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IIT-JEE PHYSICS

NEWTON'S LAWS OF MOTION

01

IIT-JEE-PHYSICS-NEWTON'S LAWS OF MOTION

PHYSICS

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IIT MAINS & ADV

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BASIC-CONCEPTS

FREE-BODY DIAGRAMS

UNDERSTANDING IMPORTANT TERMS

SAMPLE PROBLEMS

PSEUDO FORCE

NEWTON'S LAWS OF MOTION

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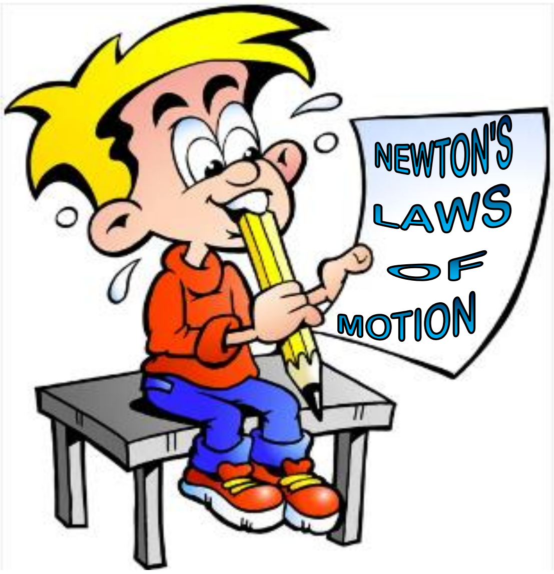
NEWTON'S LAWS OF MOTION



01

What you already know

- Motion in one dimension
- Motion in two dimension
- Relative motion

What you will learn

- What is force?
- Types of forces
- Newton's laws

02

What you already now

- Force
- Fundamental forces in nature
- Standard forces in physics
- Newton's laws of motion

What you will learn

- System
- Free-body diagram
- Equilibrium

03

What you already know

- Force
- Newton's laws of motion
- Free-body diagram

What you will learn

- Newton's second law - Quantitative analysis
- Applications of Newton's second law

04

What you already know

- Force
- FBD
- Resolving forces

What you will learn

- Force transmission through a massless body
- Spring force, spring constant, and spring balance

05

What you already know

- FBD
- Resolving forces
- Spring force

What you will learn

- Constrained motion
- String constraint
- Pulley-block system

06

What you already now

- Constrained motion
- String constraint
- Pulley-Block system
- String/Rod constrained motion

What you will learn

- What are wedge constraints
- Wedge constrained motion
- Constraint relation
- Problem solving using constraint relation

07

What you will learn

- Newton's laws on a system
- Pseudo force
- Non-inertial frames

What you already know

- Constrained motion
- Pulley-Block system
- String/Rod constrained motion
- Wedge constrained motion

NEWTON'S LAWS OF MOTION

What you already know

- Motion in one dimension
- Motion in two dimension
- Relative motion

What you will learn

- What is force?
- Types of forces
- Newton's laws

What is Force?

A **push** or **pull** that changes or tends to change the state of motion of a body.

State of motion is defined by velocity.

What can force do?

- Change the direction of motion of the body.
- Change the speed of the body.
- Change the shape of the body.

More than one of the above effects are also possible.

Force is a vector quantity.

- It has direction
- It has a magnitude
- It follows the laws of vector algebra

Fundamental forces in nature

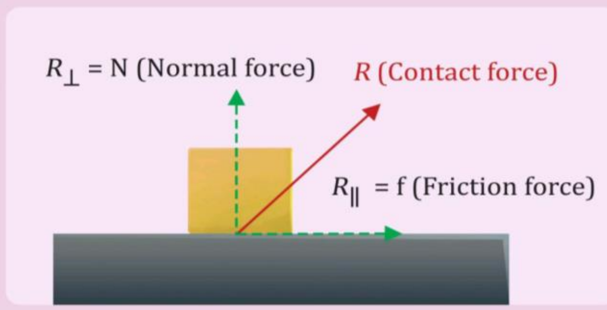
Gravitational force	Strong nuclear force	Weak nuclear force	Electromagnetic force
• Gravitational force is proportional to the product of the two masses, and inversely proportional to the square of the distance between them.	• They act between the nucleons, i.e., protons and neutrons, in a nucleus to hold them together.	• These forces are responsible for the neutron to change itself into a proton and vice versa to attain stability.	• **Electrostatic** - Force between particles due to their electric charge.

$F \propto \frac{m_1 \cdot m_2}{r^2}$ • Where m_1, m_2 are the two masses and r is the distance between them.	• Strongest of all the fundamental forces.	• Considerably short-ranged force with range much smaller than the size of a proton or a neutron.	• **Electrodynamic** - Force generated due to the motion of the charged particles.

Some standard forces

Contact force

- When two bodies are in contact with each other, they experience a force known as contact force.
- It is electromagnetic in nature.
- Its perpendicular component is known as normal force and parallel component to the surface is known as friction force.



Normal force or normal reaction

- Normal reaction is always pushing in nature.
- It is always normal (90 degrees) to the surface.

Weight

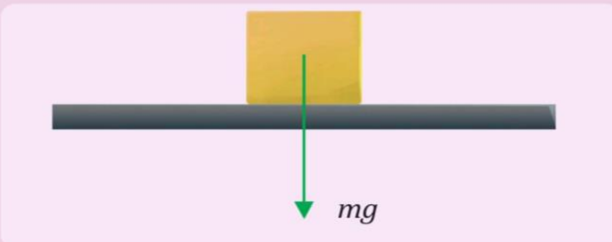
Force experienced by a body due to gravity is known as weight. It is applicable to all bodies having mass but specifically for a body near Earth's surface, we compute it as:

$$F = \frac{G \times \text{mass of Earth} \times \text{mass of the body}}{(\text{radius of Earth})^2}$$

Where,

$$\frac{G \times \text{mass of Earth}}{(\text{radius of Earth})^2} = g \text{ (constant)}$$

g = acceleration due to gravity of Earth



The weight of an object of mass m near the surface of the earth is mg directed **towards the centre of the Earth.**

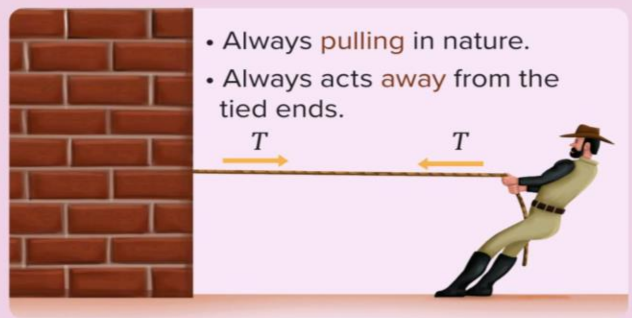
m = mass of block

g = acceleration due to gravity of Earth

Tension

When a string (or a rope) is being pulled on both ends, then its resistance against any change in its original length is known as tensile force or tension.

When the string is loose or 'slack' - No tension force appears.
When the string is stretched or 'taut' - Tension force acts.



Unless mentioned otherwise, the string is considered massless and tension force is the same throughout the string.

BOARDS

Newton's Laws of Motion

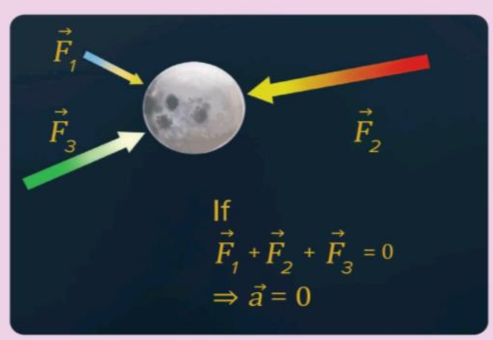
Three physical laws that laid the foundation for classical mechanics. They describe the relationship between a **body**, the forces acting upon it, and the motion of that body in response to those forces.

Newton's First Law of Motion

Every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force.

Or

If the vector sum of all the forces acting on a particle is zero, then and only then the particle remains unaccelerated (i.e., remains at rest or moves with constant velocity).



- First law is also known as the law of **inertia**.

Inertia: The resistance of a particle to change its state of rest or of uniform motion along a straight line. It could be of three types:

- Inertia of rest
- Inertia of motion
- Inertia of direction

Newton's Second Law of Motion

- Acceleration of a particle as measured from an inertial frame (**ground**) is given by the **vector sum** of all the **forces** acting on the particle divided by its **mass**.

$$\vec{a} = \frac{\vec{F}}{m}$$

or

$$\vec{F} = m\vec{a}$$

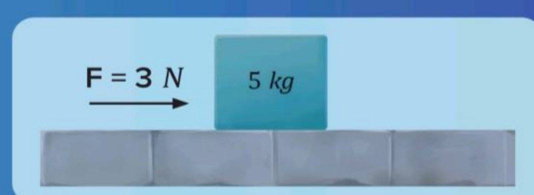
- The rate of change of momentum of an object is directly proportional to the net external force and in the direction of net force.

$$momentum = \vec{p} = m\vec{v}$$

$$\vec{F} \propto \frac{d\vec{p}}{dt} = k\frac{d}{dt}(m\vec{v}) \quad \text{(Mass is constant)}$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a} \quad ('k' = 1)$$

A block of mass 5 kg is kept on a smooth floor. A force of $3N\ \hat{i}$ is applied on it at, $t = 0$, Find the acceleration of the block.



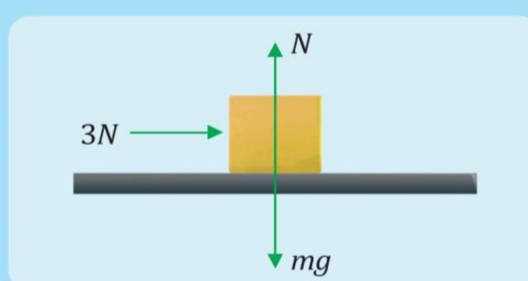
Solution

$$\vec{a} = \frac{\vec{F}}{m}$$

or

$$\vec{F} = m\vec{a}$$

$$a = \frac{3}{5} = 0.6\ ms^{-2}$$



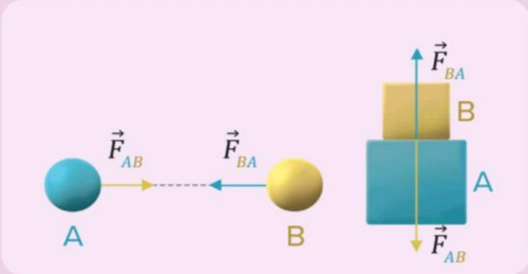
Newton's Third Law of Motion

If a body **A** exerts a force $\vec{F}$ on another body **B**, then **B** exerts a force -$\vec{F}$ on **A**, the two forces acting along the line joining the bodies.

$\vec{F}_{AB}$ = Force applied on A by B

$\vec{F}_{BA}$ = Force applied on B by A

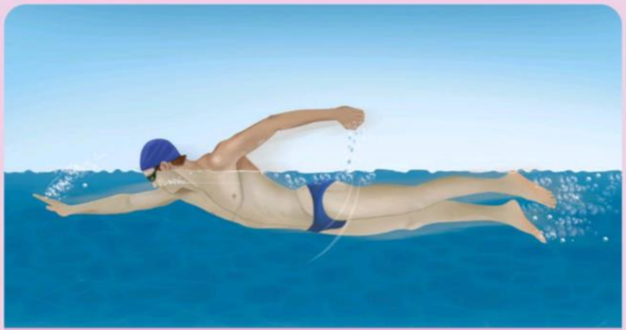
$|\vec{F}_{AB}| = |\vec{F}_{BA}|$



These two forces in Newton's third law are known as Action-Reaction pairs.

Action-Reaction pairs

Action	Reaction
Rocket pushes the burnt fuel down	The burnt fuel pushes the rocket up



Action	Reaction
Swimmer pushes the water back	The water pushes the swimmer forward

MAIN

Conditions for Action - Reaction pairs

- Occur simultaneously
- Act in opposite direction
- Act along the same line
- Act on different objects
- Have equal magnitude and same nature

NEWTON'S LAWS OF MOTION

UNDERSTANDING IMPORTANT TERMS

What you already now

- Force
- Fundamental forces in nature
- Standard forces in physics
- Newton's laws of motion

What you will learn

- System
- Free-body diagram
- Equilibrium

Interesting fact: Physics behind pushing a box

According to Newton's third law of motion, when you apply force to push a box, the box also applies equal and opposite force on you. If the forces are equal and opposite, then why don't they cancel out each other and make the resultant zero? How can the box move even when an equal and opposite force is generated? Have you ever thought about the logic behind this?

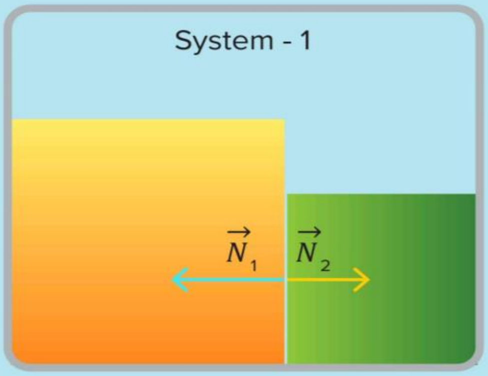
Well, from the third law, the action-reaction pair applies force on two different bodies. Here, the box applies a reaction force of the same magnitude on our hand and thus, the net reaction of forces on the box is not zero. Thus, due to the action force on it, the box moves forward.

What is System?

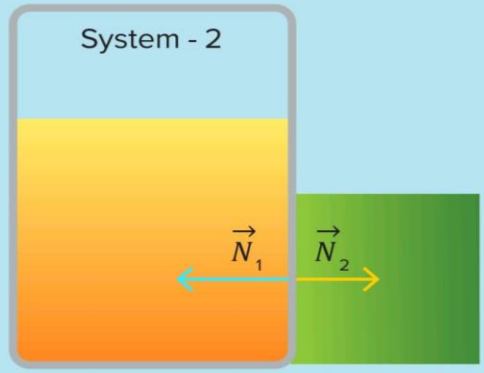
Two or more than two objects that interact with each other form a system.

Internal and external forces

If the action-reaction pair exists in the considered system, then it is known as **internal force,** otherwise it is known as **external force.**



In system 1, as shown in the given figure, both the action and the reaction forces are inside the system. Hence, the forces are known as internal forces.



In system 2, the reaction force is outside the system. Hence, force $\vec{N}_1$ is known as external force, whereas $\vec{N}_2$ force is not considered as it is not inside the system.

Free-Body Diagram (FBD)

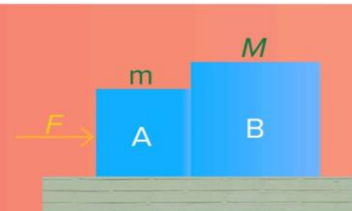
The diagrammatic representation of a body that is isolated from its surroundings, showing all the **external forces** acting on it, is known as the **free-body diagram (FBD).**

Steps for drawing the FBD

1. Isolate the free body.
2. Draw the external forces.
3. Choose the axes and resolve the forces.

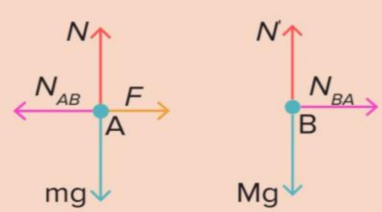
The entire Newtonian mechanics is applied on a body that is considered as a point. Hence, always take the body as a point while drawing forces in FBD. For visualisation, the body can be drawn to a shape.

Two bodies A and B of mass m and M, respectively, are being pushed by a force F towards the positive x-axis direction. Draw the free-body diagram of both the blocks.

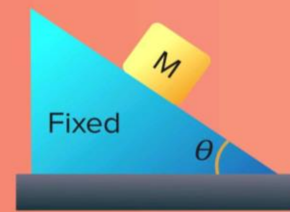


Solution

The FBD of both the blocks A and B having mass m and M, respectively, are shown in the figure.

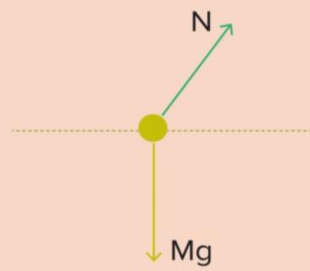


A block of mass M is placed on a wedge that is fixed to the ground as shown in the figure. Draw a free-body diagram of the block.

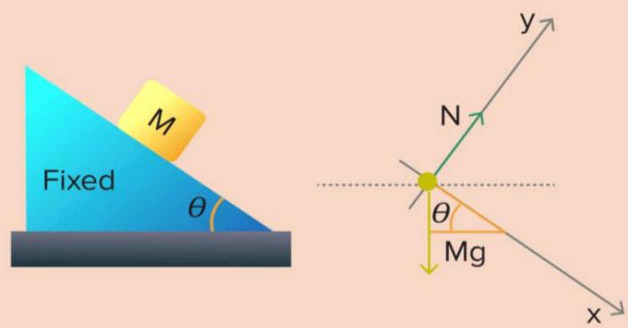


Solution

For drawing the FBD of the given problem, we must first consider the block on the wedge and apply the self-weight as well as the normal reaction on the same.



Now, the next step is to choose the axis and resolve the forces; the axes x and y are shown in the next diagram.



Components in the x direction

$F_x = Mg \sin \theta$

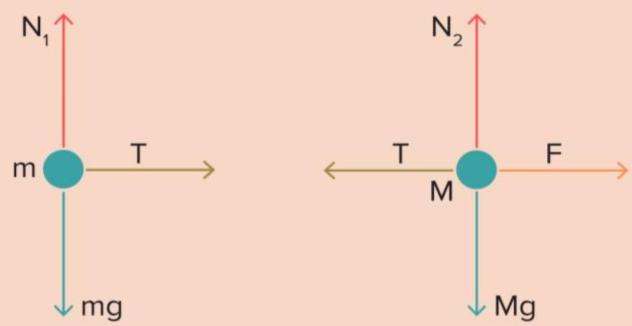
Components in the y direction

$F_y = N - Mg \cos \theta$

Two blocks of mass m and M are connected to each other with a massless rope and are being pulled by a rope with a force F. Draw the free-body diagram of both the blocks.

Solution

The FBD of both the masses have been drawn separately. The tension occurred on both the blocks because of the rope has also been shown in the individual FBD's of the masses.



Net Force

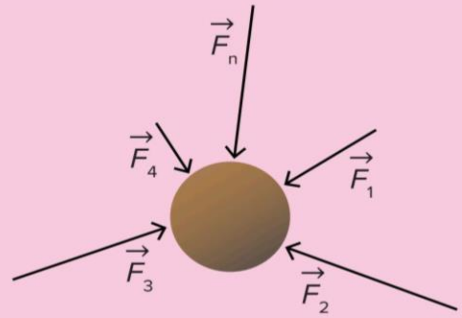
The vector sum of all the external forces that are acting on a system represents the net force on that system.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...\vec{F}_n$$

$$\vec{F}_{net} = \sum_{i=1}^{n} \vec{F}_i$$

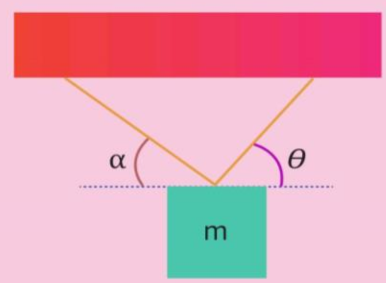
Here, the summation of all the forces gives the value of the net force acting on the body, and the vector provides the direction in which the resultant will act.

As shown in the figure, various forces are acting on the body with different magnitudes as well as in different directions, hence the resultant will give the overall effect.



Equilibrium

When the net force on an object is zero, then the object is said to be in a state of equilibrium.
In the given figure, mass m is hanging with the two tied ropes to the ceiling.



By the definition of equilibrium

$$\Sigma\left(\vec{F}_{ext}\right)_{sys} = \vec{0}$$

$$\sum \overline{F}_x = 0$$

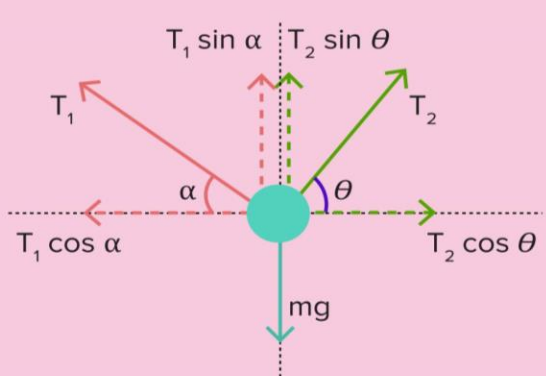
$$\sum \overline{F}_y = 0$$

$$\sum \overline{F}_z = 0$$

Steps to illustrate the equilibrium

Step 1

Draw the FBD
FBD for the system of forces given in the previous figure is



Step 2

Apply conditions of equilibrium

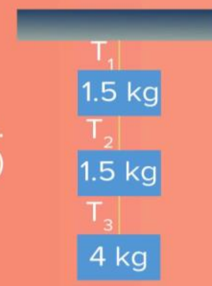
$$\sum \overline{F}_x = 0$$

$$T_2 \cos\theta - T_1 \cos\alpha = 0$$

$$\sum \overline{F}_y = 0$$

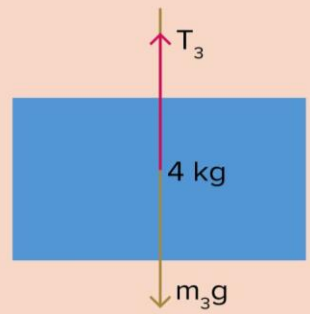
$$T_2 \sin\theta + T_1 \sin\alpha - mg = 0$$

Three blocks are suspended by a light string as shown in the given figure. What is the value of T_1 so that the system is in equilibrium? (Take $g = 10\ ms^{-2}$)



Solution

First, the FBD has to be drawn for each block separately. Then, by applying the equilibrium conditions, the value of $\boldsymbol{T_1}$ can be calculated.
For the lower block
FBD can be drawn as

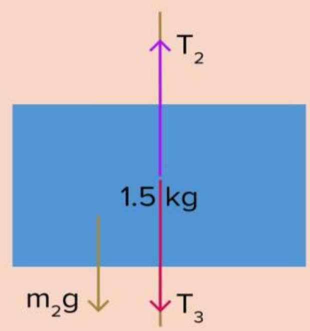


Applying the equilibrium equations
We get,

$T_3 = m_3 g$

$T_3 = 40\,N$

By putting the values of mass and gravitational acceleration, we get the value of tension.
Similarly, for the rest of the two,

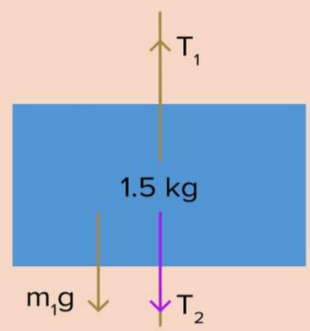


Applying the equilibrium equations, we get,

$T_2 = m_2 g + T_3$

$T_2 = 55\ N$

For the upper block, FBD can be drawn as in the adjacent figure.



Applying the equilibrium equations, we get,

$T_1 = T_2 + m_1 g$

$T_2 = 70\ N$

Hence, the value of T_1 has been calculated as **70 *N*.**

Alternative way:

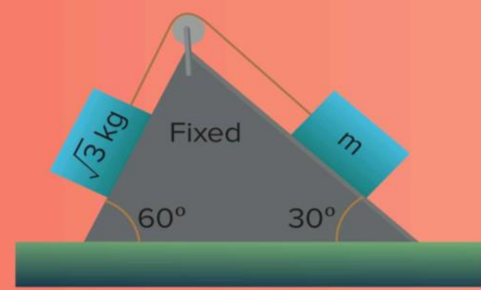
Treat all three blocks as a system.Then, the external forces are the total weight of the system acting downwards and tension on the string upwards.
By balancing forces in vertical direction,

$T_1 = m_1 g + m_2 g + m_3 g$

$T_1 = (1.5 + 1.5 + 4)\ 10$

$\Rightarrow \boldsymbol{T_1 = 70\ N}$

Two blocks are connected to a string and supported by a pulley as shown in the figure. Find the value of 'm' such that both the blocks are in equilibrium.



Solution

To get the value of the mass 'm', draw the FBD of the whole system, then apply the equilibrium conditions on the resolved force components.

For mass m,

$mg \sin 30^\circ = T$

For mass $\sqrt{3}$ kg,

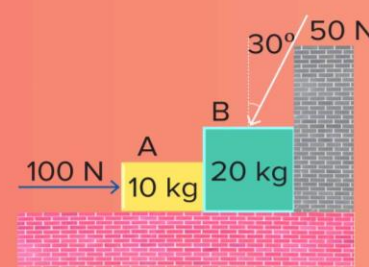
$\sqrt{3}\, g \sin 60^\circ = T$

From the two equations,

m = 3 kg

The value of the mass 'm' has been calculated as 3 kg for the blocks to be in equilibrium.

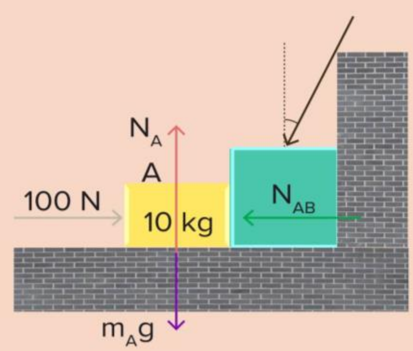
Two blocks are kept in contact as shown in the figure. Find the forces exerted by surfaces (floor and wall) on blocks, and the contact force between the two blocks. (Consider $g = 10\ ms^{-2}$)



Solution

To get the values of the forces, draw the FBD for the entire system and then apply the equilibrium conditions.

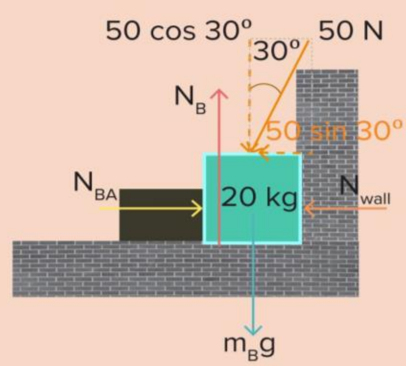
Considering block **A** of mass 10 kg, the FBD is shown as



$$\sum \vec{F}_x = 0 \qquad \Rightarrow N_{AB} = 100\,N$$

$$\sum \vec{F}_y = 0 \qquad \Rightarrow N_A = 100\,N$$

Now, considering block B of mass 20 kg,



Balancing the force in the horizontal direction,

$$\sum \vec{F}_x = 0$$

$$\Rightarrow 100 = N_{wall} + 25$$

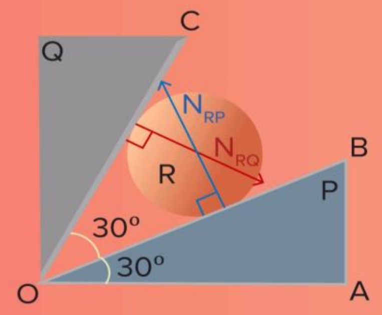
$$\Rightarrow N_{wall} = 75\,N$$

Balancing the force in the vertical direction,

$$\sum \vec{F}_y = 0$$

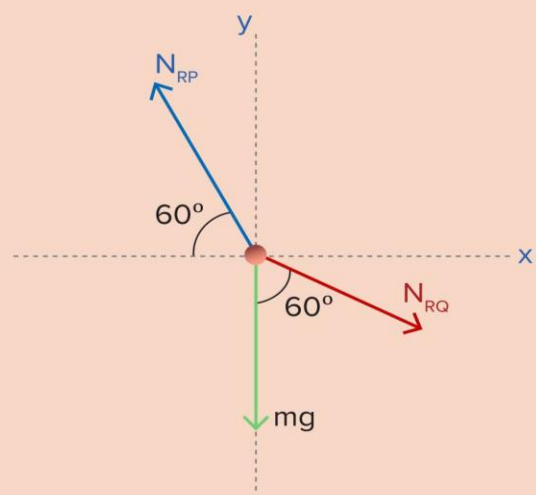
$$\Rightarrow N_B = (200 + 25\sqrt{3})N$$

A ball of mass 10 *kg* is wedged between two surfaces OB and OC as shown in the figure. What are the normal forces acting on the ball by the surfaces OB and OC?



Solution

To get the values of normal forces acting on the ball by the surfaces, draw the FBD of the ball and apply the equilibrium conditions.
The FBD of the ball is given in the adjacent figure.



Equilibrium condition in horizontal direction gives the relation in the normal forces.

$\sum \vec{F}_x = 0$

$\Rightarrow N_{RQ} \sin 60^\circ = N_{RP} \cos 60^\circ$

$\Rightarrow \frac{\sqrt{3}}{2} N_{RQ} = \frac{N_{RP}}{2}$

$\Rightarrow N_{RP} = \sqrt{3} N_{RQ}$ .... (i)

Now, applying the equilibrium condition in the vertical direction

$\sum \vec{F}_y = 0$

$\Rightarrow \frac{N_{RQ}}{2} + 100 = \frac{\sqrt{3}}{2} N_{RP}$

$\Rightarrow \sqrt{3}\, N_{RP} = N_{RQ} + 200$ .... (ii),

On solving equation (i) and (ii),

$N_{RQ} = 100\, N$

$N_{RP} = 100\sqrt{3}\, N$

NEWTON'S LAWS OF MOTION

NEWTON'S 2ND LAW APPLICATION

What you already know

- Force
- Newton's laws of motion
- Free-body diagram

What you will learn

- Newton's second law - Quantitative analysis
- Applications of Newton's second law

Recap

First law: A body maintains its state of inertia unless and until a net external force acts on it.

Second law: A net unbalanced force in magnitude is directly proportional to the rate of change of momentum of the body.

Third law: All the forces in nature are always generated in pairs.

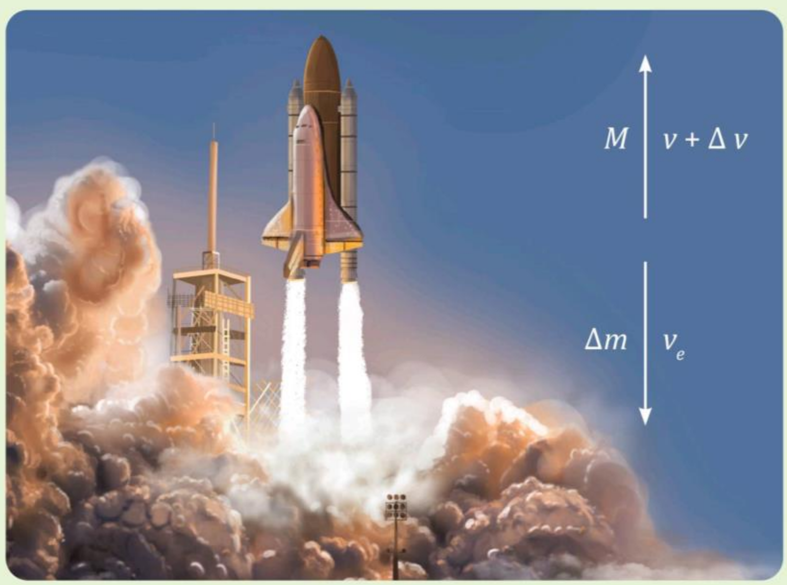
BOARDS

Quantitative Understanding of Newton's Third Law

Suppose a rocket of mass M is launched with velocity v. At an instant when its velocity increases to $v + \Delta v$, exhaust fuel gases of mass Δm will be moving in downward direction with velocity v_e.

According to Newton's third law of motion,

$$(M+\Delta m)v = M(v+\Delta v) + \Delta m(v - v_e)$$

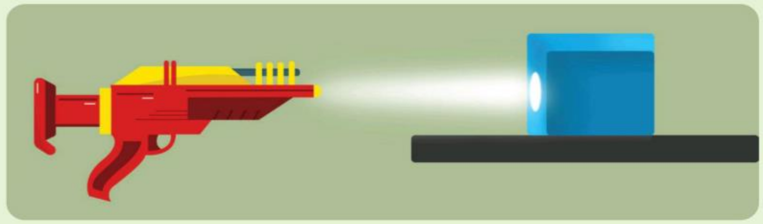


In order to understand Newton's second law, let us do some experiments.

Suppose a force gun is applying force on a block and the block moves a certain distance due to this force. Consider the instances during this motion.

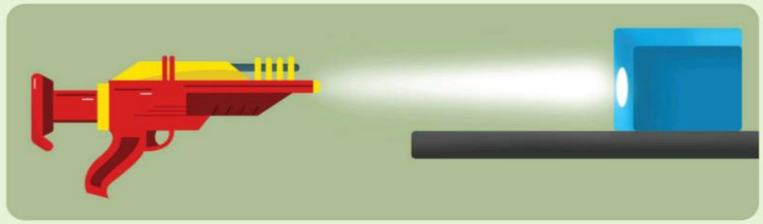
First instance

Initial velocity = $v_i \, ms^{-1}$ at $t = 0\ s$



Second instance

Final velocity = $v_f \, ms^{-1}$ at $t = T\ s$



Let,

Force applied = $F\ N$

Mass of the block = $M\ kg$

Average acceleration of the block will be,

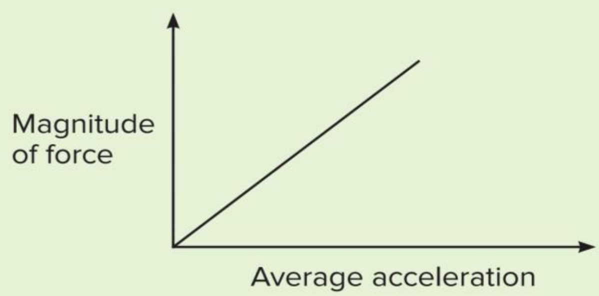
$$a_{avg} = \frac{(v_f - v_i)}{T}$$

In order to establish a relationship between force, mass, and acceleration, let us vary the parameters one by one in the form of experiments.

Experiment 1

Vary the force by keeping the mass of the block constant. Measure the average acceleration for the same time interval.

It is observed that the forces is directly proportional to the average acceleration measured. It can be plotted as shown in adjacent figure.

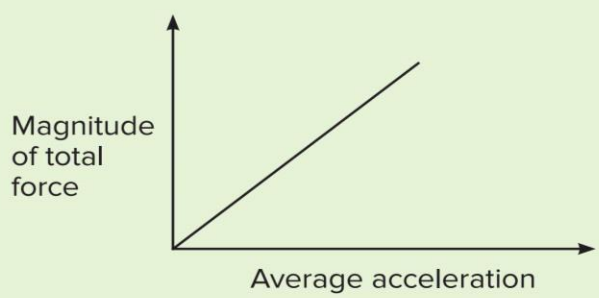


Conclusion

$F_{mag} \propto$ Average acceleration (a)

Experiment 2

Add multiple force guns from one direction. Same direction is important to add the force vectorially. Keep the mass of the block constant and measure the average acceleration for the same time interval. In this case, total force will be the vector sum of all the applied forces. It is observed that this total force is also proportional to the magnitude of average acceleration.



Conclusion

Vector sum of forces is directly proportional to the average acceleration, i.e.,

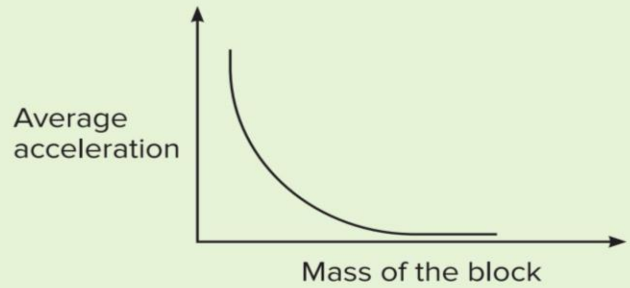
$\Sigma F_{mag} \propto$ Average acceleration (a)

Experiment 3

Keep the magnitude of force constant and vary the mass of the block. Measure the average acceleration for the same time interval. Here, we observe the following:

The average acceleration is inversely proportional to the mass of the block.

$$M_{block} \propto \frac{1}{a_{avg}}$$



Experiment 4

Vary the direction from which the force gun is fired. Keep the mass of the block constant and check the direction of acceleration. It is observed that the directions of both net force and acceleration are the same.

Summary of experiments

The summarised results of the above experiments help us to establish the relationship between force, mass, and acceleration.

$$F_{mag} \propto a_{avg}$$

$$\sum F \propto a_{avg}$$

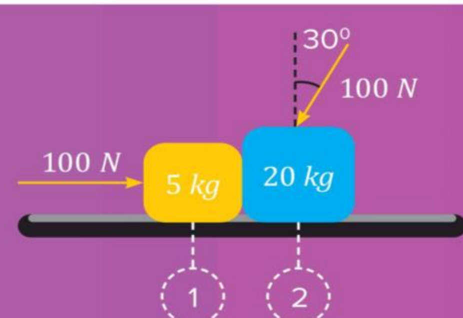
$$M_{block} \propto \frac{1}{a_{avg}}$$

The directions of both the net force and the acceleration are the same.

Thus, the relation can be written in vector form as,

$$\sum \vec{F} \propto M\vec{a}$$

Two blocks kept in contact with each other experience a force of 100 N each as shown. Find the acceleration of the 5 kg block. (Assume the surfaces to be smooth)



Solution

Here, the surface is smooth. Hence, ignore the friction.

Step 1: Draw free-body diagram (FBD),

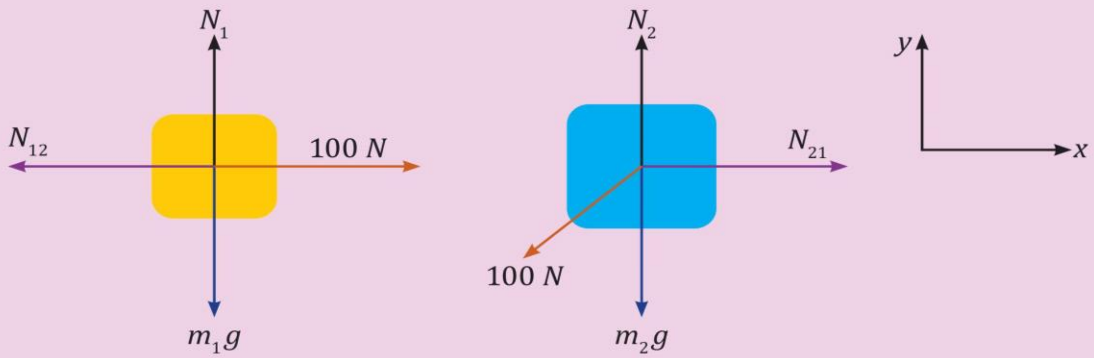
Let the 5 kg block be of mass m_1 and 20 kg block be of mass m_2.

On the 5 kg body, a total of four forces are applied.

1. Weight in downward direction

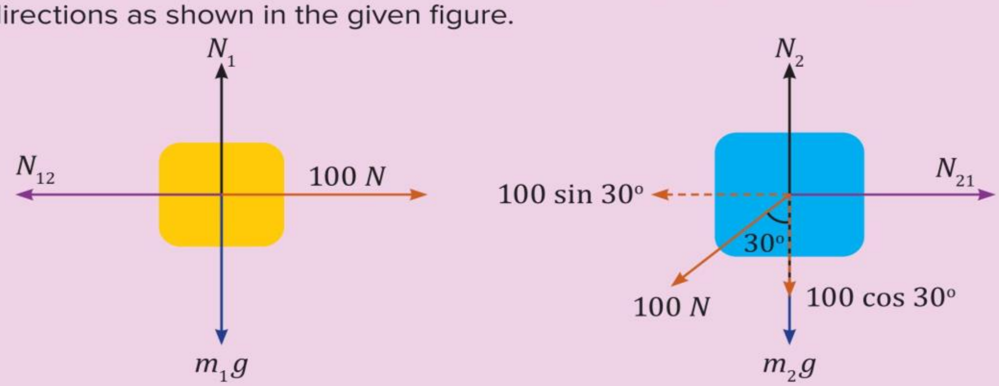
2. 100 N force in positive x direction
3. Normal reaction of body 2, N_{12}
4. There will be a normal reaction N_1 from the ground as well.

For the second body, 100 N force is applied at 30° inclined from vertical.



Step 2: Define the axes and resolve the forces into its components along the axes.

Axes are chosen so that a maximum number of forces are directed along the coordinate axes. Thus, there would be a less requirement of resolution of forces. 100 N inclined force is resolved along these two directions as shown in the given figure.



There is equilibrium in y-direction. Hence both bodies will move in x-direction with acceleration a.

Step 3: Write the equation of motion along the axes and solve them.

Note: Take the positive x-axis direction along the acceleration.

For 5 *kg* block,

Along x-direction,

$$\sum \vec{F}_x = m\overrightarrow{a_x}$$

$$\Rightarrow 100 - N_{12} = (5)a \ldots\ldots (i)$$

Along y-direction,

$$\sum \vec{F}_y = m\overrightarrow{a_y} = 0$$

$$\Rightarrow N_1 - m_1 g = 0 \ldots\ldots (ii)$$

For 20 *kg* block,

Along x-direction,

$$\sum \vec{F}_x = m\overrightarrow{a_x}$$

$$\Rightarrow N_{21} - 50 = (20)a \ldots\ldots (iii)$$

Along y-direction,

$$\sum \vec{F}_y = m\overrightarrow{a_y} = 0$$

$$\Rightarrow N_2 - m_2 g - 100\cos 30^o = 0 \ldots\ldots (iv)$$

As the two blocks apply the same normal reaction on each other,

$$|N_{12}| = |N_{21}|$$

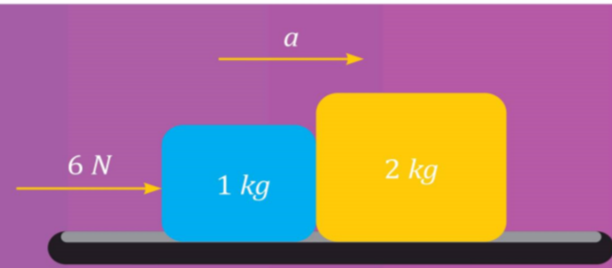
On adding equations (i) and (iii), we get,

$100 - 50 = 25a$

$\Rightarrow 50 = 25a$

$\Rightarrow a = 2\ ms^{-2}$

An arrangement of a two block system is shown. Find the net external force acting on 1 *kg* and 2 *kg* block, respectively. (Assume the surfaces to be smooth)



(A) 4 *N*, 8 *N* (B) 1 *N*, 2 *N* (C) 2 *N*, 4 *N* (D) 3 *N*, 6 *N*

Solution

As both the blocks will move together with the applied force, acceleration of both the blocks will be the same.

Step 1: Draw FBD,

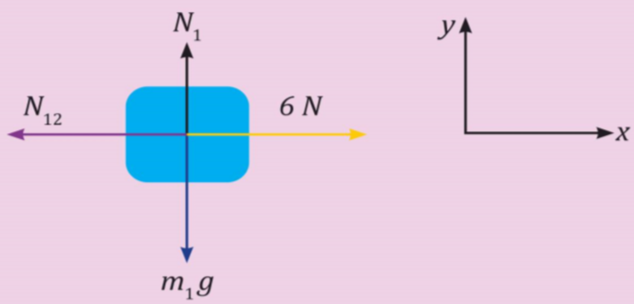
FBD for the blocks are shown in the figure.

For block 1,

Along *x*-diretion,

$\sum \vec{F}_x = m\overrightarrow{a_x}$

$\Rightarrow 6 - N_{12} = (1)a......(i)$



Along *y*-direction,

$\sum \vec{F}_y = m\overrightarrow{a_y} = 0$

$\Rightarrow N_1 = 1(10)$

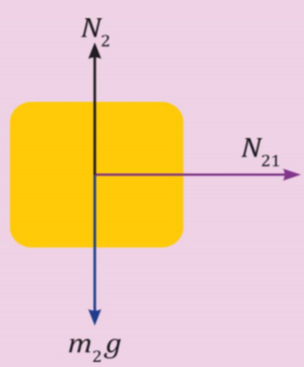
$\Rightarrow N_1 = 10\,N\ ......(ii)$

For body 2,

Along *x*-direction,

$\sum \vec{F}_x = m\overrightarrow{a_x}$

$\Rightarrow N_{21} = (2)a\ ......(iii)$



Along *y*-direction,

$\sum \vec{F}_y = m\overrightarrow{a_y} = 0$

$\Rightarrow N_2 = 20\,N ......(iv)$

As the two blocks apply the same normal reaction on each other,

$|N_{12}| = |N_{21}|$

On adding equation (*i*) and (*iii*), we get,

$6 = 3a$

$\Rightarrow a = 2\ ms^{-2}$

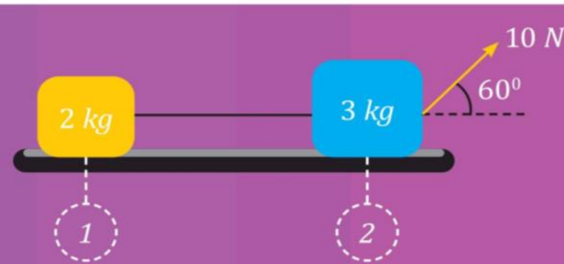
Thus,

Force on block with mass 1 *kg* = *ma* = 1 × 2 = 2 *N*

Force on block with mass 2 *kg* = *ma* = 2 × 2 = 4 *N*

Hence, option (C) is correct.

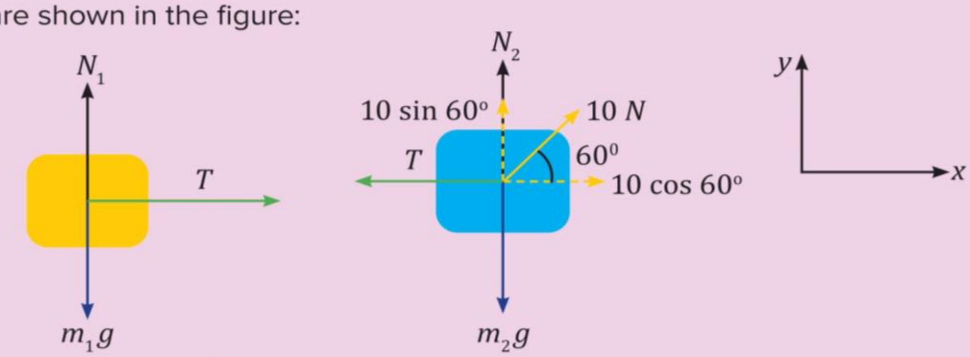
The figure shows two blocks connected by a light inextensible string. A pulling force of $10\ N$ is applied on the bigger block at 60^0 with horizontal. Find the tension in the string connecting the two masses.



(A) $5\ N$ (B) $2\ N$ (C) $1\ N$ (D) $3\ N$

Solution

The string is massless and inextensible. Hence, the string will carry equal tension along its length. FBD for the two bodies are shown in the figure:



For body 1,

Along x-direction,

$\sum \vec{F}_x = m\overrightarrow{a_x}$

$\Rightarrow T = (2)a$ ......(i)

Along y-direction,

$\sum \vec{F}_y = m\overrightarrow{a_y} = 0$

$\Rightarrow N_1 = 2(10)$

$\Rightarrow N_1 = 20\,N$ ......(ii)

For body 2,

Along x-direction,

$\sum \vec{F}_x = m\overrightarrow{a_x}$

$\Rightarrow 10\cos 60^o - T = (3)a$

$\Rightarrow 5 - T = (3)a$ ......(iii)

Along y-direction,

$\sum \vec{F}_y = m\overrightarrow{a_y} = 0$

$\Rightarrow N_2 + 10\sin 60^o = 3(10)$

$\Rightarrow N_2 + 5\sqrt{3} = 30\,N$ ......(iv)

On Adding equation (i) and (iii), we get,

$5 = 5a$

$a = 1\ ms^{-2}$

Now, from (i),

$T = 2(1) = 2\ N$

Hence, option (B) is correct.

A small block B is placed on another block of A of mass $5\ kg$ and length $20\ cm$. Initially, block B is near the right end of block A. A constant horizontal force of $10\ N$ is applied to block A. All the surfaces are assumed frictionless. Find the time elapsed before the block B separates from A.

Solution

All the surfaces are smooth.

No horizontal force on block B, hence no motion for block B. As block A moves through length 20 cm, block B will separate from block A.

FBD for block A and B is shown in the figure.

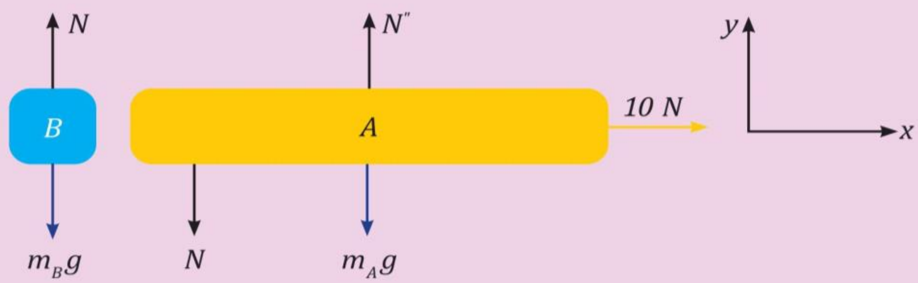
Acceleration of A,

Along x-direction,

$\sum \vec{F}_x = m\vec{a}_x$

$\Rightarrow 10 = (5)a$

$\Rightarrow a = 2\, ms^{-2}$

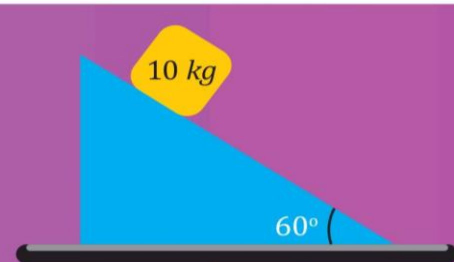


We have,

Displacement of the block,

$$x = ut + \frac{1}{2}at^2 \quad \Rightarrow \frac{20}{100} = \frac{1}{2}(2)t^2 \quad \Rightarrow t = \frac{1}{\sqrt{5}} = 0.45\, s$$

The body of mass 10 kg is placed on a wedge (fixed to the ground) inclined at 60° to the horizontal and released. What is the magnitude of velocity of the block after 2 seconds? (g = 10 ms^{-2})



(A) $10\sqrt{3} ms^{-1}$ (B) $5\sqrt{3} ms^{-1}$ (C) $5\, ms^{-1}$ (D) $10\, ms^{-1}$

Solution

FBD for 10 kg block is as follows:

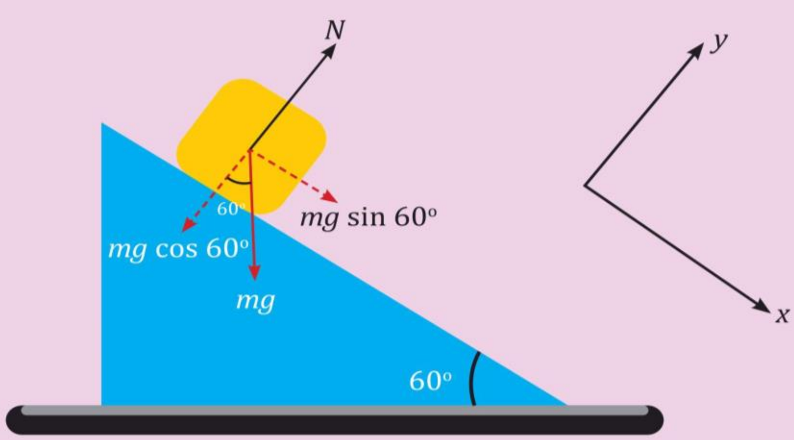
Here, x-axis is chosen along the downward inclined surface as the acceleration of the body will be along this direction.

$\sum \vec{F}_x = m\vec{a}_x$

$\Rightarrow 100 \sin 60^o = (10)a$

$\Rightarrow 50\sqrt{3} = (10)a$

$\Rightarrow a = 5\sqrt{3}\, ms^{-2}$



Here, initial velocity, u = 0

Hence,

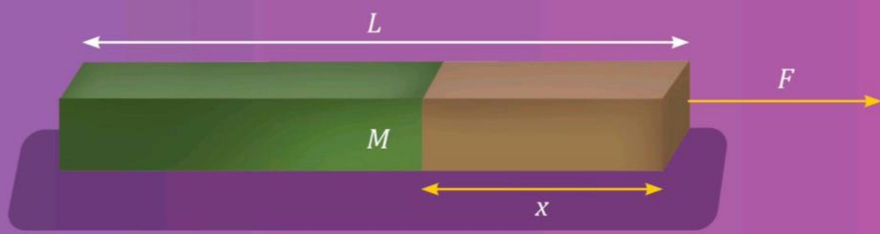
$v = u + at$

$v = 0 + (5\sqrt{3})(2)$

$v = 10\sqrt{3}\, ms^{-1}$

Hence, option (A) is correct.

A horizontal force F is applied on a uniform rod of length L and mass M. It is kept on a frictionless surface. Find the tension in the rod at a distance x from the end where the force is applied. Draw the tension vs length curve.



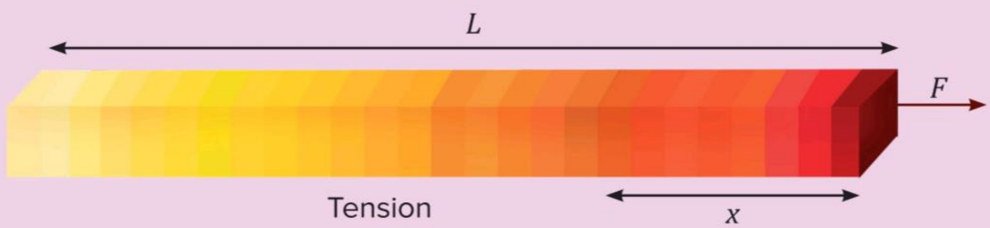
MAIN

Solution

More mass to pull, more will be tension. Hence, tension will vary uniformly along the length due to the uniform cross-section of the rod, highest from the starting element.

This is shown by colour code as red colour having maximum tension.

Let us divide the rod in two parts to break it at a distance x from the end where the force is applied.



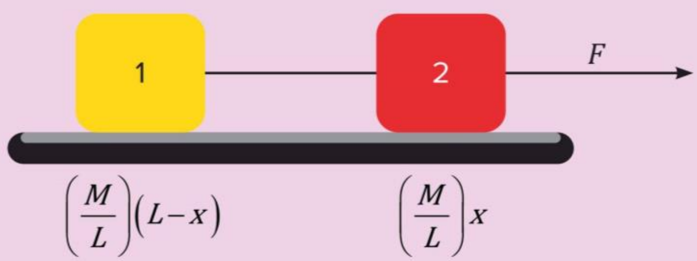
Masses of two sections of rod are:

For section with length x,

$$m_2 = \left(\frac{M}{L}\right)x$$

For section with length $(L - x)$,

$$m_1 = \left(\frac{M}{L}\right)(L-x)$$



FBD for the two sections is as follows:

For body 1,

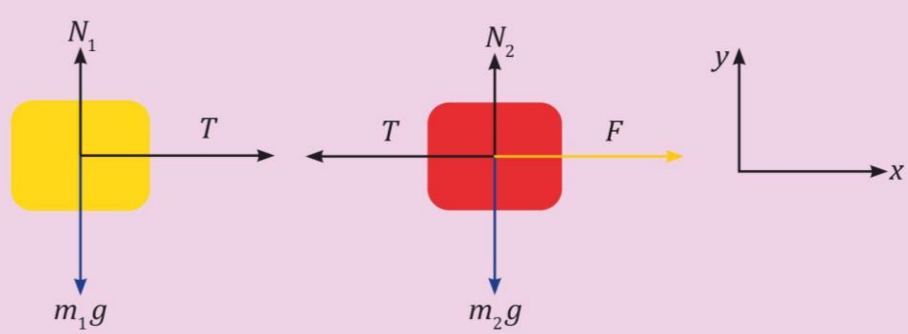
Along x-direction,

$$T = \frac{M}{L}(L-x)a ......(i)$$

For body 2,

Along x-direction,

$$F - T = \frac{M}{L}(x)a ......(ii)$$



Adding (i) and (ii),

$F = Ma$

Substitute a in equation (i),

$$T=\frac{M}{L}(L-x)\left(\frac{F}{M}\right)$$

$$\Rightarrow T=\frac{F}{L}(L-x)$$

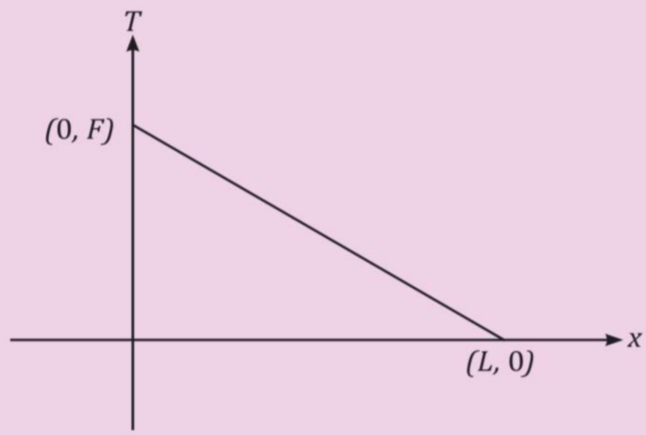
$$\Rightarrow T=F\left(1-\frac{x}{L}\right)$$

At $x = 0$, $T = F$,

At $x = L$, $T = 0$,

The tension varies linearly with the length of the rod.

Hence, the tension vs length graph can be plotted as shown in adjacent figure:



NEWTON'S LAWS OF MOTION

WEIGHING MACHINE AND SPRINGS

What you already know

- Force
- FBD
- Resolving forces

What you will learn

- Force transmission through a massless body
- Spring force, spring constant, and spring balance

Massless Body

- For massless bodies ($m \to 0$), the net force $\sum \vec{F} = 0$; however, acceleration (a) is finite.
- For massless string, the same tension is carried out throughout its length.
- 'Light-weight' can be used as a synonym for 'massless' in some questions.

Weighing Machine

It is a machine that measures the **normal force** applied on it.
Let us take an example of a raven to understand it.
Let a weighing machine be calibrated such that when cage is kept on it, it shows the $0\ kg$ reading.

Case 1:

Raven is sitting on a weighing machine and the reading is $1\ kg$.

Case 2:

Raven is fluttering in the cage and the reading is $0\ kg$.

The readings are different because the raven applies its weight on the machine in case one. In the second case, the raven does not apply any normal force on the weighing machine, hence the reading is zero. Normal reaction is applied only when two bodies are in contact.
Similarly, when you put one leg on the weighing machine and one on ground, the machine will show nearly half of the weight as reading. This shows that the machine measures the force with which you press it. So the weighing machine measures the normal reaction acting on it and not the actual weight of the body.

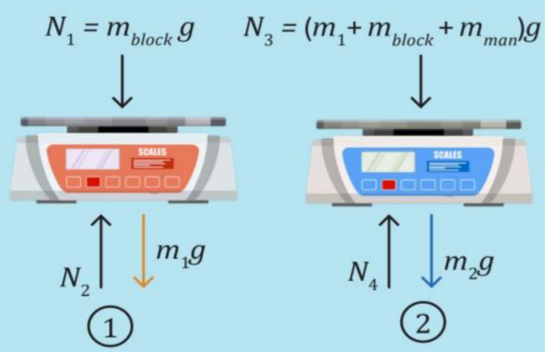
A man of mass 60 kg is standing on a weighing machine of mass 5 kg placed on the ground. Another similar weighing machine is placed over the man's head. A block of mass 50 kg is placed on the weighing machine. Calculate the readings of both the weighing machines. (g = 10 ms^{-2})

Solution

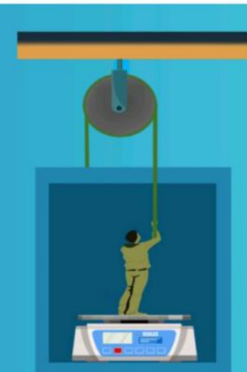
Let m_1 and m_2 be masses of weighing machine 1 and 2 respectively. Let m_{block} be mass of block and m_{man} be mass of man.

Reading of weighing machine (1)
$N_1 = (m_{block})\, g = 50g = 500\ N$
(or 50 kg, as will be displayed on weighing scale)

Reading of weighing machine (2)
$N_3 = (m_1 + m_{block} + m_{man})g = (5 + 50 + 60)\, g = 115g = 1150\ N$
(or 115 kg, as will be displayed on weighing scale)



A man of mass 60 kg is standing on a light weighing machine kept in a box of mass 30 kg. The box is hanging from a pulley fixed to the ceiling through a light rope, the other end of which is held by the man himself. If the man manages to keep the box at rest, what is the reading shown by the machine? (g = 10 ms^{-2})



(A) 30 kg (B) 15 kg (C) 45 kg (D) 60 kg

Solution

As the string is massless, it will carry the same tension throughout.
From FBD of the man,
Here, let N_1 be a normal reaction on the man applied by the machine and T be tension in the string. Then,
$N_1 + T = 600$ ....(i)
Let N be a normal reaction applied on the box by the weighing machine.

The force transmits undiminished through a massless body.
Thus, the normal force applied on the box by the weighing machine is equal to the normal reaction applied on the weighing machine by the man.

$N = N_1$

From the FBD of the box,

$T - N_1 = 300$.......(ii)

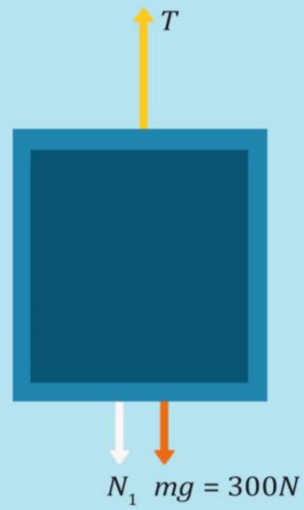
Subtracting equation (ii) from equation (i)

$2N_1 = 300\ N$

$N_1 = 150\ N$

i.e., 15 kg mass will be shown by the weighing machine.

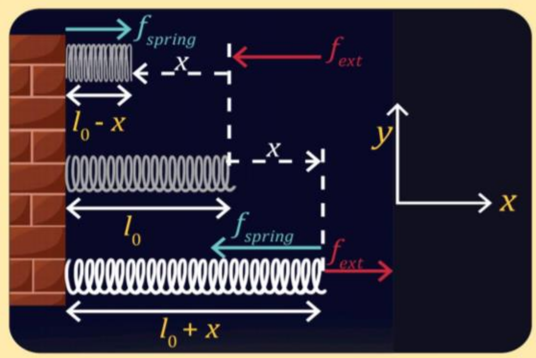
Hence, option (B) is correct.



MAIN

Ideal spring

- An ideal spring is made of an ideal string (massless and inextensible) wound in the form of a helix.
- Due to the design of spring, its length can vary on the application of the force.
- **Restoring nature:** Spring has a tendency to resist any kind of deformation.

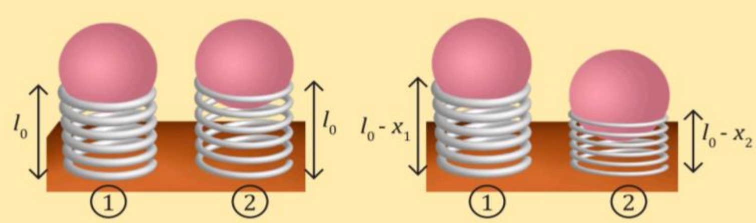


- Similarity between ideal spring and ideal string: Both are massless.
- Difference between ideal spring and ideal string: Ideal string is inextensible, while due to helical nature of spring its length can be changed.

Spring force: It is the restoring force developed in a spring.

- It is given by Hooke's law, $\vec{F} = -k\vec{x} = -kx\,\hat{x}$, where $\vec{x}$ is the change in length of the spring and k is the **spring constant.**
- Spring force is always opposite to the direction of applied force or the direction of displacement. Hence, it is taken as negative.
- For an ideal spring (massless) the restoring spring force, $F_{spring} = F_{ext}$

Spring constant (k): It denotes the **stiffness** of the spring which signifies how difficult it is to deform the spring. It is a measure of the inertia of spring to resist any kind of compression or extension.
In the figure, the same balls are kept on two springs of the same length, yet the change in length is different.



Here, spring force on spring 1 = Spring force on spring 2

$F_1 = F_2$

$k_1(x_1) = k_2(x_2)$

$\Rightarrow k_1 > k_2 \quad (\because x_1 < x_2)$

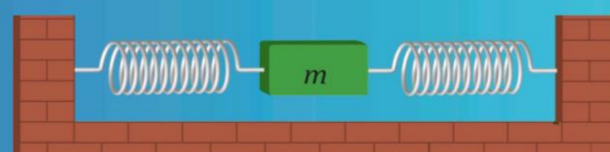
Thus, if the same force is applied on two springs of the same length, then the spring with higher spring constant will get elongated or compressed less than the spring with lower spring constant. Higher the spring constant k, more stiffer and robust the spring is, and more is the difficulty to compress or extend it.

Spring balance

It measures the restoring spring force which is equal to the tensile force on the spring. This tensile force is equal to the weight of the hanging body only under equilibrium condition.

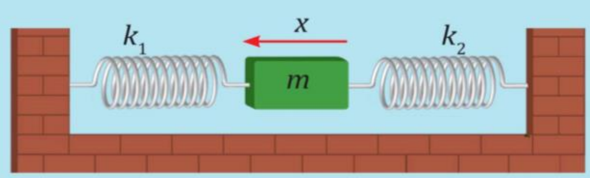
In the given figure, when the mass is hung to the vertical spring and the system comes in equilibrium, the tension in the string will be equal to the weight hung to it.

Both the springs shown in the figure are unstretched. If the block is displaced by a distance x and released, what will be the initial acceleration?

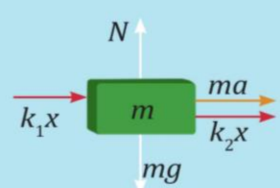


Solution

Draw FBD of mass m as shown in the figure. Here, the left spring gets compressed and hence it will apply restoring force in rightward direction to regain its shape. At the same time, the right spring will get elongated and to restore position, it will also apply force in the rightward direction.



Equating forces in horizontal direction,



$k_1 x + k_2 x = ma$

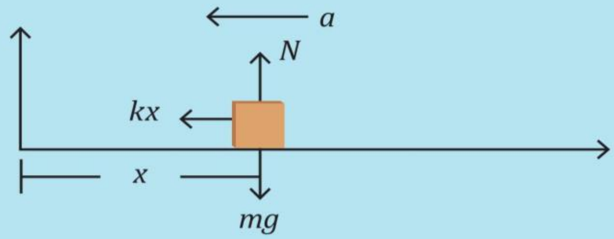
$(k_1 + k_2)x = ma$

$$a = \frac{(k_1 + k_2)x}{m}$$

A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15\ Nm^{-1}$. What will be its initial acceleration, if it is released from a point $x = 20\ cm$?

Solution

Though it is not mentioned that the particle is spring, it is specified with a spring constant and the force is also expressed in terms of spring force. Hence, we need to apply the spring concept on this particle.



From the FBD of the particle,

$$ma = kx$$

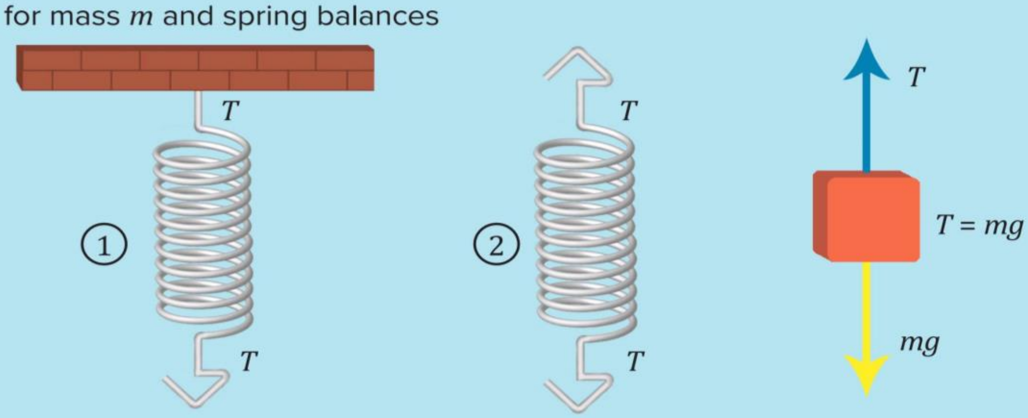
$$a = \frac{kx}{m} = \frac{15 \times 0.20}{0.3} = 10\ ms^{-2}$$

A block of mass 20 kg is suspended through two light spring balances as shown in the figure. Calculate the following:

(a) Reading in spring balance 1
(b) Reading in spring balance 2

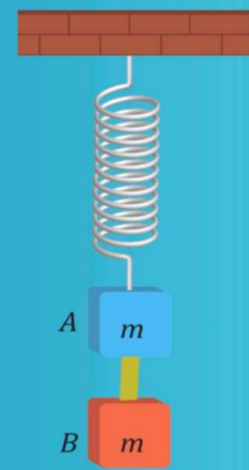
Solution

Draw FBD for mass m and spring balances



We know that the spring balance measures the external force applied on itself, i.e., tension T in this case. Thus, both the spring balances show reading $F_{spring1} = F_{spring2} = T = mg = 200\ N$ or 20 kg on scales of the spring balances.

Two blocks A and B of the same mass m attached with a light string are suspended by a spring as shown in the figure. Find the acceleration of both the blocks just after the string is cut.

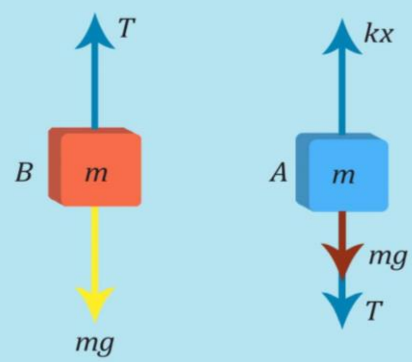


(A) $g, \frac{g}{2}$ (B) g, g (C) $2g, g$ (D) $\frac{g}{2}, \frac{g}{2}$

Solution

Here, after cutting the string, the block B will fall freely. Hence, either option (B) or (C) is correct.
Now, let us find the acceleration of block A.

Note: Restoring force in a spring does not change suddenly.

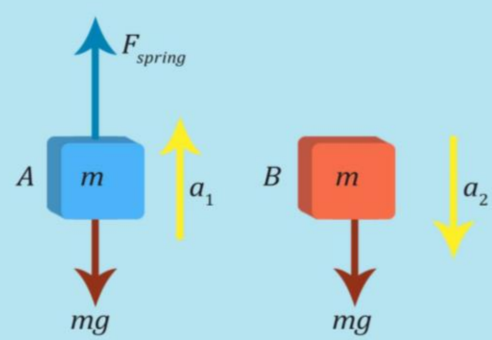


Before cutting of string,
From FBD of B,
Tension in the string is equal to the weight of block B.
$T = mg$

From FBD of A,
$F_{spring} = T + mg$
$F_{spring} = mg + mg = 2mg$

Just after cutting of string,

Restoring force will remain the same as it does not change suddenly.



From FBD of A,
$ma_1 = F_{spring} - mg$
$ma_1 = 2mg - mg$
$a_1 = g$

From FBD of B,
$ma_2 = mg$
$a_2 = g$
Hence, option (B) is correct.

A man has fallen into a ditch of width d, and two of his friends are slowly pulling him out using a light rope and two fixed pulleys. Show that the force (assumed equal for both the friends) exerted by each friend on the road increases as the man moves up. Find the force, when the man is at a depth h.

Solution

The process is very slow (quasistatic). Thus, we can assume equilibrium at every instant. Therefore, we can equate the forces for any component without considering the acceleration ($a \to 0$).

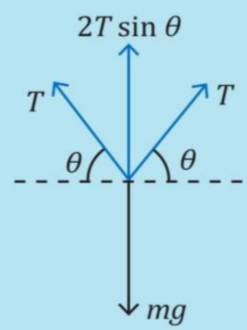
Resolution of forces on hanging man, is shown in adjacent figure:

Balancing the forces in vertical direction,

$$2T \sin\theta = mg$$

$$T = \frac{mg}{2\sin\theta}$$

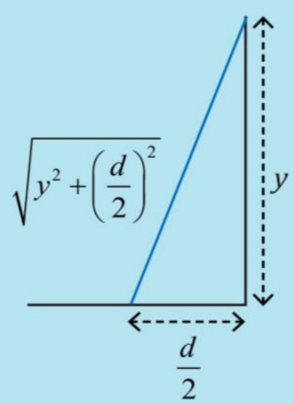
$$\sin\theta = \frac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}$$



Value of $\sin\theta$ at any instant can be found from geometry.
At any instant, let the man be at a depth y.

Since the string is massless, the tension in the string,
T = Force applied by each friend
$T = F$
Therefore, the force applied by each friend, at depth $y = h$, is:

$$F = \frac{mg}{2\sin\theta} = \frac{mg}{\dfrac{2h}{\sqrt{h^2 + \left(\frac{d}{2}\right)^2}}} = \frac{mg\sqrt{h^2 + \left(\frac{d}{2}\right)^2}}{2h}$$



As the man is pulled upwards, the angle θ decreases. Therefore the value of $\sin\theta$ also decreases, which implies that force exerted by each friend (F) increases (since F is inversely proportional to $\sin\theta$).

NEWTON'S LAWS OF MOTION

CONSTRAINED MOTION: PULLEY BLOCK SYSTEM

What you already know

- FBD
- Resolving forces
- Spring force

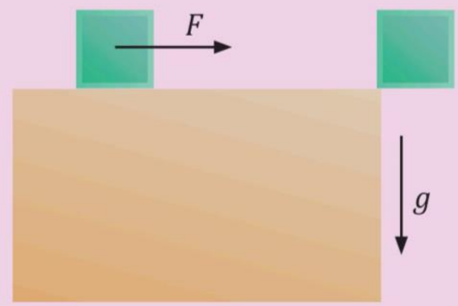
What you will learn

- Constrained motion
- String constraint
- Pulley-block system

BOARDS

Constrained Motion

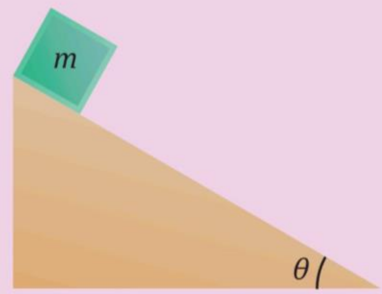
Constrained motion results when an object is forced to move in a restricted way.
Here, the object is constrained to move along the surface of the block due to the force and then falls vertically downwards due to acceleration due to gravity.



Similarly, here the block is constrained to slide along an inclined plane. The force due to which the body is kept under a constraint is known as **constraint force**.

There are two primary constraints:

1. String/Rod constraints
2. Wedge constraints



String Constraints

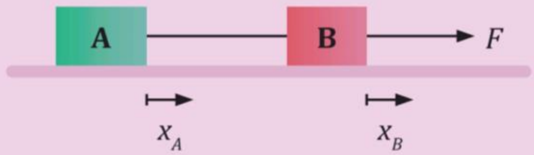
Consider objects connected through a string that has the following properties:

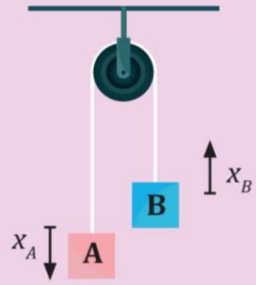
- The length of the string remains constant, i.e., string is inextensible.

Which means that if one block moves x distance, the other block also moves the same distance.
It can be mathematically written as,

$x_A = x_B$

Where x_A is displacement of block A and x_B is displacement of block B.



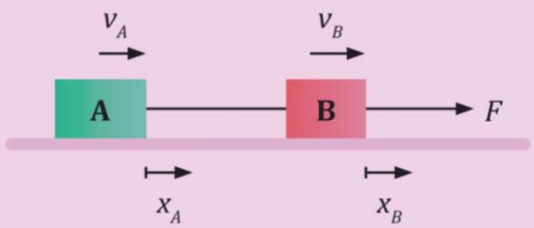


- It always remains tight and does not slack.

Parameters of the motion of such objects along the length of the string and in the direction of extension have a definite relation between them.
Here, the parameters of motion means the displacement, velocity, and acceleration.
Objects mean the bodies that are directly connected to string.
Definite relation means the constraint relation, such as $x_A = x_B$

Constraint Relations

Mathematical relations by which motion of two or more bodies within a system are related are known as constraint relations.



For the given system, as the length of string is constant,
$x_A = x_B$

Differentiating the equation with respect to time,

$$\Rightarrow \frac{dx_A}{dt} = \frac{dx_B}{dt}$$

We know that,

$$\frac{dx_A}{dt} = v_A \text{ and } \frac{dx_B}{dt} = v_B$$

$$\Rightarrow v_A = v_B$$

Differentiating the equation with respect to time again,

$$\Rightarrow \frac{dv_A}{dt} = \frac{dv_B}{dt}$$

But we know that,

$$\frac{dv_A}{dt} = a_A \text{ and } \frac{dv_B}{dt} = a_B$$

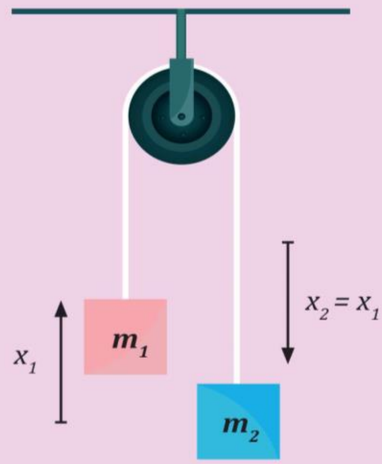
$$\Rightarrow a_A = a_B$$

Thus, we could relate the parameters using the inextensibility constraint of string.
So, constraint relations in this case are $x_A = x_B$, $v_A = v_B$ and $a_A = a_B$

Pulley-Block System

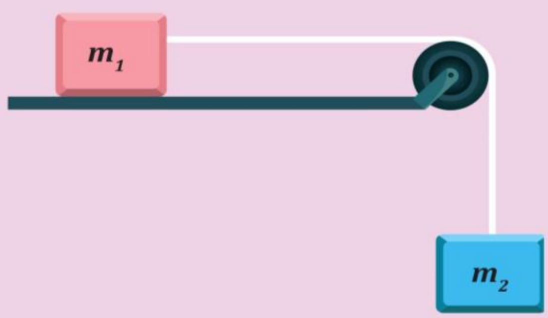
Atwood's machine

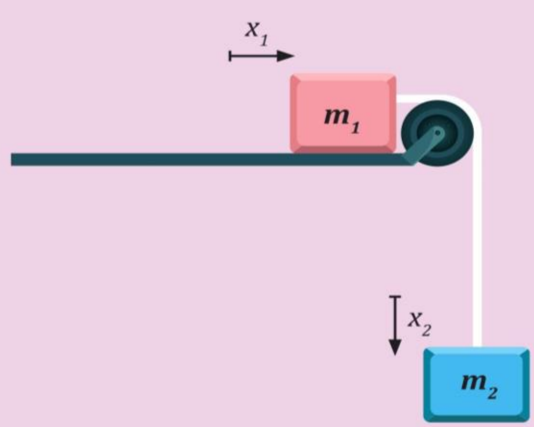
A device in which two objects are connected by a string over a pulley such that if one falls down, the other one rises, is known as **Atwood's machine**.
In this system, heavier objects will fall down and lighter objects will rise.



Consider the magnitude of displacement of two objects.
From inextensibility constraint of string,
$x_1 = x_2$
Similarly, as they are moving equal distance in equal time interval,
$v_1 = v_2$
and
$a_1 = a_2$

Example
For the two blocks shown, the string is inextensible.
Consider the magnitude of displacement of two objects,
$x_1 = x_2$
Similarly, as they are covering equal distance in an equal time interval,
$v_1 = v_2$
and
$a_1 = a_2$



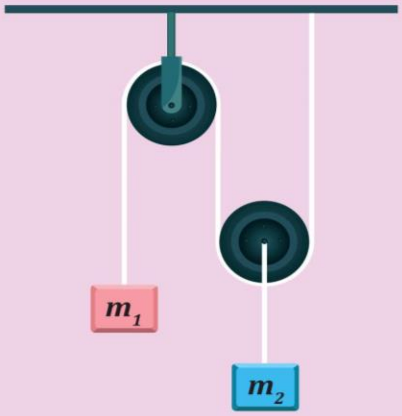


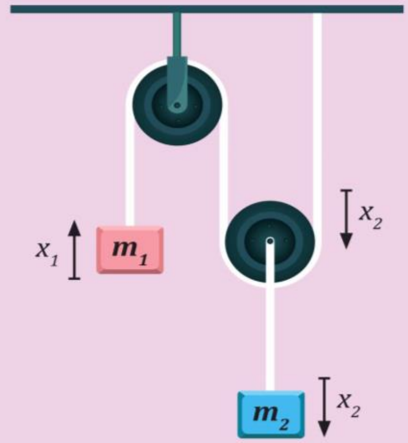
Thus, we can visually convert the system in mathematical equation form.

Example: System with movable pulley

Identify all the objects and number of strings in the problem.
Here, the distance travelled by movable pulley is equal to the distance covered by block 2 because they are connected by a single inextensible string.

As the movable pulley moves down the distance x_2, the string on both sides of this pulley should move down the same distance, i.e., x_2.
Thus, it takes a string length of $2x_2$ to lower down the movable pulley with distance x_2. This length will be completely contributed by the left side of the string as the total length is constant.



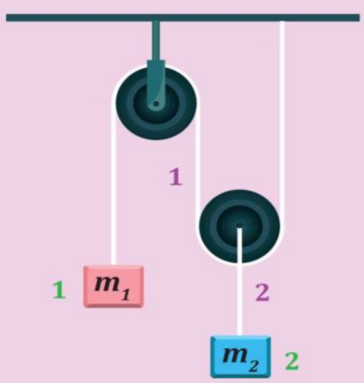


This gives a relationship between x_1 and x_2.
Thus,
$x_1 = 2x_2$
Similarly,
$v_1 = 2v_2$
and
$a_1 = 2a_2$

Steps for writing string constraint relation

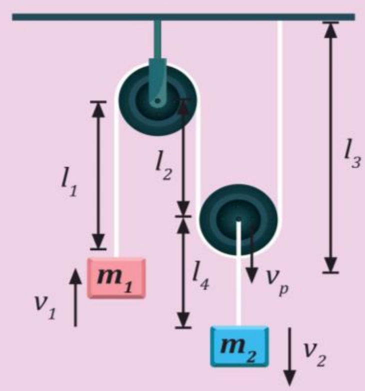
Step 1: Identify all the objects and number of strings in the problem.

The given system has two objects and two strings (as shown in figure).



Step 2: Assume variables and directions to represent the parameters of motion, such as displacement, velocity, acceleration, etc.

The two blocks, string, and one pulley are movable in the system. Let us take variables v_1 and v_2 as velocities of block 1 and 2, respectively, and v_p as the velocity of the pulley.



Step 3: Identify a string, divide it into different linear sections, and write the constraint equation.

Here, the upper string is divided into intercepts l_1, l_2, and l_3 and the lower string is of length l_4 as shown in figure.
As the total length of any inextensible string always remains constant,

$l_1 + l_2 + l_3 + l_4 + l_5 + ... = l\left(\text{Constant}\right)$

For string 1

$l_1 + l_2 + l_3 = \text{Constant}$

For string 2

$l_4 = \text{Constant}$

Step 4: Differentiate with respect to time

$l_1 + l_2 + ... = l\left(\text{Constant}\right)$

$$\frac{dl_1}{dt} + \frac{dl_2}{dt} + \frac{dl_3}{dt} = 0$$

Or

$\dot{l}_1 + \dot{l}_2 + \dot{l}_3 = 0$

$\frac{dl_1}{dt}$ = rate of increment of intercept 1

Instead of writing the differentiation symbol every time, we can simply write it as,

$$\frac{dl_1}{dt} = \dot{l}_1$$

A single dot specifies the differentiation of quantity with respect to time and two dots specify the double differentiation of quantity with respect to time.

Sign convention is followed according to the increase or decrease of length as:
Positive sign ⇒ increase in length with time
Negative sign ⇒ decrease in length with time

From the figure, we can see that length l_1 decreases with time at a rate equal to velocity of block 1 (v_1), so it is taken as negative. l_2 and l_3 increase, at a rate equal to velocity of pulley (v_p) hence, they are taken as positive.

Step 5: Write differentials in terms of velocities and solve the equations obtained.

$\dot{l}_1 + \dot{l}_2 + \dot{l}_3 = 0$

$\Rightarrow \left(-v_1\right) + v_p + v_p = 0$

$\Rightarrow v_1 = 2v_p$ ....(i)

Velocity of intercepts l_2 and l_3 is equal to the velocity of the movable pulley.

Step 6: Repeat all the steps for each string.

For string with intercept l_4,

It is the total length of the second string and hence is a constant.

$\dot{l}_4 = 0$

$\Rightarrow v_2 - v_p = 0$

$\Rightarrow v_2 = v_p$ ....(ii)

From (i) and (ii),

$v_1 = 2v_2$

Similarly,

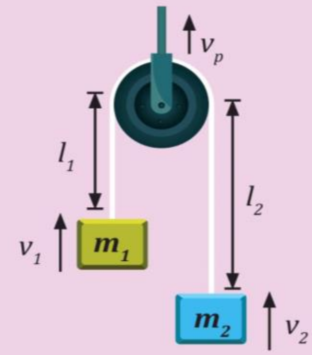
$a_1 = 2a_2$

and

$x_1 = 2x_2$

MAIN

Velocity and Acceleration of Moving Pulley



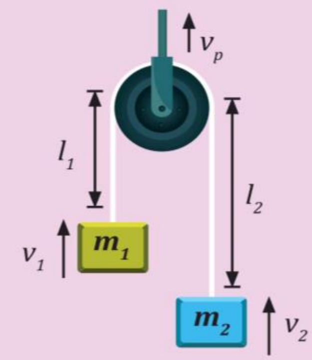
We can also get the constrained relation for moving pulley alternatively using the following vector equations.

$$\vec{v}_p = \frac{\vec{v}_1 + \vec{v}_2}{2}$$

$$\vec{a}_p = \frac{\vec{a}_1 + \vec{a}_2}{2}$$

The above equation is valid for all the cases irrespective of the direction of velocity. To prove this let us consider all the possible cases of the movable pulley

Case 1: Both masses and pulley are going upward



Step 1: Identify all the objects and number of strings in the problem. Here, two objects are present and the pulley is moving up.

Step 2: Assume variables and directions to represent the parameters of motion, such as displacement, velocity, acceleration, etc.

Here, the pulley is moving up.

Step 3: Identify a string, divide it into different linear sections, and write the constraint equation.

$l_1 + l_2 =$ Constant

Step 4: Differentiate with respect to time.

$$l_1 + l_2 = l(\text{Constant})$$

$$\frac{dl_1}{dt} + \frac{dl_2}{dt} = 0$$

Or

$$\dot{l}_1 + \dot{l}_2 = 0$$

Step 5: Write differentials in terms of velocities and solve the equations obtained.

$$\dot{l}_1 + \dot{l}_2 = 0$$

$$\Rightarrow (v_p - v_1) + (v_p - v_2) = 0$$

$$\Rightarrow 2v_p - v_1 - v_2 = 0$$

$$\Rightarrow 2v_p = v_1 + v_2$$

$$\Rightarrow v_p = \frac{v_1 + v_2}{2}$$

Alternative method

We have the vectorial formula,

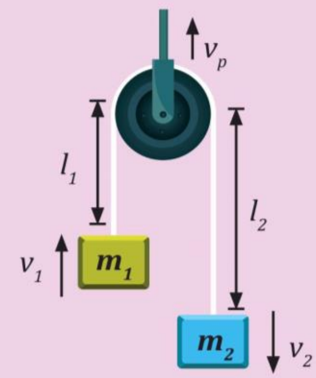
$$\vec{v}_p = \frac{\vec{v}_1 + \vec{v}_2}{2}$$

Choosing upward direction as positive, we get

$$v_p = \frac{v_1 + v_2}{2}$$

Case 2: Mass m_1 is going upward and m_2 is going downward

Here,



$$\dot{l}_1 + \dot{l}_2 = 0$$

$$\Rightarrow +\left(v_p - v_1\right) + \left(v_p + v_2\right) = 0$$

$$\Rightarrow 2v_p - v_1 + v_2 = 0$$

$$\Rightarrow 2v_p = v_1 - v_2$$

$$\Rightarrow v_p = \frac{v_1 - v_2}{2}$$

Alternative method

We have the vectorial formula,

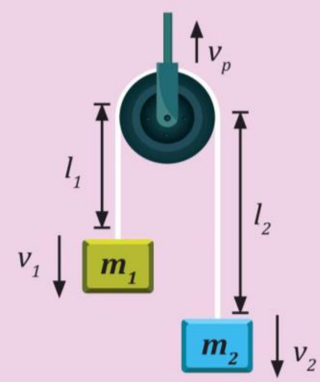
$$\vec{v}_p = \frac{\vec{v}_1 + \vec{v}_2}{2}$$

By choosing upward direction as positive we get,

$$v_p = \frac{v_1 - v_2}{2}$$

Case 3: Both masses are moving downward and pulley is moving upward

Here,



$$\dot{l}_1 + \dot{l}_2 = 0$$

$$\Rightarrow +\left(v_p + v_1\right) + \left(v_p + v_2\right) = 0$$

$$\Rightarrow 2v_p + v_1 + v_2 = 0$$

$$\Rightarrow 2v_p = -v_1 - v_2$$

$$\Rightarrow v_p = \frac{-v_1 - v_2}{2}$$

Alternative method

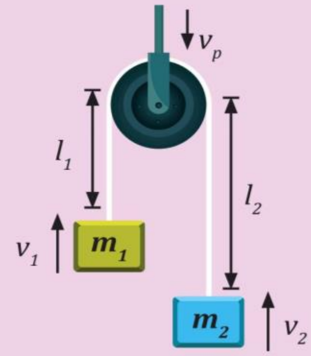
We have the vectorial formula,

$$\vec{v}_p = \frac{\vec{v}_1 + \vec{v}_2}{2}$$

By choosing upward direction as positive we get,

$$v_p = \frac{-v_1 - v_2}{2}$$

Case 4: Both masses are moving upward and pulley is moving downward



Here,

$\dot{l}_1 + \dot{l}_2 = 0$

$\Rightarrow -\left(v_p + v_1\right) - \left(v_p + v_2\right) = 0$

$\Rightarrow -2v_p - v_1 - v_2 = 0$

$\Rightarrow -2v_p = v_1 + v_2$

$\Rightarrow -v_p = \dfrac{v_1 + v_2}{2}$

Alternative method

We have the vectorial formula,

$$\vec{v}_p = \frac{\vec{v}_1 + \vec{v}_2}{2}$$

By choosing upward direction as positive we get,

$$-v_p = \frac{v_1 + v_2}{2}$$

NEWTON'S LAWS OF MOTION

WEDGE CONSTRAINTS

What you already now

- Constrained motion
- String constraint
- Pulley-Block system
- String/Rod constrained motion

What you will learn

- What are wedge constraints
- Wedge constrained motion
- Constraint relation
- Problem solving using constraint relation

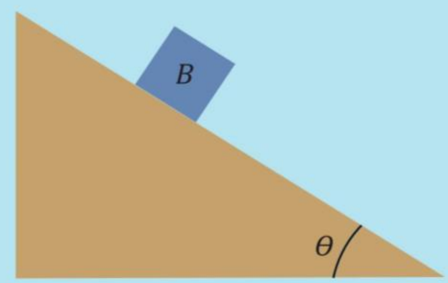
Types of constrained motion

- String/Rod constrained motion
- Wedge constrained motion

MAIN BOARDS

Wedge Constraints

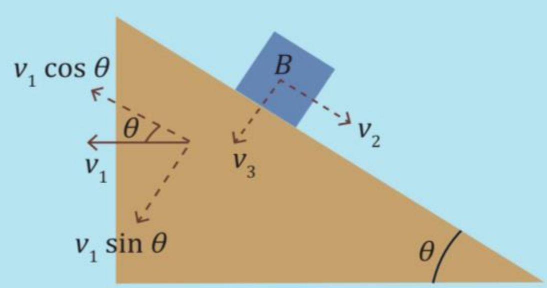
Wedge constrained motion: When a body moves over a wedge, it follows certain sliding constraints known as wedge constraints. Wedge constraint is the component of velocity and acceleration perpendicular to the contact surface (interface) of two objects. It is always equal if there is no deformation and the objects remain in contact.



Conditions of wedge constrained motion are as follows:

1. Objects are rigid.
2. The block and the wedge are always in contact.

Let v_1 be the velocity of wedge and block having the velocity v_2 and v_3 along the perpendicular direction of the interface as shown in the figure.

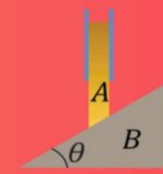


Take the component of velocity perpendicular to the interface (contact surface).

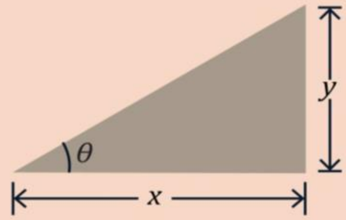
When two bodies are in contact, their velocity components perpendicular to the interface are equal.

Hence, $v_3 = v_1 \sin\theta$ (Because velocity of separation perpendicular to the interface is zero.)

A rod of mass $2m$ moves vertically downward on the surface of the wedge of mass m as shown in the figure. Find the relation between velocity of the rod and that of the wedge at any instant.



Solution



As shown in the figure, rod A can only move in the upward and downward direction, while the wedge is free to move in the horizontal direction.
Let v_A and v_B be the velocity of rod A and wedge B respectively.
Let x and y be the displacement of the wedge and rod, respectively.

$y = x\tan\theta$

Differentiating on both side with respect to time (t),

$$\frac{dy}{dt} = \frac{dx}{dt}\tan\theta$$

We know that,

$$\frac{dy}{dt} = v_B$$

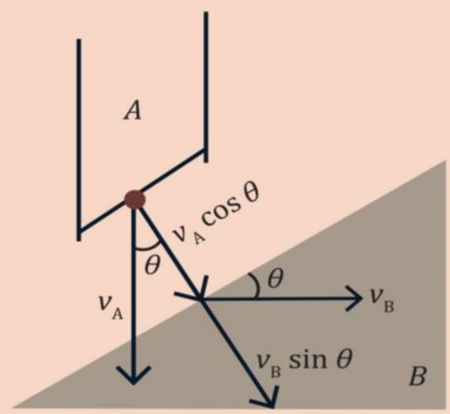
$$\frac{dx}{dt} = v_A$$

Therefore,

$$v_A = v_B\tan\theta$$

Alternative way:

We know that,
For wedge constraint, velocity along the normal to the surface is equal for the wedge and the rod
Thus, exaggerating the distance between wedge and rod, and drawing velocity components.



In normal direction,

$$v_A\cos\theta = v_B\sin\theta$$

$$\Rightarrow v_A = v_B\tan\theta$$

Find the value of velocity of block A in terms of v_B for the situation shown in the figure.

Solution

Let v_A be the velocity of block A.
As the block B moves right, block A moves downwards simultaneously.

Since, both of the blocks remain in contact with each other, the component of velocity along the perpendicular to the contact surface will be equal.
From the FBD of the blocks A and B,

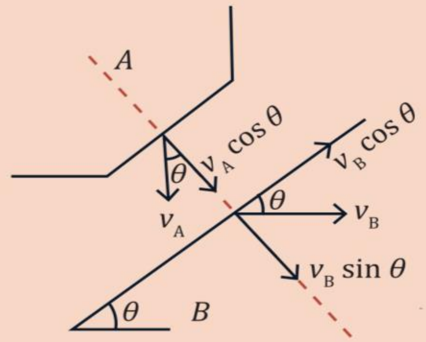
$v_A \cos\theta = v_B \sin\theta$

$v_A = v_B \tan\theta$

Put $\theta = 37^\circ$

$v_A = v_B \tan 37^\circ$

$v_A = \frac{3}{4} v_B$

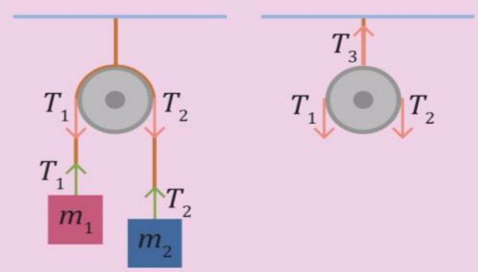


BOARDS

Ideal Pulley

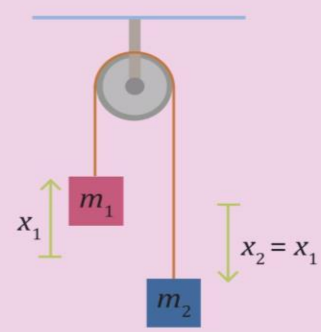
An ideal pulley is light (massless) and smooth (frictionless).

Assume the string is also light and inextensible. Due to the light (massless) string, tension in the string transmits throughout the length.
Since the pulley is frictionless, $T_1 = T_2 = T$
Since the pulley is massless, $T_3 = 2T$



There are two important things about the massless body.
1. Net force on any massless body is always zero.
2. A massless body can have finite acceleration.

Let the mass m_1 moves x_1 distance in upward direction and mass m_2 moves x_2 distance in downward direction.
String is inextensible.
So, $x_1 = x_2$ (Displacement).....(*i*)

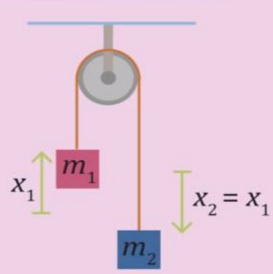


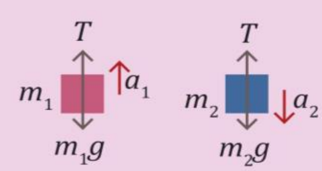
Differentiating (*i*) on both sides w.r.t time,
$v_1 = v_2$ (Velocity)......(*ii*)
Differentiating (*ii*) on both sides w.r.t time,
$a_1 = a_2$ (Acceleration)
So, there are three constraint relations.
$x_1 = x_2$
$v_1 = v_2$
$a_1 = a_2$

Steps to find out the acceleration in terms of known constraints

Step 1:

Assume the direction of accelerations and draw FBD for all masses. If required, resolve the forces along the axes.
In the figure, let the mass m_2 moves downward with acceleration a_2 and mass m_1 moves upward with acceleration a_1.





Step 2:

Write the equations using Newton's second law along each axis.

$T - m_1 g = m_1 a_1$

$m_2 g - T = m_2 a_2$

Step 3:

Write constraint equation(s).

$a_1 = a_2$

Step 4:

Solve the obtained equation(s).

Let, $a_1 = a_2 = a$

We get,

$T - m_1 g = m_1 a$ ....(*i*)

$m_2 g - T = m_2 a$ ....(*ii*)

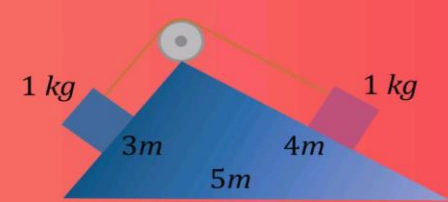
By solving equation (*i*) and (*ii*),

$$a = \left[\frac{m_2 - m_1}{m_2 + m_1}\right] g$$

Substituting *a* in (*i*),

$$T = \left[\frac{2m_1 m_2}{m_2 + m_1}\right] g$$

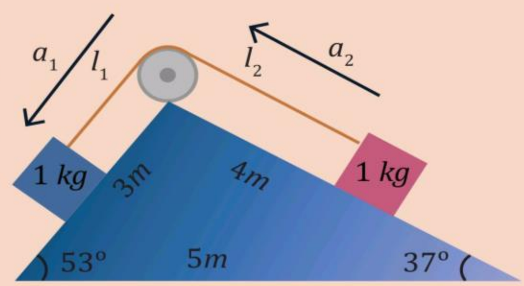
Consider the situation shown in the figure. All the surfaces are frictionless. The string and pulley are light. Find the magnitude of the acceleration of the two blocks.



Solution

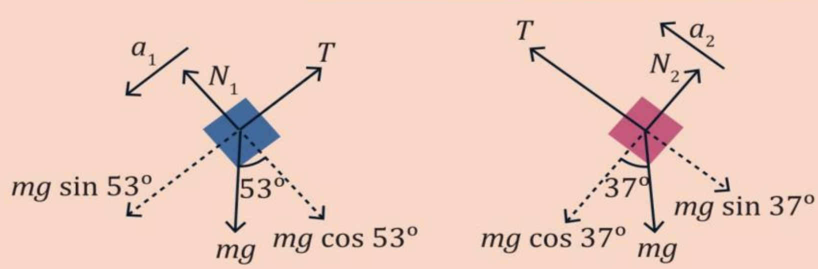
Physicality of the system explains that the mass on the left side moves downward due to steeper slope. Assume the direction of acceleration as shown.

Let the acceleration of blue block be a_1 and pink block be a_2 as shown in the figure.



FBD of the blocks is as follows:

Using Newton's second law of motion, we get,



$mg \sin 53° - T = ma_1$

$T - mg \sin 37° = ma_2$

String constraint relation

$l_1 + l_2 =$ constant

Double differentiating w.r.t. time t,

$\ddot{l}_1 + \ddot{l}_2 = 0$

$a_1 - a_2 = 0$

Here, l_1 is increasing and l_2 is decreasing.

$a_1 = a_2$

say $a_1 = a_2 = a$

We get,

$mg \sin 53° - T = ma$ (Given *m = 1 kg*)

$T - mg \sin 37° = ma$

$g \sin 53° - T = a$ ....(*i*)

$T - g \sin 37° = a$ ....(*ii*)

$\left(\sin 53° = \frac{4}{5}, \sin 37° = \frac{3}{5}\right)$

By solving equation (*i*) and (*ii*),

$a = \frac{g}{10} ms^{-2}$

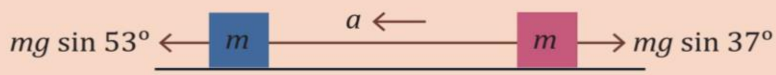
Alternative way:

Consider the whole string and masses as a system.
FBD for components of forces along the length of string is as follows:

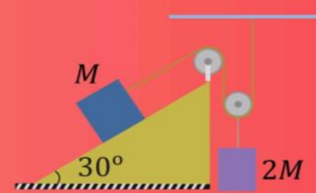
$$mg\sin 53^\circ - mg\sin 37^\circ = (m+m)a$$

$$mg\left(\frac{4}{5}-\frac{3}{5}\right) = 2ma$$

$$\Rightarrow a = \frac{g}{10}\ ms^{-2}$$



Find the acceleration of the block of mass M in the situation shown in the figure. All the surfaces are frictionless. The pulleys and string are light.



Solution

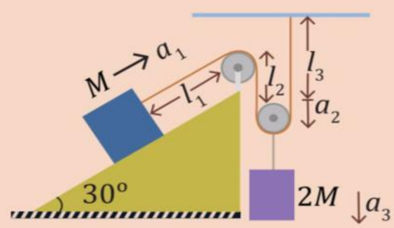
Physicality of the system explains that the $2M$ is heavier. Hence, it will move down.

Let a_1, a_2 and a_3 be the accelerations of the block of mass M, movable pulley, an d block of mass $2M$, respectively.

The movable pulley and the block of mass $2M$ are attached to the same string. So, they have the same acceleration.

$a_2 = a_3$ .....(*i*)

Divide the length of the string into three intercepts as shown in the figure.



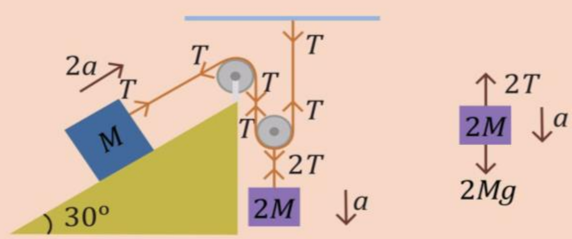
We know, from string constraints, the length of the string remains constant.

Thus,

$$l_1 + l_2 + l_3 = \text{constant}$$

After double differentiating the equation, we get,

$$\ddot{l}_1 + \ddot{l}_2 + \ddot{l}_3 = 0 \quad ....(ii)$$



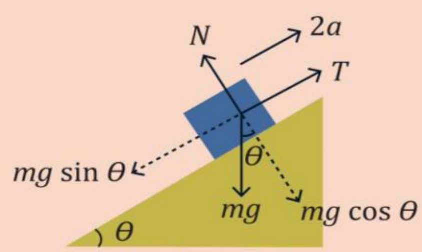
As the block of mass $2M$ moves downward, the length of l_1 decreases, and the lengths of l_2 and l_3 increase. So, length can be written in the form of acceleration from the equation (*ii*).

$$-a_1 + a_2 + a_3 = 0$$

But, $a_2 = a_3$

$a_1 = 2a_2$

Say $a_2 = a$, $a_1 = 2a$



Using Newton's second law of motion, write equations from the FBD.

$T - Mg \sin 30° = M(2a)$ ....(*iii*)

$2Mg - 2T = 2M(a)$ ....(*iv*) Or $Mg - T = M(a)$ ....(*v*)

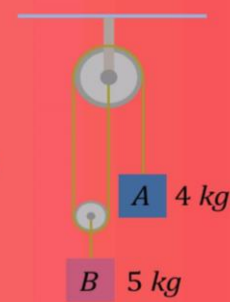
Solving equation (*iii*) and (*v*), we get,

$a = \frac{g}{6} ms^{-2}$

Acceleration of the block of mass M on the inclined plane is $2a$.

So, acceleration of the block of mass M on the inclined plane becomes $\frac{g}{3} ms^{-2}$

In the system shown in the figure, find the acceleration of blocks A and B.



Solution

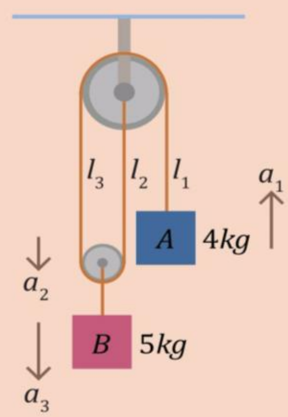
Physicality of the system explains that 5 kg is heavier. Hence, it will move down.

Let a_1, a_2 and a_3 be the acceleration of the block A of mass 4 kg, movable pulley, and block B of mass 5 kg, respectively.

The movable pulley and the block B of mass 5 kg are attached to the same string. So, they have the same acceleration.

$a_2 = a_3$ ....(*i*)

Divide the length of the string into three intercepts as shown in the figure.



We know, from string constraints, the length of the string remains constant.

$l_D + l\ + l\ =$ constant

After double differentiating the equation, we get,

$\ddot{l}_1 + \ddot{l}_2 + \ddot{l}_3 = 0$ ....(*ii*)

As the block A of mass 4 kg moves upward, the length l_1 decreases and the lengths l_2 and l_3 increases. So, double differentiation of length can be written in the form of acceleration from the equation (*ii*).

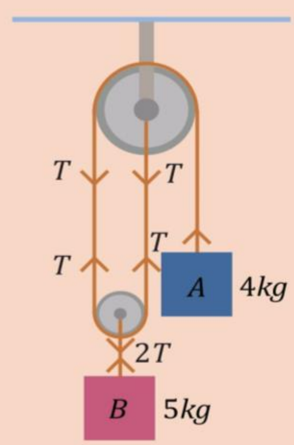
$-a_1 + a_2 + a_3 = 0$

But, $a_2 = a_3$

$a_1 = 2a_2$

Say $a_2 = a$, $a_1 = 2a$

Using Newton's second law of motion, write equations from the FBD.



FBD of block A and block B

$5g - 2T = 5a$ ....(iii)

$T - 4g = 4(2a)$ ....(iv)

Multiplying equation (iv) by 2,
We get,

$5g - 2T = 5a$ ....(v)

$2T - 8g = 16a$ ....(vi)

By solving equations (v) and (vi),
We get,

$a = \frac{-g}{7}\, ms^{-2}$

Negative sign indicates that the direction of acceleration is opposite to the direction assumed in the question.
So,
Acceleration of the block of mass 5 *kg* is $a_3 = a = \frac{g}{7} ms^{-2}$ in upward direction.

Acceleration of the block of mass 4 *kg* is $a_1 = 2a = \frac{2g}{7} ms^{-2}$ in downward direction.

NEWTON'S LAWS OF MOTION

MASTERING CONSTRAINED MOTION

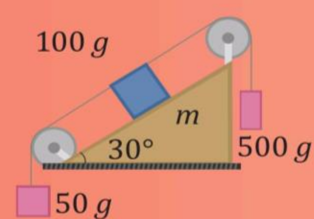
What you already know

- Constrained motion
- String and rod constraints motion
- Wedge constraints

What you will learn

- Problems on wedge constraints

Find the acceleration of the 500 g block in figure.



Solution

Step 1: Check the physicality of the problem and assume the variables.

Block with a mass 500 g is heavy. Thus, it will move down and the block with mass 50 g moves up.

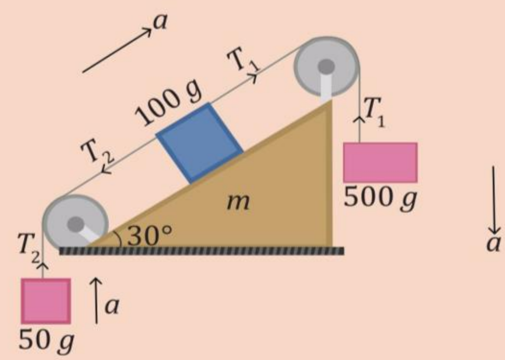
Step 2: Write the constraint relation,

Here, the magnitude of acceleration for all the blocks is the same. Let it be a.

Thus,

$a_1 = a_2 = a_3 = a$

Draw the FBD for the given system



As there are two different strings, tension in them will also be different.

To make calculations easy, let's assume masses.

$50\,g = m$

$100\,g = 2m$

$500\,g = 10m$

Now, let us draw the FBD for each masses, From FBD of mass $10m$,

$10mg - T_1 = (10m)a$ ....(i)

From FBD of mass $2m$,

$T_1 - T_2 - 2mg \sin 30° = (2m)a$ ....(ii)

From FBD of mass m,

$T_2 - mg = (m)a$ ....(iii)

Adding three equations simultaneously,

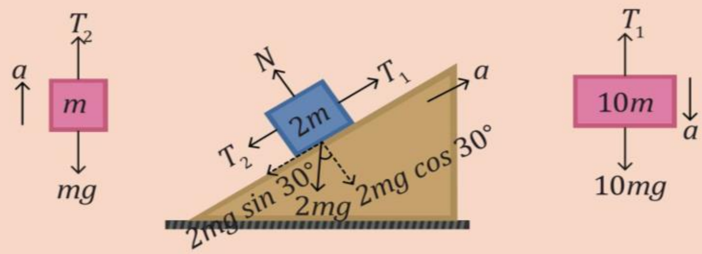
$10mg - mg - mg = 13ma$

or

$8mg = 13ma$

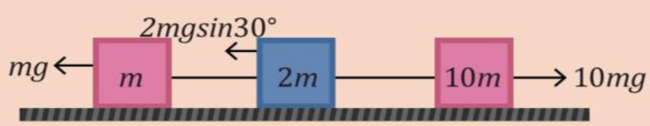
or

$a = \frac{8g}{13}$



Alternative way:

Consider the system together, as shown,
All the tensions in the strings become internal forces and hence no need to consider.



We know that,

$$\sum \vec{F}_{sys} = m_{sys}\vec{a}$$

$$\Rightarrow 10mg - 2mg \sin 30° - mg = (m + 2m + 10m)a$$

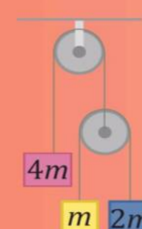
$$\Rightarrow 8mg = 13ma$$

$$\Rightarrow a = \frac{8g}{13}$$

For the given system in the figure, find the following:

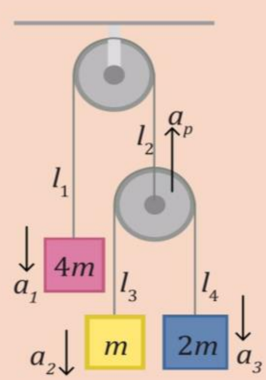
(a) Acceleration of masses m, $2m$, and $4m$.

(b) Tension in the strings. (Take $g = 10\ ms^{-2}$)



MAIN

Solution



Assume all masses going down with acceleration as shown in figure and pulley move up with acceleration a_p.

Break the string into intercepts as shown, Solving the string constraints,

$$\ddot{l}_1 + \ddot{l}_2 = 0$$
$$+a_1 - a_p = 0$$
$$\Rightarrow a_p = a_1 \quad ....(i)$$

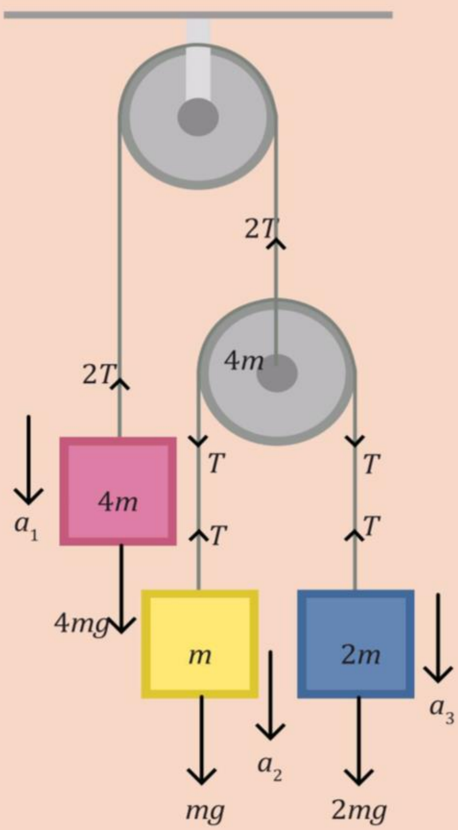
Similarly for right side string,

$$\ddot{l}_3 + \ddot{l}_4 = 0$$
$$+(a_p + a_2) + (a_p + a_3) = 0$$
$$\Rightarrow 2a_p + a_2 + a_3 = 0 \quad ....(ii)$$

From (i) and (ii)

$$2a_1 + a_2 + a_3 = 0 \quad ....(iii)$$

Here, as three terms on one side equals to zero, one or more acceleration would be negative, i.e., opposite to the direction assumed.
Let's draw the FBD for the given bodies:



From the force balance on mass $4m$,
$4mg - 2T = (4m)a_1$
or

$$g - \frac{2T}{4m} = a_1 \quad ....(iv)$$

From the force balance on mass m,
$mg - T = (m)a_2$
or

$$g - \frac{T}{m} = a_2 \quad ....(v)$$

From the force balance on mass $2m$,
$2mg - T = (2m)a_3$
or

$$g - \frac{T}{2m} = a_3 \quad ....(vi)$$

Putting (iv), (v), and (vi) in (iii),

$$2\left(g - \frac{T}{2m}\right) + \left(g - \frac{T}{m}\right) + \left(g - \frac{T}{2m}\right) = 0$$

$$\Rightarrow 4g = \frac{T}{m} + \frac{T}{m} + \frac{T}{2m} = \frac{5T}{2m}$$

$$\Rightarrow T = \frac{8mg}{5}$$

Now,
Put this value of tension in acceleration of three masses,

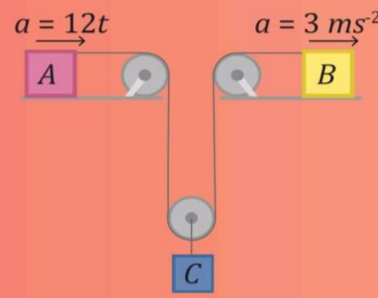
$$a_1 = g - \frac{T}{2m} = g - \frac{1}{2m}\left(\frac{8mg}{5}\right) = \frac{g}{5}$$

$$a_2 = g - \frac{T}{m} = g - \frac{1}{m}\left(\frac{8mg}{5}\right) = \frac{-3g}{5}$$

$$a_3 = g - \frac{T}{2m} = g - \frac{1}{2m}\left(\frac{8mg}{5}\right) = \frac{g}{5}$$

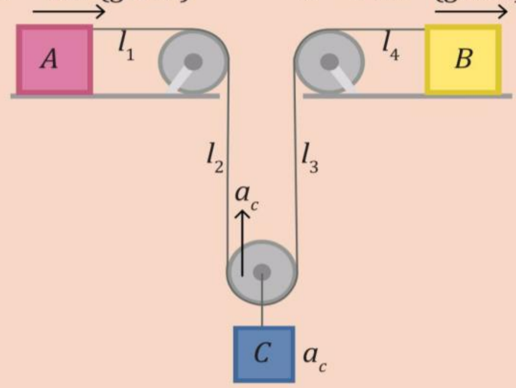
Here, as a_2 is negative, acceleration of mass m is upwards.

In the device shown, block A is given acceleration $12t$ and block B is given acceleration 3 ms^{-2}, both towards right. Where t is time in seconds. Both the blocks start from rest. Find the time when block C comes to rest.



Solution

Understand the physicality of the problem. Assign acceleration and string intercept variables as shown,



Constraint equation is,

$$\ddot{l}_1 + \ddot{l}_2 + \ddot{l}_3 + \ddot{l}_4 = 0$$

$$-a_A - a_C - a_C + a_B = 0$$

$$\Rightarrow 2a_C = a_B - a_A$$

$$\Rightarrow a_C = \frac{3-12t}{2}$$

or

$$a_C = \frac{3(1-4t)}{2} \text{.......(i)}$$

When C comes to rest, its velocity will be zero.
We have,

$$a_C = \frac{3}{2}(1-4t) = \frac{dv}{dt}$$

$$\Rightarrow dv = \frac{3}{2}(1-4t)dt$$

Integrating on both sides,
Initially, when $t = 0$, velocity of C was zero. We need to find time t at which velocity will become zero again.
Thus, using limits,

$$\int_0^0 dv = \int_0^t \frac{3}{2}(1-4t)dt$$

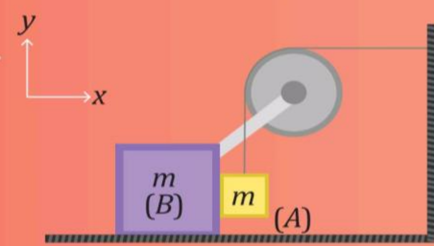
$$\Rightarrow v_0^0 = \frac{3}{2}\left[t - 2t^2\right]_0^t$$

$$\Rightarrow 0 = \frac{3}{2}\left[t - 2t^2\right]$$

Solving the quadratic equation,
$t = 0$ or $t = 0.5$
Thus, velocity of block C will be zero at initial time and at $t = 0.5\ s$

In the system shown in figure, block A is released from rest. Find the following:

(a) The acceleration of both the blocks A and B.
(b) Tension in the string.
(c) Contact force between A and B



Solution

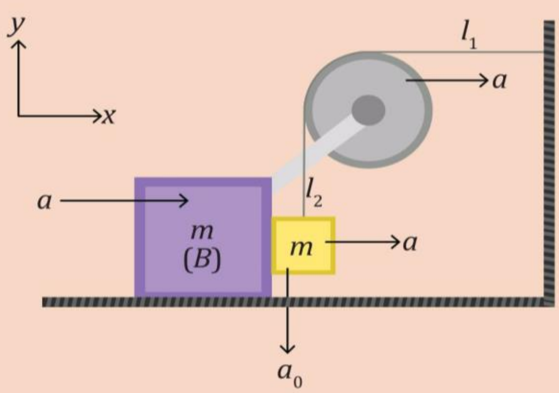
Here, block B will move in the right direction, when block A moves downwards due to self weight. While block B moves rightwards, it exerts a contact force on block A and as a result takes block A along with it. So block A has two accelerations one along downward direction and one along the rightward direction.
Understand the physicality and assume the variables for acceleration.

Constraint equation is,

$\ddot{l}_1 + \ddot{l}_2 = 0$

$-a + a_0 = 0$

$\Rightarrow a = a_0$



From FBD of block B in the horizontal direction,
$T - N = m(a)$ ....*(i)*

For block A,
From FBD of block A in the horizontal direction,
$N = ma$ ....*(ii)*

In vertical direction,
$mg - T = m(a)$........*(iii)*

Adding *(i)* and *(ii)*,
$T = 2ma$

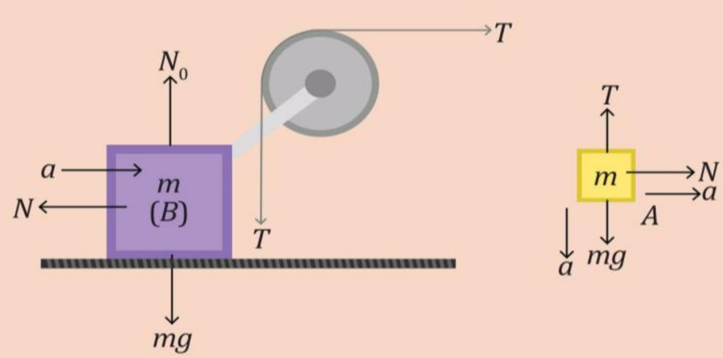
Substitute this in *(iii)*

$mg = 3ma$

$\Rightarrow a = \dfrac{g}{3}$

Putting in *(ii)*,

$N = \dfrac{mg}{3}$



Put value of N and a in *(i)*,

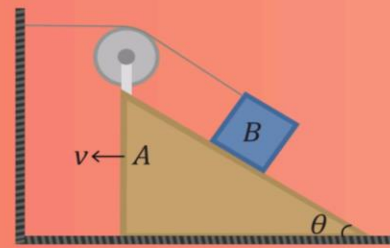
$T = m\left(\dfrac{g}{3}\right) + \dfrac{mg}{3}$

$\Rightarrow T = \dfrac{2mg}{3}$

For the given system, pulley and string are ideal, and friction is absent.

(a) Relate the speed of the wedge and the block, if the wedge moves with speed v towards the left.

(b) Find the acceleration of the wedge.

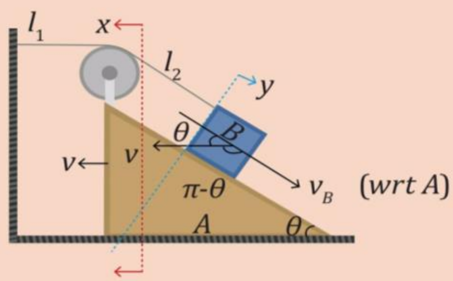


Solution

(a) In the ground frame, A moves to the left and B slides down on A and at the same time, B goes left with A. Thus, total velocity will be resultant of these two velocities.

Let v_B be the velocity of block B with respect to the wedge and v be the velocity of wedge with respect to ground.

Plot the direction of velocity of B and A as shown,



Solving the constraint equation,

$$\dot{l}_1 + \dot{l}_2 = 0$$

$$-v + v_B = 0$$

$$\Rightarrow v = v_B$$

Solving the constraint equation,

The resultant velocity will be,

$$(v_{res})_B = \sqrt{v^2 + v^2 + 2v^2 \cos(\pi - \theta)}$$

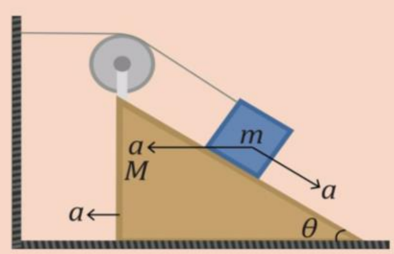
$$= \sqrt{2v^2(1 - \cos\theta)}$$

$$= \sqrt{2v^2 \times 2\sin^2\left(\frac{\theta}{2}\right)}$$

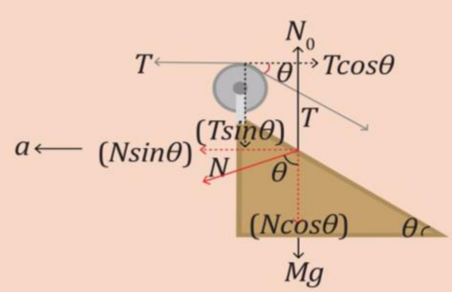
$$\Rightarrow (v_{res})_B = 2v \sin\left(\frac{\theta}{2}\right)$$

(b) Similar constraint diagram can be drawn for acceleration as that of velocity.

As the wedge moves in the left direction, the block will move downwards along the inclined direction. As the length of string is constant we can use length constraint to relate acceleration of wedge and the block. Magnitude of acceleration of the wedge and the acceleration of the block along the inclined direction is the same. Let this acceleration be a.



FBD for wedge is as follows:



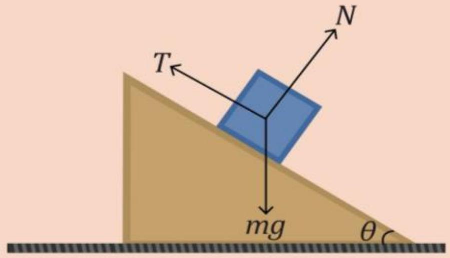
Here, self-weight is acting downward and the ground reaction is opposite to it. Tension is acting along the two directions. There is no motion of wedge along vertical direction.

From FBD of the wedge in the horizontal direction,

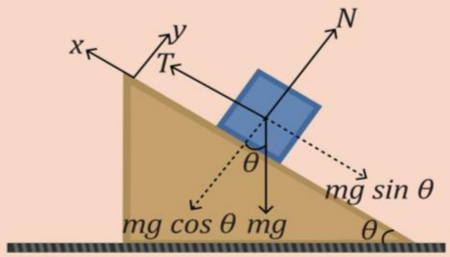
$T + N\sin\theta - T\cos\theta = Ma$

$T(1-\cos\theta) + N\sin\theta = Ma......(i)$

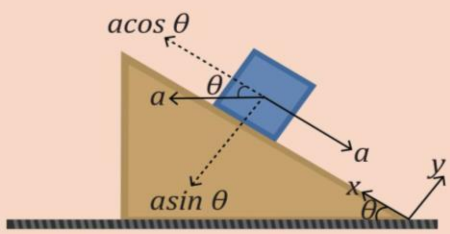
FBD of mass *m*:
Three forces acting on the block are the weight, normal reaction from wedge and tension from string.



Take the coordinate axis system as shown in figure. Resolving the components of weight of block *m* along *x* and *y* axis.



Resolve acceleration also, as shown in figure



Along *x*-direction,

$T - mg\sin\theta = m(a\cos\theta - a)$

$\Rightarrow T = mg\sin\theta - ma(1-\cos\theta)........(ii)$

Along *y*-direction,

$N - mg\cos\theta = -ma\sin\theta$

$\Rightarrow N = mg\cos\theta - ma\sin\theta.......(iii)$

Putting value of *T* and *N* from equation *(ii)* and *(iii)* respectively in equation *(i)*,

$mg\sin\theta(1-\cos\theta) - ma(1-\cos\theta)^2 + mg\cos\theta\sin\theta - ma\sin^2\theta = Ma$

$\Rightarrow mg\sin\theta - mg\sin\theta\cos\theta - ma(1+\cos^2\theta - 2\cos\theta) + mg\cos\theta\sin\theta - ma\sin^2\theta = Ma$

$\Rightarrow mg\sin\theta - ma - [\cos^2\theta + \sin^2\theta]ma + 2ma\cos\theta = Ma$

$\Rightarrow mg\sin\theta - ma - ma + 2ma\cos\theta = Ma$

$\Rightarrow mg\sin\theta = Ma + 2ma(1-\cos\theta)$

Thus,

$$a = \frac{mg\sin\theta}{[M + 2m(1-\cos\theta)]}$$

NEWTON'S LAWS OF MOTION

PSEUDO FORCE

What you already know

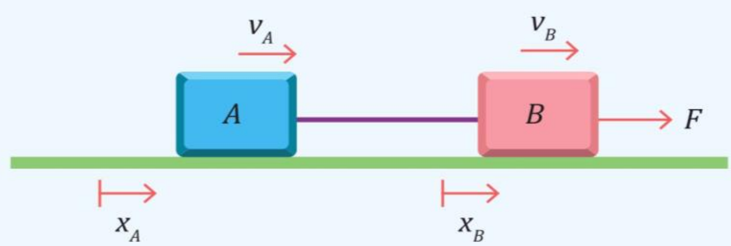
- Constrained motion
- Pulley-Block system
- String/Rod constrained motion
- Wedge constrained motion

What you will learn

- Newton's laws on a system
- Pseudo force
- Non-inertial frames

Newton's Laws on a System

Since the string is inextensible, we have constrained equation for displacement, velocity and acceleration as:



$$x_A = x_B$$

$$\Rightarrow \frac{dx_A}{dt} = \frac{dx_B}{dt} \Rightarrow v_A = v_B$$

$$\Rightarrow \frac{dv_A}{dt} = \frac{dv_B}{dt} \Rightarrow a_A = a_B$$

Considering all bodies as a system:

$$\left(\sum \vec{F}_{ext}\right)_{sys,x} = \left(m_{sys}\,\vec{a}_{sys}\right)_x$$

$$\Rightarrow F = (m_A + m_B)a$$

$$\Rightarrow a = \frac{F}{m_A + m_B}$$

Why did we choose A and B together as a system?
Because they have the same acceleration. Hence, it is advantageous to consider them as the same system.

Selection of system

Can we consider objects with different acceleration as a system?

Consider $m_1, m_2, m_3, \ldots$ are the masses of the objects of the system and $\vec{a}_1, \vec{a}_2, \vec{a}_3 \ldots$ are the acceleration of the objects respectively.
We can balance the forces on individual body as:

$$\left(\sum \vec{F}_{ext}\right)_1 = m_1 \vec{a}_1 \quad \text{........(i)}$$

$$\left(\sum \vec{F}_{ext}\right)_2 = m_2 \vec{a}_2 \quad \text{........(ii)}$$

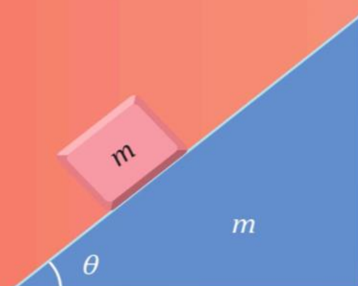
$$\left(\sum \vec{F}_{ext}\right)_3 = m_3 \vec{a}_3 \quad \text{........(iii)}$$

Adding all equations,

$$\left(\Sigma \vec{F}_{\text{ext}}\right)_{\text{sys}} = m_1 \vec{a}_1 + m_2 \vec{a}_2 + m_3 \vec{a}_3 + \cdots$$

Thus, we can consider the group of masses having different acceleration as one system.

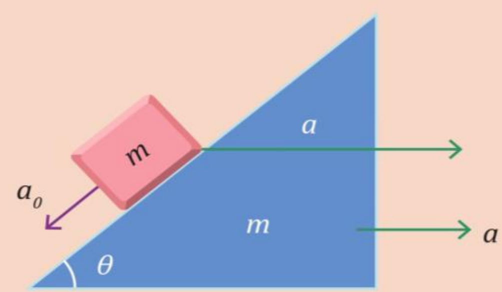
The block of mass m slides on a wedge of mass m that is free to move on the horizontal ground. Relate the accelerations of wedge and block. (All surfaces are smooth)



MAIN ADVANCED

Solution

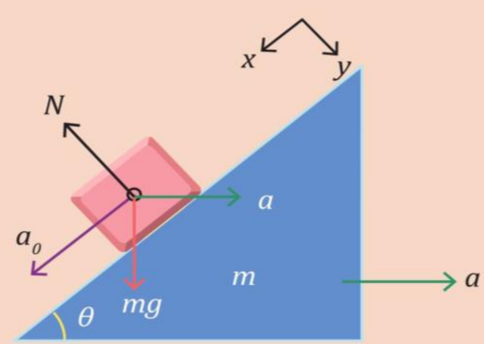
Understand the physicality. As the block slides down along the wedge surface, the wedge moves in the right direction.



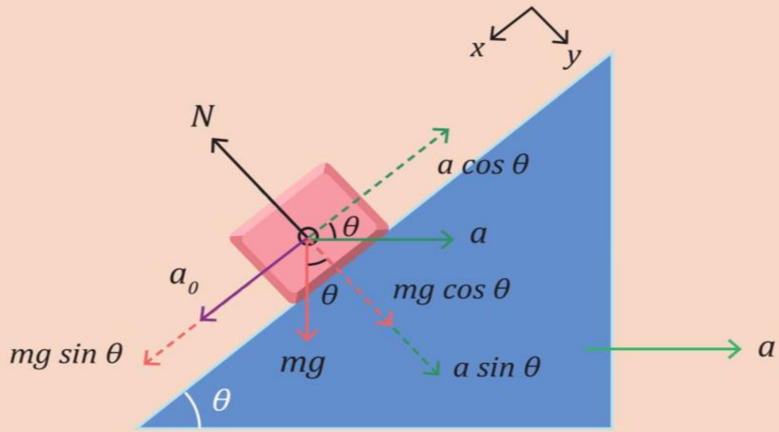
As the wedge is moving rightwards, the block will also go with it. Thus, the resultant velocity of the block will be the vector sum of these two velocities.

Let a is acceleration of the wedge with respect to the ground. And a_o be the acceleration of the block with respect to the wedge. In the ground frame, acceleration of the block is the sum of these two.

Draw FBD for the block. Forces are, weight and normal reaction on block.



Take the components of weight and acceleration in the direction of the inclined surface and normal to it.



Balancing forces in x direction,

$\Sigma \vec{F}_x = m\vec{a}_x$

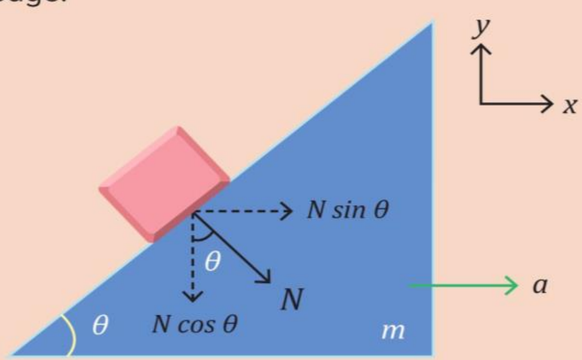
$mg \sin\theta = m(a_o - a\cos\theta)......(i)$

Balancing forces in y-direction,

$\sum \vec{F}_y = m\vec{a}_y$

$mg\cos\theta - N = m(a \sin\theta)\ ......(ii)$

Now, draw FBD for the wedge.



Balancing forces in x direction,

$\Sigma \vec{F} = m\vec{a}_x$

$N \sin\theta = m(a).......(\text{iii})$

Multiplying second equation by sinθ and add (ii) and (iii) equations,

We get,

$mg \cos\theta \sin\theta = ma(1 + \sin^2\theta)$

$g \cos\theta \sin\theta = a(1 + \sin^2\theta)\ ......(\text{iv})$

Dividing (i) by (iv),

$$\frac{mg \sin\theta}{g\cos\theta \sin\theta} = \frac{m(a_o - a\cos\theta)}{a\ 1 + \sin^2\theta}$$

Cancel out like terms and by solving,

$a\ 1+\sin^2\theta\ = a_o\cos\theta - a\cos^2\theta$

Rearranging,

$a + a\sin^2\theta + a\cos^2\theta = a_0\cos\theta$

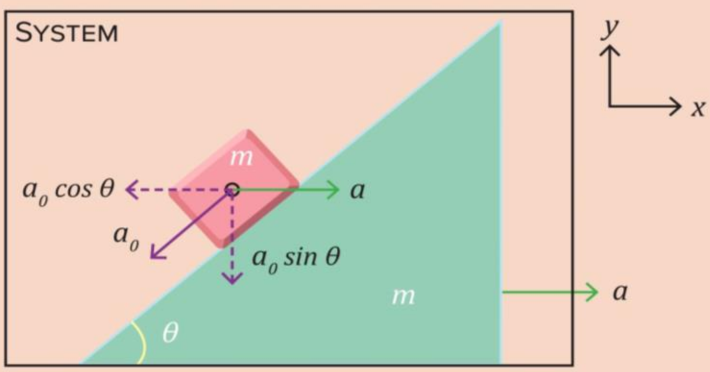
But we know that, $\sin^2\theta + \cos^2\theta = 1$

Taking a as common and using above identity,

$2a = a_o\cos\theta$

Alternate Method

Takings block and wedge together as a system,



For this system, use force balance,
As there is no external force on system in x direction,

$\Sigma\vec{F}_x = m\vec{a}_x$

$0 = ma + m(a - a_o\cos\theta)$

$\Rightarrow 2a = a_o\cos\theta$

Thus, suitable selection of the system can ease our efforts by significant times.

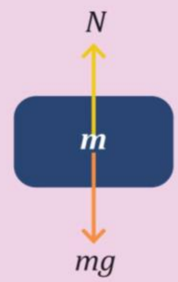
Pseudo Force

Consider a block of mass m kept on a weighing machine. Consider the system is kept in an elevator. Let observer O_1 observe this system from the ground frame. Let another observer O_2 be present inside the lift.

Case 1: Elevator is at rest

When the elevator is at rest, the reading on the weighing machine is same for both observers.

Both the observers represent system using FBD:



The forces acting on the block are weight (mg) and normal reaction (N).

Along vertical direction, $N = mg$

Case 2: Elevator is moving upwards with acceleration a

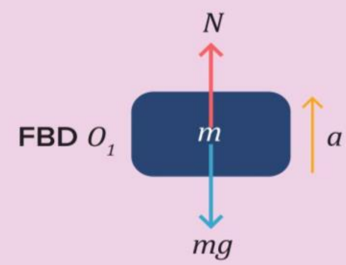
When the elevator starts moving upwards with an acceleration a, both observers observe different readings on the weighing machine than the reading when the elevator was at rest.

In order to understand why the reading is different in this case, they used concepts of physics as follows:

FBD of block for observer O_1 standing in ground frame:

This observer finds the system is moving upwards with acceleration a. Other forces acting on the block are its weight and normal reaction.

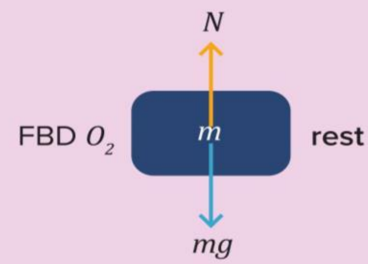
Along vertical direction,
$N - mg = ma$
$N = mg + ma$
$N = m(g + a)$
As the weighing machine measures the normal reaction acting on it, the weighing machine will show a reading corresponding to $N = m(g + a)$.



FBD of block for observer O_2 standing in lift frame:

When the observer is in the elevator, the block is at rest with respect to him. So for this case,

The forces acting on the block are weight (mg) and normal reaction (N).
Along vertical direction,
$N = mg$



However, the observer O_2 sees the reading on the weighing machine is $m(g + a)$.

And he couldn't explain his observation as his derivation was different. To find what is missing in this case, use the concept of relative motion.

Let $\vec{a}_{B,elevator}$ be the acceleration of the block with respect to elevator, $\vec{a}_{B,G}$ be the acceleration of block with respect to ground and $\vec{a}_{elevator,\,G}$ be the acceleration of the elevator with respect to ground.

$\vec{a}_{B,\,elevator} = \vec{a}_{B,G} - \vec{a}_{elevator,G}$

Multiplying both sides by m,

$m\vec{a}_{B,\,elevator} = m\vec{a}_{B,G} - m\vec{a}_{elevator,G}$

The normal reaction $\left(m\vec{a}_{B,\,elevator}\right)$ is the sum of $\left(m\vec{a}_{B,G}\right)$ and $\left(-m\vec{a}_{elevator,G}\right)$ when the elevator is accelerating.

Here, $\left(m\vec{a}_{B,G}\right)$ is the weight of block and term $\left(-m\vec{a}_{elevator,G}\right)$ appears due to the accelerated frame. This can be written in terms of force as :

$\left(\Sigma\vec{F}_P\right)_{elevator} = \left(\Sigma\vec{F}_P\right)_{Ground} + \left(-m\vec{a}_{elevator,G}\right).$

This new force term $\left(-m\vec{a}_{elevator,G}\right)$ appears due to the mass of block and acceleration of the elevator which does not make any sense physically.

MAIN

Non-Inertial Frames

The change in the net force in the ground and elevator frame is owing/attributed to the acceleration of the elevator. If the elevator was moving with uniform velocity, this term would be zero.
Thus,
It is not about movement of the elevator but acceleration of the elevator. Such frames that are accelerating are known as **non-inertial frames of reference.**

In **non-inertial frames of reference**, Newton's law seems to be failing.
Thus, Newton's law is not valid in non-inertial frames of reference.

The extra term is unreal and known as **pseudo force.**
Thus,

$$\vec{F}_{\text{pseudo}} = -m\vec{a}_{\text{elevator},G}$$

Thus for non-inertial frames, Newton's law can be modified as,

$$\left(\Sigma\vec{F}_{ext}\right)_{\text{elevator}} = \left(\Sigma\vec{F}_{ext}\right)_{\text{Ground}} + \vec{F}_{pseudo}$$

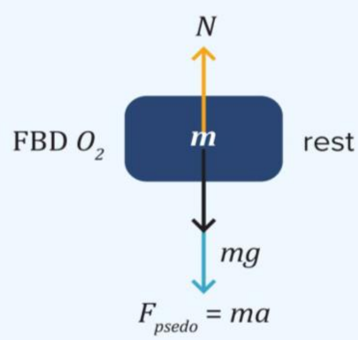
The pseudo forces are also known as **inertial forces**, although their need arises because of the use of **non-inertial frames**.

$$\vec{F}_{\text{pseudo}} = -m_{\text{sys}}\,\vec{a}_{\text{non-inertial frame}}$$

Properties of pseudo force:

- It's a fictitious force i.e., it will not have a reaction pair.
- It always acts in the direction opposite to the acceleration of the frame of reference.

Thus, the FBD for mass moving with acceleration in the elevator frame is modified as:

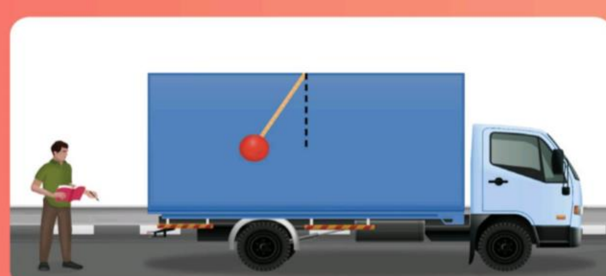


Balancing the forces in vertical direction,

$N = mg + ma$

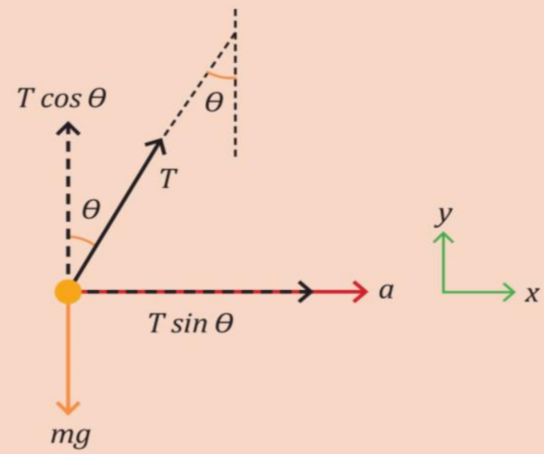
$N = m(g + a)$

Figure shows a pendulum suspended from the roof of a truck that has a constant acceleration 'a' relative to the ground. Find the deflection of the pendulum from the vertical as observed from the ground frame and from the frame attached with the truck.



Solution

FBD in ground frame:



Balancing forces,

$\Sigma \vec{F}_x = m\vec{a}_x$

$\Rightarrow T \sin\theta = ma$ ...... (i)

In y - direction,

$\Sigma \vec{F}_y = m\vec{a}_y$

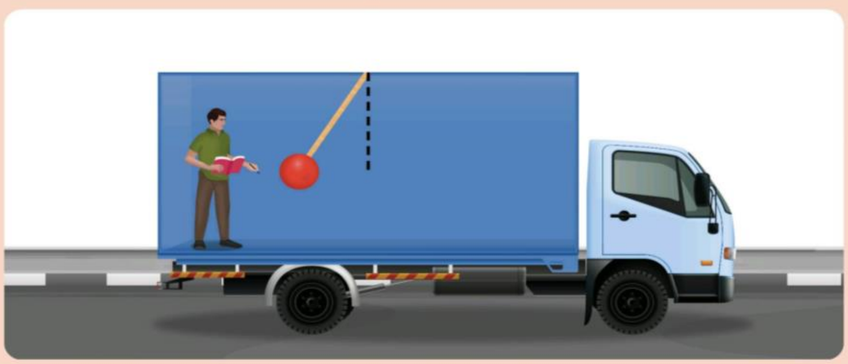
$\Rightarrow T \cos\theta = mg$ ...... (ii)

Dividing equation (i) by (ii),

$$\tan\theta = \frac{a}{g}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

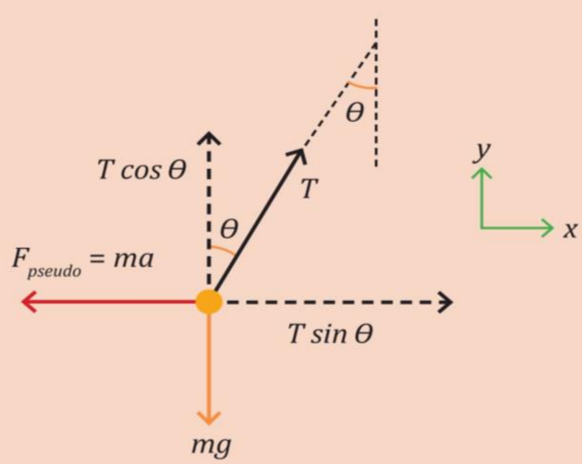
Now, if we solve the problem in truck frame, Let the observer be in truck now,



Balancing forces in horizontal direction,

$\vec{F}_{pseudo} = -m_{object}\ \vec{a}_{frame}$

Balancing forces in horizontal direction,



$\Sigma \vec{F}_x = m\vec{a}_x$

$\Rightarrow T \sin\theta = ma$ ....(i)

In y - direction,

$\Sigma \vec{F}_y = m\vec{a}_y$

$\Rightarrow T \cos\theta = mg$ ....(ii)

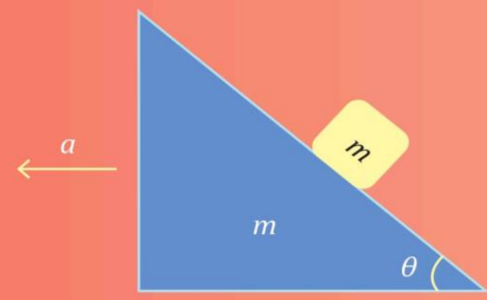
Dividing equation (i) by (ii),

$$\tan\theta = \frac{a}{g}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

So in both the frame of reference deflection θ is the same.

A block of mass m resting on a wedge of angle θ as shown in the figure. The wedge is given an acceleration 'a'. What is the minimum value of a so that the mass m falls freely?



Solution

For the block to fall freely, there should not be any normal reaction between wedge and block.
i.e., $N = 0$
Let's solve it in a non-inertial (wedge) frame.
Components shown in blue are for weight force and yellow are for pseudo force.

$$\Sigma \vec{F}_y = m\vec{a}_y$$

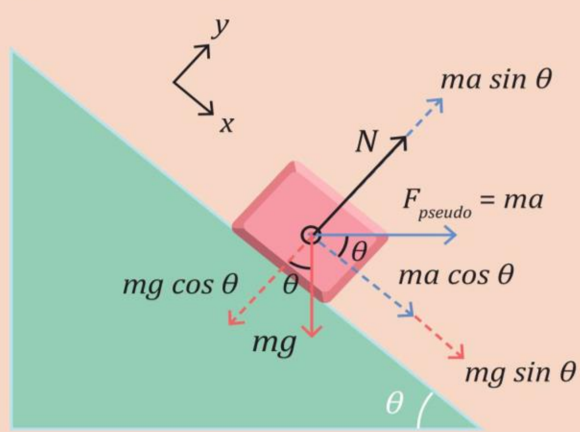
$$N + ma\sin\theta = mg\cos\theta$$

As $N = 0$,

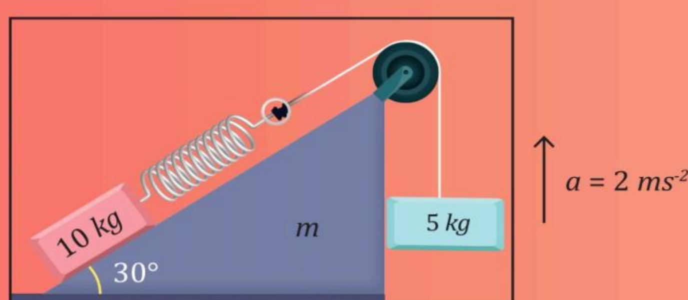
$$ma\sin\theta = mg\cos\theta$$

$$\Rightarrow a = g\frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow a = g\cot\theta$$



In the figure, the reading of the spring balanced (N) will be: (Take $g = 10\ ms^{-2}$)



Solution

Spring balance measures the force of spring. As the spring is massless here, it will measure tension in the string.

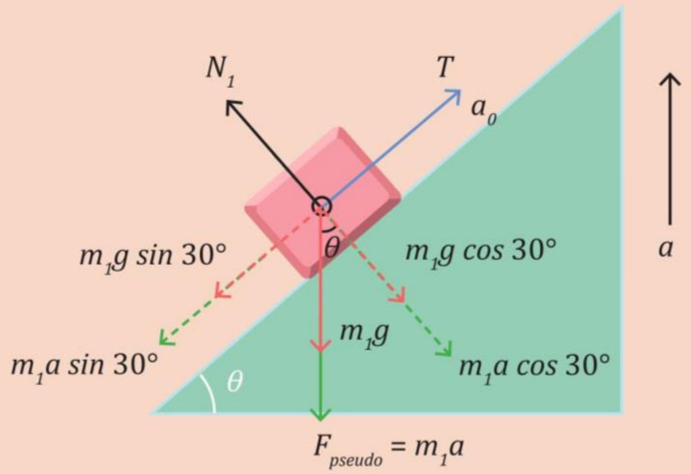
Let's solve the problem in lift frame,

Let a_o be acceleration of blocks with respect to the wedge.
When the 5 kg block goes down, the 10 kg block moves along an inclined plane with acceleration the same as a 5 kg block. (As the string is inextensible).

Tension in spring = Tension in string (As they are both ideal)

$T_{spring} = kx = T$

From the FBD of wedge and mass



Balancing force in x - direction,

$T - m_1 g \sin 30° - m_1 a \sin 30° = m_1 a_0$

$\Rightarrow T - (10)(10)\frac{1}{2} - (10)(2)\left(\frac{1}{2}\right) = 10a_0$

$T - 60 = 10a_o$ ....(i)

For $5kg$ block, along vertical direction,

$m_2 g + F_{pseudo} - T = m_2 a_o$

Where, $F_{pseudo} = m_2 a = 10N$

$\Rightarrow (50 + 10) - T = 5a_o$

$\Rightarrow 60 - T = 5a_o$ ......(ii)

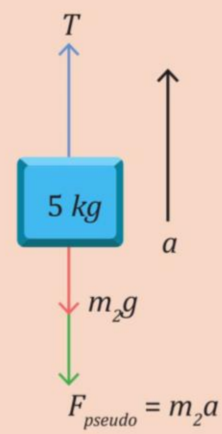
Adding equation (i) and (ii),

$0 = 15a_o$

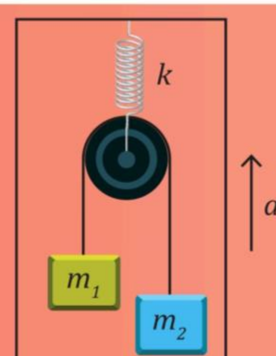
$\Rightarrow a_o = 0$

Thus,

Tension in string = Force in spring = $T = 60N$



A pulley with two blocks system is attached to the ceiling of an elevator moving upward with an acceleration a. Find the deformation in the spring.

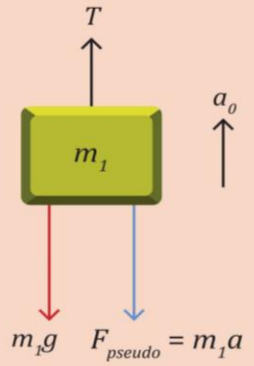


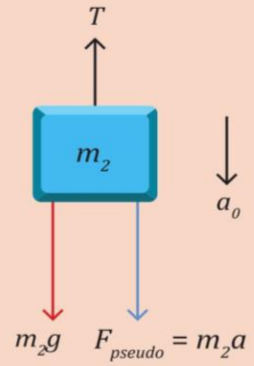
Solution

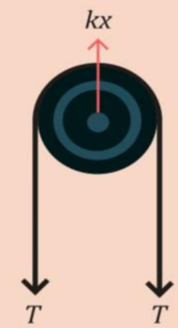
Solving in lift frame,

Let $a_o \Rightarrow$ acceleration of blocks with respect to the elevator.

FBD of masses and pulley is:







As the string and spring is lightweight, $kx = 2T$

For first block,

Balancing forces in vertical direction,

$T - m_1g - m_1a = m_1a_o$ ....(i)

For second block,

Balancing forces in vertical direction,

$m_2g + m_2a - T = m_2a_o$ ....(ii)

Adding two equations,

$(m_2 - m_1)g + (m_2 - m_1)a = (m_2 + m_1)a_o$

$$\Rightarrow \frac{(m_2 - m_1)(g + a)}{m_1 + m_2} = a_o$$

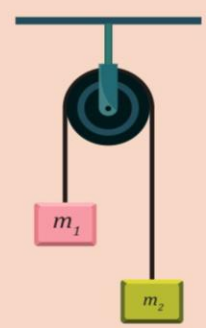
Substituting value of a_0 in equation (i),

$$T = m_1\left[\frac{(m_2 - m_1)(g + a)}{m_1 + m_2}\right] + m_1(g + a)$$

$$T = \frac{m_1m_2(g + a) - m_1^2(g + a) + m_1^2(g + a) + m_1m_2(g + a)}{m_1 + m_2}$$

$$T = \frac{2m_1m_2(g + a)}{m_1 + m_2}$$

Alternate Method



Typical atwood machine,

We have derived formula for acceleration and tension of this machine,

$$a = \frac{(m_2 - m_1)g}{(m_2 + m_1)}$$

$$T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$$ ....(iii)

We can use effective acceleration due to gravity to substitute the method of using the pseudo force concept. Effective acceleration due to gravity is as follows in different cases.

When body is moving up with acceleration a	$g_{eff} = g + a$
When body is moving down with acceleration a	$g_{eff} = g - a$

As the elevator is moving upward, acceleration would change to

$$g \rightarrow g_{eff} = g + a$$

Put in (iii), we get,

$$T = \left(\frac{2m_1m_2}{m_1+m_2}\right)(g + a)$$

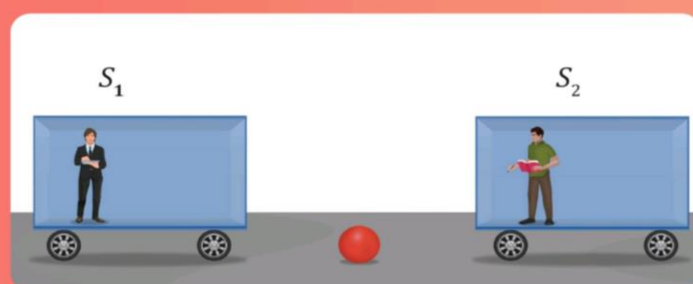
As we know,

$$2T = kx$$

$$\Rightarrow x = \frac{2T}{k}$$

$$\Rightarrow x = \frac{4m_1m_2(g+a)}{(m_1+m_2)k}$$

The accelerations of a particle as seen from two frames S_1 and S_2 have equal magnitude 4 ms^{-2}. Then,



(A) The frames must be at rest with respect to each other

(B) The frames may be moving with respect to each other but neither should be accelerated with respect to the other

(C) The acceleration of S_2 with respect to S_1 may either be zero or 8 ms^{-2}

(D) The acceleration of S_2 with respect to S_1 may be anything between zero and 8 ms^{-2}

Solution

We have,

$$\vec{a}_{p,s_1} = \vec{a}_p - \vec{a}_{s_1}$$

$$\vec{a}_{p,s_2} = \vec{a}_p - \vec{a}_{s_2}$$

Subtracting equations,

$$\vec{a}_{p,s_1} - \vec{a}_{p,s_2} = -\vec{a}_{s_1} + \vec{a}_{s_2}$$

Rearranging above equation,

$\vec{a}_{s_2,s_1,} = \vec{a}_{s_2} - \vec{a}_{s_1} = \vec{a}_{p,s_1} - \vec{a}_{p,s_2}$

As this is a vector addition, the magnitude after addition will have a minimum value of difference of magnitudes of vectors and the maximum value of the sum of magnitudes.

Thus,

$0 \le \left|\vec{a}_{s_2,s_1}\right| \le 8$

Hence option (D) is correct.

NEWTON'S LAWS OF MOTION

PRACTICING NEWTON'S LAWS

What you already know

- FBD and constrained motion
- Pulley-Block system
- String/Rod constrained motion
- Wedge constrained motion

What you will learn

- Definition of pseudo force
- Examples on Newton's laws of motion and pseudo force

Concept of Pseudo Force

What are non-inertial frame of references?

The frames that are accelerating with respect to the ground or to any inertial frame are known as the non-inertial frames of reference.

What is a pseudo force?

A pseudo force (also known as fictitious force, inertial force, or d'Alembert force) is an apparent force that acts on all the masses whose motion is described using a non-inertial frame of reference. The magnitude of the pseudo force is the mass of an object multiplied by the acceleration of the frame of reference.

After travelling through a plank of thickness h, a bullet changed its velocity from v_o to v. Find the time of motion of the bullet in the plank, assuming the resistance force to be proportional to the square of the velocity.

Solution

Resistance force, $F_R \propto v^2$

$$F_R = -kv^2$$

Here, k is proportionality constant.

Note: Negative sign in the above equation indicates that the resistance force is applied in the opposite direction of the velocity of the bullet.

We know that,

$$F_R = ma$$

$$\Rightarrow ma = -kv^2$$

Acceleration of bullet,

$$a = \frac{-kv^2}{m}$$

$$\frac{dv}{dt} = \frac{-kv^2}{m}$$

Integrating from time, $t = 0$ to $t = t$

$$\int_{v_o}^{v} \frac{dv}{v^2} = -\frac{k}{m}\int_0^t dt$$

Integrating and putting limits,

$$\frac{1}{v} - \frac{1}{v_o} = \frac{kt}{m}$$

$$t = \frac{m(v_o - v)}{kv_o v} \text{......}(i)$$

Also, acceleration can be written as follows:

$$a = v\frac{dv}{dx}$$

Note: In the given equation, acceleration is expressed in terms of change in velocity with respect to displacement.

$$a = \frac{-kv^2}{m} = v\frac{dv}{dx}$$

$$\int_{v_0}^{v} \frac{dv}{v} = -\frac{k}{m}\int_0^h dx$$

$$ln(v) - ln(v_o) = -\frac{kh}{m}$$

$$ln\left(\frac{v}{v_o}\right) = -\frac{kh}{m}$$

$$k = \frac{m}{h} ln\left(\frac{v_0}{v}\right) \text{......}(ii)$$

Put the value of k from equation (ii) into equation (i),

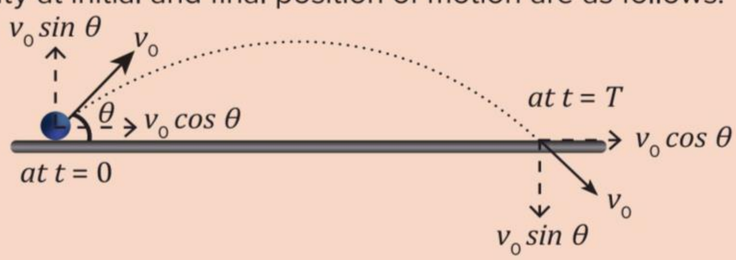
We get,

$$t = \frac{h(v_o - v)}{vv_o \, ln\left(\frac{v_o}{v}\right)}$$

A body of mass m is thrown at an angle θ to the horizontal with the initial velocity v_0. Assuming the air drag to be negligible, find the modulus of the momentum increment Δp during the total time of motion.

Solution

Components of velocity at initial and final position of motion are as follows:



Initial momentum,

$$\vec{p}_i = (mv_o \cos\theta)\hat{i} + (mv_o \sin\theta)\hat{j}$$

Final momentum,

$$\vec{p}_f = (mv_o \cos\theta)\hat{i} - (mv_o \sin\theta)\hat{j}$$

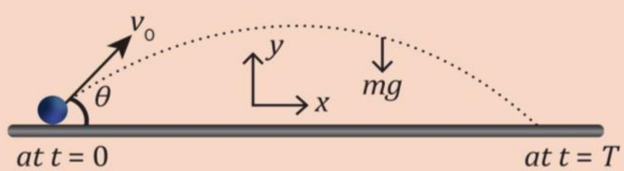
Subtracting the final momentum from initial,

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -(2mv_o \sin\theta)\hat{j}$$

Hence the magnitude:

$$|\Delta\vec{p}| = 2mv_o \sin\theta$$

Alternative way



From Newton's second law of motion,

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt}$$

$$\vec{F}_{ext} = -(mg)\hat{j}$$

Note: If F_{ext} is constant, then the average rate of change in momentum is equal to the instantaneous rate of change in momentum.

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt} = \frac{\Delta p}{\Delta t} = -mg\,\hat{j}$$

Considering the magnitude,

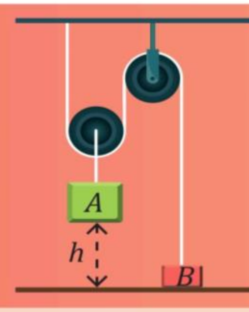
$\Delta p = mg\,\Delta t$

$\Delta p = mg\left(\frac{2v_o \sin\theta}{g}\right)$

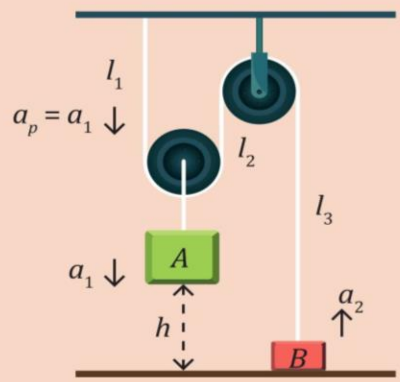
$\Delta p = 2mv_o \sin\theta$

Here, time of flight is given by, $\Delta t = \frac{2v_o \sin\theta}{g}$

In the arrangement shown, the mass of the body A is $n = 4$ times as greater than that of the body B. The height is h. Assuming ideal conditions, at a certain moment, the body B is released from the ground and the arrangement is set in motion. What is the maximum height that the body B will go up to?



Solution



Note: Assuming ideal conditions means all pulleys are ideal and all strings are ideal.
The physicality of the problem says that B is at ground level initially but due to higher weight of the block A, it moves down and block B moves up. Assume acceleration and intercepts variables for string.

Block A and pulley will fall down with the same acceleration.
Constraint relation equation,

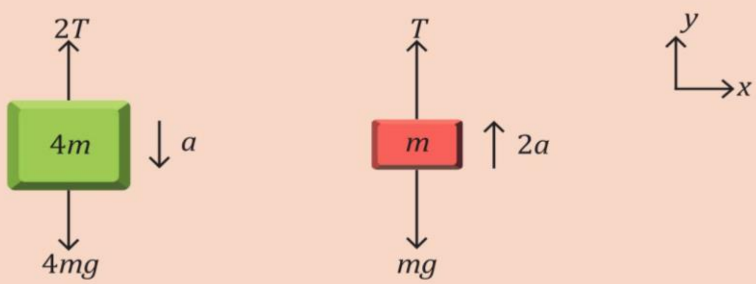
$\ddot{l}_1 + \ddot{l}_2 + \ddot{l}_3 = 0$

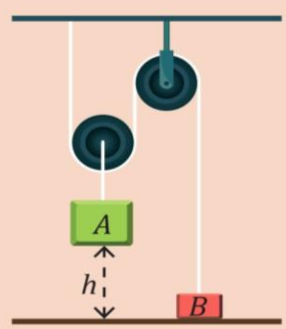
$a_1 + a_1 - a_2 = 0$

Let $a_1 = a$

$a_2 = 2a$

Now, draw FBDs to write force balance.





For block A,
Force balance in vertical direction,
$4mg - 2T = 4ma$

$\Rightarrow 2mg - T = 2ma$ ......(i)

For block B,
$T - mg = 2ma$ .... (ii)

Adding both the equation (i) and equation (ii), we get,

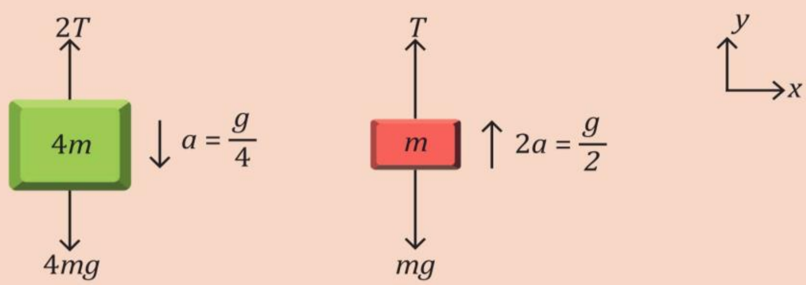
$$a = \frac{g}{4}$$

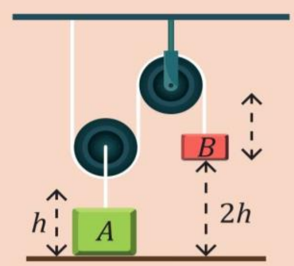
$$a_1 = \frac{g}{4}$$

$$a_2 = 2a_1 = \frac{g}{2}$$

Just before the collision of block A to the ground

Note: The constrained relation of velocity and displacement is the same as that of constrained relation of acceleration. Let v be the velocity of block A just before hitting the ground.



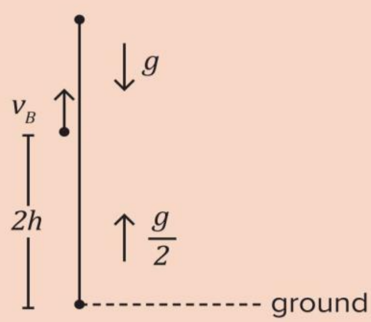


$v_A = v$

$v_B = 2v$

When block A hits the ground, $x_A = h$ and $x_B = 2h$
As A strikes the ground, B continues to go up because of inertia of block B and string gets slacked.

For B,
Maximum height of B,
$H_{B(max)} = 2h + x$



Note: Acceleration of block B up to the height ($2h$) is $\frac{g}{2}$. After height ($2h$), the string goes slack, means there is no tension in the string. Therefore, acceleration of block B after the height ($2h$) is $-g$.

Hence,
For height up to $(2h)$,

$v_B^2 = 0 + 2 \times \frac{g}{2} \times 2h$

$v_B^2 = 2gh$

Using kinematics equation,
$v^2 = u^2 + 2as$

After height $(2h)$,

$0 = v_B^2 - 2 \times g \times x$

$x = \frac{v_B^2}{2g}$

$x = \frac{2gh}{2g}$

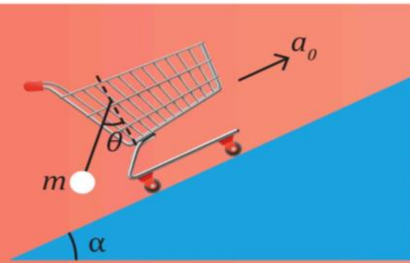
$x = h$

The maximum height the block can achieve is $x + 2h$.

$H_{B_{max}} = 2h + h$

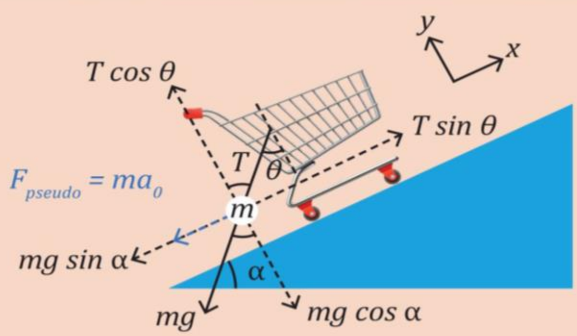
$H_{B_{max}} = 3h$

A pendulum of mass m hangs from a support fixed to a trolley. Find the direction of the string when the trolley rolls up on a plane of inclination α with acceleration a_0. (String and bob remain fixed with respect to trolley)



Solution

Let us go in the trolley frame. As it is non-inertial, we need to add pseudo force in a direction perpendicular to the acceleration of the frame.
FBD of mass m,



Trolley frame (Non-inertial frame)

Note: The pendulum is at rest or equilibrium in the trolley frame. Therefore, net force (F_{net}) is zero on the pendulum in the trolley frame.

Force balance along the x-axis,

$T \sin\theta = ma_o + mg \sin\alpha \; ......(i)$

Force balance along the y-axis,

$T \cos\theta = mg \cos\alpha \; ......(ii)$

Divide equation (i) by (ii),

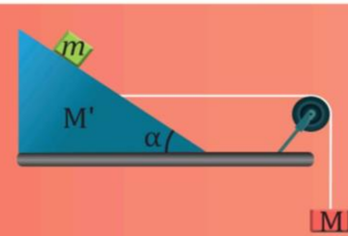
$$\frac{T \sin\theta}{T \cos\theta} = \frac{m(a_o + g \sin\alpha)}{mg \cos\alpha}$$

$$\tan\theta = \frac{a_o + g \sin\alpha}{g \cos\alpha}$$

Thus,

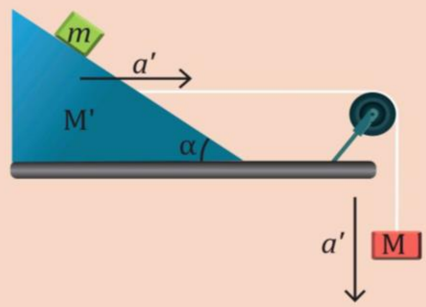
$$\theta = \tan^{-1}\left(\frac{a_o + g \sin\alpha}{g \cos\alpha}\right)$$

Find the mass M of the hanging block in figure which will prevent the smaller block from slipping over the triangular block. All the surfaces are frictionless and the strings and the pulleys are light.

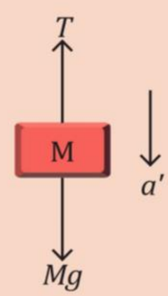


Solution

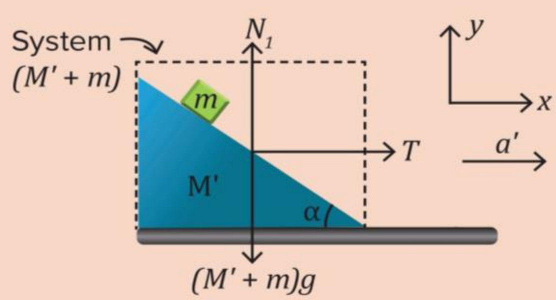
Let $a' \Rightarrow$ Acceleration of blocks M and M' with respect to the ground



FBD of the block of mass M,
Balancing forces in vertical direction,
$Mg - T = Ma' \ldots\ldots (i)$



FBD of the system,
Select the system such that the only external forces will be the tension, normal reaction and weight of the wedge and block sitting on it together.



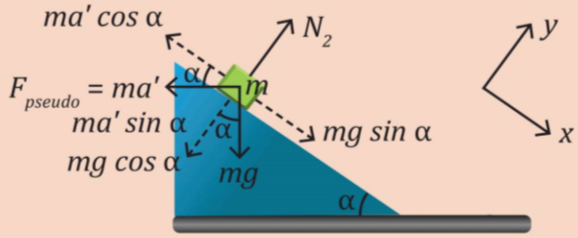
Balancing forces in horizontal direction,
$T = (M' + m)a' \ldots\ldots (ii)$

Add equation (i) and equation (ii),

$$a' = \frac{Mg}{M' + m + M} \ldots\ldots (iii)$$

Wedge frame (Non-inertial)

In order to prevent block m from slipping, F_{net} on the block along the incline should be zero.



As there is no slipping between block and wedge, the block of mass m is at rest in the wedge frame.
Balancing forces in x-direction,

$$\sum F_x = 0$$

$$ma' \cos\alpha = mg \sin\alpha$$

$$a' = g \tan\alpha$$

Putting the value of acceleration from equation (iii),

$$a' = \frac{Mg}{M + M' + m} = g \tan\alpha$$

Rearranging,

$$\frac{M}{\tan\alpha} = M + M' + m$$

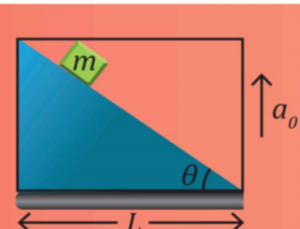
$$\Rightarrow M \cot\alpha = M + M' + m$$

$$\Rightarrow M(\cot\alpha - 1) = M' + m$$

Thus,
Mass M is written as follows:

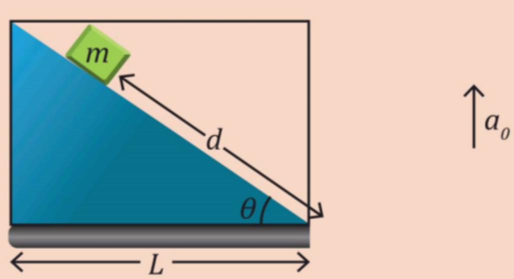
$$M = \frac{M' + m}{\cot\alpha - 1}$$

A particle slides down a smooth inclined plane of elevation θ fixed in an elevator going up with an acceleration a_0. The base of the incline has a length L. Find the time taken by the particle to reach the bottom.

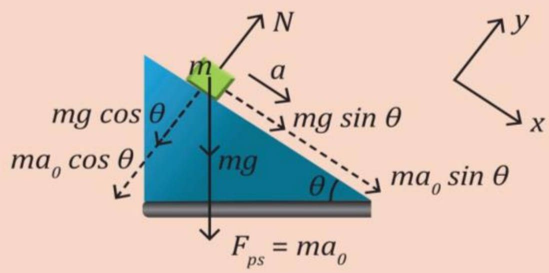


Solution

We know that the time for a motion does not depend on the frame of reference.



In elevator frame (Non-inertial frame)



$$\cos\theta = \frac{L}{d}$$

$$d = \frac{L}{\cos\theta}$$

Let a be the acceleration of block m with respect to the ground.

Balancing forces in x-direction,

$$\sum \vec{F}_x = m\vec{a}_x$$

$$mg\sin\theta + ma_o\sin\theta = ma$$

$$a = (g + a_o)\sin\theta$$

Note: Time does not depend upon the frame of reference.
The initial velocity of the block is zero.
As the acceleration is constant, kinematics equations are valid.
Thus,

$$x = ut + \frac{1}{2}at^2$$

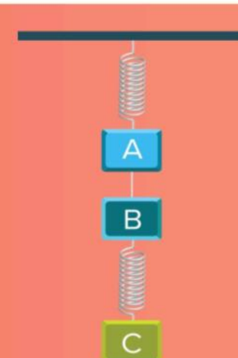
Total distance covered is as follows:

$$\frac{L}{\cos\theta} = \frac{1}{2} \times (g + a_o)\sin\theta \times t^2$$ (the block starts from rest, $u = 0$.)

$$t^2 = \frac{2L}{\sin\theta\cos\theta(g + a_o)}$$

Thus,

$$t = \left[\frac{2L}{\sin\theta\cos\theta(g + a_o)}\right]^{\frac{1}{2}}$$



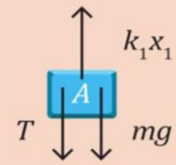
The system shown in the figure is in equilibrium. Find the acceleration of the blocks A, B, and C. At the instant when the spring between B and C is cut. (Assume springs to be ideal) mass of each black is m.

Solution

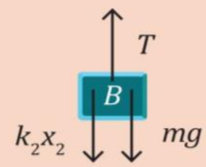
Note: Spring force experiences no change instantaneously.
Tension in string experiences instantaneous change.

Pre-spring cut

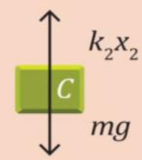
FBD of block A,
Balancing forces in vertical direction,
$k_1x_1 = T + mg$ ....(i)



FBD of block B,
Balancing forces in vertical direction,
$T = k_2x_2 + mg$.......(ii)

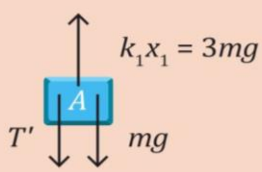


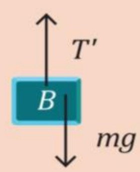
FBD of block C,
Balancing forces in vertical direction,
$k_2x_2 = mg$......(iii)

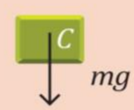


Adding equations (i), (ii), and (iii), we get,
$k_1x_1 = 3mg$ ....(iv)

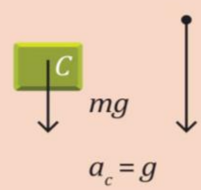
Post-spring cut







For block C,
$mg = ma_c$
$a_c = g$
Acceleration of block C is g in downward direction.



For block A and block B,
Applying force balance on the two bodies together and using $k_1x_1 = 3mg$,
$3mg - (mg + mg) = (m + m)a$
Rearranging,
$$a = \frac{g}{2}$$