a) $\frac{2\mu_0 I}{\pi d}$ into the page

(b) $\frac{\mu_0 I}{\pi d}$ out of the page

(c) $\frac{\mu_0 I}{2\pi d}$ into the page

(d) zero

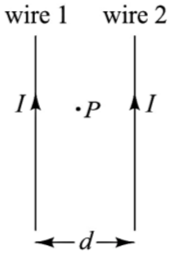


Fig. 13.63

99. In Q. 98 above, which of the four choices (a), (b), (c) and (d) of Q. 98 is correct if the direction of the current in wire 2 is reversed ?

100. A long co-axial cable consists of a solid cylindrical conductor of radius $R_1 = R$ surrounded by a thin conducting cylindrical shell of radius $R_2 = 2R$, The inner cylinder carries a current $I_1 = I$ and the outer cylindrical shell carries a current $I_2 = I/2$, in the same direction. The magnitude of the magnetic field at a point at a distance $r = 3R/2$ from the centre is

(a) $\frac{\mu_0 I}{3\pi R}$ (b) $\frac{2\mu_0 I}{3\pi R}$

(c) $\frac{3\mu_0 I}{2\pi R}$ (d) $\frac{\sqrt{3}\mu_0 I}{\pi R}$

101. In Q. 100 above, what is the magnitude of the magnetic field at a point P at a distance $r = 3R$ from centre ?

(a) $\frac{\mu_0 I}{\pi R}$ (b) $\frac{\mu_0 I}{2\pi R}$

(c) $\frac{\mu_0 I}{3\pi R}$ (d) $\frac{\mu_0 I}{4\pi R}$

102. In Q. 101 above, what is the magnitude of the magnetic field at the centre if the direction of current I_2 is reversed?

(a) $\frac{\mu_0 I}{3\pi R}$ (b) $\frac{\mu_0 I}{6\pi R}$

(c) $\frac{\mu_0 I}{9\pi R}$ (d) $\frac{\mu_0 I}{12\pi R}$

103. Figure 13.64 shows a wire $PQRS$ carrying a current I. Portions PQ and RS are straight and QR in a circular arc of radius r subtending an angle α at the centre C. The megnitude of magnetic field due to $PQRS$ at centre C is

(a) $\frac{\mu_0 I}{r}\left(1+\frac{\alpha}{2\pi}\right)$ (b) $\frac{\mu_0 I}{r}\alpha$

(c) $\frac{\mu_0 I\alpha}{2\pi r}$ (d) $\frac{\mu_0 I\alpha}{4\pi r}$

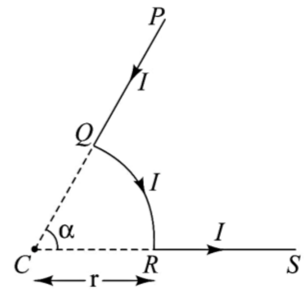


Fig. 13.64

104. A particle of charge $+q$ and mass m, after being accelerated from rest by a voltage V enters a region of a uniform magnetic field in which it describes a circular motion of radius r. How much time does the particle spend in the region of the field?

(a) $2\pi\sqrt{\frac{mr}{2qV}}$ (b) $\frac{1}{\pi r}\sqrt{\frac{2qV}{m}}$

(c) $\pi r\sqrt{\frac{m}{2qV}}$ (d) $2\pi r\sqrt{\frac{m}{2qV}}$

105. A homogeneous cylindrical rod of radius R carries a current I whose current density J (defined as current per unit cross-sectional area) is constant throughout the rod. The magnetic field at a point P at a distance $r = R/2$ from the centre of the rod is

(a) zero (b) $\frac{\mu_0 I}{\pi R}$

(c) $\frac{\mu_0 I}{2\pi R}$ (d) $\frac{\mu_0 I}{4\pi R}$

106. In Q. 105 above the magnetic field at a point P at a distance $r = 2R$ from the centre of the rod is

(a) $\frac{\mu_0 I}{2\pi R}$ (b) $\frac{\mu_0 I}{4\pi R}$

(c) $\frac{\mu_0 I}{8\pi R}$ (d) $\frac{\mu_0 I}{16\pi R}$

107. A non-homogeneous cylinder of radius R carries a current I whose current density varies with the radial distance r from the centre of the rod as $J = \sigma r$ where σ is a constant. The magnetic field at a point P at a distance $r < R$ from the centre of the rod is

(a) $\frac{\mu_0 I r^2}{2\pi R^3}$ (b) $\frac{\mu_0 I r}{2\pi R^2}$

(c) $\frac{\mu_0 I R^2}{2\pi r^3}$ (d) $\frac{\mu_0 I R}{2\pi r^2}$

108. A wire loop $PQTS$ is formed by joining two semicircular wires of radii R and $2R$ in two different ways as shown in Fig. 13.65 (a) and (b). They carry

the same current I. The ratio of magnitudes of magnetic fields at centre C in case (a) to that in case (b) is

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

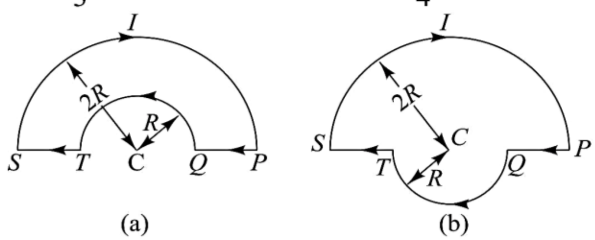


Fig. 13.65

109. In Q. 108 above, if B_1 is magnetic field in case (a) and B_2 in case (b), then

(a) B_1 is directed out of the page and B_2 into the page

(b) B_1 is directed into the page and B_2 out of the page

(c) both B_1 and B_2 are directed into the page

(d) both B_1 and B_2 are directed out of the page

110. Two coils A and B, each having n turns and radius r are held such that coil A lies in the vertical plane and coil B in the horizontal plane with their centres coinciding. The angle of dip at the place is θ. A current I_A has to be passed through coil A and a current I_B through coil B in order to nullify the earth's magnetic field at that place. Then ratio I_A/I_B must be

(a) $\sin\theta$ (b) $\cos\theta$

(c) $\tan\theta$ (d) $\cot\theta$

111. Two charged particles X and Y after being accelerated through the same potential difference V, enter a region of uniform magnetic field B and describe circular paths of radii r_1 and r_2 respectively. Then the ratio

$$\frac{\text{charge to mass ratio of } X}{\text{charge to mass ratio of } Y}, \text{ is}$$

(a) $\left(\frac{r_1}{r_2}\right)^{1/2}$ (b) $\left(\frac{r_1}{r_2}\right)$

(c) $\left(\frac{r_1}{r_2}\right)^2$ (d) $\left(\frac{r_2}{r_1}\right)^2$

112. A rectangular loop $PQRS$ carrying a current i is situated near a long straight wire AB. If a steady current I is passed through AB as shown in Fig. 13.66, the loop will

(a) rotate about an axis parallel to AB

(b) move towards AB

(c) move away from AB

(d) remain stationary

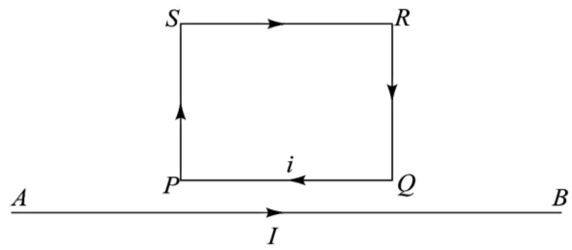


Fig. 13.66

113. Two very long parallel wires, separated by a distance d, carry equal current I in the same direction. At a certain instant of time, a point charge q is at a point P which is equidistant from the two wires, in the plane containing the two wires. If v is the velocity of the charge at this instant is perpendicular to this plane, the force due to magnetic field at P will

(a) accelerate the charged particle

(b) decelerate the charged particle

(c) make the particle oscillate between the two wires

(d) be zero

114. In Q. 113 above, if the two wires carry current I in opposite directions, the force on the charged particle at point P will be (here $B = \mu_0 I/\pi d$)

(a) zero (b) $\frac{1}{2} q\,v\,B$

(c) qvB (d) $2\,qvB$

115. Two particles 1 and 2 of masses m_1 and m_2 and having the same charge q, are moving with velocities v_1 and v_2 respectively in a plane. They describe circular paths of radii r_1 and r_2 in a uniform magnetic field **B** which is directed perpendicular to this plane as shown in Fig. 13.67. Then

(a) $m_1v_1 = m_2v_2$ (b) $m_1v_1 > m_2v_2$

(c) $m_1v_1 < m_2v_2$ (d) $\frac{m_1}{m_2} = \frac{v_1}{v_2}$

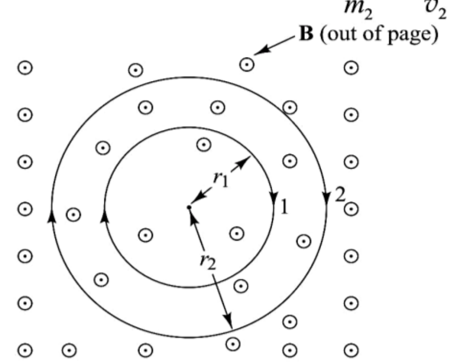


Fig. 13.67

116. A proton moving with a speed u in the $x-y$ plane along the positive x axis enters at $y = 0$ a region of uniform magnetic field **B** directed into the $x-y$ plane as shown in Fig. 13.68. After sometime, the proton leaves the region with a speed v at co-ordinate y. Then

(a) $v = u, y > 0$ (b) $v = u, y < 0$

(c) $v > u, y > 0$ (d) $v > u, y < 0$

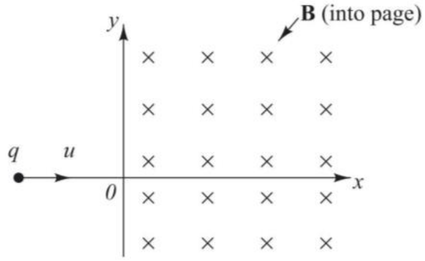


Fig. 13.68

117. A circular wire loop carrying a current I in the anti-clock wise sense is placed in a uniform magnetic field **B** directed into the plane of the coil as shown in Fig. 13.69 The loop will tend to

(a) contract

(b) expand

(c) move towards negative x – direction

(d) move towards positive y – direction

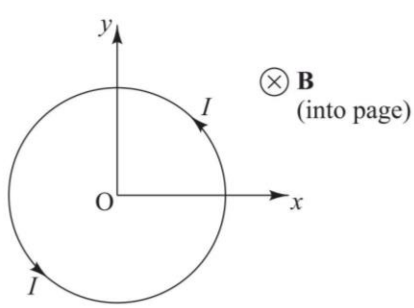


Fig. 13.69

118. Figure 13.70 shows a square wire frame $BCDE$. A current I enters at B and leaves at E. The magnitude of the magnetic field due to the current in the complete frame $ABCDEF$ at the centre O of the square is

(a) $\dfrac{\mu_0 I}{4\sqrt{2}\pi r}$ (b) $\dfrac{\mu_0 I}{2\sqrt{2}\pi r}$

(c) $\dfrac{3\mu_0 I}{4\sqrt{2}\pi r}$ (d) zero

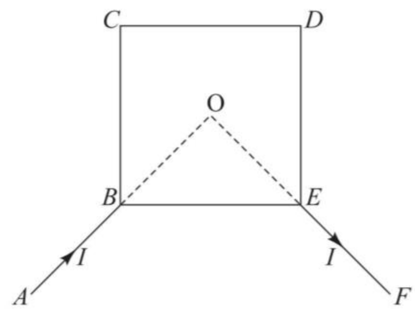


Fig. 13.70

119. Figure 13.71 shows an infinitely long wire $ABCDE$ bent such that the part BCD of the wire is a semi-circle of radius r. A current I flows in the wire as shown. The magnitude of magnetic field at centre O of the semi-circular part is

(a) $\dfrac{\mu_0 I}{\pi r}$ (b) $\dfrac{\mu_0 I}{4r}$

(c) $\dfrac{\mu_0 I}{4r}(2\pi + 1)$ (d) $\dfrac{\mu_0 I}{4r}(2\pi - 1)$

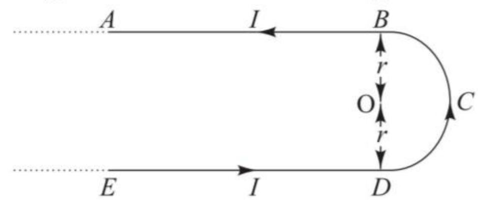


Fig. 13.71

120. In Fig. 13.72, the magnetic field at O is

(a) $\dfrac{\mu_0 I}{4\pi r}(\pi + 1)$ (b) $\dfrac{\mu_0 I}{2\pi r}(2\pi + 1)$

(c) $\dfrac{\mu_0 I}{4\pi r}$ (d) $\dfrac{\mu_0 I}{4r}$

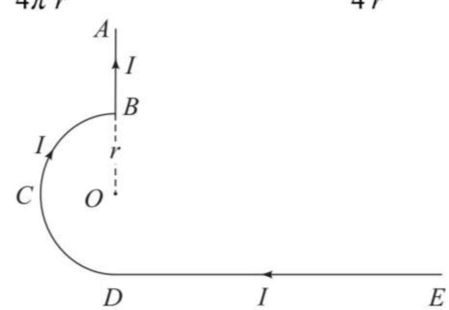


Fig. 13.72

121. An annular wire loop $ABCD$ carries a current I as shown in Fig. 13.73. O is the common centre of the curved parts AD and BC. If $OA = r$ and $OB = 2r$, The magnitude of the magnetic field at O is

(a) $\dfrac{\mu_0 I}{24r}$ (b) $\dfrac{\mu_0 I}{12r}$

(c) $\dfrac{\mu_0 I}{8r}$ (d) $\dfrac{\mu_0 I}{6r}$

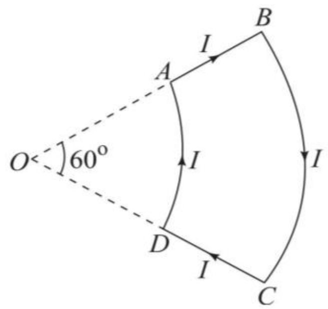


Fig. 13.73

122. The magnetic field at O due to the wire loop $ABCD$ carrying a current I as shown in Fig. 13.74 is

(a) $\frac{\mu_0 I}{4r}$ (b) $\frac{3\mu_0 I}{8r}$

(c) $\frac{5\mu_0 I}{12r}$ (d) $\frac{7\mu_0 I}{16r}$

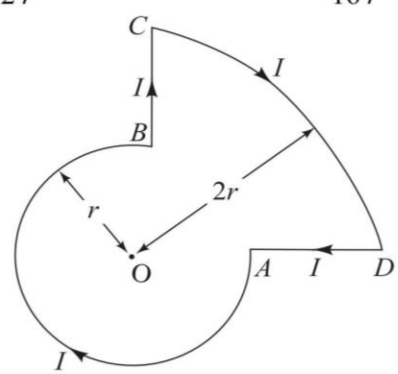


Fig. 13.74

123. The magnitude of the magnetic field at the centroid of a triangular metal loop of side a and carrying a current I is

(a) $\frac{3\mu_0 I}{4\pi a}$ (b) $\frac{3\sqrt{3}\mu_0 I}{2\pi a}$

(c) $\frac{3\sqrt{2}\mu_0 I}{\pi a}$ (d) $\frac{9\mu_0 I}{2\pi a}$

124. A charge q moves with a velocity v in a magnetic field $\mathbf{B} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$. If the acceleration of the particle at time t is $\mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - n\hat{\mathbf{k}}$, then the value of n is

(a) 1 (b) 2

(c) 3 (d) 4

125. A loop $ABCDEFGHA$ carrying a current I lies in the x–y plane as shown in Fig. 13.75. The unit vector $\hat{\mathbf{k}}$ is directed out of the plane of the page (*i.e.*, x–y plane). The magnetic moment of the current loop is

(a) $a^2 I \hat{\mathbf{k}}$ (b) $-(\sqrt{3}+2)\, a^2 I \hat{\mathbf{k}}$

(c) $-(\sqrt{3}+1)\, a^2 I \hat{\mathbf{k}}$ (d) $-\sqrt{3}\, a^2 I \hat{\mathbf{k}}$

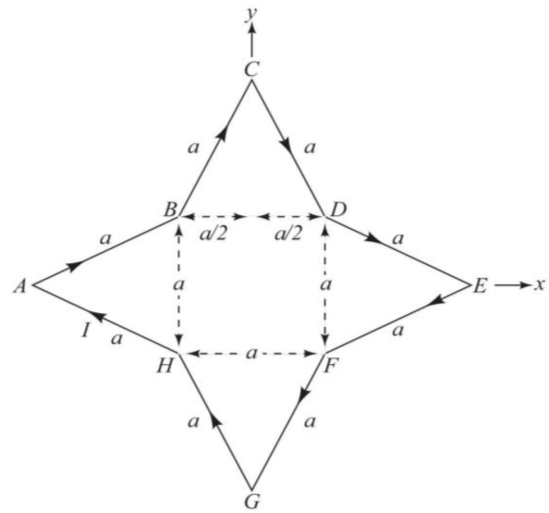


Fig. 13.75

Answers

Level A

1. (a)	2. (c)	3. (c)	4. (a)
5. (d)	6. (a)	7. (d)	8. (c)
9. (d)	10. (c)	11. (d)	12. (b)
13. (d)	14. (a)	15. (d)	16. (c)
17. (a)	18. (c)	19. (d)	20. (b)
21. (c)	22. (a)	23. (d)	24. (d)
25. (a)	26. (d)	27. (d)	28. (a)
29. (c)	30. (a)	31. (b)	32. (a)
33. (a)	34. (d)	35. (b)	36. (a)
37. (c)	38. (b)	39. (a)	40. (d)
41. (c)	42. (a)	43. (c)	44. (d)
45. (a)	46. (b)	47. (b)	48. (b)
49. (d)	50. (c)	51. (d)	52. (b)
53. (c)	54. (c)	55. (d)	

Level B

56. (d)	57. (a)	58. (c)	59. (d)
60. (c)	61. (c)	62. (a)	63. (a)
64. (a)	65. (a)	66. (a)	67. (a)
68. (c)	69. (b)	70. (d)	71. (c)
72. (b)	73. (d)	74. (c)	75. (a)
76. (b)	77. (a)	78. (b)	79. (c)
80. (c)	81. (d)	82. (d)	83. (c)
84. (c)	85. (a)	86. (a)	87. (d)

88. (b)	89. (a)	90. (b)	91. (d)
92. (a)	93. (c)	94. (a)	95. (d)
96. (d)	97. (a)	98. (d)	99. (a)
100. (a)	101. (d)	102. (d)	103. (d)
104. (c)	105. (d)	106. (b)	107. (a)
108. (c)	109. (a)	110. (d)	111. (d)
112. (c)	113. (d)	114. (a)	115. (c)
116. (a)	117. (c)	118. (d)	119. (c)
120. (a)	121. (a)	122. (d)	123. (d)
124. (b)	125. (c)		

Solutions

Level A

1. The radius of the circular loop $(r) = \dfrac{L}{2\pi}$. Therefore,

$$B = \frac{\mu_0 I}{2r} = \frac{\pi \mu_0 I}{L}$$

Hence the correct choice is (a).

2. Magnetic field at point O due to straight wire AB is

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2I}{r} = \frac{\mu_0 I}{2\pi r}$$

and that due to the circular loop is

$$B_2 = \frac{\mu_0 I}{2r}$$

Both these fields are normal to the plane of the loop and directed outside the page. Therefore, net field at point O is

$$B = B_1 + B_2 = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{2r}$$

$$= \frac{\mu_0 I}{2\pi r}(1+\pi)$$

Hence the correct choice is (c).

3. The radius of the double loop $r = R/2$. Now

$$B = \frac{\mu_0 I}{2R}$$

Magnetic field due to a loop of radius r at the centre of the loop is

$$B_1 = \frac{\mu_0 I}{r} = 2B \qquad (\because \; r = R/2)$$

Similarly for the double loop,

$$B_2 = 2B$$

Since the currents in the two loops are in the same direction, the net magnetic field at the centre = $B_1 + B_2 = 4B$. Hence the correct choice is (c).

4. Since the currents in the two loops are in opposite directions, fields B_1 and B_2 are equal and opposite. Therefore, the net magnetic field at the centre of the double loop = $B_1 - B_2 = 0$. Hence the correct choice is (a).

5. The magnetic field at a distance $x = \sqrt{3}\, r$ is

$$B' = \frac{\mu_0 I n r^2}{2(r^2+x^2)^{3/2}} = \frac{\mu_0 I n r^2}{2(r^2+3r^2)^{3/2}}$$

$$= \frac{\mu_0 I n}{16r}$$

But $\qquad B = \dfrac{\mu_0 I n}{2r}$

$\therefore$ $B' = 8B$. Hence the correct choice is (d).

6. $H = 0.314\ G = 0.314 \times 10^{-4}$ T. Let the current in coil be I. Then the magnetic field at the centre of the coil is

$$B = \frac{\mu_0 I n}{2r} = \frac{4\pi \times 10^{-7} \times I \times 50}{2 \times 5 \times 10^{-2}}$$

$$= 2\pi I \times 10^{-4}\ \text{T}$$

The value of I for which $B = H$ is given by

$$2\pi I \times 10^{-4} = 0.314 \times 10^{-4}$$

or $I = 0.05$ A

$\therefore$ Potential difference $= IR = 0.05 \times 10 = 0.5$ V. Hence the correct choice is (a).

7. Refer to Fig. 13.76. Here $r = OE = \dfrac{L}{2}$

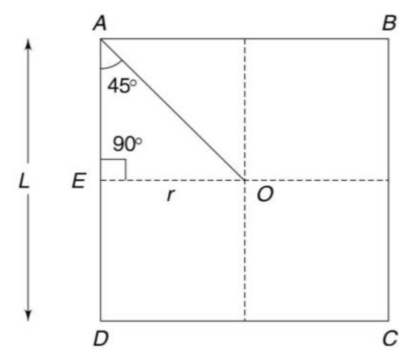


Fig. 13.76

The magnetic field at the centre O due to the current element AE is given by

$$B_{AE} = -\frac{\mu_0 I}{4\pi r}\int_{90^\circ}^{45^\circ} \sin\theta \, d\theta$$

$$= \frac{\mu_0 I}{4\pi r} \left| \cos\theta \right|_{90°}^{45°}$$

$$= \frac{\mu_0 I}{4\pi r} (\cos 45° - \cos 90°)$$

$$= \frac{\mu_0 I}{4\pi r} (\cos 45° - 0) = \frac{\mu_0 I}{4\sqrt{2}\pi r}$$

It is clear that the magnetic field at O due to current element DE is the same as that due to AE. Hence, the magnetic field at O due to one side AD is

$$B_{AD} = \frac{2\mu_0 I}{4\sqrt{2}\pi r} = \frac{\sqrt{2}\mu_0 I}{4\pi r}$$

Since the centre of the square is equidistant from the ends A, B, C and D of each side of the square and each side produces at the centre O the same magnetic field, the field due to the square is 4 times that due to one side. Hence (because $r = L/2$)

$$B = 4B_{\text{AD}} = \frac{\sqrt{2}\mu_0 I}{\pi r} = \frac{2\sqrt{2}\mu_0 I}{\pi L}$$

Hence the correct choice is (d).

8. The magnitude of the magnetic field at the centre due to each coil is B. Since the planes of the coils are at right angles to each other, the directions of the fields will be at right angles to each other. Therefore, the resultant field is

$$B_r = \sqrt{B^2 + B^2} = \sqrt{2}\, B$$

Hence the correct choice is (c).

9. The correct choice is (d) since $\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$.
10. Since the force exerted by the magnetic field is perpendicular to the direction of motion of the particle, the speed of the particle cannot change but its velocity changes. Hence the correct choice is (c).
11. Applying the left hand fist rule and noting the charge on an electron is negative, i.e. $\mathbf{F} = -e\,(\mathbf{v} \times \mathbf{B})$, it follows that the particle is an electron, which is choice (d).
12. The radius of the circular orbit is given by

$$r = \frac{\sqrt{2mK}}{qB}$$

The charge of an α–particle is twice that of a proton and its mass is four times the mass of a proton. Therefore $\sqrt{m}/q$ is the same for both. Hence r will the same for both particles. Thus the correct choice is (b).

13. The cyclotron frequency is given by

$$\nu = \frac{qB}{2\pi m}$$

It is independent of the speed of the particle and the radius of its circular path. Now $\nu \propto q/m$. The charge of a proton is half that of an α-particle and the mass of a proton is one-fourth. Therefore, ν will be doubled. Hence the correct choice is (d).

14. The correct choice is (a).
15. The correct choice is (d).
16. The velocity when the potential difference is V is

$$v = \sqrt{\frac{2eV}{m}}$$

and force $\quad F = e\,v\,B$

When the potential difference is doubled, i.e. $V' = 2V$, the velocity is

$$v' = \sqrt{\frac{2eV'}{m}} = \sqrt{\frac{2e \times 2V}{m}} = \sqrt{2}\; v$$

$\therefore$ Force $F' = e\,v'\,B = \sqrt{2}\;\; evB = \sqrt{2}\;\; F$. Hence the correct choice is (c).

17. The correct choice is (a).
18. The correct choice is (c) because the magnetic field produced by the current in the loop and the external magnetic field are along the same direction.
19. The force per unit length is

$$F = \frac{\mu_0}{4\pi} \times \frac{2I^2}{R}$$

If R is increased to $2R$ and I is reduced to $I/2$, the force per unit length becomes

$$F' = \frac{\mu_0}{4\pi} \times \frac{2(I/2)^2}{2R}$$

$$= \frac{\mu_0}{4\pi} \times \frac{2I^2}{R} \cdot \frac{1}{8} = \frac{F}{8}$$

Hence the correct choice is (d).

20. In order that the tension in the supporting wires is zero the downward gravitational force mg on the rod must be balanced by an upward force BIl due to magnetic field, i.e. $BIl = mg$

or $\quad B = \frac{mg}{Il} = \frac{50 \times 10^{-3} \times 10}{5 \times 0.5} = 0.2\ \text{T}$

Hence the correct choice is (b).

21. Speed of electron $(v) = 2.0 \times 10^6\ \text{ms}^{-1}$, radius of circular orbit $(a) = 5.0 \times 10^{-11}$ m.

The time period of electron motion in the circular orbit is

$$T = \frac{2\pi a}{v} = \frac{2\pi \times 5.0 \times 10^{-11}}{2.0 \times 10^6}$$

$$= 5\pi \times 10^{-17} \text{ s}$$

Therefore the equivalent current is

$$I = \frac{\text{charge}}{\text{time}} = \frac{e}{T} = \frac{1.6\times10^{-19}}{5\pi\times10^{-17}} = \frac{1.6}{5\pi}\times10^{-2}\text{ A}$$

Equivalent magnetic dipole moment equals current × area of circular orbit

$$= I\times(\pi a^2)$$
$$= \frac{1.6\times10^{-2}}{5\pi}\times\pi\times(5.0\times10^{-11})^2$$
$$= 8\times10^{-24}\text{ Am}^2$$

22. The magnetic field at the centre of the orbit is

$$B = \frac{\mu_0 I}{2a} = \frac{4\pi\times10^{-7}\times1.6\times10^{-2}}{2\times5\pi\times5.0\times10^{-11}} = 12.8\text{ T}$$

23. The charge crossing any point of the path per second, is n times the charge e of the electron. This constitutes the current round the orbit, i.e. $I = ne$. If A is the area of the orbit, the magnetic moment is given by the product

$$IA = ne\pi r^2$$

Hence the correct choice is (d).

24. Magnetic moment $m = AI = \pi r^2 I$, where r is the radius of the circular loop. Now, the circumference of the circle = length of the wire, i.e.

$$2\pi r = l$$

or $$r^2 = \frac{l^2}{4\pi^2}$$

Therefore, $$m = \pi r^2 I = \frac{\pi l^2 I}{4\pi^2} = \frac{l^2 I}{4\pi}$$

Hence the correct choice is (d).

25. The required speed is given by

$$v = \frac{E}{B} = \frac{5000}{0.01} = 5\times10^5\text{ ms}^{-1}$$

Hence the correct choice is (a).

26. Magnetic moment $m = IA = I\pi r^2$.

$$m = I_1\pi r_1^2 = I_2\pi r_2^2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} = \left(\frac{6}{3}\right)^2 = 4$$

Hence the correct choice is (d).

27. Force $F = qvB$

$$\therefore \text{ Acceleration} = \frac{F}{m} = \frac{qvB}{m}$$

$$= \frac{1.6\times10^{-19}\times5.0\times10^5\times0.17}{1.7\times10^{-27}}$$
$$= 8\times10^{12}\text{ ms}^{-2}$$

Hence the correct choice is (d).

28. The charge passing per second through any point of the path is ν times the charge of the electron, i.e. $I = \nu e$. If A is the area of the orbit, the magnetic moment is

$$m = IA = \nu e\pi r^2$$

Hence the correct choice is (a).

29. An electron moving in a circular orbit is equivalent to a current carrying loop. As explained above, the current is

$$I = \nu e = \frac{e}{T}$$

where T is the time period of the motion of the electron around the nucleus. If v is the speed of the electron,

$$T = \frac{2\pi r}{v}$$

$$\therefore \quad I = \frac{ev}{2\pi r} = \frac{e\omega}{2\pi} \qquad (\because v = r\omega)$$

Now, the magnetic field at the centre of the loop is

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0 e\omega}{4\pi r}$$

or $$\omega = \frac{4\pi r B}{\mu_0 e}$$

Hence the correct choice is (c).

30. We have

$$\frac{mv^2}{r} = qvB$$

or $$r = \frac{mv}{qB}$$

Frequency $= \dfrac{v}{2\pi r} = \dfrac{v}{2\pi}\times\dfrac{qB}{mv} = \dfrac{qB}{2\pi m}$ which is independent of v, the speed of the charged particle. Hence the correct choice is (a).

31. The correct choice is (b).
32. The correct choice is (a).
33. The correct choice is (a).
34. The correct choice is (d).
35. The intensity of magnetisation (M) is defined as the magnetic moment per unit volume of the material. The magnitude of M depends upon the magnetisation current which is proportional to H. Hence the correct choice is (b).
36. The correct choice is (a).

37. Ferromagnetic substances have a very high (of the order of 1000 or more) and positive susceptibility (χ). Now $\mu_r = 1 + \chi$. Hence the correct choice is (c).

38. According to Curie's law, the susceptibility χ is related to temperature T as

$$\chi = \frac{C}{T}$$

where C is curie constant. Hence the correct choice is (b).

39. When a ferromagnetic material is heated above its Curie temperature, the thermal motions at a high temperature destroy its magnetism and the material will behave as a paramagnet.

40. The correct choice is (d).

41. Susceptibility (χ) of a paramagnet is small and positive and of a diamagnet is small and negative. Now relative permeability

$$\mu_r = 1 + \chi$$

Hence μ_r is slightly greater than unity for a paramagnet and slightly less than unity for a diamagnet. Hence the correct choice is (c).

42. The magnetic moment of the solenoid is

$$M = INA = 2 \times 1000 \times 1.5 \times 10^{-4} = 0.3 \text{ Am}^2$$

Hence the correct choice is (a).

43. The potential energy of a bar magnet with its magnetic moment **M** inclined at an angle θ with magnetic field **B** is

$$U = -MB\cos\theta$$

Potential energy when $\theta = 0$ is

$$U_0 = -MB\cos 0° = -MB$$

Potential energy when $\theta = 90°$ is

$$U' = -MB\cos 90° = 0$$

$\therefore$ Work done $= U' - U_0 = 0 - (-MB) = MB = 2.0 \times 0.25 = 0.5$ J. Hence the correct choice is (c).

44. Potential energy when $\theta = 180°$ (i.e. **M** opposite to **B**) is

$$U'' = -MB\cos 180° = MB$$

$$\therefore \text{Work done} = U'' - U_0 = MB - (-MB)$$

$$= 2MB = 2 \times 2.0 \times 0.25 = 1.0 \text{ J.}$$

Hence the correct choice is (d).

45. Refer to Fig. 13.77. Let θ_1 (= 30°) be the angle between the magnetic moment vector **M** and the field vector $\mathbf{B}_1$ (= 1.5×10^{-2} T). Then, as shown in Fig. 13.58, the angle between **M** and the other field $\mathbf{B}_2$ will be $\theta_2 = 75° - 30° = 45°$.

The field $\mathbf{B}_1$ exerts a torque $\tau_1 = \mathbf{M} \times \mathbf{B}_1$ on the dipole and the field $\mathbf{B}_2$ exerts a torque $\tau_2 = \mathbf{M} \times \mathbf{B}_2$, where **M** in the magnetic moment of the dipole. Since the dipole is in stable equilibrium, the net torque τ (= $\tau_1 + \tau_2$) must be zero, i.e. the two torques must be equal and opposite. In terms of magnitudes, we have $mB_1 \sin\theta_1 = mB_2 \sin\theta_2$

or
$$B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2}$$

$$= \frac{\sqrt{2} \times 10^{-2} \times \sin 30°}{\sin 45°} = 0.01 \text{ T}$$

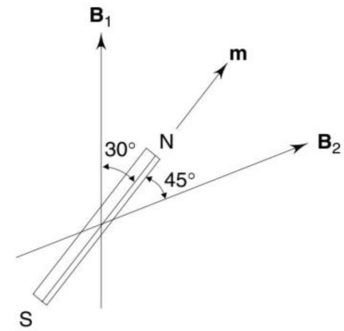


Fig. 13.77

46. $B_H = 0.20 \times 10^{-4}$ T and $\theta = 60°$. We know that $B_H = B\cos\theta$, where B is the magnitude of the total earth's field. Thus,

$$B = \frac{B_H}{\cos\theta} = \frac{0.20 \times 10^{-4}}{\cos 60°} = 0.40 \times 10^{-4} \text{ T}$$

47. Here $2a = 4$ cm, $m = 4$ JT^{-1} and $r = 2$ m. Since $a \ll r$, the magnetic field at a distance r on the equatorial line is

$$B_m = \frac{\mu_0 M}{4\pi r^3} = \frac{4\pi \times 10^{-7} \times 4}{4\pi \times (2)^3} = 5 \times 10^{-6} \text{ T}$$

48. Since $a \ll r$, the magnetic field at a distance r on the axis line is

$$B_m = \frac{2\mu_0 M}{4\pi r^3} = 10^{-5} \text{ T}$$

49. The magnetic field in the core is given by

$$B = \mu n I$$

where μ is the permeability of soft iron and n is the number of turns per unit length of the solenoid. Now

$$\mu_r = \frac{\mu}{\mu_0} \text{ and } n = \frac{3000}{2\pi r} = \frac{3000}{2\pi \times 0.1}$$

$$\therefore \quad B = \mu_r \mu_0 n I$$

$$= 2000 \times 4\pi \times 10^{-7} \times \frac{3000}{2\pi \times 0.1} \times 1 = 12 \text{ T}$$

Hence the correct choice is (d).

50. Initial total magnetic moment at temperature T = 4 K is

$$m_1 = 15\% \text{ of } (2.0 \times 10^{24} \times 1.5 \times 10^{-23}) = 4.5 \text{ JT}^{-1}$$

Now from Curie's law, we have $\chi = C/T$

or $$\frac{I}{H} = \frac{C}{T} \quad \text{(i)}$$

where I is magnetization and H is the magnetizing field. If V is the volume of the sample, then by definition, $M = I/V$ and Eq. (i) becomes

$$M = \frac{C}{V}\left(\frac{H}{T}\right) = \text{constant} \times \left(\frac{H}{T}\right) \quad \text{(ii)}$$

If M_2 is the magnetic moment at temperature T_2 = 3 K and field H_2 = 0.9 T, then from Eq. (ii) M_1 and M_2 are related as

$$\frac{M_1}{M_2} = \frac{H_1}{H_2} \cdot \frac{T_2}{T_1}$$

or $$M_2 = M_1 \times \frac{H_2}{H_1} \times \frac{T_1}{T_2}$$

$$= 4.5 \times \frac{0.9}{0.6} \times \frac{4}{3} = 9 \text{ JT}^{-1}$$

51. The correct choice is (d).
52. The correct choice is (b).
53. If a torque τ is applied, the work done is

$$W = \int_0^{\theta} \tau \, d\theta = \int_0^{\theta} mB \sin\theta \, d\theta$$

$$= -mB \left| \cos\theta \right|_0^{\theta}$$

$$= -mB(\cos\theta - 1) = mB(1 - \cos\theta)$$

Hence the correct choice is (c).

54. The correct choice is (c).
55. The correct choice is (d) since the direction of the field is parallel to the axis of a solenoid. Thus $\theta = 0$. Hence $F = 0$.

Level B

56. The correct choice is (d).
57. Electric field $E = \dfrac{V}{d}$

where V is the potential difference between the plates and d, the separation between them.

$$d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

$$E = \frac{V}{d} = \frac{600}{3 \times 10^{-3}} = 2 \times 10^5 \text{ V m}^{-1}$$

Since the electron moves undeflected between the plates, the force due to magnetic field must balance the force due to electric field. Thus

$$Bev = eE$$

or $$B = \frac{E}{v} = \frac{2 \times 10^5}{2 \times 10^6} = 0.1 \text{ T}$$

58. Wires A and B carry current I = 5 A each coming out of the plane of the page as shown in Fig. 13.78. The magnitude of magnetic field at point P due wire A is equal to that due to wire B, i.e.

$$B_A = B_B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a}$$

$$= \frac{10^{-7} \times 2 \times 5}{0.1} = 10^{-5} \text{ T}$$

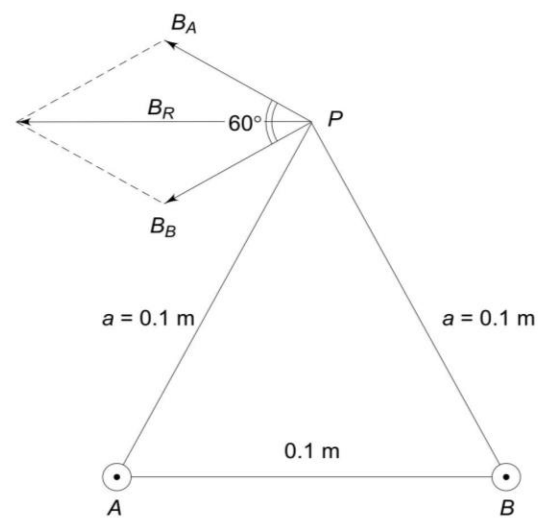


Fig. 13.78

The direction of field B_A is perpendicular to PA and that of field B_B is perpendicular to PB. Therefore, the angle between the two fields is $\theta = 60°$. The magnitude of the resultant field at P is given by

$$B_R^2 = B_A^2 + B_B^2 + 2B_A B_B \cos\theta$$

which gives $$B_R = 2B_A \cos\left(\frac{\theta}{2}\right)$$

$$= 2 \times 10^{-5} \times \frac{\sqrt{3}}{2} = \sqrt{3} \times 10^{-5} \text{ T}$$

Hence the correct choice is (c).

59. $B = \dfrac{\mu_0 I}{2r}$. For a coil of n turns, $2\pi r = n(2\pi r')$

or $r' = \dfrac{r}{n}$, where r' is the radius of the coil of n turns.

$$\therefore \quad B' = \frac{n\mu_0 I}{2r'} = \frac{n\mu_0 I}{2r/n} = n^2 B$$

Hence the correct choice is (d).

60. Since point O lies along the segments LR and MS, the magnetic field due to these segments is zero at point

O. As point O is close to R and S, the net magnetic field at O due to segments PR and QS is

$$B = B_P + B_Q$$
$$= \frac{\mu_0 I}{4\pi RO} + \frac{\mu_0 I}{4\pi SO} = \frac{\mu_0 I}{4\pi}\left(\frac{1}{d}+\frac{1}{d}\right)$$
$$= \frac{\mu_0}{4\pi}\left(\frac{2I}{d}\right) = 10^{-7} \times \frac{2\times 10}{0.02} = 10^{-4}\ \text{Wb m}^{-2}$$

The direction of this field is vertically upward, i.e. outside the plane of the paper. Hence the correct choice is (c).

61. Magnetic field $B = \frac{\mu_0}{4\pi}\cdot\frac{2I_1}{r_1} - \frac{\mu_0}{4\pi}\cdot\frac{2I_2}{r_2}$

Given $I_1 = 2.5$ A, $I_2 = 5$ A and $r_1 = r_2 = 2.5$ m. Using these values, we get $B = -\frac{\mu_0}{2\pi}$. The magnitude of B is $\mu_0/2\pi$. Hence the correct choice is (c).

62. For proton: $r = \frac{mv}{eB}$

For α-particle $r' = \frac{m'v'}{e'B} = \frac{4mv'}{2eB} = \frac{2mv'}{eB}$

Given $r = r'$. Hence $v' = \frac{v}{2}$.

Energy of proton $E = \frac{1}{2}\ mv^2$. Energy of α-particle is

$$E' = \frac{1}{2}\ m'v'^2 = \frac{1}{2} \times 4m \times \left(\frac{v}{2}\right)^2 = \frac{1}{2}\ mv^2 = E$$

Hence $E' = 1$ MeV which is choice (a).

63. $B_H = B\cos\theta$ and $B_V = B\sin\theta$. Hence

$$\frac{B_V}{B_H} = \tan\theta$$

Given $\frac{B_V}{B_H} = \frac{1}{\sqrt{3}}$. Therefore, $\tan\theta = \frac{1}{\sqrt{3}}$, i.e. $\theta = 30°$ which is choice (a).

64. The radius of the circular path is given by

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB},\ \text{where } K = \frac{1}{2}\ mv^2.$$

Thus $r \propto \frac{\sqrt{m}}{q}$. Since K and B are the same for the three particles. If m_p is the mass of a proton and q_p its charge, then $m_d = 2m_p$ and $q_d = q_p$ and $m_\alpha = 4\ m_p$ and $q_\alpha = 2q_p$. From these it follows that $r_\alpha = r_p < r_d$, which is choice (a).

65. According to Ampere's Law, the magnetic moment of a current I flowing in a circular path of area of cross-section A is given by

$$\mu_m = IA$$
$$= \frac{q}{T}\ A = \frac{q\pi(2l)^2}{T}$$

It is given that the charge q is moving in a circular path of radius $2l$. Therefore, the time period $T = 2\pi(2l)/v$. Hence

$$\mu_m = \frac{qv}{2\pi(2l)} \times \pi(2l)^2 = qvl$$

The angular momentum $L = mv(2l)$. Therefore,

$$\frac{\mu_m}{L} = \frac{qvl}{mv(2l)} = \frac{q}{2m},\ \text{which is choice (a).}$$

66. Due to electric field **E**, the force on a particle of charge q is $\mathbf{F} = q\mathbf{E}$ in the direction of the electric field. Since **E** is parallel to **B**, the velocity **v** of the particle is parallel to **B**. Hence **B** will not affect the motion of the particle since $\mathbf{v} \times \mathbf{B} = 0$. Thus, the correct choice is (a).

67. When a current is passed through the helix, the neighbouring coils of the helix attract each other due to which it contracts. As a result the contact is broken and the coils will recover their original state under the influence of a restoring force. The contract is made again and the process continues. Thus, the wire oscillates. Hence the correct choice is (a).

68. The correct choice is (c). For derivation of the expression, refer to a Textbook of Physics.

69. The radius r of the circular path is given by (see Fig. 13.79)

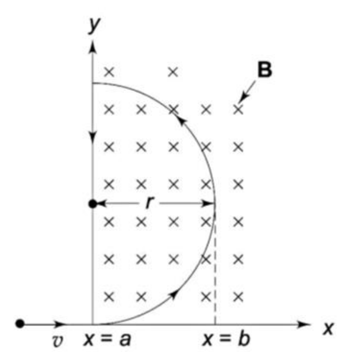


Fig. 13.79

$$\frac{mv^2}{r} = qvB$$

or $$v = \frac{qB}{m}(r)$$

$$\therefore v_{min} = \frac{qB}{m}(r_{min}) = \frac{qB}{m}(b-a),$$ which is choice (b).

70. Let m be the pole strength of each pole of the magnet (see Fig. 13.80). The magnetic field at C due to the N-pole is given by

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{(AC)^2}$$

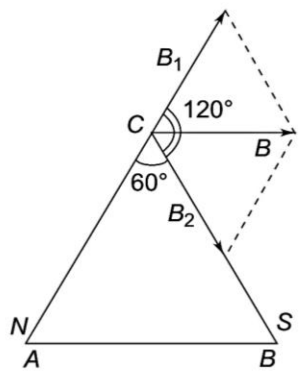


Fig. 13.80

direction along AC away from C. The magnetic field at C due to the S-pole is given by

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{m}{(BC)^2}$$

directed along CB towards B. Since $AC = BC$, $B_1 = B_2$. The resultant magnetic field at C is given by

$$B^2 = B_1^2 + B_2^2 + 2B_1 B_2 \cos 120°$$
$$= B_1^2 + B^2{}_2 - B_1 B_2$$
$$= 2B_1^2 - B_1^2 = B_1^2 \qquad (\because B_2 = B_1)$$

or $$B = B_1 = \frac{\mu_0 m}{4\pi(AC)^2} = \frac{\mu_0 m}{4\pi a^2}$$
$$= \frac{\mu_0}{4\pi}\cdot\frac{(ma)}{a^3} = \frac{\mu_0}{4\pi}\cdot\frac{M}{a^3} \qquad (1)$$

Given: $M = 1$ A m^2, $a = 10$ cm $= 0.1$ m. Also $\mu_0 = 4\pi\times10^{-7}$ T A^{-1} m. Substituting these values in (1), we get $B = 10^{-4}$ T, which is choice (d).

71. Let r be the radius of the circle. The length of the arc $= (2\pi r)\times\frac{60°}{360°} = \frac{\pi r}{3}$. Therefore, the length $2l$ of the magnet is

$$2l = \frac{\pi r}{3} \text{ or } r = \frac{6l}{\pi}$$

If m is the pole strength of each pole of the magnet, the magnetic moment of the arc $= m\times r = m\times\frac{6l}{\pi}$ $= \frac{3\times(2ml)}{\pi} = \frac{3M}{\pi}$.

Hence the correct choice is (c).

72. Refer to Fig. 13.81. Let a north pole be placed at B and a south pole at C so that they attract with a force F. A north pole placed at the third vertex A is repelled with a force F_1 by the north pole at B and attracted with a force F_2 towards the south pole at C. Since all pole strengths are equal, $F_1 = F_2 = F$. The resultant force experienced by the north pole at A is given by

$$F_r^2 = F_1^2 + F_2^2 + 2F_1F_2\cos(120°)$$
$$= F^2 + F^2 + 2F^2\times\left(-\frac{1}{2}\right) = F^2$$

or $F_r = F$, which is choice (b). $\qquad (\because F_1 = F_2 = F)$

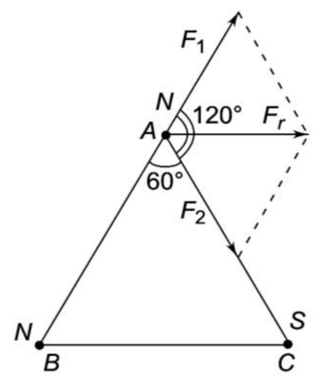


Fig. 13.81

73. The magnetic field at the centre of coil is given by

$$B = \frac{\mu_0 In}{2r}$$

Hence the correct choice is (d).

74. Given $v = (2\times10^5\,\hat{\mathbf{i}})$ ms^{-1}. The force vector is given by

$$\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$$
$$= q\{(2\times10^5\,\mathbf{i})\times(\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 3\hat{\boldsymbol{k}})\}$$
$$= 2\times10^5\times q(-4\hat{\boldsymbol{k}} + 3\hat{\mathbf{j}})$$

Therefore, the y and z components of the force are

$$F_y = 6\times10^5\times q$$

and $$F_z = -8\times10^5\times q$$

$\therefore$ Magnitude of force $= \sqrt{F_y^2 + F_z^2}$

$$= q\sqrt{(6\times10^5)^2 + (-8\times10^5)^2}$$

$= q \times 10 \times 10^5$

$= 1.6 \times 10^{-19} \times 10 \times 10^5$

$= 1.6 \times 10^{-13}$ N, which is choice (c).

75. $\frac{1}{2} mv^2 = eV$ or $v = \sqrt{\frac{2eV}{m}}$ (1)

Also $BeV = \frac{mv^2}{r}$ or $v = \frac{Ber}{m}$ (2)

Equating (1) and (2), we get

$$r = \frac{1}{B}\sqrt{\frac{2mV}{e}} \quad (3)$$

Given $e = 1.6 \times 10^{-19}$ C (singly charged ion), $B = 0.4$ T, $V = 500$ V and $m = 1.13 \times 10^{-26}$ kg. Using these values in (3), we get $r = 0.021$ m $= 2.1$ cm, which is choice (a).

76. Given $\mathbf{v} = (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ ms^{-1} and $B = (2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ tesla.

Force experienced by the proton is

$$\mathbf{F} = q(\mathbf{v} \times \boldsymbol{B}) = q\,(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \times (2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

$$= q\,(6\hat{\mathbf{i}} \times \hat{\mathbf{j}} + 9\mathbf{i} \times \hat{\mathbf{k}} + 4\hat{\mathbf{j}} \times \hat{\mathbf{j}} + 6\hat{\mathbf{j}} \times \hat{\mathbf{k}})$$

$$= q\,(6\hat{\mathbf{k}} - 9\hat{\mathbf{j}} + 0 + 6\hat{\mathbf{i}})$$

$$= 3q\,(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \text{ newton}$$

$\therefore$ Acceleration $= \frac{\boldsymbol{F}}{m} = \frac{3q}{m}(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$$= 3 \times (0.96 \times 10^8)\,(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$$

$$= 2.88 \times 10^8\,(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \text{ ms}^{-2}$$

Hence the correct choice is (b).

77. Refer to Fig. 13.82. Let us find the net magnetic field at corner C of the square $ABCD$. For the magnet at corner B, the point C is on the axial line at a distance d from the centre of the magnet. For a short magnet, the magnetic field at C is given by

$$B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

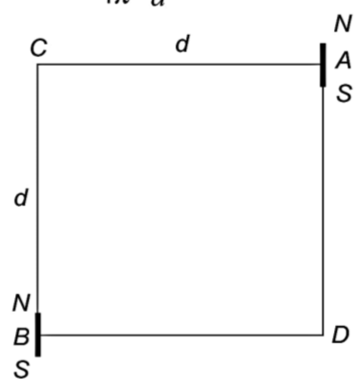


Fig. 13.82

For the magnet at corner A, the point C is on the equatorial line at a distance d from its centre. For a short magnet, the magnetic field at C due to this magnet is given by

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{M}{d^3}$$

Since their like poles are in the same direction, the net magnetic field at C is

$$B = B_1 - B_2$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{2M}{d^3} - \frac{\mu_0}{4\pi}\cdot\frac{M}{d^3} = \frac{\mu_0}{4\pi}\cdot\frac{M}{d^3}$$

Hence the correct choice is (a).

78. From Ampere's law, we have

$$\oint \boldsymbol{B}\cdot d\boldsymbol{l} = \mu_0 I$$

Since no current exists in the medium (air) inside the pipe $I = 0$. Hence $\mathbf{B} = 0$. Hence the correct choice is (b).

79. Force F on a wire of length $l = BIl$. Therefore, force per unit length of the wire $= \frac{F}{l} = BI = 10^{-4} \times 10 = 10^{-3}$ Nm^{-1}, which is choice (c).

80. The magnetic field at point P due to current I_1 in conductor AOB is

$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

and the magnetic field at point P due to current I_2 in conductor COD is

$$B_2 = \frac{\mu_0 I_2}{2\pi a}$$

Since the two conductors are perpendicular to each other, fields B_1 and B_2 will be perpendicular to each other. Therefore, the resultant field at P is

$$B = (B_1^2 + B_2^2)^{1/2} = \frac{\mu_0}{2\pi a}(I_1^2 + I_2^2)^{1/2}$$

Hence the correct choice is (c).

81. The kinetic energy of proton is

$$K = 2 \text{ MeV} = 2 \times 10^6 \text{ eV}$$

$$= 2 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} = 3.2 \times 10^{-13} \text{ J}$$

$\therefore \quad \frac{1}{2} mv^2 = 3.2 \times 10^{-13}$

Now, mass of proton is $m = 1.67 \times 10^{-27}$ kg. Therefore,

$$v^2 = \frac{2 \times 3.2 \times 10^{-13}}{1.67 \times 10^{-27}} = 3.83 \times 10^{14}$$

or $\quad v = 1.96 \times 10^7$ ms^{-1}. Now force on proton is

$$F = evB$$
$$= 1.6 \times 10^{-19} \times 1.96 \times 10^{7} \times 2.5$$
$$= 7.84 \times 10^{-12}\ \text{N}$$

Hence the closest choice is (d).

82. The point $x = +a$ lies along the line of the straight section PQ of the circuit. Hence the magnetic field at point $x = a$ is zero.

83. The radius of the circular path of a particle of mass m, charge e moving with a speed v perpendicular to a magnetic field B is given by

$$\frac{mv^2}{r} = evB \ \text{ or } \ r = \left(\frac{m}{e}\right)\frac{v}{B}$$

Thus, r is inversely proportional to $\left(\frac{e}{m}\right)$, the charge to mass ratio. Hence the correct choice is (c).

84. The total Lorentz force on the electron is

$$\mathbf{F} = -e\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

The electron will be undeflected if $\mathbf{v} \perp \mathbf{B}$. If $\mathbf{E}$ is along $+z$-direction, the force $-e\,\mathbf{E}$ will be along $-z$-direction. If $\mathbf{B}$ is along $+x$ direction, force $-e\,(\mathbf{v} \times \mathbf{B})$ will be along $+z$ direction. When $eE = evB$, the electron moves along $+y$-direction undeflected. Hence the correct choice is (c). Thus, for an electron moving along $+y$ direction, the electric field should be along $+z$ direction and magnetic field along $+x$ direction, then the electron will be undeflected.

85. Given $x = 4$ cm $= 0.04$ m and $r = 3$ cm $= 0.03$ m. The magnetic field at a point on the axis of the loop is given by

$$B = \frac{\mu_o I r^2}{2(r^2 + x^2)^{3/2}} \qquad (1)$$

Magnetic field at the centre of the coil is given by

$$B_0 = \frac{\mu_o I}{2r} \qquad (2)$$

Dividing (1) by (2), we get

$$\frac{B_0}{B} = \frac{r^3}{(r^2 + x^2)^{3/2}}$$

Substituting the values of r and x, we get

$$\frac{B_0}{B} = \frac{125}{27}$$

or $$B_0 = \frac{125}{27}B = \frac{125}{27} \times 54\ \mu\text{T} = 250\ \mu\text{T}$$

Hence the correct choice is (a).

86. Wires AB and EF experience no forces since currents in them are parallel to the magnetic field. The forces on BC and DE are equal in magnitude but are directed in opposite directions. Hence their resultant is zero. Only force acting is on CD. Hence the correct choice is (a).

87. $$\mathbf{F} = q(\boldsymbol{v} \times \boldsymbol{B}) = q\{a\hat{\mathbf{i}} \times (b\hat{\mathbf{j}} + c\hat{\mathbf{k}})\}$$
$$= q(ab\,\hat{\mathbf{i}} \times \hat{\mathbf{j}} + ac\,\hat{\mathbf{i}} \times \hat{\mathbf{k}})$$
$$= q(ab\,\hat{\mathbf{k}} - ac\,\hat{\mathbf{j}}) = qa(b\hat{\mathbf{k}} - c\hat{\mathbf{j}})$$

Magnitude of $F = qa(b^2 + c^2)^{1/2}$

The correct choice is (d).

88. Refer to Fig. 13.83. Let $AB = BC = AC = a$. Let $OD = r$.

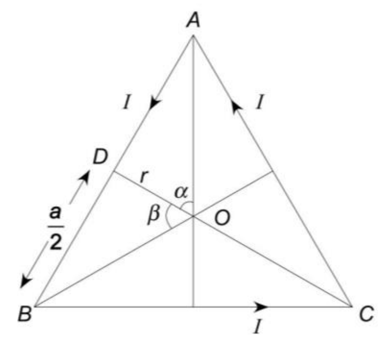


Fig. 13.83

The magnetic field at centroid O due to current I flowing in side AB of the triangle is given by

$$B_{AB} = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$$

It is clear that $\alpha = \beta = 60°$ and

$$OD = r = \frac{AD}{\tan\alpha} = \frac{a/2}{\tan 60°} = \frac{a/2}{\sqrt{3}} = \frac{a}{2\sqrt{3}}$$

$$\therefore \quad B_{AB} = \frac{\mu_0 I}{4\pi} \times \frac{2\sqrt{3}}{a} \times (\sin 60° + \sin 60°)$$
$$= \frac{3\mu_0 I}{2\pi a}$$

By symmetry, the magnetic fields due to current in sides BC and AC = that due to side AB. Hence, the magnetic field at O due to the current in the three sides of triangle ABC is

$$B = B_{AB} + B_{BC} + B_{CA} = 3B_{AB}$$

Hence the correct choice is (b).

89. Refer to Fig. 13.84. Let v be the velocity of the particle. Its kinetic energy is

$$\frac{1}{2}mv^2 = qV \text{ or } v = \left(\frac{2qV}{m}\right)^{1/2} \qquad (1)$$

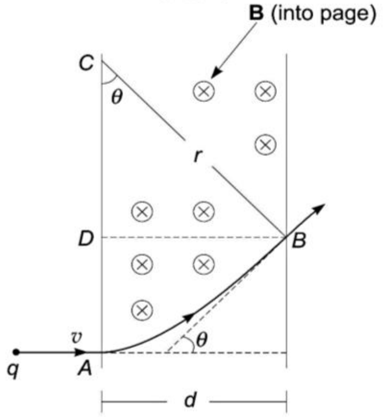


Fig. 13.84

The particle follows a circular path from A to B of radius r which is given by

$$\frac{mv^2}{r} = qvB \text{ or } r = \frac{mv}{qB} \qquad (2)$$

Using (1) and (2), we have

$$r = \frac{m}{qB}\left(\frac{2qV}{m}\right)^{1/2} = \frac{1}{B}\left(\frac{2mV}{q}\right)^{1/2}$$

I n triangle BCD, $\sin\theta = \frac{BD}{BC} = \frac{d}{r}$. Therefore,

$\sin\theta = Bd\left(\frac{q}{2mV}\right)^{1/2}$, which is choice (a).

90. Refer to Fig. 13.85. Wire PQ of length d, the spacing between rails carries a current I vertically downwards in a magnetic field pointing towards the reader and perpendicular to the length PQ of the wire. Thus angle θ between I and B is 90°. The force exerted on the wire of length d by the magnetic field is

$F = BId\sin 90° = BId$

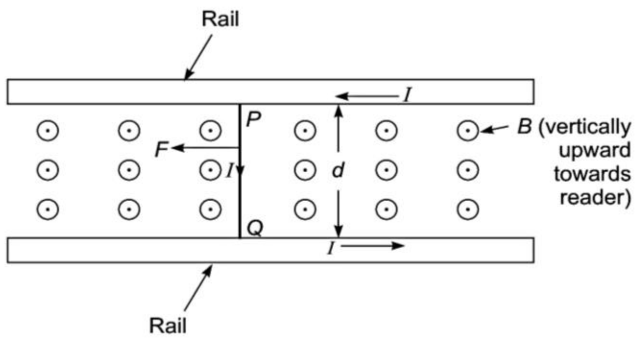


Fig. 13.85

Using Fleming's left hand rule, the direction of the force is to the left. The acceleration of the wire is

$$a = \frac{\text{force}}{\text{mass}} = \frac{F}{m} = \frac{BId}{m}$$

Now $x = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2x}{a}}$. Hence the correct choice is (b).

91. Refer to Fig. 13.86. Since the velocity of the particle is $v = a$ along the positive x-axis and the direction of the magnetic field $B = b$ in the positive z-direction, and the charge of the particle is positive, the path of the particle is a circle as shown in the figure. The radius of the circular path is

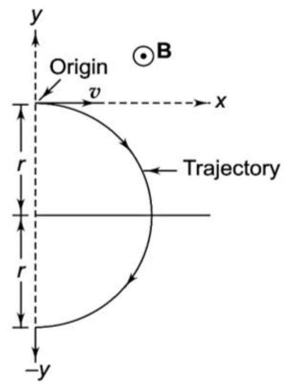


Fig. 13.86

$$r = \frac{mv}{qB} = \frac{ma}{qb}$$

Thus $\quad y = -2r = -\frac{2ma}{qb}$

So the correct choice is (d).

92. Refer to Fig. 13.87. It follows from Fleming's left hand rule that the wire segment PQ experiences a force $F_1 = B\,I\,l_1$ directed to the right and the wire segment SR experiences an equal force $F_1 = B\,I\,l_1$ but directed to the left. These forces cancel each other. The wire segment QR experiences a force $F = B\,I\,l_2$ directed vertically upward. The wire frame will remain stationary in air if F equal the weight mg of the wire frame, *i.e.*

$$B\,I\,l_2 = mg$$

$$\Rightarrow \qquad I = \frac{mg}{l_2 B}, \quad \text{which is choice (a).}$$

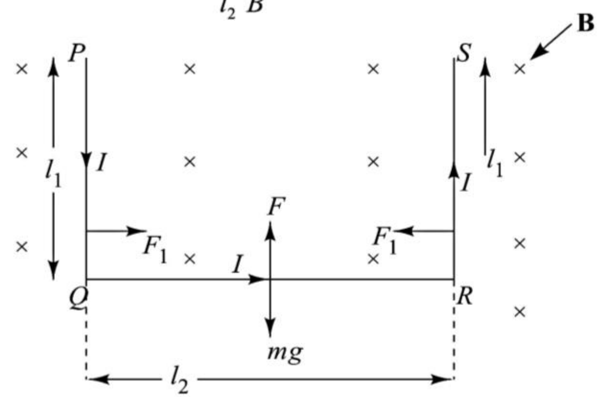


Fig. 13.87

93. Refer to Fig. 13.88.

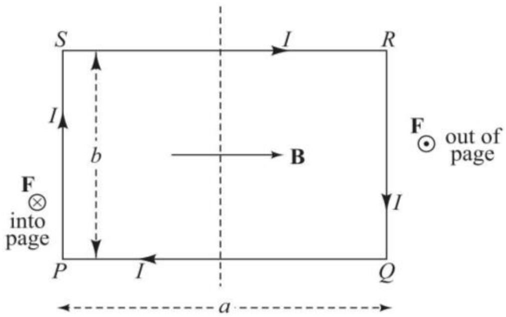


Fig. 13.88

The current element SR is parallel to **B**. Therefore $\theta = 0^0$. Hence, force exerted on arm $SR = B\,I\,a$ sin $0° = 0$. The current element QP is antiparallel to B. Therefore, $\theta = 180°$. Hence, force exerted on arm $QP = B\,I\,a \sin 180° = 0$. From Fleming's left hand rule, the force exerted on arm PS is $F = B\,I\,b \sin 90° = B\,I\,b$ directed into the page.

The force exerted on arm RQ is also $F = B\,I\,b$ but directed out of the page. These equal and opposite forces constitute a couple which exert a torque

τ = magnitude of either force × per perpendicular distance between the antiparallel forces

$= F \times PQ$

$= B\,I\,b\,a$

This torque will rotate the coil in the clockwise sense.

So the correct choice is (c).

94. Current segments PQ and ST are perpendicular to B. Therefore, the magnetic forces on PQ and ST are

$$F_1 = B\,I\,l \sin 90° = B\,I\,l$$

and $$F_2 = B\,I\,l \sin 90° = B\,I\,l$$

To find the magnetic force on the semi-circular arc QRS we divide it into a very large number of extremeley small lengths dl. Figure 13.89 shows one such element at A. The magnetic force on the element is

$\mathbf{dF} = I(\mathbf{dl} \times \mathbf{B})$. Since $\theta = 90°$, the magnitude of **dF** is

$$dF = IdlB \sin 90°$$

$$= IdlB = IBrd\theta \qquad (\because \quad dl = rd\theta)$$

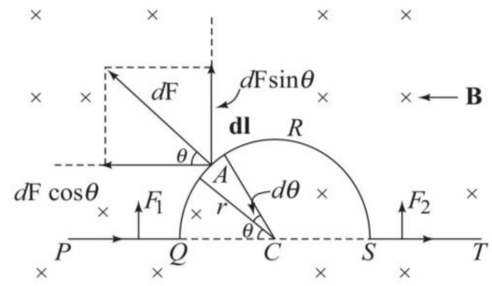


Fig. 13.89

The direction of **dF** is radially away from the centre C. The total horizontal component of force on the semi-circle is

$$(F_3)_x = \int dF \cos\theta$$

$$= I\,B\,r \int_0^{\pi} \cos\theta\, d\theta$$

$$= I\,B\,r \left|\sin\theta\right|_0^{\pi} = 0$$

The total vertical component of the force on the semi-circle is

$$(F_3)_y = \int dF \sin\theta$$

$$= I\,B\,r \int_0^{\pi} \sin\theta\, d\theta$$

$$= I\,B\,r \left|-\cos\theta\right|_0^{\pi} = 2IBr$$

$\therefore$ The total force on the wire frame is (since F_1, F_2 and $(F_3)_y$ are all directed vertically upward)

$$F = F_1 + F_2 + (F_3)_y$$

$$= BIl + BIl + 2\,BIr$$

$= 2\,BI\,(l + r)$. The direction of **F** is vertically upward.

So the correct choice is (a).

95. $\mathbf{F} = -q(\boldsymbol{v} \times \mathbf{B})$. The direction of $(\boldsymbol{v} \times \mathbf{B})$ is out of the page. Since $-(\boldsymbol{v} \times \mathbf{B}) = (\mathbf{B} \times \boldsymbol{v})$, we have

$$\mathbf{F} = q\,(\mathbf{B} \times \boldsymbol{v})$$

The direction of $(\mathbf{B} \times \boldsymbol{v})$ is into the page. So the correct choice is (d).

96. Statement I is false. The magnetic field lines due to a current carrying wires encircle the wire in closed loops. Statement II is also false because the magnetic force **F** is always perpendicular to the velocity $\boldsymbol{v}$ of the particle. Therefore power (and hence work done) $= \vec{\mathbf{F}}.\vec{v} = 0$. Since no work is done by the magnetic force, the kinetic energy of the particle cannot change. Statement III is true. Magnetic force is zero if $\boldsymbol{v}$ parallel or antiparallel to **B**. So the only correct choice is (d).

97. The magentic force provides the necessary centripetal force. Therefore,

$$qvB = \frac{mv^2}{r} \quad \Rightarrow \quad mv = qBr$$

Linear momentum $p = mv = qBr$
Angular momentum $l = pr = qBr^2$
So the correct choice is (a).

98. The magnitude of the magnetic field at a point at a distance r from a long wire carrying a current I is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

Therefore, the magnitude of the magnetic field at point P due to current in wire 1 is (since $r = d/2$).

$$B_1 = \frac{\mu_0 I}{2\pi (d/2)} = \frac{\mu_0 I}{\pi d}$$

From the right- hand thumb rule, the direction of the field is into the page. For wire 2,

$$B_2 = \frac{\mu_0 I}{\pi d} \quad \text{out of the page}$$

These two fields have equal magnitude but opposite direction. Hence they cancel each other. So the correct choice (d).

99. In this case, the two fields are both into the page and the magnitude of the field at P is

$$B = B_1 + B_2 = \frac{2\mu_0 I}{\pi d}$$

and its direction is into the page. So the correct choice is (a).

100. Figure 13.90 is the cross-sectional view of the cylinder and the shell.

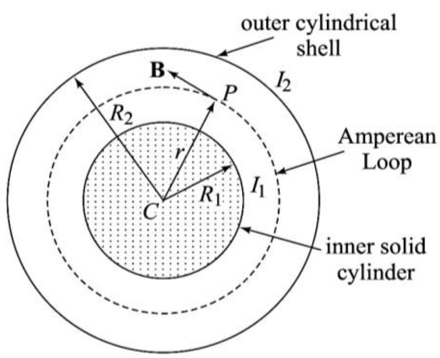


Fig. 13.90

From Ampere's Circuital law

$$\oint \mathbf{B} \cdot \mathbf{dl} = \mu_0 i$$

The current element **dl** at P is parallel to **B** at P. Hence $\theta = 0$ and

$$\oint \mathbf{B} \cdot \mathbf{dl} = \oint B\,dl \cos 0° = B \int dl = B \times 2\pi r$$

The current through the Amperean loop is

$$i = I_1$$

Therefore,

$$B \times 2\pi r = \mu_0 I_1$$

$$\Rightarrow \quad B = \frac{\mu_0 I_1}{2\pi r}$$

Putting $I_1 = I$ and $r = 3R/2$, we get

$$B = \frac{\mu_0 I}{3\pi R}, \quad \text{which is choice is (a).}$$

101. Refer to Fig. 13.91.

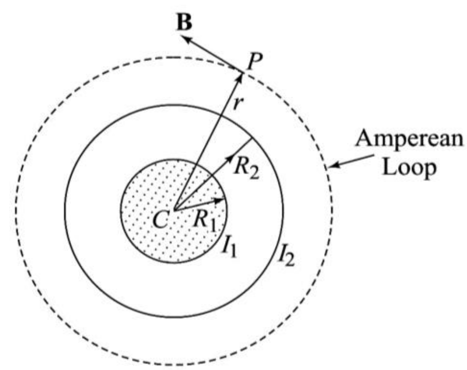


Fig. 13.91

In this case

$$B \times 2\pi r = \mu_0 i = \mu_0 (I_1 + I_2)$$

$$\Rightarrow \quad B = \frac{\mu_0 (I_1 + I_2)}{2\pi r}$$

Putting $I_1 = I$, $I_2 = I/2$ and $r = 3R$, we get

$$B = \frac{\mu_0 I}{12\pi R}$$

So the correct choice is (d).

102. In this case,

$$B \times 2\pi r = \mu_0 i = \mu_0 (I_1 - I_2)$$

$$\Rightarrow \quad B = \frac{\mu_0 (I_1 - I_2)}{2\pi r}$$

Putting $I_1 = I$, $I_2 = I/2$ and $r = 3R$, we get

$$B = \frac{\mu_0 I}{12\pi R}, \quad \text{which is choice (d).}$$

103. The straight portions PQ and RS do not produce magnetic field at C. Only the curved portion QR produces a magnetic field at C. Refer to Fig. 13.92.

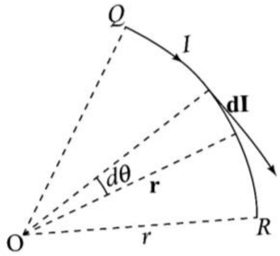


Fig. 13.92

Since the current **dl** is perpendicular to position vector **r**, the magnetic field at C by Biot-Savart law is

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin 90°}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{I\times(rd\theta)}{r^2} \qquad (\because dl = rd\theta)$$

$$= \frac{\mu_0 I}{4\pi r}d\theta$$

∴ Magnetic field at C due to arc QR is

$$B = \int dB = \frac{\mu_0 I}{4\pi r}\int_0^{\alpha} d\theta = \frac{\mu_0 I\alpha}{4\pi r}$$

So the correct choice is (d).

104. Since the particle describes a circular path, it is obvious that it enters the region of **B** with velocity $\boldsymbol{v}$ perpendicular to **B** as shown in Fig 13.93.

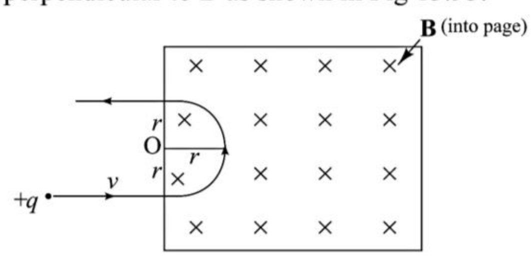


Fig. 13.93

Since the kinetic energy of the particle remains constant (because its velocity vector is always perpendicular to the force vector which is radial), it describes a semi-circle in the region of the magnetic field. Since its K.E. (and hence its speed v) remains constant, the time the particle takes to describe the semi-circle is

$$t = \frac{\pi r}{v} \qquad (1)$$

Also, kinetic energy = qV

or $$\frac{1}{2}mv^2 = qV$$

$$\Rightarrow \qquad v = \sqrt{\frac{2qV}{m}} \qquad (2)$$

Using (2) in (1) we get

$$t = \pi r\sqrt{\frac{m}{2qV}},$$ which is choice (c).

105. The current density $J = \dfrac{I}{\pi R^2}$.

For a homogeneous cylinder, the current density J is constant and does not depend on the radial distance r of the point P from centre O of the cylinder (see Fig. 13.94)

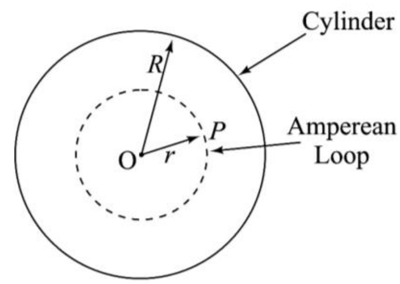


Fig. 13.94

Since the point P is inside the cylinder, $r < R$, the current enclosed by the Amperean loop is

$$i = J \times \text{area of Amperean loop}$$

$$= \frac{I}{\pi R^2}\times\pi r^2 = \frac{Ir^2}{R^2}$$

From Amperean Circuital law,

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 i$$

or $$B\times 2\pi r = \mu_0 \times \frac{Ir^2}{R^2}$$

$$\Rightarrow \qquad B = \frac{\mu_0 I r}{2\pi R^2} \qquad (1)$$

Putting $r = \dfrac{R}{2}$ in eq. (1), we get

$$B = \frac{\mu_0 I}{4\pi R},$$ which is choice (d).

106. Refer to Fig 13.95.

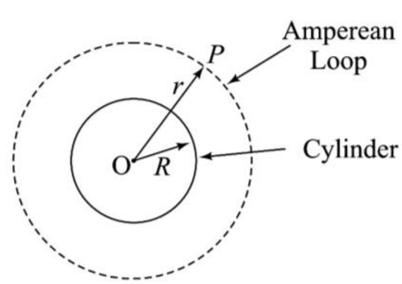


Fig. 13.95

In the case $r > R$, the current enclosed inside the Amperean loop = current on the cylinder, *i.e.* $i = I$. Hence

$$B\times 2\pi r = \mu_0 I$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

For $r = 2R$,

$$B = \frac{\mu_0 I}{4\pi R}$$, which is choice (b).

107. Since J varies with r, to find the current I through the rod, we divide it into thin cylindrical strips each of a small width dr (see Fig 13.96)

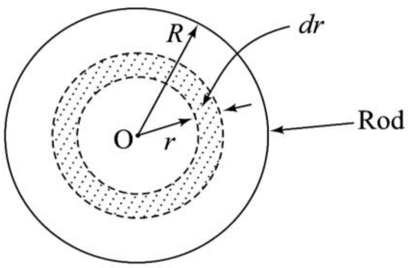


Fig 13.96

Area of strip $dA = (2\pi r)\,dr$. So the current through the strip is

$$dI = JdA = \sigma r \times (2\pi r)dr$$
$$= 2\pi\sigma r^2 dr$$

Therefore, current through the cylinder is

$$I = \int dI = 2\pi\sigma \int_0^R r^2\,dr$$

$$\Rightarrow \quad I = \frac{2}{3}\pi\sigma R^3 \qquad (1)$$

We consider Amperean loop of radius r. The current through the loop is

$$i = 2\pi\sigma \int_0^r r^2\,dr = \frac{2}{3}\pi\sigma r^3$$

From Ampere's circuital law,

$$B \times 2\pi r = \mu_0 i = \mu_0 \times \frac{2}{3}\pi\sigma r^3$$

$$\Rightarrow \quad B = \frac{1}{3}\mu_0 \sigma r^2 \qquad (2)$$

From Eq. (1)

$$\sigma = \frac{3I}{2\pi R^3} \qquad (3)$$

Using (3) in (2) we get

$$B = \frac{1}{3}\mu_0 \times \left(\frac{3I}{2\pi R^3}\right) r^2$$

$$\Rightarrow \quad B = \frac{\mu_0 I r^2}{2\pi R^3}$$

So the correct choice is (a).

Note For points outside the rod, $r > R$, the current enclosed by the Amperean loop = current through the cylinder, *i.e.* $i = I$. So

$$B \times 2\pi r = \mu_0 I$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

which is the same as the expression for a homogeneous rod for the case $r > R$.

108. The magnetic field due to straight segments PQ and TS in both cases in zero.

Case (a)

The magnetic field at C due to semi-circular loop of radius R is $B = \frac{\mu_0 I}{4R}$ directed out of the page, since the current is clockwise.

The magnetic field at C due to semi-circular loop of radius $2R$ is $B' = \frac{\mu_0 I}{4(2R)} = \frac{\mu_0 I}{8R}$ directed into the page since the current now is anticlockwise.

Since $B > B'$, the net magnetic field at C is

$$B_1 = B - B' = \frac{\mu_0 I}{4}\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{\mu_0 I}{8R}$$

directed out of the page.

Case (b)

In this case, the current in both loops is clockwise. Therefore, the magnetic field due to each semi-circular loop is directed into the page and its magnitude is

$$B_1 = B + B' = \frac{\mu_0 I}{4}\left(\frac{1}{R} + \frac{1}{2R}\right) = \frac{3\mu_0 I}{8R}$$

directed into the page.

Therefor, $\frac{B_1}{B_2} = \frac{1}{3}$, which is choice (c).

109. The correct choice is (a) as explained above.

110. Since the plane of coil A is vertical, it is perpendicular to the magnetic meridian. Hence the magnetic field produced by current I_A can neutralize the horizontal component (B_H) of earth's magnetic field. Similarly, current I_B can neutralize the vertical component (B_V). Hence

$$\frac{\mu_0 I_A n}{2r} = B_H$$

and $\frac{\mu_0 I_B n}{2r} = B_V$

Dividing we get, $\frac{I_A}{I_B} = \frac{B_H}{B_V} = \frac{B_H}{B_H \tan\theta} = \cot\theta$

So the correct choice is (d).

111. $r = \frac{\sqrt{2mqV}}{qB} = \frac{1}{B}\left(\frac{2mV}{q}\right)^{1/2}$

$\therefore \quad \frac{q}{m} = \frac{2V}{B^2 r^2}$

$\therefore \quad \frac{\left(\frac{q}{m}\right)_X}{\left(\frac{q}{m}\right)_Y} = \left(\frac{r_2}{r_1}\right)^2 \quad (\because B \text{ and } V \text{ are the same})$

So the correct choice is (d).

112. The forces acting on arms QR and PS are equal and opposite and hence cancel each other. The current in arm QP is antiparallel to current in AB. Hence QP and AB will repal each other. The current in SR is parallel to current in AB. Hence SR and AB will attract each other. Since force per unit length is $f = \frac{\mu_0 I i}{r}$ it follows that $f \propto \frac{1}{r}$.

Since arm QP is closer to AB than arm SR, it follows that the repulsive force on the coil is greater than the attractive force. So the correct choice is (c).

113. Since the currents in the two wires are in the same direction, the magnetic fields due to the current I in the wire at point P exactly mid-way between them will be equal and opposite. The net magnetic field at point P is zero. So the correct choice is (d).

114. If the two wires carry current I in opposite directions, the magnetic field at P due to current I in each wire $= \frac{\mu_0 I}{2\pi (d/2)} = \frac{\mu_0 I}{\pi d}$ directed perpendicular to the plane of the wires. The net magnetic field at P is

$$B' = \frac{\mu_0 I}{\pi d} + \frac{\mu_0 I}{\pi d} = \frac{2\mu_0 I}{\pi d} = 2B$$

Force on charge q is $\mathbf{F} = q(\boldsymbol{v} \times \mathbf{B}') = qvB' \sin\theta = 2\,qvB \sin\theta$.

Since $\mathbf{B}'$ and $\boldsymbol{v}$ are both perpendicular to the plane containing the wires, angle θ between them is zero. Hence $F = 0$ in this case also. So the correct choice is (a).

115. $\frac{mv^2}{r} = qvB \Rightarrow mv = qrB$

$\therefore \quad m_1 v_1 = q r_1 B$ and $m_2 v_2 = q r_2 B$

$\therefore \quad \frac{m_1 v_1}{m_2 v_2} = \frac{r_1}{r_2} < 1 \quad (\because r_1 < r_2)$

So the correct choice is (c).

116. $\mathbf{F} = q(\mathbf{u} \times \mathbf{B})$. Since force $\mathbf{F}$ is perpendicular to $\mathbf{u}$, it does no work on the particle. Hence the speed of the proton remains unchanged, *i.e.* $v = u$ [see Fig. 13.97] From Fleming's L.H. rule, the force is directed upwards. Hence the proton, after completing a semi-circle in the region of magnetic field emerges at positive y-coordinate.

So the correct choice is (a).

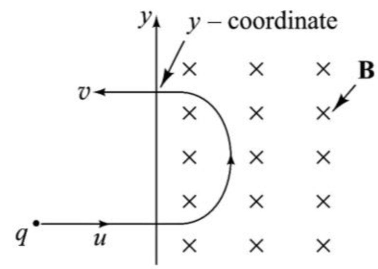


Fig. 13.97

117. Since to loop carries a current, a charge (say q) moves along the circle with a velocity (say v). The velocity v is tangential to the circle at every point. The direction of v gives the direction of the current as shown in Fig 13.98.

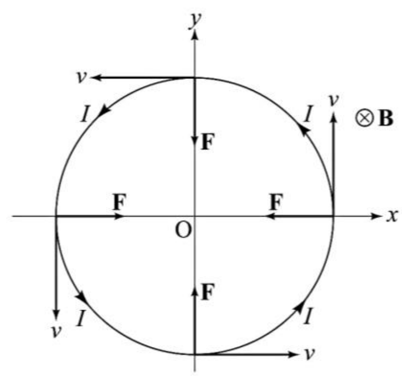


Fig. 13.98

From Fleming's left hand rule, the direction of the force $\mathbf{F}$ exerted by magnetic field is radially inwards towards O at every point. Hence the loop tends to contract. Furthermore, since $\mathbf{F}$ is perpendicular to $\boldsymbol{v}$, the force does no work on the loop. Hence it cannot have any translatory motion. Thus the correct choice is (c).

Note The loop will tend to expand if either the current in the loop is clockwise or the magnetic field points out of the plane of the coil.

118. Refer to Fig. 13.99. The magnetic field at O due to parts AB and EF of the frame is zero.

From junction rule.

$$I_1 + I_2 = I \quad (1)$$

Applying loop rule to loop $BCDEB$, we get (here R = resistance of $BC = CD = DE = BE$)

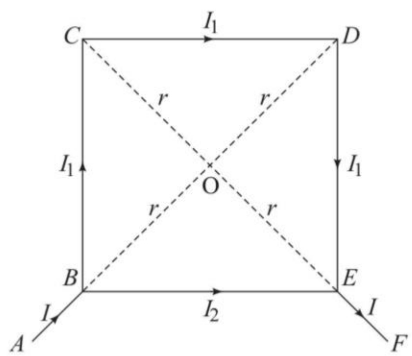


Fig. 13.99

$$3RI_1 = RI_2 \quad \Rightarrow \quad 3I_1 = I_2 \quad (2)$$

From (1) and (2), we get $I_1 = \frac{I}{4}$ and $I_2 = \frac{3I}{4}$.

Magnetic field at O due current I_1 in BC is

$$B_{BC} = \frac{\mu_0 I_1}{4\pi r}(\sin 45^0 + \sin 45^0) = \frac{\mu_0 I_1}{2\sqrt{2}\pi r}$$

But $I_1 = I/4$, therefore

$$B_{BC} = \frac{\mu_0 I}{8\sqrt{2}\pi r} \quad \text{directed into the page,}$$

Similarly,

$$B_{CD} = \frac{\mu_0 I}{8\sqrt{2}\pi r} \quad \text{directed into the page,}$$

and $$B_{DE} = \frac{\mu_0 I}{8\sqrt{2}\pi r} \quad \text{directed into the page,}$$

$$\therefore \quad B_{BCDE} = \frac{3\mu_0 I}{8\sqrt{2}\pi r} \quad (3)$$

directed into the page.

Magnetic field at O due to current I_2 in BE is

$$B_{BE} = \frac{\mu_0 I_2}{2\sqrt{2}\pi r} = \frac{3\mu_0 I}{8\sqrt{2}\pi r} \quad (4)$$

directed out of the page.

From (3) and (4) it follows that the magnetic field at O due to the complete frame is zero. So the correct choice is (d).

119. Magnetic field at O due to current I in AB is

$$B_{AB} = \frac{\mu_0 I}{4\pi r}(\sin 90° + \sin 0°) = \frac{\mu_0 I}{4\pi r}$$

directed out of the page. Similarly

$$B_{ED} = \frac{\mu_0 I}{4\pi r} \quad \text{directed out of the page.}$$

Magnetic field at O due to current I in semi-circular part BCD is

$$B_{BCD} = \frac{\mu_0 I}{4r} \quad \text{directed out of the page.}$$

$\therefore$ Magnetic field at O due to $ABCDE$ is

$$B = B_{AB} + B_{ED} + B_{BCD}$$

$$= \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r}$$

$$= \frac{\mu_0 I}{4r}(2\pi + 1) \quad \text{directed out of the page.}$$

So the correct choice is (c).

120. $B_{AB} = 0$

$$B_{DE} = \frac{\mu_0 I}{4\pi r} \quad \text{directed into the page}$$

$$B_{DCB} = \frac{\mu_0 I}{4r} \quad \text{directed into the page}$$

$$\therefore \quad B_{ABCDE} = 0 + \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r}$$

$$= \frac{\mu_0 I}{4\pi r}(\pi + 1)$$

So the correct choice is (a).

121. $B = \frac{\mu_0 I}{2r} \times \frac{\theta}{2\pi}$. Here $\theta = 60^0 = \frac{\pi}{3}$. Therefore

$$B = \frac{\mu_0 I}{12r}$$

Magnetic field at O due to current I in AD is

$$B_{AD} = \frac{\mu_0 I}{12r} \quad \text{directed out of the page.}$$

Magnetic field at O due to current I in BC is

$$B_{BC} = \frac{\mu_0 I}{12(2r)} = \frac{\mu_0 I}{24r} \quad \text{directed into the page.}$$

Therefore, Magnetic field at O due to $ABCD$ is

$$B = B_{AD} - B_{BC}$$

$$= \frac{\mu_0 I}{12r} - \frac{\mu_0 I}{24r} = \frac{\mu_0 I}{24r}$$

directed out of the page. The magnetic field at O due to straight segments AB and CD is zero. So the correct choice is (a).

122. Magnetic field at O due to straight segments BC and AD is zero. For the curved part AB, $\theta = \frac{3\pi}{2}$. Therefore,

$$B_{AB} = \frac{3\mu_0 I}{8r} \quad \text{directed into the page.}$$

For the curved part CD, $\theta = \frac{\pi}{2}$. Therefore,

$$B_{CD} = \frac{\mu_0 I}{16r} \quad \text{directed into the page.}$$

Therefore, magnetic field at O due to current I in $ABCD$ is

$$B = B_{AB} + B_{CD}$$

$$= \frac{3\mu_0 I}{8r} + \frac{\mu_0 I}{16r} = \frac{7\mu_0 I}{16r}$$

directed into the page. So the correct choice is (d).

123. Refer to Fig. 13.100.

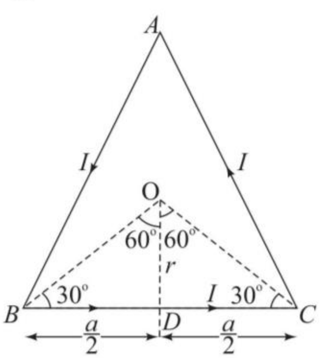


Fig. 13.100

Magnetic field at O due to current I in side BC is

$$B_{BC} = \frac{\mu_0 I}{4\pi r}\left(\sin 60^0 + \sin 60^0\right)$$

$$= \frac{\sqrt{3}\mu_0 I}{4\pi r}$$

Now $r = \frac{a}{2}\tan 30^0 = \frac{a}{2\sqrt{3}}$. Therefore,

$$B_{BC} = \frac{\sqrt{3}\mu_0 I \times 2\sqrt{3}}{4\pi \times a} = \frac{3\mu_0 I}{2\pi a}$$

directed out of the page. The magnetic field at O due to current I in sides CA and AB is the same as B_{BC} and is directed out of the page. Hence, the magnitude of magnetic field at O due to current I in ABC is

$$B = 3B_{BC} = \frac{9\mu_0 I}{2\pi a}$$

So the correct choice is (d).

124. $\mathbf{F} = q\,(\boldsymbol{v} \times \mathbf{B})$. Hence $\mathbf{F}$ is perpendicular to $\mathbf{B}$. Therefore,

$$\mathbf{F.B} = 0$$

or $\quad m\mathbf{a.B} = 0$

$\Rightarrow \quad \mathbf{a.B} = 0 \quad (\because m \neq 0)$

$\Rightarrow (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - n\hat{\mathbf{k}}) \circ (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) = 0$

$\Rightarrow \quad 2 + 6 - 4n = 0$

$\Rightarrow \quad n = 2$

So the correct choice is (b).

125. Magnetic moment $\mathbf{M} = I\mathbf{A}$

where $\mathbf{A}$ is the area vector. Since the current is clockwise the direction of $\mathbf{A}$ is along the negative z-axis, *i.e.*, along $-\hat{\mathbf{k}}$. The magnitude of $\mathbf{A}$ is

A = area of square of side a + area of four equilateral triangles each of side a

= area of $BDFH$ + 4 × area of BCD

$$= a^2 + \frac{4\sqrt{3}\,a^2}{4}$$

$$= a^2\left(\sqrt{3}+1\right)$$

$$\therefore \quad \mathbf{M} = -\left(\sqrt{3}+1\right)a^2 I\,\hat{\mathbf{k}}$$

So the correct choice is (c).

2 SECTION Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage.

Passage I

Two long parallel wires carrying currents 2.5 amperes and I ampere in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of the paper. The points P and Q are located at a distance of 5m and 2m, respectively, from a collinear point R (see Fig. 13.78).

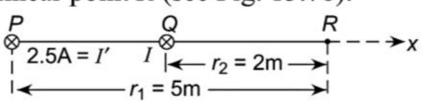


Fig. 78

An electron moving with a velocity of 4×10^5 m/s along the positive x-direction experiences a force of magnitude 3.2×10^{-20} N at the point R.

1. The magnitude of magnetic field at point R is
   (a) 2.5×10^{-7} T (b) 5.0×10^{-7} T
   (c) 5.0×10^{-6} T (d) 2.5×10^{-6} T
2. The magnitude of magnetic field at point R due to current $I' = 2.5$ A in wire P is
   (a) 1×10^{-7} T (b) 2×10^{-7} T
   (b) 3×10^{-7} T (d) 4×10^{-7} T
3. The current I in wire Q is
   (a) 1 A (b) 2 A
   (c) 3 A (d) 4 A

Solutions

1. The magnitude of the force experienced by a particle of charge q moving with a velocity v in a magnetic field B is given by
   $$F = qv\,B \sin\theta$$
   where θ is the angle between v and $\mathbf{B}$. Given $F = 3.2 \times 10^{-20}$ N, $v = 4 \times 10^5$ ms^{-1} and $\theta = 90°$. For electron $q = 1.6 \times 10^{-19}$ C. Using these value we get $B = 5 \times 10^{-7}$ T, is choice (b)
2. The magnetic field at point R due to currect I' in wire P is
   $$B_1 = \frac{\mu_0 I'}{2\pi r_1} = \frac{4\pi \times 10^{-7} \times 2.5}{2\pi \times 5} = 1 \times 10^{-7}\text{ T}$$
   The correct choice is (a).
3. The magnetic field at point R due to currect I in wire Q is
   $$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{4\pi \times 10^{-7} \times I}{2\pi \times 2} = I \times 10^{-7}\text{ T}$$
   Both fields B_1 and B_2 will be in the downward direction, parallel and collinear. Hence, the resultant magnetic field at point R is
   $$B = B_1 + B_2 = (1 + I) \times 10^{-7}\text{ T}$$
   Now $B = 5 \times 10^{-7}$ T. Therefore,
   $$(1 + I) \times 10^{-7} = 5 \times 10^{-7}$$
   or $1 + I = 5$ or $I = 4$ A.
   So the correct choice is (d).

Questions 4 to 7 are based on the following passage.

Passage II

The region between $x = 0$ and $x = L$ is filled with a uniform, steady magnetic field $B_0\hat{\mathbf{k}}$. A particle of mass m, positive charge q and velocity $v_0\hat{\mathbf{i}}$ travels along x-axis and enters the region of the magnetic field. Neglect gravity.

4. The force experienced by the charged particle in the magnetic field is
   (a) along the positive y-direction
   (b) along the negative y-direction
   (c) in the x-y plane
   (d) in the y-z plane.
5. If the particle emerges from the region of magnetic field with its final velocity at an angle of 30° to its initial velocity, the value of L is
   (a) $\frac{2mv_0}{qB_0}$ (b) $\frac{mv_0}{qB_0}$
   (c) $\frac{mv_0}{2qB_0}$ (d) $\frac{\sqrt{3}mv_0}{2qB_0}$
6. If the magnetic field now extends up to $x = 2.1\,L$, the final velocity of the particle when it emerges out of the region of magnetic field will be
   (a) $v_0\hat{\mathbf{i}}$ (b) $-v_0\hat{\mathbf{i}}$
   (c) $v_0\hat{\mathbf{j}}$ (d) $-v_0\hat{\mathbf{j}}$

7. In Q. 6, the time spent by the particle in the magnetic field is

(a) $t = \dfrac{2\pi m}{qB_0}$ (b) $t = \dfrac{\sqrt{2}\pi m}{qB_0}$

(c) $t = \dfrac{\sqrt{3}\pi m}{2qB_0}$ (d) $t = \dfrac{\pi m}{qB_0}$

Solutions

4. The force experienced by the charged particle is given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = q(v_0\,\hat{\mathbf{i}}) \times (B_0\,\hat{\mathbf{k}})$$
$$= qv_0B_0(\hat{\mathbf{i}} \times \hat{\mathbf{k}})$$
$$= qv_0\,B_0(-\hat{\mathbf{j}}) \qquad (1)$$

The force is along the negative y-direction, which is choice (b).

5. Refer to Fig. 13.101.

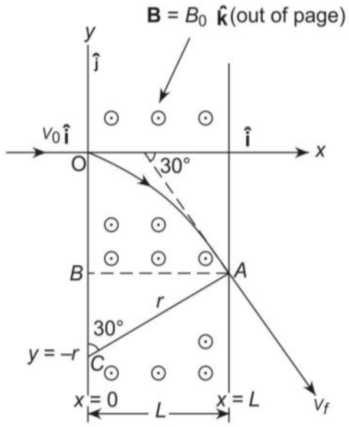


Fig. 13.101

The particle describes a circle of radius

$$r = \frac{mv_0}{qB_0} \qquad (2)$$

Since the particle emerges from the region of the magnetic field with the velocity vector making an angle of 30° with the initial vector, it follows from triangle ABC that

$$AB = AC \sin 30°$$

or $$L = r\sin 30° = \frac{mv_0 \sin 30°}{qB_0} = \frac{mv_0}{2qB_0} \qquad (3)$$

Thus the correct choice is (c).

6. Comparing (2) and (3) we find that $r = 2\ L$. Since the magnetic field now extends up to $x = 2.\ 1\ L$, the particle will continue to move in a circular path till it completes half the circular path and emerges out of the region of the magnetic field with a velocity $-v_0\,\hat{\mathbf{i}}$ moving along the negative x-axis as shown in Fig. 13.102.

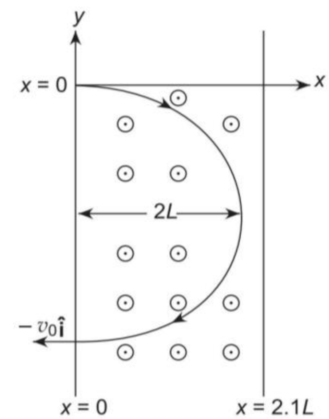


Fig. 13.102

7. Distance travelled by the particle in the magnetic field = half the circumference = πr. Therefore, time spent in the magnetic field is

$$t = \frac{\pi r}{v_0} = \frac{\pi m}{qB_0} \qquad \text{[Use Eq. (2)]}$$

So the correct choice is (d).

Questions 8 to 11 are based on the following passage.

Passage III

A wire loop consists of a straight segment AB and a circular arc ACB of radius r. The segment AB subtends an angle of 60° at the centre O of the circular arc. The wire loop carries a current I in the clockwise direction (Fig. 13.103).

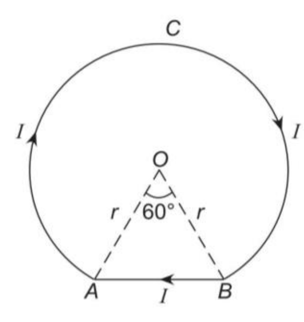


Fig. 13.103

8. The magnetic field B_1 at O due to the straight segment AB is

(a) $\dfrac{\mu_0 I}{2\pi r}$ (b) $\dfrac{\mu_0 I}{2\sqrt{2}\pi r}$

(c) $\dfrac{\mu_0 I}{2\sqrt{3}\pi r}$ (d) $\dfrac{\mu_0 I}{4\pi r}$

9. The magnetic field B_2 at O due to the circular arc ACB is

(a) $\frac{5\mu_0 I}{12r}$ (b) $\frac{\mu_0 I}{2r}$

(c) $\frac{3\mu_0 I}{8r}$ (d) $\frac{7\mu_0 I}{18r}$

10. The net magnetic field B at O due to the whole wire loop is

(a) $B = B_1 + B_2$ (b) $B = B_2 - B_1$

(c) $B = \sqrt{B_1^2 + B_2^2}$ (d) $B = \sqrt{B_2^2 - B_1^2}$

11. The direction of the magnetic field B is

(a) parallel to the plane of the coil

(b) perpendicular to the plane of the coil and directed out of the page

(c) perpendicular to the plane of the coil and directed into the page

(d) inclined at an angle of 60° with the plane of the coil

Solutions

8. As shown in Fig. 13.104, the magnetic field at O due to current I in AB is given by (use Biot-Savart law)

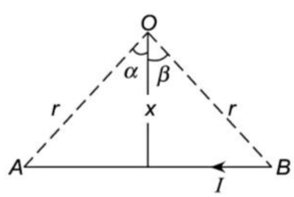


Fig. 13.104

$$B_{AB} = \frac{\mu_0 I}{4\pi x}(\sin\alpha + \sin\beta)$$

Here $\alpha = \beta = 30°$. Also $x = r\cos\alpha = r\cos 30° = \frac{\sqrt{3}r}{2}$.

Therefore,

$$B_1 = \frac{\mu_0 I}{4\pi \times \sqrt{3}r/2} \times (\sin 30° + \sin 30°)$$

$$= \frac{\mu_0 I}{2\sqrt{3}\pi r} \times (0.5 + 0.5) = \frac{\mu_0 I}{2\sqrt{3}\pi r},$$

which is choice (c).

The direction of the field is perpendicular to the plane of the paper directed into the page.

9. The magnetic field at the centre of a complete ($n = 1$ turn) circular loop of radius r and carrying a current I is

$$B = \frac{\mu_0 n I}{2r}$$

Here loop ACB is a fraction of a circle, i.e. $n < 1$. Since ACB subtends an angle (360° – 60°) = 300° at O, hence the fraction n is

$$n = \frac{300°}{360°} = \frac{5}{6}$$

Therefore, magnetic field due to arc ACB is

$$B_2 = \frac{\mu_0 \times \frac{5}{6} \times I}{2r} = \frac{5\mu_0 I}{12r}$$

As the current in ACB is clockwise, the direction of the magnetic field is perpendicular to the plane of the paper and directed into the page.

The correct choice is (a).

10. Since B_1 and B_2 are in the same direction, the net field is $B = (B_1 + B_2)$, which is choice (a).

11. The correct choice is (c).

Questions 12 to 15 are based on the following passage.

Passage IV

A moving coil galvanometer consists of a coil of N turns and area A suspended by a thin phosphor bronze strip in radial magnetic field B. The moment of inertia of the coil about the axis of rotation is I and C is the torsional constant of the phosphor bronze strip. When a current i is passed through the coil, it deflects through an angle θ (in radian).

12. Choose the correct statement from the following. The magnitude of the torque experienced by the coil is independent of

(a) N (b) B

(c) i (d) I

13. The current sensitivity of the galvanometer is increased if

(a) N, A and B are increased and C is decreased.

(b) N and A are increased and B and C are decreased

(c) N, B and C are increased and A is decreased

(d) N, A, B and C are all increased.

14. When a charge Q is passed almost instantly through the coil, the angular speed ω acquired by the coil is

(a) $\frac{NAB}{QI}$ (b) $\frac{BAQ}{NI}$

(c) $\frac{NABQ}{I}$ (d) $\frac{NAQI}{B}$

15. In Q. 14, the maximum angular deflection (in radian) of the coil is

(a) $\theta_{max} = \omega\sqrt{\frac{I}{C}}$ (b) $\theta_{max} = \frac{1}{C}\sqrt{I\omega}$

(c) $\theta_{max} = I\sqrt{\frac{\omega}{C}}$ (d) $\theta_{max} = \omega\sqrt{IC}$

Solutions

12. The magnitude of torque experienced by the coil is given by

$$\tau = iNAB \sin \alpha$$

where α is the angle which the normal to the plane of the coil makes with the direction of the magnetic field. If the magnetic field is radial, the plane of the coil is always parallel to the direction of the magnetic field, i.e. $\alpha = 90°$. Hence $\tau = iNAB = Ki$ where

$$K = NAB$$

So the correct choice is (d).

13. Let θ be the angular deflection (in radian) when a current i is passed through the coil. Then, restoring torque = $C\theta$. When the coil is in equilibrium, deflecting torque = restoring torque, i.e.

$$iNAB = C\theta$$

$\therefore$ Current sensitivity is $\dfrac{\theta}{i} = \dfrac{NAB}{C}$

Hence the correct choice is (a).

14. If ω is the angular speed acquired by the coil when a charge Q is passed through it for very short time Δt, then

$$\tau = \frac{\text{angular momentum}}{\text{time interval}} = \frac{I\omega}{\Delta t}$$

or $I\omega = \tau\Delta t = Ki\Delta t = KQ \quad \left(\because i = \dfrac{Q}{\Delta t}\right)$

or $I\omega = NABQ$ or $\omega = \dfrac{NABQ}{I}$, which is choice (c).

15. From the principle of conservation of energy, we have

$$\frac{1}{2} I\omega^2 = \frac{1}{2} C\theta_{max}^2$$

which gives $\theta_{max} = \omega\sqrt{\dfrac{I}{C}}$, which is choice (a).

3 SECTION

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by statement-2 (Reason). Each question has the following four options out of which only one choice is correct.

(a) Statement-1 is True, Statement-2 is True and Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; but Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

1. **Statement-1**

A non-uniform magnetic field that varies in magnitude from point to point but has a constant direction, is set up in a region of space. If a charged particle enters the region in the direction of the magnetic field, it will be accelerated at non-uniform rate in the region.

Statement-2

The force $\vec{F}$ experienced by a particle of charge q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F} = q(\vec{v} \times \vec{B})$.

2. **Statement-1**

A charged particle moves in a uniform magnetic field for some time. During this time, the kinetic energy of the particle cannot change but its momentum can change.

Statement-2

The magnetic force is always perpendicular to the velocity of the particle.

3. **Statement-1**

A current carrying loop is free to rotate. It is placed in a uniform magnetic field. It will attain equilibrium when its plane is perpendicular to the magnetic field.

Statement-2

The torque on the coil is zero when its plane is perpendicular to the magnetic field.

4. **Statement-1**

An electron moving in the positive x-direction enters a region where uniform electric and magnetic fields exist perpendicular to each other. The electric field is in the negative y-direction. If the electron travels undeflected in this region, the direction of the magnetic field is along the negative z-axis.

Statement-2

If a charged particle moves in a direction perpendicular to a magnetic field, the direction of the force acting on it is given by Fleming's left-hand rule.

5. **Statement-1**

If a charged particle is released from rest in a region of uniform electric and magnetic fields parallel to each other, it will move in a straight line.

Statement-2

The electric field exerts no force on the particle but the magnetic field does.

6. **Statement-1**

A proton and an alpha particle having the same kinetic energy are moving in circular paths in a uniform magnetic field. The radii of their circular paths will be equal.

Statement-2

Any two charged particles having equal kinetic energies and entering a region of uniform magnetic field $\vec{B}$ in a direction perpendicular to $\vec{B}$, will describe circular trajectories of equal radii.

7. **Statement-1**

Two particles having equal charges and masses m_1 and m_2, after being accelerated by the same potential difference (V), enter a region of uniform magnetic field and describe circular paths of radii r_1 and r_2 respectively. Then

$$\frac{m_1}{m_2} = \sqrt{\frac{r_1}{r_2}}$$

Statement-2

Gain in kinetic energy = work done to accelerate the charged particle through potential difference V.

Solutions

1. The correct choice is (d). If $\vec{v}$ is parallel to $\vec{B}$, $\vec{F}$ = 0. Hence the particle does not experience any force and is, therefore, not accelerated in the region. It will travel undeflected with a constant speed.

2. The correct choice is (a). Since the magnetic force is always perpendicular to the velocity, no work is done by a uniform magnetic field on a charged particle. Hence magnetic force cannot change the magnitude of velocity (i.e. speed); it can only change the direction of velocity. Hence kinetic energy $\left(=\frac{1}{2}mv^2\right)$ remains unchanged but momentum $\vec{p}$ $= m\vec{v}$ will change.

3. The correct choice is (a). The loop will rotate and come to rest when the torque acting on it becomes zero. The magnitude to torque acting on a loop of area A and carrying a current I in a magnetic field B is given by

$$\tau = B\,I\,A \sin\theta$$

where θ is the angle between the direction of the magnetic field and the normal to the plane of the coil. It is clear that $\tau = 0$ when $\theta = 0$, i.e. when the plane of the coil is perpendicular to the magnetic field.

4. The correct choice is (a). Because electron has a negative charge, an electric field in the negative y-direction will deflect it in the positive y-direction. It will travel undeflected if the magnetic field imparts an equal deflection in the negative y-direction. Since the magnetic force is perpendicular to the magnetic field and the charge of electron is negative, the direction of the magnetic field (according to Fleming's Left-Hand rule) should be along the negative z-direction.

5. The correct choice is (b). Due to electric field, the force is $\vec{F} = q\vec{E}$ in the direction of $\vec{E}$. Since $\vec{E}$ is parallel to $\vec{B}$, the particle velocity $\vec{v}$ (acquired due to force $\vec{F}$) is parallel to $\vec{B}$. Hence $\vec{B}$ will not exert any force since $\vec{v} \times \vec{B} = 0$ and the motion of the particle is not affected by $\vec{B}$.

6. The correct choice is (c). The radius of the circular path is given by

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}\ ;\ \text{where } K = \frac{1}{2}mv^2$$

Since K and B are the same for the two particles, $r \propto \frac{\sqrt{m}}{q}$. Now, the charge of an alpha particle is twice that of a proton and its mass is four times the mass of a proton, $\sqrt{m}/q$ will be the same for both particles. Hence r will be the same for both particles.

7. The correct choice is (d). Kinetic energy $K = qV$.

Therefore $r_1 = \frac{\sqrt{2m_1qV}}{qB}$ and $r_2 = \frac{\sqrt{2m_2qV}}{qB}$

Hence $\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}} \Rightarrow \frac{m_1}{m_2} = \left(\frac{r_1}{r_2}\right)^2$.

4 SECTION Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced)

1. If in a circular coil A of radius R, current i is flowing and in another coil B of radius $2R$, current $2i$ is flowing, then the ratio of magnetic fields B_A and B_B at their centre is

   (a) 1 (b) 2

   (c) $\frac{1}{2}$ (d) 4 **[2002]**

2. If an electron and a proton having the same momenta, enter perpendicularly to a uniform magnetic field, then

   (a) both will have the same curved path (ignoring the sense of revolution)

   (b) they will move undeflected

   (c) the path of the electron will be move curved than that of the proton

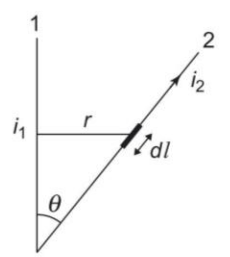
   (d) the path of the proton is more curved. **[2002]**

3. The time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of its

   (a) speed (b) mass

   (c) charge (d) magnetic field **[2002]**

4. Two wires 1 and 2 carrying currents i_1 and i_2 respectively are inclined at an angle θ as shown in the figure. What is the force on a small element dl of wire 2 at a distance r from wire 1 due to the magnetic field of wire 1?



   (a) $\frac{\mu_0}{2\pi r}(i_1 i_2\, dl \tan\theta)$ (b) $\frac{\mu_0}{2\pi r}(i_1 i_2\, dl \sin\theta)$

   (c) $\frac{\mu_0}{2\pi r}(i_1 i_2\, dl \cos\theta)$ (d) $\frac{\mu_0}{4\pi r}(i_1 i_2\, dl \sin\theta)$ **[2002]**

5. A particle of mass M and charge Q moving with velocity v describes a circular path of radius R when subjected to a uniform transverse magnetic field **B**. The work done by the field when the particle completes one full circle is

   (a) $\left(\frac{mv^2}{R}\right)2\pi R$ (b) zero

   (c) $2\pi RBQ$ (d) $2\pi RBvQ$ **[2003]**

6. A particle of charge -1.6×10^{-19} C moving with velocity 10 m s^{-1} along the x-axis enters a region where the magnetic field **B** is along the y-axis and an electric field of magnitude 10^4 V m^{-1} is along the negative z-axis. If the charged particle continuous moving along the x-axis, the magnitude of **B** is

   (a) 10^3 T (b) 10^5 T

   (c) 10^{16} T (d) 10^{-3} T **[2003]**

7. A thin rectangular magnet suspended freely has a time period of oscillation equal to T. Now it is broken into two equal halves (each having half the original length) and one piece is made to oscillate in the same field. If its period of oscillation is T', the ratio T'/T is

   (a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$

   (c) 2 (d) $\frac{1}{4}$ **[2003]**

8. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The magnitude of torque needed to maintain the needle in this position will be

   (a) $\sqrt{3}\ W$ (b) W

   (c) $\left(\frac{\sqrt{3}}{2}\right)W$ (d) $2\ W$ **[2003]**

9. The magnetic field lines inside a bar magnet

   (a) are from north pole to south pole of the magnet

   (b) do not exist

   (c) depend on the area of cross section of the magnet

   (d) are from south pole to north pole of the magnet. **[2003]**

10. Curie temperature is the temperature above which,
(a) a ferromagnetic material becomes paramagnetic
(b) a paramagnetic material becomes diamagnetic
(c) a ferromagnetic material becomes diamagnetic
(d) a paramagnetic material becomes ferromagnetic. **[2003]**

11. A current i flows along an infinitely long straight thin walled tube of radius r. The magnetic field inside the tube is
(a) infinite
(b) zero
(c) $\frac{\mu_0 i}{2\pi r}$
(d) $\frac{2i}{r}$ **[2004]**

12. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular coil of n turns. The magnetic field at the centre of the coil will be
(a) nB
(b) n^2B
(c) $2nB$
(d) $2n^2B$ **[2004]**

13. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance 4 cm from the centre is 54 μT. What will be its value at the centre of the loop?
(a) 250 μT
(b) 150 μT
(c) 125 μT
(d) 75 μT **[2004]**

14. Two long conductors, separated by a distance d, carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one wire is increased two times and its direction is reversed. The distance between wires is also increased to $3d$. The new value of force between them is
(a) $-2F$
(b) $\frac{F}{3}$
(c) $-\frac{2F}{3}$
(d) $-\frac{F}{3}$ **[2004]**

15. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2s. The magnet is cut along the length into three equal parts and the three parts are then placed on each other with their like poles together. The time period of this combination will be
(a) 2s
(b) $\frac{2}{3}$ s
(c) $2\sqrt{3}$ s
(d) $\frac{2}{\sqrt{3}}$ s **[2004]**

16. The materials suitable for making electromagnets should have
(a) high retentivity and high coercivity
(b) low retentivity and low coercivity
(c) high retentivity and low coercivity
(d) low retentivity and high coercivity **[2004]**

17. Two thin long parallel wires separated by a distance d carry a current i in the same direction. They will
(a) attract each other with a force of $\frac{\mu_0 i^2}{(2\pi d)}$
(b) repel each other with a force of $\frac{\mu_0 i^2}{(2\pi d)}$
(c) attract each other with a force of $\frac{\mu_0 i^2}{(2\pi d^2)}$
(d) repel each other with a force of $\frac{\mu_0 i^2}{(2\pi d^2)}$ **[2005]**

18. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other and carry currents of 3A and 4A. The magnetic field (in Wb m^{-2}) at the centre of the coils will be ($\mu_0 = 4\pi \times 10^{-6}$ Wb A^{-1} m^{-1})
(a) 12×10^{-5}
(b) 10^{-5}
(c) 5×10^{-5}
(d) 7×10^{-5} **[2005]**

19. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity, then
(a) its velocity will decrease
(b) its velocity will increase
(c) it will turn towards right of its initial direction of motion
(d) it will turn towards left of its initial direction of motion **[2005]**

20. A charged particle of mass m and charge q travels in a circular path of radius r that is perpendicular to magnetic field B. The time taken by the particle to complete one revolution is
(a) $\frac{2\pi mq}{B}$
(b) $\frac{2\pi q^2 B}{m}$
(c) $\frac{2\pi qB}{m}$
(d) $\frac{2\pi m}{qB}$ **[2005]**

21. A magnetic needle is kept in a non-uniform magnetic field. It experiences
(a) a torque but no force
(b) neither a torque nor a force
(c) a force as well as a torque
(d) a force but no torque **[2005]**

22. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a

(a) ellipse (b) circle

(c) helix (d) straight line **[2006]**

23. Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance, respectively. A magnet when brought close to them will

(a) attract N_1 strongly, but repel N_2 and N_3 weakly

(b) attract all three of them

(c) attract N_1 and N_2 strongly but repel N_3

(d) attract N_1 strongly, N_2 weakly and repel N_3 weakly **[2006]**

24. A long solenoid has 200 turns per cm and carries a current i. The magnetic field at its center is 6.28×10^{-2} weber/m^2. Another long solenoid has 100 turns per cm and it carries a current $i/3$. The value of the magnetic field at its center is

(a) 1.05×10^{-3} weber/m^2

(b) 1.05×10^{-4} weber/m^2

(c) 1.05×10^{-2} weber/m^2

(d) 1.05×10^{-5} weber/m^2 **[2006]**

25. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $\frac{a}{2}$ and $2a$ is

(a) $\frac{1}{4}$ (b) 4

(c) 1 (d) $\frac{1}{2}$ **[2007]**

26. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then

(a) the magnetic field is zero only on the axis of the pipe

(b) the magnetic field is different at different points inside the pipe

(c) the magnetic field at any point inside the pipe is zero

(d) the magnetic field at all points inside the pipe is the same, but not zero **[2007]**

27. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal field $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{v}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then

(a) $\vec{v} = \frac{\vec{E} \times \vec{B}}{B^2}$ (b) $\vec{v} = \frac{\vec{B} \times \vec{E}}{B^2}$

(c) $\vec{v} = \frac{\vec{E} \times \vec{B}}{E^2}$ (d) $\vec{v} = \frac{\vec{E} \times \vec{B}}{E^2}$ **[2007]**

28. A charged particle moves through a magnetic field perpendicular to its direction. Then

(a) the momentum changes but the kinetic energy is constant

(b) both momentum and kinetic energy of the particle are not constant

(c) both, momentum and kinetic energy of the particle are constant

(d) kinetic energy changes but the momentum is constant. **[2007]**

29. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by

(a) $\frac{\mu_0}{2\pi}\left(\frac{I_1 + I_2}{d}\right)^{\frac{1}{2}}$ (b) $\frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{\frac{1}{2}}$

(c) $\frac{\mu_0}{2\pi d}(I_1 + I_2)$ (d) $\frac{\mu_0}{2\pi d}(I_1{}^2 + I_2{}^2)$ **[2007]**

30. A horizontal overhead powerline is a height of 4m from the ground and carries a current of 100 A from east to west. The magnetic field directly below it on the ground is ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

(a) 2.5×10^{-7} T northward

(b) 2.5×10^{-7} T southward

(c) 5×10^{-6} T northward

(d) 5×10^{-6} T southward **[2008]**

31. Relative permittivity and permeability of a material are ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for diamagnetic material?

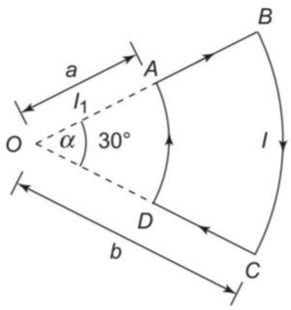
(a) $\varepsilon_r = 1.5, \mu_r = 1.5$ (b) $\varepsilon_r = 0.5, \mu_r = 1.5$

(c) $\varepsilon_r = 1.5, \mu_r = 0.5$ (d) $\varepsilon_r = 0.5, \mu_r = 0.5$

[2008]

Questions 32 and 33 are based on the following paragraph.

A current loop *ABCD* is held fixed on the plane of the paper as shown in the figure. The arcs *BC* (radius = *b*) and *DA* (radius = *a*) of the loop are joined by two straight wires *AB* and *CD*. A steady current *I* is flowing in the loop. Angle made by *AB* and *CD* at the origin *O* is 30°. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin.



[2009]

32. The magnitude of the magnetic field (*B*) due to the loop *ABCD* at the origin (*O*) is

(a) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$

(b) $\frac{\mu_0 I}{4\pi}[2(b-a)+\pi/3\,(a+b)]$

(c) zero

(d) $\frac{\mu_0 I(b-a)}{24ab}$ **[2009]**

33. Due to the presence of the current I_1 at the origin:

(a) The magnitude of the net force on the loop is given by

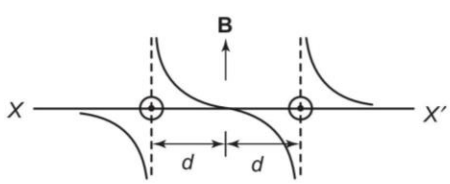
$\frac{I_1 I}{4\pi}\mu_0\,[2(b-a)+\pi/3\,(a+b)]$

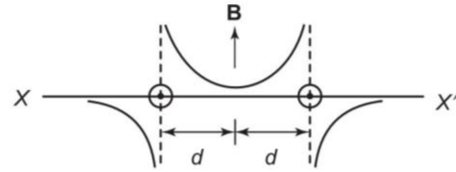
(b) The magnitude of the net force on the loop is given by $\frac{\mu_0 I I_1}{24ab}(b-a)$.

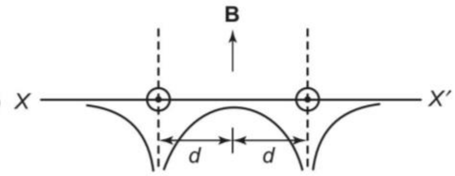
(c) The forces on *AB* and *DC* are zero.

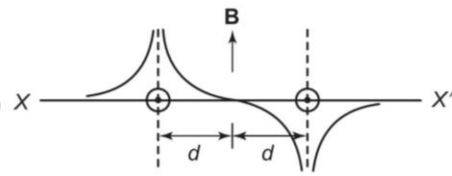
(d) The forces on *AD* and *BC* are zero. **[2009]**

34. Two long parallel wires are at a distance 2*d* apart. They carry steady equal current flowing out of the plane of the paper as shown in the figure. The variation of the magnetic field along the line *XX′* is given by

(a) 

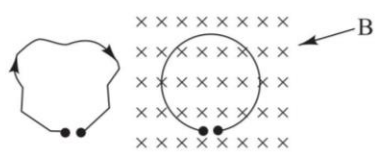
(b) 

(c) 

(d) 

[2010]

35. A thin flexible wire of length *L* is connected to two adjacent fixed points and carries a current *I* in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



(a) *IBL* (b) $\frac{IBL}{\pi}$

(c) $\frac{IBL}{2\pi}$ (d) $\frac{IBL}{4\pi}$ **[2010]**

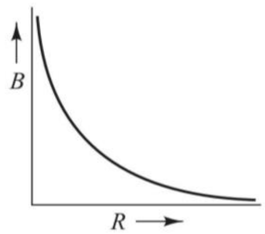
36. A current I flows in an infinitely long wire with cross-section in the form of a semicircular ring of radius *R*. The magnitude of magnetic field along its axis is

(a) $\frac{\mu_0 I}{\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi^2 R}$

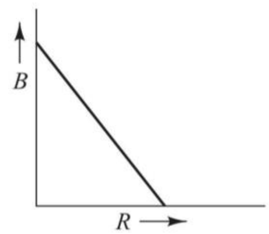
(c) $\frac{\mu_0 I}{2\pi R}$ (d) $\frac{\mu_0 I}{4\pi R}$ **[2011]**

37. A charge *Q* is uniformly distributed over the surface of non-conducting disc of radius *R*. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity *ω*. As a result of this rotation a magnetic field *B* is obtained at the centre of the disc. If we keep both the the amount

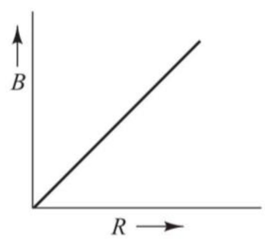
of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc, then the variation of the magnetic field at the centre of the disc will be represented by which of the following figures.



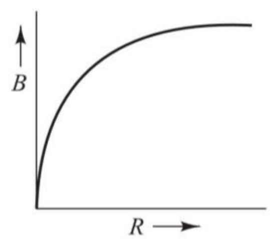
(a)



(b)



(c)



(d)

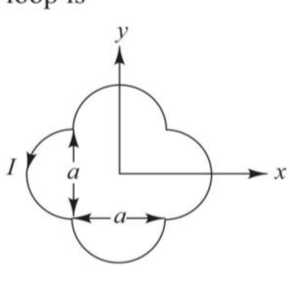
[2012]

38. Proton, deuteron and alpha particle of same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are, respectively, r_p, r_d and r_α. which one of the following relation is correct?

(a) $r_\alpha = r_p = r_d$ (b) $r_\alpha = r_p = r_d$

(a) $r_\alpha = r_p = r_d$ (b) $r_\alpha = r_p = r_d$ **[2012]**

39. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector k is coming out of the plane of the paper. The magnetic moment of the current loop is

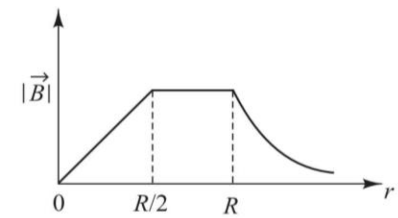


(a) $a^2 I\hat{\mathbf{k}}$ (b) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{\mathbf{k}}$

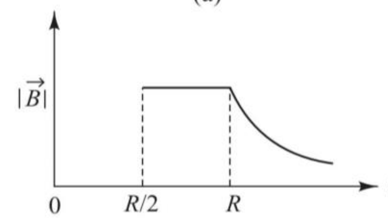
(c) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{\mathbf{k}}$ (d) $(2\pi+1)a^2 I\hat{\mathbf{k}}$

[2012]

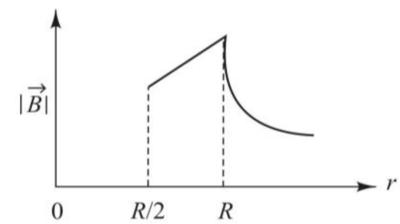
40. An infinitely long hollow conducting cylinder with inner radius $R/2$ and other radius R carries a unifrom current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by



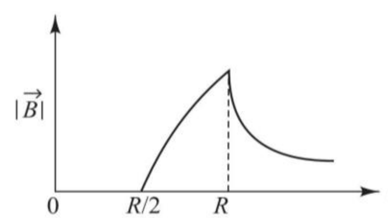
(a)



(b)



(c)



(d)

[2012]

41. This question has statement I and statement II. Of the four choices given after the statement, choose the one that best describes the two statements.

Statement-I : Higher the range, greater is the resistance of ammeter.

Statement-II : To increase the range of ammeter, additional shunt needs to be used across it.

(a) Statement-I is true, Statement-II is true, Statement-II is not the correct explanation of Statement-I.
(b) Statement-I is true, Statement-II is false.
(c) Statement-I is false, Statement-II is true.

(d) Statement-I is true, Statement-II is true, Statment-II is the correct explanation of statement-I **[2013]**

42. Two short magnets of length 1 cm each have magnetic moment 1.20 A m^2 and 1.00A m^2, respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic field at the mid-point O of the line joining their centres is close to (horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)
(a) 2.56×10^{-4} Wb/m^2
(b) 3.50×10^{-4} Wb/m^2
(c) 5.80×10^{-4} Wb/m^2
(d) 3.60×10^{-5} Wb/m^2
[2013]

43. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle take place in a plane. It follows that
(a) its velocity remains constant
(b) it moves in a straight line with a constant acceleration
(c) it moves in a circle with a constant acceleration
(d) its kinetic energy remains constant
[2014]

Answers

1. (a)	2. (a)	3. (a)	4. (c)
5. (b)	6. (a)	7. (b)	8. (a)
9. (d)	10. (a)	11. (b)	12. (b)
13. (a)	14. (c)	15. (b)	16. (c)
17. (a)	18. (c)	19. (a)	20. (d)
21. (c)	22. (d)	23. (d)	24. (c)
25. (c)	26. (c)	27. (b)	28. (a)
29. (b)	30. (d)	31. (c)	32. (d)
33. (d)	34. (a)	35. (c)	36. (b)
37. (a)	38. (b)	39. (b)	40. (d)
41. (c)	42. (a)	43. (d)	

Solutions

1. $B_A = \dfrac{\mu_0 i}{2R}$ and $B_B = \dfrac{\mu_0(2i)}{2(2R)}$

$\therefore \quad \dfrac{B_A}{B_B} = 1$

2. The radius of the circular path is

$$r = \frac{mv}{qB} \Rightarrow r = \frac{p}{qB}; \quad p = mv.$$

Since p, q, B are the same for electron and proton, the value of r will be the same for both. Since the charge of an electron is opposite in sign to that of proton, the sense of revolution will be opposite. So the correct choice is (a).

3. Time period $T = \dfrac{2\pi m}{qB}$. So the correct choice is (a).

4. The component of dl parallel to wire 1 is $dl \cos\theta$. Hence the force on the element is

$$F = B_1 i_2 \, dl \cos\theta$$

where B_1 = magnetic field at the element due to current in wire which is given by

$$B_1 = \frac{\mu_0 i_1}{2\pi r}$$

$$\therefore F = \frac{\mu_0}{2\pi r}(i_1 i_2 \, dl \cos\theta)$$

5. When a charged particle describes a circular path, the necessary centripetal force is provided by the magnetic force **F**. Since the velocity **v** is always tangential, vectors **F** and **v** are perpendicular to each other ($\theta = 90°$).

Power = $\mathbf{F} \cdot \mathbf{v} = Fv \cos\theta = Fv \cos 90° = 0$. Since power consumed is zero, the work done is zero.

6. $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

Given $\mathbf{E} = -E\hat{\mathbf{k}}$, $\mathbf{v} = v\hat{\mathbf{i}}$ and $\mathbf{B} = B\hat{\mathbf{j}}$

$$\therefore \quad \mathbf{F} = q[-E\hat{\mathbf{k}} + (v\hat{\mathbf{i}}) \times (B\hat{\mathbf{j}})]$$
$$= q(-E\hat{\mathbf{k}} + vB\hat{\mathbf{k}}) = q(-E + vB)\hat{\mathbf{k}}$$

Since the particle moves undeflected, $\mathbf{F} = 0$, i.e.

$$q(-E + vB)\hat{k} = 0$$
$$\Rightarrow \quad -E + vB = 0$$
$$\Rightarrow \quad B = \frac{E}{v} = \frac{10^4}{10} = 10^3 \text{ T}$$

7. $T = 2\pi\sqrt{\dfrac{I}{MB}}$

where I = moment of inertia of the magnet about the axis of rotation which is

$$I = \frac{mL^2}{12}; \quad L = \text{length of magnet},$$

m = mass of magnet

M = magnetic moment = pL (p = pole strength) and B is the magnetic field.

If the magnet is broken into two parts, each of length $L/2$, the pole strength p remains the same. The magnetic moment of each piece is

$$M' = p \times \frac{L}{2} = \frac{M}{2}$$

Since the mass of each piece is $m' = \frac{M}{2}$, and length $L' = \frac{L}{2}$, the moment of inertia of a piece is

$$I' = \frac{m'(L')^2}{12} = \frac{\frac{m}{2}\times\left(\frac{L}{2}\right)^2}{12} = \frac{mL^2}{8\times 12} = \frac{I}{8}$$

Time period T' is

$$T' = 2\pi\sqrt{\frac{I'}{M'B}} = 2\pi\sqrt{\frac{I/8}{BM/2}} = \frac{2\pi}{2}\sqrt{\frac{I}{MB}} \qquad (2)$$

Dividing (2) by (1)

$$\frac{T'}{T} = \frac{1}{2}$$

8. $$W = MB\,(1 - \cos\theta)$$
$$= MB\,(1 - \cos 60°) = \frac{MB}{2}$$
$$\Rightarrow \quad MB = 2\,W$$

Magnitude of torque is

$$\tau = MB \sin\theta$$
$$= 2\,W \sin 60° = 2\,W \times \frac{\sqrt{3}}{2} = \sqrt{3}\,W$$

9. Magnetic field lines form closed loop. Hence, inside the magnet, they are from south pole to north pole.
10. The correct choice is (a).
11. Consider an amperean loop inside the pipe. Since the current threading the loop is zero, from Ampere's circuital law, the magnetic field at any point inside the hollow pipe carrying a current is zero.
12. The magnetic field at the centre of a coil of n turns and radius R carrying a current I is

$$B = \frac{\mu_0 nI}{2R}$$

For $\quad n = 1, \quad B = \frac{\mu_0 I}{2R} \qquad (1)$

When the wire is bent into a circular coil of n turns, the radius of the coil becomes

$$R' = \frac{L}{2\pi n} = \frac{R}{n} \qquad (\because \quad L = 2\pi R')$$

The magnetic field at the centre of coil will be

$$B' = \frac{\mu_0 nI}{2R'}$$
$$= \frac{\mu_0 nI}{2R/n}$$
$$= \frac{\mu_0 In^2}{2R} = n^2 B$$

13. Magnetic field at a distance x from the centre of the coil of radius R and carrying a current I is

$$B = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}} \qquad (1)$$

Magnetic field at the centre of the coil is ($\because \; x = 0$)

$$B_0 = \frac{\mu_0 I}{2R} \qquad (2)$$

Dividing (2) by (1) we get

$$\frac{B_0}{B} = \frac{(R^2 + x^2)^{3/2}}{R^3} \qquad (3)$$

Given B = 54 μT, R = 3 cm = 3 × 10^{-2} m and x = 4 cm = 4 × 10^{-2} m. Substituting these values in Eq. (3), we get

$$B_0 = 250\ \mu\text{T}$$

14. If the length of each conductor is L, then

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi d} \qquad \text{(attractive)}$$

New force is

$$F' = \frac{\mu_0(-2I_1)I_2 L}{2\pi(3d)} \qquad \text{(repulsive)}$$

$$\therefore \quad \frac{F'}{F} = -\frac{2}{3} \quad \Rightarrow \quad F' = -\frac{2F}{3}.$$

15. In a vibration magnetometer, a bar magnet is suspended from a support and oscillated in a horizontal plane. The time period of the torsional oscillations is

$$T = 2\pi\sqrt{\frac{1}{MH}}\,; \quad I = \frac{mL^2}{12}$$

where m = mass of magnet, L = length of magnet and $M = pL$ is the magnetic moment; p being the pole strength and H is the horizontal component of the earth's magnetic field.

When the magnet is cut along the length into three equal parts, the moment of inertia of each part is

$$I_1 = \frac{1}{12}\left(\frac{m}{3}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{27}\left(\frac{1}{12}mL^2\right)$$

If they are placed one on top of the other with their like poles together, the moment of inertia of the combination is

$$I' = 3I_1 = \frac{1}{9}\left(\frac{1}{12}mL^2\right) = \frac{I}{9}$$

On cutting along the length, pole strength of each part is $p_1 = \frac{p}{3}$. If they are placed one on top of the other with their like poles together, the pole strength of the combination is

$$p' = 3p_1 = p$$

Magnetic moment of combination is

$$M' = p \times L = M$$

$$\therefore \quad T' = 2\pi\sqrt{\frac{I'}{M'H}} = 2\pi\sqrt{\frac{I}{9MH}}$$

$$= \frac{1}{3} \times T = \frac{1}{3} \times 2\text{s} = \frac{2}{3}\text{ s}$$

16. Electromagnets are made of soft iron because soft iron has low coercivity and high retentivity. As a result, it is easily demagnetized. So, the correct choice is (c).

17. The force per unit length between the two wires is

$$\frac{F}{l} = \frac{\mu_0 i^2}{2\pi d}$$

Since the current in the wires are in the same direction, the force between them is attractive.

18. $B_1 = \dfrac{\mu_0 I_1}{2R} = \dfrac{4\pi \times 10^{-7} \times 3}{2 \times 2\pi \times 10^{-2}} = 3 \times 10^{-5}\text{ Wb m}^{-2}$

$B_2 = \dfrac{\mu_0 I_2}{2R} = \dfrac{4\pi \times 10^{-7} \times 4}{2 \times 2\pi \times 10^{-2}} = 4 \times 10^{-5}\text{ Wb m}^{-2}$

Since the planes of the coils are perpendicular to each other, B_1 and B_2 will be perpendicular to each other. The net field is

$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \sqrt{(3\times10^{-5}) + (4\times10^{-5})^2} = 5 \times 10^{-5}\text{ Wb m}^{-2}$$

19. Force on the electron exerted by magnetic field is

$\mathbf{F}_m = q\,(\mathbf{v} \times \mathbf{B}) = 0$ since $\mathbf{v}$ is parallel to $\mathbf{B}$.

Force on the electron exerted by electric field is

$\mathbf{F}_e = q\,\mathbf{E}$

Since the charge of the electron is negative, force $\mathbf{F}_e$ will be opposite to the direction of motion of the electron. Hence, the motion of the electron will be retarded. So, the correct choice is (a).

20. The necessary centripetal force is provided by the magnetic force qvB. Hence

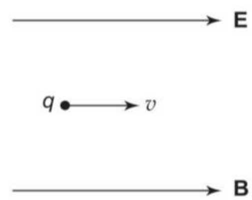
$$\frac{mv^2}{r} = qvB \quad \Rightarrow \quad r = \frac{mv}{qB}$$

Time taken by the particle to complete one revolution is

$$T = \frac{2\pi r}{v} = \frac{2\pi mv}{vqB} = \frac{2\pi m}{qB}$$

21. Since the magnetic field varies with distance, the forces on the poles of the magnet will be unequal. Hence, the magnet will experience a force as well as a torque.

22. Consider a particle of charge q in a region of parallel and uniform electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ as shown in the figure. The electric field exerts a force



$$\mathbf{F}_e = q\,\mathbf{E}$$

in the direction of the field. As a result, the particle is accelerated in the direction of the field. If $\boldsymbol{v}$ is the velocity of the particle at an instant of time, then at that instant, the force experienced by the particle due to the magnetic field is

$$\mathbf{F}_m = q\,(\boldsymbol{v} \times \boldsymbol{B})$$

Since $\boldsymbol{v}$ is parallel to $\mathbf{B}$, $\mathbf{F}_m = 0$. Hence, the particle will keep moving in a straight line in the direction of the electric field if it carries a positive charge and opposite to the direction of the electric field if it carries a negative charge.

23. The correct choice is (d).

24. For the first solenoid

$$B_1 = \mu_0\, n_1\, i_1$$

For the second solenoid

$$B_2 = \mu_0\, n_2\, i_2$$

Dividing we get,

$$\frac{B_2}{B_1} = \frac{n_2}{n_1} \times \frac{i_2}{i_1} = \frac{100}{200} \times \frac{i/3}{i} = \frac{1}{6}$$

or $B_2 = \dfrac{1}{6} B_1 = \dfrac{1}{6} \times 6.28 \times 10^{-2} \simeq 1.05 \times 10^{-2}\text{ Wb/m}^2$

25. The magnetic field at a distance r from the axis of the wire of radius a and carrying a current i for the case when r lies between 0 and a is given by

$$B_1 = \frac{\mu_0 ir}{2\pi a}; \qquad 0 \le r \le a$$

$$= \frac{\mu_0 i}{2\pi a} \times \frac{a}{2} \qquad \left(\text{for } r = \frac{a}{2}\right)$$

$$\Rightarrow \qquad B_1 = \frac{\mu_0 i}{4\pi a}$$

For the case $r > a$, the magnetic field is given by

$$B_2 = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i}{2\pi(2a)} \qquad \text{(for } r = 2a\text{)}$$

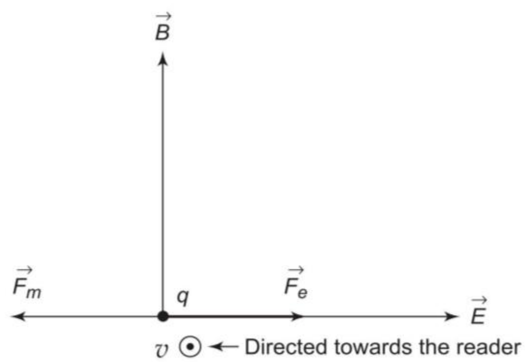
$$\therefore \qquad \frac{B_1}{B_2} = 1$$

26. The magnetic field at any point inside an infinitely long, straight and thin walled pipe carrying a current is zero. Hence the correct choice is (c).

27. The forces acting on the particle by magnetic and electric fields are

$\vec{F}_m = q(\vec{v} \times \vec{B})$ and $\vec{F}_e = q\vec{E}$

The direction of force $\vec{F}_e$ is along the direction of $\vec{E}$ and the direction of force $\vec{F}_m$ is perpendicular to vectors $\vec{v}$ and $\vec{B}$. Since the velocity of the particle remains unchanged, no net force acts on it. Hence, $F_m = F_e$ and the direction of $\vec{F}_m$ must be opposite to that of $\vec{F}_e$, as shown in the figure. From the left-hand screw rule, it follows that $\vec{v}$ should be directed perpendicular to the plane of the page and towards the reader. Thus,



$$q(\vec{v} \times \vec{B}) = -q\vec{E}$$
$$\vec{v} \times \vec{B} = -\vec{E}$$

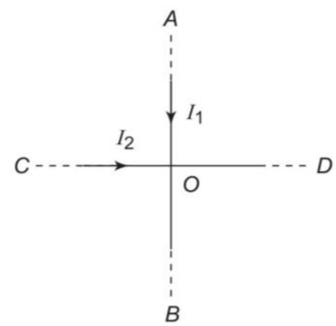
Since $\vec{v}$ is perpendicular to $\vec{B}$, the magnitude of $\vec{v}$ is

$$v = \frac{E}{B}$$

Hence choices (c) and (d) are wrong because they give $v = \dfrac{B}{E}$. Since $\vec{v}$ is directed towards the reader, it follows that the direction of vector $\vec{v}$ is the same that of vector $(\vec{E} \times \vec{B})$ and not of vector $(\vec{B} \times \vec{E})$. Hence the correct choice is (b).

28. The magnetic force experienced by the charged particle is perpendicular to its velocity. Hence, the force does no work on the particle. Therefore, the speed and hence the kinetic energy of the particle remains unchanged. Since the velocity of the particle is perpendicular to the magnetic field, it will move along a circular path in the region of the magnetic field. Therefore, its velocity and hence linear momentum will continuously change due to change in the direction of motion of the particle moving a circle. Hence the correct choice is (a).

29. The magnetic fields due to wires AOB and COD at a point P at a distance d from O respectively are (note that point P is perpendicular to the plane of the page) [see the figure]



$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

and $$B_2 = \frac{\mu_0 I_2}{2\pi d}$$

Since currents I_1 and I_2 are perpendicular to each other, fields B_1 and B_2 are also perpendicular to each other. Hence the resultant field at point P is

$$B = (B_1{}^2 + B_2{}^2)^{1/2} = \frac{\mu_0 I}{2\pi r}(I_1{}^2 + I_2{}^2)^{1/2}$$

30. $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{(4\pi \times 10^{-7}) \times 100}{2\pi \times 4} = 5 \times 10^{-6}$ T

Using the right hand thumb rule, the direction of the magnetic field will be towards south.

31. For a diamagnetic material, $\mu_r < 1$. For any material $\varepsilon_r > 1$. Hence the correct choice is (c).

32. $B_O = B_{AB} + B_{BC} + B_{CD} + B_{DA}$

$$= 0 - \frac{\mu_0 I}{4\pi b} \times \frac{\pi}{6} + 0 + \frac{\mu_0 I}{4\pi a} \times \frac{\pi}{6}$$

$$= \frac{\mu_0 I}{24}\left(\frac{1}{a} - \frac{1}{b}\right)$$

So the correct choice is (d). Magnetic field at O due to arc BC is directed into the plane of the page and away from the reader and has been taken to be negative but the magnetic field due to arc AD is directed towards the reader and is taken to be positive.

33. The direction of the magnetic field due to current I_1 at wire segments AB and DC is perpendicular to AB as well as to DC. Since the current in AB is opposite to that of in DC and the lengths of AB and DC are equal, the forces on AB and DC are equal and opposite. Hence choice (c) is wrong. The direction of the magnetic field due to current I_1 at arcs AD and BC is tangential to the arcs, i.e. $\theta = 0°$. Hence, force on arc AD and on arc BC is zero. So choice (d) is correct. Since the forces on AB and DC are equal and opposite, the net force on the loop $ABCDA$ is zero. So choices (a) and (b) are incorrect. Hence the only correct choice is (d).

34. Let I be the current in each wire. Let the left wire be located at $x = 0$, then the other wire will be at $x = 2d$. The magnetic field in the space between the wires due to the current in the wires will be in opposite directions. At a distance x from the left wire, the net magnetic field is given by

$$\mathbf{B} = \frac{\mu_0 I}{2\pi x}(\hat{\mathbf{j}}) + \frac{\mu_0 I}{2\pi(2d - x)}(-\hat{\mathbf{j}})$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} - \frac{1}{(2d-x)}\right](-\hat{\mathbf{j}})$$

At $x = 0$, $\mathbf{B} = 0$

At $x = 2d$, $\mathbf{B} \to -\infty$ and is along the negative y-direction.

For $x < d$, $\mathbf{B}$ is along the positive y-direction.

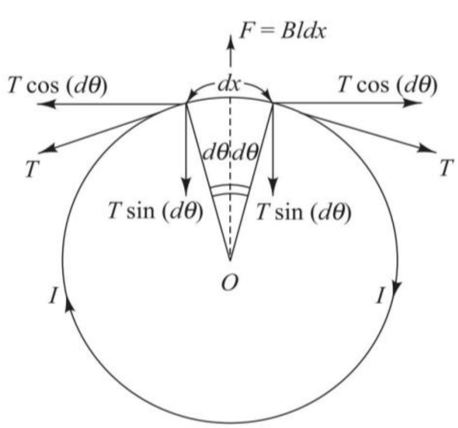
For $x > d$, $\mathbf{B}$ is along the negative y-direction.

For $x < 0$, $\mathbf{B}$ is along the negative y-direction.

For $x > 2d$, $\mathbf{B}$ is along the positive y-direction.

Hence the correct graph is (a).

35. Consider a small element of the wire of length dx. The horizontal components $T\cos(d\theta)$ cancel each other. The vertical components add up to $2T\sin(d\theta)$ because in the limit $d\theta \to 0$, they are collinear. The magnetic force an element dx is $F = B\,Idx$ vetically upward.



For equilibrium,

$$2T\sin(d\theta) = BI\,dx = B\,I\,R\,(2\,d\theta)$$

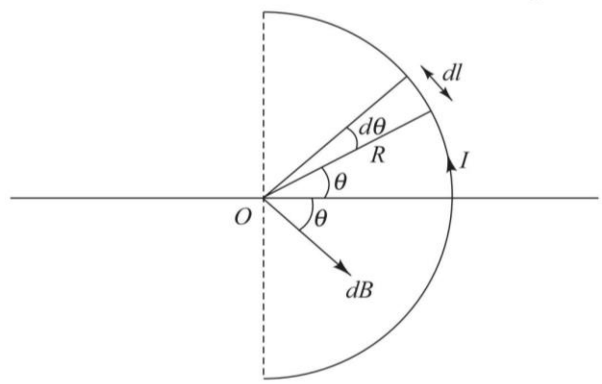
where R is the radius of the ring. For small $d\theta$, $\sin d\theta = d\theta$.

Hence $2\,T\,d\theta = 2\,B\,I\,R\,d\theta$

$$\Rightarrow \quad T = B\,I\,R = \frac{BIL}{2\pi} \qquad (\because L = 2\pi R)$$

so the correct choice is (c).

36. Divide the semi-circular ring into a large number of small elements consider an element of length dl.



$dl = R\,d\theta$. Current per unit length is $\lambda = \dfrac{I}{\pi R}$.

Therefore, current flowing through the element is $dI = \lambda dl = \lambda R d\theta$. Magnetic field at O due to the element is

$$dB = \frac{\mu_0 dI}{\pi R} = \frac{\mu_0 \lambda R d\theta}{\pi R}$$

$$= \frac{\mu_0 \lambda d\theta}{\pi}$$

The component of dB along the axis of the semi-circular wire is $dB\cos\theta$. Therefore, the magnetic field due to the complete wire is

$$B = \int_{-\pi/2}^{\pi/2} dB\cos\theta$$

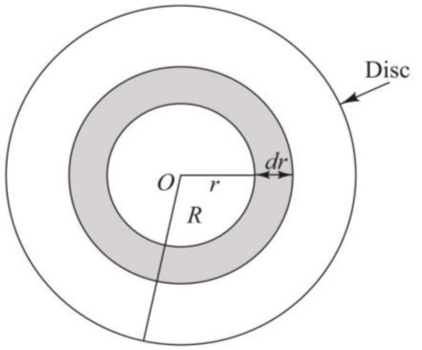
$$= \frac{\mu_0\lambda}{\pi}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta$$

$$= \frac{\mu_0\lambda}{2\pi} = \frac{\mu_0 I}{2\pi^2 R}$$

so the correct choice is (b).

37. Charge per unit area of the disc is $\sigma = \dfrac{Q}{\pi R^2}$. Divide

The disc into a large number of concentric circular elements as shown in the figure.



Area of element $= \pi(r+dr)^2 - \pi r^2 = 2\pi r dr$ $(\therefore dr << r)$

$\therefore$ Charge on the element is dq

$$= \sigma \times 2\pi r dr = \frac{Q}{\pi R^2} \times 2\pi r dr = \frac{2Q}{R^2} \times r dr$$

The magnetic at O due to the element

$$dB = \frac{\mu_0 dq}{2r} \times \frac{\omega}{2\pi} = \frac{\mu_0 \omega dq}{4\pi r} = \frac{\mu_0 \omega \times 2Q r dr}{4\pi R^2 r} = \frac{\mu_0 \omega\, Q\, dr}{2\pi R^2}$$

The magnetic field at O due to the complete disc is

$$B = \int_0^R dB = \frac{\mu_0 \omega Q}{2\pi R^2}\int_o^R dr = \frac{\mu_0 \omega Q}{2\pi R}$$

Thus $B \propto \frac{1}{R}$. so the correct graph is (a)

38. $r = \sqrt{\frac{2mk}{3q}}$; k = kinetic energy

$\Rightarrow r = k\sqrt{\frac{m}{q}}$; $k = \text{constant} = \sqrt{\frac{2k}{3}}$

$\therefore r_p = k\frac{\sqrt{m_p}}{q}$

$r_\alpha = k\frac{\sqrt{4m_p}}{2q} = k\sqrt{\frac{m_p}{q}}$

Hence $r_p = r_\alpha < r_d$, which is choice (b).

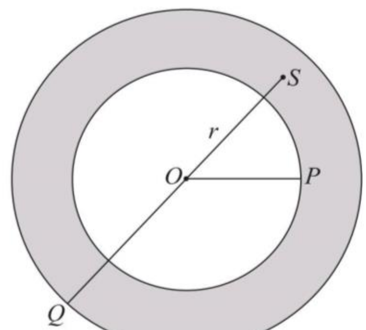
39. Magnetic Moment $\vec{M}$ = current × area of the loop $= I\vec{A}$

$= I \times \left[a^2 + \pi\left(\frac{a}{2}\right)^2 \times 2\right]\hat{\mathbf{k}}$

$= Ia^2\left(1 + \frac{\pi}{2}\right)\hat{\mathbf{k}}$

The direction of area vector $\vec{A}$ is along $\hat{\mathbf{k}}$.

40.



$$OP = \frac{R}{2},\ OQ = R,\ OS = r,$$

Inside the cavity

$\left(\text{i.e. for } r \text{ lying between zero and } \frac{R}{2}\right)$; $B = 0$

Outside the cylinder, (i.e. for $r > R$),

$$B = \frac{\mu_0 I}{2\pi r}$$

In the shaded region, $\left(\text{i. e. for } \frac{R}{2} < r < R\right)$. From Ampere's circuital law,

$B \times 2\pi r = \mu_0 I = \mu_0 J A$

where $J = \frac{I}{A}$ is the current density and A is the area of the shaded region. Now

$$A = \pi r^2 - \pi\left(\frac{R}{2}\right)^2$$

$$\therefore B \times 2\pi r = \mu_0 J\left[\pi r^2 - \frac{\pi R^2}{4}\right]$$

$$\Rightarrow B = \frac{\mu_0 J}{2}\left[\frac{r^2 - R^2/4}{r}\right] = \frac{\mu_0 J}{2}\left[r - \frac{R^2}{4r}\right]$$

Hence the correct graph is (d).

41. The resistance S to be connected in parallel with a galvanometer so that it has a range I is

$$S = \frac{I_g G}{I - I_g}$$

where G = galvanometer resistance and I_g = current for full scale deflection. So in order to increase I, the value of S must be decreased. If an additional resistance S_1 is connected across the galvanometer, the new shunt resistance becomes

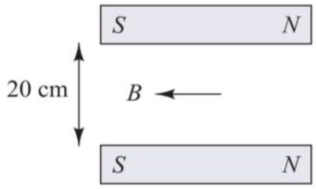
$S_2 = \dfrac{S\,S_1}{S+S_1}$ which is less then S. Hence statement-II is true.

The resistance of the ammeter after S_1 is connected is $R_1 = G + S_1$

The resistance of the ammeter after S_2 is connected is $R_2 = G + S_2$

Since $S_2 < S$; $R_2 < R_1$. Hence statement I is false

42.



Magnetic field due to two short magnetic

$$B = \frac{\mu_0}{4\pi}\frac{M_1}{r^3} + \frac{\mu_0}{4\pi}\frac{M_2}{r^3}$$

$$= \frac{4\pi \times 10^{-7}}{4\pi \times (0.1)^3}[1.2+1]$$

$$= 2.2 \times 10^{-4}\ \text{T}$$

The direction of this field is along the horizontal component of the earth's field. Thus,

$B_{net} = 2.2 \times 10^{-4} + 3.6 \times 10^{-5}$

$$= 2.56 \times 10^{-4}\ \text{T}$$

43. The correct choice is (d). The partile move in a circle with a constant speed and hence with a constant kinetic energy.