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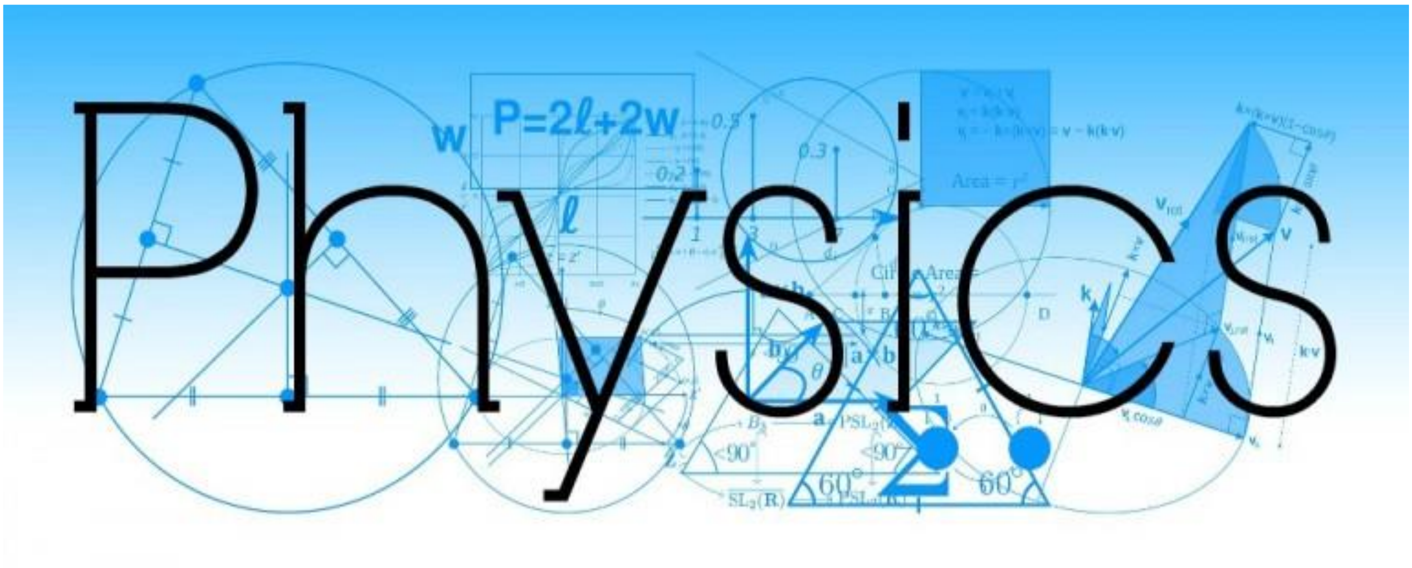

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WORK POWER ENERGY

WORK

Work is said to be done whenever a force acts on a body and the body moves through some distance.

$W = \vec{F} \cdot \vec{S} = FS\cos\theta$ (where θ is the angle between force applied $\vec{F}$ and displacement vector $\vec{S}$.)

The SI unit of work is joule (J).

Nature of Work Done

If $\theta = 0°$, $W = FS$ *i.e.*, work done is maximum.

If $\theta = 90°$, $W = 0$ *i.e.*, work done is zero.

Work Done by a Variable Force

The work done by a variable force in changing the displacement from S_1 to S_2 is $W = \int_{S_1}^{S_2} \vec{F} \cdot d\vec{S}$ = Area under the force-displacement graph

ENERGY

It is defined as the ability of a body to do work. It is measured by the amount of work that a body can do. The unit of energy used at the atomic level is electron volt (eV) and SI unit is J.

Kinetic Energy

It is the energy possessed by a body by virtue of its motion. The K.E. of a body of mass m moving with speed v is

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Potential Energy

It is the energy possessed by a body by virtue of its position (in a field) or configuration (shape or size). For a conservative force in one dimension, the potential energy function $U(x)$ may be defined as

$$F(x) = -\frac{dU(x)}{dx} \text{ or } \Delta U = U_f - U_i = -\int_{x_i}^{x_f} F(x)dx$$

Power

The rate of doing work is called power.

Average Power:

It is defined as the ratio of the small amount of work done W to the time taken t to perform the work.

$$P = \frac{W}{t}$$

The SI unit of power is watt (W).

Work Energy Theorem

The work done by the net force acting on a body is equal to the change in kinetic energy of the body.

W = Change in kinetic energy

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \Rightarrow W = \Delta K.E.$$

The work energy theorem may be regarded as the scalar form of Newton's second law of motion.

Potential Energy of a Spring

According to Hooke's law, when a spring is stretched through a distance x, the restoring force F is such that

$F \propto x$ (where k is the spring constant

or $F = -kx$ and its unit is N m^{-1}.)

The work done is stored as potential energy U of the spring

$$W = \int_0^x kx\,dx = \frac{1}{2}kx^2 \Rightarrow U = \frac{1}{2}kx^2$$

COLLISION

A collision between two bodies is said to occur if either they physically collide against each other or the path of the motion of one body is influenced by the other.

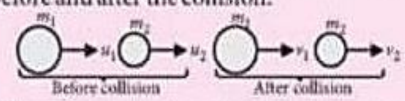
Types of Collision

Elastic collision : Both the momentum and kinetic energy of the system remain conserved.

Inelastic collision : Only the momentum of the system is conserved but kinetic energy is not conserved.

Head-on Collision or One-Dimensional Collision

It is a collision in which the colliding bodies move along the same straight line path before and after the collision.



Velocity of approach = Velocity of separation

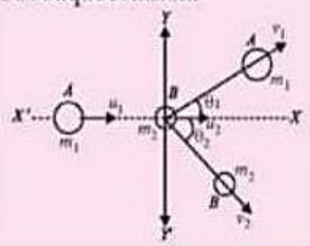
or $u_1 - u_2 = v_2 - v_1$

Also, $v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$ and

$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$

Oblique Collision

If the two bodies do not move along the same straight line path before and after the collision, the collision is said to be oblique collision.



IN PHYSICS .

"Work is said to be done by a force acting on a body provided the body is displaced actually in any direction except in a direction perpendicular to the direction of force".

IN OTHER WORDS.

Work is done by a force when the force produces a displacement in the body to which it acts in any direction except perpendicular to the direction of force".

- **For work to be done . Two conditions must be fulfilled.**

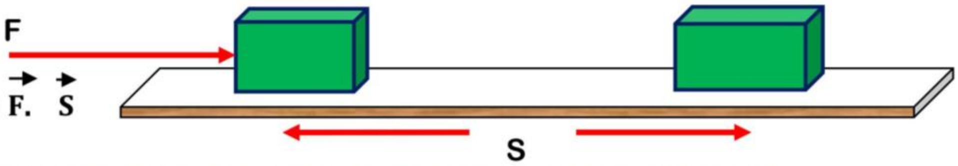
  [i] A force must be applied

  [ii] The applied force must be produce a displacement in the body in any direction except $\perp^{ar}$ to the direction of applied force.

(*a*) Suppose you are pushing the wall very hard but the wall does not move. In this case, you are doing zero work on the wall because its displacement is zero. You may feel tired after pressing hard against the wall but from physics' point of view, work done is zero.

(*b*) Suppose a man holding bucket of water is walking on a horizontal road. According to the definition of work, the man is doing no work. In this case, the man is applying an upward force $\vec{F}$ equal to the weight of the bucket. The direction of the force $\vec{F}$ he applies is perpendicular to the horizontal motion of the bucket (object). Therefore, there is no component of force $\vec{F}$ in the direction of motion (displacement). Hence work done by man on the bucket is zero.

◙ **Work done by a constant force** --- Suppose a force F is applied on a body in such a way that the body suffers a displacement S in the direction of force.



$\therefore$ work done, $W = \vec{F}\cdot\vec{S}$

◙ **Work done by a constant force acting on a body at an angle θ with the horizontal** :-

Let a force $\vec{F}$ be applied on the body such that it makes an angle θ with the horizontal direction

Now, Rect. Comp. of F

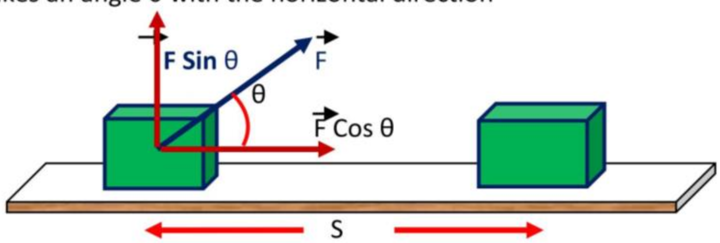
..... i) $\vec{F}\cos\theta$ (in the direction of displacement)

..... ii) $\vec{F}\cos\theta$ ($\perp^{ar}$ to the direction force)

Since the body is applied in the direction of F Cos θ

Therefore, Work done, W = (F Cos θ) S = F S Cos θ

$\vec{W} = \vec{F}\cdot\vec{S}$ **(dot)**



Thus,

"work done by a force is equal to the dot or (scalar) product of the force and the displacement of body."

Thus, **work done is scalar quantity.**

SPECIAL CASE :-- Although workdone is a scalar quantity its value may be positive, negative or even zero.

□ **Case I. Positive work :-** When $0 \le \theta < 90$

Then , **Cos θ = positive value**

$\therefore$ W = FS Cos θ = positive value

"Workdone by a force is said to be positive if the force has a component in that direction of the displacement".

□ **Case II. Zero workdone :-** When $\theta = 90^0$

Then , **$\cos 90^0 = 0$**

$\therefore$, W = F S Cos θ = Zero

"Workdone by a force is zero in the body is displaces in the direction perpendicular to the direction of the force".

□ **Case III. Negative force :-** When $90 < \theta \le 180$

Then **Cos θ = Negative value**

$\therefore$ W = F S Cos θ = Negative

"Workdone by a force is said to be negative if force has a component in a direction opposite to that of the displacement."

DIMENSION FORMULA ------ Workdone = force × distance

$W = [M^1 L^1 T^{-2}][L]$

$$W = [M^1 L^2 T^{-2}]$$

Unit of work ---- [I] Absolute unit [II] Gravitational unit

Absolute unit --- [i] joule [ii] erg

[i] Joule [in S.I] ---- from, $W = F S \cos\theta$

$J = N - m$

" Work is said to be 1 joule, when a force of 1 N moves a body actually through a distance of one meter in the direction of applied force".

[ii] erg [in cgs] ------ from, $W = F S \cos\theta$

erg = 1 dyne .1cm

"Work is said to be 1 erg when a force of 1 dyne moves a body actually through a distance of 1 cm in the direction of applied force".

Relation between joule and erg :-----------

As, $W = F \times S$

Joule = $1\ N \times 1\ m = 10^5$ dyne $\times$ 100c m $= 10^7$ dyne cm

i.e., 1 Joule = 10^7 erg.

Gravitational unit--------[i] Kilogram – meter [kg – m] [II] gram – centimeter [g – cm]

[i] Kilogram – meter [S.I] ------ from, $W = F S \cos\theta$ 1 kg – m = 1 kgf . m 1 kg m = 9.8 N . m

"Work is said to be 1 kg m when a force of 1 kgf moves a body through a distance of 1 m in the direction of the applied force".

[ii] gram centrimeter [in cgs] ---------- from, $W = F S \cos\theta$ 1 gcm = 1 gf × 1 cm = 980 dyne cm

∴ 1 gcm = 980 cm.

"Workdone is said to be 1 g – cm when a force of 1 gf moves a body through a distance of 1 cm in the direction applied force".

Relation between kg – m and g – cm : 1 kg = m = 10^3 g × 10^2 cm , 1 kg m = 10^5 g cm

Workdone in terms of rectangular component

In term of rectangular components, F and S may be written as –

$F = F_x\hat{i} + F_y\hat{j} + F_z k$

and , $S = S_x\hat{i} + S_y\hat{j} + S_z k$

W = F . S

$= [F_x\hat{i} + F_y\hat{j} + F_z k].[S_x\hat{i} + S_y\hat{j} + S_z k]$

$= [F_xS_x](\hat{i}.\hat{i}) + [F_xS_y](\hat{i}.\hat{j}) + [F_xS_z](\hat{i}.k) + [F_yS_x](\hat{j}.\hat{i}) + [F_yS_y](\hat{j}.\hat{j}) + [F_yS_z](\hat{j}.k) + [F_zS_x](k.\hat{i}) + [F_zS_y](k.\hat{j})$
$+ [F_zS_z](k.k)$

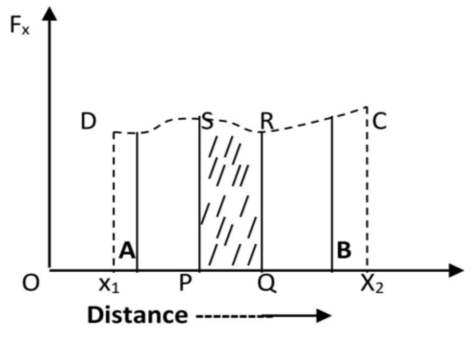
$= [F_xS_x](1) + [F_xS_y](0) + [F_xS_z](0) + [F_yS_x](0) + [F_yS_y](0) + [F_yS_z](0) + [F_zS_x](0) + [F_zS_y](0) + [F_zS_z](1)$

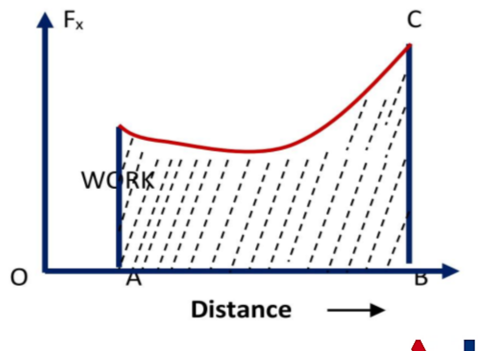
∴ $W = F.S = [F_xS_x] + [F_xS_y] + [F_zS_z]$

WORKDONE BY A VARIABLE FORCE -- It is not possible that applied force is constant in magnitude as well as direction and, there,force, we can calculate work done by a variable force.

Consider a force whose magnitude is changing acts on a body (but acting along fixed direction)

Calculation of workdone by a variable force :----





First of all, divide total displacement from X_1 to X_2 (i.e. from A to B) into a no. of infinitesimals displacement (dx).

∴ PQ = dx 4

The interval dx is so small that F_x is consider over that interval.

Therefore, small amount of workdone in moving the body from P to Q is

$dW = F \times dx = PS \times PQ$

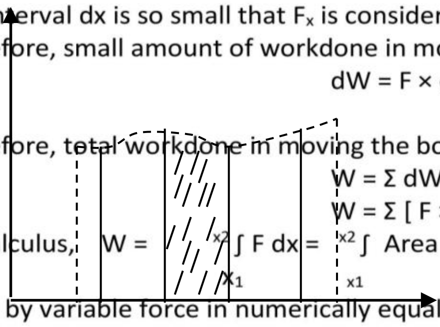
= Area of the strip PQ RS

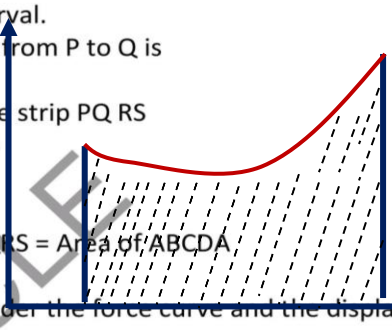
Therefore, total workdone in moving the body from A to B

$W = \Sigma\, dW$

$W = \Sigma\, [F \times dx]$

In calculus, $W = \int_{x_1}^{x_2} F\, dx = \int_{x_1}^{x_2}$ Area of the strip PQRS = Area of ABCDA





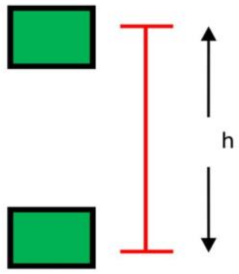
Workdone by variable force in numerically equal to the area under the force curve and the displacement acis.

CONSERVATIVE & NON CONSERVATIVE FORCES :- ----------

Conservative forces -------- ***"A force is said to be conservative if workdone by or against the force depends only on the initial & final position of the body, and not on the nature of the path followed between the initial and final position".*** Example – 'Gravitational force' is a conservative force

Calculation of amount of workdone against gravity -------- Consider a body (of mass 'm') is displace (or lift) through a height AB = h over different paths from A to B

✱Case I ---- *The body being raised vertically upwards*



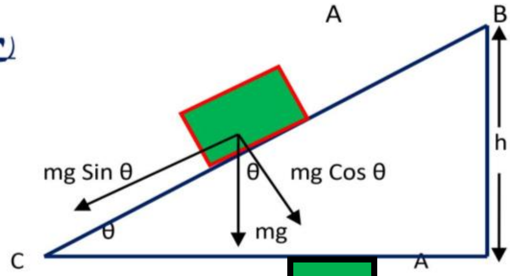
Force applied = Weight of the body

i.e. F = mg

∴ Workdone, $W_1 = F.S = F S \cos\theta = FS \cos 0^0 = mg\, h$ (1)

$\mathbf{W_1 = mgh}$ ....(i)

✱ Case II----*Body being taken along smooth inclined plane (BC)*



Force applied = mg Sin θ

F = mg Sin θ

∴ workdone = FS Cos θ

$W_2 = F\, CB \cos 0^0$

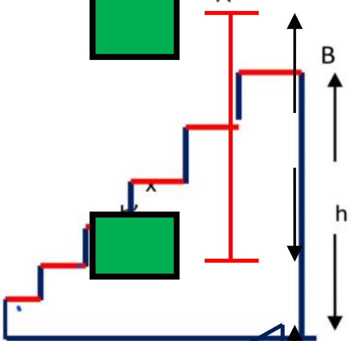
$W_2 = mg \sin\theta\, [CB \cos 0^0]$

$= mg\, AB/BC\ \ CB\ (1)$

$= mg\, AB$ [$\sin\theta = AB/BC$]

$\mathbf{W_2 = mgh}$ ----------------------------(2)

✱Case III---*Body is being taken through the height 'h' over a staircase*



Let, no. of steps = n

Height of each step = h`

Width of each step = x

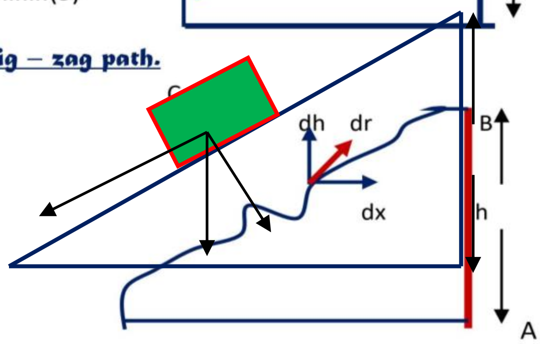
Therefore, workdone, $W_3 = n\,[mgh' \cos 0^0 + mgx \cos 90^0]$

$= n\,[mgh' + 0]$

$= n\, h'\, mg$

$\mathbf{W_3 = mgh}$ [Since nh' = h] ....(3)

✱Case III---*The body being carried through the height 'h' over Zig – zag path.*



Let dx = infinitesimally small horizontal distance

dh = Infinitesimally small vertical distance.

Workdone, $W_4 = \Sigma\, (mg)\, (dh) \cos 0^0 + \Sigma\, (mg) \cos 90^0$

$W_4 = \Sigma\, (mg)\, (dh) \cos 0^0 + 0$

$W_4 = \Sigma\, (dh)\ (mg)\ (1)$

$\mathbf{W_4 = mgh}$ -----------------(4) [$\Sigma\,(dh) = h$]

From [1], [2], [3] & [4]. we get,

$W_1 = W_2 = W_3 = W_4$

i.e. work done is the same, whatever the path followed by the body between initial and final position

Non conservative forces :- "A force is said to be non conservative if work done by or against the force in moving a body from one position to another, depends on the path followed between these two position. Example – Frictional force. 5

POWER

:- "Power is defined as the rate at which work is done".

In other word,

"Power is defined as the time rate of doing work".

i.e., POWER = Work/ Time

☻ When a body takes place lesser time to do a particular work its power is said to be greater & vice verca.

Expression Power = Work / time

$$P = W/t = \vec{F}.\vec{S}/t$$

or

$$P = \vec{F}.\vec{V} \quad (\because \vec{v} = S/t)$$

∴ **By the defⁿ of dot product**

$P = F v \cos\theta$ { where θ is the smaller angle between $\vec{F}$ & $\vec{V}$ }

Thus, ***"Power is the scalar product of force $\vec{F}$ & velocity $\vec{v}$."***

- **If both $\vec{F}$ and $\vec{v}$ point is the same direction then θ = 0**

∴ $P = F v \cos 0 = F v \times 1 = Fv$

- **Instantaneous power:**

"Instantaneous power is the power at any given instant".

Example :--- Supppose on agent does an infinitesimally small amount of work done dw in an infinitesimally small time dt.

Then, $P = dw/dt$

But, $dw = F . ds$

∴ $P = F . ds / dt$

$P = \vec{F}.\vec{v}$

Thus, Power of an agent at any instant is equal to the dot product of the force applied and the velocity at that instant. When an agent delivers power at a uniform rate, the average power is equal to the instantaneous power.

- **DIMENSION OF POWER** :- $P = W/t = ML^2T^{-2} / T = [ML^2T^3]$

- **Units of power** :- Watt (W) (SI) . (Absolute unit)

$P = W/t \quad 1 W = 1 J/ 1 sec = J/sec$

"Power is said to be 1 watt if one joule of work is done in on second by an agent".

- **Units of power** :- (i) 1 kilowatt = 10^3 watts
  1 kw = 10^3 w
  (ii)1 Megawatt = 10^6 watts
  1 Mw = 10^6 .

- **Cgs unit** 1 watt = 1 j / a
  1 W = 10^7 erg / sec (Absolute unit)

□ **Practical unit** :- **Horse power (h . p) : 1 hp = 746 W**

□ **Gravitational unit** : (i) kgf m/s (SI)

"The power of an agent is said to be 1 kgf m/s if it does 1 kgfm of work in 1 sec."

(ii) gf cm/s (in cgs)

✱Power is a scalar quantity.

ENERGY

:--- ***"The capacity or ability of the body to do work is called energy".***

- **Energy is measured by the total amount of work that a body can do.**

- It is a scalar quantity.
- **Units** -------- [S.I] ------ Joule
  ---------[cgs] ----- erg
  ---------**Practical units** --- [i] erg = 10^{-7} joule ; ----[ii] Calorie = 4.2 joule
  ----[iii] Kilowatt hour (Kwh) = 3.6×10^6 joule ; ---[iv] Electron volt (eV) = 1.6×10^{-19} joule .

"The energy [KE] acquired by an electron when it passes through a potential of volt, is called electronvolt".

$1eV = 1.6 \times 10^{19}$ C × 1 Volt
$= 1.6 \times 10^{-19}$ joule [Since 1 CV = 1 J]

▶ **Difference between power & energy** :-

[i] Power determines the rate of doing work. However,
Energy of the body shall gives us an idea of the total amount of work that the body can do.

[ii]Power depends on time in which the work is done.
Energy has nothing to do with time.

▶ **Forms of energy** :--- There are many forms of energy but in this particular chapter, we shall discuss only mechanical energy [PE + KE].

[I] KINETIC ENERGY

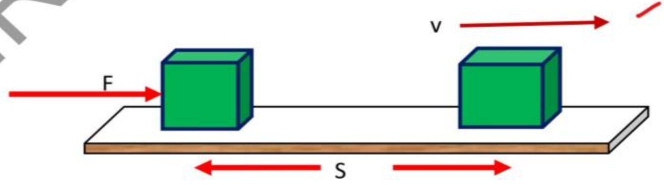
:--- [KE] ***"Kinetic energy is the energy of a body by virtue of its motion".***

A body moving with higher speed should posses more KE than a body moving with lower speed.
Ex:- Flowing water, moving air, a bullet fired from a gun. etc

Expression for KE :--- KE of a body can be obtained either form –
[i] the amount of workdone on stopping the moving body
or from – [ii] the amount of workdone in giving the present velocity to the body from the state of rest.

Non calculus method ---

Consider a body (of mass 'm') initially at rest.
Let, A constant force F be acting on the body due to which it starts moving with an acceleration 'a'
Therefore, F = ma ....[i]



Let the body acquires velocity 'v' after travelling a distance 'S'

Then from, $v^2 - u^2 = 2aS$
$v^2 = 2aS$ [Since u = 0, initially at rest]
or, $a = v^2/2S$
From [i], $\vec{F} = m\vec{v}^2/2S$
If 'W' is the workdone by the force then, $W = F \,.\, S = FS\cos\theta = FS\cos 0^0 = F\,S$
[Since F & S are in the same direction, θ = 0]

or, $W = \dfrac{mv^2 \times S}{2S}$

or , **$W = ½\, mv^2$**

This workdone on the body is a measure of KE, therefore

$KE = W = ½ \; mv^2$

CALCULUS METHOD ---

Consider a body (of mass'm') lying on a horizontal frictionless surface .
$\vec{F}$ = force applied on the body; dx = small displacement (in the direction of $\vec{F}$) ,
∴ Small amount of workdone , $dW = \vec{F} \,.\, \vec{dx} = F\,dx\cos\theta = F\,dx\cos 0^0$
$dW = F\,dx$ ---[i]

If 'a' = acceleration produced in the body , then , A/N's 2^{nd} law of motion,
$F = m\,a$
∴ $dW = ma\,dx$ [from (i)]
$dW = m\dfrac{dv}{dt}dx$
$dW = m\dfrac{dx}{dt}dv$
$dW = m\,v\,dv$ [Since dx/dt = v]

∴ Workdone by the force In increasing the velocity from zero to v

$$\int dW = \int_0^v m\,v\,dv = m\int_0^v v\,dv = m\left[\,v^2/2\,\right]_o^v = m/2\left[\,v^2 - 0\,\right] = ½\, mv^2$$

$$KE = ½\, mv^2$$

i.e. " KE of a body is numerically equals to half the product of mass & square of the velocity of the body".

Conclusion : **[from the expression , KE = ½ m v^2]**

- **KE α m , Which means , a heavier body possess greater KE and vice versa.**
- **KE α v^2 , Which means that a body moving faster possess greater KE and vice versa.**
- **KE =1/2 m v^2 Holds even when the force applied varies in magnitude or direction or in both i.e the expression is Valid irrespective of how the body acquires the velocity.**

- KE of a body is always positive , it can never be negative. 7
- Kinetic energy of the body depends upon the frame of reference.

Ex. KE of man sitting in train moving with velocity v is $\frac{1}{2} m v^2$ [in the frame of earth]
KE of same man sitting in train moving with velocity v = 0 [in the frame of the train]

RELATION BETWEEN KE AND LINEAR MOMENTUM-------

LINEAR MOMENTUM of the body (of mass 'm'), P = m v ' , where , v = velocity of the body

Also, $KE = \frac{1}{2} mv^2$

$$= \frac{1}{2}\frac{m^2 v^2}{m} = \frac{1}{2}\frac{(m\,v)^2}{m}$$

$\therefore$ $KE = \frac{P^2}{2m}$ [since P = mv]

- Case I ---- IF P = constant then, $KE \propto 1/m$
- Case II ----- IF KE = Constant then $P^2 \propto m$ or, $P \propto \sqrt{m}$
- Case III ----- IF m = constant then, $P^2 \propto KE$ or $P \propto \sqrt{KE}$

WORK ENERGY THEOREM------------

"Workdone by a force displacing a body measures the change in KE of the body".

► In simple , Work & energy[KE] are equivalent quantities.

Explanation--- When a force does some work on a body ,the KE of the body increases by the same amount .Also , when An opposing force is applied on a body , its KE decreases and **decrease in KE is equal to the workdone by the body against the retarding force.**

PROOF--- {Work energy theorem)---- Consider , a body of mass 'm' moving with velocity 'v' .

Let $\vec{F}$ be the applied force in the direction of motion.

$\vec{ds}$ = small displacement (in the direction of $\vec{F}$)

Small amount of work ,$dW = \vec{F}\cdot\vec{ds} = F\,ds \cos 0^0 = F\,ds = ma\,ds$ [Since F = ma]

$dW = m\frac{dv}{dt}(ds)$ [Since a = dv/dt]

$dW = m\frac{ds}{dt}dv = m\,v\,dv$ [Since v = ds/dt]

Total workdone by the force in increasing the velocity of the body from u to v

$$\int dW = \int_u^v mv\,dv = m\int_u^v v\,dv = m\left[\frac{v^2}{2}\right]_u^v = \frac{m}{2}[v^2 - u^2]$$

$$W = \frac{mv^2}{2} - \frac{mu^2}{2}$$

i.e. **W = Final KE -- Initial KE = Change in kinetic energy**

- **Workdone by a force is a measures of change in KE of the body**

Non-calculus method --------------------

W = FS = ma S ------------------------[i] [Since F = ma]

Let the body acquires 'v' velocity after traveling a distance 'S', then

From $v^2 - u^2 = 2aS$

$$a = \frac{v^2 - u^2}{2S}$$

Putting the value of 'a' in equation[i], we get

$$W = m\left[\frac{v^2 - u^2}{2S}\right]S = m\left[\frac{v^2 - u^2}{2}\right] = \frac{mv^2}{2} - \frac{mu^2}{2}$$

W = Final KE -- Initial KE = Change in kinetic energy

►**Conclusion----** [i] If there no change in the speed of a particle then,
Final KE – initial KE = 0
$\therefore$ W = 0

[ii] Workdone by a force is negative , if there is decrease in the KE of a particle or a body .In this case , Force and displacement are directed in opposite direction.

[iii] Workdone by the force is positive , if there is increase in the velocity and hence KE of the particle . In this case , the force and displacement are in the same direction.

POTENTIAL ENERGY -- [PE]

" Potential energy is the energy possessed by a body by virtue of its position in a field of force or by its configuration(shape)".

Or

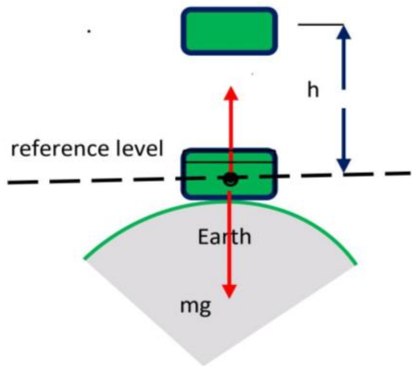
" The energy possessed by a body by virtue of its position or configuration is called potential energy".

Types of PE ⟶
- Gravitational PE
- Elastic PE
- Electrostatic PE

GRAVITATIONAL PE------ **" Gravitational PE of a body is the energy possessed by the body by virtue of its Position above the surface of earth".**

◙ The amount of workdone in carrying the body from the surface of earth to its present position against the gravitational Force of earth , is stored in the body in the form of gravitational PE.

Expression :-- Consider a Block (of mass'm') which is to be raised to a height 'h'
The force required to lift the body must be equal to its weight (i.e. mg) in upward direction .



(Assumption:--Height 'h' is not to large and the value of 'g' is Practically Constant)

∴ $W = \vec{F} \cdot \vec{S} = \vec{F} \cdot \vec{h} = F\,h\,\cos\theta = F\,h\,\cos 0^0$

$= F\,h \times 1$

$W = (mg) \times h$ $\quad [\because \vec{F} = mg]$

$= mgh.$

G P E = U = mgh

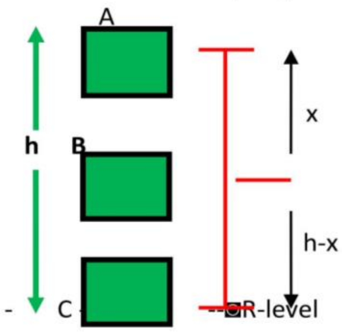
At the surface of Earth , h = 0

G P E = U = Zero

- GPE varies directly as the mass of the body .
- GPE " " " ,, height ,, ,, ,, . above the surface of the earth.
- GPE is associated with only **VERTICAL POSITION**
- PE is defined only for conservative force .
- GPE is said to be **POSITIVE** , when work is done by the body in going above reference position .
- GPE is **NEGATIVE**, When work is done on the body to bring it back to the reference position .

CONVERSION OF GPE TO KE :---

When a falls from a certain height its KE begins to increase at the expense of the GPE which is reduced. (but , total mechanical energy remains conserved .
Consider a body of mass in to be at a position A.
Let 'h' be the height of the body above reference level



At the position 'A' :--

$KE = \frac{1}{2} mv^2 = 0$ $\quad [\because v = 0$, the body is at rest.]

$PE = mgh$

∴ **Total mechanical energy (ME) = KE + PE = 0 +mgh = mgh** ----------(1)

At the position 'B' :--

Let the body be at the position 'B' after fallen through distance x :

$PE = mgh - mgx = mg(h - x)$

If v_B = velocity of the body at B then $V^2_B - 0 = 2gx$

$\therefore V^2_B = 2gx$

$\therefore KE = \frac{1}{2} m V^2_B = \frac{1}{2} m \times 2gx = mgx$

∴Total ME = KE + PE = mg (h – x) + mgx = mgh -------------(II)

At the position 'C' :- Now 'h' can be regarded as Zero. ∵ PE = mgh = 0

Also Let V_c be the velocity with which the body just touches the ground .

$\therefore V^2_c - 0 = 2gh \qquad \therefore V^2_c = 2gh$

$\therefore KE = \frac{1}{2} m V^2_c = \frac{1}{2} m \times 2gh = mgh.$

∴ Total ME = KE + PE = mgh + 0 = mgh ------------(III)

From (1) , (II) & (III) the sum of the KE & PE (i.e.ME) of freely fallen body is constant at all stages of motion.

Numerical (KE)

1.) A bullet of mass 20 g is fired from a rifle with a velocity of 800 m /s . After passing through a mud wall 100 cm thick, Velocity drops to 100m/s by the wall neglecting friction due to air .

Sol^n mass (m) = 20g = 20/1000 kg

u = 800 m/s v = 100 m/s

work done = change in KE

$F \times S = \frac{1}{2} mu^2 - \frac{1}{2} mv^2$

$F \times 100cm = \frac{1}{2} \times \frac{2}{100} = \frac{1}{2} \times \frac{2}{100} \, [(800)^2 - (100)^2]$

$F \times \frac{100}{100} m = 6300$

F = 6300N.

2.) A body of mass 5 kg initially at rest is subjected to a force of 20 N . What is the KE acquired by the at end of 10 sec.

Sol ^{n} m = 5 kg, u = 0 , f = 20 N , t = 10sec .

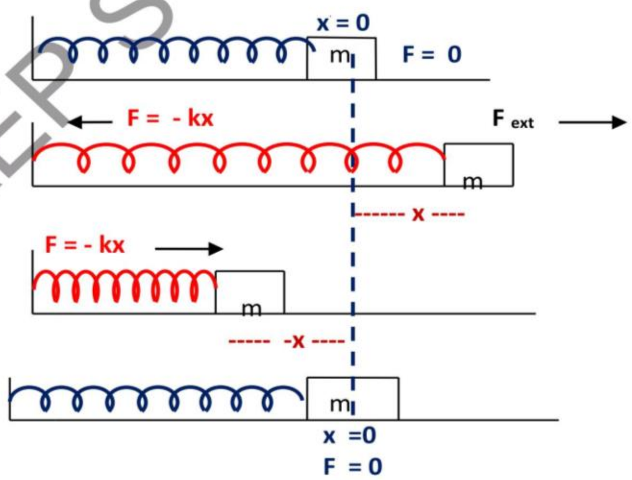
$a = f/m = \frac{20}{5} = 4 \ m/s^2$

Now , $v = u + at^2 = 0 + 4 \times 10 = 40 \ m/s$

∴ $KE = \frac{1}{2} \times 5 \times (40)^2$ = 4000joule.

Potential energy of a spring :--

Consider a spring is stretched or compressed from its normal position (say , x = 0) by a small distance x. Then , a **restoring Force** is develop in the spring to bring it to the normal position.



The Restoring force proportional to the displacement 'x' and its direction is always opposite to the displacement (where , K = spring constant or force constant)

i.e $\vec{F} \ \alpha \ -x$ | $\vec{F} = - kx$ | -----**Hooke's law**

✱**Spring constant , K = - F/x**

"Force constant (or spring constant) is defined as the retarding force per unit displacement of the spring".

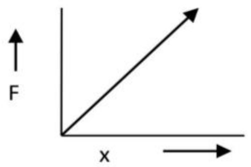
- **UNIT--- [i] In S.I** ---- K = - F/x = **N/m ;** **[ii] In cgs**----- K = - F/x = **dyne/cm**
- **DIMENSION**---- K = - F/x = **M L T^{-2}/L** = **M T^{-2}**

■ Greater the force constant smaller will be the stretch or compression of the spring for a given force.

If x = 1 then F = - u × 1 or u = -F

Hence, **force constant of a spring is numerically equal to the force required to produces unit displacement in the spring**

EXPRESSION for PE of a spring :



The external force is just equal & opposite to the restoring forces

∴ $\vec{F}(ext) = \vec{F}$

$\vec{F}$ (ext) = + Kx [F = - Kx]

Let the body be displaced further through infinitesimally an small distance 'dx' , against the restoring force.

∴ Work done in doing so is,

$dW = \vec{F}_{ext} \cdot \vec{dx}$

$= F_{ext}\, dx \cos 0^o = F_{ext}\, dx$

∴ total work done to stretch the spring through a distance x from its normal position (x = 0)

$$\int dw = \int_0^x Kx\,dx$$

$$W = K\int_0^x x\,dx$$

$$W = K\left|\frac{x^2}{2}\right|_0^x$$

$$W = \frac{K}{2}[x^2]$$

This work done is stored as PE of the stretched spring at that point.

∴ PE, $U = \frac{1}{2} Kx^2$

Thus, "PE associated with the state of compression or extension of spring (elastic object) is known as elastic PE."

Collision

"A collision is said to take place when other two bodies physically collide against each other or when the path of one body is changed by the influence of the other body."

Example:(i) A hammer striking a nail

(ii) an α - particle speeding towards nucleus of an atom gets deflected by the electrostatic force of repulsion.

- The force involved in a collision are action & reaction i.e. the internal force of the system
- Due to collision the momentum & Kinetic energy of the interacting bodies change . Since forces involved are internal Forces (action –reaction) , therefore , the total momentum is conserved. Also , the total energy is conserved.

Types of collision :- → Elastic collision → Inelastic collision

[1] ELASTIC COLLISION :--

" A collision is said to be perfectly Elastic collision if both the KE & MOMENTUM are conserved in the collision."

- This simply means , the linear momentum & the KE of the particle before and after collision are same.
- In an elastic collision ,the forces of interaction are conservative in nature.
- Perfectly elastic collisions are **extremely rare physical phenomenon**

Example :- i) Collision between atomic & sub – atomic particles are elastic collision

ii) Collision between two ivory or steel or glass ball is also nearly elastic collision.

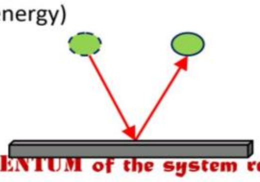
► Characteristic of an Elastic Collision:–

------O i) Linear momentum, & --------O ii) KE (in fact, total energy) is conserved.

Mechanical energy (KE) is not converted into any other forms of energy (like heat energy, sound energy)

------O iii) Forces involved are conservative .

► If a ball dropped from a certain height rebounds to its original position, then ***Collision is elastic*** because ***there is no loss of KE in this case***.

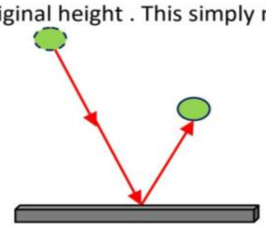


[2] INELASTIC COLLISION:---

" A collision is said to be inelastic if the LINEAR MOMENTUM of the system remains Conserved but its KE energy is not conserved."

- The KE lost in the collision appears in the form of heat energy, sound energy, and light energy etc.
- The forces of interaction in an inelastic collision are non-conservative forces.
- Most of the collisions between macroscopic bodies are inelastic collision.

Example:-If a ball is dropped from a certain height and the ball is unable to rise completely to its original height . This simply means that the ball has lost some KE and therefore the collision is inelastic.



Perfectly In elastic collision (plastic collision)

"A collision is said to be perfectly inelastic if the two bodies after collision stick together and move as one body ."

Example : When moving bullet hits the stationary wooden block then it is embedded in the wooden block and both move as one body.

▶ **CHARECTERISTICS** :

- 01) **Linear momentum is conserved.**
- 02) **KE is not conserved.**
- 03) **Total energy conserved.**
- 04) **Some of the forces are not conservative.**

◘ Coefficient of Restitution

:-- The degree of elasticity of a collision is determined by a quantity called Coefficient of Restitution .

Def^n : *"Coefficient of Restitution is defined as the ratio of relative velocity of separation after collision to the relative Velocity of approach before the collision"*

◙ **Denoted by 'e'**

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

Where , u_1 & u_2 are velocities of two bodies before collision .and v_1 & v_2 are the velocities after collision.

◙ The value of 'e' depends upon the nature of the colliding bodies .

◙ The smallest value of e can be zero and the max. value of e is 1.

✱ ***For perfectly elastic collision*** . $v_2 - v_1 = u_1 - u_2$

$$\therefore \quad e = \frac{v_2 - v_1}{u_1 - u_2} = 1$$

✱ ***For Inelastic collision*** : Relative velocity of separation after collision i.e. $v_2 - v_1 = 0$

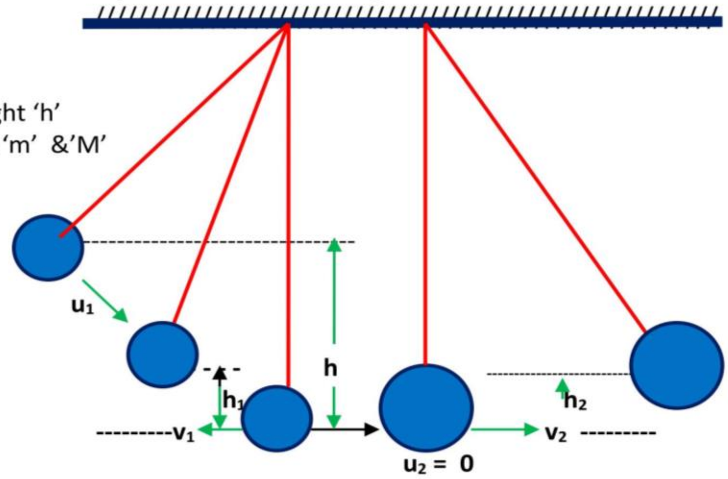
$$\therefore \quad e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{0}{u_1 - u_2} = 0$$

▶▶▶ Conclusion---- *For all other collision 'e' lies between 0 and 1* **i.e. $0 < e < 1$.**

Determination of 'e' --

Consider two sphere of masses 'm' & 'M' suspended as pendulum .Sphere of mass 'M' hangs freely At rest (.i.e. $u_2 = 0$)

Let the sphere of mass 'm' is raised through a height 'h' and then released . After collision , let the masses 'm' &'M' are rebound to heights h_1 & h_2 .



Now using the relation , $v = \sqrt{2gh}$

$$\therefore \quad u_1 = \sqrt{2gh} \quad , \quad v_1 = \sqrt{2gh_1} \quad , \quad v_2 = \sqrt{2gh_2}$$

$$\therefore \quad e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\sqrt{2gh_2} - \sqrt{2gh_1}}{\sqrt{2gh} - 0} = \frac{\sqrt{2g}\,[\sqrt{h_2} - \sqrt{h_1}]}{\sqrt{2g}\,\sqrt{h}}$$

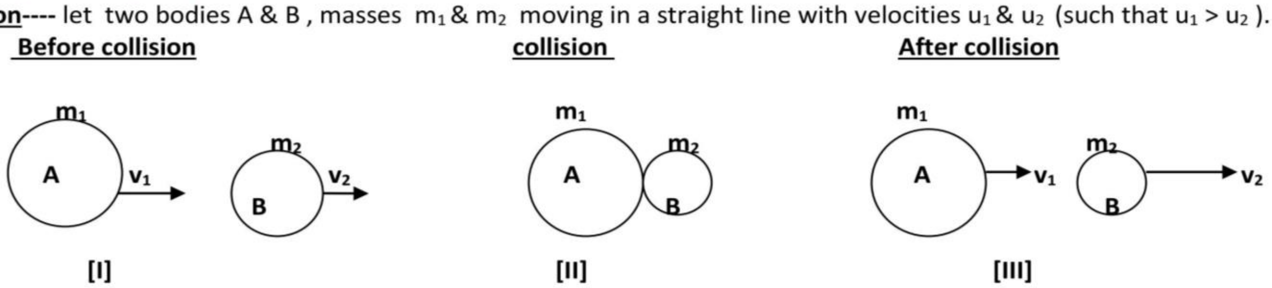
$$\therefore \quad e = \frac{[\sqrt{h_2} - \sqrt{h_1}]}{\sqrt{h}}$$

◘ *Elastic collision in one dimension [1-D]*

-- - - - - - (Means two bodies moving initially along the same straight line , strikes against each other without loss of KE and continuing to move along the same straight line after collision.)

Thus , "1-D elastic collision is that elastic collision in which the colliding bodies move along the same straight line path before and after the collision".

Expression---- let two bodies A & B , masses m_1 & m_2 moving in a straight line with velocities u_1 & u_2 (such that $u_1 > u_2$).



Relative velocity of approach ,before collision = $u_1 - u_2$ ---------------------------------[i]

∴ A & B collide , Let the collision be perfectly elastic .

Suppose after collision , their respective velocities changes to v_1 & v_2.

If $v_2 > v_1$, then, relative of separation after collision = $v_2 - v_1$ ----------------------------[ii]

According to law of conservation of linear momentum

Total linear momentum before collision = total linear momemtum after collision

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 u_2$$

$$m_1 (u_1 - v_1) = m_2 (v_2 - u_2) \quad \text{---------[A]}$$

As the collision is elastic , so KE of the system is conserve, therefore

Total KE [before] = Total KE [after]

$$\tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2$$

$$m_1 [u_1^2 - v_1^2] = m_2 [v_2^2 - u_2^2]$$

$$m_1 [(u_1 + v_1) (u_1 - v_1)] = m_2 [(v_2 - u_2) (v_2 + u_2)] \quad \text{---------[B]}$$

Dividing [B] by [A] we get,

$$\frac{m_1 [(u_1 + v_1) (u_1 - v_1)]}{m_1 (u_1 - v_1)} = \frac{m_2 [(v_2 - u_2) (v_2 + u_2)]}{m_2 (v_2 - u_2)}$$

$$(u_1 + v_1) = (v_2 + u_2)$$

$$u_1 - u_2 = v_2 - v_1 \quad \text{---------[C]}$$

i.e . **Relative** velocity of approach = Relative velocity of separation , therefore

$$e = \frac{v_2 - v_1}{u_1 - u_2} = 1$$

Thus, **Coefficient of restitution of a perfectly elastic collision in 1-D is unity{1}**

i.e. For perfectly elastic collision , e = 1

VELOCITY AFTER COLLSION ------------------

For body A --- From [C] $u_1 - u_2 = v_2 - v_1$

$$v_2 = u_1 - u_2 + v_1 \quad \text{---------[D]}$$

Putting the value of v_2 in eq. [A], we get

$$m_1 (u_1 - v_1) = m_2 [(u_1 - u_2 + v_1) - u_2]$$

$$m_1 (u_1 - v_1) = m_2 [(u_1 - 2u_2 + v_1)]$$

$$m_1 u_1 - m_1 v_1 = m_2 u_1 - 2 m_2 u_2 + m_2 v_1$$

$$v_1 (m_1 + m_2) = 2 m_2 u_2 - m_2 u_1 + m_1 u_1$$

$$v_1 (m_1 + m_2) = 2 m_2 u_2 - u_1 [m_2 + m_1]$$

$$v_1 (m_1 + m_2) = 2 m_2 u_2 + u_1 [m_1 - m_2]$$

$$v_1 = \frac{2 m_2 u_2 + u_1 [m_1 - m_2]}{(m_1 + m_2)}$$

For body B------

Putting the value of v_1 in eq. [D] we get ,

$$V_2 = (u_1 - u_2) + \frac{2 m_2 u_2 + u_1 [m_1 - m_2]}{(m_1 + m_2)}$$

Or,

$$V_2 = \frac{2 m_1 u_1 + u_2 [m_2 - m_1]}{(m_1 + m_2)}$$

Special cases ----

Case 1:- When the bodies have equal masses i.e. $m_1 = m_2$

Velocity of A after collision , $v_1 = \frac{2 m_2 u_2 + u_1 [m_1 - m_2]}{(m_1 + m_2)} = \frac{2 m u_2}{2m}$

∴ $v_1 = u_2$

i.e. Velocity of A after collision = velocity of B before collision

Velocity of **B** after collision, $v_2 = \frac{2 m_1 u_1 + u_2 [m_2 - m_1]}{(m_1 + m_2)} = \frac{2 m u_1}{2 m}$

∴ $v_2 = u_1$

i.e. **velocity of B after collision = velocity of A before collision.**

Conclusion ----------

When two bodies of equal masses suffers 1-D elastic collision , they interchange their velocities.

Case II :- When the Target B is at rest i.e. $u_2 = 0$

Velocity of A after collision $v_1 = \frac{2 m_2 \times 0 + u_1 (m_1 - m_2)}{m_1 + m_2}$

$$\therefore v_1 = \frac{u_1(m_1 - m_2)}{m_1 + m_2}$$

Velocity of B (after collision) $v_2 = \dfrac{2m_1u_1 + u_2(m_2 - m_1)}{m_1 + m_2}$

$$\therefore v_2 = \frac{2m_1u_1}{m_1 + m_2}$$

SUB CASES :-- $\boxed{u_2 = 0}$

Case (a) :-- **When both the bodies have same masses . i.e. $m_1 = m_2$**

Velocity of A

$$v_1 = \frac{2m \times 0 + u_1(m - m)}{m_1 + m_2}$$

$$\boxed{v_1 = 0}$$

Velocity of B

$$v_2 = \frac{2m u_1 + u_2(m - m)}{m + m}$$

$$v_2 = \frac{2m u_1}{2m}$$

$$v_2 = u_1$$

i.e. body A comes to rest and B starts moving with initial velocity A i.e. ***100% transfer of K.E of A to B.***

Case (b) :-- **When body B is at rest is very very heavy. i.e. $m_2 >> m_1$ (in this case m_1 can be neglected)**

Velocity of A

$$v_1 = \frac{2m_2 \times 0 + u_1(-m_2)}{m_2}$$

$$= \frac{u_1(-m_2)}{m_2}$$

$$\boxed{v_1 = -u_1}$$

i.e. A rebounds with its own velocity and B continues to be at rest (practically)

Case (c) :-- **When body B at rest has negligible mass i.e. $m_1 >> m_2$**

Velocity of A

$$v_1 = \frac{2m_2u_2 + u_1(m_1 - m_2)}{m_1 + m_2}$$

$$v_1 = \frac{2 \times 0 \times u_2 + u_1(m_1)}{m_1}$$

$$v_1 = \frac{u_1 m_1}{m_1}$$

$$\boxed{v_1 = u_1}$$

Velocity of B

$$v_2 = \frac{2m_1u_1 + u_2(0 - m_1)}{m_1}$$

$$v_2 = \frac{2m_1u_1}{m}$$

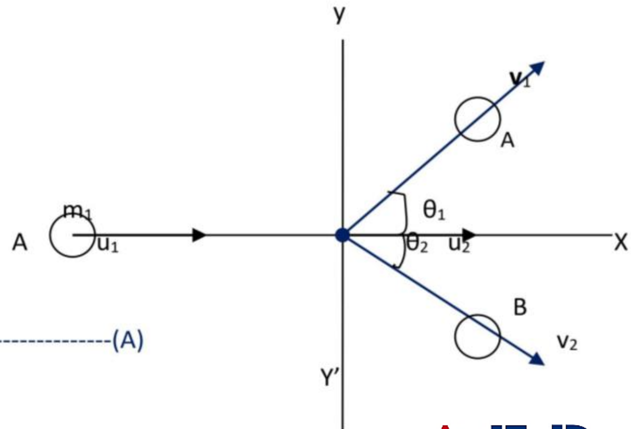
$$\boxed{v_2 = 2u_1}$$

i.e. ***A keeps on moving with the same velocity of its own and B starts moving with double the initial speed of A.***

Elastic Collision in Two Dimensions:--

(Means two bodies travelling initially along the same straight line collide without loss of KE and move along different directions in a plane after collision)

Consider a particle of mass m_1 moving X-axis with initial velocity u_1 collides elastically with another particle of mass m_2 and velocity with u_2. After collision, let the mass m_1 moves with velocity v_1 at θ_1 w.r.t the X-axis and particle m_2 moves with velocity v_2 at an angle θ_2 w.r.t X-axis



when $u_1 > u_2$, the bodies collides

Since , collision is elastic

$\therefore$ Total KE before collision = Total KE after collision

$$\tfrac{1}{2} m_1u_1^2 + \tfrac{1}{2} m_2u_2^2 = \tfrac{1}{2} m_1v_1^2 + \tfrac{1}{2} m_2 v_2^2$$

$$\mathbf{m_1 u_1^2 + m_2 u_2^2 = m_1v_1^2 + m_2 v_2^2} \text{ ------------------(A)}$$

Also linear momentum is conserved .

Along X-axis

Total L.momentum before collision = Total linear momentum after collision ,

$m_1 u_1 + m_2 u_2 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$ ---------------(B)

Along Y-axis

Total linear momentum before collision = Total linear momentum after collision.

$0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2$

(Both the bodies are moving along X-axis).

In Elastic collision in 1 –D

Two bodies of masses m_1 & m_2 moving with initial velocity u_1 & u_2 along a single axis.

when $u_1 > u_2$, the two bodies will collide involving some loss in KE.

Suppose , After collision , final velocity of $m_1 = v_1$

" " " " " $m_2 = v_2$

Since , the two bodies form one system

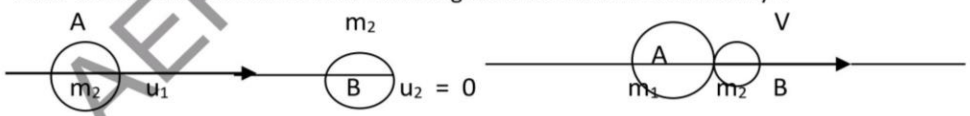
∴ Total linear momentum (B.C) = total linear momentum (A.C)

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Perfectly in elastic collision :--

Let the two Bodies of mass m_1 & m_2 where m_1 moving initially with a velocity of u_1 and m_2 is at rest i.e. ($u_2 = 0$)

After the collision the two bodies move together with a common velocity V



As momentum is conserved.

Therefore, Total linear momentum (B.C) = T. L momentum (A.C)

$m_1 u_1 + m_2 u_2 = MV$ where , $M = m_1 + m_2$

$m_1 u_1 + 0 = V(m_1 + m_2)$

$$V = \frac{m_1 u_1}{m_1 + m_2}$$

∴ $\frac{m_1}{m_1+m_2} < 1$

$$V < u_1$$

Loss in KE in perfectly inelastic Collision:--

Total KE (B .C) $= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 u_1^2 + 0$

$KE_1 = 1/2\, m_1 u_1^2$

Total KE (A.C) $= \frac{1}{2} m_2 v^2 + \frac{1}{2} m_1 v^2 = \frac{1}{2} v^2 [m_1 + m_2]$

∴ $KE_2 = \frac{1}{2} v^2 [m_1 + m_2] = \frac{1}{2} [m_1 + m_2] \frac{m_1^2 u_1^2}{[m_1 + m_2]^2}$ [since $V = \frac{m_1 u_1}{m_1 + m_2}$]

$$KE_2 = \frac{1}{2} \frac{m_1^2 u_1^2}{m_1 + m_2}$$

Loss in KE $= KE_1 - KE_2 = \frac{1}{2} m_1 u_1^2 - \frac{m_1^2 u_1^2}{2[m_1 + m_2]}$

$$= \frac{m_1 u_1^2 [m_1 + m_2] - m_1^2 u_1^2}{2[m_1 + m_2]}$$

$$= \frac{m_1^2 u_1^2 + m_1 m_2 u_1^2 - m_1^2 u_1^2}{2[m_1 + m_2]}$$

$$= \frac{m_1 m_2 u_1^2}{2[m_1 + m_2]}$$

Examples based on Work done by a Variable Force

◆ Formula Used

1. $W = \sum_i \vec{F_i} \cdot \vec{s_i}$

2. $W = \int_{S_1}^{S_2} \vec{F} \cdot d\vec{s}$

3. W = Area under the force-displacement curve between the initial and final positions of the body.

◆ Units Used

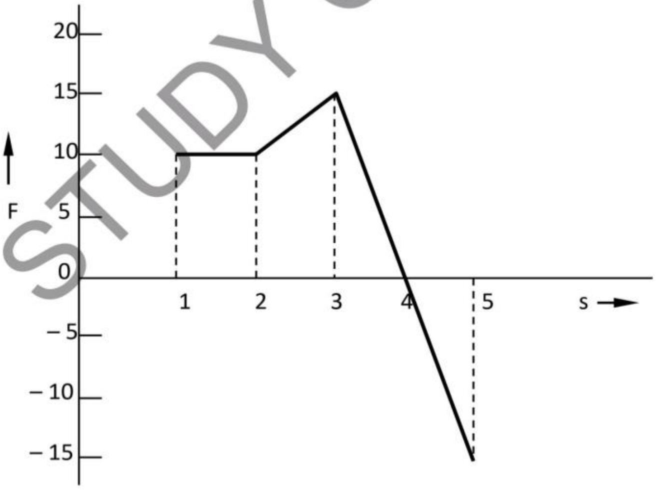
In SI, force f is in newton, distance s in metre and work done W in joule.

Q. 1. **A 2 kg particle starts at the origin and moves along the positive x-axis. The net force acting on it measured at intervals of 1 m is: 27.9, 28.3, 30.9, 34.0, 34.5, 46.9, 48.2, 50.0, 63.5, 13.6, 12.2, 32.7, 46.6 and 27.0 (in newtons). What is the total work done on the particle in this interval?**

Sol. As the forces and displacement are in same direction, so

$$W = \sum_i F_i s_i = 27.9 \times 1 + 28.3 \times 1 + 30.9 \times 1 + 34.0 \times 1 + 46.9 \times 1 + 48.2 \times 1 + 50.0 \times 1$$
$$+ 63.5 \times 1 + 13.6 \times 1 + 12.2 \times 1 + 46.6 \times 1 + 27.0 \times 1 = 496.3 \text{ J}$$

Q. 2. A body moves from point A to B under the action of a force, varying in magnitude as shown in Fig. Obtain the work done. Force is expressed in newton and displacement in metre.



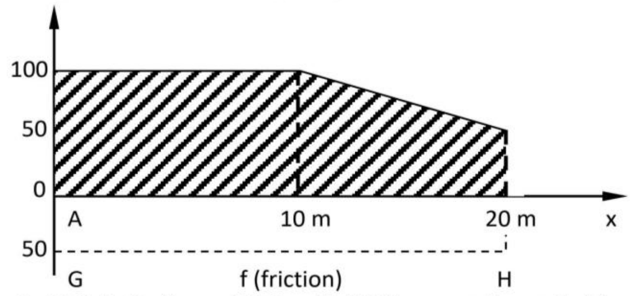
Sol. Work done = Area under F – s curve

$W_{AB} = W_{23} + W_{34} + W_{45}$ = Area under AP + Area under PQ + Area under QR – Area above RB

$= 10 \times 1 + ½ (10 + 15) \times 1 + ½ \times 1 \times 15 – ½ \times 1 \times 15 = 10 + 12.5 = 22.5$ J

Q. 3. **A woman pushes a trunk on railway platform which has a rough surface. She supplies a force of 100 N over a distance of 10 m. Thereafter she gets progressively tired and her applied force reduces linearly with distance to 50 N. the total distance by which the trunk has been moved is 20 m. Plot the force applied by the woman and the frictional force, which is 50 N. Calculate the work done by the two forces over 20 m.**

Sol. Plots of force F applied by the woman and the opposing frictional force F are shown in Fig.



Clearly, at x = 20 m, F = 50 N. As the force of friction f [= 50 N] opposes the applied force F, so it has been shown on the negative side of the force axis.

Work done by the force F applied by the woman

W_f = Area of rectangle ABCD + Are of trapezium CEID

$= 100 \times 10 + ½ (100 + 50) \times 10 = 1000 + 750 = 1750$ J

Work done by the frictional force,

W_f = Area of rectangle AGHI $= (– 50) \times 20 = 1000$ J

Q. 4.A particle moves along the X-axis from x = 0 to x = 5 m under the influence of a force given by F = 7 – 2x + 3x².
Find the work done in the process.

Sol. Work done in moving the particle from x = 0 to x = 5 m will be

$$W = \int_0^5 F dx = \int_0^5 (7 - 2x + 3x^2)\, dx$$

$$= \left(7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right)_0^5 = [7x - x^2 + x^3]_0^5 = 35 - 25 + 125 = 135 \text{ j}$$

Q. 5. A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ m^{-1/2}\ s^{-1}$. What is the work done by the net force during its displacement from x = 0 to x = 2 m.

Sol. Velocity, $v = ax^{3/2}$

Acceleration $= \frac{dv}{dt} = \frac{3}{2} ax^{1/2} \frac{dx}{dt} = \frac{3}{2} ax^{1/2} . v$

$= \frac{3}{2} ax^{1/2} . ax^{3/2}$

$= \frac{3}{2} a^2 x^2$

Force, $F = m \times$ acceleration $= \frac{3}{2} ma^2x^2$

Work done,

$W = \int_0^2 F\,dx = \frac{3}{2} \int_0^2 ma^2 x^2\,dx = \frac{3}{2} ma^2 \left[\frac{x^3}{3}\right]_0^2$

$= \frac{3 \times 0.5 \times (5)^2}{2 \times 3} [2^3 - 0^3] = 50$ J

Examples based on Work done by a Constant Force

- Formula Used

1. $W = \vec{F}.\vec{s} = Fs\cos\theta$
2. If a body of mass m is raised through height h, then W = mgh
3. If a body moves up a plane inclined at angle θ with a constant speed, then $W = mg\sin\theta \times s$

- Units Used

In SI, force F is in newton, distance s in metre and work done W in joule. In CGS system, force F is in dyne, distance s in cm and work done W in erg.

- Conversion Used

$1\ J = 10^7$ erg

Q. 1. A gardener pushes a lawn roller through a distance of 20 m. If he applies a force of 20 kg wt in a direction inclined at 60° to be ground, find the work done by him.

Sol. Here F = 20 kg wt = 20 × 9.8 N, s = 20 m, θ = 60°

$W = Fs\cos\theta = 20 \times 9.8 \times 20 \times \cos 60°$

$= 20 \times 9.8 \times 20 \times 0.5 = 1960$ J

Q. 2. A person is holding a bucket by applying a force of 10 N. He moves a horizontal distance of 5 m and then climbs up a vertical distance of 5 m and then climbs up a vertical distance of 10 m. Find the total work done by him.

Sol. For horizontal motion, the angle between force and displacement is 90°.

Here F = 10 N, s = 5 m, θ = 90°

Work done, $W_1 = F_s \cos\theta = 10 \times 5 \times \cos 90° = 0$

For vertical motion the angle between force and displacement is 0°.

Here F = 10 N, s = 10 m, θ = 0°

Work done, $W_2 = 10 \times 10 \times \cos 0° = 100$ J

Total work done $= W_1 + W_2 = 0 + 100 = 100$ J

Q. 3. A cyclist comes to a skidding stop in 10 m. During this process, the force on the cycle due to the road is 200 N and is directly opposed to the motion. (a) How much work does the road do on the cycle? (b) How much work does the cycle do on the road?

Sol. Work done on the cycle by the road is the work done by the stopping force of friction on the cycle due to the road.(a) The stopping force and the displacement make an angle of 180° with each other. Thus, work done by the road, or the work done by the stopping force is , $W_r = Fs\cos\theta = 200 \times 10 \times \cos 90° = -2000$ J

It is this negative work that brings the cycle to a halt.

(b) From Newton's Third law, an equal and opposite force acts on the road due to the cycle. Its magnitude is 200 N. However, the road undergoes no displacement.

∴ Work done by the cycle on the road = zero

Q. 4. A body constrained to move along the Z-axis of a co-ordinate system is subject to a constant force $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N, where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the X-, Y-, and Z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the Z-axis.

Sol. Here, $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N

As the body moves a distance of 4 m along Z-axis, so $\vec{s} = 4\hat{k}$ m

∴ $W = \vec{F}.\vec{s} = (-\hat{i} + 2\hat{j} + 3\hat{k}) . (4\hat{k})$

$= (-\hat{i} + 2\hat{j} + 3\hat{k}) . (0\hat{i} + 0\hat{j} + 4\hat{k})$

$= -1 \times 0 + 2 \times 0 + 3 \times 4 = 12$ J

Q. 5. A force $\vec{F} = \hat{i} + 5\hat{j} + 7\hat{k}$ acts on a particle and displaces it through $\vec{s} = 6\hat{i} + 9\hat{k}$. Calculate the work done if the force is in newton and displacement in metre.

Sol. $W = \vec{F} . \vec{S} = (\hat{i} + 5\hat{j} + 7\hat{k}) . (6\hat{i} + 0\hat{j} + 9\hat{k}) = 1 \times 6 + 5 \times 0 + 7 \times 9 = 69$ J

Q. 6. A force $\vec{F} = -K(y\hat{i} + x\hat{j})$, where K is a positive constant, acts on a particle moving in the XY plane. Starting from the origin, the particle is taken along the positive X-axis to a point (a, 0) and then parallel to the y-axis to the point (a, a). Calculate the total work done by the force on the particle.

Sol. Position vector of point (a, 0), $\vec{r_1} = a\hat{i} + o\hat{j}$

Position vector of point (a, a) $\vec{r_2} = a\hat{i} + a\hat{j}$

Displacement vector,

$\vec{r} = \vec{r_2} - \vec{r_1} = (a\hat{i} + a\hat{j}) - (a\hat{i} + o\hat{j}) = a\hat{j}$

Also, $\vec{F} = -K(y\hat{i} + x\hat{j})$

Work done, $W = \vec{F} . \vec{r} = -K(y\hat{i} + x\hat{j}) . a\hat{j} = -Kax$.

As x = a, so $W = -Ka^2$

Q. 7. A uniform chain of length 2 m is kept on a table such that as length of 60 cm hangs-freely from the edge of the table. The total mass of the chain is 4 kg. What is the workdone in pulling the entire chain on the table? Take $g = 10\ ms^{-2}$.

Sol. Mass of length 2 m of the chain = 4 kg

Mass of length 60 cm or 0.60 m of the chain $= \frac{4 \times 0.60}{2} = 1.2$ kg

Weight of the hanging part of the chain $= 1.2 \times 10 = 12$ N

As the centre of gravity of the hanging part lies at its mid-point, i.e., 30 cm or 0.30 m below the edge of the table, so the work required in pulling the hanging part on the table is W = force × distance = 12 × 0.30 = 3.6 J

Q. 8. A A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 ms^{-1}? 17

Sol. Whether the rain drop falls with decreasing acceleration or with uniform speed, the work done by gravitational force on the drop remains same.

Density of water, $\rho = 10^3$ kgm^{-3}

Here $r = 2$ mm $= 2 \times 10^{-3}$ m

Mass of rain drop, m = Volume × density $= \frac{4}{3}\pi r^3 \rho$

$= \frac{4}{3}\pi \times (2 \times 10^{-3})^3 \times 10^{-3} = \frac{32\pi}{3} \times 10^{-6}$ kg

Distance moved in each half journey, $h = \frac{500}{2} = 250$ m

$h = \frac{500}{2} = 250$ m

Work done by the gravitational force on the rain drop in each journey, $W = F \times s = mg \times h$

$= \frac{32\pi}{3} \times 10^{-6} \times 9.8 \times 250 = 0.082$ J

For entire journey,

Work done by gravitational force + Work done by resistive force = Gain in K.E.

$2 \times 0.082 + W_r = \frac{1}{2} mv^2$

or, $W_r = \frac{1}{2} \times \frac{32\pi \times 10^{-6} \times (10)^2}{3} - 0.164 = 0.0017 - 0.164 = -0.1623$ J

Q. 9. Calculate the work done in raising a stone of mass 5 kg and specific gravity 3, lying at the bed of a lake through a height of 5 m.

Sol. Specific gravity of stone $= \frac{\text{Mass of stone in air}}{\text{Mass of an equal volume of water}}$

or $3 = \frac{5\text{ kg}}{\text{Mass of an equal volume of water}}$

∴ Mass of an equal volume of of water $= \frac{5}{3}$ kg.

According to the Archimedes's principle,

loss in the weight of stone when immersed in water = weight of water displaced $= \frac{5}{3}$ kg wt

∴ Apparent weight of stone in the lake $= 5 - \frac{5}{3} = \frac{10}{3}$ kg wt

Force required to lift the stone in the lake, $F = \frac{10}{3}$ kg wt $= \frac{10 \times 9.8}{3}$ N

Work done in raising the stone through a height of 5 m, $W = F \times s = \frac{10 \times 9.8}{3} \times 5 = 163.3$ J

Q. 10. A cluster of clouds at a height of 1000 m above the earth burst and enough rain fell to cover an area of 10^6 m^2 with a depth of 2 cm. How much work would have been done in raising water to the height of clouds? Take = g = 9.8 ms^{-2} and density of water = 10^3 kgm^{-3}.

Sol. Area, $A = 10^6$ m^2 Depth, $d = 2$ cm $= 2 \times 10^{-2}$ m

∴Volume of water $= Ad = 10^6 \times 2 \times 10^{-2} = 2 \times 10^4$ m^3

Mass of water, m = Volume × density

$= 2 \times 10^4 \times 10^3 = 2 \times 10^7$ kg

Force used in raising water to the height of clouds.

F = Weight of water $= mg = 2 \times 10^7 \times 9.8$ N

Work done,

$W = Fs = 2 \times 10^7 \times 9.8 \times 1000 = 1.96 \times 10^{11}$ J

Q. 11. A locomotive of mass m starts moving so that its velocity varies according to the law v = α√s, where α is a constant and s is the distance covered. Find the total work done by all the forces acting on the locomotive during the first t seconds after the beginning of motion.

Sol. Velocity, $v = \alpha\sqrt{s} = \alpha s^{1/2}$

Acceleration, $a = \frac{dv}{dt} = \frac{1}{2}\alpha s^{-1/2} \cdot \frac{ds}{dt} = \frac{1}{2}\alpha x^{-1/2} \cdot v$

$= \frac{1}{2}\alpha s^{-1/2} \cdot \alpha s^{1/2} = \frac{1}{2}\alpha^2$

Force, $F = ma = \frac{1}{2} m\alpha^2$

Distance covered by the locomotive in first t seconds,

$s = ut + \frac{1}{2}at^2 = 0 \times t + \frac{1}{2} \times \frac{1}{2}\alpha^2 t^2 = \frac{1}{4}\alpha^2 t^2$

∴ Work done,

$W = Fs = \frac{1}{2} m\alpha^2 \times \frac{1}{4}\alpha^2 t^2 = \frac{1}{8} m\alpha^4 t^2$

Q12. A scooter skids on a road and stops after 20 m. The force on the scooter due to the road during the process of skidding is 100 N, due to which the scooter's motion is opposed. Calculate (i) The work done by the road on the scooter and (ii) Work done by the scooter on the road.

Solution:- Here, $F = 100$ N, $S = 20$m.

(i) Force F acts in a direction opposite to the displacement of the scooter.

∴ Work done by the road on the scooter, $W = FS\cos\theta = FS \cos 180^\circ = -FS$

$= -100 \times 20 = -2{,}000$ j

(ii) Scooter also exerts a force = 100 N on the road as per Newton's third law of motion. But road is not displaced from its position i.e. S = 0 ∴ Work done by the scooter on the road, $W = FS = 0$

Q13. A particle moves along x-axis from x = 0 to x = 4 m under the influence of force given by F = (5 – 3x + 2x^2) N. Calculate the work done by the force.

Solution:- Work done by a force F to displace the particle through a distance dx is given by $dW = F\,dx$ ....(1)

∴ Total work done to displace the particle from x = 0 to x = 4 m can be calculated by interpreting eqn. (1) between these limits i.e.

$$\int dW = \int_{x=0}^{x=4} F\,dx = \int_0^4 (5 - 3x + 2x^2)\,dx \quad (\text{since } F = 5 - 3x + 2x^2)$$

$$= \left[x\right]_0^4 - 3\left[\frac{x^2}{2}\right]_0^4 + 2\left[\frac{x^3}{3}\right]_0^4 = 5[4-0] - 3\left(\frac{16}{2} - 0\right) + 2\left(\frac{64}{3} - 0\right) = 20 - 24 + 42.67 = \mathbf{38.67\ j}$$

Q14. A bus weighing 10,000 kg is moving with a speed of 36 km h^{-1}. How much retarding force is required to stop this bus in a distance of 50 m? 18

Solution:- Here, m = 10,000 kg, $u = 36\ \text{km h}^{-1} = 36 \times \frac{5}{18} = 10\ \text{m s}^{-1}$, v = 0, S = 50 m.

$F \times S = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$, we get

$$F = \frac{\frac{1}{2}m(v^2 - u^2)}{S} = \frac{10{,}000(0 - 100)}{2 \times 50} = -10{,}000\ \text{N}$$

∴ Retarding force = **10,000 N.**

Conceptual tips……………………………………………………………………………………

- ☑ The work-energy theorem is not independent of Newton's second law. It may be viewed as scalar form of second law.
- ☑ The WE theorem holds in all inertial frames. It can be extended to non-inertial frames provided we include the pseudo force in the calculation of the net force acting on the body under consideration.
- ☑ When force and displacement are in same direction, the kinetic energy of the body increases. The increases in K.E. is equal to the work done on the body.
- ☑ When force and displacement are oppositely directed, the kinetic energy of the body decreases. The decrease in K.E. is equal to the work done by the body against the retarding force.
- ☑ When a body moves along a circular path with uniform speed, there is no change in its kinetic energy, By WE theorem, the work done by the centripetal force is zero.
- ☑ When K.E. increases, the work done is positive and when K.E. decreases, the work done is negative.
- ☑ In deriving the WE theorem, it has been assumed that the work done by the force is effective only in changing the K.E. of the body. However, the work done on a body may also be stored as the P.E. of the body.

Examples based on K.E. and W.E. theorem

- Formula Used
  1. Kinetic energy, $K = \frac{1}{2}mu^2$ 2. According to work-energy theorem, $W = K_f - K_i = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$
- Units Used Work done W, kinetic energies K_i and K_j are all in joule
- Conversions Used 1 eV = 1.60×10^{-19} J

Q. 1. A body of mass 4 kg initially at rest is subject to a force 16 N. What is the kinetic energy acquired by the body at the end of 10 s? [Delhi 2002]

Sol. Here m = 4 kg, F = 16 N, t = 10 s

Acceleration, $a = \frac{F}{m} = \frac{16}{4} = 4\ \text{ms}^{-2}$

∴ $v = u + at = 0\ 4 \times 10 = 40\ \text{ms}^{-1}$

The kinetic energy acquired by the body,

$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 4 \times (40)^2 = 3200$ J

Q. 2. A toy rocket of mass 0.1 kg has a small fuel of mass 0.02 kg which it burns out in 3 s. Starting from rest on a horizontal smooth track it gets a speed of 20 ms^{-1} after the fuel is burnt out. What is the approximate thrust of the rocket? What is the energy content per unit mass of the fuel? (Ignore the small mass variation of the rocket during fuel burning). [NCERT]

Sol. Here m = 0.1 kg, u = 0, v = 20 ms^{-1}, t = 3 s

Thrust of the rocket $= ma = m\frac{v-u}{t} = 0.1 \times \frac{20-0}{3} = \frac{2}{3}$ N

Kinetic energy gained by the rocket,

$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.1 \times (20)^2 = 20$ J

Energy content per unit mass of the fuel

$$\frac{\text{Total energy}}{\text{Mass of the fuel}} = \frac{20\ \text{J}}{0.02\ \text{kg}} = 1000\ \text{J kg}^{-1}$$

Q. 3. An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds. (Electron mass = 9.11×10^{-31} kg, proton mass = 1.67×10^{-27} kg, 1 eV = 1.60×10^{-19} J) [NCERT]

Sol. K.E. of the electron $= \frac{1}{2}m_e v_e^2 = 10$ ke V

K.E. of the proton $= \frac{1}{2}m_p v_p^2 = 100$ keV

∴ $\frac{\frac{1}{2}m_e v_e^2}{\frac{1}{2}m_p v_p^2} = \frac{10\ \text{ke V}}{100\ \text{ke V}} = \frac{1}{10}$

or $\frac{9.11 \times 10^{-31} \times v_e^2}{1.67 \times 10^{-27} \times v_p^2} = \frac{1}{10}$

or $\frac{v_e^2}{V_p^2} = \frac{1670}{9.11} = 183.3$

or $\frac{v_e}{v_p} = 13.53$

Thus the electron moves faster than the proton.

Q. 4. A bullet weighing 10 g is fired with a velocity of 800 ms^{-1}. After passing through a mud wall 1 m thick, its velocity decreases to 100 ms^{-1}. Find the average resistance offered by the mud wall. [NCERT]

Sol. Mass of bullet, m = 10 g = 0.01 kg

Velocity of bullet before passing through mud wall, u = 800 ms^{-1}

Velocity of bullet after passing through mud wall, v = 100 ms^{-1}

Distance covered by the bullet, s = 1 m

Let average resistance offered by the wall = F

According to work-energy theorem,

Work done by resistance offered by mud wall = Decrease in K.E.

or $Fs = \frac{1}{2}m(u^2 - v^2)$

$$\therefore F = \frac{m(u^2 - v^2)}{2s} = \frac{0.01 \times (800^2 - 100^2)}{2 \times 1} = 3150\ \text{N}$$

Q. 5. A shot travelling at the rate of 100 ms^{-1} is just able to pierce a plank 4 cm thick. What velocity is required to just pierce a plank 9 cm thick?

Sol. Here $v_1 = 100\ \text{ms}^{-1}$, $s_1 = 4$ cm, $v_2 = ?$ $s_2 = 9$ cm

K.E. lost by the shot = Work done against plank's resistance

∴ $\frac{1}{2}mv_1^2 = F \times s_1$ and $\frac{1}{2}mv_2^2 = F \times s_2$.

On dividing, $\frac{V_2^2}{V_1^2} = \frac{s_2}{s_1}$

or $\frac{v_2}{v_1} = \sqrt{\frac{s_2}{s_1}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

∴ $v_2 = \frac{3}{2} \times v_1 = \frac{3}{2} \times 100 = 150\ \text{ms}^{-1}$

Q. 6. In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with speed 200 ms^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10 % of its initial kinetic energy. What is the emergent speed of the bullet? [NCERT]

Sol. Here m = 50.0 g = 0.05 kg, u = 200 ms^{-1}

Initial K.E. $= \frac{1}{2}mu^2 = \frac{1}{2} \times 0.05 \times (200)^2 = 1000$ J

Final K.E. = 10 % of 1000 J $= \frac{10 \times 1000}{100} = 100$ J or $\frac{1}{2}mv^2 = 100$ J

$$v = \sqrt{\frac{2 \times 100}{m}} = \sqrt{\frac{2 \times 100}{0.05}} = 63.2\ \text{ms}^{-1}$$

Clearly, the speed reduces nearly by 68% and not by 90% by which the K.E. reduces.

Q. 7. ***It is well known that a raindrop or a small pebble falls under the influence of the downward gravitational force and the opposing resistive force. The latter is known to be proportional to the speed of the drop but is otherwise undetermined. Consider a drop of small pebble of mass 1.00 g falling from a cliff of height 1.00 km. It hits the ground with a speed of 50.0 ms^{-1}. What is the work done by the unknown resistive force?*** **[NCERT]**

Sol. We assume that the pebble is initially at rest on the cliff.

$\therefore$ $u = 0$, $m = 1.00\ g = 10^{-3}\ kg$, $h = 1.00\ km = 10^3\ m$, $v = 50\ ms^{-1}$.

The change in K.E. of the pebble is

$$\Delta K = \tfrac{1}{2} mv^2 - \tfrac{1}{2} mu^2 = \tfrac{1}{2} \times 10^{-3} \times (50)^2 - 0 = 1.25\ J$$

Assuming that $g = 10\ ms^{-2}$ is constant, the work done by the gravitational force is

If W_r is the work done by the resistive force on the pebble, then from the work-energy theorem,

$$\Delta K = W_g + W_r$$

or $W_r = \Delta K - W_g = 1.25 - 10.0 = -8.75\ J$

Q. 9. ***Two identical 5 kg blocks are moving with same speed of 2 ms^{-1} towards each other along a frictionless horizontal surface. The two blocks collide, stick together and come to rest. Consider the two blocks as a system. Calculate work done by (i) external forces and (ii) internal forces.*** **[CBSE 91]**

Sol. As no external forces are acting on the system, so $F_{ext} = 0$

$\therefore$ $W_{ext} = \vec{F}_{ext} \cdot \vec{s} = 0$

According to work-energy theorem,

Total work done = Change of K.E. = Final K.E. – Initial K.E.

or, $W_{ext} + W_{int} = 0 - [\ \tfrac{1}{2} mv^2 + \tfrac{1}{2} mv^2\ ] = -mv^2$

or $0 + W_{int} = -5 \times (2)^2 = -20$

$\therefore$ $W_{int} = -20\ J$

The negative sign indicates that internal forces of action and reaction act on the two blocks in a direction opposite to their motion.

Q. 10. ***If the linear momentum of a body increases by 20 %, what will be the % increase in the kinetic energy of the body?*** **[AFMC 97]**

Sol. Initial kinetic energy of the body,

$$K = \tfrac{1}{2} mv^2 = \tfrac{1}{2} \frac{m^2 v^2}{m} = \frac{p^2}{2m}$$

Increase in momentum = 20 % of p = $\frac{20}{100} \times p = \frac{p}{5}$

Final momentum = $p + \frac{p}{5} = \frac{6p}{5}$

Final kinetic energy of the body,

$$K' = \frac{(6p/5)^2}{2m} = \frac{36}{25} \frac{p^2}{2m} = \frac{36}{25} K$$

Increase in kinetic energy

$$= K' - K = \frac{36}{25} K - K = \frac{11}{25} K$$

% Increase in K.E

$$= \frac{K' - K}{K} \times 100 = \frac{11/25\ K}{K} \times 100 = 44\ \%$$

Q. 11. ***If the kinetic energy of a body increases by 300 %, by what % will the linear momentum of the body increase?*** **[Delhi 1995, 99]**

Sol. Initial kinetic energy,

$$K = \tfrac{1}{2} mv^2 = \tfrac{1}{2} \frac{m^2 v^2}{m} = \frac{p^2}{2m}$$

$\therefore$ Initial momentum, $p = \sqrt{2mK}$

Increase in kinetic energy = 300 % of K = 3 K

Final kinetic energy, $K' = K + 3K = 4\ K$

Final momentum

$$p' = \sqrt{2mK'} = \sqrt{2m \times 4K} = 2\sqrt{2mK} = 2p$$

% Increase in momentum

$$= \frac{p' - p}{p} \times 100 = \frac{2p - p}{p} \times 100 = 100\ \%$$

Q. 12. ***A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7N on a table with coefficient of kinetic friction = 0.1. Compute the (i) Work done by the applied force in 10 s, (ii) Work done by the friction in 10 s, (iii) Work done by the net force on the body in 10 s, and (iv) Change in kinetic energy of the body in 10 s. Interpret you results.*** **[NCERT]**

Sol. Here $m = 2\ kg$, $u = 0$, $F = 7\ N$, $\mu_k = 0.1$, $t = 10\ s$

Force of friction, $f_k = \mu_k R = \mu_k mg = 0.1 \times 2 \times 9.8 = 1.96\ N$

Net force with which the body moves,

$$F' = F - f_k = 7 = 1.96 = 5.04\ N$$

Acceleration, $a = \frac{F'}{m} = \frac{5.04}{2} = 2.52\ ms^{-2}$

Distance, $s = ut + \tfrac{1}{2} at^2 = 0 + \tfrac{1}{2} \times 2.52\ ms^{-2} \times (10)^2$

$= 126\ m$

(i) Work done by the applied force,

$$W_1 = Fs = 7 \times 126 = 882\ J$$

(ii) Work done by the friction,

$$W_2 = -f_k \times s = -1.96 \times 126 = -246.9\ J$$

(iii) Work done by the net force,

$$W_3 = F's = 5.04 \times 126 = 635\ J$$

(iv) final velocity acquired by the body after 10 s,

$$v = u + at = 0 + 2.52 \times 10 = 25.2\ ms^{-1}$$

Change in K.E. of the body

$$= \tfrac{1}{2} mv^2 - \tfrac{1}{2} mu^2 = \tfrac{1}{2} \times 2 \times (25.2)^2 - 0 = 635\ J$$

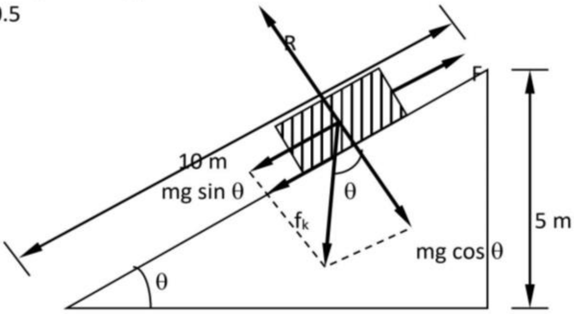
Thus, the change in K.E of the body is equal to the work done by the net force on the body.

Q. 13. ***A body of mass 0.3 kg is taken up an inclined plane to length 10 m and height 5 m, and then allowed to slide down to the bottom again. The coefficient of friction between the body and the plane is 0.15. What is the (i) Work done by the gravitational force over the round trip, (ii) Work done by the applied force over the upward journey, (iii) Work done by frictional force over the round trip, (Iv) Kinetic energy of the body at the end of the trip? How is the answer to (iv) related to the first three answers?*** **[NCERT]**

Sol. Here $m = 0.3\ kg$, $l = 10\ m$, $h = 5\ m$, $\mu_k = 0.15$

$\sin\theta = \frac{h}{l} = \frac{5}{10} = \frac{1}{2} = 0.5$

$\therefore$ $\theta = 30°$



(i) Work done by the gravitational force in moving the body up the inclined plane,

$W = Fs = -mg \sin\theta \times l = -0.3 \times 9.8 \times 0.5 \times 10 = -14.7$ J

Work done by the gravitational force in moving the body down the inclined plane,

$W' = Fs = +mg \sin\theta \times l = +14.7$ J

Work done by the gravitational force over the round trip,

$W_1 = W + W' = -14.7 + 14.7 = 0$

This is in conformity with the fact that the work done by the conservative force (e.g. gravitational force) over one round trip is zero.

(ii) Force needed to move the body up the inclined plane,

$F = mg \sin\theta + f_k = mg \sin\theta + \mu_k R$

$= mg \sin\theta + \mu_k mg \cos\theta = mg(\sin\theta + \mu_k \cos\theta)$

Work done by the applied force over the upward journey,

$W_2 = F \times l = mg(\sin\theta + \mu_k \cos\theta)\, l$

$= 0.3 \times 9.8\,(0.5 + 0.15 \times 0.866) \times 10 = 18.5$ J

(iii) Work done by the frictional force over the round trip,

$W_3 = -f_k(l + l) = -2 f_k + l$

$= -2\mu_k mg \cos\theta \times l$

$= -2 \times 0.15 \times 0.3 \times 9.8 \times \cos 30° \times 10 = -7.6$ J

(iv) Kinetic energy of the body at the end of round trip

= Work done by the net force in moving the body down the inclined plane

$= (mg \sin\theta - \mu_k \cos\theta)\, l$

$= mg(\sin\theta - \mu_k \cos\theta)\, l$

$= 0.3 \times 9.8 \times (0.5 - 0.13) = 10.9$ J

From answers to (i), (ii) and (iii), net work done on the body

$= 0 + 18.5 - 7.6 = 10.9$ J

= K.E. of the body

Thus kinetic energy of the body is equal to the net work done on the body.

Examples based on P.E. and Conservation of Energy

◆ **FORMULA USED**

1. Gravitational P.E., U = mgh
2. For a conservative force, $F = -\dfrac{dU}{dx}$
3. $\Delta U = U_f - U_i = -W = -\int_{x_i}^{x_f} F\,dx$
4. When work is done only by conservative forces only, mechanical energy is conserved.

◆ **FORMULA USED** Force is in newton and work done W, kinetic energy K and potential energy U are in Joule.

Q. 1. A vehicle of mass 15 quintal climbs up a hill 200 m high. It then moves on a level road with speed of 30 ms^{-1}. Calculate the potential energy gained by it and its total mechanical energy while running on the top of the hill.

Sol. Here m = 15 quintal = 1500 kg, g = 9.8 ms^{-2}, h = 200 m

P.E. gained, $U = mgh = 1500 \times 9.8 \times 200 = 2.94 \times 10^6$ J

When the vehicle runs on a level road with speed of 30 ms^{-1}, its K.E. is

$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 1500 \times (30)^2 = 0.675 \times 10^6$ J

Total mechanical energy, $U = K + U = (0.675 + 2.94) \times 10^6 = 3.615 \times 10^6$ J

Q. 2. A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 ms^{-1}. It hits the floor of the elevator (length of the elevator 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different, if the elevator were stationary? [NCERT]

Sol. As the elevator is moving down with a uniform speed (a = 0), so the value of g remains the same.

Here m = 0.3 kg, h = 3 m, g = 9.8 ms^{-2}

P. E. lost by the bolt = $mgh = 0.3 \times 9.8 \times 3 = 8.82$ J

As the bolt does not rebound, the energy is converted into heat.

∴ Heat produced = 8.82 J

Even if the elevator were stationary, the same amount of heat would have produced because the value of g is same in all inertial frames of reference.

Q. 3. Calculate the velocity of the bob of a simple pendulum at its mean position if it is able to rise to a vertical height of 10 cm. Take g = 9.8 ms^{-2}.

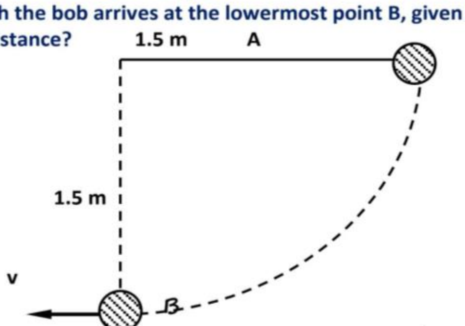
Sol. By conservation of energy,

K.E. of the bob at the mean position = P.E. of the bob at the highest position

or $\frac{1}{2}mv^2 = mgh$

∴ $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.10} = \sqrt{1.96} = 1.4\ ms^{-1}$.

Q. 4. The bob of a pendulum is released from a horizontal position A as shown. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point B, given that it dissipates 5 % of its initial energy against air resistance? [NCERT]



Sol. *Here, h = 1.5 m v = ?*

P.E. of the bob at A = mgh

K.E. of the bob at B = $\frac{1}{2}mv^2$

As 5 % of the P.E. is dissipated against air resistance, so

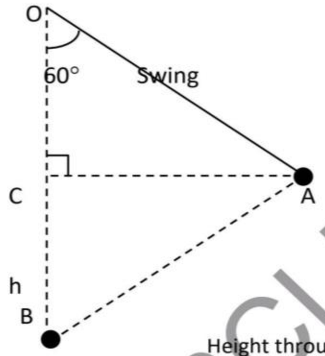
$\frac{1}{2}mv^2 = 95\%$ of mgh or $\frac{1}{2}mv^2 = \frac{95}{100} \times mgh$

or $\sqrt{\dfrac{2 \times 95 \times gh}{100}} = \sqrt{\dfrac{2 \times 95 \times 9.8 \times 1.5}{100}}$

$= \sqrt{27.93} = 5.3\ ms^{-1}$

Q. 5. A girl of mass 40 kg sits in a swing formed by a rope of 6 m length. A person pulls the swing to a side so that the rope makes an angle of 60° with the vertical. What is the gain in potential energy of the girl?

Sol. Here, m = 40 Kg, OA = OB = 6 m



From right ΔOCA,

$\frac{OC}{OA} = \cos 60°$

OC = OA cos 60° = 6 × ½ = 3 m

Height through which the girl is raised,

h = CB = OB – OC = 6 – 3 = 3 m

∴ P.E. gained by the girl

= mgh = 40 × 9.8 × 3 = 1176 J

Q. 6. A ball at rest is dropped from a height of 12 m. It loses 25 % of its kinetic energy in striking the ground, find the height to which it bounces. How do you account for the loss in kinetic energy?

Sol. K.E. gained by the ball in falling down = P.E. Lost by the ball in falling down = mgh

On bouncing upwards, the ball loses 25 % of its kinetic energy and the remaining 75 % changes back into potential energy. If the ball bounces to height h', then

mgh' = 75 % of mgh

∴ $mgh' = \frac{75}{100} \times mgh$

or h' = 0.75 h = 0.75 × 12 = 9 m.

The loss in kinetic energy of the ball occurs because a part of it changes into sound and heat.

Q. 7. A bullet of mass 0.012 kg and horizontal speed 70 ms⁻¹ strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also estimate the amount of heat produced in the block. [NCERT]

Sol. Mass of bullet, m = 0.012 kg, Speed of bullet, v = 70 ms^{-1}, Mass of block, M = 0.4 kg

If V is the velocity of the combination after collision, then from the law of conservation of momentum,

mv = (M + m) V

or $V = \frac{mv}{M+m} = \frac{0.012 \times 70}{0.4 + 0.012} = \frac{0.84}{0.412} = 2.04\ ms^{-1}$

Let h be the height through which the block rises. Then from the conservation of energy,

P.E. of the combination = K.E. of the combination

(M + m) gh = ½ (M + m) V²

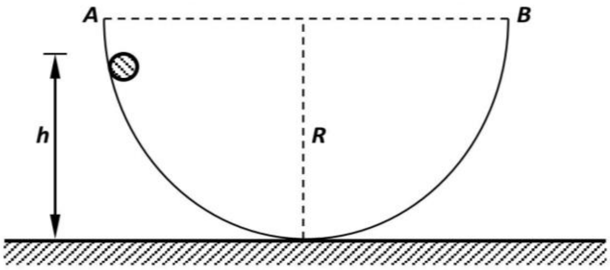
or $h = \frac{V^2}{2g} = \frac{(2.04)^2}{2 \times 9.8} = 0.212$ m

Amount of heat produced = Loss of K.E. of the bullet

= Initial K.E. of the bullet – K.E. of the combination= ½ mv² – ½ (M + m) V²

= ½ × 0.012 × (70)² – ½ × 0.412 × (2.04)² = 29.4 – 0.86 = 28.54 J

Q. 8. Fig. shows a frictionless hemispherical bowl of radius R. A ball of mass m is pushed down the wall from a point A. It just rises up to the edge of the bowl. Calculate the speed with which the ball is pushed down along the wall.



Sol. Let the ball be pushed down along the wall with a speed v. According to law of conservation of energy,

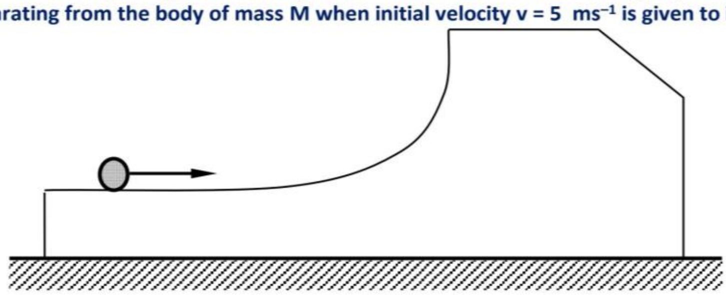
Total energy at A = Total energy at B

or ½ mv² + mgh = 0 + mgR

or v² = 2g (R – h)

or $v = \sqrt{2g(R-h)}$

Q. 9. A body of mass M = 9.8 kg with a small disc of mass m = 0.2 kg placed on its horizontal surface ab, rests on a smooth horizontal plane, as shown in Fig. The disc can move freely along the smooth groove abc of mass M. To what height (relative to the initial position) will the disc rise after separating from the body of mass M when initial velocity v = 5 ms⁻¹ is given to it in the horizontal direction?



Sol. Let V be the velocity with which the two bodies together move. By law of conservation of momentum,

$(M + m) V = mv$ or $V = \frac{mv}{M + m}$

But $m = 0.2$ kg, $M = 9.8$ kg, $v = 5\ ms^{-1}$

$\therefore$ $V = \frac{0.2 \times 5}{9.8 + 0.2} = 0.1\ ms^{-1}$

K.E. lost by the disc = ½ mv^2 = ½ $(M + m) V^2$

$= ½ \times 0.2 \times 25 - ½ (9.8 + 0.2) \times 0.01 = 2.45$ J

Suppose the disc rises to height h after separating from the body of mass M. Then

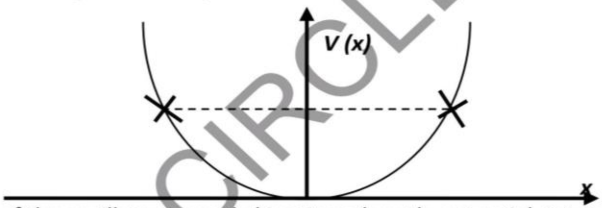
P.E. gained by the disc = mgh = $0.2 \times 9.8 \times h$ joule

By conservation of energy, P.E. gained = K.E. lost

$\therefore$ $0.2 \times 9.8 \times h = 2.45$

or $h = \frac{2.45}{0.2 \times 9.8} = 1.25$ m

Q. 10. ***The potential energy function for a particle executive linear simple harmonic motion is given by V (x) = kx²/2, where k is the force constant of the oscillator. For k = 0.5 Nm⁻¹, the graph of V (x) versus x is shown in Fig. Show that a particle of total energy 1 J moving under this potential must "turn back" when it reaches x = ±2 m.*** **[NCERT]**



Sol. At any instant, the energy of the oscillator is partly kinetic and partly potential. It total energy is

$E = K + V$ or $K = ½ mv^2 + ½ kx^2$

An oscillating particle turns back at the point where its instantaneous velocity is zero i.e., the particle will turn back at such a point x where v = 0

$\therefore$ $E = 0 + ½ kx^2$

But E = 1 J, $K = 0.5\ Nm^{-1}$

$\therefore$ $1 = ½ \times 0.5 \times x^2$ or $x^2 = 4$

or $x = \pm 2$m

Q. 11. ***A person trying to lose weight (dieter) lifts a 10 kg mass 0.5 m, 1000 times. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force? (b) Fat supplies 3.8 × 10⁷ J of energy per kilogram which is converted to mechanical energy with a 20 % efficiency rate. How much fat will the dieter use up?*** **[NCERT]**

Sol. (a) Here m = 10 kg, h = 0.5 m, n = 1000, $g = 9.8\ ms^{-2}$

Work done against the gravitational force, $W = n \times mgh = 1000 \times 10 \times 9.8 \times 0.5 = 49{,}000$ J

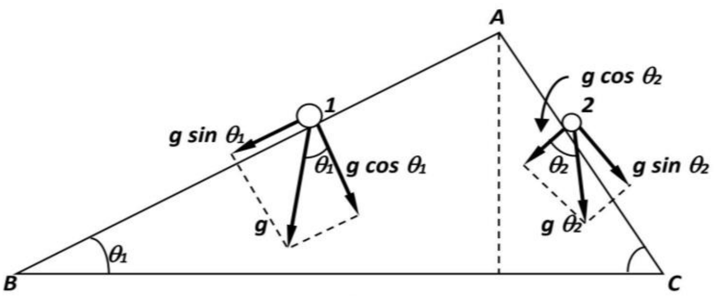
(b) Mechanical energy supplied by 1 kg of fat = 20 % of 3.8×10^7 of 3.8×10^7 J

$= 20 \times 3.8 \times 10^7 = 76 \times 10^5$ J

$\therefore$ Fat consumed for 76×10^5 J of energy = 1 kg

$= \frac{1 \times 49{,}000}{76 \times 10^5} = 6.45 \times 10^{-3}$ kg

Q. 12. ***Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track. Will the stones reach the bottom at the same time? Will they reach there with the same speed? Explain. Given θ₁ = 30°, θ₁ = 30°, θ₂ = 60° and h = 10 m, what are the speeds and times taken by the two stones?***



Sol. Let a_1 be the acceleration of the stone 1 down the inclined track AB, Then

$a_1 = g \sin \theta$

If the stone 1 takes time t_1 slide down the track AB, then

$AB = 0 + ½ a_1 t_1^2$ $[s = ut + ½ at^2]$

$\frac{h}{\sin \theta_1} = ½ g \sin \theta_1 t_1^2$ or $t_1^2 = \frac{2h}{g \sin^2 \theta_1}$

or $t_1 = \frac{1}{\sin \theta_1}\sqrt{\frac{2h}{g}}$

Similarly, for stone 2, we can write

$t_2 = \frac{1}{\sin \theta_2}\sqrt{\frac{2h}{g}}$

For both the stones, h is same.

As $\theta_1 < \theta_2$ $\therefore$ $\sin \theta_1 < \sin \theta_2$

Consequently, $t_1 > t_2$

Given $\theta_1 = 30°$, $\theta_2 = 60°$, h = 10 m, $g = 10\ ms^{-2}$

$\therefore$ $t_1 = \frac{1}{\sin \theta_1}\sqrt{\frac{2h}{g}} = \frac{1}{\sin 30°}\sqrt{\frac{2 \times 10}{10}} = 2\sqrt{2}$ s

$t_2 = \frac{1}{\sin \theta_2}\sqrt{\frac{2h}{g}} = \frac{1}{\sin 60°}\sqrt{\frac{2 \times 10}{10}} = 2\sqrt{\frac{2}{3}}$ s

$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 10} = \sqrt{2} \times 10$

$= 1.414 \times 10 = 14.14\ ms^{-1}$.

Thus, the stones 2 on the steeper plane AC reaches the bottom earlier than stone 1.

As both the stones are initially at the same height h, so

P.E. at A = K.E. at B or C

$mgh = ½ mv^2$; $v = \sqrt{2gh}$ i.e., both the stones will reach the bottom with the same speed.

Q. 15. A ball falls under gravity from a height of 10 m with an initial downward velocity u. It collides with the ground, loses 50 % of its energy in collision and then rises back to the same height. Find the initial velocity u. [IIT]

Sol. If m is the mass of the ball, then its total initial energy at height h = $\frac{1}{2}mu^2 + mgh$

Energy after collision = 50 % of $[\frac{1}{2}mu^2 + mgh] = \frac{1}{2}[\frac{1}{2}mu^2 + mgh]$

As the ball rebounds to height h, so

$$\frac{1}{2}[\frac{1}{2}mu^2 + mgh] = mgh \quad \text{or} \quad \frac{1}{4}mu^2 = \frac{1}{2}mgh$$

or $u = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 10} = 14\ ms^{-1}$

Q. 16. A particle of mass m is moving in a horizontal circle of radius r, under a centripetal force equal to $-(K/r^2)$, where K is constant. What is the total energy of the particle? [IIT]

Sol. As the particle is moving in horizontal circle, so

Centripetal force, $F = \frac{mv^2}{r} = \frac{K}{r^2}$

This gives $mv^2 = \frac{K}{r}$

∴ K.E. of the particle, $K = \frac{1}{2}mv^2 = \frac{K}{2r}$

As $F = -\frac{dU}{dr}$

∴ Potential energy,

$$U = -\int_{\infty}^{r} F dr = -\int_{\infty}^{r}\left(-\frac{K}{r^2}\right)dr = K\int_{\infty}^{r} r^{-2}\, dr = -\frac{K}{r}$$

Total energy = K + U = $\frac{K}{2r} - \frac{K}{r} = -\frac{K}{2r}$

CONCEPTUAL TIPS..

☑ The notion of potential energy applies to only those forces where the work done against the force gets stored up as energy by virtual of position or configuration of the body. When external constraints are removed, this energy appears as kinetic energy.

☑ The potential energy of a body subjected to a conservation force is uncertain upto a certain limit. This is because the point of zero potential energy is a matter of choice.

☑ For the gravitational P.E., the zero of potential energy is chosen to be the ground.

☑ For the spring potential energy $\frac{1}{2}kx^2$, the zero of the potential energy is the equilibrium position of the oscillating mass.

☑ Every mechanical force is not associated with a potential energy. The work done by friction over a closed path is not zero because no potential energy can be associated with friction.

Examples based on Potential Energy of a Spring

◆ **FORMULA USED**

1. According to Hooke's law, $F = -kx$ 2. Force constant, $k = \frac{F}{x}$

3. Work done on a spring or P.E. of a spring stretched through distance x, $W = U = \frac{1}{2}kx^2$

◆ **UNITS USED** Force F is in newton, distance x in metre, potential energy U in joule and force constant k in Nm^{-1}.

Q. 1. Two springs have force constants k_1 and k_2 ($k_1 > k_2$). On which spring is more work done, if (i) they are stretched by the same force and (ii) they are stretched by the same amount?

Sol. (i) Suppose the two springs get stretched by distance x_1 and x_2 by the same force F. Then

$$F = k_1 x_1 = k_2 x_2$$

$$\frac{W_1}{W_2} = \frac{\frac{1}{2}k_1 x_1^2}{\frac{1}{2}k_2 x_2^2} = \frac{k_1 x_1 . x_1}{k_2 x_2 . x_2} = \frac{F . x_1}{F . x_2} = \frac{x_1}{x_2} = \frac{k_2}{k_1}$$

As $k_1 > k_2$ ∴ $W_1 < W_2$ or $W_2 > W_1$

(ii) Suppose the two springs are stretched by the same distance x. Then

$$\frac{W_1}{W_2} = \frac{\frac{1}{2}k_1 x^2}{\frac{1}{2}k_2 x^2} = \frac{k_1}{k_2}$$

As $K_1 > K_2$ ∴ $W_1 > W_2$

Q. 2. The length of a steel wire increases by 0.5 cm when it is loaded with a weight of 5 kg. Calculate (i) force constant of the wire and (ii) work done in stretching the wire.

Sol. If a force F applied to a wire increases its length by x, then accordingly to Hooke's law,

$F = kx$

Where k is force constant

Given $F = mg = 5 \times 10 = 50$ N,

$x = 0.5$ cm $= 0.5 \times 10^{-2}$ lm

∴ $k = F = \frac{50}{0.5 \times 10^{-2}} = 1.0 \times 10^4\ Nm^{-1}$

(ii) Work done in stretching the wire,

$W = \frac{1}{2}kx^2 = \frac{1}{2} \times 1.0 \times 10^4 \times (0.5 \times 10^{-2})^2$

$= 0.125$ J

Q. 3. The potential energy of a spring when stretched through a distance x is 10 J. What is the amount of work done on the same spring to stretch it through an additional distance x? [AFMC 91]

Sol. P.E. of the spring when stretched through a distance x,

$U = \frac{1}{2}kx^2 = 10$ J

When x becomes 2x, the potential energy will be

$U' = \frac{1}{2}k(2x)^2 = 4 \times \frac{1}{2}kx^2 = 4 \times 10 = 40$ J

∴ Work done = U′ – U = 40 – 10 = 30 J

Q. 4. To simulate car accidents, auto manufacturers study the collisions of moving cars with mounted springs of different springs constants. consider a typical simulation with a car of mass 1000 kg moving with a speed 18.0 kmh^{-1} on a smooth road and colliding with a horizontally mounted spring of spring constant $6.25 \times 10^3\ Nm^{-1}$. What is the maximum compression of the spring? [NCERT]

Sol. Here m = 1000 kg, $k = 6.25 \times 10^3\ Nm^{-1}$, $v = 18\ kmh^{-1} = 18 \times 5\ ms^{-1} = 5\ ms^{-1}$

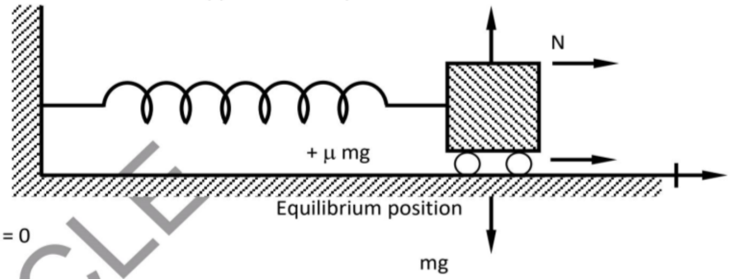
At the maximum compression x_m, the kinetic energy of the car is converted entirely into the potential energy of the spring. Therefore, Gain of P.E. of the spring = Loss in K.E. of the car

or $\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2$

or $x_m^2 = \frac{mv^2}{k} = \frac{1000 \times 5 \times 5}{6.25 \times 10^3} = 4$ ∴ $x_m = 2.0$ m

Q. 5. Consider example 50 taking the coefficient of friction, μ, to be 0.5 and calculate the maximum compression of the spring. [NCERT]

Sol. In presence of friction, both the spring force and the frictional force act so as to oppose the compression of the spring, as shown in Fig.



The change in K.E. of the car is

$\Delta K = K_j - K_i = 0 - ½\, mv^2$

The work done by the two opposing forces is

$W = ½\, kx_m^2 - \mu\, mgx_m$

By work-energy theorem, W = ΔK

∴ $½\, kx_m^2 + \mu\, mg\, x_m = ½\, mv^2$

or $kx_m^2 + 2\mu mgx_m - mv^2 = 0$

or $6.25 \times 10^3 x_m^2 + 2 \times 0.5 \times 1000 \times 10x_m - 1000\,(5)^2 = 0$

or $5x_m^2 + 8x_m - 20 = 0$

∴ $x_m = \dfrac{-8 \pm \sqrt{64 + 400}}{10}$

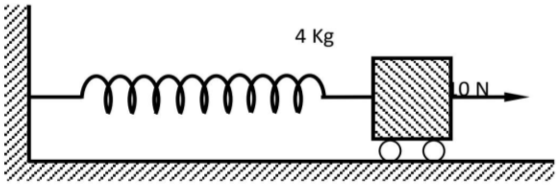
As x_m is positive, so

$x_m = \dfrac{-8 + 21.54}{10} = 1.354$ m

As expected, this value is less than the value obtained in the above example.

Q. 6. The spring shown in Fig. has a force constant of 24 Nm⁻¹. The mass of the block attached to the spring is 4 kg. Initially the block is at rest and spring is unstretched. The horizontal surface is frictionless. If a constant horizontal force of 10 N is applied on the block, then what is the speed of the block when it has been moved through a distance of 0.5 m?

Sol. Here $k = 24$ Nm^{-1}, m = 4 kg, x = 0.5 m, F = 24 N



By the law of conservation of energy,

Work done on the spring = Gain in K.E. + Gain in P.E.

or $Fx = ½\, mv^2 + ½\, kx^2$

or $10 \times 0.5 = ½ \times 4 \times v^2 + ½ \times 24 \times (0.5)^2$

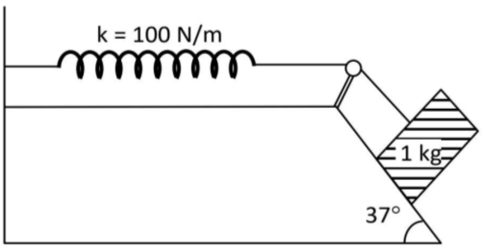
or $5 = 2v^2 + 3$ or $v^2 = 1$

∴ $v = 1$ ms^{-1}

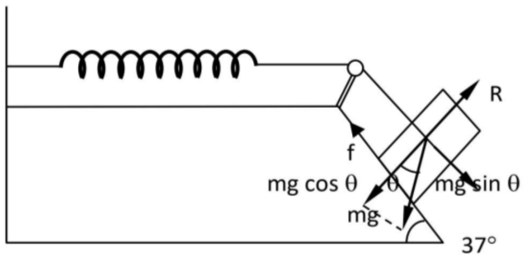
Q. 7. A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 N.m⁻¹ as shown in Fig. (a). the block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that spring has negligible mass and the pulley is frictionless. [NCERT]

Sol. Here m = 1 kg, k = 100 Nm^{-1}, g = 10 ms^{-2}

Clearly, from Fig. (b), we have $R = mg\cos\theta$; $f = \mu R = \mu mg\cos\theta$



(a)



(b)

Net force on the block down the incline

$= mg\sin\theta - f = mg\sin\theta - \mu\, mg\cos\theta$

$= mg(\sin\theta - \mu\cos\theta)$

Distance moved, x = 10 cm = 0.1 m

In equilibrium Work done = P.E. of stretched spring ∴

$mg(\sin\theta - \mu\cos\theta)\, x = ½\, kx^2$

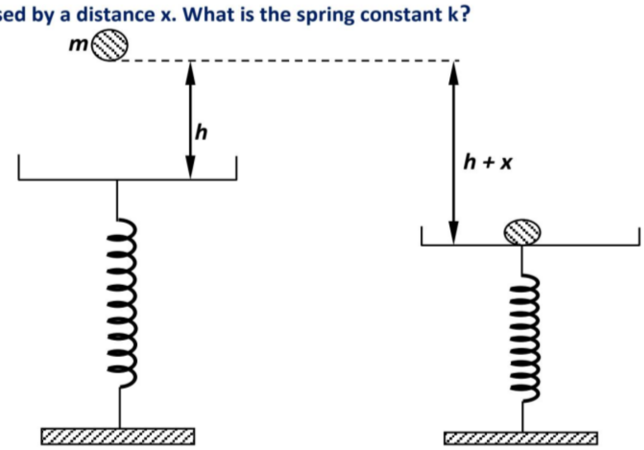
or $2\, mg(\sin\theta - \mu\cos\theta) = kx$

or $2 \times 1 \times 10\,(\sin 37° - \mu\cos 37°) = 100 \times 0.1$

or $20\,(0.601 - \mu \times 0.798) = 10$

$\mu = 0.126$

Q. 8. A ball of mass m is dropped from a height h on a platform fixed at a top of a vertical spring, as shown in Fig. The platform is depressed by a distance x. What is the spring constant k?



Sol. The ball falls through a total distance of (h + x)

∴ P.E. lost by the ball = mg (h + x)

Work done on the spring = $½\, kx^2$

By conservation of energy,

$½\, kx^2 = mg\,(h + x)$ ∴ $k = \dfrac{2\, mg\,(h + x)}{x^2}$

Q. 9. **A block of mass 2 kg initially at rest is dropped from a height of 1 m into a vertical spring having force constant 490 Nm^{-1}. Calculate the maximum distance through which the spring will be compressed.** 25

Sol. Here m = 2 kg, h = 1 m, k = 490 Nm^{-1}.

As shown in Fig. let the spring be compressed through distance x. Then the block falls through a height h + x

Get in P.E. of the spring = Loss in P.E. of the block

$\frac{1}{2}kx^2 = mg(h + x)$

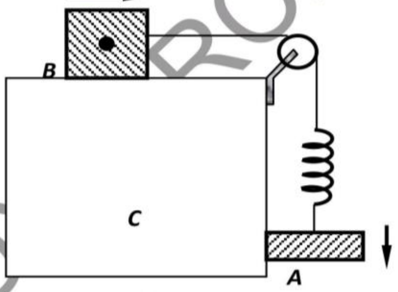
or $\frac{1}{2} \times 490 \times x^2 = 2 \times 9.8 \times (1 + x)$

or $12.5x^2 - x - 1 = 0$

∴ $x = \frac{1 \pm \sqrt{1 + 4 \times 12.5}}{2 \times 12.5} = \frac{1 \pm \sqrt{51}}{} = 0.3256$ m

Q. 10. **Two blocks A and B are connected to each other as shown in Fig. The string and spring is massless and pulley frictionless. Block B slides over the horizontal top surface of stationary block C and the block A slides along the vertical side of C both with same uniform speed. The coefficient of friction between the blocks is 0.2 and the spring constant of spring is 1960 Nm^{-1}. If mass of block A is 2 kg, calculate (i) the mass of block B and (ii) energy stored in spring.**

[IIT 82]



Sol. Various forces acting on the blocks A and B are shown in Fig.

Let mass of block B = m

Tension in the string = T

For block A: T = 2 g [∵ m = 2 kg]

For block B: T = f = μR = μmg = 0.2 × mg

∴ 0.2 × mg = 2g

or $m = \frac{2}{0.2} = 10$ kg

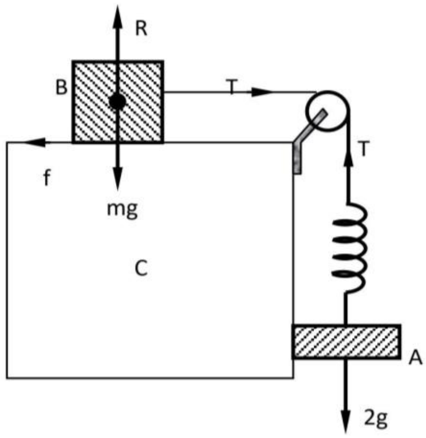
Also T = 2g = 2 × 9.8 = 19.6 N.

Let x be the extension of the spring due to the tension T. Then T = kx

or $x = \frac{T}{k} = \frac{19.6 \text{ N}}{1960 \text{ Nm}^{-1}} = 0.01$ m

Energy stored in the spring

$U = \frac{1}{2}kx^2 = \frac{1}{2} \times 1960 \times (0.01)^2 = 0.098$ J



Conceptual tips..

☑ **In the principle of conservation of energy, we include mass into total energy, because mass can be converted into energy.**

☑ **The principle of conservation of energy cannot be proved mathematically, but is an empirical principle. The deductions made on the basis of this principle are found to be true.**

Examples based on Mass-Energy Equivalence

◆ Formula Used

According to Einstein, energy equivalent of mass m is $E = mc^2$, where c = speed of light in free space = 3×10^8 ms^{-1}

◆ Units Used

Mass m is in kg and energy E in joule.

◆ Conversions Used

1. eV = 1.6×10^{-19} J   2. 1 MeV = 10^6 eV = 1.6×10^{-13} J   3. 1 amu= 931 MeV

Q. 1. **Express:**

(a) The energy required to break one bond (10^{-20} J) in DNA in eV. **(b) The kinetic energy of an air molecule (10^{-21}J) in eV.**

(c) The daily intake of a human adult (10^7 J) in kilocalories.

Sol. (a) 1 eV = 1.6×10^{-19} J

Energy required to break one bond in DNA

$= 10^{-20}$ J $= \frac{10^{-20}}{1.6 \times 10^{-19}}$ eV = 0.1 eV.

(b) 1 eV = 1.6×10^{-19} J

Kinetic energy of an air molecule

$= 10^{-21}$ J $= \frac{10^{-21}}{1.6 \times 10^{-19}} = 0.01$ eV.

(c) 1 Kcal = 4186 J

The average daily human consumption

$= 10^{-21}$ J $= \frac{10^7}{4186}$ kcal = 2389 k cal ≃ 2400 kcal

Q. 2. **How much mass is converted into energy per day in Tarapur nuclear power plant operated at 10^7 kW?**

Sol. Power, P = 10^7 kW = 10^{10} W = 10^{10} Js^{-1}

Time, t = 1 day = 24 × 60 × 60 s

Energy produced per day,

E = Pt = $10^{10} \times 24 \times 60 \times 60 = 864 \times 10^{12}$ J

As $E = mc^2$

∴ $M = \frac{E}{c^2} = \frac{864 \times 10^{12}}{(3 \times 10^8)^2}$

$= 9.6 \times 10^{-3}$ kg = 9.6 g.

Q. 3. **If 1000 kg of water is heated from 0° C to 100° C, calculate the increase in the mass of water.**

Sol. Here m = 1000 kg = 10^6 g, θ = 100 – 0 = 100° C

Specific heat of water, s = 1 cal g^{-1} °C^{-1}

Heat gained by water = ms θ × 10^6 × 1 × 100 = 10^8 cal = 4.2 × 10^8 J [∵ 1 cal = 4.2 J] 26

Increase in mass, $\Delta m = \frac{4.2 \times 10^8}{c^2} = \frac{4.2 \times 10^8}{(3 \times 10^8)^2} = 0.466 \times 10^{-8}$ kg.

Q. 4. Calculate the energy in MeV equivalent to the rest mass of an electron. Given that the rest mass of an electron, $m_0 = 9.1 \times 10^{-31}$ kg, 1 MeV = 1.6×10^{-13} J and speed of light, $c = 3 \times 10^8$ ms^{-1}

Sol. $E = m_0 c^2 = 9.1 \times 10^{-31} \times (3 \times 10^8)^2$

$= 81.9 \times 10^{-15}$ J

$= \frac{81.1 \times 10^{-15}}{1.6 \times 10^{-13}} = 0.512$ MeV.

Q. 5. Estimate the amount of energy released in the following nuclear fusion reaction:

$${}_1H^2 + {}_1H^2 \rightarrow {}_2He^3 + {}_0n^1$$

Given mass of ${}_1H^2$ = 2.0141 amu, mass of ${}_2He^3$ = 3.0160 amu, mass of ${}_0n^1$ = 1.0087 amu and 1 amu = 1.661×10^{-27} kg. Express you answer in units of MeV.

Sol. ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^3 + {}_0n^1$

Total initial mass (${}_1H^2 + {}_1H^2$) = 2.0141 + 2.0141 = 4.0282 amu

Total final mass (${}_2He^3 + {}_0n^1$) = 3.0160 + 1.0087 = 4.0247 amu

Decrease in mass,

$\Delta m = 4.0282 - 4.0247 = 0.0035$ amu

$= 0.0035 \times 1.661 \times 10^{-27}$ kg

∴ Energy released

$= \Delta m \times c^2 = 0.0035 \times 1.661 \times 10^{-27} \times (3 \times 10^8)^2$

$= 5.232 \times 10^{-13}$ J $= \frac{5.232 \times 10^{-13}}{1.6 \times 10^{-13}} = 3.27$ MeV

Q. 6. When slow neutrons are incident on a target containing ${}_{92}U^{235}$, a possible fission reaction is ${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_0n^1$ Estimate the amount of energy released using the following data: Given, mass of ${}_{92}U^{235}$ = 235.04 amu, mass of ${}_0n^1 \neq 1.0087$ amu, mass of ${}_{56}Ba^{141}$ = 140.91 amu, mass of ${}_{36}Kr^{92}$ = 91.926 amu and energy equivalent to 1 amu = 931 MeV [NCERT]

Sol. ${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_0n^1$

Total initial mass (${}_{92}U^{235} + {}_0n^1$) = 235.04 + 1.0087 = 236.0487 amu

Total final mass (${}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1$) = 140.91 + 91.926 + 3 × 1.0087 = 235.8621 amu

Decrease in mass, Δm = 236.0487 – 235.8621 = 0.1866 amu

Energy released = Δm × 931 = 0.1866 × 931 = 173.725 MeV.

Examples based on Power

- Formula Used

  1. Power = $\frac{Work}{Time}$ or $P = \frac{W}{t}$

  2. Also $P = \vec{F} \cdot \vec{v}$

  When θ = 0°, P = Fv

- Units Used: Work W is in joule, force F in newton, time t in second, velocity v in ms^{-1}, Power P in watt.
- Conversions Used: 1 Kilowatt = 1000 watt or 1 kW = 1000 W

  1 horsepower = 746 watt or 1 hp = 746 = 746 W.

Q. 1. A man weighing 60 kg climbs up a staircase carrying a load of 20 kg on his head. The stair case has 20 steps each of height 0.2 m. If he takes 10 s to climb, find his power.

Sol. Here m = 60 + 20 = 80 kg, h = 20 × 0.2 = 4 m ; g = 9.8 ms^{-2}, t = 10 s

$P = \frac{W}{t} = \frac{mgh}{t} = \frac{80 \times 9.8 \times 4}{10} = \frac{3136}{10} = 313.6$ W.

Q. 2. A car of mass 2000 kg is lifted up a distance of 30 m by a crane in 1 min. A second crane does the same job in 2 min. Do the cranes consume the same or different amounts of fuel? What is the power supplied by each crane? Neglect power dissipation against friction.

Sol. Here m = 2000 kg, s = 30 m, t_1 = 1 min = 60 s, t_2 = 2 min = 120 s

Work done by each crane, W = Fs = mgs = 2000 × 9.8 × 30 = 5.88 × 10^5 J

As both the cranes do same amount of work, so both consume same amount of fuel.

Power supplied by first crane,

$P_1 = \frac{W}{t_1} = \frac{5.88 \times 10^5}{60} = 9800$ W

Power supplied by second crane,

$P_2 = \frac{W}{t_2} = \frac{5.88 \times 10^5}{120} = 4900$ W

Q. 3. A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m³ in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 3 %, how much electric power is consumed by the pump?

Sol. Mass of water = Volume × density

$= 30 \times 1000 = 3 \times 10^4$ kg

∴ Output power = $\frac{\text{Work done}}{\text{Time}} = \frac{mgh}{t}$

$= \frac{3 \times 10^4 \times 9.8 \times 40}{15 \times 60} = \frac{39200}{3}$ W

As Efficiency = $\frac{\text{Output Power}}{\text{Input power}} \times 100$

∴ Input power = $\frac{\text{Output power}}{\text{Efficiency}} \times 100$

$= \frac{39200}{3 \times 30} \times 100$

$= 43.6 \times 10^3$ W = 43.6 kW

Q. 4. The human heart discharges 75 ml of blood at each beat against a pressure of 0.1 m of Hg. Calculate the power of heart assuming that pulse frequency is 80 beats per minute. Density of Hg = 13.6 × 10³ kgm⁻³.

Sol. Volume of blood discharged per beat,

V = 75 ml = 75 × 10^{-6} m^{-3}

Pressure, P = 0.1 m of Hg

= 0.1 × 13.6 × 10^3 × 9.8 Nm^{-2} [∵ P = hρg]

Work done per beat = PV

Work done in 80 beats = 80 × PV

Time, t = 1 min = 60 s

Power = $\frac{Work}{Time} = \frac{80 \times PV}{t}$

$= \frac{80 \times 0.1 \times 13.6 \times 10^3 \times 9.8 \times 75 \times 10^{-6}}{60}$

= 1.33 W.

Q. 5. An electric motor is used to left an elector and its load (total mass = 1500 kg) to a height of 20 m. The time taken for the job is 20 s. What is the work done? What is the rate at which work is done? If the efficiency of the motor is 75%, at which rate is the energy supplied to the motor?

Sol. Here m = 1500 kg, h = 20 m, η = 75 %, t = 20 s

Work done,

W = mgh = 1500 × 9.8 × 20 = 2.94 × 10^5 J

Rate of doing work = $\frac{W}{t} = \frac{2.94 \times 10^5}{20} = 1.47 \times 10^4$ W

As $\eta = \frac{\text{Output power}}{\text{Input power}}$

∴ $\frac{75}{100} = \frac{1.47 \times 10^4}{\text{Input power}}$

Input power or the rate at which energy is supplied

$= \frac{1.47 \times 10^4 \times 100}{75} = 1.96 \times 10^4$ W.

Q. 6. Calculate the horse power of a man who can chew ice at the rate of 30 g per minute. Given 1 hp = 746 W and J = 4.2 J cal^{-1}. 27

Sol. Mass of ice chewed by man, m = 30 g
Latent heat of ice, L = 80 cal g^{-1}
Heat required to melt ice, H = mL = 30 × 80 cal
Work done, W = JH = 4.2 × 30 × 80 cal
Time taken, t = 1 min = 60 s

Power, $P = \frac{W}{t} = \frac{4.2 \times 30 \times 80}{60} = 168$ W

$= \frac{168}{746} = 0.225$ hp.

Q. 7. **A machine gun fires 60 bullets per minute with a velocity of 700 ms^{-1}. If each bullet has mass of 50 g, find the power developed by the gun.**

Sol. Mass of 60 bullets = 60 × 50 = 3000 g = 3 kg
v = 700 ms^{-1}, t = 1 min = 60 s

Power $= \frac{W}{t} = \frac{K.E.}{t} = \frac{1}{2} \cdot \frac{mv^2}{t} = \frac{3 \times (700)^2}{2 \times 60} = 12250$ W

Q. 8. **An elevator which can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of 2 ms^{-1}. The frictional force opposing the motion is 4000 N. Determine the minimum power delivered by the motor to the elevator in watts as well as in horse** **[NCERT Delhi 03C]**

Sol. The downward force on the elevator is $F = mg + F_f = 1800 \times 10 + 4000 = 22000$ N
The motor must supply enough power to balance this force. Hence,
P = Fv = 22000 × 2 = 44000 W
$= \frac{44000}{746}$ hp ≃ 59 hp

Q. 9. **The blades of a windmill sweep out a circle of area A. (a) if the wind flows at a velocity v perpendicular to the circle, whatis the mass of the air passing through it in time t? (b) What is the kinetic energy of the air? (c) Assume that the windmill converts25 % of the wind's energy into electrical energy, and that A = 30 m^2, v = 36 km/h and the density of air is 1.2 kg m^{-3}. What is the electrical power produced?** **[NCERT]**

Sol. (a) Volume of the air passing through the windmill in time t
= Area of circle × distance covered by wind in time t
= A × vt = Avt
Mass of the air passing through the windmill in time t,
m = Density × volume = ρAvt.
(b) Kinetic energy of the air is
$K = \frac{1}{2} mv^2 = \frac{1}{2} \rho A v t \times v^2 = \frac{1}{2} \rho A v^3 t$.

(c) K.E. of air converted into electrical energy in time t
$K' = 25\%$ of $K = \frac{25}{100} \times \frac{1}{2} \rho A v^3 t = \frac{1}{8} \rho A v^3 t$
Electrical power produced
$= \frac{K'}{t} = \frac{1}{8} \rho a v^3$
$= 1/8 \times 1.2 \times 30 \times (10)^3$

Q. 10. ***A large family uses 8 kW of power (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20 % of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW? (b) Compare this area to that of the roof of a house constructed on a plot of size 20 m × 15 m with a permission to cover upto 70 %*** **[NCERT, Delhi 2003]**

Sol. (a) Let the area needed to supply 8 kW = A m^2
Energy incident per unit area = 200 W
Energy incident on area A = 200 × A W
Energy converted into useful electrical energy = 20 % of 200 × A = 40 A W
But 40 A W = 80 kW = 8000 W
or $A = \frac{8000}{40} = 200$ m^2

(b) Area of the roof of the given house,
$A' = 70\%$ of 20 m × 15 m $= \frac{70 \times 20 \times 15}{100} = 210$ m^2
Required ratio $= \frac{A}{A'} = \frac{200}{210} = 20 : 21$

Q. 11A well 20 m deep and 3 m in diameter contains water to a depth of 14 metre. How long will a 5 hp engine take a empty it?

Sol. Radius of the well, $r = \frac{3}{2}$ m

Area of cross-section of the well $= \pi r^2 \frac{22}{7} \times \left(\frac{3}{2}\right)^2 = \frac{99}{14}$ m^2

Volume of water in the well = Are of cross-section × depth $= \frac{99}{14} \times 14 = 99$ m^3

∴ Mass of water in the well = Volume × density
$= 99 \times 10^3$ kg

As the well is emptied, the height through which water has to be raised by the engine chages from (20 – 14) m in the beginning to (20 – 0) m at the end.

∴ Average height raised $= \frac{6 + 20}{2} = 13$ m

Work required to empty the well, $W = mgh = 99 \times 10^3 \times 9.8 \times 13 = 12612600$ J

Power, P = 5 hp = 5 × 746 W

Required time, $t = \frac{W}{P} = \frac{12612600}{5 \times 746} = 3381.6$ s

Q. 12. ***The turbine pits at the Niagra falls are 50 m deep. The average horse power developed is 500. If the efficiency of the generator is 85%; how much water passes through the turbines per minute? Take g = 10 ms^{-2}.***

Sol. Useful power developed = 5000 hp
Efficiency = 85 %
∴ Total power generated
$= \frac{100}{85} \times 5000$ hp
$= \frac{100 \times 5000 \times 746}{85}$ W

Total work done by the falling water in 1 min or 60 s,
$W = Pt = \frac{100 \times 5000 \times 746}{85} \times 60 = 26.94 = 10^2$ J

Now, mgh = W
∴ $m = \frac{W}{gh} = \frac{26.94 \times 10^7}{10 \times 50} = 5.39 \times 10^5$ kg.

Q. 13. ***A man cycles up a hill, whose slope is 1 in 20 with a velocity of 6.4 kmh^{-1} along the hill. The weight of the mass and the cycle is 98 kg. What work per minute is he doing? What is his horse power?*** ***28***

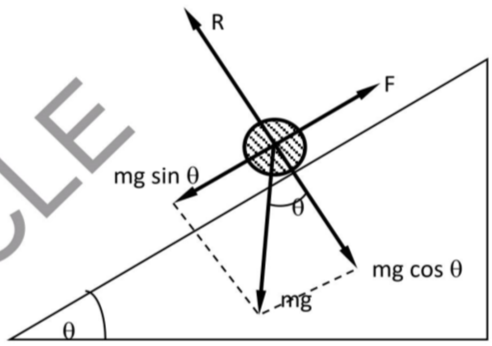
Sol. Refer to Fig. given below. If the inclination of the hill with the horizontal is θ, then

$\sin\theta = \frac{1}{20}$

$v = 6.4 \text{ kmh}^{-1} = \frac{6.4 \times 5}{18} \text{ ms}^{-1}$

$= \frac{16}{9} \text{ ms}^{-1}$

m = 98 kg, t = 1 min = 60 s



As the velocity of the cyclist is uniform, so the only force he has to exert is against gravity. It is given by

$F = mg \sin\theta$

Power of the man,

$P = Fv = mg \sin\theta \times v$

$= 98 \times 9.8 \times \frac{1}{20} \times \frac{16}{9}$ W

$= \frac{98 \times 9.6 \times 16}{746 \times 20 \times 9}$ hp = 0.144 hp

Work done per minute, $W = Pt = \frac{98 \times 9.8 \times 16}{20 \times 9} \times 60 = 5122.1$ J

conceptual tips..

☑Total linear momentum is conserved at each instant of every collision.
☑Total energy is conserved in all collisions.
☑The total kinetic energy may or may not be conserved during a collision.
☑Even for an elastic collision, the kinetic energy conservation holds after the collision is over an does not hold at every instant of the collision.
☑When two bodies collide; they get deformed and may be momentarily at rest with respect to each other.
☑The impact and deformation during a collision may convert part of the initial kinetic energy into heat and sound.
☑At reach instant of the collision, the total kinetic energy and total linear momentum are both conserved in elastic as well as inelastic collisions.
☑In an elastic collision, the kinetic energy conservation holds only after the collision is over. It does not hold during the short duration of actual collision.
☑At the time of collisions, the two colliding objects are deformed and may be momentarily at rest with respect to each other.
☑When two equal masses suffer a glancing collision with one of them at rest, after the collision. the two masses move at right angles to each other.

Examples based on Collisions

◆ **FORMULA USED**

1. Linear momentum is conserved both in elastic and inelastic collisions/ $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$
2. Kinetic energy is conserved in elastic collisions. $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 r_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$
3. In one-dimensional elastic collision, velocities after the collision are given by

$v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2 m_2}{m_1 + m_2} \cdot u_2$

$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 + m_1}{m_1 + m_2} \cdot u_2$

4. Coefficient of restitution for a collision is given by $e = -\frac{v_1 - v_2}{u_1 - u_2} = \frac{|v_1 - v_2|}{|u_1 - u_2|}$
5. For a ball rebounding from a floor, $e = \frac{v}{u}$
6. For an elastic collision (involving no loss of K.E.), e = 1
7. For an inelastic collision (involving loss of K.E.), e < 1

◆ **UNITS USED**

Masses m_1, m_2 are in kg, velocities u_1, u_2, v_1, v_2 are in ms^{-1}, linear momenta in kg ms^{-1}, kinetic energy in joule and coefficient of restitution 'e' has no units.

Q. 1. ***Two bodies of masses 5 kg and 3 kg moving in the same direction along the same straight line with velocities 5 ms^{-1} and 3 ms^{-1} respectively suffer one-dimensional elastic collision. Find their velocities after the collisions.***

Sol. Here m_1 = 5 kg, u_1 = 5 ms^{-1}, m_2 = 3 kg, u_2 = 3 ms^{-1}

∴ $v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$

$= \frac{5 - 3}{5 + 3} \times 5 + \frac{2 \times 3}{5 + 3} \times 3$

$= \frac{2}{8} \times 5 + \frac{6}{8} \times 3 = \frac{5}{4} + \frac{9}{4} = \frac{14}{4} = 3.5 \text{ ms}^{-1}$.

$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$

$= \frac{2 \times 5}{5 + 3} \times 5 + \frac{3 - 5}{5 + 3} \times 3$

$= \frac{50}{8} - \frac{6}{8} = \frac{44}{8} = 5.5 \text{ ms}^{-1}$

Q. 2. ***A 10 kg ball and 20 kg ball approach each other with velocities 20 ms⁻¹ and 10 ms⁻¹ respectively. What are their velocities after collision if the collision is perfectly elastic?***

Sol. Here $m_1 = 10$ kg, $m_2 = 20$ kg, $u_1 = 20\ ms^{-1}$, $u_2 = -10\ ms^{-1}$

$u_1 = 20\ ms^{-1}$, $u_2 = -10\ ms^{-1}$

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} . u_1 + \frac{2m_2}{m_1 + m_2} . u_2$$

$$= \frac{10 - 20}{10 + 20} \times 20 + \frac{2 \times 20}{10 + 20} \times (-10)$$

$$= -\frac{20}{3} - \frac{40}{30} = -\frac{60}{3} = -20\ ms^{-1}$$

$$v_2 = \frac{2m_1}{m_1 + m_2} . u_1 + \frac{m_2 - m_1}{m_1 + m_2} . u_2$$

$$= \frac{2 \times 10}{10 + 20} \times 20 + \frac{20 - 10}{10 + 20} \times (-10)$$

$$= \frac{40}{3} - \frac{10}{3} = \frac{30}{3} = 10\ ms^{-1}$$

Q. 3. ***Two ball bearings of mass m each moving in opposite directions with equal speeds v collide head on with each other. Predict the outcome of the collision, assuming it to be perfectly elastic.*** **[NCERT]**

Sol. Here $m_1 = m_2 = m$ (say), $u_1 = v$, $u_2 = -v$

As the collision is perfectly elastic, velocities after the collision will be

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} . u_1 + \frac{2m_2}{m_1 + m_2} . u_2$$

$$= \frac{m - m}{m + m} . v + \frac{2m}{m + m} (-v)$$

$$= 0 - v = -v$$

$$v_2 = \frac{2m_1}{m_1 + m_2} . u_1 + \frac{m_2 - m_1}{m_1 + m_2} . u_2$$

$$= \frac{2m}{m + m} . v + \frac{m - m}{m + m} . (-v)$$

$$= v + 0 = v$$

Thus the two balls bounce back with equal speeds after the collision

Q. 4. ***A trolley of mass 200 kg moves with a uniform speed of 36 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 m s⁻¹ relative to a trolley in a direction opposite to the trolley's motion, and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?*** **[NCERT]**

Sol. The child gives an impulse to the trolley at the start and then runs with a constant relative velocity of 4 ms⁻¹ with respect to the trolley's new velocity.

Total initial momentum, $p_i = (m_1 + m_2)\, u_1$

$$= (20 + 200) \times \frac{36 \times 5}{18} = 2200\ kg\ ms^{-1}$$

Let new velocity of the trolley = v_2

Child's velocity relative to the trolley in opposite direction = 4 ms⁻¹

Child actual velocity (relative to ground) = $v_2 - 4$

Total final momentum, $p_f = m_1 v_1 + m_2 v_2$

$= 20\,(v_2 - 4) + 200\, v_2 = 220\, v_2 - 80$

By conservation of linear momentum,

$p_f = p_i$

$220\, v_2 - 80 = 2200$

$\therefore \quad v_2 = \frac{2280}{220} = 10.36\ ms^{-1}$

Time taken by the child to cover length of the trolley

$$= \frac{10\ m}{4\ ms^{-1}} = 2.5\ s$$

Distance covered by the trolley in 2.5 s

$= \quad 10.36 \times 2.5 = 25.9$ m.

Q. 5. ***A railway carriage of mass 9000 kg moving with a speed of 36 kmh⁻¹ collides with a stationary carriage of the same mass. After the collision, the carriages get coupled and move together. What is their common speed after collision? What type of collision is this?***

Sol. Here $m_1 = 9000$ kg, $u_1 = 36\ kmh^{-1} = 10\ ms^{-1}$

$m_2 = 9000$ kg, $u_2 = 0$, $v_1 = v_2 = v = ?$

By conservation of momentum,

$m_1 u_1 + m_2 u_2 = (m_1 + m_2)\, v$

$9000 \times 10 + 0 = (9000 + 9000)\, v$

or $\quad v = \frac{90000}{18000} = 5\ ms^{-1}$

$= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$

$= \frac{1}{2} \times 9000 \times 10 \times 10 + 0$

$= 450000$ J

Total K.E. after collision = $\frac{1}{2} (m_1 + m_2)\, v^2$

$= \frac{1}{2} \times 2 \times 9000 \times 5^2$

$= 225000$ J

Thus total K.E. after collision < Total K.E. before collision. Hence, the collision is inelastic.

Q. 6. ***What percentage of kinetic energy of a moving particle is transferred to a stationary particle, when moving particle strikes with a stationary particle of mass (i) 9 times in mass (ii) equal in mass and (iii) 1/19th of its mass?***

Sol. For the moving particle, $m_1 = m$ (say), initial vel. = u_1

For the stationary particle, $m_2 = xm$ (say), initial vel. = $u_2 = 0$

As $v_2 = \frac{2m_1}{m_1 + m_2} . u_1 + \frac{m_2 - m_1}{m_1 + m_2} . u_2$

$\therefore \quad v_2 = \frac{2m}{m + xm} . u_1 + 0 = \frac{2u_1}{1 + x}$

K.E. of the moving particle before collision,

$K_1 = \frac{1}{2} m_1 u_1^2 = \frac{1}{2} m u_1^2$

K.E. of the stationary particle after collision,

$$K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} . mx . \frac{4u_1^2}{(1 + x)^2}$$

$$= \frac{4x}{(1 + x)^2} . \frac{1}{2} m u_1^2 = \frac{4x}{(1 + x)^2} . K_1$$

% of K.E. transferred

$$= \frac{K_2}{K_1} \times 100 = \frac{4x\, K_1}{(1 + x^2)} \times \frac{100}{K_1} = \frac{4x}{(1 + x)^2} \times 100\ \%$$

(i) When moving particle strikes with a stationary particle 9 times in mass, x = 9

$$\%\ \text{of K.E. transferred} = \frac{4 \times 9}{(1 + 9)^2} \times 100 = 36\ \%$$

(ii) When moving particle strikes with a stationary particle of equal mass, x = 1

$$\%\ \text{of K.E. transferred} = \frac{4 \times 1}{(1 + 1)^2} \times 100 = 100\ \%$$

(iii) When moving particles strikes a stationary particle 1/19th of its mass, x = 1/19

$$\%\ \text{of K.E. transferred} = \frac{4 \times (1/19)}{(1 + 1/19)^2} \times 100 = 19\ \%$$

Q. 7. ***Slowing down of neutrons: In a nuclear reactor a neutron of high speed (typically 10^7 ms^{-1}) must be slowed to 10^3 ms^{-1} so that it can have a high probability of interacting with isotope $^{235}_{92}U$ and causing it to fission. Show that a neutron can lose most of its kinetic energy in an elastic collision with a light nucleus like deuterium or carbon which has a mass of only a few times the neutron mass. The material making up the light nuclei, usually heavy water (D_2O) or graphite is called a moderator. Or [NCERT] 30***
A body of mass M at rest is struck by a moving body of mass m. Prove that fraction of the initial K.E. of the mass m transferred to the struck body is 4 m M/(m + M)² in an elastic collision.

Sol. Here m_1 = mass of neutron = m

m_2 = mass of target nucleus = M

$u_1 = u$ and $u_2 = 0$

Now $v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$

$= \frac{2m}{m + M} \cdot u + 0 = \frac{2mu}{m + M}$

Initial K.E. of mass m, $K_1 = ½\, m_1u_1^2 = ½\, mu^2$

Final K.E. of mass M,

$K_2 = ½\, m_2v_2^2 = ½\, M\left(\frac{2mu}{m + M}\right)^2 = \frac{2\,Mm^2\,u^2}{(m + M)^2}$

Fraction of the Initial K.E. transferred,

$f = \frac{K_2}{K_1} = \frac{2\,Mm^2\,u^2}{(m + M)^2} \times \frac{2}{mu^2} = \frac{4\,mM}{(m + M)^2}$

(i) For denterium, M = 2 n, therefore,

$f = \frac{4m \times 2m}{(m + 2m)} = \frac{8}{9} \simeq 0.9$

About 90 % of the neutron's energy is transferred to deuterium.

(ii) For carbon, M = 12 m, therefore

$f = \frac{4m \times 12\, m}{(m + 12m)^2} = 0.284$

About 28.4 % of the neutron's energy is transferred to carbon.

Q. 8. ***A ball is dropped to the ground from a height of 2 m. The coefficient of restitution is 0.6. To what height will the ball rebound?***

Sol. As the ball falls to the ground, its potential energy mgh_1, changes into kinetic energy $½\, mv^2_1$.

∴ $mgh_1 = ½\, mv^2_1$ ... (i)

After rebounding, its kinetic energy $½\, mv_2^2$ changes into potential energy mgh_2.

∴ $mgh_2 = ½\, mv_2^2$ ... (ii)

Dividing (ii) by (i), we get

$\frac{h_2}{h_1} = \left(\frac{v_2}{v_1}\right)^2$

But $h_1 = 2$ m, $e = \frac{v_2}{v_1} = 0.6$

∴ $\frac{h_2}{2\text{ m}} = (0.6)^2 = 0.36$ or $h_2 = 0.72$ m.

Q. 9. ***A ball is dropped vertically from a height of 3.6 m. It rebounds from a horizontal surface to a height of 1.6 m. Find the coefficient of restitution of the material of the ball.***

Sol. Here $h_1 = 3.6$ m, $h_2 = 1.6$ m

Velocity of the ball with which it reaches the horizontal surface,

$u = \sqrt{2gh_1} = \sqrt{2 \times 9.8 \times 3.6} = 8.4$ ms^{-1}

Velocity of the ball with which it rebounds,

$v = \sqrt{2gh_2} = \sqrt{2 \times 9.8 \times 1.6} = 5.6$ ms^{-1}

Coefficient of restitution, $e = \frac{v}{u} = \frac{5.6}{8.4} = 0.667$

Q. 10. ***A ball is dropped from a height h. It rebounds from the ground a number of times. Given that the coefficient of restitution is e, to what height does it go after the nth rebounding?***

Sol. Let v_0 be the velocity with which the ball strikes the ground first time and v_n the velocity after n^{th} rebounding. Then the coefficient of restitution will be

$e = \frac{v_1}{v_0} = \frac{v_2}{v_1} = \frac{v_3}{v_2} = \text{.........} = \frac{v_n}{v_n - 1}$

∴ $e^n = \frac{v_1}{v_0} \times \frac{v_2}{v_1} \times \frac{v_3}{v_2} \times \ldots \times \frac{v_n}{v_n - 1} = \frac{v_n}{v_0}$

But $v_0 = \sqrt{2gh}$ and $v_n = \sqrt{2gH}$

Where H is the height to which the ball rises after nth rebounding.

Hence, $e^n = \frac{v_n}{v_0} = \sqrt{\frac{2gH}{2gh}} = \sqrt{\frac{H}{h}}$

or $H = he^{2n}$

Q. 11. ***A sphere of mass m moving with a velocity u hits another stationary sphere of same mass. If e is the coefficient of restitution, what is the ratio of the velocities of two spheres after the collision?***

Sol. Here $u_1 = u$, $u_2 = 0$

∴ $e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{u - 0}$

or $v_2 - v_1 = e\,u$ ... (i)

By the law of conservation of momentum,

$mu + m \times 0 = mv_1 + mv_2$

or $v_1 + v_2 = eu$ ... (ii)

Adding (i) and (ii), $2v_2 = u + eu = u\,(1 + e)$

or $v_2 = \frac{u\,(1 + e)}{2}$

Again, from (ii),

$v_1 = u - v_2 = u - \frac{u\,(1 + e)}{2} = \frac{u\,(1 - e)}{2}$

Hence, $\frac{v_2}{v_1} = \frac{1 + e}{1 - e}$

Q. 12. ***A ball moving with a speed of 9 ms^{-1} strikes an identical ball such that after the collision the direction of each ball makes an angle 30° with the original line of motion. Find the speeds of the two balls after the collision. Is the kinetic energy conserved in the collision process?***

Sol. Let m be the mass of each ball and v_1 and v_2 be their final velocities after collision.

Here $u_1 = 9$ ms^{-1}, $u_2 = 0$

Initial momentum = Final momentum along the original line of motion

∴ $m \times 9 + m \times 0 = m\, v_1 \cos 30° + m\, v_2 \cos 30°$

or $m \times 9 = m\, \frac{\sqrt{3}}{2}\, (v_1 + v_2)$

or $v_1 + v_2 = \frac{18}{\sqrt{3}} = \frac{6 \times 3}{\sqrt{3}} = 6\sqrt{3}$ ... (1)

By conservation of momentum along a direction perpendicular to the original line, we have

$m \times 0 + m \times 0 = mv_1 \sin 30° - mv_2 \sin 30°$

or $0 = m\,(v_1 - v_2) \times ½$

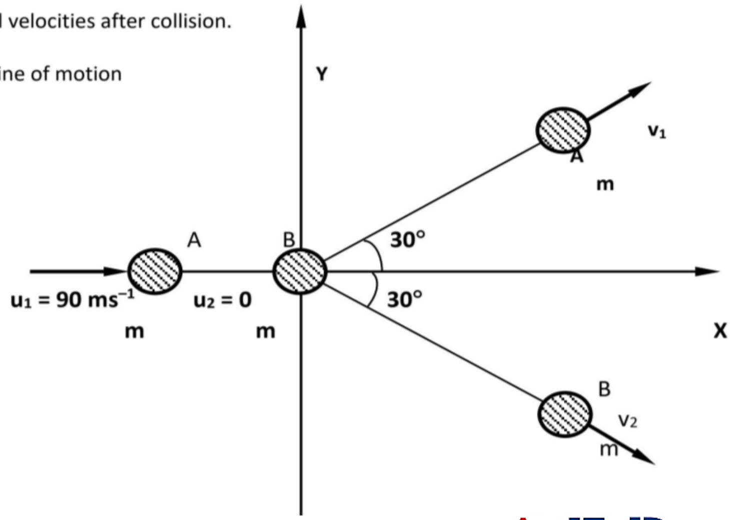
or $v_1 - v_2 = 0$

or $v_1 = v_2$ ... (2)

From (1) and (2), we have

$v_1 = v_2 = 3\sqrt{3}$ ms^{-1}

Total K.E. before collision



$= \frac{1}{2} \times m \times (9)^2 + 0 = \frac{81}{2} m = 40.5$ m

Total K.E. after collision

$= \frac{1}{2} \times m \times (3\sqrt{3})^2 + \frac{1}{2} \times m \times (3 \times \sqrt{3})^2 = 27$ m

i.e. Total K.E. after collision < Total K.E. before collision ; Hence, K.E. is not conserved in the collision process.

Q. 13. ***A ball moving on a horizontal frictionless plane hits an identical ball at rest with a velocity of 0.5 ms⁻¹. If the collision is elastic, calculate the speed imparted to the target ball if the speed of the projectile after the collision is 30 cm s⁻¹. Show that the two balls will move at right angles to each other after the collision.***

Sol. The situation is shown in Fig. Let m be the mass of each ball.

As the collision is elastic, so K.E. is conserved.

$\frac{1}{2} m u_1^2 + \frac{1}{2} m u_2^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2$

or $u_1^2 + u_2^2 = v_1^2 + v_2^2$

or $(0.5)^2 + 0 - (0.3)^2 + v_2^2$

or $v_2 = 0.4\ ms^{-1}$

Applying law of conservation of momentum along X-axis, we get

$0.5\ m + 0 = 0.3\ m \cos\theta_1 + 0.4\ m \cos\theta_2$

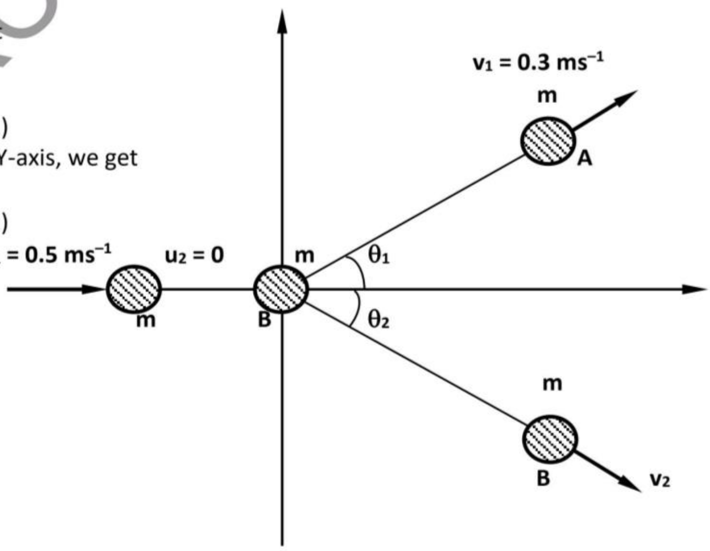
or $5 = 3\cos\theta_1 + 4\cos\theta_2$

or $3\cos\theta_1 = 5 - 4\cos\theta_2$ ... (1)

Again, applying law of conservation of momentum along Y-axis, we get

$0.3\ m \sin\theta_1 = 0.4\ m \sin\theta_2$

or $3\sin\theta_1 = 4\sin\theta_2$ ... (2)



Squaring and adding equations (1) and (2), we get

$9(\cos^2\theta_1 + \sin^2\theta_1) = (5 - 4\cos\theta_2)^2 + (4\sin\theta_2)^2$

or $9 = 16(\cos^2\theta_2 + \sin^2\theta_2) + 25 - 40\cos\theta_2$

or $\cos\theta_2 = \frac{4}{5}$

From (1), $\cos\theta_1 = \frac{1}{3}\left(5 - 4 \times \frac{4}{5}\right) = \frac{3}{5}$

Hence, $\sin\theta_1 = \frac{4}{5}$ and $\sin\theta_2 = \frac{3}{5}$

Now, $\sin(\theta_1 + \theta_2) = \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2$

$= \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5} = 1$

∴ $\theta_1 + \theta_2 = 90°$; Hence, after the collision, the two balls will move off at right angles to each other.

Q. 14. ***Consider the collision depicted in Fig. to be between two billiard balls with equal masses $m_1 = m_2$. The first ball is called the cue while the second ball is called the target. The billiard player wants to "sink" the target ball in a corner pocket, which is at an angle $\theta_2 = 37°$. Assume that the collision is elastic and that friction and rotational motion are not important. Obtain θ_1.*** **[NCERT]**

Sol. By conservation of momentum, we have

$m u_1 + 0 = m v_1 + m v_2$ or $u_1 = v_1 + v_2$ ... (i)

By conservation of energy, we have

$\frac{1}{2} m u_1^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2$ or $u_1^2 = v_1^2 + v_2^2$ ... (ii)

From (i), we have

$\vec{u_1} \cdot \vec{u_1} = (\vec{v_1} + \vec{v_2}) \cdot (\vec{v_1} + \vec{v_2})$

$= \vec{v_1} \cdot \vec{v_1} + \vec{v_1} \cdot \vec{v_2} + \vec{v_2} \cdot \vec{v_1} + \vec{v_2} \cdot \vec{v_2}$

or $u_1^2 = v_1^2 + v_2^2 + 2\, v_1 \cdot v_2$

or $u_1^2 = u_1^2 + 2\,\vec{v_1} \cdot \vec{v_2}$ [Using (ii)]

or $\vec{v_1} \cdot \vec{v_2} = 0$

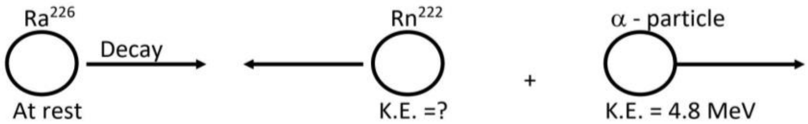
Thus the angle between $\vec{v_1}$ and $\vec{v_2}$ is 90°.

or $\theta_1 + \theta_2 = 90°$

∴ $\theta_1 = 90° - \theta_2 = 90 - 37° = 53°$

Q. 15. ***A nucleus of radium ($_{88}Ra^{226}$) decays to $_{86}Rn^{222}$ by the emission of α-particle ($_2He^4$) of energy 4.8 MeV. If mass of $_{86}Rn^{222}$ = 222.0 amu and mass of $_2He^4$ = 4.003 amu. then calculate the recoil energy of the daughter nucleus $_{86}Rn^{22}$*** [NCERT]

Sol. The nuclear decay may be represented as follows:



The kinetic energy of a particle is given by $K = \frac{p^2}{2m}$ ∴ $p = \sqrt{2mK}$

As momentum is conserved in the absence of an external force, so mK = constant

or $m_{Rn} K_{Rn} = m_\alpha K_\alpha$

or $K_{Rn} = \frac{m_\alpha K_\alpha}{m_{Rn}} = = \frac{4.003 \times 4.8}{222} = 0.0866$ MeV.

Q. 16. ***The nucleus Fe^{57} emits a γ-ray of energy 14.4 keV. If the mass of the nucleus is 56.935 amu, calculate the recoil energy of the nucleus. Take 1 amu = 1.66×10^{-27} kg.*** ***32***

Sol. The nuclear decay may be represented as follows:

$Fe^{57} \rightarrow Fe^{57} + h\nu$ (γ-ray photon)

Exited state Ground state

According to de-Broglie hypothesis, momentum of a photon of energy E is

$$p = \frac{E}{c} = \frac{14.4\text{ keV}}{c} = \frac{14.4 \times 1.6 \times 10^{-16}\text{ J}}{3 \times 10^{8}\text{ ms}^{-1}} = 7.68 \times 10^{-24}\text{ kg ms}^{-1}$$

By conservation of momentum, the momentum of daughter nucleus,

p = momentum of γ-ray photon $= 7.68 \times 10^{-24}$ kg ms^{-1}

The recoil energy of the nucleus will be

$$K = \frac{p^2}{2m} = \frac{(7.68 \times 10^{-24})^2}{2 \times 56.935 \times 1.66 \times 10^{-27}} = 0.312 \times 10^{-21}\text{ J}$$

$$= \frac{0.312 \times 10^{-21}}{1.6 \times 10^{-16}}\text{ keV} = 1.95 \times 10^{-6}\text{ keV}.$$

... End.